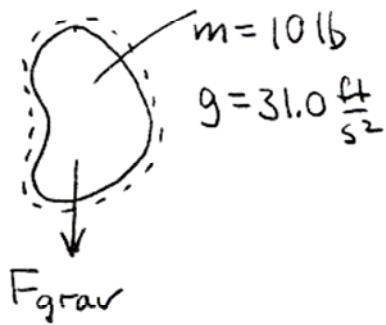


PROBLEM 1.4

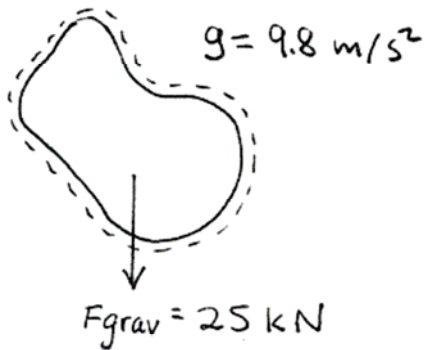


$$F_{grav} = mg = (10 \text{ lb}) \left(31.0 \frac{\text{ft}}{\text{s}^2} \right) \left| \frac{1 \text{ lb}_f}{32.2 \text{ lb ft/s}^2} \right|$$

$\nwarrow$ rounded F_{grav}

$$= 9.63 \text{ lb}_f \leftarrow$$

PROBLEM 1.5



$$m = \frac{F_{\text{grav}}}{g} = \left(\frac{25 \text{ kN}}{9.8 \text{ m/s}^2} \right) \left| \frac{1000 \text{ N}}{\text{kN}} \right| \left| \frac{1 \text{ kg m/s}^2}{1 \text{ N}} \right|$$
$$= 2551 \text{ kg} \quad \leftarrow m$$

PROBLEM 1.6

(a) $F_{\text{grav}} = mg = (100 \text{ kg})(25 \frac{\text{m}}{\text{s}^2}) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m}/\text{s}^2} \right| = 2500 \text{ N} \leftarrow F_{\text{grav}}$

(b) Mass value remains the same. So, on Earth

$$F_{\text{grav}} = mg = (100 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2}) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m}/\text{s}^2} \right| = 981 \text{ N} \leftarrow F_{\text{grav}}$$

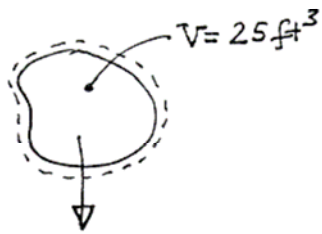
PROBLEM 1.7

$$(a) \quad g = \frac{F_{\text{grav}}}{m} = \frac{144.4 \text{ lbf}}{150 \text{ lb}} \left| \frac{32.174 \text{ lb} \cdot \text{ft}/\text{s}^2}{1 \text{ lbf}} \right| = 30.97 \frac{\text{ft}}{\text{s}^2} \longleftarrow g$$

(b) Mass value remains the same. So

$$F_{\text{grav}} = m g = (150 \text{ lb})(32.174 \text{ ft}/\text{s}^2) \left| \frac{1 \text{ lbf}}{32.174 \text{ lb} \cdot \text{ft}/\text{s}^2} \right| = 150 \text{ lbf} \longleftarrow F_{\text{grav}}$$

PROBLEM 1.8



$$F_{\text{grav}} = 3.5 \text{ lbf}$$

$$g_{\text{moon}} = 5.47 \text{ ft/s}^2$$

$$g_{\text{mars}} = 12.86 \text{ ft/s}^2$$

Accordingly

$$(F_{\text{grav}})_{\text{mars}} = \left(\frac{g_{\text{mars}}}{g_{\text{moon}}} \right) (F_{\text{grav}})_{\text{moon}}$$

$$= \left(\frac{12.86 \text{ ft/s}^2}{5.47 \text{ ft/s}^2} \right) (3.5 \text{ lbf}) = 8.23 \text{ lbf} \quad \leftarrow F_{\text{grav, mars}}$$

The density is $\rho = m/V$. Applying Eq. (*) with data on mars

$$m = \left(\frac{8.23 \text{ lbf}}{12.86 \text{ ft/s}^2} \right) \left| \frac{\overset{\text{rounded}}{32.2 \text{ lb} \cdot \text{ft/s}^2}}{1 \text{ lbf}} \right| = 20.61 \text{ lb}$$

Then

$$\rho = \frac{20.61 \text{ lb}}{25 \text{ ft}^3} = 0.824 \frac{\text{lb}}{\text{ft}^3} \quad \leftarrow \rho$$

PROBLEM 1.9

$$\begin{aligned} F &= ma = m(60g) = 60mg \\ &= 60(50\text{ lb})(32.2 \frac{\text{ft}}{\text{s}^2}) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft}/\text{s}^2} \right| = 3000 \text{ lbf} \quad \leftarrow F \end{aligned}$$

↑ rounded

PROBLEM 1.10.

Eq. 1.8 is used in both parts: $n = m/M$, where M is from Tables A-1.

(a) $m = M n$, $n = 10 \text{ kmol}$

Air: $m = (28.97 \text{ kg/kmol})(10 \text{ kmol}) = 289.7 \text{ kg}$

H_2O : $m = (18.02 \text{ kg/kmol})(10 \text{ kmol}) = 180.2 \text{ kg}$

Cu: $m = (63.54 \text{ kg/kmol})(10 \text{ kmol}) = 635.4 \text{ kg}$

SO_2 : $m = (64.06 \text{ kg/kmol})(10 \text{ kmol}) = 640.6 \text{ kg}$

(b) $n = m/M$, $m = 20 \text{ lb}$

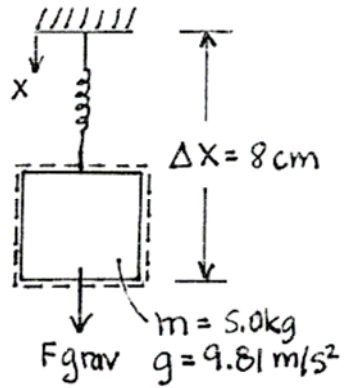
Air: $n = (20 \text{ lb}) / (39.94 \text{ lb/lbmol}) = 0.501 \text{ lbmol}$

H_2 : $n = (20 \text{ lb}) / (2.016 \text{ lb/lbmol}) = 9.921 \text{ lbmol}$

N_2 : $n = (20 \text{ lb}) / (28.01 \text{ lb/lbmol}) = 0.714 \text{ lbmol}$

C: $n = (20 \text{ lb}) / (12.01 \text{ lb/lbmol}) = 1.665 \text{ lbmol}$

PROBLEM 1.11



$$F_{\text{spring}} = K(\Delta X) \quad \text{and} \quad F_{\text{spring}} = F_{\text{grav}} = mg$$

Thus
 $K(\Delta X) = mg$

and

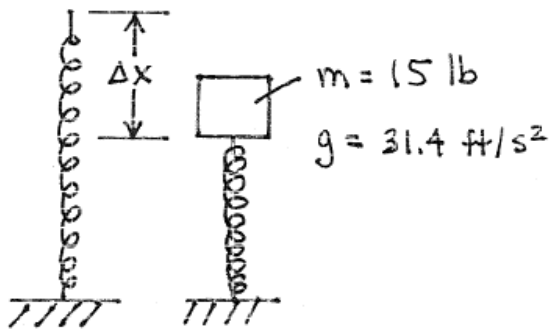
$$K = \frac{mg}{\Delta X}$$

$$= \frac{(5.0 \text{ kg})(9.81 \text{ m/s}^2)}{(8 \text{ cm})} \left| \frac{1 \text{ N}}{\text{kg} \cdot \text{m/s}^2} \right|$$

$$= 6.131 \text{ N/cm} \leftarrow$$

proportionality
constant

PROBLEM 1.12



The spring is known to deflect 0.12 in for every 1 lbf of applied force. Thus, we begin by determining the weight of the object

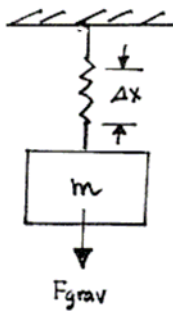
$$\begin{aligned}
 F_{\text{grav}} &= mg \\
 &= (15 \text{ lb})(31.4 \text{ ft/s}^2) \left| \frac{1 \text{ lbf}}{32.2 \text{ ft} \cdot \text{lb/s}^2} \right| \\
 &= 14.63 \text{ lbf}
 \end{aligned}$$

↖ rounded

The deflection is

$$\Delta x = (0.12 \text{ in/lbf})(14.63 \text{ lbf}) = 1.76 \text{ in} \leftarrow \Delta x$$

PROBLEM 1.13



For a linear spring, $F_{\text{spring}} = K(\Delta x)$, where Δx is the spring extension. Since $F_{\text{spring}} = F_{\text{grav}} = mg$, we have $K(\Delta x) = mg$. Since m and K are independent of location, the local acceleration of gravity is proportional to the deflection. Thus

Mars:

$$\frac{g_{\text{mars}}}{g_{\text{earth}}} = \frac{(\Delta x)_{\text{mars}}}{(\Delta x)_{\text{earth}}} = \frac{0.116 \text{ in}}{0.291 \text{ in}} \Rightarrow g_{\text{mars}} = \left(\frac{0.116}{0.291} \right) \left(32.174 \frac{\text{ft}}{\text{s}^2} \right) = 12.825 \frac{\text{ft}}{\text{s}^2} \leftarrow$$

Moon:

$$\frac{g_{\text{moon}}}{g_{\text{earth}}} = \frac{(\Delta x)_{\text{moon}}}{(\Delta x)_{\text{earth}}} \Rightarrow (\Delta x)_{\text{moon}} = \left(\frac{g_{\text{moon}}}{g_{\text{earth}}} \right) (\Delta x)_{\text{earth}} = \left(\frac{5.471}{32.174} \right) (0.291 \text{ in}) = 0.049 \text{ in} \swarrow$$

PROBLEM 1.14

Deceleration occurs from 10 mi/h to rest in 0.1 s. The average acceleration magnitude is

$$|a|_{\text{avg}} = \frac{\Delta V}{\Delta t} = \left(\frac{10 \text{ mi/h} - 0}{0.1 \text{ s}} \right) \left| \frac{5280 \text{ ft}}{1 \text{ mi}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right|$$

$$= 146.7 \text{ ft/s}^2$$

or, in g's

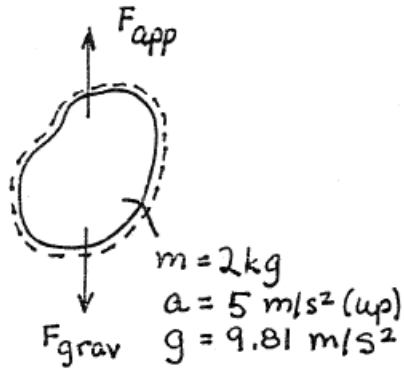
$$|a|_{\text{avg}} = \left(\frac{146.7 \text{ ft/s}^2}{32.2 \text{ ft/s}^2} \right) = 4.6 \text{ g's} \leftarrow \text{deceleration (in g's)}$$

Thus, the magnitude of the average force applied is

$$|F|_{\text{avg}} = m |a|_{\text{avg}} = (200 \text{ lb}) (146.7 \frac{\text{ft}}{\text{s}^2}) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right|$$

$$= 911.2 \text{ lbf} \leftarrow \text{rounded } |F|_{\text{avg}}$$

PROBLEM 1.15



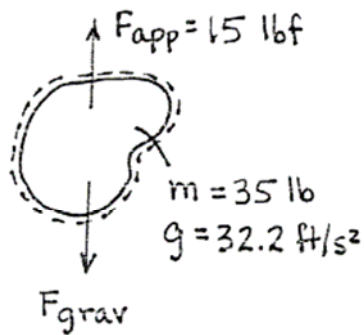
$$F_{app} - F_{grav} = m a$$

$$F_{app} = m a + m g = m (a + g)$$

$$= (2 \text{ kg})(5 + 9.81) \frac{\text{m}}{\text{s}^2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right|$$

$$= 29.62 \text{ N} \leftarrow F_{app}$$

PROBLEM 1.16



$$F_{app} - F_{grav} = ma$$

$$a = \frac{F_{app} - F_{grav}}{m} = \frac{F_{app} - mg}{m}$$

$$= \frac{F_{app}}{m} - g$$

$$= \left(\frac{15 \text{ lbf}}{35 \text{ lb}} \right) \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right| - 32.2 \text{ ft/s}^2$$

(rounded)

$$= -18.4 \text{ ft/s}^2 \text{ (downward)} \quad \leftarrow a$$

PROBLEM 1.17

$$F_{\text{grav}, E} = m g_E \quad (\text{on Earth})$$

$$F_{\text{grav}, S} = m g_S \quad (\text{in Space Station})$$

Mass remains the same. So,

$$\frac{F_{\text{grav}, S}}{F_{\text{grav}, E}} = \frac{g_S}{g_E}$$

$$\Rightarrow F_{\text{grav}, S} = F_{\text{grav}, E} \left(\frac{g_S}{g_E} \right) = 700 \text{ N} \left(\frac{6 \text{ m/s}^2}{9.81 \text{ m/s}^2} \right)$$

$$= 428.1 \text{ N} \quad \leftarrow F_{\text{grav}, S}$$

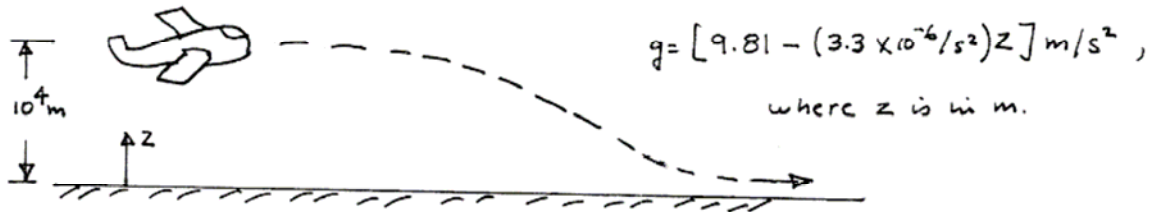
Also,

$$F_{\text{grav}, E} = 700 \text{ N} \left| \frac{0.22481 \text{ lbf}}{1 \text{ N}} \right| = 157.4 \text{ lbf}$$

$$F_{\text{grav}, S} = 428.1 \text{ N} \left| \frac{0.22481 \text{ lbf}}{1 \text{ N}} \right| = 96.2 \text{ lbf}$$

← In lbf.

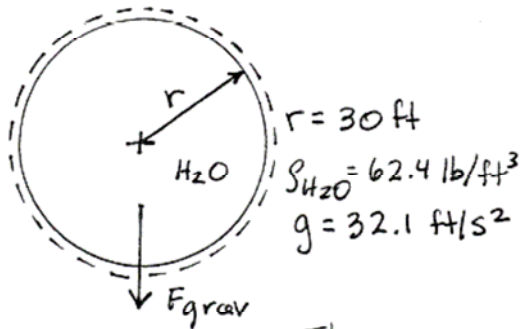
PROBLEM 1.18



Since weight is $W = mg$, the percent change in weight on landing is

$$\begin{aligned} \% \text{ change} &= \left[\frac{W|_{z=0} - W}{W} \right] (100) \\ &= \left[\frac{[9.81 \text{ m/s}^2] - [(9.81 \text{ m/s}^2) - (3.3 \times 10^{-6}/s^2)(10^4 \text{ m})]}{[9.81 - (3.3 \times 10^{-6})(10^4)]} \right] (100) = \left[\frac{3.3 \times 10^{-2}}{9.777} \right] (100) \\ &= +0.34\% \end{aligned}$$

PROBLEM 1.19



The mass of water is $m = \gamma V$, where V is the volume of the spherical tank. Thus

$$V = \frac{4}{3} \pi r^3 = \left(\frac{4}{3}\right)(\pi)(30 \text{ ft})^3 = 1.131 \times 10^5 \text{ ft}^3$$

and the mass is

$$m = \gamma V = (62.4 \frac{\text{lb}}{\text{ft}^3})(1.131 \times 10^5 \text{ ft}^3) = 7.06 \times 10^6 \text{ lb} \leftarrow m$$

The weight is

$$F_{\text{grav}} = mg = (7.06 \times 10^6 \text{ lb})(32.1 \frac{\text{ft}}{\text{s}^2}) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| = 7.04 \times 10^6 \text{ lbf} \leftarrow F_{\text{grav}}$$

(rounded)

PROBLEM 1.20

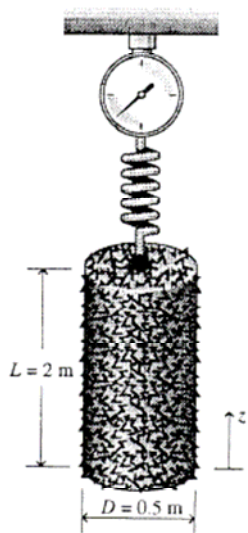


Figure P1.20

$$g = 9.78 \text{ m/s}^2$$

$$\rho = 7800 - 360(z/L)^2 \text{ kg/m}^3,$$

where z is in m .

The scale records the weight, $F_{\text{grav}} = mg$, where mass m is given by

$$\begin{aligned} m &= \int_{\text{Vol}} \rho dV = \int_0^L \rho A dz \\ &= A \int_0^L [7800 - 360(z/L)^2] dz \end{aligned}$$

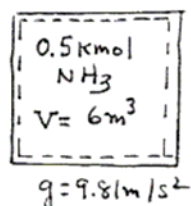
For simplicity, introduce a new variable: $\hat{z} = z/L$, so $dz = L d\hat{z}$ and the expression for mass reads

$$\begin{aligned} m &= AL \int_0^1 [7800 - 360(\hat{z})^2] d\hat{z} \\ &= AL \left[7800\hat{z} - 360 \frac{(\hat{z})^3}{3} \right]_0^1 \\ &= \left(\frac{\pi D^2}{4} \right) L \left[7680 \frac{\text{kg}}{\text{m}^3} \right] \\ &= \frac{\pi}{4} (0.5 \text{ m})^2 (2 \text{ m}) \left[7680 \frac{\text{kg}}{\text{m}^3} \right] = 3016 \text{ kg} \end{aligned}$$

Finally,

$$F_{\text{grav}} = mg = (3016 \text{ kg}) \left(9.78 \frac{\text{m}}{\text{s}^2} \right) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| = 29,496 \text{ N} \leftarrow$$

PROBLEM 1.21



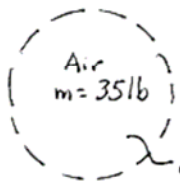
(a) $F_{\text{grav}} = m g$

Using Eq. 1.8, $m = n M = 0.5 \text{ kmol} \left(17.03 \frac{\text{kg}}{\text{kmol}} \right) = 8.52 \text{ kg}$ ↖ Table A-1

$\therefore F_{\text{grav}} = (8.52 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| = 83.58 \text{ N} \longleftarrow \vec{F}_{\text{grav}}$

(b) $\bar{V} = \frac{V}{n} = \frac{6 \text{ m}^3}{0.5 \text{ kmol}} = 12 \frac{\text{m}^3}{\text{kmol}}, \quad v = \frac{V}{m} = \frac{6 \text{ m}^3}{8.52 \text{ kg}} = 0.704 \frac{\text{m}^3}{\text{kg}} \longleftarrow \bar{V}, v$

PROBLEM 1.22


 $g = 31.0 \text{ ft/s}^2$

(a) $V = \frac{V}{m}$ where $V = \frac{\pi d^3}{6} = \frac{\pi (10)^3 \text{ ft}^3}{6} = 523.6 \text{ ft}^3$

$\therefore v = \frac{523.6 \text{ ft}^3}{35 \text{ lb}} = 14.96 \frac{\text{ft}^3}{\text{lb}}$ $\leftarrow v$

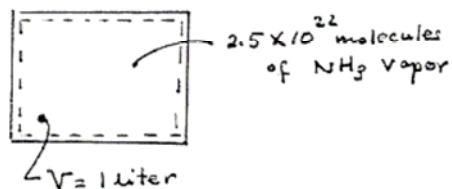
Using Eq. 1.9, $\bar{V} = M v = \left(28.97 \frac{\text{lb}}{\text{lbmol}} \right) \left(14.96 \frac{\text{ft}^3}{\text{lb}} \right)$ $\leftarrow \bar{V}$

$= 433.39 \frac{\text{ft}^3}{\text{lbmol}}$

(b) $F_{\text{grav}} = m g = (35 \text{ lb}) \left(31.0 \frac{\text{ft}}{\text{s}^2} \right) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| = 33.7 \text{ lbf} \leftarrow F_{\text{grav}}$

$\nwarrow$ rounded

PROBLEM 1.23



(a) From Sec. 1.5, the number of molecules in a gram mole (mol) is 6.022×10^{23} (Avogadro's number).

$$\text{Thus } n = \frac{2.5 \times 10^{22} \text{ molecules}}{6.022 \times 10^{23} \text{ molecules/mol}} = 4.15 \times 10^{-2} \text{ mols}$$

Converting to kmol,

$$n = 4.15 \times 10^{-2} \text{ mol} \left| \frac{1 \text{ kmol}}{10^3 \text{ mol}} \right| = 4.15 \times 10^{-5} \text{ kmol} \leftarrow$$

Using Eq. 1.8, $m = nM$, so

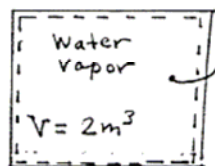
$$m = 4.15 \times 10^{-5} \text{ kmol} \left(\overset{\text{Table A-1}}{17.03 \frac{\text{kg}}{\text{kmol}}} \right) = 7.07 \times 10^{-4} \text{ kg} \leftarrow$$

Then

$$v = \frac{V}{m} = \frac{(1 \text{ liter})}{7.07 \times 10^{-4} \text{ kg}} \left| \frac{10^{-3} \text{ m}^3}{1 \text{ liter}} \right| = 1.41 \frac{\text{m}^3}{\text{kg}} \leftarrow$$

$$\bar{v} = \frac{V}{n} = \frac{10^{-3} \text{ m}^3}{4.15 \times 10^{-5} \text{ kmol}} = 24.1 \frac{\text{m}^3}{\text{kmol}} \leftarrow$$

PROBLEM 1.24



$$p = 0.3 \text{ MPa}$$

$$T = 160^\circ \text{C}$$

$$v = 0.651 \text{ m}^3/\text{kg}$$

$$(a) \quad v = \frac{V}{m} \Rightarrow m = \frac{V}{v}$$

$$\therefore m = \frac{2 \text{ m}^3}{0.651 \text{ m}^3/\text{kg}} = 3.07 \text{ kg}$$

Eg. 1.8,

$$n = \frac{m}{M} = \frac{3.07 \text{ kg}}{18.02 \text{ kg/kmol}} = 0.17 \text{ kmol}$$

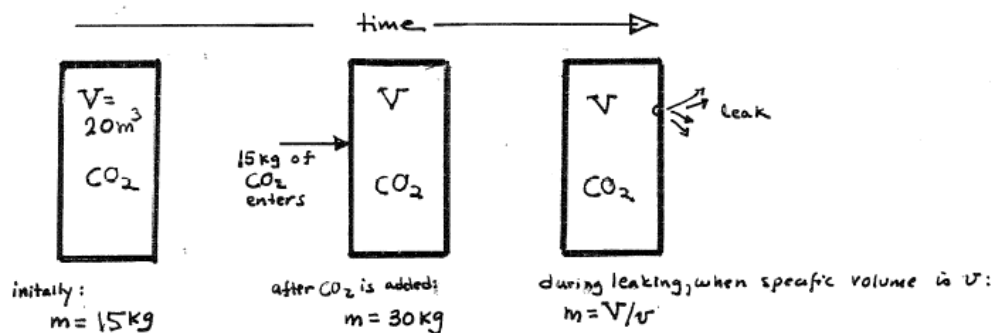
↑ Table A-2

(b) From Sec. 1.5, the number of molecules in a gram mole (mol) is 6.022×10^{23} (Avogadro's number). Thus,

$$\left[\begin{array}{c} \text{Number of molecules} \\ \text{in the vessel} \end{array} \right] = (0.17 \text{ kmol}) \left| \frac{10^3 \text{ mol}}{\text{kmol}} \right| \left(6.022 \times 10^{23} \frac{\text{molecules}}{\text{mol}} \right)$$

$$= 1.02 \times 10^{26}$$

PROBLEM 1.25



(a) Using $v = V/m$

initial specific volume: $v = \frac{20 \text{ m}^3}{15 \text{ kg}} = 1.33 \frac{\text{m}^3}{\text{kg}}$

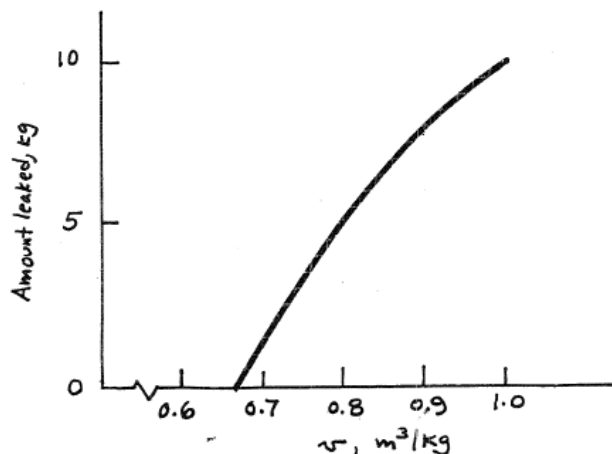
after an additional 15 kg enters: $v = \frac{20 \text{ m}^3}{30 \text{ kg}} = 0.67 \frac{\text{m}^3}{\text{kg}}$

(b) Beginning with 30 kg of CO_2 in the cylinder, the amount of mass that has leaked is

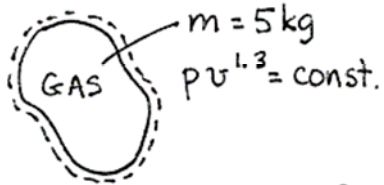
$$\begin{aligned} [\text{Amount leaked}] &= [30 \text{ kg}] - \left[\text{mass in cylinder when the specific volume in the cylinder is } v \right] \\ &= [30 \text{ kg}] - \left[\frac{20 \text{ m}^3}{v} \right] \end{aligned}$$

In writing this, we assume that the specific volume does not vary with location within the cylinder: the gas leaks slowly, so the specific volume within the cylinder varies only with time.

Since the specific volume v varies from $0.67 \text{ m}^3/\text{kg}$, when 30 kg is in the cylinder, to the specified upper limit specified: $1.0 \text{ m}^3/\text{kg}$, the plot is



PROBLEM 1.27



$$P_1 = 1 \text{ bar}, v_1 = 0.2 \text{ m}^3/\text{kg}$$

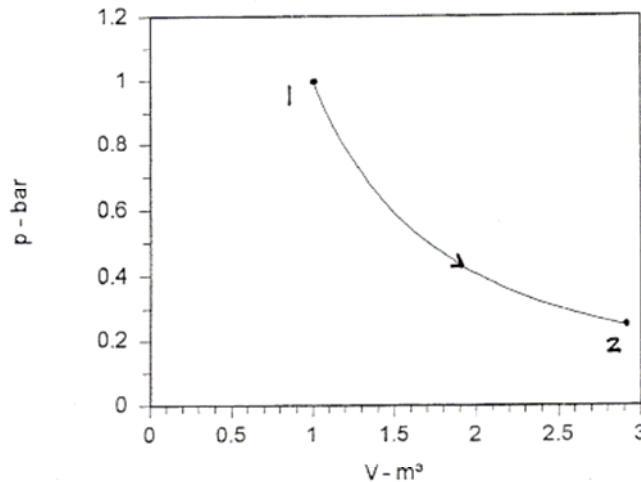
$$P_2 = 0.25 \text{ bar}$$

From the pressure-specific volume relation

$$v_2 = \left(\frac{P_1}{P_2} \right)^{\frac{1}{1.3}} v_1 = \left(\frac{1}{0.25} \right)^{\frac{1}{1.3}} (0.2 \text{ m}^3/\text{kg})$$

$$= 0.5810 \text{ m}^3/\text{kg}$$

$$V_2 = v_2 m = 2.905 \text{ m}^3 \leftarrow V_2$$



PROBLEM 1.28

$m = 2 \text{ lb}$
 $PV^n = \text{constant}$
 $P_1 = 20 \text{ lbf/in}^2$
 $V_1 = 10 \text{ ft}^3$
 $P_2 = 100 \text{ lbf/in}^2$
 $V_2 = 2.9 \text{ ft}^3$

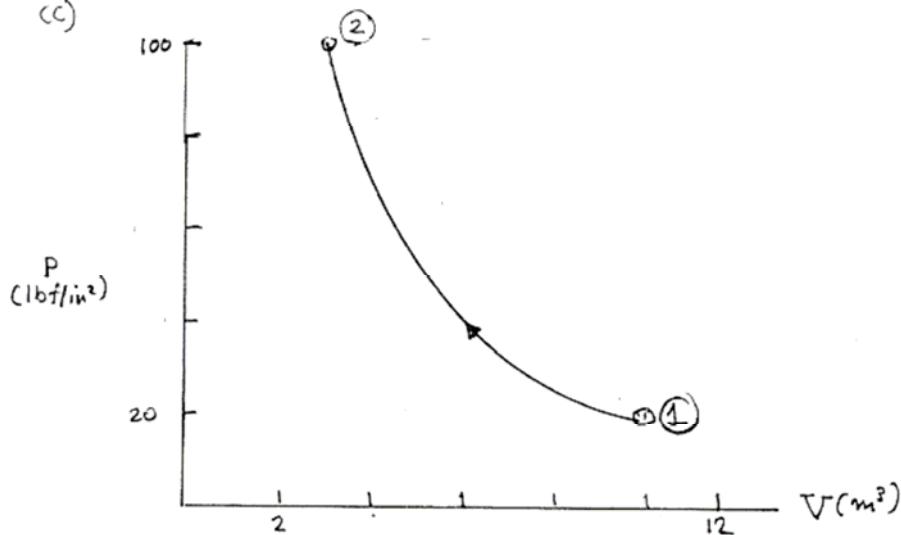
$(a) \quad \left. \begin{array}{l} P_1 V_1^n = \text{constant} \\ P_2 V_2^n = \text{constant} \end{array} \right\} \Rightarrow P_1 V_1^n = P_2 V_2^n$

$\Rightarrow \frac{P_1}{P_2} = \left(\frac{V_2}{V_1} \right)^n \quad \text{or} \quad \frac{20}{100} = \left(\frac{2.9}{10} \right)^n$

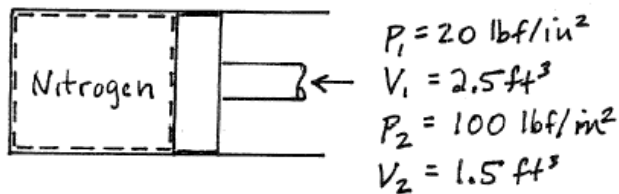
Solving, $n = 1.3$

$(b) \quad v_1 = \frac{V_1}{m} = \frac{10 \text{ ft}^3}{2 \text{ lb}} = 5 \frac{\text{ft}^3}{\text{lb}}, \quad v_2 = \frac{V_2}{m} = \frac{2.9 \text{ ft}^3}{2 \text{ lb}} = 1.45 \frac{\text{ft}^3}{\text{lb}}$

(c)



PROBLEM 1.29



The pressure-volume relation is linear during the process. Thus

$$P = P_1 + \left(\frac{P_2 - P_1}{V_2 - V_1} \right) (V - V_1)$$

Or, using given data

$$P = 20 \frac{\text{lbf}}{\text{in}^2} + \frac{(100 - 20) \text{ lbf/in}^2}{(1.5 - 2.5) \text{ ft}^3} (V - 2.5) \text{ ft}^3$$

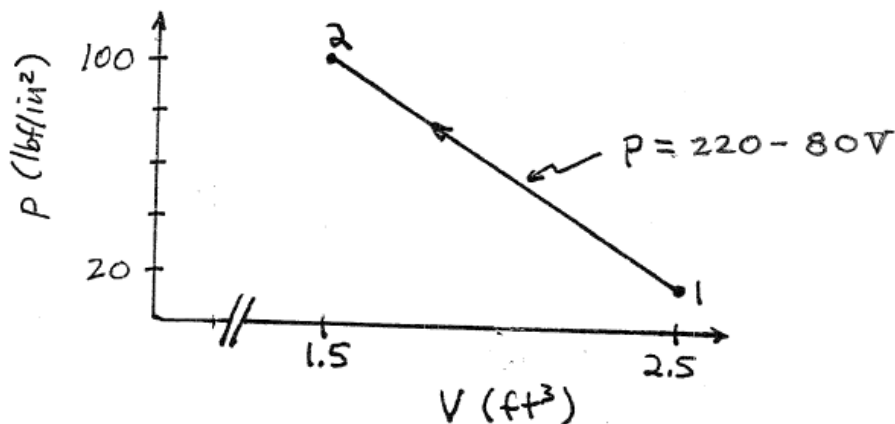
$$= 220 - 80V$$

UNITS: $\frac{\text{lbf/in}^2}{\text{ft}^3}$

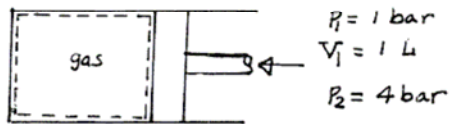
When $V = 2.1 \text{ ft}^3$

$$P = 220 \frac{\text{lbf}}{\text{in}^2} - 80 \frac{\text{lbf/in}^2}{\text{ft}^3} (2.1 \text{ ft}^3) = 52 \text{ lbf/in}^2$$

On p - V coordinates



PROBLEM 1.30



- (a) The process is described by $pV = \text{constant}$. The constant can be evaluated using data at state 1:

$$\begin{aligned} pV &= \text{constant} \\ &= p_1 V_1 \\ &= (1 \text{ bar})(1 \text{ L}) = 1 \text{ bar} \cdot \text{L} \end{aligned}$$

So, for every state during the process, we have the relation $pV = 1 \text{ bar} \cdot \text{L}$

When $p = 3 \text{ bar}$,

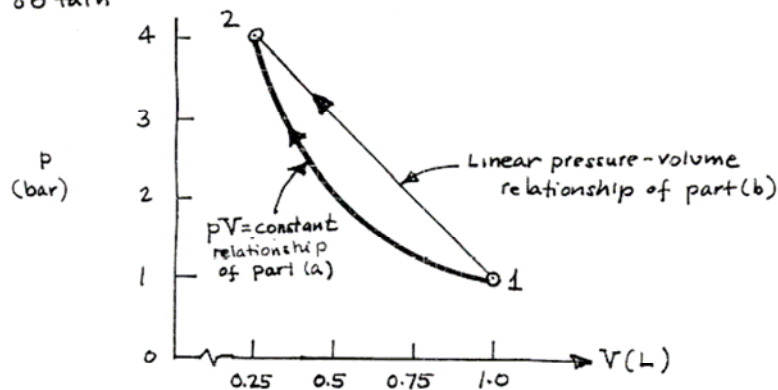
$$V = \frac{1 \text{ bar} \cdot \text{L}}{3 \text{ bar}} = 0.33 \text{ L}$$

When $p = 4 \text{ bar}$, $V_2 = \frac{1 \text{ bar} \cdot \text{L}}{4 \text{ bar}} = 0.25 \text{ L}$

Plotting the relation on pressure-volume coordinates we use

$$p = \frac{1 \text{ bar} \cdot \text{L}}{V}$$

to obtain



- (b) For comparison, the linear pressure-volume relationship is shown on the plot above. The volume corresponding to $p = 3 \text{ bar}$ can be obtained simply using the slope of the straight line between 1 and 2:

$$|\text{slope}| = \frac{(4 - 1) \text{ bar}}{(1.0 - 0.25) \text{ L}} = \frac{(3 - 1)}{(1.0 - V)} \Rightarrow V = 0.5 \text{ L}$$

This value also can be read from the plot.

PROBLEM 1.31

Three processes:



① 1-2: $pV = \text{constant}$

$$P_1 = 1 \text{ bar}, V_1 = 1 \text{ m}^3, V_2 = 0.2 \text{ m}^3$$

② 2-3 $p = \text{constant}, V > V_2$ (expansion), $V_3 = 1.0 \text{ m}^3$

③ 3-1 $V = \text{constant}$

For process 1-2, $pV = \text{constant}$. The constant can be evaluated using data at state 1:

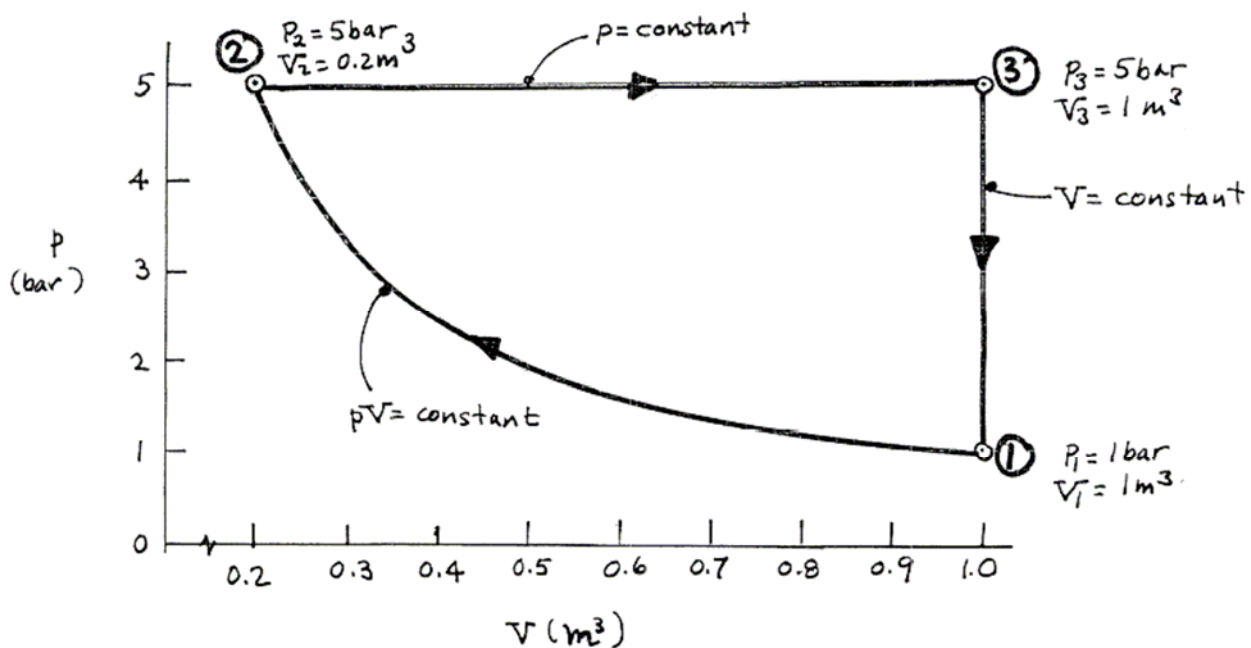
$$\begin{aligned} pV &= \text{constant} \\ &= P_1 V_1 \\ &= (1 \text{ bar})(1 \text{ m}^3) = 1 \text{ bar} \cdot \text{m}^3 \end{aligned}$$

Accordingly, on a pressure-volume plot process 1-2 is described by

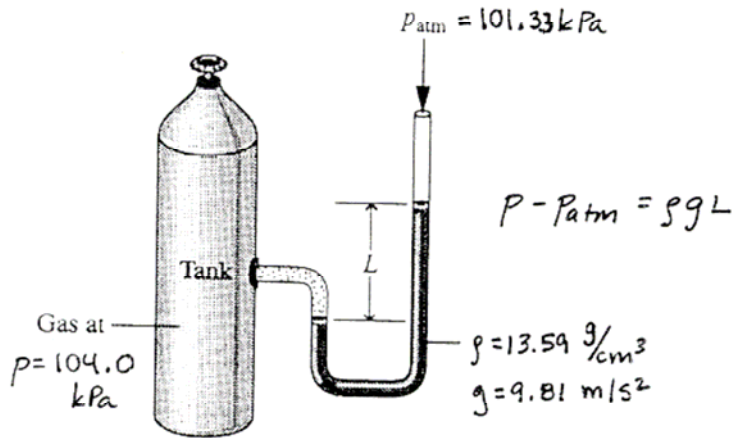
$$p = \frac{1 \text{ bar} \cdot \text{m}^3}{V}$$

In particular, when $V_2 = 0.2 \text{ m}^3$, $p = 5 \text{ bar}$.

Sketching the processes in series,



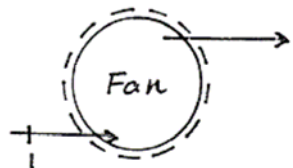
PROBLEM 1.32



$$\begin{aligned}
 (a) \quad L &= \frac{P - P_{\text{atm}}}{\rho g} \\
 &= \frac{(104.0 - 101.33) \text{ kPa}}{(13.59 \text{ g/cm}^3)(9.81 \text{ m/s}^2)} \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{10^3 \text{ g}}{1 \text{ kg}} \right| \left| \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \\
 &= 0.02 \text{ m} = 2 \text{ cm} \quad \leftarrow L
 \end{aligned}$$

$$(b) \quad P_{\text{gage}} = P - P_{\text{atm}} = 104.0 - 101.33 = 2.67 \text{ kPa} \quad \leftarrow P_{\text{gage}}$$

PROBLEM 1.33



$$P_{i,vac} = 4.2 \text{ in. H}_2\text{O}$$

$$P_{atm} = 14.5 \text{ lbf/in}^2$$

$$g = 32.2 \text{ ft/s}^2$$

$$\rho_f = 49.94 \text{ lb/ft}^3$$

$$P_{i,vac} = \rho_f g L$$

$$= (49.94 \frac{\text{lb}}{\text{ft}^3}) (32.2 \frac{\text{ft}}{\text{s}^2}) (\frac{4.2}{12} \text{ ft}) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right|$$

$$= 0.1214 \text{ lbf/in}^2$$

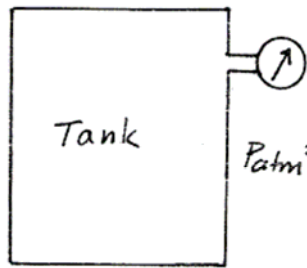
← rounded

$$P_{i,abs} = P_{atm} - P_{i,vac}$$

$$= 14.5 - 0.1214 = 14.379 \frac{\text{lbf}}{\text{in}^2}$$

$P_{i,abs}$

PROBLEM 1.34



$$P_{atm} = 98 \text{ kPa}$$

$$P_{abs} = 0.4 \text{ bar} = 40 \text{ kPa}$$

$$P_{abs} < P_{atm} \Rightarrow \text{vacuum}$$

$$P_{vac} = P_{atm} - P_{abs}$$

$$= 98 - 40 = 58 \text{ kPa} \leftarrow P_{vac}$$

PROBLEM 1.35

See Fig. P 1.35. Applying the principles discussed in Sec. 1.6.1, the atmospheric pressure is

$$\begin{aligned}
 P_{\text{atm}} = \rho_m g L &\Rightarrow L = \frac{P_{\text{atm}}}{\rho_m g} = \frac{100 \times 10^3 \text{ N/m}^2}{\left(13.59 \frac{\text{g}}{\text{cm}^3}\right) \left|\frac{1 \text{ kg}}{10^3 \text{ g}}\right| \left|\frac{10^2 \text{ cm}^3}{1 \text{ m}^3}\right| (9.81 \frac{\text{m}}{\text{s}^2})} \left|\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right| \\
 &= 0.75 \text{ m} \left|\frac{10^3 \text{ mm}}{1 \text{ m}}\right| \\
 &= 750 \text{ mm Hg} \quad \leftarrow
 \end{aligned}$$

Converting to in Hg,

$$L = 750 \text{ mm Hg} \left|\frac{1 \text{ cm}}{10 \text{ mm}}\right| \left|\frac{1 \text{ in}}{2.54 \text{ cm}}\right| = 29.53 \text{ in Hg} \quad \leftarrow$$

PROBLEM 1.36

$$p_{\text{atm}} = 14.7 \text{ lbf/in}^2$$

$$g = 32.0 \text{ ft/s}^2$$

Water
 $\nu = 0.01604 \text{ ft}^2/\text{s}$

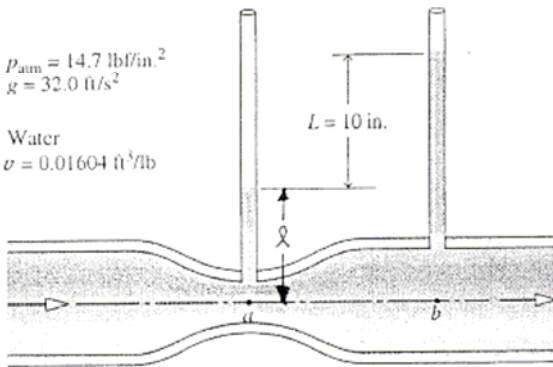


Fig. P1.36

$$p_a = p_{\text{atm}} + \rho g l$$

$$p_b = p_{\text{atm}} + \rho g (l + L)$$

$$\therefore (p_b - p_a) = \rho g L$$

$$= \left(\frac{1}{0.01604 \text{ ft}^2/\text{s}} \right) \left(\frac{32.0 \text{ ft}}{\text{s}^2} \right) \left(\frac{10}{12} \text{ ft} \right) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{s}^2/\text{ft}} \right|$$

$$= + 51.63 \frac{\text{lbf}}{\text{ft}^2} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right|$$

$$= + 0.36 \text{ lbf/in}^2$$

← increases

← rounded

PROBLEM 1.37

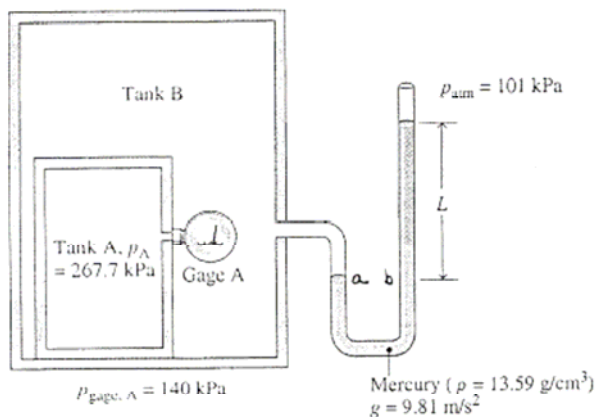


Fig. P1.37

Solving,

$$L = \frac{p_B - p_{\text{atm}}}{\rho g} = \frac{(127.7 \text{ kPa} - 101 \text{ kPa}) \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|}{(13.59 \frac{\text{g}}{\text{cm}^3}) \left| \frac{1 \text{ kg}}{10^3 \text{ g}} \right| \left| \frac{10^2 \text{ cm}}{1 \text{ m}} \right|^3 (9.81 \frac{\text{m}}{\text{s}^2})} = 0.2 \text{ m}$$

$$= (0.2 \text{ m}) \left| \frac{10^2 \text{ cm}}{1 \text{ m}} \right| = 20 \text{ cm}$$

Looking at gage A,

$$p_{\text{gage, A}} = p_A - p_B$$

$$\Rightarrow p_B = p_A - p_{\text{gage, A}}$$

$$= 267.7 \text{ kPa} - 140 \text{ kPa}$$

$$= 127.7 \text{ kPa}$$

The pressures at a and b are equal. At a, the pressure is p_B .

At b, the pressure is

$$p_b = p_{\text{atm}} + \rho g L$$

$$\text{So, } p_B = p_{\text{atm}} + \rho g L$$

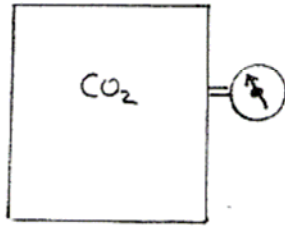
PROBLEM 1.38

See Fig. P1.38.

The pressure acting on the vehicle at a depth $L = 1000 \text{ ft}$ is

$$\begin{aligned}
 P &= P_{\text{atm}} + \rho g L \\
 &= 1 \text{ atm} + \left(62.4 \frac{\text{lb}}{\text{ft}^3} \right) \left(32.2 \frac{\text{ft}}{\text{s}^2} \right) (1000 \text{ ft}) \left| \frac{1 \text{ atm}}{14.696 \text{ lbf/in}^2} \right| \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| \left| \frac{1 \text{ lbf}}{32.2 \text{ lb-ft/s}^2} \right| \\
 &= 1 \text{ atm} + 29.49 \text{ atm} = 30.49 \text{ atm} \quad \leftarrow \text{rounded}
 \end{aligned}$$

PROBLEM 1.39



$$P_{\text{atm}} = 750 \text{ mmHg}, \rho_m = 13.598 / \text{cm}^3, g = 9.81 \text{ m/s}^2$$

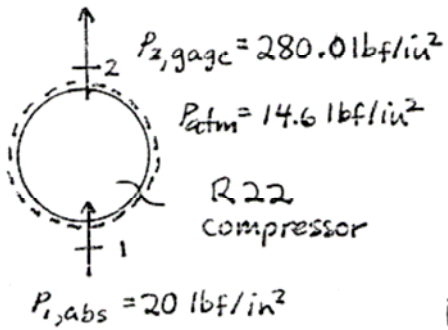
$$P_{\text{gauge}} = 10 \text{ kPa (vacuum)}$$

The atmospheric pressure, in kPa, can be found using the "for example" of Sec. 1.6.1. The calculation is the same as used there, giving $P_{\text{atm}} = 10^5 \text{ N/m}^2$.

$$\text{Thus, } P_{\text{atm}} = 10^5 \frac{\text{N}}{\text{m}^2} \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| = 100 \text{ kPa}. \text{ Then, with Eq. 1.15}$$

$$P_{\text{CO}_2} = 100 \text{ kPa} - 10 \text{ kPa} = 90 \text{ kPa}$$

PROBLEM 1.40

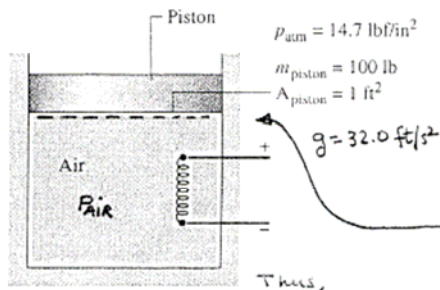


$$\begin{aligned}\Delta P_{abs} &= P_{2,abs} - P_{1,abs} \\ &= (P_{2,gage} + P_{atm}) - P_{1,abs} \\ &= (280.0 + 14.6) - 20 \\ &= 274.6 \text{ lbf/in}^2\end{aligned}$$

$$P_{2,abs}/P_{1,abs} = (280.0 + 14.6)/20 = 14.73$$

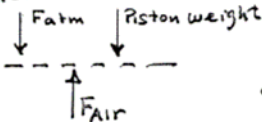
$\xleftarrow{\Delta P_{abs}} \quad \xleftarrow{P_2/P_1}$

PROBLEM 1.41



We assume the air expands slowly. Thus, at the interface between the lower piston surface and the air static equilibrium exists. Moreover, as the piston moves slowly in the cylinder friction between the piston and cylinder can be ignored.

Force balance:



$$\Rightarrow P_{AIR} A_{pist} = P_{atm} A_{pist} + m_{pist} g$$

$$\therefore P_{AIR} = P_{atm} + \frac{m_{pist} g}{A_{pist}}$$

Thus,

$$P_{AIR} = 14.7 \frac{\text{lbf}}{\text{in}^2} + \left(\frac{100 \text{ lb} \times 32.0 \text{ ft/s}^2}{1 \text{ ft}^2} \right) \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| = 15.39 \frac{\text{lbf}}{\text{in}^2} \text{ or } P_{AIR} = 0.69 \frac{\text{lbf}}{\text{in}^2} \text{ (gage)}$$

Fig. P1.41

PROBLEM 1.42

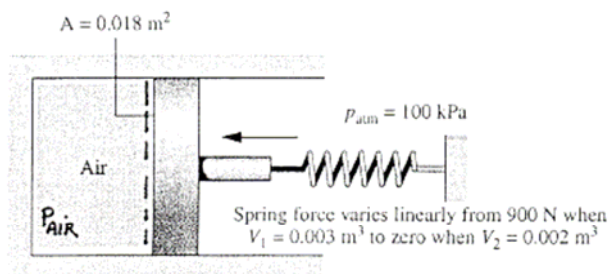


Fig. P1.42

At the initial and final states static equilibrium with no effect of friction (between the piston and cylinder) can be assumed. Thus, looking at the interface between the air and the piston (shown dotted) we have

$$P_{AIR} A \leftarrow \begin{array}{c} \leftarrow P_{atm} A \\ \leftarrow F_{spring} \end{array} \Rightarrow P_{AIR} = P_{atm} + \frac{F_{spring}}{A}$$

Initially, $F_{\text{spring}} = 900 \text{ N}$. So, $P_1 = 100 \text{ kPa} + \frac{900 \text{ N}}{0.018 \text{ m}^2} \left[\frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right] = 150 \text{ kPa}$. $\leftarrow$

$$P_1 = 150 \times P_a \left| \frac{1 \text{ atm}}{101.325 \text{ kPa}} \right| = 1.48 \text{ atm.}$$

Finally, $F_{\text{spring}} = 0 \text{ N}$. So, $P_2 = 100 \text{ kPa}$.

$$P_2 = 100 \text{ kPa} \left| \frac{1 \text{ atm}}{101.325 \text{ kPa}} \right| = 0.99 \text{ atm.}$$

PROBLEM 1.43

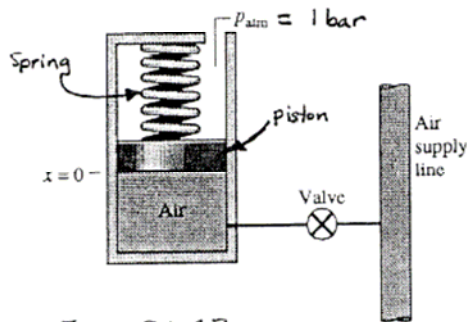


Figure P1.43

$$g = 9.81 \text{ m/s}^2$$

$$F_{\text{spring}} = kx, \text{ where } k = 10,000 \text{ N/m}$$

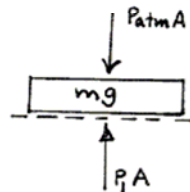
For the piston:

$$m = 10 \text{ kg}$$

$$A = 7.8 \times 10^{-3} \text{ m}^2$$

$$\text{For the air: } \Delta V = 3.9 \times 10^{-4} \text{ m}^3$$

Initially, $x=0$ and there is no spring force acting on the piston. Also, friction between the piston and the cylinder wall can be ignored. Accordingly, the force exerted by the air within the cylinder on the bottom of piston is equal to the weight of the piston plus the force exerted by the atmosphere on the top of the piston:



$$\sum F_x = 0:$$

$$P_1 A = P_{\text{atm}} A + mg$$

$$P_1 = P_{\text{atm}} + \frac{mg}{A}$$

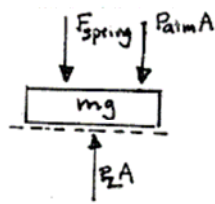
$$P_1 = 1 \text{ bar} + \left[\frac{(10 \text{ kg})(9.81 \text{ m/s}^2)}{7.8 \times 10^{-3} \text{ m}^2} \right] \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right|$$

$$P_1 = 1.126 \text{ bar} \quad \leftarrow P_1$$

Finally, the force exerted by the air within the cylinder on the bottom of the piston is equal to the weight of the piston plus the force exerted by the atmosphere on the top of the piston plus the force exerted by the spring on the top of the piston:

Continued on next slide

Problem 1-43 continued



Ignoring friction,

$$\Sigma F_x = 0:$$

$$P_2 A = P_{atm} A + mg + F_{spring}$$

$$P_2 = P_{atm} + \frac{mg}{A} + \frac{F_{spring}}{A}$$

For the spring, $F_{spring} = kx$, where x is found using the increase in volume of the air:

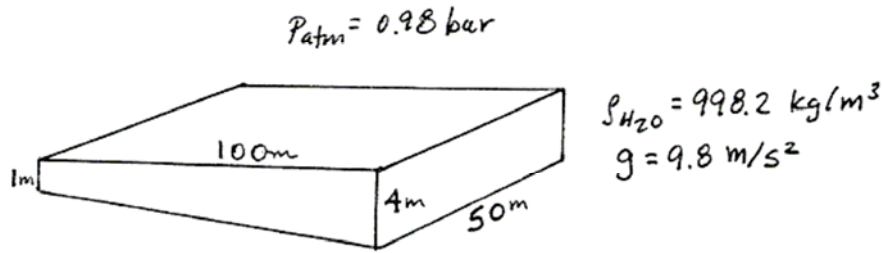
$$x = \frac{\Delta V}{A} = \frac{3.9 \times 10^{-4} \text{ m}^3}{7.8 \times 10^{-3} \text{ m}^2} = 0.05 \text{ m}$$

Collecting results

$$\begin{aligned} P_2 &= P_{atm} + \frac{mg}{A} + \left[\frac{(10,000 \text{ N/m})(0.05 \text{ m})}{(7.8 \times 10^{-3} \text{ m}^2)} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| \right] \\ &= 1.126 \text{ bar} + 0.641 \text{ bar} \\ &= 1.767 \text{ bar} \end{aligned}$$

← P_2

PROBLEM 1.44



The total force on the bottom of the pool is the sum of the weight of the water and the downward force of atmospheric pressure on the surface of the water;

$$F_{tot} = F_{grav} + F_{atm} \quad (a)$$

To get the weight of the water, first find the mass, as follows

$$m = \rho V = (998.2 \frac{\text{kg}}{\text{m}^3}) \left[(1)(100)(50) + \frac{1}{2}(100)(3)(50) \right] \text{m}^3 \\ = (998.2)(12500) = 1.25 \times 10^7 \text{ kg}$$

Thus

$$F_{grav} = mg = (1.25 \times 10^7 \text{ kg})(9.81 \text{ m/s}^2) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kN}}{10^3 \text{ N}} \right| = 1.226 \times 10^5 \text{ kN}$$

Or

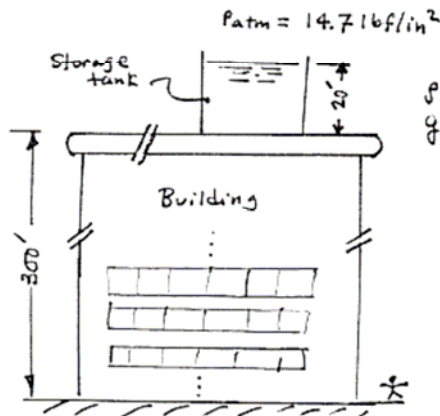
$$F_{atm} = P_{atm} A_{surface} = (0.98 \text{ bar})(100 \times 50) \text{m}^2 \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kN}}{10^3 \text{ N}} \right| = 4.9 \times 10^5 \text{ kN}$$

$$F_{tot} = 1.226 \times 10^5 + 4.9 \times 10^5 = 6.126 \times 10^5 \text{ kN} \leftarrow F_{tot}$$

The depth at the center of the pool is $h = 2.5 \text{ m}$. Thus

$$P = P_{atm} + \rho g h \\ = (0.98 \text{ bar}) \left| \frac{100 \text{ kPa}}{1 \text{ bar}} \right| + (998.2 \frac{\text{kg}}{\text{m}^3})(9.8 \frac{\text{m}}{\text{s}^2})(2.5 \text{ m}) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| = 122.5 \text{ kPa} \quad P$$

PROBLEM 1.45



$$\rho = 62.2 \text{ lb/ft}^3$$

$$\gamma = 32.0 \text{ ft/s}^2$$

The pressure at the bottom of the storage tank is

$$p = P_{atm} + \rho g L$$

$$= 14.7 \frac{\text{lbf}}{\text{in}^2} + \left(62.2 \frac{\text{lb}}{\text{ft}^3} \right) \left(32.0 \frac{\text{ft}}{\text{s}^2} \right) \left(20 \text{ ft} \right) \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| \left| \frac{1 \text{ lbf}}{32.2 \text{ lb ft/s}^2} \right|$$

$$= 14.7 \frac{\text{lbf}}{\text{in}^2} + 8.6 \frac{\text{lbf}}{\text{in}^2} = 23.3 \frac{\text{lbf}}{\text{in}^2}$$

rounded

PROBLEM 1.46

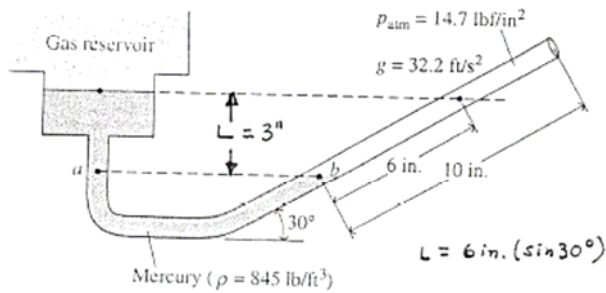


Fig. P1.46

That is,

$$\begin{aligned}
 p_{\text{gas}} &= 14.7 \frac{\text{lbf}}{\text{in}^2} - \left(845 \frac{\text{lb}}{\text{ft}^3} \right) \left(32.2 \frac{\text{ft}}{\text{s}^2} \right) \left(\frac{3}{12} \text{ft} \right) \left| \frac{1 \text{ft}^2}{144 \text{in}^2} \right| \left| \frac{1 \text{lbf}}{32.2 \text{lb} \cdot \text{ft}/\text{s}^2} \right| \\
 &= 14.7 \frac{\text{lbf}}{\text{in}^2} - 1.47 \frac{\text{lbf}}{\text{in}^2} = 13.23 \frac{\text{lbf}}{\text{in}^2} \quad (\text{absolute})
 \end{aligned}$$

- (b) Since the gas pressure is less than the atmospheric pressure, we have $p_{\text{gas}} = 1.47 \frac{\text{lbf}}{\text{in}^2}$ (vacuum)

- (c) Because the liquid level change is small for small pressure variations about atmospheric pressure, a U-tube manometer may be difficult to read accurately. The inclined manometer has greater sensitivity and is easier to read accurately. See Introduction to Fluid Mechanics, 5th Edition, Sec. 3-3, by R.W. Fox and A.T. McDonald, J. Wiley & Sons, Inc.

(a)

$$p_a = p_b, \text{ where } p_b = 14.7 \frac{\text{lbf}}{\text{in}^2}$$

Also,

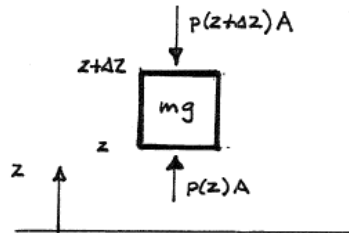
$$p_a = p_{\text{gas}} + \rho g L$$

So,

$$p_{\text{gas}} = 14.7 \frac{\text{lbf}}{\text{in}^2} - \rho g L$$

PROBLEM 1.47

Consider the variation of pressure owing to the effect of gravity. For an element in a motionless gas or liquid, the forces acting are the forces of pressure on the upper and lower surfaces and the weight of the system:



$$\Sigma F_z = 0: \quad 0 = p(z+\Delta z)A + mg - p(z)A$$

where

$$\begin{aligned} m &= \rho V \\ &= \rho[A\Delta z] \\ &= \frac{[A\Delta z]}{v} \end{aligned}$$

Rearranging

$$A[p(z+\Delta z) - p(z)] = -\left(\frac{A\Delta z}{v}\right)g$$

or

$$\frac{p(z+\Delta z) - p(z)}{\Delta z} = -\frac{g}{v}$$

In the limit as $\Delta z \rightarrow 0$. This gives

$$\frac{dp}{dz} = -\frac{g}{v} \quad (*)$$

(a) Atmosphere. If $v = C/p$, where $C = 72.435(\text{m}^3/\text{kg})(\text{kPa}) = 72,435 \frac{\text{N}\cdot\text{m}}{\text{kg}}$, is inserted in Eq. (*), we have

$$\frac{dp}{dz} = -\frac{gp}{C} \Rightarrow \frac{d \ln p}{dz} = -g/C \Rightarrow \ln p = -\frac{gz}{C} + \ln K$$

or

$$p = K \exp\left(-\frac{gz}{C}\right)$$

When $z=0$, $p = p_0$ (atm), giving $p = p_0 \exp(-gz/C)$.

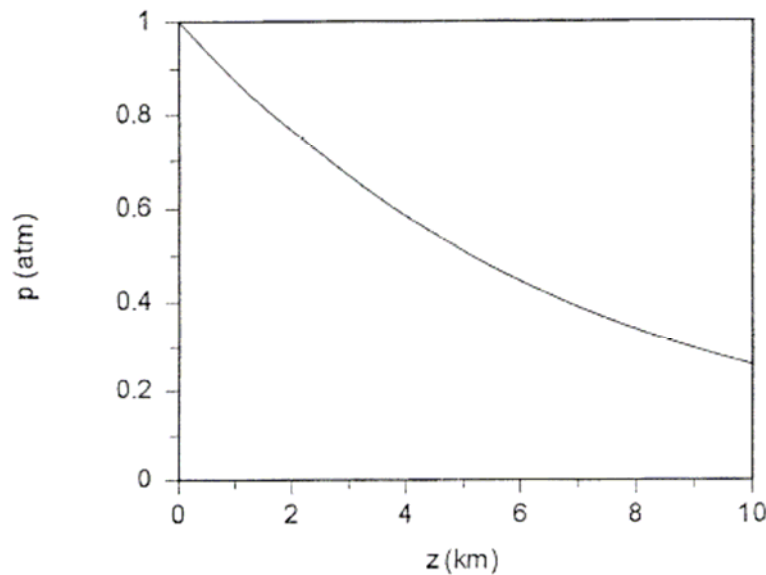
Inserting known values gives p in atm when z is in km

$$p = p_0 \exp\left(\frac{-(9.81 \text{ m/s}^2)(z \text{ km})}{72,435 \frac{\text{N}\cdot\text{m}}{\text{kg}}} \left| \frac{10^3 \text{ m}}{1 \text{ km}} \right| \left| \frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2} \right| \right) = p_0 \exp(-0.135z)$$

This relationship is shown in the accompanying plot.

Continued on next slide

Problem 1-47 continued



(b) Ocean. Letting $\hat{z} (= -z)$ denote depth, Eq. (*) reads

$$\frac{dp}{d\hat{z}} = \frac{g}{v}$$

Then, assuming v is constant

$$p = \left(\frac{g}{v}\right)\hat{z} + K$$

When $\hat{z} = 0$, $p = p_0$ (1 atm), giving $p = \left(\frac{g}{v}\right)\hat{z} + p_0$.

Inserting known values gives p in atm when $\hat{z}$ is in km

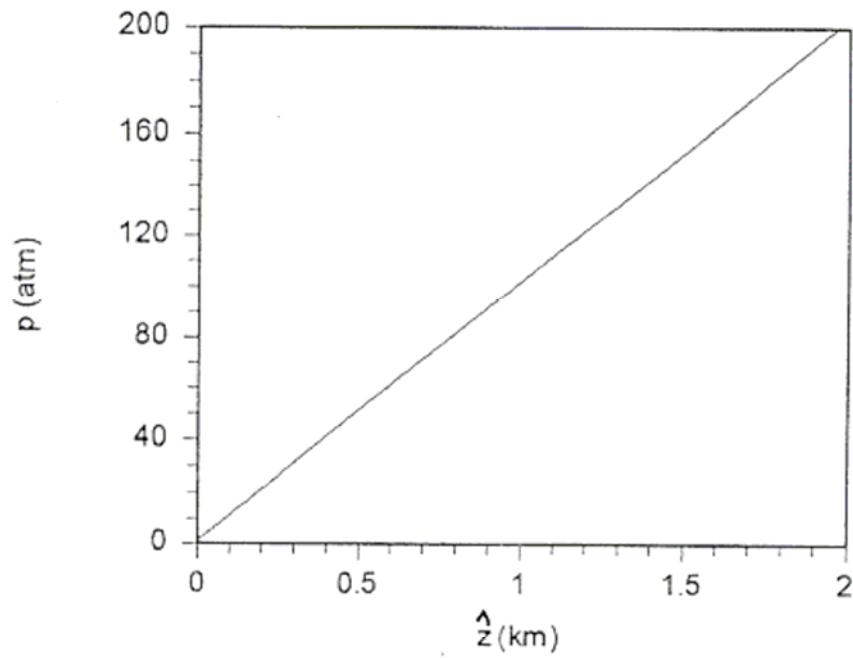
$$p = p_0 + \left[\frac{(9.81 \text{ m/s}^2) \hat{z}(\text{km})}{0.956 \times 10^{-3} \text{ m}^3/\text{kg}} \left| \frac{10^3 \text{ m}}{1 \text{ km}} \right| \left| \frac{1 \text{ atm}}{1.01325 \times 10^5 \text{ N/m}^2} \right| \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \right]$$

$$= p_0 + 101.3 \hat{z}$$

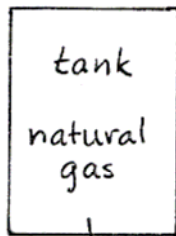
This relationship is shown in the accompanying plot.

Continued on next slide

Problem 1-47 continued



PROBLEM 1.48



$m = 1000 \text{ kg}$

For the gas in the tank, the p - v - T relation is

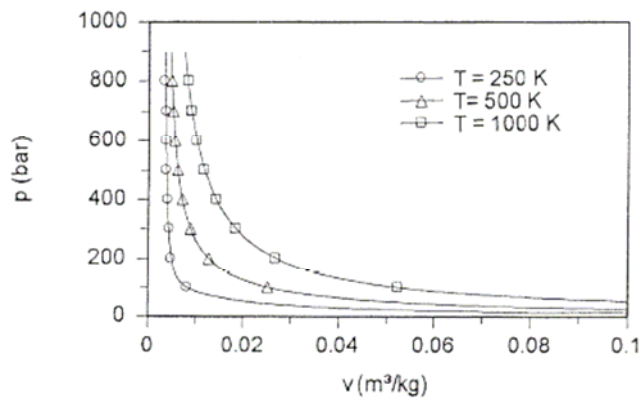
$$p = [(5.18 \times 10^{-3}) T / (v - 0.002668)] - (8.91 \times 10^3) / v^2$$

Solving iteratively for v at $p = 100 \text{ bar}$, $T = 255 \text{ K}$ we get

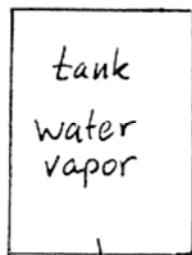
$$v = 0.00884 \text{ m}^3/\text{kg}$$

$$V = m v = 8.84 \text{ m}^3$$

The following plots can be constructed for $T = 250, 500$, and 1000 K :



Problem 1-49



$$V = 82.3 \text{ ft}^3$$

For the water vapor in the tank, the p - v - T relation is

$$p = [(0.5954)T / (v - 0.2708)] - 63.36 / v^2$$

where v is in ft^3/lb , T is in $^\circ\text{R}$, and p is in lb/ft^2

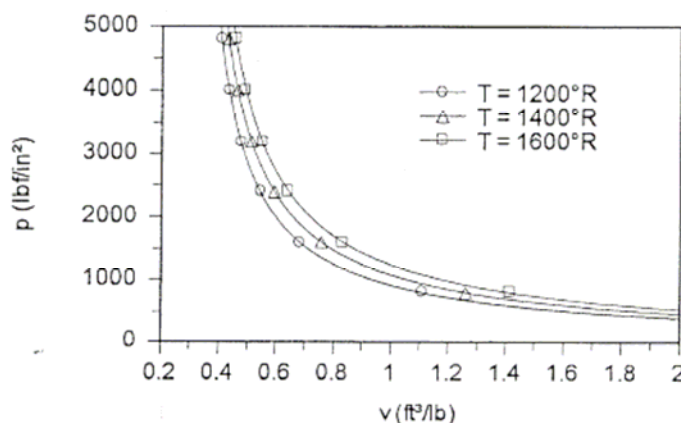
Solving iteratively for v at $p = 1500 \text{ lb}/\text{ft}^2$, $T = 1140^\circ\text{R}$ we get

$$v = 0.686 \text{ ft}^3/\text{lb}$$

Thus, with the given value for V

$$m = V/v = 82.3 / 0.686 = 120 \text{ lb} \quad \underline{\hspace{1cm}} m$$

The following plots can be constructed for $T = 1200, 1400, 1600^\circ\text{R}$:



PROBLEM 1.50

Using Eqs. 1.18, 1.19

$$T(^{\circ}\text{F}) = 1.8 T(^{\circ}\text{C}) + 32, \quad T(^{\circ}\text{F}) = T(^{\circ}\text{R}) - 459.67$$

(a) $T(^{\circ}\text{C}) = 21$

$$T(^{\circ}\text{F}) = (1.8)(21) + 32 = 69.8$$

$$T(^{\circ}\text{R}) = T(^{\circ}\text{F}) + 459.67 = 529.47$$

(b) $T(^{\circ}\text{C}) = -40$

$$T(^{\circ}\text{F}) = (1.8)(-40) + 32 = -40$$

$$T(^{\circ}\text{R}) = -40 + 459.67 = 419.67$$

(c) $T(^{\circ}\text{C}) = 500$

$$T(^{\circ}\text{F}) = (1.8)(500) + 32 = 932$$

$$T(^{\circ}\text{R}) = 932 + 459.67 = 1391.67$$

(d) $T(^{\circ}\text{C}) = 0$

$$T(^{\circ}\text{F}) = (1.8)(0) + 32 = 32$$

$$T(^{\circ}\text{R}) = 32 + 459.67 = 491.67$$

(e) $T(^{\circ}\text{C}) = 100$

$$T(^{\circ}\text{F}) = (1.8)(100) + 32 = 212$$

$$T(^{\circ}\text{R}) = 212 + 459.67 = 671.67$$

(f) $T(^{\circ}\text{C}) = -273.15$

$$T(^{\circ}\text{F}) = (1.8)(-273.15) + 32 = -459.67$$

$$T(^{\circ}\text{R}) = -459.67 + 459.67 = 0$$

PROBLEM 1.51

Using Eq. 1.19 expressed as

$$T(^{\circ}\text{C}) = \frac{T(^{\circ}\text{F})}{1.8} - \frac{32}{1.8}$$

$$= \frac{T(^{\circ}\text{F})}{1.8} - 17.78$$

together with Eq. 1.17,

(a) $T(^{\circ}\text{F}) = 68$

$$T(^{\circ}\text{C}) = \frac{68}{1.8} - 17.78 = 20$$

$$T(\text{K}) = 20 + 273.15 = 293.15$$

(b) $T(^{\circ}\text{F}) = -40$

$$T(^{\circ}\text{C}) = \frac{-40}{1.8} - 17.78 = -40$$

$$T(\text{K}) = -40 + 273.15 = 233.15$$

(c) $T(^{\circ}\text{F}) = 500$

$$T(^{\circ}\text{C}) = \frac{500}{1.8} - 17.78 = 260$$

$$T(\text{K}) = 260 + 273.15 = 533.15$$

(d) $T(^{\circ}\text{F}) = 0$

$$T(^{\circ}\text{C}) = \frac{0}{1.8} - 17.78 = -17.78$$

$$T(\text{K}) = -17.78 + 273.15 = 255.37$$

(e) $T(^{\circ}\text{F}) = 212$

$$T(^{\circ}\text{C}) = \frac{212}{1.8} - 17.78 = 100$$

$$T(\text{K}) = 100 + 273.15 = 373.15$$

(f) $T(^{\circ}\text{F}) = -459.67$

$$T(^{\circ}\text{C}) = \frac{-459.67}{1.8} - 17.78 = -273.15$$

$$T(\text{K}) = 0$$

PROBLEM 1.52

$$T_{\text{gas}} = 1985^{\circ}\text{C}$$

Using Eq. 1.17

$$T(\text{K}) = 1985^{\circ}\text{C} + 273.15 = 2258.15 \text{ K}$$



Using Eq. 1.16

$$T(^{\circ}\text{R}) = 1.8 (2258.15) = 4064.67^{\circ}\text{R}$$



Using Eq. 1.18

$$T(^{\circ}\text{F}) = 4064.67 - 459.67 = 3605^{\circ}\text{F}$$



PROBLEM 1.53

$$T_M = \text{Measured temperature} = 40^\circ\text{C}$$

$$T_N = \text{Normal temperature} = 37^\circ\text{C}$$

Using Eq. 1.19

$$T_M = 1.8(40) + 32 = 104^\circ\text{F}$$

$$T_N = 1.8(37) + 32 = 98.6^\circ\text{F}$$



PROBLEM 1.55

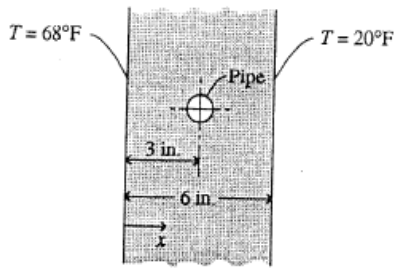


Figure P1.55

Since the temperature variation is linear

$$T = mx + b$$

where m is the slope and b is the value when $x = 0$.

Slope:

$$m = \left[\frac{20^\circ\text{F} - 68^\circ\text{F}}{6 \text{ in.}} \right] = -8^\circ\text{F/in.}$$

$$\text{When } x = 0, T = 68^\circ\text{F}$$

Accordingly, the temperature variation through the wall is

$$T = -(8^\circ\text{F/in.})x + 68^\circ\text{F}$$

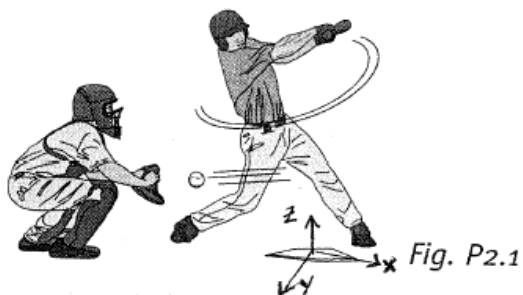
Checking the temperature at the pipe: $x = 3 \text{ in.}$

$$\begin{aligned} T &= -(8^\circ\text{F/in.})(3 \text{ in.}) + 68^\circ\text{F} \\ &= 44^\circ\text{F} \end{aligned}$$

There is no danger of freezing.



PROBLEM 2.1



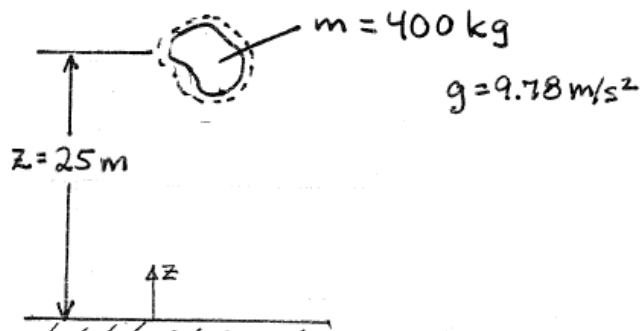
$$\begin{aligned}
 KE &= \frac{1}{2} m V^2 \\
 &\quad \uparrow \text{relative to home plate} \\
 &= \frac{1}{2} (0.31b) \left\{ \frac{94 \text{ miles}}{h} \left| \frac{1 h}{3600 s} \right| \left| \frac{5280 \text{ ft}}{1 \text{ mile}} \right| \right\}^2 \\
 &= 2851.1 \frac{lb \cdot ft}{s^2} \left| \frac{1 lbf}{32.2 lb \cdot ft/s^2} \right| \left| \frac{1 Btu}{778 \text{ ft} \cdot lbf} \right| \\
 &\quad \leftarrow \text{rounded} \rightarrow \\
 KE &= 0.114 \text{ Btu}
 \end{aligned}$$

PROBLEM 2.2

KNOWN: An object of known mass is located at a specified elevation relative to the surface of the earth.

FIND: Determine gravitational potential energy of the object.

ENGR. MODEL: (1) The object is a closed system. (2) The acceleration of gravity is constant.

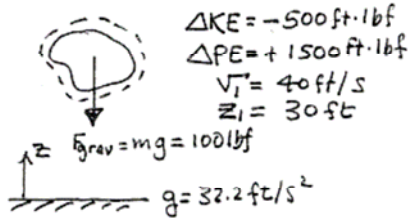


ANALYSIS: The gravitational potential energy is

$$PE = m g z$$

$$\begin{aligned} &= (400 \text{ kg})(9.78 \frac{\text{m}}{\text{s}^2})(25 \text{ m}) \left| \frac{1 \text{ N}}{\text{kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kN}}{1000 \text{ N}} \right| \left| \frac{1 \text{ kJ}}{1 \text{ kN} \cdot \text{m}} \right| \\ &= 97.8 \text{ kJ} \leftarrow \text{PE} \end{aligned}$$

PROBLEM 2.3



(a) $\Delta KE = \frac{1}{2} m [v_2^2 - v_1^2]$, $m = \frac{F_{\text{grav}}}{g} = \frac{100 \text{ lbf}}{(32.2 \text{ ft/s}^2)} \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right| = 100 \text{ lb}$
 Solving for v_2 ,

$$v_2 = \left[\frac{2 \Delta KE}{m} + v_1^2 \right]^{\frac{1}{2}} = \left[\frac{2(-500 \text{ ft} \cdot \text{lbf})}{100 \text{ lb}} \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right| + (40 \frac{\text{ft}}{\text{s}})^2 \right]^{\frac{1}{2}}$$

$$= 35.75 \text{ ft/s}$$

(b) $\Delta PE = m g (z_2 - z_1) \Rightarrow 1500 \text{ ft} \cdot \text{lbf} = 100 \text{ lbf} (z_2 - 30 \text{ ft})$

$$\uparrow F_{\text{grav}}$$

 Solving,

$$z_2 = 45 \text{ ft}$$

PROBLEM 2.4

KNOWN: A brick of known volume and density experiences a given decrease in elevation.

$$V = 2.5 \times 3.5 \times 6 \text{ in}^3$$

FIND: Determine the change in potential energy.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The brick is a closed system. (2) The acceleration of gravity is constant. (3) The density of the body is uniform throughout.

ANALYSIS:

$$V = (2.5 \text{ in})(3.5 \text{ in})(6 \text{ in}) \left| \frac{1 \text{ ft}^3}{12^3 \text{ in}^3} \right| = 0.03038 \text{ ft}^3$$

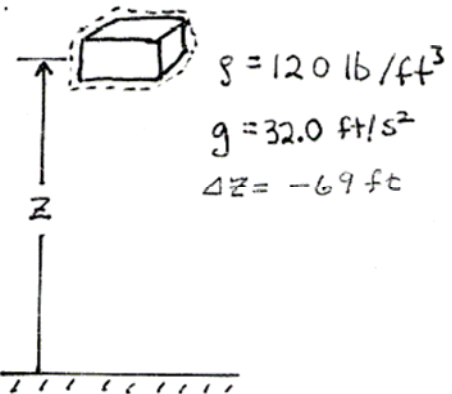
Based on assumption (3)

$$m = \rho V = (120 \text{ lb/ft}^3)(0.03038 \text{ ft}^3) = 3.65 \text{ lb}$$

The change in potential energy and the elevation are related by

$$\Delta PE = mg \Delta z$$

$$= (3.65 \text{ lb}) \left(32.0 \frac{\text{ft}}{\text{s}^2} \right) (-69 \text{ ft}) \left| \frac{1 \text{ lb}_f}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| = -250.3 \text{ ft} \cdot \text{lb}_f$$

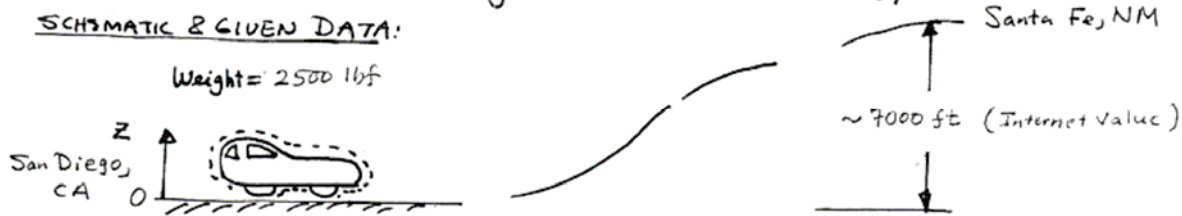


PROBLEM 2.5

KNOWN: An automobile of known weight travels from sea level to a known elevation.

FIND: Determine the change in potential energy.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. As shown in the schematic, the automobile is the closed system. 2. The acceleration of gravity is constant.

ANALYSIS: The change in potential energy is

$$\Delta PE = mg(z_2 - z_1)$$

The quantity mg is recognized as the vehicle weight. Thus, inserting known values

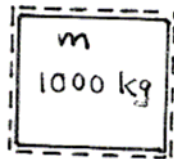
$$\Delta PE = (2500 \text{ lbf})(7000 \text{ ft}) \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 2.25 \times 10^4 \text{ Btu}$$

PROBLEM 2.6

KNOWN: An object of known mass decelerates from a given initial velocity to a known final velocity.

FIND: Determine the change in kinetic energy of the object.

SCHEMATIC & GIVEN DATA:



$$V_1 = 100 \text{ m/s}$$

$$V_2 = 20 \text{ m/s}$$

ENGR. MODEL: The object is a closed system.

ANALYSIS: The change in kinetic energy is

$$\Delta KE = \frac{1}{2}m[V_2^2 - V_1^2]$$

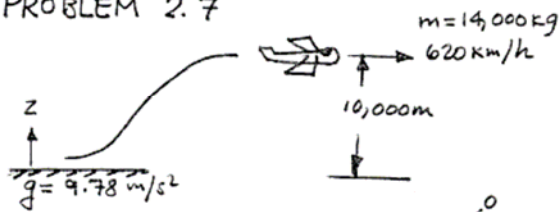
Inserting known values and converting units

$$\Delta KE = \frac{1}{2}(1000 \text{ kg})[20^2 - 100^2] \frac{\text{m}^2}{\text{s}^2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= -4800 \text{ kJ}$$

ΔKE

PROBLEM 2.7



$$\Delta KE = \frac{1}{2} m [v_2^2 - \cancel{v_1^2}]$$

$$= \frac{1}{2} (14000 \text{ kg}) \left[620 \frac{\text{km}}{\text{h}} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{1000 \text{ m}}{\text{km}} \right| \right]^2 \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ J} \cdot \text{m}} \right|$$

$$= 207,623 \text{ kJ}$$

←

$$\Delta PE = m g (z_2 - \cancel{z_1})$$

$$= (14000 \text{ kg}) (9.78 \text{ m/s}^2) (10,000 \text{ m}) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ J} \cdot \text{m}} \right|$$

$$= 1,369,200 \text{ kJ}$$

←

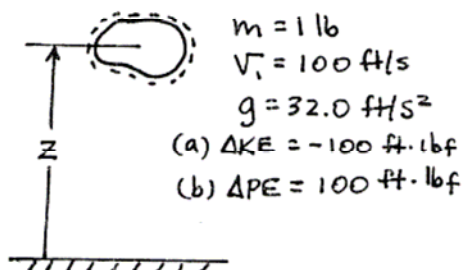
PROBLEM 2.8

KNOWN: An object of known mass moves with a given velocity.

FIND: Determine (a) the final velocity for a given change in kinetic energy, and (b) the change in elevation for a given change in potential energy.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The object is a closed system. (2) The acceleration of gravity is constant.



ANALYSIS: (a) The change in kinetic energy is related to the initial and final velocities by

$$\Delta KE = \frac{1}{2} m [V_2^2 - V_1^2]$$

Thus, solving for the final velocity V_2

$$V_2 = \sqrt{\frac{2 \Delta KE}{m} + V_1^2}$$

Inserting values and converting units accordingly

$$V_2 = \sqrt{\frac{(2)(-100 \text{ ft} \cdot \text{lbf})}{(1 \text{ lb})} \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right| + 100^2 \frac{\text{ft}^2}{\text{s}^2}}$$

$$\textcircled{1} \quad = 59.67 \text{ ft/s} \quad \longleftarrow V_2$$

(b) The change in potential energy is related to the change in elevation by

$$\Delta PE = mg \Delta z$$

Thus, the change in elevation is

$$\begin{aligned} \Delta z &= \frac{\Delta PE}{mg} \\ &= \frac{(100 \text{ ft} \cdot \text{lbf})}{(1 \text{ lb})(32.0 \text{ ft/s}^2)} \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right| \end{aligned}$$

$$\textcircled{2} \quad = 100.6 \text{ ft} \quad \longleftarrow \Delta z$$

1. The velocity decreases, as expected.
2. The elevation increases, as expected.

PROBLEM 2.9

KNOWN: An object of known mass accelerates from a given initial velocity to a given final velocity due to the action of a resultant force.

FIND: Determine the work done by the resultant force.

SCHEMATIC & GIVEN DATA:



$$m = 2 \text{ kg}$$

$$V_1 = 200 \text{ m/s}$$

$$V_2 = 500 \text{ m/s}$$

ENGR. MODEL: (1) The object is a closed system. (2) The resultant force is the only interaction between the object and its surroundings.

ANALYSIS: By assumption (2), the work of the resultant force must equal the change in kinetic energy. Thus, using Eq. 2.6

$$\textcircled{1} \quad \text{work} = \frac{1}{2} m (V_2^2 - V_1^2)$$

$$= \frac{1}{2} (2 \text{ kg}) (500^2 - 200^2) \frac{\text{m}^2}{\text{s}^2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 210 \text{ kJ} \leftarrow$$

work

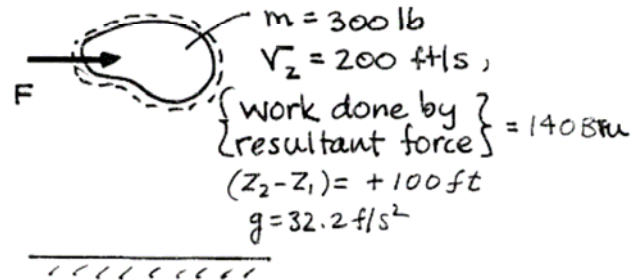
-
1. The increase in kinetic energy of the object is the result of energy transferred to it by the work of the resultant force.

PROBLEM 2.10

KNOWN: An object of known mass undergoes a change of kinetic energy due to the action of a resultant force. The final velocity, the work done by the force, and the change in elevation are given.

FIND: Determine the initial velocity.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The object is a closed system. (2) The force of gravity acts on the object. (3) The resultant force accounts for all other forces acting on the object.

ANALYSIS: By assumption (3), the work of the resultant force must equal the sum of the changes in kinetic energy and gravitational potential energy. Thus, with Eq. 2.9

$$\text{work} = \frac{1}{2} m (V_2^2 - V_1^2) + m g (Z_2 - Z_1)$$

Solving for V_1^2 , inserting values, and converting units

$$\begin{aligned}
 V_1^2 &= \frac{2[mg(Z_2 - Z_1) - \text{work}]}{m} + V_2^2, \text{ where } mg(Z_2 - Z_1) = (300)(32.2)(100) \left| \frac{1}{32.2} \right| \left| \frac{1}{778} \right| \\
 &= - \frac{2(101.4 \text{ Btu})}{(300 \text{ lb})} \left| \frac{778 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}} \right| \left| \frac{32.2 \text{ lb}\cdot\text{ft/s}^2}{1 \text{ lbf}} \right| + 200^2 \frac{\text{ft}^2}{\text{s}^2} \\
 &= 23065 \text{ ft}^2/\text{s}^2
 \end{aligned}$$

or

$$\textcircled{1} \quad V_1 = 151.9 \text{ ft/s} \leftarrow \text{-----} V_2$$

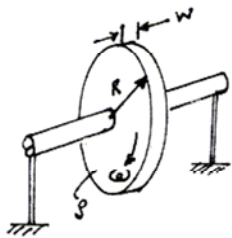
1. The increase in velocity reflects the increase in kinetic energy of the object as a result of energy transferred to it by the work of the resultant force.

PROBLEM 2.11

KNOWN: Data are provided for a disk-shaped flywheel.

FIND: (a) Obtain appropriate expressions for the moment of inertia and the kinetic energy. (b) Using given data, determine the kinetic energy and mass for a steel flywheel. (c) Using results from part (b), determine the radius and mass of an aluminum flywheel.

SCHEMATIC & GIVEN DATA:



Steel flywheel:

$$\omega = 3000 \text{ RPM}$$

$$R = 0.38 \text{ m}$$

$$W = 0.025 \text{ m}$$

Aluminum flywheel:

$$\omega = 3000 \text{ RPM}$$

$$W = 0.025 \text{ m}$$

ENGR. MODEL: 1. The flywheel is the closed system. 2. Motion is relative to the flywheel support structure.

ANALYSIS:

(a) Evaluating the moment of inertia

$$I = \int_{\text{vol}} \rho r^2 dV$$

For the disk, $dV = (2\pi r dr)W$. Thus, since ρ is constant

$$I = \rho (2\pi)W \int_0^R r^3 dr = \rho \pi W R^4 / 2 \quad \leftarrow I$$

The kinetic energy is

$$KE = \int_{\text{vol}} \left(\frac{1}{2} \rho V^2 \right) dV$$

and $V = r\omega$, so

$$\begin{aligned} KE &= \int_0^R \left(\frac{1}{2} \rho r^2 \omega^2 \right) (2\pi r dr) W \\ &= \frac{1}{2} \rho \omega^2 (2\pi) W \int_0^R r^3 dr \\ &= \frac{1}{2} \left(\rho \pi \frac{R^4}{2} W \right) \omega^2 \\ &\quad \leftarrow I \end{aligned}$$

$$= \frac{1}{2} I \omega^2 \quad \leftarrow KE$$

(b) From Table A-19, the density of steel is $\rho = 8060 \text{ kg/m}^3$. Thus, the mass is

$$\begin{aligned} m &= \rho V = \rho [W \cdot \pi R^2] \\ &= \left(8060 \frac{\text{kg}}{\text{m}^3} \right) [(0.025 \text{ m}) \cdot \pi \cdot (0.38 \text{ m})^2] = 91.41 \text{ kg} \quad \leftarrow m \end{aligned}$$

Using the result of part (a), $KE = \frac{1}{2} I \omega^2$, where

$$I = \pi \rho W \frac{R^4}{2} = \frac{\pi}{2} \left(8060 \frac{\text{kg}}{\text{m}^3} \right) (0.025 \text{ m}) (0.38 \text{ m})^4 = 6.6 \text{ kg} \cdot \text{m}^2$$

Continued on next slide

Problem 2-11 continued

Accordingly,

$$KE = \frac{1}{2} I \omega^2 = \frac{1}{2} (6.6 \text{ kg} \cdot \text{m}^2) \left(3000 \frac{\text{REV}}{\text{min}} \left| \frac{2\pi \text{ rad}}{\text{REV}} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \right)^2 \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right|$$

$$= 32.57 \times 10^4 \text{ N} \cdot \text{m} \quad \leftarrow KE$$

- (c) If ω , w , and KE are the same for the aluminum flywheel as for the steel flywheel

$$(KE)_{AL} = (KE)_{ST}$$

$$\left(\frac{1}{2} I \omega^2 \right)_{AL} = \left(\frac{1}{2} I \omega^2 \right)_{ST} \Rightarrow I_{AL} = I_{ST}$$

or

$$\left(\pi \rho w \frac{R^4}{2} \right)_{AL} = \left(\pi \rho w \frac{R^4}{2} \right)_{ST}$$

$$\Rightarrow (\rho R^4)_{AL} = (\rho R^4)_{ST}$$

$$R_{AL} = \left(\frac{\rho_{ST}}{\rho_{AL}} \right)^{1/4} R_{ST}$$

With ρ_{AL} from Table A-19, $\rho_{AL} = 2700 \text{ kg/m}^3$

$$R_{AL} = \left(\frac{8060}{2700} \right)^{1/4} (0.38 \text{ m})$$

$$= 0.5 \text{ m} \quad \leftarrow R_{AL}$$

Then, the mass of the aluminum flywheel is

$$m = \rho V = \rho [w \pi R^2]$$

$$= \left(2700 \frac{\text{kg}}{\text{m}^3} \right) (0.025 \text{ m}) (\pi) (0.5 \text{ m})^2 = 53.01 \text{ kg} \quad \leftarrow m$$

PROBLEM 2.12

From Problem 2.11, $KE = \frac{1}{2} I \omega^2 \Rightarrow \omega = \left(\frac{2 KE}{I} \right)^{1/2}$ where $I = 200 \text{ lb} \cdot \text{ft}^2$.

The change in potential energy of a 100 lb mass raised 30 ft is

$$\Delta PE = mg(z_2 - z_1) = (100 \text{ lb}) \left(32.2 \frac{\text{ft}}{\text{s}^2} \right) (30 \text{ ft}) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft}/\text{s}^2} \right| = 3000 \text{ ft} \cdot \text{lbf}.$$

Thus, with $KE = 3000 \text{ ft} \cdot \text{lbf}$,

$$\omega = \left(\frac{2 (3000 \text{ ft} \cdot \text{lbf})}{200 \text{ lb} \cdot \text{ft}^2} \left| \frac{32.2 \text{ lb} \cdot \text{ft}/\text{s}^2}{1 \text{ lbf}} \right| \right)^{1/2} = 31.08 \frac{1}{\text{s}}$$

In terms of RPM

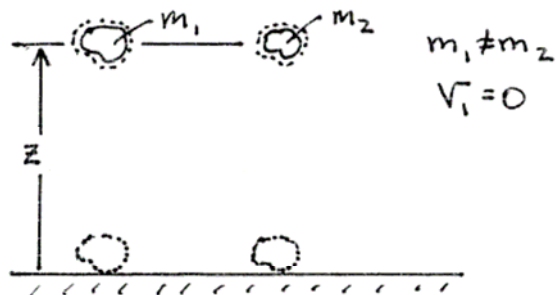
$$\omega = \left(31.08 \frac{1}{\text{s}} \right) \left| \frac{1 \text{ rev}}{2\pi} \right| \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 297 \text{ rev/min} \quad \leftarrow$$

PROBLEM 2.13

KNOWN: Two objects fall freely under the influence of gravity from rest and the same initial elevation.

FIND: Show that the magnitudes of the velocities are equal at the moment just before they strike the earth.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) An object in free fall is a closed system. (2) The acceleration of gravity is constant. (3) There is no effect of air resistance. (4) The only force acting is that due to gravity.

ANALYSIS: For an object falling freely under the influence of gravity, Eq 2.11 applies

$$\frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1) = 0$$

For $V_1 = 0$ and $z_2 = 0$

$$\frac{1}{2} m V_2^2 = m g z_1$$

Thus
$$V_2 = \sqrt{2 g z_1}$$

Since the final velocity doesn't depend on mass, both objects will have identical velocities at the moment just before they strike the earth.

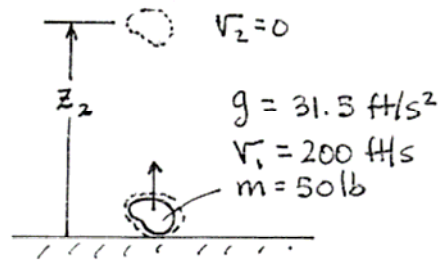
PROBLEM 2.14

KNOWN: An object of known mass is projected upward from the surface of the earth with a known initial velocity. The only force acting on the object is the force of gravity.

FIND: Plot the velocity of the object versus elevation and determine the elevation when its velocity reaches zero.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The object is a closed system. (2) The acceleration of gravity is constant. (3) The only force acting on the object is the force of gravity.



ANALYSIS: Since the only force acting on the body is the force of gravity, Eq. 2.11 applies. Thus, the velocity and elevation are related to the initial condition by

$$\frac{1}{2} m V^2 + m g z = \frac{1}{2} m V_1^2 + m g z_1$$

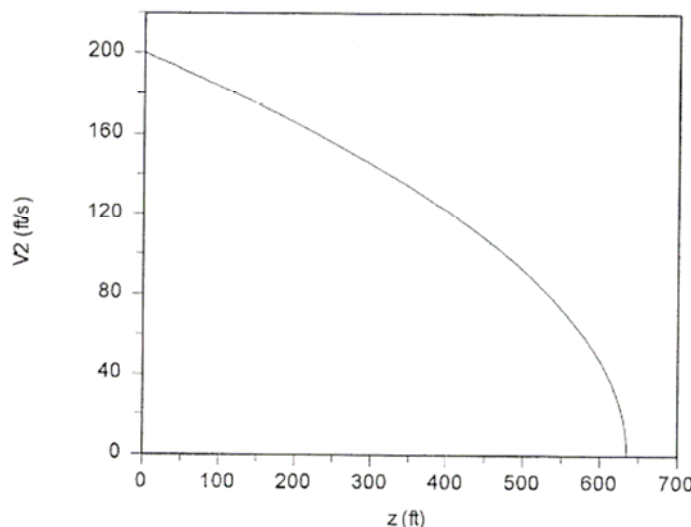
Thus, solving for V

$$V = \sqrt{V_1^2 - 2gz} \quad \leftarrow \text{Expression for } V \text{ in terms of } z$$

When $V_2 = 0$, z_2 is

$$z_2 = \frac{V_1^2}{2g} = \frac{200^2 \text{ ft}^2/\text{s}^2}{2(31.5 \text{ ft/s}^2)} = 634.92 \text{ ft} \quad \leftarrow z_2$$

Plotting the above relationship

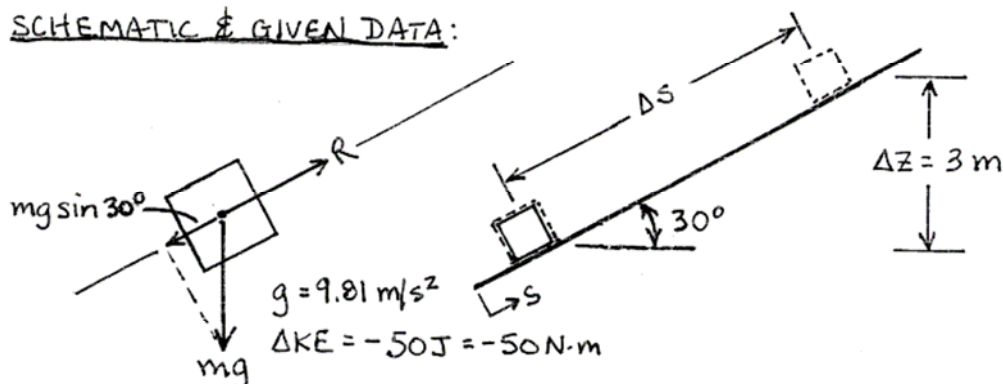


PROBLEM 2.15

KNOWN: A block of known mass moves along an inclined surface. The change in elevation and the change in kinetic energy of the block are given. The block is acted upon by a force parallel to the incline and the force of gravity.

FIND: Determine the magnitude and direction of applied force.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The block is a closed system. (2) The acceleration of gravity is constant. (3) The force R is constant. (4) There is no frictional force between the block and the incline.

ANALYSIS: Beginning with Eq. 2.6

$$\int \underline{F} \cdot d\underline{s} = \Delta KE$$

From the free body diagram, the dot product $\underline{F} \cdot d\underline{s}$ can be expressed as

$$\underline{F} \cdot d\underline{s} = (R - mg \sin 30^\circ) ds$$

So

$$\int_{s_1}^{s_2} (R - mg \sin 30^\circ) ds = \Delta KE$$

Noting that $mg \sin 30^\circ ds = mg \Delta z$, the integral becomes

$$R \Delta s = \Delta KE + mg \Delta z$$

Evaluating Δs

$$\Delta s = \frac{\Delta z}{\sin 30^\circ} = \frac{3 \text{ m}}{\sin 30^\circ} = 6 \text{ m}$$

Thus

$$R = \frac{(-50 \text{ N}\cdot\text{m}) + (10 \text{ kg})(9.81 \text{ m/s}^2)(3 \text{ m})}{(6 \text{ m})} \left| \frac{\text{N}}{\text{kg}\cdot\text{m/s}^2} \right|$$

$$= 40.72 \text{ N}$$

$\xrightarrow{\quad R \quad}$

The positive value denotes that the direction is the same as indicated on the above figure.

PROBLEM 2.16

KNOWN: Beginning from rest, and object of known mass slides down an inclined plane. The length of the ramp is given.

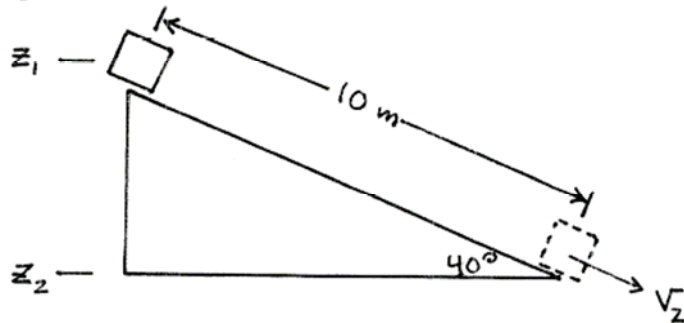
FIND: Determine the velocity of the object at the bottom of the ramp.

SCHEMATIC & GIVEN DATA:

$$m = 200 \text{ kg}$$

$$g = 9.81 \text{ m/s}^2$$

$$V_1 = 0$$



ENGR. MODEL: (1) The mass is a closed system. (2) There is no friction between the mass and the ramp, and air resistance is negligible. (3) The acceleration of gravity is constant.

ANALYSIS: By assumption (2), the only force acting on the system is the force of gravity. Thus, Eq. 2.11 applies

$$\textcircled{1} \quad \frac{1}{2} m (\cancel{V_2^2} - \cancel{V_1^2}) + mg(z_2 - z_1) = 0$$

Solving for V_2

$$V_2 = \sqrt{2g(z_1 - z_2)}$$

From trigonometric relationships

$$z_1 - z_2 = (10 \text{ m}) \sin 40^\circ$$

Thus

$$V_2 = \sqrt{2(9.81 \text{ m/s}^2)(10 \text{ m}) \sin 40^\circ}$$

$$= 11.23 \text{ m/s} \quad \leftarrow \quad V_2$$

1. Even though the object travels along an inclined path, the vertical distance appears in this expression.

PROBLEM 2.17



Fig. P2.17

- ⊙ Exercise value = 620 kcal
- ⊙ Calorie value, 1 cup of vanilla ice cream = 264 kcal (Internet)

To break even calorie-wise, Jack may have

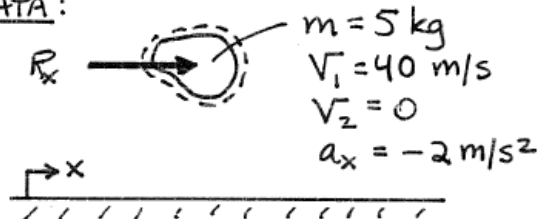
$$\frac{620 \text{ kcal}}{264 \text{ kcal/cup}} = 2.35 \text{ cups}$$

PROBLEM 2.18

KNOWN: A system of known mass and a given initial velocity experiences a constant deceleration due to the action of a resultant force and comes to rest.

FIND: Determine the length of time the force is applied and the work.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system is closed. (2) The horizontal deceleration is constant.

ANALYSIS: To find the time, use the fact that the acceleration is constant, as follows

$$a_x = \frac{dV}{dt} \Rightarrow dV = a_x dt$$

$$\int_{V_1}^{V_2} dV = \int_{t_1}^{t_2} a_x dt$$

or $V_2 - V_1 = a_x \Delta t$

Thus

$$\Delta t = \frac{-V_1}{a_x} = \frac{(-40 \text{ m/s})}{(-2 \text{ m/s}^2)} = 20 \text{ s} \leftarrow \Delta t$$

The work of the force R_x is found from Eq. 2.9

$$\text{Work} = \frac{1}{2} m (V_2^2 - V_1^2)$$

$$= -\frac{1}{2} (5 \text{ kg}) (40^2) \frac{\text{m}^2}{\text{s}^2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$\textcircled{1} \quad = -4 \text{ kJ} \leftarrow \text{work}$$

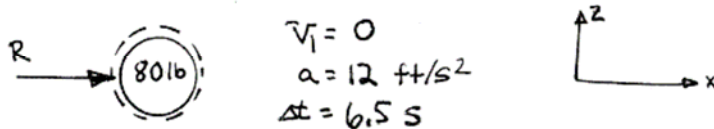
1. The negative denotes energy transfer out of the system.

PROBLEM 2.19

KNOWN: An object of known mass, acted on by a resultant force for a specified time, experiences a constant acceleration from a known initial velocity.

FIND: Determine the work of the resultant force.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The object is the closed system. 2. Motion is horizontal, so the system experiences no change in potential energy. 3. The horizontal acceleration is constant.

ANALYSIS: Applying Eq. 2.9 with assumption 2, the work of the force R is given by

$$\text{Work} = \frac{1}{2} m (V_2^2 - V_1^2)$$

The final velocity V_2 can be determined using assumption 3

$$\frac{dV}{dt} = a \quad (\text{constant})$$

Thus

$$dV = a dt \Rightarrow V_2 - V_1 = a (t_2 - t_1)$$

Or

$$V_2 = \overset{0}{V_1} + a (t_2 - t_1)$$

$$= 12 \frac{\text{ft}}{\text{s}^2} (6.5 \text{ s}) = 78 \text{ ft/s}$$

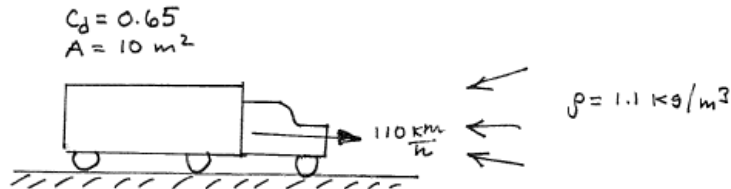
$$\begin{aligned} \text{Then work} &= \frac{1}{2} (80 \text{ lb}) \left(78 \frac{\text{ft}}{\text{s}} \right)^2 \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \\ &= 7558 \text{ ft} \cdot \text{lbf} \quad \leftarrow \text{work (ft} \cdot \text{lbf)} \\ &= (7558 \text{ ft} \cdot \text{lbf}) \left| \frac{1 \text{ Btu}}{778.2 \text{ ft} \cdot \text{lbf}} \right| = 9.71 \text{ Btu} \quad \leftarrow \text{work (Btu)} \end{aligned}$$

PROBLEM 2.20

KNOWN: The drag force on a truck is known as a function of velocity and other parameters.

FIND: Determine the power required by the truck to overcome drag.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The truck is the system.

ANALYSIS: Applying Eq. 2.13, the power required to overcome drag is

$$\begin{aligned}
 \dot{W}_d &= \mathbf{F}_d \cdot \mathbf{V} = \left(\frac{1}{2} C_d A \rho V^2 \right) V \\
 &= \frac{1}{2} C_d A \rho V^3 \\
 &= \frac{1}{2} (0.65) (10 \text{ m}^2) (1.1 \frac{\text{kg}}{\text{m}^3}) \left[\left(110 \frac{\text{km}}{\text{h}} \right) \left| \frac{10^3 \text{ m}}{\text{km}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \right]^3 \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\
 &= 102 \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\
 &= 102 \text{ kW}
 \end{aligned}$$

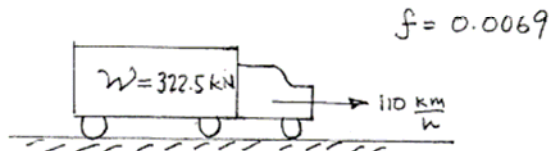
$\leftarrow \dot{W}_d$

PROBLEM 2.21

KNOWN: The force associated with the rolling resistance of the tires of a truck is known as a function of the truck weight.

FIND: Determine the power required by the truck to overcome rolling resistance.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The truck is the system.

ANALYSIS: Applying Eq. 2.13, the power required to overcome rolling resistance is

$$\begin{aligned}\dot{W}_r &= F_r \cdot V = (fW)V \\ &= (0.0069) \left(322.5 \text{ kN} \left| \frac{10^3 \text{ N}}{1 \text{ kN}} \right| \right) \left(110 \frac{\text{km}}{\text{h}} \left| \frac{10^3 \text{ m}}{1 \text{ km}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \right) \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 68 \text{ kW}\end{aligned}$$

$\longleftarrow \dot{W}_r$

PROBLEM 2.22

KNOWN: The drag force and the force associated with rolling resistance are known as functions of variables associated with a vehicle in motion.

FIND: (a) Determine the power required to overcome drag and rolling resistance when the vehicle is moving at 55 mi/h. (b) Plot the quantities of part (a) and their sum versus vehicle velocity ranging from 0 to 75 mi/h. Discuss the implication for vehicle fuel economy.

$$W = 3550 \text{ lbf}$$

$$A = 23.3 \text{ ft}^2$$

$$C_d = 0.34$$

$$f = 0.02$$



ENGR. MODEL: The vehicle is the system.

ANALYSIS: Applying Eq. 2.13, the power required to overcome drag is

$$\begin{aligned} \dot{W}_d &= F_d \cdot V = \left(\frac{1}{2} C_d A \rho V^2 \right) V = \frac{1}{2} C_d A \rho V^3 \\ &= \frac{1}{2} (0.34) (23.3 \text{ ft}^2) (0.08 \frac{\text{lb}}{\text{ft}^3}) \left[V \left(\frac{\text{mi}}{\text{h}} \right) \left| \frac{5280 \text{ ft}}{\text{mi}} \right| \left| \frac{\text{h}}{3600 \text{ s}} \right| \right]^3 \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{\text{hp}}{550 \text{ ft} \cdot \text{lbf/s}} \right| \\ &= 5.65 \times 10^{-5} [V (\text{mi/h})]^3 \text{ hp} \quad (*) \end{aligned}$$

The power required to overcome rolling resistance is

$$\begin{aligned} \dot{W}_r &= F_r \cdot V = (f W) V = (0.02) (3550 \text{ lbf}) \left[V \left(\frac{\text{mi}}{\text{h}} \right) \left| \frac{5280 \text{ ft}}{\text{mi}} \right| \left| \frac{\text{h}}{3600 \text{ s}} \right| \right] \left| \frac{\text{hp}}{550 \text{ ft} \cdot \text{lbf/s}} \right| \\ &= 0.189 V \left(\frac{\text{mi}}{\text{h}} \right) \text{ hp} \quad (**) \end{aligned}$$

(a) When $V = 55 \text{ mi/h}$, we have

$$\dot{W}_d = 5.65 \times 10^{-5} [55]^3 = 9.4 \text{ hp}$$

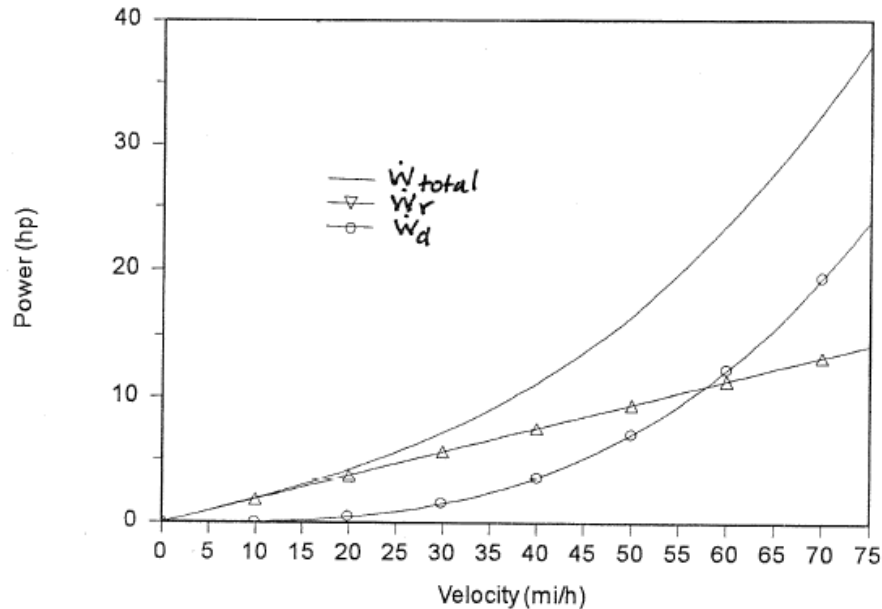
$$\dot{W}_r = 0.189 [55] = 10.4 \text{ hp}$$

← $\dot{W}_d, \dot{W}_r$

Continued on next slide

Problem 2-22 continued

- (b) Letting V range from 0 to 75 mi/h, the accompanying plots can be developed.



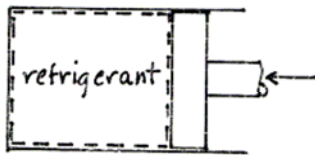
We see from the plots that up to about 50 mi/h, the power required to overcome rolling resistance is more significant than the power to overcome drag. At higher speeds, the drag effect becomes dominant because of the V^3 term in the expression for W_d . Since the power to overcome these effects is developed by the engine from the fuel stored on board the vehicle, high-speed driving has an especially significant effect on fuel consumption.

PROBLEM 2.23

KNOWN: Measured data for pressure versus volume during the compression of a refrigerant are given.

FIND: (a) Determine n for a fit of the data by $pV^n = \text{constant}$. (b) Use the result of part (a) to evaluate the work done during the compression. (c) Evaluate the work using graphical or numerical integration of the data, (d) compare and discuss parts (b) and (c).

SCHEMATIC & GIVEN DATA:



Data Point	p (lbf/in. ²)	V (in. ³)
1	112	13.0
2	131	11.0
3	157	9.0
4	197	7.0
5	270	5.0
6	424	3.0

ENGR. MODEL: 1. The refrigerant in the piston-cylinder assembly form a closed system. 2. The pressure values provided approximate the pressure at the piston face.

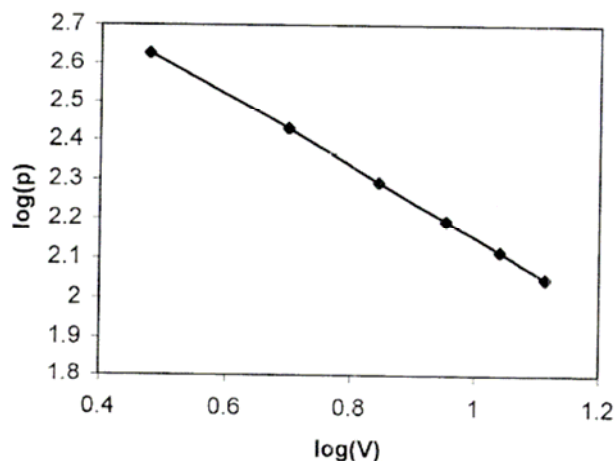
ANALYSIS:

(a) One approach to find n is to begin with $pV^n = \text{constant}$. Taking the log of both sides of this equation

$$\log p + n \log V = \log c$$

or
$$\log p = (-n) \log V + \log c$$

Thus, $(-n)$ corresponds to the slope of a plot of $\log p$ vs. $\log V$. Using ① a spreadsheet program to obtain the plot and the least squares best fit curve:



From the curve fit
 $(-n) = -0.90887$
 or $n = 0.90887$ ← n
 Thus
 $pV^{0.90887} = \text{constant}$

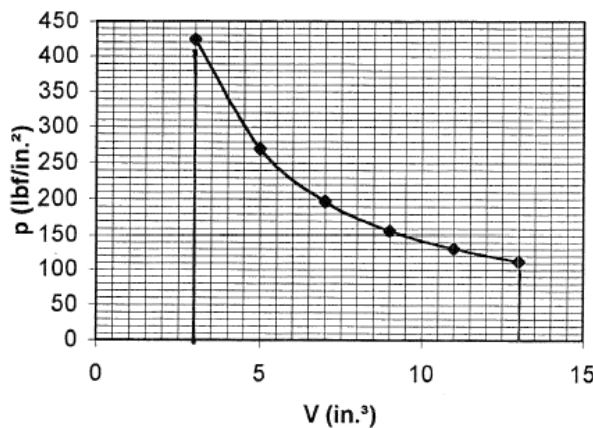
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Problem 2-23 continued

(b) Using the results of part (a) and the procedure of Example 2.1, the work is

$$\begin{aligned}
 W &= \int_{V_1}^{V_2} p \, dV = \frac{P_2 V_2 - P_1 V_1}{(1-n)} \\
 &= \frac{(424 \text{ lbf/in}^2)(3.0 \text{ in}^3) - (112)(13.0)}{(1 - 0.90887)} \left| \frac{1 \text{ ft}}{12 \text{ in}} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\
 &= -0.2163 \text{ Btu} \longleftarrow W
 \end{aligned}$$

② (c) A graphical evaluation of the work involves a plot of the tabulated data and a smooth curve drawn through the data points:



Each elemental rectangle in the plot contributes the following to the area under the curve:

$$\left(10 \frac{\text{lbf}}{\text{in}^2} \right) (0.5 \text{ in}^3) \left| \frac{1 \text{ ft}}{12 \text{ in}} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 5.356 \times 10^{-4} \text{ Btu}$$

The number of rectangles is approximately 401.1, thus

$$W \approx (401.1)(5.356 \times 10^{-4}) = -0.2148 \text{ Btu} \longleftarrow W$$

(d) The results obtained in parts (b) and (c) are in good agreement. Each should be considered a plausible estimate for the reasons presented in Sec. 2.2.4 in the discussion of actual expansion and compression processes.

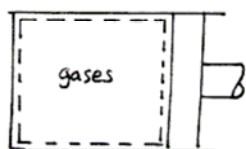
1. The software IT could be used to obtain the least squares curve fit by programming the equations for curve fitting. It is easier to use a spreadsheet program in this instance, however.
2. The only measured data are the tabulated data points, shown as filled circles. The smooth curve does not necessarily represent the actual pressure at the piston face for the corresponding volume.

PROBLEM 2.24

KNOWN: Measured pressure-volume data for an expansion of gases within the cylinder of an internal combustion engine are given.

FIND: (a) Determine n for a fit of the data by $pV^n = \text{constant}$. (b) Use the result of part (a) to evaluate the work done in the expansion. (c) Evaluate the work done using graphical or numerical integration of the data. (d) Compare and discuss parts (c), (d).

SCHEMATIC & GIVEN DATA:



Data Point	p (bar)	V (cm ³)
1	15	300
2	12	361
3	9	459
4	6	644
5	4	903
6	2	1608

ENGR. MODEL: 1. As shown in the schematic, the gases within the piston-cylinder form the closed system. 2. The pressure values provided approximate the pressure at the piston face.

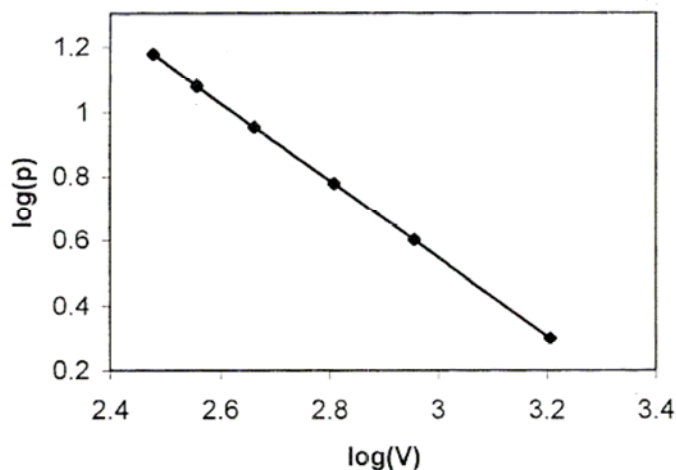
ANALYSIS (a) One approach to find n is to begin with $pV^n = \text{constant}$. Taking the log of both sides of this equation

$$\log p + n \log V = \log c$$

or

$$\log p = (-n) \log V + \log c$$

Thus, $(-n)$ corresponds to the slope of a plot of $\log p$ vs. $\log V$. Using a spreadsheet program to obtain the plot and the least squares best fit curve:



From the curve fit

$$(-n) = -1.1996$$

or

$$n = 1.1996$$

Thus

$$pV^{1.1996} = \text{constant}$$

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Problem 2-24 continued

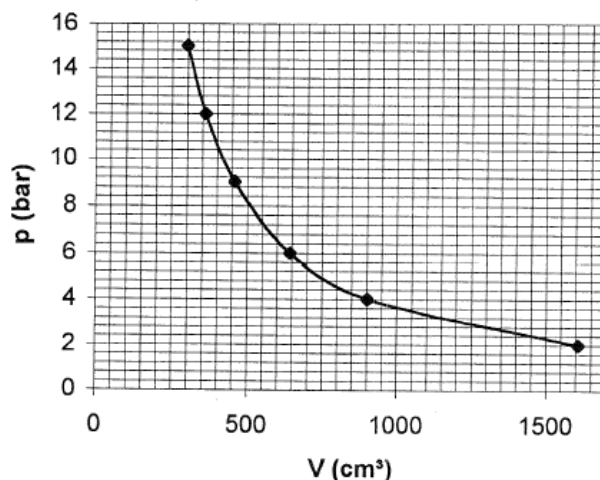
(b) Using the results of part (a) and the procedure of Example 2.1, the work is

$$W = \int_{V_1}^{V_2} p dV = \frac{P_2 V_2 - P_1 V_1}{(1-n)}$$

$$= \frac{(2 \text{ bar})(1608 \text{ cm}^3) - (15)(300)}{(1-1.1996)} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$= 0.643 \text{ kJ} \leftarrow W$$

② (c) A graphical evaluation of the work involves a plot of the tabulated data and a smooth curve drawn through the data points:



Each elemental rectangle in the plot contributes the following to the area under the curve:

$$(.4 \text{ bar})(50 \text{ cm}^3) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 0.002 \text{ kJ}$$

The number of rectangles is approximately 324, thus

$$W \approx (324)(0.002) = 0.648 \text{ kJ} \leftarrow W$$

(d) The results obtained in parts (b) and (c) are in good agreement. Each should be considered a plausible estimate for the reasons presented in Sec. 2.2.4 in the discussion of actual expansion and compression processes.

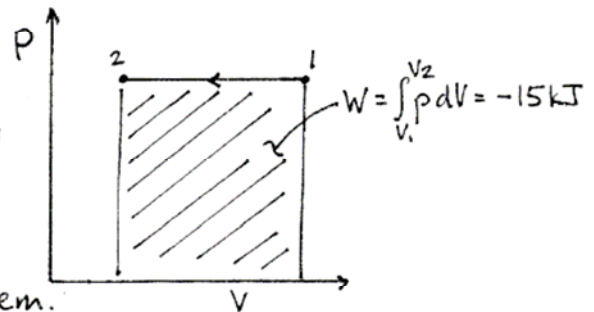
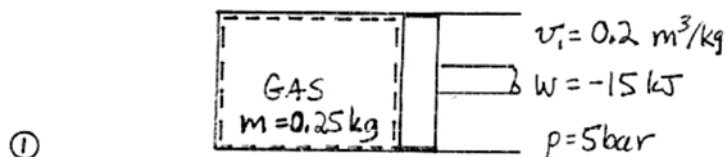
1. The software IT could be used to obtain the least squares curve fit by programming the equations for curve fitting. It is easier to use a spreadsheet program in this instance, however.
2. The only measured data are the tabulated data points, shown as filled circles. The smooth curve does not necessarily represent the actual pressure at the piston face for the corresponding volume.

PROBLEM 2.25

KNOWN: A known amount of gas undergoes a constant-pressure process in a piston-cylinder assembly beginning at a specified specific volume. The work is known.

FIND: Determine the final volume.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The gas is a closed system.
 (2) Pressure is constant during the process.
 (3) Volume change is the only work mode.

ANALYSIS: Using Eq. 2.17

$$W = \int_{V_1}^{V_2} p dV = p(V_2 - V_1)$$

$$= p(V_2 - m v_i)$$

Solving for V_2 and inserting values

$$V_2 = \frac{W}{p} + m v_i$$

$$= \frac{(-15 \text{ kJ})}{(5 \text{ bar})} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| + (0.25 \text{ kg})(0.2 \text{ m}^3/\text{kg})$$

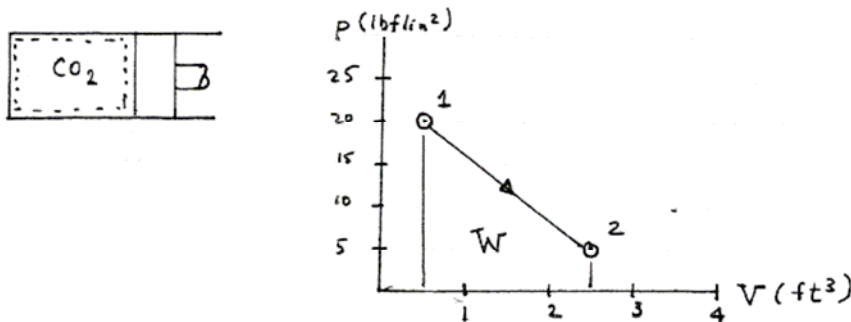
$$= 0.02 \text{ m}^3 \longleftarrow V_2$$

PROBLEM 2.26

KNOWN: CO_2 gas within a piston-cylinder assembly undergoes an expansion where the p - V relation is $p = A + BV$.

FIND: Evaluate A and B and the work, all in specified units.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The CO_2 is the closed system. (2) The p - V relation during expansion is linear. (3) Volume change is the only work mode.

ANALYSIS:

Part (b)

$$p_1 = A + BV_1 \Rightarrow (20 \text{ lbf/in}^2) = B(0.5 \text{ ft}^3) + A$$

$$p_2 = A + BV_2 \Rightarrow (5 \text{ lbf/in}^2) = B(2.5 \text{ ft}^3) + A$$

Solving, $B = -7.5 \frac{\text{lbf/in}^2}{\text{ft}^3}$, $A = 23.75 \text{ lbf/in}^2$ ←

observe that A and B have numerical values and units.

Part (a) Since volume change is the work mode here, Eq. 2.17 applies. Thus

$$W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} [A + BV] dV = \left[AV + \frac{BV^2}{2} \right]_{V_1}^{V_2}$$

$$= A[V_2 - V_1] + \frac{B}{2}[V_2^2 - V_1^2]$$

$$= 23.75 \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| [2.5 - 0.5] \text{ ft}^3 + \left(\frac{-7.5}{2} \right) \frac{\text{lbf/in}^2}{\text{ft}^3} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| [(2.5)^2 - (0.5)^2] \text{ ft}^3$$

$$= 6840 \text{ ft} \cdot \text{lbf} - 3240 \text{ ft} \cdot \text{lbf} = 3600 \text{ ft} \cdot \text{lbf} \quad \leftarrow$$

$$= (3600 \text{ ft} \cdot \text{lbf}) \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 4.63 \text{ Btu} \quad \leftarrow$$

Alternative solution of part (a):

Since the p - V relation is linear, W can be easily evaluated geometrically as the area under the process line:

$$W = p_{\text{ave}}(V_2 - V_1) = \left(\frac{p_1 + p_2}{2} \right) (V_2 - V_1) = \left(\frac{20 + 5}{2} \right) \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| (2.5 - 0.5) \text{ ft}^3$$

$$= 3600 \text{ ft} \cdot \text{lbf} \quad \leftarrow$$

PROBLEM 2.27

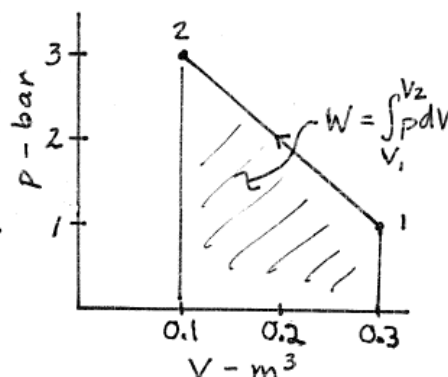
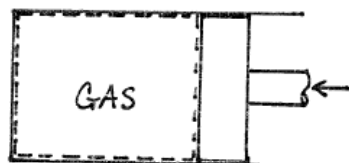
KNOWN: A gas undergoes a compression process. Pressure and volume are given at the initial and final states. Pressure and volume are related linearly during the process.

FIND: Determine the work.

SCHEMATIC & GIVEN DATA:

$$P_1 = 1 \text{ bar}, V_1 = 0.3 \text{ m}^3$$

$$P_2 = 3 \text{ bar}, V_2 = 0.1 \text{ m}^3$$



ENGR. MODEL: (1) The gas is a closed system. (2) The p-V relation during compression is linear. (3) Volume change is the only work mode.

ANALYSIS: Based on the given data, the p-V relation can be expressed as

$$p = (4 \text{ bar}) - (10 \frac{\text{bar}}{\text{m}^3}) V$$

where p is in bar and V is in m^3 . The work is determined using Eq. 2.17

$$W = \int_{V_1}^{V_2} p dV$$

Inserting the p-V relation and integrating

$$W = \int_{V_1=0.3 \text{ m}^3}^{V_2=0.1 \text{ m}^3} [4 - 10 V] dV$$

$$= \left[4V - \left(\frac{10}{2} \right) V^2 \right] \bigg|_{V_1=0.3}^{V_2=0.1}$$

$$= \left[(4 \text{ bar})(0.1 - 0.3) \text{ m}^3 - (5 \frac{\text{bar}}{\text{m}^3})((0.1)^2 - (0.3)^2) (\text{m}^3)^2 \right] \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

① ②

$$= -40 \text{ kJ} \leftarrow$$

W

1. The negative sign for work denotes energy transfer to the system.
2. Since the p-V relation is linear, W can be easily evaluated in terms the average pressure as follows:

$$W = P_{\text{ave}} \Delta V = \left(\frac{P_1 + P_2}{2} \right) (V_2 - V_1) = -(2 \text{ bar})(0.1 - 0.3) \text{ m}^3 \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

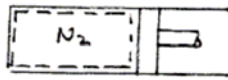
$$= -40 \text{ kJ}$$

PROBLEM 2.28

KNOWN: N_2 gas within a piston-cylinder assembly undergoes a compression where the p - V relation is $pV^{1.35} = \text{constant}$.

FIND: Determine the volume at the final state and the work.

SCHMATIC & GIVEN DATA:



$$pV^{1.35} = \text{constant}$$

$$P_1 = 0.2 \text{ MPa}, V_1 = 2.75 \text{ m}^3$$

$$P_2 = 2 \text{ MPa}$$

ENGR. MODEL:

1. The N_2 is the closed system.
2. The p - V relation during compression is specified.
3. Volume change is the only work mode.

ANALYSIS: (a) $P_1 V_1^n = P_2 V_2^n \Rightarrow V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} V_1$, $n = 1.35$. Thus,

$$V_2 = \left(\frac{0.2 \text{ MPa}}{2 \text{ MPa}}\right)^{\frac{1}{1.35}} (2.75 \text{ m}^3) = 0.5 \text{ m}^3 \quad \leftarrow$$

(b) Since volume change is the work mode, Eq. 2.17 applies. Following the procedure of part (a) of Example 2.1 we have

$$W = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{(2 \text{ MPa})(0.5 \text{ m}^3) - (0.2 \text{ MPa})(2.75 \text{ m}^3)}{1-1.35} \left| \frac{10^6 \text{ N/m}^2}{1 \text{ MPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

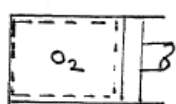
$$= -1285.7 \text{ kJ} \quad \leftarrow$$

PROBLEM 2.29

KNOWN: O_2 gas within a piston-cylinder assembly undergoes an expansion when the p - V relation is $p = AV^{-1} + B$.

FIND: Determine the initial and final pressures and the work.

SCHEMATIC & GIVEN DATA:



$$p = AV^{-1} + B$$

$$A = 0.06 \text{ bar} \cdot \text{m}^3$$

$$B = 3.0 \text{ bar}$$

$$V_1 = 0.01 \text{ m}^3, V_2 = 0.03 \text{ m}^3$$

ENER. MODEL:

1. The O_2 is the closed system
2. The p - V relation during expansion is specified.
3. Volume change is the only work mode.

ANALYSIS:

$$(a) \quad P_1 = [(0.06 \text{ bar} \cdot \text{m}^3) / 0.01 \text{ m}^3] + 3.0 \text{ bar} \quad P_2 = [(0.06 \text{ bar} \cdot \text{m}^3) / (0.03 \text{ m}^3)] + 3.0 \text{ bar}$$

$$\therefore P_1 = 9.0 \text{ bar} \quad \therefore P_2 = 5.0 \text{ bar} \quad \leftarrow$$

(b) Since volume change is the work mode, Eq. 2.17 applies. That is,

$$W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \left[\frac{A}{V} + B \right] dV = A \ln \frac{V_2}{V_1} + B(V_2 - V_1)$$

$$= (0.06 \text{ bar} \cdot \text{m}^3) \ln \left(\frac{0.03 \text{ m}^3}{0.01 \text{ m}^3} \right) + (3.0 \text{ bar}) [0.03 - 0.01] \text{ m}^3$$

$$= [0.0659 + 0.06] \text{ bar} \cdot \text{m}^3 \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

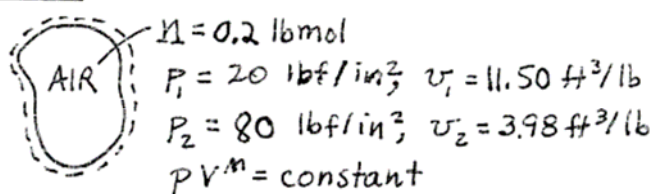
$$= 12.59 \text{ kJ} \quad \leftarrow$$

PROBLEM 2.30

KNOWN: Air undergoes a polytropic process between two specified states.

FIND: Determine the work.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The air is a closed system. (2) The system undergoes a polytropic process: $pV^n = \text{constant}$.

ANALYSIS: From the pressure-volume relation for a polytropic process

$$P_1 V_1^n = P_2 V_2^n \Rightarrow P_1 v_1^n = P_2 v_2^n \quad (\text{since mass is constant})$$

Solving for n

$$n = \frac{\log(P_1/P_2)}{\log(v_2/v_1)} = \frac{\log(20/80)}{\log(3.98/11.50)} = 1.307$$

Now, using Eq. 2.17 to determine work and with the molecular weight of air from Table A-1E

$$\begin{aligned} W &= \int_{V_1}^{V_2} p dV = m \int_{v_1}^{v_2} p dv = m(\text{const.}) \int_{v_1}^{v_2} \frac{dv}{v} \\ &= m \left[\frac{(P_2 v_2^n) v_2^{1-n} - (P_1 v_1^n) v_1^{1-n}}{1-n} \right] = m \left(\frac{P_2 v_2 - P_1 v_1}{1-n} \right) \end{aligned}$$

Now $m = (0.2 \text{ lbmol})(28.97 \text{ lb/lbmol}) = 5.794 \text{ lb}$

and

$$W = (5.794 \text{ lb}) \left[\frac{(80 \text{ lbf/in}^2)(3.98 \text{ ft}^3/\text{lb}) - (20)(11.50)}{(1 - 1.307)} \right] \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

① $= -308.8 \text{ Btu} \leftarrow W$

- The negative sign for work denotes energy transfer into the system.

PROBLEM 2.31

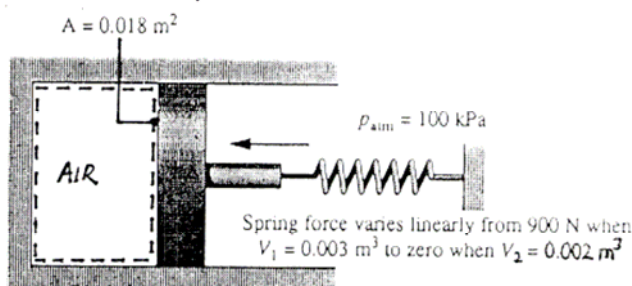
KNOWN: Warm air cools slowly in a piston-cylinder assembly from a known initial volume to a known final volume. During the process, a spring exerts a force on the piston that varies linearly from a known initial value to a final value of zero.

FIND: Determine the initial and final pressures of the air, and the work.

SCHEMATIC & GIVEN DATA:

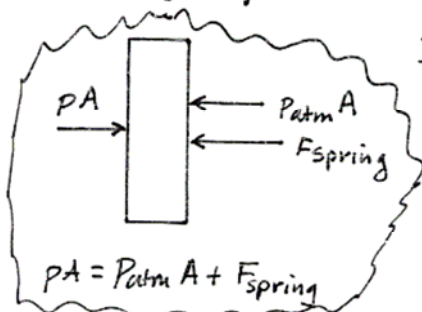
$$V_1 = 0.003 \text{ m}^3$$

$$V_2 = 0.002 \text{ m}^3$$



ENGR. MODEL: (1) The air is a closed system. (2) The process occurs slowly, so there is no acceleration of the piston. (3) There is no friction between the piston and the cylinder wall. (4) The spring force varies linearly with volume.

ANALYSIS: The initial and final pressures of the air are determined from a free-body diagram of the piston, as follows. That is, $\sum F = 0$, so



Initially: $F_{\text{spring}} = 900 \text{ N}$

$$P_1 = P_{\text{atm}} + \frac{F_{\text{spring},1}}{A}$$

$$= 100 \text{ kPa} + \frac{(900 \text{ N})}{(0.018 \text{ m}^2)} \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| = 150 \text{ kPa}$$

Finally: $F_{\text{spring}} = 0 \Rightarrow P_2 = 100 \text{ kPa}$

Now, the work is determined using Eq. 2.17, $W = \int_{V_1}^{V_2} p dV$, but from above

$$p = P_{\text{atm}} + \frac{F_{\text{spring}}}{A}$$

Since the spring force varies linearly from 900 N to zero as volume goes from $V_1 = 0.003 \text{ m}^3$ to $V_2 = 0.002 \text{ m}^3$

$$F_{\text{spring}} = \left(\frac{900 \text{ N}}{0.001 \text{ m}^3} \right) (V - 0.002)$$

and

$$W = \int_{V_1}^{V_2} \left(P_{\text{atm}} + \frac{F_{\text{spring}}}{A} \right) dV = \int_{V_1}^{V_2} \left[100 + \frac{(900)}{(0.001)(0.018)} (V - 0.002) \right] dV$$

Units: $\frac{\text{N}}{\text{m}^2}$ Units: m^3

$$= \int_{V_1}^{V_2} [100 + 50000V - 100] dV = \left(\frac{50000}{2} \right) V^2 \Big|_{V_1=0.003 \text{ m}^3}^{V_2=0.002 \text{ m}^3}$$

Converts units to kPa

$$\textcircled{1} \quad = -0.125 \text{ kPa} \cdot \text{m}^3 \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = -0.125 \text{ kJ} \quad W$$

1. The negative sign denotes that the piston does work on the air as the air cools. Also, the atmosphere and the spring do work on the piston.

PROBLEM 2.32

KNOWN: Air within a piston-cylinder assembly undergoes two processes in series.

FIND: Determine the total work.

SCHEMATIC & GIVEN DATA:

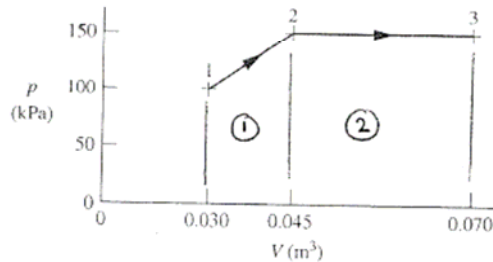


Fig. P2.32

ENGINEER MODEL:

1. The air within the piston-cylinder assembly is the closed system.
2. The two-step p - V relation during expansion is specified.

ANALYSIS: Since volume change is the work mode, Eq. 2.17 applies. Furthermore the integral can be evaluated geometrically in terms of the total area under the process line:

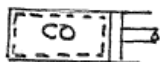
$$\begin{aligned}
 W &= \int_{V_1}^{V_2} p dV = p_{ave}^{(1)} (V_2 - V_1) + p_2^{(2)} (V_3 - V_2) = \left(\frac{p_1 + p_2}{2} \right) (V_2 - V_1) + p_2 (V_3 - V_2) \\
 &= \left[\left(\frac{150 + 100}{2} \right) (\text{kPa}) [0.045 - 0.030] \text{ m}^3 + (150 \text{ kPa}) (0.070 - 0.045) \text{ m}^3 \right] \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\
 &= 1.875 \text{ kJ} + 3.75 \text{ kJ} = 5.625 \text{ kJ}
 \end{aligned}$$

PROBLEM 2.33

KNOWN: Carbon monoxide gas within a piston-cylinder assembly undergoes three processes in series.

FIND: Sketch the processes in series in a p-V diagram and evaluate the work for each.

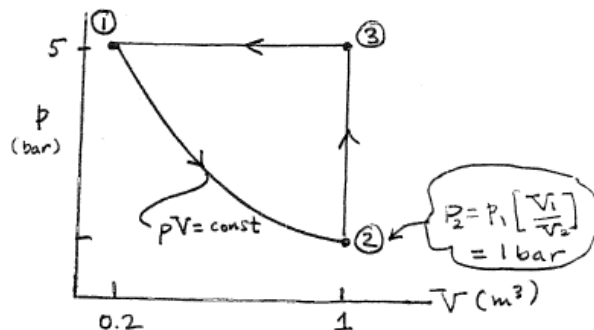
Schematic & Given Data:



Process 1-2: Expansion from $p_1 = 5 \text{ bar}$, $V_1 = 0.2 \text{ m}^3$ to $V_2 = 1 \text{ m}^3$, during which the pressure-volume relationship is $pV = \text{constant}$.

Process 2-3: Constant-volume heating from state 2 to state 3, where $p_3 = 5 \text{ bar}$.

Process 3-1: Constant-pressure compression to the initial state.



ENGR. MODEL:

1. The gas is the closed system.
2. Volume change is the only work mode.
3. Each of the three processes is specified.

ANALYSIS: Since volume change is the work mode, Eq. 2.17 applies.

$$\begin{aligned} \text{Process 1-2: } W_{12} &= \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{C}{V} dV = C \ln \frac{V_2}{V_1} = p_1 V_1 \ln \frac{V_2}{V_1} \\ &= (5 \text{ bar})(0.2 \text{ m}^3) \ln \left(\frac{1 \text{ m}^3}{0.2 \text{ m}^3} \right) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = +160.94 \text{ kJ} \end{aligned}$$

Process 2-3: The piston does not move ($V = \text{constant}$). Thus, $W_{23} = 0$

$$\begin{aligned} \text{Process 3-1: } W_{31} &= \int_{V_3}^{V_1} p dV = p_3 (V_1 - V_3) = 5 \text{ bar} (0.2 - 1.0) \text{ m}^3 \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= -400 \text{ kJ} \end{aligned}$$

constant at p_3

PROBLEM 2.34

KNOWN: Air within a piston-cylinder assembly undergoes three processes in series.

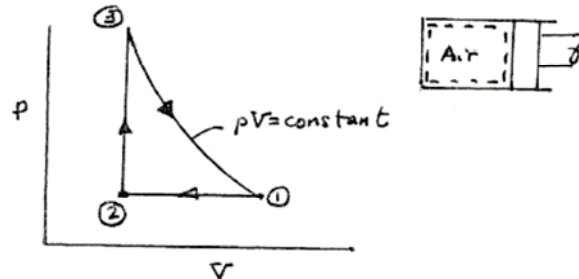
FIND: Sketch the processes in series on p - V coordinates, evaluate volume at state 2, and evaluate the work for each process.

SCHEMATIC & GIVEN DATA

Process 1-2: Compression at constant pressure from $p_1 = 10 \text{ lbf/in}^2$, $V_1 = 4 \text{ ft}^3$ to state 2.

Process 2-3: Constant-volume heating to state 3, where $p_3 = 50 \text{ lbf/in}^2$.

Process 3-1: Expansion to the initial state, during which the pressure-volume relationship is $pV = \text{constant}$.



ENGR. MODEL:

1. The air is the closed system.
2. Each of the three processes is specified.
3. Volume change is the only work mode.

ANALYSIS: Since volume change is the work mode, Eq. 2.17 applies.

Process 1-2: $W_{12} = \int_{V_1}^{V_2} p dV = p[V_2 - V_1]$. Volume V_2 is needed.

Note that $V_2 = V_3$ and $p_1 V_1 = p_3 V_3 \Rightarrow V_3 = (p_1/p_3) V_1 = \left(\frac{10 \text{ lbf/in}^2}{50 \text{ lbf/in}^2}\right)(4 \text{ ft}^3) = 0.8 \text{ ft}^3$

Thus, $W_{12} = 10 \frac{\text{lbf}}{\text{in}^2} [0.8 \text{ ft}^3 - 4 \text{ ft}^3] \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = -5.92 \text{ Btu}$

Process 2-3: The piston does not move ($V = \text{constant}$). Thus, $W_{23} = 0$.

Process 3-1: $W_{31} = \int_{V_3}^{V_1} p dV = \int_{V_3}^{V_1} \frac{C}{V} dV = C \ln\left(\frac{V_1}{V_3}\right) = p_1 V_1 \ln\left(\frac{V_1}{V_3}\right)$

$$= \left(10 \frac{\text{lbf}}{\text{in}^2}\right)(4 \text{ ft}^3) \ln\left(\frac{4 \text{ ft}^3}{0.8 \text{ ft}^3}\right) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = +11.92 \text{ Btu}$$

PROBLEM 2.35

KNOWN: Operating data is provided for a belt sander.

FIND: Evaluate the power transmitted by the belt to the surface and work done in one minute of sanding.

SCHEMATIC & GIVEN DATA:

Belt speed =
1500 ft/min
Normal force
on sander =
15 lbf

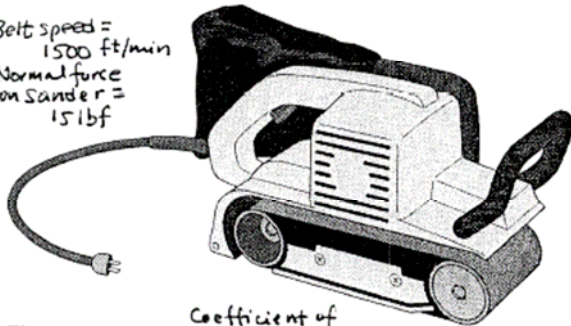


Fig. P2.35

Coefficient of
friction is 0.2

(b) In one minute of sanding, the work done on the surface is

$$W = (0.096 \frac{\text{Btu}}{\text{s}}) \left| \frac{60 \text{ s}}{\text{min}} \right| (1 \text{ min})$$

$$= 5.76 \text{ Btu} \quad \leftarrow$$

ENGR. MODEL

- The force exerted by the belt is related to the normal force, F_N , by the coefficient of friction:

$$F = (\text{coeff. of friction}) F_N = 0.2 F_N$$

ANALYSIS: (a) Using Eq. 2.13, the power, $\dot{W}$, transmitted is

$$\dot{W} = F \cdot V = 0.2 F_N V$$

or

$$\dot{W} = 0.2 (15 \text{ lbf}) (1500 \frac{\text{ft}}{\text{min}}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= 0.096 \frac{\text{Btu}}{\text{s}} \quad \leftarrow$$

or

$$\dot{W} = 0.096 \frac{\text{Btu}}{\text{s}} \left| \frac{3600 \text{ s}}{\text{h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

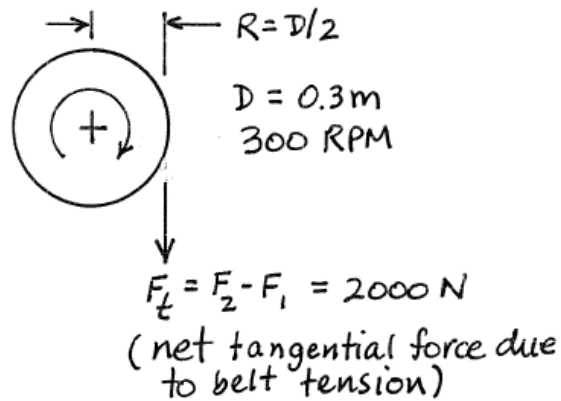
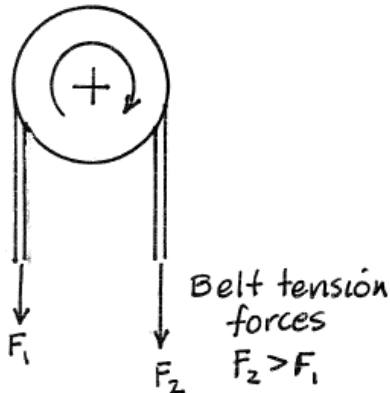
$$= 0.136 \text{ hp} \quad \leftarrow$$

PROBLEM 2.36

KNOWN: The rotational speed and diameter of a drive shaft pulley are known. The net force applied by the belt on the pulley is also given.

FIND: Determine the applied torque and the power transmitted.

SCHEMATIC & GIVEN DATA:



ANALYSIS: The torque is calculated using the tangential force and the radius at which it acts

$$\begin{aligned} \mathcal{J} &= F_t \cdot R \\ &= (2000 \text{ N}) \left(\frac{0.3}{2} \text{ m} \right) = 300 \text{ N} \cdot \text{m} \end{aligned}$$

Thus, with Eq. 2.20 the power transmitted is

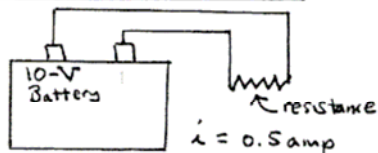
$$\begin{aligned} \dot{W}_{\text{shaft}} &= \mathcal{J} \cdot \omega \\ &= (300 \text{ N} \cdot \text{m}) \left(300 \frac{\text{rev}}{\text{min}} \right) \left(2\pi \frac{1}{\text{rev}} \right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ J}}{1 \text{ N} \cdot \text{m}} \right| \left| \frac{1 \text{ kW}}{10^3 \text{ J/s}} \right| \\ &= 9.42 \text{ kW} \end{aligned}$$

PROBLEM 2.37

KNOWN: Operating data are given for a 10-V battery providing current to a resistance.

FIND: Determine the resistance, in ohms, and the amount of energy transfer by work, in kJ.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

$$\text{Resistance} = \frac{\text{Voltage}}{\text{Current}} = \frac{10 \text{ volts}}{0.5 \text{ amp}} \left| \frac{1 \text{ ohm}}{1 \text{ volt/amp}} \right| = 20 \text{ ohm} \quad \leftarrow$$

With Eq. 2.1 applied to the battery, which is discharging,

$$\dot{W} = (\text{voltage})(\text{current}) = (10 \text{ volt})(0.5 \text{ amp}) \left| \frac{1 \text{ Watt/amp}}{1 \text{ volt}} \right| = 5 \text{ Watt}$$

Then, for 30 minutes of operation,

$$W = \int \dot{W} dt = (5 \text{ watt})(30 \text{ min.}) \left| \frac{60 \text{ s}}{1 \text{ min}} \right| \left| \frac{1 \text{ J/s}}{1 \text{ watt}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ J}} \right| = 9 \text{ kJ} \quad \leftarrow$$

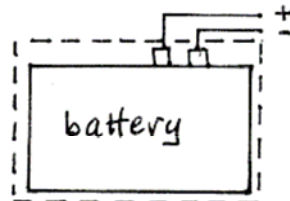
$\uparrow$ Constant

PROBLEM 2.38

KNOWN: An electric storage battery is charged with a constant current for a known length of time.

FIND: Determine the cost of electricity to charge the battery.

SCHEMATIC & GIVEN DATA:



12 volt

2 amp

$\Delta t = 24 \text{ h}$

electricity cost = \$0.08/kW·h

ENGR. MODEL: The voltage and current are constant.

ANALYSIS: The electric power is obtained using Eq. 2.21

$$|\dot{W}| = \mathcal{E} i$$

Since the voltage and current are constant, the electricity used in 24 h is

$$|W| = \int_{t_1}^{t_2} \dot{W} dt = |\dot{W}| \Delta t = \mathcal{E} i \Delta t$$

$$= (2 \text{ amp})(12 \text{ volt}) \left| \frac{1 \text{ Watt/amp}}{1 \text{ volt}} \right| (24 \text{ h}) \left| \frac{1 \text{ kW}}{10^3 \text{ Watt}} \right|$$

$$= 0.576 \text{ kW·h}$$

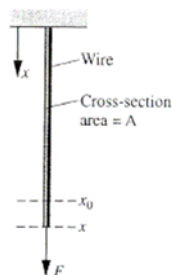
The cost is

$$\text{cost} = (0.576 \text{ kW·h})(\$0.08/\text{kW·h}) \approx \$0.05 \leftarrow \text{electric cost}$$

PROBLEM 2.39

KNOWN: A wire suspended vertically is stretched by an applied force.
FIND: Obtain an expression for the work done on the wire and evaluate the work for a given set of data.

SCHEMATIC & GIVEN DATA:



normal stress strain
 $\sigma = C \epsilon$
 $\epsilon = \frac{x - x_0}{x_0}$
 C : Young's modulus

Data set:
 $x_0 = 10 \text{ ft}$
 $x = 10.01 \text{ ft}$
 $C = 2.5 \times 10^7 \frac{\text{lb f}}{\text{in}^2}$
 $A = 0.1 \text{ in}^2$

ENER. MODEL:

1. The wire is the closed system.
2. The moving boundary is the only work mode.
3. The change in area A is negligible.
4. The normal stress and thus the applied force varies linearly with strain.

Fig. P2.39

ANALYSIS: (a) The work done on the wire is given by Eq. 2.18

$$W = - \int_{x_0}^x \sigma A dx$$

From the given stress-strain relation

$$\sigma = C \epsilon = C \left(\frac{x - x_0}{x_0} \right)$$

Where C is a constant (Young's modulus). From this expression

$$d\epsilon = \frac{dx}{x_0} \Rightarrow dx = x_0 d\epsilon$$

Substituting into the work expression

$$W = - \int_0^{\epsilon} (C\epsilon) A (x_0 d\epsilon) = - C A x_0 \int_0^{\epsilon} \epsilon d\epsilon$$

Finally $W = - \frac{C A x_0 \epsilon^2}{2}$ ← work expression

(b) Substituting given data into the work expression

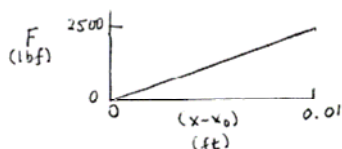
$$W = - \frac{(2.5 \times 10^7 \frac{\text{lb f}}{\text{in}^2})(0.1 \text{ in}^2)(10 \text{ ft}) \left[\frac{0.01}{10} \right]^2}{2} = -12.5 \text{ ft} \cdot \text{lb f}$$

The downward force varies with strain according to

$$F = \sigma A = C \epsilon A = A C \left[\frac{x - x_0}{x_0} \right]$$

When $x = x_0$, $F = 0$. When $x - x_0 = 0.01 \text{ ft}$,

$$F = (0.1 \text{ in}^2)(2.5 \times 10^7 \frac{\text{lb f}}{\text{in}^2}) \left(\frac{0.01 \text{ ft}}{10 \text{ ft}} \right) = 2500 \text{ lb f}$$



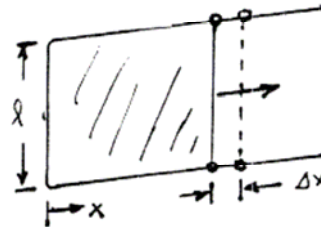
PROBLEM 2.40

KNOWN: A soap film on a wire frame is stretched.

FIND: Determine the work done.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The film is a closed system. (2) The moving boundary is the only work mode. (3) The surface tension is constant, acting on both sides of the film.



ANALYSIS: (a) The work is determined using Eq. 2.19

$$W = - \int_{A_1}^{A_2} \tau dA = - \int_{x_1}^{x_2} \tau 2l dx$$

For constant surface tension τ

$$W = - \tau 2l \Delta x$$

(b) If $l = 5 \text{ cm}$, $\Delta x = 0.5 \text{ cm}$, $\tau = 25 \times 10^{-5} \text{ N/cm}$,

$$W = - (25 \times 10^{-5} \text{ N/cm}) (2) (5 \text{ cm}) (0.5 \text{ cm}) \left| \frac{1 \text{ m}}{10^2 \text{ cm}} \right| \left| \frac{1 \text{ J}}{1 \text{ N} \cdot \text{m}} \right| = -1.25 \times 10^{-5} \text{ J} \leftarrow$$

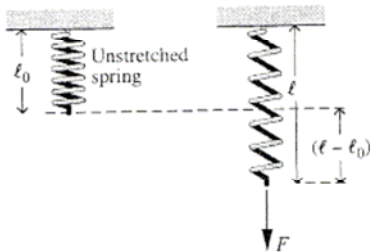
The negative sign denotes work done on the film. Note the small magnitude of the work required to stretch the film.

PROBLEM 2.41

KNOWN: Data is provided for a spring stretched by a force applied at its end.

FIND: Obtain an expression for the work done in stretching the spring and evaluate the work using given data.

SCHEMATIC & GIVEN DATA:



$$F = k(l - l_0)$$

$$l_0 = 3 \text{ cm}$$

$$l_1 = 6 \text{ cm}$$

$$l_2 = 10 \text{ cm}$$

$$k = 10^4 \text{ N/m}$$

Fig. P2.41

ENGR. MODEL:

1. The spring is the closed system.
2. The moving boundary is the only work mode.
3. Hooke's law applies.

ANALYSIS: (a) The work done in stretching the spring is given by

$$W = - \int_1^2 F dl$$

Letting $x = l - l_0$, this becomes

$$\begin{aligned} W &= - \int_1^2 kx dx = -k \left[\frac{x^2}{2} \right]_1^2 \\ &= -\frac{k}{2} [(l_2 - l_0)^2 - (l_1 - l_0)^2] \end{aligned}$$

(b) When $(l_1 - l_0) = 3 \text{ cm}$ and $(l_2 - l_0) = 7 \text{ cm}$,

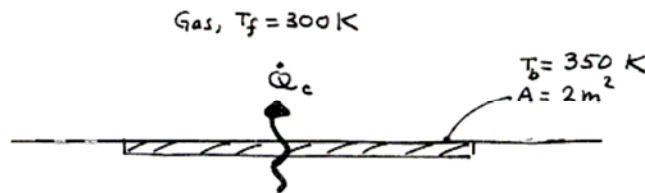
$$\begin{aligned} W &= \left(-\frac{10^4 \text{ N/m}}{2} \right) [(7 \text{ cm})^2 - (3 \text{ cm})^2] \left| \frac{1 \text{ m}}{100 \text{ cm}} \right|^2 \left| \frac{1 \text{ J}}{1 \text{ N}\cdot\text{m}} \right| \\ &= -20 \text{ J} \end{aligned}$$

PROBLEM 2.42

KNOWN: Data are provided for a flat surface cooled convectively by a gas.

FIND: Determine the range of the heat transfer rate for cooling by free convection. By forced convection.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: Heat transfer between the surface and the gas is by convection only.

ANALYSIS: Using Eq. 2.34

$$\begin{aligned}\dot{Q}_c &= h A (T_b - T_f) \\ &= h (2 \text{ m}^2) (350 - 300) \text{ K} \\ &= h (100 \text{ m}^2 \cdot \text{K})\end{aligned}\quad (*)$$

With data from Table 2.1

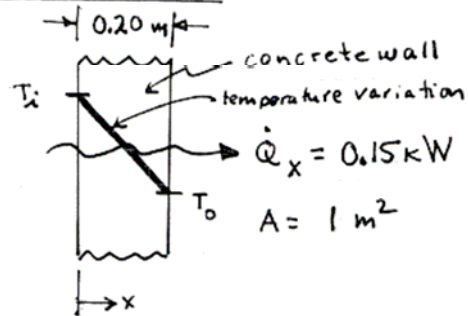
	$h \text{ (W/m}^2 \cdot \text{K)}$	$\dot{Q}_c \text{ from Eq (*) , in kW}$	
free convection:	2 - 25	0.2 - 2.5	←
forced convection:	25 - 250	2.5 - 25	←

PROBLEM 2.43

KNOWN: Data are provided for a concrete wall at steady state.

FIND: Determine the temperature difference across the wall.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The wall is at steady state. 2. The temperature varies linearly through the wall. 3. Energy transfer is by conduction.

ANALYSIS: Using Eq. 2.31 together with assumption 2

$$\dot{Q}_x = -kA \frac{dT}{dx} \quad , \quad \frac{dT}{dx} = \frac{T_o - T_i}{L}$$

where $L = 0.20 \text{ m}$ and $(T_o - T_i)$ is the temperature difference across the wall.

Thus

$$\dot{Q}_x = -kA \left[\frac{T_o - T_i}{L} \right]$$

$$\Rightarrow (T_o - T_i) = - \left[\frac{\dot{Q}_x L}{kA} \right]$$

With $k = 1.4 \text{ W/m} \cdot \text{K}$ from Table A-19

$$(T_o - T_i) = - \left[\frac{(150 \text{ W})(0.20 \text{ m})}{(1.4 \frac{\text{W}}{\text{m} \cdot \text{K}})(1 \text{ m}^2)} \right]$$

$$= -21.4 \text{ K}$$

$$\longleftarrow T_o - T_i$$

where the minus sign is required because $T_o < T_i$.

PROBLEM 2.44

KNOWN: Data are provided for an exterior wall of a building at steady state.

FIND: Determine the outer surface temperature and the heat transfer rate.

SCHEMATIC & GIVEN DATA:

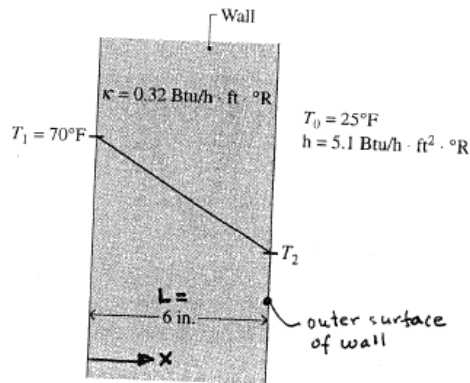


Fig. P2.44

ENGR. MODEL:

1. The wall is at steady state.
2. The temperature varies linearly through the wall.
3. Heat transfer at the outer wall surface is by convection only.

ANALYSIS:

Using Eq. 2.31 together with assumption 2

$$\begin{aligned}\dot{Q}_x &= -kA \frac{dT}{dx} \\ &= -kA \left[\frac{T_2 - T_1}{L} \right] \quad (*)\end{aligned}$$

At steady state, the rate of heat transfer by conduction to the outer surface of the wall equals the rate of heat transfer by convection from the outer surface, where convection is given by Eq. 2.34: $\dot{Q}_c = hA[T_2 - T_0]$. Thus, at $x = L$

$$\begin{aligned}\dot{Q}_x &= \dot{Q}_c \\ -kA \left[\frac{T_2 - T_1}{L} \right] &= hA[T_2 - T_0]\end{aligned}$$

Solving

$$\begin{aligned}T_2 &= \frac{hLT_0 + kT_1}{hL + k} = \frac{(5.1 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{R}})(0.5 \text{ ft})(485^\circ\text{R}) + (0.32 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot ^\circ\text{R}})(530^\circ\text{R})}{(5.1 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{R}})(0.5 \text{ ft}) + (0.32 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot ^\circ\text{R}})} \\ &= 490^\circ\text{R} \quad (30^\circ\text{F})\end{aligned}$$

Then, using Eq. (*), we get

$$\begin{aligned}\frac{\dot{Q}_x}{A} &= -k \left[\frac{T_2 - T_1}{L} \right] \\ &= -0.32 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot ^\circ\text{R}} \left[\frac{490^\circ\text{R} - 530^\circ\text{R}}{0.5 \text{ ft}} \right] = 25.6 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2} \leftarrow\end{aligned}$$

PROBLEM 2.45

KNOWN: Steady-state data are provided for a composite wall formed from a steel layer and a brick layer.

FIND: Determine the minimum thickness of the brick layer to keep the outer surface temperature of the brick $\leq 105^\circ\text{F}$.

SCHMATIC & GIVEN DATA:

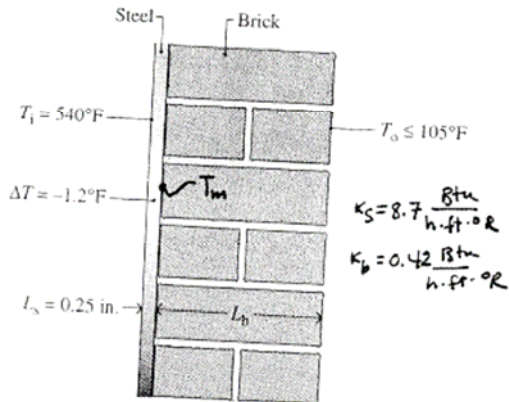


Fig. P2.45

ENGR. MODEL:

1. The wall is at steady state.
2. The temperature varies linearly through each layer.

ANALYSIS:

Using Eq. 2.31 together with assumption 2

$$\left(\frac{\dot{Q}}{A}\right)_{\text{steel}} = -k_s \left[\frac{T_m - T_1}{L_s} \right]$$

$$\left(\frac{\dot{Q}}{A}\right)_{\text{brick}} = -k_b \left[\frac{T_o - T_m}{L_b} \right]$$

where T_m denotes the temperature at the steel-brick interface.

At steady state, the rate of conduction to the steel-brick interface must equal the rate of conduction from that interface. Thus

$$\left(\frac{\dot{Q}}{A}\right)_{\text{steel}} = \left(\frac{\dot{Q}}{A}\right)_{\text{brick}}$$

$$-k_s \left[\frac{T_m - T_1}{L_s} \right] = -k_b \left[\frac{T_o - T_m}{L_b} \right]$$

Solving

$$L_b = \frac{k_b}{k_s} \left[\frac{T_o - T_m}{T_m - T_1} \right] L_s$$

$\downarrow (540 - 1.2) = 538.8^\circ\text{F}$
 $\downarrow = -1.2^\circ\text{F}$

$$L_b = \left(\frac{0.42 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{R}}{8.7 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{R}} \right) \left[\frac{538.8 - T_o}{-1.2^\circ\text{F}} \right] (0.25 \text{ in})$$

Since $T_o \leq 105^\circ\text{F}$,

$$L_b \geq \left(\frac{0.42}{8.7} \right) \left[\frac{538.8 - 105}{-1.2^\circ\text{F}} \right] (0.25 \text{ in})$$

$$L_b \geq 4.36 \text{ in.}$$

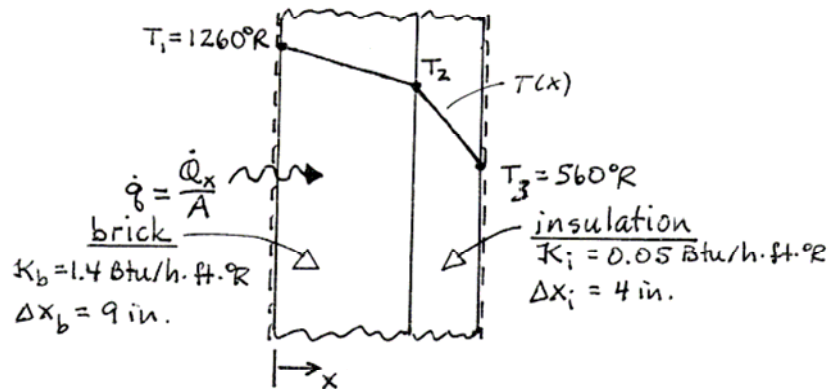


PROBLEM 2.46

KNOWN: Energy transfer occurs by conduction through a composite plane wall consisting of two layers.

FIND: Determine the steady-state heat flux and the temperature at the interface between the layers.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The wall is a closed system at steady state. (2) The temperature distributions are linear in both layers. (3) The two layers are in perfect thermal contact. (4) The thermal conductivities of both layers are uniform.

ANALYSIS: With Eq. 2.31 and recognizing that at steady state the rate at which energy is conducted must be the same in each layer,

$$\dot{q} = -k_b \left. \frac{dT}{dx} \right|_{\text{brick}} = -k_i \left. \frac{dT}{dx} \right|_{\text{insul.}}, \quad \text{where } \dot{q} \equiv \dot{Q}/A.$$

Since the temperature distributions are linear

$$\dot{q} = k_b \frac{T_1 - T_2}{\Delta x_b} = k_i \frac{T_2 - T_3}{\Delta x_i}$$

$$\text{or} \quad \dot{q} = \frac{T_1 - T_2}{R_b} = \frac{T_2 - T_3}{R_i} \quad (*)$$

where $R_b = \Delta x_b / k_b$ and $R_i = \Delta x_i / k_i$. From the first of Eqs. (*)

$$① \quad T_2 = T_1 - \dot{q} R_b \quad (**)$$

Combining Eq. (**) with the second of Eqs. (*)

$$\dot{q} = \frac{T_1 - T_3}{R_b + R_i} \quad (***)$$

Inserting values

$$R_b = \frac{(9/12) \text{ ft}}{(1.4 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{R})} = 0.5357 \frac{\text{h} \cdot ^\circ\text{R}}{\text{Btu}}; \quad R_i = \frac{4/12}{0.05} = 6.667 \frac{\text{h} \cdot ^\circ\text{R}}{\text{Btu}}$$

Continued on next slide

Problem 2-46 continued

Thus

$$\dot{q} = \frac{1260 - 560}{(0.5357 + 6.667)} = 97.19 \text{ Btu/h} \leftarrow \dot{q}$$

Now, inserting values in (**)

$$\begin{aligned} T_2 &= 1260 - (97.19)(0.5357) \\ &= 1208^\circ\text{R} \leftarrow T_2 \end{aligned}$$

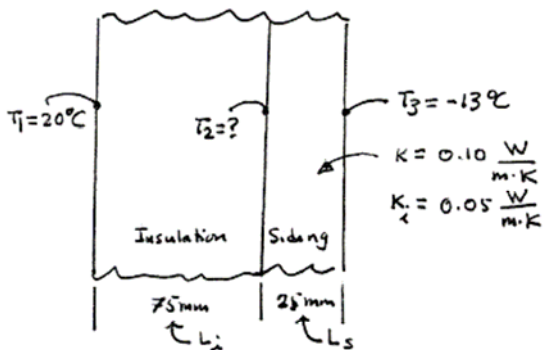
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1. The form of (***) illustrates the analogy between heat conduction through a composite wall and electric current flow through a series of resistances. The temperature difference in the numerator is analogous to a voltage difference, and the value of R_b and R_i are "thermal resistances" analogous to the electrical resistances.

PROBLEM 2.47

KNOWN: Energy transfer occurs by conduction through a composite plane wall consisting of two layers.

FIND: Determine the temperature at the interface between the two layers and the rate of heat transfer through the wall.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The wall is at steady state.
2. The temperature varies linearly through each layer.

ANALYSIS: With Eq. 2.3.1 and recognizing that at steady state rate at which energy is conducted must be the same in each layer,

$$\frac{\dot{Q}}{A} = -k_i \left[\frac{T_2 - T_1}{L_i} \right] = -k \left[\frac{T_3 - T_2}{L_s} \right] \quad (*)$$

Solving for T_2 ,

$$T_2 = \frac{\left(\frac{k_i}{L_i}\right)T_1 + \left(\frac{k}{L_s}\right)T_3}{\left(\frac{k_i}{L_i}\right) + \left(\frac{k}{L_s}\right)} \quad , \quad \frac{k_i}{L_i} = \frac{0.10 \text{ W/m}\cdot\text{K}}{0.075 \text{ m}} = \frac{2}{3} \frac{\text{W}}{\text{m}^2\cdot\text{K}}$$

$$\frac{k}{L_s} = \frac{0.10 \text{ W/m}\cdot\text{K}}{0.025 \text{ m}} = 4 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$$

$$= \frac{\left(\frac{2}{3} \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)(293 \text{ K}) + \left(4 \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)(260 \text{ K})}{\left(\frac{2}{3} + 4\right) \frac{\text{W}}{\text{m}^2\cdot\text{K}}} = 264.7 \text{ K } (-8^\circ\text{C}) \leftarrow$$

Then, Eq (*) gives

$$\frac{\dot{Q}}{A} = -\frac{k_i}{L_i} [T_2 - T_1] = -\frac{2}{3} \frac{\text{W}}{\text{m}^2\cdot\text{K}} [264.7 - 293] \text{ K} = 18.9 \frac{\text{W}}{\text{m}^2}$$

Alternatively

$$\frac{\dot{Q}}{A} = -\frac{k}{L_s} [T_3 - T_2] = -4 \frac{\text{W}}{\text{m}^2\cdot\text{K}} [260 - 264.7] \text{ K} = 18.8 \frac{\text{W}}{\text{m}^2}$$

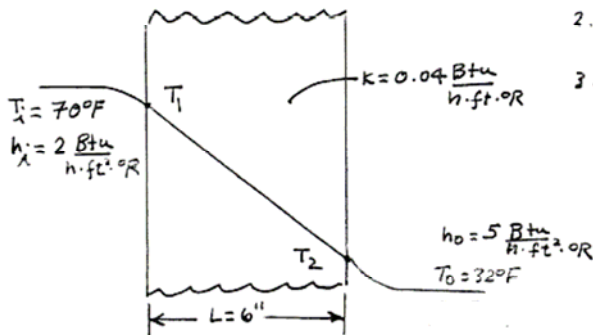
which checks to within round-off.

PROBLEM 2.48

KNOWN: Steady-state data are provided for the outer wall of a house.

FIND: Determine the rate of heat transfer through the wall.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The wall is at steady state.
2. Temperature varies linearly through the wall.
3. At the inner and outer surfaces, convection is the only heat transfer mode.

ANALYSIS: At steady state, the rates of energy transfer to the wall by convection, through the wall by conduction, and from the wall by convection are all equal. That is,

$$\frac{\dot{Q}}{A} = \underbrace{h_i(T_i - T_1)}_{\text{convection to wall}} = \underbrace{-k \left[\frac{T_2 - T_1}{L} \right]}_{\text{conduction through wall}} = \underbrace{h_o(T_2 - T_o)}_{\text{convection from wall}} \quad (*)$$

Solving the first and third of Eqs. (*),

$$T_1 = T_i - \frac{\dot{Q}/A}{h_i}, \quad T_2 = T_o + \frac{\dot{Q}/A}{h_o}$$

Substituting into the second of Eqs. (*) and simplifying

$$\textcircled{1} \quad \frac{\dot{Q}}{A} = -k \left[\frac{(T_o + \frac{\dot{Q}/A}{h_o}) - (T_i - \frac{\dot{Q}/A}{h_i})}{L} \right] = \frac{T_i - T_o}{\underbrace{\left(\frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o} \right)}}_{\text{thermal "resistances"}} \quad (**)$$

Inserting values into Eq. (**)

$$\begin{aligned} \frac{\dot{Q}}{A} &= \frac{[530^\circ\text{R} - 492^\circ\text{R}] \text{ (Btu/h}\cdot\text{ft}^2\cdot\text{°R)}}{\left[\frac{1}{2} + \frac{0.5}{0.04} + \frac{1}{5} \right]} \\ &\quad \uparrow \quad \uparrow \quad \uparrow \\ &\quad \textcircled{0.5} \quad \textcircled{12.5} \quad \textcircled{0.2} \\ &= 2.88 \frac{\text{Btu/h}}{\text{ft}^2} \end{aligned}$$

1. The form of Eq. (**) illustrates the analogy between heat transfer and electric current flow through resistances in series. The temperature difference in the numerator is analogous to the voltage difference, and the terms in the denominator account for "thermal resistances" analogous to electrical resistances. In this case, the insulated wall is the greatest of the three resistances, as shown by the calculation below.

PROBLEM 2.49

KNOWN: Steady-state data is provided for a flat surface covered with insulation.

FIND: Obtain an expression for the temperature of the outer surface of the insulation, T_i , in a specified form. Sketch and discuss the relation.

SCHEMATIC, GIVEN DATA:

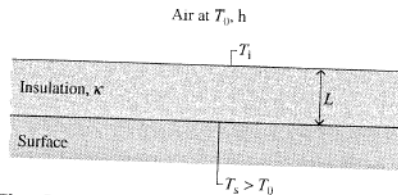


Fig. P2.49

ENGR. MODEL:

1. The composite of surface, insulation, and adjacent air is at steady state.
2. The temperature variation in the insulation is linear.
3. Energy transfer between the insulation and air is by convection only.

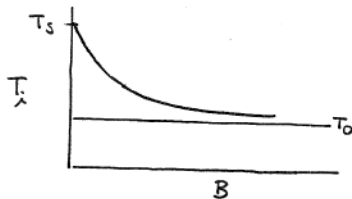
ANALYSIS: (a) At steady state, the rate at which energy is conducted to the outer surface of the insulation equals the rate energy is convected away from the surface. Thus, with Eq. 2.3.1 (with assumption 2) and Eq. 2.3.4,

$$\frac{\dot{Q}}{A} = -\kappa \frac{[T_i - T_s]}{L} = h [T_i - T_0]$$

Solving for T_i :

$$(1) (*) \quad T_i = \frac{B T_0 + T_s}{B + 1}, \quad \text{where } B = \frac{hL}{\kappa}$$

(b) For fixed T_s, T_0 :



That is, for B sufficiently large, T_i approaches the air temperature; and for B sufficiently small, T_i approaches the surface temperature.

(c) Let $T_0 = 30^\circ\text{C}$, $T_s = 300^\circ\text{C}$ and $T_i \leq 60^\circ\text{C}$. Then Eq. (*) gives

$$\frac{B(30^\circ\text{C}) + 300^\circ\text{C}}{B + 1} \leq 60^\circ\text{C} \Rightarrow B \leq 8$$

If $h = 4 \text{ W/m}^2\cdot\text{K}$ and $L = 0.06 \text{ m}$, then

$$B = \frac{hL}{\kappa} = \frac{(4 \text{ W/m}^2\cdot\text{K})(0.06 \text{ m})}{\kappa} \geq 8 \Rightarrow \kappa \leq 0.03 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

If $h = 20 \text{ W/m}^2\cdot\text{K}$ and $L = 0.06 \text{ m}$, then

$$B = \frac{hL}{\kappa} = \frac{(20 \text{ W/m}^2\cdot\text{K})(0.06 \text{ m})}{\kappa} \geq 8 \Rightarrow \kappa \leq 0.15 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

Thus, for a given thickness of insulation, a wider range of potential insulating materials is allowed if the convection coefficient h is greater.

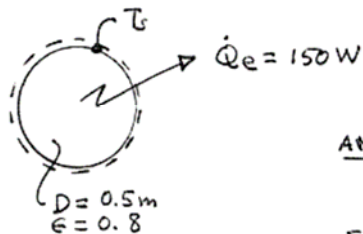
1. Observe that B is dimensionless — that is, without units. For example, if $h: \frac{\text{W}}{\text{m}^2\cdot\text{K}}$, $L: \text{m}$, $\kappa: \frac{\text{W}}{\text{m}\cdot\text{K}}$, then $B: \frac{(\text{W/m}^2\cdot\text{K})(\text{m})}{(\text{W/m}\cdot\text{K})} \Rightarrow B$ is a numerical value without units.

PROBLEM 2.50

KNOWN: Steady-state operating data are provided for a spherical interplanetary probe.

FIND: Determine the surface temperature of the sphere, in K.

SCHEMATIC & GIVEN DATA:



Thus,

ENGR. MODEL:

1. The probe is at steady state.
2. The probe emits but does not receive radiation.

ANALYSIS: In this case, Eq. 2.32 applies:

$\dot{Q}_e = \epsilon \sigma A T_s^4$, where $A = \pi D^2$ and σ is the Stefan-Boltzmann constant, $5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$. Solving for T_s

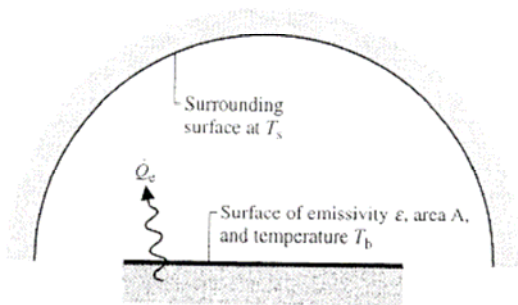
$$T_s = \left[\frac{\dot{Q}_e}{\epsilon \pi D^2 \sigma} \right]^{1/4} = \left[\frac{150 \text{ W}}{0.8 \pi (0.5 \text{ m})^2 (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)} \right]^{1/4} = 255 \text{ K} \leftarrow$$

PROBLEM 2.51

KNOWN: Data are provided for a body placed in a large, evacuated chamber.

FIND: Determine the rate at which radiation is emitted from the surface and the net rate at which radiation is exchanged between the body and chamber.

SCHEMATIC & GIVEN DATA:



$$A = 0.5 \text{ m}^2, \epsilon = 0.8, T_b = 423 \text{ K}, T_s = 298 \text{ K}$$

ENGR. MODEL:

1. The area of the enclosed surface is much less than that of the chamber walls.
2. The chamber is evacuated.

ANALYSIS:

- (a) The rate radiation is emitted from the surface is given by Eq. 2.32, where σ is the Stefan-Boltzmann constant. That is

$$\begin{aligned} \dot{Q}_e &= \epsilon \sigma A T_b^4 = 0.8(0.5 \text{ m}^2) \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} \right) (423 \text{ K})^4 \\ &= 726 \text{ W} \end{aligned}$$

- (b) The net rate at which radiation is transferred from the surface to the chamber walls is given by Eq. 2.33. That is

$$\begin{aligned} (\dot{Q}_e)_{\text{net}} &= \epsilon \sigma A [T_b^4 - T_s^4] = 0.8(0.5 \text{ m}^2) \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} \right) \left((423 \text{ K})^4 - (298 \text{ K})^4 \right) \\ &= 547 \text{ W} \end{aligned}$$

PROBLEM 2.52

KNOWN: Data are provided for a grill hood.

FIND: Determine the net rate of heat transfer between the hood and the surroundings by convection and radiation, per unit area of hood surface.

SCHEMATIC & GIVEN DATA:

$$T_0 = 27^\circ\text{C}$$

$$h = 10 \text{ W/m}^2 \cdot \text{K}$$

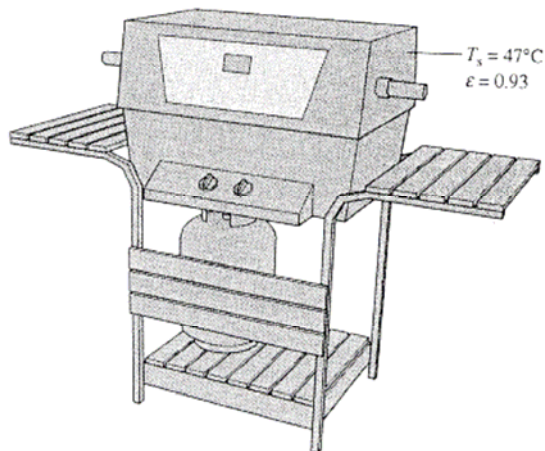


Fig. P2.52

ENGR. MODEL:

1. Radiative heat transfer between the hood and surroundings is modeled as an exchange between a surface at T_s and a much larger surrounding surface at T_0 .

ANALYSIS:

Using Eq. 2.34, convection heat transfer is

$$\frac{\dot{Q}_c}{A} = h[T_s - T_0] = 10 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} [20 \text{ K}] \left| \frac{1 \text{ kW}}{10^3 \text{ W}} \right|$$

$$= 0.2 \text{ kW/m}^2 \quad \leftarrow$$

Using Eq. 2.33, radiative heat transfer is

$$\frac{\dot{Q}_e}{A} = \epsilon \sigma [T_s^4 - T_0^4]$$

$$= 0.93 [5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}] [(320 \text{ K})^4 - (300 \text{ K})^4]$$

$$= 125.8 \frac{\text{W}}{\text{m}^2} \left| \frac{1 \text{ kW}}{10^3 \text{ W}} \right| = 0.13 \text{ kW/m}^2 \quad \leftarrow$$

$$\text{Total} = (\dot{Q}_c/A) + (\dot{Q}_e/A) = 0.33 \text{ kW/m}^2 \quad \leftarrow$$

PROBLEM 2.53

KNOWN: Data is provided for $Q, W, E_1, E_2, \Delta E$ for several processes of a closed system.
FIND: Determine values of missing table data.

Process	Q	W	E_1	E_2	ΔE
a	+50	-20	-20	+50	+70
b	+50	+20	+20	+50	+30
c	-40	-60	+40	+60	+20
d	-90	-90	+50	+50	0
e	+50	+150	+20	-80	-100

$$\left. \begin{array}{l} \text{a} \\ \text{b} \\ \text{c} \\ \text{d} \\ \text{e} \end{array} \right\} \Delta E = Q - W$$

Process a:

$$\begin{aligned} \Delta E &= Q - W \\ \therefore Q &= \Delta E + W = 70 + (-20) = \boxed{+50} \\ \Delta E &= E_2 - E_1 = +70 \Rightarrow E_1 = E_2 - 70 \\ E_1 &= 50 - 70 = \boxed{-20} \end{aligned}$$

Process b: $\Delta E = E_2 - E_1 = 50 - 20 = \boxed{30}$

$$\begin{aligned} \Delta E &= Q - W \\ \therefore W &= Q - \Delta E = +50 - 30 = \boxed{20} \end{aligned}$$

Process c:

$$\begin{aligned} \Delta E &= E_2 - E_1 \Rightarrow E_1 = E_2 - \Delta E \\ &= 60 - 20 = \boxed{40} \\ \Delta E &= Q - W \Rightarrow Q = \Delta E + W \\ \therefore Q &= 20 + (-60) = \boxed{-40} \end{aligned}$$

Process d:

$$\begin{aligned} \Delta E &= E_2 - E_1 \Rightarrow E_1 = E_2 - \Delta E \\ \therefore E_1 &= 50 - 0 = \boxed{50} \\ \Delta E &= Q - W \Rightarrow Q = \Delta E + W \\ \therefore Q &= 0 + (-90) = \boxed{-90} \end{aligned}$$

Process e:

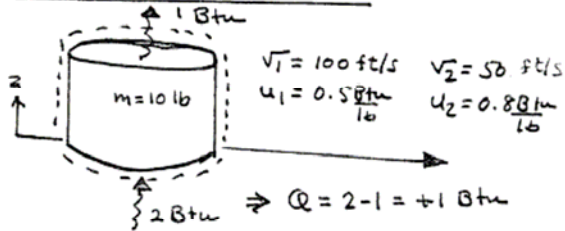
$$\begin{aligned} \Delta E &= Q - W = 50 - 150 = \boxed{-100} \\ \Delta E &= E_2 - E_1 \Rightarrow E_2 = \Delta E + E_1 \\ E_2 &= -100 + 20 = \boxed{-80} \end{aligned}$$

PROBLEM 2.54

KNOWN: Data is provided for a vertical cylindrical mass undergoing a process.

FIND: Determine for the mass the change in kinetic energy and the work.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The mass is the closed system.
2. The system experiences no elevation change.
3. Heat transfer occurs only through the bottom and top in the directions of the arrows.

ANALYSIS:

$$(a) \quad \Delta KE = \frac{1}{2} m [V_2^2 - V_1^2] = \frac{1}{2} (10 \text{ lb}) \left[\left(\frac{50 \text{ ft}}{\text{s}} \right)^2 - \left(\frac{100 \text{ ft}}{\text{s}} \right)^2 \right] \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = -1.5 \text{ Btu} \leftarrow$$

$$(b) \quad \Delta U + \Delta KE + \Delta PE = Q - W \Rightarrow W = Q - \Delta U - \Delta KE. \quad \text{Thus, with } Q = (2 \text{ Btu} - 1 \text{ Btu}) = +1 \text{ Btu, we get}$$

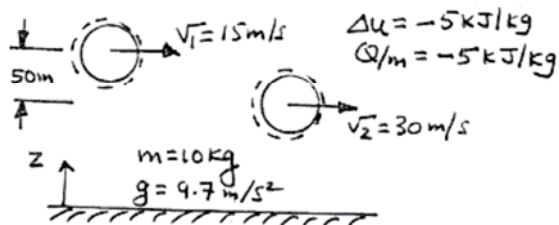
$$W = (+1 \text{ Btu}) - 10 \text{ lb} \left[0.8 - 0.5 \right] \frac{\text{Btu}}{\text{lb}} - (-1.5 \text{ Btu}) = -0.5 \text{ Btu} \leftarrow$$

PROBLEM 2.55

KNOWN: Data is provided for a 10 kg mass undergoing a process.

FIND: Determine the work for the process of the mass.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The 10 kg mass is the closed system.
2. The acceleration of gravity is constant.

ANALYSIS:

$$\Delta U + \Delta KE + \Delta PE = Q - W \Rightarrow W = Q - \Delta U - \Delta KE - \Delta PE$$

$$\checkmark \quad Q = m \left[\frac{Q}{m} \right] = 10 \text{ kg} \left[-5 \frac{\text{kJ}}{\text{kg}} \right] = -50 \text{ kJ}$$

$$\checkmark \quad \Delta U = m \Delta u = 10 \text{ kg} \left[-5 \frac{\text{kJ}}{\text{kg}} \right] = -50 \text{ kJ}$$

$$\checkmark \quad \Delta KE = \frac{m}{2} [V_2^2 - V_1^2] = \frac{10 \text{ kg}}{2} \left[\left(30 \frac{\text{m}}{\text{s}} \right)^2 - \left(15 \frac{\text{m}}{\text{s}} \right)^2 \right] \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = +3.38 \text{ kJ}$$

$$\checkmark \quad \Delta PE = mg(z_2 - z_1) = (10 \text{ kg})(9.7 \text{ m/s}^2)(-50 \text{ m}) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = -4.85 \text{ kJ}$$

$$\therefore W = (-50 \text{ kJ}) - (-50 \text{ kJ}) - (+3.38 \text{ kJ}) - (-4.85 \text{ kJ})$$

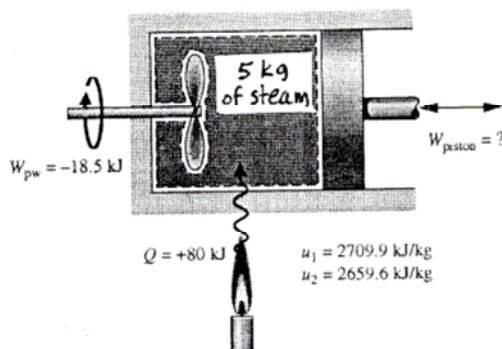
$$= +1.47 \text{ kJ} \quad \leftarrow$$

PROBLEM 2.56

KNOWN: Five kg of steam undergo an expansion in a piston-cylinder assembly from state 1 to state 2. During the process there is a known heat transfer to the steam and a known work transfer of energy to the steam by a paddle wheel. The change in specific internal energy of the steam is also known.

FIND: Determine the amount of energy transfer by work from the steam to the piston during the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The steam is the closed system. 2. There is no change in the kinetic or potential energy from state 1 to state 2.

ANALYSIS: The net work can be determined from an energy balance. That is, with assumption 2

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

or

$$W = Q - \Delta U$$

The net work is the sum of the work associated with the paddlewheel W_{pw} and the work done on the piston W_{piston} :

$$W = W_{pw} + W_{piston}$$

From the given information $W_{pw} = -18.5 \text{ kJ}$, where the minus sign is required because the paddle wheel transfers energy to the system. Collecting results

$$W_{pw} + W_{piston} = Q - \Delta U$$

or

$$\begin{aligned} W_{piston} &= Q - \Delta U - W_{pw} \\ &= Q - m(u_2 - u_1) - W_{pw} \\ &= 80 \text{ kJ} - 5 \text{ kg} \left(\frac{2659.6 - 2709.9 \text{ kJ}}{\text{kg}} \right) - (-18.5 \text{ kJ}) \\ &= +350 \text{ kJ} \end{aligned}$$

where the positive sign indicates that energy is transferred from the steam to the piston as the steam expands during the process. W_{piston}

PROBLEM 2.57

KNOWN: A gas within a piston-cylinder assembly undergoes two different processes between the same end states.

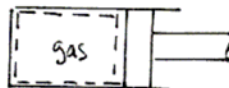
FIND: Sketch the processes on p-V coordinates. For each process evaluate W and Q .

SCHEMATIC & GIVEN DATA:

Data: $p_1 = 10 \text{ bar}$, $V_1 = 0.1 \text{ m}^3$, $U_1 = 400 \text{ kJ}$ and $p_2 = 1 \text{ bar}$, $V_2 = 1.0 \text{ m}^3$, $U_2 = 200 \text{ kJ}$.

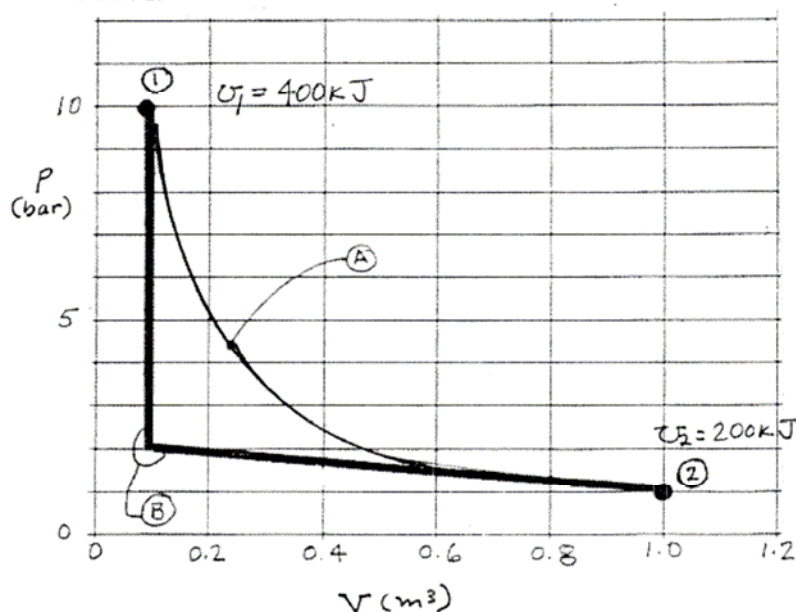
Process A: Process from 1 to 2 during which the pressure-volume relation is $pV = \text{constant}$

Process B: Constant-volume process from state 1 to a pressure of 2 bar, followed by a linear pressure-volume process to state 2.



ENGR. MODEL:

1. The gas is the closed system.
2. Kinetic and potential energy effects can be ignored.
3. The p-V relation is specified for each process.
4. The moving boundary is the only work mode.



ANALYSIS: For Process A, $W_A = \int_1^2 p dV = \int_{V_1}^{V_2} \frac{C}{V} dV = C \ln \frac{V_2}{V_1} = p_1 V_1 \ln \frac{V_2}{V_1}$

$$\text{Thus, } W_A = (10 \text{ bar})(0.1 \text{ m}^3) \ln \left[\frac{1.0 \text{ m}^3}{0.1 \text{ m}^3} \right] \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = +230.26 \text{ kJ} \quad \leftarrow W_A$$

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_A - W_A \Rightarrow Q_A = \Delta U + W_A = (200 - 400) \text{ kJ} + 230.26 \text{ kJ} = +30.26 \text{ kJ} \quad \leftarrow Q_A$$

For Process B, the piston does not move for the first step (constant volume) and thus there is no work. The work can be evaluated geometrically for the second step, during which the p-V relation is linear.

$$W_B = p_{\text{ave}}[V_2 - V_1] = \left[\frac{2 \text{ bar} + 1 \text{ bar}}{2} \right] [1.0 \text{ m}^3 - 0.1 \text{ m}^3] \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 135 \text{ kJ} \quad \leftarrow W_B$$

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_B - W_B \Rightarrow Q_B = \Delta U + W$$

Note: Since U is a property, ΔU is the same for each process

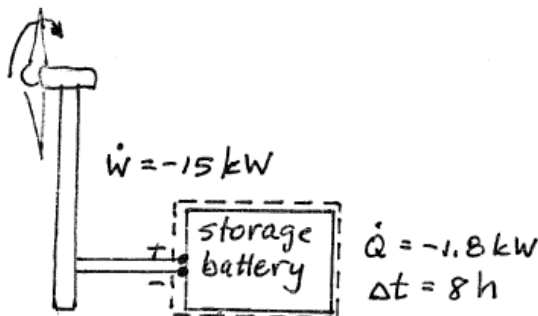
$$Q_B = (200 - 400) \text{ kJ} + 135 \text{ kJ} = -65 \text{ kJ} \quad \leftarrow Q_B$$

PROBLEM 2.58

KNOWN: A storage battery is charged by the electric power output from a windmill. The work and heat transfer rates are known.

FIND: Determine for 8 h of operation (a) the total amount of energy stored.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The battery is a closed system. (2) The work and heat transfer rates are constant.

ANALYSIS: (a) The amount of energy stored is found from an energy balance

$$\Delta E = Q - W$$

(*)

To evaluate Q and W , respectively

$$\begin{aligned} Q &= \int_{t_1}^{t_2} \dot{Q} dt = \dot{Q} \Delta t = (-1.8 \text{ kW})(8 \text{ h}) = -14.4 \text{ kW} \cdot \text{h} \\ &= (-14.4 \text{ kW} \cdot \text{h}) \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \\ &= -51,840 \text{ kJ} \end{aligned}$$

$$\begin{aligned} W &= \int_{t_1}^{t_2} \dot{W} dt = \dot{W} \Delta t = (-15 \text{ kW})(8 \text{ h}) = -120 \text{ kW} \cdot \text{h} \\ &= (-120) \left| \frac{1}{1} \right| \left| \frac{3600}{1} \right| = -4.32 \times 10^5 \text{ kJ} \end{aligned}$$

Inserting these results in (*)

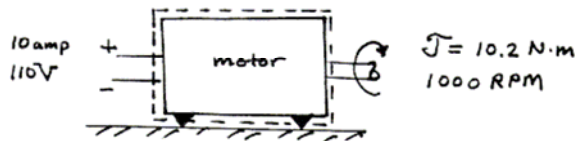
$$\Delta E = (-51,840) - (-4.32 \times 10^5) = 3.8 \times 10^5 \text{ kJ} \leftarrow \Delta E$$

PROBLEM 2.59

KNOWN: Operating data are provided for an electric motor at steady state.

FIND: Determine the electric power required by the motor and the power developed by the output shaft. Determine the rate of heat transfer for the motor.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. As shown in the schematic, the motor is the closed system.
2. The system is at steady state.

ANALYSIS: (a) Using Eq. 2.21,

$$\begin{aligned}\dot{W}_{\text{electric}} &= -(\text{Voltage})(\text{Current}) \\ &= -(110 \text{ volts})(10 \text{ amp}) \left| \frac{1 \text{ watt/amp}}{1 \text{ volt}} \right| = -1100 \text{ watt} \left| \frac{1 \text{ kW}}{10^3 \text{ W}} \right| \\ &= -1.1 \text{ kW}\end{aligned}$$

$\longleftarrow \dot{W}_{\text{electric}}$

(b) Using Eq. 2.20,

$$\begin{aligned}\dot{W}_{\text{shaft}} &= (\text{Torque})(\text{angular velocity}) \\ &= (10.2 \text{ N}\cdot\text{m}) \left(1000 \frac{\text{Rev}}{\text{min}} \left| \frac{2\pi \text{ rad}}{\text{Rev}} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \right) \left| \frac{1 \text{ kW}}{10^3 \text{ N}\cdot\text{m/s}} \right| \\ &= 1.07 \text{ kW}\end{aligned}$$

$\longleftarrow \dot{W}_{\text{shaft}}$

Thus for the motor,

$$\begin{aligned}\dot{W} &= \dot{W}_{\text{electric}} + \dot{W}_{\text{shaft}} \\ &= (-1.1 \text{ kW}) + (1.07 \text{ kW}) = -0.03 \text{ kW}\end{aligned}$$

$\longleftarrow \dot{W}$

(c) An energy rate balance for the motor at steady state reads

$$\textcircled{1} \quad \frac{dE}{dt} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W}$$

$\longleftarrow \dot{Q}$

$= -0.03 \text{ kW}$

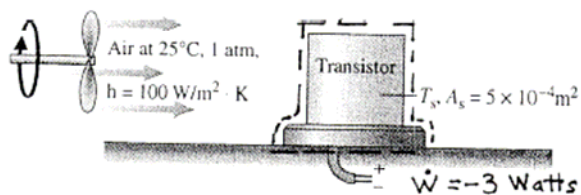
-
1. Owing to the effects of electrical resistance and friction within the motor, the power out is less than the power in. At steady state, the difference in these power quantities is carried out of the motor by heat transfer.

PROBLEM 2.60

KNOWN: Steady-state data are provided for a transistor cooled convectively.

FIND: Determine for the transistor the heat transfer rate and the outer surface temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The transistor is the closed system.
2. The system is at steady state.
3. No heat transfer occurs through the base of the transistor.

ANALYSIS: (a). An energy rate balance reads $\frac{dE}{dt} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W} = -3 \text{ Watts}$ ←

(b) Since cooling occurs convectively, $\dot{Q} = -hA[T_s - T_{\text{air}}]$, where the minus sign is introduced because heat transfer is from the transistor and $T_s > T_{\text{air}}$. Solving

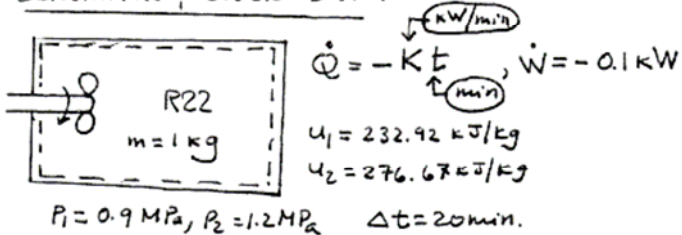
$$T_s = T_{\text{air}} - \left[\frac{\dot{Q}}{hA} \right] = 298 \text{ K} - \left[\frac{-3 \text{ W}}{\left(100 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \right) \left(5 \times 10^{-4} \text{ m}^2 \right)} \right] = 358 \text{ K} \quad (85^\circ \text{C}) \quad \leftarrow$$

PROBLEM 2.61

KNOWN: Data are provided for a rigid, closed tank fitted with a paddle wheel. The tank contains Refrigerant 22.

FIND: (a) Q and W for the refrigerant. (b) Evaluate the constant K in the heat transfer relation provided.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The Refrigerant 22 is the closed system.
2. No overall changes in kinetic or potential energy occur.

ANALYSIS: (a) $W = \int_1^2 \dot{W} dt = -(0.1 \text{ kW})(20 \text{ min}) \left| \frac{60 \text{ s}}{1 \text{ min}} \right| \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = -120 \text{ kJ}$ ←

Energy Balance:

$$\Delta U + \Delta KE + \Delta PE = Q - W \Rightarrow Q = \Delta U + W \Rightarrow Q = m(u_2 - u_1) + W$$

$$\therefore Q = 1 \text{ kg} (276.67 - 232.92) + (-120 \text{ kJ}) = -76.25 \text{ kJ}$$
 ←

(b) $Q = \int_1^2 \dot{Q} dt = \int_0^t -Kt dt = -\frac{Kt^2}{2} \Rightarrow$

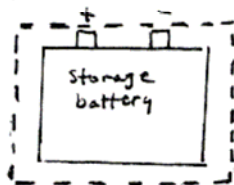
$$K = -\frac{2Q}{t^2} = -\frac{2(-76.25 \text{ kJ})}{(20 \text{ min})^2} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right|$$

$$= 6.354 \times 10^{-3} \frac{\text{kW}}{\text{min}}$$
 ←

PROBLEM 2.62

KNOWN: Data are provided for the charging and discharging of a storage battery.
FIND: Determine the time rate of change of energy during charging and discharging periods. Determine the overall change in energy of the battery.

SCHEMATIC & GIVEN DATA:



Charging:
 1st 12 hrs. $\dot{W} = -15 \text{ kW}$
 $\dot{Q} = -1.5 \text{ kW}$

Discharging:
 2nd 12 hrs. $\dot{W} = +5 \text{ kW}$
 $\dot{Q} = -0.5 \text{ kW}$

ENGR. MODEL

1. The storage battery is the closed system.
2. The $\dot{W}$ and $\dot{Q}$ rates are each constant with time.

ANALYSIS:

$$(a) \quad \frac{dE}{dt} = \dot{Q} - \dot{W}$$

$$= -1.5 \text{ kW} - (-15 \text{ kW}) = +13.5 \text{ kW} \quad \leftarrow$$

$$(b) \quad \frac{dE}{dt} = \dot{Q} - \dot{W}$$

$$= -0.5 \text{ kW} - (5 \text{ kW}) = -5.5 \text{ kW} \quad \leftarrow$$

(c) The change in system energy in the charging phase is

$$\Delta E = \int \frac{dE}{dt} dt = (+13.5 \text{ kW})(12 \text{ h}) \left| \frac{1 \text{ kW/s}}{1 \text{ kW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = 5.832 \times 10^5 \text{ kJ}$$

The change in system energy in the discharging phase is

$$\Delta E = \int \frac{dE}{dt} dt = (-5.5 \text{ kW})(12 \text{ h}) \left| \frac{1 \text{ kW/s}}{1 \text{ kW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = -2.376 \times 10^5 \text{ kJ}$$

$$\text{So, the overall change in energy over 24 hours} = (5.832 - 2.376) \times 10^5 \text{ kJ}$$

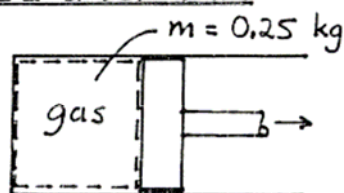
$$= 3.456 \times 10^5 \text{ kJ} \quad \leftarrow$$

PROBLEM 2.63

KNOWN: A gas of known mass expands in a piston-cylinder from a specified initial state to a known final pressure. The pressure-volume for the process is given. Also, the specific internal energy change is known.

FIND: Determine the heat transfer for the process.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} P_1 &= 8 \text{ bar}, V_1 = 0.02 \text{ m}^3 \\ P_2 &= 2 \text{ bar} \\ pV^{1.2} &= \text{constant} \\ \Delta u &= -55 \text{ kJ/kg} \end{aligned}$$

ENGR. MODEL: (1) The gas is a closed system. (2) Kinetic and potential energy effects are negligible. (3) The process is described by $pV^{1.2} = \text{constant}$. (4) Volume change is the only work mode.

ANALYSIS: The heat transfer can be found using an energy balance. First, find the work using Eq. 2.17, as follows:

$$W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{\text{const.}}{V^{1.2}} dV$$

Integrating and simplifying (See Example 2.1 for details),

$$W = \frac{P_2 V_2 - P_1 V_1}{(1-1.2)}$$

The volume at the final state is $V_2 = (P_1/P_2)^{1/1.2} V_1 = 0.0635 \text{ m}^3$. Thus

$$\begin{aligned} W &= \frac{(2 \text{ bar})(0.0635 \text{ m}^3) - (8)(0.02)}{(1-1.2)} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= 16.5 \text{ kJ} \end{aligned}$$

Now, to get the heat transfer, begin with the energy balance

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$$

Solving for Q and noting that $\Delta U = m \Delta u$

$$\begin{aligned} Q &= m \Delta u + W \\ &= (0.25 \text{ kg})(-55 \text{ kJ/kg}) + (16.5 \text{ kJ}) \end{aligned}$$

$$\textcircled{1} \quad = 2.75 \text{ kJ} \quad \leftarrow Q$$

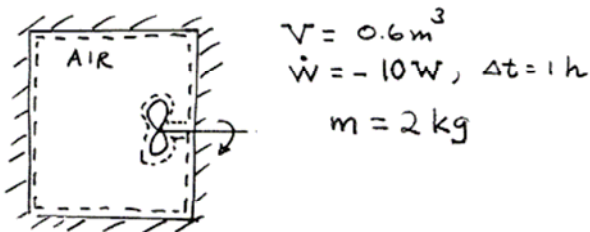
1. The heat transfer is positive, denoting energy transfer by heat to the gas as it expands.

PROBLEM 2.64

KNOWN: Air contained in a rigid well-insulated tank receives energy at a specified rate from a paddle wheel.

FIND: Determine the specific volume at the final state, the energy transfer by work, and the change in specific internal energy of the air.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The air is the closed system. 2. For the system, $Q = 0$ and there are no changes in kinetic energy and potential energy.

ANALYSIS: (a) Since the mass and volume are each constant, the specific volume at states 1 and 2 are the same:

$v_2 = v_1$. Thus

$$v_2 = V/m = (0.6 \text{ m}^3)/(2 \text{ kg}) = 0.3 \frac{\text{m}^3}{\text{kg}}$$

← v_2

(b) To evaluate W , integrate

$$W = \int_0^{1 \text{ h}} \dot{W} dt = \int_0^{1 \text{ h}} (-10 \text{ W}) dt = (-10 \text{ W})(1 \text{ h}) \left| \frac{1 \text{ J/s}}{1 \text{ W}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left| \frac{\text{kJ}}{10^3 \text{ J}} \right|$$

$$= -36 \text{ kJ}$$

← W

(c) The change in specific internal energy can be found from an energy balance:

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = \cancel{Q} - W \Rightarrow m \Delta u = -W$$

or

$$u_2 - u_1 = \frac{(-W)}{m}$$

$$= \frac{-(-36 \text{ kJ})}{2 \text{ kg}} = 18 \frac{\text{kJ}}{\text{kg}}$$

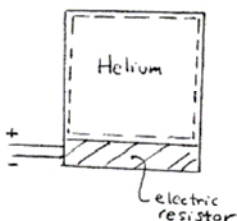
← $u_2 - u_1$

PROBLEM 2.65

KNOWN: Data are provided for helium contained in a closed, rigid tank fitted with an electrical resistance.

FIND: Plot the change in energy of the helium, in kJ, for $t \geq 0$ and comment.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The helium is the system.
2. For the system, $\dot{W} = 0$.

ANALYSIS: An energy rate balance reads

$$\frac{dE}{dt} = \dot{Q} - \dot{W}^o$$

As the system receives energy by heat transfer from the resistor at a rate of 1 kW and loses energy by heat transfer to its surroundings at the rate of 5t W,

$$\dot{Q} = [1000 - 5t] \text{ W}$$



Thus,

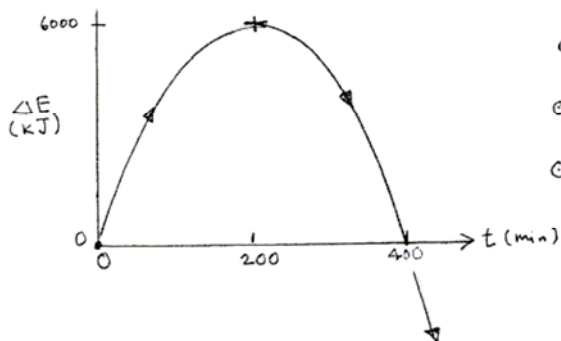
$$\frac{dE}{dt} = 1000 - 5t$$

and

$$\Delta E = \int_0^t \frac{dE}{dt} dt = \int_0^t (1000 - 5t) dt = \left[1000t - \frac{5t^2}{2} \right] \text{ W} \cdot \text{min}$$

$$= \left[1000t - \frac{5t^2}{2} \right] \text{ W} \cdot \text{min} \left| \frac{1 \text{ kJ/s}}{10^3 \text{ W}} \right| \left| \frac{60 \text{ s}}{\text{min}} \right| = \left[60t - 0.15t^2 \right] \text{ kJ}$$

(t in min.)



$$\Delta E = E(t) - E(0) = E(t) - E_0$$

- $0 \rightarrow 200 \text{ min}$. Energy increases from the initial value at $t=0$: E_0 .
- $200 \rightarrow 400 \text{ min}$. Energy decreases to its initial value, E_0 .
- $400 \text{ min} \rightarrow$. Energy decreases from its initial value, E_0 .

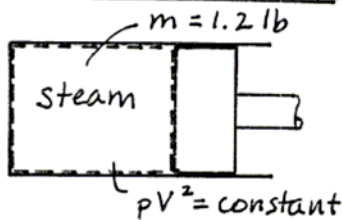
Note that since any arbitrary value E_0 can be assigned to the energy of the system at $t=0$, no particular significance can be attached to the value of energy at the initial state or at any other state. Only changes in the energy of the system have significance.

PROBLEM 2.66

KNOWN: A known mass of steam undergoes a polytropic process in a piston-cylinder assembly. The initial and final states and the heat transfer are specified.

FIND: Determine the work and the final specific volume.

SCHEMATIC & GIVEN DATA:



State 1: $p_1 = 500 \text{ lbf/in}^2$, $v_1 = 1.701 \text{ ft}^3/\text{lb}$

$u_1 = 1363.3 \text{ Btu/lb}$

State 2: $u_2 = 990.58 \text{ Btu/lb}$

$Q = -342.9 \text{ Btu}$

ENGR. MODEL: (1) The steam is a closed system. (2) Kinetic and potential energy effects are negligible. (3) The process is polytropic with $n=2$.

ANALYSIS: To determine the work, begin with the energy balance

$$\Delta \overset{\circ}{K}E + \Delta \overset{\circ}{P}E + \Delta U = Q - W$$

With $\Delta U = m(u_2 - u_1)$, and solving for W

$$W = Q - m(u_2 - u_1)$$

$$= (-342.9 \text{ Btu}) - (1.2 \text{ lb})(990.58 - 1363.3) \text{ Btu/lb}$$

$$= 104.4 \text{ Btu} \leftarrow W$$

To get v_2 , begin with Eq. 2.17. Introduce $pV^2 = C$, where $C = p_1 v_1^2$, and integrate:

$$\begin{aligned} W &= \int_{v_1}^{v_2} p dv = m \int_{v_1}^{v_2} p dv = m(p_1 v_1^2) \int_{v_1}^{v_2} \frac{dv}{v^2} \\ &= -m(p_1 v_1^2) \left(\frac{1}{v_2} - \frac{1}{v_1} \right) \end{aligned}$$

Solving for $1/v_2$

$$\frac{1}{v_2} = \frac{1}{v_1} - \frac{W}{m p_1 v_1^2}$$

$$\begin{aligned} &= \left(\frac{1}{1.701 \text{ ft}^3/\text{lb}} \right) - \frac{(104.4 \text{ Btu})}{(1.2 \text{ lb})(500 \text{ lbf/in}^2)(1.701 \text{ ft}^3/\text{lb})^2} \left| \frac{778 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}} \right| \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| \\ &= 0.2630 \end{aligned}$$

① Thus $v_2 = \frac{1}{0.2630} = 3.802 \text{ ft}^3/\text{lb} \leftarrow v_2$

1. The increase in volume is consistent with the positive sign for work, denoting energy transfer out of the system by work.

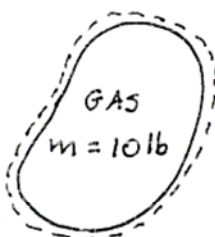
PROBLEM 2.67

KNOWN: A known mass of gas undergoes a polytropic process between two specified states. The relationship between pressure, volume, and internal energy is known for the gas.

FIND: Determine the heat transfer.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The gas is a closed system. (2) Kinetic and potential energy effects are negligible. (3) The process is polytropic with $n = 1.3$.



$$P_1 = 60 \text{ lbf/in}^2, v_1 = 6.0 \text{ ft}^3/\text{lb}$$

$$P_2 = 20 \text{ lbf/in}^2$$

$$P v^{1.3} = \text{constant}$$

For the gas

$$u = (0.2651) P v - 95.436$$

(where v is in ft^3/lb , P is in lbf/in^2 , and u is in Btu/lb)

ANALYSIS: The heat transfer is determined from an energy balance. First, determine the work beginning with Eq. 2.17 (see Example 2.1 for details)

$$W = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{\text{const.}}{V^{1.3}} dV = \left(\frac{P_2 V_2 - P_1 V_1}{1 - 1.3} \right) = m \left(\frac{P_2 v_2 - P_1 v_1}{1 - 1.3} \right)$$

Evaluating v_2

$$v_2 = \left(\frac{P_1}{P_2} \right)^{\frac{1}{1.3}} v_1 = \left(\frac{60}{20} \right)^{\frac{1}{1.3}} (6.0) = 13.97 \text{ ft}^3/\text{lb}$$

Thus

$$W = (10 \text{ lb}) \left[\frac{(20 \text{ lbf/in}^2)(13.97 \text{ ft}^3/\text{lb}) - (60)(6.0)}{(1 - 1.3)} \right] \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= 497.3 \text{ Btu}$$

Now, writing the energy balance

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

Noting that $\Delta U = m(u_2 - u_1)$ and solving for Q

$$Q = m(u_2 - u_1) + W$$

Evaluating $(u_2 - u_1)$

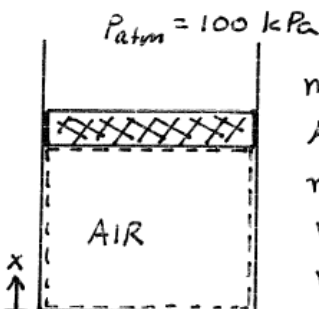
PROBLEM 2.68

KNOWN: A known quantity of air undergoes a process in a vertical piston-cylinder assembly. The initial and final volumes are given, and the change in specific internal energy is specified.

FIND: Determine the heat transfer.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The air is a closed system. (2) Kinetic and potential energy effects are negligible for the air. (3) There is no friction between the piston and the cylinder wall. (4) The process occurs slowly with no acceleration of the piston. (5) The acceleration of gravity is constant, $g = 9.81 \text{ m/s}^2$.



$$\begin{aligned} m_{\text{pist}} &= 50 \text{ kg} \\ A_{\text{pist}} &= 0.01 \text{ m}^2 \\ m_{\text{air}} &= 5 \text{ g} = 0.005 \text{ kg} \\ V_1 &= 5 \text{ L} = 0.005 \text{ m}^3 \\ V_2 &= 0.002 \text{ m}^3 \\ \Delta u &= -260 \text{ kJ/kg} \end{aligned}$$

ANALYSIS: For the piston, $\Sigma F_x = 0$. Thus, if p is the pressure exerted by the air at each state of the process, we get

$$\begin{aligned} P_{\text{atm}} A_{\text{pist}} &= p A_{\text{pist}} + m_{\text{pist}} g \\ p &= P_{\text{atm}} + \frac{m_{\text{pist}} g}{A_{\text{pist}}} \\ &= 100 \text{ kPa} + \frac{(50 \text{ kg})(9.81 \text{ m/s}^2)}{(0.01 \text{ m}^2)} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| \\ &= 149.05 \text{ kPa} \end{aligned}$$

To find the work for the process, use Eq. 2.17. Noting that the pressure is constant

$$\begin{aligned} W &= \int_{V_1}^{V_2} p dV = p(V_2 - V_1) \\ &= (149.05 \text{ kPa})(0.002 - 0.005) \text{ m}^3 \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= -0.447 \text{ kJ} \end{aligned}$$

Now, the energy balance reduces to $\Delta \overset{0}{K} + \Delta \overset{0}{P} + \Delta U = Q - W$. Thus, with $\Delta U = m_{\text{air}} \Delta u$

$$Q = m_{\text{air}} \Delta u + W = (0.005 \text{ kg})(-260 \frac{\text{kJ}}{\text{kg}}) + (-0.447 \text{ kJ})$$

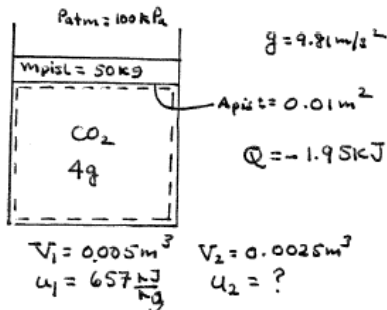
$$Q = -1.747 \text{ kJ} \quad \leftarrow Q$$

PROBLEM 2.69

KNOWN: Data are provided for CO₂ gas contained in a piston-cylinder assembly.

FIND: For the CO₂ determine the pressure and the final specific internal energy.

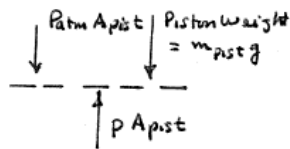
SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The CO₂ is the closed system.
2. The moving boundary is the only work mode.
3. As cooling occurs slowly there is no acceleration of the piston. The value of g remains constant.
4. Friction between the piston and cylinder can be ignored.
5. For the CO₂, $\Delta KE = 0$ and ΔPE can be ignored.

ANALYSIS: (a) Since there is no friction and the piston is not accelerated, the force exerted by the CO₂ in the cylinder on the bottom of the piston is equal to the weight of the piston plus the force exerted by the atmosphere on the top of the piston:



$$\Rightarrow p A_{pist} = P_{atm} A_{pist} + m_{pist} g$$

$$\begin{aligned} p &= P_{atm} + \frac{m_{pist} g}{A_{pist}} \\ &= 100 \text{ kPa} + \frac{(50 \text{ kg})(9.81 \text{ m/s}^2)}{0.01 \text{ m}^2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| \\ &= 149.1 \text{ kPa} \left| \frac{1 \text{ bar}}{10^2 \text{ kPa}} \right| = 1.491 \text{ bar} \quad \leftarrow p \end{aligned}$$

(b) The work can be evaluated using Eq. 2.17. Since the pressure remains constant

$$\begin{aligned} W &= \int_1^2 p dV = p [V_2 - V_1] = 1.491 \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| [0.0025 - 0.005] \text{ m}^3 \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= -0.373 \text{ kJ} \end{aligned}$$

An energy balance for the CO₂ reads,

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$$

$$\Delta U = -1.95 \text{ kJ} - (-0.373 \text{ kJ}) = -1.577 \text{ kJ}$$

Then, with $\Delta U = m(u_2 - u_1)$,

$$u_2 = \frac{\Delta U}{m} + u_1$$

$$= \frac{-1.577 \text{ kJ}}{(4/1000) \text{ kg}} + 657 \frac{\text{kJ}}{\text{kg}}$$

$$= 262.8 \frac{\text{kJ}}{\text{kg}}$$

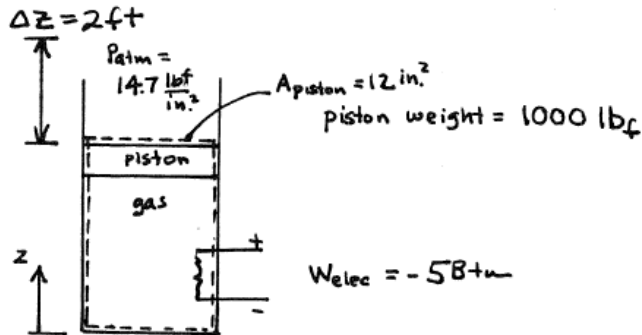
$\leftarrow u_2$

PROBLEM 2.70

KNOWN: An electrical resistor transfers energy to a gas contained in a vertical piston-cylinder assembly.

FIND: Determine the change in internal energy of the gas

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system is the piston plus the gas. 2. For the piston, $\Delta U = 0$, $\Delta KE = 0$. For the gas, $\Delta PE = 0$, $\Delta KE = 0$. 3. For the system, $Q = 0$. 4. Friction between piston and cylinder can be ignored. (5) The mass of the electrical resistor is negligible. So $\Delta U = 0$ for the resistor.

ANALYSIS: An energy balance for the system reads

$$\cancel{\Delta KE} + [(\Delta PE)_{\text{piston}} + (\Delta PE)_{\text{gas}}] + [(\cancel{\Delta U})_{\text{piston}} + (\Delta U)_{\text{gas}} + (\cancel{\Delta U})_{\text{resistor}}] = \cancel{Q} - W$$

$$\text{or } (\Delta PE)_{\text{piston}} + (\Delta U)_{\text{gas}} = -W$$

For the piston

$$(\Delta PE)_{\text{piston}} = mg(z_2 - z_1)$$

For the system consisting of freely-moving piston (assumption 4) and gas, the net work is the sum of the electrical work and the work done at the top of the piston in displacing the surrounding atmosphere:

$$W = W_{\text{elect}} + \int_{z_1}^{z_2} (P_{\text{atm}} A_{\text{piston}}) dz \Rightarrow W = W_{\text{elect}} + (P_{\text{atm}} A_{\text{piston}})(z_2 - z_1)$$

Collecting results

$$mg(z_2 - z_1) + (\Delta U)_{\text{gas}} = -[W_{\text{elect}} + (P_{\text{atm}} A_{\text{piston}})(z_2 - z_1)]$$

$$\begin{aligned} \text{Thus } (\Delta U)_{\text{gas}} &= -W_{\text{elec}} - P_{\text{atm}} A_{\text{piston}} (z_2 - z_1) - mg(z_2 - z_1) \\ &= -(-5 \text{ Btu}) - (14.7 \frac{\text{lbf}}{\text{in}^2})(12 \text{ in}^2)(2 \text{ ft}) \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| - (1000 \text{ lbf})(2 \text{ ft}) \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\ &= 1.98 \text{ Btu} \leftarrow \Delta U \end{aligned}$$

PROBLEM 2.71

KNOWN: A system undergoes a thermodynamic cycle consisting of four processes in series.

FIND: Complete the table of energy values provided for the cycle and determine whether the cycle is a power cycle or a refrigeration cycle.

SCHEMATIC & GIVEN DATA:

Process	ΔU	ΔKE	ΔPE	ΔE	Q	W
1-2	72	-5	-6	d	0	j
2-3	64	0	c	e	90	k
3-4	-97	b	0	f	h	92
4-1	a	0	3	g	i	0

ANALYSIS: For a cycle, the overall change in every property value is zero.

$$a. \quad \Sigma(\Delta U) = 0: 72 + 64 - 97 + a = 0 \Rightarrow a = -39$$

$$b. \quad \Sigma(\Delta KE) = 0: -5 + 0 + b + 0 = 0 \Rightarrow b = 5$$

$$c. \quad \Sigma(\Delta PE) = 0: -6 + c + 0 + 3 = 0 \Rightarrow c = 3$$

$$\Delta E = \Delta U + \Delta KE + \Delta PE:$$

$$d. \quad d = 72 - 5 - 6 \Rightarrow d = 61$$

$$e. \quad e = 64 + 0 + c \Rightarrow e = 67$$

$$f. \quad f = -97 + b + 0 \Rightarrow f = -92$$

$$g. \quad g = a + 0 + 3 \Rightarrow g = -36$$

$$\Delta E = Q - W:$$

$$h. \quad f = h - 92 \Rightarrow h = 0$$

$$i. \quad g = i - 0 \Rightarrow i = -36$$

$$j. \quad d = 0 - j \Rightarrow j = -61$$

$$k. \quad e = 90 - k \Rightarrow k = 23$$

For the overall cycle, $W_{\text{cycle}} = Q_{\text{cycle}}$

$$\left. \begin{aligned} W_{\text{cycle}} &= -61 + 23 + 92 + 0 = 54 \\ Q_{\text{cycle}} &= 0 + 90 + 0 - 36 = 54 \end{aligned} \right\} \begin{array}{l} \text{checks the above} \\ \text{calculations. Also,} \\ \text{note that } \Sigma(\Delta E) = 0 \end{array}$$

Since $W_{\text{cycle}} > 0$, the cycle is a power cycle.

PROBLEM 2.72

KNOWN: A system undergoes a power cycle consisting of four processes in series.

FIND: Complete the table of energy values provided for the cycle and evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:

Process	ΔU	ΔKE	ΔPE	ΔE	Q	W
1-2	950	50	0	+1000 (c)	1000	0 (e)
2-3	-500 (a)	0	50	-450	0 (f)	450
3-4	-650	+50 (b)	0	-600	-600 (g)	0
4-1	200	-100	-50	+50 (d)	0	-50 (h)

ANALYSIS:

(a) For a cycle, the overall changes in U , KE , PE and E are zero:

$$\Sigma(\Delta U) = 950 + (a) - 650 + 200 = 0 \Rightarrow (a) = -500$$

$$\Sigma(\Delta KE) = 50 + 0 + (b) - 100 = 0 \Rightarrow (b) = +50$$

$$\Sigma(\Delta PE) = 0 + 50 + 0 - 50 = 0 \quad \checkmark$$

$$\Sigma(\Delta E) = (c) - 450 - 600 + (d) = 0 \Rightarrow (c) + (d) = 1050$$

For Process 1-2, $\Delta E = \Delta U + \Delta KE + \Delta PE$
 $= 950 + 50 + 0 = 1000$ $\checkmark$ (c)
 So, (d) = +50

Also, for Process 1-2,
 $\Delta E = Q - W \Rightarrow W = \Delta E - Q$
 $= 1000 - 1000 = 0$ (e)

For Process 2-3,
 $\Delta E = Q - W \Rightarrow Q = \Delta E + W$
 $= -450 + 450 = 0$ (f)

For Process 3-4,
 $\Delta E = Q - W \Rightarrow Q = \Delta E + W$
 $= -600 + 0 = -600$ (g)

For Process 4-1,
 $\Delta E = Q - W \Rightarrow W = Q - \Delta E$
 $= 0 - 50 = -50$ (h)

(b) For any power cycle,

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}}$$

Here, $W_{\text{cycle}} = 0 + 450 + 0 - 50 = 400$

$Q_{\text{in}} = 1000$

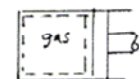
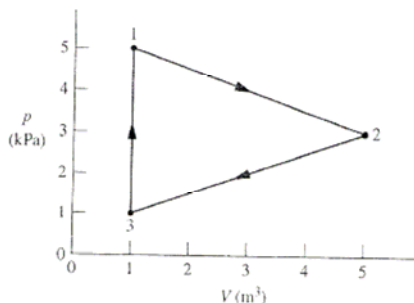
$$\Rightarrow \eta = \frac{400}{1000} = 0.40 \quad (40\%)$$

PROBLEM 2.73

KNOWN: Data are provided for a power cycle executed by a gas in a piston-cylinder assembly.

FIND: For each process evaluate W . Find Q for processes 1-2, 2-3. Evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} U_2 - U_1 &= 15 \text{ kJ} \\ Q_{31} &= 10 \text{ kJ} \end{aligned}$$

ENGR. MODEL

1. The gas is the closed system.
2. Volume change is the only work mode.
3. For each process, $\Delta KE = \Delta PE = 0$.

Fig. P2.73

ANALYSIS: (a) The work can be evaluated from Eq. 2.17. For Process 3-1, the piston does not move (volume is constant). Thus, $W_{31} = 0$. For Processes 1-2 and 2-3, the work can be evaluated geometrically. That is,

$$\begin{aligned} W_{12} &= P_{ave} [V_2 - V_1] = \left(\frac{P_1 + P_2}{2} \right) (V_2 - V_1) = \left[\left(\frac{5+3}{2} \right) \text{ kPa} \right] [5-1] \text{ m}^3 \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{1000 \text{ N}\cdot\text{m}} \right| \\ &= 16 \text{ kJ} \\ W_{23} &= P_{ave} [V_3 - V_2] = \left(\frac{P_2 + P_3}{2} \right) (V_3 - V_2) = \left[\left(\frac{3+1}{2} \right) \text{ kPa} \right] [1-5] \text{ m}^3 \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{1000 \text{ N}\cdot\text{m}} \right| \\ &= -8 \text{ kJ} \end{aligned}$$

(b) Q_{31} is given. For Process 1-2, $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$

$$\Rightarrow Q_{12} = \Delta U + W_{12} = 15 \text{ kJ} + 16 \text{ kJ} = 31 \text{ kJ}$$

For Process 2-3, $\Delta U + \Delta KE + \Delta PE = Q_{23} - W_{23}$

$$\Rightarrow Q_{23} = (U_3 - U_2) + W_{23}$$

To find $(U_3 - U_2)$, note that since internal energy is a property

$$\begin{aligned} (U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) &= 0 \\ \Rightarrow (U_3 - U_2) &= - \underbrace{(U_2 - U_1)}_{15 \text{ kJ}} = \underbrace{(U_1 - U_3)}_{10 \text{ kJ}} \end{aligned}$$

Energy balance for Process 3-1:
 $(U_1 - U_3) = Q_{31} - W_{31} = 10 \text{ kJ}$

$$\therefore (U_3 - U_2) = -15 \text{ kJ} - 10 \text{ kJ} = -25 \text{ kJ}$$

$$\textcircled{1} \quad \therefore Q_{23} = -25 \text{ kJ} + (-8 \text{ kJ}) = -33 \text{ kJ}$$

(c) For any power cycle, the thermal efficiency is $\eta = \frac{W_{cycle}}{Q_{in}}$

$$\text{Here, } W_{cycle} = W_{12} + W_{23} + W_{31} = 16 - 8 + 0 = 8 \text{ kJ}$$

$$Q_{in} = Q_{12} + Q_{31} = 31 + 10 = 41 \text{ kJ}$$

$$\therefore \eta = \frac{8 \text{ kJ}}{41 \text{ kJ}} = 0.195 \text{ (19.5\%)} \quad \leftarrow \eta$$

1. Also, note that for any cycle, $W_{cycle} = Q_{cycle}$ (Eq. 2.46). Thus:

$$\begin{aligned} W_{12} + W_{23} + W_{31} &= Q_{12} + Q_{23} + Q_{31} \Rightarrow Q_{23} = W_{12} + W_{23} + W_{31} - Q_{12} - Q_{31}, \text{ or} \\ Q_{23} &= 16 + (-8) + 0 - 31 - 10 = -33 \text{ kJ}. \end{aligned}$$

PROBLEM 2.74

KNOWN: A gas within a piston-cylinder assembly undergoes a thermodynamic cycle consisting of three processes in series.

FIND: Determine Q_{12} , Q_{31} , U_3 . Determine if the cycle can be a power cycle.

SCHEMATIC & GIVEN DATA:

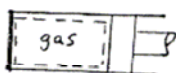
Process 1-2: Compression with $pV = \text{constant}$, $W_{12} = -104 \text{ kJ}$,
 $U_1 = 512 \text{ kJ}$, $U_2 = 690 \text{ kJ}$.

Process 2-3: $W_{23} = 0$, $Q_{23} = -150 \text{ kJ}$.

Process 3-1: $W_{31} = +50 \text{ kJ}$.

ENGR. MODEL:

1. The gas is the closed system.
2. Volume change is the only work mode.
3. For each process, $\Delta KE = \Delta PE = 0$.



ANALYSIS: (a) Process 1-2, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12} \Rightarrow$

$$Q_{12} = [U_2 - U_1] + W_{12} = (690 - 512) \text{ kJ} + (-104 \text{ kJ}) = +74 \text{ kJ} \quad \leftarrow Q_{12}$$

For any cycle, $W_{\text{cycle}} = Q_{\text{cycle}}$ (Eq. 2.40). Thus

$$\begin{aligned} W_{12} + W_{23} + W_{31} &= Q_{12} + Q_{23} + Q_{31} \\ \Rightarrow Q_{31} &= W_{12} + W_{23} + W_{31} - Q_{12} - Q_{23} \\ &= (-104) + 0 + 50 - 74 - (-150) = +22 \text{ kJ} \quad \leftarrow Q_{31} \end{aligned}$$

Process 3-1: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{31} - W_{31} \Rightarrow U_1 - U_3 = Q_{31} - W_{31}$

$$\textcircled{1} \Rightarrow U_3 = U_1 - Q_{31} + W_{31} = 512 - 22 + 50 = 540 \text{ kJ} \quad \leftarrow U_3$$

(b) A power cycle is one for which $W_{\text{cycle}} > 0$. For the current cycle,

$$\begin{aligned} W_{\text{cycle}} &= W_{12} + W_{23} + W_{31} \\ &= +(-104) + (0) + (50) = -54 \text{ kJ} \end{aligned}$$

No. This cycle cannot be a power cycle. $\leftarrow$

1. As checks on these calculations, note that

$$\begin{aligned} \text{Process 2-3: } (U_3 - U_2) &= Q_{23} - W_{23} \\ &= (-150 \text{ kJ}) - 0 \Rightarrow U_3 - U_2 = -150 \text{ kJ} \\ &\Rightarrow U_3 = 690 - 150 = 540 \text{ kJ} \end{aligned}$$

Since U is a property,

$$\begin{aligned} (U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) &= 0 \\ (690 - 512) + (U_3 - 690) + (512 - U_3) &= 0 \\ 178 + \textcircled{540} + (-22) &= 0 \quad \checkmark \end{aligned}$$

PROBLEM 2.75

KNOWN: A gas undergoes a thermodynamic cycle consisting of three processes.

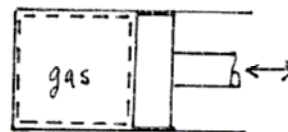
FIND: Determine the heat transfer and work for process 2-3, and whether the cycle is a power cycle or a refrigeration cycle.

SCHEMATIC & GIVEN DATA: The following data are given for each process:

Process 1-2: compression with $pV = \text{const.}$
 $p_1 = 1 \text{ bar}$, $V_1 = 1.6 \text{ m}^3$ to $V_2 = 0.2 \text{ m}^3$
 $U_2 - U_1 = 0$

Process 2-3: constant pressure to $V_3 = V_1$

Process 3-1: constant volume, $U_1 - U_3 = -3549 \text{ kJ}$



ENGR. MODEL: (1) The gas is a closed system. (2) Neglect kinetic and potential energy changes. (3) Volume change is the only work mode.

ANALYSIS: To find the work for Process 2-3, use Eq. 2.17, with constant pressure

$$W_{23} = \int_{V_2}^{V_3} p dV = p_2 (V_3 - V_2)$$

Using the p - V relation for process 1-2; $p_2 = \left(\frac{V_1}{V_2}\right) p_1 = 8 \text{ bar}$. Thus, with $V_3 = V_1$

$$W_{23} = (8 \text{ bar})(1.6 - 0.2) \text{ m}^3 \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 1120 \text{ kJ} \leftarrow W_{23}$$

The energy balance for process 2-3 reduces to

$$Q_{23} = (U_3 - U_2) + W_{23}$$

To get $U_3 - U_2$, note that for the cycle, $\Delta U_{\text{cycle}} = 0$. Thus

$$(U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0 \Rightarrow (U_3 - U_2) = -(U_1 - U_3) = 3549 \text{ kJ}$$

Finally, for process 2-3

$$Q_{23} = (3549 \text{ kJ}) + (1120 \text{ kJ}) = 4669 \text{ kJ} \leftarrow Q_{23}$$

Next, for process 1-2, $\Delta U = 0$, so $Q_{12} = W_{12}$. Using Eq. 2.17

$$Q_{12} = W_{12} = \int_{V_1}^{V_2} p dV = p_1 V_1 \ln \frac{V_2}{V_1} = (1)(1.6) \ln \left(\frac{0.2}{1.6} \right) \left| \frac{10^5}{10^3} \right| = -332.7 \text{ kJ}$$

And, for process 3-1: $W_{31} = 0$, and $Q_{31} = U_1 - U_3 = -3549 \text{ kJ}$. Collecting various results

$$W_{\text{cycle}} = W_{12} + W_{23} + W_{31} = (-332.7) + (1120) = +787.3 \text{ kJ}$$

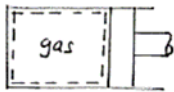
Since $W_{\text{cycle}} > 0$, the cycle is a power cycle.

PROBLEM 2.76

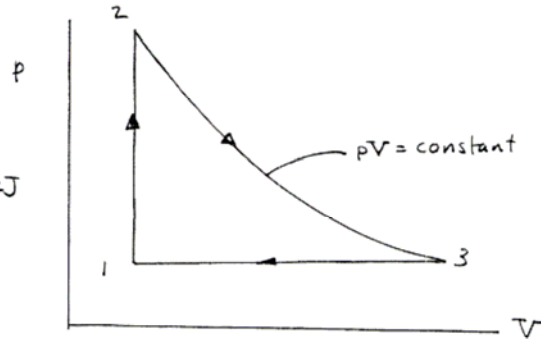
KNOWN: A gas undergoes a thermodynamic cycle consisting of three processes.

FIND: Sketch the cycle on a p - V diagram. Calculate W_{cycle} , Q_{23} , Q_{31} , and determine whether the cycle is a power or refrigeration cycle.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} 1-2: & V = 0.028 \text{ m}^3, \quad u_2 - u_1 = 26.4 \text{ kJ} \\ 2-3: & pV = \text{constant}, \quad u_3 = u_2 \\ 3-1: & p = 1.4 \text{ bar}, \quad W_{31} = -10.5 \text{ kJ} \end{aligned}$$



ENGR. MODEL: 1. The gas is the closed system. 2. For the system, $\Delta KE = \Delta PE = 0$. 3. Volume change is the only work mode.

ANALYSIS: (b) $W_{\text{cycle}} = W_{12} + W_{23} + W_{31}$. Since volume is constant and volume change is the only work mode, $W_{12} = 0$. To find W_{23}

$$W_{23} = \int_{V_2}^{V_3} p dV = \int_{V_2}^{V_3} \frac{C}{V} dV = C \ln \frac{V_3}{V_2} = p_3 V_3 \ln \frac{V_3}{V_2} \quad \leftarrow V_2 = V_1$$

To evaluate V_3

$$W_{31} = \int_{V_3}^{V_1} p dV = p(V_1 - V_3) \Rightarrow V_3 = V_1 - W_{31}/p$$

$$\text{or} \quad V_3 = 0.028 \text{ m}^3 - \frac{(-10.5 \text{ kJ})}{(1.4 \text{ bar})} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}} \right| = 0.103 \text{ m}^3$$

Then

$$W_{23} = (1.4 \text{ bar}) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| (0.103 \text{ m}^3) \ln \left(\frac{0.103}{0.028} \right) \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 18.78 \text{ kJ}$$

Finally,

$$W_{\text{cycle}} = 0 + 18.78 + (-10.5) = 8.28 \text{ kJ} \quad \leftarrow W_{\text{cycle}}$$

(c) An energy balance for 2-3 gives Q_{23} : $\Delta U + \Delta KE + \Delta PE = Q_{23} - W_{23} \Rightarrow Q_{23} = W_{23} + \Delta U$

(d) To evaluate Q_{31} begin with an energy balance together with assumption 2

$$u_1 - u_3 = Q_{31} - W_{31} \Rightarrow Q_{31} = (u_1 - u_3) + W_{31}$$

Since there is no overall change in internal energy for the cycle, $\Sigma(\Delta U) = 0$

$$\begin{aligned} (u_2 - u_1) + (u_3 - u_2) + (u_1 - u_3) &= 0 \Rightarrow \\ (u_1 - u_3) &= -26.4 - (0) = -26.4 \text{ kJ} \end{aligned}$$

Then

$$\textcircled{1} \quad Q_{31} = (-26.4) + (-10.5) = -36.9 \text{ kJ} \quad \leftarrow Q_{31}$$

1. As a check, note that $Q_{\text{cycle}} = W_{\text{cycle}}$, $Q_{\text{cycle}} = Q_{12} + Q_{23} + Q_{31}$. An energy balance for 1-2 gives $Q_{12} = 26.4 \text{ kJ}$. So $Q_{\text{cycle}} = 26.4 + 18.78 + (-36.9) = 8.28 \text{ kJ}$, which checks the result of part (b).

PROBLEM 2.77

KNOWN: A gas within a piston-cylinder assembly undergoes a thermodynamic cycle consisting of three processes in series.

FIND: Determine V_1 , W_{12} , and Q_{12} . Determine if the cycle can be power cycle or a refrigeration cycle.

SCHEMATIC & GIVEN DATA:

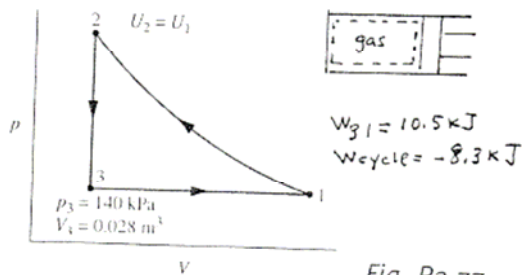


Fig. P2.77

ENGR. MODEL:

1. The gas is the closed system.
2. Volume change is the only work mode.
3. For each process, $\Delta KE = \Delta PE = 0$.

ANALYSIS:

(a) To find V_1 , note that

$$W_{31} = \int_3^1 p dV = p [V_1 - V_3] \Rightarrow V_1 = V_3 + \frac{W_{31}}{p}$$

$\uparrow$ 140 kPa

or

$$V_1 = 0.028 \text{ m}^3 + \frac{10.5 \text{ kJ}}{140 \text{ kPa}} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| = 0.103 \text{ m}^3 \quad \leftarrow V_1$$

(b) To find W_{12} , write $W_{\text{cycle}} = W_{12} + W_{23} + W_{31}$. Since the only work mode is volume change, the work is given by Eq. 2.17. Since the piston does not move in process 2-3 (volume is constant), $W_{23} = 0$. Thus

$$W_{12} = W_{\text{cycle}} - \cancel{W_{23}}^0 - W_{31}$$

$$= -8.3 \text{ kJ} - 10.5 \text{ kJ} = -18.8 \text{ kJ} \quad \leftarrow W_{12}$$

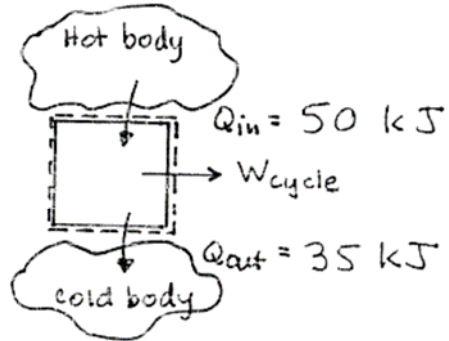
A power cycle is one for which $W_{\text{cycle}} > 0$. Here, we have $W_{\text{cycle}} = -8.3 \text{ kJ}$. So, the cycle cannot be a power cycle, but it can be a refrigeration (or heat pump) cycle.

To find Q_{12} , write an energy balance: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$

$$\Rightarrow Q_{12} = \underbrace{\Delta U}_{U_2 = U_1} + W_{12}$$

$$\Rightarrow Q_{12} = -18.8 \text{ kJ} \quad \leftarrow Q_{12}$$

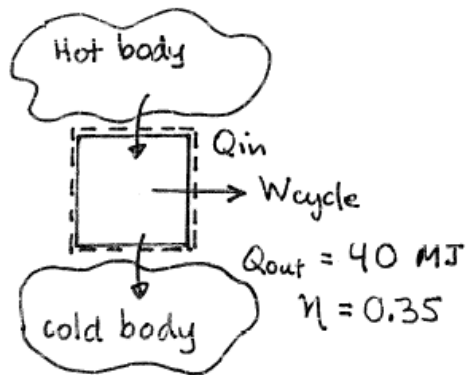
PROBLEM 2.78



$$W_{cycle} = Q_{in} - Q_{out}$$
$$= 50 - 35 = 15 \text{ kJ} \leftarrow W_{cycle}$$

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{15}{50} = 0.3 (30\%) \leftarrow \eta$$

PROBLEM 2.79



$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Solving for Q_{in}

$$Q_{in} = \frac{Q_{out}}{1 - \eta} = \frac{40}{(1 - .35)}$$

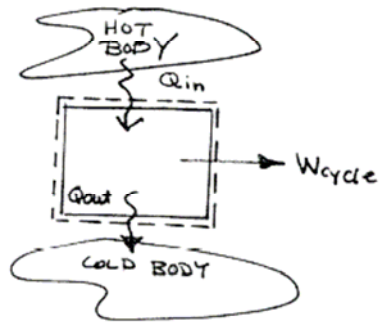
$$= 61.54 \text{ MJ} \leftarrow Q_{in}$$

Thus

$$W_{cycle} = \eta Q_{in} = (.35)(61.54)$$

$$= 21.54 \text{ MJ} \leftarrow W_{cycle}$$

PROBLEM 2.80



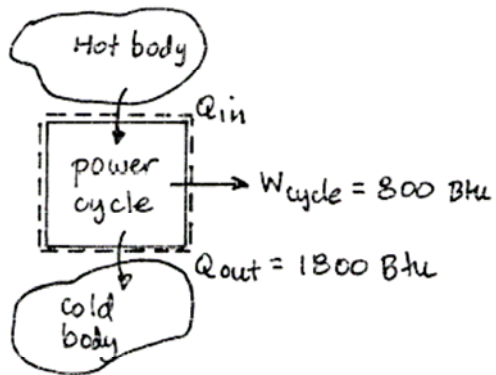
$$Q_{in} = 2600 \text{ Btu}$$

$$Q_{out} = 1800 \text{ Btu}$$

$$\text{Energy Balance: } W_{cycle} = Q_{cycle} = Q_{in} - Q_{out} = 800 \text{ Btu}$$

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{800 \text{ Btu}}{2600 \text{ Btu}} = 0.308 (30.8\%)$$

PROBLEM 2.81



Energy balance: $W_{cycle} = Q_{in} - Q_{out}$

$$Q_{in} = W_{cycle} + Q_{out} \\ = 800 + 1800 = 2600 \text{ Btu}$$

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{800}{2600}$$

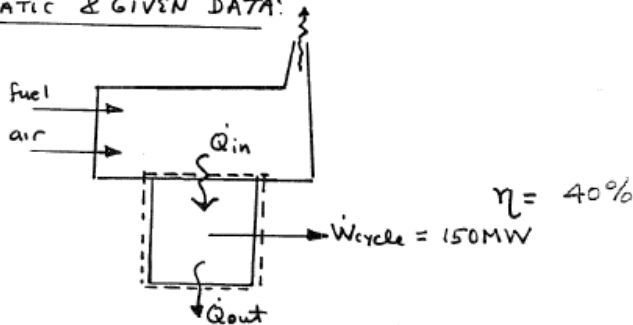
$$= 0.308 \text{ (30.8\%)} \quad \leftarrow \eta$$

PROBLEM 2.82

KNOWN: Operating data are provided for a power cycle.

FIND: Determine the net rate energy is added by heat transfer, the net work output annually, and the value of the net work, in \$/year.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system undergoes a power cycle. 2. The cycle operates steadily for 8000 h annually. 3. The value of the net work is \$0.08/kW·h

ANALYSIS: (a) To determine $\dot{W}_{cycle}$

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} \Rightarrow \dot{Q}_{in} = \frac{\dot{W}_{cycle}}{\eta} = \frac{150 \text{ MW}}{0.40} = 375 \text{ MW} \quad \leftarrow \dot{Q}_{in}$$

(b) With assumption 2

$$W_{cycle} = (150 \text{ MW}) \left| \frac{10^3 \text{ kW}}{1 \text{ MW}} \right| \left(\frac{8000 \text{ h}}{\text{year}} \right) = 1.2 \times 10^9 \frac{\text{kW} \cdot \text{h}}{\text{year}} \quad \leftarrow W_{cycle}$$

(c) With assumption 3

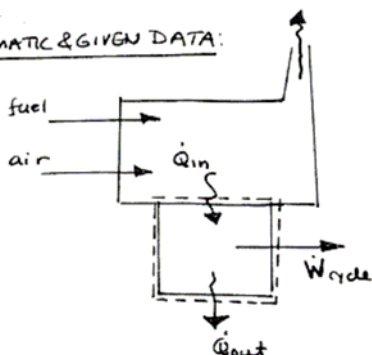
$$\$ = (1.2 \times 10^9 \frac{\text{kW} \cdot \text{h}}{\text{year}}) \left(\frac{\$0.08}{\text{kW} \cdot \text{h}} \right) = \$96 \text{ M/year} \quad \leftarrow \$$$

PROBLEM 2.83

KNOWN: Operating data are provided for a power cycle.

FIND: Determine, in \$/year, the value of the power generated and the cost of $\dot{Q}_{in}$.

SCHEMATIC & GIVEN DATA:



$$\eta = 40\%$$

$$\dot{W}_{cycle} = 100 \text{ MW}$$

ASSUMPTIONS: 1. The system undergoes a thermodynamic cycle. 2. The power cycle operates steadily for 8000 h per year. 3. Electricity is valued at \$0.08/kW·h and $\dot{Q}_{in}$ at \$4.50/GJ

ANALYSIS: (a) Using assumptions 2, 3

$$\dot{\$}_{\text{electricity}} = (100 \text{ MW}) \left| \frac{10^3 \text{ kW}}{1 \text{ MW}} \right| \left(\frac{8000 \text{ h}}{\text{year}} \right) \left(\frac{\$0.08}{\text{kW} \cdot \text{h}} \right) = \$64 \times 10^6 / \text{year} \leftarrow \dot{\$}_{\text{electricity}}$$

(b) With

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} \Rightarrow \dot{Q}_{in} = \frac{\dot{W}_{cycle}}{\eta} = \frac{100 \text{ MW}}{0.40} = 250 \text{ MW}$$

$$\dot{\$}_{\dot{Q}_{in}} = (250 \text{ MW}) \left| \frac{1 \text{ GJ/s}}{10^3 \text{ MW}} \right| \left| \frac{3600 \text{ s}}{\text{h}} \right| \left(\frac{8000 \text{ h}}{\text{year}} \right) \left(\frac{\$4.50}{\text{GJ}} \right) = \$32.4 \text{ M/year} \leftarrow \dot{\$}_{\dot{Q}_{in}}$$

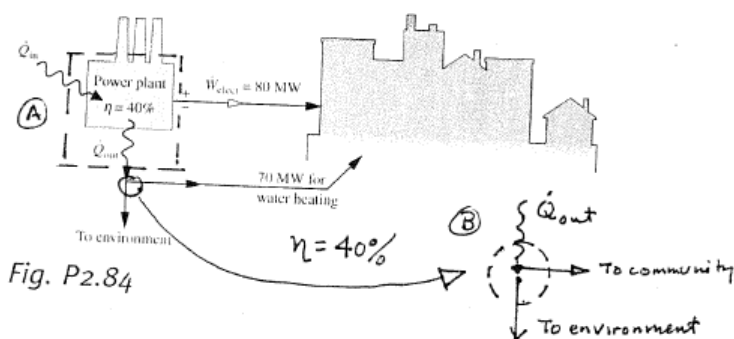
(c) No. Part (b) gives just the annual fuel cost. But for a power plant there are other costs. An important one is the cost of the plant, itself, which is a capital cost.

PROBLEM 2.84

KNOWN: Operating data are provided for a cogeneration power plant operating in a thermodynamic cycle at steady state.

FIND: Determine the rates at which energy is added by heat transfer and discarded to the environment. Also determine the value of the electricity generated.

SCHEMATIC & GIVEN DATA:



ENR. MODEL

1. As shown in the schematic, two systems — A and B — are considered.
2. Each system is at steady state. System A operates on a thermodynamic cycle.
3. Electricity is valued at \$0.08 per kW·h.

ANALYSIS:

$$(a) \quad \eta = \frac{W_{\text{elect}}}{\dot{Q}_{\text{in}}} \Rightarrow \dot{Q}_{\text{in}} = \frac{W_{\text{elect}}}{\eta} = \frac{80 \text{ MW}}{0.40} = 200 \text{ MW}$$

← $\dot{Q}_{\text{in}}$

For the power plant, $\dot{W}_{\text{cycle}} = \dot{Q}_{\text{cycle}}$. That is

$$W_{\text{elect}} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} \Rightarrow \dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - W_{\text{elect}} = (200 - 80) \text{ MW} = 120 \text{ MW}$$

(b) Considering system B,

$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{to environment}} + \dot{Q}_{\text{community}} \quad (\text{All terms are positive in the directions of the arrows.})$$

$$\Rightarrow \dot{Q}_{\text{to environment}} = \dot{Q}_{\text{out}} - \dot{Q}_{\text{community}} = 120 \text{ MW} - 70 \text{ MW} = 50 \text{ MW}$$

← $\dot{Q}_{\text{to environment}}$

$$(c) \quad \left[\text{Annual Value of Electricity} \right] = (80 \text{ MW}) \left| \frac{10^3 \text{ kW}}{1 \text{ MW}} \right| \left(365 \times 24 \frac{\text{h}}{\text{year}} \right) \left(\frac{\$0.08}{\text{kW}\cdot\text{h}} \right) = \$56.1 \text{ M/year}$$

← Annual value

PROBLEM 2.85

(a) Window air conditioner.

cold body : environment inside the building served

hot body : outside environment

(b) Nuclear power plant.

hot body : coolant circulated through the reactor core

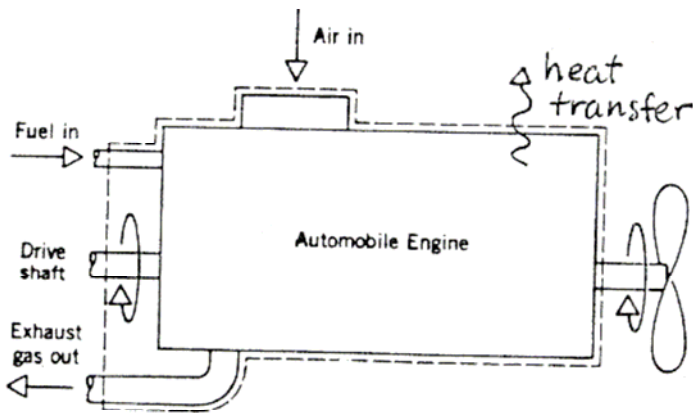
cold body : sea water

(c) Ground source heat pump.

hot body : environment inside the building served

cold body : ground

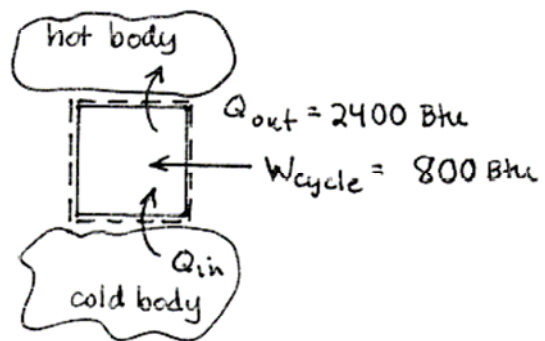
PROBLEM 2.86



ASSUMPTION: Engine operates at steady state.

An engine operating at steady state has continuous flows of fuel and air in as well as exhaust gas out. It also develops a steady power output. Although the operation of an engine does not conform to the strict definition of a thermodynamic cycle, there are similarities between an engine and a power cycle as illustrated schematically in Fig. 2.15(a). Specifically, we can think of the hot gases formed during combustion as playing the role of the hot body, and the surroundings, which interact with the engine through heat transfer and the discharge of hot exhaust gas, can be viewed as the cold body.

PROBLEM 2.87



$$\beta = \frac{Q_{in}}{W_{cycle}}$$

$$W_{cycle} = Q_{out} - Q_{in}$$

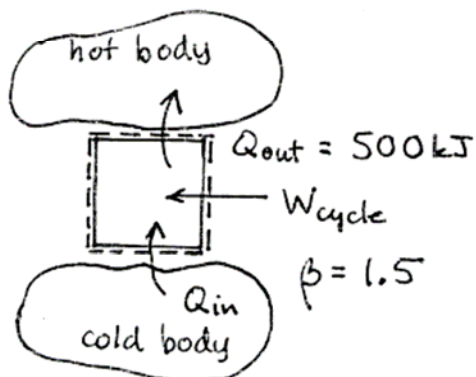
$$\Rightarrow Q_{in} = Q_{out} - W_{cycle}$$

$$= 2400 - 800$$

$$= 1600 \text{ Btu}$$

$$\beta = \frac{1600}{800} = 2.0 \leftarrow \beta$$

PROBLEM 2.88



$$\beta = \frac{Q_{in}}{W_{cycle}} = \frac{Q_{in}}{Q_{out} - Q_{in}}$$

Solving for Q_{in}

$$Q_{in} = Q_{out} \left(\frac{\beta}{1 + \beta} \right)$$

$$= (500) \left(\frac{1.5}{1 + 1.5} \right) = 300 \text{ kJ} \leftarrow Q_{in}$$

$$W_{cycle} = \frac{Q_{in}}{\beta} = \frac{300}{1.5}$$

$$= 200 \text{ kJ} \leftarrow W_{cycle}$$

PROBLEM 2. 89

KNOWN: Steady-state operating data are provided for a household refrigerator,

FIND: Determine the cost of operating the refrigerator in a month when it operates 360 hours.

SCHEMATIC & GIVEN DATA:

- $\beta = 2.4$
- $\dot{Q}_{in} = 600 \text{ Btu/h}$
operation for 360 h
- Value of electricity
= \$0.08 per kW·h

ANALYSIS:

For a refrigeration cycle,

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}}$$

where $\dot{Q}_{in}$ is the rate energy is removed from the refrigerated space.

$$\text{Thus, } \dot{W}_{cycle} = \frac{\dot{Q}_{in}}{\beta} = \frac{600 \text{ Btu/h}}{2.4} = 250 \frac{\text{Btu}}{\text{h}}$$

$$\dot{\$} = \left(250 \frac{\text{Btu}}{\text{h}} \right) \left(360 \frac{\text{h}}{\text{month}} \right) \left(\frac{\$0.08}{\text{kW}\cdot\text{h}} \right) \left| \frac{1 \text{ kW}}{3413 \text{ Btu/h}} \right| = \$2.11 / \text{month}$$

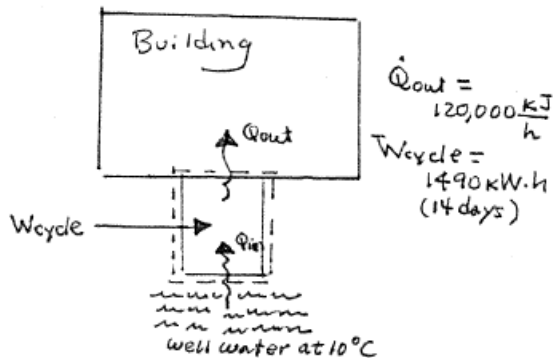
←

PROBLEM 2.90

KNOWN: Operating data are provided for a heat pump cycle operating at steady state while receiving energy by heat transfer from well water.

FIND: Determine the amount of energy received from well water and the heat pump's coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENER. MODEL:

1. The heat pump is the system.
2. The building and well water play the roles of hot and cold bodies, respectively.
3. The heat pump operates at steady state.
4. All energy transfers are positive in the directions of the arrows.

ANALYSIS:

(a) Applying an energy balance using assumption 4 (Eq. 2.44),

$$W_{cycle} = Q_{out} - Q_{in} \Rightarrow Q_{in} = Q_{out} - W_{cycle}$$

$\uparrow (Q_{out} \Delta t)$

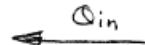
$$Q_{out} = \left(120,000 \frac{kJ}{h} \right) \left(\frac{24h}{day} \right) (14 \text{ days})$$

$$= 40.32 \times 10^6 \text{ kJ}$$

$$W_{cycle} = (1490 \text{ kW} \cdot h) \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = 5.364 \times 10^6 \text{ kJ}$$

Thus

$$Q_{in} = (40.32 - 5.364) \times 10^6 \text{ kJ} = 34.956 \times 10^6 \text{ kJ}$$



(b) Apply Eq. 2.47

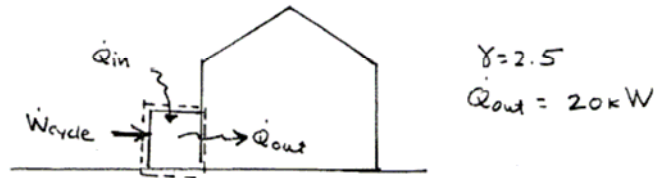
$$\gamma = \frac{Q_{out}}{W_{cycle}} = \frac{40.32 \times 10^6 \text{ kJ}}{5.364 \times 10^6 \text{ kJ}} = 7.52$$



PROBLEM 2.91

KNOWN: Operating data are provided for a heat pump.

FIND: Determine the net power required to operate the heat pump and its monthly cost.



ASSUMPTIONS: 1. The system undergoes a heat pump cycle. 2. The cycle operates steadily for 200 h monthly. 3. Electricity is valued at \$0.08/kW·h.

ANALYSIS: (a) The coefficient of performance for the heat pump is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_{cycle}} \Rightarrow \dot{W}_{cycle} = \frac{\dot{Q}_{out}}{\gamma} = \frac{20 \text{ kW}}{2.5} = 8 \text{ kW} \quad \leftarrow \gamma$$

(b) With assumptions 2, 3

$$\dot{\$} = (8 \text{ kW}) \left(\frac{200 \text{ h}}{\text{month}} \right) \left(\frac{\$0.08}{\text{kW} \cdot \text{h}} \right) = \$128/\text{month} \quad \leftarrow \dot{\$}$$

PROBLEM 2.92

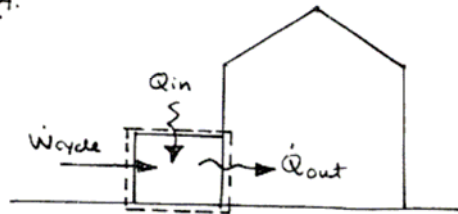
KNOWN: Operating data are provided for a heat pump.

FIND: Determine the coefficient of performance and the monthly cost to operate the heat pump.

SCHEMATIC & GIVEN DATA:

$$\dot{W}_{\text{cycle}} = 7.8 \text{ hp}$$

$$\dot{Q}_{\text{out}} = 60,000 \text{ Btu/h}$$



ASSUMPTIONS: 1. The system undergoes a heat pump cycle. 2. The cycle operates steadily for 200 h monthly. 3. Electricity is valued at \$0.08/kW·h

ANALYSIS: (a) The coefficient of performance for a heat pump is

$$\gamma = \frac{\dot{Q}_{\text{out}}}{\dot{W}_{\text{cycle}}} = \frac{60,000 \text{ Btu/h}}{(7.8 \text{ hp}) \left| \frac{2545 \text{ Btu/h}}{\text{hp}} \right|} = 3.02 \quad \leftarrow \gamma$$

(b) Using assumptions 2, 3

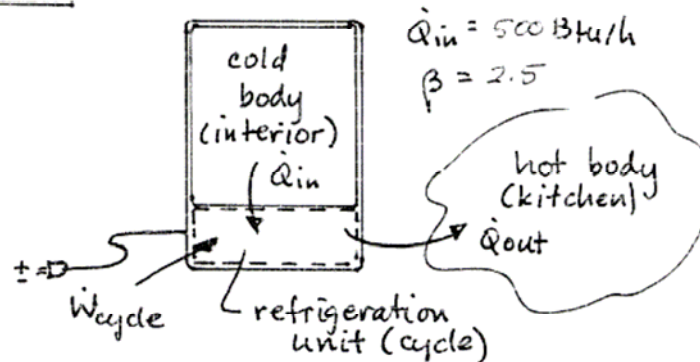
$$\text{\$} = (7.8 \text{ hp}) \left| \frac{1 \text{ kW}}{1.341 \text{ hp}} \right| \left(\frac{200 \text{ h}}{\text{month}} \right) \left(\frac{\text{\$} 0.08}{\text{kW} \cdot \text{h}} \right) = \text{\$} 93 / \text{month} \quad \leftarrow \text{\$}$$

PROBLEM 2.93

KNOWN: Operating data are given for a household refrigerator.

Find: Determine the cost of operating the refrigerator for 360 h per month.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The refrigeration unit operates as a refrigeration cycle. (2) The cycle operates steadily for 300 h monthly. (3) Electricity is valued at \$ 0.12 / kWh.

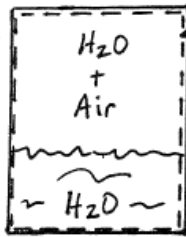
ANALYSIS: To determine the electric power input, begin with the expression for coefficient of performance: $\beta = \dot{Q}_{in} / \dot{W}_{cycle}$. Solving for the power

$$\begin{aligned}\dot{W}_{cycle} &= \dot{Q}_{in} / \beta \\ &= \left(\frac{500 \text{ Btu/h}}{2.5} \right) \left| \frac{1 \text{ kW}}{3413 \text{ Btu/h}} \right| = 0.059 \text{ kW}\end{aligned}$$

The monthly cost rate, $\dot{\$}$, is

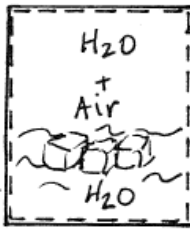
$$\begin{aligned}\dot{\$} &= (0.059 \text{ kW})(300 \text{ h/month})(\$0.12 / \text{kW}\cdot\text{h}) \\ &= \$2.12 / \text{month}\end{aligned}$$

PROBLEM 3.1



← system boundary

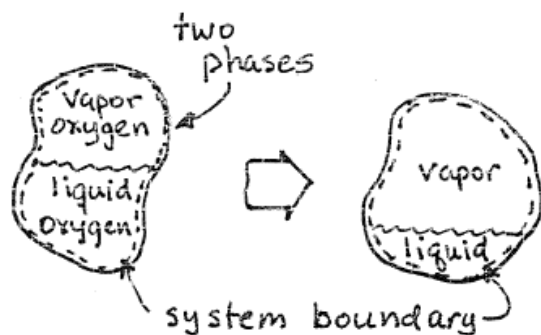
- two phases are present (liquid and gas).
- not a pure substance because composition is different in each phase.



← system boundary

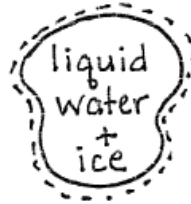
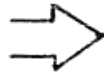
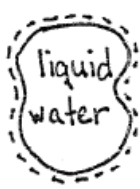
- three phases are present (solid, liquid, and gas).
- not a pure substance because composition of gas phases is different than that of the solid and liquid phases.

PROBLEM 3.2



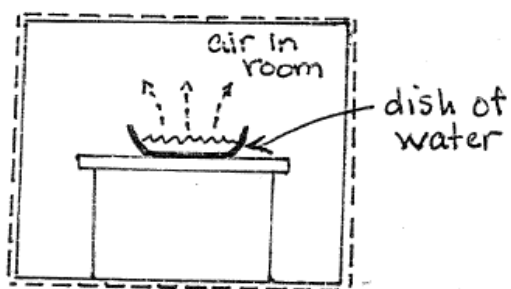
The system is a pure substance. Although the liquid is vaporized, the system remains fixed in chemical composition and is chemically homogeneous.

PROBLEM 3.3



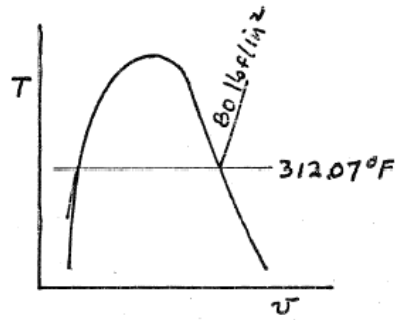
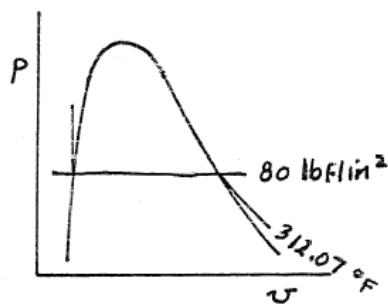
The system is a pure substance. Although the phases change, the system remains of fixed chemical composition and is chemically homogeneous.

PROBLEM 3.4

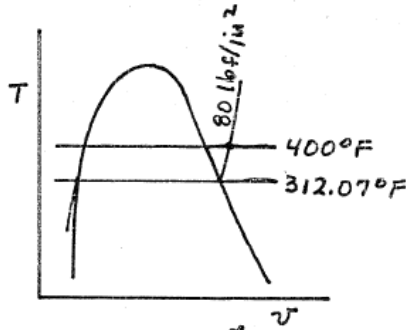
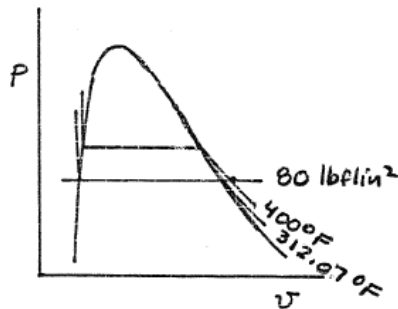


The system is not a pure substance during the process since the composition of the gas phase changes as water evaporates into the air. Once all of the water evaporates, the gas phase comes to equilibrium and the composition becomes homogeneous. At this point, the gas phase can be treated as a pure substance.

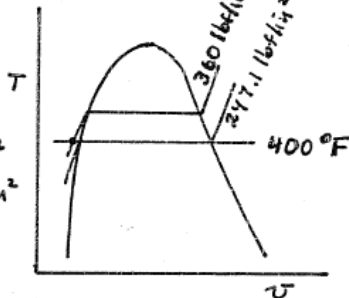
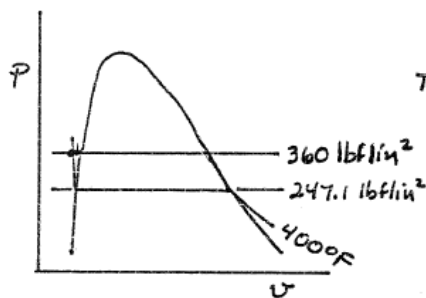
PROBLEM 3.5



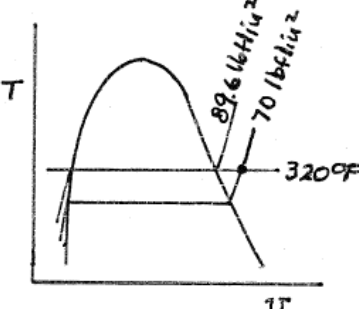
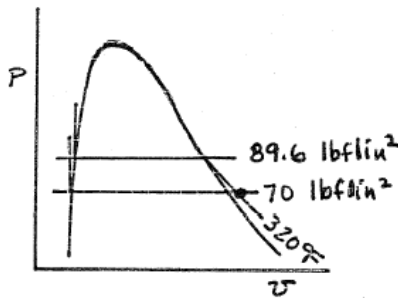
$p = 80 \text{ lbf/in}^2$
 $T = 312.07^\circ\text{F}$
 two-phase liquid-vapor mixture



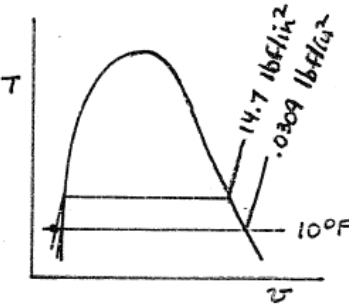
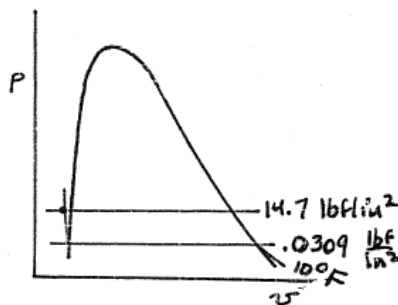
$p = 80 \text{ lbf/in}^2$
 $T = 400^\circ\text{F}$
 superheated vapor



$T = 400^\circ\text{F}$
 $p = 360 \text{ lbf/in}^2$
 subcooled (compressed) liquid

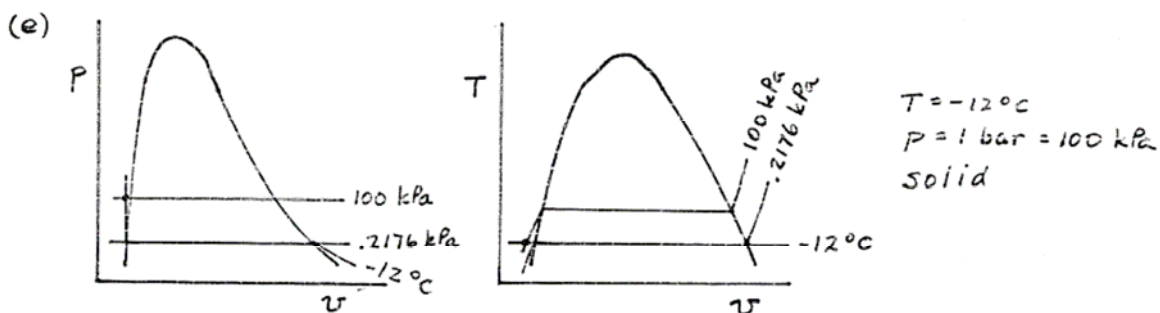
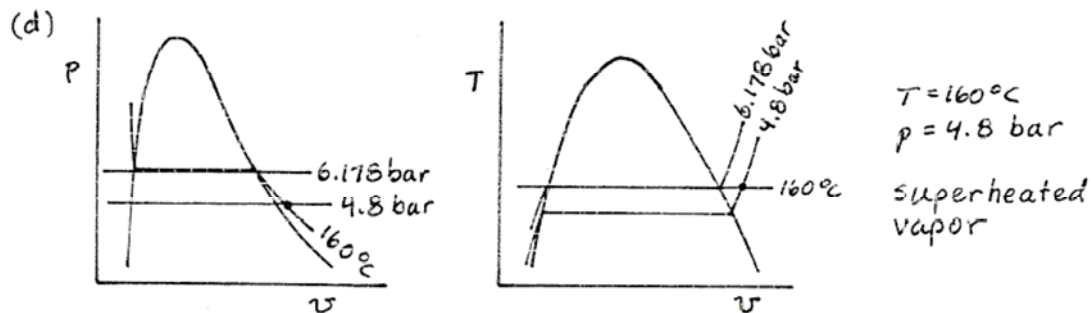
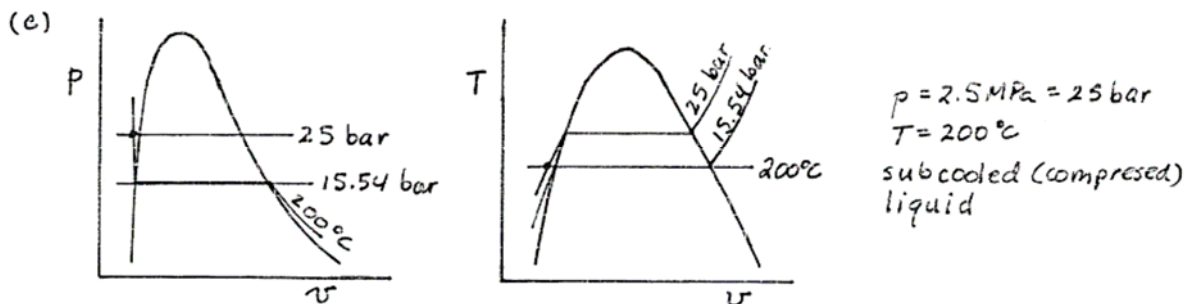
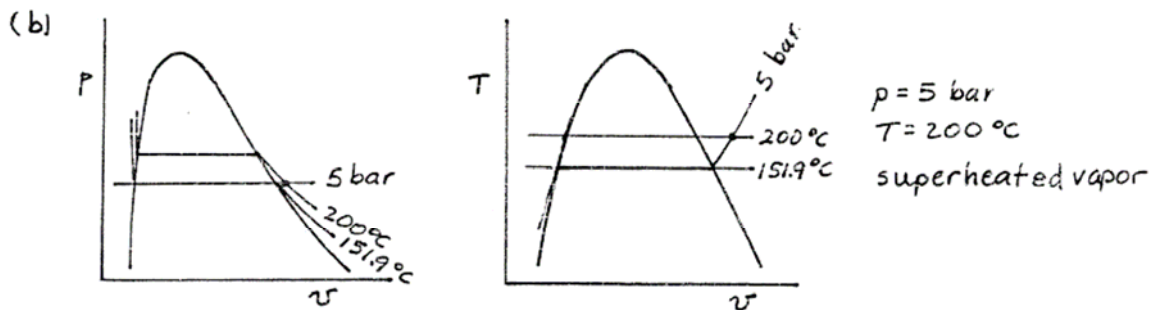
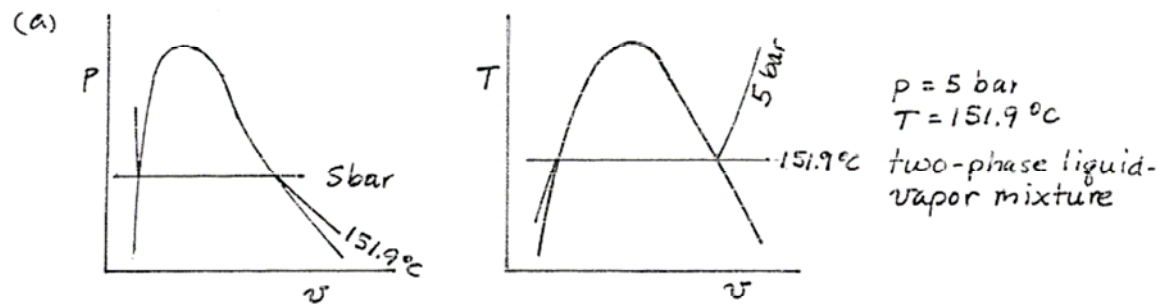


$T = 320^\circ\text{F}$
 $p = 89.6 \text{ lbf/in}^2$
 superheated vapor



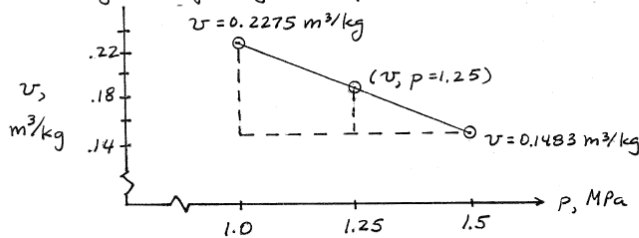
$T = 10^\circ\text{F}$
 $p = 14.7 \text{ lbf/in}^2$
 solid

PROBLEM 3.6



PROBLEM 3.7

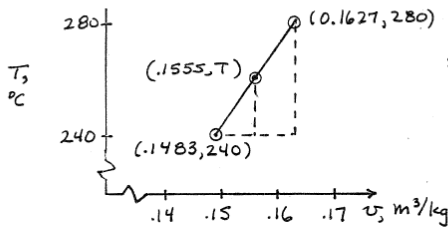
- (a) At a temperature of 240°C , the specified pressure of 1.25 MPa falls between the table values of 1.0 and 1.5 MPa . To determine the specific volume corresponding to 1.25 MPa , we think of the slope of a straight line joining the adjacent table states, as follows:



similar triangles:

$$\left| \text{slope} \right| = \frac{v - 0.1483}{1.5 - 1.0} = \frac{0.2275 - 0.1483}{1.5 - 1.0} \Rightarrow v = 0.1483 + \left(\frac{0.25}{0.50} \right) (0.2275 - 0.1483) = 0.1879 \text{ m}^3/\text{kg} \quad (a)$$

- (b) At a pressure of 1.5 MPa , the given specific volume of $0.1555\text{ m}^3/\text{kg}$ falls between the table values of 240 and 280°C . To determine the temperature corresponding to the given specific volume, we think of the slope of a straight line joining the adjacent table states, as follows:



$$\text{slope} = \frac{T - 240}{0.1555 - 0.1483} = \frac{280 - 240}{0.1627 - 0.1483} \\ \Rightarrow T = 240 + \left[\frac{0.1555 - 0.1483}{0.1627 - 0.1483} \right] (40) = 260^\circ\text{C} \quad (b)$$

- (c) In this case, the specified pressure falls between the table values of 1.0 and 1.5 MPa and the specified temperature falls between the table values of 200 and 240°C . Thus, double interpolation is required.

- At 220°C , the specific volume at each pressure is simply the average over the interval:

$$\text{at } 1.0\text{ MPa, } 220^\circ\text{C}; v = \frac{0.2060 + 0.2275}{2} = 0.21675 \text{ m}^3/\text{kg}$$

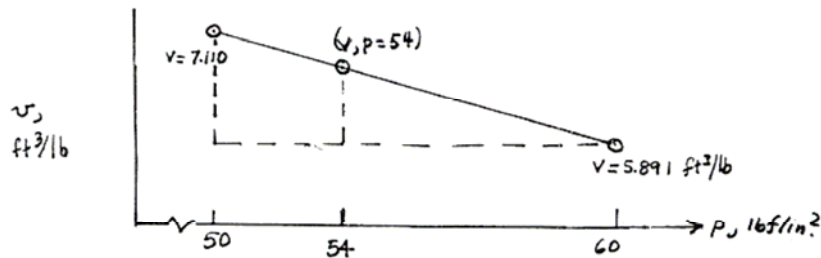
$$\text{at } 1.5\text{ MPa, } 220^\circ\text{C} \quad v = \frac{0.1325 + 0.1483}{2} = 0.1404 \text{ m}^3/\text{kg}$$

- Thus, with the same approach as in (a)

$$\frac{v - 0.1404}{1.5 - 1.0} = \frac{0.21675 - 0.1404}{1.5 - 1.0} \Rightarrow v = 0.1404 + \left(\frac{0.1}{0.5} \right) (0.21675 - 0.1404) = 0.15567 \text{ m}^3/\text{kg} \quad (c)$$

PROBLEM 3.8

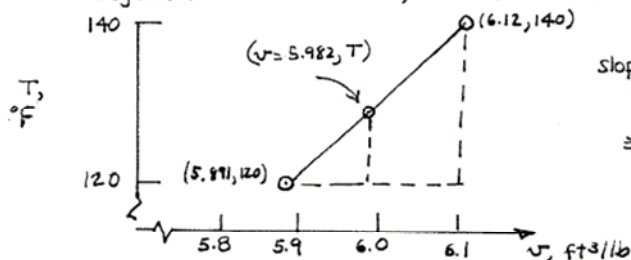
- (a) At a temperature of 120°C , the specified pressure of 54 lbf/in^2 falls between the table values of 50 and 60 lbf/in^2 . To determine the specific volume corresponding to 54 lbf/in^2 , we think of the slope of a straight line joining the adjacent table states, as follows:



similar triangles:

$$|\text{slope}| = \frac{v - 5.891}{60 - 54} = \frac{7.110 - 5.891}{60 - 50} \Rightarrow v = 5.891 + \frac{6}{10}(7.110 - 5.891) = 6.622 \frac{\text{ft}^3}{\text{lb}} \quad (a)$$

- (b) At a pressure of 60 lbf/in^2 , the given specific volume of $5.982\text{ ft}^3/\text{lb}$ falls between the table values of 120 and 140°F . To determine the temperature corresponding to the given specific volume, we think of the slope of a straight line joining the adjacent table states, as follows:



$$\text{slope} = \frac{T - 120}{5.982 - 5.891} = \frac{140 - 120}{6.12 - 5.891}$$

$$\Rightarrow T = 120 + \left[\frac{5.982 - 5.891}{6.12 - 5.891} \right] (20) = 127.9^\circ\text{F} \quad (b)$$

- (c) In this case, the specified pressure falls between the table values of 50 and 60 lbf/in^2 and the specified temperature falls between the table values of 100 and 120°F . Thus, double interpolation is required.

- At 110°F , the specific volume at each pressure is simply the average over the interval:

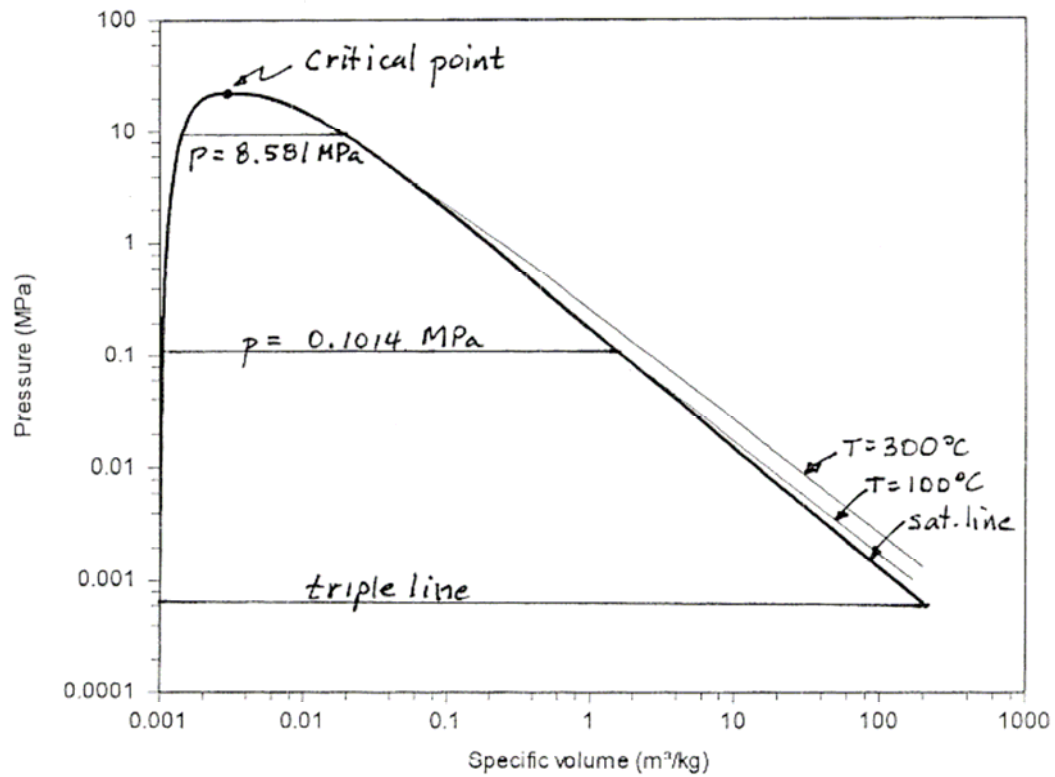
$$\text{at } 50 \frac{\text{lbf}}{\text{in}^2}, 110^\circ\text{F}; v = \frac{7.110 + 6.836}{2} = 6.973 \frac{\text{ft}^3}{\text{lb}}$$

$$\text{at } 60 \frac{\text{lbf}}{\text{in}^2}, 110^\circ\text{F}; v = \frac{5.891 + 5.659}{2} = 5.775 \frac{\text{ft}^3}{\text{lb}}$$

- Then, with the same approach as in (a)

$$\frac{v - 5.775}{60 - 58} = \frac{6.973 - 5.775}{60 - 50} \Rightarrow v = 5.775 + \frac{2}{10}(6.973 - 5.775) = 6.015 \frac{\text{ft}^3}{\text{lb}} \quad (c)$$

PROBLEM 3.9



PROBLEM 3.10 For water...

(a) $p = 300 \text{ kPa}$, $v = 0.5 \text{ m}^3/\text{kg}$. Find T in $^\circ\text{C}$.

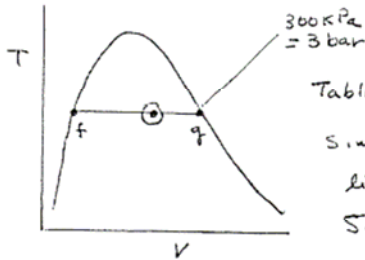


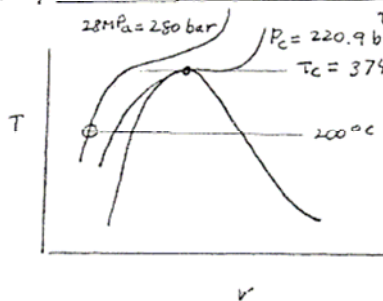
Table A-3, $v_f = 1.0732/10^3 \text{ m}^3/\text{kg}$, $v_g = 0.6058 \text{ m}^3/\text{kg}$.

Since $v_f < v < v_g$, the state is in the two-phase, liquid-vapor region, as shown on the T-v diagram.

So, $T = T_{\text{sat}}(3 \text{ bar}) = 133.6^\circ\text{C}$.

← T

(b) $p = 28 \text{ MPa}$, $T = 200^\circ\text{C}$. Find v in m^3/kg .



State is in liquid region. From Table A-5,

280 bar

300 bar

@ 200°C $v = \frac{1.1344}{10^3}$

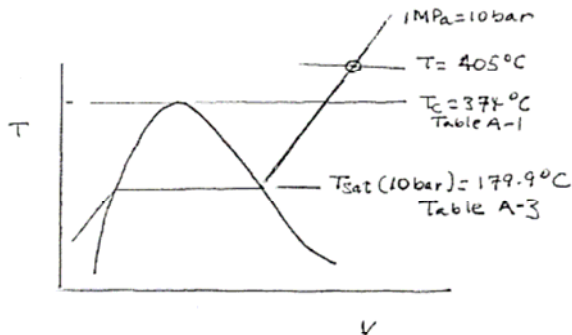
$v = \frac{1.1302}{10^3}$

So, at 280 bar

$v \approx \frac{1.1319}{10^3} \frac{\text{m}^3}{\text{kg}}$

← v

(c) $p = 1 \text{ MPa}$, $T = 405^\circ\text{C}$. Find v in m^3/kg .

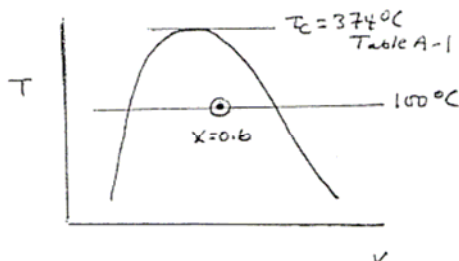


From Table A-4 at 10 bar, interpolation gives

$v = 0.309 \text{ m}^3/\text{kg}$

← v

(d) $T = 100^\circ\text{C}$, $x = 60\%$. Find v in m^3/kg .



Eq. 3.2:

$$v_x = (1-x)v_f + xv_g$$

with data from Table A-2 at 100°C,

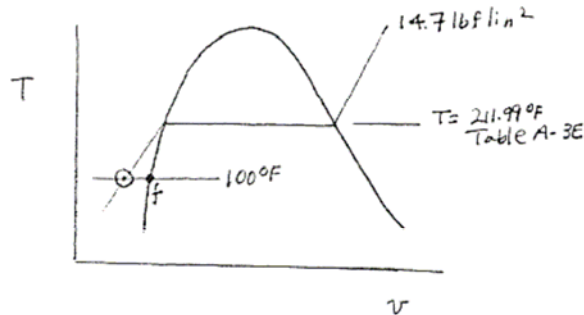
$$v_x = 0.4 \left[\frac{1.0435}{10^3} \right] + 0.6 \left[\frac{1.673}{10^3} \right]$$

$$= 1.0042 \text{ m}^3/\text{kg}$$

← v

PROBLEM 3.11

(a) Water. $p = 14.7 \text{ lbf/in}^2$, $T = 100^\circ\text{F}$. Find v in ft^3/lb .



State is in the liquid region. Since $p < 500 \text{ lbf/in}^2$, Table A-5E does not provide the needed data. So, use Eq. 3.11:

$$v \approx v_f(100^\circ\text{F}) = 0.01613 \frac{\text{ft}^3}{\text{lb}} \leftarrow v$$

(Table A-2E)

(b) Ammonia. $T = -30^\circ\text{C}$, $x = 50\%$. Find v in m^3/kg .

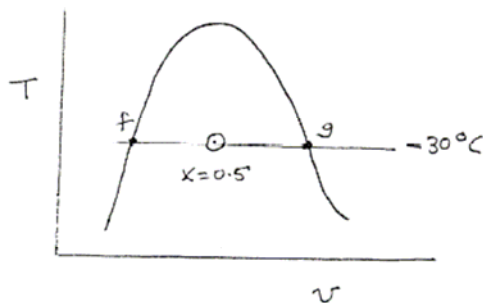


Table A-13 gives at -30°C

$$v_f = 1.4757 \frac{\text{m}^3}{\text{kg}}, v_g = 0.9634 \frac{\text{m}^3}{\text{kg}}$$

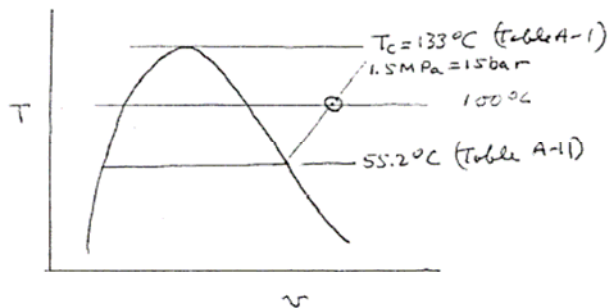
Then, with Eq. 3.2

$$v_x = (1-x)v_f + x v_g$$

$$= 0.5 \left[1.4757 + 0.9634 \right]$$

$$= 0.4824 \frac{\text{m}^3}{\text{kg}} \leftarrow v$$

(c) Refrigerant 134a. $p = 1.5 \text{ MPa}$, $T = 100^\circ\text{C}$. Find v in m^3/kg .



From Table A-12, interpolation gives

$$v = 0.0174 \frac{\text{m}^3}{\text{kg}} \leftarrow v$$

PROBLEM 3.12

(a) Water. $v = 0.5 \text{ m}^3/\text{kg}$, $p = 3 \text{ bar}$. Find T in $^\circ\text{C}$.

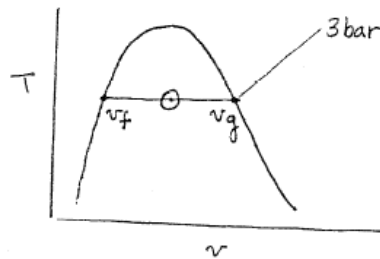


Table A-3, $v_f = 1.0732/10^3 \text{ m}^3/\text{kg}$, $v_g = 0.6058 \text{ m}^3/\text{kg}$.

Since $v_f < v < v_g$, the state is in the two-phase, liquid-vapor region — see T-v diagram.

Thus, $T = T_{\text{sat}}(3 \text{ bar}) = 133.6^\circ\text{C}$

← T

(b) Ammonia. $p = 11 \text{ lbf/in}^2$, $T = -20^\circ\text{F}$. Find v in ft^3/lb .

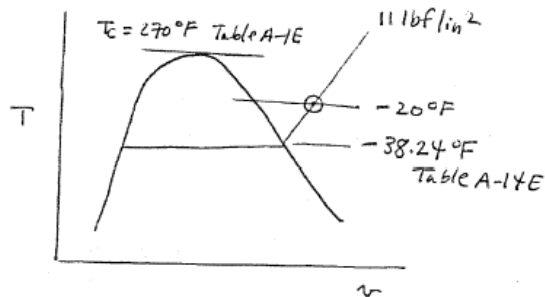


Table A-15E

10 lbf/in^2 -20°F $v = 27.21 \frac{\text{ft}^3}{\text{lb}}$ 12 lbf/in^2 $v = 22.621 \frac{\text{ft}^3}{\text{lb}}$

$\Rightarrow$ at 11 lbf/in^2

$v = 24.931 \frac{\text{ft}^3}{\text{lb}}$ ← v

(c) Propane. $p = 1 \text{ MPa}$, $T = 85^\circ\text{C}$. Find v in m^3/kg .

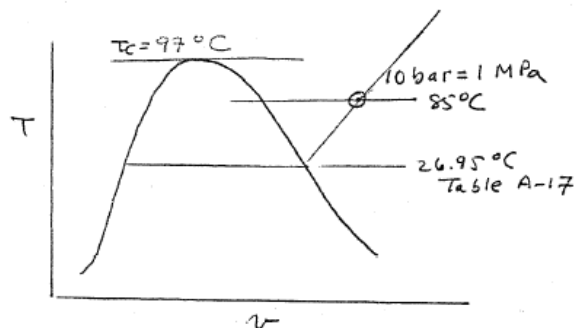
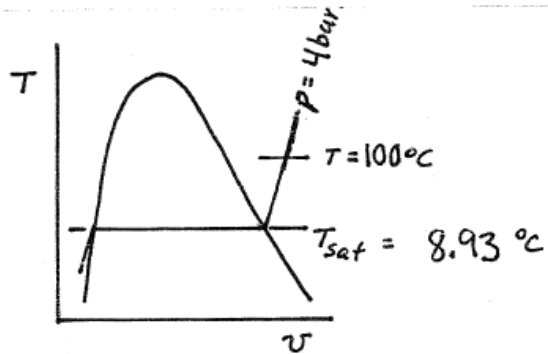


Table A-18 at 10 bar:

10 bar
 80°C $v = 0.05992 \text{ m}^3/\text{kg}$
 85°C — — —
 90°C $v = 0.06226 \text{ m}^3/\text{kg}$

$\Rightarrow v = 0.06109 \frac{\text{m}^3}{\text{kg}}$ ← v

PROBLEM 3.13



Refrigerant 134a:

$$p = 4\text{ bar} , T = 100\text{ }^{\circ}\text{C}$$

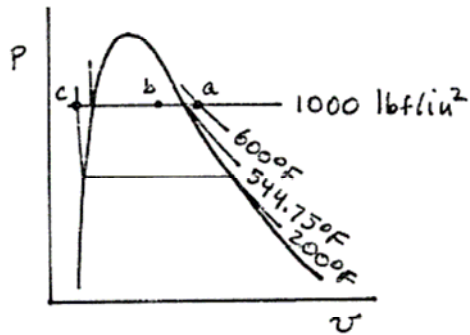
Table A-12: superheated vapor

$$v = 0.07327\text{ m}^3/\text{kg}$$

$$m = \frac{V}{v} = \frac{(0.1\text{ m}^3)}{(0.07327\text{ m}^3/\text{kg})}$$

$$= 1.36\text{ kg} \leftarrow m$$

PROBLEM 3.14



H_2O
 $m = 2 \text{ lb}$
 $V = ?$
 $p = 1000 \text{ lbf/in}^2$

H_2O :

(a) $T = 600^\circ F \Rightarrow$ superheated vapor

Table A-4E: $v = 0.514 \text{ ft}^3/\text{lb}$

$$V = m v = (2 \text{ lb})(0.514 \text{ ft}^3/\text{lb}) = 1.028 \text{ ft}^3 \leftarrow V$$

(b) $x = 0.8 \Rightarrow$ two-phase liquid-vapor mixture

Table A-3E: $v_f = 0.02159 \text{ ft}^3/\text{lb}$

$$v_g = 0.446 \text{ ft}^3/\text{lb}$$

$$v = v_f + x(v_g - v_f) = 0.3612 \text{ ft}^3/\text{lb}$$

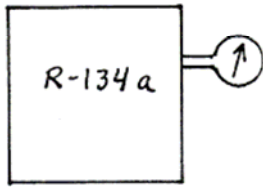
$$V = (2 \text{ lb})(0.3612 \text{ ft}^3/\text{lb}) = 0.7224 \text{ ft}^3 \leftarrow V$$

(c) $T = 200^\circ F \Rightarrow$ subcooled (compressed) liquid

Table A-5E: $v = 0.016580 \text{ ft}^3/\text{lb}$

$$V = (2)(0.016580) = 0.0332 \text{ ft}^3 \leftarrow V$$

PROBLEM 3.15



$$V = 2 \text{ ft}^3$$

$$m = 5 \text{ lb}$$

$$P_{\text{gage}} = 71.39 \text{ lbf/in}^2$$

$$P_{\text{atm}} = 14.4 \text{ lbf/in}^2$$

$$T = ?$$

First determine the absolute pressure p

$$P = P_{\text{gage}} + P_{\text{atm}}$$

$$= 71.39 + 14.4 = 85.79 \text{ lbf/in}^2$$

$$\text{Now, } v = V/m = (2 \text{ ft}^3)/(5 \text{ lb})$$

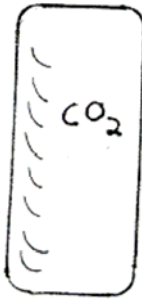
$$= 0.4 \text{ ft}^3/\text{lb}$$

From Table A-10E at $p \approx 85.79$

$v_f < v < v_g \Rightarrow$ two-phase liquid-vapor mixture

$$\text{Thus, } T = T_{\text{sat}} \approx 70^\circ\text{F} \leftarrow T$$

PROBLEM 3.16



$$\begin{aligned}T &= -40\text{ K} \\m &= 2\text{ kg} \\V &= 0.05\text{ m}^3 \\v_f &= 0.896 \times 10^{-3} \frac{\text{m}^3}{\text{kg}} \\v_g &= 3.824 \times 10^{-2} \frac{\text{m}^3}{\text{kg}}\end{aligned}$$

First, find the specific volume

$$v = \frac{V}{m} = \frac{0.05\text{ m}^3}{2\text{ kg}} = 0.025\text{ m}^3/\text{kg}$$

Now, the quality is

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.025 - 0.896 \times 10^{-3}}{3.824 \times 10^{-2} - 0.896 \times 10^{-3}}$$

$$= 0.645 \text{ (64.5 \%)} \quad \leftarrow x$$

PROBLEM 3.17

(a) H_2O ; $T = 20^\circ\text{C}$, $v = 20 \text{ m}^3/\text{kg}$

Table A-2: $v_f = 1.0018 \times 10^{-3} \text{ m}^3/\text{kg}$, $v_g = 57.791 \text{ m}^3/\text{kg}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{20 - 1.0018 \times 10^{-3}}{57.791 - 1.0018 \times 10^{-3}} = 0.346 \text{ (34.6\%)} \leftarrow x$$

(b) Propane; $p = 15.00 \text{ bar}$, $v = 0.02997 \text{ m}^3/\text{kg}$

Table A-17: $v_g = 0.02997 \text{ m}^3/\text{kg}$

$$\Rightarrow x = 1 \text{ (100\%)} \leftarrow x$$

(c) Refrigerant 134a; $T = 60^\circ\text{C}$, $v = 0.001 \text{ m}^3/\text{kg}$

Table A-10: $v_f = 0.9488 \times 10^{-3} \text{ m}^3/\text{kg}$, $v_g = 0.0114 \text{ m}^3/\text{kg}$

$$x = \frac{0.001 - 0.9488 \times 10^{-3}}{0.0114 - 0.9488 \times 10^{-3}} = 0.0049 \text{ (0.49\%)} \leftarrow x$$

(d) Ammonia; $p = 1 \text{ MPa} = 10 \text{ bar}$, $v = 0.1 \text{ m}^3/\text{kg}$

Table A-14: $v_f = 1.6584 \times 10^{-3} \text{ m}^3/\text{kg}$, $v_g = 0.1285 \text{ m}^3/\text{kg}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.1 - 1.6584 \times 10^{-3}}{0.1285 - 1.6584 \times 10^{-3}} = 0.775 \text{ (77.5\%)} \leftarrow x$$

PROBLEM 3.18

(a) H_2O ; $p = 14.696 \text{ lb}_f/\text{in}^2$, $v = 25 \text{ ft}^3/\text{lb}$

Table A-3E: $v_f = 0.01672 \text{ ft}^3/\text{lb}$, $v_g = 26.80 \text{ ft}^3/\text{lb}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{25 - 0.01672}{26.80 - 0.01672} = 0.933 (93.3\%) \leftarrow x$$

(b) Propane; $T = -80^\circ\text{F}$, $v = 0.02653 \text{ ft}^3/\text{lb}$

Table A-16E: $v = v_f = 0.02653 \text{ ft}^3/\text{lb}$

saturated liquid $\Rightarrow x = 0 (0\%) \leftarrow x$

(c) Refrigerant 134a; $p = 50 \text{ lb}_f/\text{in}^2$, $v = 0.5 \text{ ft}^3/\text{lb}$

Table A-11E: $v_f = 0.01252 \text{ ft}^3/\text{lb}$, $v_g = 0.9422 \text{ ft}^3/\text{lb}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.5 - 0.01252}{0.9422 - 0.01252} = 0.524 (52.4\%) \leftarrow x$$

(d) Ammonia; $T = -40^\circ\text{F}$, $v = 20 \text{ ft}^3/\text{lb}$

Table A-13E: $v_f = 0.02322 \text{ ft}^3/\text{lb}$, $v_g = 24.8672 \text{ ft}^3/\text{lb}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{20 - 0.02322}{24.8672 - 0.02322} = 0.804 (80.4\%) \leftarrow x$$

PROBLEM 3.19

$$v_1 = v_2 = 1.0 \text{ ft}^3/\text{lb}$$

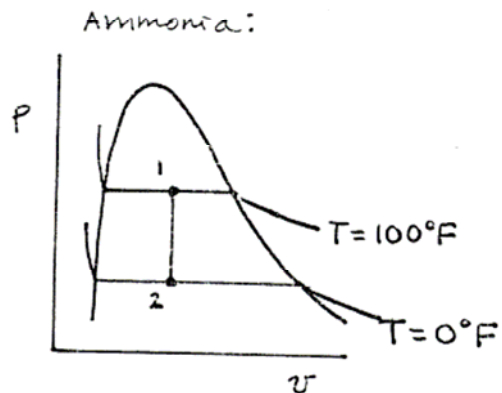
Table A-13E: $v_{f2} = 0.02419 \text{ ft}^3/\text{lb}$, $v_{g2} = 9.1100 \text{ ft}^3/\text{lb}$

$$v_2 = 1.0 \text{ ft}^3/\text{lb} \Rightarrow$$

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}}$$

$$= \frac{1.0 - 0.02419}{9.1100 - 0.02419}$$

$$= 0.107 \text{ (10.7\%)}$$



← x_2

PROBLEM 3.20



sat. vapor, $m_g = 4.2 \text{ kg}$

$P = 150 \text{ bar}$

$V = 0.2 \text{ m}^3$

sat. liquid, $m_f = 3.8 \text{ kg}$

For the two-phase liquid-vapor mixture, the specific volume is

$$v = \frac{V}{m_f + m_g}$$

$$= \frac{0.2}{(3.8 + 4.2)}$$

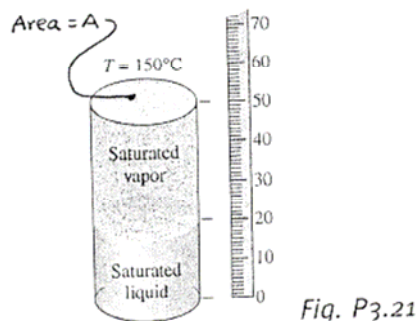
$$= 0.025 \text{ m}^3/\text{kg} \leftarrow v$$

PROBLEM 3.21

KNOWN: A closed, rigid container holds different volumes of saturated liquid water and saturated vapor water.

FIND: Determine the quality of the mixture.

SCHEMATIC & GIVEN DATA:



ANALYSIS: $x = \frac{m_{\text{vap}}}{m_{\text{vap}} + m_{\text{liq}}}$, $m = V/v$. Thus, $m_{\text{vap}} = \frac{V_{\text{vap}}}{v_g}$, $m_{\text{liq}} = \frac{V_{\text{liq}}}{v_f}$

$$x = \frac{V_{\text{vap}}/v_g}{(V_{\text{vap}}/v_g) + (V_{\text{liq}}/v_f)}$$

$V_{\text{vap}} = 30A$ and $V_{\text{liq}} = 20A$, where area A is in the same units as the vertical measure shown. Then

$$x = \frac{(30A/v_g)}{(30A/v_g) + (20A/v_f)} = \frac{1}{1 + \frac{20}{30} \left(\frac{v_g}{v_f} \right)}$$

Since ratios appear in the last expression, the quantities can be in any consistent units.

① Using v_f and v_g from Table A-2 at 150°C ,

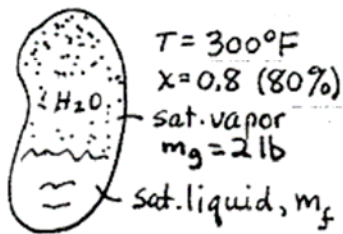
$$v_f = 1.0905 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$v_g = 0.3928 \text{ m}^3/\text{kg}$$

$$x = \frac{1}{1 + \frac{20}{30} \left(\frac{0.3928}{1.0905 \times 10^{-3}} \right)} = 0.0041 \quad (0.41\%) \quad \leftarrow x$$

1. Using v_f and v_g at 302°F (150°C) from Table A-2E: $v_f = 0.017468 \text{ ft}^3/\text{lb}$,
 $v_g = 6.292 \text{ ft}^3/\text{lb}$ gives the same value for x , as can be verified.

PROBLEM 3.22



The total mass is $m = m_f + m_g$.

With the definition of quality: $x = m_g/m$

$$m_f = \left(\frac{1-x}{x}\right) m_g = \left(\frac{1-0.8}{0.8}\right)(2 \text{ lb}) = 0.5 \text{ lb} \leftarrow m_f$$

The total mass m is

$$m = m_f + m_g = 0.5 + 2 = 2.5 \text{ lb}$$

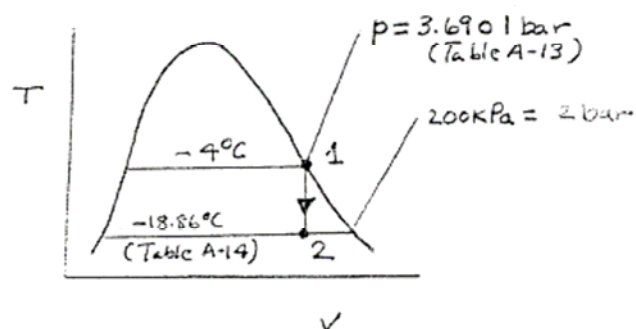
Using data from Table A-2E at 300°F :

$$v = v_f + x(v_g - v_f) = 0.01745 + (0.8)(6.472 - 0.01745) \\ = 5.1811 \text{ ft}^3/\text{lb}$$

$$V = m v = (2.5 \text{ lb})(5.1811 \text{ ft}^3/\text{lb}) \\ = 12.95 \text{ ft}^3 \leftarrow V$$

PROBLEM 3.23

Ammonia:



As shown by the T-v diagram, state 2 is in the two-phase liquid-vapor region at 2 bar. Table A-14 gives $T_2 = -18.86^\circ\text{C}$ ← T_2

Then, with $v_1 = 0.334 \text{ m}^3/\text{kg}$ from Table A-13 and $v_2 = v_1$,

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.334 - (1.5071/10^3)}{0.5946 - (1.5071/10^3)} = 0.561 \text{ (56.1\%)} \leftarrow x_2$$

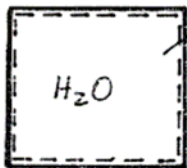
where v_f and v_g are from Table A-14 at 2 bar.

PROBLEM 3.24

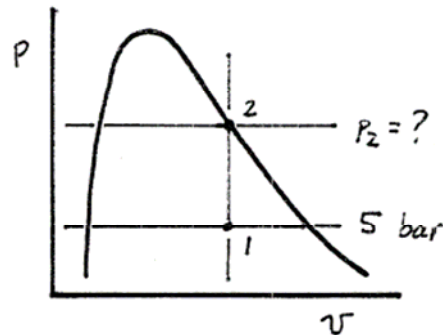
KNOWN: A two-phase liquid-vapor mixture of water, initially at a given pressure and quality, is heated in a closed rigid tank until only saturated water vapor remains. The volume is known.

FIND: Determine the mass of the water in the tank and the final pressure.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} V &= 0.2 \text{ m}^3 \\ P_1 &= 5 \text{ bar} \\ x_1 &= 0.5 \\ x_2 &= 1.0 \end{aligned}$$



ENGR. MODEL: (1) The water is a closed system. (2) The volume and mass are constant.

ANALYSIS: Using data from Table A-3, the initial specific volume is

$$\begin{aligned} v_1 &= v_f + x_1 (v_g - v_f) \\ &= 1.0926 \times 10^{-3} + (0.5)(0.3749 - 1.0926 \times 10^{-3}) \\ &= 0.1880 \text{ m}^3/\text{kg} \end{aligned}$$

Thus, the mass is

$$m = \frac{V}{v_1} = \frac{(0.2 \text{ m}^3)}{(0.1880 \text{ m}^3/\text{kg})} = 1.064 \text{ kg} \leftarrow m$$

By assumptions (1) and (2), $v_2 = v_1$. Thus

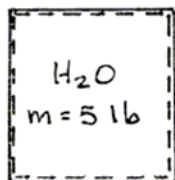
$$P_2 \approx P_{\text{sat}} @ v_2 = 10.5 \text{ bar} \leftarrow P_2$$

PROBLEM 3.25

KNOWN: A specified amount of water is heated in a closed, rigid tank from a known initial state to a specified final temperature.

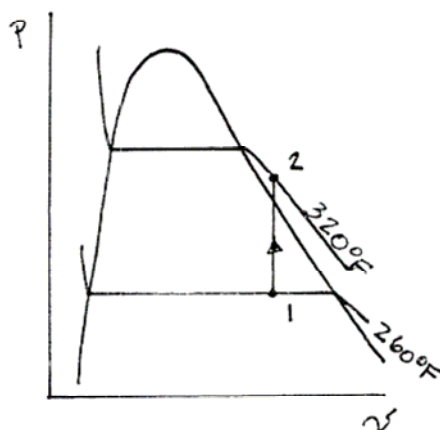
FIND: Determine the mass of vapor initially present and the final pressure.

SCHEMATIC & GIVEN DATA:



State 1: $T_1 = 260^\circ\text{F}$, $x_1 = 0.6$

State 2: $T_2 = 320^\circ\text{F}$



ENGR. MODEL: (1) The water is the closed system. (2) The volume is constant.

ANALYSIS: The initial mass of vapor in the tank is found using the known total mass and the quality x_1 at the initial state

$$m_{g1} = x_1 m = (0.6)(5 \text{ lb}) = 3 \text{ lb} \quad \xrightarrow{\quad \quad \quad} m_{g1}$$

Now, since the volume and total mass are constant during the process, the specific volumes at states 1 and 2 are equal: $v_2 = v_1$. From Table A-2E at 260°F , $v_{f1} = 0.01708 \text{ ft}^3/\text{lb}$ and $v_{g1} = 11.77 \text{ ft}^3/\text{lb}$. Thus

$$\begin{aligned} v_2 = v_1 &= v_{f1} + x_1(v_{g1} - v_{f1}) \\ &= 0.01708 + (0.6)(11.77 - 0.01708) = 7.0688 \text{ ft}^3/\text{lb} \end{aligned}$$

Now, from Table A-2E at 320°F , $v_g = 4.919 \text{ ft}^3/\text{lb}$. Since $v_2 > v_{g@320^\circ\text{F}}$, State 2 is in the superheated vapor region. Thus, interpolating in Table A-4 at 320°F , $7.0688 \text{ ft}^3/\text{lb}$

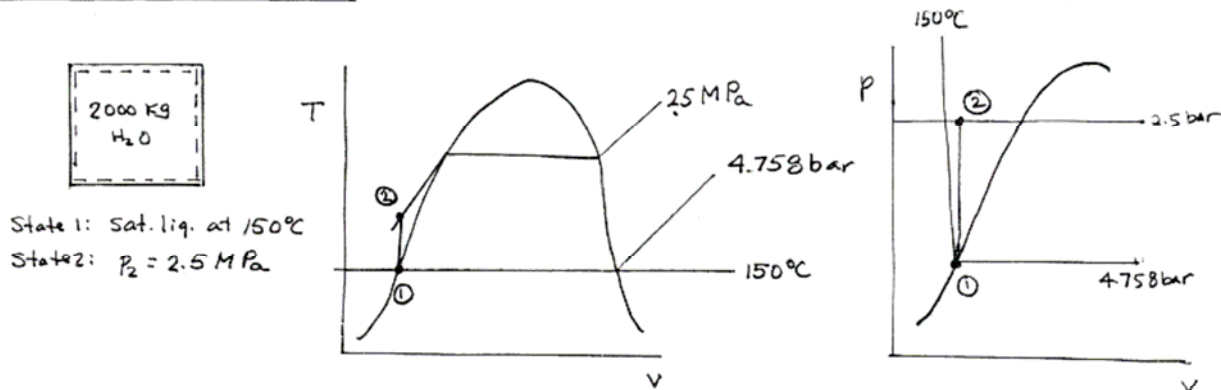
$$P_2 \approx 64.27 \text{ lbf/in.}^2 \quad \xrightarrow{\quad \quad \quad} P_2$$

PROBLEM 3.26

KNOWN: A specified amount of water is heated in a closed rigid tank from a known initial state to a final pressure.

FIND: Sketch the process on T-v and p-v diagrams. Determine the volume of the tank and the temperature at the final state.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The water is the closed system. 2. Volume remains constant.

ANALYSIS: Since volume and total mass remain constant during the process, the specific volume at states 1, 2 are equal:

$v_2 = v_1$. From Table A-2 at 150°C

$$v_1 = v_f(150^\circ\text{C}) = 1.0905 \times 10^{-3} \text{ m}^3/\text{kg}$$

As shown by the T-v and p-v diagrams, state 2 is in the liquid region. Interpolation in Table A-5 gives $T_2 = 150.15^\circ\text{C}$

The total volume is

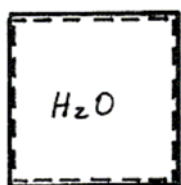
$$V = m v = (2000 \text{ kg}) \left(\frac{1.0905}{10^3} \frac{\text{m}^3}{\text{kg}} \right) = 2.181 \text{ m}^3$$

PROBLEM 3.27

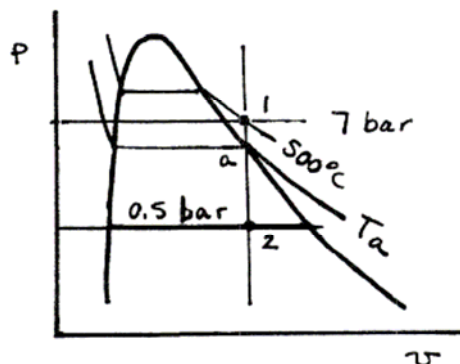
KNOWN: Steam is cooled in a closed rigid container from a known initial state to a known final pressure.

FIND: Determine the temperature at which condensation first occurs, the fraction of the total mass that condenses, and the volume occupied by saturated liquid at the final state.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} P_1 &= 7 \text{ bar} \\ T_1 &= 500^\circ\text{C} \\ V &= 1 \text{ m}^3 \end{aligned}$$



ENGR. MODEL ; (1) The steam is a closed system. (2) The volume is constant.

ANALYSIS: By assumptions (1) and (2), the specific volume is constant. Thus, using data from Table A-4

$$v_1 = v_a = v_2 = 0.5070 \text{ m}^3/\text{kg}$$

Then from Table A-2

$$T_a = T_{\text{sat}} @ v_a = 140^\circ\text{C} \leftarrow T_a$$

Next, the fraction of the total mass that condenses is

$$\text{fraction condensed} = \frac{m_{f2}}{m} = 1 - x_2$$

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}} = \frac{0.5070 - 1.0300 \times 10^{-3}}{3.420 - 1.0300 \times 10^{-3}} = 0.1480$$

Thus

$$\text{fraction condensed} = 1 - 0.1480 = 0.8520 \leftarrow$$

$$\begin{aligned} \text{The total mass of the system is } m &= \frac{V}{v_1} = \frac{(1 \text{ m}^3)}{(0.5070 \text{ m}^3/\text{kg})} \\ &= 1.972 \text{ kg} \end{aligned}$$

Then, the volume of the liquid at state 2 is

$$V_{f2} = m v_{f2} = (1.972 \text{ kg})(1.0300 \times 10^{-3} \text{ m}^3/\text{kg})$$

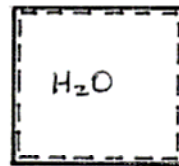
$$V_{f2} = 0.00203 \text{ m}^3 \leftarrow V_{f2}$$

PROBLEM 3.28

KNOWN: Water vapor is heated in a closed, rigid tank from saturated vapor at a known temperature to a known final temperature.

FIND: Determine the initial and final pressures and sketch the process on T - v and p - v diagrams.

ENGR. MODEL: (1) The water vapor is a closed system. (2) The volume is constant.



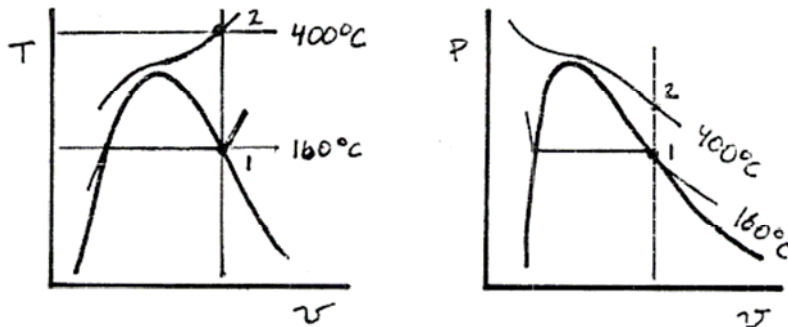
State 1: $T_1 = 160^\circ\text{C}$
sat. vapor

State 2: $T_2 = 400^\circ\text{C}$

ANALYSIS: Using data from Table A-2, $p_1 = 6.178 \text{ bar}$ ← P_1

With assumptions (1) and (2), $v_2 = v_1 = 0.3071 \text{ m}^3/\text{kg}$. Interpolating in Table A-3, $p_2 \approx 9.99 \text{ bar}$ ← P_2

Thus



PROBLEM 3.29

KNOWN: Ammonia undergoes an isothermal process from a known initial state to the saturated vapor state.

FIND: Determine the initial and final pressures, and sketch the process on $T-v$ and $p-v$ diagrams.

ENGR. MODEL: (1) The ammonia is a closed system. (2) The process is isothermal.

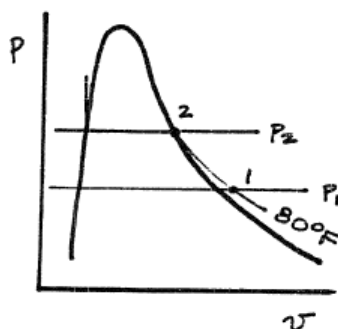
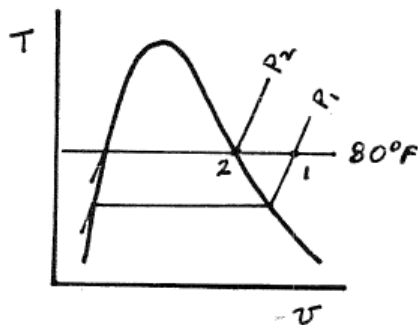


State 1: $T_1 = 80^\circ\text{F}$
 $v_1 = 10 \text{ ft}^3/\text{lb}$

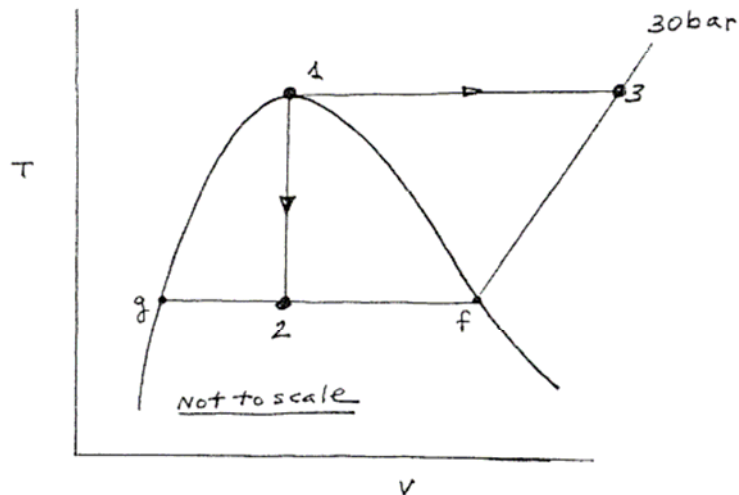
State 2: $T_2 = T_1$
 Sat. vapor

ANALYSIS: Interpolating in Table A-15E, $P_1 \approx 33.86 \text{ lbf/in}^2$ $\xrightarrow{\quad\quad\quad} P_1$
 Using data from Table A-13E, $P_2 = 153.13 \text{ lbf/in}^2$ $\xrightarrow{\quad\quad\quad} P_2$

Thus



PROBLEM 3.30



- 1-2: Constant-volume cooling
- 1-3: constant-temperature expansion

State 1. Critical point. From the last table entry of Table A-2, the critical point of water is described by

$$T_c = 374.14^\circ\text{C}$$

$$P_c = 220.9 \text{ bar}$$

$$v_c = 3.155 \times 10^{-3} \text{ m}^3/\text{kg}$$

State 2. $P_2 = 30 \text{ bar}$, $v_2 = v_c = 3.155 \times 10^{-3} \text{ m}^3/\text{kg}$. With data from Table A-3 at 30 bar

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{(3.155 - 1.2165) \times 10^{-3}}{(66.68 - 1.2165) \times 10^{-3}} = 0.0296 \text{ (2.96\%)} \leftarrow x_2$$

State 3 $P_3 = 30 \text{ bar}$, $T_3 = 374.14^\circ\text{C}$. Interpolation in Table A-4 gives

$$v_3 = 0.0948 \text{ m}^3/\text{kg} \leftarrow v_3$$

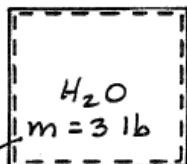
PROBLEM 3.31

KNOWN: A specified amount of water is heated in a closed rigid tank. The volume is known, and data are given for the initial and final states.

FIND: Determine the pressures at the initial and final states.

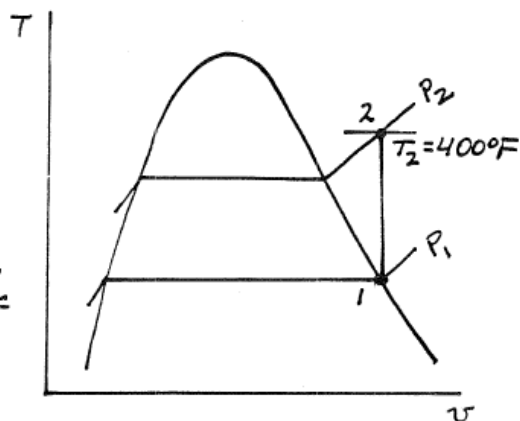
SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The water is the closed system. (2) The volume is constant.



$V = 13.3 \text{ ft}^3$ state 1: sat. vap.
state 2: $T_2 = 400^\circ\text{F}$

ANALYSIS: The T - v diagram is shown to the right. To determine the pressures, first fix state 2. Since the volume and mass are constant, $v_1 = v_2$. Thus



$$v_2 = V/m = (13.3 \text{ ft}^3)/(3 \text{ lb}) = 4.4333 \text{ ft}^3/\text{lb}$$

Now, interpolating in Table A-4E at 400°F , $4.4333 \text{ ft}^3/\text{lb}$; $P_2 \approx 111.7 \text{ lbf/in}^2$

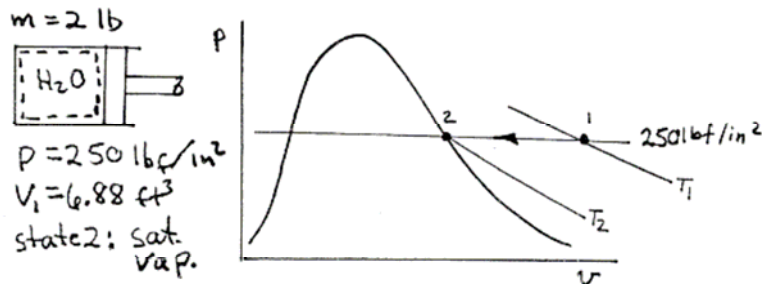
Then, from Table A-3E, at $v_g = 4.4333 \text{ ft}^3/\text{lb}$; $P_1 = P_{\text{sat}} \approx 100 \text{ lbf/in}^2$

PROBLEM 3.32

KNOWN: Water is compressed at a constant pressure between two specified states.

FIND: Determine the temperatures at the initial and final states, and the work for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The given quantity of water is the closed system. 2. Volume change is the only work mode. 3. The process occurs at constant pressure.

ANALYSIS: The initial specific volume is

$$v_1 = \frac{V_1}{m} = \frac{6.88 \text{ ft}^3}{2 \text{ lb}} = 3.44 \text{ ft}^3/\text{lb}$$

From Table A-4E, with $v_1 = 3.44 \text{ ft}^3/\text{lb}$ and $p_1 = 250 \text{ lbf/in}^2$

$$T_1 = 1000^\circ \text{F} \quad \leftarrow T_1$$

The final specific volume is v_g at 250 lbf/in^2 .

From Table A-2E

$$v_2 = v_g @ 250 \text{ psi} = 1.845 \text{ ft}^3/\text{lb}$$

$$\text{Thus } T_2 = 401.04^\circ \text{F} \quad \leftarrow T_2$$

The final volume is

$$V_2 = m v_2 = (2 \text{ lb})(1.845 \frac{\text{ft}^3}{\text{lb}}) = 3.69 \text{ ft}^3$$

To evaluate the work

$$W = \int_1^2 p dV = p(V_2 - V_1)$$

$$= (250 \frac{\text{lbf}}{\text{in}^2}) \left(\frac{144 \text{ in}^2}{1 \text{ ft}^2} \right) [3.69 - 6.88] \text{ ft}^3 \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= -147.6 \text{ Btu} \quad \leftarrow W$$

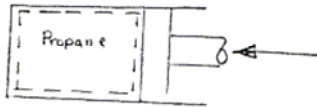
Water is compressed, thus the work is negative.

PROBLEM 3.33

KNOWN: Propane in a piston-cylinder assembly undergoes a constant-pressure process for which data are provided.

FIND: Determine specified data at the final state.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} m &= 7 \text{ lb} \\ P_1 &= 200 \text{ lbf/in}^2 \\ T_1 &= 200^\circ\text{F} \\ W &= -88.84 \text{ Btu} \end{aligned}$$

ENGR. MODEL:

1. The given mass of propane is the closed system.
2. Volume change is the only work mode.
3. The process occurs at constant pressure.

ANALYSIS: Two properties are required to fix the final state. One of these is the pressure: $P_2 = 200 \text{ lbf/in}^2$. The other is the specific volume found from the given value for work:

Since volume change is the only work mode and pressure remains constant,

$$W = \int p dV = m p (v_2 - v_1)$$

Solving for v_2

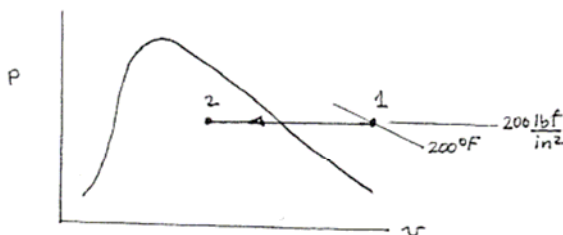
$$v_2 = \frac{W}{m p} + v_1$$

From Table A-18E, $v_1 = 0.7025 \text{ ft}^3/\text{lb}$. Thus

$$\begin{aligned} v_2 &= \frac{(-88.84 \text{ Btu})}{(7 \text{ lb})(200 \frac{\text{lbf}}{\text{in}^2})} \left| \frac{798 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}} \right| \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| + 0.7025 \frac{\text{ft}^3}{\text{lb}} \\ &= -0.3428 \frac{\text{ft}^3}{\text{lb}} + 0.7025 \frac{\text{ft}^3}{\text{lb}} = 0.3597 \frac{\text{ft}^3}{\text{lb}} \end{aligned}$$

From Table A-17E at 200 lbf/in^2 , we see $v_f < v_2 < v_g$. Thus state 2 is a two-phase liquid-vapor state. Calculating quality

$$\begin{aligned} x_2 &= \frac{v_2 - v_f}{v_g - v_f} = \frac{0.3597 - 0.03432}{0.5261 - 0.03432} \\ &= 0.662 \quad (66.2\%) \end{aligned}$$

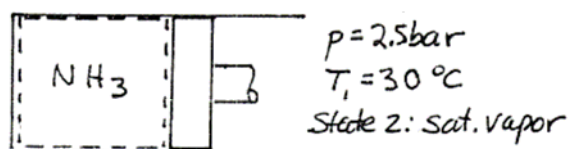


PROBLEM 3.34

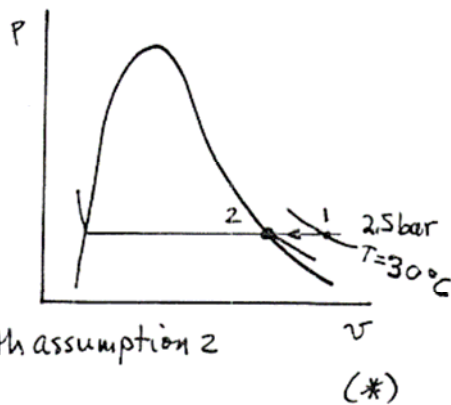
KNOWN: Ammonia undergoes a constant pressure process. The initial and final states are fixed.

FIND: Determine the work for the process, per unit mass of ammonia.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The refrigerant is the closed system. (2) The pressure remains constant.



ANALYSIS: The work is determined using Eq. 2.17 with assumption 2

$$W = \int p dV = m \int_{v_1}^{v_2} p dv = mp(v_2 - v_1) \quad (*)$$

From Table A-15 at 2.5 bar, 30 °C; $v_1 = 0.57745 \text{ m}^3/\text{kg}$. From Table A-14, $v_2 = v_g @ 2.5 \text{ bar} = 0.4821 \text{ m}^3/\text{kg}$. Thus, from (*)

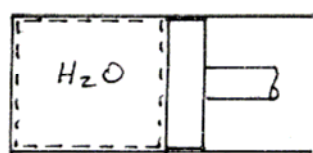
$$\begin{aligned} \frac{W}{m} &= p(v_2 - v_1) = (2.5 \text{ bar})(0.4821 - 0.57745) \frac{\text{m}^3}{\text{kg}} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \\ &= -23.84 \text{ kJ/kg} \end{aligned}$$

PROBLEM 3.35

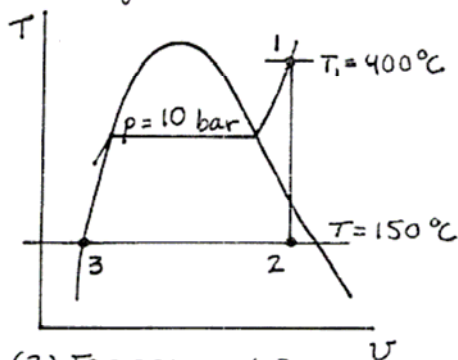
KNOWN: Water vapor undergoes a constant volume cooling process followed by isothermal condensation to saturated liquid.

FIND: Evaluate the work per unit mass.

SCHEMATIC & GIVEN DATA:



$P_1 = 10 \text{ bar}$
 $T_1 = 400^\circ\text{C}$
 $T_2 = 150^\circ\text{C}$
 State 3: Sat. liquid at 150°C



ENGR. MODEL: (1) The water is the closed system. (2) For process 1-2, the volume is constant. (3) For process 2-3, the temperature is constant. (4) Volume change is the only work mode.

ANALYSIS: By assumptions (2) and (4), only process 2-3 involves work. To evaluate the work, begin with Eq. 2.17

$$W = \int_{v_1}^{v_2} p dv \Rightarrow \frac{W}{m} = \int_{v_1}^{v_2} p dv$$

Next, we fix each state and obtain relevant data. Since the mass and volume are constant to process 1-2, $v_2 = v_1$. From Table A-4; $v_1 = 0.3066 \text{ m}^3/\text{kg}$.

With $v_2 = v_1 = 0.3066$, we see in Table A-2 at 150°C that $v_f < v_2 < v_g$. Thus, state 2 is in the two-phase, liquid-vapor region. Since the temperature is constant for process 2-3, so is the pressure. That is

$$P_2 = P_3 = P_{\text{sat}@150^\circ\text{C}} = 4.758 \text{ bar}$$

Finally, from Table A-2, $v_3 = v_f @ 150^\circ\text{C} = 1.0905 \times 10^{-3} \text{ m}^3/\text{kg}$. Thus

$$\begin{aligned} \frac{W}{m} &= \int_{v_1}^{v_2} p dv = p_2 (v_3 - v_2) \\ &= (4.758 \text{ bar}) (1.0905 \times 10^{-3} - 0.3066) \frac{\text{m}^3}{\text{kg}} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \end{aligned}$$

$$\textcircled{1} \quad = -145.4 \text{ kJ/kg} \leftarrow \frac{W}{m}$$

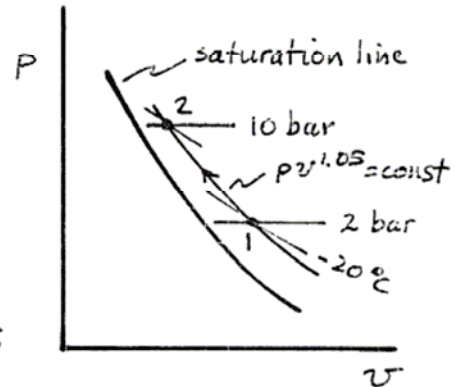
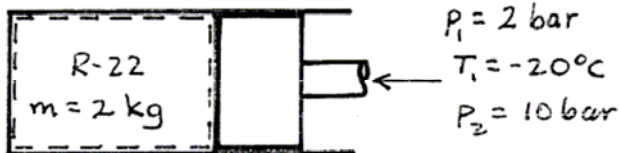
1. The negative sign denotes energy transfer to the system by work during process 2-3.

PROBLEM 3.36

KNOWN: Refrigerant 22 undergoes a process with a known pressure-volume relation from a given initial state to a given final pressure.

FIND: Calculate the work for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL : (1) The refrigerant is a closed system. (2) The process is polytropic, with $n = 1.05$

ANALYSIS: The work is obtained by using Eq. 2.17. First, determine the specific volumes. From Table A-9, $v_1 = 0.11520 \text{ m}^3/\text{kg}$, and

$$v_2 = \left(\frac{P_1}{P_2} \right)^{\frac{1}{1.05}} v_1 = \left(\frac{2}{10} \right)^{\frac{1}{1.05}} (0.11520) = 0.024875 \text{ m}^3/\text{kg}$$

The work is

$$W = \int_{v_1}^{v_2} p dv = m \int_{v_1}^{v_2} p dv = m \int_{v_1}^{v_2} \frac{\text{const.}}{v^{1.05}} dv$$

$$= \frac{m(p_2 v_2 - p_1 v_1)}{(1-1.05)}$$

$$= \frac{(2 \text{ kg}) [(10 \text{ bar})(0.024875 \text{ m}^3/\text{kg}) - (2)(0.11520)]}{(1 - 1.05)} \left[\frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right] \left[\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right]$$

$$\textcircled{1} \quad = -73.4 \text{ kJ} \quad \leftarrow \quad W$$

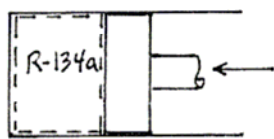
1. The negative value for work indicates that the process is a compression process

PROBLEM 3.37

KNOWN: Refrigerant 134a undergoes a polytropic process from a specified initial state to a given final temperature.

FIND: Determine the final pressure and the work per unit mass.

SCHEMATIC & GIVEN DATA:



$$P_1 = 200 \text{ kPa} = 2 \text{ bar}$$

$$T_1 = -10^\circ\text{C}$$

$$T_2 = 50^\circ\text{C}$$

$$p v^{1.05} = \text{constant.}$$

ENGR. MODEL: (1) The refrigerant is the closed system. (2) The process is polytropic, with $n = 1.05$.

ANALYSIS: State 1 is fixed by $T_1 = -10^\circ\text{C}$, $p_1 = 2 \text{ bar}$. From Table A-12; $v_1 = 0.09938 \text{ m}^3/\text{kg}$. Thus, with the polytropic process expression

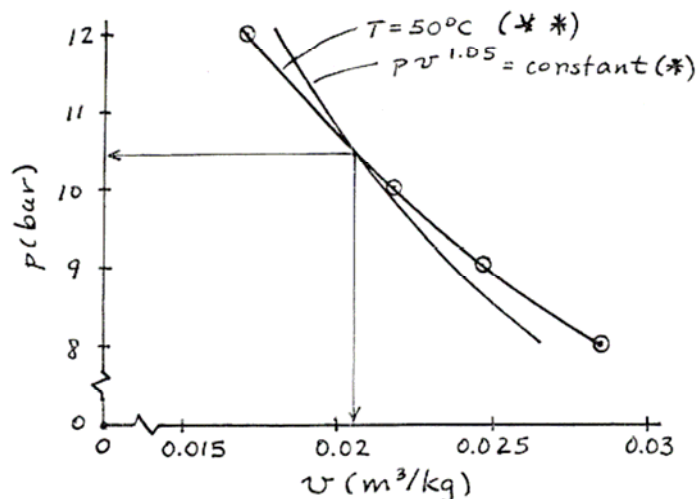
$$v_2 = (P_1/P_2)^{1/1.05} v_1 \quad (*)$$

In Table A-12 at $T_2 = 50^\circ\text{C}$, there is a unique v associated with each pressure, that is

$$v_{\text{table}} = v(P, T_2 = 50^\circ\text{C}) \quad (**)$$

The following table of data and plot are obtained using (*) and (**). Where the lines intersect, both expressions are satisfied simultaneously.

$P(\text{bar})$	(*)	(**)
8	0.02654	0.02846
9	0.02372	0.02472
10	0.02146	0.02171
11	0.01960	---
12	0.01804	0.01712



From the plot:

$$P_2 = 10.4 \text{ bar}$$

$$v_2 = 0.0205 \text{ m}^3/\text{kg}$$

Continued on next slide

Problem 3-37 continued

The expressions (*) and (**) can be solved much more easily using the following IT code:

```
p1 = 2 // bar
T1 = -10 // °C
T2 = 50 // °C

v1 = v_PT("R134A", p1, T1) // m³/kg
p2 * v2^n = p1 * v1^n
n = 1.05
v2 = v_PT("R134A", p2, T2)
```

The results from the computer solution are $p_2 = 10.38 \text{ bar}$, $v_2 = 0.0207 \text{ m}^3/\text{kg}$, which agree well with the graphical results.

Now, the work is determined using Eq. 2.17 and the procedure of Example 2.1 for the polytropic process to get

$$\frac{W}{m} = \int_{v_1}^{v_2} p dv = \left(\frac{p_2 v_2 - p_1 v_1}{1-n} \right)$$

Using the values from the computer solution

$$\frac{W}{m} = \frac{(10.38 \text{ bar})(0.0207 \text{ m}^3/\text{kg}) - (2)(0.09938)}{(1 - 1.05)} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

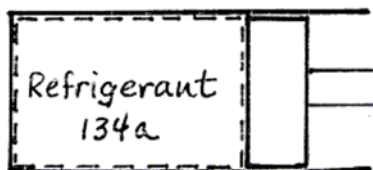
$$= -32.21 \text{ kJ} \leftarrow \text{w/m}$$

PROBLEM 3.38

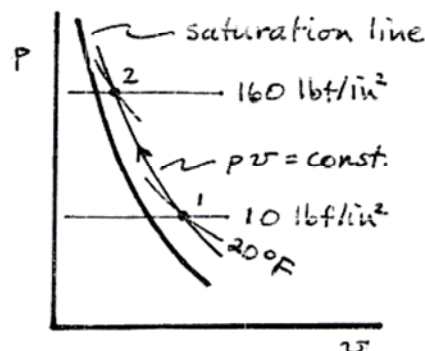
KNOWN: Refrigerant 134a is compressed in a piston-cylinder assembly from a known initial state to a known final pressure. The pressure-specific volume relationship for the process is given.

FIND: Determine the work.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} m &= 0.04 \text{ lb} \\ P_1 &= 10 \text{ lbf/in}^2 \\ T_1 &= 20^\circ\text{F} \\ P_2 &= 160 \text{ lbf/in}^2 \\ p v &= \text{const.} \end{aligned}$$



ENGR. MODEL: (1) The refrigerant is a closed system. (2) The process is polytropic with $p v = \text{const.}$

ANALYSIS: The work is obtained by using Eq. 2.17. First, determine the specific volumes. From Table A-12E, $v_1 = 4.9297 \text{ ft}^3/\text{lb}$, and

$$v_2 = \left(\frac{P_1}{P_2}\right) v_1 = \left(\frac{10}{160}\right)(4.9297) = 0.3081 \text{ ft}^3/\text{lb}$$

The work is

$$W = \int_{v_1}^{v_2} p dv = m \int_{v_1}^{v_2} p dv = m \int_{v_1}^{v_2} \frac{\text{const.}}{v} dv$$

$$= m(p_1 v_1) \ln\left(\frac{v_2}{v_1}\right) = m(p_1 v_1) \ln\left(\frac{P_1}{P_2}\right)$$

$$= (0.04 \text{ lb})(10 \text{ lbf/in}^2)(4.9297 \frac{\text{ft}^3}{\text{lb}}) \ln\left(\frac{10}{160}\right) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

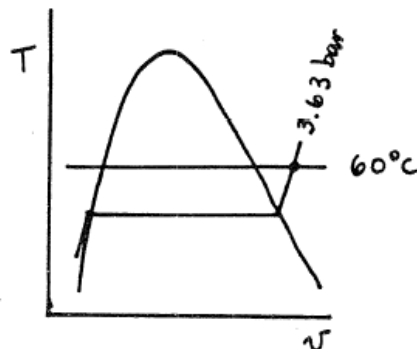
①

$$= -1.012 \text{ Btu} \quad \underline{\hspace{10em}} \quad W$$

1. The work is negative for the compression, as expected.

PROBLEM 3.39

- (a) Refrigerant 134a; $T = 60^\circ\text{C}$, $v = 0.072 \text{ m}^3/\text{kg}$
 from Table A-10 at 60°C ; $v_g = 0.0114 \text{ m}^3/\text{kg}$
 $v > v_g \Rightarrow$ superheated vapor
 Interpolating in Table A-12; $p = 3.63 \text{ bar}$
 $= 363 \text{ kPa}$
 $h = \underline{302.1 \text{ kJ/kg}}$



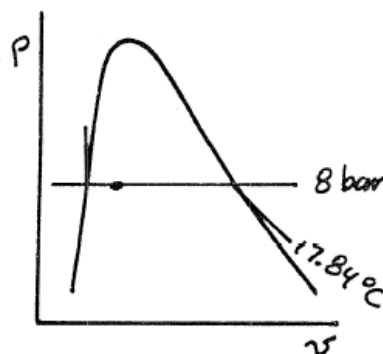
- (b) Ammonia; $p = 8 \text{ bar}$, $v = 0.005 \text{ m}^3/\text{kg}$
 from Table A-14 at 8 bar; $v_g = 0.1596 \text{ m}^3/\text{kg}$
 $v_f < v < v_g \Rightarrow$ two-phase, liquid-vapor mixture
 $T = T_{\text{sat}} = 17.84^\circ\text{C}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.005 - 1.6302 \times 10^{-3}}{0.1596 - 1.6302 \times 10^{-3}} = 0.02133$$

$$u = u_f + x(u_g - u_f)$$

$$= 262.64 + (0.02133)(1330.64 - 262.64)$$

$$= \underline{285.4 \text{ kJ/kg}}$$



- (c) Refrigerant 22; $T = -10^\circ\text{C}$, $u = 200 \text{ kJ/kg}$
 From Table A-7 at -10°C , $u_f < u < u_g$
 $\Rightarrow$ 2-phase liquid-vapor mixture
 $p = p_{\text{sat}} = 3.8485 \text{ bar}$

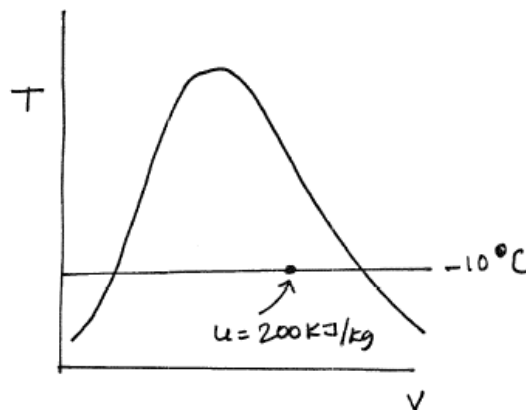
$$x = \frac{u - u_f}{u_g - u_f} = \frac{200 - 33.27}{223.02 - 33.27}$$

$$= 0.879$$

$$\therefore v = v_f + x(v_g - v_f)$$

$$= \frac{0.7606}{10^3} + 0.879(0.0652 - \frac{0.7606}{10^3})$$

$$= 0.0574 \text{ m}^3/\text{kg}$$



PROBLEM 3.40

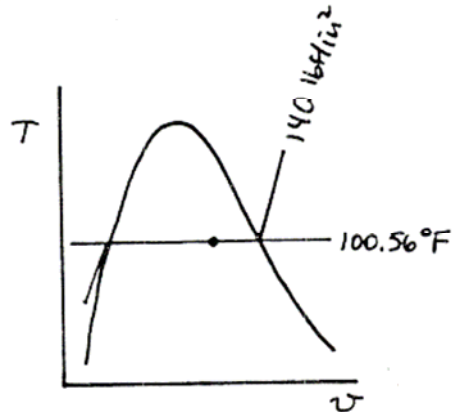
(a) Refrigerant 134a; $p = 140 \text{ lbf/in}^2$, $h = 100 \text{ Btu/lb}$

from Table A-11E; $h_f < h < h_g$
 $\Rightarrow$ two-phase, liquid-vapor mixture

$$x = \frac{h - h_f}{h_{fg}} = \frac{100 - 44.43}{70.52} = 0.788$$

$$\begin{aligned} v &= v_f + x(v_g - v_f) \\ &= 0.01386 + (0.788)(0.3358 - 0.01386) \\ &= \underline{0.2675 \text{ ft}^3/\text{lb}} \end{aligned}$$

$$T = T_{\text{sat}} = \underline{100.56^\circ\text{F}}$$

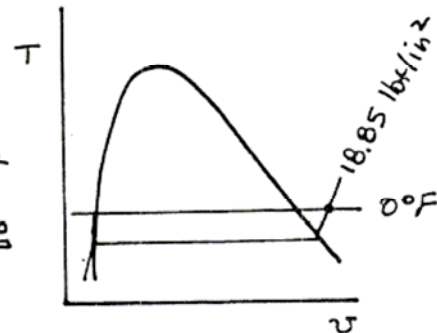


(b) Ammonia; $T = 0^\circ\text{F}$, $v = 15 \text{ ft}^3/\text{lb}$

from Table A-13E; $v_g = 9.1100 \text{ ft}^3/\text{lb}$

$v > v_g \Rightarrow$ superheated vapor

Interpolating in Table A-15E: $\underline{p = 18.85 \text{ lbf/in}^2}$
 $\underline{h = 615.2 \text{ Btu/lb}}$

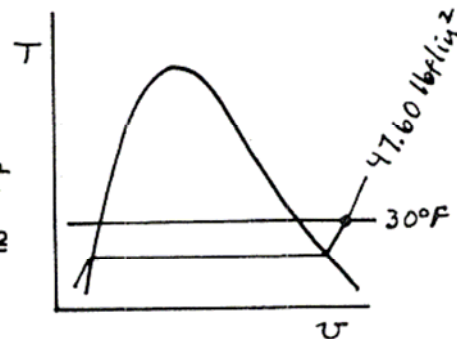


(c) Refrigerant 22; $T = 30^\circ\text{F}$, $v = 1.2 \text{ ft}^3/\text{lb}$

from Table A-7E, $v_g = 0.7804 \text{ ft}^3/\text{lb}$

$v > v_g \Rightarrow$ superheated vapor

Interpolating in Table A-9E, $\underline{p = 47.60 \text{ lbf/in}^2}$
 $\underline{h = 108.80 \text{ Btu/lb}}$



PROBLEM 3.41

- (a) See Solution to Problem 3.39 for table-lookup data. Use these to check IT results.
- (b) See solution to Problem 3.40 for table-lookup data. Use these to check IT results.

PROBLEM 3.42 Water is the substance

(a) $p = 3 \text{ bar}$, $T = 240^\circ\text{C}$

Table A-4

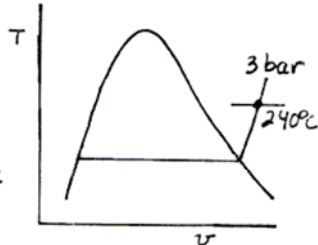
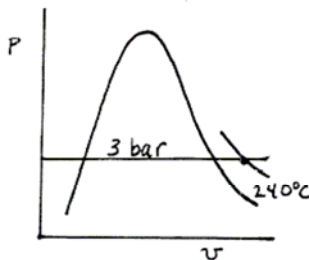
$v = 0.781 \text{ m}^3/\text{kg}$

$u = 2713.1 \text{ kJ/kg}$

IT Results

$v = 0.7805 \text{ m}^3/\text{kg}$

$u = 2713 \text{ kJ/kg}$



(b) $p = 3 \text{ bar}$, $v = 0.5 \text{ m}^3/\text{kg}$

Table A-3; $v_g < v < v_f$

$\Rightarrow T = 133.6^\circ\text{C}$

$x = \frac{v - v_f}{v_g - v_f}$

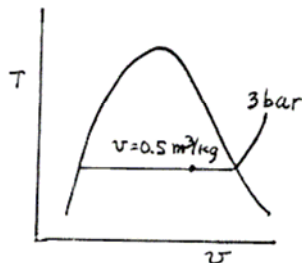
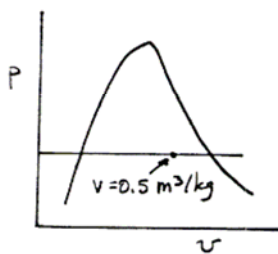
$= \frac{0.5 - 1.0732 \times 10^{-3}}{0.6058 - 1.0732 \times 10^{-3}}$

$= 0.825$

$\therefore u = u_f + x(u_g - u_f)$

$= 561.15 + (0.825)(2543.6 - 561.5)$

$= 2196.7 \text{ kJ/kg}$



IT Results

$x = 0.825$

2196 kJ/kg

(c) $T = 400^\circ\text{C}$, $p = 10 \text{ bar}$

Table A-4

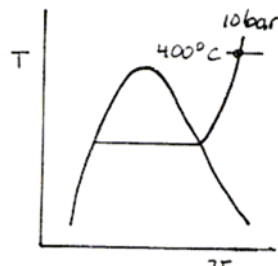
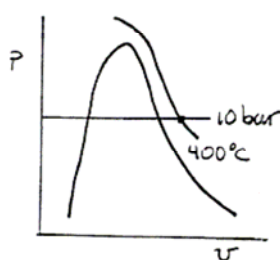
$v = 0.3066 \text{ m}^3/\text{kg}$

$h = 3263.9 \text{ kJ/kg}$

IT Results

$v = 0.3066 \text{ m}^3/\text{kg}$

$h = 3263 \text{ kJ/kg}$



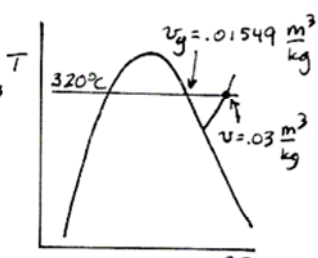
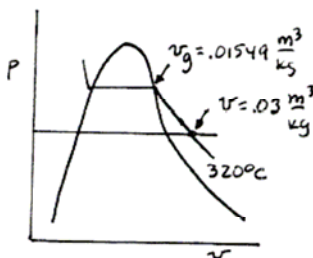
(d) $T = 320^\circ\text{C}$, $v = 0.03 \text{ m}^3/\text{kg}$

Table A-2; $v > v_g$ at 320°C

$\Rightarrow$ Table A-4; At 320°C the state falls between 60 and 80 bar. Interpolating,

$P = 74.67 \text{ bar} = 7.467 \text{ MPa}$

$h = 2678.0 \text{ kJ/kg}$



IT Results; $p = 7.356 \text{ MPa}$, $u = 2682 \text{ kJ/kg}$

(e) $p = 28 \text{ MPa} = 280 \text{ bar}$, $T = 520^\circ\text{C}$

Table A-2

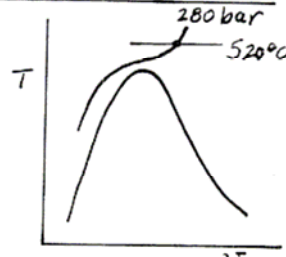
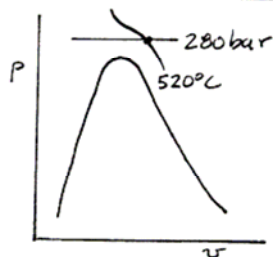
$v = 0.01020 \text{ m}^3/\text{kg}$

$h = 3192.3 \text{ kJ/kg}$

IT Results

$v = 0.0102 \text{ m}^3/\text{kg}$

$h = 3192 \text{ kJ/kg}$



Continued on next slide

Problem 3-42 continued

(f) $T = 100^\circ\text{C}$, $x = 0.60$

Table A-2

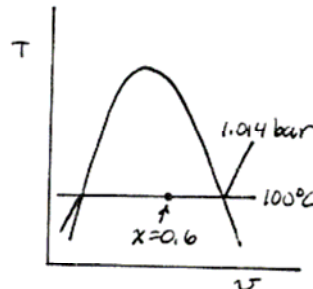
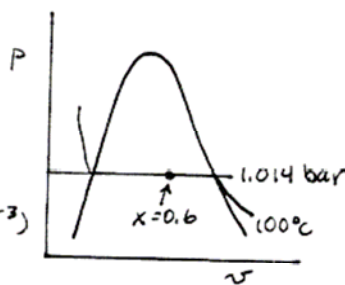
$p = 1.014 \text{ bar}$

$v = v_f + x(v_g - v_f)$

$= 1.0435 \times 10^{-3} + (0.60)(1.673 - 1.0435 \times 10^{-3})$

$= 1.0042 \text{ m}^3/\text{kg}$

IT Results; $p = 1.013 \text{ bar}$, $v = 1.004 \text{ m}^3/\text{kg}$



(g) $T = 10^\circ\text{C}$, $v = 100 \text{ m}^3/\text{kg}$

Table A-2; $v_f < v < v_g$ at 10°C .

Thus, $p = 0.01228 \text{ bar} = 1.228 \text{ kPa}$

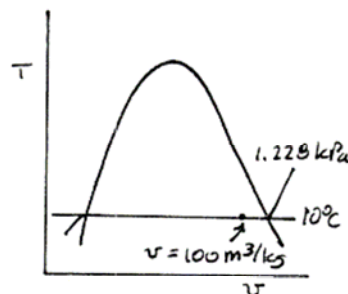
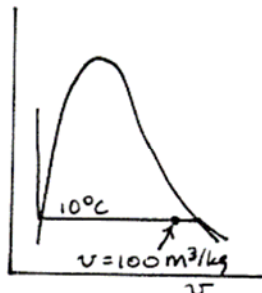
$x = \frac{v - v_f}{v_g - v_f} = \frac{100 - 1.0004 \times 10^{-3}}{106.379 - 1.0004 \times 10^{-3}}$

$= 0.94$

$h = h_f + x h_{fg}$

$= 42.01 + (0.94)(2477.7) = 2371 \text{ kJ/kg}$

IT Results; $x = 0.94$, $h = 2371 \text{ kJ/kg}$



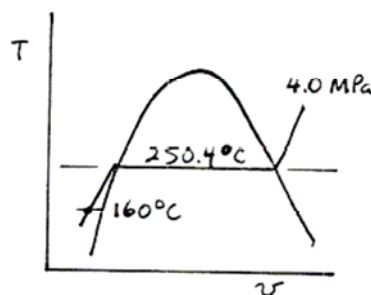
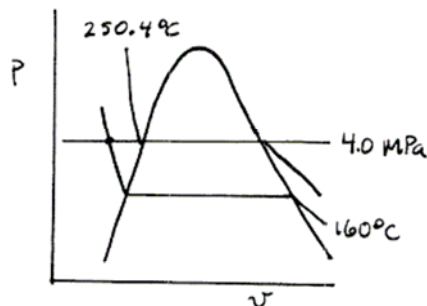
(h) $p = 4 \text{ MPa} = 40 \text{ bar}$, $T = 160^\circ\text{C}$

Table A-3 at 40 bar ; $T_{\text{sat}} = 250.4^\circ\text{C}$

$\Rightarrow$ liquid state ($T < T_{\text{sat}}$)

Table A-5; double interpolation

	$p = 2.5 \text{ MPa}$	$p = 5.0 \text{ MPa}$
$T = 140^\circ\text{C}$	$v = 1.0784 \times 10^{-3}$ $u = 587.82$	1.0768×10^{-3} 586.76
$T = 180^\circ\text{C}$	$v = 1.1261 \times 10^{-3}$ $u = 761.16$	1.1240×10^{-3} 759.63
	at $p = 4.0 \text{ MPa}$	
140°C	$v = 1.0774 \times 10^{-3}$ $u = 587.18$	
180°C	$v = 1.1248 \times 10^{-3}$ $u = 760.24$	



Then, at 4.0 MPa , 160°C

$v = 1.1011 \times 10^{-3} \text{ m}^3/\text{kg}$

$u = 673.71 \text{ kJ/kg}$

IT Results

$v = 1.102 \times 10^{-3} \text{ m}^3/\text{kg}$

$u = 670.7 \text{ kJ/kg}$

PROBLEM 3.43 Water is the substance.

(a) $p = 20 \text{ lbf/in}^2$, $T = 400^\circ\text{F}$

Table A-4E

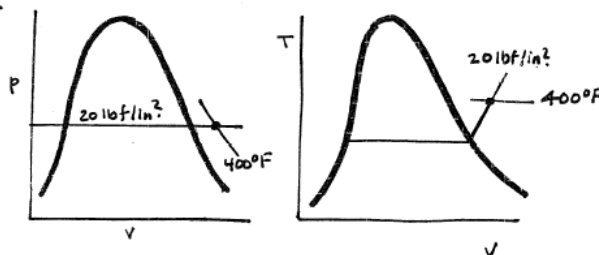
$v = 25.43 \text{ ft}^3/\text{lb}$

$u = 1145.18 \text{ Btu/lb}$

IT Results

$v = 25.43 \text{ ft}^3/\text{lb}$

$u = 1145 \text{ Btu/lb}$



(b) $p = 20 \text{ lbf/in}^2$, $v = 16 \text{ ft}^3/\text{lb}$

Table A-3E $v_f < v < v_g$

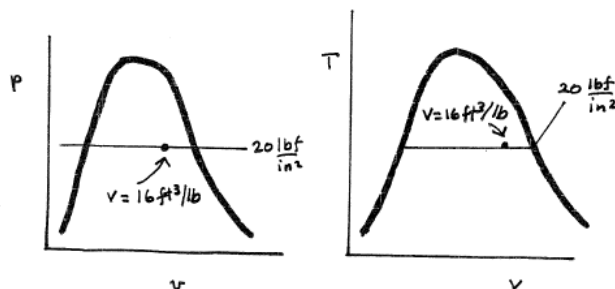
$\Rightarrow T = 227.96^\circ\text{F}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{16 - 0.01683}{20.09 - 0.01683} = 0.796$$

$\therefore u = u_f + x(u_g - u_f)$

$= 196.19 + 0.796(1082 - 196.19)$

$= 901.29 \text{ Btu/lb}$



IT Results: $T = 228^\circ\text{F}$, $x = 0.7962$, $u = 901.3 \text{ Btu/lb}$

(c) $T = 900^\circ\text{F}$, $p = 170 \text{ lbf/in}^2$

Table A-4E, interpolate at 900°F

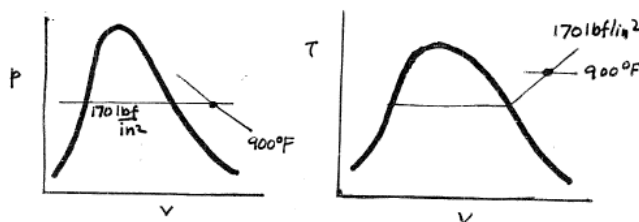
$v = 4.734 \text{ ft}^3/\text{lb}$

$h = 1478.05 \text{ Btu/lb}$

IT Results

$v = 4.718 \text{ ft}^3/\text{lb}$

$h = 1478 \text{ Btu/lb}$



(d) $T = 600^\circ\text{F}$, $v = 0.6 \text{ ft}^3/\text{lb}$

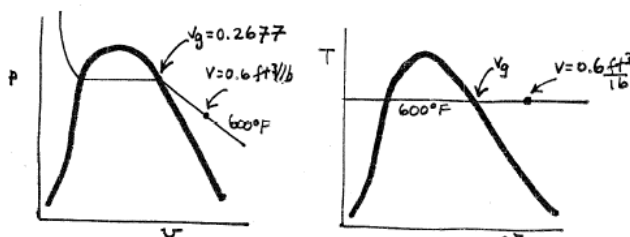
Table A-2E, $v > v_g$ at 600°F .

$\Rightarrow$ Table A-4E. At 600°F the state falls between 800 and 900 lbf/in².

Interpolating,

$p = 885.6 \text{ lbf/in}^2$

$u = 1163.34 \text{ Btu/lb}$



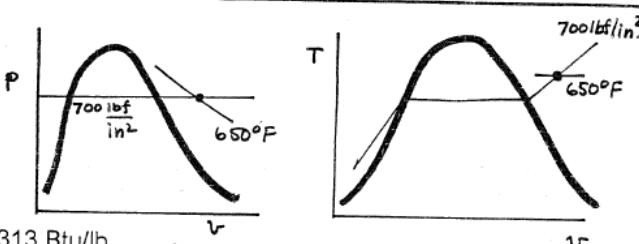
IT Results: $p = 884.3 \text{ lbf/in}^2$, $u = 1163 \text{ Btu/lb}$

(e) $p = 700 \text{ lbf/in}^2$, $T = 650^\circ\text{F}$

Table A-4E, interpolation at 700 lbf/in^2

$v = 0.85 \text{ ft}^3/\text{lb}$

$h = 1312.3 \text{ Btu/lb}$



IT Results: $v = 0.852 \text{ ft}^3/\text{lb}$, $h = 1313 \text{ Btu/lb}$

Continued on next slide

Problem 3-43 continued

(f) $T = 400^\circ\text{F}$, $x = 90\%$

Table A-2E

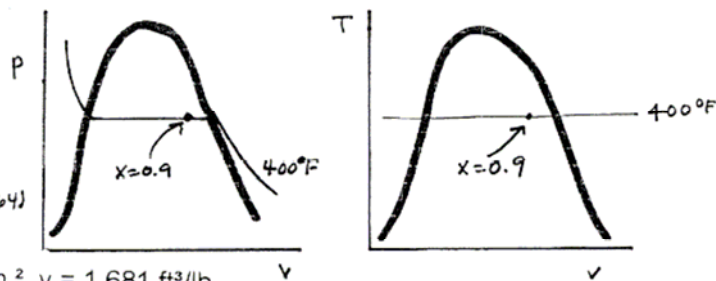
$p = 247.1 \text{ lbf/in}^2$

$$v = v_f + x v_{fg}$$

$$= 0.01864 + .9(1.866 - 0.01864)$$

$$= 1.6813 \text{ ft}^3/\text{lb}$$

IT Results: $p = 247.1 \text{ lbf/in}^2$, $v = 1.681 \text{ ft}^3/\text{lb}$



(g) $T = 40^\circ\text{F}$, $v = 1950 \frac{\text{ft}^3}{\text{lb}}$

Table A-2E $v_f < v < v_g$ at 40°F . Thus, $p = 0.1217 \text{ lbf/in}^2$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{1950 - 0.016}{2441 - 0.016}$$

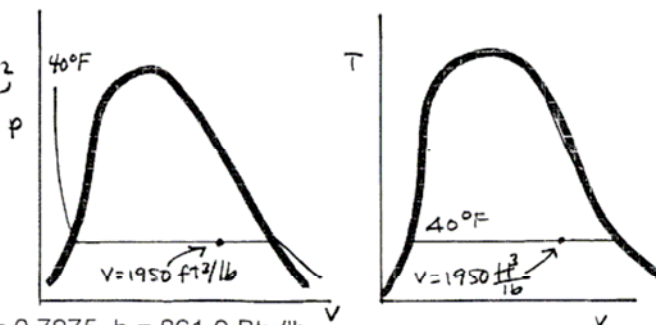
$$= 0.798$$

$$h = h_f + x h_{fg}$$

$$= 802 + 0.798(1070.9)$$

$$= 862.6 \text{ Btu/lb}$$

IT Results: $p = 0.1217 \text{ lbf/in}^2$, $x = 0.7975$, $h = 861.9 \text{ Btu/lb}$



(h) $p = 600 \text{ lbf/in}^2$, $T = 320^\circ\text{F}$

Table A-3E at 600 lbf/in^2

$T_{\text{sat}} = 486.33^\circ\text{F}$

$\Rightarrow$ liquid state

Table A-5E - double interpolation

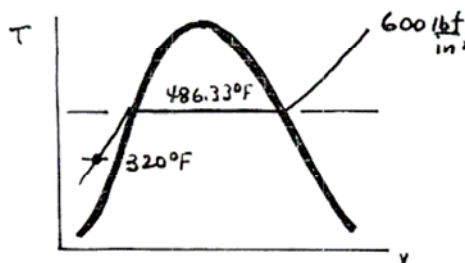
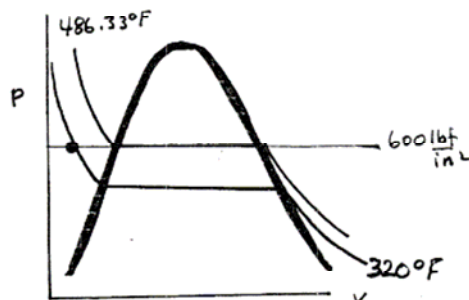
	$p = 500 \text{ lbf/in}^2$	$p = 1000$
$T = 300^\circ\text{F}$	$v = 0.017416$ $u = 268.92$	0.017379 268.24
$T = 400$	$v = 0.018608$ $u = 373.68$	0.018550 372.55

	at 600 lbf/in^2
300°F	$v = 0.017409$ $u = 268.78$
400°F	$v = 0.018596$ $u = 373.45$

Then, at 600 lbf/in^2 , 320°F

$$v = 0.017646 \text{ ft}^3/\text{lb}$$

$$u = 289.71 \text{ Btu/lb}$$



IT Results

$v = 0.01766 \text{ ft}^3/\text{lb}$
 $u = 288.3 \text{ Btu/lb}$

PROBLEM 3.44

(a) H_2O at $p = 15 \text{ MPa} = 150 \text{ bar}$, $T = 100^\circ\text{C}$

from Table A-5; $v = 1.0361 \times 10^{-3} \text{ m}^3/\text{kg}$, $h = 430.28 \text{ kJ/kg}$

(b) Using saturated liquid data from Table A-2;

$$v \approx v_{f@100^\circ\text{C}} = 1.0435 \times 10^{-3} \text{ m}^3/\text{kg}$$

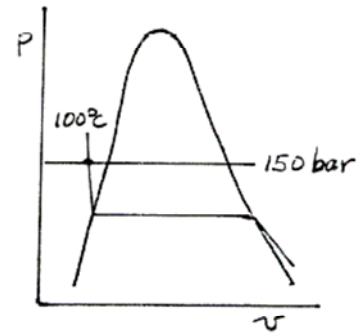
Now, using Eq. 3.13 to estimate specific enthalpy

$$h \approx h_{f@100^\circ\text{C}} + v_{f@100^\circ\text{C}} (p - p_{\text{sat}@100^\circ\text{C}})$$

$$= 419.04 \frac{\text{kJ}}{\text{kg}} + 1.0435 \times 10^{-3} \frac{\text{m}^3}{\text{kg}} (150 - 1.014) \text{ bar}$$

$$\left(\frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right) \left(\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right)$$

$$\textcircled{1} \quad = 419.04 + 15.55 = 434.59 \text{ kJ/kg}$$



1. Note that $h_{f@100^\circ\text{C}}$ is a plausible estimate for h in some applications (within about 2.6% of the value from Table A-5).

PROBLEM 3.45

H₂O at $T = 400^\circ\text{F}$, $p = 3000 \text{ lbf/in}^2$

from Table A-5E: $v = 0.018334 \text{ ft}^3/\text{lb}$

$$h = 378.50 \text{ Btu/lb}$$

The specific volume and enthalpy values can also be estimated using saturation data from Table A-2E and Eqs. 3.11 and 3.14, respectively.

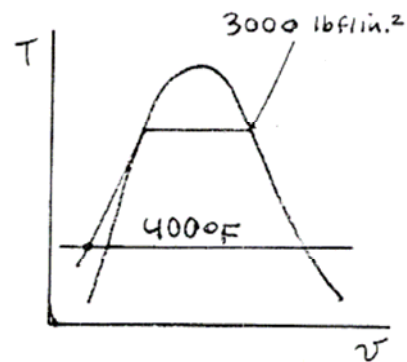
$$v \approx v_f(400^\circ\text{F}) = \underline{0.01864 \text{ ft}^3/\text{lb}}$$

and

$$h \approx h_f(400^\circ\text{F}) + v_f(400^\circ\text{F}) [p - p_{\text{sat}}@400^\circ\text{F}]$$

$$= 375.1 \frac{\text{Btu}}{\text{lb}} + 0.01864 \frac{\text{ft}^3}{\text{lb}} \left[3000 \frac{\text{lbf}}{\text{in}^2} - 247.1 \frac{\text{lbf}}{\text{in}^2} \right] \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= \underline{384.6 \text{ Btu/lb}}$$

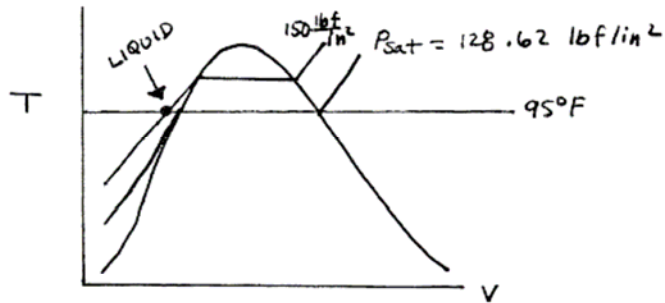


PROBLEM 3.45

Refrigerant 134a at 95°F , 150 lbf/in^2

Table A-10E at 95°F ; $P_{\text{sat}} = 128.62 \text{ lbf/in}^2$

$\Rightarrow$ Liquid state when $p = 150 \text{ lbf/in}^2$



Using Eq. 3.11, $v \approx v_f(95^\circ\text{F}) = \underline{0.01371 \text{ ft}^3/\text{lb}}$ $\leftarrow v$

Using Eq. 3.14, $h \approx h_f(95^\circ\text{F}) = \underline{42.47 \text{ Btu/lb}}$ $\leftarrow h$

PROBLEM 3.47

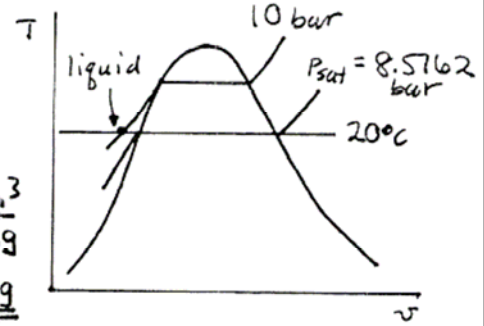
Ammonia at 20°C , $1\text{ MPa} = 10\text{ bar}$

Table A-13 at 20°C ; $P_{\text{sat}} = 8.5762\text{ bar}$

$\Rightarrow$ liquid state when $p = 10\text{ bar}$

Using Eq. 3.11; $v \approx v_f(20^\circ\text{C}) = \underline{\underline{1.6386 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}}}$

Using Eq. 3.14; $h \approx h_f(20^\circ\text{C}) = \underline{\underline{274.26\text{ kJ/kg}}}$



PROBLEM 3.48

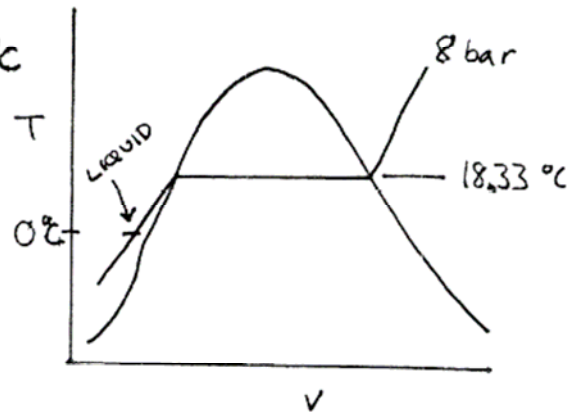
Propane at 800 kPa, 0°C

Table A-17 at 8 bar, $T_{\text{sat}} = 18.33^{\circ}\text{C}$

$\Rightarrow$ liquid state

Using Eq. 3.11, $v \approx v_f(0^{\circ}\text{C})$
 $= \underline{\underline{1.890 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}}}$

Using Eq. 3.14, $h \approx h_f(0^{\circ}\text{C})$
 $= \underline{\underline{95.1 \text{ kJ/kg}}}$



PROBLEM 3.49

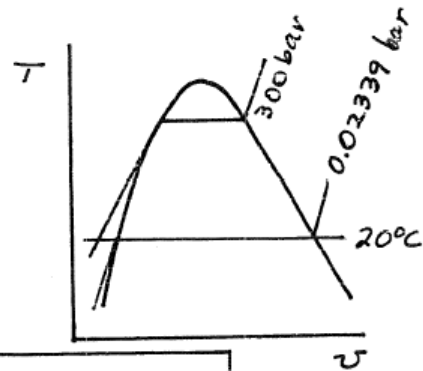
H_2O at $T=20^\circ C$, $P_{sat} \leq p \leq 300 \text{ bar}$

from Table A-2: $P_{sat} = 0.02339 \text{ bar}$

$$v_f = 1.0018 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$u_f = 83.95 \text{ kJ/kg}$$

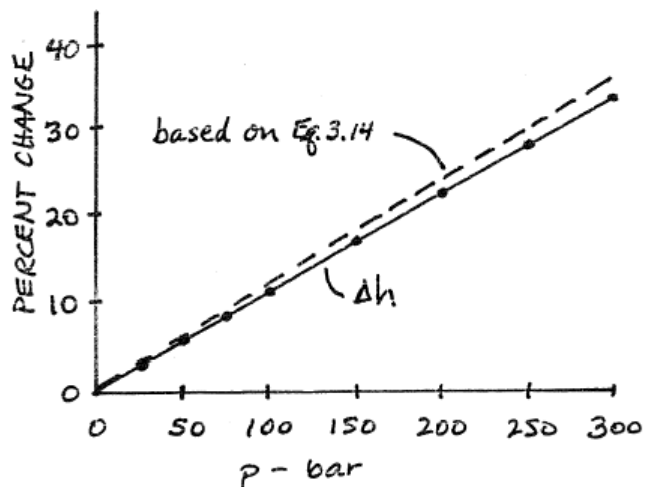
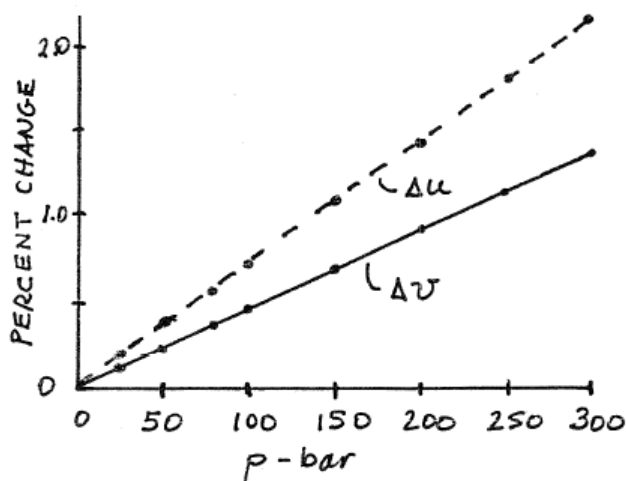
$$h_f = 83.96 \text{ kJ/kg}$$



from Table A-5:

P - bar ($T=20^\circ C$).								
	25	50	75	100	150	200	250	300
$v \times 10^3$	1.0006	.9995	.9984	.9972	.9950	.9928	.9907	.9886
u	83.80	83.65	83.50	83.36	83.06	82.77	82.47	82.17
h	86.30	88.65	90.99	93.33	97.99	102.62	107.24	111.84

	25	50	75	100	150	200	250	300
$\Delta v (\%)$	0.12	0.23	0.34	0.46	0.68	0.90	1.11	1.32
$\Delta u (\%)$	0.18	0.36	0.54	0.70	1.06	1.41	1.76	2.12
$\Delta h (\%)$	2.79	5.59	8.37	11.12	16.69	22.22	27.73	33.21



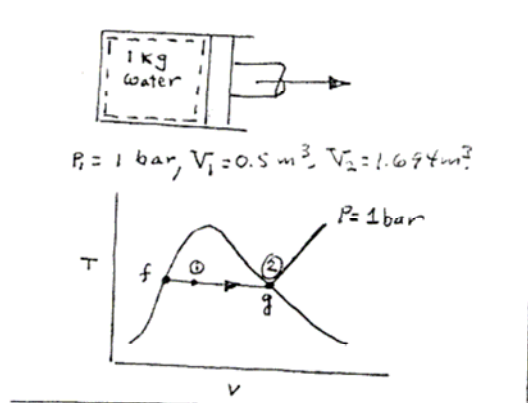
COMMENTS: The changes in v and u are small over the pressure range in this problem. Thus, v_f and u_f are reasonable approximations, respectively. However, h deviates by over 30% from h_f , and the approximation $h \approx h_f$ is not reasonable. Note that Eq. 3.14 provides a plausible estimate of h for compressed liquids when tabular data are not available.

PROBLEM 3.50

KNOWN: Data are provided for a process of water contained in a piston-cylinder assembly.

FIND: Show the process on a T-v diagram and evaluate W and Q for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The water in the piston-cylinder assembly is the closed system.
2. The water expands at constant temperature.
3. Volume change is the only work mode.
4. Kinetic and potential energy effects play no role.

ANALYSIS: The initial state is fixed by $P_1 = 1 \text{ bar}$ and $v_1 = \frac{V_1}{m} = \frac{0.5 \text{ m}^3}{1 \text{ kg}} = 0.5 \frac{\text{m}^3}{\text{kg}}$.

From Table A-3 at 1 bar we see $v_f < v_1 < v_g$. So state 1 is in the two-phase liquid-vapor region. Moreover, $v_2 = \frac{V_2}{m} = 1.694 \text{ m}^3/\text{kg}$ corresponds to the saturated vapor state at 1 bar. Expansion at fixed temperature corresponds, therefore, to an expansion at fixed pressure, as shown on the T-v diagram.

Since volume change is the only work mode, $W_{12} = \int_1^2 p dV = p(V_2 - V_1)$. Thus

$$W_{12} = \left(10^5 \frac{\text{N}}{\text{m}^2} \right) (1.694 - 0.5) \text{ m}^3 \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 119.4 \text{ kJ} \quad \leftarrow W_{12}$$

An energy balance reduces as follows: $\Delta U + \Delta KE + \Delta PE = Q - W$, giving

$$Q_{12} = \Delta U + W_{12} = m(u_2 - u_1) + W_{12}$$

where $u_1 = (1-x)u_f + xu_g$. To find x, note that $v_1 = (1-x)v_f + xv_g$, giving

$$x = \frac{v_1 - v_f}{v_g - v_f} = \frac{0.5 - (1.0432/10^3)}{1.694 - (1.0432/10^3)} = 0.295$$

where v_f and v_g are from Table A-3 at 1 bar. Then,

$$u_1 = (0.705)(417.36) + (0.295)(2506.1) = 1033.5 \text{ kJ/kg}$$

Finally, with $u_2 = u_g$ at 1 bar,

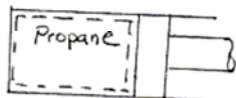
$$\begin{aligned} Q_{12} &= 1 \text{ kg} (2506.1 - 1033.5) + 119.4 \\ &= 1592 \text{ kJ} \end{aligned} \quad \leftarrow Q_{12}$$

PROBLEM 3.51

KNOWN: Data are provided for a process of propane contained in a piston-cylinder assembly.

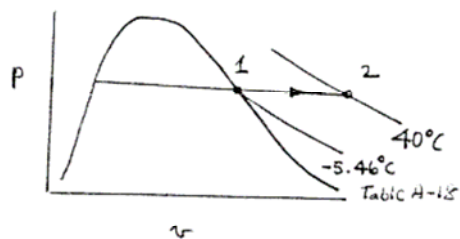
FIND: Show the process on a p-v diagram and evaluate W/m and Q/m for the process.

SCHEMATIC & GIVEN DATA:



1: Saturated vapor at 400 kPa.

2: $P_2 = P_1$, $T_2 = 40^\circ\text{C}$



ENGR. MODEL:

1. The propane in the piston-cylinder assembly is the closed system.
2. The propane expands at constant pressure.
3. Volume change is the only work mode.
4. Kinetic and potential energy effects play no role.

ANALYSIS: The process is shown in the p-v diagram. Since volume change is the only work mode

$$W_{12} = \int_1^2 p \, dV \Rightarrow (W_{12}/m) = p(v_2 - v_1)$$

With data from Table A-18 at 4 bar,

$$\frac{W_{12}}{m} = (4 \times 10^5 \frac{\text{N}}{\text{m}^2})(0.1392 - 0.1137) \frac{\text{m}^3}{\text{kg}} \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 10.2 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow \frac{W_{12}}{m}$$

An energy balance reduces to $\Delta U + \Delta KE + \Delta PE = Q - W$, or

$$\frac{Q_{12}}{m} = (u_2 - u_1) + \frac{W_{12}}{m}$$

$$= (488.1 - 418.0) + 10.2$$

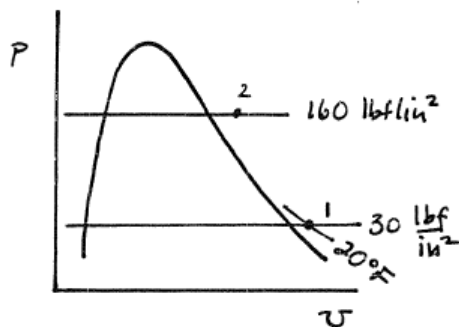
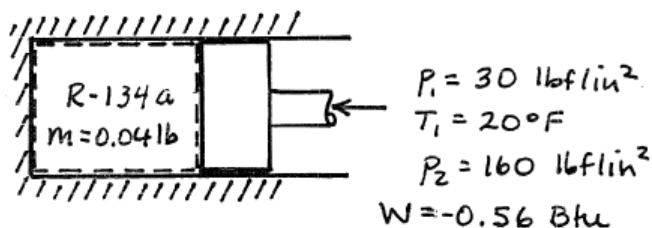
$$= 80.3 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow \frac{Q_{12}}{m}$$

PROBLEM 3.52

KNOWN: Refrigerant 134a is compressed from a given initial state to a given final pressure. There is no heat transfer, and the work is known.

FIND: Determine the final temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The R-134a is a closed system. (2) There is no heat transfer. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The final state is fixed by determining u_2 using the energy balance.

$$\Delta \overset{\circ}{KE} + \Delta \overset{\circ}{PE} + \Delta U = \overset{\circ}{Q} - W$$

or, with $\Delta U = m(u_2 - u_1)$

$$u_2 = -\frac{W}{m} + u_1$$

Referring to Table A-12E; $u_1 = 96.26 \text{ Btu/lb}$. Thus

$$\begin{aligned}
 u_2 &= -\frac{(-0.56 \text{ Btu})}{(0.04 \text{ lb})} + 96.26 \text{ Btu/lb} \\
 &= 110.26 \text{ Btu/lb}
 \end{aligned}$$

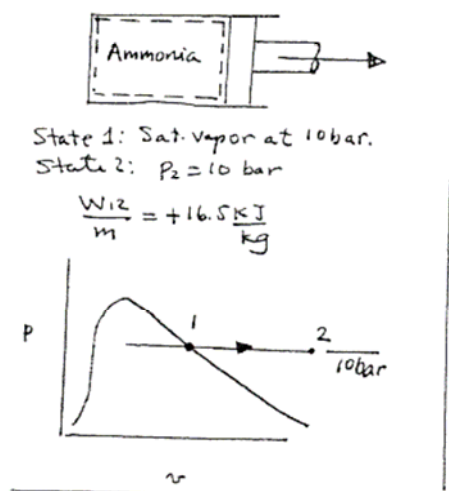
Referring again to Table A-12E; $T_2 \approx 121.6^\circ\text{F}$

PROBLEM 3.53

KNOWN: Data are provided for a process of ammonia contained in a piston-cylinder assembly.

FIND: Determine the final temperature and Q/m for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The ammonia in the piston-cylinder assembly is the closed system.
2. The expansion occurs at constant pressure.
3. Volume change is the only work mode.
4. Changes in kinetic and potential energy are negligible.

ANALYSIS: Two property values are required to fix state 2. One is the pressure and the other is specific volume found from W_{12}/m .

$$\begin{aligned} \frac{W_{12}}{m} &= \int_1^2 p dv = p(v_2 - v_1) \\ \Rightarrow v_2 &= \frac{W_{12}/m}{p} + v_1 \\ &= \left(\frac{16.5 \text{ kJ/kg}}{10 \times 10^5 \text{ N/m}^2} \right) \left| \frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}} \right| + 0.1285 \text{ m}^3/\text{kg} \quad (\text{Table A-15}) \\ &= 0.1450 \text{ m}^3/\text{kg} \\ \text{So, from Table A-15 at } 10 \text{ bar, } v_2 &= 0.1450 \text{ m}^3/\text{kg}, \\ T_2 &= 50^\circ\text{C} \quad \leftarrow T_2 \end{aligned}$$

An energy balance reduces to $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$, or

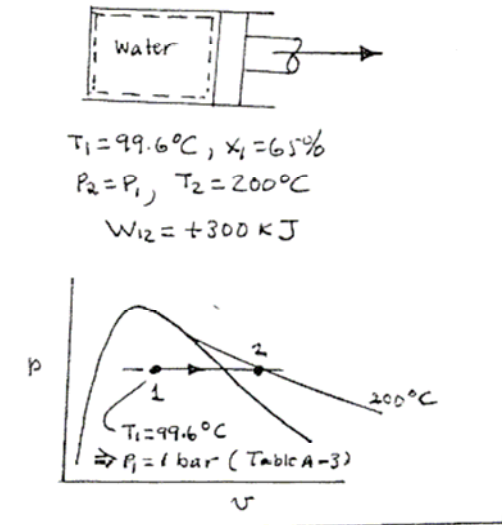
$$\begin{aligned} \frac{Q_{12}}{m} &= u_2 - u_1 + \frac{W_{12}}{m} \\ &= (1391.07 - 1334.66) + 16.5 \\ &= 72.91 \text{ kJ/kg} \quad \leftarrow \frac{Q_{12}}{m} \end{aligned}$$

PROBLEM 3.54

KNOWN: Data are provided for a process of water contained in a piston-cylinder assembly.

FIND: Determine the mass of water and Q for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The water in the piston-cylinder assembly is the closed system.
2. The process occurs at constant pressure.
3. Volume change is the only work mode.
4. Changes in kinetic and potential energy are negligible.

ANALYSIS: (a) Since volume change is the only work mode, $W_{12} = \int p dV$ or

$$W_{12} = m p (v_2 - v_1).$$

Thus

$$m = \frac{W_{12}}{p(v_2 - v_1)} = \frac{300 \text{ kJ}}{(105 \frac{\text{N}}{\text{m}^2})(2.172 - 1.1015) \frac{\text{m}^3}{\text{kg}}} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| = 2.80 \text{ kg} \quad \leftarrow m$$

$$\begin{aligned} \text{where } v_1 &= v_f + x_1(v_g - v_f) = \left(\frac{1.0432}{103} \right) + 0.65 \left(1.694 - \frac{1.0432}{103} \right) \\ &= 1.1015 \frac{\text{m}^3}{\text{kg}} \quad (\text{Table A-3 data}) \\ v_2 &= 2.172 \frac{\text{m}^3}{\text{kg}} \quad (\text{Table A-4 data}) \end{aligned}$$

(b) An energy balance reduces to $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$ or

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

$$\begin{aligned} \text{where } u_1 &= u_f + x_1(u_g - u_f) = 417.36 + 0.65(2506.1 - 417.36) \\ &= 1775.04 \frac{\text{kJ}}{\text{kg}} \quad (\text{Table A-3 data}) \end{aligned}$$

$$u_2 = 2658.1 \frac{\text{kJ}}{\text{kg}} \quad (\text{Table A-4 data})$$

$$\begin{aligned} \therefore Q_{12} &= 300 \text{ kJ} + 2.8 \text{ kg} [2658.1 - 1775.04] \frac{\text{kJ}}{\text{kg}} \\ &= 2772.57 \text{ kJ} \end{aligned}$$

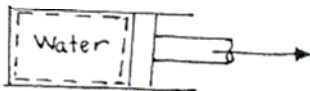
$\leftarrow Q_{12}$

PROBLEM 3.55

KNOWN: Data are provided for a process of water contained in a piston-cylinder assembly. The process involves three parts.

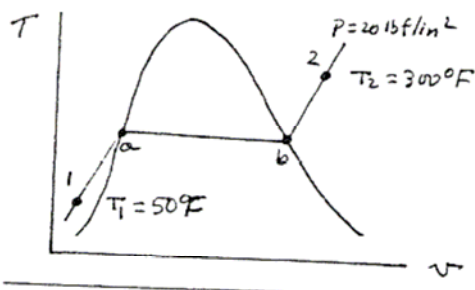
FIND: Determine $\frac{Q}{m}$ and $\frac{W}{m}$ for each of the three parts of the overall process.

SCHEMATIC & GIVEN DATA:



Constant pressure = $20.16 \frac{\text{bf}}{\text{in}^2}$

- 1 $\rightarrow$ a: $T_1 = 50^\circ\text{F}$ to sat. liquid
- b $\rightarrow$ c: Sat. Liquid to Sat. Vapor
- c $\rightarrow$ d: Sat. Vapor to $T_2 = 300^\circ\text{F}$



① 1 $\rightarrow$ a:

$$\begin{aligned} \frac{W_{1a}}{m} &= p(v_a - v_1) \\ &= \left(20.16 \frac{\text{bf}}{\text{in}^2}\right) (0.01683 - 0.01602) \left(\frac{\text{ft}^3}{\text{lb}}\right) \left|\frac{144 \text{ in}^2}{\text{ft}^2}\right| \left|\frac{18 \text{ lbm}}{778 \text{ ft}^3 \text{ lbf}}\right| \\ &= 0.003 \frac{\text{Btu}}{\text{lb}} \end{aligned}$$

$$\frac{Q_{1a}}{m} = (196.19 - 18.06) + 0.003 = 178.13 \frac{\text{Btu}}{\text{lb}}$$

② a $\rightarrow$ b:

$$\frac{W_{ab}}{m} = 20(20.09 - 0.01683) \left|\frac{144}{778}\right| = 74.31 \frac{\text{Btu}}{\text{lb}}$$

$$\frac{Q_{ab}}{m} = (1082.0 - 196.19) + 74.31 = 960.12 \frac{\text{Btu}}{\text{lb}}$$

③ b $\rightarrow$ 2:

$$\frac{W_{b2}}{m} = 20(22.36 - 20.09) \left|\frac{144}{778}\right| = 8.40 \frac{\text{Btu}}{\text{lb}}$$

$$\frac{Q_{b2}}{m} = (1108.7 - 1082.0) + 8.40 = 35.10 \frac{\text{Btu}}{\text{lb}}$$

ENGR. MODEL:

1. The water in the piston-cylinder assembly is the closed system.
2. The process occurs at constant pressure.
3. Volume change is the only work mode.
4. Kinetic and potential energy effects are negligible.

ANALYSIS: For each process we use

$$W = \int p dV = p \Delta V \Rightarrow \frac{W}{m} = p \Delta v \quad (1)$$

and the following expression obtained from an energy balance:

$$\frac{Q}{m} = \Delta u + \frac{W}{m} = \Delta u + p \Delta v \quad (2)$$

Data from Tables A-2, A-3, A-4.

Note: At 1, $v_1 \approx v_f(T_1)$
 $u_1 \approx u_f(T_1)$

	$v(\text{ft}^3/\text{lb})$	$u(\text{Btu}/\text{lb})$
1	0.01602	18.06
a	0.01683	196.19
b	20.09	1082.0
2	22.36	1108.7

PROBLEM 3.56

KNOWN: Data are provided for a process of propane contained within a piston-cylinder assembly.

FIND: Determine the initial and final temperatures and W and Q for the process.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL:

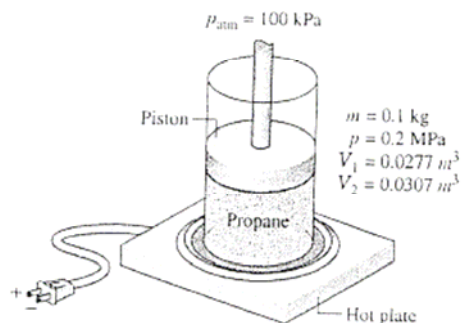


Fig. P3.56

1. The propane within the piston-cylinder assembly is the closed system.
2. Friction between the piston and cylinder is negligible and the expansion occurs slowly at a constant pressure of 0.2 MPa .
3. Volume change is the only work mode.
4. For the system there are no significant kinetic and potential energy effects.

ANALYSIS: (a) To find T_1 and T_2 requires two property values to fix each state.

Since pressure is constant, it is one of the properties. The specific volume provides the other: $v_1 = \frac{V_1}{m} = \frac{0.0277 \text{ m}^3}{0.1 \text{ kg}} = 0.277 \frac{\text{m}^3}{\text{kg}}$, $v_2 = \frac{V_2}{m} = \frac{0.0307 \text{ m}^3}{0.1 \text{ kg}} = 0.307 \frac{\text{m}^3}{\text{kg}}$.

Thus, from Table A-18 at $0.2 \text{ MPa} = 2 \text{ bar}$, $T_1 = 30^\circ\text{C}$, $T_2 = 60^\circ\text{C}$. $\leftarrow T_1, T_2$

(b) Since volume change is the only work mode and pressure is constant,

$$\begin{aligned} W_{12} &= \int_1^2 p dV = p(V_2 - V_1) \\ &= 0.2 \text{ MPa} [0.0307 - 0.0277] \text{ m}^3 \left| \frac{10^6 \text{ N/m}^2}{1 \text{ MPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \\ &= 0.6 \text{ kJ} \end{aligned}$$

(c) An energy balance reduces to give, $\Delta U + \cancel{\Delta K E} + \cancel{\Delta P E} = Q_{12} - W_{12}$ on

$$Q_{12} = W_{12} + m(U_2 - U_1)$$

Use data from Table A-18 at $0.2 \text{ MPa} = 2 \text{ bar}$,

$$Q_{12} = 0.6 \text{ kJ} + 0.1 \text{ kg} (524.3 - 476.3) \frac{\text{kJ}}{\text{kg}}$$

$$= 5.4 \text{ kJ}$$

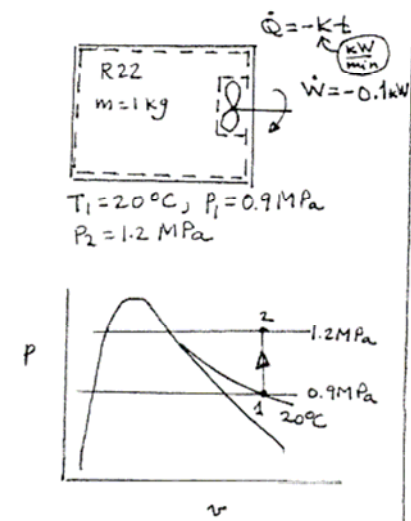
$\longleftarrow Q_{12}$

PROBLEM 3.57

KNOWN: Data are provided for Refrigerant 22 contained in a closed, rigid tank fitted with a paddle wheel.

FIND: For the Refrigerant 22, determine Q and W . Also evaluate the constant K in the given expression for the heat transfer rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The Refrigerant 22 in the tank is the closed system.
2. For the process of the R22, there are no overall changes in kinetic or potential energy.

ANALYSIS: Since the system volume and system mass do not change, the specific volume of the R22 at states 1 and 2 are the same: $v_2 = v_1$. Thus, at state 1, Table A-9 gives $v_1 = 0.0263 \text{ m}^3/\text{kg}$
 $u_1 = 232.92 \text{ kJ/kg}$.

Then with $v_2 = 0.0263 \text{ m}^3/\text{kg}$ at $P_2 = 1.2 \text{ MPa}$,

$$u_2 = 276.67 \text{ kJ/kg}.$$

States 1 and 2 are shown on the p - v diagram.

(a) After 20 min. of stirring

$$W_{12} = \int_0^{20} \dot{W} dt = -0.1 \text{ kW} [20 \text{ min}] \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| \left| \frac{60 \text{ s}}{1 \text{ min}} \right|$$

$$= -120 \text{ kJ}$$

(b) For the process, an energy balance reads $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$. Or

$$Q_{12} = m(u_2 - u_1) + W_{12} = (1 \text{ kg})(276.67 - 232.92) \frac{\text{kJ}}{\text{kg}} + (-120 \text{ kJ})$$

$$= -76.25 \text{ kJ}$$

(c) Also,

$$Q_{12} = \int_0^{20 \text{ min}} \dot{Q} dt = \int_0^{20 \text{ min}} (-Kt) dt = \left[-\frac{Kt^2}{2} \right]_0^{20 \text{ min}}, \text{ where } K \text{ is in } \frac{\text{KW}}{\text{min}}.$$

From part (b), $Q_{12} = -76.25 \text{ kJ}$. Collecting results,

$$-76.25 \text{ kJ} = -K \frac{(20 \text{ min})^2}{2}$$

$$\Rightarrow K = \frac{2(76.25 \text{ kJ})}{400 (\text{min})^2} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right|$$

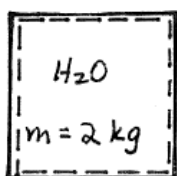
$$= 6.354 \times 10^{-3} \frac{\text{KW}}{\text{min}}$$

PROBLEM 3.58

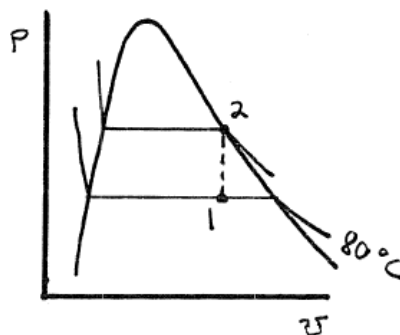
KNOWN: Saturated water vapor is contained in a rigid tank at a known initial temperature. The pressure drops as a result of heat transfer to a known final value.

FIND: Determine the amount of energy transfer by heat.

SCHEMATIC & GIVEN DATA:



$T_1 = 80^\circ\text{C}$
 $x_1 = 0.60$
 state 2: sat. vapor



- ① **ENGR. MODEL:** (1) The H₂O is a closed system. (2) For the process, $v_2 = v_1$. (3) For the system, $W = 0$. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The heat transfer is determined using the energy balance

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - \cancel{W}$$

or, with $\Delta U = m(u_2 - u_1)$

$$Q = m(u_2 - u_1)$$

Using data from Table A-2

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 334.86 + (0.60)(2482.2 - 334.86) = 1623.26 \text{ kJ/kg}$$

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = 1.0291 \times 10^{-3} + (0.60)(3.407 - 1.0291 \times 10^{-3}) = 2.0446 \text{ m}^3/\text{kg}$$

Then, using Table A-2 with $v_g = v_2 = v_1 = 2.0446 \frac{\text{m}^3}{\text{kg}}$

$$T \approx 93.5^\circ\text{C}$$

$$\text{and } u_2 = u_{g@93.5^\circ\text{C}} = 2498.8 \text{ kJ/kg}$$

$$\text{Finally } Q = (2 \text{ kg})(2498.8 - 1623.26) \text{ kJ/kg}$$

$$= 1751.1 \text{ kJ} \leftarrow Q$$

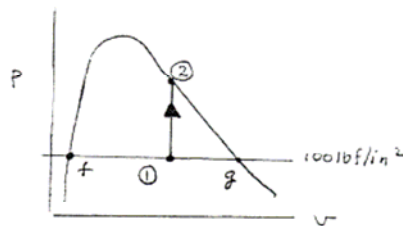
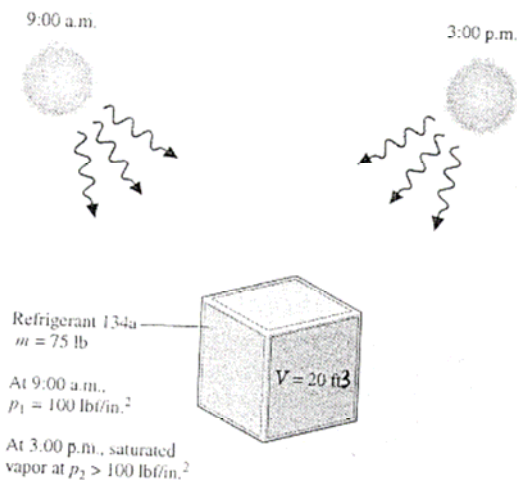
1. Since volume and mass are each constant, there is no change in specific volume for the process.

PROBLEM 3.59

KNOWN: Data are provided for Refrigerant 134a in a closed, rigid tank exposed to solar radiation.

FIND: For the process of the refrigerant, determine the initial temperature, final pressure, and Q .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The R134a in the tank is the closed system.
2. For the system, $W = 0$.
3. Kinetic and potential energy effects play no role.

Fig. P3.59

ANALYSIS: At the initial state, $v_1 = \frac{V}{m} = \frac{20 \text{ ft}^3}{75 \text{ lb}} = 0.2667 \frac{\text{ft}^3}{\text{lb}}$. Since $v_f < v_1 < v_g$, the initial state is in the two-phase, liquid-vapor region. Further, since mass and volume are each constant, there is no change in specific volume for the process: $v_2 = v_1$.

(a) Since the initial state is a two-phase, liquid-vapor mixture at 100 lbf/in.^2 , the initial temperature is the corresponding saturation temperature. From Table A-11E, $T_1 = 79.17^\circ\text{F}$.

(b) Interpolating in Table A-11E with $v_2 = v_g = 0.2667 \text{ ft}^3/\text{lb}$, we get $p_2 = 174.4 \text{ lbf/in.}^2$ and $u_2 = 107.93 \text{ Btu/lb}$.

(c) Reducing an energy balance $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$, we get

$$Q_{12} = m(u_2 - u_1)$$

Finding u_1 requires the quality at state 1. That is, with v_f and v_g from Table A-11E at 100 lbf/in.^2 ,

$$x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{0.2667 - 0.01332}{0.4747 - 0.01332} = 0.549$$

Then, $u_1 = u_f + x_1(u_g - u_f) = 36.75 + (0.549)(103.68 - 36.75) = 73.49 \text{ Btu/lb}$

Finally,

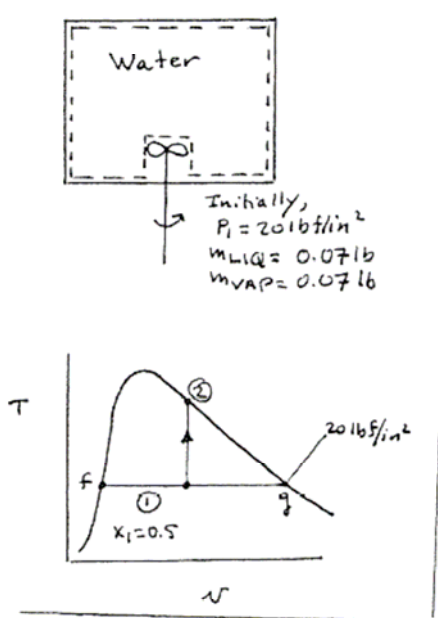
$$Q_{12} = 75 \text{ lb}(107.93 - 73.49) \frac{\text{Btu}}{\text{lb}} = 2583 \text{ Btu}$$

PROBLEM 3.60

KNOWN: Data are provided for water contained in a rigid, insulated tank fitted with a paddle wheel.

FIND: For the water determine the volume, initial temperature, final pressure, and work.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The water in the tank is the closed system.
2. For the system, $Q = 0$
3. For the system there is no overall change in kinetic energy or potential energy.

ANALYSIS: Initially, the system consists of 0.07 lb of sat. liquid and 0.07 lb of saturated vapor. Thus,

$$x_1 = \frac{0.07 \text{ lb}}{0.14 \text{ lb}} = 0.5$$

Moreover, the initial temperature corresponds to the saturation temperature corresponding to the pressure of 20 lbf/in². That is, $T_1 = 227.96^\circ\text{F}$

The total volume occupied is found using

$$\begin{aligned} V &= m_{\text{LIQ}} v_f + m_{\text{VAP}} v_g \\ &= (0.07)(0.01683) + (0.07)(20.09) \\ &= 1.407 \text{ ft}^3 \end{aligned}$$

All property data used thus far is from Table A-3E.

Since total volume and total mass are each constant, the specific volume at state 2 equals the specific volume at state 1: $v_2 = v_1$, where $v_1 = \frac{V}{m} = \frac{1.407 \text{ ft}^3}{0.14 \text{ lb}} = 10.05 \frac{\text{ft}^3}{\text{lb}}$.

Then, interpolating with $v_2 = v_g$ in Table A-3E gives

$$\begin{aligned} P_2 &= 42 \text{ lbf/in}^2 \\ u_2 &= 1093 \text{ Btu/lb} \end{aligned}$$

An energy balance reduces to read, $\Delta U + \Delta KE + \Delta PE = Q - W_{12}$, or

$$W_{12} = m(u_2 - u_1)$$

where $u_1 = u_f + x_1(u_g - u_f) = 196.19 + 0.5(1082.196.19) = 639.1 \text{ Btu/lb}$. Thus

$$W_{12} = 0.14 \text{ lb} (1093 - 639.1) \frac{\text{Btu}}{\text{lb}} = 63.5 \text{ Btu}$$

PROBLEM 3.61

KNOWN: A two-phase liquid-vapor mixture of water in a closed, rigid container is heated on a hot plate.

FIND: Referring to Fig. E 3.2, determine for a specified heat transfer rate (a) the time to bring the mixture from State 1 to State 2, (b) the time to bring the mixture from State 1 to State 3.

ENGINEERING MODEL: See Example 3.2. There is no change in kinetic or potential energy. $W = 0$.

ANALYSIS: An energy balance reduces to read $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \dot{Q} - \dot{W}$, or $\dot{Q} = \Delta U$. Since the heat transfer rate is constant, $\dot{Q} = \dot{Q} \Delta t$. Collecting results

$$\Delta t = \frac{m \Delta u}{\dot{Q}} \quad (1)$$

where $m = 0.59 \text{ kg}$, $x_1 = 0.5$ and $x_2 = 0.731$ (from the solution to Example 3.2). Thus,

$$\text{at 1 bar: } u_1 = u_f + x_1(u_g - u_f) = 417.36 + 0.5(2526.1 - 417.36) = 1461.73 \frac{\text{kJ}}{\text{kg}}$$

$$\text{at 1.5 bar: } u_2 = u_f + x_2(u_g - u_f) = 466.94 + 0.731(2579.7 - 466.94) = 1967.51 \frac{\text{kJ}}{\text{kg}}$$

State 3 is saturated vapor; so, interpolating with $v_g = 0.8475 \text{ m}^3/\text{kg}$, $u_g = 2531.26 \frac{\text{kJ}}{\text{kg}}$

Then, with Eq. (1), the time required for Process 1-2 is

$$\begin{aligned} \Delta t &= \frac{0.59 \text{ kg} (1967.51 - 1461.73) \text{ kJ/kg}}{(0.1 \text{ kW}) \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right|} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \\ &= 0.83 \text{ h} \end{aligned} \quad \leftarrow (a)$$

With Eq. (1), the time required for Process 2-3 is

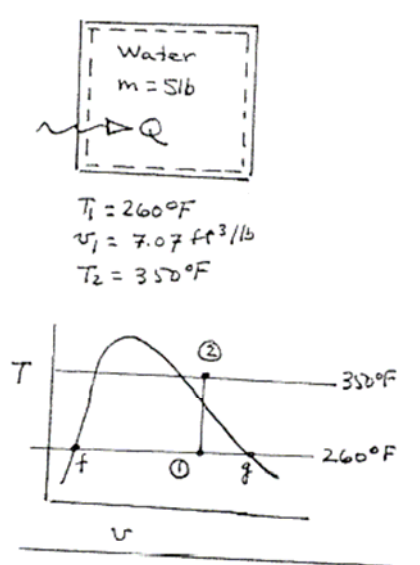
$$\begin{aligned} \Delta t &= \frac{0.59 \text{ kg} (2531.26 - 1461.73) \text{ kJ/kg}}{0.1 \text{ kW} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right|} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \\ &= 1.75 \text{ h} \end{aligned} \quad \leftarrow (b)$$

PROBLEM 3.62

KNOWN: Data are provided for a process of water contained in a rigid tank.

FIND: For the water, determine the initial and final pressure and Q for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The water in the tank is the closed system.
2. For the system, $W = 0$ and no kinetic and potential energy changes occur.

ANALYSIS: At 260°F , $v_f < v_1 < v_g$. So the system is a two-phase liquid-vapor mixture. The pressure corresponds to the saturation pressure at 260°F . That is, $P_1 = 35.42 \text{ lbf/in}^2$.

Moreover,

$$x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{7.07 - 0.01708}{11.77 - 0.01708} = 0.6$$

All data used thus far is from Table A-2E.

Since the mass and volume are each constant, there is no change in specific volume during the process: $v_2 = v_1$. Accordingly, interpolation in Table A-4E at 350°F with $v_2 = 7.07 \text{ ft}^3/\text{lb}$, gives

$$P_2 = 67.43 \text{ lbf/in}^2$$

$$u_2 = 1120.32 \text{ Btu/lb}$$

For this process, an energy balance reduces to read, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$ or

$$Q_{12} = m(u_2 - u_1)$$

$$= 5 \text{ lb} \left[1120.32 - 745.74 \right] \frac{\text{Btu}}{\text{lb}} = 1872.9 \text{ Btu}$$

where with data from Table A-2E at 260°F

$$u_1 = u_f + x_1(u_g - u_f)$$

$$= 228.6 + 0.6(1090.5 - 228.6)$$

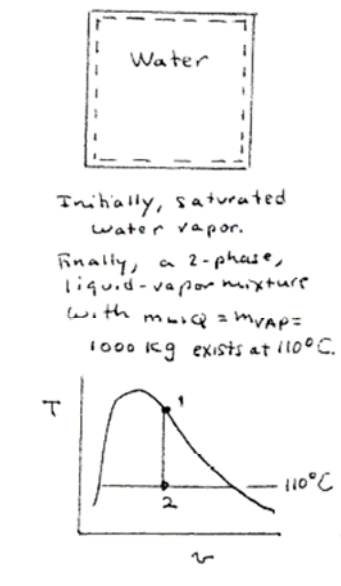
$$= 745.74 \text{ Btu/lb}$$

PROBLEM 3.63

KNOWN: Data are provided for a process of water contained in a closed, rigid tank.

FIND: Determine Q for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The water contained in the tank is the closed system.
2. For the system, $W = 0$ and there is no change in kinetic or potential energy.

ANALYSIS: At state 2, $m_{\text{vap}} = m_{\text{liq}} = 1000 \text{ kg}$. Thus $x_2 = 0.5$.

Since mass and volume are each constant, there is no change in specific volume for the process: $v_2 = v_1$.

With data from Table A-2 at 110°C ,

$$\begin{aligned} v_2 &= v_f + x_2 (v_g - v_f) \\ &= \frac{1.0516}{10^3} + 0.5 \left[1.210 - \frac{1.0516}{10^3} \right] \\ &= 0.6053 \text{ m}^3/\text{kg} \end{aligned}$$

$$\begin{aligned} u_2 &= u_f + x_2 (u_g - u_f) \\ &= 461.14 + 0.5 (2518.1 - 461.14) = 1489.62 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

Also, interpolating in Table A-2 with $v_1 = v_g = 0.6053 \text{ m}^3/\text{kg}$ gives $u_1 = 2543.89 \frac{\text{kJ}}{\text{kg}}$.

Reducing an energy balance for the process, $\Delta U + \Delta KE + \Delta PE = Q - W$ on

$$\begin{aligned} Q_{12} &= m(u_2 - u_1) \\ &= 2000 \text{ kg} [1489.62 - 2543.89] \frac{\text{kJ}}{\text{kg}} \\ &= -2.109 \times 10^6 \text{ kJ} \end{aligned}$$

$\leftarrow Q_{12}$

1 The volume of the tank is

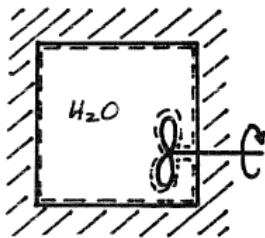
$$\begin{aligned} V &= m v = 2000 \text{ kg} \left(0.6053 \frac{\text{m}^3}{\text{kg}} \right) \\ &= 1211 \text{ m}^3 \end{aligned}$$

PROBLEM 3.64

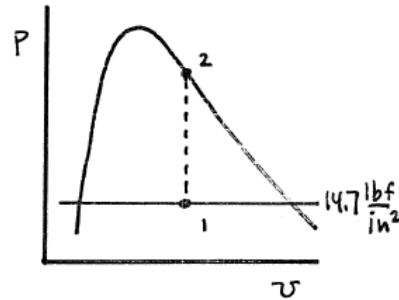
KNOWN: A two-phase, liquid-vapor mixture of H_2O is stirred in a rigid, well-insulated tank until only saturated vapor remains.

FIND: Determine the amount of energy transfer by work.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} V_{f1} &= 0.005 \text{ ft}^3 \\ V_{g1} &= 1.2 \text{ ft}^3 \\ P_1 &= 14.7 \text{ lbf/in}^2 \end{aligned}$$



ENGR. MODEL: (1) The H_2O is a closed system. (2) There is no heat transfer. (3) The volume is constant. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The work is determined using the energy balance

$$\Delta \overset{\circ}{K}E + \Delta \overset{\circ}{P}E + \Delta U = \overset{\circ}{Q} - W$$

or, with $\Delta U = m(u_2 - u_1)$

$$W = m(u_1 - u_2)$$

Using data from Table A-3E

$$m_{f1} = \frac{V_{f1}}{v_{f1}} = \frac{(0.005 \text{ ft}^3)}{(0.01672 \text{ ft}^3/\text{lb})} = 0.299 \text{ lb}$$

$$m_{g1} = \frac{V_{g1}}{v_{g1}} = \frac{(1.2 \text{ ft}^3)}{(26.80 \text{ ft}^3/\text{lb})} = 0.045 \text{ lb}$$

$$x_1 = m_{g1} / (m_{f1} + m_{g1}) = 0.045 / (0.299 + 0.045) = 0.131$$

$$\begin{aligned} u_1 &= u_{f1} + x_1(u_{g1} - u_{f1}) = 180.10 + (0.131)(1077.6 - 180.10) \\ &= 297.7 \text{ Btu/lb} \end{aligned}$$

$$v_2 = v_1 = (V_{f1} + V_{g1}) / (m_{f1} + m_{g1}) = (1.205 \text{ ft}^3) / (0.299 + 0.045) \text{ lb} = 3.503 \frac{\text{ft}^3}{\text{lb}}$$

Interpolating in Table A-3E with $v_g = 3.503 \text{ ft}^3/\text{lb} \Rightarrow u_2 = 1109.2 \frac{\text{Btu}}{\text{lb}}$

Finally

$$W = (0.299 + 0.045)(297.7 - 1109.2)$$

①

$$= -279.16 \text{ Btu} \leftarrow$$

W

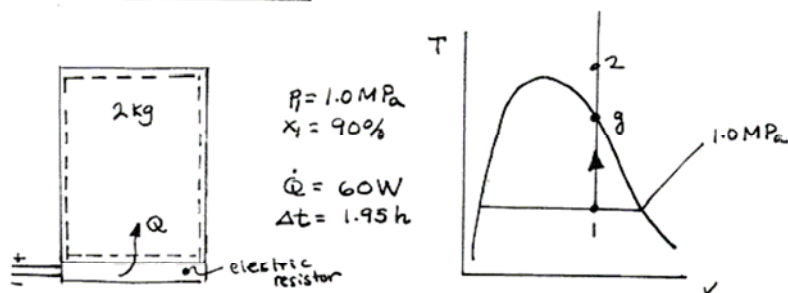
1. The negative sign for work denotes energy transfer into the system, as expected.

PROBLEM 3.65

KNOWN: Water contained in a rigid, insulated tank is heated from a specified initial state.

FIND: Determine the final temperature of the water.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The water is the closed system. 2. For the system, $W = 0$, and $\Delta KE = \Delta PE = 0$. 3. Volume remains constant. 4. The initial and final states are equilibrium states.

ANALYSIS: The values of two independent intensive properties are required to fix the final state. Since the total volume and mass do not change, one of these is the final specific volume: $v_2 = v_f$. Using data from Table A-3,

$$v_f = v_f + x(v_g - v_f) = \frac{1.1273}{10^3} + 0.9 \left[0.1944 - \frac{1.1273}{10^3} \right] = 0.1751 \text{ m}^3/\text{kg}$$

The other property is u_2 , found from an energy balance. With assumption 2,

$$\Delta U + \Delta KE + \Delta PE = Q - W$$

$$\Rightarrow m(u_2 - u_f) = Q \Rightarrow u_2 = u_f + Q/m \quad (*)$$

With data from Table A-3

$$u_f = u_f + x(u_g - u_f) = 761.68 + 0.9(2583.6 - 761.68) = 2401.4 \text{ kJ/kg}$$

Thus, since $Q = \int \dot{Q} dt = \dot{Q} \Delta t$,

$$u_2 = 2401.4 + \frac{(60 \text{ W})(1.95 \text{ h})}{2 \text{ kg}} \left| \frac{1 \text{ J/s}}{1 \text{ W}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left| \frac{\text{kJ}}{10^3 \text{ J}} \right| = 2612 \frac{\text{kJ}}{\text{kg}}$$

Using state g shown on the T-v diagram for reference; $v_g = v_f$. Interpolating in Table A-3 gives, $u_g = 2587 \text{ kJ/kg}$. Since $u_2 > u_g$, state 2 is in the superheated vapor region, as shown on the figure.

Thus, state 2 is fixed by $u_2 = 2612 \text{ kJ/kg}$, $v_2 = 0.1751 \text{ m}^3/\text{kg}$. Since interpolating in Table A-4 with u_2 , v_2 is inconvenient, we resort to using Interactive Thermodynamics: IT to determine T_2 , as follows:

Continued on next slide

Problem 3-65 continued

The IT code is

```
p1 = 10 // bar
x1 = 0.9
Qdot = 60 // W
delt = 1.95 // h
m = 2 // kg
v1 = vsat_Px("Water/Steam", p1, x1) // m³/kg
u1 = usat_Px("Water/Steam", p1, x1) // kJ/kg
m * (u2 - u1) = Q
Q = Qdot * delt * (3600 / 1000)
v2 = v1
v2 = v_PT("Water/Steam", p2, T2)
u2 = u_PT("Water/Steam", p2, T2)
```

IT Results

T2 198.7 °C
p2 11.59 bar

T₂

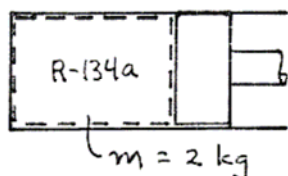


PROBLEM 3.66

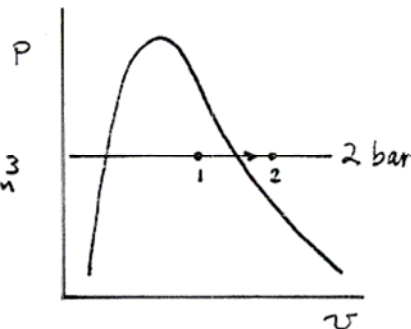
KNOWN: Refrigerant 134a undergoes a constant-pressure process. The pressure and the initial and final volumes are specified.

FIND: Determine the work and heat transfer for the process.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} P_1 &= P_2 = 2 \text{ bar} \\ V_1 &= 0.12 \text{ m}^3 \\ V_2 &= 2.0 V_1 = 0.24 \text{ m}^3 \end{aligned}$$



ENGR. MODEL: (1) The refrigerant is a closed system. (2) The pressure is constant. (3) Kinetic and potential energy effects are constant.

ANALYSIS: The work is determined using Eq. 2.17

$$\begin{aligned} W &= \int_{V_1}^{V_2} p dV = p(V_2 - V_1) \\ &= (2 \text{ bar})(0.24 - 0.12) \text{ m}^3 \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= 24 \text{ kJ} \end{aligned}$$

The heat transfer is found using the energy balance

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

or, with $\Delta U = m(u_2 - u_1)$

$$Q = m(u_2 - u_1) + W$$

From Table A-11, with $v_1 = V_1/m = 0.12/2 = 0.06 \text{ m}^3/\text{kg}$ and $p = 2 \text{ bar}$

$$x_1 = \frac{v_1 - v_{f,1}}{v_{g,1} - v_{f,1}} = \frac{0.06 - 0.7532 \times 10^{-3}}{0.0993 - 0.7532 \times 10^{-3}} = 0.6012$$

$$\begin{aligned} u_1 &= u_{f,1} + x_1(u_{g,1} - u_{f,1}) = 36.69 + (0.6012)(221.43 - 36.69) \\ &= 147.76 \text{ kJ/kg} \end{aligned}$$

And, from Table A-12, at $v_2 = 0.12 \text{ m}^3/\text{kg}$ and $p = 2 \text{ bar}$

$$u_2 = 255.66 \text{ kJ/kg}$$

Thus

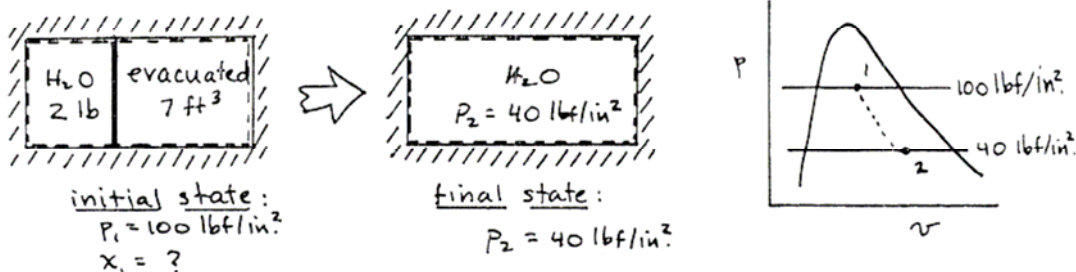
$$\begin{aligned} Q &= (2 \text{ kg})(255.66 - 147.76) \text{ kJ/kg} + (24 \text{ kJ}) \\ &= 239.8 \text{ kJ} \end{aligned}$$

PROBLEM 3.57

KNOWN: A known amount of H_2O at a specified initial pressure is confined to one side of a rigid, well insulated container by a partition. The partition is removed and the H_2O expands into the initially evacuated side, reaching a specified pressure when equilibrium is attained.

FIND: Determine the initial quality and the overall volume of the container.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The contents of the tank are the closed system. (2) The expansion occurs with no work or heat transfer. (3) The volume of the tank is constant. (4) There are no changes in kinetic or potential energy.

ANALYSIS: First, we solve this problem analytically. Then, we use the software Interactive Thermodynamics: IT. Begin with the total volume

$$V = m v_1 + 7 \text{ ft}^3$$

where $m = 2 \text{ lb}$. Also, noting that $V = m v_2$, we get

$$v_2 = \frac{V}{m} = v_1 + \frac{7 \text{ ft}^3}{2 \text{ lb}} \Rightarrow v_2 = v_1 + 3.5 \text{ ft}^3/\text{lb} \quad (1)$$

① Now, v_1 and v_2 can be expressed in terms of the qualities x_1 and x_2 , respectively

$$v_1 = v_{f1} + x_1 (v_{g1} - v_{f1})$$

$$v_2 = v_{f2} + x_2 (v_{g2} - v_{f2})$$

From (1)

$$v_{f2} + x_2 (v_{g2} - v_{f2}) = v_{f1} + x_1 (v_{g1} - v_{f1}) + 3.5$$

Using data from Table A-3E

$$0.01715 + x_2 (10.50 - 0.01715) = 0.01774 + x_1 (4.434 - 0.01774) + 3.5$$

$$\text{or} \quad x_2 (10.483) = x_1 (4.4163) + 3.50059 \quad (2)$$

Now, an overall energy balance reads $\Delta KE + \Delta PE + \Delta U = Q - W \Rightarrow \Delta U = 0$, or $u_2 = u_1$. In terms of saturated liquid and saturated vapor data

$$u_{f2} + x_2 (u_{g2} - u_{f2}) = u_{f1} + x_1 (u_{g1} - u_{f1})$$

Inserting values from Table A-3E

Continued on next slide

Problem 3-67 continued

$$236.03 + x_2(1092.3 - 236.03) = 298.3 + x_1(1105.8 - 298.3)$$

or $x_2(856.27) = x_1(807.5) + 62.27$ (3)

Expressions (2) and (3) each involve the unknown qualities x_1 and x_2 .
Solving simultaneously

$$\left. \begin{array}{l} x_1 = 0.5006 \\ x_2 = 0.5448 \end{array} \right\} \text{analytical results}$$

Using these results gives $v_1 = 2.2284 \text{ ft}^3/\text{lb}$, $v_2 = 5.7284 \text{ ft}^3/\text{lb}$, and

$$V = 11.457 \text{ ft}^3$$

The IT code is much less involved than the analytical solution. Seven expressions are written that involve the seven unknowns: $V, v_1, v_2, x_1, x_2, u_1, u_2$. The equation solver solves automatically, without the need to further manipulate the expressions algebraically. The code follows

```
p1 = 100 // lbf/in.2
p2 = 40
m = 2 // lb
V = m * v1 + 7 // ft3
v2 = v1 + 3.5
v1 = vsat_Px("Water/Steam", p1, x1)
v2 = vsat_Px("Water/Steam", p2, x2)
u1 = usat_Px("Water/Steam", p1, x1)
u1 = u2
u2 = usat_Px("Water/Steam", p2, x2)
```

② $\left. \begin{array}{l} V \quad 11.46 \\ x1 \quad 0.5007 \end{array} \right\} \text{IT Results}$

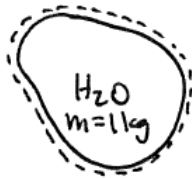
1. This assumes that state 2 is in the two-phase liquid vapor region. If state 2 is actually in the superheated vapor region, x_1 and/or x_2 would turn out to be physically unreasonable, and it would be necessary to modify the solution method.
2. There is excellent agreement between the analytical and computer results.

PROBLEM 3-68

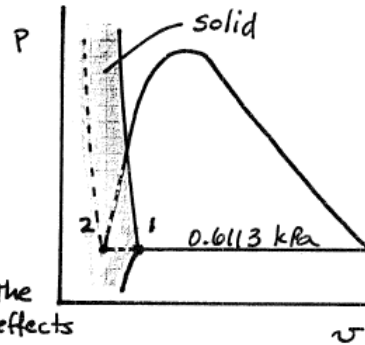
KNOWN: Saturated solid water is heated at constant pressure to saturated liquid.

END: Determine the work and heat transfer. Show that the heat transfer equals the change in enthalpy.

SCHEMATIC & GIVEN DATA:



State 1: triple point
sat. solid
State 2: sat. liquid



ENGR. MODEL: (1) The H₂O is a closed system. (2) The pressure is constant. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The work is determined using Eq. 2.17

$$W = \int_{v_1}^{v_2} p \, dv = m p (v_2 - v_1) \quad (1)$$

From Table A-6; $v_1 = v_i @ T_{tp} = 1.0908 \times 10^{-3} \text{ m}^3/\text{kg}$. Also, from Table A-2;

$v_2 = v_f @ T_{tp} = 1.002 \times 10^{-3} \text{ m}^3/\text{kg}$. Thus

$$W = (1 \text{ kg}) (0.6113 \text{ kPa}) (1.002 \times 10^{-3} - 1.0908 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} \left| \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{1 \text{ kN} \cdot \text{m}} \right|$$

$$= -5.428 \times 10^{-5} \text{ kJ} \leftarrow W$$

The heat transfer is found using an energy balance.

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

or, with $\Delta U = m(u_2 - u_1)$

$$Q = m(u_2 - u_1) + W \quad (2)$$

From Table A-6; $u_1 = -333.40 \text{ kJ/kg}$, and from Table A-2; $u_2 = 0.00$. Thus

$$Q = (1 \text{ kg}) [0 - (-333.40)] \frac{\text{kJ}}{\text{kg}} + (-5.428 \times 10^{-5} \text{ kJ})$$

$$= 333.4 \text{ kJ} \leftarrow Q$$

Combining Eqs. (1) and (2)

$$Q = m(u_2 - u_1) + m p (v_2 - v_1)$$

$$= m [(u_2 + p v_2) - (u_1 + p v_1)]$$

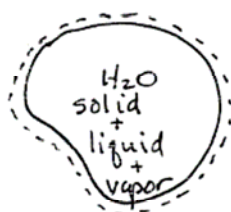
$$= m (h_2 - h_1) \leftarrow$$

PROBLEM 3.69

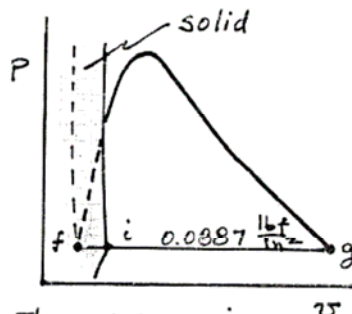
KNOWN: A three-phase, solid-liquid-vapor mixture of H_2O at the triple point is heated at constant pressure to saturated vapor.

FIND: Determine the work and heat transfer.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} m_{i1} &= 1 \text{ lb} \\ m_{f1} &= 1 \text{ lb} \\ m_{g1} &= 0.2 \text{ lb} \\ m_{g2} &= 2.2 \text{ lb} \\ \text{triple point} \end{aligned}$$



ENGR. MODEL: (1) The H_2O is a closed system. (2) The pressure is constant. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The work is determined using Eq. 2.17

$$W = \int_{V_1}^{V_2} p \, dV = p(V_2 - V_1)$$

From Table A-6E; $p = 0.0887 \text{ lbf/in}^2$, $T = 32.018^\circ\text{F}$. Also

$$v_i = 0.01747 \text{ ft}^3/\text{lb}, \quad v_g = 3302 \text{ ft}^3/\text{lb}$$

From Table A-2E; $v_f = 0.01602 \text{ ft}^3/\text{lb}$. Thus

$$\begin{aligned} V_1 &= m_{i1} v_i + m_{f1} v_f + m_{g1} v_g \\ &= (1)(0.01747) + (1)(0.01602) + (0.2)(3302) = 660.4 \text{ ft}^3 \end{aligned}$$

$$V_2 = (2.2)(3302) = 7264.4 \text{ ft}^3$$

and

$$\begin{aligned} W &= (0.0887 \frac{\text{lbf}}{\text{in}^2})(7264.4 - 660.4) \text{ ft}^3 \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\ &= 108.4 \text{ Btu} \end{aligned}$$

To determine the heat transfer use the energy balance

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

or,

$$Q = \Delta U + W$$

Again, with data from the tables

$$\begin{aligned} U_1 &= m_{i1} u_i + m_{f1} u_f + m_{g1} u_g = (1)(-143.34) + (1)(0) + (0.2)(1021.2) \\ &= 60.9 \text{ Btu} \end{aligned}$$

$$U_2 = m_{g2} u_g = (2.2)(1021.2) = 2246.6 \text{ Btu}$$

Finally

$$Q = (2246.6 - 60.9) + (108.4) = 2294.1 \text{ Btu}$$

PROBLEM 3.70

KNOWN: Data are provided for water in a piston-cylinder assembly undergoing two processes in series: constant-pressure followed by constant-volume.

FIND: For the overall process find W and Q .

SCHEMATIC & GIVEN DATA:

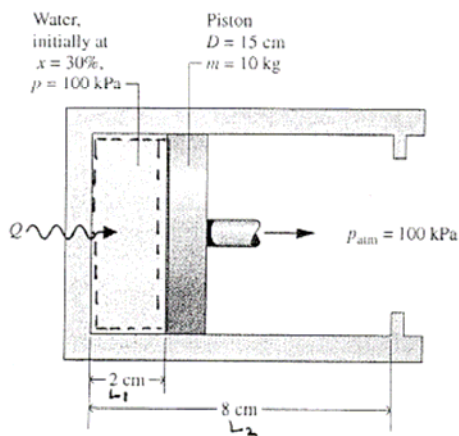
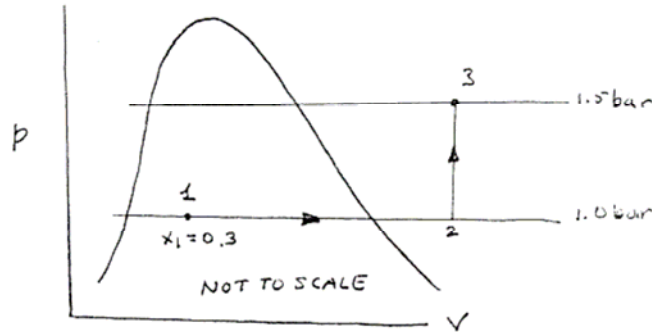


Fig. P3.70



ENGR. MODEL:

1. The water in the piston-cylinder assembly is the system.
2. Friction and kinetic and potential energy effects are absent. The piston is not accelerated during the process.
3. Volume change is the only work mode.

ANALYSIS:

Since volume change is the only work mode, work can be evaluated from Eq. 2.17.

Thus
$$W_{12} = \int_1^2 p dV = p[V_2 - V_1] \quad (1)$$

Note that $W_{23} = 0$ because there is no piston motion (volume is constant).

An energy balance for the overall process reads.

$$(u_3 - u_1) + \Delta KE + \Delta PE = Q_{13} - W_{13} \Rightarrow Q_{13} = m(u_3 - u_1) + W_{13} \quad (2)$$

Need V_1, V_2, m, u_1, u_3

$$\begin{aligned} \Rightarrow V_1 &= \frac{\pi D^2}{4} L_1 = \frac{\pi (0.15 \text{ m})^2}{4} (0.02 \text{ m}) \\ &= 3.53 \times 10^{-4} \text{ m}^3 \\ \Rightarrow V_2 &= \frac{\pi D^2}{4} L_2 = \frac{\pi (0.15 \text{ m})^2}{4} (0.08 \text{ m}) \\ &= 1.414 \times 10^{-3} \text{ m}^3 \end{aligned}$$

With data from Table A-3,

$$\begin{aligned} v_1 &= v_f + x(v_g - v_f) \\ &= \frac{1.0432}{10^3} + 0.3 \left[\frac{1.694 - 1.0432}{10^3} \right] \\ &= 0.50893 \text{ m}^3/\text{kg} \end{aligned}$$

$$\begin{aligned} u_1 &= u_f + x(u_g - u_f) \\ &= 417.36 + 0.3(2506.1 - 417.36) \\ &= 1043.98 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

$$\Rightarrow m = \frac{V_1}{v_1} = 6.94 \times 10^{-4} \text{ kg}$$

$$v_3 = v_2 = \frac{V_2}{m} = \frac{1.414 \times 10^{-3}}{6.94 \times 10^{-4}} = 2.0376 \frac{\text{m}^3}{\text{kg}} \quad \text{Interpolating in Table A-4 at 1.5 bar, } u_3 = 2952.1 \frac{\text{kJ}}{\text{kg}}$$

Using Eq. (1),

$$W_{12} = p[V_2 - V_1] = (1 \text{ bar}) \left[\frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right] (1.414 - 3.53) \times 10^{-4} \text{ m}^3 \left[\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right] = 0.106 \text{ kJ} \quad \leftarrow W$$

Using Eq. (2)

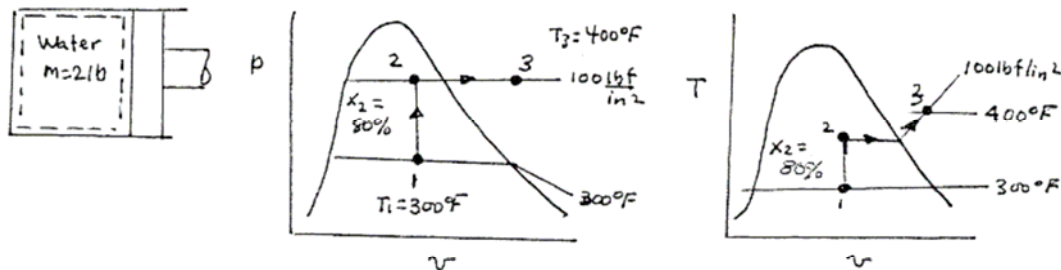
$$Q_{13} = m(u_3 - u_1) + W_{13} = (6.94 \times 10^{-4} \text{ kg})(2952.1 - 1043.98) \frac{\text{kJ}}{\text{kg}} + 0.106 \text{ kJ} = 1.324 + 0.106 = 1.43 \text{ kJ} \quad \leftarrow Q$$

PROBLEM 3.71

KNOWN: Water contained in a piston-cylinder assembly undergoes two processes in series: constant-volume heating followed by a constant-pressure process.

FIND: Sketch T-v and p-v diagrams of the processes in series. For each process find Q and W.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The water in the piston-cylinder assembly is a closed system.
2. Neglect kinetic and potential energy effects.
3. Volume change is the only work mode.

ANALYSIS: For process 1-2, the piston does not move (volume is constant). So, $W_{12} = 0$.

Reducing an energy balance, $\Delta U + \Delta KE + \Delta PE = Q - W$, we get

$$Q_{12} = m(u_2 - u_1)$$

With data from Table A-3E at 100 lbf/in²

$$u_2 = u_f + x_2(u_g - u_f) = 298.3 + 0.8(1105.8 - 298.3) = 944.3 \text{ Btu/lb}$$

$$v_2 = v_f + x_2(v_g - v_f) = 0.01774 + 0.8(4.434 - 0.01774) = 3.551 \text{ ft}^3/\text{lb}$$

Since $v_2 = v_1$, the quality at state 1 is obtained using data from Table A-2E at 300°F

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{3.551 - 0.01745}{6.472 - 0.01745} = 0.5475$$

Thus,

$$u_1 = u_f + x(u_g - u_f) = 269.5 + 0.5475(1100.0 - 269.5) = 724.2 \text{ Btu/lb}$$

$$\therefore Q_{12} = 2 \text{ lb} (944.3 - 724.2) \frac{\text{Btu}}{\text{lb}} = 440.2 \text{ Btu}$$

For process 2-3 the pressure is constant, thus

$$W_{23} = \int_2^3 p dV = m p (v_3 - v_2) = 2 \text{ lb} (100 \times 144 \frac{\text{lbf}}{\text{ft}^2}) (4.934 - 3.551) \frac{\text{ft}^3}{\text{lb}} \left| \frac{18 \text{ in}}{12 \text{ ft}} \right| \frac{1 \text{ ft}^2}{144 \text{ in}^2} = 51.2 \text{ Btu}$$

where v_3 is from Table A-4 at 100 lbf/in², 400°F. An energy balance reduces to give

$$Q_{23} = m(u_3 - u_2) + W_{23} = 2 \text{ lb} (1136.2 - 944.3) \frac{\text{Btu}}{\text{lb}} + 51.2 \text{ Btu} = 435.0 \text{ Btu}$$

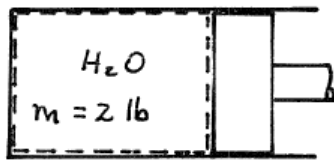
where u_3 is from Table A-4 at state 3.

PROBLEM 3.72

KNOWN: A system consisting of H_2O is compressed isothermally and then is heated at constant volume. Data are known at each of the principal states, and the work is known for process 1-2.

FIND: Determine the heat transfer for each process.

SCHEMATIC & GIVEN DATA:



State 1: $T_1 = 300^\circ F$

$V_1 = 20 \text{ ft}^3$

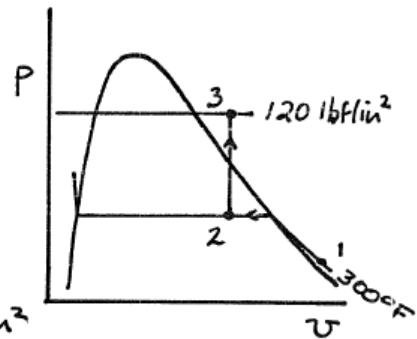
$W_{12} = -90.8 \text{ Btu}$

State 2: $T_2 = T_1$

$V_2 = 9.05 \text{ ft}^3$

State 3: $V_3 = V_2$

$P_3 = 120 \text{ lbf/in}^2$



ENGR. MODEL: (1) The H_2O is a closed system. (2) Process 1-2 occurs at constant temperature and process 2-3 occurs at constant volume. (3) Kinetic and potential energy effects are negligible. (4) Volume change is the only work mode.

ANALYSIS: First, fix each of the principal states.

State 1: $T_1 = 300^\circ F$, $v_1 = V_1/m = 10 \text{ ft}^3/\text{lb}$. From Table A-4E; $u_1 = 1104.1 \text{ Btu/lb}$

State 2: $T_2 = 300^\circ F$, $v_2 = V_2/m = 9.05/2 = 4.525 \text{ ft}^3/\text{lb} \Rightarrow$ two-phase (Table A-2E)

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}} = \frac{4.525 - 0.01745}{6.472 - 0.01745} = 0.6984$$

$$u_2 = u_{f2} + x_2(u_{g2} - u_{f2}) = 269.5 + (0.6984)(1100.0 - 269.5) = 849.5 \text{ Btu/lb}$$

State 3: $P_3 = 120 \text{ lbf/in}^2$, $v_3 = 4.525$. Interpolating in Table A-4E; $u_3 = 1166.3 \text{ Btu/lb}$

Now, using energy balances for each process

Process 1-2: $\Delta K\dot{E}^0 + \Delta P\dot{E}^0 + \Delta U = Q_{12} - W_{12}$

$$Q_{12} = m(u_2 - u_1) + W_{12}$$

$$= (2 \text{ lb})(849.5 - 1104.1) \text{ Btu/lb} + (-90.8 \text{ Btu})$$

$$= -600 \text{ Btu} \leftarrow Q_{12}$$

Process 2-3: $\Delta K\dot{E}^0 + \Delta P\dot{E}^0 + \Delta U = Q_{23} - W_{23}$

$$Q_{23} = m(u_3 - u_2)$$

$$= (2)(1166.3 - 849.5)$$

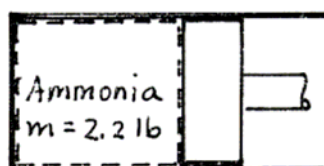
$$= 633.6 \text{ Btu} \leftarrow Q_{23}$$

PROBLEM 3.73

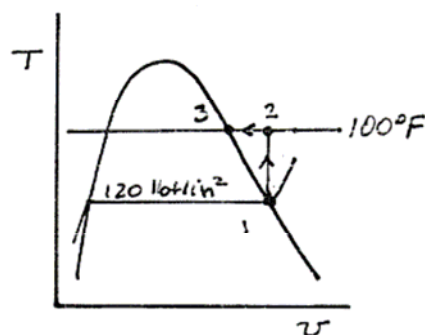
KNOWN: Ammonia undergoes two processes in series in a piston-cylinder assembly; the first at constant volume and the second at constant temperature. Data are known at each of three end states and the heat transfer is given for the second process.

FIND: Determine (a) the heat transfer for the first process, and (b) the work for the second process.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} P_1 &= 120 \text{ lbf/in}^2 \\ x_1 &= 1.0 \\ T_2 &= 100^\circ\text{F} \\ v_2 &= v_1 \\ T_3 &= T_2 \\ x_3 &= 1.0 \\ Q_{23} &= -98.9 \text{ Btu} \end{aligned}$$



ENGR. MODEL: (1) The ammonia is closed system. (2) The processes occur at constant volume and constant temperature, respectively. (3) Kinetic and potential energy effects are negligible. (4) Volume change is the only work mode.

ANALYSIS: The heat transfer Q_{12} and work W_{23} are found using energy balances. First, fix each of the principal states.

State 1: $P_1 = 120 \text{ lbf/in}^2$, $x_1 = 1.0 \Rightarrow u_1 = 572.73 \text{ Btu/lb}$ (Table A-14E)
 $v_1 = 2.4745 \text{ ft}^3/\text{lb}$

State 2: $v_2 = v_1$, $T_2 = 100^\circ\text{F}$; Interpolating in Table A-15E
 $u_2 \approx 589.17 \text{ Btu/lb}$

State 3: $T_3 = 100^\circ\text{F}$, $x_3 = 1.0 \Rightarrow u_3 = 576.51 \text{ Btu/lb}$ (Table A-13E)

(a) The energy balance for process 1-2 reduces to

$$\cancel{\Delta K\dot{E}} + \cancel{\Delta P\dot{E}} + m(u_2 - u_1) = Q_{12} - W_{12}$$

and

$$\begin{aligned} Q_{12} &= m(u_2 - u_1) \\ &= (2.2 \text{ lb})(589.17 - 572.73) \text{ Btu/lb} = 36.17 \text{ Btu} \leftarrow Q_{12} \end{aligned}$$

(b) Similarly, for process 2-3

$$\cancel{\Delta K\dot{E}} + \cancel{\Delta P\dot{E}} + m(u_3 - u_2) = Q_{23} - W_{23}$$

$$\begin{aligned} W_{23} &= Q_{23} - m(u_3 - u_2) \\ &= (-98.9 \text{ Btu}) - (2.2 \text{ lb})(576.51 - 589.17) \text{ Btu/lb} \\ &= -71.05 \text{ Btu} \leftarrow W_{23} \end{aligned}$$

PROBLEM 3.74

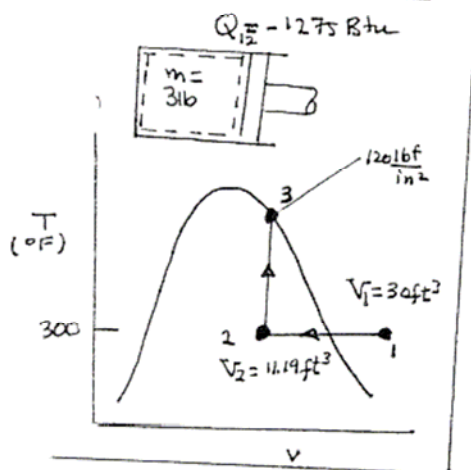
KNOWN: Water contained within a piston-cylinder assembly undergoes two processes in series:

Process 1-2: Constant-temperature compression to $V_2 = 11.19 \text{ ft}^3$, during which there is an energy transfer by heat from the water of 1275 Btu.

Process 2-3: Constant-volume heating to $p_3 = 120 \text{ lbf/in}^2$.

FIND: Sketch the processes in series on a T-v diagram. Find W_{12} and Q_{23} .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The water within the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. Kinetic and potential energy effects are negligible.

ANALYSIS: State 1 is fixed by $T_1 = 300^\circ\text{F}$ and

$$v_1 = v_f/m = \frac{3.0 \text{ ft}^3}{3 \text{ lb}} = 1.0 \frac{\text{ft}^3}{\text{lb}}. \text{ Interpolating in Table A-4E gives } u_1 = 1104.06 \text{ Btu/lb.}$$

State 2 is fixed by $T_2 = 300^\circ\text{F}$ and $v_2 = \frac{v_g}{m}$ or $v_2 = \frac{11.19 \text{ ft}^3}{3 \text{ lb}} = 3.73 \frac{\text{ft}^3}{\text{lb}}$. The quality at 2 is obtained with data at 300°F from Table A-2E

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{3.73 - 0.01745}{6.472 - 0.01745} = 0.5752$$

$$\text{Then, } u_2 = u_f + x_2(u_g - u_f) = 269.5 + 0.5752(1100.0 - 269.5) = 747.2 \text{ Btu/lb}$$

State 3 is fixed by $v_3 = v_2$ and $p_3 = 120 \text{ lbf/in}^2$. From Table A-3E, we see state 3 is saturated vapor. So, $u_3 = u_g = 1108.3 \text{ Btu/lb}$.

An energy balance for Process 1-2 reads $\Delta U + \Delta KE + \Delta PE = Q - W$. Thus

$$W_{12} = Q_{12} - m(u_2 - u_1) = -1275 \text{ Btu} - 3 \text{ lb}(747.2 - 1104.06) \frac{\text{Btu}}{\text{lb}} = -204.42 \text{ Btu}$$

$\leftarrow W_{12}$

Since the piston does not move (volume is constant), $W_{23} = 0$. An energy balance for Process 2-3 then reads $\Delta U + \Delta KE + \Delta PE = Q - W$ or

$$Q_{23} = m(u_3 - u_2) = 3 \text{ lb}(1108.3 - 747.2) \frac{\text{Btu}}{\text{lb}} = 1083.3 \text{ Btu}$$

$\leftarrow Q_{23}$

PROBLEM 3.75

KNOWN: Water contained in a piston-cylinder assembly undergoes two processes in series.

FIND: Evaluate Q and W for each process. Sketch the two processes in series on a $p-v$ diagram.

SCHEMATIC & GIVEN DATA:

Process 1-2: Constant-pressure cooling until the piston face rests against the stops. The volume occupied by the water is then one-half its initial volume.

Process 2-3: With the piston face resting against the stops, the water cools to 25°C .

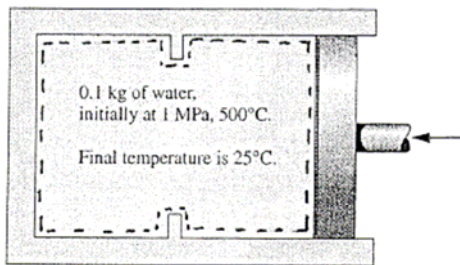
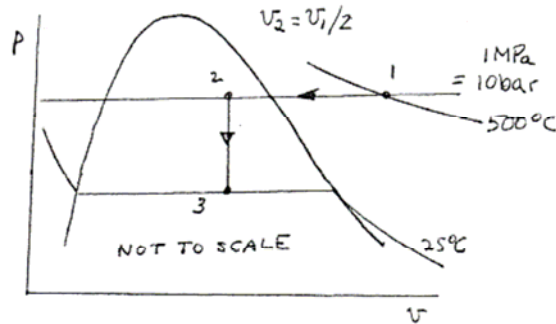


Fig. P3.75



ENGR. MODEL:

1. The water in the piston cylinder assembly is the system.
2. Volume change is the only work mode.
3. Kinetic and potential energy changes are absent.

ANALYSIS: Since volume change is the only work mode, work can be evaluated from Eq. 2.17. Thus, $W_{23} = 0$ since the piston does not move (volume is constant) and

$$W_{12} = \int_1^2 p dV = m p (V_2 - V_1)$$

From Table A-4 at 10 bar, 500°C , $V_1 = 0.3541 \text{ m}^3/\text{kg}$, $u_1 = 3124.4 \text{ kJ/kg}$. At state 2,

$$V_2 = V_1/2 = \frac{0.3541 \text{ m}^3/\text{kg}}{2} = 0.17705 \text{ m}^3/\text{kg} \quad \text{Then } x_2 = \frac{V_2 - V_f}{V_g - V_f} = \frac{(0.17705 - 0.0029/10^3)}{(0.1944 - 0.0029/10^3)} = 0.91$$

(Data from Table A-2)

$$\text{Thus, } u_2 = u_f + x_2(u_g - u_f) = 761.68 + 0.91[2583.6 - 761.68] = 2419.63 \text{ kJ/kg}$$

Calculating,

$$W_{12} = m p (V_2 - V_1) = (0.1 \text{ kg})(1 \text{ MPa}) \left| \frac{10^6 \text{ N/m}^2}{1 \text{ MPa}} \right| \left| (0.17705 - 0.3541) \frac{\text{m}^3}{\text{kg}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = -17.71 \text{ kJ} \quad \leftarrow W_{12}$$

$$\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1) + W_{12} = 0.1 \text{ kg} \left(\frac{2419.63 - 3124.4 \text{ kJ}}{\text{kg}} \right) - 17.71 \text{ kJ}$$

$$\therefore Q_{12} = -88.19 \text{ kJ} \quad \leftarrow Q_{12}$$

State 3 is fixed by $V_3 = V_2$ and $T_3 = 25^\circ\text{C}$. Then $x_3 = \frac{V_3 - V_f}{V_g - V_f} = \frac{0.17705 - (0.0029/10^3)}{(0.1944 - (0.0029/10^3))} = 0.0041$
(Data from Table A-2)

$$\text{Thus, } u_3 = u_f + x_3(u_g - u_f) = 104.88 + 0.0041(2469.8 - 104.88) = 114.33 \text{ kJ/kg}$$

For process 2-3,

$$\Delta U + \Delta KE + \Delta PE = Q_{23} - W_{23}$$

$$\Rightarrow Q_{23} = m(u_3 - u_2) = 0.1 \text{ kg} \left(\frac{114.33 - 2419.63 \text{ kJ}}{\text{kg}} \right) = -230.53 \text{ kJ} \quad \leftarrow Q_{23}$$

PROBLEM 3.76

KNOWN: A system consisting of ammonia undergoes a cycle composed of three processes.

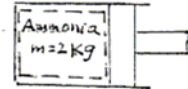
FIND: Sketch p - v and T - v diagrams. Determine the cycle net work and the heat transfer for each process.

SCHEMATIC & GIVEN DATA:

Process 1-2: constant volume from $p_1 = 10 \text{ bar}$, $x_1 = 0.6$ to saturated vapor

Process 2-3: constant temperature, $Q_{23} = 228 \text{ kJ}$

Process 3-1: constant pressure



ENGR. MODEL: (1) Closed system. (2) Neglect kinetic and potential energy effects. (3) Volume change is the only work mode.

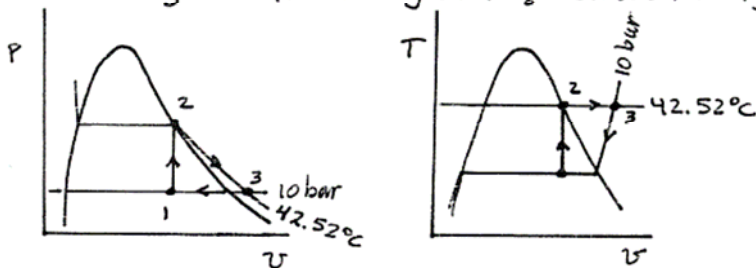
ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 10 \text{ bar}$, $x_1 = 0.6$. Thus $u_1 = u_f + x_1(u_g - u_f) = 296.10 + (0.6)(1334.66 - 296.10) = 919.24 \text{ kJ/kg}$ (Table A-14)

$$v_1 = v_f + x_1(v_g - v_f) = 0.07776 \text{ m}^3/\text{kg}$$

State 2: Sat. vapor, $v_2 = v_1$. Interpolating in Table A-13, at $v_g = 0.07776 \text{ m}^3/\text{kg}$ gives $u_2 \approx 1341.26 \text{ kJ/kg}$ and $T_2 = 42.52^\circ\text{C}$.

State 3: $p_3 = p_1 = 10 \text{ bar}$, $T_3 = T_2 = 42.52^\circ\text{C}$. Interpolating in Table A-15 $u_3 = 1374.95 \text{ kJ/kg}$ and $v_3 \approx 0.14027 \text{ m}^3/\text{kg}$



The required energy transfers are determined by using energy balances.

$$\text{Process 1-2: } \Delta KE + \Delta PE + m(u_2 - u_1) = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1) = 844.0 \text{ kJ} \quad Q_{12}$$

$$\text{Process 2-3: } \Delta KE + \Delta PE + m(u_3 - u_2) = Q_{23} - W_{23} \Rightarrow W_{23} = Q_{23} - m(u_3 - u_2) = 160.6 \text{ kJ}$$

$$\begin{aligned} \text{Process 3-1: Using Eq. 2.17; } W_{31} &= \int_{v_3}^{v_1} p \, dv = \int_{v_3}^{v_1} m \, p \, dv = m \, p \, (v_1 - v_3) \\ &= (2 \text{ kg})(10 \text{ bar})(0.07776 - 0.14027) \frac{\text{m}^3}{\text{kg}} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \\ &= -125.0 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \text{and } \Delta KE + \Delta PE + m(u_1 - u_3) &= Q_{31} - W_{31} \Rightarrow Q_{31} = m(u_1 - u_3) + W_{31} \\ &= -1036.4 \text{ kJ} \quad Q_{31} \end{aligned}$$

$$\textcircled{1} \textcircled{2} \quad W_{\text{cycle}} = W_{12} + W_{23} + W_{31} = 0 + 160.6 + (-125.0) = 35.6 \text{ kJ} \quad W_{\text{cycle}}$$

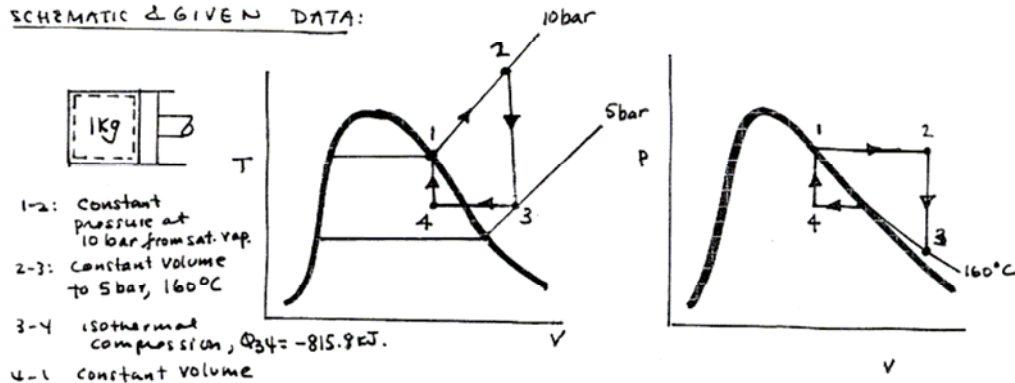
1. The positive value for W_{cycle} indicates the cycle is a power cycle.
2. A check of the net heat transfer confirms that $Q_{\text{cycle}} = W_{\text{cycle}}$.

PROBLEM 3.77

KNOWN: One kg of water undergoes a thermodynamic cycle composed of four processes.

FIND: Determine the thermal efficiency

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The water is the closed system. 2. Volume change is the only work mode. 3. Kinetic and potential energy effects can be ignored.

ANALYSIS: The thermal efficiency of a power cycle is $\eta = W_{\text{cycle}}/Q_{\text{in}}$ (see discussion of Eqs. 2.42, 4.3), where $W_{\text{cycle}} = W_{12} + W_{23} + W_{34} + W_{41}$.

Process 1-2: state 1 is fixed. State 2 is fixed by $P_2 = 10 \text{ bar}$, $V_2 = V_3$. From Table A-4

$$V_3 = 0.3835 \frac{\text{m}^3}{\text{kg}}, \quad u_3 = 2575.2 \text{ kJ/kg}$$

$$\Rightarrow W_{12} = \int_1^2 p dV = p m (v_2 - v_1) = (10 \text{ bar}) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| (1 \text{ kg}) (0.3835 - 0.1944) \frac{\text{m}^3}{\text{kg}} \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 189.1 \text{ kJ}$$

An energy balance reduces to give $Q = \Delta U + W$, or

$$Q_{12} = m(u_2 - u_1) + W = (1 \text{ kg}) [3231.8 - 2583.6] + 189.1 \text{ kJ} = 837.3 \text{ kJ}$$

where u_2 is obtained from Table A-3 using $P_2, v_2 (=v_3)$.

Process 2-3: $W_{23} = 0$

$$Q_{23} = m(u_3 - u_2) + W_{23} = (1 \text{ kg}) [2575.2 - 3231.8] = -656.6 \text{ kJ}$$

Process 3-4: $Q_{34} = -815.8 \text{ kJ}$ (given). Then, $\Delta U = Q_{34} - W_{34}$ gives

$$W_{34} = Q_{34} - \Delta U = Q_{34} - m(u_4 - u_3)$$

State 4 is fixed by $T_4 = 160^\circ\text{C}$, $V_4 = V_1$

$$\therefore x_4 = \frac{v_4 - v_f}{v_g - v_f} = \frac{0.1944 - 1.102/10^3}{0.3071 - 1.102/10^3} = 0.6317$$

$$\Rightarrow u_4 = u_f + x_4(u_g - u_f) = 674.86 + (0.6317)(2568.4 - 674.86) = 1871 \text{ kJ/kg}$$

$$\Rightarrow W_{34} = -815.8 - (1)[1871 - 2575.2] = -111.6 \text{ kJ}$$

Continued on next slide

Problem 3-77 continued

Process 4-1: $W_{41} = 0$, and

$$\begin{aligned} Q_{41} &= \Delta U + \cancel{W_{41}}^0 = m(u_1 - u_4) \\ &= (1 \text{ kg})[2583.6 - 1871] \frac{\text{kJ}}{\text{kg}} = 712.6 \text{ kJ} \end{aligned}$$

The net work is then

$$W_{\text{cycle}} = W_{12} + W_{23} + W_{34} + W_{41}$$

$$\textcircled{1} \quad = 189.1 + 0 + (-111.6) + 0 = 77.5 \text{ kJ}$$

To obtain Q_{in} ,

$$Q_{\text{in}} = Q_{12} + Q_{41} = 837.3 + 712.6 = 1549.9 \text{ kJ}$$

Then

$$\eta = \left(\frac{77.5}{1549.9} \right) = 0.05 \quad (5\%)$$

① As a check, note that for every cycle $Q_{\text{cycle}} = W_{\text{cycle}}$.

$$Q_{\text{cycle}} = Q_{12} + Q_{23} + Q_{34} + Q_{41}$$

$$= 837.3 + (-656.6) + (-815.8) + 712.6 = 77.5 \text{ kJ}$$

which agrees with W_{cycle} calculated using the work quantities.

PROBLEM 3.78

KNOWN: A system consisting of Refrigerant 22 undergoes a cycle composed of three processes.

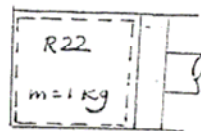
FIND: Sketch p-v and T-v diagrams. Determine the cycle net work and the heat transfer for each process.

SCHEMATIC & GIVEN DATA:

Process 1-2: Constant pressure from $p_1 = 30 \text{ lbf/in}^2$
 $x_1 = 0.95$ to $T_2 = 40^\circ\text{F}$

Process 2-3: isothermal with $W_{23} = -11.82 \text{ Btu}$ to saturated vapor

Process 3-1: Adiabatic expansion; $Q_{31} = 0$



ENGR. MODEL: (1) The R-22 is a closed system. (2) kinetic and potential energy effects are negligible. (3) Volume change is the only work mode.

ANALYSIS: First, using data from the refrigerant tables, fix each state.

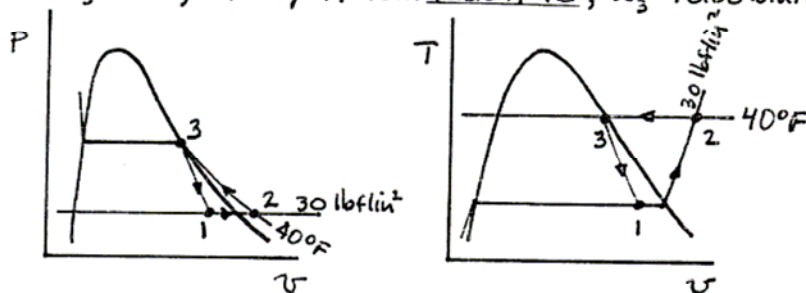
State 1: Table A-8E at $p_1 = 30 \text{ lbf/in}^2$ and $x_1 = 0.95$

$$u_1 = u_f + x_1(u_g - u_f) = 7.38 + (0.95)(93.67 - 7.38) = 89.36 \text{ Btu/lb}$$

$$v_1 = v_f + x_1(v_g - v_f) = 0.01178 + (0.95)(1.7430 - 0.01178) = 1.6564 \text{ ft}^3/\text{lb}$$

State 2: from Table A-9E at $p_2 = p_1$, $T_2 = 40^\circ\text{F}$; $u_2 = 100.40 \text{ Btu/lb}$, $v_2 = 1.9858 \text{ ft}^3/\text{lb}$

State 3: $T_3 = 40^\circ\text{F}$, sat. vapor. From Table A-7E; $u_3 = 98.08 \text{ Btu/lb}$



The required energy transfers are determined energy balances.

Process 1-2: Using Eq. 2.17, $W_{12} = \int_{v_1}^{v_2} p \, dv = m p_1 (v_2 - v_1)$

$$W_{12} = (1 \text{ lb})(30 \text{ lbf/in}^2)(1.9858 - 1.6564) \text{ ft}^3/\text{lb} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 1.829 \text{ Btu}$$

$$\Delta \cancel{KE} + \cancel{APE} + m(u_2 - u_1) = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1) + W_{12} = 12.87 \text{ Btu} \leftarrow Q_{12}$$

Process 2-3: $W_{23} = -11.82$, $\Delta \cancel{KE} + \cancel{APE} + m(u_3 - u_2) = Q_{23} - W_{23}$

$$Q_{23} = m(u_3 - u_2) + W_{23} = -14.14 \text{ Btu} \leftarrow Q_{23}$$

Process 3-1: $\Delta \cancel{KE} + \cancel{APE} + m(u_1 - u_3) = Q_{31} - W_{31}$

$$W_{31} = -m(u_1 - u_3) = 8.72 \text{ Btu}$$

$$\textcircled{1} \textcircled{2} W_{\text{cycle}} = W_{12} + W_{23} + W_{31} = (1.829) + (-11.82) + (8.72) = -1.27 \text{ Btu} \leftarrow W_{\text{cycle}}$$

1. The negative value for W_{cycle} indicates the cycle is a refrigeration cycle.
2. A check of the net heat transfer confirms that $Q_{\text{cycle}} = W_{\text{cycle}}$.

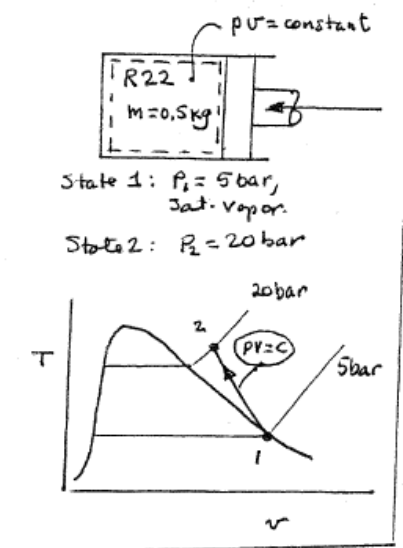
PROBLEM 3.79

KNOWN: Data are provided for a process of Refrigerant 22 contained in a piston-cylinder assembly.

FIND: For the process, find W and Q .

SCHEMATIC & GIVEN DATA:

ENGR. MODEL:



1. The R22 contained in the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The process is described by $pv = \text{constant}$.
4. Kinetic and potential energy effects can be ignored.

ANALYSIS: The work is given by

$$W_{12} = \int_1^2 p dV = m \int_1^2 p dv = m c \ln \frac{v_2}{v_1}$$

or

$$W_{12} = m(p_1 v_1) \ln \frac{v_2}{v_1}$$

From Table A-8, at 5 bar , $v_1 = 0.0469 \text{ m}^3/\text{kg}$. Then, since $p_1 v_1 = p_2 v_2$, $v_2 = \left(\frac{p_1}{p_2}\right) v_1$. That is, $v_2 = \left(\frac{5 \text{ bar}}{20 \text{ bar}}\right)(0.0469 \frac{\text{m}^3}{\text{kg}}) = 0.0117 \text{ m}^3/\text{kg}$. Accordingly

$$W_{12} = (0.5 \text{ kg}) \left(5 \times 10^5 \frac{\text{N}}{\text{m}^2} \right) (0.0469 \frac{\text{m}^3}{\text{kg}}) \ln(0.25) \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= -16.25 \text{ kJ}$$

$\leftarrow W_{12}$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$, or

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

From Table A-8, $u_1 = 226.54 \text{ kJ/kg}$. Interpolating in Table A-9 at 20 bar and $v_2 = 0.0117 \text{ m}^3/\text{kg}$, $u_2 = 243.53 \text{ kJ/kg}$. Then

$$Q_{12} = -16.25 \text{ kJ} + 0.5 \text{ kg} (243.53 - 226.54) \frac{\text{kJ}}{\text{kg}}$$

$$= -7.76 \text{ kJ}$$

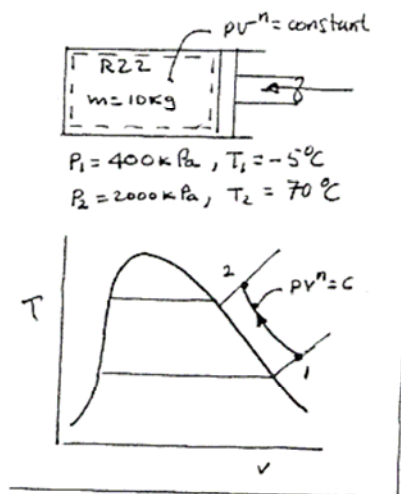
$\leftarrow Q_{12}$

PROBLEM 3.80

KNOWN: Data are provided for a process of Refrigerant 22 contained in a piston-cylinder assembly.

FIND: For the process, find W and Q .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The R22 contained in the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The process is described by $pv^n = \text{constant}$.
4. Kinetic and potential energy effects can be ignored.

ANALYSIS: Work is found using

$$W_{12} = \int_1^2 p dV = m \int_1^2 p dv \quad \left(= \frac{C}{v^n} \right)$$

$$W_{12} = m \left[\frac{P_2 V_2 - P_1 V_1}{1-n} \right] \quad (\text{See Example 2.1}) \quad (1)$$

The value for the exponent n can be obtained with P_1 , P_2 and V_1 , V_2 from Table A-9: At 4 bar, -5°C , $V_1 = 0.0586 \text{ m}^3/\text{kg}$. At 20 bar, 70°C , $V_2 = 0.013 \text{ m}^3/\text{kg}$. Then, using $pv^n = \text{constant}$, we get $P_1 V_1^n = \text{constant}$ and $P_2 V_2^n = \text{constant}$, giving

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^n \Rightarrow \frac{20}{4} = \left(\frac{0.0586}{0.013} \right)^n. \text{ Solving, } n = 1.069.$$

Equation (1) gives

$$W_{12} = (10 \text{ kg}) \left[\frac{(20 \text{ bar})(0.013 \frac{\text{m}^3}{\text{kg}}) - (4 \text{ bar})(0.0586 \frac{\text{m}^3}{\text{kg}})}{1 - 1.069} \right] \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$= -371 \text{ kJ}$$

$\leftarrow W_{12}$

An energy balance reduces to read, $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$, or

$$\begin{aligned} Q_{12} &= W_{12} + m(u_2 - u_1) \\ &= -371 \text{ kJ} + 10 \text{ kg} (255.35 - 225.16) \frac{\text{kJ}}{\text{kg}} \\ &= -69.1 \text{ kJ} \end{aligned}$$

$\leftarrow Q_{12}$

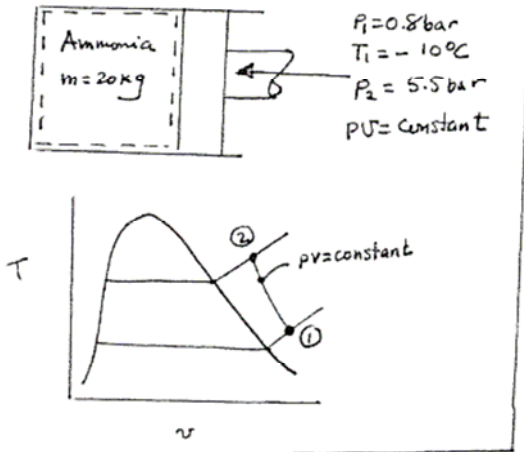
where u_1 and u_2 are from Table A-9.

PROBLEM 3.81

KNOWN: Data are provided for a process of ammonia contained within a piston-cylinder assembly.

FIND: For the process find W and Q .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The ammonia contained in the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The process is described by $PV = \text{constant}$.
4. Kinetic and potential energy effects can be ignored.

ANALYSIS: The work is given by

$$W_{12} = \int_1^2 P dV = m \int_1^2 P dv = m C \ln \frac{v_2}{v_1}$$

or

$$W_{12} = m(P_1 v_1) \ln \frac{v_2}{v_1} \quad (1)$$

where $C = \frac{P_1 v_1}{\ln \frac{v_2}{v_1}}$ and $L = P_1 v_1$.

From Table A-15 at $P_1 = 0.8 \text{ bar}$, $T_1 = -10^\circ\text{C}$, $v_1 = 1.5834 \text{ m}^3/\text{kg}$, $u_1 = 1325.85 \text{ kJ/kg}$. Since $PV = \text{constant}$, $P_2 v_2 = P_1 v_1$ or $v_2 = \left(\frac{P_1}{P_2}\right) v_1 = \left(\frac{0.8}{5.5}\right)(1.5834) = 0.2303 \text{ m}^3/\text{kg}$. Interpolating in Table A-15 with v_2 gives $u_2 = 1326.21 \text{ kJ/kg}$.

Using Eq. (1) we get

$$W_{12} = (20 \text{ kg})(0.8 \text{ bar})(1.5834 \frac{\text{m}^3}{\text{kg}}) \ln \left(\frac{0.8}{5.5} \right) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$= -4884 \text{ kJ}$$

Reducing an energy balance, $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$,

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

$$= -4884 \text{ kJ} + 20 \text{ kg} (1326.21 - 1325.85) \frac{\text{kJ}}{\text{kg}}$$

$$= -4877 \text{ kJ}$$

$\leftarrow W_{12}$

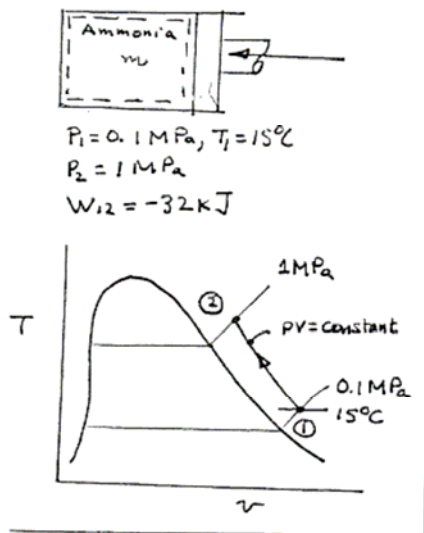
$\leftarrow Q_{12}$

PROBLEM 3.82

KNOWN: Data are provided for a process of ammonia contained within a piston-cylinder assembly.

FIND: For the process, find Q .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The ammonia contained in the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The process is described by $PV = \text{constant}$.
4. Kinetic and potential energy effects can be ignored.

ANALYSIS: The mass, m , of ammonia is required. This can be found from the given value for W_{12} .

The work can be evaluated from

$$W_{12} = \int_1^2 p dV = m \int_1^2 p dV = m C \ln \frac{V_2}{V_1} \quad \left(C = P_1 V_1 \right)$$

Thus, $W_{12} = m(P_1 V_1) \ln \frac{V_2}{V_1}$. Solving

$$m = \frac{W_{12}}{(P_1 V_1) \ln V_2/V_1}$$

From Table A-15 at 0.1 MPa , 15°C , $v_1 = 1.39 \text{ m}^3/\text{kg}$, $u_1 = 1365.95 \text{ kJ/kg}$.

$$\Rightarrow m = \left[\frac{-32 \text{ kJ}}{(0.1 \text{ MPa})(1.39 \frac{\text{m}^3}{\text{kg}}) \ln \left(\frac{0.1}{1.0} \right) \left| \frac{10^6 \text{ N/m}^2}{1 \text{ MPa}} \right| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}} \right] = 0.1 \text{ kg}$$

$\left(\frac{P_1}{P_2} = \frac{V_2}{V_1} \right)$

Reducing an energy balance, $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$, or

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

$$\begin{aligned} &= -32 \text{ kJ} + (0.1 \text{ kg})(1370.61 - 1365.95) \frac{\text{kJ}}{\text{kg}} \\ &= -31.53 \text{ kJ} \end{aligned}$$

← Q_{12}

where u_2 is obtained from Table A-15 by interpolating at $P_2 = 1 \text{ MPa}$ using

$$v_2 = v_1(P_1/P_2) = 0.139 \text{ m}^3/\text{kg}.$$

PROBLEM 3.83

KNOWN: Water : contained within a piston-cylinder assembly fitted with a spring undergoes a process from state 1 to state 2.

FIND: $P_1, P_2, m, W_{12}, Q_{12}$.

SCHEMATIC & GIVEN DATA:

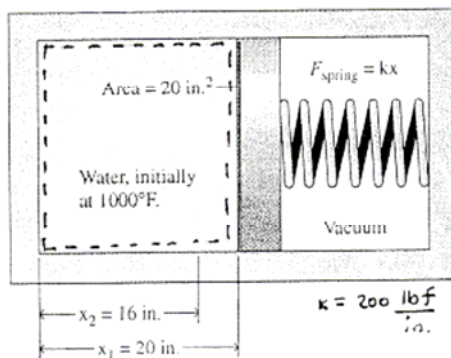


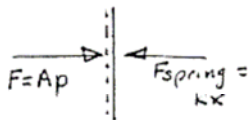
Fig. P3.83

ENGR. MODEL:

1. The water is the system.
2. Changes in kinetic and potential energy are absent. Friction between the piston and cylinder is negligible.
3. Cooling occurs slowly so no piston acceleration occurs.
4. Volume change is the only work mode.

ANALYSIS:

(a) To evaluate pressure consider a force balance at the piston face:



$$\therefore Ap = kx \text{ or } p = kx/A$$

$$\text{Then } P_1 = \frac{(200 \text{ lbf/in.})(20 \text{ in.})}{20 \text{ in}^2} = 200 \text{ lbf/in}^2$$

$$P_2 = \frac{(200 \text{ lbf/in.})(16 \text{ in.})}{20 \text{ in}^2} = 160 \text{ lbf/in}^2$$

$\leftarrow P_1, P_2$

(b) To find the mass, m , write $m = V_1/v_1$, where v_1 is obtained from Table A-4E at $P_1 = 200 \text{ lbf/in}^2$, $T_1 = 1000^\circ\text{F}$: $v_1 = 4.31 \text{ ft}^3/\text{lb}$. Thus,

$$m = \frac{(20 \text{ in}^2)(20 \text{ in.})}{4.31 \text{ ft}^3/\text{lb}} \left| \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3 \right| = 0.054 \text{ lb}$$

$\leftarrow m$

$$(c) \quad W_{12} = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} kx dx = \frac{k}{2} [x_2^2 - x_1^2] = \left(\frac{200 \text{ lbf/in.}}{2} \right) [(16 \text{ in.})^2 - (20 \text{ in.})^2] \left| \frac{1 \text{ ft}}{12 \text{ in.}} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= -1.54 \text{ Btu}$$

$\leftarrow W_{12}$

$$(d) \quad \Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1) + W_{12}$$

From Table A-4E at $P_1 = 200 \text{ lbf/in}^2$, $T_1 = 1000^\circ\text{F}$, $u_1 = 1369.8 \text{ Btu/lb}$.
State 2 is fixed by $P_2 = 160 \text{ lbf/in}^2$ and v_2 , where $m = \frac{v_1}{v_2} = \frac{v_1}{v_2} \Rightarrow v_2 = \frac{v_1}{m} v_1$, or
 $v_2 = \frac{(20 \text{ in}^2)(16 \text{ in.})}{(20 \text{ in}^2)(20 \text{ in.})} (4.31 \frac{\text{ft}^3}{\text{lb}}) = 3.448 \frac{\text{ft}^3}{\text{lb}}$. Interpolating in Table A-4E, $u_2 = 1171.98 \frac{\text{Btu}}{\text{lb}}$.

$$\text{Thus, } Q_{12} = 0.054 [1171.98 - 1369.8] \frac{\text{Btu}}{\text{lb}} + (-1.54 \text{ Btu})$$

$$= -12.22 \text{ Btu}$$

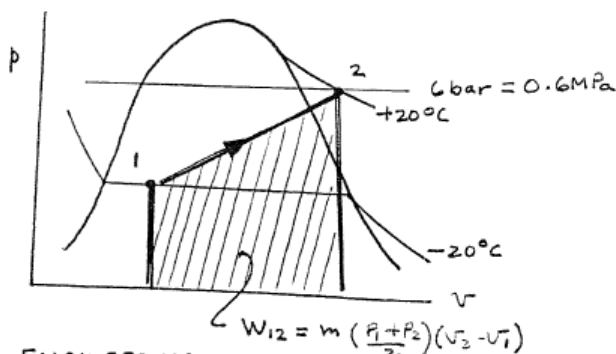
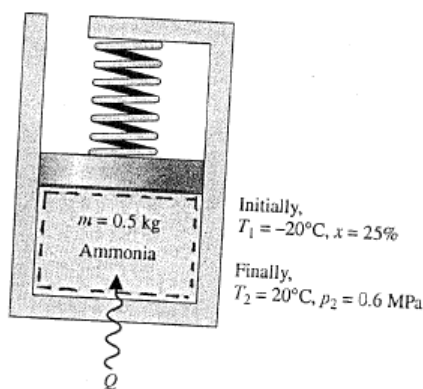
$\leftarrow Q_{12}$

PROBLEM 3.84

KNOWN: Ammonia contained in a piston-cylinder assembly undergoes a process during which it is heated.

FIND: For the process, find Q and W .

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The ammonia is the system.
2. Volume change is the only work mode. As shown by the p - v diagram, pressure varies linearly with volume.
3. There are no significant kinetic and potential energy effects.

ANALYSIS: Since volume change is the only work mode, $W_{12} = \int_1^2 p dV$, or
 $W_{12} = m \int_{v_1}^{v_2} p dv$. This can be evaluated geometrically as, $W_{12} = m p_{ave} (v_2 - v_1)$.

Thus,

$$W_{12} = m \left[\frac{p_1 + p_2}{2} \right] (v_2 - v_1) = 0.5 \text{ kg} \left[\frac{1.9019 + 6}{2} \right] \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| (0.22155 - 0.157) \frac{\text{m}^3}{\text{kg}} \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 12.75 \text{ kJ}$$

Using x at the initial state, $v_1 = v_f + x(v_g - v_f) = \frac{1.5038}{10^3} + 0.25(0.6233 - \frac{1.5038}{10^3}) = 0.157 \frac{\text{m}^3}{\text{kg}}$

Data from Table A-13

Then, from Table A-15, $v_2 = 0.22155 \text{ m}^3/\text{kg}$.

Writing an energy balance,

$$\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1) + W_{12}$$

Using Table A-13 data, $u_1 = u_f + x(u_g - u_f) = 88.4 + 0.25(1299.23 - 88.40) = 391.11 \text{ kJ/kg}$.

From Table A-15, $u_2 = 1347.62 \text{ kJ/kg}$. Thus,

$$Q_{12} = 0.5 \text{ kg} (1347.62 - 391.11) \frac{\text{kJ}}{\text{kg}} + 12.75 \text{ kJ}$$

$$= 491.01 \text{ kJ}$$

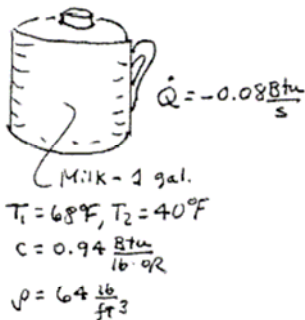
$\leftarrow Q_{12}$

PROBLEM 3.85

KNOW IV: Data are provided for a gallon of milk placed in a refrigerator.

FIND: Determine the time, in min., for the milk to cool from 68°F to 40°F.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The milk is the closed system.
2. For the system, $\dot{W} = 0$ and kinetic and potential energy play no role.
3. The milk is modeled as incompressible with constant specific heat, C .

ANALYSIS: An energy rate balance reduces as follows, $\frac{dU}{dt} + \cancel{\frac{dKE}{dt}} + \cancel{\frac{dPE}{dt}} = \dot{Q} - \dot{W}$,

or

$$\frac{dU}{dt} = \dot{Q} \Rightarrow \Delta U = \int \dot{Q} dt \Rightarrow m(u_2 - u_1) = \dot{Q} \Delta t.$$

In this expression, $m = \rho V = \left(64 \frac{\text{lb}}{\text{ft}^3} \right) \left(1 \text{ gal.} \right) \left| \frac{0.13368 \text{ ft}^3}{1 \text{ gal.}} \right| = 8.56 \text{ lb.}$ Also,

with Eq. 3.20a $(u_2 - u_1) = C(T_2 - T_1)$. Collecting results

$$\Delta t = \frac{m c (T_2 - T_1)}{\dot{Q}} = \frac{(8.56 \text{ lb}) (0.94 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}) (-28^\circ\text{R})}{(-0.08 \text{ Btu/s})} \left| \frac{1 \text{ min}}{60 \text{ s}} \right|$$

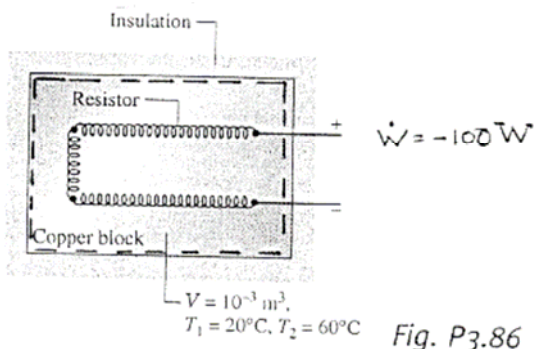
$= 47 \text{ min}$ Δt

PROBLEM 3.86

KNOWN: Data are provided for an insulated copper block receiving energy from an embedded resistor.

FIND: Determine the time, in min., for the temperature of the block to increase to 60°C from an initial temperature of 20°C .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The copper block is the closed system. The mass of the resistor is ignored.
2. For this system, $\dot{Q} = 0$ and kinetic and potential energy play no role.
3. The copper block is modeled as incompressible with constant specific heat, C .

ANALYSIS: An energy rate balance reduces as follows, $\frac{dU}{dt} + \frac{dKE}{dt} + \frac{dPE}{dt} = \dot{Q} - \dot{W}$
 $\Rightarrow \frac{dU}{dt} = -\dot{W}$. Integrating, $\Delta U = -\int_0^t \dot{W} dt$. With Eq. 3.20a, $\Delta U = mc(T_2 - T_1)$.
 Collecting results, $mc(T_2 - T_1) = -\int_0^t \dot{W} dt$. Since $\dot{W}$ is constant, $mc(T_2 - T_1) = -\dot{W}t$.

From Table A-19, for copper $\rho = 8930 \text{ kg/m}^3$, $C = 0.385 \text{ kJ/kg}\cdot\text{K}$. Finally

$$t = \frac{mc(T_2 - T_1)}{-\dot{W}} = \frac{(8930 \text{ kg/m}^3)(10^{-3} \text{ m}^3)(0.385 \text{ kJ/kg}\cdot\text{K})(40 \text{ K})}{(100 \text{ W})} \left| \frac{1 \text{ W}}{1 \text{ J/s}} \right| \left| \frac{10^3 \text{ J}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right|$$

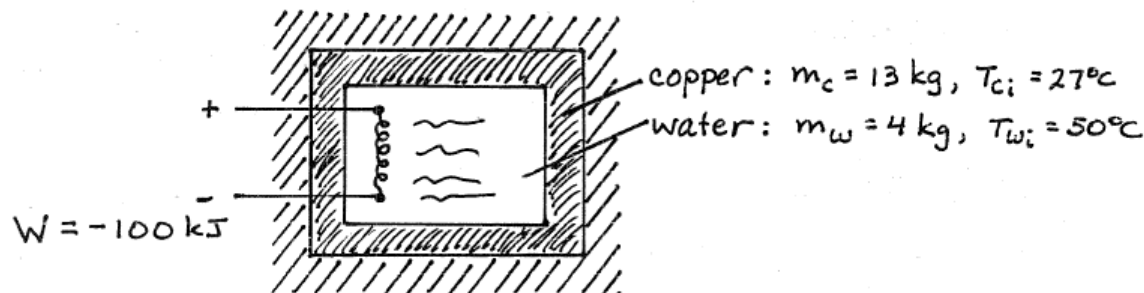
$$= 22.9 \text{ min}$$

PROBLEM 3.87

KNOWN: An electric resistor transfers energy to water in a well-insulated copper tank.

FIND: Determine the final equilibrium temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The copper tank and water are a closed system. (2) There is no heat transfer. (3) The copper and water behave as incompressible substances with constant specific heats. (4) Kinetic and potential energy effects are negligible. (5) No energy is stored in the resistor; $\Delta U_{res} = 0$

ANALYSIS: The energy balance reduces as follows

$$\Delta KE + \Delta PE + \Delta U = \dot{Q} - W \Rightarrow \Delta U = -W$$

or $\Delta U_{res} + \Delta U_c + \Delta U_w = -W$

$$m_c c_c (T_f - T_{ci}) + m_w c_w (T_f - T_{wi}) = -W$$

Solving for the final temperature, T_f

$$T_f = \frac{m_c c_c T_{ci} + m_w c_w T_{wi} - W}{m_c c_c + m_w c_w}$$

Using Table A-19, the specific heat of copper is $c_c = 0.385 \text{ kJ/kg} \cdot \text{K}$. And for water at 325 K, $c_w = 4.179 \text{ kJ/kg} \cdot \text{K}$. Thus

$$\textcircled{1} \quad T_f = \frac{(13 \text{ kg})(0.385 \text{ kJ/kg} \cdot \text{K})(27 + 273) \text{ K} + (4)(4.179)(50 + 273) - (-100)}{(13 \text{ kg})(0.385 \text{ kJ/kg} \cdot \text{K}) + (4)(4.179)}$$

$$= 322.3 \text{ K} = 49.3^\circ\text{C} \longleftarrow T_f$$

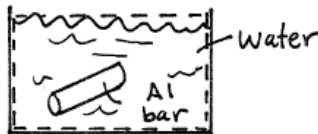
1. Note that temperature in $^\circ\text{C}$ could have been used directly in this expression.

PROBLEM 3-88

KNOWN: A hot aluminum bar is placed in an open tank of cool water. The bar and the water come to equilibrium with no heat transfer with the surroundings.

FIND: Determine the final temperature.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} V_a &= 0.2 \text{ ft}^3 \\ T_{a,i} &= 200^\circ \text{F} \\ V_w &= 4 \text{ ft}^3 \\ T_{w,i} &= 70^\circ \text{F} \end{aligned}$$

ENGR. MODEL: (1) The Al bar and water are the closed system. (2) The aluminum and water behave as incompressible substances with constant specific heats. (3) Kinetic and potential energy effects are neglected. (4) There is no heat transfer between the tank and its surroundings, and $W=0$.

ANALYSIS: The energy balance reduces as follows:

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = \cancel{Q} - \cancel{W} \Rightarrow \Delta U = 0$$

or $\Delta U_a + \Delta U_w = 0$. Thus, if T_f is the final equilibrium temperature

$$\textcircled{1} \quad m_a c_a (T_f - T_{a,i}) + m_w c_w (T_f - T_{w,i}) = 0$$

Solving for T_f

$$T_f = \frac{m_a c_a T_{a,i} + m_w c_w T_{w,i}}{m_a c_a + m_w c_w} \quad (*)$$

To evaluate m_w , use $v_f @ 70^\circ \text{F}$ from Table A-2E to get

$$m_w = \frac{V_w}{v_f} = \frac{4 \text{ ft}^3}{0.01605 \text{ ft}^3/\text{lb}} = 249.2 \text{ lb}$$

To evaluate m_a , using data from Table A-19E

$$m_a = \rho_a V_a = (169 \frac{\text{lb}}{\text{ft}^3})(0.2 \text{ ft}^3) = 33.8 \text{ lb}$$

Now, with $C_a = 0.216 \text{ Btu/lb}^\circ \text{R}$ and $C_w = 0.998 \text{ Btu/lb}^\circ \text{R}$ (at 540°R) from Table A-19E, we can evaluate (*)

$$\begin{aligned} T_f &= \frac{(33.8 \text{ lb})(0.216 \text{ Btu/lb}^\circ \text{R})(200 + 460)^\circ \text{R} + (249.2)(0.998)(70 + 460)^\circ \text{R}}{(33.8 \text{ lb})(0.216 \text{ Btu/lb}^\circ \text{R}) + (249.2)(0.998)} \\ &= 533.7^\circ \text{R} = 73.7^\circ \text{F} \leftarrow T_f \end{aligned}$$

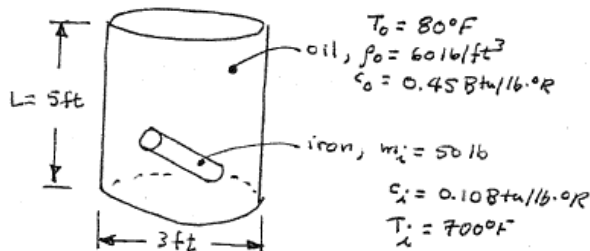
1. Note that temperature in $^\circ \text{F}$ could be used in this expression because it involves temperature differences.

PROBLEM 3.89

KNOWN: Data are provided for an iron casting quenched in oil.

FIND: Determine the final equilibrium temperature of the iron and oil.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The closed system is the oil plus iron.
2. For the system, $Q = 0$, $W = 0$ and there are no effects of kinetic and potential energy.
3. The oil and iron are each modeled as incompressible with constant specific heat, C .
4. The volume occupied by the iron can be ignored.

ANALYSIS: The energy balance reduces as follows: $\Delta U + \Delta KE + \Delta PE = Q - W$, or $\Delta U]_{\text{oil}} + \Delta U]_{\text{iron}} = 0$. Using Eq. 3.26a, $\Delta U = mc\Delta T$, we get

$$m_o c_o [T_f - T_o] + m_i c_i [T_f - T_i] = 0$$

Solving for T_f

$$T_f = \frac{m_o c_o T_o + m_i c_i T_i}{m_o c_o + m_i c_i} \quad (1)$$

The mass of the oil is found using $m_o = \rho_o V_o$, where V_o denotes the volume occupied by the oil. Using $\rho_o = 49.1 \text{ lb/ft}^3$ from Table A-19E, the volume of the iron is $V_i = \left(\frac{50 \text{ lb}}{49.1 \text{ lb/ft}^3}\right) = 0.1 \text{ ft}^3$, which can be ignored (assumption 4). The mass of the oil is then

$$m_o = \rho_o V = \rho_o \left(\frac{\pi D^2}{4}\right) L = 60 \frac{\text{lb}}{\text{ft}^3} \left(\frac{\pi (3 \text{ ft})^2}{4}\right) 5 \text{ ft} = 2121 \text{ lb}$$

Substituting values into Eq (1) gives

$$T_f = \frac{(2121 \text{ lb}) \left(0.45 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}\right) (540^\circ\text{R}) + (50 \text{ lb}) \left(0.1 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}\right) (1160^\circ\text{R})}{(2121 \text{ lb}) \left(0.45 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}\right) + (50 \text{ lb}) \left(0.1 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}\right)}$$

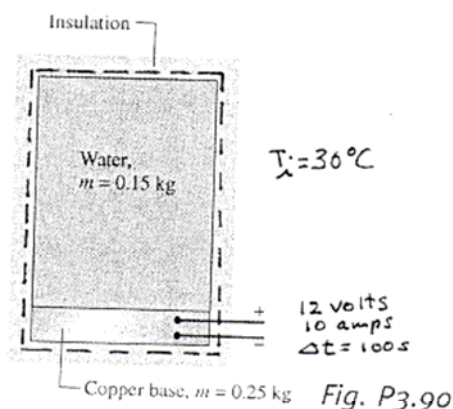
$$= 543^\circ\text{R} (83^\circ\text{F})$$

PROBLEM 3.90

KNOWN: Data are provided for a closed, insulated tank containing liquid water. The tank has a copper base in which a heating element is embedded.

FIND: Determine the final temperature when equilibrium is reached.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The tank and its contents are the closed system.
2. For the system, $Q=0$ and there are no effects of kinetic and potential energy.
3. The water and copper base are each modeled as incompressible with constant specific heat, c .
4. The mass of the tank walls is negligible.

ANALYSIS: An energy balance reduces to read $\Delta E_{\text{system}} = Q - W$, where $\Delta U = \Delta U_{\text{tank}} + \Delta U_{\text{water}} + \Delta U_{\text{copper}}$. By assumption 4, $\Delta U_{\text{tank}} = 0$. Also, with Eq. 3.20a, $\Delta U = mc\Delta T$. Collecting results

$$m_w c_w [T_f - T_i] + m_{cu} c_{cu} [T_f - T_i] = -W$$

Solving

$$T_f = T_i + \frac{-W}{m_w c_w + m_{cu} c_{cu}} \quad (1)$$

where

$$W = -(120 \text{ W})(100 \text{ s}) \left| \frac{1 \text{ J/s}}{1 \text{ W}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ J}} \right| = -12 \text{ kJ}$$

From Table A-19, $c_{cu} = 0.385 \text{ kJ/kg} \cdot \text{K}$, $c_w = 4.18 \text{ kJ/kg} \cdot \text{K}$. Inserting values in Eq. (1)

$$\begin{aligned} T_f &= 303.15 \text{ K} + \frac{12 \text{ kJ}}{(0.15 \text{ kg})(4.18 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) + (0.25 \text{ kg})(0.385 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})} \\ &= 319.74 \text{ K} \quad (46.6^\circ \text{C}) \end{aligned}$$

PROBLEM 3.91

H₂O at $p = 200 \text{ bar}$, $T = 470^\circ\text{C} = 743 \text{ K}$

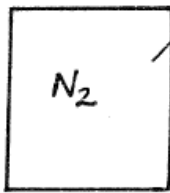
$$(a) \left. \begin{aligned} P_R &= P/P_c = 200 \text{ bar} / 220.9 \text{ bar} = 0.905 \\ T_R &= T/T_c = 743 \text{ K} / 647.3 \text{ K} = 1.15 \end{aligned} \right\} \begin{array}{l} \text{Fig A-1} \\ Z \approx 0.79 \end{array} \xleftarrow{\text{Z chart}}$$

$$(b) \text{ Table A-4 at } p = 200 \text{ bar}, T = 470^\circ\text{C} \Rightarrow v = 0.1355 \text{ m}^3/\text{kg}$$

$$Z = \frac{Pv}{RT} = \frac{(200 \text{ bar})(0.1355 \text{ m}^3/\text{kg})}{\left(\frac{8.314}{18.02}\right) \frac{\text{kJ}}{\text{kg K}} (743 \text{ K})} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$= 0.791 \xleftarrow{\text{Z table}}$$

PROBLEM 3.92



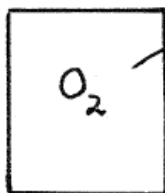
$m = 40 \text{ kg}$
 $p = 17 \text{ MPa}$
 $T = 180 \text{ K}$
 $V = ?$

Determine z : $P_R = 170/33.9 = 5.02$ } Fig. A-2
 $T_R = 180/126.2 = 1.43$ } $z \approx 0.82$

$$V = \frac{zmRT}{P} = \frac{(0.82)(40 \text{ kg}) \left(\frac{8.314 \text{ kJ}}{28.01 \text{ kg} \cdot \text{K}} \right) (180 \text{ K}) \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right|}{(17 \times 10^6 \text{ N/m}^2)}$$

$$= 0.103 \text{ m}^3 \leftarrow V$$

PROBLEM 3.93



$$V = 5 \text{ ft}^3$$

$$T = 400^\circ \text{R}$$

$$P = 4000 \text{ lb}_f/\text{in}^2$$

$$m = ?$$

Determine z :

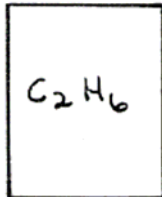
$$P_R = \frac{4000/14.7}{49.8} = 5.5 \quad \text{Fig A-2}$$

$$T_R = 400/278 = 1.44 \quad \left. \begin{array}{l} P_R = 5.5 \\ T_R = 1.44 \end{array} \right\} z \approx 0.84$$

$$m = \frac{PV}{zRT} = \frac{(4000 \text{ lb}_f/\text{in}^2)(5 \text{ ft}^3)}{(0.84) \left(\frac{1545}{32.00} \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m \cdot ^\circ \text{R}} \right) (400 \text{ R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|$$

$$= 177.5 \text{ lb} \quad \underline{\hspace{10em}} \quad m$$

PROBLEM 3.94



$$T = 115^\circ F = 575^\circ R$$

$$v = 0.30 \text{ ft}^3/\text{lb}$$

$$p = ?$$

Determine z : $T_R = \frac{575}{549} = 1.05$

$$v'_R = \frac{v P_c}{R T_c}$$

$$= \frac{(0.30 \text{ ft}^3/\text{lb})(48.2 \text{ atm})}{\left(\frac{1545}{30.07} \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot ^\circ R}\right)(575^\circ R)} \left| \frac{14.7 \frac{\text{lb}_f}{\text{in}^2}}{1 \text{ atm}} \right| \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|$$

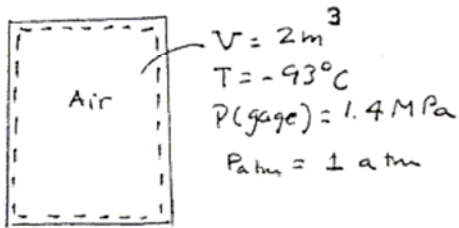
$$= 1.04$$

Figure A-1: $z \approx 0.74$

$$p = \frac{z R T}{v} = \frac{(0.74) \left(\frac{1545}{30.07} \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot ^\circ R}\right)(575^\circ R)}{(0.30 \text{ ft}^3/\text{lb})} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right|$$

$$= 506 \text{ lb}_f/\text{in}^2 \leftarrow \underline{\hspace{10em}} P$$

PROBLEM 3.95



Determine Z :

$$P = \overset{P_{\text{atm}}}{1.01325 \text{ bar}} + \overset{P(\text{gage})}{14 \text{ bar}}$$

$$= 15.01325 \text{ bar}$$

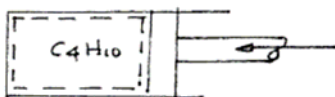
$$P_R = \frac{P}{P_c} = \frac{15.01325}{37.7} = 0.40$$

$$T_R = \frac{180 \text{ K}}{133 \text{ K}} = 1.35$$

Fig A-1 $Z \sim 0.955$

$$m = \frac{P V}{Z R T} = \frac{(15.01325 \times 10^5 \text{ N/m}^2)(2 \text{ m}^3)}{0.955 \left(\frac{8314 \text{ N}\cdot\text{m}}{28.97 \text{ kg}\cdot\text{K}} \right) 180 \text{ K}} = 60.9 \text{ kg} \quad \leftarrow$$

PROBLEM 3.96



$$T_1 = 173^\circ\text{C} = 446\text{K}$$

$$P_1 = 1.9\text{ MPa}$$

$$P_2 = 2.5\text{ MPa}$$

Compressibility chart data:

$$T_R = \frac{446\text{ K}}{425\text{ K}} = 1.05$$

$$P_{R1} = \frac{1.9\text{ bar}}{3.8\text{ bar}} = 0.5$$

$$P_{R2} = \frac{2.5\text{ bar}}{3.8\text{ bar}} = 0.66$$

From Fig. A-1,

$$Z_1 \approx 0.835$$

$$Z_2 \approx 0.775$$

The variation of Z with P_R for $T_R = 1.05$ is closely linear. Thus,

$$Z_{\text{ave}} \approx 0.81$$

$$W = \int_1^2 p \, dV$$

$$\text{Using } p = m \left(\frac{ZRT}{V} \right),$$

$$\frac{W}{m} = RT \int_1^2 \frac{Z}{V} \, dV$$

$$= RT \int_1^2 Z \, d \ln V$$

$$\frac{W}{m} = RT Z_{\text{ave}} \ln \left[\frac{V_2}{V_1} \right]$$

$$\text{Noting that } V = Z m RT / P,$$

$$V_2 = Z_2 m RT / P_2$$

$$V_1 = Z_1 m RT / P_1$$

Collecting results

$$\frac{W}{m} = RT Z_{\text{ave}} \ln \left[\frac{Z_2}{Z_1} \cdot \frac{P_1}{P_2} \right]$$

Inserting values,

$$\frac{W}{m} = \left(\frac{8.314 \text{ kJ}}{58.12 \text{ kg/kmol}} \right) (446\text{ K}) (0.81) \ln \left[\frac{0.775}{0.835} \cdot \frac{1.9}{2.5} \right]$$

$$= -18 \frac{\text{kJ}}{\text{kg}}$$

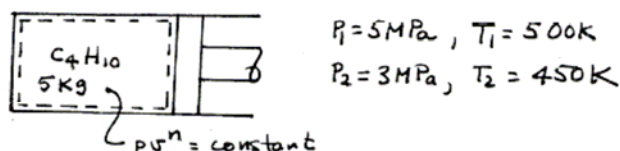
← W/m

PROBLEM 3.97

KNOWN: Five kg of C_4H_{10} contained in a piston-cylinder arrangement undergo a process between known states for which $pV = \text{constant}$.

FIND: Determine the work.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The Butane is a closed system. 2. The process is described by $pV^n = \text{constant}$. 3. Volume change is the only work mode.

ANALYSIS: The work is given by

$$W = \int_1^2 p dV = m \int_1^2 \frac{C}{V^{1/n}} dV = m \frac{(P_2 V_2 - P_1 V_1)}{1-n}$$

To evaluate W requires V_1, V_2 and n . The compressibility chart can be used to obtain V_1 and V_2 : From Table A-1, $P_c = 38 \text{ bar}, T_c = 425 \text{ K}, M = 58.12$.

$$\begin{aligned} \bullet \quad P_{R1} &= \frac{P_1}{P_c} = \frac{50 \text{ bar}}{38 \text{ bar}} = 1.32 \\ T_{R1} &= \frac{T_1}{T_c} = \frac{500 \text{ K}}{425 \text{ K}} = 1.18 \end{aligned} \quad \left. \vphantom{\begin{aligned} P_{R1} &= \frac{P_1}{P_c} \\ T_{R1} &= \frac{T_1}{T_c} \end{aligned}} \right\} \text{Fig. A-2: } V_{R1}' = 0.6$$

$$\Rightarrow V_1 = V_{R1}' \left(\frac{R T_c}{P_c} \right) = 0.6 \left(\frac{8314 \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}}{58.12} \right) (425 \text{ K}) = 0.0096 \frac{\text{m}^3}{\text{kg}}$$

$$\begin{aligned} \bullet \quad P_{R2} &= \frac{P_2}{P_c} = \frac{30}{38} = 0.79 \\ T_{R2} &= \frac{T_2}{T_c} = \frac{450}{425} = 1.06 \end{aligned} \quad \left. \vphantom{\begin{aligned} P_{R2} &= \frac{P_2}{P_c} \\ T_{R2} &= \frac{T_2}{T_c} \end{aligned}} \right\} \text{Fig. A-1 } V_{R2}' = 0.99$$

$$\Rightarrow V_2 = \frac{(0.99) (8314/58.12) (425)}{(38 \times 10^5)} = 0.0158 \frac{\text{m}^3}{\text{kg}}$$

To find n , use $P_2 V_2^n = P_1 V_1^n \Rightarrow \left(\frac{V_2}{V_1} \right)^n = \frac{P_1}{P_2} \Rightarrow n = \frac{\ln(P_1/P_2)}{\ln(V_2/V_1)}$

$$\therefore n = \frac{\ln(5/3)}{\ln(0.0158/0.0096)} = 1.025$$

Thus, the work is

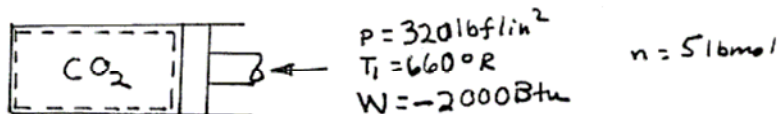
$$\begin{aligned} W &= \frac{m(P_2 V_2 - P_1 V_1)}{1-n} = (5 \text{ kg}) \frac{[(3 \text{ MPa})(0.0158 \text{ m}^3/\text{kg}) - (5)(0.0096)] \left| \frac{10^6 \text{ N/m}^2}{1 \text{ MPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|}{1-1.025} \\ &= 120 \text{ kJ} \end{aligned}$$

PROBLEM 3.98

KNOWN: Five lbmol of CO_2 undergo a constant-pressure compression process in a piston-cylinder assembly. The work, pressure, and initial gas temperature are known.

FIND: Evaluate the final temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The CO_2 is a closed system. 2. Pressure remains constant during the compression process.

ANALYSIS: The final state of the gas is fixed by pressure, 320 lbf/in^2 , and the specific volume $\bar{v}_2$, which can be evaluated from the work.

$$\bar{W} = \int_1^2 p dV = p(\bar{V}_2 - \bar{V}_1) = n p (\bar{v}_2 - \bar{v}_1) \Rightarrow \bar{v}_2 = \bar{v}_1 + \frac{\bar{W}}{n p} \quad (*)$$

$\bar{v}_1$ can be found from the compressibility chart. From Table A-15, $P_c = 72.9 \text{ atm}$, $T_c = 548^\circ\text{R}$. Then

$$P_R = \frac{(320/14.7) \text{ atm}}{72.9 \text{ atm}} \approx 0.30 \quad T_{R1} = \frac{660^\circ\text{R}}{548^\circ\text{R}} \approx 1.2$$

Using these, Fig. A-1 gives $\bar{v}_R' = 4.0$. Then

$$\bar{v}_1 = \bar{v}_R' \left(\frac{\bar{R} T_c}{P_c} \right) = 4.0 \left(\frac{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lbmol} \cdot ^\circ\text{R}})(548^\circ\text{R})}{(72.9 \times 14.7) \text{ lbf/in}^2} \right) \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 21.95 \text{ ft}^3/\text{lbmol}$$

Returning to Eq. (*)

$$\begin{aligned} \bar{v}_2 &= \bar{v}_1 + \frac{\bar{W}}{n p} = 21.95 \frac{\text{ft}^3}{\text{lbmol}} + \frac{(-2000 \text{ Btu}) \left| \frac{778 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}} \right|}{(5 \text{ lbmol})(320 \times 144 \text{ lbf/ft}^2)} \\ &= 15.21 \text{ ft}^3/\text{lbmol} \end{aligned}$$

So, at state 2, $P_{R2} = P_{R1} = 0.30$ and $\bar{v}_{R2}' = \bar{v}_2 P_c / \bar{R} T_c$

$$\bar{v}_{R2}' = \frac{(15.21 \text{ ft}^3/\text{lbmol})(72.9)(14.7)(144) \text{ lbf/ft}^2}{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lbmol} \cdot ^\circ\text{R}})(548^\circ\text{R})} = 2.77$$

Returning to Fig. A-1, $T_{R2} = 0.95$. Thus

$$T_2 = T_{R2} T_c = (0.95)(548^\circ\text{R}) = 521^\circ\text{R}$$

PROBLEM 3.99

The ideal gas equation is accurate for ranges of pressure and temperature for which $z \approx 1$. We might arbitrarily consider the ideal gas model to be satisfactory if $0.95 \leq z \leq 1.05$ (5% deviation). These limits are illustrated on the compressibility chart below.

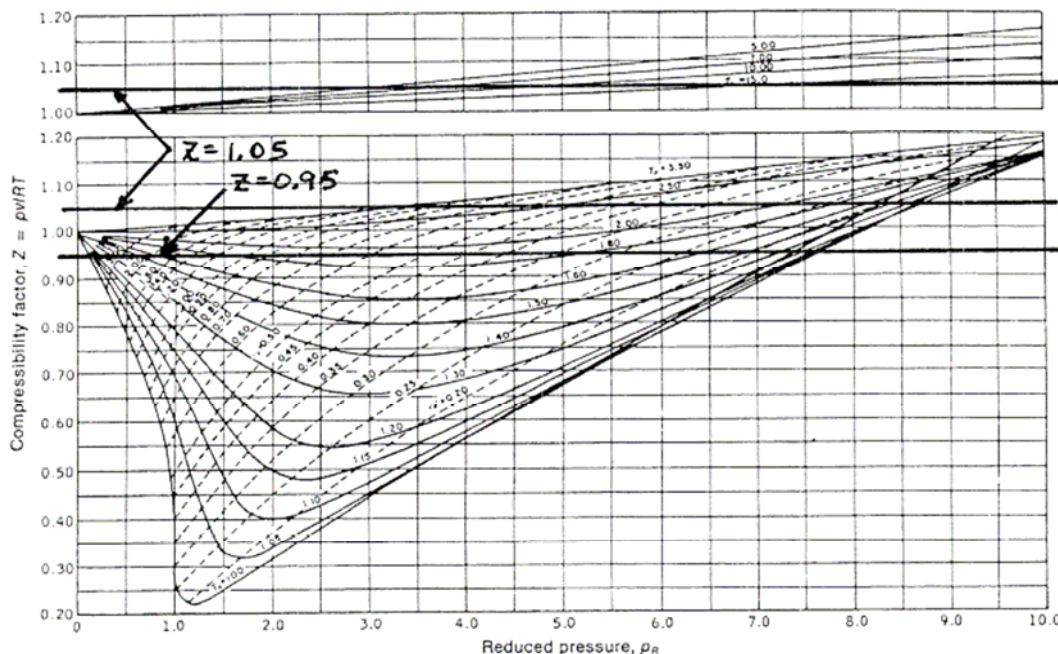


Figure A-2 Generalized compressibility chart, $p_R \leq 10.0$. Source: E. F. Obert, *Concepts of Thermodynamics*, McGraw-Hill, New York, 1960.

For air, Table A-1 gives; $T_c = 133 \text{ K}$, $p_c = 37.7 \text{ bar}$. The following observations can be made from the chart. The ideal gas model is reasonable when

- $p_R < 0.1$, for all temperatures ($p < 3.77 \text{ bar}$)
- for $T_R = 2.0$ ($T = 266 \text{ K}$); $p_R < 7.2$ ($p < 271 \text{ bar}$)
- for $T_R = 2.5$ ($T = 333 \text{ K}$); $p_R < 5.0$ ($p < 189 \text{ bar}$)
- for $T_R = 3.5$ ($T = 466 \text{ K}$); $p_R < 3.5$ ($p < 132 \text{ bar}$)
- for $T_R > 5.0$ ($T = 665 \text{ K}$); $p_R < 3.2$ ($p < 121 \text{ bar}$)
- for $T_R > 15$ ($T = 1995 \text{ K}$); $p_R < 7.0$ ($p < 264 \text{ bar}$)
- for $T_R > 2.0$ ($T = 266 \text{ K}$); $p_R < 3.2$ ($p < 121 \text{ bar}$)

Thus, for many common applications, air can be reasonably modeled as an ideal gas.

PROBLEM 3.100

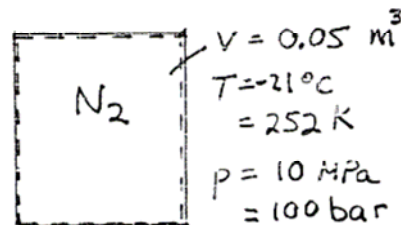
(a) Using the ideal gas equation of state

$$m = \frac{pV}{RT}$$

$$= \frac{(10 \text{ MPa})(0.05 \text{ m}^3)}{\left(\frac{8.314}{28.01} \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(252 \text{ K})} \left| \frac{10^6 \text{ N/m}^2}{1 \text{ MPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$m(\text{ideal gas})$

$$= 6.685 \text{ kg} \leftarrow$$



(b) Using data from Table A-1

$$\left. \begin{aligned} P_R = p/p_c &= \frac{100 \text{ bar}}{33.9 \text{ bar}} = 2.95 \\ T_R = T/T_c &= \frac{252 \text{ K}}{126 \text{ K}} = 2.00 \end{aligned} \right\} \begin{array}{l} \text{Fig. A-2} \\ v_R' \approx 0.65 \end{array}$$

Thus

$$v = v_R' \left(\frac{RT_c}{p_c} \right) = (0.65) \left[\frac{\left(\frac{8.314}{28.01}\right)(126)}{(33.9)} \right] \left(\frac{1}{10^2} \right) = 7.171 \times 10^{-3} \text{ m}^3/\text{kg}$$

and

$$\textcircled{1} \quad m = \frac{V}{v} = \frac{0.05 \text{ m}^3}{7.171 \times 10^{-3} \text{ m}^3/\text{kg}} = 6.973 \text{ kg} \leftarrow m(\text{chart})$$

1. Assuming that the chart value is correct, the ideal gas model under-predicts the mass by about 4.1%.

PROBLEM 3.101

(a) Water vapor at 2000 lbf/in², 700°F

From Table A-4E: $v_{\text{table}} = 0.249 \text{ ft}^3/\text{lb}$

Ideal gas model:

$$v_{\text{ideal gas}} = \frac{\left(\frac{1545}{18.02} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)(1160^\circ\text{R})}{(2000 \text{ lbf/in}^2) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|} = 0.3453 \text{ ft}^3/\text{lb}$$

$$\% \text{ error} = \frac{0.3453 - 0.249}{0.249} \times 100 = 38.7\% \leftarrow (a)$$

(b) Water vapor at 1 lbf/in², 200°F

Table A-4E: $v_{\text{table}} = 392.5 \text{ ft}^3/\text{lb}$

Ideal gas model:

$$v_{\text{ideal gas}} = \frac{\left(\frac{1545}{18.02}\right)(660)}{(1) \left| 144 \right|} = 393.0 \text{ ft}^3/\text{lb}$$

$$\% \text{ error} = \frac{393.0 - 392.5}{392.5} \times 100 = 0.13\% \leftarrow (b)$$

(c) Ammonia at 60 lbf/in², 160°F

Table A-15E: $v_{\text{table}} = 6.3458 \text{ ft}^3/\text{lb}$

Ideal gas model:

$$v_{\text{ideal gas}} = \frac{\left(\frac{1545}{17.04}\right)(620)}{(60)(144)} = 6.5063 \text{ ft}^3/\text{lb}$$

$$\% \text{ error} = \frac{6.5063 - 6.3458}{6.3458} \times 100 = 2.53\% \leftarrow (c)$$

(d) Air at 1 atm, 2000°R

$$P_R = P/P_c = 1 \text{ atm}/37.2 \text{ atm} = 0.027 \left. \begin{array}{l} \text{Fig. A-2} \\ Z \approx 1.0 \end{array} \right\}$$

$$T_R = T/T_c = 2000^\circ\text{R}/239^\circ\text{R} = 8.37$$

Ideal gas $\leftarrow (d)$

(e) R-22 at 300 lbf/in², 140°F

Table A-9E: $v_{\text{table}} = 0.1849 \text{ ft}^3/\text{lb}$

Ideal gas model:

$$v_{\text{ideal gas}} = \frac{\left(\frac{1545}{86.48}\right)(600)}{(300) \left| 144 \right|} = 0.2481 \text{ ft}^3/\text{lb}$$

$$\% \text{ error} = \frac{0.2481 - 0.1849}{0.1849} \times 100 = 34.2\% \leftarrow (e)$$

PROBLEM 3.102

Check the applicability of the ideal gas model for

(a) Water at 700°F and $P_1 = 1600 \text{ lbf/in}^2$, $P_2 = 160 \text{ lbf/in}^2$

① Method 1. Use Steam Table data.

For $T = 700^\circ\text{F}$, $P_1 = 1600 \text{ lbf/in}^2$, Table A-4E gives $v = 0.342 \text{ ft}^3/\text{lb}$.

Using the ideal gas model

$$v = \frac{RT}{P} = \left(\frac{1545 \text{ ft} \cdot \text{lbf}}{18.02 \text{ lb} \cdot ^\circ\text{R}} \right) \left(\frac{1160^\circ\text{R}}{1600 \text{ lbf/in}^2} \right) \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 0.432 \text{ ft}^3/\text{lb}$$

No. The ideal gas model is not applicable.

For $T = 700^\circ\text{F}$, $P_2 = 160 \text{ lbf/in}^2$, Table A-4E gives $v = 4.243 \text{ ft}^3/\text{lb}$.

Using the ideal gas model

$$v = \frac{RT}{P} = \left(\frac{1545}{18.02} \right) \left(\frac{1160}{160} \right) \left| \frac{1}{144} \right| = 4.32 \text{ ft}^3/\text{lb}$$

The ideal gas value is about 2% greater than the steam table value.
For many applications, this would be acceptable.

② Method 2. Use the compressibility chart. $T_c = 1165^\circ\text{R}$, $P_c = 218 \text{ atm}$.

$$T_R = \frac{T}{T_c} = \frac{700 + 460}{1165} \approx 1.0$$

$$P_{R1} = \frac{P_1}{P_c} = \frac{(1600 \frac{\text{lbf}}{\text{in}^2}) \left| \frac{1 \text{ atm}}{14.7 \text{ lbf/in}^2} \right|}{218 \text{ atm}} = 0.5 \Rightarrow Z_1 \approx 0.815$$

No. Not applicable

$$P_{R2} = \frac{P_2}{P_c} = \left(\frac{160}{218} \right) \left| \frac{1}{14.7} \right| = 0.05 \Rightarrow Z_2 \approx 0.98$$

For many applications, this would be acceptable.

(b) CO_2 at 865 K and $P_1 = 75 \text{ bar}$, $P_2 = 15 \text{ bar}$.

Using the compressibility chart, $T_c = 304 \text{ K}$, $P_c = 73.9 \text{ bar}$.

$$T_R = \frac{865}{304} = 2.85$$

$$P_{R1} = \frac{75}{73.9} \approx 1.0 \Rightarrow Z_1 \approx 1.0$$

$$P_{R2} = \frac{15}{73.9} = 0.2 \Rightarrow Z_2 \approx 1.0$$

Ideal gas model is acceptable for many applications at these states.

PROBLEM 3.103

KNOWN: Oxygen is at a state fixed by pressure and specific volume.

FIND: Determine the temperature using generalized compressibility data and compare with the value calculated from the ideal gas model.

ANALYSIS: $p = 250 \text{ bar}$, $v = 0.003 \text{ m}^3/\text{kg}$. With $T_c = 154 \text{ K}$ and $p_c = 50.5 \text{ bar}$ from Table A-1

$$p_R = \frac{250}{50.5} = 4.95, \quad v_R = \frac{v p_c}{R T_c} = \frac{(0.003 \text{ m}^3/\text{kg})(50.5 \times 10^5 \text{ N/m}^2)}{\left(\frac{8314}{32.00} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}\right)(154 \text{ K})} = 0.379$$

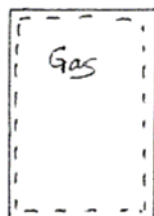
From Fig. A-2, $T_R = 2.0$. Thus $T = T_c T_R = (154 \text{ K})(2.0) = 308 \text{ K}$ $\longleftarrow T$

Using the ideal gas equation of state

$$T = \frac{p v}{R} = \frac{(250 \times 10^5 \text{ N/m}^2)(0.003 \text{ m}^3/\text{kg})}{\left(\frac{8314}{32.00} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}\right)} = 289 \text{ K}$$

This value is 6 % less than the result obtained with compressibility data.

PROBLEM 3.104



$$\begin{aligned} T_1 &= 27^\circ\text{C} \\ P_1 &= 300 \text{ kPa (gage)} \\ T_2 &= 77^\circ\text{C} \\ P_2 &= ? \\ P_{\text{atm}} &= 1 \text{ atm} \end{aligned}$$

Using the ideal gas model,

$$\begin{aligned} P_1 V &= m R T_1 \\ P_2 V &= m R T_2 \end{aligned}$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$P_2 = \left(\frac{T_2}{T_1} \right) P_1$$

In this expression, the temperatures must be in K and the pressures must be on an absolute basis. Thus, $T_1 = 300 \text{ K}$, $T_2 = 350 \text{ K}$ and

$$\begin{aligned} P_1 &= P_{\text{atm}} + P_1(\text{gage}) = 101.325 \text{ kPa} + 300 \text{ kPa} \\ &= 401.325 \text{ kPa} \end{aligned}$$

Finally

$$\begin{aligned} P_2 &= \left(\frac{350}{300} \right) (401.325 \text{ kPa}) \\ &= 468.213 \text{ kPa} \quad (366.89 \text{ kPa (gage)}) \end{aligned}$$

PROBLEM 3.105

KNOWN: A tank of known mass contains air at a specified temperature and pressure.

FIND: Determine the volume of the air. Verify that the ideal gas model applies.

ANALYSIS: $m = 10 \text{ lb}$, $T = 70^\circ\text{F} = 530^\circ\text{R}$, $P = 30 \text{ lbf/in}^2$. With T_c , P_c from Table A-1E
 $T_c = 239^\circ\text{R}$, $P_c = 37.2 \text{ atm}$

$$T_R = \frac{T}{T_c} = \frac{530^\circ\text{R}}{239^\circ\text{R}} = 2.22, \quad P_R = \frac{P}{P_c} = \left(\frac{30/14.7}{37.2} \right) = 0.055$$

Referring to Fig. A-1, $Z \approx 1$. Thus, use of the ideal gas equation of state is justified: $PV = mRT$, or

$$V = \frac{mRT}{P} = \frac{(10 \text{ lb}) \left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right) (530^\circ\text{R})}{(30 \times 144 \text{ lbf/ft}^2)} = 65.4 \text{ ft}^3 \longleftarrow m$$

PROBLEM 3.106

For an ideal gas: $s = p / \left(\frac{\bar{R}}{M} \right) T = pM / \bar{R}T$

For helium and air, each at the same pressure and temperature

$$s_{\text{He}} = pM_{\text{He}} / \bar{R}T$$

$$s_{\text{air}} = pM_{\text{air}} / \bar{R}T$$

Thus

$$\frac{s_{\text{He}}}{s_{\text{air}}} = \frac{M_{\text{He}}}{M_{\text{air}}} = \frac{4.003}{28.97} = 0.138$$

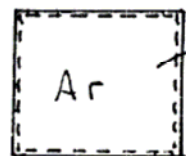
PROBLEM 3.107

- ① Using the ideal gas model to determine the volume

$$V = \frac{n \bar{R} T}{P}$$

$$= \frac{(1 \text{ lbmol}) \left(1545 \frac{\text{ft} \cdot \text{lb}_f}{\text{lbmol} \cdot ^\circ \text{R}} \right) (550 ^\circ \text{R})}{(100 \text{ lb}_f/\text{in}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right|$$

$$= 59.01 \text{ ft}^3$$



$$\begin{aligned} n &= 1 \text{ lbmol} \\ P &= 100 \text{ lb}_f/\text{in}^2 \\ T &= 550 ^\circ \text{R} \end{aligned}$$

-
1. The applicability of the ideal gas model can be verified by referring to the compressibility chart.

PROBLEM 3.108

From Table A-21, $\bar{c}_p / \bar{R} = 5/2$. Then, with Eq. 3.45,
 $\bar{c}_v = \bar{c}_p - \bar{R}$. Accordingly,

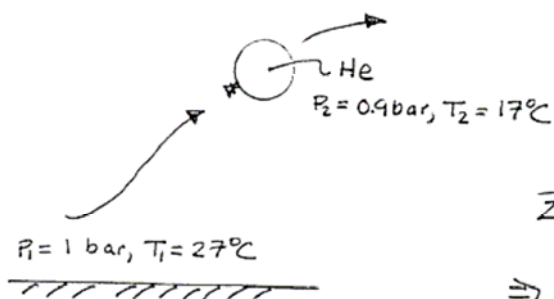
$$\frac{\bar{c}_v}{\bar{R}} = \frac{\bar{c}_p}{\bar{R}} - 1 = \frac{5}{2} - 1 = \frac{3}{2}$$

Finally

$$k = \frac{\bar{c}_p}{\bar{c}_v} = \frac{5/2}{3/2} = 5/3$$

← monatomic gases
adhering to the
ideal gas model

PROBLEM 3.109



$$Z = \frac{pV}{mRT}$$

$$\Rightarrow V = \frac{Z(mRT)}{P}$$

$$\therefore \frac{V_2}{V_1} = \left[\frac{Z_2(mRT_2)/P_2}{Z_1(mRT_1)/P_1} \right]$$

$$= \left(\frac{Z_2}{Z_1} \right) \left(\frac{T_2}{T_1} \right) \left(\frac{P_1}{P_2} \right) \quad (1)$$

For Helium, $T_c = 5.2 \text{ K}$, $P_c = 2.3 \text{ bar}$. Thus, the ideal gas model applies because T_R is very large at both conditions:

$$\begin{aligned} P_{R1} &= \frac{P_1}{P_c} = \frac{1 \text{ bar}}{2.3 \text{ bar}} = 0.43 \\ T_{R1} &= \frac{T_1}{T_c} = \frac{300}{5.2} = 57.7 \end{aligned} \quad \left. \vphantom{\begin{aligned} P_{R1} &= \frac{P_1}{P_c} \\ T_{R1} &= \frac{T_1}{T_c} \end{aligned}} \right\} Z_1 \sim 1.0$$

$$\begin{aligned} P_{R2} &= \frac{P_2}{P_c} = \frac{0.9}{2.3} = 0.39 \\ T_{R2} &= \frac{T_2}{T_c} = \frac{290}{5.2} = 55.8 \end{aligned} \quad \left. \vphantom{\begin{aligned} P_{R2} &= \frac{P_2}{P_c} \\ T_{R2} &= \frac{T_2}{T_c} \end{aligned}} \right\} Z_2 \sim 1.0$$

Finally, inserting values, Eq. (1) gives

$$\frac{V_2}{V_1} = \left(\frac{T_2}{T_1} \right) \left(\frac{P_1}{P_2} \right) = \left(\frac{290 \text{ K}}{300 \text{ K}} \right) \left(\frac{1 \text{ bar}}{0.9 \text{ bar}} \right) = 1.074$$

$$\% = \frac{V_2 - V_1}{V_1} = +0.074 \quad (+7.4\%)$$



PROBLEM 3.110

KNOWN: Methane undergoes a process between two states, each fixed by known values of pressure and temperature.

FIND: Determine Δh by integrating $\bar{c}_p(T)$ from Table A-1 and check the result using $\bar{T}$.

ANALYSIS: $T_1 = 320 \text{ K}$, $P_1 = 2 \text{ bar}$; $T_2 = 800 \text{ K}$, $P_2 = 10 \text{ bar}$. From Table A-1, $T_c = 191 \text{ K}$, $P_c = 46.4 \text{ bar}$. Thus

$$T_{R1} = \frac{320}{191} = 1.68 \quad T_{R2} = \frac{800}{191} = 4.19$$

$$P_{R1} = \frac{2}{46.4} = 0.04 \quad P_{R2} = \frac{10}{46.4} = 0.22$$

Referring to Fig. A-1 at P_{R1}, T_{R1} and P_{R2}, T_{R2} , the ideal gas model is applicable. Thus, $\Delta h = \int_{T_1}^{T_2} \bar{c}_p dT$, where $\bar{c}_p(T)$ is obtained from Table A-21. That is

$$\bar{c}_p(T) = R(\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4)$$

$$\Rightarrow h(T_2) - h(T_1) = \int_{T_1}^{T_2} \bar{c}_p(T) dT = R \int_{T_1}^{T_2} (\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4) dT$$

$$= R \left[\alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5) \right]$$

Inserting values

$$h(T_2) - h(T_1) = \left(\frac{8.314 \text{ kJ}}{\text{kmol} \cdot \text{K}} \right) \left[\frac{3.826}{16.04 \frac{\text{kg}}{\text{kmol}}} (800 - 320) + \left(\frac{-3.979 \times 10^{-3}}{2} \right) (800^2 - 320^2) \right.$$

$$+ \left(\frac{24.558 \times 10^{-6}}{3} \right) (800^3 - 320^3) + \left(\frac{-22.733 \times 10^{-9}}{4} \right) (800^4 - 320^4)$$

$$\left. + \left(\frac{6.963 \times 10^{-12}}{5} \right) (800^5 - 320^5) \right]$$

$$= 1489.3 \text{ kJ/kg} \quad \Delta h$$

IT Code

T1 = 320 // K
T2 = 800 // K
h1 = h_T("CH4", T1) // kJ/kg
h2 = h_T("CH4", T2) // kJ/kg
delh = h2 - h1

IT Result

$\Delta h = 1489 \text{ kJ/kg}$

Alternative solution :

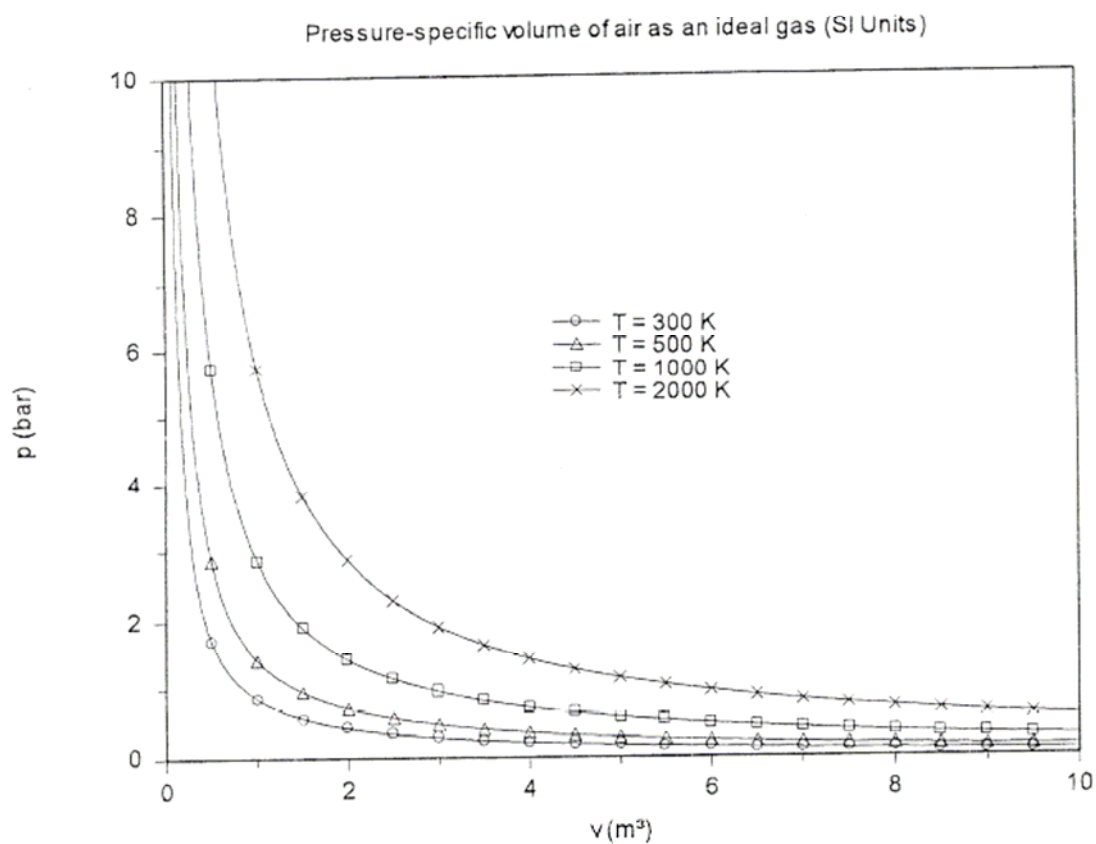
cp = cp_T("CH4", T)

dh = Integral(cp, T)

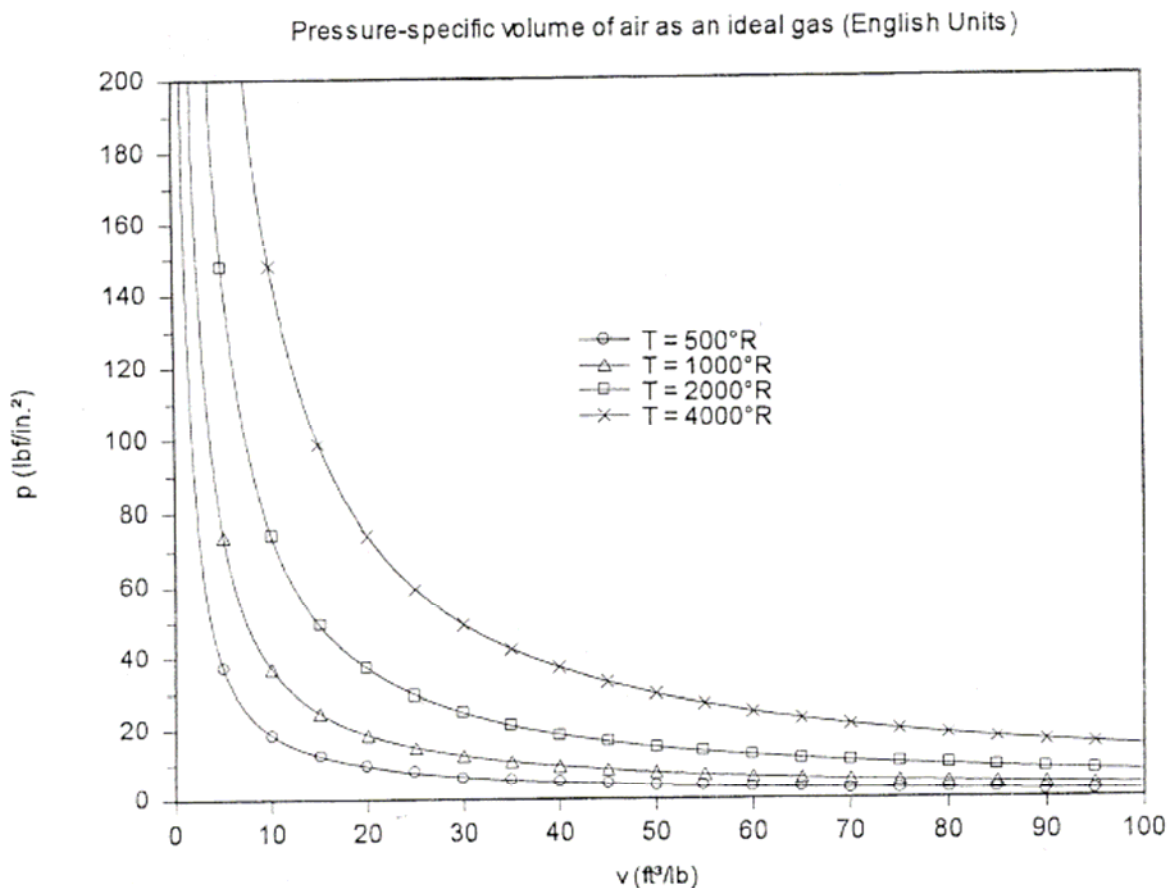
Using Explore button, sweep T from 320 K to 800 K in steps of 1.

The analytical and IT results compare very favorably.

PROBLEM 3.111



PROBLEM 3.112

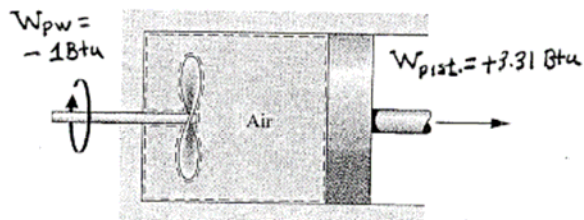


PROBLEM 3.113

KNOWN: Data are provided for air contained in a piston-cylinder assembly fitted with a paddle wheel.

FIND: For the process of the air, find temperature at the final state and Q .

SCHEMATIC & GIVEN DATA:



Initially, $p_1 = 30 \text{ lbf/in}^2$, $T_1 = 540^\circ\text{F}$, $V_1 = 4 \text{ ft}^3$.

Finally, $p_2 = 20 \text{ lbf/in}^2$, $V_2 = 4.5 \text{ ft}^3$.

Fig. P3.113

ENGR. MODEL

1. The air within the piston-cylinder assembly is the closed system.
2. The changes in kinetic and potential energy for the process play no role.
3. Air is modeled as an ideal gas.

ANALYSIS: (a) Using the ideal gas equation of state, $p_1 V_1 = m R T_1$ and $p_2 V_2 = m R T_2$,

$$\frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1} = \left(\frac{4.5 \text{ ft}^3}{4 \text{ ft}^3} \right) \left(\frac{20 \text{ lbf/in}^2}{30 \text{ lbf/in}^2} \right) = 0.75 \Rightarrow T_2 = 0.75 (1000^\circ\text{R}) = 750^\circ\text{R} \leftarrow T_2$$

(b) Reducing an energy balance, $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$, giving

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

where $W_{12} = W_{pw} + W_{pist.} = (-1 \text{ Btu}) + (+3.31 \text{ Btu}) = 2.31 \text{ Btu}$. The specific internal energy values are obtained from Table A-22E: $u_1 = 172.43 \text{ Btu/lb}$, $u_2 = 128.25 \text{ Btu/lb}$. The mass m , is obtained from the ideal gas equation of state,

$$m = \frac{p_1 V_1}{(\bar{R}/M) T_1} = \frac{(30 \times 144 \frac{\text{lbf}}{\text{ft}^2})(4 \text{ ft}^3)}{(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}})(1000^\circ\text{R})} = 0.324 \text{ lb}$$

Finally

$$Q_{12} = 2.31 \text{ Btu} + (0.324 \text{ lb})(128.25 - 172.43) \frac{\text{Btu}}{\text{lb}} = -12 \text{ Btu}$$

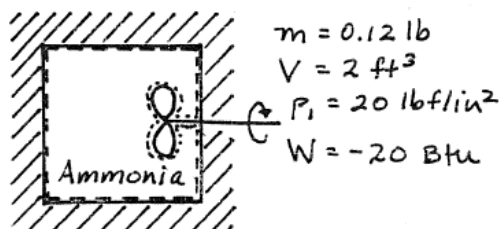
$\leftarrow Q_{12}$

PROBLEM 3.114

KNOWN: A paddle wheel transfers energy to ammonia contained in a closed, rigid, well-insulated tank.

FIND: Determine the final temperature of the ammonia.

SCHEMATIC & GIVEN DATA:



- ENGR. MODEL:** (1) The ammonia is a closed system. (2) The ammonia behaves as an ideal gas. (3) There is no heat transfer. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: To determine the final temperature, begin with the energy balance

$$\Delta \overset{\circ}{K}E + \Delta \overset{\circ}{P}E + \Delta U = \overset{\circ}{Q} - W \quad (*)$$

Evaluating ΔU

$$\Delta U = m \Delta u = m \int_{T_1}^{T_2} c_v(T) dT$$

From Table A-21E: $c_p = R(3.591 + 0.274 \times 10^{-3}T + 2.576 \times 10^{-6}T^2 - 1.437 \times 10^{-9}T^3 + 0.2601 \times 10^{-12}T^4)$

with $c_v = c_p - R$

$$\begin{aligned} \Delta U &= mR \int_{T_1}^{T_2} (2.591 + 0.274 \times 10^{-3}T + 2.576 \times 10^{-6}T^2 - 1.437 \times 10^{-9}T^3 + 0.2601 \times 10^{-12}T^4) dT \\ &= (0.12 \text{ lb}) \left(\frac{1545 \text{ ft} \cdot \text{lbf}}{17.04 \text{ lb} \cdot ^\circ\text{R}} \right) \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \left[2.591 (T_2 - T_1) + \frac{0.274 \times 10^{-3}}{2} (T_2^2 - T_1^2) \right. \\ &\quad \left. + \frac{2.576 \times 10^{-6}}{3} (T_2^3 - T_1^3) - \frac{1.437 \times 10^{-9}}{4} (T_2^4 - T_1^4) + \frac{0.2601 \times 10^{-12}}{5} (T_2^5 - T_1^5) \right] \end{aligned} \quad (**)$$

The initial temperature is

$$T_1 = \frac{P_1 V}{mR} = \frac{(20 \text{ lbf/in}^2)(2 \text{ ft}^3)}{(0.12 \text{ lb}) \left(\frac{1545 \text{ ft} \cdot \text{lbf}}{17.04 \text{ lb} \cdot ^\circ\text{R}} \right)} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 529.4^\circ\text{R}$$

Combining the starred relations and inserting values, T_2 becomes the only unknown. Using an equation solver such as IT,

$$T_2 = 920.1^\circ\text{R} \leftarrow T_2$$

1. For state 1, $P_{R1} = (20)/(111.3 \cdot 14.7) = 0.00122$, $T_{R1} = (529.4)/(830) = 0.72$. From Fig. A-1; $z_1 \approx 1$. Thus, assumption 2 is reasonable for state 1. By similar considerations, it can be concluded that the ideal gas model is reasonable for state 2 as well.

PROBLEM 3.115

KNOWN: Data are provided for nitrogen contained within a piston-cylinder assembly and undergoing a process.

FIND: For the process, evaluate Q .

SCHEMATIC & GIVEN DATA:

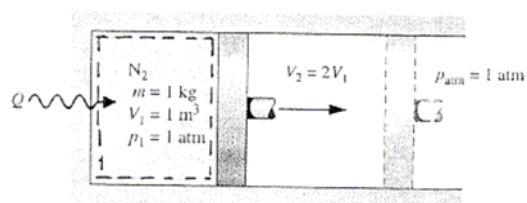


Fig. P3.115

ENGR. MODEL:

1. The nitrogen is the system.
2. The heating occurs slowly, so there is no acceleration of the piston.
3. Friction between the piston and cylinder can be ignored.
4. For the process $\Delta KE = \Delta PE = 0$.
5. The specific heat ratio is constant, and the ideal gas model applies.

ANALYSIS: Since $P_R = P/P_c = 1/37.2 = 0.03$, the ideal gas model is applicable (Sec. 3.11). Also, with Assumptions 2 and 3, the process occurs at a constant pressure of 1 atm. Applying an energy balance, $\Delta U + \Delta KE + \Delta PE = Q - W$. So $Q = \Delta U + W$. In this case, W can be found from Eq. 2.17:

$$W = \int_1^2 p dV = p \Delta V. \text{ Collecting results}$$

$$Q = m(u_2 - u_1) + p(V_2 - V_1) \quad (1)$$

With Eq. 3.50, $(u_2 - u_1) = c_v(T_2 - T_1)$. Thus

$$Q = m c_v(T_2 - T_1) + p(V_2 - V_1) \quad (1)$$

Since $P_1 V_1 = m R T_1$ and $P_2 V_2 = m R T_2$, $T_2 = \left(\frac{P_2 V_2 / m R}{P_1 V_1 / m R} \right) T_1 = \frac{V_2}{V_1} T_1$. Thus

$T_2 = 2T_1$, where

$$T_1 = \frac{P_1 V_1}{m R} = \frac{(1.01325 \times 10^5 \text{ N/m}^2)(1 \text{ m}^3)}{(1 \text{ kg}) \left(\frac{8314 \text{ N} \cdot \text{m}}{28.01 \text{ kg} \cdot \text{K}} \right)} = 341.4 \text{ K}$$

$$\text{So, } T_2 = 2(341.4 \text{ K}) = 682.8 \text{ K}.$$

The specific heat c_v can be found from Eq. 3.47b,

$$c_v = \frac{R}{k-1} = \left(\frac{8.314 \text{ kJ}}{28.01 \text{ kg} \cdot \text{K}} \right) \left(\frac{1}{1.4-1} \right) = 0.742 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Substituting data into Eq. (1) gives

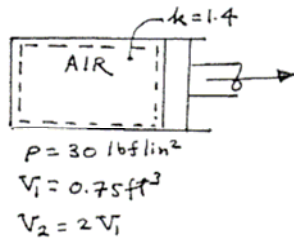
$$\begin{aligned} Q &= (1 \text{ kg}) \left(0.742 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (682.8 - 341.4) \text{ K} + \left(1.01325 \times 10^5 \frac{\text{N}}{\text{m}^2} \right) (2 \text{ m}^3 - 1 \text{ m}^3) \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= 354.6 \text{ kJ} \end{aligned}$$

PROBLEM 3.116

KNOWN: Data are provided for air contained within a piston-cylinder assembly.

FIND: For the process of the air, find W and Q .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The air in the piston-cylinder assembly is the closed system.
2. For the air, the ideal gas model with $k=1.4$ applies.
3. No change in kinetic or potential energy occurs during the process.
4. Volume change is the only work mode.

ANALYSIS:

- (a) The work is evaluated from $W_{12} = \int_1^2 p dV$. Since pressure is constant

$$W_{12} = p[V_2 - V_1] = 30 \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| [0.75 \text{ ft}^3] \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= 4.16 \text{ Btu} \quad \leftarrow W_{12}$$

- (b) Reducing an energy balance, $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$, or

$$\textcircled{1} \quad Q_{12} = W_{12} + m(u_2 - u_1) \quad (1)$$

With Eqs. 3.47b and 3.50, $c_v = R/(k-1)$ and $(u_2 - u_1) = c_v[T_2 - T_1]$, giving

$$Q_{12} = W_{12} + m \left[\frac{R}{k-1} \right] (T_2 - T_1) \quad (2)$$

The ideal gas equation of state also gives $pV_1 = mRT_1$ and $pV_2 = mRT_2$. Thus, $p(V_2 - V_1) = mR(T_2 - T_1)$, and Eq. 2 becomes

$$Q_{12} = W_{12} + \frac{p(V_2 - V_1)}{k-1}$$

$$= 4.16 \text{ Btu} + \frac{4.16 \text{ Btu}}{1.4-1}$$

$$= 14.56 \text{ Btu} \quad \leftarrow Q_{12}$$

1. Since $W_{12} = m p(V_2 - V_1)$, Eq. (1) can be expressed as

$$Q_{12} = m[(u_2 - u_1) + p(V_2 - V_1)]$$

$$= m[(u_2 + pV_2) - (u_1 + pV_1)]$$

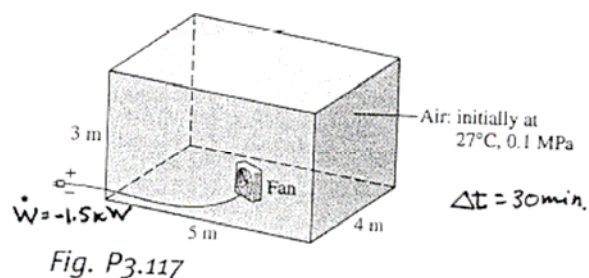
$$= m[h_2 - h_1]$$

PROBLEM 3.117

KNOWN: Data are provided for air within an enclosure fitted with a fan.

FIND: For the air, determine the mass, final temperature, and final pressure.

SCHEMATIC & GIVEN DATA:



ENG. MODEL

1. The contents of the enclosure are the closed system.
2. The volume occupied by the fan can be ignored and the fan stores no energy.
3. The air can be modeled as an ideal gas.
4. For the system, $\dot{Q} = 0$ and there are no overall changes in kinetic or potential energy.

ANALYSIS:

(a) Using the ideal gas equation of state with assumption 2, $P_1 V = m R T_1$, or

$$m = \frac{P_1 V}{R T_1} = \frac{(0.1 \text{ MPa})(60 \text{ m}^3)}{\left(\frac{8.314 \text{ N}\cdot\text{m}}{28.97 \text{ kg}\cdot\text{K}}\right)(300 \text{ K})} \left| \frac{10^6 \text{ N/m}^2}{1 \text{ MPa}} \right| = 69.69 \text{ kg} \quad \leftarrow m$$

(b) Reducing an energy balance, $\Delta U + \Delta \cancel{E_{\text{fan}}} + \Delta \cancel{KE} + \Delta \cancel{PE} = \cancel{Q} - W$.
Thus

$$-W = m[u_2 - u_1] \Rightarrow u_2 = u_1 - \frac{W}{m} \quad (1)$$

where $W = \int \dot{W} dt = \dot{W} \Delta t$, or

$$W = \dot{W} \Delta t = (-1.5 \text{ kW})(30 \text{ min}) \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = -2700 \text{ kJ}$$

Also, from Table A-22, $u_1 = 214.07 \text{ kJ/kg}$. Thus, Eq. (1) gives

$$u_2 = 214.07 \frac{\text{kJ}}{\text{kg}} - \frac{(-2700 \text{ kJ})}{69.69 \text{ kg}} = 252.81 \frac{\text{kJ}}{\text{kg}}$$

Interpolating in Table A-22, $T_2 = 354 \text{ K} (81^\circ \text{C})$ $\leftarrow T_2$

(c) Using the ideal gas equation of state again, $P_1 V = m R T_1$ and

$P_2 V = m R T_2$,

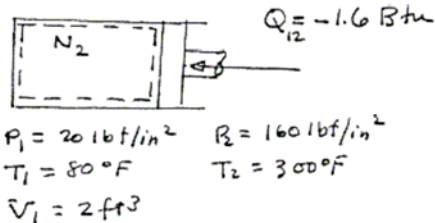
$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow P_2 = \frac{T_2}{T_1} P_1 = \left(\frac{354}{300}\right)(0.1 \text{ MPa})$$

$$= 0.12 \text{ MPa} \quad \leftarrow P_2$$

PROBLEM 3.118

KNOWN: Data are provided for nitrogen contained within a piston-cylinder assembly.
FIND: Determine the mass of the nitrogen and W for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The nitrogen in the piston-cylinder assembly is the closed system.
2. For the nitrogen the ideal gas model applies.
3. The changes in kinetic and potential energy play no role.

ANALYSIS:

(a) Using the ideal gas equation of state

$$m = \frac{P_1 V_1}{R T_1} = \frac{(20 \times 144 \frac{\text{lbf}}{\text{ft}^2})(2 \text{ ft}^3)}{\left(\frac{1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot \text{mol} \cdot \text{R}}}{28.01 \frac{\text{lb}}{\text{lbmol}}}\right)(540^\circ\text{R})} = 0.193 \text{ lb} \quad \leftarrow m$$

(b) Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$, or

$$W_{12} = Q_{12} + m(u_2 - u_1)$$

The specific internal energy values can be obtained from Table A-23E, where they are given on a molar basis. Thus

$$\begin{aligned}
 W_{12} &= Q_{12} - m \left[\frac{\bar{u}_2 - \bar{u}_1}{M} \right] \\
 &= -1.6 \text{ Btu} - 0.193 \text{ lb} \left[\frac{(3774.9 - 2678) \text{ Btu/lbmol}}{28.01 \text{ lb/lbmol}} \right] \\
 &= -9.16 \text{ Btu} \quad \leftarrow W_{12}
 \end{aligned}$$

PROBLEM 3.119

KNOWN: Data are provided for helium contained in a piston-cylinder assembly.

FIND: Determine the energy transfer by heat sufficient for the piston to start rising from rest on a set of stops.

SCHEMATIC & GIVEN DATA:

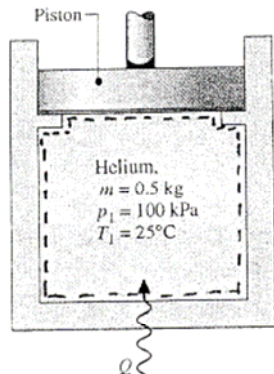


Fig. P3.119

ENGR MODEL:

1. The helium is the closed system.
2. For the system, $W=0$ and $\Delta KE = \Delta PE = 0$.
3. The ideal gas model applies for the helium.

ANALYSIS: Heating occurs until the pressure of the helium becomes 500 kPa.

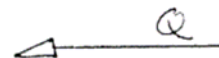
The heating occurs at constant volume. Thus, $p_1 V = m R T_1$ and $p_2 V = m R T_2$, giving $T_2/T_1 = p_2/p_1$. That is, $T_2 = T_1 \left(\frac{p_2}{p_1} \right) = 298 \text{ K} \left(\frac{500 \text{ kPa}}{100 \text{ kPa}} \right) = 1490 \text{ K}$.

An energy balance reads, $\Delta U + \Delta KE + \Delta PE = Q - W$ or

$Q = m(u_2 - u_1)$. Since helium is a monatomic gas, its specific heat c_p is constant: $c_p = 5/2 R$ (see Table A-21). Then, with Eq. 3.44, $c_v = 3/2 R$.

Thus,

$$\begin{aligned} Q &= m c_v (T_2 - T_1) = m \left(\frac{3}{2} R \right) (T_2 - T_1) \\ &= (0.5 \text{ kg}) \left(\frac{3}{2} \cdot \frac{8.314}{4.003} \right) (1490 - 298) \text{ K} \\ &\quad \uparrow \text{M from Table A-1} \\ &= 1857 \text{ kJ} \end{aligned}$$

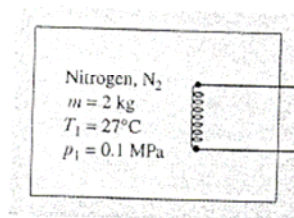


PROBLEM 3.120

KNOWN: Data are provided for nitrogen contained in a tank fitted with an electrical resistor.

FIND: For the nitrogen, determine the final temperature and pressure.

SCHEMATIC & GIVEN DATA:



$$Q = -12.59 \text{ kJ}$$

$$\dot{W} = -0.12 \text{ kW}$$

$$\Delta t = 10 \text{ min}$$

Fig. P3.120

ENGR. MODEL:

1. The nitrogen within the tank is the closed system.
2. The resistor has negligible mass.
3. The nitrogen can be modeled as an ideal gas.
4. For the process of the nitrogen kinetic and potential energy play no role.

ANALYSIS: Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$.

Thus

$$\Delta U = Q - W \Rightarrow n(\bar{u}_2 - \bar{u}_1) = Q - W$$

where n denotes the amount of mass on a molar basis: $n = m/M$. Solving for $\bar{u}_2$

$$\bar{u}_2 = \bar{u}_1 + \frac{Q - W}{(m/M)} \quad (1)$$

where $\bar{u}_1 = 6229 \text{ kJ/kmol}$ from Table A-23 and $W = \int \dot{W} dt = \dot{W} \Delta t$:

$$W = (-0.12 \text{ kW})(10 \text{ min}) \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = -72 \text{ kJ}$$

Inserting values in Eq. (1)

$$\begin{aligned} \bar{u}_2 &= 6229 \frac{\text{kJ}}{\text{kmol}} + \frac{(-12.59 \text{ kJ}) - (-72 \text{ kJ})}{\left(\frac{2 \text{ kg}}{28.01 \text{ kg/kmol}} \right)} \\ &= 7061 \frac{\text{kJ}}{\text{kmol}} \end{aligned}$$

From Table A-23, we get $T_2 = 340 \text{ K} (67^\circ \text{C})$

T_2

Using the ideal gas equation of state, $P_1 V = m R T_1$, $P_2 V = m R T_2$.

Thus,

$$\begin{aligned} \frac{P_2}{P_1} &= \frac{T_2}{T_1} \Rightarrow P_2 = P_1 \left(\frac{T_2}{T_1} \right) \\ &= (0.1 \text{ MPa}) \left(\frac{340 \text{ K}}{300 \text{ K}} \right) \\ &= 0.11 \text{ MPa} \end{aligned}$$

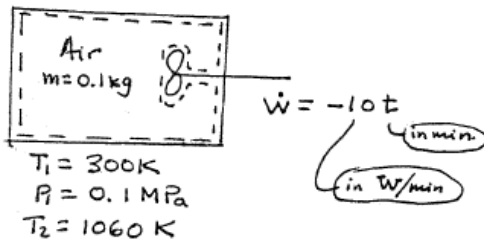
P_2

PROBLEM 3.121

KNOWN: Data are provided for air contained within a piston-cylinder assembly fitted with a paddlewheel.

FIND: For the air, find the final pressure, W and Q .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The air in the piston-cylinder assembly is the closed system.
2. The air is modeled as an ideal gas.
3. No changes in kinetic or potential energy occur.

ANALYSIS: (a) Using the ideal gas equation of state $P_1 V = m R T_1$ and $P_2 V = m R T_2$, we get

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow P_2 = P_1 \left(\frac{T_2}{T_1} \right) = (0.1 \text{ MPa}) \left(\frac{1060 \text{ K}}{300 \text{ K}} \right) = 0.3531 \text{ MPa} \leftarrow P_2$$

(b)

$$\begin{aligned} W_{12} &= \int_0^{t=20 \text{ min}} \dot{W} dt = \int_0^{t=20 \text{ min}} (-10t) dt = -\frac{10t^2}{2} \Big|_0^{t=20 \text{ min}} \\ &= -\frac{10 \text{ W/min}}{2} [20 \text{ min}]^2 = -2000 \text{ W} \cdot \text{min} \left| \frac{60 \text{ s}}{1 \text{ min}} \right| \left| \frac{1 \text{ J/s}}{1 \text{ W}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ J}} \right| = -120 \text{ kJ} \leftarrow W_{12} \end{aligned}$$

(c) Reducing an energy balance $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$, or

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

with data from Table A-22, $u_1 = 214.07 \text{ kJ/kg}$, $u_2 = 810.62 \text{ kJ/kg}$,

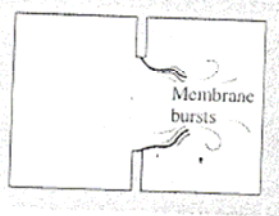
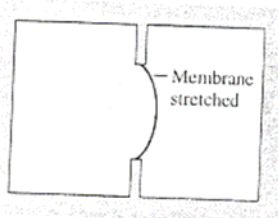
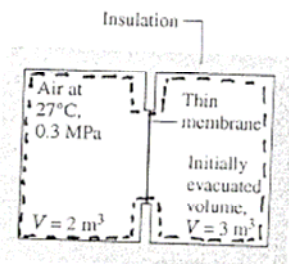
$$\begin{aligned} Q_{12} &= -120 \text{ kJ} + 0.1 \text{ kg} (810.62 - 214.07) \frac{\text{kJ}}{\text{kg}} \\ &= -60.35 \text{ kJ} \leftarrow Q_{12} \end{aligned}$$

PROBLEM 3.122

KNOWN: Data is provided for air on one side of a rigid, insulated container. The other side of the container is initially evacuated.

FIND: For the air, determine the mass and the final temperature and pressure.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

(a) Using the ideal gas model equation of state,

$$m = \frac{P_1 V_1}{R T_1} = \frac{(0.3 \text{ MPa})(2 \text{ m}^3)}{\left(\frac{8314 \text{ N}\cdot\text{m}}{28.97 \text{ kg}\cdot\text{K}}\right)(300 \text{ K})} \left| \frac{10^6 \text{ N/m}^2}{1 \text{ MPa}} \right| = 6.97 \text{ kg} \leftarrow m$$

(b) Reducing an energy balance,

$$\Delta U + \cancel{\Delta U_{\text{Membrane}}} + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$$

$$\Rightarrow \Delta U = m(u_2 - u_1) = 0$$

$$\Rightarrow u_2 = u_1$$

Since internal energy depends on temperature alone for an ideal gas,

$$T_2 = T_1 = 300 \text{ K} \leftarrow T_2$$

(c) Using the ideal gas model equation of state again,

$$P_1 V_1 = m R T$$

$$P_2 V_2 = m R T$$

$$\Rightarrow P_2 V_2 = P_1 V_1$$

$$\text{or } P_2 = P_1 \left[\frac{V_1}{V_2} \right]$$

$$= 0.3 \text{ MPa} \left[\frac{2 \text{ m}^3}{5 \text{ m}^3} \right]$$

$$= 0.12 \text{ MPa} \leftarrow P_2$$

ENGR. MODEL:

1. The closed system is region within the container.
2. The air is modeled as an ideal gas.
3. The mass of the membrane can be ignored.
4. There are no significant kinetic or potential energy effects.

PROBLEM 3.123

KNOWN: Data are provided for air on one side of a rigid container. The other side of the container is initially evacuated.

FIND: For the air, determine the final temperature and Q .

SCHEMATIC & GIVEN DATA:

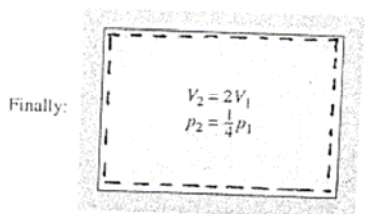
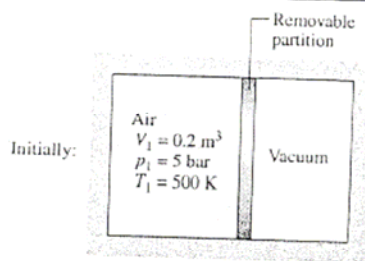


Fig. P3.123

ENGR. MODEL:

1. The closed system is the region within the container, ignoring the partition.
2. The air is modeled as an ideal gas.
3. There are no overall changes in kinetic or potential energy.
4. $W = 0$.

ANALYSIS:

- (a) Using the ideal gas model equation of state, $p_1 V_1 = m R T_1$,
 $p_2 V_2 = m R T_2$. Thus,

$$T_2 = T_1 \left[\frac{p_2 V_2}{p_1 V_1} \right] = 500 \text{ K} \left[\frac{1}{4} \right] [2] = 250 \text{ K} \quad \leftarrow T_2$$

- (b) An energy balance reduces to $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$, or

$$Q = m (u(T_2) - u(T_1))$$

where

$$m = \frac{p_1 V_1}{R T_1} = \frac{(5 \times 10^5 \text{ N/m}^2)(0.2 \text{ m}^3)}{\left(\frac{8314}{28.97} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (500 \text{ K})} = 0.7 \text{ kg}$$

So, with data from Table A-22

$$Q = 0.7 \text{ kg} (178.28 - 359.49) \frac{\text{kJ}}{\text{kg}}$$

$$= -126.8 \text{ kJ} \quad \leftarrow Q$$

PROBLEM 3.124

KNOWN: Air and carbon monoxide are contained to opposite sides of a rigid, insulated container by a partition free to move.

FIND: Determine the final equilibrium temperature and pressure. Also, find the volume occupied by each gas finally.

SCHEMATIC & GIVEN DATA:

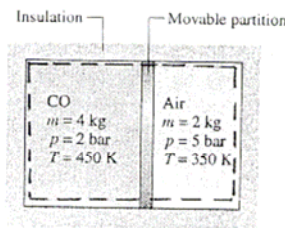


Fig. P3.124

ENGR. MODEL:

1. The system is the container contents.
2. For the system, $W = Q = 0$ and there is no effect of kinetic and potential energy.
3. The partition is free to move, allows conduction from one gas to the other, and experiences no change in energy.
4. Each gas is modeled as an ideal gas with $k = 1.395$.

ANALYSIS: (a) An energy balance reads $\Delta U + \Delta KE + \Delta PE = Q - W$, or $\Delta U = 0$, where $\Delta U = [\Delta U]_{CO} + [\Delta U]_{Air} + [\Delta U]_{partition}$. That is,

$$[\Delta U]_{CO} + [\Delta U]_{Air} = 0$$

With Eq. 3.50 this becomes

$$(m c_v)_{CO} [T - 450 K] + (m c_v)_{Air} [T - 350 K] = 0$$

Using Eq. 3.47b

$$c_{v, CO} = \frac{R}{k-1} = \frac{(8.314/28.01) \text{ kJ/kg} \cdot \text{K}}{1.395-1} = 0.751 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$c_{v, Air} = \frac{R}{k-1} = \frac{(8.314/28.97) \text{ kJ/kg} \cdot \text{K}}{1.395-1} = 0.727 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Solving,

$$T = \frac{(m c_v)_{CO} (450 K) + (m c_v)_{Air} (350 K)}{(m c_v)_{CO} + (m c_v)_{Air}}$$

$$= \frac{(4 \text{ kg})(0.751 \text{ kJ/kg} \cdot \text{K})(450 K) + (2 \text{ kg})(0.727 \text{ kJ/kg} \cdot \text{K})(350 K)}{(4 \text{ kg})(0.751 \text{ kJ/kg} \cdot \text{K}) + (2 \text{ kg})(0.727 \text{ kJ/kg} \cdot \text{K})} = 417.4 \text{ K}$$

(b) The total volume remains constant, where $V = V_{CO} + V_{Air} = 3.07 \text{ m}^3$, and

$$V_{Air} = \left(\frac{m R T}{P} \right)_{Air} = \frac{(2 \text{ kg}) \left(\frac{8.314}{28.97} \right) \left(\frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (350 \text{ K})}{(5 \times 10^5 \text{ N/m}^2)} = 0.4 \text{ m}^3$$

$$V_{CO} = \left(\frac{m R T}{P} \right)_{CO} = \frac{(4) \left(\frac{8.314}{28.01} \right) (450)}{(2 \times 10^5)} = 2.67 \text{ m}^3$$

At equilibrium, each gas is at $T = 417.4 \text{ K}$ and pressure P . With $V = m R T / P$,

$$3.07 \text{ m}^3 = \frac{(m R)_{Air} T}{P} + \frac{(m R)_{CO} T}{P} \Rightarrow P = \frac{[(m R)_{Air} + (m R)_{CO}] T}{3.07 \text{ m}^3}$$

Calculating

$$P = \frac{\left[\frac{2 \text{ kg} \left(\frac{8.314}{28.97} \right) \left(\frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) + 4 \text{ kg} \left(\frac{8.314}{28.01} \right) \left(\frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) \right] 417.4 \text{ K}}{3.07 \text{ m}^3} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right|$$

$$= 2.39 \text{ bar}$$

Continued on next slide

Problem 3-124 continued

(c) The final volumes occupied by the gas are

$$\bar{V}_{\text{Air}} = \frac{m_{\text{Air}} R T}{P} = \frac{(2 \text{ kg}) \left(\frac{8314 \text{ N}\cdot\text{m}}{28.97 \text{ kg}\cdot\text{K}} \right) (417.4 \text{ K})}{2.39 \times 10^5 \text{ N/m}^2} = 1 \text{ m}^3$$

$$\bar{V}_{\text{CO}} = \frac{m_{\text{CO}} R T}{P} = \frac{(4 \text{ kg}) \left(\frac{8314 \text{ N}\cdot\text{m}}{28.01 \text{ kg}\cdot\text{K}} \right) (417.4 \text{ K})}{2.39 \times 10^5 \text{ N/m}^2} = 2.07 \text{ m}^3$$

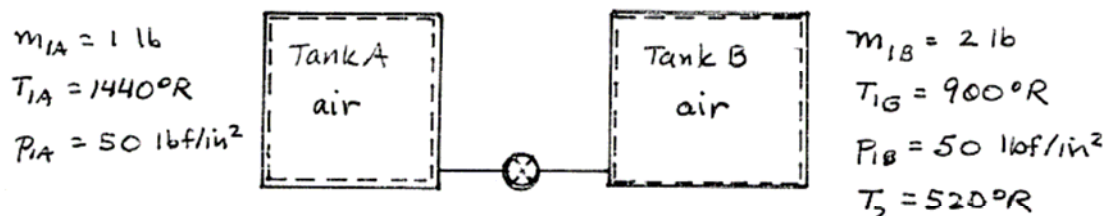
Note, $V_{\text{Air}} + V_{\text{CO}} = 3.07 \text{ m}^3$, the total volume occupied initially.

PROBLEM 3.125

KNOWN: The contents of two uninsulated tanks are allowed to mix, and equilibrium is attained at the temperature of the surroundings.

FIND: Determine the amount of energy transfer by heat and the final pressure.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The contents of the two tanks are a closed system. (2) There is no work. (3) The air behaves as an ideal gas. (4) Kinetic and potential energy effects can be neglected.

ANALYSIS: To find the heat transfer, begin with the energy balance.

$$\Delta \overset{\circ}{KE} + \Delta \overset{\circ}{PE} + \Delta U = Q - \overset{\circ}{W}$$

Since the final state is an equilibrium state

$$\Delta U = (m_A + m_B) u_2 - (m_A u_A + m_B u_B)$$

Thus

$$Q = (m_A + m_B) u_2 - (m_A u_A + m_B u_B)$$

Using data from Table A-22E

$$Q = (3 \text{ lb})(88.62 \text{ Btu/lb}) - [(1)(254.66) + (2)(154.57)] = -297.9 \text{ Btu} \leftarrow Q$$

To find the final pressure, first determine the volume of each tank

$$V_A = \frac{m_A R T_A}{P_A} = \frac{(1 \text{ lb}) \left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right) (1440^\circ\text{R})}{(50 \text{ lbf/in}^2) \left[144 \text{ in}^2 / 1 \text{ ft}^2 \right]} = 10.66 \text{ ft}^3$$

$$V_B = \frac{(2) \left(\frac{1545}{28.97} \right) (900)}{(50) \left[144 / 1 \right]} = 13.33 \text{ ft}^3$$

Thus, at the final state

$$P_2 = \frac{m R T_2}{(V_A + V_B)} = \frac{(3) \left(\frac{1545}{28.97} \right) (520)}{(10.66 + 13.33) \left[144 / 1 \right]} = 24.07 \text{ lbf/in}^2 \leftarrow P_2$$

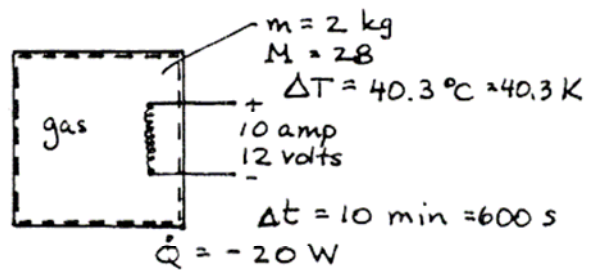
PROBLEM 3.126

KNOWN: A gas is contained in a closed rigid tank fitted with an electric resistor. The constant current and voltage and the constant heat transfer rate are known.

FIND: Determine an average value of specific heat c_p .

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The gas is a closed system. (2) The gas behaves as an ideal gas. (3) Kinetic and potential energy effects are negligible.



ANALYSIS: To determine the specific heat, begin with the energy balance

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

with $\Delta U = m c_v \Delta T = m (c_p - R) \Delta T$, we get

$$c_p = \left(\frac{Q - W}{m \Delta T} \right) + R \quad \text{using Eq. 3.44}$$

Next, evaluate the heat transfer and work, respectively

$$Q = \int_{t_1}^{t_2} \dot{Q} dt = \dot{Q} \Delta t = (-20 \text{ W})(600 \text{ s}) \left| \frac{1 \text{ kJ/s}}{10^3 \text{ W}} \right|$$

$$= -12 \text{ kJ}$$

$$W = \int_{t_1}^{t_2} \dot{W} dt = \dot{W} \Delta t$$

$$= -(10 \text{ amp})(12 \text{ volts}) \left| \frac{1 \text{ W/amp}}{1 \text{ volt}} \right| (600 \text{ s}) \left| \frac{1 \text{ kJ/s}}{10^3 \text{ W}} \right|$$

$$= -72 \text{ kJ}$$

Thus

$$c_p = \left(\frac{(-12 \text{ kJ}) - (-72 \text{ kJ})}{(2 \text{ kg})(40.3 \text{ K})} \right) + \left(\frac{8.314}{28} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right)$$

$$= 1.041 \text{ kJ/kg} \cdot \text{K} \quad \leftarrow c_p$$

PROBLEM 3.127

KNOWN: Data are provided for CO_2 contained in an assembly consisting of a piston-cylinder connected to a tank.

FIND: For the CO_2 , evaluate Q and W .

SCHEMATIC & GIVEN DATA:

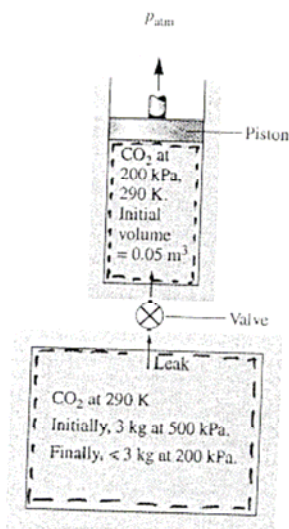


Fig. P3.127

ENGR MODEL:

1. The total amount of CO_2 is the closed system.
2. The CO_2 is model as an ideal gas.
3. Volume change is the only work mode.
4. The CO_2 temperature remains constant.
5. The pressure of the CO_2 in the cylinder remains constant.
6. For the CO_2 there is no overall change in kinetic or potential energy.

ANALYSIS: Since volume change is the only work mode, $W = \int p dV$, where p is the pressure acting on the piston by the CO_2 , which is 200 kPa. Thus

$$W = p \Delta V = (200 \times 10^3 \frac{\text{N}}{\text{m}^2}) \Delta V \quad (1)$$

where ΔV is the change in volume of the CO_2 .

Initial Volume: $V_{\text{initial}} = V_{\text{cylinder}} + V_{\text{tank}} = 0.05 \text{ m}^3 + \frac{m_T RT}{P_T}$

$$= 0.05 \text{ m}^3 + \left(\frac{8314 \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}}{44.01 \frac{\text{kg}}{\text{kmol}}} \right) \frac{(3 \text{ kg})(290 \text{ K})}{(500 \text{ kPa}) \left| \frac{10^3 \text{ N/m}^2}{\text{kPa}} \right|}$$

$$= 0.379 \text{ m}^3$$

Final Volume: $V_{\text{final}} = \frac{m \bar{R} T}{P}$, where m is the total mass of CO_2 . The initial mass in the tank is 3 kg. The initial mass in the cylinder is

$$m_{\text{cyl}} = \frac{P V_{\text{cyl}}}{R T} = \frac{(200 \times 10^3 \text{ N/m}^2)(0.05 \text{ m}^3)}{\left(\frac{8314 \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}}{44.01 \frac{\text{kg}}{\text{kmol}}} \right) (290 \text{ K})} = 0.183 \text{ kg.}$$

So, $m = 3 + 0.183 = 3.183 \text{ kg}$. And

$$V_{\text{final}} = \frac{(3.183 \text{ kg}) \left(\frac{8314 \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}}{44.01 \frac{\text{kg}}{\text{kmol}}} \right) (290 \text{ K})}{(200 \times 10^3 \text{ N/m}^2)} = 0.872 \text{ m}^3$$

Thus, Eq. (1) gives

$$W = (200 \times 10^3 \frac{\text{N}}{\text{m}^2}) (0.872 \text{ m}^3 - 0.379 \text{ m}^3) \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 98.6 \text{ kJ} \quad \leftarrow W$$

An energy balance reads, $\Delta U + \Delta KE + \Delta PE = Q - W$, where $\Delta U = 0$ because the temperature of the CO_2 remains constant and the internal energy depends on temperature alone for an ideal gas. Thus

$$Q = W = 98.6 \text{ kJ} \quad \leftarrow Q$$

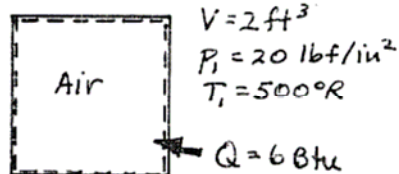
PROBLEM 3.128

KNOWN: Heat transfer occurs to air in a rigid tank. The initial state is specified.

FIND: Determine the final temperature and pressure using (a) a constant specific value, (b) a specific heat function, (c) the air tables.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The air is a closed system. (2) The air behaves as an ideal gas. (3) There is no work. (4) Kinetic and potential energy effects are negligible.



ANALYSIS: To determine the final temperature, begin with the energy balance

$$\cancel{\Delta K\dot{E}} + \cancel{\Delta P\dot{E}} + \Delta U = Q - \cancel{W}$$

$$\text{or} \quad m(u_2 - u_1) = Q \Rightarrow u_2 - u_1 = Q/m \quad (*)$$

Evaluating the mass

$$m = \frac{PV}{RT} = \frac{(20 \text{ lbf/in}^2)(2 \text{ ft}^3)}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)(500^\circ\text{R})} \left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right) = 0.216 \text{ lb}$$

(a) For a constant specific heat c_v

$$u_2 - u_1 = c_v(T_2 - T_1)$$

Thus, with (*)

$$T_2 = \frac{Q}{mc_v} + T_1$$

From Table A-20E, at $T = 100^\circ\text{F}$ (600°R), $c_v = 0.172 \text{ Btu/lb} \cdot ^\circ\text{R}$. Thus

$$T_2 = \frac{(6 \text{ Btu})}{(0.216 \text{ lb})(0.172 \text{ Btu/lb} \cdot ^\circ\text{R})} + 500^\circ\text{R}$$

$$= 661.5^\circ\text{R} \quad \leftarrow (a) T_2$$

Accordingly, for this temperature interval the value used for c_v is appropriate. The pressure P_2 is

$$P_2 = \frac{mRT_2}{V} = \frac{(0.216)\left(\frac{1545}{28.97}\right)(661.5)}{(2)(144)} = 26.46 \text{ lbf/in}^2 \quad \leftarrow (a) P_2$$

① (b) From Table A-21E

$$c_p = R(\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4)$$

$$\text{with } c_p = c_v + R$$

$$c_v = R(\alpha - 1 + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4)$$

Continued on next slide

Problem 3-128 continued

Thus

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v dT$$

$$= R \left[(\alpha - 1)(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5) \right]$$

Inserting values

$$u_2 - u_1 = \left(\frac{1.986 \text{ Btu/lb mol} \cdot ^\circ\text{R}}{28.97 \text{ lb/lb mol}} \right) \left[2.653(T_2 - 500) - \left(\frac{0.742 \times 10^{-3}}{2} \right) (T_2^2 - 500^2) \right.$$

$$\left. + \left(\frac{1.017 \times 10^{-6}}{3} \right) (T_2^3 - 500^3) - \left(\frac{0.328 \times 10^{-9}}{4} \right) (T_2^4 - 500^4) + \left(\frac{0.02632 \times 10^{-12}}{5} \right) (T_2^5 - 500^5) \right]$$

(**)

Combining the starred relations and inserting values, T_2 becomes the only unknown. Solving iteratively

$$T_2 = 661.8^\circ\text{R} \leftarrow (b) T_2$$

and

$$P_2 = 26.47 \text{ lbf/in}^2 \leftarrow (b) P_2$$

(c) From Table A-22E, $u_1 = 85.20 \text{ Btu/lb}$. From (*)

$$u_2 = Q/m + u_1$$

$$= \left(\frac{6 \text{ Btu}}{0.216 \text{ lb}} \right) + 85.20 = 112.98 \text{ Btu/lb}$$

Interpolating in Table A-22E

$$T_2 = 661.8^\circ\text{R} \leftarrow (c) T_2$$

and

$$P_2 = 26.47 \text{ lbf/in}^2 \leftarrow (c) P_2$$

① IT solution (based on part b)

$$T_1 = 500 \text{ // } ^\circ\text{R}$$

$$p_1 = 20 \text{ // lbf/in}^2$$

$$V = 2 \text{ // ft}^3$$

$$Q = 6 \text{ // Btu}$$

$$\text{delu} = Q/m$$

$$m = V/v_1$$

$$v_1 = v_{\text{TP}}(\text{"Air"}, T_1, p_1)$$

$$\text{delu} = R * ((\alpha - 1)(T_2 - T_1) + (\beta/2)(T_2^2 - T_1^2) + (\gamma/3)(T_2^3 - T_1^3) + (\delta/4)(T_2^4 - T_1^4) + (\epsilon/5)(T_2^5 - T_1^5))$$

$$R = 1.986/28.97 \text{ // Btu/lb} \cdot ^\circ\text{R}$$

$$\alpha = 3.653$$

$$\beta = -0.742\text{E-}3$$

$$\gamma = 1.017\text{E-}6$$

$$\delta = -0.328\text{E-}9$$

$$\epsilon = 0.02632\text{E-}12$$

$$v_2 = v_1$$

$$v_2 = v_{\text{TP}}(\text{"Air"}, T_2, p_2)$$

IT Results

$$T_2 = 661.8^\circ\text{R}$$

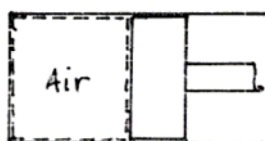
$$p_2 = 26.47 \text{ bar}$$

PROBLEM 3.129

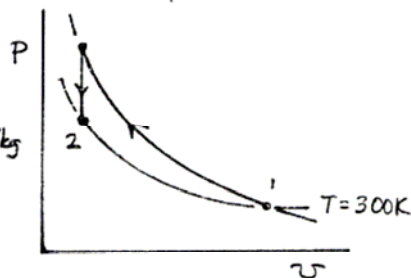
KNOWN: Air undergoes an adiabatic compression followed by constant volume cooling.

FIND: Determine the work for the first process and the heat transfer for the second one, using (a) air table data, (b) constant specific heats.

SCHEMATIC & GIVEN DATA:



Process 1-2: adiabatic
 $P_1 = 1 \text{ bar}$, $T_1 = 300 \text{ K}$
 $P_2 = 15 \text{ bar}$, $v_2 = .1227 \text{ m}^3/\text{kg}$
 Process 2-3: Constant volume
 $T_3 = 300 \text{ K}$



ENGR. MODEL: (1) The air is a closed system. (2) There is no heat transfer for the first process and no work for the second one. (3) The air behaves as an ideal gas. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, using the ideal gas equation of state to determine T_2

$$T_2 = \frac{P_2 v_2}{R} = \frac{(15 \text{ bar})(.1227 \text{ m}^3/\text{kg})}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|}$$

$$= 641.3 \text{ K}$$

(a) Using the energy balance for each process

Process 1-2: $\cancel{\Delta KE} + \cancel{\Delta PE} + m(u_2 - u_1) = \cancel{Q_{12}} - W_{12}$

$$\frac{W_{12}}{m} = u_1 - u_2$$

From Table A-22, $u_1 = 214.07 \text{ kJ/kg}$ and, interpolating, $u_2 = 466.5 \text{ kJ/kg}$. Thus

$$\frac{W_{12}}{m} = 214.07 - 466.5 = -252.4 \text{ kJ/kg} \leftarrow \frac{W_{12}}{m}$$

Process 2-3: $\cancel{\Delta KE} + \cancel{\Delta PE} + m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}$

$$\frac{Q_{23}}{m} = u_3 - u_2$$

Since $T_3 = T_1$, $u_3 = u_1$, and $Q_{23}/m = W_{12}/m = -252.4 \text{ kJ/kg} \leftarrow \frac{Q_{23}}{m}$

(b) Assuming a constant value for c_v

$$\frac{W_{12}}{m} = c_v (T_1 - T_2)$$

From Table A-20, at 300 K, $c_v = 0.718 \text{ kJ/kg} \cdot \text{K}$. Thus

$$\frac{W_{12}}{m} = (0.718 \text{ kJ/kg} \cdot \text{K})(300 - 641.3) \text{ K}$$

$$= -245 \text{ kJ/kg} \leftarrow \frac{W_{12}}{m}$$

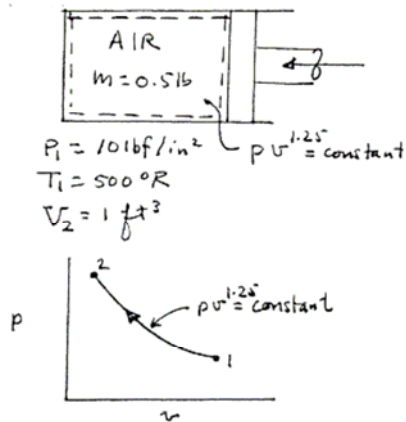
and $\frac{Q_{23}}{m} = \frac{W_{12}}{m} = -245 \text{ kJ/kg} \leftarrow \frac{Q_{23}}{m}$

PROBLEM 3.130

KNOWN: Data are provided for air contained in a piston-cylinder assembly.

FIND: Using constant specific heats and data from Table A-22E, find W and Q .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The air within the piston-cylinder assembly is the closed system.
2. The process is described by $p v^{1.25} = \text{constant}$.
3. Volume change is the only work mode.
4. Kinetic and potential energy play no role.
5. The air is modeled as an ideal gas: Part (a). Specific heats are regarded as constant at their values at 500 R. Part (b). Table A-22E data is used.

ANALYSIS: Work is evaluated from $W_{12} = \int_1^2 p dV = \frac{m(P_2 V_2 - P_1 V_1)}{1-n}$ (see

Example 2.1 for details). Using the ideal gas equation of state,

$$v_1 = \frac{RT}{P_1} = \frac{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)(500^\circ\text{R})}{(10)(144) \frac{\text{lbf}}{\text{ft}^2}} = 18.52 \text{ ft}^3/\text{lb}.$$

From given data, $v_2 = 2 \text{ ft}^3/\text{lb}$. Since $p v^{1.25} = \text{constant}$, the pressure at state 2 is

$$P_2 = \left(\frac{v_1}{v_2}\right)^n P_1 = \left(\frac{18.52}{2}\right)^{1.25} (10 \text{ lbf/in}^2) = 161.5 \text{ lbf/in}^2$$

The temperature at state 2 is found from the ideal gas equation of state:

$$T_2 = \frac{P_2 v_2}{R} = \frac{(161.5)(144) \frac{\text{lbf}}{\text{ft}^2} (2 \text{ ft}^3/\text{lb})}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)} = 872^\circ\text{R}.$$

Thus, the work is

$$W_{12} = \frac{m(P_2 V_2 - P_1 V_1)}{1-n} = \frac{(0.5 \text{ lb}) \left[(161.5)(144)(2) - (10)(144)(18.52) \right] \frac{\text{ft} \cdot \text{lbf}}{\text{lb}}}{1-1.25} \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = -51.0 \text{ Btu}$$

Reducing an energy balance gives $Q_{12} = W_{12} + m(u_2 - u_1)$.

Constant c_v : $(u_2 - u_1) = c_v(T_2 - T_1)$. Taking c_v at 500 R from Table A-20E, $c_v = 0.171 \text{ Btu/lb} \cdot ^\circ\text{R}$. Thus

$$Q_{12} = -51.0 + 0.5 \text{ lb} \left(0.171 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (872 - 500)^\circ\text{R} = -19.2 \text{ Btu}$$

Table A-22E: $u_1 = 85.2 \text{ Btu/lb}$, $u_2 = 149.61 \text{ Btu/lb}$,

$$Q_{12} = -51.0 + 0.5(149.61 - 85.2) = -18.8 \text{ Btu}$$

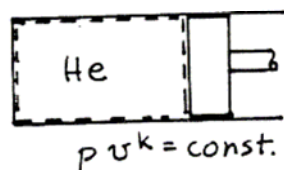
In this case, the assumption of constant c_v results in a value for Q that departs by about 2% from that obtained using the ideal gas table, A-22E. For a similar process evaluation at a higher temperature at state 2, a greater departure can be expected, however.

PROBLEM 3.131

KNOWN: Helium gas undergoes a process for which $p v^k = \text{constant}$ from a given initial state to a specified final pressure.

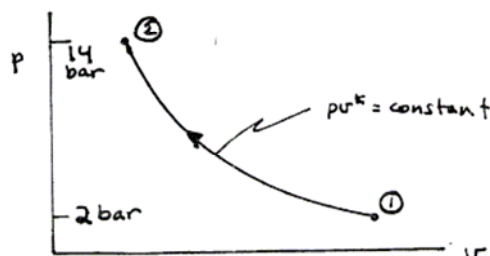
FIND: Determine the work and heat transfer, each per unit of mass.

SCHEMATIC & GIVEN DATA:



$$P_1 = 2 \text{ bar}, T_1 = 200 \text{ K}$$

$$P_2 = 14 \text{ bar}$$



ENGR. MODEL: 1. The helium is a closed system. 2. The process is described by $p v^k = \text{constant}$. 3. The helium behaves as an ideal gas. 4. Kinetic and potential energy changes are ignored.

ANALYSIS: From Table A-21, $\bar{c}_p = 2.5 \bar{R}$. Using Eq. 3.45, $\bar{c}_v = \bar{c}_p - \bar{R} = 1.5 \bar{R}$. Thus $k = \bar{c}_p / \bar{c}_v = 2.5 / 1.5 = 1.667$.

With assumptions 2 and 3,

$$W = \int_1^2 p dv = m \int_1^2 \frac{C}{v^k} dv \Rightarrow \frac{W}{m} = \frac{P_2 v_2 - P_1 v_1}{1-k} = \frac{R(T_2 - T_1)}{1-k} \quad (\text{Eq. 3.57})$$

To find T_2 , use Eq. 3.56 with $n=k$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = 200 \text{ K} \left(\frac{14}{2} \right)^{0.667/1.667} = 575 \text{ K}$$

Inserting values,

$$\frac{W}{m} = \frac{R(T_2 - T_1)}{(1-k)}$$

$$= \frac{(8.314) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (575 - 200) \text{ K}}{(1 - 1.667)} = -1167.7 \text{ kJ} \leftarrow \frac{W}{m}$$

Next, the heat transfer is

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

$$Q = m(u_2 - u_1) + W$$

$$\text{or} \quad \frac{Q}{m} = (u_2 - u_1) + \frac{W}{m}$$

For a monatomic gas, c_v is constant. Thus

$$\frac{Q}{m} = c_v(T_2 - T_1) + \frac{R(T_2 - T_1)}{1-k}$$

With $c_v = R/(k-1)$ (Eq. 3.47b)

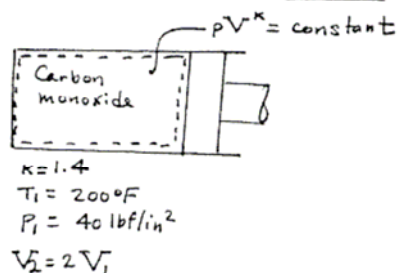
$$\frac{Q}{m} = \left(\frac{R}{k-1} \right) (T_2 - T_1) + \left(\frac{R}{1-k} \right) (T_2 - T_1) = 0 \leftarrow \frac{Q}{m}$$

PROBLEM 3.132

KNOWN: Data are provided for CO undergoing a process while contained in a piston-cylinder assembly.

FIND: For the CO, determine the final temperature and pressure. Also evaluate Q/m and W/m for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The CO is the closed system.
2. The process of the CO is described by $pV^k = \text{constant}$.
3. For the process, $\Delta KE = \Delta PE = 0$.
4. For CO the ideal gas model with constant specific heat ratio $k = 1.4$ applies.
5. Volume change is the only work mode.

ANALYSIS:

- (a) For CO modeled as an ideal gas undergoing a process described by $pV^k = \text{constant}$, we have from Eqs. 3.56,

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^k \quad \text{and} \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \quad \text{Thus,}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^k = 40 \frac{\text{lbf}}{\text{in}^2} (0.5)^{1.4} = 15.16 \frac{\text{lbf}}{\text{in}^2} \quad \leftarrow P_2$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{k-1} = 660 \text{ K} (0.5)^{(1.4-1)} = 500.2^\circ\text{R} \quad (40^\circ\text{F}) \quad \leftarrow T_2$$

- (b) Since volume change is the only work mode, $W_{12} = \int_1^2 p dV$, which with Eq. 3.57 gives

$$\frac{W_{12}}{m} = \frac{R(T_2 - T_1)}{1 - k} = \frac{(1.986 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}})}{28.01 \frac{\text{lb}}{\text{lb-mol}}} \frac{(500.2 - 660) \text{ K}}{1 - 1.4} = 28.33 \frac{\text{Btu}}{\text{lb}} \quad \leftarrow \frac{W_{12}}{m}$$

An energy balance reduces to $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$

$$\Rightarrow Q_{12} = \Delta U + W_{12} = m(u_2 - u_1) + W_{12} \quad \left(\frac{W_{12}}{m} = \frac{R(T_2 - T_1)}{1 - k} \right)$$

With Eq. 3.47b, $c_v = R/(k-1)$. Thus, $(u_2 - u_1) = [R/(k-1)](T_2 - T_1)$, and we get

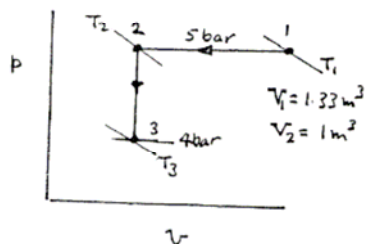
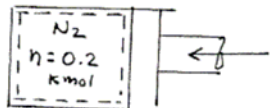
$$\frac{Q_{12}}{m} = \frac{R(T_2 - T_1)}{k-1} + \frac{R(T_2 - T_1)}{1-k} = 0 \quad \leftarrow \frac{Q_{12}}{m}$$

PROBLEM 3.133

KNOWN: Data are provided for N_2 contained in a piston-cylinder assembly. The N_2 undergoes two processes in series.

FIND: For each process find W and Q

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The N_2 contained in the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The N_2 is modeled as an ideal gas.
4. Kinetic and potential energy effects play no role.

ANALYSIS: Process 1-2 occurs at constant pressure. Thus $W_{12} = \int_1^2 p dV = p[V_2 - V_1]$.

That is

$$W_{12} = (5 \text{ bar}) [1 \text{ m}^3 - 1.33 \text{ m}^3] \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = -165 \text{ kJ} \quad \leftarrow W_{12}$$

Process 2-3 occurs at constant volume. Thus $W_{23} = 0$.

$\leftarrow W_{23}$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$. Thus

$Q = W + n\Delta\bar{u}$. This requires T_1, T_2, T_3 . To find T_1 use the ideal gas equation of state:

$$T_1 = \frac{P_1 V_1}{nR} = \frac{(5 \times 10^5 \frac{\text{N}}{\text{m}^2})(1.33 \text{ m}^3)}{(0.2 \text{ kmol})(\frac{8314 \text{ N}\cdot\text{m}}{\text{kmol}\cdot\text{K}})} = 400 \text{ K}.$$

For process 1-2, $p = \text{constant}$; so the ideal gas equation of state gives

$$pV_1 = nRT_1, \quad pV_2 = nRT_2, \quad \text{or}$$

$$T_2 = T_1 \left[\frac{V_2}{V_1} \right] = 400 \left[\frac{1 \text{ m}^3}{1.33 \text{ m}^3} \right] = 301 \text{ K}$$

For process 2-3, $V = \text{constant}$; so $p_2 V = nRT_2$, $p_3 V = nRT_3$ or

$$T_3 = \left(\frac{p_3}{p_2} \right) T_2 = \left(\frac{4 \text{ bar}}{5 \text{ bar}} \right) (301 \text{ K}) = 241 \text{ K}$$

For process 1-2, with $\bar{u}_1$ and $\bar{u}_2$ from Table A-23

$$Q_{12} = W_{12} + n[\bar{u}_2 - \bar{u}_1] = -165 \text{ kJ} + 0.2 \text{ kmol} [6250 - 8314] \frac{\text{kJ}}{\text{kmol}} = -578 \text{ kJ} \quad \leftarrow Q_{12}$$

$$\text{For process 2-3 } Q_{23} = \cancel{W_{23}} + n[\bar{u}_3 - \bar{u}_2] = 0.2 \text{ kmol} [5000 - 6250] \frac{\text{kJ}}{\text{kmol}} = -250 \text{ kJ} \quad \leftarrow Q_{23}$$

PROBLEM 3.134

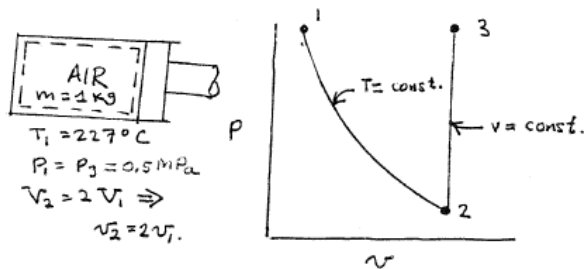
KNOWN: Data are provided for air in a piston-cylinder assembly. The air undergoes two processes in series:

Process 1-2: Constant-temperature expansion until the volume is twice the initial volume.

Process 2-3: Constant-volume heating until the pressure is again 0.5 MPa.

FIND: Sketch the processes in series on a p - v diagram. Determine P_2 , T_3 , and W and Q for each process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The air within the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The air is modeled as an ideal gas.
4. Kinetic and potential energy play no significant role.

ANALYSIS: (a) For process 1-2, $P_1 v_1 = RT_1$ and $P_2 v_2 = RT_2$, where $v_2 = 2v_1$ and $T_2 = T_1$.

Thus, $P_2 v_2 = P_1 v_1$ and

$$P_2 = P_1 \left(\frac{v_1}{v_2} \right) = 0.5 \text{ MPa} \left(\frac{1}{2} \right) = 0.25 \text{ MPa} \quad \leftarrow P_2$$

(b) For process 2-3, $P_2 v_2 = RT_2$ and $P_3 v_3 = RT_3$, where $v_3 = v_2$. Thus,

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} \Rightarrow T_3 = T_2 \left(\frac{P_3}{P_2} \right) = 500 \text{ K} \left(\frac{0.5 \text{ MPa}}{0.25 \text{ MPa}} \right) = 1000 \text{ K} \quad \leftarrow T_3$$

(c) For process 1-2, $W_{12} = \int_1^2 p dv = m \int_1^2 p dv = mRT \int_1^2 \frac{1}{v} dv$. Thus

$$W_{12} = mRT \ln \frac{v_2}{v_1} = (1 \text{ kg}) \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (500 \text{ K}) \ln 2 = 99.46 \text{ kJ} \quad \leftarrow W_{12}$$

Reducing an energy balance, $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$, where

$\Delta U = m(u(T_2) - u(T_1)) = 0$, since $T_1 = T_2$. Accordingly, $Q_{12} = W_{12} = 99.46 \text{ kJ} \quad \leftarrow Q_{12}$

(d) For process 2-3, the piston does not move (volume is constant). Thus, $W_{23} = 0$.

Reducing an energy balance, $\Delta U + \Delta KE + \Delta PE = Q_{23} - W_{23}$. Or

$$Q_{23} = m[u(T_3) - u(T_2)]$$

With data from Table A-22,

$$Q_{23} = (1 \text{ kg}) [758.94 - 359.49] \frac{\text{kJ}}{\text{kg}} = 399.45 \text{ kJ} \quad \leftarrow Q_{23}$$

PROBLEM 3.135

KNOWN: Air contained within a piston-cylinder assembly undergoes a thermodynamic cycle consisting of four processes in series.

FIND: Sketch the cycle on p - v coordinates. Evaluate the work and heat transfer for each process and the thermal efficiency of the overall cycle.

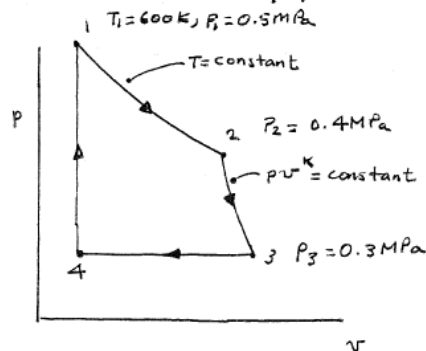
SCHEMATIC & GIVEN DATA:

Process 1-2: Constant-temperature expansion at 600 K from $p_1 = 0.5$ MPa to $p_2 = 0.4$ MPa.

Process 2-3: Polytropic expansion with $n = k$ to $p_3 = 0.3$ MPa.

Process 3-4: Constant-pressure compression to $V_4 = V_1$.

Process 4-1: Constant-volume heating.



ENGR. MODEL:

1. The air is the closed system.
2. The air is modeled as an ideal gas with constant specific heat ratio, $k=1.4$.
3. For each process: $\Delta KE = \Delta PE = 0$.
4. Volume change is the only work mode.

ANALYSIS: (a) Each process is considered in turn:

Process 1-2: $\Delta U + \Delta KE + \Delta PE = Q - W$. Since the temperature remains constant and $u = u(T)$ for an ideal gas, $\Delta U = 0$, leaving $Q = W$. Using Eq. 2.17

$$W_{12} = \int_1^2 p \, dV = \int_1^2 \frac{mRT}{V} \, dV = mRT \ln \frac{V_2}{V_1}. \text{ Thus, using } V = mRT/p$$

$$\begin{aligned} \frac{Q_{12}}{m} &= \frac{W_{12}}{m} = RT \ln \frac{V_2}{V_1} = RT \ln \left[\frac{mRT/p_2}{mRT/p_1} \right] = RT \ln \frac{p_1}{p_2} \\ &= \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (600 \text{ K}) \ln \frac{0.5 \text{ MPa}}{0.4 \text{ MPa}} = 38.42 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow W_{12}, Q_{12} \end{aligned}$$

Process 2-3: When $n = k$, integration of Eq. 2.17 gives $W = \frac{p_2 V_3 - p_3 V_2}{1-k}$ (see Example 2.1).

Then, with $pV = mRT$, this gives

$$\frac{W_{23}}{m} = \frac{R(T_3 - T_2)}{1-k}, \text{ where (Eq. 3.52)} \quad \frac{T_2}{T_3} = \left(\frac{p_2}{p_3} \right)^{\frac{k-1}{k}} \Rightarrow T_3 = 600 \text{ K} \left(\frac{0.3}{0.4} \right)^{\frac{1.4-1}{1.4}} = 553 \text{ K}$$

Then

$$\frac{W_{23}}{m} = \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) \left(\frac{553 - 600}{1-1.4} \right) \text{ K} = 33.72 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow W_{23}$$

Energy balance: $\Delta U + \Delta KE + \Delta PE = Q - W \Rightarrow Q = \Delta U + W$.

Since k is constant, c_v is constant (Eq. 3.47a). Then, with Eq. 3.50 and the expression for W_{23}/m above,

$$\begin{aligned} \frac{Q_{23}}{m} &= c_v(T_3 - T_2) + \frac{R(T_3 - T_2)}{1-k} \equiv 0 \\ &= \left(\frac{R}{k-1} \right) (T_3 - T_2) \end{aligned} \quad \leftarrow Q_{23}$$

Continued on next slide

Problem 3-135 continued

Process 3-4. Since pressure is constant, $W_{34} = P_3 [V_4 - V_3]$. Using $PV = mRT$, this becomes

$$\frac{W_{34}}{m} = R[T_4 - T_3]$$

$$\text{Also, } \left. \begin{array}{l} P_4 V_4 = mRT_4 \\ P_1 V_1 = mRT_1 \end{array} \right\} V_4 = V_1 \Rightarrow \frac{T_4}{T_1} = \frac{P_4}{P_1} \Rightarrow T_4 = \left(\frac{P_4}{P_1}\right) T_1 = 600 \text{ K} \left(\frac{0.3 \text{ MPa}}{0.5 \text{ MPa}}\right) = 360 \text{ K}$$

$$\text{So, } \frac{W_{34}}{m} = \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}}\right)(360 - 553) \text{ K} = -55.39 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow W_{34}$$

Energy Balance: $\Delta U = Q - W \Rightarrow Q = \Delta U + W$. Since c_v is constant

$$\frac{Q_{34}}{m} = c_v(T_4 - T_3) + (W_{34}/m)$$

$$= \frac{R}{\gamma - 1} = \left(\frac{8.314/28.97}{1.4 - 1}\right) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.717 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$= \left(0.717 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(360 - 553) \text{ K} - 55.39 \frac{\text{kJ}}{\text{kg}} = -193.77 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow Q_{34}$$

Process 4-1. Since the piston does not move (volume is constant), $W_{41} = 0$. $\leftarrow W_{41}$

Energy Balance: $\Delta U = Q - W \Rightarrow Q = \Delta U$. Since c_v is constant

$$\frac{Q_{41}}{m} = c_v(T_1 - T_4) = 0.717 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}(600 - 360) = 172.08 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow Q_{41}$$

(b) For any cycle, the thermal efficiency is found from $\eta = W_{\text{cycle}}/Q_{\text{in}}$. Here,

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{(W_{12} + W_{23} + W_{34} + W_{41})/m}{(Q_{12} + Q_{41})/m}$$

$$= \frac{38.42 + 33.72 - 55.39 + 0}{38.42 + 172.08} = \frac{16.75}{210.5} = 0.08 \text{ (8\%)} \quad \leftarrow \eta$$

①

1 For any cycle, $W_{\text{cycle}} = Q_{\text{cycle}}$. In this case, $\frac{W_{\text{cycle}}}{m} = 16.75 \text{ kJ/kg}$.

And

$$Q_{\text{cycle}} = Q_{12} + Q_{23} + Q_{34} + Q_{41} = 38.42 + 0 + (-193.77) + 172.08$$

$$= 16.73 \frac{\text{kJ}}{\text{kg}}$$

which agrees within round off and provides a check on the calculations performed for the four processes.

PROBLEM 3.136

KNOWN: Air undergoes a power cycle consisting of three processes.

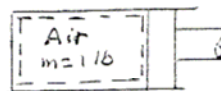
FIND: Sketch the cycle on a p - v diagram and determine (a) the pressure at state 2, (b) the temperature at state 3, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA: The following data are given for each process:

Process 1-2: constant volume from $p_1 = 20 \text{ lbf/in}^2$,
 $T_1 = 500^\circ\text{R}$ to $T_2 = 820^\circ\text{R}$

Process 2-3: Adiabatic expansion to $v_3 = 1.4 v_2$

Process 3-1: Constant-pressure compression.



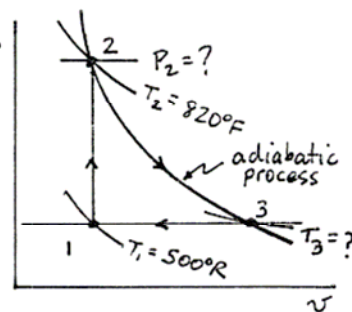
ENGR. MODEL: (1) The air is the closed system. (2) The air behaves as an ideal gas. (3) Kinetic and potential energy effects are neglected. (4) Volume change is the only work mode.

ANALYSIS: (a) For process 1-2, the volume is constant. Thus, using the ideal gas equation of state

$$\begin{aligned} p_1 v &= m R T_1 \\ p_2 v &= m R T_2 \end{aligned} \Rightarrow \frac{p_1}{p_2} = \frac{T_1}{T_2} \Rightarrow p_2 = \frac{T_2}{T_1} p_1$$

$$= \frac{820}{500} (20 \frac{\text{lbf}}{\text{in}^2})$$

$$= 32.8 \frac{\text{lbf}}{\text{in}^2}$$



(b) Again, using the ideal equation of state

$$\frac{v_3}{v_2} = 1.4 = \frac{m R T_3 / p_3}{m R T_2 / p_2} = \frac{T_3}{T_2} \cdot \frac{p_2}{p_3} \Rightarrow T_3 = (1.4) \frac{p_3}{p_2} T_2 = (1.4) \frac{p_1}{p_2} T_2$$

$$\text{Thus } T_3 = (1.4) \left(\frac{20}{32.8} \right) (820) = 700^\circ\text{R}$$

(c) The thermal efficiency is $\eta = W_{\text{cycle}} / Q_{\text{in}}$. Analyze each process in turn.

Process 1-2: const. volume $\Rightarrow W_{12} = 0$.

$$\text{Energy Balance: } \Delta KE + \Delta PE + \Delta U = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1)$$

With data from Table A-22

$$Q_{12} = (1 \text{ lb}) (140.47 - 85.20) \text{ Btu/lb} = 55.27 \text{ Btu}$$

Process 2-3: Adiabatic: $m(u_3 - u_2) = Q_{23} - W_{23}$

$$W_{23} = -m(u_3 - u_2) = - (1 \text{ lb}) (119.58 - 140.47) \text{ Btu/lb} = 20.89 \text{ Btu}$$

Process 3-1: $p = \text{constant}$. Using Eq. 2.17 $W = \int_{v_3}^{v_1} p dv = m p (v_1 - v_3) = m R (T_1 - T_3)$

$$W_{31} = (1 \text{ lb}) \left(\frac{1545}{28.97} \right) \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| (500 - 700)^\circ\text{R} = -13.71 \text{ Btu}$$

$$\text{Energy Balance: } m(u_1 - u_3) = Q_{31} - W_{31} \Rightarrow Q_{31} = m(u_1 - u_3) + W_{31}$$

$$Q_{31} = (1 \text{ lb}) (85.20 - 119.58) \text{ Btu/lb} + (-13.71 \text{ Btu}) = -48.09 \text{ Btu}$$

$$\text{Thus } W_{\text{cycle}} = W_{12} + W_{23} + W_{31} = 0 + 20.89 + (-13.71) = 7.18 \text{ Btu}$$

$$Q_{\text{in}} = Q_{12} = 55.27 \text{ Btu}$$

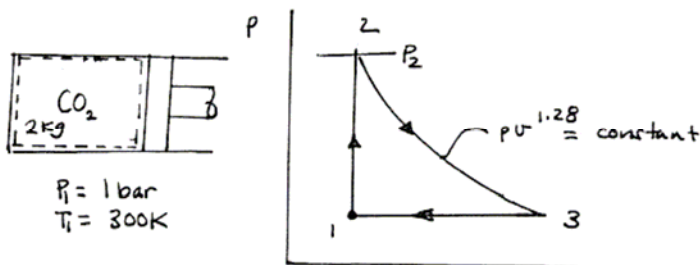
$$\text{Finally } \eta = W_{\text{cycle}} / Q_{\text{in}} = 7.18 / 55.27 = 0.1299 \text{ (12.99\%)} \quad \eta$$

PROBLEM 3.137

KNOWN: A system consisting of 2 kg of CO_2 undergoes a power cycle consisting of three processes.

FIND: Sketch the cycle on a p - v diagram, and plot the thermal efficiency versus P_2/P_1 ranging from 1.05 to 4.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The CO_2 is a closed system. 2. The CO_2 behaves as an ideal gas. 3. Process 2-3 is described by $p v^{1.28} = \text{constant}$. 4. Kinetic and potential energy effects are negligible. 5. Volume change is the only work mode.

ANALYSIS: Begin by developing the key expressions describing the cycle. The thermal efficiency is given by $\eta = W_{\text{net}}/Q_{\text{in}}$, where $W_{\text{net}} = W_{12} + W_{23} + W_{31}$. Since volume is constant $W_{12} = 0$. For process 2-3 and process 3-1

$$W_{23} = m \int_2^3 p dv = m \frac{(P_3 v_3 - P_2 v_2)}{1-n} = m R \frac{(T_3 - T_2)}{1-n} \quad (1)$$

$$W_{31} = m \int_3^1 p dv = m (P_1 v_1 - P_3 v_3) = m R (T_1 - T_3) \quad (2)$$

Applying an energy balance:

$$\text{Process 1-2: } m(u_2 - u_1) = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1) \quad (3)$$

$$\text{Process 2-3: } m(u_3 - u_2) = Q_{23} - W_{23} \Rightarrow Q_{23} = m(u_3 - u_2) + W_{23} \quad (4)$$

$$\text{Process 3-1: } m(u_1 - u_3) = Q_{31} - W_{31} \Rightarrow Q_{31} = m(u_1 - u_3) + W_{31} \quad (5)$$

Additionally, the ideal gas equation of state gives

$$\frac{P_1 V}{P_2 V} = \frac{m R T_1}{m R T_2} \Rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow T_2 = \left(\frac{P_2}{P_1}\right) T_1 \quad (6)$$

and with Eq. 3.56 for process 2-3 ($P_3 = P_1$)

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{(n-1)/n} \Rightarrow T_3 = T_2 \left(\frac{P_1}{P_2}\right)^{(n-1)/n} \quad (7)$$

SAMPLE CALCULATION: $P_2/P_1 = 4$

If $P_2/P_1 = 4$, Eq. (6) gives $T_2 = 4(300) = 1200 \text{ K}$, and Eq. (7) gives

$$T_3 = (1200 \text{ K}) \left(\frac{1}{4}\right)^{.28/1.28} = 886 \text{ K}$$

Eq. (1) then gives

$$W_{23} = \frac{(2 \text{ kg}) \left(\frac{8.314 \text{ kJ}}{44.01 \text{ kg} \cdot \text{K}}\right) (886 - 1200) \text{ K}}{1 - 1.28} = 423.7 \text{ kJ}$$

Continued on next slide

Problem 3-137 continued

Eg. (2) gives

$$W_{31} = (2 \text{ kg}) \left(\frac{8.314 \text{ kJ}}{44.01 \text{ kg} \cdot \text{K}} \right) (300 - 886) \text{ K} = -221.4 \text{ kJ}$$

With data from Table A-23, Eg. (3) gives

$$Q_{12} = \left(\frac{2 \text{ kg}}{44.01 \text{ kJ/kmol}} \right) (43,871 - 6939) \frac{\text{kJ}}{\text{kmol}} = 1678.3 \text{ kJ}$$

Eg. (4) gives

$$Q_{23} = \left(\frac{2}{44.01} \right) (29,298 - 43,871) + 423.7 = -238.6 \text{ kJ}$$

Eg. (5) gives

$$Q_{31} = \left(\frac{2}{44.01} \right) (6939 - 29,298) - 221.4 = -1237.5 \text{ kJ}$$

Since Q_{23} and Q_{31} are both negative, we conclude that $Q_{in} = Q_{12}$. Thus, for the case $p_2/p_1 = 4$

$$\eta = \frac{423.7 + (-221.4)}{1678.3} = 0.121 \text{ (12.1 \%)} \quad \leftarrow \eta_{(p_2/p_1=4)}$$

Turning now to IT:

$$T1 = 300 \text{ // K}$$

$$p1 = 1 \text{ // bar}$$

$$m = 2 \text{ // kg}$$

$$M = 44.01 \text{ // kg/kmol}$$

$$R = 8.314 / M$$

$$W_{23} = m \cdot R \cdot (T3 - T2) / (1 - 1.28)$$

$$W_{31} = m \cdot R \cdot (T1 - T3)$$

$$m \cdot (u2 - u1) = Q_{12}$$

$$W_{cycle} = W_{23} + W_{31}$$

$$\eta = W_{cycle} / Q_{12}$$

$$p2 / p1 = T2 / T1$$

$$p2 = r \cdot p1$$

$$r = 4$$

$$p3 = p1$$

$$T3 / T2 = (p3 / p2)^{(1.28-1) / 1.28}$$

$$u1 = u_T(\text{"CO2"}, T1)$$

$$u2 = u_T(\text{"CO2"}, T2)$$

$$u3 = u_T(\text{"CO2"}, T3)$$

/* Using the Explore button, sweep r from 1.05 to 4 in steps of 0.01. */

IT Results (r=4)

$$Q_{in} = 1678 \text{ kJ}$$

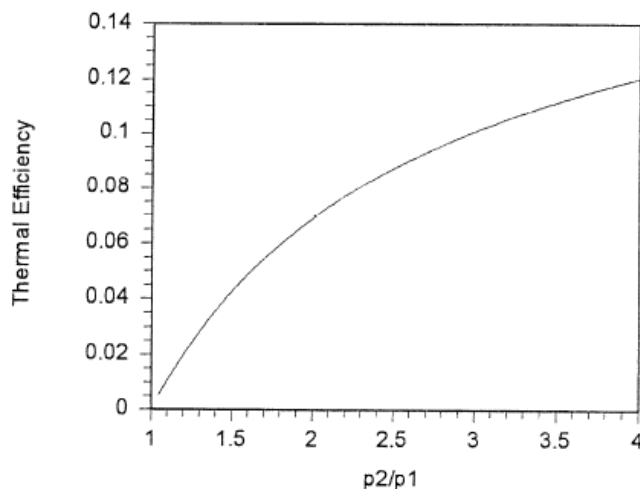
$$T_2 = 1200 \text{ K}$$

$$T_3 = 886.1 \text{ K}$$

$$W_{23} = 423.6 \text{ kJ}$$

$$W_{31} = -221.4 \text{ kJ}$$

$$\eta = 0.1205$$

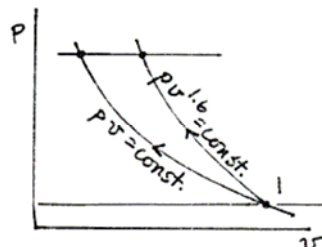
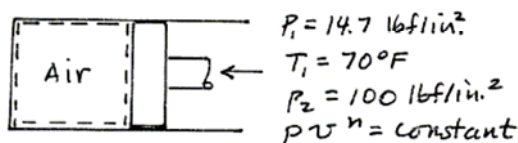


PROBLEM 3.138

KNOWN: Air undergoes a polytropic process from a known initial state to a given final pressure.

FIND: Plot the heat transfer and work for $1.0 \leq n \leq 1.6$. Investigate the error in heat transfer introduced by assuming constant c_v evaluated at T_1 .

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The air is a closed system. (2) The process is polytropic. (3) The air behaves as an ideal gas. (4) Neglect kinetic and potential energy effects. (4) Volume change is the only work mode.

Begin by fixing state 2. From the polytropic process relation

$$P_1 v_1^n = P_2 v_2^n \Rightarrow v_2 = (P_1 / P_2)^{1/n} v_1$$

Thus, with $v_1 = RT_1 / P_1$, and $T_2 = P_2 v_2 / R$, state 2 is determined for any value of n . These expressions are evaluated using IT as follows:

$$\begin{aligned} p_1 &= 14.7 \text{ // lbf/in.}^2 \\ T_1 &= 70 \text{ // } ^\circ\text{F} \\ p_2 &= 100 \text{ // lbf/in.}^2 \\ m &= 1 \text{ // lb} \\ n &= 1.6 \end{aligned}$$

IT Results ($n = 1.6$)

$$\begin{aligned} p_1 * v_1^n &= p_2 * v_2^n \\ v_1 &= v_TP(\text{"Air"}, T_1, p_1) \\ v_2 &= v_TP(\text{"Air"}, T_2, p_2) \end{aligned}$$

$$\begin{aligned} T_2 &= 627.4 \text{ } ^\circ\text{F} \\ v_1 &= 13.34 \text{ ft}^3/\text{lb} \\ v_2 &= 4.026 \text{ ft}^3/\text{lb} \end{aligned}$$

The work is evaluated using Eq. 2.17 and Eq. 3.54

$$W = \int_{v_1}^{v_2} p dv = m \int_{v_1}^{v_2} p dv = \frac{m (P_2 v_2 - P_1 v_1)}{(1-n)} \quad (n \neq 1)$$

Or, as expressed using IT

$$W/m = (p_2 * v_2 - p_1 * v_1) / (1-n) * (144 / 778) \text{ // Btu/lb}$$

To find Q , use the energy balance: $\Delta KE + \Delta PE + \Delta U = Q - W$. or

$$m \Delta u = Q - W$$

Method 1

To evaluate Δu , use data from Table A-22

$$\Delta u = u(T_2) - u(T_1)$$

From IT:

$$\begin{aligned} m * \Delta u_1 &= Q_1 - W \\ u_1 &= u_T(\text{"Air"}, T_1) \\ u_2 &= u_T(\text{"Air"}, T_2) \\ \Delta u_1 &= u_2 - u_1 \end{aligned}$$

Continued on next slide

Problem 1-138 continued

Method 2

An alternative method to evaluate Δu is to use a constant c_v value from Table A-20 determined at $T_1 = 70^\circ\text{F}$. That is

$$\Delta u = c_v (T_2 - T_1)$$

From 1T:

$$\begin{aligned} m \cdot \Delta u_2 &= Q_2 - W \\ c_v &= c_{v,T}(\text{"Air", } T_1) \\ \Delta u_2 &= c_v (T_2 - T_1) \end{aligned}$$

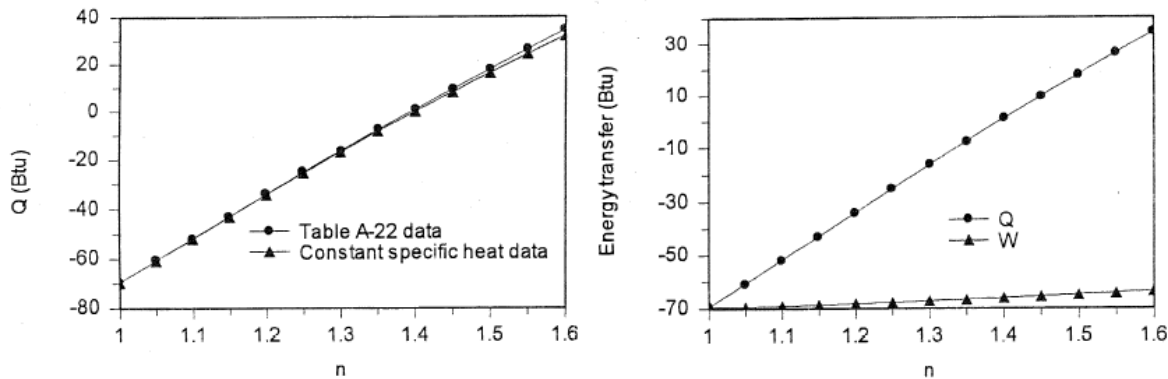
Now, solving gives

$$W = -63.68 \text{ Btu}$$

$$\begin{aligned} u_1 &= 90.38 \text{ Btu/lb} \\ u_2 &= 188.3 \text{ Btu/lb} \\ \Delta u_1 &= 97.88 \text{ Btu} \\ Q_1 &= 34.19 \text{ Btu} \end{aligned}$$

$$\begin{aligned} c_v &= 0.1711 \text{ Btu/lb}\cdot^\circ\text{R} \\ \Delta u_2 &= 95.38 \text{ Btu} \\ Q_2 &= 31.7 \text{ Btu} \end{aligned}$$

- ① Using the Explore button, sweep n from 1.001 to 1.6 in steps of 0.01. The data obtained give the following graphs:



From the graph of Q and W vs. n , we can see several effects:

- Q and W become equal as $n \rightarrow 1$, since for $n=1$ the temperature is constant, and hence $\Delta u = 0$.
- Q changes sign, going from negative (heat out) to positive (heat in) at about $n=1.4$. W is negative (work in) for all cases.
- W increases only slightly, whereas Q changes significantly.

From the graph of Q vs. n we see that the solutions become identical as $n \rightarrow 1$ (constant temperature) and diverge slightly as $n \rightarrow 1.6$. For $n=1.6$, the percentage deviation in Q is about 7.3%.

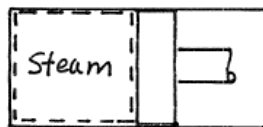
1. By using $n=1.001$, we avoid the case of $n=1$ for which Eq. 3.54 cannot be used. However, the slight deviation from $n=1$ cannot be seen on the plots.

PROBLEM 3.139

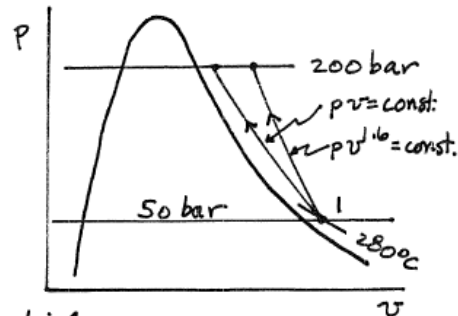
KNOWN: Steam undergoes a polytropic process from a known initial state to a given final pressure.

FIND: Plot the heat transfer per unit mass of steam for $1.0 \leq n \leq 1.6$. Investigate the error in heat transfer introduced by assuming the ideal gas model.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} P_1 &= 5 \text{ MPa} = 50 \text{ bar} \\ T_1 &= 280^\circ\text{C} \\ P_2 &= 20 \text{ MPa} = 200 \text{ bar} \\ p v^n &= \text{constant} \end{aligned}$$



ENGR. MODEL: (1) The steam is a closed system. (2) The process is polytropic. (3) Kinetic and potential energy effects are neglected. (4) Volume change is the only work mode.

ANALYSIS: The following relations are used to determine the heat transfer, Q/m :

$$\text{Process relation: } P_1 v_1^n = P_2 v_2^n \quad (1)$$

$$\text{Work expression (Eqs. 2.17 and 3.54): } \frac{W}{m} = \frac{(P_2 v_2 - P_1 v_1)}{1-n} \quad (2)$$

$$\begin{aligned} \text{Energy balance: } \Delta KE + \Delta PE + \Delta U &= Q - W \\ \Rightarrow (u_2 - u_1) &= \frac{Q}{m} - \frac{W}{m} \end{aligned} \quad (3)$$

When using steam table data (IT functions or Table A-4):

- $P_1 = 50 \text{ bar}, T_1 = 280^\circ\text{C} \Rightarrow v_1, u_1$
- with $P_2 = 200 \text{ bar}$ and (2), v_2 can be determined.
- $P_2, v_2 \Rightarrow u_2, T_2$

When using the ideal gas model (IT "H₂O" functions or Table A-23):

- $P_1 = 50 \text{ bar}, T_1 = 280^\circ\text{C} \Rightarrow v_1 = \frac{RT_1}{P_1}$
- $T_1 = 280^\circ\text{C} \Rightarrow u(T_1)$
- with $P_2 = 200 \text{ bar}$ and (2), v_2 can be determined.
- $P_2, v_2 \Rightarrow T_2 = \frac{P_2 v_2}{R}$
- $T_2 \Rightarrow u(T_2)$

With either of these schemes, it is then possible to use (2) and (3) to solve for W/m and Q/m .

The corresponding IT code is given on the next page.

Continued on next slide

Problem 1-139 continued

```

p1 = 50 // bar
T1 = 280 // °C
p2 = 200 // bar
m = 1 // kg
n = 1.6
p1 * v1^n = p2 * v2^n
W / m = ((p2 * v2 - p1 * v1) / (1 - n)) * 100
m * (u2 - u1) = Q - W

```

```

// Using steam table data:
v1 = v_PT("Water/Steam", p1, T1)
u1 = u_PT("Water/Steam", p1, T1)
v2 = v_PT("Water/Steam", p2, T2)
u2 = u_PT("Water/Steam", p2, T2)

```

```

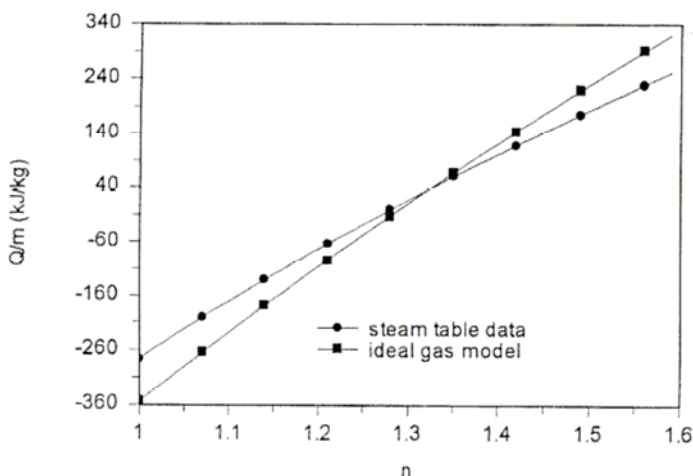
// Using the ideal gas model:
v1 = v_TP("H2O", T1, p1)
u1 = u_T("H2O", T1)
v2 = v_TP("H2O", T2, p2)
u2 = u_T("H2O", T2)

```

IT Results for n = 1.6

quantity	steam table data	Ideal gas model
Q/m (kJ/kg)	251.9	320.8
T ₂ (°C)	583.7	652.1
W/m (kJ/kg)	-240.8	-291
u ₁ (kJ/kg)	2645	-1.319E4
u ₂ (kJ/kg)	3138	-1.257E4
v ₁ (m ³ /kg)	0.04224	0.05104
v ₂ (m ³ /kg)	0.01766	0.02135

- ① Using the Explore button, sweep n from 1.001 to 1.6 in steps of 0.01. The data obtained give the following plot:



Discussion:

- Q/m varies significantly with n.
- Q/m changes sign, going from negative (heat out) to positive (heat in).
- The steam table and ideal gas solutions exhibit the same general trends. However, the values for Q/m differ significantly in some ranges of n.

1. By using n = 1.001, we avoid the case of n = 1 for which (2) above cannot be used. However, the slight deviation from n = 1 cannot be seen on the plot.

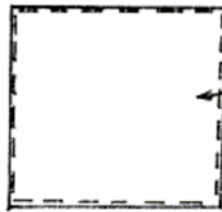
PROBLEM 4.1

KNOWN: Air leaks into an initially evacuated tank at a constant mass flow rate

FIND: Determine the pressure in the tank after 30 s.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The control volume has one inlet and no exits. (2) The air behaves as an ideal gas. (3) The mass flow rate is constant.



$$\dot{m}_i = 0.004 \text{ lb/s}$$

$$m(t=0) = 0$$

$$V = 5 \text{ ft}^3$$

$$T_2 = 70^\circ\text{F} = 530^\circ\text{R}$$

ANALYSIS: The mass rate balance, Eq. 4.2, reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i \Rightarrow dm_{cv} = \dot{m}_i dt$$

Integrating

$$\int_0^m dm_{cv} = \int_0^{30} \dot{m}_i dt = \int_0^{30} (0.004) dt$$

$$m(t=30) = (0.004)(30) = 0.12 \text{ lb}$$

Now, using the ideal gas model

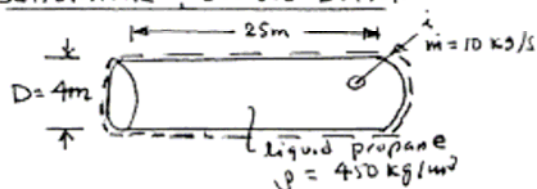
$$P_2 = \frac{mRT_2}{V} = \frac{(0.12 \text{ lb}) \left(\frac{1545 \text{ ft} \cdot \text{lb}_f}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right) (530^\circ\text{R})}{(5 \text{ ft}^3)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right|$$

$$= 4.711 \text{ lb}_f/\text{in}^2 \leftarrow P_2$$

PROBLEM 4.2

Liquid propane enters an initially-empty cylindrical storage tank at a mass flow rate of 10 kg/s. The tank is 25-m long and has a 4-m diameter. The density of the liquid propane is 450 kg/m³. Determine the time, in h, to fill the tank.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume has one inlet and no exits. (2) The mass flow rate is constant. (3) The propane density is constant.

ANALYSIS:

The mass rate balance, Eq. 4.2, reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i \Rightarrow m_{cv}(t_f) - m_{cv}(0) = \int_0^{t_f} \dot{m}_i dt$$

Since the tank is initially empty and the mass flow rate is constant, this becomes

$$m_{cv}(t_f) = \dot{m}_i t_f \Rightarrow t_f = \frac{m_{cv}(t_f)}{\dot{m}_i}$$

where

$$m_{cv}(t_f) = \rho V = (450 \frac{\text{kg}}{\text{m}^3}) \left(\pi \left(\frac{4\text{m}}{4} \right)^2 (25\text{m}) \right) = 141,372 \text{ kg}$$

Finally

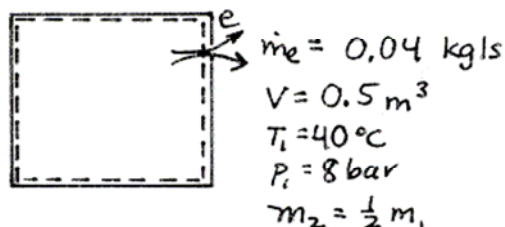
$$t_f = \frac{141,372 \text{ kg}}{(10 \text{ kg/s})} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 3.93 \text{ h} \leftarrow$$

PROBLEM 4.3

KNOWN: A tank containing Ammonia develops a leak. The ammonia escapes at a constant mass flow rate.

FIND: Determine the time at which half the mass has escaped and the pressure in the tank at that time.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is shown on the accompanying diagram. (2) The temperature in the tank remains constant. (3) The volume is constant.

ANALYSIS: Applying the mass rate balance

$$\frac{dm_{cv}}{dt} = -\dot{m}_e = -0.04 \text{ kg/s}$$

Integrating over time

$$\int_{m_{cv}(0)}^{m_{cv}(t)} dm_{cv} = - \int_0^t \dot{m}_e dt = -0.04 t$$

Solving for t

$$t = \frac{m_{cv}(t) - m_{cv}(0)}{(-0.04 \text{ kg/s})}$$

From Table A-15, $v_i = 0.17720 \text{ m}^3/\text{kg}$. Thus, the initial mass is

$$m_{cv}(0) = \frac{V}{v_i} = 2.822 \text{ kg}$$

and

$$m_{cv}(t) = \frac{1}{2} m_{cv}(0) = 1.411 \text{ kg}$$

Thus

$$t = \frac{(1.411 - 2.822) \text{ kg}}{(-0.04 \text{ kg/s})} = 35.3 \text{ s} \leftarrow t$$

Now, to get P_2 , determine v_2

$$v_2 = \frac{V}{m_{cv}(t)} = 0.3544 \text{ m}^3/\text{kg}$$

Interpolating in Table A-15 at $T_2 = 40^\circ\text{C}$, $v_2 = 0.3544 \text{ m}^3/\text{kg}$

$$P_2 \approx 4.17 \text{ bar} \leftarrow P_2$$

PROBLEM 4.4

KNOWN: Data are provided for a crude oil storage tank.

FIND: After 24h, determine the mass and volume of oil in the tank.

SCHEMATIC & GIVEN DATA:

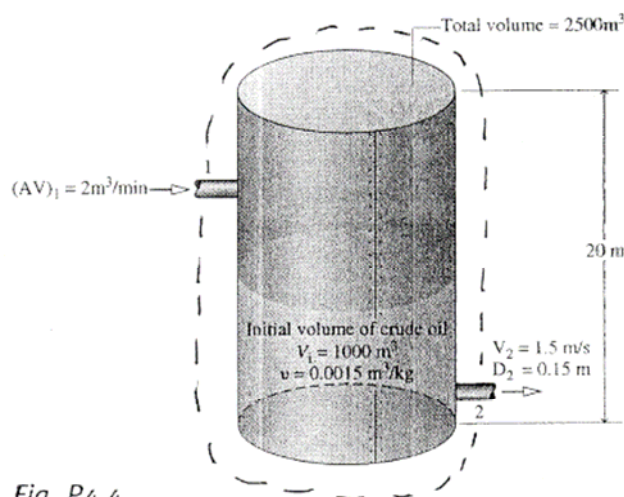


Fig. P4.4

ENGR. MODEL

1. As shown by the sketch, a control volume encloses the storage tank.
2. The specific volume of the oil is constant.

(a) Mass rate balance: $\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2$, where

$$\dot{m}_1 = \frac{(AV)_1}{v} = \left(\frac{2 \text{ m}^3/\text{min}}{0.0015 \text{ m}^3/\text{kg}} \right) \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 8 \times 10^4 \frac{\text{kg}}{\text{h}}$$

$$\dot{m}_2 = \frac{A_2 V_2}{v} = \frac{(\pi D_2^2/4)(V_2)}{v} = \frac{\pi (0.15 \text{ m})^2 (1.5 \text{ m/s})}{4 (0.0015 \text{ m}^3/\text{kg})} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = 6.36 \times 10^4 \frac{\text{kg}}{\text{h}}$$

$$\therefore \frac{dm_{cv}}{dt} = 1.64 \times 10^4 \frac{\text{kg}}{\text{h}}$$

Integrating

$$\Rightarrow m_{cv} - m_{cv(0)} = (1.64 \times 10^4 \frac{\text{kg}}{\text{h}})(24 \text{ h}) = 39.36 \times 10^4 \text{ kg}$$

$$\begin{aligned} \left[\frac{V_1}{v} = \frac{1000 \text{ m}^3}{0.0015 \text{ m}^3/\text{kg}} \right] \\ = 66.67 \times 10^4 \text{ kg} \end{aligned}$$

So,

$$m_{cv}(24 \text{ h}) = (66.67 + 39.36) \times 10^4 \text{ kg} = 1.06 \times 10^6 \text{ kg} \quad \leftarrow m_{cv}$$

(b)

$$\begin{aligned} V(24 \text{ h}) &= v m_{cv}(24 \text{ h}) = \left(0.0015 \frac{\text{m}^3}{\text{kg}} \right) (1.06 \times 10^6 \text{ kg}) \\ &= 1590 \text{ m}^3 \end{aligned} \quad \leftarrow V$$

PROBLEM 4.5

KNOWN: Data are provided for a vegetable oil-filled spray can.

FIND: Determine the mass flow rate per spray, and the mass remaining in the can after a specified number of sprays.

SCHEMATIC & GIVEN DATA:



- 560 sprays, each lasting 0.25 s and having a mass of 0.25 g
- initial mass of vegetable oil in the can is 170 g

ENGR. MODEL: The control volume is shown in the accompanying schematic.

ANALYSIS: (a) Since each spray has a duration of 0.25 s and consists of 0.25 g

$$\dot{m}_e = \frac{0.25 \text{ g}}{0.25 \text{ s}} = 1 \text{ g/s}$$

(b) The mass rate balance, Eq. 4.2, gives on integration

$$m_{cv}(t + \Delta t) - m_{cv}(0) = \cancel{m_i^0} - m_e$$

where $m_{cv}(0)$ is the initial amount of mass within the can and m_e is the amount of mass that exits. Thus, with $m_{cv}(0) = 170 \text{ g}$ and

$$m_e = (560 \text{ sprays}) \left(\frac{0.25 \text{ g}}{\text{spray}} \right) = 140 \text{ g}$$

The mass of vegetable oil remaining in the can after 560 sprays is

$$\begin{aligned} m_{cv} &= m_{cv}(0) - m_e \\ &= 170 \text{ g} - 140 \text{ g} = 30 \text{ g} \end{aligned}$$

PROBLEM 4.6

Figure P4.6 shows a mixing tank initially containing 3000 lb of liquid water. The tank is fitted with two inlet pipes, one delivering hot water at a mass flow rate of 0.8 lb/s and the other delivering cold water at a mass flow rate of 1.3 lb/s. Water exits through a single exit pipe at a mass flow rate of 2.6 lb/s. Determine the amount of water, in lb, in the tank after one hour.

SCHEMATIC & GIVEN DATA:

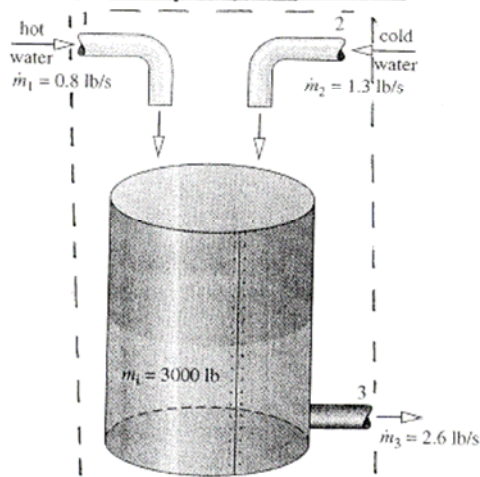


Fig. P4.6

ENGR. MODEL:

1. The control volume has two inlets and one exit, as shown in the sketch.
2. The mass flow rates are constant.

ANALYSIS:

The mass rate balance, Eq. 4.2, reads

$$\begin{aligned}\frac{dm_{cv}}{dt} &= \dot{m}_1 + \dot{m}_2 - \dot{m}_3 \\ &= (0.8 + 1.3 - 2.6) \text{ lb/s} \\ &= -0.5 \text{ lb/s}\end{aligned}$$

Integrating from $t = 0$ to $t = 1 \text{ h} (3600 \text{ s})$,

$$\begin{aligned}m_{cv}(1 \text{ h}) - m_{cv}(0) &= (-0.5 \frac{\text{lb}}{\text{s}})(3600 \text{ s}) \\ &= -1800 \text{ lb}\end{aligned}$$

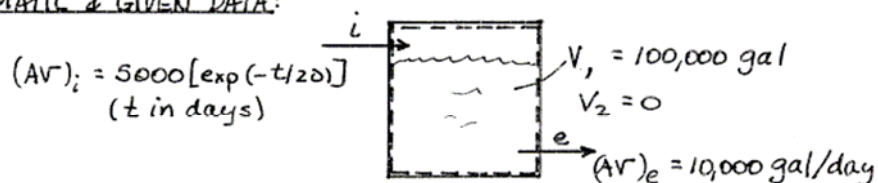
$$\begin{aligned}\therefore m_{cv}(1 \text{ h}) &= m_{cv}(0) - 1800 \text{ lb} \\ &= 3000 \text{ lb} - 1800 \text{ lb} = 1200 \text{ lb} \quad \leftarrow\end{aligned}$$

PROBLEM 4.7

KNOWN: A water storage tank contains a known volume of water. The volume flow rates in and out of the tank are given.

FIND: Determine how many days the tank will contain water.

SCHEMATIC & GIVEN DATA:



ENGR MODEL: (1) The control volume is as shown on the above sketch. (2) The water is modeled as incompressible.

ANALYSIS: The mass rate balance reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$

With $m_{cv} = \rho V$, and $\dot{m}_i = \rho (AV)_i$ and $\dot{m}_e = \rho (AV)_e$

$$\rho \frac{dV}{dt} = \rho (AV)_i - \rho (AV)_e$$

$$\Rightarrow \frac{dV}{dt} = (AV)_i - (AV)_e = 5000[\exp(-t/20)] - 10,000$$

Integrating from $t=0$ to $t=t_f$

$$V_2 - V_1 = \int_0^{t_f} (5000[\exp(-t/20)] - 10,000) dt$$

$$= \left(\frac{5000}{(-1/20)} \exp(-t/20) - 10,000 t \right) \Big|_0^{t_f}$$

$$\textcircled{1} \quad = [100,000 \exp(-t_f/20) - 1] - 10,000 t_f \quad (*)$$

With $V_1 = 100,000 \text{ gal}$ and for $V_2 = 0$

$$0 = 200,000 - 100,000 \exp(-t_f/20) - 10,000 t_f$$

Using IT to solve we get:

IT solution:

$$0 = 200000 - 100000 \cdot \exp(-t_f/20) - 10000 \cdot t_f$$

$$// t_f = 15.36 \text{ days} \quad \leftarrow t_f$$

1. To obtain V vs. t using IT:

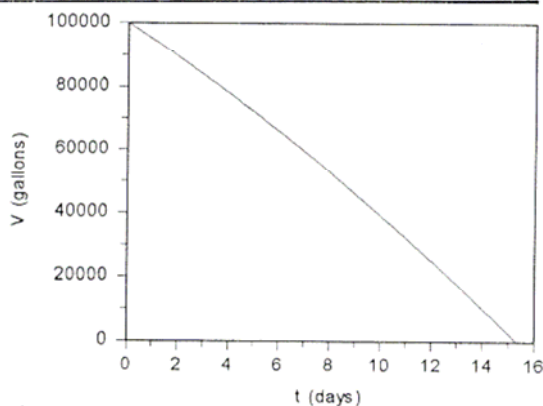
$$\text{der}(V,t) = AV_1 - AV_2$$

$$AV_1 = 5000 \cdot (\exp(-t/20))$$

$$AV_2 = 10000$$

// Use the Explore button to sweep t from

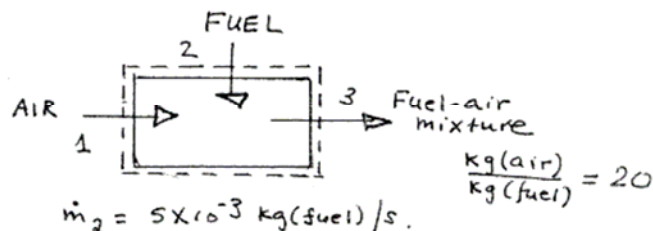
// 0 to 15.36 days in steps of 0.05.



PROBLEM 4.8

A carburetor in an internal combustion engine mixes air with fuel to achieve a combustible mixture in which the *air-to-fuel ratio* is 20 kg (air) per kg (fuel). For a fuel mass flow rate of 5×10^{-3} kg/s, determine the mass flow rate of the mixture, in kg/s.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. All injected fuel exits at 3, mixed with air.

ANALYSIS:

At steady state, for each substance individually it is necessary for the incoming and outgoing mass flow rates to be equal. That is,

$$\text{AIR: } \dot{m}_1 = \dot{m}_3$$

$$\text{FUEL: } \dot{m}_2 = \dot{m}_3$$

We know that

$$\frac{\dot{m}_3}{\dot{m}_2} = 20 \frac{\text{kg(air)}}{\text{kg(fuel)}}$$

$$\Rightarrow \dot{m}_3 = 20 \frac{\text{kg(air)}}{\text{kg(fuel)}} \dot{m}_2 = 20 \frac{\text{kg(air)}}{\text{kg(fuel)}} (5 \times 10^{-3} \frac{\text{kg(fuel)}}{\text{s}})$$

$$= 0.10 \frac{\text{kg(air)}}{\text{s}}$$

The mass flow rate of the mixture is then

$$\dot{m}_3 = \dot{m}_3 + \dot{m}_2$$

$$= 0.10 \frac{\text{kg(air)}}{\text{s}} + 0.005 \frac{\text{kg(fuel)}}{\text{s}}$$

$$= 0.105 \frac{\text{kg(mixture)}}{\text{s}}$$



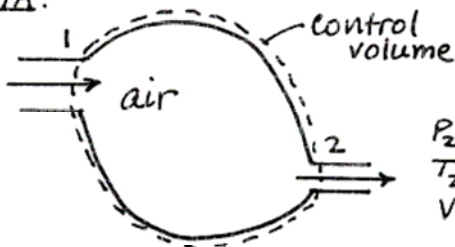
PROBLEM 4.9

KNOWN: Air flows through a one-inlet, one-exit control volume. Data are known at the inlet and exit.

FIND: Determine (a) the mass flow rate, (b) the exit area.

SCHEMATIC & GIVEN DATA:

$$\begin{aligned} P_1 &= 8 \text{ bar} \\ T_1 &= 600 \text{ K} \\ V_1 &= 40 \text{ m/s} \\ A_1 &= 20 \text{ cm}^2 \\ &= 0.002 \text{ m}^2 \end{aligned}$$



$$\begin{aligned} P_2 &= 2 \text{ bar} \\ T_2 &= 400 \text{ K} \\ V_2 &= 350 \text{ m/s} \end{aligned}$$

ENGR. MODEL: (1) The control volume is at steady state. (2) The air behaves as an ideal gas. (3) The flow is one-dimensional at the inlet and exit.

ANALYSIS: Beginning with the mass rate balance

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

Using data at the inlet and the ideal gas equation of state

$$\begin{aligned} \text{(a)} \quad \dot{m} &= \rho_1 A_1 V_1 = \left(\frac{P_1}{RT_1} \right) A_1 V_1 \\ &= \frac{(8 \text{ bar})}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (600 \text{ K})} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| (0.002 \text{ m}^2) (40 \text{ m/s}) \\ &= 0.3717 \text{ kg/s} \leftarrow \dot{m} \end{aligned}$$

(b) From $\dot{m}_1 = \dot{m}_2$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Thus

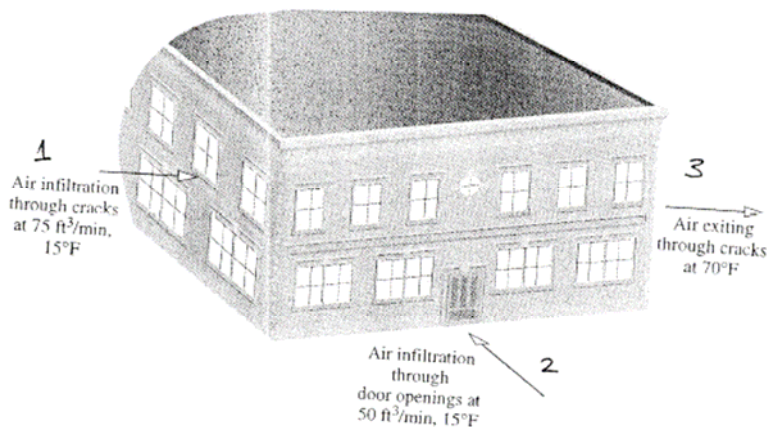
$$A_2 = \left(\frac{\rho_1}{\rho_2} \right) \left(\frac{V_1}{V_2} \right) A_1$$

With $\rho = P/RT$

$$\begin{aligned} A_2 &= \left(\frac{P_1}{P_2} \right) \left(\frac{T_2}{T_1} \right) \left(\frac{V_1}{V_2} \right) A_1 \\ &= \left(\frac{8}{2} \right) \left(\frac{400}{600} \right) \left(\frac{40}{350} \right) (20 \text{ cm}^2) \\ &= 6.095 \text{ cm}^2 \leftarrow A_2 \end{aligned}$$

PROBLEM 4.10

The small two-story office building shown in Fig. P4.10 has 36,000 ft³ of occupied space. Due to cracks around windows and outside doors, air leaks in on the windward side of the building and leaks out on the leeward side of the building. Outside air also enters the building when outer doors are opened. On a particular day, tests were conducted. The outdoor temperature was measured to be 15°F. The inside temperature was controlled at 70°F. Keeping the doors closed, the infiltration rate through the cracks was determined to be 75 ft³/min. The infiltration rate associated with door openings, averaged over the work day, was 50 ft³/min. The pressure difference was negligible between the inside and outside of the building. (a) Assuming ideal gas behavior, determine at steady state the volumetric flow rate of air exiting the building, in ft³/min. (b) When expressed in terms of the volume of the occupied space, determine the number of building air changes per hour.



ENGR. MODEL:

1. A control volume encloses the building.
2. The control volume is at steady state.
3. The air is modeled as an ideal gas.
4. Pressure is constant.

ANALYSIS: (a) At steady state $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$, where

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{(AV)_1}{(RT_1/P)}, \quad \dot{m}_2 = \frac{(AV)_2}{v_2} = \frac{(AV)_2}{(RT_2/P)}, \quad \dot{m}_3 = \frac{(AV)_3}{v_3} = \frac{(AV)_3}{RT_3/P}$$

Accordingly,

$$\begin{aligned} \frac{P(AV)_3}{RT_3} &= \frac{P(AV)_1}{RT_1} + \frac{P(AV)_2}{RT_2} \Rightarrow (AV)_3 = \frac{T_3}{T_0} [(AV)_1 + (AV)_2] \\ &= \frac{530^\circ R}{473^\circ R} \left[75 \frac{\text{ft}^3}{\text{min}} + 50 \frac{\text{ft}^3}{\text{min}} \right] \\ &= 139.5 \frac{\text{ft}^3}{\text{min}} \end{aligned} \quad \leftarrow (a)$$

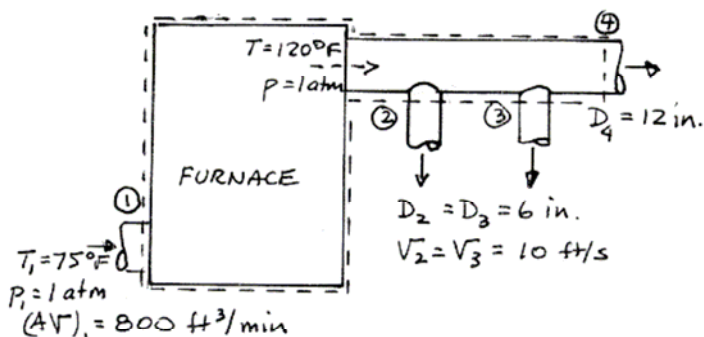
$$\begin{aligned} (b) \quad \left[\begin{array}{c} \text{Building Air} \\ \text{Changes} \\ \text{per hour} \end{array} \right] &= \frac{139.5 \frac{\text{ft}^3}{\text{min}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right|}{36,000 \text{ ft}^3/\text{air change}} \\ &= 0.23 \text{ air changes/h} \end{aligned} \quad \leftarrow (b)$$

PROBLEM 4.11:

KNOWN: Air enters a furnace operating at steady state and is to a duct system consisting of three ducts. Data are known at the inlet and each of the discharge ducts.

FIND: Determine (a) the mass flow rate entering the furnace, (2) the volumetric flow rate in each of the 6-in. exit ducts, (c) the velocity in the 12-in. exit duct.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control shown on the accompanying sketch is at steady state. (2) The air behaves as an ideal gas. (3) The temperature and pressure in each duct are the same as the temperature and pressure of the air delivered to the duct system.

ANALYSIS: (a) Using Eq. 4.4b with the ideal gas equation

$$\begin{aligned} \dot{m}_1 &= \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1} \\ &= \frac{(1 \text{ atm})(800 \text{ ft}^3/\text{min})}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot ^\circ\text{R}}\right)(535^\circ\text{R})} \left| \frac{14.696 \text{ lb}_f/\text{in}^2}{1 \text{ atm}} \right| \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \\ &= 0.989 \text{ lb/s} \end{aligned}$$

(b) Since $D_2 = D_3$ and $V_2 = V_3$,

$$\begin{aligned} (AV)_2 &= (AV)_3 = \left(\frac{\pi D^2}{4}\right)V = \left(\frac{\pi (\frac{6}{12} \text{ ft})^2}{4}\right) \left(10 \frac{\text{ft}}{\text{s}}\right) \left|\frac{60 \text{ s}}{1 \text{ min}}\right| \\ &= 117.8 \text{ ft}^3/\text{min} \end{aligned}$$

(c) Applying the mass balance to get $\dot{m}_4$

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 - \dot{m}_4 \Rightarrow \dot{m}_4 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \quad (*)$$

where

$$\dot{m}_2 = \dot{m}_3 = \frac{(AV)}{v} = \frac{P(AV)}{RT} = \frac{(14.696)(117.8)}{\left(\frac{1545}{28.97}\right)(580)} \left| \frac{144}{60} \right| = 0.1343 \text{ lb/s}$$

From (*)

$$\dot{m}_4 = 0.989 - 2(0.1343) = 0.7204 \text{ lb/s}$$

Finally, from $\dot{m}_4 = \frac{A_4 V_4}{v_4}$

$$V_4 = \frac{v_4 \dot{m}_4}{A_4} = \frac{(RT_4)(\dot{m}_4)}{(P_4)\left(\frac{\pi D_4^2}{4}\right)} = \frac{\left(\frac{1545}{28.97}\right)(580)(0.7204)}{(14.696) \frac{\pi (12)^2}{4}} = 13.41 \text{ ft/s}$$

PROBLEM 4.12

KNOWN: Refrigerant 134a flows through a refrigeration condenser. Data are known at the inlet and exit. The mass flow at the inlet is given.

FIND: Determine (a) the inlet velocity, (b) the diameter of the exit pipe.

SCHEMATIC & GIVEN DATA:

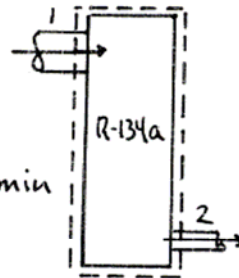
ENGR. MODEL: (1) The control volume is at steady state.
(2) The flow is one-dimensional at the inlet and exit.

$$P_1 = 9 \text{ bar}$$

$$T_1 = 50^\circ\text{C}$$

$$d_1 = 2.5 \text{ cm}$$

$$\dot{m}_1 = 6 \text{ kg/min}$$



$$P_2 = 9 \text{ bar}$$

$$T_2 = 30^\circ\text{C}$$

$$V_2 = 2.5 \text{ m/s}$$

ANALYSIS: (a) Solving Eq. 4.4b for V_1

$$V_1 = \frac{\dot{m}_1 V_1}{A_1} = \frac{\dot{m}_1 V_1}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{4 \dot{m}_1 V_1}{\pi d_1^2}$$

From Table A-12, $v_f = 0.02472 \text{ m}^3/\text{kg}$. Thus

$$V_1 = \frac{(4)(6 \text{ kg/min})(0.02472 \text{ m}^3/\text{kg})}{\pi (2.5)^2 \text{ cm}^2} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{10^4 \text{ cm}^2}{1 \text{ m}^2} \right|$$

$$= 5.04 \text{ m/s} \longleftarrow V_2$$

(b) To find the diameter of the exit pipe, begin with mass rate balance

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_2 = \dot{m}_1$$

Thus, with $\dot{m}_2 = A_2 V_2 / v_2$

$$A_2 = \frac{\dot{m}_2 v_2}{V_2}$$

From Table A-11, $T_2 < T_{\text{sat}} @ 9 \text{ bar}$. Thus, the refrigerant is a sub-cooled liquid. From Table A-10,

$$v_2 \approx v_f @ 30^\circ\text{C} = 0.8417 \times 10^{-3} \text{ m}^3/\text{kg}$$

Inserting values

$$A_2 = \frac{(6 \text{ kg/min})(0.8417 \times 10^{-3} \text{ m}^3/\text{kg})}{(2.5 \text{ m/s})} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 3.367 \times 10^{-5} \text{ m}^2$$

Finally, with $A = \pi d^2/4$

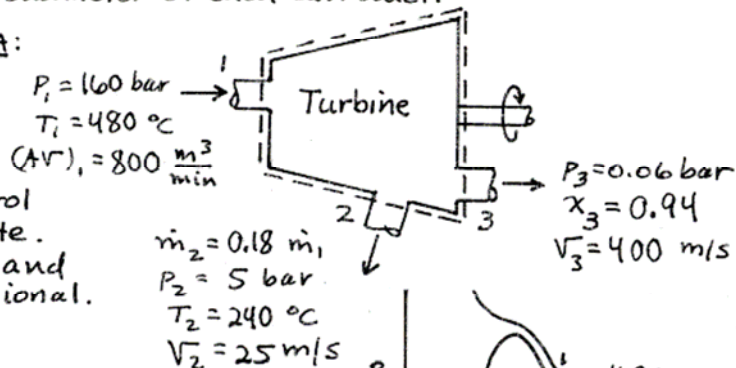
$$d_2 = \left(\frac{4A_2}{\pi}\right)^{1/2} = 0.0065 \text{ m} = 0.65 \text{ cm} \longleftarrow d_2$$

PROBLEM 4.13

KNOWN: Data are given for steam flowing through a turbine with one inlet and two exits.

FIND: Determine the diameter of each exit duct.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) The flow at the inlet and each exit is one-dimensional.

ANALYSIS: The mass flow rate at the inlet is

$$\dot{m}_1 = \frac{(AV)_1}{v_1}$$

From Table A-4, $v_1 = 0.01842 \text{ m}^3/\text{kg}$. Thus

$$\dot{m}_1 = \frac{(800 \text{ m}^3/\text{min}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right|}{(0.01842 \text{ m}^3/\text{kg})} = 723.9 \text{ kg/s}$$

and $\dot{m}_2 = 0.18 \dot{m}_1 = 130.3 \text{ kg/s}$

Applying the mass rate balance

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 593.6 \text{ kg/s}$$

Now, with $\dot{m} = AV/v$, and from Table A-4; $v_2 = 0.4646 \text{ m}^3/\text{kg}$

$$A_2 = \frac{\dot{m}_2 v_2}{V_2} = \frac{(130.3 \text{ kg/s})(0.4646 \text{ m}^3/\text{kg})}{(25 \text{ m/s})} = 2.421 \text{ m}^2$$

Noting that $A = \pi d^2/4$

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = 1.756 \text{ m} \leftarrow d_2$$

From Table A-3, at $P_3 = 0.06 \text{ bar}$ and $x_3 = 0.94$

$$\begin{aligned} v_3 &= v_{f3} + x_3(v_{g3} - v_{f3}) \\ &= 1.0064 \times 10^{-3} + (0.94)(23.739 - 1.0064 \times 10^{-3}) = 22.315 \text{ m}^3/\text{kg} \end{aligned}$$

Thus

$$A_3 = \frac{\dot{m}_3 v_3}{V_3} = \frac{(593.6)(22.315)}{(400)} = 33.115 \text{ m}^2$$

and

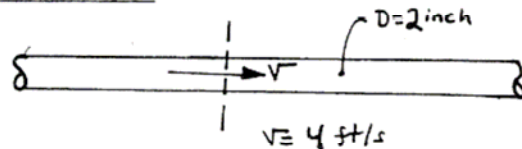
$$d_3 = \sqrt{\frac{4A_3}{\pi}} = 6.49 \text{ m} \leftarrow d_3$$

PROBLEM 4.14

KNOWN: Data are provided for substances flowing through a pipe of known diameter.

FIND: Determine the mass flow rate for each of three specified substances.

SCHEMATIC & GIVEN DATA:



ANALYSIS: With Eq. 4.4b

$$\dot{m} = \frac{A V}{v} = \frac{(\pi D^2/4) V}{v} = \frac{\pi \left[\left(\frac{1}{12} \text{ ft} \right)^2 / 4 \right] [4 \text{ ft/s}]}{v} = \frac{0.02182 \text{ ft}^3/\text{s}}{v}$$

(a) water at 50 lbf/in², 80°F

with $v \approx v_f(80^\circ\text{F}) = 0.01607 \text{ ft}^3/\text{lb}$ (Table A-2E)

$$\therefore \dot{m} = \frac{0.02182 \text{ ft}^3/\text{s}}{0.01607 \text{ ft}^3/\text{lb}} = 1.35760 \text{ lb/s} \leftarrow \text{(a)}$$

(b) Nitrogen as an ideal gas at 50 lbf/in², 80°F = 540°R

$$v = \frac{RT}{P} = \frac{\left(\frac{1545}{28.01} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right) (540^\circ\text{R})}{(50 \text{ lbf/in}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 4.137 \text{ ft}^3/\text{lb}$$

$$\therefore \dot{m} = \frac{0.02182 \text{ ft}^3/\text{s}}{4.137 \text{ ft}^3/\text{lb}} = 0.00527 \text{ lb/s} \leftarrow \text{(b)}$$

(c) R22 at 50 lbf/in², 80°F

$v = 1.2716 \text{ ft}^3/\text{lb}$ from Table A-9E

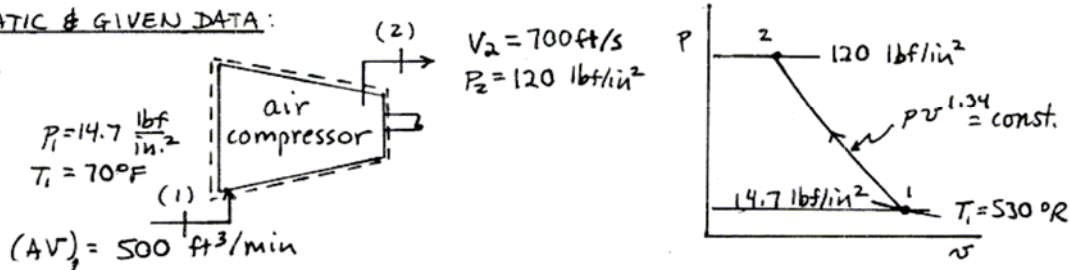
$$\therefore \dot{m} = \frac{0.02182 \text{ ft}^3/\text{s}}{1.2716 \text{ ft}^3/\text{lb}} = 0.01716 \text{ lb/s} \leftarrow \text{(c)}$$

PROBLEM 4.15

KNOWN: Data are known at the inlet and exit of an air compressor operating at steady state. Each unit of mass passing through undergoes a polytropic process.

FIND: Determine the temperature and diameter at the exit.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Each unit of mass undergoes a process described by $p v^{1.34} = \text{constant}$. (3) The air behaves as an ideal gas, as can be verified by reference to the compressibility chart.

ANALYSIS: The mass rate balance reads,

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \Rightarrow \frac{(AV)_1}{v_1} = \frac{(AV)_2}{v_2} \quad (*)$$

Evaluate v_1 using the ideal gas equation

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)(530^\circ\text{R})}{(14.7 \text{ lbf/in}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 13.35 \text{ ft}^3/\text{lb}$$

With assumption (2),

$$v_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.34}} v_1 = \left(\frac{14.7}{120}\right)^{\frac{1}{1.34}} (13.35 \frac{\text{ft}^3}{\text{lb}}) = 3.192 \text{ ft}^3/\text{lb}$$

The exit temperature is determined using the ideal gas equation of state.

$$T_2 = \frac{P_2 v_2}{R} = \frac{(120 \text{ lbf/in}^2)(3.192 \text{ ft}^3/\text{lb})}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right|$$

$$= 1034^\circ\text{R} = 574^\circ\text{F} \leftarrow T_2$$

Solving (*) for A_2 and inserting values

$$A_2 = \frac{(AV)_1 v_2}{V_2 v_1}$$

$$= \frac{(500 \text{ ft}^3/\text{min})(3.192 \text{ ft}^3/\text{lb})}{(700 \text{ ft/s})(13.35 \text{ ft}^3/\text{lb})} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 0.002846 \text{ ft}^2$$

$$= 0.410 \text{ in}^2$$

Using $A_2 = \pi d_2^2/4$

$$d_2 = \left(\frac{4A_2}{\pi}\right)^{1/2} = \left(\frac{(4)(0.410 \text{ in}^2)}{\pi}\right)^{1/2}$$

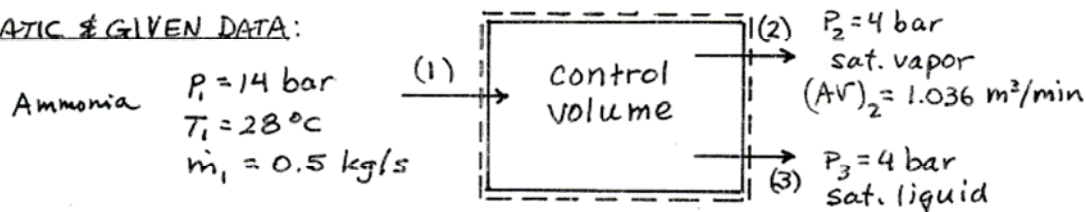
$$d_2 = 0.722 \text{ in} \leftarrow d_2$$

PROBLEM 4.16

KNOWN: Ammonia flows through a control volume at steady state. The control volume has one inlet and two exit, and data are known at each flow boundary.

FIND: Determine (a) the minimum inlet diameter so the ammonia velocity does not exceed 20 m/s. (b) the volumetric flow rate of the second exit stream.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) The flow at the inlet is one-dimensional.

ANALYSIS: (a) To relate velocity and pipe diameter at the inlet, use Eq. 4.4b

$$V_1 = \frac{\dot{m}_1 v_1}{A_1} = \frac{\dot{m}_1 v_1}{\left(\frac{\pi d_1^2}{4}\right)}$$

Thus, velocity varies inversely with diameter. The minimum diameter corresponds to $V_1 = 20$ m/s.

To get v_1 , note from Table A-14 that $T_1 = 28^\circ\text{C}$ is less than T_{sat} at 14 bar. Hence, from Table A-13, $v_1 \approx v_f @ 28^\circ\text{C} = 1.6714 \times 10^{-3} \text{ m}^3/\text{kg}$, and

$$(d_1)_{\min} = \sqrt{\frac{4 \dot{m}_1 v_1}{\pi V_1}} = \sqrt{\frac{(4)(0.5 \text{ kg/s})(1.6714 \times 10^{-3} \text{ m}^3/\text{kg})}{\pi (20 \text{ m/s})}}$$

$$= 0.00729 \text{ m} = 0.729 \text{ cm} \leftarrow (d_1)_{\min}$$

(b) To find $(AV)_3$, begin with the mass rate balance

$$\frac{dm_{\text{cv}}}{dt} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2$$

With $\dot{m} = (AV)/v$

$$(AV)_3 = v_3 \left[\dot{m}_1 - (AV)_2 / v_2 \right]$$

From Table A-14 at 4 bar; $v_2 = 0.3094 \text{ m}^3/\text{kg}$ and $v_3 = 1.5597 \times 10^{-3} \text{ m}^3/\text{kg}$. Thus

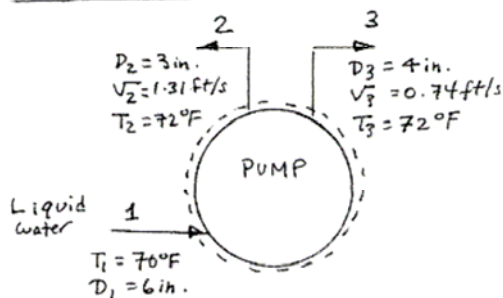
$$(AV)_3 = (1.5597 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}) \left[(0.5 \frac{\text{kg}}{\text{s}}) (\frac{60 \text{ s}}{1 \text{ min}}) - \frac{(1.036 \text{ m}^3/\text{min})}{(0.3094 \text{ m}^3/\text{kg})} \right]$$

$$= 0.0416 \text{ m}^3/\text{min} \leftarrow (AV)_3$$

PROBLEM 4.17

Liquid water at 70°F enters a pump through an inlet pipe having a diameter of 6 in. The pump operates at steady state and supplies water to two exit pipes having diameters of 3 in. and 4 in., respectively. The velocity of the water exiting the 3-in. pipe is 1.31 ft/s. At the exit of the 4-in. pipe the velocity is 0.74 ft/s. The temperature of the water in each exit pipe is 72°F. Determine (a) the mass flow rate, in lb/s, in the inlet pipe and each of the exit pipes, and (b) the volumetric flow rate at the inlet, in ft³/min.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The control volume shown in the sketch is at steady state.
2. For the liquid water, $v \approx v_f(T)$.

ANALYSIS: (a) The mass flow rates at 2 and 3 are given by $\dot{m} = \frac{A \dot{V}}{v}$. That is,

$$\dot{m}_2 = \frac{A_2 \dot{V}_2}{v_f(T_2)} = \frac{\left(\frac{\pi (3/12)^2 \text{ ft}^2}{4} \right) (1.31 \text{ ft/s})}{(0.01606 \text{ ft}^3/\text{lb})} = 4.14 \text{ lb/s}$$

$$\dot{m}_3 = \frac{A_3 \dot{V}_3}{v_f(T_3)} = \frac{\left(\frac{\pi (4/12)^2 \text{ ft}^2}{4} \right) (0.74 \text{ ft/s})}{(0.01606 \text{ ft}^3/\text{lb})} = 4.02 \text{ lb/s}$$

A mass rate balance gives at steady state,

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 = 4.14 \frac{\text{lb}}{\text{s}} + 4.02 \frac{\text{lb}}{\text{s}} = 8.02 \frac{\text{lb}}{\text{s}}$$

(b) The volumetric flow rate at 1 is

$$\begin{aligned} (A \dot{V})_1 &= \dot{m}_1 v_f(T_1) \\ &= \left(8.02 \frac{\text{lb}}{\text{s}} \right) \left(0.01605 \frac{\text{ft}^3}{\text{lb}} \right) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) \\ &= 7.72 \text{ ft}^3/\text{min} \end{aligned}$$



PROBLEM 4.18

KNOWN: Separate streams of water and ethylene glycol (glycol) mix to form a single mixture that is half glycol by mass. Data are given for the inlet flows.

FIND: Determine (a) the molar and volumetric flow rates of the entering glycol. (b) the diameters of the supply pipes.

SCHEMATIC & GIVEN DATA:

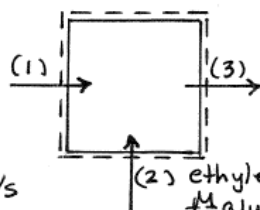
Water:

$$T_1 = 20^\circ\text{C}$$

$$P_1 = 1 \text{ bar}$$

$$\dot{n}_1 = 4.2 \frac{\text{kmol}}{\text{min}}$$

$$V_1 = V_2 = 2.5 \text{ m/s}$$



Mixture: 50% glycol by mass

$$M_{\text{glycol}} = 62.07 \text{ kg/kmol}$$

$$S_{\text{glycol}} = 1.115 S_{\text{H}_2\text{O}}$$

ENGR. MODEL: (1) The control volume is at steady state. (2) The water and glycol are each incompressible substances.

ANALYSIS: (a) To find the glycol flow rates, begin with a mass rate balance.

At steady state: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$. For the mixture

$$\left. \begin{aligned} 50\% \text{ glycol} &\Rightarrow \frac{\dot{m}_2}{\dot{m}_3} = 0.5 \\ 50\% \text{ water} &\Rightarrow \frac{\dot{m}_1}{\dot{m}_3} = 0.5 \end{aligned} \right\} \Rightarrow \dot{m}_1 = \dot{m}_2$$

Now

$$\dot{m}_1 = \dot{n}_1 M_{\text{H}_2\text{O}} = (4.2 \frac{\text{kmol}}{\text{min}})(18.02 \frac{\text{kg}}{\text{kmol}}) = 75.68 \text{ kg/min}$$

$$\dot{n}_2 = \frac{\dot{m}_2}{M_{\text{glycol}}} = \frac{75.68 \text{ kg/min}}{62.07 \text{ kg/kmol}} = 1.219 \text{ kmol/min} \leftarrow \dot{n}_2$$

Also, with $\dot{m} = \rho(AV)$

$$(AV)_2 = \frac{\dot{m}_2}{S_{\text{glycol}}} = \frac{\dot{m}_2}{1.115 S_{\text{H}_2\text{O}}}$$

From Table A-2, $\nu_{\text{H}_2\text{O}} \approx \nu_f @ 20^\circ\text{C} = 1.0018 \times 10^{-3} \text{ m}^3/\text{kg}$ and $S_{\text{H}_2\text{O}} = 1/\nu_{\text{H}_2\text{O}} = 998.2 \text{ kg/m}^3$.

Thus

$$(AV)_2 = \frac{75.68 \text{ kg/min}}{1.115 (998.2) \text{ kg/m}^3} = 0.068 \text{ m}^3/\text{min} \leftarrow (AV)_2$$

(b) The area of the water supply pipe is

$$A_1 = \frac{\dot{m}_1}{S_{\text{H}_2\text{O}} V_1} = \frac{(75.68 \text{ kg/min})}{(998.2 \text{ kg/m}^3)(2.5 \text{ m/s})} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{10^4 \text{ cm}^2}{1 \text{ m}^2} \right| = 5.054 \text{ cm}^2$$

With $A = \pi d^2/4$

$$d_1 = \sqrt{\frac{4A_1}{\pi}} = \sqrt{\frac{(4)(5.054 \text{ cm}^2)}{\pi}} = 2.54 \text{ cm} \leftarrow d_1$$

Similarly for the glycol stream

$$A_2 = \frac{75.68}{(1.115)(998.2)(2.5)} \left| \frac{10^4}{60} \right| = 4.533 \text{ cm}^2$$

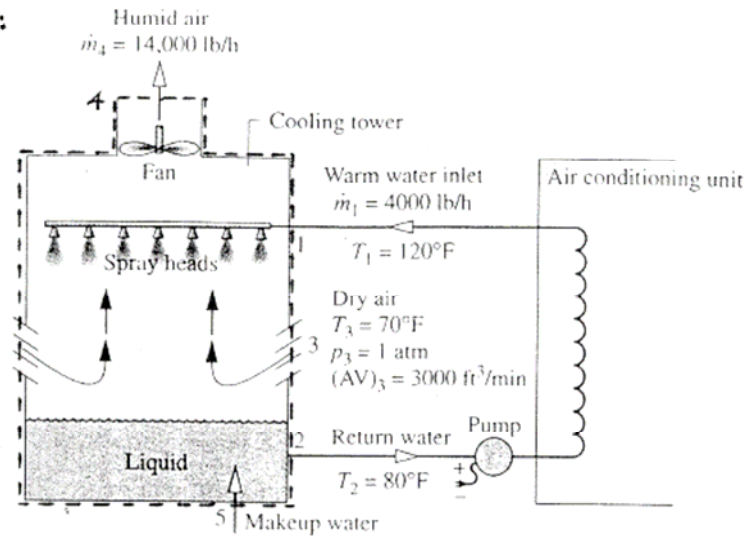
$$d_2 = \sqrt{\frac{(4)(4.533)}{\pi}} = 2.4 \text{ cm} \leftarrow d_2$$

PROBLEM 4.19

KNOWN: Data are known for inlet and exit streams of an air conditioner-cooling tower operating at steady state.

FIND: Determine the mass flow rate of the makeup water.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) The dry air stream at location 3 behaves as an ideal gas. (this can be checked using the compressibility chart.)

ANALYSIS: The mass rate balance for the control reduces as follows:

$$\frac{dm_{cv}}{dt} = \dot{m}_1 + \dot{m}_3 + \dot{m}_5 - \dot{m}_2 - \dot{m}_4$$

$$\dot{m}_5 = \dot{m}_2 - \dot{m}_1 + \dot{m}_4 - \dot{m}_3$$

From the schematic, we see that circuit through the air conditioning unit is closed. Therefore, at steady state $\dot{m}_1 = \dot{m}_2$. Thus

$$\dot{m}_5 = \dot{m}_4 - \dot{m}_3 \quad (*)$$

To get $\dot{m}_3$, use Eq. 4.4b and the ideal gas equation

$$\dot{m}_3 = \frac{(AV)_3}{v_3} = \frac{P_3 (AV)_3}{R T_3}$$

$$= \frac{(1 \text{ atm})(3000 \text{ ft}^3/\text{min})}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot ^\circ \text{R}}\right)(530^\circ \text{R})} \left| \frac{14.696 \text{ lbf/in}^2}{1 \text{ atm}} \right| \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{60 \text{ min}}{1 \text{ h}} \right|$$

$$= 13480 \text{ lb/h}$$

Finally, from (*)

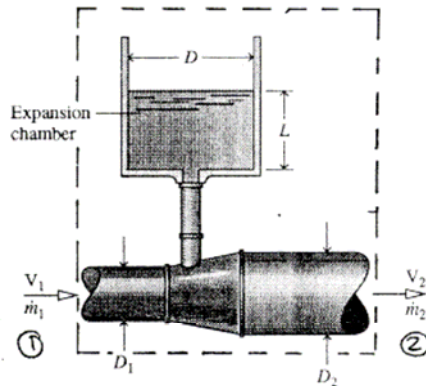
$$\dot{m}_5 = 14,000 - 13,480 = 520 \text{ lb/h} \quad \leftarrow \dot{m}_5$$

PROBLEM 4.20

KNOWN: A pipe carrying an incompressible liquid contains an expansion chamber.

FIND: (a) Develop an expression for the rate of change of liquid level in the chamber in terms of certain quantities. (b) Compare the relative magnitudes of the mass flow rates for specified values for dL/dt , the rate of change of liquid level.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume is shown on the accompanying schematic. 2. The liquid is modeled as incompressible. 3. Flow is one-dimensional at 1, 2.

ANALYSIS: (a) The mass rate balance for the control volume is

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2$$

with

$$m_{cv} = \rho \left(\frac{\pi D^2}{4} \right) L$$

$$\dot{m}_1 = \rho \left(\frac{\pi D_1^2}{4} \right) V_1$$

$$\dot{m}_2 = \rho \left(\frac{\pi D_2^2}{4} \right) V_2$$

the mass rate balance becomes

$$\rho \left(\frac{\pi D^2}{4} \right) \frac{dL}{dt} = \rho \left(\frac{\pi D_1^2}{4} \right) V_1 - \rho \left(\frac{\pi D_2^2}{4} \right) V_2$$

or, solving for dL/dt and simplifying

$$\frac{dL}{dt} = \frac{D_1^2 V_1 - D_2^2 V_2}{D^2} \quad \leftarrow \frac{dL}{dt}$$

(b) The mass flow rate expressions indicate $\dot{m}_1 \sim D_1^2 V_1$, $\dot{m}_2 \sim D_2^2 V_2$.

Thus,

$$\frac{dL}{dt} > 0 \Rightarrow D_1^2 V_1 > D_2^2 V_2 \Rightarrow \dot{m}_1 > \dot{m}_2$$

$$\frac{dL}{dt} = 0 \Rightarrow D_1^2 V_1 = D_2^2 V_2 \Rightarrow \dot{m}_1 = \dot{m}_2$$

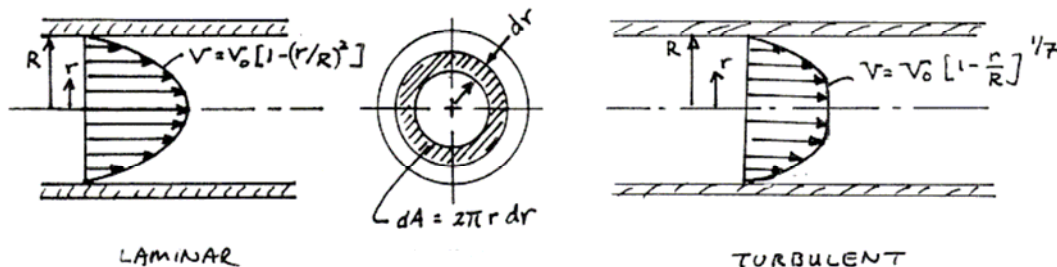
$$\frac{dL}{dt} < 0 \Rightarrow D_1^2 V_1 < D_2^2 V_2 \Rightarrow \dot{m}_1 < \dot{m}_2$$

PROBLEM 4.21

KNOWN: Velocity distributions are given for laminar and turbulent flow of an incompressible liquid in a circular pipe.

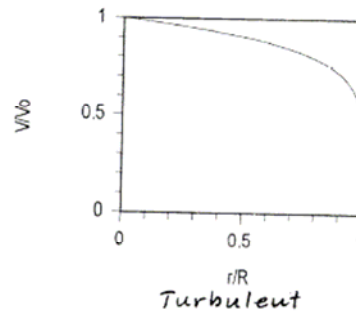
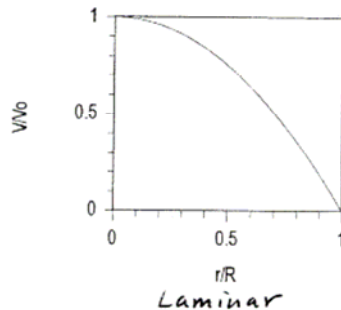
FIND: For each distribution, plot V/V_0 vs. r/R , derive expressions for the mass flow rate, average flow velocity, and specific kinetic energy. Determine the percent error if the specific kinetic energy is evaluated in terms of average velocity. Discuss.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The liquid is modeled as incompressible.

ANALYSIS: (a)



(b) Using Eq. 4.3, the mass flow rate for laminar flow is

$$\begin{aligned}
 \dot{m} &= \int_A \rho V dA \\
 &= \int_0^R \rho V_0 \left[1 - \left(\frac{r}{R}\right)^2\right] 2\pi r dr \\
 &= \rho V_0 2\pi \int_0^R \left[r - \frac{r^3}{R^2}\right] dr \\
 &= \rho V_0 2\pi \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right] \bigg|_{r=0}^{r=R} = \rho V_0 2\pi \left[\frac{R^2}{2} - \frac{R^2}{4}\right] \\
 &= \rho V_0 \pi \frac{R^2}{2}
 \end{aligned}$$

Using Eq. 4.4a, the average velocity is

$$\begin{aligned}
 V_{ave} &= \frac{\dot{m}}{\rho A} = \frac{\rho V_0 \pi R^2 / 2}{\rho (\pi R^2)} \\
 &= \frac{V_0}{2}
 \end{aligned}$$

Continued on next slide

Problem 4-21 continued

Using Eq. 4.3, the mass flow rate for turbulent flow is

$$\begin{aligned}\dot{m} &= \int_A \rho V dA \\ &= \int_0^R \rho V_0 \left[1 - \frac{r}{R}\right]^{\frac{1}{7}} 2\pi r dr \\ &= \rho V_0 2\pi \int_0^R r \left[1 - \frac{r}{R}\right]^{\frac{1}{7}} dr\end{aligned}$$

To evaluate the integral, let $u = 1 - r/R$ and $dr = -R du$. Then

$$\begin{aligned}\dot{m} &= \rho V_0 2\pi \int_1^0 R(1-u) u^{1/7} (-R du) = \rho V_0 2\pi \left[-R^2 \int_1^0 (u^{1/7} - u^{8/7}) du \right] \\ &= \rho V_0 2\pi \left[-R^2 \left(\frac{7}{8} u^{8/7} - \frac{7}{15} u^{15/7} \right) \right]_{u=1}^{u=0} \\ &= \rho V_0 2\pi \left[R^2 \frac{49}{120} \right] = \rho V_0 \pi \left(\frac{49}{60} \right) R^2\end{aligned}$$

Using Eq. 4.4a, the average velocity is

$$V_{ave} = \frac{\dot{m}}{\rho A} = \frac{\rho V_0 \pi \left(\frac{49}{60} \right) R^2}{\rho (\pi R^2)} = \frac{49}{60} V_0$$

- (c) The specific kinetic energy (kinetic energy per unit mass) carried through an area normal to the flow is

$$ke = \frac{\int_A \frac{1}{2} \rho V^2 dA}{\dot{m}} = \frac{\rho \int_A \frac{1}{2} V^2 dA}{\rho \int_A V dA} = \frac{\int_A \frac{1}{2} V^2 dA}{\int_A V dA} = \frac{\int_A \frac{1}{2} V^2 dA}{V_{ave} A} \quad (*)$$

Forming the ratio of Eq. (*) to the specific kinetic energy calculated as $V_{ave}^2/2$, we have the kinetic energy coefficient (or kinetic energy correction factor):

$$\alpha = \frac{\int_A \frac{1}{2} V^2 dA}{\frac{1}{2} V_{ave}^2 A} = \frac{\int_A V^2 dA}{V_{ave}^2 A}$$

For Laminar Flow: $\alpha = 2 \Rightarrow \% \text{ ERROR} = 50$

① For Turbulent Flow: $\alpha = 1.058 \Rightarrow \% \text{ ERROR} = 5.5$

The flatter turbulent velocity profile adheres most closely to the idealization of one-dimensional flow.

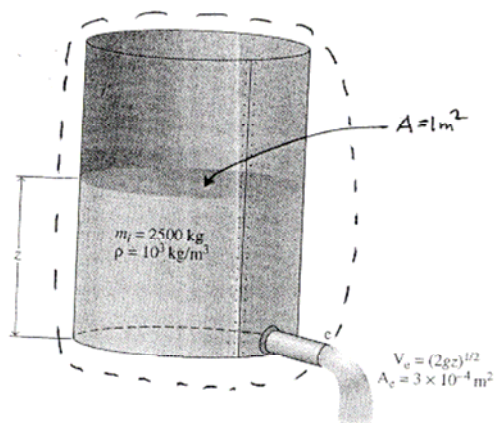
-
1. For further discussion, see R. W. Fox and A. T. McDonald, Introduction to Fluid Mechanics, 5th ed., J. Wiley & Sons, New York, pp 354-356.

PROBLEM 4.22

KNOWN: Data are provided for a cylindrical tank being drained of water.

FIND: Determine the time, in min., when the tank holds 900 kg of water.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. As shown in the sketch, a control volume enclosing the tank is being considered.
2. $g = 9.81 \text{ m/s}^2$
3. The density of water is 10^3 kg/m^3 .

Fig. P4.22

ANALYSIS: Mass rate balance: $\frac{dm_{cv}}{dt} = -\dot{m}_e$, where $\dot{m}_e$ is the mass flow rate at the exit. That is, $\dot{m}_e = \rho_e A_e V_e$ (Eq. 4.4a). Accordingly

$$\begin{aligned} \frac{dm_{cv}}{dt} &= -\rho A_e \sqrt{v_e}, \text{ where } v_e = (2gz)^{1/2} \\ &= -\rho A_e (2gz)^{1/2} \end{aligned} \quad (1)$$

Also, note that $m_{cv}(t) = \rho A z$, where A is the area of the water free surface. Thus, $z = m_{cv}(t)/\rho A$ and Eq. (1) becomes

$$\frac{dm_{cv}}{dt} = -\rho A_e \left[\frac{2g m_{cv}(t)}{\rho A} \right]^{1/2} = - \left[\frac{2g \rho A_e^2}{A} \right]^{1/2} (m_{cv}(t))^{1/2}$$

Thus,
$$\frac{1}{(m_{cv})^{1/2}} \frac{dm_{cv}}{dt} = - \left[\frac{2g \rho A_e^2}{A} \right]^{1/2}$$

Integrating from $t=0$ to t_f , we get $2(m_{cv})^{1/2} \Big|_0^{t_f} = - \left[\frac{2g \rho A_e^2}{A} \right]^{1/2} t_f$. Or

$$t_f = - \left[\frac{2A}{g \rho A_e^2} \right]^{1/2} \left[(m_{cv}(t_f))^{1/2} - (m_{cv}(0))^{1/2} \right] \quad (2)$$

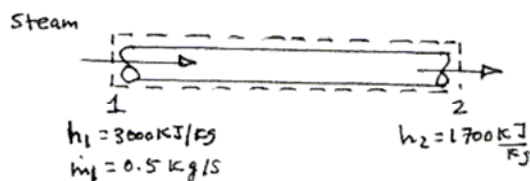
Inserting $A = 1 \text{ m}^2$, $g = 9.81 \text{ m/s}^2$, $\rho = 10^3 \text{ kg/m}^3$, $A_e = 3 \times 10^{-4} \text{ m}^2$, $m_{cv}(0) = 2500 \text{ kg}$, and $m_{cv}(t_f) = 900 \text{ kg}$, we get

$$\begin{aligned} t_f &= - \left[\frac{2(1 \text{ m}^2)}{(9.81 \frac{\text{m}}{\text{s}^2})(10^3 \frac{\text{kg}}{\text{m}^3})(3 \times 10^{-4} \text{ m}^2)^2} \right]^{1/2} \left[(900 \text{ kg})^{1/2} - (2500 \text{ kg})^{1/2} \right] \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \\ &= 15.86 \text{ min} \end{aligned}$$

PROBLEM 4.23

Steam enters a horizontal pipe operating at steady state with a specific enthalpy of 3000 kJ/kg and a mass flow rate of 0.5 kg/s. At the exit, the specific enthalpy is 1700 kJ/kg. If there is no significant change in kinetic energy from inlet to exit, determine the rate of heat transfer between the pipe and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} \equiv 0$, there is no significant change in kinetic energy from inlet to exit, and $\Delta PE = 0$ (horizontal).

ANALYSIS: Reducing Eq. 4.20a

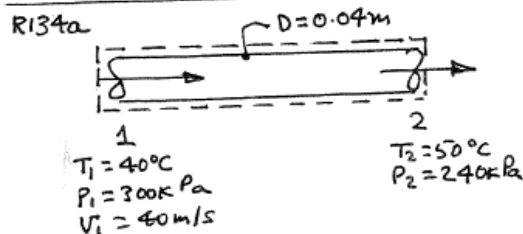
$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)} \right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m} [h_2 - h_1] = (0.5 \text{ kg/s})(1700 - 3000) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -650 \text{ kW} \leftarrow$$

PROBLEM 4.24

Refrigerant 134a enters a horizontal pipe operating at steady state at 40°C, 300 kPa and a velocity of 40 m/s. At the exit, the temperature is 50°C and the pressure is 240 kPa. The pipe diameter is 0.04 m. Determine (a) the mass flow rate of the refrigerant, in kg/s, (b) the velocity at the exit, in m/s, and (c) the rate of heat transfer between the pipe and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} \equiv 0$ and $\Delta p_e = 0$ (horizontal).

ANALYSIS: (a) Using Eq. 4.4a,

$$\dot{m}_1 = \frac{A V_1}{v_1} = \frac{\left(\frac{\pi (0.04\text{ m})^2}{4}\right) (40\frac{\text{m}}{\text{s}})}{0.08089\frac{\text{m}^3}{\text{kg}}}$$

Table A-12

$$\dot{m}_1 = 0.621\text{ kg/s} \quad \leftarrow (a)$$

(b) $\dot{m}_1 = \dot{m}_2$ (steady state)

$$\Rightarrow \frac{A V_1}{v_1} = \frac{A V_2}{v_2} \Rightarrow v_2 = \frac{v_2}{v_1} V_1$$

$$\therefore v_2 = \left(\frac{0.10562}{0.08089}\right) (40\text{ m/s}) = 52.23\text{ m/s}$$

(b)

(c) Reducing Eq. 6.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m} \left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \right]$$

$$= 0.621\frac{\text{kg}}{\text{s}} \left[\left(294.47 - 284.05 \right) \frac{\text{kJ}}{\text{kg}} + \left[\frac{(52.23)^2 - (40)^2}{2} \right] \left(\frac{\text{m}^2}{\text{s}^2} \right) \left| \frac{1\text{ N}}{1\text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N} \cdot \text{m}} \right| \right]$$

Unit conversions on K.E. term

$$= 0.621\frac{\text{kg}}{\text{s}} \left[10.42 + 0.56 \right] \frac{\text{kJ}}{\text{kg}} \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right| = +6.82\text{ kW} \quad \leftarrow (c)$$

PROBLEM 4.25

As shown in Fig. P4.25, air enters a pipe at 25°C, 100 kPa with a volumetric flow rate of 23 m³/h. On the outer pipe surface is an electrical resistor covered with insulation. With a voltage of 120 V, the resistor draws a current of 4 amps. Assuming the ideal gas model with $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$ for air and ignoring kinetic and potential energy effects, determine (a) the mass flow rate of the air, in kg/h, and (b) the temperature of the air at exit, in °C.

SCHEMATIC & GIVEN DATA:

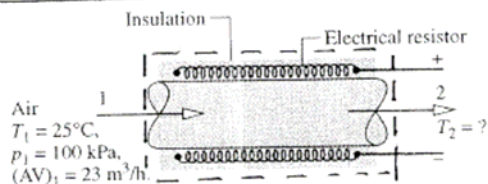


Fig. P4.25

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects can be ignored.
3. The air can be modeled as an ideal gas with constant specific heat c_p .

ANALYSIS:

$$(a) \quad \dot{m} = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{R T_1} = \frac{(10^5 \text{ N/m}^2)(23 \text{ m}^3/\text{h})}{\left(\frac{8314 \text{ N} \cdot \text{m}}{28.97 \text{ kg} \cdot \text{K}}\right)(298 \text{ K})} = 26.89 \frac{\text{kg}}{\text{h}} \quad \leftarrow (a)$$

(b) Reducing Eq. 4.20a using listed assumptions,

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \cancel{\frac{v_1^2 - v_2^2}{2}} + g(\cancel{z_1 - z_2}) \right]$$

where $(h_1 - h_2) = c_p (T_1 - T_2)$ and

$$\begin{aligned} \dot{W}_{cv} &= -(\text{voltage})(\text{current}) \\ &= -(120 \text{ volts})(4 \text{ amps}) \left| \frac{1 \text{ watt/amp}}{1 \text{ volt}} \right| \left| \frac{1 \text{ kW}}{10^3 \text{ watt}} \right| \\ &= -0.48 \text{ kW} \end{aligned}$$

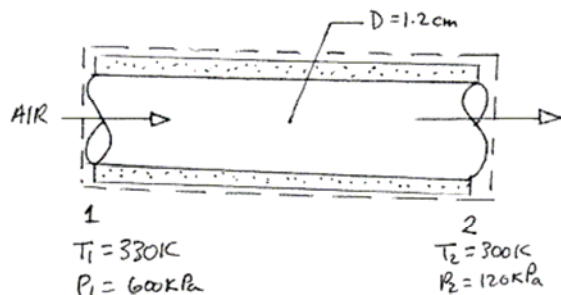
Collecting results, and solving for T_2 ,

$$\begin{aligned} T_2 &= T_1 - \frac{\dot{W}_{cv}}{\dot{m} c_p} \\ &= 298 - \frac{(-0.48 \text{ kW})}{(26.89 \frac{\text{kg}}{\text{h}})(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| \\ &= 362 \text{ K} (89^\circ\text{C}) \quad \leftarrow (b) \end{aligned}$$

PROBLEM 4.27

Air at 600 kPa, 330 K enters a well-insulated, horizontal pipe having a diameter of 1.2 cm and exits at 120 kPa, 300 K. Applying the ideal gas model for air, determine at steady state (a) the inlet and exit velocities, each in m/s, and (b) the mass flow rate, in kg/s.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer can be ignored. Also, $\Delta pe = 0$ (horizontal); $\dot{W}_{cv} = 0$.
3. The air can be modeled as an ideal gas.

ANALYSIS: (a) Mass rate balance, $\dot{m}_1 = \dot{m}_2$. That is

$$\frac{A_1 \bar{V}_1}{v_1} = \frac{A_2 \bar{V}_2}{v_2} \Rightarrow \bar{V}_1 = \frac{v_1}{v_2} \bar{V}_2 = \frac{(RT_1/P_1)}{(RT_2/P_2)} \bar{V}_2$$

$$\bar{V}_1 = \left(\frac{P_2}{P_1}\right) \left(\frac{T_1}{T_2}\right) \bar{V}_2 = \left(\frac{120 \text{ kPa}}{600 \text{ kPa}}\right) \left(\frac{330 \text{ K}}{300 \text{ K}}\right) \bar{V}_2$$

$$\bar{V}_1 = 0.22 \bar{V}_2 \quad (1)$$

Energy rate balance,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{\bar{V}_1^2 - \bar{V}_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow 0 = (h_1 - h_2) + \frac{\bar{V}_1^2 - \bar{V}_2^2}{2} \Rightarrow 0 = (h_1 - h_2) + \frac{(0.22 \bar{V}_2)^2 - \bar{V}_2^2}{2}$$

Or, with data from Table A-22 for h_1 and h_2 ,

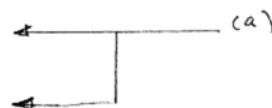
$$\frac{0.9516 \bar{V}_2^2}{2} = (330.34 - 300.19) \frac{\text{kJ}}{\text{kg}}$$

$$\Rightarrow \bar{V}_2 = \sqrt{\frac{2(330.34 - 300.19) \text{ kJ/kg}}{0.9516} \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right|}$$

$$= 251.73 \text{ m/s}$$

And with Eq. (1)

$$\bar{V}_1 = 0.22 \bar{V}_2 = 55.38 \text{ m/s}$$



(b) Then, with

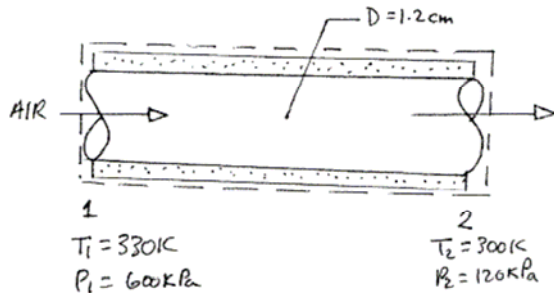
$$\dot{m} = \frac{A_2 \bar{V}_2}{v_2} = \frac{P_2 A_2 \bar{V}_2}{RT_2}$$

$$= \frac{(120 \times 10^3 \frac{\text{N}}{\text{m}^2}) \left(\frac{\pi}{4} \left(\frac{1.2 \text{ m}}{100} \right)^2 \right) (251.73 \text{ m/s})}{\left(\frac{8314}{28.97} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (300 \text{ K})} = 0.04 \frac{\text{kg}}{\text{s}} \quad (b)$$

PROBLEM 4.27

Air at 600 kPa, 330 K enters a well-insulated, horizontal pipe having a diameter of 1.2 cm and exits at 120 kPa, 300 K. Applying the ideal gas model for air, determine at steady state (a) the inlet and exit velocities, each in m/s, and (b) the mass flow rate, in kg/s.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer can be ignored. Also, $\Delta p_e = 0$ (horizontal); $\dot{W}_{cv} = 0$.
3. The air can be modeled as an ideal gas.

ANALYSIS: (a) Mass rate balance, $\dot{m}_1 = \dot{m}_2$. That is

$$\frac{A_1 \bar{V}_1}{v_1} = \frac{A_2 \bar{V}_2}{v_2} \Rightarrow \bar{V}_1 = \frac{v_1}{v_2} \bar{V}_2 = \frac{(RT_1/P_1)}{(RT_2/P_2)} \bar{V}_2$$

$$\bar{V}_1 = \left(\frac{P_2}{P_1}\right) \left(\frac{T_1}{T_2}\right) \bar{V}_2 = \left(\frac{120 \text{ kPa}}{600 \text{ kPa}}\right) \left(\frac{330 \text{ K}}{300 \text{ K}}\right) \bar{V}_2$$

$$\bar{V}_1 = 0.22 \bar{V}_2 \quad (1)$$

Energy rate balance,

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow 0 = (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} \Rightarrow 0 = (h_1 - h_2) + \frac{(0.22 \bar{V}_2)^2 - \bar{V}_2^2}{2}$$

Or, with data from Table A-22 for h_1 and h_2 ,

$$\frac{0.9516 \bar{V}_2^2}{2} = (330.34 - 300.19) \frac{\text{kJ}}{\text{kg}}$$

$$\Rightarrow \bar{V}_2 = \sqrt{\frac{2(330.34 - 300.19) \text{ kJ/kg}}{0.9516} \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right|}$$

$$= 251.73 \text{ m/s}$$

And with Eq. (1)

$$\bar{V}_1 = 0.22 \bar{V}_2 = 55.38 \text{ m/s}$$



(b) Then, with

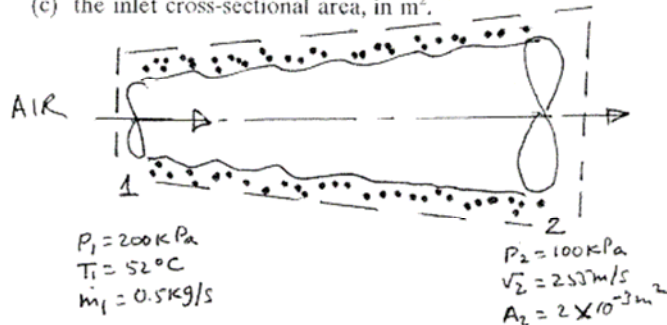
$$\dot{m} = \frac{A_2 \bar{V}_2}{v_2} = \frac{P_2 A_2 \bar{V}_2}{RT_2}$$

$$= \frac{(120 \times 10^3 \frac{\text{N}}{\text{m}^2}) \left(\frac{\pi}{4} (1.2 \times 10^{-2} \text{ m})^2\right) (251.73 \text{ m/s})}{\left(\frac{8314}{28.97} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}\right) (300 \text{ K})} = 0.04 \frac{\text{kg}}{\text{s}} \quad (b)$$

PROBLEM 4.28

At steady state, air at 200 kPa, 52°C and a mass flow rate of 0.5 kg/s enters an insulated duct having differing inlet and exit cross-sectional areas. At the duct exit, the pressure is 100 kPa, the velocity is 255 m/s, and the cross-sectional area is $2 \times 10^{-3} \text{ m}^2$. Assuming the ideal gas model, determine

- the temperature of the air at the exit, in °C.
- the velocity of the air at the inlet, in m/s.
- the inlet cross-sectional area, in m^2 .



ENGR. MODEL:

- The control volume shown in the schematic is at steady state.
- For the control volume, $\dot{W}_{cv} = 0$ and stray heat transfer can be ignored. Also, $\Delta p_e = 0$.
- The air is modeled as an ideal gas.

ANALYSIS: (a) Mass rate balance, $\dot{m}_2 = \dot{m}_1$. Thus, with $\dot{m}_2 = A_2 V_2 / v_2$ and

$v_2 = RT_2 / P_2$, we get

$$T_2 = \frac{A_2 V_2 P_2}{\dot{m} R} = \frac{(2 \times 10^{-3} \text{ m}^2)(255 \text{ m/s})(10^5 \text{ N/m}^2)}{(0.5 \text{ kg/s}) \left(\frac{8314 \text{ N}\cdot\text{m}}{28.97 \text{ kg}\cdot\text{K}} \right)} = 355.4 \text{ K} \quad (82^\circ\text{C}) \quad \leftarrow (a)$$

(b) Energy rate balance,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow V_1 = \sqrt{V_2^2 + 2(h_2 - h_1)}$$

with specific enthalpy data from Table A-22

$$V_1 = \sqrt{\left(255 \frac{\text{m}}{\text{s}} \right)^2 + 2 \left(355.4 - 325.31 \right) \frac{\text{kJ}}{\text{kg}} \left| \frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}} \right|} = 355 \text{ m/s} \quad \leftarrow (b)$$

(c) Since $\dot{m}_1 = A_1 V_1 / v_1 = \frac{P_1 A_1 V_1}{R T_1}$,

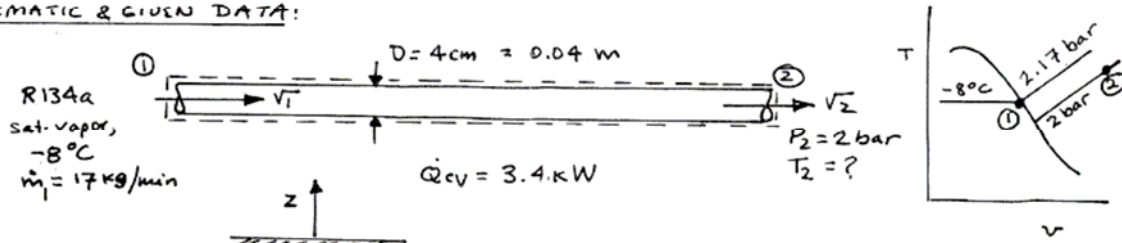
$$A_1 = \frac{\dot{m}_1 R T_1}{P_1 V_1} = \frac{(0.5 \text{ kg/s}) \left(\frac{8314 \text{ N}\cdot\text{m}}{28.97 \text{ kg}\cdot\text{K}} \right) (325 \text{ K})}{(200 \times 10^3 \frac{\text{N}}{\text{m}^2}) (355 \text{ m/s})} = 6.57 \times 10^{-4} \text{ m}^2 \quad \leftarrow (c)$$

PROBLEM 4.29

KNOWN: Refrigerant 134a flows through a horizontal pipe at steady state, for which operating data are provided.

FIND: Determine the exit temperature and velocity, and the inlet velocity.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $W_{cv} = 0$ and there is no change in potential energy from inlet to exit.

ANALYSIS: At steady state, the mass rate balance reduces to $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. The inlet velocity is found as follows:

$$\dot{m} = \frac{A_1 v_1}{v_1} \Rightarrow v_1 = \frac{\dot{m} v_1}{A_1} = \frac{4 \dot{m} v_1}{\pi d^2}$$

With v_1 from Table A-10

$$v_1 = \frac{4(17 \text{ kg/min}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (0.0919 \text{ m}^3/\text{kg})}{\pi (0.04 \text{ m})^2} = 20.72 \text{ m/s}$$

Similarly, v_2 is found from $\dot{m}_1 = \dot{m}_2$ as follows:

$$\dot{m}_1 = \dot{m}_2 \Rightarrow \frac{A_1 v_1}{v_1} = \frac{A_2 v_2}{v_2} \Rightarrow v_2 = \frac{v_2}{v_1} v_1 \quad (*)$$

In this expression, v_1 and v_1 are known. However, $v_2 = v(T_2, P_2)$ is unknown.

Another relation is obtained using the energy rate balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{v_1^2 - v_2^2}{2} + g(z_1 - z_2) \right]$$

$$= \dot{Q}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{v_1^2 - v_2^2}{2} \right]$$

Inserting known values, including h_1 from Table A-10

$$0 = (3.4 \text{ kW}) \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| + (17 \text{ kg/min}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left[(242.54 - h_2) \frac{\text{kJ}}{\text{kg}} + \frac{(20.72 \text{ m/s})^2 - v_2^2}{2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \right] \quad (**)$$

where $h_2 = h(T_2, P_2)$.

Equations (*) and (**) can be solved simultaneously by referring to Table A-12 for v_2 and h_2 as functions of T_2 and P_2 . The process is iterative. To avoid iteration, IT can be used effectively, as follows:

Continued on next slide

Problem 4-29 continued

IT Code

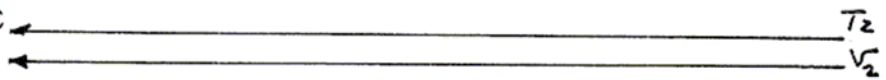
```
// Data
T1 = -8 // °C
x1 = 1
p2 = 2 // bar
d = 4 // cm
mdot = 17 // kg/min
Qdot = 3.4 // kW

// Determine mdot
p1 = Psat_T("R134A", T1)
v1 = vsat_Px("R134A", p1, x1)
h1 = hsat_Px("R134A", p1, x1)
mdot = ((pi * (d / 100)^2 / 4) * V1 / v1) * 60

// Find exit state
0 = Qdot + (mdot / 60) * ((h1 - h2) + (V1^2 - V2^2) / (2 * 1000))
V2 / v2 = V1 / v1
h2 = h_PT("R134A", p2, T2)
v2 = v_PT("R134A", p2, T2)
```

IT Results

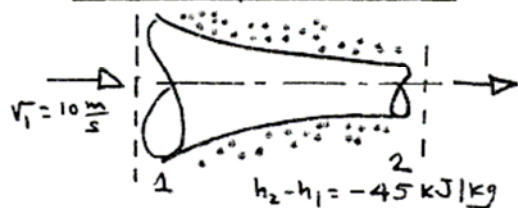
① $T_2 = 4.976^\circ\text{C} \approx 5^\circ\text{C}$
 $V_2 = 24.08 \text{ m/s}$
 $V_1 = 20.71 \text{ m/s}$



PROBLEM 4.30

Steam enters a well-insulated, horizontal nozzle operating at steady state with a velocity of 10 m/s. If the specific enthalpy decreases by 45 kJ/kg from inlet to exit, determine the velocity at the exit, in m/s.

Schematic & Given Data:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and $\Delta p_e = 0$.

ANALYSIS: Reducing Eq. 4.20a

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m} \left[h_1 - h_2 + V_1^2 - V_2^2 + g(z_1 - z_2) \right] \Rightarrow V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

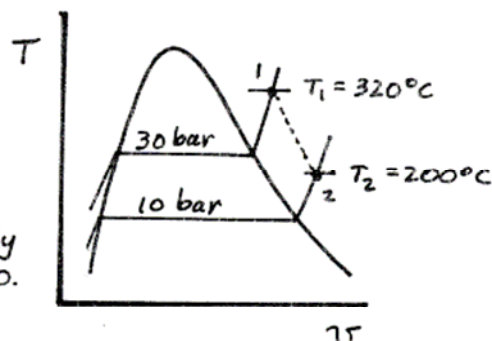
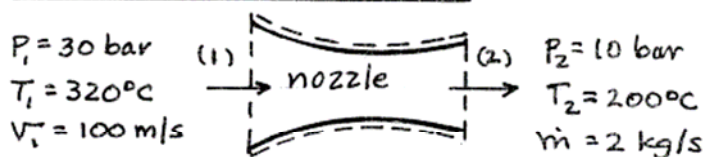
$$\therefore V_2 = \sqrt{\left(10 \frac{\text{m}}{\text{s}}\right)^2 + 2 \left(45 \frac{\text{kJ}}{\text{kg}}\right) \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|} = 300 \text{ m/s} \quad \leftarrow$$

PROBLEM 4.31

KNOWN: Steam flows through a nozzle with known conditions at the inlet and exit. The mass flow rate is given.

FIND: Determine (a) the exit velocity, (b) the inlet and exit flow areas.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible.

ANALYSIS: (a) The velocity of steam at the exit is found from the steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for V_2

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

From Table A-4, $h_1 = 3043.4 \text{ kJ/kg}$ and $h_2 = 2827.9 \text{ kJ/kg}$. Thus

$$V_2 = \sqrt{2(3043.4 - 2827.9) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kg} \cdot \text{m}^2/\text{s}^2}{1 \text{ N}} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| + (100^2) \text{ m}^2/\text{s}^2}$$

$$= 664.1 \text{ m/s} \leftarrow V_2$$

(b) To find the inlet and exit flow areas, use $\dot{m} = (A\dot{v})/v$. Solving

$$A_1 = \frac{\dot{m} v_1}{V_1} \quad \text{and} \quad A_2 = \frac{\dot{m} v_2}{V_2}$$

From Table A-4, $v_1 = 0.0850 \text{ m}^3/\text{kg}$ and $v_2 = 0.2060 \text{ m}^3/\text{kg}$. Thus

$$A_1 = \frac{(2 \text{ kg/s})(0.0850 \text{ m}^3/\text{kg})}{(100 \text{ m/s})} \left| \frac{10^4 \text{ cm}^2}{1 \text{ m}^2} \right| = 17 \text{ cm}^2 \leftarrow A_1$$

and

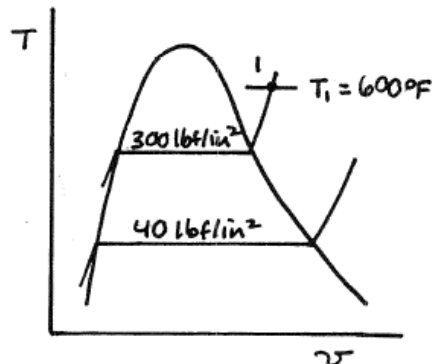
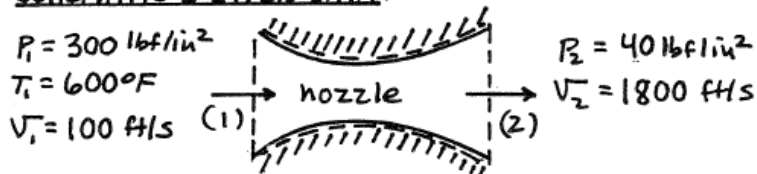
$$A_2 = \frac{(2)(0.2060)}{(664.1)} \left| \frac{10^4}{1} \right| = 6.2 \text{ cm}^2 \leftarrow A_2$$

PROBLEM 4.32

KNOWN: Steam flows through a well-insulated nozzle with known conditions at the inlet and exit.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) There is no heat transfer, and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible.

ANALYSIS: The pressure is known at the exit. The state is fixed by determining h_2 using the steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2} \right)$$

From Table A-4E, $h_1 = 1314.5 \text{ Btu/lb}$. Thus

$$\begin{aligned}
 h_2 &= (1314.5 \text{ Btu/lb}) + \left(\frac{100^2 - 1800^2}{2} \right) \frac{\text{ft}^2}{\text{s}^2} \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\
 &= 1250.03 \text{ Btu/lb}
 \end{aligned}$$

Interpolating in Table A-4E at $P_2 = 40 \text{ lbf/in}^2$, $h_2 = 1250.03 \text{ Btu/lb}$ gives

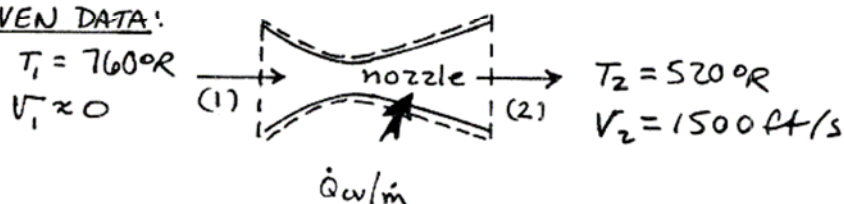
$$T_2 = 428^\circ\text{F} \leftarrow T_2$$

PROBLEM 4.33

KNOWN: Air flows through a nozzle with known conditions at the inlet and exit. Heat transfer occurs from the air to the surroundings.

FIND: Determine the heat transfer per lb of air flowing.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) For the control volume, $\dot{W}_{cv} = 0$. (3) The air behaves as an ideal gas. (4) The inlet kinetic energy and potential energy effects are negligible.

ANALYSIS: To determine the heat transfer, begin with the steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2) \right]$$

Solving for $\dot{Q}_{cv}/\dot{m}$

$$\frac{\dot{Q}_{cv}}{\dot{m}} = h_2 - h_1 + \frac{V_2^2}{2}$$

From Table A-22E, $h_1 = 182.08 \text{ Btu/lb}$ and $h_2 = 124.27 \text{ Btu/lb}$. Thus

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}} &= (124.27 - 182.08) \frac{\text{Btu}}{\text{lb}} + \frac{(1500 \frac{\text{ft}}{\text{s}})^2}{2} \left| \frac{1 \text{ lb} \cdot \text{ft}}{32.2 \text{ ft} \cdot \text{lb/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lb}} \right| \\ \frac{\dot{Q}_{cv}}{\dot{m}} &= -12.9 \text{ Btu/lb} \leftarrow \dot{Q}/\dot{m} \end{aligned}$$

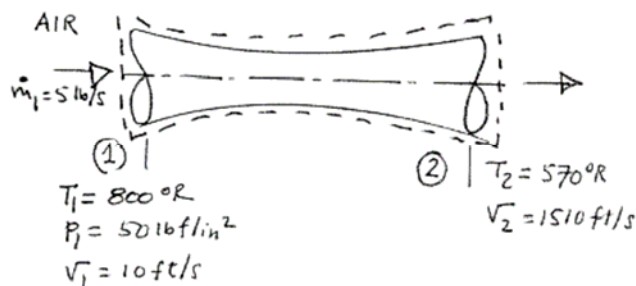
The negative sign indicates that the heat transfer is from the nozzle to the surroundings.

PROBLEM 4.34

KNOWN: Steady-state operating data are provided for a nozzle.

FIND: Evaluate the area at the inlet and $\dot{Q}_{cv}/\dot{m}$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. As shown by the sketch, a control volume encloses the nozzle.
2. The control volume is at steady state.
3. There is no change in potential energy from inlet to exit (horizontal).
4. Air is modeled as an ideal gas.

ANALYSIS: (a) Mass rate balance: $\dot{m}_2 = \dot{m}_1$, where $\dot{m}_1 = 5 \text{ lb/s}$. Using

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} \Rightarrow A_1 = \frac{\dot{m}_1 v_1}{V_1} = \frac{\dot{m}_1 R T_1}{P_1 V_1} = \frac{(5 \text{ lb/s}) \left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right) (800^\circ\text{R})}{(50 \times 144 \frac{\text{lbf}}{\text{ft}^2}) (10 \frac{\text{ft}}{\text{s}})} = 2.96 \text{ ft}^2 \leftarrow A_1$$

(b) Energy rate balance: $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$

$$\Rightarrow \frac{\dot{Q}_{cv}}{\dot{m}} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2}$$

Data from Table A-22E, $h_1 = 191.81 \text{ Btu/lb}$, $h_2 = 136.26 \text{ Btu/lb}$. Then

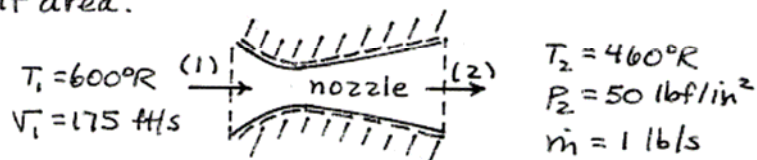
$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}} &= (136.26 - 191.81) \frac{\text{Btu}}{\text{lb}} + \left[\frac{(1510 \text{ ft/s})^2 - (10 \text{ ft/s})^2}{2} \right] \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\ &= -10.04 \text{ Btu/lb} \end{aligned} \quad \leftarrow \frac{\dot{Q}_{cv}}{\dot{m}}$$

PROBLEM 4.35

KNOWN: Helium flows through a well-insulated nozzle with known conditions at the inlet and exit. The mass flow rate is given.

FIND: Determine the exit area.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) The helium behaves as an ideal gas. (4) Potential energy effects are negligible.

ANALYSIS: From the mass rate balance, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. With $\dot{m} = (\dot{A}V)/v$

$$A_2 = \frac{\dot{m} v_2}{V_2}$$

Introducing the ideal gas equation of state

$$A_2 = \frac{\dot{m} R T_2}{V_2 P_2}$$

The exit velocity is found using the steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} [(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)]$$

From Table A-21, the specific heat c_p of helium is

$$c_p = \left(\frac{5}{2}\right) R = \left(\frac{5}{2}\right) \frac{(1.986 \text{ Btu/lb mol} \cdot ^\circ\text{R})}{(4.003 \text{ lb/lb mol})} = 1.24 \text{ Btu/lb} \cdot ^\circ\text{R}$$

Since c_p is a constant $\Delta h = c_p \Delta T$, and

$$\begin{aligned} V_2 &= \sqrt{2 c_p (T_1 - T_2) + V_1^2} \\ &= \sqrt{2(1.24 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}})(600 - 460)^\circ\text{R} \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right| \left| \frac{178 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}} \right| + (175^2) \frac{\text{ft}^2}{\text{s}^2}} \\ &= 2954 \text{ ft/s} \end{aligned}$$

Finally

$$\begin{aligned} A_2 &= \frac{(1 \text{ lb/s}) \left(\frac{1545 \text{ ft} \cdot \text{lbf}}{4.003 \text{ lb} \cdot ^\circ\text{R}} \right) (460^\circ\text{R}) \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right|}{(2954 \text{ ft/s}) (50 \text{ lbf/in}^2)} \\ &= 8.35 \times 10^{-3} \text{ ft}^2 \end{aligned}$$

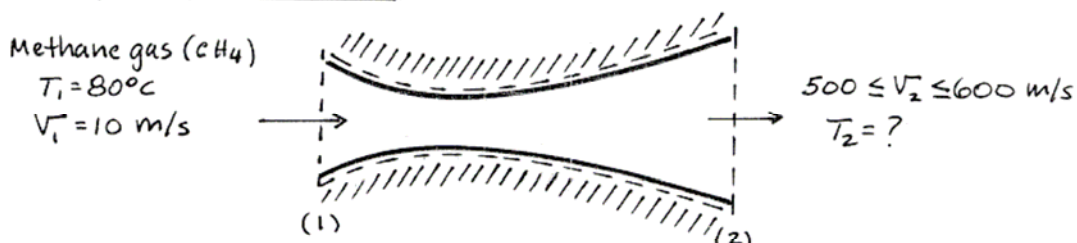
A_2

PROBLEM 4.36

KNOWN: Methane gas flows through a nozzle with known inlet conditions and a specified range of exit velocities.

FIND: Plot exit temperature versus exit velocity.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (3) Potential energy effects can be ignored. (4) The methane behaves as an ideal gas.

ANALYSIS: Since $h = h(T)$ for the ideal gas, the exit temperature is determined by evaluating h_2 using steady state mass and energy balances:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Thus, with $h = h(T)$

$$0 = h(T_1) - h(T_2) + \left(\frac{V_1^2 - V_2^2}{2} \right) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \quad (*)$$

for h in kJ/kg and V in m/s.

Using IT, the functions $h(T)$ for methane (CH_4) as an ideal gas are accessed readily, and data for T_2 is obtained using the following code. The results are shown on the accompanying plot.

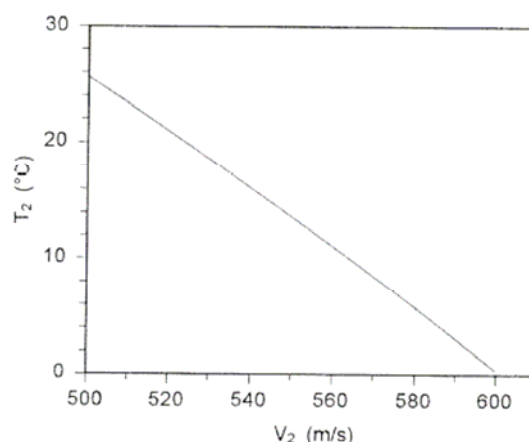
IT Code

```
T1 = 80 // °C
V1 = 10 // m/s
V2 = 550 // m/s
```

```
0 = (h1 - h2) + ((V1^2 - V2^2) / 2) * (1 / 10^3) // kJ/kg
h1 = h_T("CH4", T1)
h2 = h_T("CH4", T2)
```

```
// Using the Explore button, sweep V2 from
// 500 to 600 in steps of 0.1.
```

① Result for $V_2 = 550$: $T_2 = 13.63^\circ\text{C}$



From (*), $h(T_2) = h(T_1) + \left(\frac{V_1^2 - V_2^2}{2} \right)$. Thus, as V_2 increases, $h(T_2)$ decreases. Therefore, T_2 decreases as expected.

1. A sample calculation using the c_p function from Table A-21 to evaluate the enthalpy change in (*) confirms this result. With IT, this integration is not necessary.

PROBLEM 4.37

KNOWN: Steady-state operating data are provided for a jet engine.
FIND: Determine the velocity at the diffuser exit.

SCHEMATIC & GIVEN DATA:

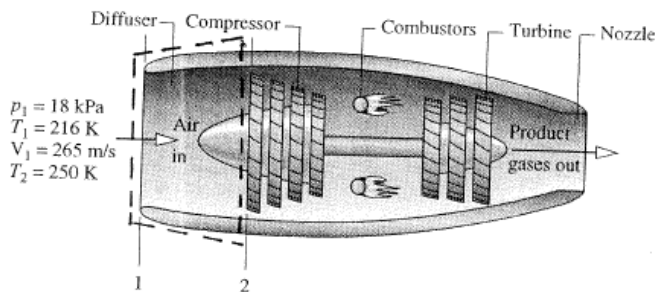


Fig. P4.37

ENGR. MODEL

1. As shown in the sketch, a control volume encloses the diffuser.
2. The control volume is at steady state.
3. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$ and potential energy effects can be ignored.
4. The air is modeled as an ideal gas.

ANALYSIS: For the control volume, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_a$. An energy rate balance reads

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_a \left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$

Accordingly,

$$V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

$$= \sqrt{\left(265 \frac{\text{m}}{\text{s}}\right)^2 + 2 \underbrace{(215.97 - 250.05)}_{\text{Table A-22}} \frac{\text{KJ}}{\text{kg}} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ KJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|}$$

$$= 45 \text{ m/s}$$

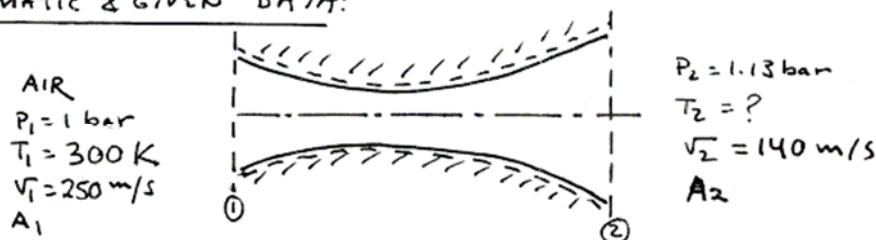
$$\leftarrow V_2$$

PROBLEM 4.38

KNOWN: Data is provided for a diffuser at steady state, through which air is flowing.

FIND: Determine the ratio of the exit flow area to the inlet flow area, and the exit temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and potential energy effects can be ignored. 3. Air is modeled as an ideal gas.

ANALYSIS: The mass rate balance reads $\dot{m}_2 = \dot{m}_1$, or

$$\frac{A_2 V_2}{v_2} = \frac{A_1 V_1}{v_1} \Rightarrow \frac{A_2 V_2}{(RT_2/P_2)} = \frac{A_1 V_1}{(RT_1/P_1)} \Rightarrow \frac{A_2}{A_1} = \left(\frac{P_1}{P_2}\right) \left(\frac{T_2}{T_1}\right) \left(\frac{V_1}{V_2}\right) \quad (1)$$

The exit temperature, T_2 , can be obtained using an energy rate balance:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$

$$h_2 = h_1 + \frac{V_1^2}{2} - \frac{V_2^2}{2}$$

Using data from Table A-22

$$h_2 = \left(300.19 \frac{\text{kJ}}{\text{kg}}\right) + \left(\frac{250^2 - 140^2}{2}\right) \frac{\text{m}^2}{\text{s}^2} \left| \frac{1 \text{ N}}{1 \text{ kg m/s}^2} \right| \left| \frac{1 \text{ kJ}}{1000 \text{ N m}} \right|$$

$$= 321.64 \text{ kJ/kg}$$

Then, interpolating in Table A-22 for $h_2 = 321.64 \text{ kJ/kg}$

$$T_2 = 321.3 \text{ K} \leftarrow T_2$$

Returning to Eq. (1)

$$\frac{A_2}{A_1} = \left(\frac{1 \text{ bar}}{1.13 \text{ bar}}\right) \left(\frac{321.3 \text{ K}}{300 \text{ K}}\right) \left(\frac{250 \text{ m/s}}{140 \text{ m/s}}\right)$$

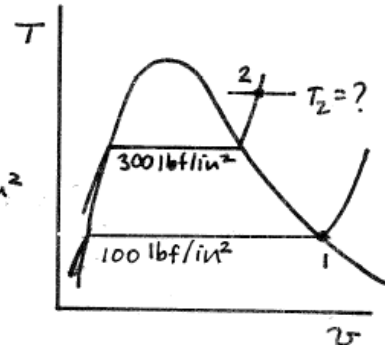
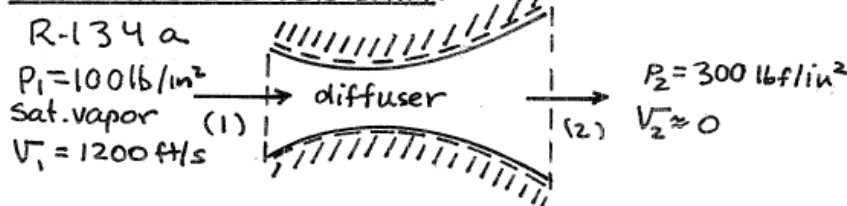
$$= 1.692 \quad A_2/A_1$$

PROBLEM 4.39

KNOWN: Refrigerant 134a passes through an insulated diffuser. Data are known at the inlet and exit.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) The heat transfer is negligible and $W_{cv} = 0$. (3) Potential energy effects and kinetic energy at exit can be neglected.

ANALYSIS: The exit pressure is given. The state is fixed by determining h_2 using a steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2

$$h_2 = h_1 + \frac{V_1^2}{2}$$

From Table A-11E, at $P_1 = 100 \frac{\text{lbf}}{\text{in}^2}$, $h_1 = h_g = 112.46 \text{ Btu/lb}$. Thus

$$\begin{aligned} h_2 &= 112.46 \text{ Btu/lb} + \frac{1}{2} (1200^2 \text{ ft}^2/\text{s}^2) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft}/\text{s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\ &= 141.20 \text{ Btu/lb} \end{aligned}$$

Interpolating in Table A-12E at $P_2 = 300 \text{ lbf/in}^2$, $h_2 = 141.20 \text{ Btu/lb}$ gives

$$T_2 \approx 222.9^\circ\text{F} \leftarrow T_2$$

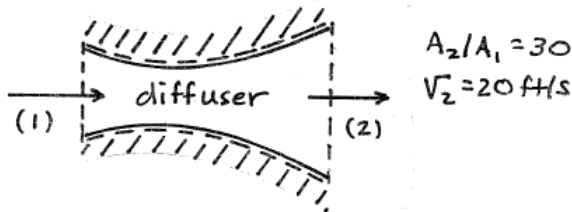
PROBLEM 4.40

KNOWN: Carbon dioxide flows through a well-insulated diffuser with known conditions at the inlet and exit.

FIND: Determine the exit temperature and pressure and the mass flow rate.

SCHEMATIC & GIVEN DATA:

$$\begin{aligned} \text{CO}_2 \\ P_1 &= 20 \text{ lbf/in}^2 \\ T_1 &= 500^\circ\text{R} \\ V_1 &= 800 \text{ ft/s} \\ A_1 &= 1.4 \text{ in}^2 \end{aligned}$$



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible. (4) The carbon dioxide behaves as an ideal gas.

ANALYSIS: Since $h = h(T)$ for an ideal gas, the exit temperature can be found by evaluating h_2 . Beginning with the steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2 , and noting that $\bar{h} = h/M$

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2} \right) \Rightarrow \bar{h}_2 = \bar{h}_1 + \left(\frac{V_1^2 - V_2^2}{2} \right) M$$

where M is the molecular weight. From Table A-23E, $\bar{h}_1 = 3706.2 \text{ Btu/lbmol}$ and

$$\begin{aligned} \bar{h}_2 &= 3706.2 \frac{\text{Btu}}{\text{lbmol}} + \left(\frac{800^2 - 20^2}{2} \right) \frac{\text{ft}^2}{\text{s}^2} \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{44.01 \text{ lb}}{1 \text{ lbmol}} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\ &= 4268 \text{ Btu/lbmol} \end{aligned}$$

Interpolating in Table A-23E; $T_2 \approx 563.6^\circ\text{R}$ ← T_2

To get P_2 , begin with $\dot{m}_1 = \dot{m}_2$. Thus

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

with $p v = RT$

$$\frac{P_1}{RT_1} (A_1 V_1) = \frac{P_2}{RT_2} (A_2 V_2)$$

or

$$\begin{aligned} P_2 &= P_1 \left(\frac{T_2}{T_1} \right) \left(\frac{A_1}{A_2} \right) \left(\frac{V_1}{V_2} \right) = (20 \frac{\text{lbf}}{\text{in}^2}) \left(\frac{563.6}{500} \right) \left(\frac{1}{30} \right) \left(\frac{800}{20} \right) \\ &= 30.06 \text{ lbf/in}^2 \leftarrow P_2 \end{aligned}$$

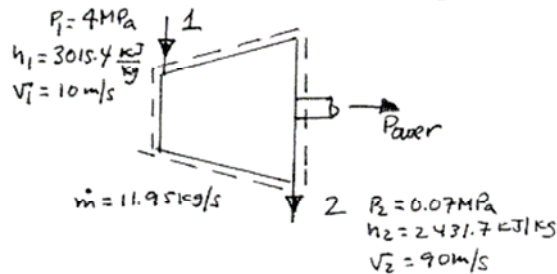
Finally, the mass flow rate is

$$\begin{aligned} \dot{m} &= \frac{(A_1 V_1) P_1}{RT_1} = \frac{(1.4 \text{ in}^2)(800 \text{ ft/s})(20 \text{ lbf/in}^2)}{\left(\frac{1545}{44.01} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right)(500^\circ\text{R})} \\ &= 1.276 \text{ lb/s} \leftarrow \dot{m} \end{aligned}$$

PROBLEM 4.41

Steam enters a well-insulated turbine operating at steady state at 4 MPa with a specific enthalpy of 3015.4 kJ/kg and a velocity of 10 m/s. The steam expands to the turbine exit where the pressure is 0.07 MPa, specific enthalpy is 2431.7 kJ/kg, and the velocity is 90 m/s. The mass flow rate is 11.95 kg/s. Neglecting potential energy effects, determine the power developed by the turbine, in kW.

SCHEMATIC & GIVEN DATA:



ENER. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$ and $\Delta pe \approx 0$.

ANALYSIS: Reducing Eq. 4.20a

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right)$$

$$\Rightarrow \dot{W}_{cv} = \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} \right]$$

$$= (11.95 \frac{\text{kg}}{\text{s}}) \left[(3015.4 \frac{\text{kJ}}{\text{kg}} - 2431.7 \frac{\text{kJ}}{\text{kg}}) + \left[\frac{(10 \frac{\text{m}}{\text{s}})^2 - (90 \frac{\text{m}}{\text{s}})^2}{2} \right] \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \right]$$

$$= 11.95 \frac{\text{kg}}{\text{s}} \left[583.7 \frac{\text{kJ}}{\text{kg}} - 4 \frac{\text{kJ}}{\text{kg}} \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

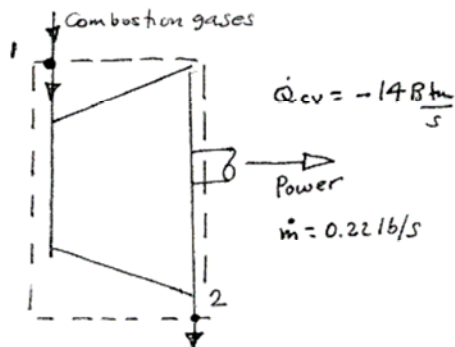
$$= 6927 \text{ kW}$$



PROBLEM 4.42

Hot combustion gases, modeled as air behaving as an ideal gas, enter a turbine at 145 lbf/in.^2 , 2700°R with a mass flow rate of 0.22 lb/s and exit at 29 lbf/in.^2 and 1620°R . If heat transfer from the turbine to its surroundings occurs at a rate of 14 Btu/s , determine the power output of the turbine, in hp.

SCHEMATIC & GIVEN DATA:



ENGINEER MODEL

1. The control volume shown in the figure is at steady state.
2. Kinetic and potential energy effects are ignored.
3. The combustion gases are modeled as air as an ideal gas.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}(h_1 - h_2)$$

with enthalpy data from Table A-22E,

$$\begin{aligned} \dot{W}_{cv} &= -14 \frac{\text{Btu}}{\text{s}} + 0.22 \frac{\text{lb}}{\text{s}} [703.35 - 401.10] \frac{\text{Btu}}{\text{lb}} \\ &= 52.495 \frac{\text{Btu}}{\text{s}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| \\ &= 74.26 \text{ hp} \end{aligned}$$

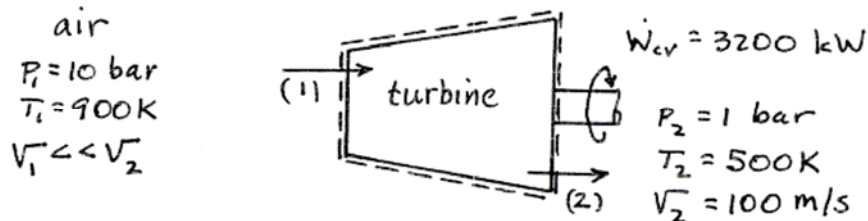


PROBLEM 4.43

KNOWN: Air expands through a turbine with known conditions at the inlet and exit. The power developed is known.

FIND: Determine the mass flow rate and the exit area.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Potential energy effects and kinetic energy at the inlet can be neglected. (4) The air behaves as an ideal gas.

ANALYSIS: Begin with a steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cv}}{(h_1 - h_2) - \frac{V_2^2}{2}}$$

From Table A-22; $h_1 = 932.93 \text{ kJ/kg}$ and $h_2 = 503.02 \text{ kJ/kg}$. Thus

$$\begin{aligned} \dot{m} &= \frac{(3200 \text{ kW}) \left(\frac{1 \text{ kJ/s}}{1 \text{ kW}} \right)}{(932.93 - 503.02) \text{ kJ/kg} - \left(\frac{100^2 \text{ m}^2/\text{s}^2}{2} \right) \left| \frac{1 \text{ N}}{\text{kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|} \\ &= 7.53 \text{ kg/s} \end{aligned}$$

The exit area is

$$\begin{aligned} A_2 &= \frac{v_2 \dot{m}}{V_2} = \frac{RT_2 \dot{m}}{P_2 V_2} \\ &= \frac{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (500 \text{ K}) (7.53 \text{ kg/s}) \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right|}{(1 \text{ bar}) (100 \text{ m/s})} \\ &= 0.108 \text{ m}^2 \end{aligned}$$

1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.

PROBLEM 4-44

KNOWN: Air expands through a turbine with known conditions at the inlet and exit. The inlet mass flow rate and the power developed are given.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:



- ENGR. MODEL:** (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas.

ANALYSIS: Since $h = h(T)$ for an ideal gas, the exit temperature can be found by evaluating h_2 . Beginning with the steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and with assumption (3) we get

$$0 = -\dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

$$\text{or} \quad h_2 = -\dot{W}_{cv}/\dot{m} + h_1 \quad (1)$$

Using data from Table A-22E for h_1 and inserting values into (1)

$$h_2 = - \frac{(2550 \text{ hp})}{(10.5 \text{ lb/s})} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right| + 385.08 \frac{\text{Btu}}{\text{lb}}$$

$$= 213.39 \text{ Btu/lb}$$

Interpolating in Table A-22E

$$T_2 = 888.3^\circ\text{R} \quad \leftarrow T_2$$

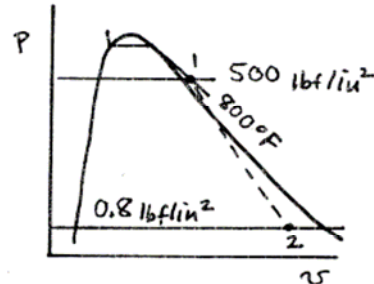
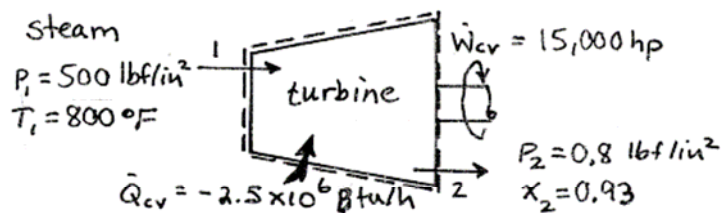
1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.

PROBLEM 4.45

KNOWN: A steam turbine operates at steady state with known inlet and exit conditions. The power developed and the heat transfer rate between the turbine and its surroundings are specified.

FIND: Determine the volumetric flow rate of steam at the inlet.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: To calculate the volume flow rate, begin with mass and energy rate balances for the one-inlet, one-exit control volume at steady state

$$0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

Solving for the mass flow rate

$$\dot{m} = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{(h_2 - h_1)}$$

From Table A-4E, at $P_1 = 500 \text{ lbf/in}^2$, $T_1 = 800^\circ\text{F}$; $h_1 = 1412.1 \text{ Btu/lb}$. From Table A-3E, at $P_2 = 0.8 \text{ lbf/in}^2$

$$\begin{aligned} h_2 &= h_{f2} + x_2 h_{fg2} \\ &= (62.41) + (0.93)(1040.2) = 1029.80 \text{ Btu/lb} \end{aligned}$$

Thus

$$\dot{m} = \frac{(-2.5 \times 10^6 \text{ Btu/h}) - (15,000 \text{ hp}) \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right|}{(1029.80 - 1412.1) \text{ Btu/lb}}$$

$$= 10.64 \times 10^4 \text{ lb/h}$$

Then, from Table A-4E at P_1 and T_1 ; $v_1 = 1.441 \text{ ft}^3/\text{lb}$.

The volumetric flow rate at the inlet is then

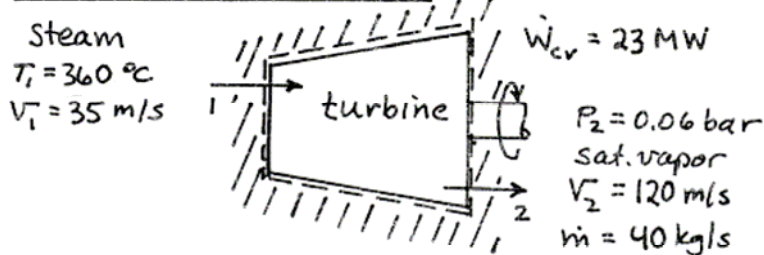
$$\begin{aligned} (AV)_1 &= \dot{m} v_1 = (10.64 \times 10^4 \frac{\text{lb}}{\text{h}}) (1.441 \frac{\text{ft}^3}{\text{lb}}) \\ &= 1.53 \times 10^5 \text{ ft}^3/\text{h} \leftarrow (AV)_1 \end{aligned}$$

PROBLEM 4.46

KNOWN: A well-insulated steam turbine operates at steady-state with known inlet and exit conditions. The power output and mass flow rate are specified.

FIND: Determine the inlet pressure.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer can be neglected. (3) The potential energy change from inlet to exit can be neglected.

ANALYSIS: The temperature is known at the inlet. To fix the inlet state, h_1 is determined using mass and energy balances. For the control volume

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

and

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2) \right]$$

Solving for h_1 ,

$$h_1 = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 + \left(\frac{V_2^2 - V_1^2}{2} \right)$$

From Table A-3 for saturated vapor at $p_2 = 0.06\text{ bar}$; $h_2 = 2567.4\text{ kJ/kg}$. Thus

$$\begin{aligned} h_1 &= \frac{(23\text{ MW})}{(40\text{ kg/s})} \left| \frac{10^3\text{ kW}}{1\text{ MW}} \right| + \left(2567.4 \frac{\text{kJ}}{\text{kg}} \right) + \left(\frac{120^2 - 35^2}{2} \right) \frac{\text{m}^2}{\text{s}^2} \left| \frac{1\text{ N}}{1\text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N} \cdot \text{m}} \right| \\ &= 3149.0\text{ kJ/kg} \end{aligned}$$

Interpolating in Table A-4, with $T_1 = 360^\circ\text{C}$ and $h_1 = 3149.0\text{ kJ/kg}$ gives

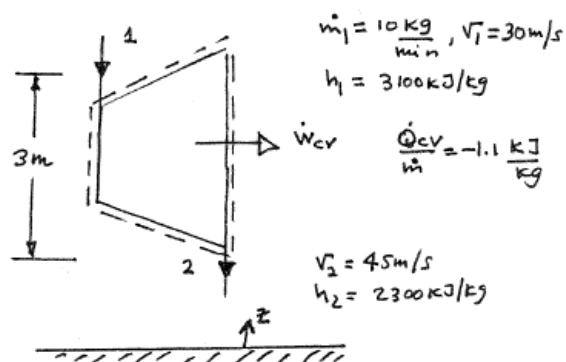
$$P_1 \approx 25.0\text{ bar} \leftarrow P_1$$

PROBLEM 4.47

KNOWN: Steady-state data are provided for a steam turbine.

FIND: Determine the power developed by the turbine.

SCHEMATIC & GIVEN DATA



ENGR. MODEL:

1. As shown in the sketch, a control volume encloses the turbine.
2. The control volume is at steady state.
3. $g = 9.81 \text{ m/s}^2$.

ANALYSIS: Mass rate balance at steady state, $\dot{m}_2 = \dot{m}_1$. Energy rate balance at steady state,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

where

$$\checkmark \quad \frac{\dot{Q}_{cv}}{\dot{m}} = -1.1 \frac{\text{kJ}}{\text{kg}}$$

$$\checkmark \quad (h_1 - h_2) = (3100 - 2300) \frac{\text{kJ}}{\text{kg}} = 800 \frac{\text{kJ}}{\text{kg}}$$

$$\checkmark \quad \frac{V_1^2 - V_2^2}{2} = \left[\frac{(30 \text{ m/s})^2 - (45 \text{ m/s})^2}{2} \right] \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = -0.56 \frac{\text{kJ}}{\text{kg}}$$

$$\checkmark \quad g(z_1 - z_2) = (9.81 \frac{\text{m}}{\text{s}^2})(3 \text{ m}) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 0.03 \frac{\text{kJ}}{\text{kg}}$$

Collecting results,

$$\begin{aligned} \frac{\dot{W}_{cv}}{\dot{m}} &= [-1.1 + 800 - 0.56 + 0.03] \frac{\text{kJ}}{\text{kg}} \\ &= 798.37 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

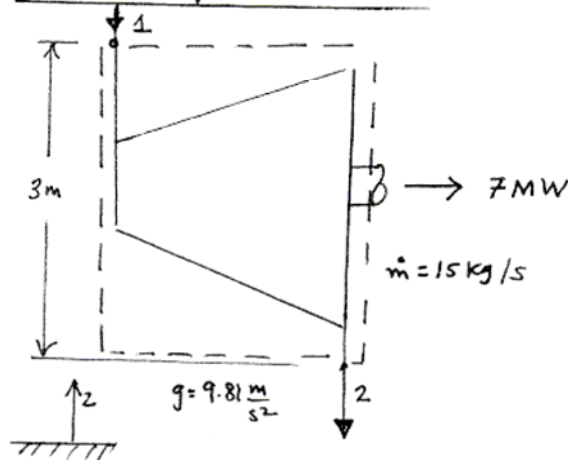
$$\begin{aligned} \Rightarrow \dot{W}_{cv} &= \left(10 \frac{\text{kg}}{\text{min}} \right) \left(798.37 \frac{\text{kJ}}{\text{kg}} \right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 133.1 \text{ kW} \end{aligned}$$

$\leftarrow \dot{W}_{cv}$

PROBLEM 4.48

Steam enters a turbine operating at steady state at 2 MPa, 360°C with a velocity of 100 m/s. Saturated vapor exits at 0.1 MPa and a velocity of 50 m/s. The elevation of the inlet is 3 m higher than at the exit. The mass flow rate of the steam is 15 kg/s, and the power developed is 7 MW. Let $g = 9.81 \text{ m/s}^2$. Determine (a) the area at the inlet, in m^2 , and (b) the rate of heat transfer between the turbine and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. $g = 9.81 \text{ m/s}^2$

ANALYSIS:

$$(a) \quad \dot{m} = \frac{A_1 V_1}{v_1} \Rightarrow A_1 = \frac{\dot{m} v_1}{V_1} = \frac{(15 \text{ kg/s})(0.1411 \text{ m}^3/\text{kg})}{(100 \text{ m/s})} = 0.021 \text{ m}^2 \quad (a)$$

(b) Considering Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m} \left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] \quad (1)$$

where, with data from Tables A-3 and A-4,

$$h_2 - h_1 = (2675.5 - 3159.3) \frac{\text{kJ}}{\text{kg}} = -483.8 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{V_2^2 - V_1^2}{2} = \left[\frac{(50 \text{ m/s})^2 - (100 \text{ m/s})^2}{2} \right] \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = -3.75 \frac{\text{kJ}}{\text{kg}}$$

$$g(z_2 - z_1) = (9.81 \frac{\text{m}}{\text{s}^2})(-3 \text{ m}) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = -0.03 \frac{\text{kJ}}{\text{kg}}$$

Inserting values in Eq. (1)

$$\begin{aligned} \dot{Q}_{cv} &= 7000 \text{ kW} + 15 \frac{\text{kg}}{\text{s}} \left[-483.8 - 3.75 - 0.03 \right] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= -313.7 \text{ kW} \quad (b) \end{aligned}$$

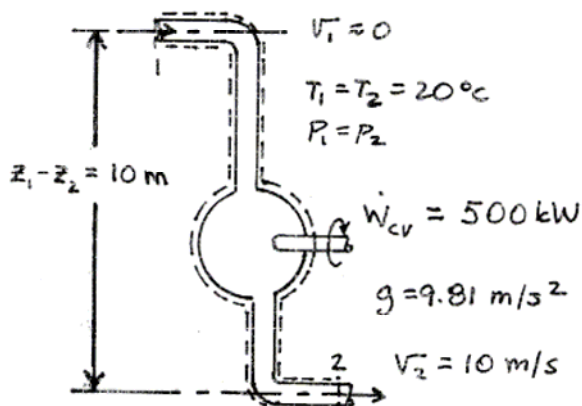
PROBLEM 4.49

KNOWN: Water flows through a hydraulic turbine with known conditions at the inlet and exit. The power output is specified.

FIND: Determine the mass flow rate.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL : (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Changes in temperature and pressure from inlet to exit are negligible. (4) Kinetic energy can be neglected at the inlet. (5) The acceleration of gravity is constant; $g = 9.81 \text{ m/s}^2$.



ANALYSIS: To find the mass flow rate, begin with steady state mass and energy rate balances

$$0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

and

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

where the enthalpy term is cancelled because of assumption (3). Solving

$$\dot{m} = \frac{\dot{W}_{cv}}{-\frac{V_2^2}{2} + g(z_1 - z_2)}$$

Inserting values

$$\begin{aligned} \dot{m} &= \frac{(500 \text{ kW}) \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right|}{\left[\left(-\frac{10^2}{2} \right) \frac{\text{m}^2}{\text{s}^2} + (9.81 \frac{\text{m}}{\text{s}^2})(10 \text{ m}) \right] \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|} \\ &= 10,400 \text{ kg/s} \end{aligned}$$

PROBLEM 4.50

KNOWN: Steady-state operating data are provided for a two-stage turbine with a reheater.

FIND: Determine the steam mass flow rate, the total power developed, and rate of heat transfer for the steam flowing through the reheater.

SCHEMATIC & GIVEN DATA:

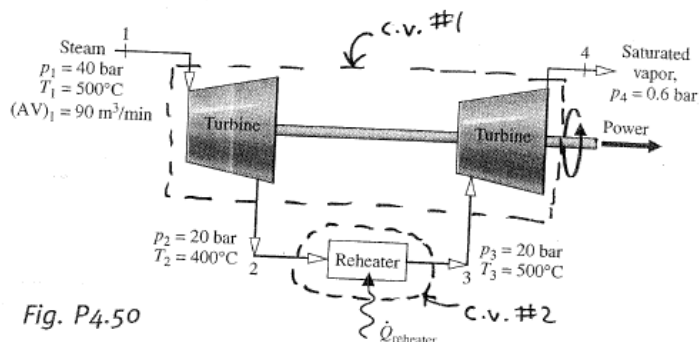


Fig. P4.50

ENGR. MODEL:

1. As shown in the sketch, two control volumes are under consideration.
2. Each control volume is at steady state.
3. Kinetic and potential energy effects can be ignored.
4. For control volume #1, stray heat transfer can be ignored.

ANALYSIS: (a) The mass flow rate at inlet 1 is found from $\dot{m}_1 = \frac{(AV)_1}{v_1}$.

From Table A-4 at 40 bar, 500°C, $v_1 = 0.08643 \text{ m}^3/\text{kg}$. Then,

$$\dot{m}_1 = \frac{90 \text{ m}^3/\text{min}}{0.08643 \text{ m}^3/\text{kg}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 6.248 \times 10^4 \frac{\text{kg}}{\text{h}} \quad \leftarrow \dot{m}$$

(b) An energy rate balance for control volume #1 reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 [h_1 - h_2] + \dot{m}_1 [h_3 - h_4]$$

$$\Rightarrow \dot{W}_{cv} = \dot{m}_1 [(h_1 - h_2) + (h_3 - h_4)]$$

With data from Tables A-3 and A-4, $h_1 = 3445.3 \text{ kJ/kg}$, $h_2 = 3247.6 \text{ kJ/kg}$, $h_3 = 3467.6 \text{ kJ/kg}$, $h_4 = 2653.5 \text{ kJ/kg}$.

$$\begin{aligned} \Rightarrow \dot{W}_{cv} &= \dot{m}_1 [(3445.3 - 3247.6) + (3467.6 - 2653.5)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 6.248 \times 10^4 \frac{\text{kg}}{\text{h}} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 17.36 \frac{\text{kg}}{\text{s}} \\ &= 17,365 \text{ kW} \quad \leftarrow \dot{W}_{cv} \end{aligned}$$

(c) An energy rate balance for control volume #2 reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 (h_2 - h_3) \Rightarrow \dot{Q}_{cv} = \dot{m}_1 (h_3 - h_2)$$

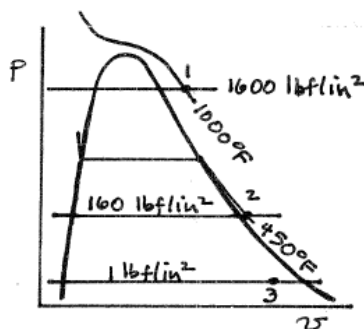
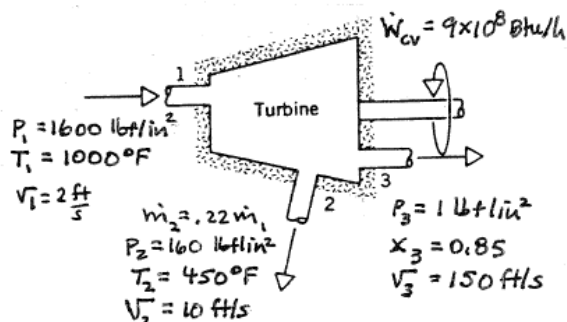
$$\begin{aligned} \Rightarrow \dot{Q}_{cv} &= 17.36 \frac{\text{kg}}{\text{s}} (3467.6 - 3247.6) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 3,819 \text{ kW} \quad \leftarrow \dot{Q}_{cv} \end{aligned}$$

PROBLEM 4.51

KNOWN: Steam passes through an extraction turbine operating at steady state with known inlet and exit conditions. The power output is specified.

FIND: Determine (a) the inlet mass flow rate, (b) the diameter of the extraction duct.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. A control volume enclosing the turbine is at steady state. 2. For the control volume, heat transfer and potential energy effects are negligible.

ANALYSIS: (a) To find $\dot{m}_1$, apply a mass rate balance: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$. Then, since $\dot{m}_2/\dot{m}_1 = 0.22$, we have $\dot{m}_3/\dot{m}_1 = 0.78$. Next, apply an energy rate balance:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left[h_1 + \frac{V_1^2}{2} \right] - \dot{m}_2 \left[h_2 + \frac{V_2^2}{2} \right] - \dot{m}_3 \left[h_3 + \frac{V_3^2}{2} \right]$$

where the potential energy terms are omitted by assumption 2. Solving

$$\dot{m}_1 = \frac{\dot{W}_{cv}}{\left[h_1 + \frac{V_1^2}{2} \right] - \frac{\dot{m}_2}{\dot{m}_1} \left[h_2 + \frac{V_2^2}{2} \right] - \frac{\dot{m}_3}{\dot{m}_1} \left[h_3 + \frac{V_3^2}{2} \right]}$$

From Table A-4E, $h_1 = 1487 \text{ Btu/lb}$, $h_2 = 1246.1 \text{ Btu/lb}$. With Table A-3E data $h_3 = h_{f3} + x_3 (h_{g3} - h_{f3}) = 69.74 + 0.85 (1036) = 950.3 \text{ Btu/lb}$. Thus

$$\dot{m}_1 = \frac{(9 \times 10^8 \text{ Btu/h})}{\left[1487 \frac{\text{Btu}}{\text{lb}} + \frac{(2 \text{ ft/s})^2}{2} \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \right] - 0.22 \left[1246.1 + \frac{(10)^2}{2} \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lb}} \right| \right] - 0.78 \left[950.3 + \frac{(150)^2}{2} \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lb}} \right| \right]}$$

$$= 1.91 \times 10^6 \text{ lb/h}$$

(b) Using $v_2 = 3.228$ from Table A-4E

$$A_2 = \frac{\dot{m}_2 v_2}{V_2} = \frac{(0.22)(1.91 \times 10^6 \text{ lb/h})(3.228 \text{ ft}^3/\text{lb})}{(10 \text{ ft/s})} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 37.68 \text{ ft}^2$$

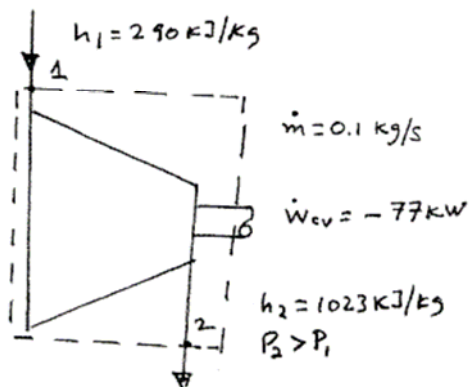
Since $A_2 = \pi d_2^2/4$

$$d_2 = \sqrt{\frac{4 A_2}{\pi}} = \sqrt{\frac{(4)(37.68 \text{ ft}^2)}{\pi}} = 6.93 \text{ ft}$$

PROBLEM 4.52

Air enters a compressor operating at steady state at 1 atm with a specific enthalpy of 290 kJ/kg and exits at a higher pressure with a specific enthalpy of 1023 kJ/kg. The mass flow rate is 0.1 kg/s. If the compressor power input is 77 kW, determine the rate of heat transfer between the compressor and its surroundings, in kW. Neglect kinetic and potential energy effects.

SCHEMATIC & GIVEN DATA



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Kinetic and potential energy effects can be neglected.

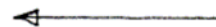
ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)} \right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m} [h_2 - h_1]$$

$$= -77 \text{ kW} + 0.1 \text{ kg/s} [1023 - 290] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

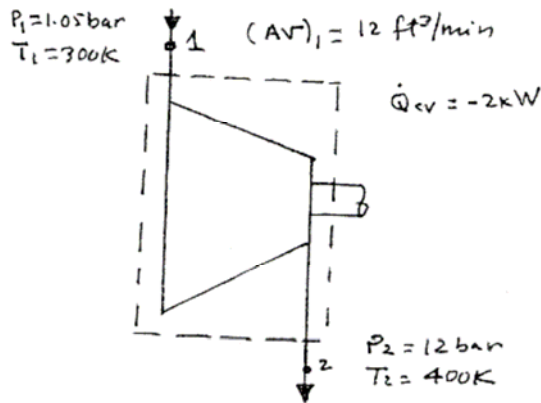
$$= -3.7 \text{ kW}$$



PROBLEM 4.53

Air enters a compressor operating at steady state at 1.05 bar, 300 K, with a volumetric flow rate of 12 m³/min and exits at 12 bar, 400 K. Heat transfer occurs at a rate of 2 kW from the compressor to its surroundings. Assuming the ideal gas model for air and neglecting kinetic and potential energy effects, determine the power input, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. The air is modeled as an ideal gas.
3. Kinetic and potential energy effects are neglected.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + g(\cancel{z_1 - z_2}) \right]$$

$$\Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m} [h_1 - h_2] \quad (1)$$

where

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{R T_1} = \frac{(12 \text{ ft}^3/\text{min})(1.05 \times 10^5 \text{ N/m}^2)}{\left(\frac{8314}{28.97} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right)(300 \text{ K})} \left| \frac{1 \text{ min}}{60 \text{ s}} \right|$$

$$= 0.2439 \frac{\text{kg}}{\text{s}}$$

Then, with enthalpy data from Table A-22, Eq. (1) gives

$$\dot{W}_{cv} = -2 \text{ kW} + (0.2439 \frac{\text{kg}}{\text{s}}) [300.19 - 400.98] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

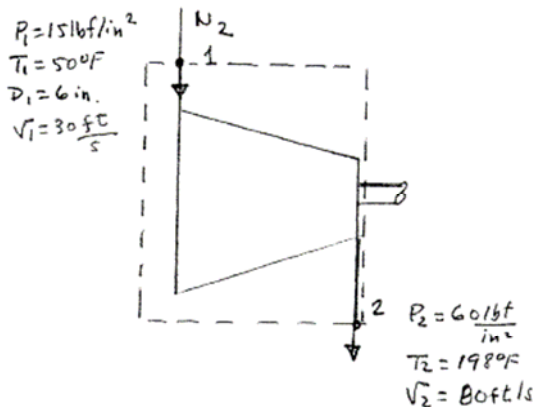
$$= -26.6 \text{ kW}$$



PROBLEM 4.54

Nitrogen is compressed in an axial-flow compressor operating at steady state from a pressure of 15 lbf/in^2 and a temperature of 50°F to a pressure 60 lbf/in^2 . The gas enters the compressor through a 6-in.-diameter duct with a velocity of 30 ft/s and exits at 198°F with a velocity of 80 ft/s . Using the ideal gas model, and neglecting stray heat transfer and potential energy effects, determine the compressor power input, in hp.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and potential energy effects can be ignored.
3. The nitrogen is modeled as an ideal gas.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \dot{W}_{cv} = \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} \right]$$

where

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(\pi D_1^2/4) V_1}{R T_1 / P_1} = \frac{P_1 (\pi D_1^2/4) V_1}{R T_1} = \frac{(15 \times 144 \frac{\text{lbf}}{\text{ft}^2}) (\frac{\pi}{4} (0.5 \text{ ft})^2) (30 \frac{\text{ft}}{\text{s}})}{(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.01 \text{ lb} \cdot ^\circ\text{R}}) (510^\circ\text{R})}$$

$$= 0.45 \text{ lb/s}$$

Then with specific enthalpies on a molar basis from Table A.23E,

$$\dot{W}_{cv} = (0.45 \frac{\text{lb}}{\text{s}}) \left[\frac{(3541.8 - 4571.9) \text{ Btu}}{28.01 \text{ lb}} + \left(\frac{(30)^2 - (80)^2}{2} \right) \left(\frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right) \left(\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right) \right]$$

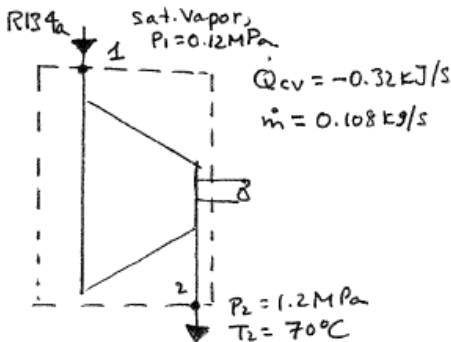
$$= (0.45 \frac{\text{lb}}{\text{s}}) \left[-36.78 - 0.11 \right] \frac{\text{Btu}}{\text{lb}} \left| \frac{3600 \text{ s}}{\text{h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

$$= -23.48 \text{ hp}$$

PROBLEM 4.55

Refrigerant 134a enters a compressor operating at steady state as saturated vapor at 0.12 MPa and exits at 1.2 MPa and 70°C at a mass flow rate of 0.108 kg/s. As the refrigerant passes through the compressor, heat transfer to the surroundings occurs at a rate of 0.32 kJ/s. Determine at steady state the power input to the compressor, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, kinetic and potential energy effects can be ignored.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m} [h_1 - h_2]$$

With data from Tables A-11 and A-12,

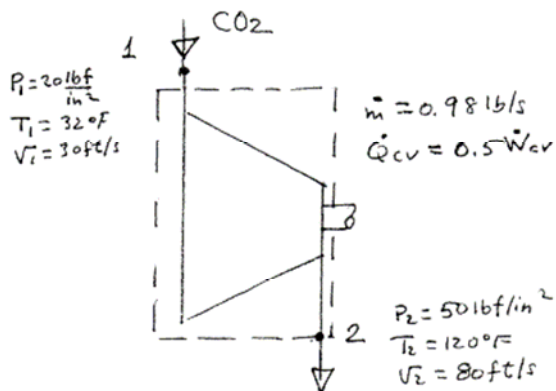
$$\begin{aligned} \dot{W}_{cv} &= -0.32 \frac{\text{kJ}}{\text{s}} + 0.108 \frac{\text{kg}}{\text{s}} [233.86 - 298.96] \frac{\text{kJ}}{\text{kg}} \\ &= -7.35 \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -7.35 \text{ kW} \end{aligned}$$



PROBLEM 4.56

Carbon dioxide gas is compressed at steady state from a pressure of 20 lbf/in.² and a temperature of 32°F to a pressure of 50 lbf/in.² and a temperature of 120°F. The gas enters the compressor with a velocity of 30 ft/s and exits with a velocity of 80 ft/s. The mass flow rate is 0.98 lb/s. The magnitude of the heat transfer rate from the compressor to its surroundings is 5% of the compressor power input. Using the ideal gas model with $c_p = 0.21$ Btu/lb · °R and neglecting potential energy effects, determine the compressor power input, in horsepower.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, potential energy effects can be neglected.
3. The CO₂ is modeled as an ideal gas with constant $c_p = 0.21$ Btu/lb · °R.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\downarrow \dot{Q}_{cv} = 0.5 \dot{W}_{cv}$$

$$\Rightarrow \dot{W}_{cv} = \frac{\dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} \right]}{0.95} = \frac{\dot{m} \left[c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2} \right]}{0.95}$$

Inserting values,

$$\dot{W}_{cv} = \frac{(0.98 \frac{\text{lb}}{\text{s}}) \left[0.21 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} (-88^\circ\text{R}) + \left[\frac{(30)^2 - (80)^2}{2} \left(\frac{\text{ft}^2}{\text{s}^2} \right) \left| \frac{1 \text{ lbf}}{32.174 \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \right] \right]}{0.95}$$

$$= \frac{(0.98 \frac{\text{lb}}{\text{s}}) \left[-18.48 \frac{\text{Btu}}{\text{lb}} - 0.11 \frac{\text{Btu}}{\text{lb}} \right]}{0.95} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

$$= -27.1 \text{ hp}$$

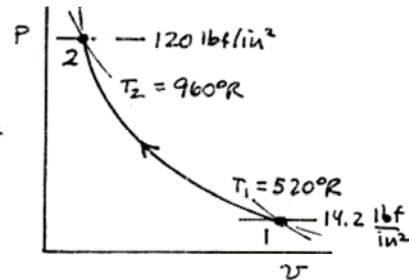
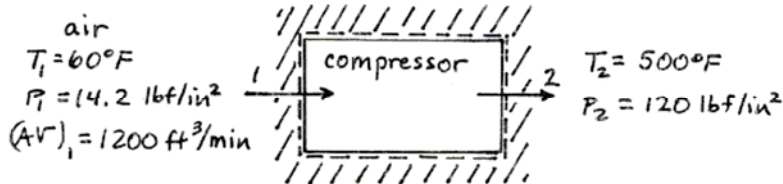
The power input is 27.1 hp.

PROBLEM 4.57

KNOWN: A well-insulated air compressor operates with known inlet and exit states. The inlet volumetric flow rate is also known.

FIND: Determine the compressor power and the exit volumetric flow rate.

SCHEMATIC & GIVEN DATA:



- ENGR. MODEL:** (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) The air behaves as an ideal gas. (4) Kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: To find the power, begin with steady state mass and energy balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \dot{m} [h_1 - h_2]$$

To evaluate $\dot{m}$, use Eq. 4.4b and the ideal gas equation of state

$$\begin{aligned} \dot{m} &= \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1} \\ &= \frac{(14.2 \text{ lbf/in}^2)(1200 \text{ ft}^3/\text{min})}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)(520^\circ\text{R})} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 5309 \text{ lb/h} \end{aligned}$$

Thus, with $h_1 = 124.27 \text{ Btu/lb}$ and $h_2 = 231.06 \text{ Btu/lb}$ from Table A-22E

$$\begin{aligned} \dot{W}_{cv} &= (5309 \frac{\text{lb}}{\text{h}})(124.27 - 231.06) \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| \\ &= -222.8 \text{ hp} \end{aligned}$$

$\dot{W}_{cv}$

The exit volumetric flow rate is

$$\begin{aligned} (AV)_2 &= \dot{m} v_2 = \dot{m} \left(\frac{RT_2}{P_2} \right) \\ &= (5309 \text{ lb/h}) \left| \frac{1 \text{ h}}{60 \text{ min}} \right| \frac{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right)(960^\circ\text{R})}{(120 \text{ lbf/in}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| \\ &= 262 \text{ ft}^3/\text{min} \end{aligned}$$

$(AV)_2$

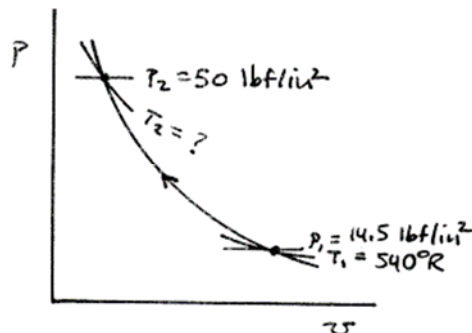
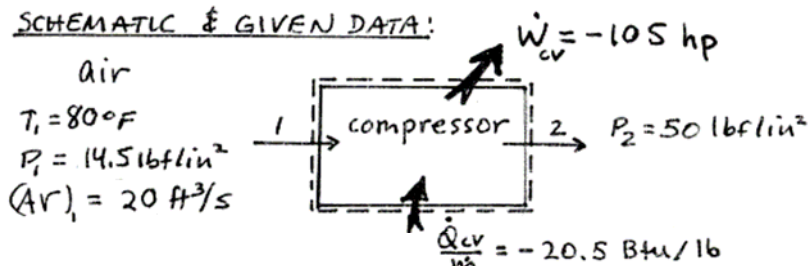
1. The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart.
2. The negative sign for power denotes energy transfer into the control volume, as expected.

PROBLEM 4.58

KNOWN: An air compressor operates with known power, inlet conditions, and exit pressure. The inlet volumetric flow rate and the heat transfer rate per unit mass of air flow are also specified.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:



- ENGR. MODEL:** (1) The control volume is at steady state. (2) Kinetic and potential energy changes from inlet to exit are negligible. (3) The air behaves as an ideal gas.

ANALYSIS: To find the compressor power, begin with steady state mass and energy rate balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} [h_1 - h_2] + \left(\frac{\dot{m} v_1^2}{2} - \frac{\dot{m} v_2^2}{2} \right) + \dot{m} g (z_1 - z_2)$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2

$$h_2 = h_1 + \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}}$$

To evaluate $\dot{m}$, use Eq. 4.4b and the ideal gas equation of state

$$\begin{aligned} \dot{m} &= \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1} \\ &= \frac{(14.5 \text{ lbf/in}^2)(20 \text{ ft}^3/\text{s}) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|}{\left(\frac{1545 \text{ ft} \cdot \text{lb}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right) (540^\circ\text{R})} = 1.44 \text{ lb/s} \end{aligned}$$

Using data from Table A-22E

$$\begin{aligned} h_2 &= \left(129.06 \frac{\text{Btu}}{\text{lb}} \right) + (-20.5 \frac{\text{Btu}}{\text{lb}}) - \left(\frac{-105 \text{ hp}}{1.44 \text{ lb/s}} \right) \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right| \\ &= 160.1 \text{ Btu/lb} \end{aligned}$$

Interpolating in Table A-22E with $h_2 = 160.1 \text{ Btu/lb}$

$$T_2 = 669^\circ\text{R} = 209^\circ\text{F} \leftarrow T_2$$

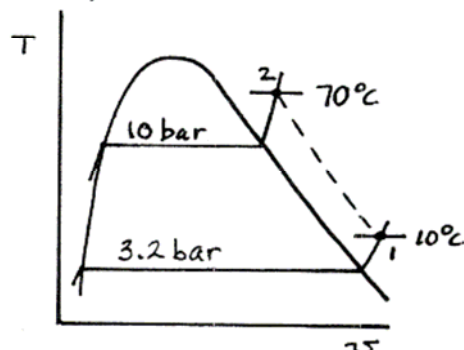
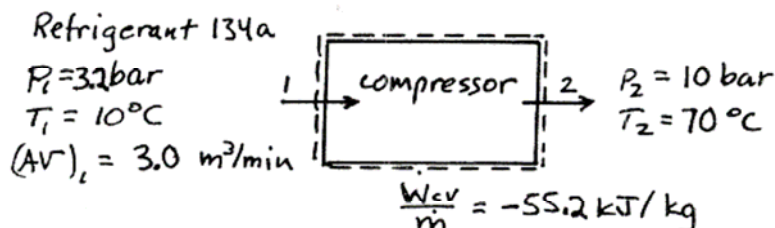
1. The applicability of the ideal gas model can be checked using the Compressibility Chart.

PROBLEM 4.59

KNOWN: A Refrigerant 134a compressor has known conditions at the inlet and exit. The inlet volumetric flow rate and the compressor power per unit mass of refrigerant flowing are also specified.

FIND: Determine the heat transfer rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: To find the heat transfer rate, begin with steady state energy and mass rate balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving

$$\dot{Q}_{cv} = \dot{m} \left[\left(\frac{W_{cv}}{\dot{m}} \right) + (h_2 - h_1) \right]$$

To evaluate $\dot{m}$, use Eq. 4.4b and data for v_1 from Table A-12

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{3.0 \text{ m}^3/\text{min}}{0.06576 \text{ m}^3/\text{kg}} = 45.62 \text{ kg/min}$$

From Table A-12, $h_1 = 255.65 \text{ kJ/kg}$, and $h_2 = 302.34 \text{ kJ/kg}$
 Thus

$$\begin{aligned} \dot{Q}_{cv} &= \left(45.62 \frac{\text{kg}}{\text{min}} \right) \left[(-55.2) + (302.34 - 255.65) \right] \frac{\text{kJ}}{\text{kg}} \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{1 \text{ kW}}{1 \text{ kJ/s}} \right) \\ &= -0.14 \text{ kW} \end{aligned}$$

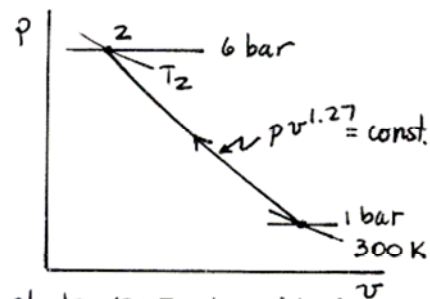
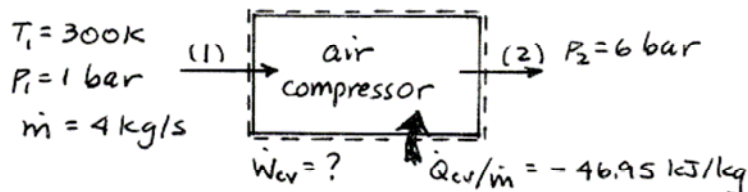
The negative sign indicates that the heat transfer is from the system.

PROBLEM 4.60

KNOWN: Air is compressed at steady state from a given initial state to a given final pressure. The mass flow is known, and each unit of mass undergoes a specified process in going from inlet to exit.

FIND: Determine the compressor power.

SCHEMATIC & GIVEN DATA:



- ENGR. MODEL:** (1) The control volume is at steady state. (2) Each unit of mass undergoes a process described by $Pv^{1.27} = \text{constant}$. (3) The air can be modeled as an ideal gas. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: To determine the power, begin with mass and energy rate balances at steady state

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$ and the indicated terms are deleted by assumption 4. Rearranging and solving for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \dot{m} \left[(\dot{Q}_{cv}/\dot{m}) + (h_1 - h_2) \right] \quad (*)$$

Now, specific enthalpy h_1 is read from Table A-22 at 300 K: $h_1 = 300.19 \frac{\text{kJ}}{\text{kg}}$.

To get T_2 , we use Eq. 3.56 with $n = 1.27$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \Rightarrow T_2 = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} T_1 = \left(\frac{6}{1} \right)^{\frac{1.27-1}{1.27}} (300 \text{ K}) = 439.1 \text{ K}$$

Interpolating in Table A-22; $h_2 = 440.7 \text{ kJ/kg}$. Inserting values in (*)

$$\dot{W}_{cv} = \left(4 \frac{\text{kg}}{\text{s}} \right) \left[(-46.95 \frac{\text{kJ}}{\text{kg}}) + (300.19 - 440.7) \frac{\text{kJ}}{\text{kg}} \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

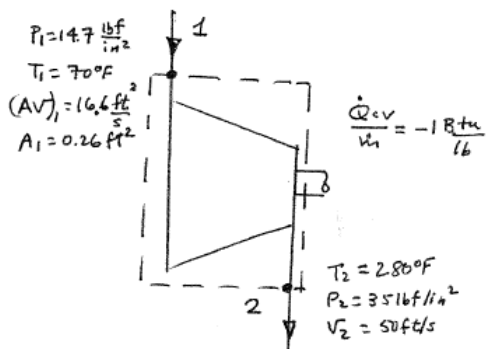
$$\textcircled{2} \quad = -750 \text{ kW} \quad \dot{W}_{cv}$$

1. The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart.
2. The negative sign for power denotes energy transfer by work into the control volume, as expected.

PROBLEM 4.61

Air enters a compressor operating at steady state with a pressure of 14.7 lbf/in.^2 and a temperature of 70°F . The volumetric flow rate at the inlet is $16.6 \text{ ft}^3/\text{s}$, and the flow area is 0.26 ft^2 . At the exit, the pressure is 35 lbf/in.^2 , the temperature is 280°F , and the velocity is 50 ft/s . Heat transfer from the compressor to its surroundings occurs at a rate of 1.0 Btu per lb of air flowing. Potential energy effects are negligible, and the ideal gas model can be assumed for the air. Determine (a) the velocity of the air at the inlet, in ft/s , (b) the mass flow rate, in lb/s , and (c) the compressor power, in Btu/s and hp .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, potential energy effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS:

(a) Using given data at the inlet,

$$V_1 = \frac{(AV)_1}{A_1} = \frac{16.6 \text{ ft}^3/\text{s}}{0.26 \text{ ft}^2} = 63.85 \frac{\text{ft}}{\text{s}} \quad \leftarrow \text{ca)}$$

$$(b) \quad \dot{m}_1 = \frac{(AV)_1}{V_1} = \frac{P_1 (AV)_1}{RT_1} = \frac{(14.7 \times 144 \frac{\text{lbf}}{\text{ft}^2})(16.6 \frac{\text{ft}^3}{\text{s}})}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}}\right)(530^\circ\text{R})} = 1.24 \frac{\text{lb}}{\text{s}} \quad \leftarrow \text{cb)}$$

(c) Reducing Eq. 4.20a

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \dot{W}_{cv} = \dot{m} \left[\frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} \right]$$

Inserting values from Table A-22E and other data,

$$\dot{W}_{cv} = 1.24 \frac{\text{lb}}{\text{s}} \left(-1 \frac{\text{Btu}}{\text{lb}} + (126.67 - 177.23) \frac{\text{Btu}}{\text{lb}} + \left[\frac{(63.85)^2 - (50)^2}{2} \right] \frac{(\text{ft}^2/\text{s}^2)}{32.2 \text{ ft} \cdot \text{s}^2/\text{ft} \cdot \text{s}^2} \left\| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right\| \left\| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right\| \right)$$

$$= 1.24 \frac{\text{lb}}{\text{s}} \left[-1 - 50.56 + 0.03 \right] \frac{\text{Btu}}{\text{lb}}$$

$$= -63.9 \frac{\text{Btu}}{\text{s}} \quad \leftarrow$$

or

$$\dot{W}_{cv} = -63.9 \frac{\text{Btu}}{\text{s}} \left\| \frac{3600 \text{ s}}{1 \text{ h}} \right\| \left\| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right\|$$

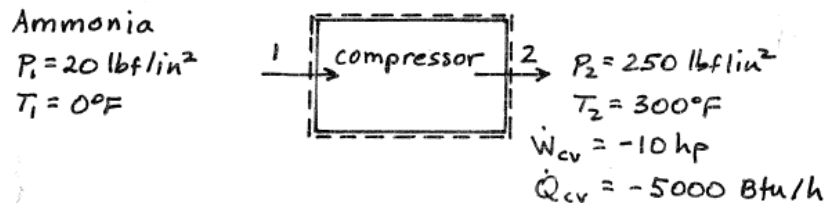
$$= -90.4 \text{ hp} \quad \leftarrow$$

PROBLEM 4.62

KNOWN: An ammonia compressor has known conditions at the inlet and exit. The compressor power and the heat transfer rate are also specified.

FIND: Determine the inlet volumetric flow rate using ammonia tables and ideal gas relationships. Discuss.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Kinetic and potential energy changes from inlet to exit are negligible. (3) For the second part, the ammonia behaves as an ideal gas.

ANALYSIS: To begin, determine the mass flow rate by using the steady-state mass and energy balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving

$$\dot{m} = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{(h_2 - h_1)} \quad (*)$$

From Table A-15E; $h_1 = 614.84 \text{ Btu/lb}$ and $h_2 = 760.39 \text{ Btu/lb}$. Thus

$$\dot{m} = \frac{(-5000 \text{ Btu/h}) - (-10 \text{ hp}) \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right|}{(760.39 - 614.84) \text{ Btu/lb}} \left| \frac{1 \text{ h}}{60 \text{ min}} \right| = 2.342 \text{ lb/min}$$

Now, using $v_1 = 14.078 \text{ ft}^3/\text{lb}$ from Table A-15E and Eq. 4-11b

$$(\dot{A}V)_1 = \dot{m} v_1 = (2.342)(14.078) = 32.97 \text{ ft}^3/\text{min} \quad \leftarrow (\dot{A}V)_1 \text{ (Ammonia table)}$$

To evaluate $h_2 - h_1$ for an ideal gas, integrate the specific heat function $c_p(T)$ for ammonia from Table A-21E

$$\begin{aligned} h_2 - h_1 &= \frac{\bar{R}}{\bar{M}} \int_{T_1}^{T_2} (\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4) dT \\ &= \frac{\bar{R}}{\bar{M}} \left[\alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5) \right] \end{aligned}$$

With $T_1 = 460^\circ\text{R}$, $T_2 = 760^\circ\text{R}$, and coefficient values from Table A-21E

$$\begin{aligned} h_2 - h_1 &= \frac{(1.986 \text{ Btu/lb mol} \cdot ^\circ\text{R})}{(17.04 \text{ lb/lb mol})} [1329 ^\circ\text{R}] \\ &= 154.9 \text{ Btu/lb} \end{aligned}$$

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Using this result with (*)

$$\dot{m} = \frac{(-5000) - (-10)(2545)}{(154.9)} \left| \frac{1}{60} \right| = 2.2 \text{ lb/min}$$

Incorporating the ideal gas equation of state into Eq. 4-11b

$$\begin{aligned} (AV)_1 &= \dot{m} v_1 = \dot{m} \left(\frac{RT_1}{P_1} \right) \\ &= \left(2.2 \frac{\text{lb}}{\text{min}} \right) \frac{\left(\frac{1545}{17.04} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right) (460^\circ\text{R})}{(20 \text{ lbf/in}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| \\ &= 31.86 \text{ ft}^3/\text{min} \end{aligned}$$

(AV)₁
(ideal gas)

Discussion: The % deviation in assuming ideal gas behavior is

$$\begin{aligned} \% \text{ deviation} &= \left[\frac{(AV)_{\text{tables}} - (AV)_{\text{ideal gas}}}{(AV)_{\text{tables}}} \right] \times 100 \\ &= \left[\frac{32.97 - 31.86}{32.97} \right] \times 100 = 3.4\% \end{aligned}$$

Thus, the ideal gas model is reasonably accurate. To explore this further, consider

$$\begin{aligned} \% \text{ deviation} &= \left[\frac{\Delta h_{\text{ideal gas}} - \Delta h_{\text{tables}}}{\Delta h_{\text{tables}}} \right] \times 100 \\ &= \left[\frac{154.9 - 145.55}{145.55} \right] \times 100 = 6.4\% \end{aligned}$$

The applicability of the ideal gas model can also be checked by determining the compressibility factor. For state 1

$$Z_1 = \frac{P_1 v_1}{R T_1} = \frac{(20)(144)(14.078)}{\left(\frac{1545}{17.04} \right) (460)} = 0.972$$

For state 2, $v_2 = 1.8191 \text{ ft}^3/\text{lb}$ from Table A-15E. Thus

$$Z_2 = \frac{(250)(144)(1.8191)}{\left(\frac{1545}{17.04} \right) (760)} = 0.95$$

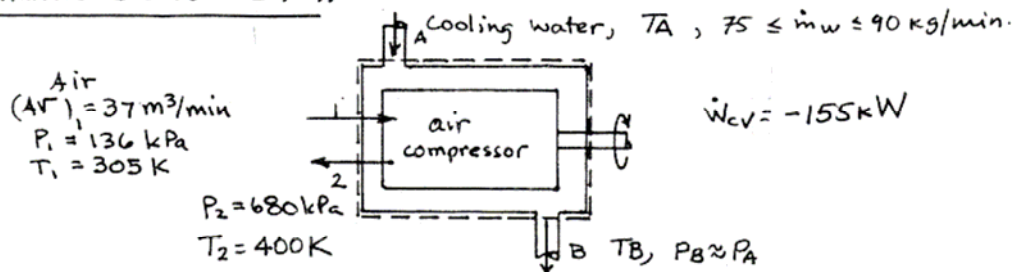
Both of these values are reasonably close to unity.

PROBLEM 4.63

KNOWN: Data are provided for a water-jacketed air compressor operating at steady state.

FIND: Determine the cooling water temperature increase for a specified cooling water mass flow rate. Plot the cooling water temperature increase versus cooling water mass flow rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer from the outside of the cooling water jacket is negligible. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas. (5) The cooling water is incompressible with $c_p = 4.179 \text{ kJ/kg} \cdot \text{K}$ from Table A-19.

ANALYSIS: (a) The steady-state energy balance for the overall compressor is

$$0 = \dot{Q}_{cv}^o - \dot{W}_{cv} + \dot{m}_1 \left(h_1 + \frac{V_1^2}{2} + g z_1 \right) - \dot{m}_2 \left(h_2 + \frac{V_2^2}{2} + g z_2 \right) + \dot{m}_A \left(h_A + \frac{V_A^2}{2} + g z_A \right) - \dot{m}_B \left(h_B + \frac{V_B^2}{2} + g z_B \right)$$

where the indicated terms drop out by assumptions (2) and (3). Since the water and air streams are separate

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{\text{air}}$$

$$\dot{m}_A = \dot{m}_B \equiv \dot{m}_w$$

Thus
$$0 = -\dot{W}_{cv} + \dot{m}_{\text{air}} (h_1 - h_2) + \dot{m}_w (h_A - h_B) \quad (1)$$

With assumption 5, the enthalpy change of the cooling water can be found using Eq. 3.20b:

$$h_B - h_A = c (T_B - T_A) + v (P_B - P_A) = c (T_B - T_A)$$

Accordingly, Eq. (1) gives

$$T_B - T_A = \frac{[-\dot{W}_{cv} + \dot{m}_{\text{air}} (h_1 - h_2)]}{c \dot{m}_w}$$

Using the data at location 1 to evaluate $\dot{m}_{\text{air}}$

$$\dot{m}_{\text{air}} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{R T_1} = \frac{(37 \text{ m}^3/\text{min}) (136 \text{ kPa})}{(8.314 \text{ kJ}) / (28.97 \text{ kg} \cdot \text{K}) (305 \text{ K})} \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 57.49 \text{ kg/min}$$

Also, from Table A-22, $h_1 = 305.22 \text{ kJ/kg}$, $h_2 = 400.48 \text{ kJ/kg}$

Collecting results

$$T_B - T_A = \frac{[-(-155 \text{ kJ/s}) (60 \text{ s/min}) + (57.49 \text{ kg/min}) (305.22 - 400.48) \text{ kJ/kg}]}{(4.179 \text{ kJ/kg} \cdot \text{K}) \dot{m}_w}$$

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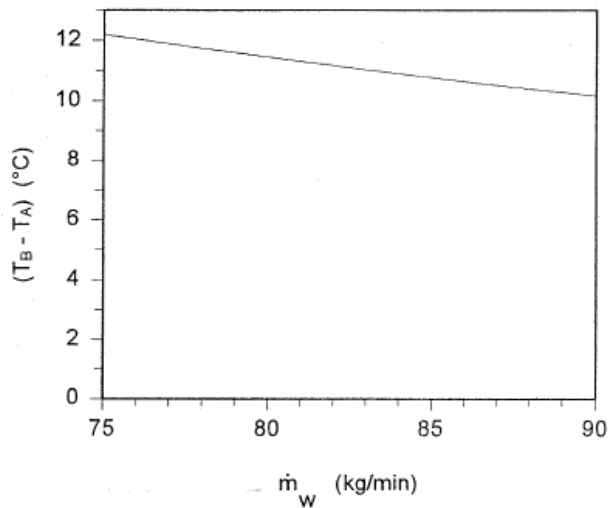
Problem 4-63 continued

or

$$\begin{aligned}(T_B - T_A) &= \frac{3823.5 \text{ kJ/min}}{(4.179 \text{ kJ/kg} \cdot \text{K}) \dot{m}_w} \\ &= \frac{914.93 \text{ kg} \cdot \text{K/min}}{\dot{m}_w} \quad (*)\end{aligned}$$

For the case of $\dot{m}_w = 82 \text{ kg/min}$; $T_B - T_A = 11.2 \text{ K}$ ← $\frac{\Delta T_w}{(\text{part a})}$

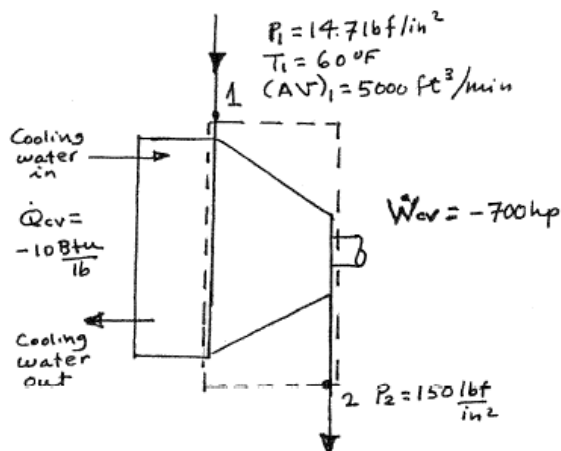
(b) Equation (*) can be plotted readily using a spreadsheet, plotting program, or IT. The IT plot follows:



PROBLEM 4.64

Air enters a compressor operating at steady state at 14.7 lbf/in^2 and 60°F and is compressed to a pressure of 150 lbf/in^2 . As the air passes through the compressor, it is cooled at a rate of 10 Btu per lb of air flowing by water circulated through the compressor casing. The volumetric flow rate of the air at the inlet is $5000 \text{ ft}^3/\text{min}$, and the power input to the compressor is 700 hp . The air behaves as an ideal gas, there is no stray heat transfer, and kinetic and potential effects are negligible. Determine (a) the mass flow rate of the air, lb/s , and (b) the temperature of the air at the compressor exit, in $^\circ\text{F}$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS: (a) The mass flow rate is

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{R T_1} = \frac{(14.7 \times 144 \text{ lbf/ft}^2)(5000 \text{ ft}^3/\text{min})}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)(520^\circ\text{R})} \left| \frac{60 \text{ s}}{1 \text{ min}} \right|$$

$$= 6.36 \frac{\text{lb}}{\text{s}} \quad \leftarrow (a)$$

(b) Reducing an energy rate balance,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)} \right]$$

$$\Rightarrow h_2 = h_1 + \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}}$$

With h_1 from Table A-22E and other known data,

$$h_2 = 124.27 \frac{\text{Btu}}{\text{lb}} - 10 \frac{\text{Btu}}{\text{lb}} - \frac{[-700 \text{ hp}]}{6.36145} \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right|$$

$$= 192.08 \text{ Btu/lb}$$

Interpolation in Table A-22E gives

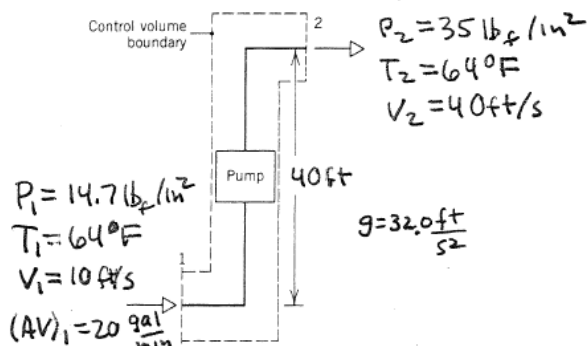
$$T_2 = 801^\circ\text{R} \quad (341^\circ\text{F}) \quad \leftarrow (b)$$

PROBLEM 4.65

KNOWN: Water is steadily pumped through a piping arrangement to a higher elevation where it is discharged with a known mass flow rate.

FIND: Determine the power required by the pump.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} \approx 0$.
3. The water is modeled as incompressible.
4. $g = 32.0 \text{ ft/s}^2$

ANALYSIS: Reducing mass and energy rate balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow (-\dot{W}_{cv}) = \dot{m} \left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$

With assumption 3 and Eq. 3.20b

$$\begin{aligned} (h_2 - h_1) &= c(T_2 - T_1) + v(P_2 - P_1) = v(P_2 - P_1) \\ &= (0.01604 \frac{\text{ft}^3}{\text{lb}}) (35 - 14.7) \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{\text{ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 0.060 \frac{\text{Btu}}{\text{lb}} \end{aligned}$$

where $v = v_f(64^\circ\text{F})$ from Table A-2E. The change in specific kinetic energy is

$$\frac{V_2^2 - V_1^2}{2} = \left[\frac{(40 \text{ ft/s})^2 - (10 \text{ ft/s})^2}{2} \right] \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 0.0299 \frac{\text{Btu}}{\text{lb}}$$

and

$$g(z_2 - z_1) = (32.0 \frac{\text{ft}}{\text{s}^2}) (40 \text{ ft}) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 0.0511 \frac{\text{Btu}}{\text{lb}}$$

The mass flow rate is calculated using

$$\dot{m} = \frac{(AV)_1}{v} = \frac{(20 \text{ gal/min})}{(0.01604 \text{ ft}^3/\text{lb})} \left| \frac{0.13368 \text{ ft}^3}{1 \text{ gal}} \right| \left| \frac{1 \text{ min}}{60 \text{ sec}} \right| = 2.78 \text{ lb/s}$$

Collecting results

$$\begin{aligned} (-\dot{W}_{cv}) &= (2.78 \frac{\text{lb}}{\text{s}}) [0.060 + 0.0299 + 0.0511] \frac{\text{Btu}}{\text{lb}} \\ &= 0.392 \text{ Btu/s} \leftarrow \\ &= (0.392 \frac{\text{Btu}}{\text{s}}) \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 0.55 \text{ hp} \leftarrow (-\dot{W}_{cv}) \end{aligned}$$

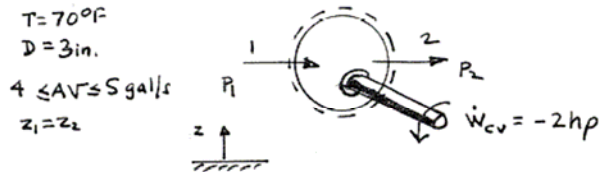
1. Alternatively, Eq. 3.13 can be used to evaluate $(h_2 - h_1)$.

PROBLEM 4.66

KNOWN: Data are provided for a water pump operating at steady state.

FINDS: Plot the pressure rise from inlet to exit versus volumetric flow.

SCHEMATIC & GIVEN DATA:



- ① ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} \approx 0$. 3. Water is modeled as incompressible.

ANALYSIS: The mass rate balance is $\dot{m}_2 = \dot{m}_1$. Since the inlet and exit pipe diameters are the same and the water is incompressible, $V_1 = V_2$, and so the kinetic term vanishes in the energy rate balance, which reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] \quad (\text{no elevation change})$$

- ② With Eq. 3.20b,
- $$h_2 - h_1 = c(T_2 - T_1) + v(P_2 - P_1)$$

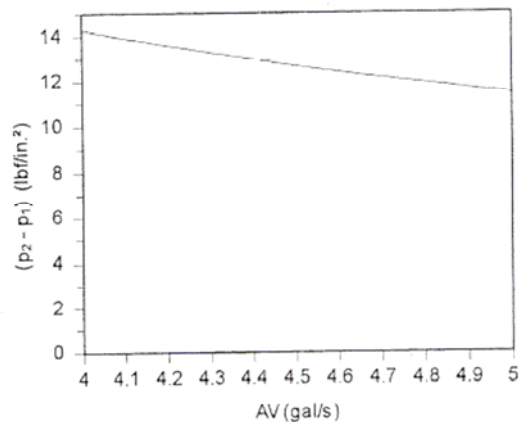
Thus

$$(-\dot{W}_{cv}) = \dot{m} [v(P_2 - P_1)] \Rightarrow P_2 - P_1 = \frac{(-\dot{W}_{cv})}{\dot{m} v}$$

With $\dot{m} = \frac{AV}{v} \Rightarrow AV = \dot{m} v$,

$$\begin{aligned}
 (P_2 - P_1) &= \frac{(-\dot{W}_{cv})}{(AV)} \\
 &= \frac{(2\text{ hp}) \left| \frac{2545\text{ Btu/h}}{1\text{ hp}} \right| \left| \frac{1\text{ h}}{3600\text{ s}} \right| \left| \frac{778\text{ ft}\cdot\text{lbf}}{1\text{ Btu}} \right| \left| \frac{1\text{ ft}^2}{144\text{ in}^2} \right|}{(AV)(\text{gal/s}) \left| \frac{0.13368\text{ ft}^3}{1\text{ gal}} \right|} \\
 &= \frac{57.14}{(AV)} \frac{\text{lbf}}{\text{in}^2} \quad (*)
 \end{aligned}$$

Eq. (*) can be plotted readily using a spreadsheet, plotting program, or IT. The IT plot is shown at the right.



1. Since the water is at 70°F , only a small temperature difference with normal surroundings would be observed. So, heat transfer can be ignored.
2. Alternatively, Eq. 3.13 can be used to obtain the same result.

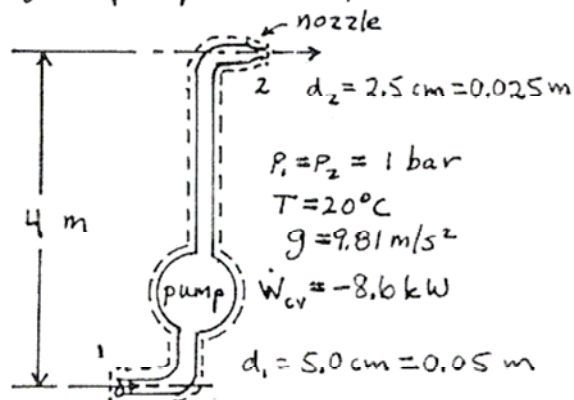
PROBLEM 4.67

KNOWN: Water flows through pumping system with known inlet and exit conditions. The power required by the pump is also specified.

FIND: Determine the mass flow rate.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) The water behaves as an incompressible liquid. (4) The acceleration of gravity is constant at $g = 9.81 \text{ m/s}^2$. (5) The temperature and pressure are nearly constant throughout.



ANALYSIS: To find $\dot{m}$, begin with steady-state mass and energy rate balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$= -\dot{W}_{cv} + \dot{m} \left[\frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] \quad (*)$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, and the specific enthalpy term is eliminated based on assumption (3) and Eq. 3.20b.

From Eq. 5.3a, $V = \dot{m}v/A$, and (*) becomes

$$0 = -\dot{W}_{cv} + \dot{m} \left[\frac{(\dot{m}v/A_1)^2 - (\dot{m}v/A_2)^2}{2} + g(z_1 - z_2) \right]$$

$$= -\dot{W}_{cv} + \frac{\dot{m}^3 v^2}{2} \left(\frac{1}{A_1^2} - \frac{1}{A_2^2} \right) + \dot{m}g(z_1 - z_2) \quad (**)$$

From $A = \pi d^2/4$; $A_1 = 0.000491 \text{ m}^2$ and $A_2 = 0.001964 \text{ m}^2$. Now, with $v \approx v_f @ 20^\circ\text{C} = 1.0018 \times 10^{-3} \text{ m}^3/\text{kg}$ from Table A-2, we can insert values in (**)

$$0 = -(-8.6 \text{ kW}) \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| + \dot{m}^3 \frac{(1.0018 \times 10^{-3} \frac{\text{m}^3}{\text{kg}})^2}{2} \left[\frac{1}{0.000491^2} - \frac{1}{0.001964^2} \right] \frac{1}{\text{m}^4}$$

$$\sqrt{\left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| + \dot{m} (9.81 \frac{\text{m}}{\text{s}^2}) (-4 \text{ m}) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|}$$

or $0 = 8.6 - 1.9513 \times 10^{-3} \dot{m}^3 - 0.03924 \dot{m}$ (where $\dot{m}$ is in kg/s)

This equation is cubic in $\dot{m}$. The solution is

$$\dot{m} = 15.98 \text{ kg/s}$$

1. Eq. 3.20b: $h_1 - h_2 = c(T_1 - T_2) + v(P_1 - P_2)$. Here, $T_1 = T_2$, $P_1 = P_2$; so $(h_1 - h_2) = 0$.

PROBLEM 4.68

As shown in Fig. P4.68, a power washer used to clean the siding of a house has water entering through a hose at 20°C, 1 atm and a velocity of 0.2 m/s. A jet of water exits with a velocity of 20 m/s at an average elevation of 5 m with no significant change in temperature or pressure. At steady state, the magnitude of the heat transfer rate from the power washer to the surroundings is 10% of the electrical power input. Evaluating electricity at 8 cents per kW·h, determine the cost of the power required, in cents per liter of water delivered. Compare with the cost of water, assuming 0.05 cents per liter, and comment.

SCHEMATIC & GIVEN DATA:

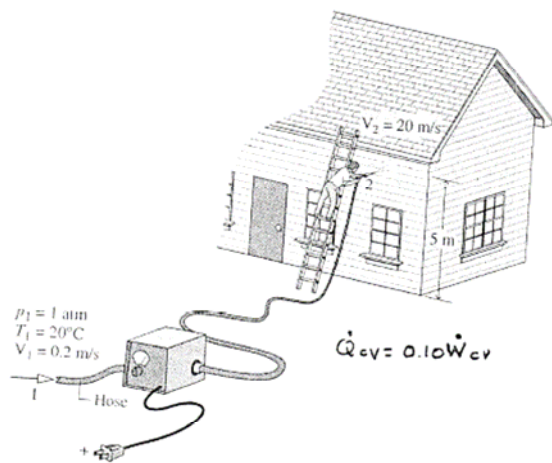


Fig. 4.68

ENGR. MODEL:

1. A control volume encloses the power washer, including the inlet and delivery hoses.
2. The control volume is at steady state.
3. For liquid water, $v \approx v_f(T)$ and $h \approx h_f(T)$.
4. There is no significant change in temperature or pressure from inlet to exit.
5. The cost of electricity is 8 cents per kW·h. The cost of water is 0.05 cents per liter.
6. $g = 9.8 \text{ m/s}^2$.

ANALYSIS: Reducing an energy rate balance,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2) \right]$$

$\dot{Q}_{cv} = 0.1 \dot{W}_{cv}$ $\dot{m} = \frac{(AV)_1}{v_1}$ ≈ 0 by assumptions 3 and 4

$$\Rightarrow \frac{\dot{W}_{cv}}{(AV)_1} = \frac{1}{0.9 v_1} \left[\left(\frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2) \right]$$

Inserting data, including $v_1 \approx v_f(20^\circ\text{C}) = 1.0018 \times 10^{-3} \text{ m}^3/\text{kg}$ from Table A-2,

$$\begin{aligned} \frac{\dot{W}_{cv}}{(AV)_1} &= \frac{10^3}{0.9 (1.0018) \text{ m}^3/\text{kg}} \left[\frac{(0.2)^2 - (20)^2}{2} \left(\frac{\text{m}}{\text{s}} \right)^2 + 9.8 \frac{\text{m}}{\text{s}^2} (-5 \text{ m}) \right] \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= -276 \frac{\text{kJ}}{\text{m}^3} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \left| \frac{10^{-3} \text{ m}^3}{1 \text{ L}} \right| = -7.7 \times 10^{-5} \frac{\text{kW} \cdot \text{h}}{\text{L}} \end{aligned}$$

Costing:

$$\left[\text{Electricity per Liter} \right] = 7.7 \times 10^{-5} \frac{\text{kW} \cdot \text{h}}{\text{L}} \left(\frac{8 \text{ cents}}{\text{kW} \cdot \text{h}} \right) = 6.2 \times 10^{-4} \frac{\text{cents}}{\text{L}}$$

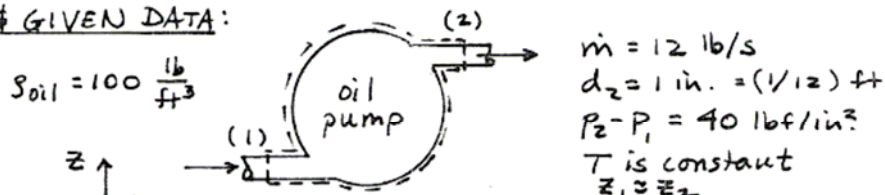
Comment: The cost of the water is significantly greater than the cost of electricity to deliver it:
 $(0.05 \text{ cents/L}) / (6.2 \times 10^{-4} \text{ cents/L}) = 81$.

PROBLEM 4.69

KNOWN: An oil pump operating at steady state delivers oil with a known mass flow rate and pressure rise from inlet to exit.

FIND: If pumps are available in 1/4-horsepower increments, determine the horsepower rating of the pump needed for this application.

SCHEMATIC & GIVEN DATA:



- ENGR. MODEL:** (1) The control volume is at steady state. (2) For the control volume, $\dot{Q}_{cv} \approx 0$. (3) The oil is incompressible. (4) The potential energy change from inlet to exit and the inlet kinetic energy are negligible.

ANALYSIS: To determine the power required, begin with steady state forms of the mass and energy rate balances, as follows:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$ and assumptions (2) and (4) have been applied. Thus

$$\dot{W}_{cv} = \dot{m} \left[(h_1 - h_2) - \frac{V_2^2}{2} \right]$$

With Eq. 3.20b

$$(2) \quad h_1 - h_2 = c(T_1 - T_2) + v(P_1 - P_2) = \frac{P_1 - P_2}{\rho}$$

Thus

$$\dot{W}_{cv} = \dot{m} \left[\frac{P_1 - P_2}{\rho} - \frac{V_2^2}{2} \right]$$

Using Eq. 4.4b with $A = \pi d^2/4$

$$V_2 = \frac{4\dot{m}}{\rho \pi d^2} = \frac{4(12 \text{ lb/s})}{(100 \text{ lb/ft}^3) \pi (1/12)^2 \text{ ft}^2} = 22 \text{ ft/s}$$

Inserting values and noting that $P_1 - P_2 = -(P_2 - P_1)$

$$\dot{W}_{cv} = (12 \frac{\text{lb}}{\text{s}}) \left[\frac{-40 \text{ lbf/in}^2}{100 \text{ lb/ft}^3} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| - \frac{(22 \text{ ft/s})^2}{2} \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \right]$$

$$= (-781.4 \frac{\text{ft} \cdot \text{lbf}}{\text{s}}) \left| \frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lbf/s}} \right| = -1.42 \text{ hp}$$

pump size = 1.5 hp ← hp rating

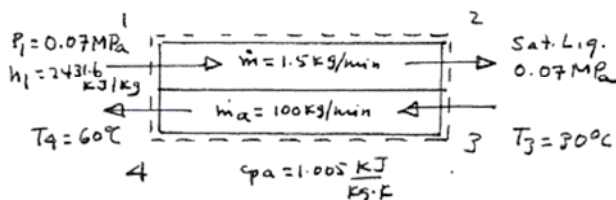
1. It is assumed that there is a small temperature difference between the pump and the surroundings, so heat transfer can be ignored.

2. Alternatively, Eq. 3.13 can be used to obtain the same result.

PROBLEM 4.70

Steam enters a heat exchanger operating at steady state at 0.07 MPa with a specific enthalpy of 2431.6 kJ/kg and exits at the same pressure as saturated liquid. The steam mass flow rate is 1.5 kg/min. A separate stream of air with a mass flow rate of 100 kg/min enters at 30°C and exits at 60°C. The ideal gas model with $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$ can be assumed for air. Kinetic and potential energy effects are negligible. Determine (a) the quality of the entering steam and (b) the rate of heat transfer between the heat exchanger and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and all kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas with constant c_p .

ANALYSIS:

(a) From Table A-3 at 0.07 MPa, $h_f = 376.70 \text{ kJ/kg}$, $h_g = 2660.0 \text{ kJ/kg}$.
Thus, the steam is a two-phase liquid-vapor mixture at 1. Then,

$$x_1 = \frac{h_1 - h_f}{h_g - h_f} = \frac{2431.6 - 376.70}{2660.0 - 376.70} = 0.9$$

(b) Reducing Eq. 4.18,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] + \dot{m}_a \left[h_3 - h_4 + \frac{V_3^2 - V_4^2}{2} + g(z_3 - z_4) \right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m} [h_2 - h_1] + \dot{m}_a \left[\frac{h_4 - h_3}{c_p (T_4 - T_3)} \right]$$

$$= \dot{m} [h_2 - h_1] + \dot{m}_a c_p (T_4 - T_3)$$

Inserting values,

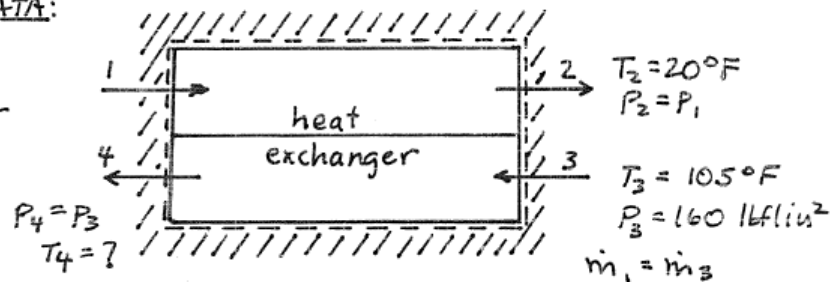
$$\begin{aligned} \dot{Q}_{cv} &= \left(1.5 \frac{\text{kg}}{\text{min}} \right) \left[376.7 - 2431.6 \right] \frac{\text{kJ}}{\text{kg}} + \left(100 \frac{\text{kg}}{\text{min}} \right) \left(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (+30 \text{ K}) \\ &= -3082.4 \frac{\text{kJ}}{\text{min}} + 3015 \frac{\text{kJ}}{\text{min}} \\ &= -67.35 \frac{\text{kJ}}{\text{min}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= -1.12 \text{ kW} \end{aligned}$$

PROBLEM 4.71

KNOWN: Separate vapor and liquid streams of Refrigerant 134a pass in counter flow through a well-insulated heat exchanger. Data are known at the inlets and exits.

FIND: Determine the exit temperature of the liquid stream.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer between the heat exchanger and the surroundings can be neglected and $\dot{W}_{cv} = 0$. (3) Kinetic and potential energy changes from inlet to exit are negligible.

ANALYSIS: To fix state 4, begin with steady state mass and energy balances to determine h_4

$$\begin{aligned}\dot{m}_1 &= \dot{m}_2 \\ \dot{m}_3 &= \dot{m}_4\end{aligned}$$

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1 \left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + g(z_1 - z_2) \right] + \dot{m}_3 \left[(h_3 - h_4) + \cancel{\frac{V_3^2 - V_4^2}{2}} + g(z_3 - z_4) \right]$$

with $\dot{m}_1 = \dot{m}_3$

$$h_4 = (h_1 - h_2) + h_3$$

From Table A-10E; $h_1 = 101.75$ Btu/lb and $p_1 = 21.203$ lbf/in². Interpolating in Table A-12E; $h_2 = 105.76$ Btu/lb.

States 3 and 4 are both subcooled liquid states. The following approximations are reasonable

$$h_3 \approx h_f @ T_3$$

$$h_4 \approx h_f @ T_4$$

with $h_3 = 46.01$ Btu/lb from Table A-10E

$$h_4 = (101.75 - 105.76) + 46.01 = 42 \text{ Btu/lb}$$

Interpolating in Table A-10E

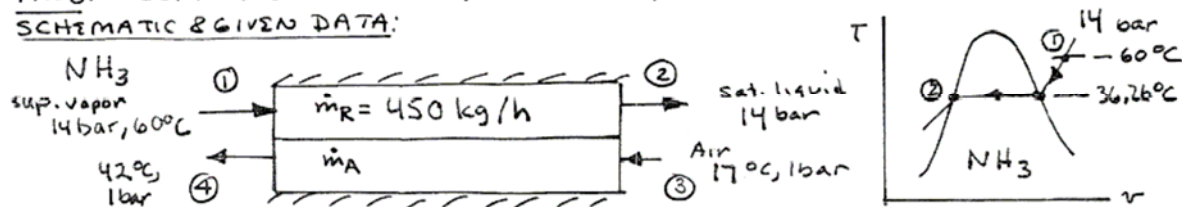
$$T_4 \approx 93.7^\circ\text{F} \leftarrow T_4$$

PROBLEM 4.72

KNOWN: Ammonia and air pass in separate streams through a heat exchanger at steady state, for which data are provided.

FIND: Determine the mass flow rate of the air.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. A control volume enclosing the heat exchanger is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$, heat transfer can be ignored, and kinetic/potential energy effects are negligible. ③ Air is modeled as an ideal gas.

ANALYSIS: Since the streams flow separately, the conservation of mass principle indicates at steady state: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_R$ and $\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_A$. An energy rate balance reads

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_R \left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)} \right] + \dot{m}_A \left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)} \right]$$

$\Rightarrow$

$$\dot{m}_A = \dot{m}_R \left[\frac{h_1 - h_2}{h_4 - h_3} \right]$$

From Table A-15, $h_1 = 1542.89 \text{ kJ/kg}$. From Table A-14, $h_2 = 352.97 \text{ kJ/kg}$. From Table A-22, $h_3 = 290.16 \text{ kJ/kg}$, $h_4 = 315.27 \text{ kJ/kg}$. Then

$$\dot{m}_A = (450) \frac{\text{kg}}{\text{h}} \left[\frac{1542.89 - 352.97}{315.27 - 290.16} \right] \left| \frac{1 \text{ h}}{60 \text{ min}} \right| = 355.4 \frac{\text{kg}}{\text{min}} \leftarrow \dot{m}_A$$

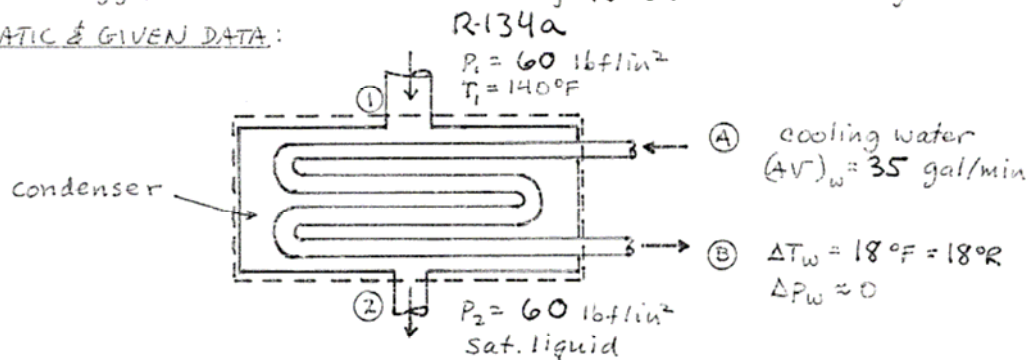
1. The validity of the ideal gas model is readily checked using the generalized compressibility chart.

PROBLEM 4.73

KNOWN: Refrigerant R-134a and cooling water pass in separate streams through a condenser (heat exchanger). The volumetric flow rate of cooling water and other data are given at the inlets and exits.

FINID: Determine (a) the mass flow rate of R-134a, and (b) the rate of energy transfer from the condensing R-134a to the cooling water.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer from the outside of the condenser is negligible. (3) Kinetic and potential energy changes from inlet to exit are negligible. (4) The cooling water is modeled as an incompressible liquid with constant specific heat.

ANALYSIS: (a) Since the R-134a and cooling water are separate streams

$$\text{R-134a: } \dot{m}_1 = \dot{m}_2 = \dot{m}_R$$

$$\text{Water: } \dot{m}_A = \dot{m}_B = \dot{m}_w$$

The mass flow rate of R-134a is found from the steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] + \dot{m}_w \left[(h_A - h_B) + \frac{V_A^2 - V_B^2}{2} + g(z_A - z_B) \right]$$

Or

$$\dot{m}_R = \frac{\dot{m}_w (h_B - h_A)}{(h_1 - h_2)}$$

For the water, using Eq. 3.20b

$$h_B - h_A = c (T_B - T_A) + v (P_B - P_A)$$

and

$$\dot{m}_w = \frac{(AV)_w}{v_w}$$

From Table A-19E; $c \approx 1 \text{ Btu/lb} \cdot ^\circ\text{R}$, and $v_w = 1/5 = 0.0161 \text{ ft}^3/\text{lb}$.

For the R-134a, $h_1 = 129.53 \text{ Btu/lb}$ from Table A-12E and $h_2 = 27.24 \text{ Btu/lb}$ from Table A-11E. Inserting values

$$\dot{m}_R = \left(\frac{(35 \text{ gal/min}) \left(\frac{0.13368 \text{ ft}^3}{1 \text{ gal}} \right)}{(0.0161 \text{ ft}^3/\text{lb})} \right) \frac{(1 \text{ Btu/lb} \cdot ^\circ\text{R})(18^\circ\text{R})}{(129.53 - 27.24) \text{ Btu/lb}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right|$$

$$= 3068.3 \text{ lb/h} \quad \dot{m}_R$$

Continued on next slide

Problem 4-73 continued

(b) For a control volume enclosing only the R-134a

$$0 = \dot{Q}_R - \dot{W}_R^o + \dot{m}_R [(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)]$$

where $\dot{Q}_R$ denotes the heat transfer rate for the R-134a only. Thus

$$\begin{aligned}\dot{Q}_R &= \dot{m}_R (h_2 - h_1) \\ &= (3068.3 \text{ lb/h}) (27.24 - 129.53)\end{aligned}$$

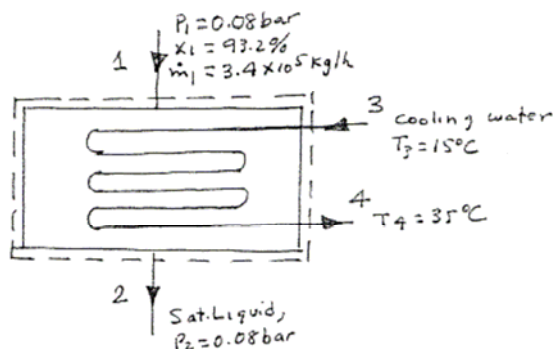
$$\textcircled{1} \quad = -3.140 \times 10^5 \text{ Btu/h} \quad \leftarrow \dot{Q}_R$$

1. The negative value for $\dot{Q}_R$ denotes energy transfer heat from the R-134a to the cooling water, as expected.

PROBLEM 4.74

Steam at a pressure of 0.08 bar and a quality of 93.2% enters a shell-and-tube heat exchanger where it condenses on the outside of tubes through which cooling water flows, exiting as saturated liquid at 0.08 bar. The mass flow rate of the condensing steam is 3.4×10^5 kg/h. Cooling water enters the tubes at 15°C and exits at 35°C with negligible change in pressure. Neglecting stray heat transfer and ignoring kinetic and potential energy effects, determine the mass flow rate of the cooling water, in kg/h, for steady-state operation.

SCHEMATIC & GIVEN DATA:



ENG. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} \approx 0$, and kinetic and potential energy effects can be ignored.
3. For the cooling water, $h \approx h_f(T)$.

ANALYSIS: Reducing Eq. 4.18,

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_1 \left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)} \right] + \dot{m}_3 \left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)} \right]$$

$$\Rightarrow \dot{m}_3 = \dot{m}_1 \frac{[h_1 - h_2]}{[h_4 - h_3]} \quad (1)$$

where with data from Table A-3,

$$h_1 = h_f + x_1(h_g - h_f) = 173.88 + 0.932(2403.1) = 2413.6 \frac{\text{kJ}}{\text{kg}}$$

And with data from Table A-2, $h_3 \approx h_f(15^\circ\text{C}) = 62.99 \frac{\text{kJ}}{\text{kg}}$, $h_4 \approx h_f(35^\circ\text{C}) = 146.68 \frac{\text{kJ}}{\text{kg}}$.

Inserting values into Eq. (1), we get

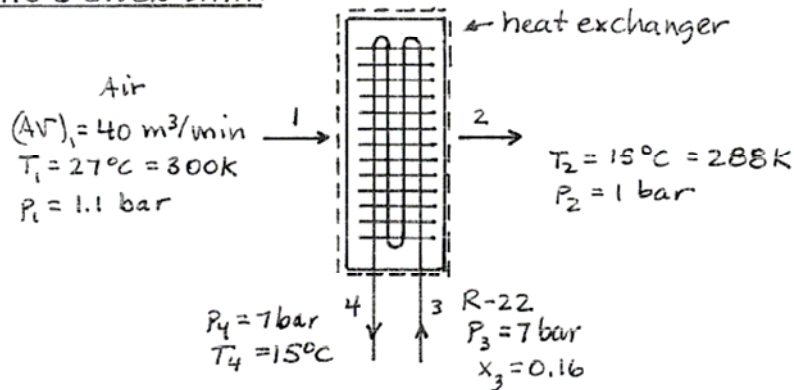
$$\begin{aligned} \dot{m}_3 &= 3.4 \times 10^5 \frac{\text{kg}}{\text{h}} \left[\frac{2413.6 - 173.88}{146.68 - 62.99} \right] \\ &= 9.06 \times 10^6 \frac{\text{kg}}{\text{h}} \end{aligned}$$

PROBLEM 4.75

KNOWN: Air and Refrigerant 22 pass in separate streams through a heat exchanger. Data are known at the inlet and exit of each stream.

FIND: Determine (a) the mass flow rate of refrigerant and (b) the rate of energy transfer from the air to the refrigerant.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer from the outside of the heat exchanger is negligible, and $\dot{W}_{cv} = 0$. (3) Kinetic and potential effects can be neglected. (4) The air behaves as an ideal gas, as can be verified by reference to the compressibility chart.

ANALYSIS: (a) The mass flow rate of refrigerant is determined using Steady-State mass and energy balances. First, since the air and refrigerant flow as separate streams

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{\text{air}}$$

$$\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_{\text{R-22}}$$

Thus, the energy rate balance reduces as follows

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_{\text{air}} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] + \dot{m}_{\text{R-22}} \left[(h_3 - h_4) + \frac{V_3^2 - V_4^2}{2} + g(z_3 - z_4) \right]$$

and

$$\dot{m}_{\text{R-22}} = \dot{m}_{\text{air}} \left(\frac{h_1 - h_2}{h_4 - h_3} \right)$$

The mass flow rate of air is found using data at the inlet and the ideal gas equation of state

$$\begin{aligned} \dot{m}_{\text{air}} &= \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1} \\ &= \frac{(1.1 \text{ bars})(40 \text{ m}^3/\text{min})}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right)(300 \text{ K})} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= 51.11 \text{ kg/min} \end{aligned}$$

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Problem 4-75 continued

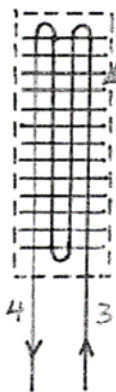
From Table A-22 ; $h_1 = 300.19 \text{ kJ/kg}$ and $h_2 = 288.15 \text{ kJ/kg}$. Further, using data from Table A-8

$$h_3 = h_{f3} + x_3 h_{fg3} = 58.04 + (0.16)(195.60) = 89.34 \text{ kJ/kg}$$

And, from Table A-9 ; $h_4 = 256.86 \text{ kJ/kg}$. Thus

$$\begin{aligned} \dot{m}_{R-22} &= (51.11 \text{ kg/min}) \left(\frac{300.19 - 288.15}{256.86 - 89.34} \right) \\ &= 3.673 \text{ kg/min} \end{aligned}$$

(b) Consider a control volume enclosing only the refrigerant stream



$$0 = \dot{Q}_{R-22} + \dot{m}_{R-22} (h_3 - h_4)$$

$$\dot{Q}_{R-22} = \dot{m}_{R-22} (h_4 - h_3)$$

$$= (3.673 \text{ kg/min})(256.86 - 89.34) \text{ kJ/kg}$$

$$= 615.3 \text{ kJ/min}$$

COMMENT: For a control volume enclosing only the air stream

$$\dot{Q}_{air} = \dot{m}_{air} (h_2 - h_1)$$

$$= (51.11 \text{ kg/min})(288.15 - 300.19) \text{ kJ/kg}$$

$$= -615.3 \text{ kJ/min}$$

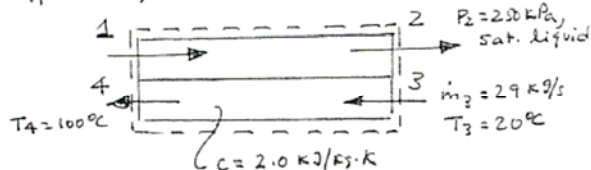
Thus, $\dot{Q}_{R-22} = -\dot{Q}_{air}$, as expected.

PROBLEM 4-76

Steam enters a heat exchanger operating at steady state at 250 kPa and a quality of 90% and exits as saturated liquid at the same pressure. A separate stream of oil with a mass flow rate of 29 kg/s enters at 20°C and exits at 100°C with no significant change in pressure. The specific heat of the oil is $c = 2.0 \text{ kJ/kg} \cdot \text{K}$. Kinetic and potential energy effects are negligible. If heat transfer from the heat exchanger to its surroundings is 10% of the energy required to increase the temperature of the oil, determine the steam mass flow rate, in kg/s.

SCHEMATIC & GIVEN DATA:

$$P_1 = 250 \text{ kPa}, x = 90\%$$



$$\dot{Q}_{cv} = -0.10 [\dot{m}_3 (h_4 - h_3)] \quad (1)$$

①
Energy required to increase the oil temperature.

ANALYSIS: Reducing Eq. 4.18,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] + \dot{m}_3 \left[h_3 - h_4 + \frac{V_3^2 - V_4^2}{2} + g(z_3 - z_4) \right]$$

$$\Rightarrow \dot{m}_1 = \frac{-\dot{Q}_{cv} + \dot{m}_3 [h_4 - h_3]}{h_1 - h_2}$$

Introducing Eq. (1),

$$\begin{aligned} \dot{m}_1 &= \frac{+0.10 \dot{m}_3 [h_4 - h_3] + \dot{m}_3 [h_4 - h_3]}{h_1 - h_2} \\ &= \frac{(1.10) \dot{m}_3 [h_4 - h_3]}{h_1 - h_2} \quad (2) \end{aligned}$$

With data from Table A-3, $h_1 = h_f + x_1 h_{fg}$, $h_2 = h_f \Rightarrow h_1 - h_2 = x_1 h_{fg}$. That is,
 $h_1 - h_2 = 0.90(2181.5 \text{ kJ/kg}) = 1963.4 \text{ kJ/kg}$.

Inserting values into Eq. (2), using Eq. 3.206: $(h_4 - h_3) = c(T_4 - T_3) + v(P_4 - P_3)$, we get

$$\dot{m}_1 = \frac{1.10(29 \text{ kg/s})(2.0 \text{ kJ/kg} \cdot \text{K})(80 \text{ K})}{1963.4 \text{ kJ/kg}} = 2.6 \text{ kg/s}$$

1. Consider a control volume consisting of the oil side of the heat exchanger. Assume steady state with $\dot{W}_{cv} = 0$ and ignore kinetic and potential energy effects. An energy rate balance gives $\dot{Q}_{oil} = \dot{m}_3 (h_4 - h_3)$, which is the energy that would be required just to increase the oil temperature.

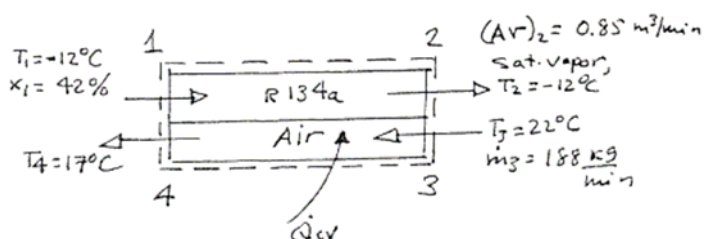
ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. The oil is modeled as incompressible with $c = 2.0 \text{ kJ/kg} \cdot \text{K}$ and no significant change in pressure.

PROBLEM 4.77.

Refrigerant 134a enters a heat exchanger at -12°C and a quality of 42% and exits as saturated vapor at the same temperature with a volumetric flow rate of $0.85\text{ m}^3/\text{min}$. A separate stream of air enters at 22°C with a mass flow rate of $188\text{ kg}/\text{min}$ and exits at 17°C . Assuming the ideal gas model for air and ignoring kinetic and potential energy effects, determine (a) the mass flow rate of the Refrigerant 134a, in kg/min , and (b) the heat transfer between the heat exchanger and its surroundings, in kJ/min .

SCHEMATIC & GIVEN DATA:



ENER. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS:

$$(a) \dot{m}_2 = \frac{(\dot{A}V)_2}{v_2} = \frac{0.85\text{ m}^3/\text{min}}{0.1068\text{ m}^3/\text{kg}} = 7.96\text{ kg}/\text{min} \quad (= \dot{m}_1) \quad \leftarrow (a)$$

(Table A-10)

(b) Reducing Eq. 4.18,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left[h_1 - h_2 + \frac{v_1^2 - v_2^2}{2} + g(z_1 - z_2) \right] + \dot{m}_3 \left[h_3 - h_4 + \frac{v_3^2 - v_4^2}{2} + g(z_3 - z_4) \right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}_1 [h_2 - h_1] + \dot{m}_3 [h_4 - h_3] \quad (1)$$

With data from Table A-10, $h_2 = 240.15\text{ kJ/kg}$ and $h_1 = h_f + x_1 h_{fg} = 34.39 + 0.42(205.77) = 120.81\text{ kJ/kg}$. Also, from Table A-22, $h_3 = 295.17\text{ kJ/kg}$ and $h_4 = 290.16\text{ kJ/kg}$.

Substituting values into Eq. (1), we get

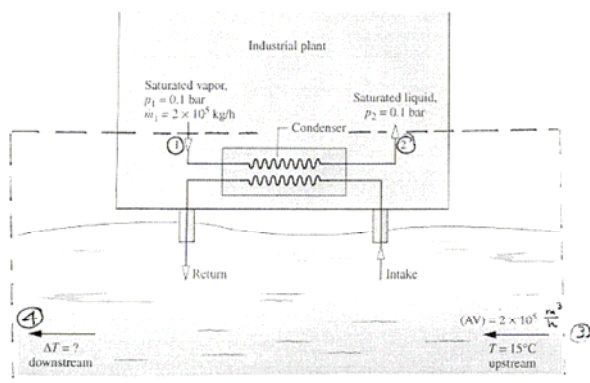
$$\begin{aligned} \dot{Q}_{cv} &= 7.96\text{ kg}/\text{min} \left(240.15 - 120.81 \right) \frac{\text{kJ}}{\text{kg}} + 188\text{ kg}/\text{min} \left(290.16 - 295.17 \right) \frac{\text{kJ}}{\text{kg}} \\ &= 949.95\text{ kJ}/\text{min} - 941.88\text{ kJ}/\text{min} \\ &= +8.1\text{ kJ}/\text{min} \quad \leftarrow (b) \end{aligned}$$

1. A positive sign here seems reasonable owing to the low interior temperature of the heat exchanger relative to the temperature of the entering air.

PROBLEM 4.78

As sketched in Fig. P4.78, a condenser using river water to condense steam with a mass flow rate of $2 \times 10^5 \text{ kg/h}$ from saturated vapor to saturated liquid at a pressure of 0.1 bar is proposed for an industrial plant. Measurements indicate that several hundred meters upstream of the plant, the river has a volumetric flow rate of $2 \times 10^5 \text{ m}^3/\text{h}$ and a temperature of 15°C . For operation at steady state and ignoring changes in kinetic and potential energy, determine the river-water temperature rise, in $^\circ\text{C}$, downstream of the plant traceable to use of such a condenser, and comment.

SCHEMATIC & GIVEN DATA:



ENER. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, there is no net heat transfer, and kinetic and potential energy effects can be ignored.
3. At 3 and 4, $v \approx v_f(T)$, $h \approx h_f(T)$.

ANALYSIS: Reducing Eq. 4.18,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left[h_1 - h_2 + \frac{v_1^2 - v_2^2}{2} + g(z_1 - z_2) \right] + \dot{m}_2 \left[h_3 - h_4 + \frac{v_3^2 - v_4^2}{2} + g(z_3 - z_4) \right]$$

$$\Rightarrow h_4 = h_3 + \frac{\dot{m}_1}{\dot{m}_3} [h_1 - h_2]$$

where

$$\dot{m}_3 = \frac{(AV)_3}{v_3} = \frac{2 \times 10^5 \text{ m}^3/\text{h}}{\left(\frac{1.0009 \text{ m}^3}{10^3 \text{ kg}} \right)} = 2 \times 10^8 \frac{\text{kg}}{\text{h}}$$

(Table A-2)

Then, with $h_1 - h_2 = h_{fg} @ 0.1 \text{ bar}$, $h_1 - h_2 = 2392.8 \frac{\text{kJ}}{\text{kg}}$ (Table A-3). Also, from Table A-2, $h_3 = 62.99 \frac{\text{kJ}}{\text{kg}}$.

$$h_4 = 62.99 + \left(\frac{2 \times 10^5}{2 \times 10^8} \right) (2392.8) = 65.28 \frac{\text{kJ}}{\text{kg}}$$

Interpolation in Table A-3 with $h_4 \approx h_f(T_4)$ gives $T_4 = 15.6^\circ\text{C}$

$\Rightarrow$ Temperature rise $= 0.6^\circ\text{C}$

Comment: As discussed in Sec. 2.6.2, adverse environmental consequences can result when large quantities of warm water are returned to a river or lake.

PROBLEM 4.79

Figure P4.79 shows a solar collector panel embedded in a roof. The panel, which has a surface area of 24 ft^2 , receives

SCHEMATIC & GIVEN DATA:

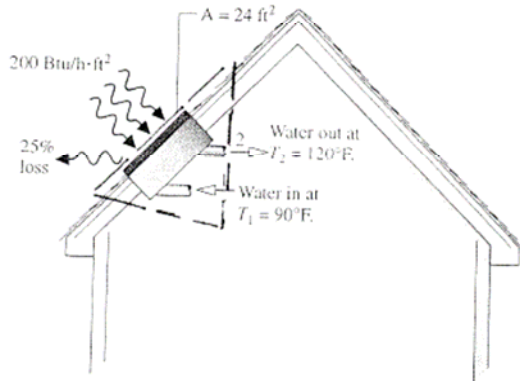


Fig. P4.79

energy from the sun at a rate of $200 \text{ Btu/h per ft}^2$ of collector surface. Twenty-five percent of the incoming energy is lost to the surroundings. The remaining energy is used to heat domestic hot water from 90 to 120°F . The water passes through the solar collector with a negligible pressure drop. Neglecting kinetic and potential effects, determine at steady state how many gallons of water at 120°F the collector generates per hour.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy can be neglected.
3. For liquid water, $v \approx v_f(T)$ and $h \approx h_f(T)$.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{v_1^2 - v_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \dot{m} = \frac{\dot{Q}_{cv}}{h_2 - h_1}$$

(1)

where

$$\dot{Q}_{cv} = (0.75)(200 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2})(24 \text{ ft}^2) = 3600 \text{ Btu/h}$$

Then, with $h_1 \approx h_f(90^\circ\text{F}) = 58.07 \text{ Btu/lb}$, $h_2 \approx h_f(120^\circ\text{F}) = 88.0 \text{ Btu/lb}$ from Table A-2E.

Inserting values in Eq. (1),

$$\dot{m} = \frac{3600 \text{ Btu/h}}{(88.0 - 58.07) \text{ Btu/lb}} = 120.3 \frac{\text{lb}}{\text{h}}$$

The volumetric flow rate of the water delivered at 120°F is then

$$\begin{aligned} (AV)_2 &= v_2 \dot{m} = v_f(120^\circ\text{F}) \dot{m} \\ &= (0.01621 \frac{\text{ft}^3}{\text{lb}}) (120.3 \frac{\text{lb}}{\text{h}}) \\ &= 1.95 \text{ ft}^3/\text{h} \end{aligned}$$

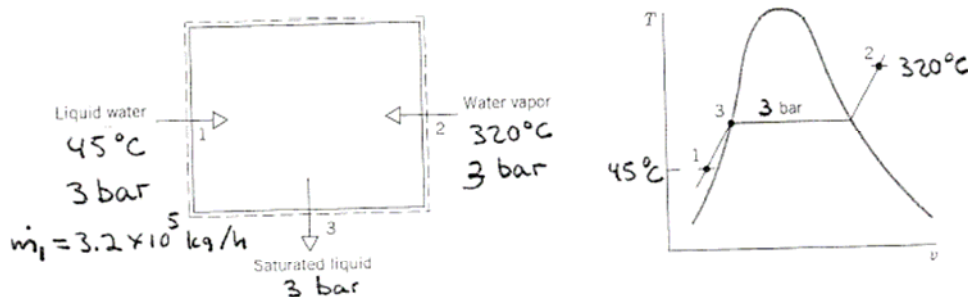
Converting to gallons/h,

$$(AV)_2 = 1.95 \frac{\text{ft}^3}{\text{h}} \left| \frac{1 \text{ gal}}{0.13368 \text{ ft}^3} \right| = 14.59 \frac{\text{gal}}{\text{h}}$$

PROBLEM 4.80

KNOWN: Data are provided for a feed water heater at steady state.
FIND: Determine the mass flow rate of the incoming vapor stream, $\dot{m}_2$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state.
 2. For the control volume, $\dot{W}_{cv} = 0$, and heat transfer with the surroundings can be ignored. Kinetic and potential energy effects also can be neglected.

ANALYSIS: At steady state the mass rate balance reads

$$0 = \dot{m}_1 + \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

The energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left(h_1 + \frac{V_1^2}{2} + gz_1 \right) + \dot{m}_2 \left(h_2 + \frac{V_2^2}{2} + gz_2 \right) - \dot{m}_3 \left(h_3 + \frac{V_3^2}{2} + gz_3 \right)$$

$$\Rightarrow 0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Introducing $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3 \Rightarrow \dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3} \right]$$

From Table A-4, $h_2 = 3110.1 \text{ kJ/kg}$. From Table A-3, $h_3 = 561.47 \text{ kJ/kg}$.
 With Eq. 3.14 and data from Table A-2

$$\textcircled{1} \quad h_1 \approx h_f(45^\circ\text{C}) = 188.45 \text{ kJ/kg}$$

$$\text{Thus } \dot{m}_2 = (3.2 \times 10^5 \frac{\text{kg}}{\text{h}}) \left[\frac{561.47 - 188.45}{3110.1 - 561.47} \right] = 0.468 \times 10^5 \frac{\text{kg}}{\text{h}} \leftarrow \dot{m}_2$$

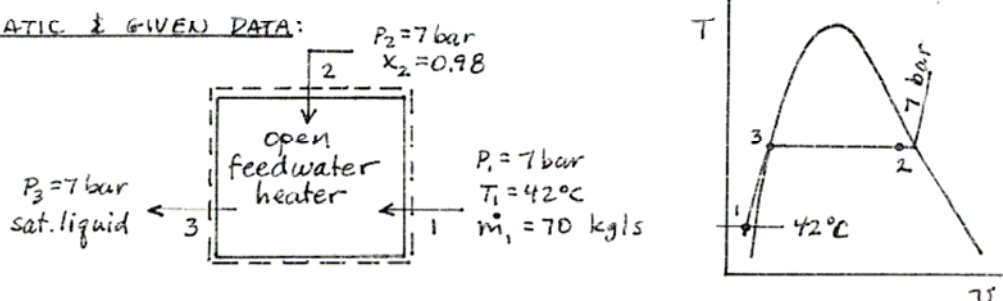
1. Using Eq. 3.13, $h_1 \approx 188.74 \text{ kJ/kg}$ and $\dot{m}_2 = 0.468 \times 10^5 \text{ kg/h}$

PROBLEM 4.81

KNOWN: An open feedwater heater operates with known inlet and exit conditions. The mass flow rate at one inlet is given.

FIND: Determine the mass flow rate at the second inlet.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer with the surroundings is negligible, and $\dot{W}_{cv} = 0$. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: To find $\dot{m}_2$, begin with the steady-state mass and energy balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left(h_1 + \frac{V_1^2}{2} + g z_1 \right) + \dot{m}_2 \left(h_2 + \frac{V_2^2}{2} + g z_2 \right) - \dot{m}_3 \left(h_3 + \frac{V_3^2}{2} + g z_3 \right)$$

With $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$ and assumption (3)

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

or

$$\dot{m}_2 = \dot{m}_1 \left(\frac{h_1 - h_3}{h_3 - h_2} \right)$$

From Table A-3, $T_1 < T_{sat}$ at 7 bar. Hence, state 1 is compressed liquid. Using Eq. 3.13 and data from Table A-2 at 42°C

$$\begin{aligned} \textcircled{1} \quad h_1 &\approx h_f(T_1) + v_f(T_1) [p - p_{sat}(T_1)] \\ &= 175.9 \text{ kJ/kg} + (1.0086 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}) [7 - 0.08268] \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= 176.6 \text{ kJ/kg} \end{aligned}$$

Further, from Table A-3

$$\begin{aligned} h_2 &= h_{f2} + x_2 h_{fg2} = 697.22 + (0.98)(2066.3) = 2722.2 \text{ kJ/kg} \\ h_3 &= 697.22 \text{ kJ/kg} \end{aligned}$$

Finally

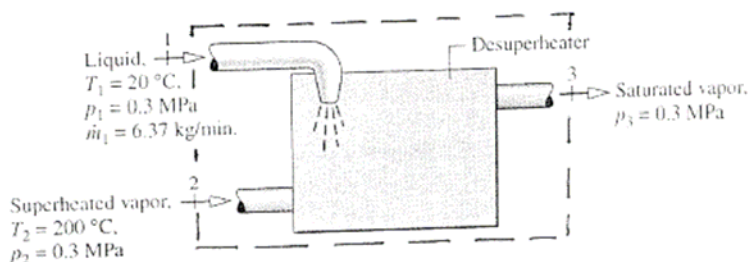
$$\begin{aligned} \dot{m}_2 &= (70 \text{ kg/s}) \left(\frac{176.6 - 697.22}{697.22 - 2722.2} \right) \\ &= 18.0 \text{ kg/s} \end{aligned}$$

1. Alternatively, Eq. 3.14 can be used: $h_1 \approx h_f(T_1) = 175.9 \text{ kJ/kg}$. The result is $\dot{m}_2 = 18.02 \text{ kg/s}$.

PROBLEM 4.82

For the *desuperheater* shown in Fig. P4.82, liquid water at state 1 is injected into a stream of superheated vapor entering at state 2. As a result, saturated vapor exits at state 3. Data for steady state operation are shown on the figure. Ignoring stray heat transfer and kinetic and potential energy effects, determine the mass flow rate of the incoming superheated vapor, in kg/min.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects can be ignored. $\dot{W}_{cv} \equiv 0$.
3. At state 1, $h_1 \approx h_f(T_1)$.

ANALYSIS:

$$\text{Mass rate balance: } \dot{m}_3 = \dot{m}_1 + \dot{m}_2 \quad (1)$$

Energy rate balance,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1 \left[h_1 + \frac{V_1^2}{2} + gz_1 \right] + \dot{m}_2 \left[h_2 + \frac{V_2^2}{2} + gz_2 \right] - \dot{m}_3 \left[h_3 + \frac{V_3^2}{2} + gz_3 \right] \quad (2)$$

$$\Rightarrow 0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Combining Eqs. (1), (2)

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$\Rightarrow \dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3} \right] \quad (3)$$

From Table A-2, $h_1 \approx 83.96 \text{ kJ/kg}$. From Table A-3, $h_3 = 2725.3 \text{ kJ/kg}$.
From Table A-4, $h_2 = 2865.5 \text{ kJ/kg}$. Inserting values in Eq. (3),

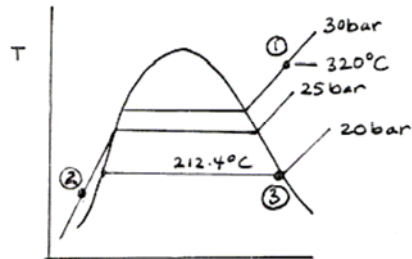
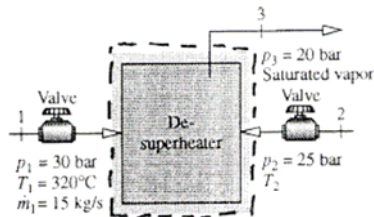
$$\dot{m}_2 = 6.37 \frac{\text{kg}}{\text{min}} \left[\frac{2725.3 - 83.96}{2865.5 - 2725.3} \right] = 120.01 \frac{\text{kg}}{\text{min}} \quad \leftarrow$$

PROBLEM 4.83

KNOWN: Data are provided for a desuperheater operating at steady state.

FIND: (a) For a specified temperature for the entering liquid, determine the liquid mass flow rate. (b) Plot the liquid mass flow rate versus the liquid temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. A control volume enclosing the desuperheater with inlets at 1 and 2 and an exit at 3 is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$ and heat transfer with the surroundings can be ignored. Kinetic and potential energy effects can be ignored.

ANALYSIS: The mass rate balance at steady state reads $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$.

The energy rate balance at steady state reduces as follows:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1 \left[h_1 + \frac{V_1^2}{2} + gz_1 \right] + \dot{m}_2 \left[h_2 + \frac{V_2^2}{2} + gz_2 \right] - \dot{m}_3 \left[h_3 + \frac{V_3^2}{2} + gz_3 \right]$$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

Solving for $\dot{m}_2$

$$\dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3} \right] \quad (1)$$

(a) From Table A-4; $h_1 = 3043.4$ kJ/kg. From Table A-3; $h_3 = 2799.5$ kJ/kg. Further, at $T_2 = 200^\circ\text{C}$, $p_2 = 25$ bar; Table A-5 gives $h_2 = 852.8$ kJ/kg. Thus

$$\dot{m}_2 = (15 \frac{\text{kg}}{\text{s}}) \left[\frac{2799.5 - 3043.4}{852.8 - 2799.5} \right] = 1.88 \text{ kg/s} \quad \leftarrow \dot{m}_2 \text{ (part a)}$$

(b) The following IT code is used to develop data to construct a plot of $\dot{m}_2$ vs. T_2 :

IT Code

```
p1 = 30 // bar
T1 = 320 // °C
mdot1 = 15 // kg/s
p2 = 25 // bar
T2 = 20 // °C
p3 = 20 // bar
```

```
h1 = h_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
h3 = hsat_Px("Water/Steam", p3, 1)
```

```
mdot2 = mdot1 * ((h3 - h1) / (h2 - h3))
```

Continued on next slide

Problem 4-83 continued

IT Result for $T_2 = 200^\circ\text{C}$

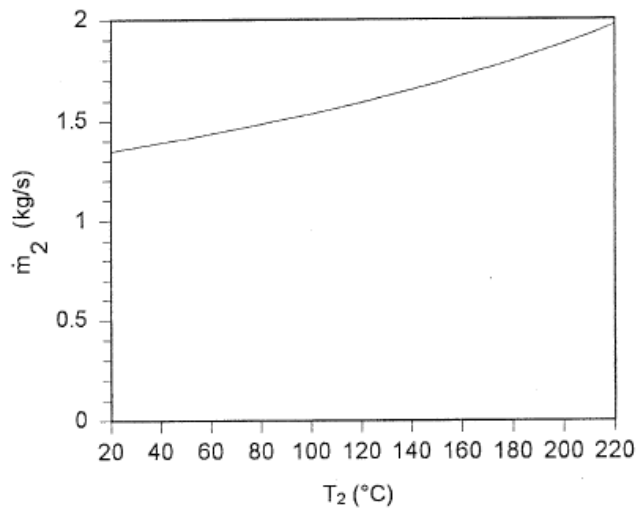
$$h_1 = 3043 \text{ kJ/kg}$$

$$h_2 = 852.4 \text{ kJ/kg}$$

$$h_3 = 2799 \text{ kJ/kg}$$

$$\dot{m}_2 = 1.879 \text{ kg/s}$$

This result compares very favorably with the result of part (a). Thus, with the computer solution validated, use the Explore button and sweep T_2 from 20 to 220°C in steps of 10. The resulting data are used to construct the following plot:



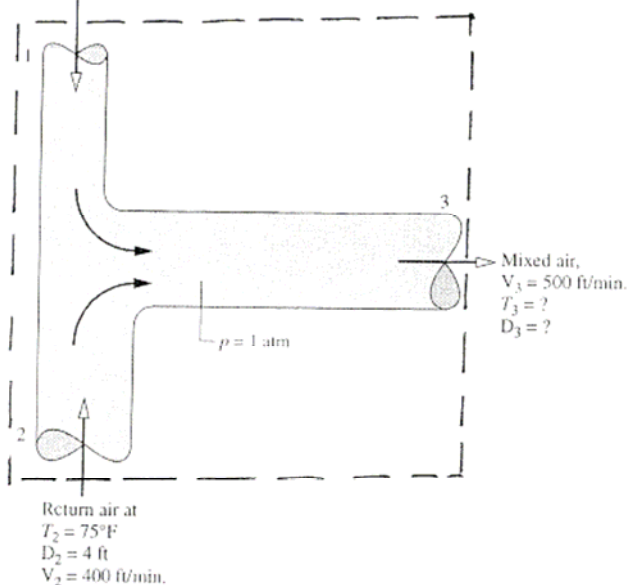
1. Note that IT uses the approximation of Eq. 3.14 for liquid enthalpies.

PROBLEM 4.84

Figure P4.84 provides steady-state data for the ducting ahead of the chiller coils in an air conditioning system. Outside air at 90°F is mixed with return air at 75°F. Stray heat transfer is negligible, kinetic and potential energy effects can

SCHEMATIC & GIVEN DATA:

Outside air at
 $T_1 = 90^\circ\text{F}$
 $V_1 = 600 \text{ ft}^3/\text{min}$
 $(AV)_1 = 2000 \text{ ft}^3/\text{min}$



be ignored, and the pressure throughout is 1 atm. Modeling the air as an ideal gas with $c_p = 0.24 \text{ Btu/lb} \cdot \text{R}$, determine (a) the mixed-air temperature, in °F, and (b) the diameter of the mixed-air duct, in ft.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and kinetic and potential energy effects are negligible. $\dot{W}_{cv} = 0$.
3. The air is modeled as an ideal gas with $c_p = 0.24 \text{ Btu/lb}$.

ANALYSIS: Mass rate balance: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$, where

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{R T_1} = \frac{(14.7 \times 144 \frac{\text{lb}_f}{\text{ft}^2})(2000 \frac{\text{ft}^3}{\text{min}})}{(1545 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot \text{R}})(530^\circ\text{R})} = 144.33 \frac{\text{lb}}{\text{min}}$$

$$\dot{m}_2 = \frac{A_2 V_2}{v_2} = \frac{(\frac{\pi D_2^2}{4})(V_2)}{R T_2 / P_2} = \frac{P_2 (\frac{\pi D_2^2}{4})(V_2)}{R T_2} = \frac{(14.7 \times 144 \frac{\text{lb}_f}{\text{ft}^2})(\frac{\pi}{4}(4 \text{ ft})^2)(400 \frac{\text{ft}}{\text{min}})}{(1545 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot \text{R}})(535^\circ\text{R})} = 372.92 \frac{\text{lb}}{\text{min}}$$

(a) Energy rate balance:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 (h_1 + \frac{V_1^2}{2} + g z_1) + \dot{m}_2 (h_2 + \frac{V_2^2}{2} + g z_2) - (\dot{m}_1 + \dot{m}_2) (h_3 + \frac{V_3^2}{2} + g z_3)$$

$$\Rightarrow 0 = \dot{m}_1 [h_1 - h_3] + \dot{m}_2 [h_2 - h_3] \Rightarrow 0 = \dot{m}_1 c_p (T_1 - T_3) + \dot{m}_2 c_p (T_2 - T_3)$$

$$\Rightarrow T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} = \frac{(144.33)(530^\circ\text{R}) + (372.92)(535^\circ\text{R})}{(144.33 + 372.92)} = 539^\circ\text{R} (79^\circ\text{F}) \quad \leftarrow (a)$$

(b) $\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 517.25 \frac{\text{lb}}{\text{min}}$. Also, $\dot{m}_3 = \frac{A_3 V_3}{v_3} = \frac{A_3 V_3 P_3}{R T_3}$ and $A_3 = \frac{\pi D_3^2}{4}$. Collecting results

$$D_3 = \sqrt{\frac{4}{\pi} \frac{\dot{m}_3 R T_3}{P_3 V_3}} = \sqrt{\frac{4}{\pi} \frac{(517.25 \frac{\text{lb}}{\text{min}})(1545 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot \text{R}})(539^\circ\text{R})}{(14.7 \times 144 \frac{\text{lb}_f}{\text{ft}^2})(500 \frac{\text{ft}}{\text{min}})}} = 4.23 \text{ ft} \quad \leftarrow (b)$$

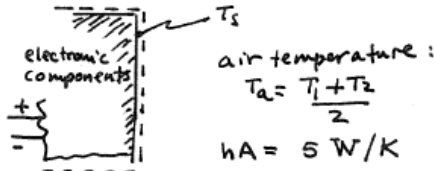
PROBLEM 4.85

KNOWN: The electronic components of Example 4.8 are cooled by air flowing through the electronics enclosure.

FIND: Determine the largest average surface temperature of the components for which specified limits are met.

SCHEMATIC & GIVEN DATA:

Also See Fig. E 4.8



ENGR. MODEL: 1. The electronic components form the system, which is at steady state.

ANALYSIS: The energy transfer from the electronic components to the air by convection is

$$\dot{Q}_c = hA[T_s - T_a]$$

- ① At steady state, the electric power provided to the electric components equals the energy removed by heat transfer: $\dot{Q}_c = 80 \text{ W}$. Also, $hA = 5 \text{ W/K}$. Solving for T_s , noting that $T_1 = 293 \text{ K}$ and $T_2 \leq 305 \text{ K} (22^\circ \text{C})$

$$T_s = T_a + \frac{\dot{Q}_c}{hA}$$

$$T_s = \left(\frac{T_1 + T_2}{2} \right) + \frac{\dot{Q}_c}{hA}$$

②
$$T_s \leq \left(\frac{293 + 305}{2} \right) \text{ K} + \frac{80 \text{ W}}{5 \text{ W/K}} = 315 \text{ K} (42^\circ \text{C}) \quad \leftarrow$$

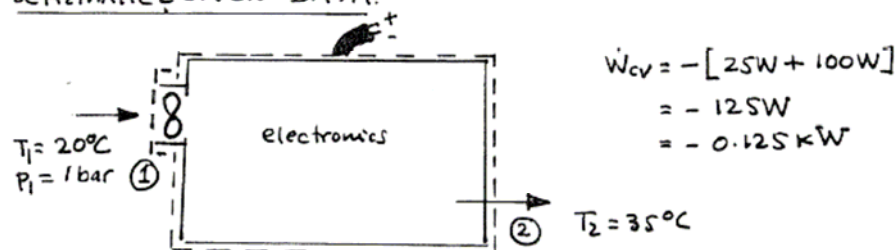
1. This can be obtained formally by applying an energy rate balance to a system consisting of the electronic components.
2. For reliability, excessive temperatures within the electronics are avoided by controlling the surface temperature T_s .

PROBLEM 4.86

KNOWN: Data are provided for an electronics enclosure cooled by an air flow induced by a fan.

FIND: Determine the volumetric flow rate of the air entering the fan.

SCHEMATIC & GIVEN DATA:



ENG. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, heat transfer with the surroundings and kinetic/potential energy effects can be ignored. 3. Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. The energy rate balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

or

$$\dot{m} = \frac{(-\dot{W}_{cv})}{h_2 - h_1}$$

with $\dot{m} = (AV)_1 / v_1$ and using the ideal gas equation of state

$$\dot{m} = \frac{(AV)_1 P_1}{RT_1}$$

Collecting results and inserting data, including enthalpy values from Table A-22

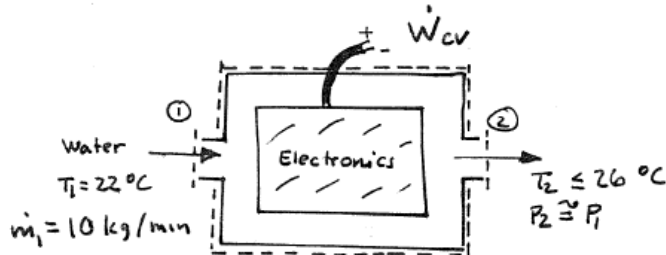
$$\begin{aligned} (AV)_1 &= \left(\frac{RT_1}{P_1} \right) \left[\frac{(-\dot{W}_{cv})}{h_2 - h_1} \right] \\ &= \frac{\left(\frac{8314 \text{ N} \cdot \text{m}}{28.97 \text{ kg} \cdot \text{K}} \right) (293 \text{ K})}{(10^5 \text{ N/m}^2)} \left[\frac{0.125 \text{ kJ/s}}{(308.2 - 293.2) \frac{\text{kJ}}{\text{kg}}} \right] \\ &= 7 \times 10^{-3} \frac{\text{m}^3}{\text{s}} \end{aligned}$$

PROBLEM 4.87

KNOWN: Data are provided for a water-jacketed housing filled with electronic components, which is at steady state.

FIND: Determine the maximum power input to satisfy a limit on the temperature of the water exiting the enclosure.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, heat transfer with the surroundings can be ignored, as can kinetic/potential energy effects. 3. For the water entering and exiting the housing $h \approx h_f(T)$.

①

ANALYSIS: At steady state, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. An energy rate balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$

giving

$$\begin{aligned} \dot{W}_{cv} &= \dot{m} (h_1 - h_2) \\ &= \dot{m} (h_f(T_1) - h_f(T_2)) \end{aligned}$$

Since $T_2 \leq 26^\circ\text{C}$

$$\begin{aligned} \dot{W}_{cv} &\geq \dot{m} (h_f(22^\circ\text{C}) - h_f(26^\circ\text{C})) \\ &\geq (10 \frac{\text{kg}}{\text{min}}) (92.33 - 109.07) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \end{aligned}$$

②

$$\dot{W}_{cv} \geq -2.79 \text{ kW} \leftarrow$$

1. Alternatively, Eq. 3.20b with c from Table A-19 can be used.

2. By the usual sign convention, $\dot{W}_{cv}$ represents power output. Hence, the constraint on power input can be expressed alternatively in terms of the magnitude of $\dot{W}_{cv}$ as

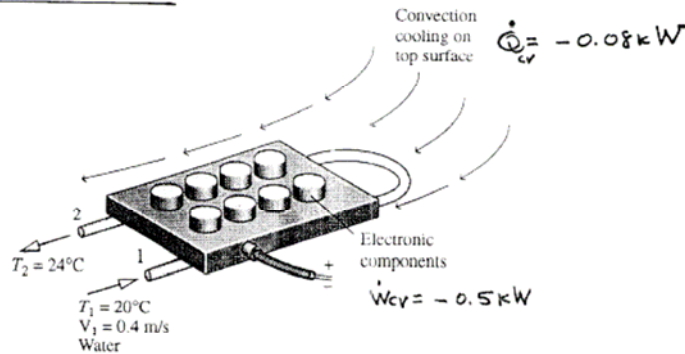
$$0 \leq |\dot{W}_{cv}| \leq 2.79 \text{ kW}$$

PROBLEM 4.88

KNOWN: Data are provided for electronic components mounted on a plate that are cooled by convection to the surroundings and water circulating through a tube bonded to the plate. Operation is at steady state.

FIND: Determine the tube diameter.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. A control volume encloses the plate-mounted electronic components with an inlet at 1 and an exit at 2. 2. The control volume is at steady state. 3. For the water entering and exiting, $h \sim h_f(T)$, $v \sim v_f(T)$. 4. Kinetic and potential energy effects can be ignored.

ANALYSIS: At steady state, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Also,

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(\pi D^2/4) V_1}{v_f(T_1)}$$

An energy rate balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$

or

$$\dot{m} = \frac{[\dot{Q}_{cv} - \dot{W}_{cv}]}{h_2 - h_1} = \frac{[\dot{Q}_{cv} - \dot{W}_{cv}]}{h_f(T_2) - h_f(T_1)}$$

Collecting results

$$D = \sqrt{\frac{4 v_f(T_1)}{\pi V_1} \left[\frac{(\dot{Q}_{cv} - \dot{W}_{cv})}{h_f(T_2) - h_f(T_1)} \right]}$$

With data from Table A-2: $v_f(20^\circ\text{C}) = (1.0018/10^3) \text{ m}^3/\text{kg}$, $h_f(T_1) = 83.96 \text{ kJ/kg}$, $h_f(T_2) = 100.7 \text{ kJ/kg}$

$$\begin{aligned} D &= \sqrt{\frac{(4)(1.0018/10^3) \text{ m}^3/\text{kg}}{\pi (0.4 \text{ m/s})} \left[\frac{(-0.08 - (-0.5)) \text{ kJ/s}}{(100.7 - 83.96) \text{ kJ/kg}} \right]} \\ &= 0.0089 \text{ m} \left| \frac{10^2 \text{ cm}}{\text{m}} \right| \\ &= 0.89 \text{ cm} \end{aligned}$$

1. Alternatively, the incompressible model can be used, with Eq 3.20b and c from Table A-19.

PROBLEM 4.89

KNOWN: Ammonia expands through a valve from a known pressure and temperature to a given final pressure.

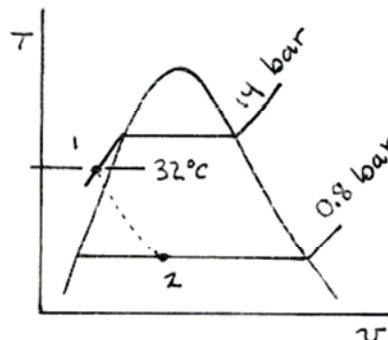
FIND: Determine the exit quality.

SCHEMATIC & GIVEN DATA:

$$P_1 = 1.4 \text{ MPa} \\ = 14 \text{ bar} \\ T_1 = 32^\circ\text{C}$$



$$P_2 = 0.08 \text{ MPa} \\ = 0.8 \text{ bar} \\ x_2 = ?$$



ENGR. MODEL: (1) A control volume enclosing the valve is at steady state. (2) The refrigerant undergoes a throttling process; $h_1 = h_2$.

ANALYSIS: According to data from Table A-14, at $P_1 = 14 \text{ bar}$; $T_{\text{sat}} = 36.26^\circ\text{C}$. Since $T_1 < T_{\text{sat}}$, state 1 is in the compressed liquid region. For simplicity,

① we use Eq. 3.14 to evaluate h_1 , as follows:

$$h_1 \approx h_f(T_1) = 332.17 \text{ kJ/kg}$$

where the value is obtained from Table A-13.

By assumption (2)

$$h_1 = h_2 \\ = h_{f2} + x_2 h_{fg2}$$

Solving and inserting data from Table A-14 at $P_2 = 0.8 \text{ bar}$

$$x_2 = \frac{h_1 - h_{f2}}{h_{fg2}} \\ = \frac{332.17 \text{ kJ/kg} - 9.04 \text{ kJ/kg}}{1382.73 \text{ kJ/kg}} \\ = 0.2337 \text{ (23.37\%)} \quad x_2$$

1. Here, we have ignored the effect of pressure on the specific enthalpy of liquid refrigerant. We could have used Eq. 3.13 to estimate the effect of pressure. In that case, h_1 would have been 332.44 kJ/kg , and the exit quality would have been $x_2 = 0.2339$ (23.39%). Thus, using Eq. 3.14 is very accurate in this case, and the approximation of Eq. 3.14 is commonly used when refrigeration systems are analyzed.

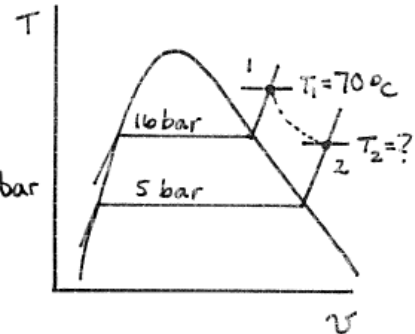
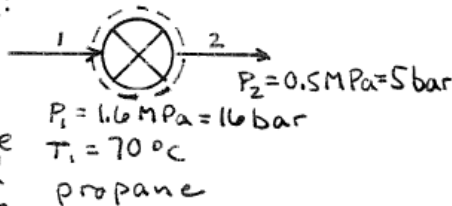
PROBLEM 4.90

KNOWN: Propane expands through a valve from a known inlet state to a known exit pressure.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The control volume is at steady state.
(2) Heat transfer is negligible and $W_{cv} = 0$. (3) kinetic and potential energy effects are negligible. (throttling process)



ANALYSIS: In accordance with the assumptions for a throttling process, $h_1 = h_2$. Using data from Table A-18

$$h_2 = h_1 = 568.5 \text{ kJ/kg}$$

Interpolating in Table A-18 at $p_2 = 5 \text{ bar}$, $h_2 = 568.5 \text{ kJ/kg}$

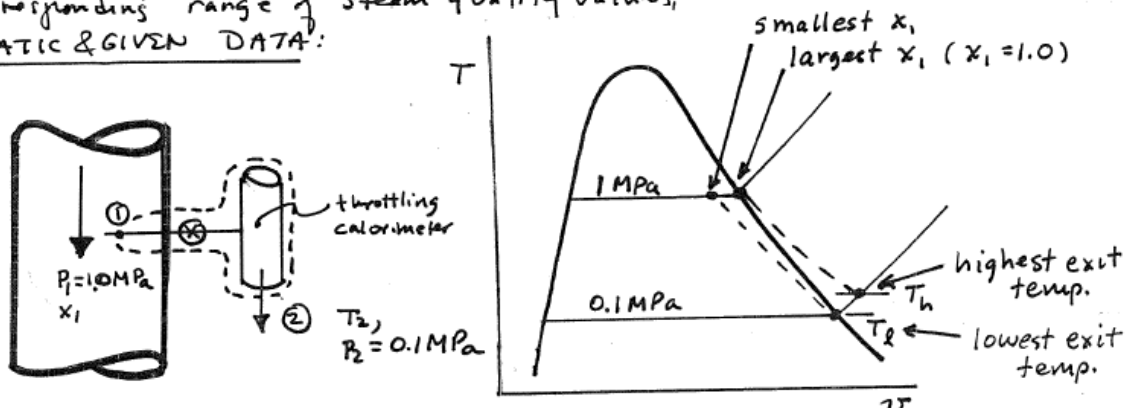
$$T_2 \approx 54.7 \text{ } ^\circ\text{C} \longleftarrow T_2$$

PROBLEM 4.91

KNOWN: Data are provided for a throttling calorimeter attached to a large pipe carrying a two-phase liquid-vapor mixture. Operation is at steady state. The substance is H_2O .

FIND: Determine the range of throttling calorimeter exit temperatures for which the device can determine the quality of steam in the pipe, and the corresponding range of steam quality values.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the figure is at steady state. 2. The expansion through the calorimeter adheres to the throttling process model: $h_2 \approx h_1$, where $h_1 = h_{f1} + x_1(h_{g1} - h_{f1})$.

ANALYSIS: With assumption 2

$$h(T_2, P_2) = h_{f1} + x_1(h_{g1} - h_{f1}) \quad (1)$$

where the state at the exit is fixed by T_2, P_2 , and thus must be superheated vapor or, in the limit, saturated vapor. Also, the quality x_1 is required to be less than or, in the limit, equal to 1.0.

Referring to the $T-v$ diagram, the highest exit temperature, T_h , corresponds to a steam quality of 1.0. That is, with data from Table A-3, Eq. (1) gives

$$\begin{aligned} h(T_h, P_2) &= h_g(P_1) \\ &= 2778.1 \text{ kJ/kg} \end{aligned}$$

Interpolating in Table A-4 at 0.1 MPa gives $T_h = 151^\circ\text{C}$.

The lowest exit temperature, T_l , corresponds to saturated vapor exiting the calorimeter, for which $h_2 = 2675.5 \text{ kJ/kg}$ and $T_l = 99.6^\circ\text{C}$. Eq. (1) gives

$$x_1 = \frac{h_2 - h_{f1}}{h_{g1} - h_{f1}} = \frac{2675.5 - 762.81}{2015.3} = 0.949$$

In summary

$$\begin{aligned} 99.6 &\leq T_2 \leq 151^\circ\text{C} \\ 0.949 &\leq x_1 \leq 1.0 \end{aligned}$$

PROBLEM 4.92

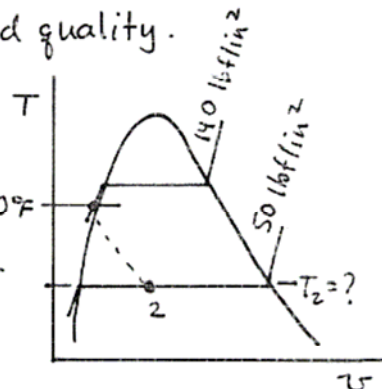
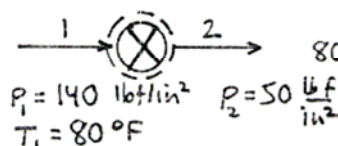
KNOWN: Refrigerant 134a expands through a valve from a known inlet state to a known final pressure.

FIND: Determine the exit temperature and quality.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The control volume is at steady state. (2) Heat transfer is negligible, and $W_{cv} = 0$. (3) Kinetic and potential energy effects are negligible. (throttling process)

R-134a



ANALYSIS: In accordance with the assumptions for a throttling process, $h_1 = h_2$. From Table A-11E; $T_1 < T_{sat} @ 140 \text{ lbf/in}^2$. Thus, the inlet state is in the compressed liquid region. Ignoring the effect of pressure on the enthalpy of the liquid, Table A-10E gives

$$h_1 \approx h_f(80^\circ\text{F}) = 37.27 \text{ Btu/lb}$$

Since $h_2 = h_1 = h_{f2} + x_2 h_{fg2}$, data from Table A-11E at 50 lbf/in² give

$$\begin{aligned} x_2 &= \frac{h_2 - h_{f2}}{h_{fg2}} \\ &= \frac{37.27 - 24.14}{83.29} \end{aligned}$$

$$= 0.158 \quad (15.8\%) \quad \leftarrow x_2$$

and

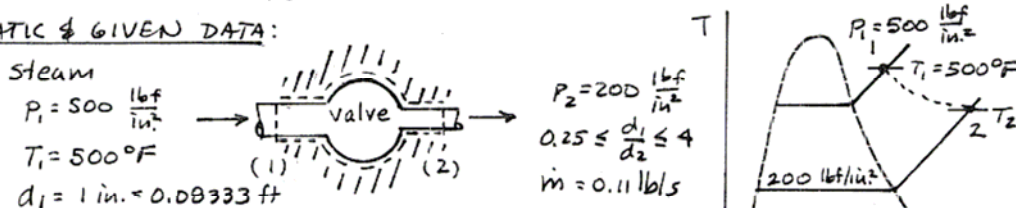
$$T_2 = 40.27^\circ\text{F} \quad \leftarrow T_2$$

PROBLEM 4.93

KNOWN: Steam flows through a well-insulated valve from specified inlet conditions to a known exit pressure.

FIND: (a) Determine the exit velocity and exit temperature for a given ratio of inlet-to-exit pipe diameters, d_1/d_2 . (b) Plot the exit velocity, temperature, and specific enthalpy versus d_1/d_2 ranging from 0.25 to 4.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible.

ANALYSIS: (a) To determine the exit velocity, begin with the mass balance and Eq. 4.4b: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, and

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \Rightarrow V_2 = \frac{A_1}{A_2} \cdot \frac{V_1}{v_1} \cdot v_2 = \left(\frac{d_1}{d_2}\right)^2 \cdot \frac{V_1}{v_1} \cdot v_2$$

Now, $V_1/v_1 = \dot{m}/A_1 = 4\dot{m}/\pi d_1^2$. Inserting values

$$\frac{V_1}{v_1} = \frac{(4)(0.11 \text{ lb/s})}{\pi (0.08333 \text{ ft})^2} = 20.17$$

Thus

$$V_2 = 20.17 \left(\frac{d_1}{d_2}\right)^2 \cdot v(T_2, P_2) \quad (1)$$

and, with $v_1 = 0.992 \text{ ft}^3/\text{lb}$ from Table A-4E

$$V_1 = (20.17)(0.992) = 20.01 \text{ ft/s}$$

From (1) we see that it is necessary to fix state 2 to evaluate V_2 . Another relation is obtained from the energy balance at steady state.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] \quad (2)$$

or

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

with h_1 from Table A-4E

$$V_2 = \sqrt{2 \left[1231.5 - h(T_2, P_2) \right] \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| + (20.01 \text{ ft/s})^2} \quad (3)$$

Equations (1) and (3) can be solved simultaneously using data from Table A-4E and an iterative process. The results are, for $d_1/d_2 = 0.25$

$$T_2 = 434.6^\circ\text{F}$$

$$V_2 = 3.140 \text{ ft/s}$$

Continued on next slide

(b) The following IT code can be used to solve Eqs. (1) and (2) with the associated data for steam:

```
p1 = 500 // lbf/in.2
T1 = 500 // °F
mdot = 0.11 // lb/s
d1 = 1/12 // ft.
p2 = 200 // lbf/in.2
dratio = d1 / d2
dratio = .25
```

```
A1 = pi * d1^2 / 4
mdot = A1 * V1 / v1
V2 / V1 = (d1 / d2)^2 * (v2 / v1)
```

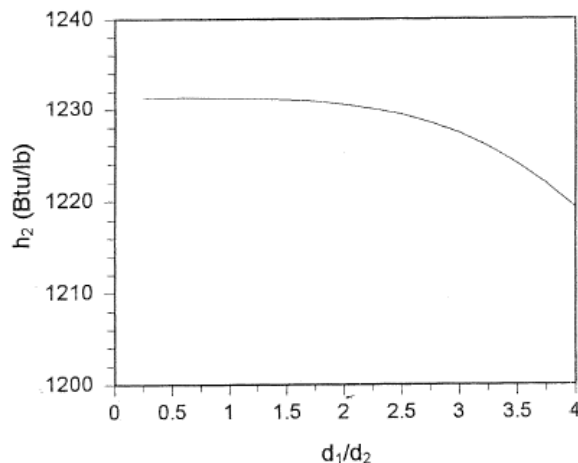
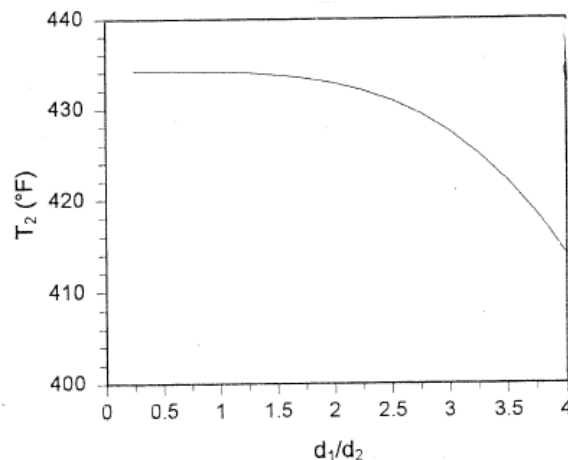
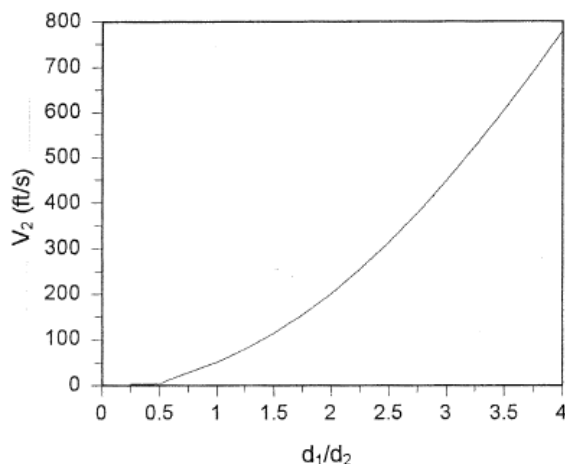
```
h1 = h_PT("Water/Steam", p1, T1)
v1 = v_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
v2 = v_PT("Water/Steam", p2, T2)
```

```
0 = (h1 - h2) + ((V1^2 - V2^2) / (2 * 32.2 * 778))
```

IT Results for the sample
case of $d_1/d_2 = 0.25$

```
T2 = 434.2 °F
V2 = 3.139 ft/s
h2 = 1231 Btu/lb
V1 = 20.01 ft/s
v2 = 2.490 ft3/lb
```

Using the Explore button, sweep dratio from 0.25 to 4 in steps of 0.25. Then, construct the following plots:



COMMENT:

Note that as d_1/d_2 becomes large, the throttling process assumptions (Sec. 4.10) of negligible kinetic energy and $h_2 \approx h_1$ are not valid.

PROBLEM 4.94

KNOWN: Steady-state operating data are provided for a turbine powering a compressor and electric generator.

FIND: For the turbine determine (a) the volumetric flow rate of the air at the exit and (b) the heat transfer rate.

SCHEMATIC & GIVEN DATA:

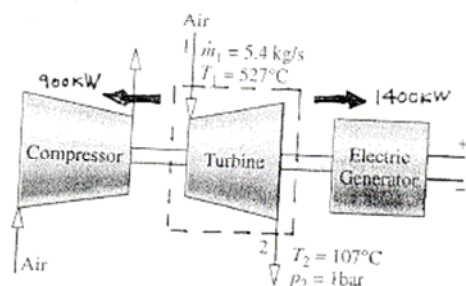


Fig. P4.94

ENGINEERING MODEL:

1. As shown in the sketch, a control volume encloses the turbine.
2. The control volume is at steady state.
3. The air can be modeled as an ideal gas, and kinetic and potential energy changes are negligible.

ANALYSIS: (a) At steady state, $\dot{m}_1 = \dot{m}_2$. Then, with $\dot{m}_2 = (\dot{A}V)_2 / v_2$,

$$(\dot{A}V)_2 = \dot{m}_2 v_2 = \dot{m}_2 \left[\frac{RT_2}{P_2} \right] = \left(5.4 \frac{\text{kg}}{\text{s}} \right) \left[\frac{\left(\frac{8314 \text{ N} \cdot \text{m}}{\text{kmol} \cdot \text{K}} \right) (380 \text{ K})}{10^5 \text{ N/m}^2} \right] = 5.89 \frac{\text{m}^3}{\text{s}} \leftarrow$$

(b) An energy rate balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + g(\cancel{z_1 - z_2}) \right]$$

where $\dot{W}_{cv} = 900 \text{ kW} + 1400 \text{ kW} = 2300 \text{ kW}$. With h_1 and h_2 from Table A-22,

$$\begin{aligned} \dot{Q}_{cv} &= \dot{W}_{cv} + \dot{m}(h_2 - h_1) \\ &= 2300 \text{ kW} + 5.4 \frac{\text{kg}}{\text{s}} \left(380.77 - 821.95 \right) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= -82 \text{ kW} \end{aligned} \leftarrow$$

PROBLEM 4.95

KNOWN: Steady-state operating data are provided for a throttling valve in series with a heat exchanger.

FIND: Determine the pressure at the valve exit and the mass flow rate of the liquid stream.

SCHEMATIC & GIVEN DATA:

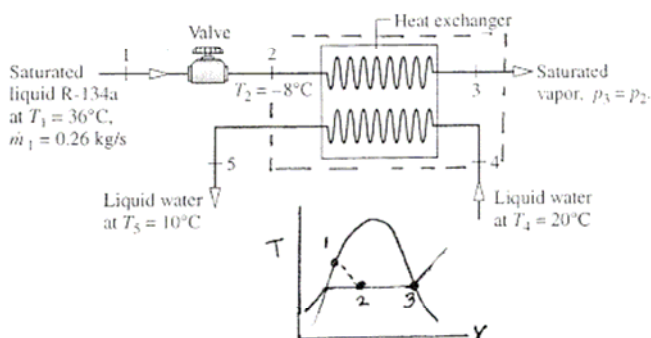


Fig. P4.95

ENGR. MODEL

1. As shown in the sketch, a control volume encloses the heat exchanger.
2. The control volume is at steady state.
3. For the control volume, stray heat transfer and kinetic and potential energy effects can be ignored.
4. The expansion through the valve is a throttling process: $h_2 \approx h_1$, $p_3 \approx p_2$ and $p_5 \approx p_4$.

ANALYSIS:

- (a) Since the expansion through the valve is a throttling process, $h_2 \approx h_1$. From Table A-10, $h_1 = 100.25 \text{ kJ/kg}$. Then, inspection of Table A-10 at -8°C with $h = 100.25 \text{ kJ/kg}$ shows state 2 is in the two-phase region. The pressure is the saturation pressure at -8°C : $p_2 = 2.1704 \text{ bar} = 217.04 \text{ kPa}$. $\leftarrow p_2$

- (b) Mass and energy rate balances for the control volume read

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_2 [h_2 - h_3] + \dot{m}_4 [h_4 - h_5]$$

$$\Rightarrow \dot{m}_4 = \frac{\dot{m}_2 [h_3 - h_2]}{[h_4 - h_5]}$$

$\dot{m}_2 = 0.26 \text{ kg/s}$

Since $p_3 = p_2$, $h_3 = h_g(-8^\circ\text{C}) = 242.54 \text{ kJ/kg}$. (Table A-10). For liquid water, $h_4 \approx h_f(T_4)$, $h_5 \approx h_f(T_5)$. Then, with data from Table A-2, $h_4 = 83.96 \text{ kJ/kg}$ and $h_5 = 42.01 \text{ kJ/kg}$.

$$\therefore \dot{m}_4 = (0.26 \text{ kg/s}) \left[\frac{242.54 - 100.25}{83.96 - 42.01} \right]$$

$$= 0.88 \text{ kg/s}$$

$\leftarrow \dot{m}_4$

PROBLEM 4.96

KNOWN: A steam turbine at steady state is operated at part load by throttling the steam to lower pressure before it enters the turbine.

FIND: For the turbine, determine the inlet temperature and the power developed per lb of steam flowing.

SCHEMATIC & GIVEN DATA:

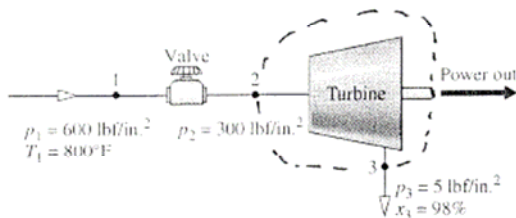


Fig. P4.96

ENGR. MODEL:

1. The control volume enclosing the turbine shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ and all kinetic and potential energy effects can be ignored.
3. The expansion across the valve is a throttling process: $h_2 = h_1$.

ANALYSIS: (a) Since $h_2 = h_1$, state 2 is fixed by $p_2 = 300 \text{ lbf/in.}^2$ and $h_2 = h_1 = 1407.6 \text{ Btu/lb}$ (from Table A-4E). Interpolating in Table A-4E then gives $T_2 = 774.6^\circ\text{F}$.

(b) Energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m} \left(h_2 - h_3 + \frac{\cancel{V_2^2 - V_3^2}}{2} + g(\cancel{z_2 - z_3}) \right)$$

$$\Rightarrow \dot{W}_{cv}/\dot{m} = h_2 - h_3$$

With data from Table A-3E,

$$h_3 = h_f + x_3(h_g - h_f) = 130.17 + 0.98(1000.9) = 1111.05 \text{ Btu/lb}$$

Finally

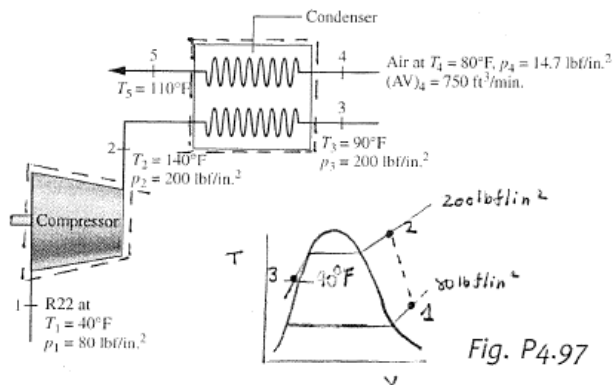
$$\begin{aligned} \dot{W}_{cv}/\dot{m} &= (1407.6 - 1111.05) \text{ Btu/lb} \\ &= 296.6 \text{ Btu/lb} \end{aligned}$$

PROBLEM 4.97

KNOWN: Steady-state operating data are provided for a compressor in series with a condenser.

FIND: Determine the mass flow rate of the Refrigerant 22 and the compressor power.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. As shown in the sketch, two control volumes are under consideration.
2. Each control volume operates at steady state.
3. For each control volume stray heat transfer and kinetic and potential energy effects can be ignored. For the condenser, $\dot{W}_{cv} = 0$.
4. The air is modeled as an ideal gas.

ANALYSIS:

- (a) Mass and energy balances for the control volume enclosing the condenser read

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_R [h_2 - h_3] + \dot{m}_A [h_4 - h_5]$$

$$\Rightarrow \dot{m}_R = \dot{m}_A \left[\frac{h_5 - h_4}{h_2 - h_3} \right]$$

where

$$\dot{m}_A = \frac{[AV]_4}{v_4} = \frac{P_4 [AV]_4}{R T_4} = \frac{(750 \text{ ft}^3/\text{min})(14.7 \times 144 \text{ lbf/ft}^2)}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot \text{R}} \right) (540 \text{ R})} = 55.13 \frac{\text{lb}}{\text{min}}$$

$$v_4 = \frac{R T_4}{P_4}$$

Then, with $h_2 = 121.25 \text{ Btu/lb}$ from Table A-9E and $h_3 \approx h_f(90^\circ\text{F}) = 36.32 \text{ Btu/lb}$ from Table A-7E, together with $h_4 = 129.06 \text{ Btu/lb}$ and $h_5 = 136.26 \text{ Btu/lb}$ from Table A-22E, we get

$$\dot{m}_R = 55.13 \frac{\text{lb}}{\text{min}} \left[\frac{136.26 - 129.06}{121.25 - 36.32} \right] = 4.67 \frac{\text{lb}}{\text{min}} \quad \leftarrow \dot{m}_R$$

- (b) Mass and energy rate balances for the control volume enclosing the compressor reduce to

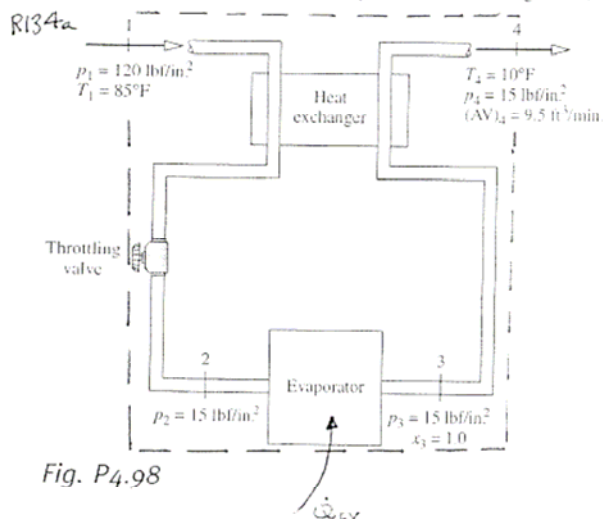
$$\dot{W}_{cv} = \dot{m}_R [h_1 - h_2]$$

$$= 4.67 \frac{\text{lb}}{\text{min}} \left(108.42 - 121.25 \right) \frac{\text{Btu}}{\text{lb}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

$$= -1.41 \text{ hp} \quad \leftarrow \dot{W}_{cv}$$

PROBLEM 4.98

4.98 Fig. P4.98 shows part of a refrigeration system consisting of a heat exchanger, an evaporator, a throttling valve,



and associated piping. Data for steady-state operation with Refrigerant 134a are given in the figure. There is no significant heat transfer to or from the heat exchanger, valve, and piping. Kinetic and potential energy effects are negligible. Determine the rate of heat transfer between the evaporator and its surroundings, in Btu/h.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. There is no significant heat transfer to or from the heat exchanger, valve, and piping.

ANALYSIS: Reducing an energy rate balance,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_4 + \frac{V_1^2 - V_4^2}{2} + g(z_1 - z_4) \right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}(h_4 - h_1) \quad (1)$$

In light of assumption 3, $\dot{Q}_{cv}$ can only account for heat transfer between the evaporator and surroundings.

The required mass flow rate is

$$\dot{m} = \frac{(AV)_4}{v_4} = \frac{9.5 \text{ ft}^3/\text{min}}{3.166 \text{ ft}^3/\text{lb}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 179.92 \frac{\text{lb}}{\text{h}}$$

where v_4 is from Table A-12E. Also from Table A-12E, $h_4 = 104.38 \text{ Btu/lb}$.

Referring to Table A-11E, at 120 lbf/in² the saturation temperature is 90.54 °F. Accordingly, at state 1 R134a is a liquid. Using Table A-10E,

$$h_1 \approx h_f(T_1) = 38.99 \text{ Btu/lb}.$$

Substituting values into Eq. (1)

$$\dot{Q}_{cv} = \left(179.92 \frac{\text{lb}}{\text{h}} \right) (104.38 - 38.99) \frac{\text{Btu}}{\text{lb}}$$

①

$$= 11,765 \text{ Btu/h}$$

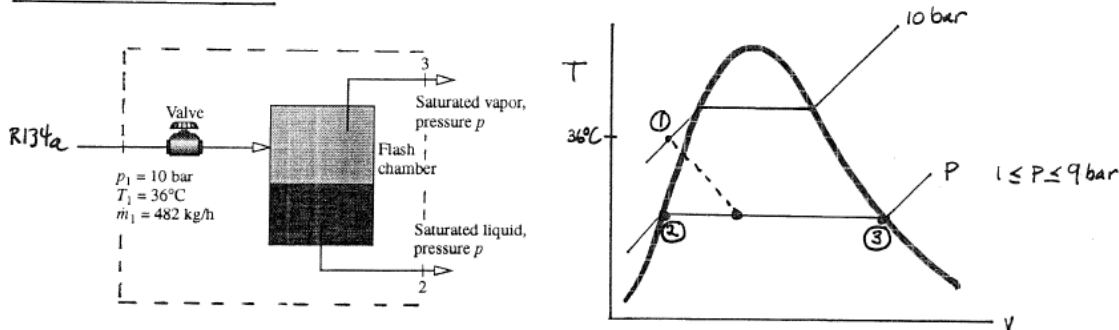
1. Refrigerant flowing through the evaporator of a refrigeration system receives energy by heat transfer from the refrigerated space. See Chap. 10 for discussion.

PROBLEM 4.99

KNOWN: Data are provided for a flash chamber operating at steady state. Saturated vapor and saturated liquid streams exit at pressure p .

FIND: If $p = 4$ bar, determine the mass flow rates of the exiting streams. Plot the mass flow rates of the exiting streams versus pressure p , $1 \leq p \leq 9$ bar.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the accompanying figure is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$, heat transfer with the surroundings is negligible, and kinetic/potential energy effects can be ignored. 3. For the liquid entering at 1, $h_1 \approx h_f(T_1)$.

ANALYSIS: For the control volume, the mass rate balance at steady state gives

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3, \text{ or } \dot{m}_3 = \dot{m}_1 - \dot{m}_2.$$

An energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left[h_1 + \frac{V_1^2}{2} + gz_1 \right] - \dot{m}_2 \left[h_2 + \frac{V_2^2}{2} + gz_2 \right] - \dot{m}_3 \left[h_3 + \frac{V_3^2}{2} + gz_3 \right]$$

$$\Rightarrow 0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \text{ or } 0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - (\dot{m}_1 - \dot{m}_2) h_3$$

$$\Rightarrow \dot{m}_2 = \dot{m}_1 \left[\frac{h_1 - h_3}{h_2 - h_3} \right]$$

(a) From Table A-10; $h_1 = h_f(36^\circ\text{C}) = 100.25 \text{ kJ/kg}$. Also, at $p = 4$ bar, Table A-11 gives; $h_2 = h_f(4 \text{ bar}) = 62 \text{ kJ/kg}$ and $h_3 = h_g(4 \text{ bar}) = 252.32 \text{ kJ/kg}$. Thus

$$\dot{m}_2 = \left(482 \frac{\text{kg}}{\text{h}} \right) \left[\frac{(100.25) - (252.32)}{(62) - (252.32)} \right] = 385.1 \frac{\text{kg}}{\text{h}} \leftarrow \dot{m}_2$$

From above

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 482 - 385.1 = 96.9 \text{ kg/h} \leftarrow \dot{m}_3$$

(b) The following IT code can be used to generate data for plots of $\dot{m}_2$ and $\dot{m}_3$ versus p :

IT Code

```
p1 = 10 // bar
T1 = 36 // °C
mdot1 = 482 // kg/h
p = 4 // bar
x2 = 0
x3 = 1
mdot1 = mdot2 + mdot3
0 = mdot1 * h1 - mdot2 * h2 - mdot3 * h3
h1 = h_PT("R134A", p1, T1)
h2 = hsat_Px("R134A", p, x2)
h3 = hsat_Px("R134A", p, x3)
```

Continued on next slide

Problem 4-99 continued

Using $p = 4$ bar as a sample case

IT Results ($p = 4$ bar)

$$\dot{m}_2 = 385.1 \text{ kg/h}$$

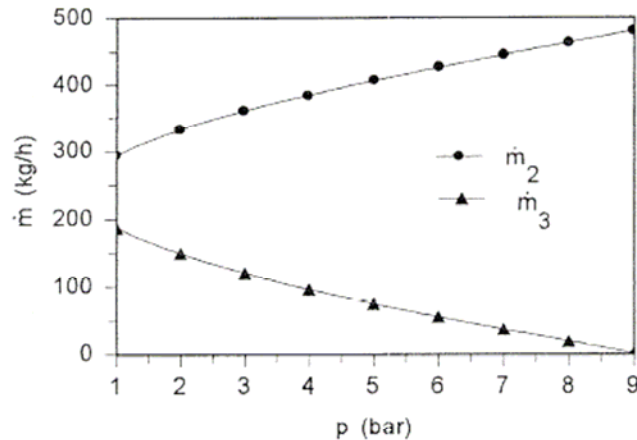
$$\dot{m}_3 = 96.87 \text{ kg/h}$$

$$h_1 = 100.3 \text{ kJ/kg}$$

$$h_2 = 62 \text{ kJ/kg}$$

$$h_3 = 252.3 \text{ kJ/kg}$$

These results compare very favorable with those of part(a). Now, using the Explore button, sweep p from 1 to 9 bar in steps of 0.1 bar. Then, the following plot can be constructed:

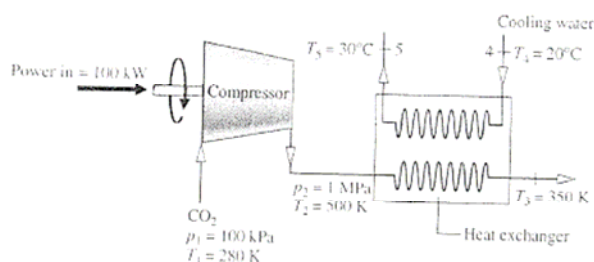


These plots imply that as the pressure in the flash chamber, p , is decreased a greater fraction of the incoming liquid flow "flashes" to saturated vapor.

PROBLEM 4.100

Carbon dioxide (CO_2) modeled as an ideal gas flows through the compressor and heat exchanger shown in Fig. P4.100. The power input to the compressor is 100 kW. A separate liquid cooling water stream flows through the heat exchanger. All data are for operation at steady state. Stray heat transfer with the surroundings can be neglected, as can all kinetic and potential energy changes. Determine (a) the mass flow rate of the CO_2 , in kg/s, and (b) the mass flow rate of the cooling water, in kg/s.

SCHEMATIC, GIVEN DATA:



ANALYSIS:

- (a) An energy rate balance for the compressor reduces to read

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \dot{m}_1 = -\frac{\dot{W}_{cv}}{h_2 - h_1} \quad (1)$$

With enthalpy data on a molar basis from Table A-23

$$h_2 - h_1 = \frac{\bar{h}_2 - \bar{h}_1}{M} = \left(\frac{17,678 - 8697}{44.01} \right) \frac{\text{kJ}}{\text{kg}} = 204.1 \text{ kJ/kg}$$

Then, Eq. (1) gives

$$\dot{m}_1 = \frac{-(-100 \text{ kW})}{204.1 \text{ kJ/kg}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 0.49 \text{ kg/s} \quad \leftarrow (a)$$

- (b) An energy rate balance for the heat exchanger reduces to read

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_2 [h_2 - h_3] + \dot{m}_4 [h_4 - h_5]$$

($\dot{m}_2 = \dot{m}_1$)

$$\Rightarrow \dot{m}_4 = \left(\frac{h_2 - h_3}{h_5 - h_4} \right) \dot{m}_1 \quad (2)$$

where $h_4 \approx h_f(T_4) = 83.96 \text{ kJ/kg}$, $h_5 \approx h_f(T_5) = 125.79 \text{ kJ/kg}$ from Table A-2. Also, from Table A-23

$$h_2 - h_3 = \frac{\bar{h}_2 - \bar{h}_3}{M} = \frac{17,678 - 11,351}{44.01} = 143.8 \text{ kJ/kg}$$

Then Eq. (2) gives

$$\dot{m}_4 = \left(\frac{143.8 \text{ kJ/kg}}{41.83 \text{ kJ/kg}} \right) (0.49 \text{ kg/s}) = 1.68 \text{ kg/s} \quad \leftarrow (b)$$

ENGR. MODEL

1. Control volumes at steady state enclose the compressor and heat exchanger.
2. Stray heat transfer and kinetic and potential energy effects are neglected. For the heat exchanger, $\dot{W}_{cv} = 0$.
3. CO_2 is modeled as an ideal gas.

PROBLEM 4.101

KNOWN: Steady-state operating data are provided for a waste heat recovery-steam generator-turbine system.

FIND: On the basis of an appropriate analysis, indicate whether or not, the system should be implemented.

SCHEMATIC & GIVEN DATA:

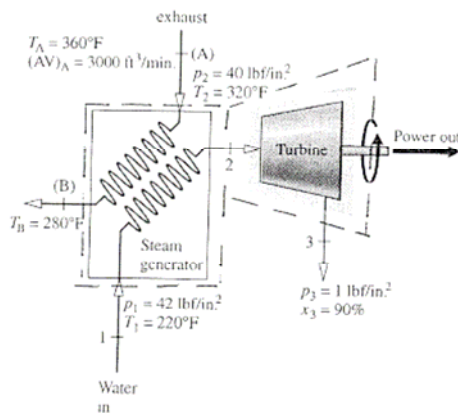


Fig. P4.101

ENGR. MODEL: (1) Both control volumes are at steady state. (2) Heat transfer is negligible between each control volume and its surroundings, and $\dot{W}_{cv} = 0$ for the heat exchanger. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas, at a pressure of 14.7 lbf/in². (5) Power is evaluated at 8 cents per kW·h.

ANALYSIS: The turbine power can be determined from an energy balance once the steam flow rate is determined. Begin with a steady-state energy balance on the steam generator

$$0 = \dot{m}_A - \dot{m}_B \Rightarrow \dot{m}_A = \dot{m}_B \equiv \dot{m}_{gas}$$

$$0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{st}$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_{gas} [(h_A - h_B) + \frac{V_A^2 - V_B^2}{2} + g(z_A - z_B)] + \dot{m}_{st} [(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)]$$

$$\dot{m}_{st} = \dot{m}_{gas} \frac{(h_A - h_B)}{(h_2 - h_1)}$$

The gas flow rate is

$$\dot{m}_{gas} = \frac{(AV)_A}{v_A} = \frac{p_A (AV)_A}{RT_A}$$

$$= \frac{(14.7 \text{ lbf/in}^2)(3000 \text{ ft}^3/\text{min})}{(154.5 \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}})(820^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 14.52 \text{ lb/min}$$

From Table A-22E; $h_A = 196.69 \text{ Btu/lb}$ and $h_B = 177.23 \text{ Btu/lb}$. For the water, from Table A-2E; $p_1 > p_{sat@220^\circ\text{F}} \Rightarrow$ compressed liquid. Thus, $h_1 \approx h_f@220^\circ\text{F} = 188.2 \text{ Btu/lb}$. At 2, using Table A-4E; $h_2 = 1196.8 \text{ Btu/lb}$. The flow rate is

$$\dot{m}_{st} = (14.52 \text{ lb/min}) \frac{(196.69 - 177.23)}{(1196.8 - 188.2)} = 2.8 \text{ lb/min}$$

Continued on next slide

Problem 4-101 continued

Turning now to the control volume enclosing the turbine

$$0 = \dot{Q}_{cv} - \dot{W}_t + \dot{m}_s \left[(h_2 - h_3) + \frac{\cancel{V_2^2} - \cancel{V_3^2}}{2} + g(\cancel{z_2} - \cancel{z_3}) \right]$$

$$\dot{W}_t = \dot{m}_s (h_2 - h_3)$$

Using data from Table A-3E

$$h_3 = h_f + x_3 (h_{fg}) = 69.7 + 0.9 (1036) = 1002.1 \text{ Btu/lb}$$

Thus

$$\dot{W}_t = \left(2.8 \frac{\text{lb}}{\text{min}} \right) (1196.8 - 1002.1) \frac{\text{Btu}}{\text{lb}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

$$= 12.85 \text{ hp}$$

Evaluating power at 8 cents per kW·h, the value for 24 × 365 = 8760 hours of operation annually is

$$\dot{\$} = (12.85 \text{ hp}) \left| \frac{0.7457 \text{ kW}}{1 \text{ hp}} \right| \left(\frac{8760 \text{ h}}{\text{year}} \right) \left(\frac{\$ 0.08}{\text{kW} \cdot \text{h}} \right)$$

$$= \$ 6715/\text{year}$$

It is unlikely that this value would cover the capital and operating costs associated with the overall system. Accordingly, even though the oven exhaust has a thermodynamic potential for use, the potential is not likely to be exploited by the proposed means owing to economic considerations.

PROBLEM 4.102

KNOWN: Steady-state operating data are provided for a simple steam power plant.

FIND: Determine the thermal efficiency and the mass flow rate of the condenser cooling water.

SCHEMATIC & GIVEN DATA:

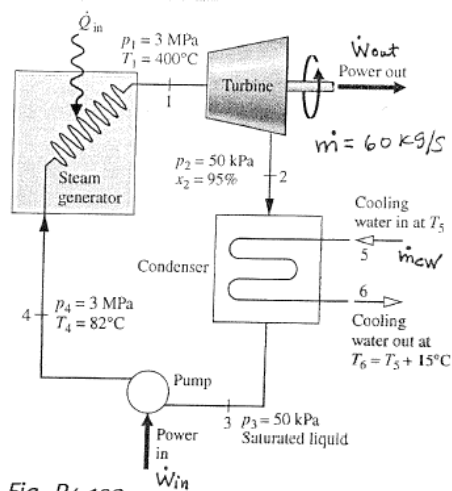


Fig. P4.102

ENGR. MODEL:

1. Control volumes at steady state enclose each of the four components.
2. For each control volume, stray heat transfer and kinetic and potential energy effects are negligible.
3. Energy transfers are in the direction of the arrows.
4. Model the cooling water as incompressible with $(p_5 - p_6) \approx 0$.

ANALYSIS: (a) For any power cycle, the thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}}$$

where $\dot{W}_{\text{cycle}} = \dot{W}_{\text{out}} - \dot{W}_{\text{in}}$. In this case, mass and energy rate balances for control volumes enclosing the turbine and pump reduce to read

$$\dot{W}_{\text{out}} = \dot{m} [h_1 - h_2], \quad \dot{W}_{\text{in}} = \dot{m} (h_4 - h_3)$$

where $h_1 = 3230.9 \frac{\text{kJ}}{\text{kg}}$ (from Table A-4), $h_2 = h_f + x_2(h_g - h_f) = 340.49 + 0.95(2305.4) = 2530.6 \text{ kJ/kg}$ and $h_3 = 340.9 \text{ kJ/kg}$ (Data from Table A-3), $h_4 = 345.66 \text{ kJ/kg}$ (Table A-5).

With these values,

$$\dot{W}_{\text{out}} = \dot{m} (h_1 - h_2) = 60 \text{ kg/s} (3230.9 - 2530.6) \frac{\text{kJ}}{\text{kg}} = 42018 \text{ kJ/s}$$

$$\dot{W}_{\text{in}} = \dot{m} (h_4 - h_3) = 60 \text{ kg/s} (345.66 - 340.9) \frac{\text{kJ}}{\text{kg}} = 310 \text{ kJ/s}$$

Mass and energy rate balances for a control volume enclosing the steam generator reduce to give $\dot{Q}_{\text{in}} = \dot{m} (h_1 - h_4) = 60 \text{ kg/s} (3230.9 - 345.66) \frac{\text{kJ}}{\text{kg}} = 173,114 \text{ kJ/s}$. The thermal efficiency is then

$$\eta = \frac{42,018 - 310}{173,114} = 0.241 \text{ (24.1\%)} \quad \leftarrow \eta$$

(b) Mass and energy rate balances for a control volume enclosing the condenser reduce to give

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m} [h_2 - h_3] + \dot{m}_{\text{cw}} [h_5 - h_6]$$

$\Rightarrow$

$$\dot{m}_{\text{cw}} = \frac{\dot{m} [h_2 - h_3]}{[h_6 - h_5]}$$

where, with assumption 4 and Eq. 3.20b, $(h_6 - h_5) \approx c(T_6 - T_5) + v(P_6 - P_5)$. Using $c = 4.2 \text{ kJ/kg} \cdot \text{K}$ from Table A-19, we get

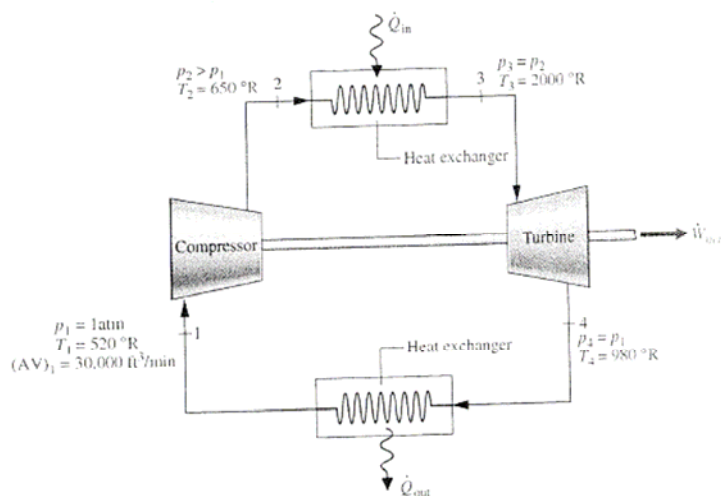
$$\dot{m}_{\text{cw}} = 60 \frac{\text{kg}}{\text{s}} \left[\frac{2530.6 - 340.49}{(4.2)(15)} \right]$$

$$= 2086 \text{ kg/s} \quad \leftarrow \dot{m}_{\text{cw}}$$

PROBLEM 4.103

A simple gas turbine power cycle operating at steady state with air as the working substance is shown in Fig. P4.103. The cycle components include an air compressor mounted on the same shaft as the turbine. The air is heated in the high-pressure heat exchanger before entering the turbine. The air exiting the turbine is cooled in the low-pressure heat exchanger before returning to the compressor. Kinetic and potential effects are negligible. The compressor and turbine are adiabatic. Using the ideal gas model for air, determine the (a) power required for the compressor, in hp, (b) power output of turbine, in hp, and (c) thermal efficiency of the cycle.

SCHEMATIC & GIVEN DATA:



ENGINEER MODEL:

1. Control volumes at steady state enclose the compressor, turbine and high-pressure heat exchanger.
2. For the compressor and turbine, $\dot{Q}_{cv} = 0$. For the heat exchanger, $\dot{W}_{cv} = 0$.
3. Kinetic and potential energy effects are negligible.
4. The air is modeled as an ideal gas.

ANALYSIS: Find the mass flow rate as follows:

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1} = \frac{(1.47 \times 10^4 \frac{\text{lbf}}{\text{in}^2})(30,000 \text{ ft}^3/\text{min})}{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot \text{R}})(520 \text{ °R})} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 137,394 \frac{\text{lb}}{\text{h}}$$

(a) An energy rate balance for the compressor reduces to read

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (137,394 \frac{\text{lb}}{\text{h}})(124.27 - 155.51) \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = -1687 \text{ hp} \quad \leftarrow (a)$$

where specific enthalpies are from Table A-22E.

(b) An energy rate balance for the turbine reduces to read

$$\dot{W}_t = \dot{m}(h_3 - h_4) = (137,394 \frac{\text{lb}}{\text{h}})(509.71 - 236.02) \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 14,505 \text{ hp} \quad \leftarrow (b)$$

$$(c) \quad \eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_t - |\dot{W}_c|}{\dot{Q}_{in}}$$

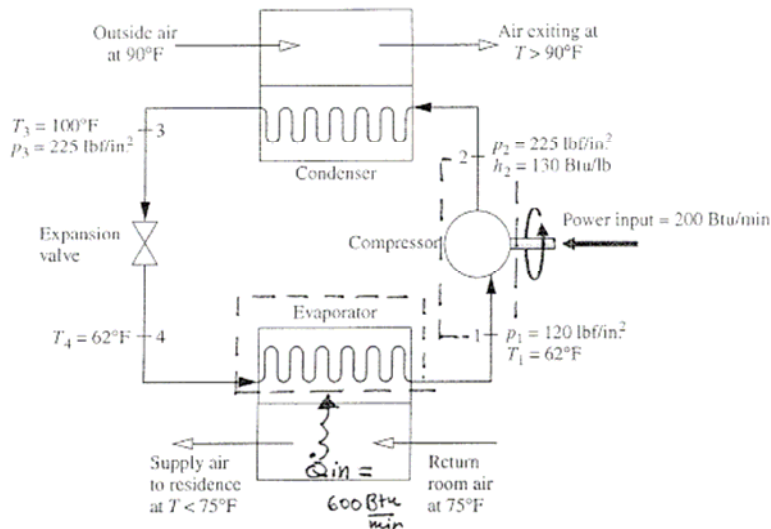
$$= \frac{\dot{m}(h_3 - h_4) - \dot{m}(h_2 - h_1)}{\dot{m}(h_3 - h_2)}$$

$$= \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = \frac{268.69 - 31.24}{349.20} = 0.68 \text{ (68\%)} \quad \leftarrow (c)$$

PROBLEM 4.104

A residential air conditioning system operates at steady state, as shown in Fig. P4.104. Refrigerant 22 circulates through the components of the system. Property data at key locations are given on the figure. If the evaporator removes energy by heat transfer from the room air at a rate of 600 Btu/min, determine (a) the rate of heat transfer between the compressor and the surroundings, in Btu/min, and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. Control volumes at steady state enclose the compressor and the refrigerant side of the evaporator.
2. Kinetic and potential energy effects can be ignored. For the evaporator, $\dot{W}_{ev} = 0$.
3. The expansion through the valve is a throttling process:
 $h_4 = h_3$.
4. At state 3,
 $h_3 \approx h_f(T_3)$

ANALYSIS: (a) To find the mass flow rate of the refrigerant, write an energy rate balance for the control volume enclosing the refrigerant side of the evaporator:

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) \Rightarrow \dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} \quad (i)$$

From Table A-9E, $h_1 = 109.88 \text{ Btu/lb}$ (saturated vapor value). Using $h_4 = h_3$, Table A-7E gives $h_3 \approx h_f(T_3) = 39.41 \text{ Btu/lb}$. Then, Eq. (i) yields

$$\dot{m} = \frac{600 \text{ Btu/min}}{(109.88 - 39.41)} = 8.5 \frac{\text{lb}}{\text{min}}$$

An energy rate balance for the compressor reduces to give

$$\begin{aligned} \dot{Q}_{cv} &= \dot{W}_{cv} + \dot{m}(h_2 - h_1) = -200 \frac{\text{Btu}}{\text{min}} + 8.5 \frac{\text{lb}}{\text{min}} (130 - 109.88) \frac{\text{Btu}}{\text{lb}} \\ &= -29 \text{ Btu/min} \end{aligned} \quad \leftarrow (a)$$

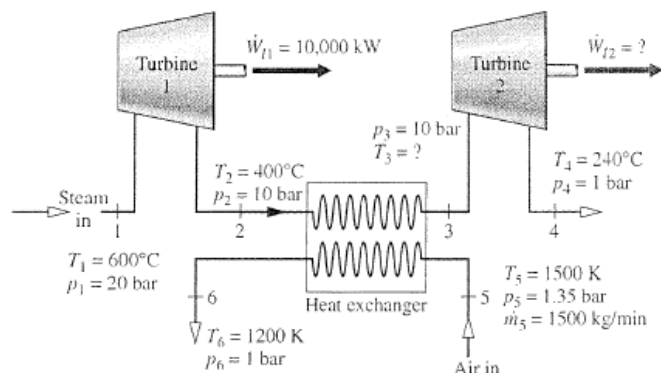
(b) The coefficient of performance is given by (see Sec. 2.6.3)

$$\beta = \frac{\dot{Q}_{in}}{|\dot{W}_{comp}|} = \frac{600 \text{ Btu/min}}{200 \text{ Btu/min}} = 3.0 \quad \leftarrow (b)$$

PROBLEM 4.105

Separate streams of steam and air flow through the turbine and heat exchanger arrangement shown in Fig. P4.105. Steady-state operating data are provided on the figure. Heat transfer with the surroundings can be neglected, as can all kinetic and potential energy effects. Determine (a) T_3 , in K, and (b) the power output of the second turbine, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. Consider a control volume at steady state enclosing each of the three components.
2. For each control volume, ignore stray heat transfer and kinetic and potential energy effects.
3. The ideal gas model applies for the air. (This can be verified from the compressibility chart.)

ANALYSIS:

- (a) To determine the steam mass flow rate write an energy rate balance for turbine 1 and use data from Table A-4:

$$\dot{W}_{t1} = \dot{m}_1 (h_1 - h_2) \Rightarrow \dot{m}_1 = \frac{\dot{W}_{t1}}{h_1 - h_2} = \frac{10,000 \text{ kW} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right|}{(3690.1 - 3263.9) \text{ kJ/kg}}$$

$$= 23.46 \text{ kg/s}$$

Next, an energy rate balance for the heat exchanger reduces to

$$0 = \dot{m}_2 [h_2 - h_3] + \dot{m}_5 [h_5 - h_6]$$

with data from Table A-22,

$$\Rightarrow h_3 = h_2 + \frac{\dot{m}_5}{\dot{m}_2} [h_5 - h_6] = 3263.9 \frac{\text{kJ}}{\text{kg}} + \frac{(1500/60) \text{ kg/s} (1685.97 - 1277.79) \frac{\text{kJ}}{\text{kg}}}{23.46 \text{ kg/s}}$$

$$= 3645.6 \frac{\text{kJ}}{\text{kg}}$$

Interpolating in Table A-4 at 10 bar gives, $T_3 = 576^\circ \text{C}$

← (a)

- (b) An energy rate balance for turbine 2 is

$$\dot{W}_{t2} = \dot{m}_3 (h_3 - h_4) = 23.46 \frac{\text{kg}}{\text{s}} (3645.6 - 2957.5) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 16,213 \text{ kW}$$

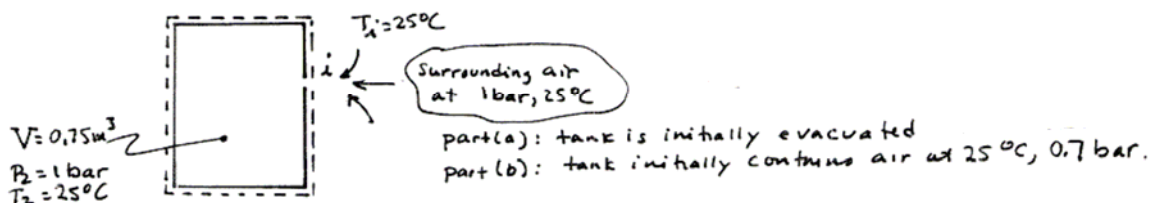
← (b)

PROBLEM 4.106

KNOWN: A pinhole leak develops in the wall of a rigid tank of known volume, and air from the surroundings enters at a known temperature and pressure. The temperature of the air within the tank remains constant at the temperature of the surroundings due to heat transfer from the tank contents.

FIND: Determine the heat transfer between the tank contents and surroundings if (a) the tank is initially evacuated, (b) the tank initially contains air at a specified condition.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. For the control volume shown in the accompanying figure, $\dot{W}_{cv} = 0$, and kinetic/potential energy effects are negligible. 2. The air is modeled as an ideal gas. 3. The condition of the air leaking into the tank at i remains constant at 1 bar, 25°C. 4. The temperature of the air within the tank is 25°C.

ANALYSIS: The mass rate balance for the control volume takes the form $dm_{cv}/dt = \dot{m}_i$. With indicated assumptions, the energy rate balance reduces to

$$\frac{d\bar{U}_{cv}}{dt} = \dot{Q}_{cv} + \dot{m}_i h_i \Rightarrow \frac{d\bar{U}_{cv}}{dt} = \dot{Q}_{cv} + h_i \frac{dm_{cv}}{dt}$$

With assumption 3, h_i remains constant. Thus, on integration

$$\Delta \bar{U}_{cv} = \dot{Q}_{cv} + \int h_i dm_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = \dot{Q}_{cv} + h_i (m_2 - m_1)$$

Introducing $h_i = u_i + (pv)_i$, and collecting like terms

$$\dot{Q}_{cv} = \underbrace{m_2(u_2 - u_i)}_{(1)} - \underbrace{m_1(u_1 - u_i)}_{(2)} - (m_2 - m_1)(pv)_i \quad (1)$$

Since the specific internal energy of an ideal gas depends on temperature only, term (1) vanishes because $T_2 = T_i$. Term (2) vanishes in part (a) because $m_1 = 0$, and in part (b) because $T_1 = T_i$. Accordingly, Eq. (1) reduces in each part to the following working equation: (2a)

$$\dot{Q}_{cv} = -(m_2 - m_1)(pv)_i$$

That is, the heat transfer from the tank removes the energy entering the tank at i by flow work (see Sec. 4.4.2). Since $pv_i = RT_i$, Eq. (2a) becomes alternatively (2b)

$$\dot{Q}_{cv} = -(m_2 - m_1)RT_i$$

(a) $m_1 = 0$. Using the ideal gas equation of state, $m_2 = P_2 V / RT_2 = P_2 V / RT_i$. So

$$\dot{Q}_{cv} = -P_2 V = -(1 \text{ bar}) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| (0.75 \text{ m}^3) \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = -75 \text{ kJ} \quad \leftarrow$$

(b) $m_1 \neq 0$. Since $m_1 = P_1 V / RT_1 = P_1 V / RT_i$, $m_2 = P_2 V / RT_i$,

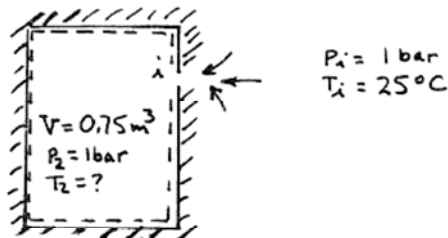
$$\begin{aligned} \dot{Q}_{cv} &= -(P_2 - P_1)V = -(1 \text{ bar} - 0.7 \text{ bar}) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| (0.75 \text{ m}^3) \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= -22.5 \text{ kJ} \quad \leftarrow \end{aligned}$$

PROBLEM 4.107

KNOWN: A pinhole develops in the wall of an initially evacuated tank, and air enters from the surroundings until the pressure is 1 bar.

FIND: Determine the final temperature of the air in the tank in the absence of heat transfer between the tank contents and surroundings.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume is shown in the accompanying figure. 2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects are negligible. 3. The air is modeled as an ideal gas. 4. The condition of the air entering the tank remain constant at 1 bar, 25°C .

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_i$. The energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i$$

Combining the mass and energy rate balance, and integrating using assumption 4

$$\Delta U_{cv} = \int_1^2 \dot{m}_i h_i dt \Rightarrow \Delta U_{cv} = h_i \int_1^2 \dot{m}_i dt \Rightarrow \Delta U_{cv} = h_i (m_2 - m_1)$$

or since the tank is initially evacuated

$$m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1) \Rightarrow u_2 = h_i \quad (1)$$

From Table A-22 at $T_1 = 298 \text{ K } (25^\circ\text{C})$, $h_i = 298.18 \text{ kJ/kg}$. Then,

① interpolating with $u_2 = 298.18 \text{ kJ/kg}$, $T_2 = 416.5 \text{ K } (143.5^\circ\text{C})$ ←

1. To interpret the temperature increase of the air within the tank during the filling process, rewrite Eq. (1) with $h_i = u_i + (pv)_i$ to read

$$u_2 = u_i + \underline{(pv)_i}$$

where the underlined term is the flow work discussed in Sec. 4.4.2. Thus, the temperature of air in the tank increases during the process because work is done by the surroundings -- flow work -- on the matter entering the tank.

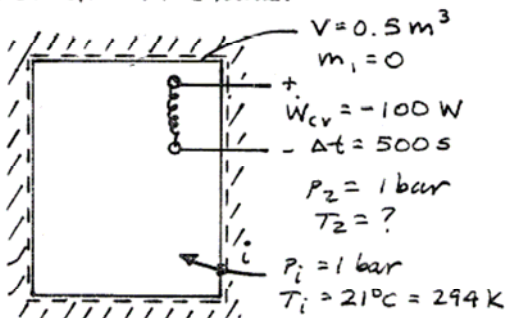
PROBLEM 4.108

KNOWN: An electric resistor transfers energy to air from the surroundings entering a rigid, well-insulated tank. The final pressure is known.

FIND: Determine the final temperature of air in the tank.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The control volume is shown in the accompanying figure. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) The condition of the air entering the tank remains constant. (5) For the resistor, $\Delta U_{\text{resistor}} = 0$.



ANALYSIS: The mass rate balance reduces to $dm_{cv}/dt = \dot{m}_i$, and the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i$$

Combining and integrating using assumption (4)

$$\begin{aligned} \Delta U_{cv} &= - \int_{t_1}^{t_2} \dot{W}_{cv} dt + \int_{t_1}^{t_2} \dot{m}_i h_i dt \\ &= - \dot{W}_{cv} \Delta t + h_i \int_{t_1}^{t_2} \dot{m}_i dt = - \dot{W}_{cv} \Delta t + h_i (m_2 - m_1) \end{aligned}$$

With $\Delta U_{cv} = m_2 u_2 - m_1 u_1$,

$$m_2 u_2 - m_1 u_1 = - \dot{W}_{cv} \Delta t + (m_2 - m_1) h_i$$

or

$$m_2 u_2 = - \dot{W}_{cv} \Delta t + m_2 h_i \Rightarrow u_2 = \frac{- \dot{W}_{cv} \Delta t}{m_2} + h_i$$

By assumption (3), $m_2 = P_2 V / RT_2$, and

$$u_2 = \left(\frac{RT_2}{P_2 V} \right) (- \dot{W}_{cv} \Delta t) + h_i \quad (1)$$

Inserting values

$$- \dot{W}_{cv} \Delta t = - (-100 \text{ W})(500 \text{ s}) \left| \frac{1 \text{ kJ/s}}{10^3 \text{ W}} \right| = 50 \text{ kJ}$$

$$\begin{aligned} \frac{RT_2}{P_2 V} &= \frac{\left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (T_2 \text{ in K})}{(1 \text{ bar})(0.5 \text{ m}^3)} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \\ &= (5.74 \times 10^{-3}) T_2 \end{aligned}$$

From Table A-22; $h_i = 294.17 \text{ kJ/kg}$. Thus

$$u_2 = (5.74 \times 10^{-3} \frac{1}{\text{kg} \cdot \text{K}}) (50 \text{ kJ}) T_2 + 294.17 \text{ kJ/kg} \quad (2)$$

Eq. (2) can be solved for T_2 using an iterative procedure with data for u_2 from Table A-22. The result is

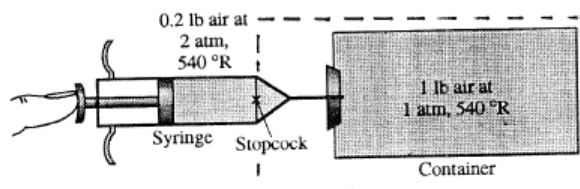
$$T_2 = 665.3 \text{ K} = 392.3^\circ \text{C}$$

PROBLEM 4.109

KNOWN: A small amount of air at a given state is injected into a container initially holding air at a different state.

FIND: Determine the final temperature and pressure of the air in the container.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The control volume is shown in the accompanying schematic. 2. The state of the air remains constant until it passes the stopcock. 3. For the control volume, $\dot{W}_{cv} = 0$, heat transfer with the surroundings can be ignored, and kinetic/potential energy effects are negligible. 4. The ideal gas model applies for the air.

ANALYSIS: The mass rate balance reads $dm_{cv}/dt = \dot{m}_i$. The energy rate balance reduces with given assumptions to

$$\textcircled{1} \quad \frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i \Rightarrow \frac{dU_{cv}}{dt} = h_i \frac{dm_{cv}}{dt}$$

With assumption 2, h_i remains constant. Thus, integration over time gives

$$\Delta U_{cv} = \int h_i dm_{cv} \Rightarrow \Delta U_{cv} = h_i \Delta m_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1)$$

where 1 and 2 denote the initial and final states within the container, respectively. That is, $m_1 = 1 \text{ lb}$ and $m_2 = 1 \text{ lb} + 0.2 \text{ lb} = 1.2 \text{ lb}$. Solving for u_2 , and inserting data from Table A-22E for u_1 and h_i

$$u_2 = \frac{m_1 u_1 + (m_2 - m_1) h_i}{m_2} = \frac{(1 \text{ lb})[92.04 \frac{\text{Btu}}{\text{lb}}] + (0.2 \text{ lb})[129.06 \frac{\text{Btu}}{\text{lb}}]}{1.2 \text{ lb}} = 98.21 \frac{\text{Btu}}{\text{lb}}$$

Interpolating in Table A-22E with u_2 gives $T_2 = 576^\circ \text{R}$ (116°F).

Using the ideal gas equation of state, $P_2 = m_2 R T_2 / V$, where $V = m_1 R T_1 / P_1$, the final pressure is

$$P_2 = \left(\frac{m_2}{m_1} \right) \left(\frac{T_2}{T_1} \right) P_1 = \left(\frac{1.2 \text{ lb}}{1 \text{ lb}} \right) \left(\frac{576^\circ \text{R}}{540^\circ \text{R}} \right) (14.7 \frac{\text{lbf}}{\text{in}^2}) = 18.8 \frac{\text{lbf}}{\text{in}^2}$$

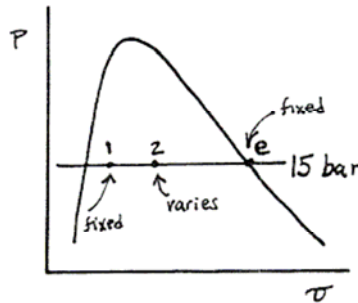
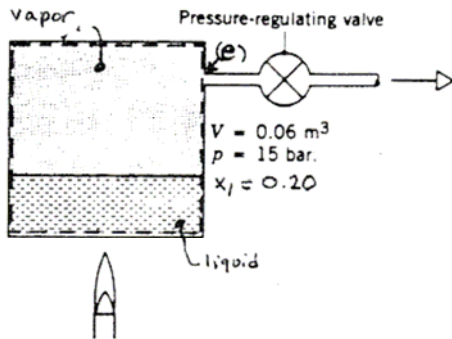
-
1. The plunger does not directly enter the analysis -- it is located in the surroundings of the control volume. However, there is flow work where air enters the control volume. This is incorporated in the $(pu)_i$ term of the specific enthalpy at the inlet, h_i . As shown by the following analysis, the final temperature of the air in the container is higher than the initial temperature. The higher temperature is due to the flow work effect.

PROBLEM 4.110

KNOWN: A rigid tank of known volume initially containing a two-phase liquid-vapor mixture at a known state is heated, allowing saturated vapor to escape while maintaining constant pressure in the tank until a specified final quality is attained.

FIND: (a) If the final quality is 0.5, determine the mass in the tank and the amount of heat transfer. (b) Plot the mass in the tank and the heat transfer versus final quality ranging from 0.2 to 1.0.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume is shown in the schematic. 2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic/potential energy effects are negligible. 3. At exit e the state remains constant, as shown in the p-v diagram.

ANALYSIS: The mass rate balance reads $dm_{cv}/dt = -\dot{m}_e$. With the indicated assumptions, the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e \Rightarrow \frac{dU_{cv}}{dt} = \dot{Q}_{cv} + h_e \frac{dm_{cv}}{dt}$$

Then, with assumption 3, integration over time results in

$$\Delta U_{cv} = \dot{Q}_{cv} + h_e \int_1^2 dm_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = \dot{Q}_{cv} + h_e [m_2 - m_1], \text{ or}$$

$$\dot{Q}_{cv} = m_2 u_2 - m_1 u_1 - h_e [m_2 - m_1] \quad (1)$$

where $h_e = 2792.2 \text{ kJ/kg}$, and

$$v_1 = v_f + x_1 (v_g - v_f)$$

$$= \left(\frac{1.1539}{10^3} \right) + 0.2 \left[0.1318 - \frac{1.1539}{10^3} \right] = 0.02728 \frac{\text{m}^3}{\text{kg}} \Rightarrow m_1 = \frac{V}{v_1} = \frac{0.06 \text{ m}^3}{0.02728 \text{ m}^3/\text{kg}} = 2.199 \text{ kg}$$

$$u_1 = u_f + x_1 (u_g - u_f) = 843.16 + 0.2 [2594.5 - 843.16] = 1193.4 \text{ kJ/kg}$$

With the same values for v_f, v_g, u_f and u_g , $v_2 = v_f + x_2 (v_g - v_f)$ and $u_2 = u_f + x_2 (u_g - u_f)$. Also, $m_2 = V/v_2$.

(a) $x = 0.5$ Then,

$$v_2 = \left(\frac{1.1539}{10^3} \right) + 0.5 \left[0.1318 - \frac{1.1539}{10^3} \right] = 0.06648 \frac{\text{m}^3}{\text{kg}} \Rightarrow m_2 = \frac{0.06 \text{ m}^3}{0.06648 \text{ m}^3/\text{kg}} = 0.903 \text{ kg} \leftarrow m_2$$

$$u_2 = 843.16 - 0.5 [2594.5 - 843.16] = 1718.8 \text{ kJ/kg}$$

$$\Rightarrow \dot{Q}_{cv} = (0.903)(1718.8) - (2.199)(1193.4) - 2792.2(0.903 - 2.199) \\ = 2546.5 \text{ kJ} \leftarrow \dot{Q}_{cv}$$

Continued on next slide

Problem 4-110 continued

(b) Data are obtained to make the required plots using IT, as follows:

IT Code

```
V = 0.06 // m³
p = 15 // bar
x1 = 0.2
x2 = 0.5
```

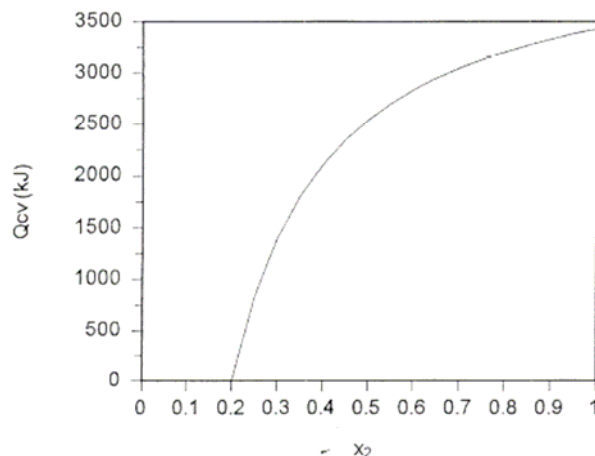
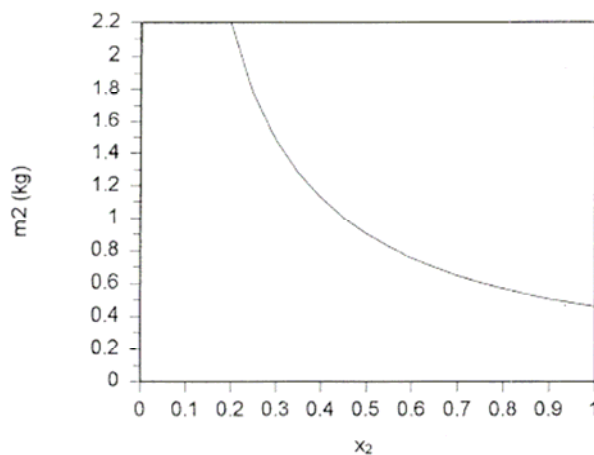
```
m1 = V / v1
m2 = V / v2
Qcv = m2 * u2 - m1 * u1 - he * (m2 - m1)
```

```
he = hsat_Px("Water/Steam", p, 1)
u1 = usat_Px("Water/Steam", p, x1)
u2 = usat_Px("Water/Steam", p, x2)
v1 = vsat_Px("Water/Steam", p, x1)
v2 = vsat_Px("Water/Steam", p, x2)
```

IT Result ($x_2 = 0.5$)

$Q_{cv} = 2547 \text{ kJ}$

The result for $x_2 = 0.5$ compares very favorably with the result of part (a). Now, using the **Exlope** button, sweep x_2 from 0.2 to 1.0 in steps of 0.05. The following plots are constructed from the data:



We see from the plots that the heat transfer increases rapidly with x_2 and that the mass in the tank drops rapidly as vapor escapes.

PROBLEM 4.111

As shown in Fig. P4.111, a 300-ft³ tank contains H₂O initially at 30 lbf/in.² and a quality of 80%. The tank is connected to a large steam line carrying steam at 200 lbf/in.², 450°F. Steam flows into the tank through a valve until the tank pressure reaches 100 lbf/in.² and the temperature is 400°F, at which time the valve is closed. Determine the amount of mass, in lb, that enters the tank and the heat transfer between the tank and its surroundings, in Btu.

SCHEMATIC & GIVEN DATA:

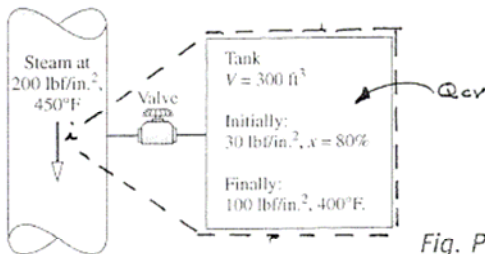


Fig. P4.111

ENGR. MODEL:

1. The control volume is shown on the sketch.
2. Conditions within the steam line remain constant.
3. $\dot{W}_{cv} = 0$. Kinetic and potential energy effects can be neglected.

ANALYSIS: Applying the mass rate balance

$$\frac{dm_{cv}}{dt} = \dot{m}_i \Rightarrow m_2 - m_1 = \int_{t_1}^{t_2} \dot{m}_i dt = \left\{ \begin{array}{l} \text{amount of mass that} \\ \text{enters the control volume} \end{array} \right\}$$

In this expression, $m = V/v$. At the initial state, with data from Table A-3E,

$$v_1 = v_f + x v_{fg} = 0.0017 + 0.8(13.75 - 0.0017) = 11.0 \text{ ft}^3/\text{lb}$$

$$\Rightarrow m_1 = \frac{300 \text{ ft}^3}{11.0 \text{ ft}^3/\text{lb}} = 27.27 \text{ lb}$$

At the final state, $v_2 = 4.934 \text{ ft}^3/\text{lb}$ (Table A-4E). So,

$$m_2 = \frac{300 \text{ ft}^3}{4.934 \text{ ft}^3/\text{lb}} = 60.8 \text{ lb}$$

$$\Rightarrow (m_2 - m_1) = (60.8 \text{ lb} - 27.27 \text{ lb}) = 33.53 \text{ lb} \quad \leftarrow \begin{array}{l} \text{mass that} \\ \text{enters the tank} \end{array}$$

The energy rate balance reduces as follows:

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i \left(h_i + \frac{v_i^2}{2} + gz_i \right) \Rightarrow \frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \dot{m}_i h_i$$

Integrating and noting that h_i remains constant

$$m_2 u_2 - m_1 u_1 = \dot{Q}_{cv} - h_i \int_{t_1}^{t_2} \dot{m}_i dt \Rightarrow \dot{Q}_{cv} = m_2 u_2 - m_1 u_1 - h_i (m_2 - m_1)$$

Using Table A-3E data, $u_1 = u_f + x(u_g - u_f) = 218.84 + 0.8(1088 - 218.84) = 914.17 \text{ Btu/lb}$.

And from Table A-4E, $u_2 = 1136.2 \text{ Btu/lb}$, $h_i = 1240.7 \text{ Btu/lb}$.

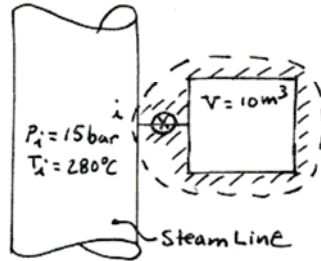
$$\begin{aligned} \text{Finally } \dot{Q}_{cv} &= (60.8 \text{ lb})(1136.2 \text{ Btu/lb}) - (27.27 \text{ lb})(914.17 \text{ Btu/lb}) - 33.53 \text{ lb}(1240.7 \text{ Btu/lb}) \\ &= -2551 \text{ Btu} \end{aligned} \quad \leftarrow \dot{Q}_{cv}$$

PROBLEM 4.112

KNOWN: An initially-evacuated, well-insulated tank is filled from a large steam line until the pressure in the tank attains a specified value, p .

FIND: (a) Determine the mass and temperature in the tank when $p = 15$ bar.
(b) Plot the mass and temperature in the tank versus p ranging from 0.1 to 15 bar.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume is shown in the schematic. 2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects are negligible. 3. At inlet i , the state remains constant.

ANALYSIS: The mass rate balance reads $d\dot{m}_{cv}/dt = \dot{m}_i$. With the indicated assumptions, the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i \Rightarrow \frac{dU_{cv}}{dt} = h_i \frac{dm_{cv}}{dt}$$

Since h_i is constant (assumption 3), integration over time gives

$$\Delta U_{cv} = h_i \Delta m_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1) \Rightarrow u_2 = h_i \quad (1)$$

(a) $p = 15$ bar. From Table A-4, $h_i = 2992.7$ kJ/kg. Then, interpolation at 15 bar with $u_2 = 2992.7$ kJ/kg; $T_2 = 425^\circ\text{C}$, $v_2 = 0.211$ m³/kg. Thus, the final mass is

$$m_2 = \frac{V}{v_2} = \frac{10 \text{ m}^3}{0.211 \text{ m}^3/\text{kg}} = 47.39 \text{ kg}$$

(b) IT is used as follows:

IT Code

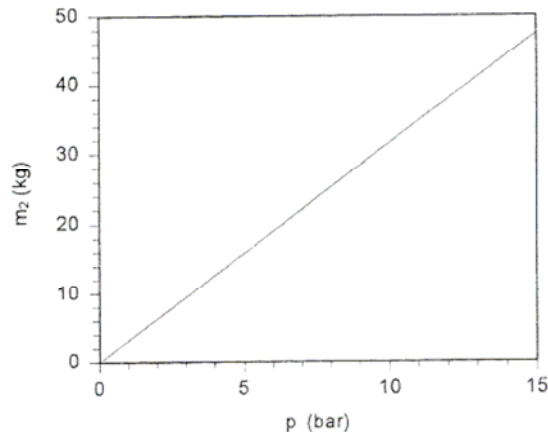
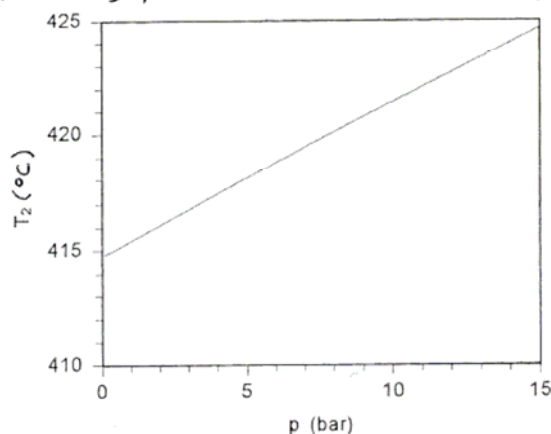
```
V = 10 // m^3
pline = 15 // bar
Tline = 280 // °C
p = 15 // bar
u2 = u_PT("Water/Steam", p, T2)
hi = h_PT("Water/Steam", pline, Tline)
u2 = hi
v2 = v_PT("Water/Steam", p, T2)
m2 = V / v2
```

IT Results (p = 15 bar)

```
T2 = 424.7 °C
m2 = 47.39 kg
v2 = 0.211 m^3/kg
```

These results compare very favorably with the results of part (a).

Now, using the Explore button, sweep p from 0.1 to 15 bar, in steps of 0.1. The following plots are constructed from the data:



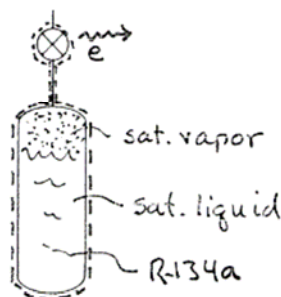
PROBLEM 4.113

KNOWN: A storage tank containing Refrigerant 134a develops a leak allowing saturated vapor to escape until the level of saturated liquid in the tank has dropped to a known value.

FIND: Determine the mass of refrigerant that has escaped and the heat transfer.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL : (1) For the control volume, $\dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The pressure is constant.



$$\begin{aligned} P &= 100 \text{ lbf/in}^2 \\ V &= 2 \text{ ft}^3 \\ V_{f1} &= 1.6 \text{ ft}^3 \\ V_{f2} &= 0.8 \text{ ft}^3 \end{aligned}$$

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. Integrating

$$\int_1^2 \dot{m}_e dt = m_1 - m_2 \quad (\text{mass escaped})$$

To get the masses, use the given volumes and data from Table A-11E at a pressure of 100 lbf/in²

$$m_{f1} = \frac{V_{f1}}{v_f} = \frac{(1.6 \text{ ft}^3)}{(0.01332 \text{ ft}^3/\text{lb})} = 120.12 \text{ lb}$$

$$m_{g1} = \frac{V_{g1}}{v_g} = \frac{V - V_{f1}}{v_g} = \frac{2 - 1.6}{0.4747} = 0.84 \text{ lb}$$

$$m_1 = m_{f1} + m_{g1} = 120.96 \text{ lb}$$

Similarly for state 2

$$m_{f2} = 60.06 \text{ lb}, m_{g2} = 2.53 \text{ lb}; m_2 = 62.59 \text{ lb}$$

and

$$\int_1^2 \dot{m}_e dt = 120.96 - 62.59 = 58.37 \text{ lb} \quad \leftarrow \text{mass escaped}$$

With assumptions (1) and (2), the energy rate balance reduces to

$$\frac{d\dot{U}_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Combining the mass and energy rate balances and integrating

$$\dot{Q}_{cv} = m_2 u_2 - m_1 u_1 - h_e (m_2 - m_1)$$

From above, $x_1 = m_{g1}/m_1 = 0.00694$ and $x_2 = m_{g2}/m_2 = 0.04042$. Thus, with data from Table A-11E

$$u_1 = u_f + x_1 (u_g - u_f) = 37.21 \text{ Btu/lb}; u_2 = 39.46 \text{ Btu/lb}$$

$$\text{and } h_e = h_g @ 100 \text{ lbf/in}^2 = 112.46 \text{ Btu/lb}$$

Thus

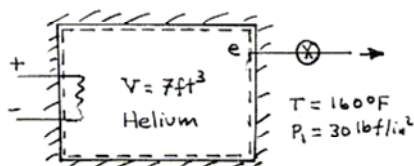
$$\begin{aligned} \dot{Q}_{cv} &= (62.59)(39.46) - (120.96)(37.21) - (112.46)(62.59 - 120.96) \\ &= 4533.2 \text{ Btu} \quad \leftarrow \dot{Q}_{cv} \end{aligned}$$

PROBLEM 4.114

KNOWN: Helium is withdrawn slowly from a well-insulated tank of known volume until the pressure within the tank is p . The temperature within the tank is kept constant by an electrical resistor.

FIND: (a) When $p = 18 \text{ lbf/in}^2$, determine the mass of helium withdrawn and the energy input to the resistor. (b) Plot the quantities of part (a) versus p ranging from 15 to 30 lbf/in^2 .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume is shown in the schematic. 2. For the control volume, $Q_{cv} = 0$, and kinetic/potential energy effects can be ignored. 3. Helium is modeled as an ideal gas. 4. The mass of the resistor is small enough to be ignored.

ANALYSIS: The mass of helium withdrawn over a time interval equals the difference between the initial amount of mass in the tank and the mass in the tank at the later time: (Amount withdrawn) = $m_1 - m_2$. Since $T_1 = T_2$, the ideal gas equation of state can be used to rewrite this as follows, where $p_2 = p$:

$$(\text{Amount withdrawn}) = \frac{P_1 V}{RT} - \frac{p V}{RT} = (P_1 - p) \frac{V}{RT} \quad (1)$$

$$= (30 - p) \left(\frac{\text{lbf}}{\text{in}^2} \right) \left(\frac{144 \text{ in}^2}{1 \text{ ft}^2} \right) \left(\frac{7 \text{ ft}^3}{\left[\frac{154.5 \text{ ft} \cdot \text{lbf}}{4.003 \text{ lb} \cdot \text{R}} \right] (620 \text{ R})} \right)$$

$$= (30 - p) (4.21 \times 10^{-3}) \text{ lb} \quad (2)$$

The mass rate balance is $dm_{cv}/dt = -\dot{m}_e$. The energy rate balance reduces to $dU_{cv}/dt = \dot{Q}_{cv} - \dot{W}_{cv} - \dot{m}_e h_e$. Introducing the mass rate balance, and noting that h_e remains constant because temperature is constant,

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + h_e \frac{dm_{cv}}{dt} \Rightarrow \Delta U_{cv} = -W_{cv} + h_e \Delta m_{cv} \Rightarrow (-W_{cv}) = m_2 u_2 - m_1 u_1 - (m_2 - m_1) h_e$$

Since temperature is constant, $u_2 = u(T)$, $u_1 = u(T)$, $h_e = u(T) + (pV)_e$. Thus

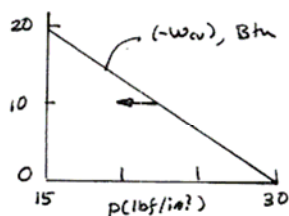
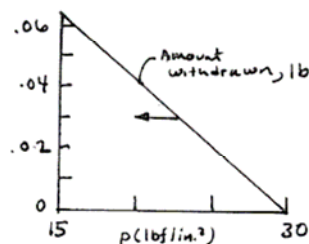
$$\textcircled{1} \quad (-W_{cv}) = (m_1 - m_2)(pV)_e \Rightarrow \text{with the ideal gas equation of state, } (-W_{cv}) = (m_1 - m_2)RT$$

By Eq. (1) $(m_1 - m_2) = (P_1 - p)V/RT$, giving $(-W_{cv}) = (P_1 - p)V$. That is,

$$(-W_{cv}) = (30 - p) \left(\frac{\text{lbf}}{\text{in}^2} \right) \left(\frac{144 \text{ in}^2}{1 \text{ ft}^2} \right) (7 \text{ ft}^3) \left(\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right) = 1.296 (30 - p) \text{ Btu} \quad (3)$$

(a) When $p = 18 \text{ lbf/in}^2$, Eq. (2) gives (Amt. withdrawn) = 0.0505 kg, Eq. (3) gives $(-W_{cv}) = 15.55 \text{ Btu}$.

(b) PLOTS



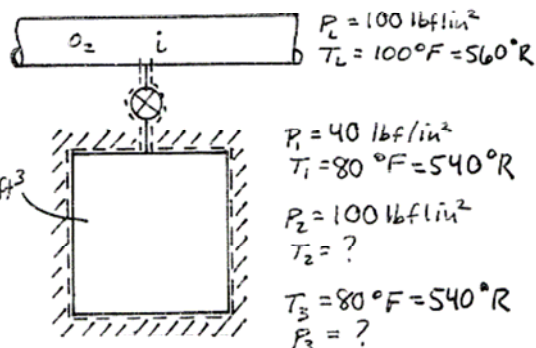
1. To maintain the temperature within the tank constant, the resistor must provide energy to the helium in the tank equal to the energy carried out by flow work at e .

PROBLEM 4.115

KNOWN: An insulated tank containing oxygen (O_2) is connected to a supply line carrying oxygen. A valve between the tank and the line is opened and gas flows into the tank until the pressure reaches that of the line. The valve is closed and eventually the tank contents cool back to their initial temperature.

FIND: Determine (a) the tank temperature when the valve closes and (b) the final pressure in the tank.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) For the control volume shown, $W_{cv} = 0$. (2) There is no heat transfer during the filling process. (3) Kinetic and potential energy effects are negligible. (4) The state of the O_2 in the supply line remains constant. (5) The O_2 behaves as an ideal gas.

ANALYSIS: (a) Consider first the filling process. The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_i$ and with assumptions (1), (2), and (3), the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i$$

The specific enthalpy in the supply line is constant. Combining the mass and energy rate balances, and integrating

$$m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1)$$

Using the ideal gas equation of state

$$m_1 = \frac{P_1 V}{RT_1}, \quad m_2 = \frac{P_2 V}{RT_2}$$

Combining and rearranging gives

$$u_2 = h_i + \frac{T_2}{T_1} \frac{P_1}{P_2} (u_1 - h_i)$$

From Table A-23E

$$h_i = \left(\frac{3886.6 \text{ Btu/lb mol}}{32.00 \text{ lb/lb mol}} \right) = 121.46 \text{ Btu/lb}$$

$$u_1 = 2673.8 / 32.00 = 83.56 \text{ Btu/lb}$$

Inserting values

$$u_2 = 121.46 - (0.02807) T_2$$

Since u_2 depends on T_2 , the final temperature can be found by iteration using data from Table A-23E and this expression. The result is

$$T_2 \approx 661^\circ\text{R} = 201^\circ\text{F} \leftarrow T_2$$

Continued on next slide

Problem 4-115 continued

- (b) To determine the pressure after the contents of the tank cool off, first find the mass in the tank after the valve closes

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(100 \text{ lbf/in}^2)(5 \text{ ft}^3)}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{32.00 \text{ lb} \cdot ^\circ\text{R}}\right)(661 ^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|$$
$$= 2.256 \text{ lb}$$

Thus, the final pressure is

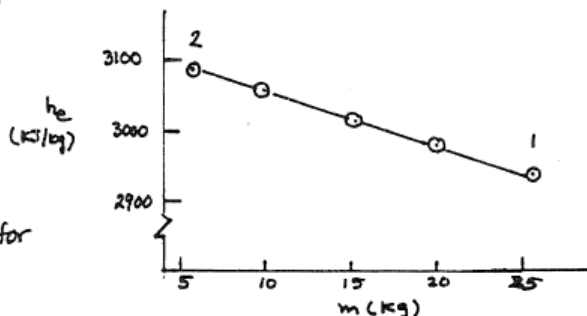
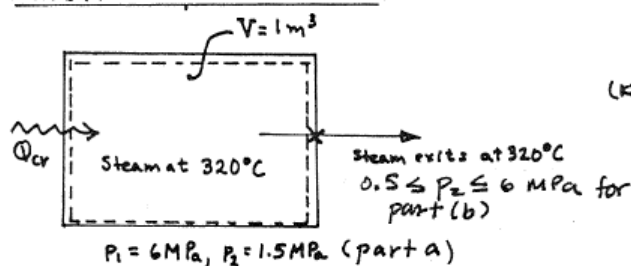
$$P_3 = \frac{m_2 R T_3}{V}$$
$$= \frac{(2.256) \left(\frac{1545}{32.00}\right)(540)}{(5) | 144 |} = 81.7 \text{ lbf/in}^2 \leftarrow P_2$$

PROBLEM 4.116

KNOWN: A tank of known volume initially contains steam at a specified state. Steam is withdrawn slowly until the pressure drops to pressure p . The temperature is kept constant by heat transfer to the tank contents.

FIND: (a) Determine the heat transfer for $p = 1.5 \text{ MPa}$. (b) Plot the heat transfer versus p ranging from 0.5 to 6 MPa .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) For the control volume shown in the figure, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be ignored. (2) At each instant, pressure is uniform throughout the steam.

ANALYSIS: (a) The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. Using the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Introducing the mass rate balance and integrating

$$\Delta U_{cv} = Q_{cv} + \int_1^2 h_e dm \Rightarrow Q_{cv} = m_2 u_2 - m_1 u_1 - \int_1^2 h_e dm \quad (1)$$

where m denotes the mass contained within the tank. At any instant, $m = V/v$ where v is specific volume at that instant determined by 320°C and the tank pressure. Initially, $p_1 = 6 \text{ MPa}$, so Table A-4 gives $v_1 = 0.03876 \text{ m}^3/\text{kg}$, $u_1 = 2720 \text{ kJ/kg}$. Finally, $p_2 = 1.5 \text{ MPa}$, so $v_2 = 0.1765 \text{ m}^3/\text{kg}$, $u_2 = 2817.1 \text{ kJ/kg}$. Thus

$$m_1 = \frac{V}{v_1} = \frac{1 \text{ m}^3}{0.03876 \text{ m}^3/\text{kg}} = 25.8 \text{ kg}, \quad m_2 = \frac{V}{v_2} = \frac{1 \text{ m}^3}{0.1765 \text{ m}^3/\text{kg}} = 5.67 \text{ kg} \quad (2)$$

For each of several pressures in the interval $1.5 \text{ MPa} < p < 6.0 \text{ MPa}$, the mass within the tank can be calculated in like manner and the specific enthalpy h_e determined from Table A-4 at 320°C and the specified pressure. These values allow the plot of h_e vs m given above to be constructed. The area under the line from 1 to 2 equals the term $-\int_1^2 h_e dm$ of Eq. (1). As this variation is very nearly linear, the average value of h_e can be used to evaluate this term:

$$-\int_1^2 h_e dm = \left(\frac{h_{e1} + h_{e2}}{2} \right) (m_1 - m_2) = \left(\frac{3081.9 + 2952.6}{2} \right) (25.8 - 5.67) \\ = (3017.25 \frac{\text{kJ}}{\text{kg}}) (20.13 \text{ kg}) = 60737 \text{ kJ}$$

Finally, inserting values into Eq. (1)

$$Q_{cv} = (5.67 \text{ kg})(2817.1 \frac{\text{kJ}}{\text{kg}}) - (25.8 \text{ kg})(2720 \frac{\text{kJ}}{\text{kg}}) + 60737 \text{ kJ} \\ = 6534 \text{ kJ} \quad \leftarrow Q_{cv}$$

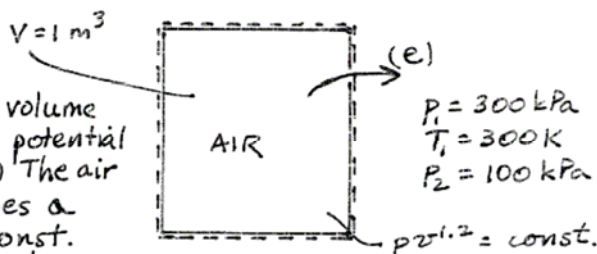
PROBLEM 4.117

KNOWN: Air escapes slowly from a tank. The initial state and final pressure are known. The air that remains in the tank undergoes a polytropic process.

FIND: Determine the heat transfer for a control volume enclosing the tank.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) For the control volume shown, $\dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The air remaining in the tank undergoes a process described by $p v^{1.2} = \text{const.}$ (4) The air is modeled as an ideal gas with constant specific heats.



ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. With the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e \quad (1)$$

Since the process occurs slowly, the state of the air in the tank is uniform at any time. Thus, $U_{cv} = m u$, and $h_e = u + RT$. Inserting these expressions into (1) along with the mass rate balance

$$m \frac{du}{dt} + u \frac{dm}{dt} = \dot{Q}_{cv} + (u + RT) \frac{dm}{dt}$$

$$\text{Thus } \dot{Q}_{cv} dt = m du - RT dm = m c_v dT - RT dm \quad (2)$$

Each term on the right side of (2) can be expressed in terms of pressure, as follows. First, from $p v^{1.2} = \text{constant}$

$$m = \frac{V}{v} = \left(\frac{p}{\text{const}} \right)^{1/1.2} V = C p^{1/1.2}$$

$$\text{and } dm = C \left(\frac{1}{1.2} \right) p^{(1/1.2 - 1)} dp \quad (3)$$

$$\text{Further } RT = \frac{pV}{m} = \frac{pV}{C p^{1/1.2}} = \frac{V}{C} p^{(1 - 1/1.2)} \quad (4)$$

$$\text{and } dT = \frac{V}{RC} \left(1 - \frac{1}{1.2} \right) p^{(-1/1.2)} dp \quad (5)$$

Substituting (3), (4) and (5) into (2) gives

$$\begin{aligned} \dot{Q}_{cv} dt &= [C p^{1/1.2}] c_v \left[\frac{V}{RC} \left(1 - \frac{1}{1.2} \right) p^{(-1/1.2)} dp \right] \\ &\quad - \left[\frac{V}{C} p^{(1 - 1/1.2)} \right] [C \left(\frac{1}{1.2} \right) p^{(1/1.2 - 1)} dp] \\ &= \left[\frac{c_v}{R} \left(1 - \frac{1}{1.2} \right) - \left(\frac{1}{1.2} \right) \right] V dp \end{aligned}$$

Integrating

$$\dot{Q}_{cv} = \left[\frac{c_v}{R} \left(1 - \frac{1}{1.2} \right) - \left(\frac{1}{1.2} \right) \right] V (P_2 - P_1)$$

Continued on next slide

Problem 4-117 continued

From Table A-20, $c_v \approx 0.717 \text{ kJ/kg}\cdot\text{K}$. Thus

$$Q_{cv} = \left[\frac{(0.717 \text{ kJ/kg}\cdot\text{K})}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg}\cdot\text{K}} \right)} \left(1 - \frac{1}{1.2} \right) - \left(\frac{1}{1.2} \right) \right] (1 \text{ m}^3) (100 - 300) \text{ kPa} \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$\textcircled{1} \quad = 83.39 \text{ kJ} \quad \underline{\hspace{10cm}} \quad Q_{cv}$$

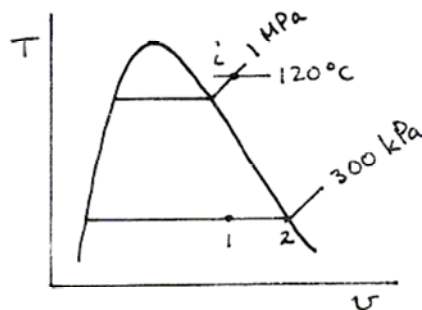
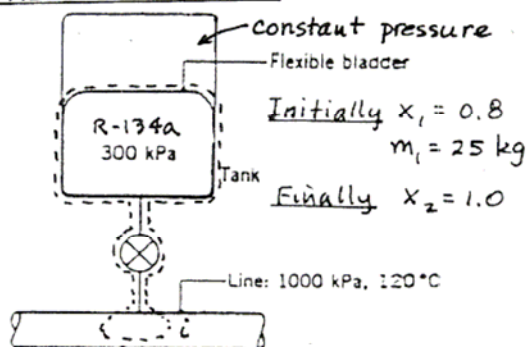
1. The positive sign denotes that the energy transfer is into the control volume.

PROBLEM 4.118

KNOWN: A well-insulated tank containing R-134a is connected to a supply line. As refrigerant is allowed to flow into the tank, a flexible bladder in the tank expands to maintain the refrigerant in the tank at constant pressure.

FIND: Determine the amount of mass admitted to the tank between the initial time and the instant when all the liquid in the tank is vaporized.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is shown with $\dot{Q}_{cv} = 0$. (2) Conditions in the supply line remain constant. (3) The pressure remains constant in the tank. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The mass rate balance takes the form; $dm_{cv}/dt = \dot{m}_i$. With the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i$$

The specific enthalpy h_i is constant by assumption (2). Thus, combining the mass and energy rate balances and integrating

$$\Delta U_{cv} = -W_{cv} + \int_{t_1}^{t_2} \dot{m}_i h_i dt$$

Since h_i is constant

$$\Delta U_{cv} = -W_{cv} + h_i \int_{t_1}^{t_2} \dot{m}_i dt = -W_{cv} + h_i (m_2 - m_1) \quad (1)$$

To evaluate the work, note that the pressure in the tank is constant. Thus

$$W_{cv} = \int p dV = p(V_2 - V_1) = p(m_2 v_2 - m_1 v_1) \quad (2)$$

Combining (1) and (2), and noting that $\Delta U_{cv} = m_2 u_2 - m_1 u_1$,

$$m_2 u_2 - m_1 u_1 = -p(m_2 v_2 - m_1 v_1) + h_i (m_2 - m_1)$$

or

$$m_2 [u_2 + p v_2 - h_i] = m_1 [u_1 + p v_1 - h_i]$$

$$m_2 [h_2 - h_i] = m_1 [h_1 - h_i]$$

Continued on next slide

Problem 4-118 continued

Solving for m_2

$$m_2 = m_1 \left(\frac{h_1 - h_i}{h_2 - h_i} \right)$$

Using data from Table A-11 at 3 bar: $h_f = 50.85$, $h_g = 247.59$ kJ/kg

$$h_1 = (50.85) + (0.8) [247.59 - 50.85] = 208.24 \text{ kJ/kg}$$

$$h_2 = 247.59 \text{ kJ/kg}$$

From Table A-12, $h_i = 356.52$ kJ/kg. Thus

$$m_2 = 25 \text{ kg} \left(\frac{208.24 - 356.52}{247.59 - 356.52} \right) = 34.03 \text{ kg}$$

Finally,

$$\begin{aligned} \Delta m &= m_2 - m_1 \\ &= 34.03 - 25 = 9.03 \text{ kg} \end{aligned}$$



PROBLEM 4.119

KNOWN: Air is admitted slowly into an insulated piston-cylinder assembly from a supply line until the volume inside the cylinder has doubled.

FIND: Plot the final temperature and mass inside the cylinder for a given range of supply temperatures.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) For the control volume shown, $\dot{Q}_{cv} = 0$. (2) Kinetic and potential energy effects can be neglected. (3) The weight of the piston and friction between the piston and cylinder wall can be neglected. (4) The air behaves as an ideal gas.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_i$. With the assumptions listed, the energy balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i$$

The line conditions are constant, so h_i is constant. Combining the mass and energy rate balances and integrating

$$m_2 u_2 - m_1 u_1 = -W_{cv} + h_i (m_2 - m_1) \quad (1)$$

To evaluate the work, note that the pressure in the cylinder is always atmospheric since the process is slow and since assumption (3) applies.

Thus

$$W_{cv} = \int_{V_1}^{V_2} p dV = p(V_2 - V_1)$$

From the given data

$$V_1 = \left(\frac{\pi d_{pist}^2}{4} \right) L_1 = \left(\frac{\pi (.3^2) m^2}{4} \right) (0.5 m) = 0.03534 m^3$$

$$\text{and } V_2 = 0.07068 m^3$$

$$W_{cv} = (1 \text{ bar})(0.07068 - 0.03534) m^3 \left| \frac{10^5 N/m^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 N \cdot m} \right| = 3.534 \text{ kJ}$$

From Table A-22, at $T_1 = 300 \text{ K}$; $u_1 = 214.07 \text{ kJ/kg}$. Also, $h_i = h_i(T_{supply})$. For $T_{supply} = 300 \text{ K}$, $h_i = 300.19 \text{ K}$.

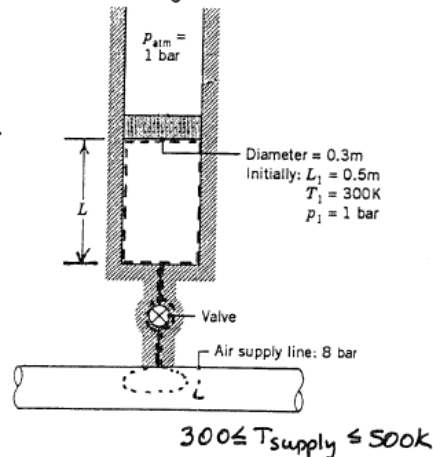
Using the ideal gas model

$$m_1 = \frac{P_1 V_1}{R T_1} = \frac{(1 \text{ bar})(0.03534 m^3)}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (300 \text{ K})} \left| \frac{10^5 N/m^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 N \cdot m} \right| = 0.04105 \text{ kg}$$

$$\text{and } m_2 = \frac{P_2 V_2}{R T_2} = \frac{(1)(0.07068)(100)}{\left(\frac{8.314}{28.97} \right) (T_2)} = \frac{24.628}{T_2} \quad (2)$$

Incorporating these results into (1) and rearranging

Continued on next slide



Problem 4-119 continued

$$m_2 u_2 - h_i (m_2 - m_1) = m_1 u_1 - W_{cv}$$

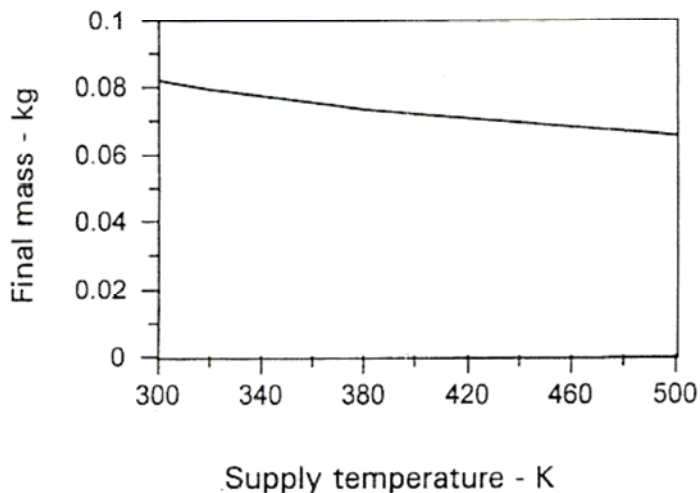
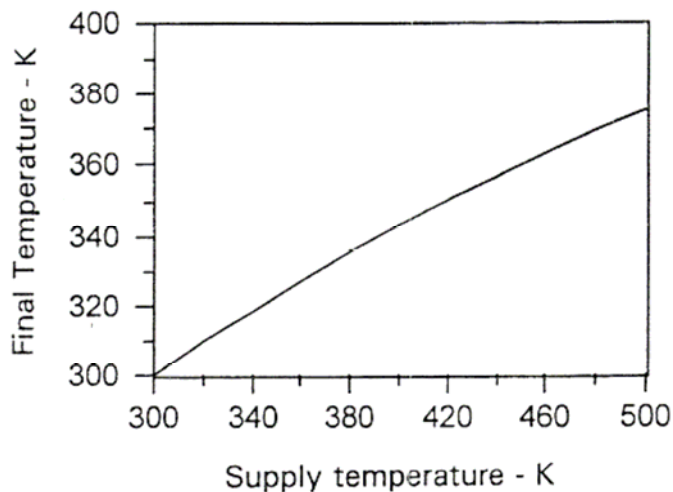
$$\frac{24.628}{T_2} u_2 - h_i \left(\frac{24.628}{T_2} - 0.04105 \right) = 5.254 \quad (3)$$

Equations (2) and (3) can be solved for given values of $h_i (T_{\text{supply}})$ by an iterative procedure and data from Table A-22. For $T_{\text{supply}} = 300\text{K}$,

$$T_2 = 300\text{K}$$

$$m_2 = 0.0821 \text{ kg}$$

Plotting for the range of T_{supply} values



PROBLEM 4.120

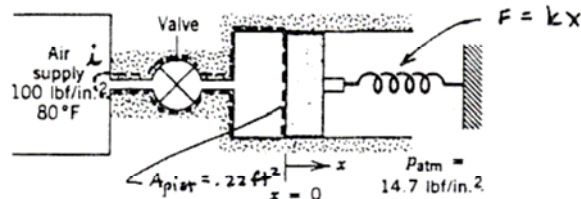
KNOWN: A well-insulated piston-cylinder assembly is connected by a valve to an air supply. The supply conditions and the initial state of air inside the cylinder. Air is admitted slowly causing the piston to compress a spring. The initial and final volumes within the cylinder are known.

FIND: Plot the final pressure and final temperature within the cylinder versus spring constant k varying from 650 to 750 lbf/ft.

SCHEMATIC & GIVEN DATA:

Initially: $P_1 = 14.7 \text{ lbf/in}^2$
 $T_1 = 80^\circ\text{F}$
 $V_1 = 0.1 \text{ ft}^3$

Finally: $V_2 = 0.4 \text{ ft}^3$



ENGR. MODEL: (1) The control volume is shown on the accompanying diagram, with $\dot{Q}_{cv} = 0$. (2) Conditions in the air supply remain constant. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas. (5) There is no friction between the piston and cylinder wall.

ANALYSIS: The final pressure is found by applying Newton's Law to the piston. Since air is admitted slowly; $\sum F_x = 0$. Thus

$$P A_{pist} = P_{atm} A_{pist} + kx$$

With $V - V_1 = x A_{pist}$, the pressure is

$$P = P_{atm} + \frac{k(V - V_1)}{A_{pist}^2} \quad (a)$$

At $V = V_2$

$$P_2 = 14.7 \frac{\text{lbf}}{\text{in}^2} + \frac{(k \text{ lbf/ft})(0.4 - 0.1) \text{ ft}^3}{(0.22^2) \text{ ft}^4} \left(\frac{1 \text{ ft}^2}{144 \text{ in}^2} \right)$$

$$= [14.7 + 0.043k] \text{ lbf/in}^2 \quad (1)$$

The mass rate balance takes the form; $dm_{cv}/dt = \dot{m}_i$. With assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i$$

The specific enthalpy h_i is constant by assumption (3). Thus, combining the mass and energy rate balances and integrating

$$\Delta U_{cv} = -W_{cv} - \int_1^2 h_i dm_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = -W_{cv} + h_i (m_2 - m_1) \quad (2)$$

Since the process occurs slowly, $W_{cv} = \int p dV$. With Eq.(a) above

$$W_{cv} = \int_{V_1}^{V_2} \left[\left(P_{atm} - \frac{kV_1}{A_{pist}^2} \right) + \frac{kV}{A_{pist}^2} \right] dV$$

$$= \left(P_{atm} - \frac{kV_1}{A_{pist}^2} \right) (V_2 - V_1) + \frac{k(V_2^2 - V_1^2)}{2 A_{pist}^2}$$

Continued on next slide

Problem 4-120 continued

$$\begin{aligned}
 \therefore W_{cv} &= P_{atm}(V_2 - V_1) + \frac{K}{(A_{pist})^2} \left[\frac{V_2^2 - V_1^2}{2} - V_1(V_2 - V_1) \right] \\
 &= P_{atm}(V_2 - V_1) + \frac{K}{2(A_{pist})^2} [V_2 - V_1]^2 \\
 &= \left[14.7 \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| (0.3 \text{ ft}^3) + \frac{K (\text{lbf/ft})}{2(0.22 \text{ ft}^2)^2} [0.3 \text{ ft}^2]^2 \right] \left| \frac{1 \text{ Btu}}{778 \text{ ft lbf}} \right| \\
 &= [0.816 + (1.195 \times 10^{-3}) K] \text{ Btu} \quad (3)
 \end{aligned}$$

Also, with the ideal gas model equation of state

$$\begin{aligned}
 m_1 &= \frac{P_1 V_1}{R T_1} = \frac{(14.7 \times 144 \text{ lbf/ft}^2)(0.1 \text{ ft}^3)}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ \text{R}} \right) (540^\circ \text{R})} = 7.35 \times 10^{-3} \text{ lb} \\
 m_2 &= \frac{P_2 V_2}{R T_2}, \text{ where } P_2 \text{ is given by Eq. (1)} \quad (4)
 \end{aligned}$$

Since $T_1 = T_2 = 540^\circ \text{R}$ (80°F), $u_1 = u(540^\circ \text{R})$, $h_1 = h(540^\circ \text{R})$.

Accordingly, T_2 can be determined by solving Eq. (2), together with Eqs. (1), (3), (4) and known values of V_2 , u_1 , and h_1 .

This scheme is implemented via the following IT code:

IT Code

```

p1 = 14.7 // lbf/in.2
T1 = 80 // °F
Ti = T1
V1 = 0.1 // ft3
V2 = 0.4 // ft3
K = 650 // lbf/ft

p2 = 14.7 + 0.043 * K
m2 * u2 - m1 * u1 = -Wcv + hi * (m2 - m1)
Wcv = 0.816 + 1.195E-3 * K
m1 = V1 / v1
m2 = V2 / v2

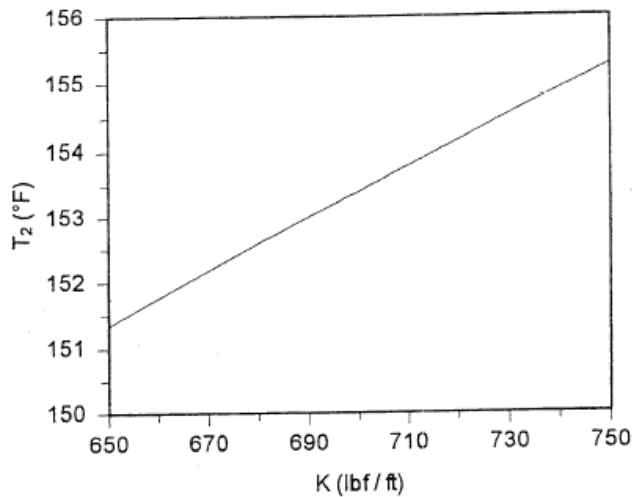
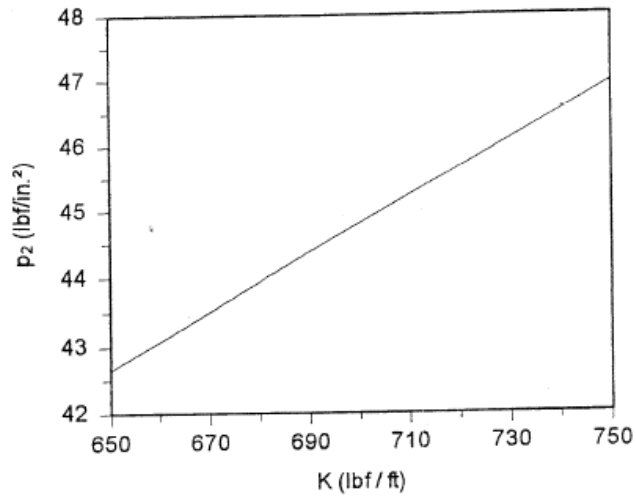
u1 = u_T("Air", T1)
v1 = v_TP("Air", T1, p1)
u2 = u_T("Air", T2)
v2 = v_TP("Air", T2, p2)
hi = h_T("Air", Ti)

```

Now, using the Explore button, sweep K from 650 to 750 lbf/ft in steps of 1, to get the following plots:

Continued on next slide

Problem 4-120 continued



Note that the pressure plot is linear, as indicated by Eq. (1), and could have been readily plotted by hand. However, the temperature plot is non-linear and would be quite difficult to obtain without the use of IT or other software.

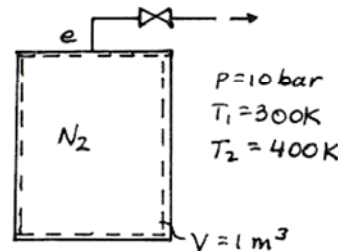
PROBLEM 4.121

KNOWN: Heat transfer occurs to nitrogen gas contained in a rigid tank. Gas escapes through a pressure relief valve, maintaining constant pressure in the tank. The initial and final temperatures are specified.

FIND: Determine the mass of nitrogen that escapes and the amount of energy transfer by heat.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) For the control volume shown, $\dot{W}_{cv} = 0$. (2) The state in the control volume can be assumed to be uniform at any time during the process. (3) Kinetic and potential energy effects can be neglected. (4) The nitrogen behaves as an ideal gas with constant specific heats evaluated at 350K.



ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. Thus the mass that escapes is

$$\int_1^2 \dot{m}_e dt = -\int_{m_1}^{m_2} dm_{cv} = m_1 - m_2$$

With the ideal gas equation of state

$$\begin{aligned} \int_1^2 \dot{m}_e dt &= \frac{PV}{R} \left[\frac{1}{T_1} - \frac{1}{T_2} \right] \\ &= \frac{(10 \text{ bar})(1 \text{ m}^3)}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right)} \left[\frac{1}{300 \text{ K}} - \frac{1}{400 \text{ K}} \right] \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= 2.904 \text{ kg} \quad \leftarrow \text{mass escaped} \end{aligned}$$

With the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Noting that $U_{cv} = mu$ and $h_e = u + RT$, and with the mass rate balance

$$\begin{aligned} m \frac{du}{dt} + u \frac{dm}{dt} &= \dot{Q}_{cv} + (u + RT) \frac{dm}{dt} \\ \dot{Q}_{cv} &= m \frac{du}{dt} - RT \frac{dm}{dt} \end{aligned}$$

For an ideal gas, $du = c_v dT$. Also, with $m = PV/RT$

$$dm = \left(\frac{PV}{R} \right) \frac{dT}{(-T^2)}$$

Thus

$$\dot{Q}_{cv} = \left(\frac{PV}{RT} \right) c_v \frac{dT}{dt} + \left(\frac{PV}{RT} \right) R \frac{dT}{dt} = \frac{PV}{R} (c_v + R) \frac{1}{T} \frac{dT}{dt}$$

Continued on next slide

Problem 4-121 continued

Noting that $c_v + R = c_p$

$$\dot{Q}_{cv} dt = \left(\frac{PVc_p}{R} \right) \frac{dT}{T}$$

From Table A-20, $c_p = 1.041 \text{ kJ/kg} \cdot \text{K}$. Integrating and inserting values

$$\int_1^2 \dot{Q}_{cv} dt = \left(\frac{PVc_p}{R} \right) \int_{T_1}^{T_2} \frac{dT}{T}$$

$$Q_{cv} = \left(\frac{PVc_p}{R} \right) \ln \frac{T_2}{T_1}$$

$$= \frac{(10 \text{ bar})(1 \text{ m}^3)(1.041 \text{ kJ/kg} \cdot \text{K})}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right)} \ln \left(\frac{400}{300} \right) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 1043 \text{ kJ} \longleftarrow Q_{cv}$$

PROBLEM 4.122

KNOWN: The air supply to an office is shut off overnight and the room temperature drops. In the morning, the thermostat is reset and a known volumetric flow rate of heated air is supplied.

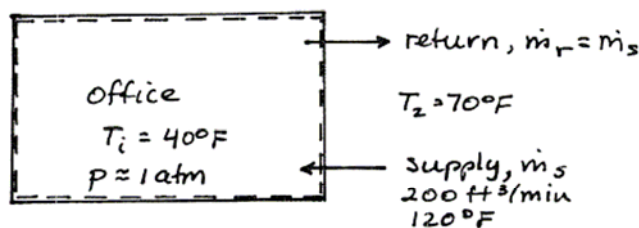
FIND: Estimate the time it takes for the room to reach 70°F, and plot room temperature as a function of time.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) For the control volume shown, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$.

(2) The air behaves as an ideal gas with constant specific heats.

(3) The pressure is taken as 1 atm everywhere. (4) Kinetic and potential energy effects can be neglected. (5) The room air is well-mixed.



ANALYSIS: The mass rate balance takes the form $dm/dt = \dot{m}_s - \dot{m}_r$. With $\dot{m}_s = \dot{m}_r$, $dm/dt = 0$. Thus, the mass in the room is constant with time. The energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_s h_s - \dot{m}_r h_r$$

or, with $U_{cv} = m u$ and $h_r = h(T)$

$$m \frac{du}{dt} + u \frac{dm}{dt} = \dot{m}_s (h_s - h(T))$$

Further, $du = c_v dT$ and $h_s - h(T) = c_p (T_s - T)$. Thus

$$m c_v \frac{dT}{dt} = \dot{m}_s c_p (T_s - T)$$

With $dT = -d(T_s - T)$ and $c_p/c_v = k$

$$- \frac{m d(T_s - T)}{k (T_s - T)} = \dot{m}_s dt \quad (1)$$

Evaluating $\dot{m}_s$

$$\begin{aligned} \dot{m}_s &= \frac{(AV)_s}{v_s} = \frac{P(AV)_s}{RT_s} \\ &= \frac{(14.7 \text{ lbf/in}^2)(200 \text{ ft}^3/\text{min})}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}}\right)(580^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 13.69 \text{ lb/min} \end{aligned}$$

and

$$m = \frac{PV}{RT} = \frac{(14.7)(20,000)(144)}{\left(\frac{1545}{28.97}\right)(515)} = 154 \text{ lb}$$

where the average of the initial and final temperatures is used to estimate the mass. Also, from A-20E, $k = 1.4$.

Continued on next slide

Problem 4-122 continued

Integrating Eq. (1) from $t=0$ ($T=T_i = 40^\circ\text{F}$) to any time t

$$-\left(\frac{m}{k \cdot \dot{m}_s}\right) \ln\left(\frac{T_s - T}{T_s - T_i}\right) = t$$

or, solving for T

$$T = T_s - (T_s - T_i) \exp\left[\left(-\frac{\dot{m}_s k}{m}\right)t\right]$$

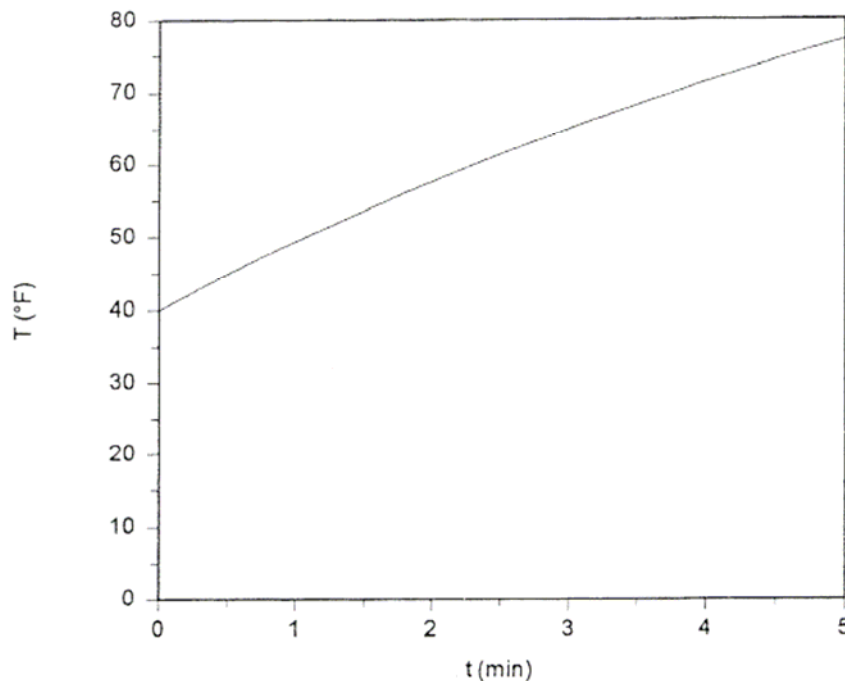
Inserting values

$$T = 120 - 80 \exp\left[(-0.1245)t\right] \quad (2)$$

Solving for t_2 when $T_2 = 70^\circ\text{F}$ gives

$$t_2 = 3.79 \text{ min} \longleftarrow t_2$$

Eq. (2) can readily be plotted using software. The following plot was constructed using IT:



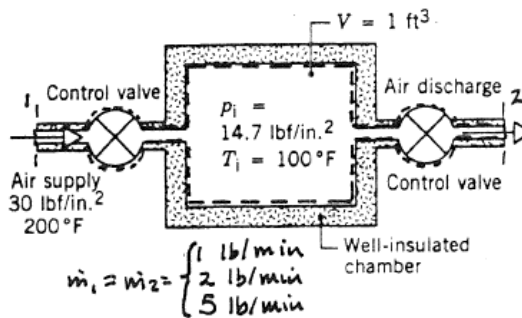
PROBLEM 4.123

KNOWN: Air flows through a well-insulated chamber. The initial conditions in the chamber, the supply conditions, and the inlet and exit mass flow rates are specified.

FIND: Determine the temperature and pressure of the air in the chamber as functions of time and plot for various mass flow rates.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) For the control volume shown, $\dot{Q}_{cv} = 0$ and $\dot{W}_{cv} = 0$. (2) The temperature and pressure in the chamber are uniform throughout at any time. (3) Kinetic and potential energy effects are negligible. (4) The air is modeled as an ideal gas with constant specific heats.



ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_1 - \dot{m}_2$. With $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, $dm_{cv}/dt = 0$. Thus, the amount of mass contained within the control volume is constant. Using assumptions listed, the energy balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

or, with $U_{cv} = m_u$

$$m_{cv} \frac{du}{dt} + u \frac{dm_{cv}}{dt} = \dot{m}[h_1 - h(t)]$$

where $h_2 = h(t)$ by assumption (2). Further, $du = c_v dT$ and $h(t) = c_p T(t)$, so

$$m_{cv} c_v \frac{dT}{dt} = \dot{m}[h_1 - c_p T(t)]$$

or

$$\frac{dT}{dt} + \left(\frac{\dot{m} c_p}{m_{cv} c_v} \right) T = \frac{\dot{m} c_p T_i}{m_{cv} c_v}$$

Introducing $k = c_p/c_v$

$$\frac{dT}{dt} + \left(\frac{\dot{m} k}{m_{cv}} \right) T = \left(\frac{\dot{m} k T_i}{m_{cv}} \right)$$

The solution of this differential equation takes the form

$$T(t) = C \exp \left[- \left(\frac{\dot{m} k}{m_{cv}} \right) t \right] + T_i$$

With $T(0) = T_i$; $C = T_i - T_i$, so

$$T(t) = (T_i - T_i) \exp \left[- \left(\frac{\dot{m} k}{m_{cv}} \right) t \right] + T_i \quad (1)$$

Continued on next slide

Problem 4-123 continued

From the given data

$$m_{cv} = \frac{P_i V_i}{RT_i} = \frac{(14.7 \text{ lbf/in}^2)(1 \text{ ft}^3)}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)(560^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 0.0709 \text{ lb}$$

From Table A-20E; $k = 1.4$. Also, $T_i = 100^\circ\text{F}$ and $T_\infty = 200^\circ\text{F}$. Thus, from (1)

$$\dot{m} = 1 \text{ lb/s} \quad T(t) = -100 e^{-0.3291 t} + 200$$

$$\dot{m} = 2 \text{ lb/s} \quad T(t) = -100 e^{-0.6582 t} + 200$$

$$\dot{m} = 5 \text{ lb/s} \quad T(t) = -100 e^{-1.6455 t} + 200$$

The pressure is $p = \frac{m R T(t)}{V}$. Thus

$$p = \left[\frac{(0.0709 \text{ lb}) \left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right)}{(1 \text{ ft}^3) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|} \right] [T(t) + 460]$$

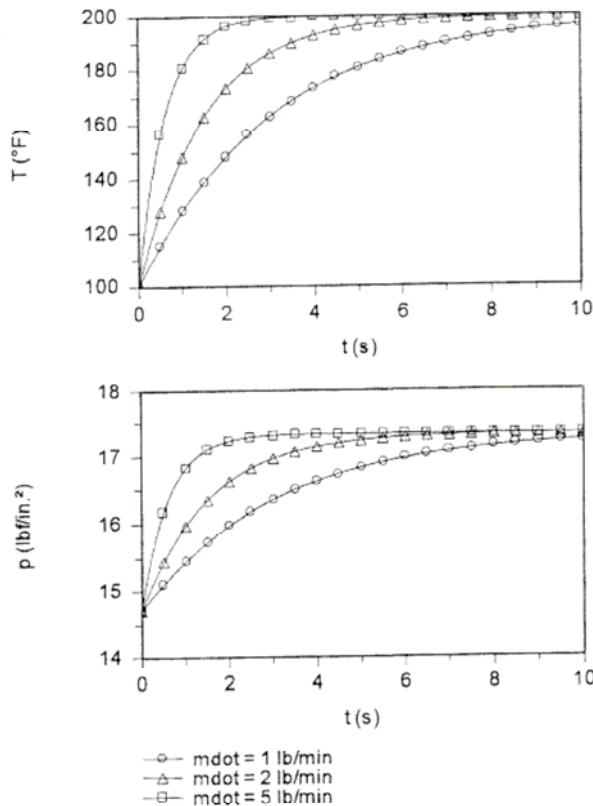
Finally

$$\dot{m} = 1 \text{ lb/s} \quad p(t) = -2.6258 e^{-0.3291 t} + 17.3304$$

$$\dot{m} = 2 \text{ lb/s} \quad p(t) = -2.6258 e^{-0.6582 t} + 17.3304$$

$$\dot{m} = 5 \text{ lb/s} \quad p(t) = -2.6258 e^{-1.6455 t} + 17.3304$$

Using IT to plot, we get

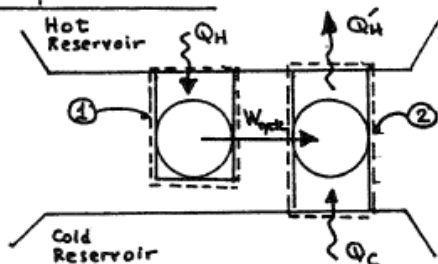


PROBLEM 5.1

KNOWN: A system undergoes a cycle in violation of the Kelvin-Planck statement of the second law.

FIND: Show that a violation of the Clausius statement of the second law is a consequence.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) System 1 undergoes a cycle while receiving energy Q_H from the hot reservoir and developing work W_{cycle} . (This is in violation of the Kelvin-Planck statement of the second law.) (2) System 2 undergoes a cycle while removing energy Q_C from the cold reservoir and discharging energy Q_H' to the hot reservoir. The work developed by System 1 is used to drive System 2.

ANALYSIS: An energy balance for system 1 reduces to

$$W_{\text{cycle}} = Q_H$$

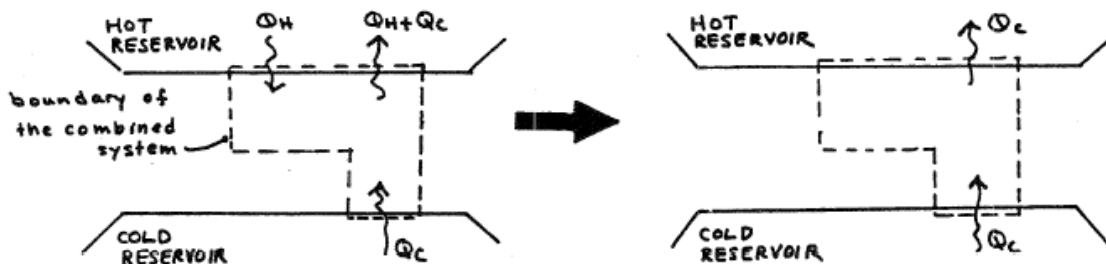
For System 2 an energy balance reads

$$W_{\text{cycle}} = Q_H' - Q_C$$

Combining these expressions

$$Q_H' = Q_H + Q_C$$

Next, consider a combined system consisting of Systems 1 and 2:



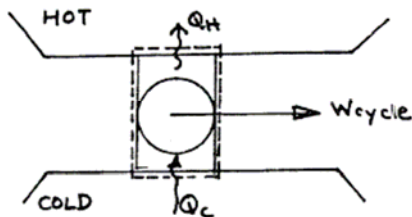
The sole result of the combined system is that a net amount of energy Q_C is transferred from the cold reservoir to the hot reservoir. This is a violation of the Clausius statement of the second law.

PROBLEM 5.2

KNOWN: A system undergoes a cycle while communicating with two reservoirs. The system develops work while receiving Q_c from the cold reservoir and discharging Q_H to the hot reservoir.

FIND: Evaluate the claimed performance using (a) the Clausius statement of the second law, (b) the Kelvin-Planck statement of the second law.

SCHEMATIC & GIVEN DATA:



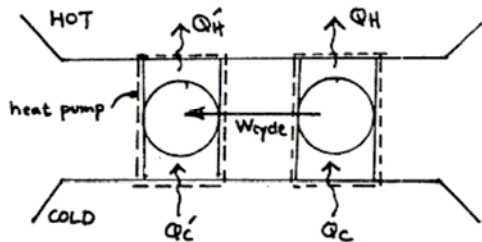
ENGR. MODEL: The system shown in the accompanying figure undergoes a power cycle while communicating thermally with two reservoirs as shown.

ANALYSIS: An energy balance for the system reads

$$W_{\text{cycle}} = Q_c - Q_H$$

The claim can be evaluated using either the Clausius or the Kelvin-Planck statement:

(a) **Clausius.** As shown in the figure below, let the given cycle drive a heat pump cycle.



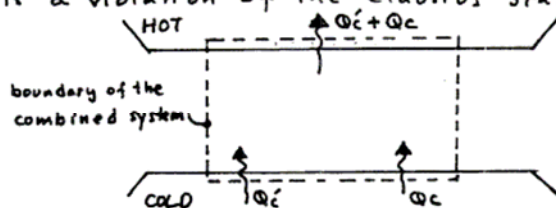
An energy balance for the heat pump reads

$$Q'_H = Q'_c + W_{\text{cycle}}$$

or, with $W_{\text{cycle}} = Q_c - Q_H$

$$Q'_H = Q'_c + Q_c - Q_H$$

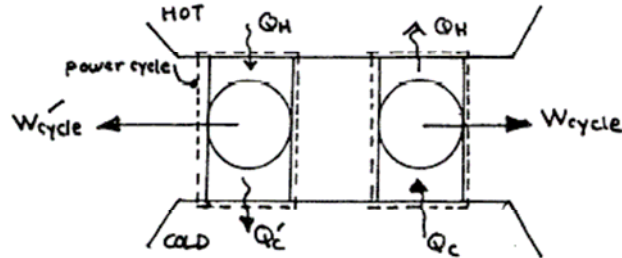
Next, consider the combined system of the two systems together. The sole result of the combined system is that a net amount of energy $Q'_c + Q_c$ is transferred from the cold reservoir to the hot reservoir. This is a violation of the Clausius statement of the second law, so the original system cannot operate as assumed. The inventor's claim is invalid.



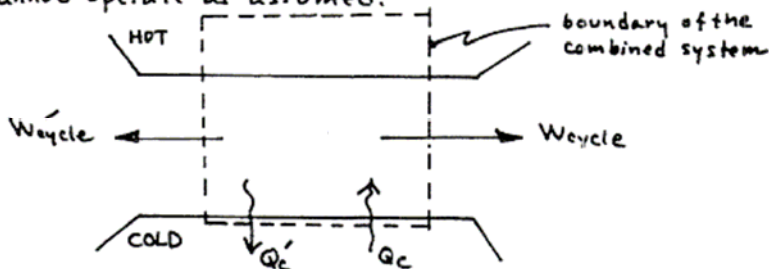
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Problem 5-2 continued

(b) Kelvin-Planck. As shown in the figure below, let Q_H be supplied to a system undergoing a power cycle.



Next, consider the combined system formed by the two systems and the hot reservoir. Observe that the hot reservoir experiences no net change as it receives Q_H from one cycle and supplies Q_H to the other. Thus, since the combined system is made up of parts that undergo cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the cold reservoir, while producing a net amount of work. However, this is a violation of the Kelvin-Planck statement of the second law, so the original system cannot operate as assumed.



PROBLEM 5.3

Classify the following processes of a closed system as possible, impossible, or indeterminate.

	Entropy Change	Entropy Transfer	Entropy Production
(a)	>0	0	
(b)	<0		>0
(c)	0	>0	
(d)	>0	>0	
(e)	0	<0	
(f)	>0		<0
(g)	<0	<0	

Eq. 5.2 for a closed system:

$$[\text{Entropy Change}] = [\text{Entropy Transfer}] + [\text{Entropy Production}]$$

(a) $[>0] = [0] + [\text{Entropy Production}] \Rightarrow [\text{Entropy Production}] > 0$. Possible.

(b) $[\text{Entropy Production}] > 0$. Possible.

(c) $[0] = [>0] + [\text{Entropy Production}] \Rightarrow [\text{Entropy Production}] < 0$. Impossible.

(d) $[>0] = [>0] + [\text{Entropy Production}]$. No conclusion about $[\text{Entropy Production}]$ can be reached without actual values for the other terms. Indeterminate.

(e) $[0] = [<0] + [\text{Entropy Production}] \Rightarrow [\text{Entropy Production}] > 0$. Possible.

(f) $[\text{Entropy Production}] < 0$. Impossible.

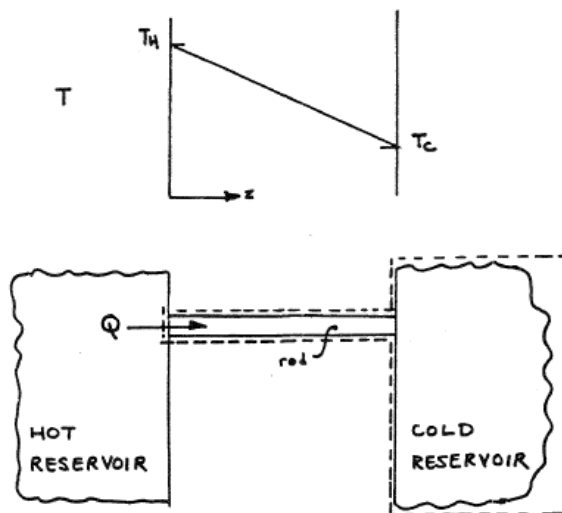
(g) $[<0] = [<0] + [\text{Entropy Production}]$. No conclusion about $[\text{Entropy Production}]$ can be reached without actual values for the other terms. Indeterminate.

PROBLEM 5.4

KNOWN: Energy transfer by conduction from a hot reservoir to a cold reservoir takes place spontaneously.

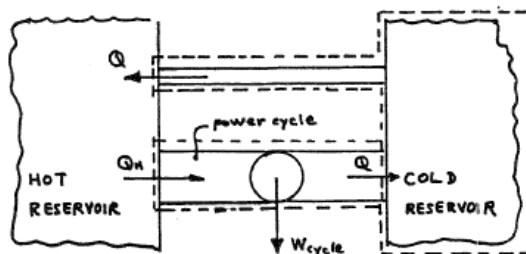
FIND: Using the Kelvin - Planck statement of the second law, show that such a process is irreversible.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the accompanying figure receives energy Q from the hot reservoir which passes by conduction through a rod at steady state to the cold reservoir. (2) A power cycle is available for use in demonstrating that the process is irreversible.

ANALYSIS: The objective in this proof by contradiction is to devise a system that undergoes a cycle which develops work while the system communicates thermally with a single reservoir, thereby violating the Kelvin-Planck statement.



Note: Q represents the energy transferred from the cold reservoir to the hot reservoir without any other effect.

Note: Q represents the energy discharged to the cold reservoir from the power cycle.

The system shown above consists of the original system plus a system capable of undergoing a power cycle (assumption 2). This combined system undergoes a cycle as follows. (1) An amount of energy Q_H is transferred from the hot reservoir to the power cycle, producing work W_{cycle} and discharging energy Q to the cold reservoir. (2) An amount of energy Q is transferred from the cold reservoir to the hot reservoir without any other effects. (This would be possible only if the process described in assumption 1 were reversible.)

Since Q is added to the cold reservoir by the power cycle in the first

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Problem 5-4 continued

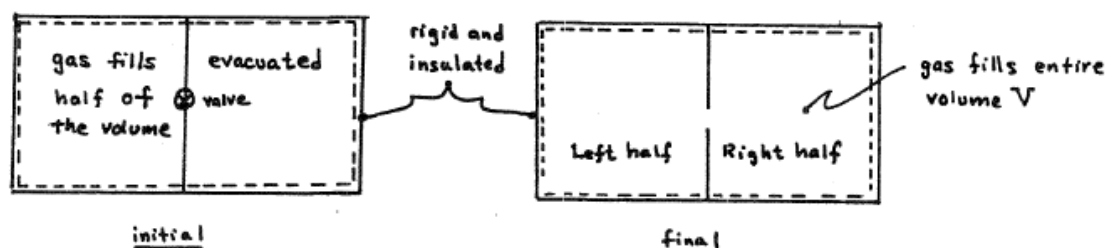
process of the cycle and the same amount of energy is removed from the cold reservoir in the second process of the cycle, this reservoir experiences no net change in its condition. The power cycle enclosed within the combined system also undergoes a cycle. Accordingly, the combined system undergoes a cycle in which work W_{cycle} is produced while exchanging energy by heat transfer with a single reservoir (the hot reservoir). Such a cycle violates the Kelvin-Planck statement of the second law, and thus is impossible. It follows that one, or both, of the processes making up the cycle executed by the combined system must be impossible. However, as the first process involving the power cycle can occur, it is the second process that must be impossible: energy Q cannot be transferred from the cold to the hot reservoir without other effects. By definition, then, the transfer of energy Q by conduction from the hot to the cold reservoir is irreversible.

PROBLEM 5.5

KNOWN: When an interconnecting valve is opened, a gas expands from one half of a tank to the other half, which is initially evacuated.

FIND: Using the Kelvin-Planck statement of the second law, show that such a process is irreversible.

SCHEMATIC & GIVEN DATA:



ENER. MODEL: (1) The system shown in the accompanying figure undergoes a process in which the gas expands spontaneously to fill the entire volume V . (2) During the process $Q = W = 0$. (3) The initial and final states are equilibrium states. There is no change in kinetic or potential energy between these states. (4) A turbine and a thermal reservoir are available for use in demonstrating that the process is irreversible.

ANALYSIS: The object in this proof by contradiction is to devise a system that undergoes a power cycle while the system communicates thermally with a single reservoir, thereby violating the Kelvin-Planck statement of the second law.

Before considering such a system, note that an energy balance for the spontaneous process is

$$U_{\text{final}} - U_{\text{initial}} = \cancel{Q} - \cancel{W}$$

where assumptions 2 and 3 have been used. Accordingly,

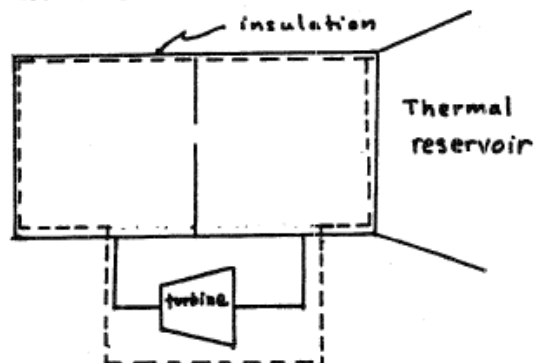
$$U_{\text{final}} = U_{\text{initial}}$$

That is, the internal energy of the system does not change in the free expansion.

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Problem 5-5 continued

As shown in the figure below, modify the system to include a turbine and introduce a thermal reservoir in the surroundings (assumption 4).



Starting with the gas filling the entire volume V , let the modified system undergo a cycle consisting of three processes.

Process 1. Let the reverse of the free expansion occur without any other effects. That is, the gas passes spontaneously from the right half of the tank until it fills only the left half. (This would be possible only if the process described in assumption 1 were reversible.)

Process 2. Let part of the gas expand through the turbine into the right half of the tank until the pressure in both halves is the same. In expanding through the turbine the gas does work so that its internal energy is decreased: $U < U_{\text{initial}}$.

Process 3. Remove part of the tank insulation and add energy by heat transfer from the thermal reservoir until the internal energy of the gas is restored to its initial value. Thus, a cycle is completed.

The net result of this cycle is to draw energy from a single reservoir by heat transfer and produce an equivalent amount of work. Such a cycle violates the Kelvin-Planck statement of the second law, and thus is impossible. Since both the heating of the gas by the reservoir (process 3) and the development of work as gas passes through the turbine (process 2) are possible, it can be concluded that it is process 1 that is impossible. Since process 1 is the reverse of the original free expansion, it follows that the original process is irreversible.

PROBLEM 5.6

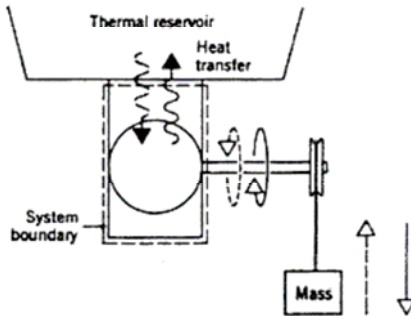
- (a) False. The first and second laws of thermodynamics are independent principles. See discussion of Sec. 5.1.
- (b) False. For any process of a closed system, internally reversible or otherwise, $\Delta E = Q - W$, where Q and W are independent means of energy transfer. Unless $\Delta E = 0$, the magnitudes and directions of Q and W are not coordinated.
- (c) False. See Eq. 5.2. For example, when there is entropy production but no entropy transfer, entropy change is positive.
- (d) True. Entropy is a property. Thus, the value of entropy change is determined by the end states and not the nature of the process.

PROBLEM 5.7

KNOWN: A system undergoes a cycle reversibly while communicating thermally with a single reservoir.

FIND: Show that $W_{\text{cycle}} = 0$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the accompanying figure undergoes a reversible cycle. (2) During the cycle the system communicates thermally only with a single reservoir.

ANALYSIS: Let the system undergo one cycle. According to the Kelvin-Planck statement of the second law, the work for the cycle is zero or negative in value: $W_{\text{cycle}} \leq 0$.

As the cycle is reversible, it is possible to return both the system and its surroundings to their initial states. Accordingly, there would be no net change in the condition of the reservoir or the elevation of the mass used to store energy in the surroundings.

① The above statements are consistent only if the sign of equality is used: $W_{\text{cycle}} = 0$.

1. In sec. 5.4, the converse is demonstrated: If the sign of equality applies: $W_{\text{cycle}} = 0$, then the cycle is reversible. Taken together, these two demonstrations establish the following proposition: If, and only if, the sign of equality applies in Eq. 5.1, the cycle is reversible.

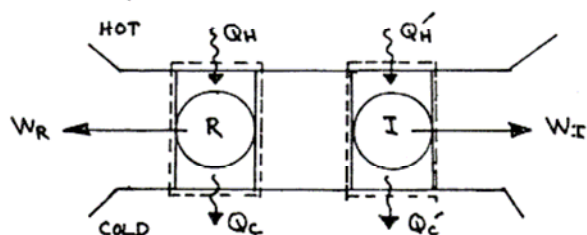
PROBLEM 5.8

KNOWN: A reversible power cycle R and an irreversible power cycle I operate between the same two reservoirs.

FIND: (a) If each cycle receives the same amount of energy from the hot reservoir, show that I discharges more energy to the cold reservoir.

(b) If each cycle develops the same net work, show that I receives more energy from the hot reservoir.

SCHEMATIC & GIVEN DATA:



To show:

(a) $Q_H = Q'_H \Rightarrow Q'_C > Q_C$

(b) $W_R = W_I \Rightarrow Q'_H > Q_H$

ENGR. MODEL: The system denoted by R in the accompanying figure undergoes a reversible power cycle while system I undergoes an irreversible power cycle.

ANALYSIS: (a) By the first Carnot Corollary, $\eta_R > \eta_I$. Since both cycles receive the same amount of energy Q_H , it follows that $W_R > W_I$.

An energy balance for R and I read, respectively

$$W_R = Q_H - Q_C$$

$$W_I = Q'_H - Q'_C$$

Collecting results

$$Q_H - Q_C > Q'_H - Q'_C$$

Accordingly

$$Q'_C > Q_C$$

as was to be demonstrated.

Thus, not only do actual cycles develop less work they also discharge more energy by heat transfer to their surroundings, thereby increasing the effect of thermal pollution.

(b) Since $\eta_R > \eta_I$ and $W_R = W_I \equiv W$,

$$\frac{W}{Q_H} > \frac{W}{Q'_H} \Rightarrow Q'_H > Q_H$$

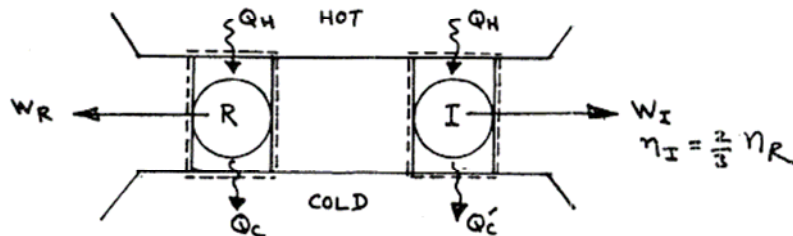
If the hot reservoir were maintained by, say, energy from the combustion of a fossil fuel, cycle I would have the greater fuel requirement. Also, note that cycle I also would have the greater energy discharge to the cold reservoir, so the comments of (a) would also be applicable.

PROBLEM 5.9

KNOWN: A power cycle I and a reversible power cycle R operate between the same two reservoirs. For these cycles, $\eta_I = \frac{2}{3} \eta_R$.

FIND: Show that cycle I must be irreversible.

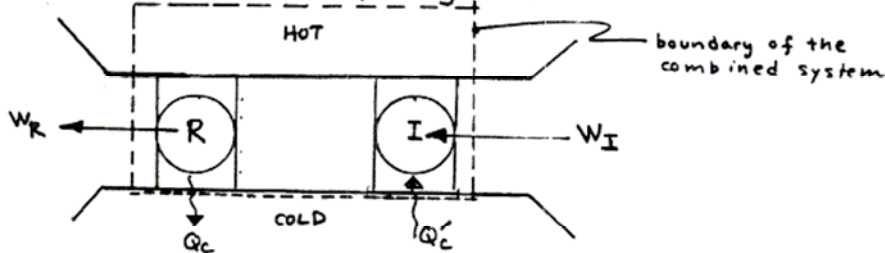
SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system denoted by R in the accompanying figure undergoes a reversible power cycle while system I undergoes a power cycle such that $\eta_I = \frac{2}{3} \eta_R$. (2) Both cycles receive the same energy Q_H from the hot reservoir.

ANALYSIS: If $\eta_I = \frac{2}{3} \eta_R$ and both cycles receive Q_H from the hot reservoir, $W_I = \frac{2}{3} W_R < W_R$.

In this proof by contradiction, assume I is reversible. Then, if I operates in the opposite direction as a refrigeration (or heat pump) cycle, the magnitudes of the energy transfers Q_H , Q'_C , and W_I would remain the same but would be oppositely directed as shown in the figure below. With I operating in the opposite direction, the hot reservoir would experience no net change since it would receive Q_H from I while passing Q_H to R.



As the combined system shown in the figure above is made up of parts that execute cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the cold reservoir. Accordingly, the combined system must satisfy Eq. 5.1 expressed as $W_{\text{cycle}} = 0$, where the sign of equality is used because R is reversible and I has been assumed reversible. Evaluating W_{cycle} for the combined system in terms of the work amounts W_R and W_I , $W_{\text{cycle}} = W_R - W_I$, it follows that

$$W_I = W_R$$

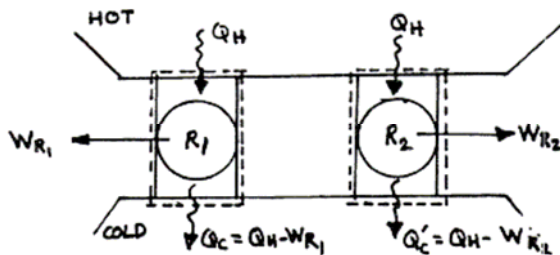
As this conclusion is not in agreement with the requirement that $W_I < W_R$, it can be concluded that the hypothesis is false and I must be irreversible.

PROBLEM 5.10

KNOWN: The Kelvin-Planck statement of the second law: $W_{\text{cycle}} \leq 0$ (single reservoir).

FIND: Show that all reversible power cycles operating between the same two reservoirs have the same thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The systems shown in the accompanying figure each undergo reversible power cycles while receiving the same amount of energy Q_H from the hot reservoir.

ANALYSIS: Let cycle R_1 operate as a reversible refrigeration (or heat pump cycle). The magnitudes of W_{R1} , Q_H , and Q_C remain the same but are now oppositely directed. Further, with R_1 working in the opposite direction, the hot reservoir would experience no net change in its condition since it would receive Q_H from R_1 while passing Q_H to R_2 .

The demonstration is completed by considering the combined system consisting of the two cycles and the hot reservoir. Since its parts execute cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the cold reservoir. Accordingly, the combined system must satisfy the Kelvin-Planck statement expressed as

$$W_{\text{cycle}} = 0$$

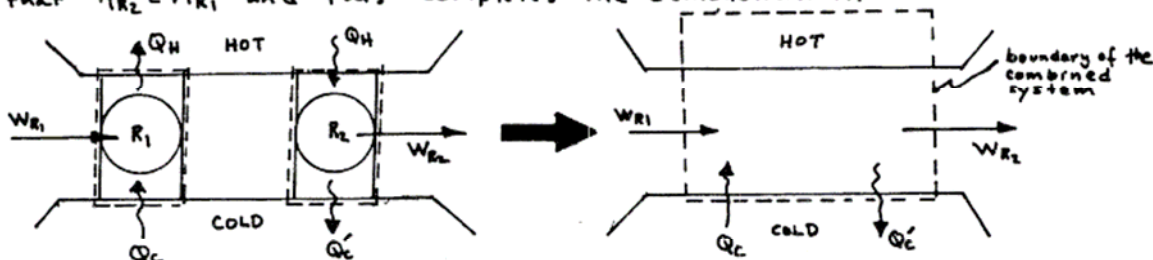
where the equality is used because all parts of the combined system are free of irreversibilities. Evaluating W_{cycle} in terms of the work amounts W_{R1} and W_{R2}

$$W_{R2} - W_{R1} = 0$$

or

$$W_{R2} = W_{R1}$$

Since each of the power cycles receives the same energy input Q_H , it follows that $\eta_{R2} = \eta_{R1}$, and this completes the demonstration.

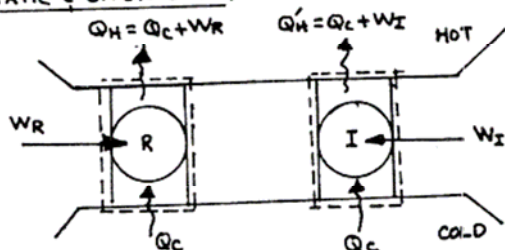


PROBLEM 5.11

KNOWN: The Kelvin-Planck statement of the second law: $W_{\text{cycle}} \leq 0$ (single reservoir).

FIND: Show that (a) the coefficient of performance of an irreversible refrigeration cycle is less than the coefficient of performance of a reversible refrigeration cycle when both exchange energy by heat transfer with the same two reservoirs, (b) all reversible refrigeration cycles operating between the same two reservoirs have the same coefficient of performance. For parts (c), (d), see next page.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The systems shown in the accompanying figure undergo refrigeration cycles. Each cycle removes energy Q_c from the cold reservoir. R accomplishes this reversibly while I is irreversible.

ANALYSIS: (a) Let R operate as a reversible power cycle. The magnitudes of W_R , Q_c and Q_H remain the same but are now oppositely directed. Further, with R working in the opposite direction, the cold reservoir would experience no net change in its condition since it would receive Q_c from R while passing Q_c to I.

The demonstration is completed by considering the combined system consisting of the two cycles and the cold reservoir. Since its parts execute cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the hot reservoir. Accordingly, the combined system must satisfy the Kelvin-Planck statement expressed as

$$W_{\text{cycle}} < 0$$

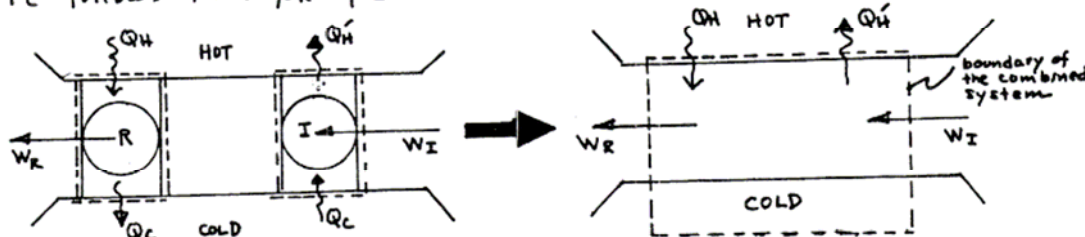
where the inequality is used because irreversible cycle I is included. Evaluating W_{cycle} in terms of the work amounts W_R and W_I

$$W_R - W_I < 0$$

or

$$W_R < W_I$$

Since each of the refrigeration cycles receives the same energy input Q_c , it follows that $\beta_R > \beta_I$ and this completes the demonstration.



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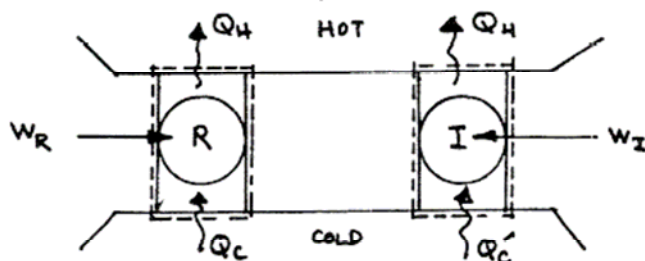
Problem 5-11 continued

ANALYSIS (b) This proposition can be demonstrated in a parallel way by considering two reversible refrigeration cycles R_1 and R_2 operating between the same two reservoirs. Then, letting R_1 play the role of R and R_2 the role of I in the previous development, a combined system consisting of the two cycles and the hot reservoir may be formed which must satisfy $W_{\text{cycle}} = 0$, where the equality is used because all parts of the combined system are free of irreversibilities. Thus, it can be concluded that $W_{R_1} = W_{R_2}$, and therefore $\beta_{R_1} = \beta_{R_2}$.

Parts (c), (d):

FIND: Show that (c) the coefficient of performance of an irreversible heat pump cycle is less than the coefficient of performance a reversible heat pump cycle when both exchange energy by heat transfer with the same two reservoirs, (d) all reversible heat pump cycles operating between the same two reservoirs have the same coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The systems shown in the accompanying figure undergo heat pump cycles. (2) Each cycle provides energy Q_H to the hot reservoir. R accomplishes this reversibly while I is irreversible. (3) Both cycles operate between the same two reservoirs.

ANALYSIS: (c) The demonstration parallels the approach used in part (a). However, in the present case the combined system would consist of the two cycles and the hot reservoir. (b). The demonstration parallels the approach used in part (b).

PROBLEM 5.12

KNOWN: The symbol Θ denotes temperature on Kelvin's logarithmic scale.

FIND: (a) Show that $\Theta = \ln T + C$, where T is temperature on the Kelvin scale. (b) Determine the range of temperature values on the logarithmic scale. (c) Obtain an expression for the thermal efficiency of a reversible power cycle operating between reservoirs at Θ_H and Θ_C on the logarithmic scale.

ANALYSIS: The two scales arise from different specifications of the function Ψ in Eq.(a) of Sec. 5.8.1

$$\left(\frac{\Theta_C}{\Theta_H}\right)_{\text{rev cycle}} = \Psi$$

That is

$$\left(\frac{\Theta_C}{\Theta_H}\right)_{\text{rev cycle}} = \frac{T_C}{T_H} \quad \text{Kelvin scale}$$

$$\left(\frac{\Theta_C}{\Theta_H}\right)_{\text{rev cycle}} = \frac{\exp \Theta_C}{\exp \Theta_H} \quad \text{Logarithmic scale}$$

(a) By comparison of the last two equations

$$\frac{T_C}{T_H} = \frac{\exp \Theta_C}{\exp \Theta_H} = \exp [\Theta_C - \Theta_H]$$

Thus

$$\ln T_C - \ln T_H = \Theta_C - \Theta_H$$

and therefore

$$\Theta = \ln T + C \quad \leftarrow \text{(a)}$$

where C is a constant determining the level of temperature corresponding to zero on the logarithmic scale.

(b) Temperatures on the Kelvin scale vary from 0 to $+\infty$. With the relationship of part (a), temperatures on the logarithmic scale vary from $-\infty$ to $+\infty$. (b)

(c) Use of

$$\left(\frac{\Theta_C}{\Theta_H}\right)_{\text{rev cycle}} = \frac{\exp \Theta_C}{\exp \Theta_H}$$

in Eq 5.4 gives an expression for the thermal efficiency of a reversible power cycle while operating between reservoirs at temperatures Θ_H and Θ_C on the logarithmic scale

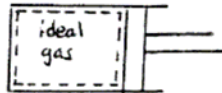
$$\eta = 1 - \frac{\exp \Theta_C}{\exp \Theta_H} \quad \leftarrow \text{(c)}$$

PROBLEM 5.13

KNOWN: The gas temperature scale introduced in Sec. 5.8.2 and the Kelvin scale defined in Sec. 5.8.1

FIND: Demonstrate that the two scales are identical.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the figure consists of a gas obeying $pV = RT'$, where T' denotes temperature on the gas scale. (2) The system undergoes a reversible cycle while communicating thermally with reservoirs at T_H' and T_C' . (3) The cycle consists of four processes in series: 1-2 isothermal at T_H' , 2-3 adiabatic, 3-4 isothermal at T_C' , 4-1 adiabatic. (4) Kinetic and potential energy effects are absent.

ANALYSIS: With assumption (4), an energy balance in differential form reads

$$du = \delta(Q/m) - \delta(W/m)$$

Then, since the processes are internally reversible, $\delta(W/m) = p dv$. Also, for the ideal gas, $du = c_v dT'$. Collecting results

$$c_v dT' = \delta(Q/m) - p dv \Rightarrow c_v dT' = \delta(Q/m) - \frac{RT'}{v} dv \Rightarrow \underline{c_v dT' = \delta(Q/m) - RT' \frac{dv}{v}}$$

Process 1-2: The gas temperature is constant at T_H' , so

$$\int_{1-2} c_v dT' = \delta(Q/m)_{1-2} - RT_H' \ln(v_2/v_1) \Rightarrow (Q/m)_{1-2} = RT_H' \ln(v_2/v_1) \quad (1)$$

Process 2-3: There is no heat transfer, and the gas temperature varies from T_H' to T_C' , so

$$c_v dT' = \delta(Q/m) - RT' \frac{dv}{v} \Rightarrow \int_{T_H'}^{T_C'} \frac{c_v dT'}{T'} = -R \ln\left(\frac{v_3}{v_2}\right) \quad (2)$$

Process 3-4: The gas temperature is constant at T_C' , and the heat rejected in the process is

$$(Q/m)_{3-4} = RT_C' \ln(v_4/v_3) \Rightarrow |(Q/m)_{3-4}| = RT_C' \ln\left(\frac{v_3}{v_4}\right) \quad (3)$$

Process 4-1: There is no heat transfer, and the gas temperature varies from T_C' to T_H' , so

$$\int_{T_C'}^{T_H'} \frac{c_v dT'}{T'} = -R \ln\left(\frac{v_1}{v_4}\right) \Rightarrow \int_{T_H'}^{T_C'} \frac{c_v dT'}{T'} = -R \ln\left(\frac{v_4}{v_1}\right) \quad (4)$$

From Equations (2) and (4), $v_3/v_2 = v_4/v_1$ or $v_3/v_4 = v_2/v_1$. Using this result together with Equations (1) and (3) gives

$$\frac{|(Q/m)_{3-4}|}{(Q/m)_{1-2}} = \frac{T_C'}{T_H'}$$

which corresponds to Equation 5.7 underlying the Kelvin scale. This implies $T' \equiv T$, where T denotes temperature on the Kelvin scale.

1. The reversible cycle considered here corresponds to the Carnot cycle discussed in Sec. 5.10.

PROBLEM 5.15

The expression for resistance is

$$R = R_0 \exp \left[\beta \left(\frac{1}{T} - \frac{1}{T_0} \right) \right]$$

where $R_0 = 2.2 \, \Omega$ and $T_0 = 310 \, \text{K}$.

Since $R = 0.31 \, \Omega$ at $T = 422 \, \text{K}$

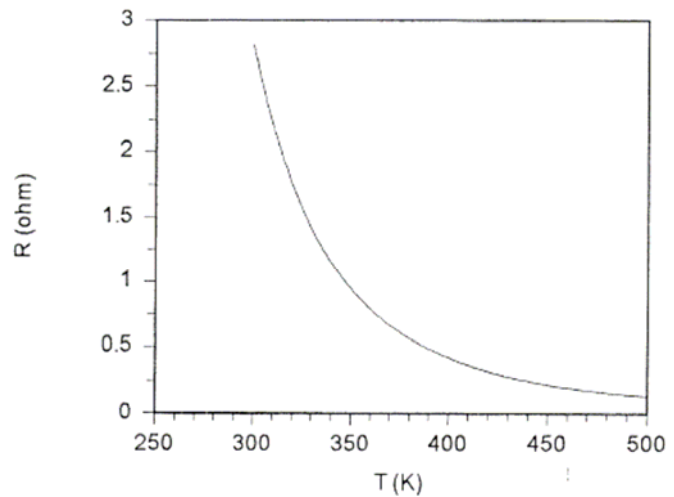
$$.31 = (2.2) \exp \left[\beta \left(\frac{1}{422} - \frac{1}{310} \right) \right]$$

Solving for β

$$\beta = \frac{\ln(.31/2.2)}{\left(\frac{1}{422} - \frac{1}{310} \right)} = 2288.9 \, \text{K}$$

Thus

$$R = 2.2 \exp \left[2288.9 \left(\frac{1}{T} - \frac{1}{310} \right) \right]$$



PROBLEM 5.16

GIVEN DATA:

	$T (^{\circ}\text{F})$	$R (\Omega)$
Test 1	32	51.39
Test 2	196	51.72

$$R = R_0 [1 + \alpha(T - T_0)]$$

$$\text{Let } T_0 = 32^{\circ}\text{F}$$

From the data at $T = 32^{\circ}\text{F}$

$$51.39 = R_0 [1 + \alpha(32 - 32)] \Rightarrow R_0 = 51.39 \Omega$$

At $T = 196^{\circ}\text{F}$, $R = 51.72$. Thus

$$51.72 = 51.39 [1 + \alpha(196 - 32)] \Rightarrow \alpha = 3.9155 \times 10^{-5} (^{\circ}\text{F})^{-1}$$

$$\text{Finally, } R = 51.39 [1 + 3.9155 \times 10^{-5} (T - 32)]$$

$$R = 51.326 + 2.0122 \times 10^{-3} T$$

$$T = 496.97 (R - 51.326)$$

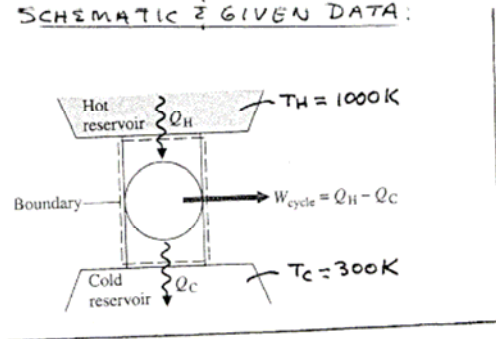
$$T (R = 51.47) = 71.6^{\circ}\text{F} \leftarrow$$

PROBLEM 5.17

KNOWN: Data are provided for a power cycle operating between hot and cold reservoirs.

FIND: In each case, determine whether the cycle operates reversibly, operates irreversibly, or is impossible.

SCHEMATIC & GIVEN DATA:



(a) $Q_H = 600\text{ kJ}, W_{\text{cycle}} = 300\text{ kJ}, Q_C = 300\text{ kJ}$.

Check energy balance:
 $W_{\text{cycle}} = Q_H - Q_C = 600 - 300 = 300\text{ kJ}$ (OK)

With the given data,

$$\eta = \frac{W_{\text{cycle}}}{Q_H} = \frac{300\text{ kJ}}{600\text{ kJ}} = 0.5$$

The maximum thermal efficiency is
 $\eta_{\text{MAX}} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{1000} = 0.7$

Since $\eta < \eta_{\text{MAX}}$, the cycle operates irreversibly.

(b) $Q_H = 400\text{ kJ}, W_{\text{cycle}} = 280\text{ kJ}, Q_C = 120\text{ kJ}$.

Check energy balance:
 $W_{\text{cycle}} = Q_H - Q_C = 400 - 120 = 280\text{ kJ}$ (OK)

With the given data

$$\eta = \frac{W_{\text{cycle}}}{Q_H} = \frac{280}{400} = 0.7$$

From part (a), $\eta_{\text{MAX}} = 0.7$.

Since $\eta = \eta_{\text{MAX}}$, the cycle operates reversibly.

(c) $Q_H = 700\text{ kJ}, W_{\text{cycle}} = 300\text{ kJ}, Q_C = 500\text{ kJ}$.

Check energy balance:

$$W_{\text{cycle}} = Q_H - Q_C = 700 - 500 = 200\text{ kJ}$$

This does not agree with the claimed value. The cycle cannot operate as claimed. Impossible.

(d) $Q_H = 800\text{ kJ}, W_{\text{cycle}} = 600\text{ kJ}, Q_C = 200\text{ kJ}$.

Check energy balance:

$$W_{\text{cycle}} = Q_H - Q_C = 800\text{ kJ} - 200\text{ kJ} = 600\text{ kJ}$$
 (OK)

With the given data

$$\eta = \frac{W_{\text{cycle}}}{Q_H} = \frac{600}{800} = 0.75$$

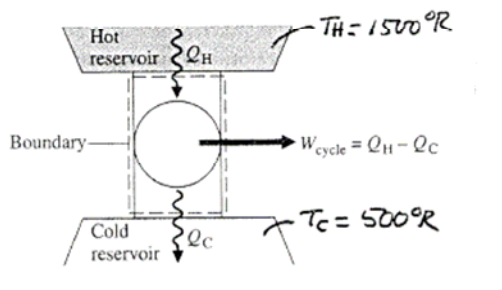
From part (a), $\eta_{\text{MAX}} = 0.70$. Since $\eta > \eta_{\text{MAX}}$, the cycle cannot operate as claimed. Impossible.

PROBLEM 5.18

KNOWN: Data are provided for a power cycle operating between hot and cold reservoirs.

FIND: In each case, determine whether the cycle operates reversibly, operates irreversibly, or is impossible.

SCHEMATIC & GIVEN DATA:



(a) $Q_H = 900 \text{ Btu}, W_{\text{cycle}} = 450 \text{ Btu}$

With the given data,

$$\eta = \frac{W_{\text{cycle}}}{Q_H} = \frac{450 \text{ Btu}}{900 \text{ Btu}} = 0.5$$

The maximum thermal efficiency is

$$\eta_{\text{MAX}} = 1 - \frac{T_C}{T_H} = 1 - \frac{500^\circ R}{1500^\circ R} = 0.667$$

Since $\eta < \eta_{\text{MAX}}$, the cycle operates irreversibly.

(b) $Q_H = 900 \text{ Btu}, Q_C = 300 \text{ Btu}$

From the energy balance

$$W_{\text{cycle}} = Q_H - Q_C = 900 - 300 = 600 \text{ Btu}$$

With these data, the thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_H} = \frac{600}{900} = 0.667$$

Since $\eta = \eta_{\text{MAX}}$, the cycle operates reversibly.
(Part (a))

(c) $W_{\text{cycle}} = 600 \text{ Btu}, Q_C = 400 \text{ Btu}$

From the energy balance

$$W_{\text{cycle}} = Q_H - Q_C$$

$$\Rightarrow Q_H = W_{\text{cycle}} + Q_C = 600 + 400 = 1000 \text{ Btu}$$

With these data, the thermal efficiency is

$$\eta = \frac{600}{1000} = 0.60$$

Since $\eta < \eta_{\text{MAX}}$, the cycle
(Part (a))
operates irreversibly.

(d) $\eta = 70\%$

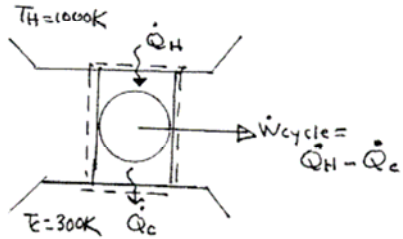
Since $\eta > \eta_{\text{MAX}} = 0.667$ (Part (a)),
this cycle cannot operate as
claimed. Impossible.

PROBLEM 5.19

KNOWN: Data are provided for a power cycle operating between hot and cold reservoirs. The data are for steady-state operation.

FIND: In each case, determine whether the cycle operates reversibly, operates irreversibly, or is impossible.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

Using Eq. 5.9, the maximum theoretical thermal efficiency any such cycle can achieve is

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{1000} = 0.7 (70\%)$$

(a) $\dot{Q}_H = 500 \text{ kW}, \dot{Q}_C = 100 \text{ kW}$

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = 1 - \frac{\dot{Q}_C}{\dot{Q}_H} = 1 - \frac{100 \text{ kW}}{500 \text{ kW}} = 0.8 (80\%)$$

Since $\eta > \eta_{MAX}$, this case is impossible.

(b) $\dot{Q}_H = 500 \text{ kW}, \dot{W}_{cycle} = 250 \text{ kW}, \dot{Q}_C = 200 \text{ kW}$

Check energy balance:

$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C = 500 \text{ kW} - 200 \text{ kW} = 300 \text{ kW}$$

Since $\dot{W}_{cycle}$ is given as 250 kW, the energy balance is not satisfied. This case is impossible.

(c) $\dot{W}_{cycle} = 350 \text{ kW}, \dot{Q}_C = 150 \text{ kW}$

Using the energy balance,

$$\dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_C = 350 \text{ kW} + 150 \text{ kW} = 500 \text{ kW}$$

Then

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{350 \text{ kW}}{500 \text{ kW}} = 0.7 (70\%)$$

Since $\eta = \eta_{MAX}$, this case corresponds to reversible operation.

(d) $\dot{Q}_H = 500 \text{ kW}, \dot{Q}_C = 200 \text{ kW}$

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = 1 - \frac{\dot{Q}_C}{\dot{Q}_H} = 1 - \frac{200 \text{ kW}}{500 \text{ kW}} = 0.6 (60\%)$$

Since $\eta < \eta_{MAX}$, this case corresponds to irreversible operation.

PROBLEM 5.20

KNOWN: Data are provided for four cases involving reversible power cycles operating between hot and cold reservoirs.

SCHEMATIC & GIVEN DATA:

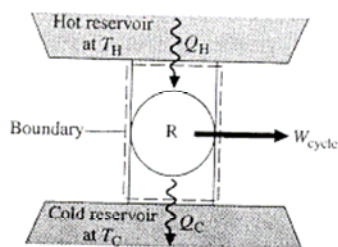


Fig. P5.20

(a) $T_H = 1200\text{ K}$, $T_C = 300\text{ K}$. Find η .

With Eq. 5.9, $\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{300\text{ K}}{1200\text{ K}} = 0.75 \text{ (75\%)} \quad \leftarrow (a)$

(b) $T_H = 500^\circ\text{C}$, $T_C = 20^\circ\text{C}$, $W_{\text{cycle}} = 1000\text{ kJ}$. Find Q_C and Q_H .

With Eq. 5.9 and $\eta = W_{\text{cycle}}/Q_H$, which applies for any power cycle,

$$\frac{W_{\text{cycle}}}{Q_H} = 1 - \frac{T_C}{T_H} \Rightarrow Q_H = \frac{W_{\text{cycle}}}{1 - \frac{T_C}{T_H}} = \frac{1000\text{ kJ}}{1 - \frac{293\text{ K}}{773\text{ K}}} = 1610\text{ kJ}$$

For any power cycle, $W_{\text{cycle}} = Q_H - Q_C$; thus,

$$Q_C = Q_H - W_{\text{cycle}} = 1610\text{ kJ} - 1000\text{ kJ} = 610\text{ kJ}$$

(c) $\eta = 60\%$, $T_C = 40^\circ\text{F}$. Find T_H .

With Eq. 5.9,

$$\eta = 1 - \frac{T_C}{T_H} \Rightarrow 0.60 = 1 - \frac{500^\circ\text{R}}{T_H} \Rightarrow T_H = 1250^\circ\text{R} \text{ (790}^\circ\text{F)} \quad \leftarrow (c)$$

(d) $\eta = 40\%$, $T_H = 727^\circ\text{C}$. Find T_C .

With Eq. 5.9,

$$\eta = 1 - \frac{T_C}{T_H} \Rightarrow 0.40 = 1 - \frac{T_C}{1000\text{ K}} \Rightarrow T_C = 600\text{ K} \text{ (327}^\circ\text{C)} \quad \leftarrow (d)$$

PROBLEM 5.21

KNOWN: Data are provided for a power cycle operating between hot and cold reservoirs.

FIND: Determine (a) η and (b) the temperature T_C of the cold reservoir.

ANALYSIS: $Q_H = 100 \text{ kJ}$, $T_H = 327^\circ\text{C}$, $Q_C = 40 \text{ kJ}$

(a) $\eta = \frac{W_{\text{cycle}}}{Q_H} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{40 \text{ kJ}}{100 \text{ kJ}} = 0.6 (60\%)$ $\longleftarrow$

(b) Since the cycle operates reversibly, $\eta = \eta_{\text{MAX}} = 1 - \frac{T_C}{T_H}$. Thus, with $T_H = 327^\circ\text{C} + 273 = 600 \text{ K}$

$$0.6 = 1 - \frac{T_C}{T_H} = 1 - \frac{T_C}{600}$$

Solving, $T_C = 240 \text{ K} (-33^\circ\text{C})$ $\longleftarrow$

PROBLEM 5.22

KNOWN: A power cycle operates between hot and cold reservoirs at 600°C and 112°C , respectively.

FIND: Determine the maximum theoretical thermal efficiency for any such cycle.

ANALYSIS: The maximum theoretical thermal efficiency is given by Eq. 5.9.

$$\eta_{\text{MAX}} = 1 - \frac{T_c}{T_h} = 1 - \frac{(112 + 273)}{(602 + 273)} = 1 - \frac{385\text{K}}{875\text{K}} = 0.56 \text{ (56\%)} \leftarrow$$

PROBLEM 5.23

KNOWN: Data are provided for a reversible power cycle operating as in Fig. 5.5: $T_C = 40^\circ\text{F}$, $W_{\text{cycle}} = 3Q_C$.

FIND: Determine η and T_H .

ANALYSIS: (a) $W_{\text{cycle}} = Q_H - Q_C \Rightarrow Q_H = W_{\text{cycle}} + Q_C = 3Q_C + Q_C = 4Q_C$

$$\eta = \frac{W_{\text{cycle}}}{Q_H} = \frac{3Q_C}{4Q_C} = 0.75 \text{ (75\%)} \quad \leftarrow$$

(b) Since the power cycle operates reversibly,

$$\eta_{\text{MAX}} = 1 - \frac{T_C}{T_H} \Rightarrow 0.75 = 1 - \frac{(40 + 460)}{T_H} \quad \leftarrow$$

Solving $T_H = 2000^\circ\text{R} \text{ (1540}^\circ\text{F)}$

PROBLEM 5.24

KNOWN: A reversible power cycle has the same thermal efficiency for each of two sets of conditions: $T_H = T$, $T_C = 500\text{K}$ and $T_H = 2000\text{K}$, $T_C = 1000\text{K}$.

FIND: Determine T .

ANALYSIS: $\eta = 1 - \frac{500\text{K}}{T}$ and $\eta = 1 - \frac{1000\text{K}}{2000\text{K}} \Rightarrow T = 1000\text{K} \quad \leftarrow$

PROBLEM 5.25

KNOWN: Data are provided for two reversible power cycle in series that produce the same net work.

FIND: Determine (a) the intermediate temperature T and the thermal efficiency for each of the two power cycles. (b) For a single cycle, determine the thermal efficiency and net work.

SCHEMATIC & GIVEN DATA:

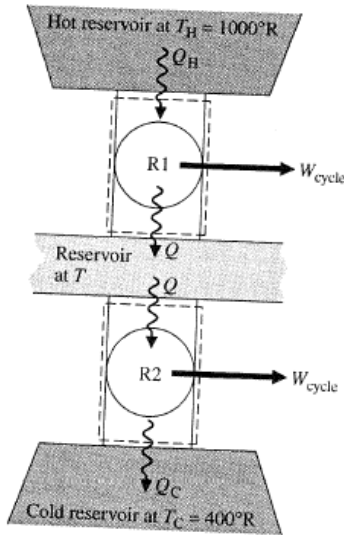


Fig. P5.25

ANALYSIS:

(a) Energy balances: Since W_{cycle} is the net work developed by each cycle

$$W_{\text{cycle}} = Q_H - Q = Q - Q_C$$

$$\Rightarrow Q = \frac{Q_H + Q_C}{2} \quad (1)$$

Since the cycles are reversible, Eq. 5.7 applies to each:

$$\frac{Q}{Q_H} = \frac{T}{T_H}, \quad \frac{Q_C}{Q} = \frac{T}{T_C}$$

$$\Rightarrow Q_H = \frac{T_H}{T} Q$$

$$Q_C = \frac{T_C}{T} Q$$

Substituting these results into Eq. (1)

$$Q = \frac{\frac{T_H}{T} Q + \frac{T_C}{T} Q}{2}$$

$$\Rightarrow T = \frac{T_H + T_C}{2} = \frac{1000^\circ\text{R} + 400^\circ\text{R}}{2} = 700^\circ\text{R}$$

Then, for R1:

$$\eta_1 = 1 - \frac{T}{T_H} = 1 - \frac{700^\circ\text{R}}{1000^\circ\text{R}} = 0.3 \quad (30\%)$$

For R2:

$$\eta_2 = 1 - \frac{T_C}{T} = 1 - \frac{400^\circ\text{R}}{700^\circ\text{R}} = 0.43 \quad (43\%)$$

(b) For the single reversible cycle operating between $T_H = 1000^\circ\text{R}$ and $T_C = 400^\circ\text{R}$,

$$\eta_{\text{MAX}} = 1 - \frac{T_C}{T_H} = 1 - \frac{400^\circ\text{R}}{1000^\circ\text{R}} = 0.6 \quad (60\%)$$

Also, the net work of this cycle is

$$W_{\text{cycle}} = Q_H - Q_C$$

$$= (Q_H - Q) + (Q - Q_C)$$

$$= 2 W_{\text{cycle}}$$

PROBLEM 5.26

KNOWN: Data are provided for a reversible power cycle: $T_H = 1000 \text{ K}$ and $T_C = 300 \text{ K}$. $Q_H/\text{cycle} = 100 \text{ kJ/cycle}$.

FIND: For 10 cycles of operation, find W_{cycle} .

ANALYSIS: Since the cycle is reversible, $\eta = \eta_{\text{max}}$, giving

$$\frac{W_{\text{cycle}}}{Q_H} = 1 - \frac{T_C}{T_H} \quad , \quad \text{where } Q_H = \left(\frac{100 \text{ kJ}}{\text{cycle}} \right) (10 \text{ cycles}) = 1000 \text{ kJ}$$

Thus,

$$W_{\text{cycle}} = 1000 \text{ kJ} \left[1 - \frac{300 \text{ K}}{1000 \text{ K}} \right] = 700 \text{ kJ}$$

PROBLEM 5.27

KNOWN: Data are provided for a reversible power cycle: $T_H = 1040^\circ\text{F}$, $T_C = 40^\circ\text{F}$
and $W_{\text{cycle/cycle}} = 600 \text{ Btu/cycle}$.

FIND: For 3 cycles of operation, find Q_H .

ANALYSIS: Since the cycle is reversible, $\eta = \eta_{\text{MAX}}$, giving

$$\frac{W_{\text{cycle}}}{Q_H} = 1 - \frac{T_C}{T_H}, \quad \text{where } W_{\text{cycle}} = \left(600 \frac{\text{Btu}}{\text{cycle}}\right)(3 \text{ cycles}) = 1800 \text{ Btu}.$$

Thus

$$\frac{1800 \text{ Btu}}{Q_H} = 1 - \frac{(40 + 460)}{(1040 + 460)} = \frac{2}{3} \Rightarrow Q_H = 2700 \text{ Btu} \quad \leftarrow$$

PROBLEM 5.28

KNOWN: An inventor claims to have a power cycle operating between hot and cold reservoirs at T_H and $T_C = 300\text{K}$, respectively, for which $\eta = 40\%$

FIND: Evaluate the claim if T_H is (a) 600K , (b) 500K , (c) 400K .

ANALYSIS: For the claim to be possible, $\eta \leq \eta_{\text{MAX}}$. That is, $\eta \leq 1 - \frac{T_C}{T_H}$ or $.40 \leq 1 - \frac{300\text{K}}{T_H}$.

(a) $T_H = 600\text{K}$:

$$\eta_{\text{MAX}} = 1 - \frac{300\text{K}}{600\text{K}} = 0.5 (50\%). \quad \text{Possible. The cycle would be operating irreversibly.}$$

(b) $T_H = 500\text{K}$

$$\eta_{\text{MAX}} = 1 - \frac{300\text{K}}{500\text{K}} = 0.4 (40\%). \quad \text{Possible but the cycle would be operating reversibly. Thus, the claim is unlikely.}$$

(c) $T_H = 400\text{K}$

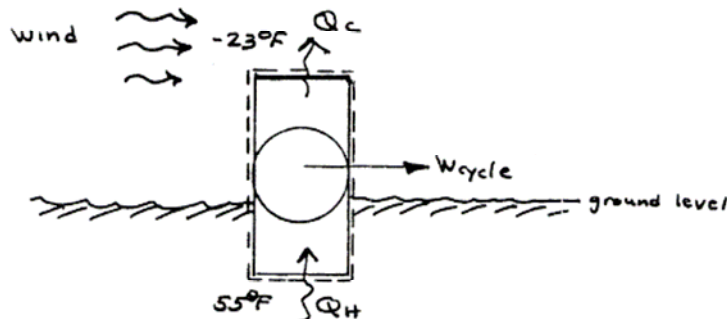
$$\eta_{\text{MAX}} = 1 - \frac{300\text{K}}{400\text{K}} = 0.25 (25\%) \quad \text{Impossible. The claimed performance is greater than the maximum theoretical value.}$$

PROBLEM 5.29

KNOWN: A power cycle operates between the ground at a temperature of 55°F and the atmosphere at a temperature of -23°F . A thermal efficiency of 14% is claimed.

FIND: Evaluate this claim.

SCHEMATIC & GIVEN DATA:



ENGINE MODEL: (1) The system shown in the figure undergoes a power cycle. (2) The ground at 55°C and the atmosphere at -23°F play the roles of hot and cold reservoirs, respectively.

ANALYSIS: The maximum thermal efficiency any power cycle may have when operating between hot and cold reservoirs at T_H and T_C , respectively, is given by Eq. 5.9:

$$\eta_{\text{MAX}} = 1 - \frac{T_C}{T_H}$$

$$= 1 - \frac{(460 - 23)^\circ\text{R}}{(460 + 55)^\circ\text{R}} = 0.151 \quad (15.1\%)$$

The claimed performance is in agreement with the requirements of the second law. Still, the efficiency value is so low as to raise concerns over economic viability.

PROBLEM 5.30

KNOWN: An inventor claims to have developed a power cycle operating between hot and cold reservoirs at 1000K and 250K that develops net work equal to $W_{\text{cycle}} = N Q_c$.

FIND: Determine the maximum theoretical value of N for any such cycle.

ANALYSIS: Any such cycle must satisfy $\eta \leq \eta_{\text{MAX}} = 1 - T_c/T_H$,

where $\eta = W_{\text{cycle}}/Q_H$. That is

$$\frac{W_{\text{cycle}}}{Q_H} \leq 1 - \frac{250\text{K}}{1000\text{K}} = 0.75$$

An energy balance gives $Q_H = W_{\text{cycle}} + Q_c$. Thus

$$\frac{W_{\text{cycle}}}{W_{\text{cycle}} + Q_c} \leq 0.75$$

Since $W_{\text{cycle}} = N Q_c$, this becomes

$$\frac{N Q_c}{N Q_c + Q_c} \leq 0.75 \Rightarrow \frac{N}{N+1} \leq 0.75$$

Solving, $N \leq 3$. $N_{\text{MAX}} = 3$.

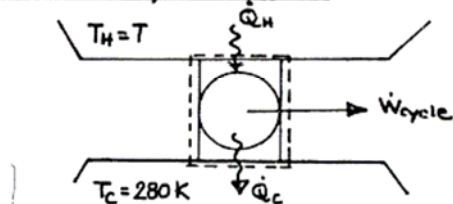


PROBLEM 5.31

KNOWN: Steady state operating data are provided for a system undergoing a power cycle while receiving and discharging energy by heat transfer with a hot and cold reservoir at temperatures T and 280 K , respectively

FIND: Determine the minimum theoretical value for T , in K .

SCHEMATIC & GIVEN DATA:



$$\begin{aligned}\dot{W}_{\text{cycle}} &= 40\text{ kW} \\ \dot{Q}_C &= 1000\text{ kJ/min}\end{aligned}$$

ENGR. MODEL: (1) The system is shown on the accompanying figure. (2) The system undergoes a power cycle. (3) The data provided is for operation at steady state.

ANALYSIS: An energy rate balance gives

①

or

$$\dot{W}_{\text{cycle}} = \dot{Q}_H - \dot{Q}_C$$

$$\begin{aligned}\dot{Q}_H &= \dot{W}_{\text{cycle}} + \dot{Q}_C \\ &= 40\text{ kW} \left(\frac{1\text{ kJ/s}}{1\text{ kW}} \right) + 1000 \frac{\text{kJ}}{\text{min}} \left(\frac{\text{min}}{60\text{ s}} \right) \\ &= 56.67\text{ kJ/s}\end{aligned}$$

We know that $\eta \leq \eta_{\text{max}}$; that is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_H} \leq 1 - \frac{T_C}{T}$$

Inserting values

$$\frac{40\text{ kJ/s}}{56.67\text{ kJ/s}} \leq 1 - \frac{280\text{ K}}{T}$$

$$\Rightarrow \frac{280}{T} \leq 1 - \frac{40}{56.67} = 0.294$$

$$\Rightarrow \underbrace{952\text{ K}}_{T_{\text{min}}} \leq T \quad \leftarrow T_{\text{min}}$$

- At steady state, the cycle energy balance and thermal efficiency can be expressed in terms of rates:

$$\begin{aligned}\dot{W}_{\text{cycle}} &= \dot{Q}_H - \dot{Q}_C \\ \eta &= \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_H}\end{aligned}$$

PROBLEM 5.32

KNOWN: steady-state data are provided for a power cycle developed by an inventor.

FIND: Evaluate the inventor's performance claim.

ANALYSIS: The inventor claims the power cycle will develop $\dot{W}_{\text{cycle}} = 100 \text{ hp}$ for $\dot{Q}_H = 5.1 \times 10^5 \text{ Btu/h}$, while operating between hot and cold reservoirs at 1000 K and 500 K . Any such cycle must satisfy

$$\eta \leq \eta_{\text{MAX}} = 1 - \frac{T_C}{T_H} = 1 - \frac{500 \text{ K}}{1000 \text{ K}} = 0.5$$

where $\eta = \dot{W}_{\text{cycle}} / \dot{Q}_H$. That is,

$$\frac{\dot{W}_{\text{cycle}}}{\dot{Q}_H} = \frac{100 \text{ hp}}{5.1 \times 10^5 \text{ Btu/h}} \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right| \leq 0.5$$

$= 0.499$

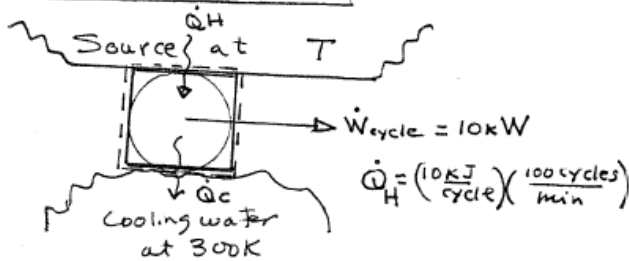
Accordingly, the inventor's claim corresponds to nearly reversible operation. While this is not impossible, it is highly unlikely.

PROBLEM 5.33

KNOWN: Steady-state operating data are provided for a power cycle receiving energy by heat transfer from a source at temperature T and rejecting energy by heat transfer to cooling water at 300 K .

FIND: Determine the minimum theoretical value for T , in K .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The system shown in the sketch undergoes a power cycle.
2. The data provided are for steady-state operation.
3. The source and cooling water play the roles of hot and cold reservoirs, respectively.

ANALYSIS:

The power developed must be less than, or equal to, the power developed by a reversible power cycle operating between thermal reservoirs at the specified temperatures. That is, with Eq. 5.9

$$\eta \leq \eta_{\text{MAX}} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{T}$$

where

$$\eta = \frac{W_{\text{cycle}}}{Q_H} = \frac{10 \text{ kW}}{(10 \frac{\text{kJ}}{\text{cycle}})(100 \frac{\text{cycles}}{\text{min}})(\frac{1 \text{ min}}{60 \text{ s}})} \bigg| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \bigg|$$

$$= 0.6$$

$$\therefore 0.6 \leq 1 - \frac{300}{T}$$

$$\Rightarrow T \geq 750 \text{ K}$$

The minimum theoretical value of T is 750 K .

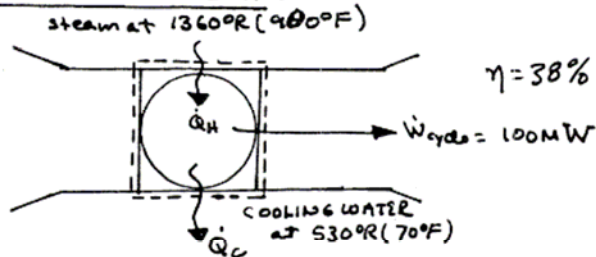


PROBLEM 5.34

KNOWN: Steady-state operating data are provided for a system undergoing a power cycle wh. receiving and discharging energy by heat transfer at 900 and 70°F, respectively.

FIND: Determine the rate energy is discharged and compare with the minimum theoretical rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system shown in the schematic undergoes a power cycle. 2. Operation is at steady state. 3. The steam and cooling water play the roles of hot and cold reservoirs, respectively.

ANALYSIS: (a) For any cycle at steady state, $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$, or $\dot{Q}_C = \dot{Q}_H - \dot{W}_{cycle}$. Also, $\eta = \dot{W}_{cycle} / \dot{Q}_H \Rightarrow \dot{Q}_H = \dot{W}_{cycle} / \eta$. Collecting results

$$\dot{Q}_C = \dot{W}_{cycle} \left[\frac{1}{\eta} - 1 \right] = 100 \text{ MW} \left[\frac{1}{0.38} - 1 \right] = 163.2 \text{ MW} \left| \frac{3.413 \times 10^6 \text{ Btu/h}}{1 \text{ MW}} \right| = 5.57 \times 10^8 \frac{\text{Btu}}{\text{h}}$$

(b) The thermal efficiency must be less than, or equal to, the thermal efficiency of a reversible power cycle operating between reservoirs at $T_H = 1360^\circ\text{R}$, $T_C = 530^\circ\text{R}$:

$$\eta \leq \left[1 - \frac{T_C}{T_H} \right], \text{ where } \eta = \frac{\dot{W}_{cycle}}{\dot{W}_{cycle} + \dot{Q}_C}. \text{ That is}$$

$$\textcircled{1} \left(\frac{100 \text{ MW}}{100 \text{ MW} + \dot{Q}_C} \right) \leq 1 - \frac{530}{1360} = 0.61 \Rightarrow \frac{100 \text{ MW}}{0.61} \leq 100 \text{ MW} + \dot{Q}_C \Rightarrow$$

$$\dot{Q}_C \geq 63.9 \text{ MW} \left| \frac{3.413 \times 10^6 \text{ Btu/h}}{1 \text{ MW}} \right| = 2.18 \times 10^8 \frac{\text{Btu}}{\text{h}}$$

Thus, the minimum theoretical rate at which energy could be discharged to the cooling water is $2.18 \times 10^8 \text{ Btu/h}$.

Comparing the values for $\dot{Q}_C$

$$\frac{\dot{Q}_C}{(\dot{Q}_C)_{\text{MIN}}} = \frac{5.57 \times 10^8 \text{ Btu/h}}{2.18 \times 10^8 \text{ Btu/h}} = 2.56$$

Thus, the actual heat discharge rate is over 2½ times the minimum theoretical rate. This illustrates that irreversibilities within the system during each cycle of operation contribute significantly to thermal pollution.

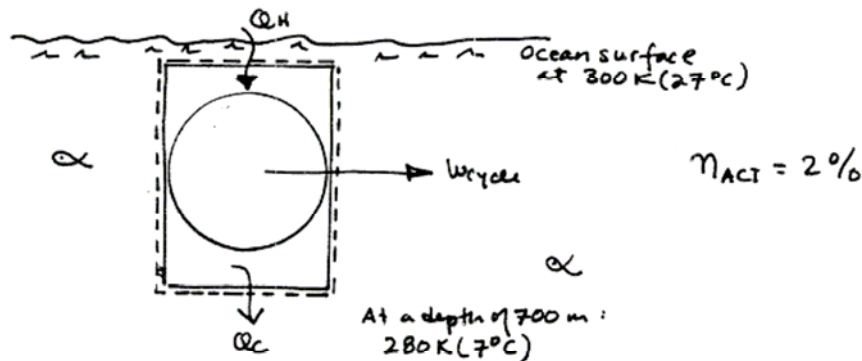
1. The actual thermal efficiency, 38%, is considerably less than the maximum value, 61%, owing to the effect of irreversibilities.

PROBLEM 5.35

KNOWN: A power cycle operates between surface ocean water at 27°C and water at a depth of 700 m where the temperature is 7°C .

FIND: Determine the maximum theoretical thermal efficiency for any such cycle and compare with the observed thermal efficiency of OTEC power plants.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system shown in the schematic undergoes a power cycle. 2. The ocean surface water and water at a depth of 700 m play the roles of hot and cold reservoirs, respectively.

ANALYSIS: (a) The thermal efficiency of any power cycle under these conditions must be less than, or equal to, the thermal efficiency of a reversible power cycle operating between reservoirs at $T_H = 300\text{K}$, $T_C = 280\text{K}$, respectively. That is

$$\eta \leq \left[1 - \frac{T_C}{T_H}\right] = \left[1 - \frac{280}{300}\right] = 0.067 \quad (6.7\%) \Rightarrow \eta_{\text{MAX}} = 6.7\%$$

(b) If $\eta_{\text{act}} = 2\%$, then

$$\frac{\eta_{\text{act}}}{\eta_{\text{MAX}}} = \frac{2\%}{6.7\%} = 0.3$$

That is, for the assumed conditions, actual OTEC cycles operate at 30% of the maximum theoretical value. This indicates that there may be some scope for improving actual performance,

by a few percentage points, via conventional engineering measures. Still, the characteristically low value of η_{MAX} for OTEC plants suggests that power development by such means may not be competitive economically with other types of power plants in use today.

PROBLEM 5.36

KNOWN: A system undergoes a power cycle while receiving energy by heat transfer from condensing steam and discharging energy by heat transfer to a lake.

FIND: Determine the minimum theoretical steam mass flow rate for a net power output of 1 MW.

SCHEMATIC & GIVEN DATA:

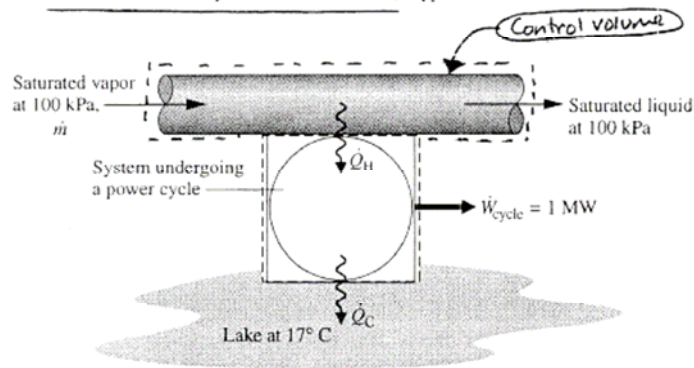


Fig. P5.36

ENGR. MODEL:

1. As shown by the sketch, two systems are under consideration, each operating at steady state.
2. The condensing steam and the lake play the roles of hot and cold reservoirs, respectively.
3. For the control volume, kinetic and potential energy effects are ignored. Steam condenses at 100 kPa.

ANALYSIS: For the system undergoing the power cycle, we have

$$\frac{\dot{W}_{\text{cycle}}}{\dot{Q}_H} \leq 1 - \frac{T_C}{T_H} \quad , \quad \text{where } T_C = 290 \text{ K and } T_H = T_{\text{sat}}(100 \text{ kPa}) = 99.63^\circ\text{C} = 373 \text{ K} \quad (\text{Table A-3 at } 100 \text{ kPa})$$

An energy rate balance for the control volume gives $\dot{Q}_H = \dot{m}[h_g - h_f]$, where $h_g - h_f = 2258 \text{ kJ/kg}$. Collecting results

$$\frac{1 \text{ MW} \left| \frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right|}{\dot{m} (2258 \text{ kJ/kg})} \leq 1 - \frac{290 \text{ K}}{373 \text{ K}} = 0.223$$

Thus,

$$\frac{10^3 \text{ kJ/s}}{(2258 \text{ kJ/kg})(0.223)} \leq \dot{m}$$

$$\Rightarrow \dot{m} \geq 1.99 \text{ kg/s}$$

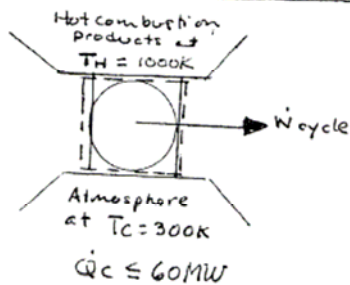
$$\leftarrow \dot{m}_{\text{MIN}}$$

PROBLEM 5.37

KNOWN: Steady-state operating and cert data are provided for a power cycle.

FIND: Determine, in \$/year, the maximum value of the power developed and the corresponding fuel cost.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The system shown by the dashed line undergoes a power cycle.
2. The system operates at steady state.
3. The hot combustion gases and atmosphere play the roles of hot and cold reservoirs, respectively.
3. The cycle operates 8000 h/year. The value of the power developed is \$0.08 per kW·h. The cost to provide the heat transfer is \$4.50 per GJ.

ANALYSIS: For any such cycle the maximum power developed occurs when the cycle operates reversibly. Accordingly, the thermal efficiency is given by Eq. 5.9. Then, with $\dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_C$ from an energy balance, we have

$$\frac{\dot{W}_{cycle}}{\dot{W}_{cycle} + \dot{Q}_C} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{1000} = 0.7 \Rightarrow \dot{W}_{cycle} = 0.7(\dot{W}_{cycle} + \dot{Q}_C) \text{ or}$$

$\dot{W}_{cycle} = \frac{0.7}{0.3} \dot{Q}_C \leq \frac{0.7}{0.3} (60 \text{ MW}) = 140 \text{ MW}$. It follows that the value of the maximum power that can be developed is

$$\dot{\$} \dot{W}_{cycle} = (140 \text{ MW}) \left| \frac{10^3 \text{ kW}}{1 \text{ MW}} \right| \left(\frac{8000 \text{ h}}{\text{year}} \right) \left| \frac{\$ 0.08}{\text{kW} \cdot \text{h}} \right| = \$ 89.6 \text{ M/year}$$

The corresponding value of $\dot{Q}_H$ is $\dot{Q}_H = 140 \text{ MW} + 60 \text{ MW}$. Accordingly, the corresponding cost is

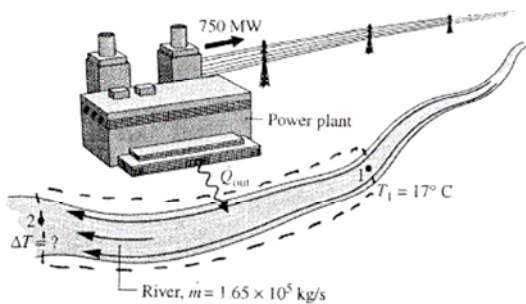
$$\begin{aligned} \dot{\$} \dot{Q}_H &= (200 \text{ MW}) \left| \frac{1 \text{ GJ/s}}{10^3 \text{ MW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left(\frac{8000 \text{ h}}{\text{year}} \right) \left(\frac{\$ 4.50}{\text{GJ}} \right) \\ &= \$ 25.92 \text{ M/year} \end{aligned}$$

PROBLEM 5.38

KNOWN: Steady-state operating data are provided for a power plant discharging energy by heat transfer to a river.

FIND: Determine the increase in the temperature of the river traceable to such heat transfer for each of two cases.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The power plant is modeled as a power cycle operating at steady state.
2. The combustion products provide energy by heat transfer to the power cycle. Energy is discharged by heat transfer to the river. These are the only heat transfers.
3. The river water is modeled as incompressible with constant specific heat c .

ANALYSIS:

For a control volume enclosing a section of the river, as shown in the sketch, an energy rate balance reduces to give

$$\dot{Q}_{out} = \dot{m} [h_2 - h_1]$$

where $\dot{Q}_{out}$ is a positive number denoting the energy discharged from the power cycle to the river. With Eq. 3.20b, this becomes

$$\dot{Q}_{out} = \dot{m} c [T_2 - T_1] \quad (1)$$

For the power cycle, $\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}}$, where $\dot{Q}_{in} = \dot{W}_{cycle} + \dot{Q}_{out}$. That is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{W}_{cycle} + \dot{Q}_{out}} \Rightarrow \dot{Q}_{out} = \dot{W}_{cycle} \left[\frac{1}{\eta} - 1 \right] \quad (2)$$

Combining Eqs. (1) and (2),

$$(T_2 - T_1) = \frac{\dot{W}_{cycle} \left[\frac{1}{\eta} - 1 \right]}{\dot{m} c} \quad (3)$$

where $c = 4.2 \text{ kJ/kg} \cdot \text{K}$ from Table A-19.

(a) $\eta = \eta_{MAX} = \left[1 - \frac{T_c}{T_H} \right] = \left[1 - \frac{290}{590} \right] = 0.508$. Inserting values in Eq. (3)

$$(T_2 - T_1) = \frac{(750 \text{ MW}) \left[\frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right] \left[\frac{1}{0.508} - 1 \right]}{(1.65 \times 10^5 \text{ kg/s})(4.2 \text{ kJ/kg} \cdot \text{K})} = 1.05 \text{ K} (= 1.05^\circ \text{C}) \quad (a)$$

(b) $\eta = \frac{2}{3} \eta_{MAX} = 0.339$

$$(T_2 - T_1) = \frac{(750) \left[10^3 \right] \left[\frac{1}{0.339} - 1 \right]}{(1.65 \times 10^5)(4.2)} = 2.11 \text{ K} (= 2.11^\circ \text{C}) \quad (b)$$

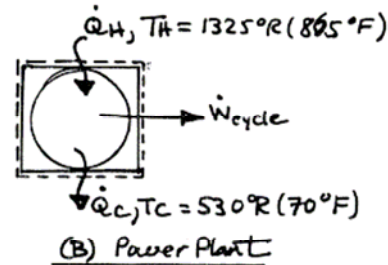
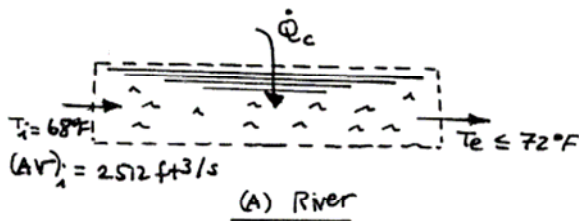
Comment: An effect of irreversibilities within the power plant is to increase the amount of energy discharged by heat transfer to the river and thus increase the temperature rise of the river traceable to such heat transfer.

PROBLEM 5.39

KNOWN: Operating data are provided for a power plant that discharges energy by heat transfer to a river. For environmental reasons, the temperature of the river downstream of the power plant is restricted.

FIND: Estimate the maximum theoretical power that can be developed subject to the restriction on river temperature.

SCHEMATIC & GIVEN DATA



ENGR. MODEL: 1. The two systems shown in the schematic are at steady state. In each case, $\dot{Q}_c$ is in the direction of the arrow. 2. For system (A), effects of kinetic and potential energy can be ignored. 3. System (B) undergoes a power cycle that receives energy from a reservoir at T_H and discharges energy to a reservoir at T_C , which equals the average river temperature.

ANALYSIS: An energy rate balance for control volume (A) reduces to give

$$\dot{Q}_c = \dot{m} [h_e - h_i] = \frac{(\dot{A}V)_i}{v_i} [h_e - h_i]$$

Then, with $v_i = v_f(T_i) = 0.01605 \text{ ft}^3/\text{lb}$, $h_i = h_f(T_i) = 36.09 \text{ Btu/lb}$, and $h_e \leq h_f(72^\circ\text{F}) = 40.09 \text{ Btu/lb}$ from Table A-2E

$$\dot{Q}_c \leq \left(\frac{2512 \text{ ft}^3/\text{s}}{0.01605 \text{ ft}^3/\text{lb}} \right) (40.09 - 36.09) \left(\frac{\text{Btu}}{\text{lb}} \right) = 6.26 \times 10^5 \text{ Btu/s}$$

Thus, to meet the environmental constraint, the power cycle cannot discharge energy to the river at a rate greater than $(\dot{Q}_c)_{\text{MAX}} = 6.26 \times 10^5 \text{ Btu/s}$.

Turning to system (B), an energy balance reduces to $\dot{Q}_H = \dot{W}_{\text{cycle}} + \dot{Q}_c$. Moreover, the thermal efficiency must be less than, or equal to, the thermal efficiency of a reversible power cycle operating between reservoirs at T_H, T_C : $\eta \leq [1 - \frac{T_C}{T_H}]$, where $\eta = \dot{W}_{\text{cycle}} / \dot{Q}_H$. Collecting results

$$\frac{\dot{W}_{\text{cycle}}}{\dot{W}_{\text{cycle}} + \dot{Q}_c} \leq \left[1 - \frac{T_C}{T_H} \right] \Rightarrow \dot{W}_{\text{cycle}} \leq \left[1 - \frac{T_C}{T_H} \right] (\dot{W}_{\text{cycle}} + \dot{Q}_c)$$

$$\Rightarrow \dot{W}_{\text{cycle}} \leq \left[\frac{T_H}{T_C} - 1 \right] (\dot{Q}_c) \Rightarrow \dot{W}_{\text{cycle}} \leq \left[\frac{T_H}{T_C} - 1 \right] (\dot{Q}_c)_{\text{MAX}} \quad (1)$$

Inserting values, Eq. (1) gives

$$\dot{W}_{\text{cycle}} \leq \left[\frac{1325^\circ\text{R}}{530^\circ\text{R}} - 1 \right] (6.26 \times 10^5 \frac{\text{Btu}}{\text{s}}) = (1.5) (6.26 \times 10^5 \frac{\text{Btu}}{\text{s}}) = 9.39 \times 10^5 \frac{\text{Btu}}{\text{s}}$$

$$\leq (9.39 \times 10^5 \frac{\text{Btu}}{\text{s}}) \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left| \frac{1 \text{ MW}}{3.413 \times 10^6 \text{ Btu/h}} \right| = 990.4 \text{ MW}$$

or

$$\textcircled{1} \quad (\dot{W}_{\text{cycle}})_{\text{MAX}} = 990.4 \text{ MW}$$

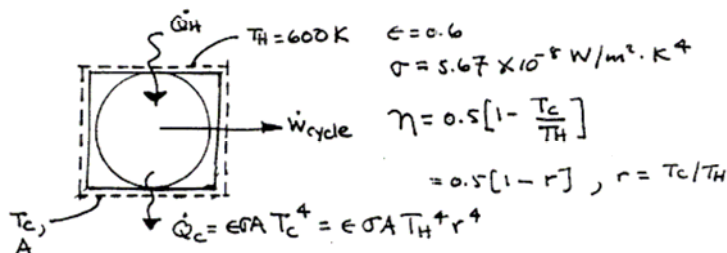
1. Owing to the effect of irreversibilities, an actual power plant would develop much less power than the maximum theoretical value determined here.

PROBLEM 5.40

KNOWN: A power cycle receives energy by heat transfer at a specified temperature and rejects energy by heat transfer at temperature T_c . The thermal efficiency is one-half that of a reversible cycle operating between reservoirs at the respective temperatures.

FIND: (a) For $T_c = 400\text{K}$, determine the net power developed per unit of radiator surface area and the thermal efficiency. (b) Plot these quantities versus T_c and determine the maximum value of the net power developed per unit area. (c) Determine the range of T_c for which the net power per unit area is within 2% of the maximum.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the schematic undergoes a power cycle and operates at steady state. (2) Heat transfer at T_c is only by thermal radiation, as indicated on the schematic. The surface receives no radiation. (3) For the power cycle, $\eta = 0.5 \eta_{\text{max}}$.

ANALYSIS: The thermal efficiency of a power cycle operating at steady state is $\eta = W_{\text{cycle}} / \dot{Q}_H$. Introducing the energy rate balance, $W_{\text{cycle}} = \dot{Q}_H - \dot{Q}_C$

$$\eta = \frac{W_{\text{cycle}}}{W_{\text{cycle}} + \dot{Q}_C} \Rightarrow W_{\text{cycle}} = \left(\frac{\eta}{1 - \eta} \right) \dot{Q}_C$$

Now, from above $\eta = 0.5(1 - r)$ and $\dot{Q}_C = \epsilon \sigma A T_H^4 r^4$. Thus

$$W_{\text{cycle}} = \left(\frac{0.5(1 - r)}{1 - 0.5(1 - r)} \right) \epsilon \sigma A T_H^4 r^4$$

Simplifying and solving for W_{cycle}/A

$$W_{\text{cycle}}/A = (\epsilon \sigma T_H^4) \left[\frac{(1 - r)}{(1 + r)} r^4 \right] \quad (1)$$

(a) $T_c = 400\text{K}$. In this case, $r = 400/600 = 0.6667$. Thus

$$\begin{aligned} \frac{W_{\text{cycle}}}{A} &= (0.6)(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4})(600\text{K})^4 \left[\frac{(1 - 0.6667)}{(1 + 0.6667)} (0.6667)^4 \right] \left| \frac{1\text{ kW}}{10^3 \text{ W}} \right| \\ &= 0.174 \frac{\text{kW}}{\text{m}^2} \end{aligned}$$

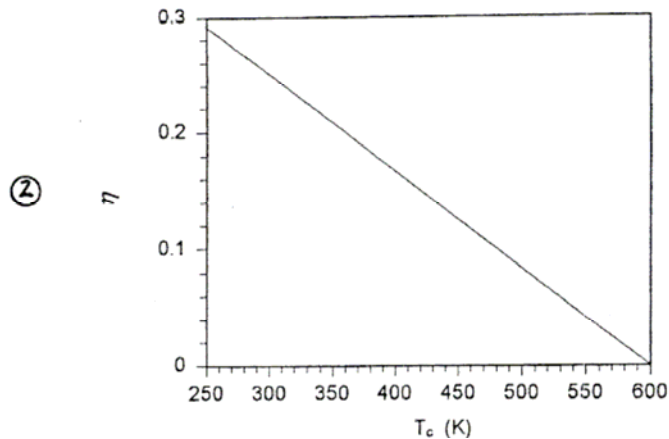
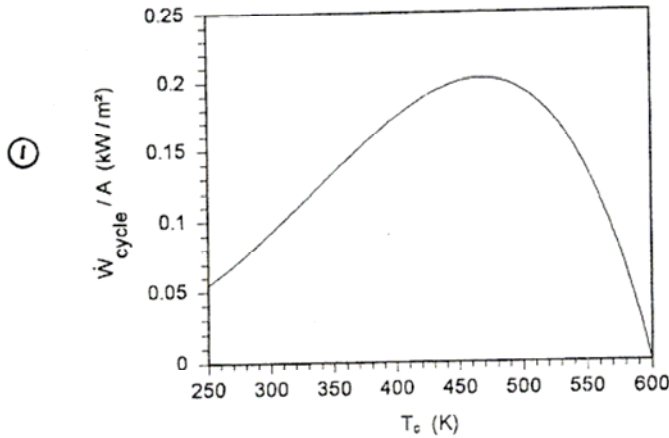
Also

$$\eta = 0.5(1 - r) = 0.1667 \quad (16.67\%)$$

Continued on next slide

Problem 5-40 continued

(b) Plots The following plots are constructed using IT:



(c) For the net power per unit area within 2% of the maximum value,

$$0.98 (W_{cycle}/A)_{max} = (0.98)(0.2017) = 0.1977 \text{ kW/m}^2$$

By inspection of the computer data: $441 \text{ K} < T_c < 491 \text{ K}$ ← $T_c \text{ range}$

Again, this is confirmed by inspection of the graph.

1. Alternatively, the value of r for which W_{cycle}/A is maximum could be obtained by differentiating (1) with respect to r , setting the resulting expression equal to zero, and solving for r . The result is $r = 0.781$, and $(W_{cycle}/A)_{max} = 0.2017$. This confirms the computer solution.

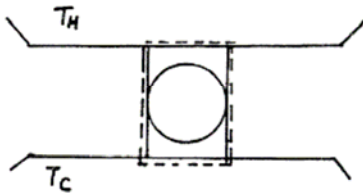
2. Note that the thermal efficiency corresponding to $(W_{cycle}/A)_{max}$ is 0.1095 (10.95%). In this application, maximizing η may not be the objective. The size of the radiator surface may be a more significant consideration.

PROBLEM 5.41

KNOWN: A system undergoes a reversible power cycle while receiving and discharging energy by heat transfer with two thermal reservoirs.

FIND: To increase the thermal efficiency, determine whether it would be better to increase T_H or decrease T_C .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The system shown in the accompanying figure undergoes a reversible power cycle while receiving and discharging energy by heat transfer with two thermal reservoirs.

ANALYSIS: Trends can be determined by differentiation of Eq. 5.9, which is applicable in this case

$$\left(\frac{\partial \eta}{\partial T_H}\right)_{T_C} = \frac{T_C}{(T_H)^2} \Rightarrow \text{As } T_H \text{ increases, } \eta \text{ increases.}$$

$$\left(\frac{\partial \eta}{\partial T_C}\right)_{T_H} = -\frac{1}{T_H} \Rightarrow \text{As } T_C \text{ decreases, } \eta \text{ increases.}$$

Quantitative evaluations also can be had using Eq. 5.9

$$\eta = 1 - \frac{T_C}{T_H}$$

If T_C is decreased by ϵ degrees, the thermal efficiency is increased

$$\eta_{(-)} = 1 - \frac{(T_C - \epsilon)}{T_H} = \eta + \frac{\epsilon}{T_H} \quad (1)$$

If T_H is increased by ϵ degrees, the thermal efficiency is increased

$$\begin{aligned} \eta_{(+)} &= 1 - \frac{T_C}{T_H + \epsilon} \\ &= \frac{T_H - T_C + \epsilon}{T_H + \epsilon} = \frac{\eta T_H + \epsilon}{T_H + \epsilon} = \frac{\eta + \epsilon/T_H}{1 + \epsilon/T_H} \end{aligned} \quad (2)$$

Combining Eqs. (1), (2)

$$\eta_{(+)} = \frac{\eta_{(-)}}{1 + \epsilon/T_H} \Rightarrow \eta_{(+)} < \eta_{(-)}$$

This indicates that to increase the thermal efficiency it is better to decrease T_C .

- ① The possibility of increasing thermal efficiency by reducing T_C below that of the environment is not practical; however, for maintaining T_C below the ambient temperature would require a refrigerator that would have to be supplied work to operate. An increase in thermal efficiency by increasing T_H is limited by the properties of the materials used to fabricate the system undergoing the cycle.

1. A discussion of the influence of the temperatures of heat addition and heat rejection on the thermal efficiency of practical power cycles is provided in Sec. 8.2.3.

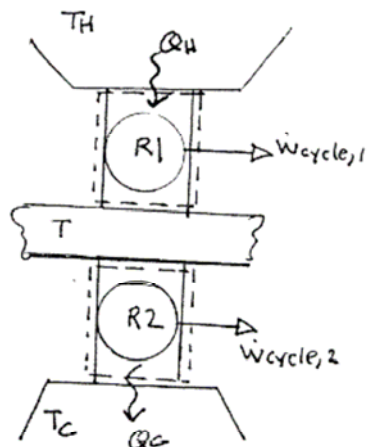
PROBLEM 5.42

KNOWN: Two reversible power cycles are arranged in series, where $T_H > T > T_C$.

FIND: (a) Obtain an expression for η for a single power cycle operating between hot and cold reservoirs at T_H and T_C in terms of the thermal efficiencies of the two cycles.

(b) Obtain an expression for the intermediate temperature T for the special case where the thermal efficiencies of the two cycles are equal.

SCHEMATIC & GIVEN DATA:



$$(a) \quad \eta_1 = 1 - \frac{T}{T_H} \Rightarrow \frac{T}{T_H} = 1 - \eta_1$$

$$\eta_2 = 1 - \frac{T_C}{T} \Rightarrow \frac{T_C}{T} = 1 - \eta_2$$

$$\eta = 1 - \frac{T_C}{T_H} \Rightarrow \frac{T_C}{T_H} = 1 - \eta$$

Then,

$$1 = \left(\frac{T}{T_H} \right) \left(\frac{T_C}{T} \right) \left(\frac{T_H}{T_C} \right) = \frac{(1 - \eta_1)(1 - \eta_2)}{(1 - \eta)}$$

$$\Rightarrow 1 = \frac{1 - \eta_1 - \eta_2 + \eta_1 \eta_2}{1 - \eta}$$

$$\Rightarrow \eta = \eta_1 + \eta_2 - \eta_1 \eta_2$$

← (a)

$$(b) \quad \eta_1 = 1 - \frac{T}{T_H}, \quad \eta_2 = 1 - \frac{T_C}{T} \quad \text{If } \eta_1 = \eta_2,$$

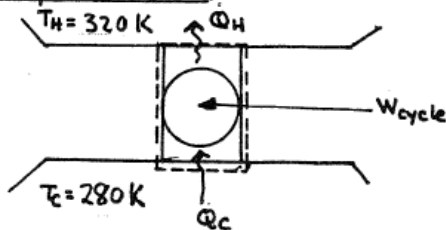
$$1 - \frac{T}{T_H} = 1 - \frac{T_C}{T} \Rightarrow T^2 = T_C T_H \quad \text{or} \quad T = \sqrt{T_C T_H} \quad \leftarrow (b)$$

PROBLEM 5.43

KNOWN: Operating data are provided for a system undergoing a refrigeration cycle while receiving and discharging energy by heat transfer with two thermal reservoirs at specified temperatures.

FIND: For each of four sets of data determine if the cycle is reversible, irreversible, or impossible.

SCHEMATIC & GIVEN DATA:



Energy balance:

$$W_{\text{cycle}} = Q_H - Q_C$$

Definition:

$$\beta = \frac{Q_C}{W_{\text{cycle}}}$$

ENGR. MODEL: The system shown in the accompanying figure undergoes a refrigeration cycle.

ANALYSIS:

(a) $Q_C = 1500 \text{ kJ}$, $W_{\text{cycle}} = 150 \text{ kJ}$. The coefficient of performance is

$$\beta = \frac{Q_C}{W_{\text{cycle}}} = \frac{1500 \text{ kJ}}{150 \text{ kJ}} = 10.0$$

The maximum coefficient of performance under the stated conditions is given by Eq. 5.10

$$\beta_{\text{max}} = \frac{T_C}{T_H - T_C} = \frac{280 \text{ K}}{(320 - 280) \text{ K}} = 7.0$$

Accordingly, as $\beta > \beta_{\text{max}}$, the cycle is impossible. ← (a)

(b) $Q_C = 1400 \text{ kJ}$, $Q_H = 1600 \text{ kJ}$. With the energy balance, the coefficient of performance can be written as

$$\beta = \frac{Q_C}{Q_H - Q_C} = \frac{1400 \text{ kJ}}{(1600 - 1400) \text{ kJ}} = 7.0$$

As $\beta = \beta_{\text{max}}$, this cycle is reversible. ← (b)

(c) $Q_H = 1600 \text{ kJ}$, $W_{\text{cycle}} = 400 \text{ kJ}$. With the energy balance, the coefficient of performance can be written as

$$\beta = \frac{Q_H - W_{\text{cycle}}}{W_{\text{cycle}}} = \frac{(1600 - 400)}{400} = 3.0$$

As $\beta < \beta_{\text{max}}$, this cycle is irreversible. ← (c)

(d) $\beta = 5$. As $\beta < \beta_{\text{max}}$, this cycle is irreversible. ← (d)

PROBLEM 5.44

KNOWN: A reversible refrigeration cycle operates between cold and hot reservoirs at T_c and T_H .

FIND: For each of 5 sets of data, obtain a specified result.

ANALYSIS:

(a) $\beta = 3.5$, $T_H = 80^\circ\text{F}$, find T_c . Since the cycle is reversible, Eq. 5.10 applies:

$$\beta_{\text{MAX}} = \frac{T_c}{T_H - T_c} = 3.5 \Rightarrow \frac{T_c}{540^\circ\text{R} - T_c} = 3.5 \Rightarrow T_c = 420^\circ\text{R} \quad (-40^\circ\text{F}) \quad \leftarrow (a)$$

(b) $T_c = -30^\circ\text{C}$, $T_H = 30^\circ\text{C}$, find β .

$$\beta = \beta_{\text{MAX}} = \frac{T_c}{T_H - T_c} = \frac{243\text{K}}{(303 - 243)\text{K}} = 4.05 \quad \leftarrow (b)$$

(c) $Q_c = 500\text{Btu}$, $Q_H = 800\text{Btu}$, $T_c = 20^\circ\text{F}$, find T_H .

With Eqs. 5.5 and 5.10,

$$\beta = \beta_{\text{MAX}} \Rightarrow \frac{Q_c}{Q_H - Q_c} = \frac{T_c}{T_H - T_c}$$

$$\frac{500\text{Btu}}{800\text{Btu} - 500\text{Btu}} = \frac{480^\circ\text{R}}{T_H - 480^\circ\text{R}} \Rightarrow T_H = 768^\circ\text{R} \quad (308^\circ\text{F}) \quad \leftarrow (c)$$

(d) $T_c = 30^\circ\text{F}$, $T_H = 100^\circ\text{F}$, find β

$$\beta = \beta_{\text{MAX}} = \frac{T_c}{T_H - T_c} = \frac{490^\circ\text{R}}{560^\circ\text{R} - 490^\circ\text{R}} = 7 \quad \leftarrow (d)$$

(e) $\beta = 8.9$, $T_c = -5^\circ\text{C}$, find T_H .

$$\beta_{\text{MAX}} = \frac{T_c}{T_H - T_c} = 8.9$$

$$\Rightarrow \frac{268\text{K}}{T_H - 268\text{K}} = 8.9 \Rightarrow T_H = 298\text{K} \quad (25^\circ\text{C}) \quad \leftarrow (e)$$

PROBLEM 5.45

KNOWN: Data are provided for a reversible heat pump operating at steady state.

FIND: For each of 3 sets of data, obtain a specified result.

ANALYSIS: In these cases, Eq. 5.10 is applicable.

(a) $T_H = 21^\circ\text{C}$, $T_C = 7^\circ\text{C}$, find β .

$$\beta = \beta_{\text{MAX}} = \frac{T_C}{T_H - T_C} = \frac{294}{294 - 280} = 21 \quad \leftarrow \text{(a)}$$

(b) $\dot{Q}_H = 10.5 \text{ kW}$, $\dot{Q}_C = 8.75 \text{ kW}$, $T_C = 0^\circ\text{C}$, find T_H .

With Eqs. 5.5 and 5.10,

$$\beta = \beta_{\text{MAX}} \Rightarrow \frac{\dot{Q}_C}{\dot{Q}_H - \dot{Q}_C} = \frac{T_C}{T_H - T_C}$$

$$\frac{8.75 \text{ kW}}{10.5 \text{ kW} - 8.75 \text{ kW}} = \frac{273 \text{ K}}{T_H - 273 \text{ K}} \Rightarrow T_H = 328 \text{ K} \quad \leftarrow \text{(b)}$$

(55°C)

(c) $\beta = 10$, $T_H = 27^\circ\text{C}$, find T_C .

$$\beta = \beta_{\text{MAX}} \Rightarrow 10 = \frac{T_C}{T_H - T_C} \Rightarrow 10 = \frac{T_C}{300 \text{ K} - T_C}$$

$$T_C = 273 \text{ K} \quad \leftarrow \text{(c)}$$

(0°C)

PROBLEM 5.46

KNOWN: Two reversible cycles operate between hot and cold reservoirs at T_H and T_C , respectively.

- (a) If one is a power cycle and the other is a heat pump cycle, what is the relation between the coefficient of performance of the heat pump cycle and the thermal efficiency of the power cycle?

Power cycle: $\eta = \frac{T_H - T_C}{T_H}$ (Eq. 5.9)

Heat Pump: $\gamma = \frac{T_H}{T_H - T_C}$ (Eq. 5.11)

By inspection,

$$\Rightarrow \gamma = \frac{1}{\eta}$$

← (a)

- (b) If one is a refrigeration cycle and the other is a heat pump cycle, what is the relation between their coefficients of performance?

Refrigeration cycle: $\beta = \frac{T_C}{T_H - T_C}$ (Eq. 5.10)

Heat pump cycle: $\gamma = \frac{T_H}{T_H - T_C}$ (Eq. 5.11)

Multiply each expression by $(T_H - T_C)$ and subtract,

$$\beta(T_H - T_C) = T_C$$

$$\gamma(T_H - T_C) = T_H$$

$$\Rightarrow (T_H - T_C)(\gamma - \beta) = (T_H - T_C)$$

$$\Rightarrow \gamma - \beta = 1 \Rightarrow \gamma = \beta + 1$$

← (b)

PROBLEM 5.47

A refrigeration cycle rejects $Q_H = 500$ Btu per cycle to a hot reservoir at $T_H = 540^\circ\text{R}$, while receiving $Q_C = 375$ Btu per cycle from a cold reservoir at temperature T_C . For 10 cycles of operation, determine (a) the net work input, in Btu, and (b) the minimum theoretical temperature T_C , in $^\circ\text{R}$.

ANALYSIS:

(a) Applying Eq. 2.44

$$\begin{aligned} W_{\text{cycle}} &= Q_H - Q_C = [500 - 375] \frac{\text{Btu}}{\text{cycle}} (10 \text{ cycles}) \\ &= 1250 \text{ Btu} \end{aligned}$$

← (a)

(b) For any refrigeration cycle,

$$\beta \leq \beta_{\text{MAX}} = \frac{T_C}{T_H - T_C}$$

With Eq. 5.5

$$\frac{Q_C}{Q_H - Q_C} \leq \frac{T_C}{T_H - T_C}$$

$$\frac{3750 \text{ Btu}}{1250 \text{ Btu}} \leq \frac{T_C}{540^\circ\text{R} - T_C} \Rightarrow 405^\circ\text{R} \leq T_C$$

← (b)

PROBLEM 5.48

A reversible heat pump cycle operates as in Fig. 5.7 between hot and cold reservoirs at $T_H = 27^\circ\text{C}$ and $T_C = -3^\circ\text{C}$, respectively. Determine the fraction of the heat transfer Q_H discharged at T_H provided by (a) the net work input, (b) the heat transfer Q_C from the cold reservoir T_C .

ANALYSIS:

(a) $\beta = \beta_{\text{MAX}}$

$$\frac{Q_H}{W_{\text{cycle}}} = \frac{T_H}{T_H - T_C} \Rightarrow \frac{W_{\text{cycle}}}{Q_H} = \frac{T_H - T_C}{T_H} = \frac{300\text{K} - 270\text{K}}{300\text{K}} = 0.1$$

$$\therefore \frac{W_{\text{cycle}}}{Q_H} = 0.1 \quad (10\%) \quad \leftarrow (a)$$

(b) $Q_H = W_{\text{cycle}} + Q_C$

$$\Rightarrow 1 = W_{\text{cycle}} + \frac{Q_C}{Q_H} \Rightarrow \frac{Q_C}{Q_H} = 0.9 \quad (90\%)$$

$\begin{matrix} \nearrow \\ = 0.1 \\ \text{part (a)} \end{matrix}$

$$\leftarrow (b)$$

PROBLEM 5.49

A reversible power cycle and a reversible heat pump cycle operate between hot and cold reservoirs at temperature $T_H = 1000^\circ\text{R}$ and T_C , respectively. If the thermal efficiency of the power cycle is 60%, determine (a) T_C in $^\circ\text{R}$, and (b) the coefficient of performance of the heat pump.

ANALYSIS:

$$(a) \quad \eta = \eta_{\text{MAX}} = \frac{T_H - T_C}{T_H} \quad (\text{Eq. 5.9})$$

$$0.60 = \frac{1000^\circ\text{R} - T_C}{1000^\circ\text{R}} \Rightarrow T_C = 400^\circ\text{R}$$

← (a)

$$(b) \quad \gamma = \gamma_{\text{MAX}} = \frac{T_H}{T_H - T_C} \quad (\text{Eq. 5.11})$$

By inspection,

$$\gamma = \frac{1}{\eta} = \frac{1}{0.6} = 1.67$$

← (b)

PROBLEM 5.50

An inventor has developed a refrigerator capable of maintaining its freezer compartment at 20°F while operating in a kitchen at 70°F, and claims the device has a coefficient of performance of (a) 10, (b) 9.6, (c) 4. Evaluate the claim in each of the three cases.

ANALYSIS:

$$\beta \leq \beta_{\max} = \frac{T_c}{T_H - T_c} \quad (\text{Eq. 5.10})$$

$$\Rightarrow \beta \leq \frac{480^\circ\text{R}}{500^\circ\text{R}} = 9.6$$

- (a) $\beta = 10$. Not a valid claim. β must be less than 9.6
- (b) $\beta = 9.6$. Claim allowed by the second law of thermodynamics only if the cycle can operate reversibly. Unlikely.
- (c) $\beta = 4$. Claim allowed by second law.

PROBLEM 5.51

PROBLEM 5.51

An inventor claims to have developed a refrigerator that at steady state requires a net power input of 0.7 horsepower to remove 12,000 Btu/h of energy by heat transfer from the freezer compartment at 0°F and discharge energy by heat transfer to a kitchen at 70°F. Evaluate this claim.

ANALYSIS:

$$\beta \leq \beta_{\max} = \frac{T_c}{T_H - T_c} \Rightarrow \frac{\dot{Q}_c}{W_{\text{cycle}}} \leq \frac{T_c}{T_H - T_c}$$

$$\frac{12,000 \frac{\text{Btu}}{\text{h}}}{0.7 \text{ hp}} \left| \frac{1 \text{ hp}}{2545 \frac{\text{Btu}}{\text{h}}} \right| < \frac{460^\circ \text{R}}{70^\circ \text{R}}$$

$= 6.74$
 $= 6.57$

No. The claim cannot be allowed. The claimed β exceeds the maximum theoretical value.

PROBLEM 5.53

Data are provided for two reversible refrigeration cycles. One cycle operates between hot and cold reservoirs at 27°C and -8°C , respectively. The other cycle operates between the same hot reservoir at 27°C and a cold reservoir at -28°C . If each refrigerator removes the same amount of energy by heat transfer from its cold reservoir, determine the ratio of the net work input values of the two cycles.

ANALYSIS:

$$\text{Refrigeration:} \quad \frac{Q}{W_{\text{cycle},1}} = \frac{T_c}{T_H - T_c} = \frac{265\text{K}}{35\text{K}} = 7.57$$

$$\text{Refrigeration:} \quad \frac{Q}{W_{\text{cycle},2}} = \frac{T_c'}{T_H - T_c'} = \frac{245\text{K}}{55\text{K}} = 4.45$$

$$\Rightarrow \frac{Q/W_{\text{cycle},1}}{Q/W_{\text{cycle},2}} = \frac{7.57}{4.45} \Rightarrow \frac{W_{\text{cycle},2}}{W_{\text{cycle},1}} = 1.7 \quad \leftarrow$$

PROBLEM 5.54

By removing energy by heat transfer from its freezer compartment at a rate of 1.25 kW, a refrigerator maintains the freezer at -26°C on a day when the temperature of the surroundings is 22°C . Determine the minimum theoretical power, in kW, required by the refrigerator at steady state.

ANALYSIS:

$$\beta \leq \beta_{\max} = \frac{T_c}{T_H - T_c} \Rightarrow \frac{\dot{Q}_c}{\dot{W}_{\text{cycle}}} \leq \frac{T_c}{T_H - T_c}$$

$$\left(\frac{\dot{Q}_c}{\dot{W}_{\text{cycle}}} \right)$$

$$\Rightarrow \dot{Q}_c \left[\frac{T_H - T_c}{T_c} \right] \leq \dot{W}_{\text{cycle}}$$

$$\therefore 1.25 \text{ kW} \left[\frac{48 \text{ K}}{247 \text{ K}} \right] \leq \dot{W}_{\text{cycle}}$$

$$\Rightarrow \dot{W}_{\text{cycle}} \geq 0.243 \text{ kW}$$

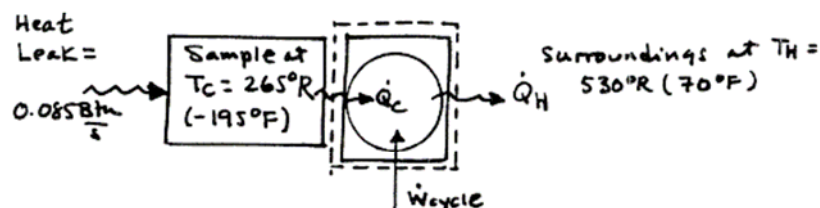


PROBLEM 5.55

KNOWN: A cryogenic sample must be kept at -195°F by a refrigeration system.

FIND: Determine the minimum theoretical power required by the refrigerator.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system shown in the schematic undergoes a refrigeration cycle at steady state. 2. The sample is also at steady state. 3. The sample and surroundings play the role of cold and hot reservoirs, respectively.

ANALYSIS: The coefficient of performance of the refrigerator must be less than, or equal to, the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at T_c, T_H . Thus,

$$\beta \leq \beta_{\max} \Rightarrow \frac{\dot{Q}_c}{\dot{W}_{\text{cycle}}} \leq \frac{T_c}{T_H - T_c} \Rightarrow \dot{Q}_c \left[\frac{T_H - T_c}{T_c} \right] \leq \dot{W}_{\text{cycle}}$$

At steady state, the rate energy must be removed from the sample equals the rate energy leaks into the sample. Thus, $\dot{Q}_c = 0.085 \text{ Btu/s}$, and

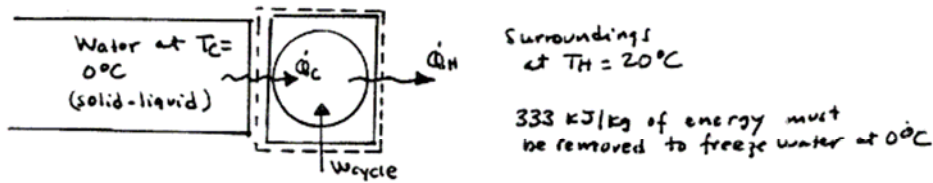
$$0.085 \frac{\text{Btu}}{\text{s}} \left[\frac{530 - 265}{265} \right] \leq \dot{W}_{\text{cycle}} \quad \text{or} \quad \dot{W}_{\text{cycle}} \geq 0.085 \frac{\text{Btu}}{\text{s}} \quad \leftarrow \text{MIN. POWER}$$

$= 1$

PROBLEM 556

KNOWN: Steady-state operating data are provided for an ice maker.

FIND: Determine the maximum rate ice can be formed at 0°C , per kW of power input.



ENGR. MODEL: 1. The system shown in the schematic undergoes a refrigeration cycle at steady state. 2. The water and the surroundings play the role of cold and hot reservoirs.

ANALYSIS: The coefficient of performance of the refrigeration cycle must be less than, or equal to, the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at T_c , T_h . Thus,

$$\beta \leq \beta_{\text{MAX}} \Rightarrow \frac{\dot{Q}_c}{\dot{W}_{\text{cycle}}} \leq \frac{T_c}{T_h - T_c}$$

At steady state, the energy removed while freezing water is $\dot{Q}_c = (333 \text{ kJ/kg}) \dot{M}$, where $\dot{M}$ is the rate water freezes, in kg/h. Collecting results

$$\frac{(333 \text{ kJ/kg}) \dot{M} (\text{kg/h})}{\dot{W}_{\text{cycle}} (\text{kJ/s})} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \leq \frac{273 \text{ K}}{(293 - 273 \text{ K})} \Rightarrow \frac{\dot{M}}{\dot{W}_{\text{cycle}}} \leq \left(\frac{273}{20} \right) \frac{13600}{333} = 147.6 \frac{\text{kg/h}}{\text{kW}}$$

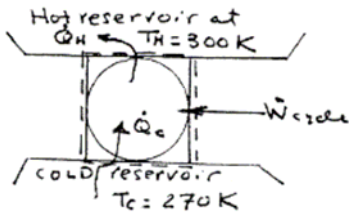
MAX. RATE

PROBLEM 5.57

KNOWN: Steady-state operating data are provided for a refrigeration cycle operating between hot and cold reservoirs.

FIND: Determine the minimum theoretical power input, in kW, per kW of heat transfer from the cold reservoir.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The system shown in the sketch undergoes a refrigeration cycle.
2. Operation is at steady state.
3. Heat transfer occurs only with the hot and cold reservoirs.

ANALYSIS:

With Eqs. 5.5 and 5.10, we have

$$\beta \leq \beta_{MAX}$$

$$\frac{\dot{Q}_C}{\dot{W}_{cycle}} \leq \frac{T_C}{T_H - T_C}$$

$$\Rightarrow \frac{T_H - T_C}{T_C} \leq \frac{\dot{W}_{cycle}}{\dot{Q}_C}$$

$$\frac{300\text{ K} - 270\text{ K}}{270} \leq \frac{\dot{W}_{cycle}}{\dot{Q}_C}$$

$$\frac{1}{9} \leq \frac{\dot{W}_{cycle}}{\dot{Q}_C}$$

The minimum theoretical power input in kW per kW of heat transfer from the cold reservoir is $1/9$ (0.111).

PROBLEM 5.58

At steady state, a refrigeration cycle operating between hot and cold reservoirs at 300 K and 275 K, respectively, removes energy by heat transfer from the cold reservoir at a rate of 600 kW.

- (a) If the cycle's coefficient of performance is 4, determine the power input required, in kW.
- (b) Determine the minimum theoretical power required, in kW, for any such cycle.

ANALYSIS:

$$(a) \quad \beta = \frac{\dot{Q}_c}{\dot{W}_{cycle}} \Rightarrow \dot{W}_{cycle} = \frac{\dot{Q}_c}{\beta} = \frac{600 \text{ kW}}{4} = 150 \text{ kW} \quad \leftarrow (a)$$

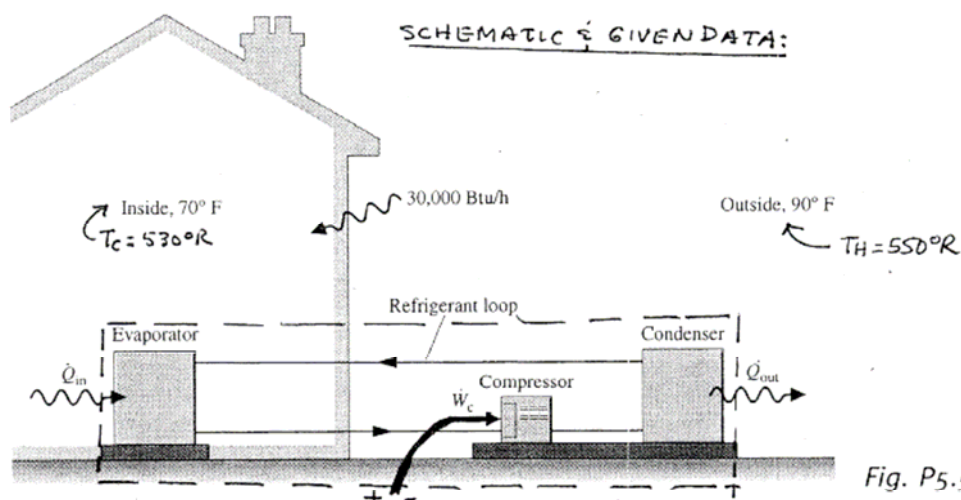
$$(b) \quad \beta \leq \beta_{max} \Rightarrow \frac{\dot{Q}_c}{\dot{W}_{cycle}} \leq \frac{T_c}{T_H - T_c} \Rightarrow \dot{Q}_c \left[\frac{T_H - T_c}{T_c} \right] \leq \dot{W}_{cycle}$$
$$600 \text{ kW} \left[\frac{25 \text{ K}}{275 \text{ K}} \right] \leq \dot{W}_{cycle}$$
$$\Rightarrow \dot{W}_{cycle} \geq 54.5 \text{ kW} \quad \leftarrow (b)$$

PROBLEM 5.59

As shown in Fig P5.59, an air conditioner operating at steady state maintains a dwelling at 70°F on a day when the outside temperature is 90°F. If the rate of heat transfer into the dwelling through the walls and roof is 30,000 Btu/h, might a net power input to the air conditioner compressor of 3 hp be sufficient? If yes, determine the coefficient of performance. If no, determine the minimum theoretical power input, in hp.

ENGR. MODEL:

1. The dashed line defines the air conditioning (refrigeration system).
2. The system is at steady state.
3. The inside and outside air play the roles of the cold and hot reservoirs, respectively.



ANALYSIS: To maintain the dwelling at 70°F, the air conditioning system must remove energy by heat transfer at a rate of 30,000 Btu/h. Thus, $\dot{Q}_{in} = 30,000 \text{ Btu/h}$.

Also,

$$\beta \leq \beta_{max} = \frac{T_c}{T_H - T_c} = \frac{530^\circ R}{20^\circ R} = 26.5$$

$$\Rightarrow \frac{\dot{Q}_{in}}{\dot{W}_c} \leq 26.5 \Rightarrow \dot{W}_c \geq \frac{30,000 \text{ Btu/h}}{26.5} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 0.445 \text{ hp} \quad \text{MIN. POWER Required}$$

Yes, a power input of 3 hp might be sufficient. The corresponding coefficient of performance is then

$$\beta = \frac{\dot{Q}_c}{\dot{W}_{cycle}} = \left(\frac{30,000 \text{ Btu/h}}{3 \text{ hp}} \right) \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 3.93.$$

PROBLEM 5.60

A heat pump cycle is used to maintain the interior of a building at 20°C . At steady state, the heat pump receives energy by heat transfer from well water at 10°C and discharges energy by heat transfer to the building at a rate of $120,000 \text{ kJ/h}$. Over a period of 14 days, an electric meter records that $1490 \text{ kW} \cdot \text{h}$ of electricity is provided to the heat pump. Determine

- (a) the amount of energy that the heat pump receives over the 14-day period from the well water by heat transfer, in kJ.
- (b) the heat pump's coefficient of performance.
- (c) the coefficient of performance of a reversible heat pump cycle operating between hot and cold reservoirs at 20°C and 10°C .

For parts (a), (b) see the solution to Problem 2.90.

Part (c):

$$\gamma_{\text{MAX}} = \frac{T_H}{T_H - T_C} = \frac{(273 + 20) \text{ K}}{10 \text{ K}} = 29.3$$

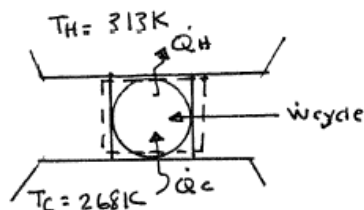
where the building interior and well water play the roles of the hot and cold reservoirs, respectively.

← (a), (b)

← (c)

PROBLEM 5.61

5.61 A refrigeration cycle has a coefficient of performance equal to 75% of the value for a reversible refrigeration cycle operating between cold and hot reservoirs at -5°C and 40°C , respectively. For operation at steady state, determine the net power input, in kW per kW of cooling, required by (a) the actual refrigeration cycle and (b) the reversible refrigeration cycle. Compare values.



$$(a) \quad \beta = 0.75 \beta_{\max} = 0.75 \left[\frac{T_C}{T_H - T_C} \right] = 0.75 \left[\frac{268 \text{ K}}{45 \text{ K}} \right] = 4.47$$

$$\left(\frac{\dot{Q}_C}{\dot{W}_{\text{cycle}}} \right)$$

$$\Rightarrow \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{1}{4.47} = 0.224$$

(a) ←

$$(b) \quad \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{1}{5.96} = 0.168$$

(b) ←

Comparing values

$$\frac{(\dot{W}_{\text{cycle}}/\dot{Q}_{\text{in}})_{(a)}}{(\dot{W}_{\text{cycle}}/\dot{Q}_{\text{in}})_{(b)}} = \frac{0.224}{0.168} = 1.33 \Rightarrow 33\% \text{ more work required in the actual device, per unit of cooling, owing to the effect of internal irreversibilities.}$$

PROBLEM 5.62

By removing energy by heat transfer from a room, a window air conditioner maintains the room at 22°C on a day when the outside temperature is 32°C .

- Determine, in kW per kW of cooling, the *minimum* theoretical power required by the air conditioner.
- To achieve required rates of heat transfer with practical-sized units, air conditioners typically receive energy by heat transfer at a temperature *below* that of the room being cooled and discharge energy by heat transfer at a temperature *above* that of the surroundings. Consider the effect of this by determining the *minimum* theoretical power, in kW per kW of cooling, required when $T_C = 18^\circ\text{C}$ and $T_H = 36^\circ\text{C}$, and compare with the value found in part (a).

Assuming the room interior and the outside are playing the roles of the cold and hot reservoirs, respectively,

$$(a) \quad \beta \leq \beta_{\max} = \frac{T_C}{T_H - T_C} = \frac{(22 + 273)\text{K}}{10\text{K}} = 29.5$$

$$\left(\frac{\dot{Q}_C}{\dot{W}_{\text{cycle}}} \right)_{\min} = \frac{1}{29.5} = 3.39 \times 10^{-2}$$

$$(b) \quad \beta \leq \beta_{\max} = \frac{T_C'}{T_H' - T_C'} = \frac{(18 + 273)\text{K}}{18\text{K}} = 16.17$$

$$\Rightarrow \left(\frac{\dot{W}_{\text{cycle}}}{\dot{Q}_C} \right)_{\min} = \frac{1}{16.17} = 6.18 \times 10^{-2}$$

Comparing values,

$$\frac{(\dot{W}_{\text{cycle}}/\dot{Q}_C)_{(b)}}{(\dot{W}_{\text{cycle}}/\dot{Q}_C)_{(a)}} = \frac{6.18 \times 10^{-2}}{3.39 \times 10^{-2}} = 1.82$$

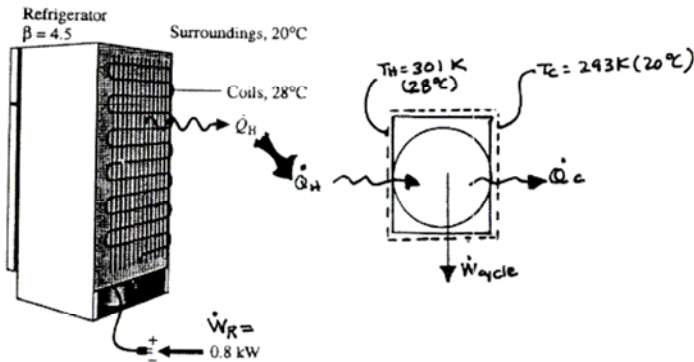
$\Rightarrow$ 82% more work required owing to the effect of "external" irreversibilities related to heat transfer to and from the refrigerator.

PROBLEM 5.63

KNOWN: Steady-state operating data are provided for a refrigerator.

FIND: Determine (a) the rate energy is rejected from the refrigerator to the surroundings by heat transfer, (b) the lowest theoretical temperature inside the refrigerator, and (c) the maximum theoretical power that could be developed by a power cycle from the energy rejected. Discuss.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) Operation is at steady state. (2) In part (b) the interior of the refrigerator and the surroundings play the roles of cold and hot reservoirs, respectively. (3) In part (c), the coils and surroundings play the roles of the hot and cold bodies, respectively.

ANALYSIS: (a) With Eq. 5.5, $\beta = \dot{Q}_{in} / \dot{W}_R$. An energy balance for the refrigerator reads $\dot{Q}_H = \dot{Q}_{in} + \dot{W}_R$. In these expressions, $\dot{Q}_{in}$ represent the energy into the circulating refrigerant from the food and other contents inside the refrigerator. Collecting results

$$\beta = \frac{\dot{Q}_H - \dot{W}_R}{\dot{W}_R} \Rightarrow \dot{Q}_H = (1 + \beta) \dot{W}_R = (1 + 4.5)(0.8 \text{ kW}) = 4.4 \text{ kW} \leftarrow$$

(b) From Sec. 5.9.2, we know that $\beta \leq \beta_{MAX}$, where β_{MAX} is given by Eq. 5.10. Thus, with $T_H = 293 \text{ K} (20^\circ\text{C})$ and T_C denoting the temperature inside the refrigerator,

$$\textcircled{1} \quad 4.5 \leq \frac{T_C}{T_H - T_C} \Rightarrow 4.5 T_H \leq 5.5 T_C \Rightarrow \left(\frac{4.5}{5.5}\right) T_H \leq T_C$$

Solving, $T_C \geq 239.7 \text{ K}$. Thus, the lowest theoretical temperature inside the refrigerator is 239.7 K . $\leftarrow$

(c) For the power cycle shown in the sketch, the thermal efficiency is $\eta = \dot{W}_{cycle} / \dot{Q}_H$. From Sec. 5.9.1, we know $\eta \leq \eta_{MAX}$, where η_{MAX} is given by Eq. 5.9. Then, with $T_C = 293 \text{ K} (20^\circ\text{C})$ and $T_H = 301 \text{ K} (28^\circ\text{C})$, we get

Continued on next slide

Problem 5-63 continued

$$\frac{\dot{W}_{\text{cycle}}}{\dot{Q}_H} \leq 1 - \frac{T_C}{T_H} \Rightarrow \dot{W}_{\text{cycle}} \leq \left[1 - \frac{T_C}{T_H} \right] \dot{Q}_H$$

$$\dot{W}_{\text{cycle}} \leq \left[1 - \frac{293\text{K}}{301\text{K}} \right] (4.4\text{kW})$$

$$\textcircled{2} \quad \leq 0.12\text{kW}$$

The maximum theoretical power that could be developed from the energy rejected is 0.12 kW. $\leftarrow$

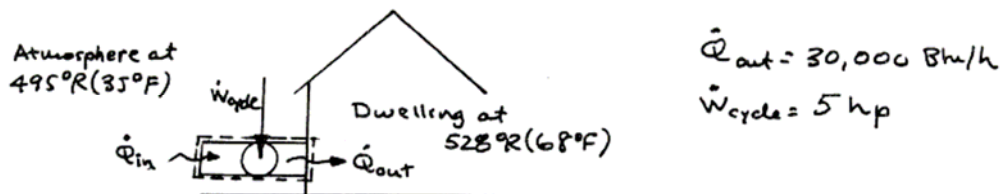
-
1. Note that the surroundings is the natural hot reservoir for the refrigerator, and thus we use $T_H = 293\text{K} (20^\circ\text{C})$ and not the coil temperature, which would reduce to 293K if the refrigerator were unplugged.
 2. Although the refrigerator rejects energy at an appreciable rate, 4.4kW, the thermodynamic value of that energy as measured by the power that might be developed from it is relatively small. Accordingly, there is little incentive for attempting to exploit this heat transfer, and $\dot{Q}_H$ is simply discharged to the surroundings without use.

PROBLEM 5.54

KNOWN: A heat pump provides heating to a dwelling. Operating data are provided.

FIND: Determine the actual operating cost and compare with the minimum theoretical operating cost for each day of operation.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the figure undergoes a heat cycle. (2) All data are for operation at steady state. (3) The atmosphere and the dwelling play the roles of cold and hot reservoirs, respectively. (4) The cost of electricity is 8 cents per kW.

ANALYSIS: Using given data, the actual operating cost is

$$(\text{Cost per day}) = (5 \text{ hp}) \left| \frac{1 \text{ kW}}{1.341 \text{ hp}} \right| \left| \frac{24 \text{ h}}{1 \text{ day}} \right| \left(\frac{0.08 \$}{\text{kW} \cdot \text{h}} \right) = \$7.16/\text{day}$$

The minimum theoretical operating cost corresponds to the minimum theoretical power requirement. Since $\beta < \beta_{\text{MAX}}$,

$$\Rightarrow \frac{\dot{Q}_{\text{out}}}{W_{\text{cycle}}} \leq \frac{T_H}{T_H - T_C}$$

or

$$\frac{30,000 \text{ Btu/h}}{W_{\text{cycle}}} \leq \frac{528^\circ\text{R}}{(528 - 495)^\circ\text{R}} = 16$$

$$\Rightarrow \frac{30,000 \text{ Btu/h}}{16} \leq W_{\text{cycle}}$$

$$1875 \frac{\text{Btu}}{\text{h}} \leq W_{\text{cycle}}$$

The minimum cost is then

$$(\text{Minimum cost per day}) = \left(1875 \frac{\text{Btu}}{\text{h}} \right) \left(\frac{1 \text{ kW}}{3413 \text{ Btu/h}} \right) \left(\frac{24 \text{ h}}{1 \text{ day}} \right) \left(\frac{0.08 \$}{\text{kW} \cdot \text{h}} \right)$$

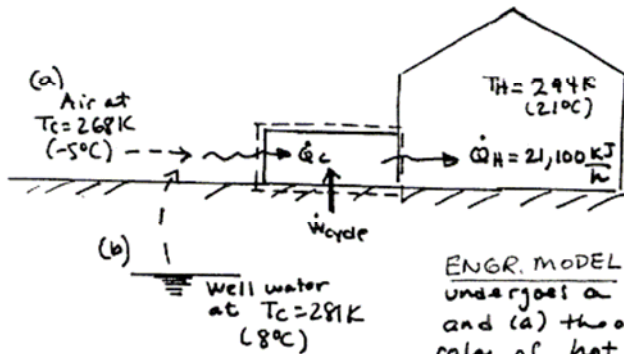
$$= \$1.05/\text{day}$$

The actual operating cost is about 7 times higher than the minimum theoretical cost.

PROBLEM 5.65

KNOWN: Operating data are provided for a heat pump receiving energy by heat transfer from the outdoor air. From well water.

FIND: For each case, determine the minimum theoretical daily cost to operate.



ENGR. MODEL: 1. The system shown in the schematic undergoes a heat pump cycle. 2. The dwelling and (a) the outside air, (b) the well water play the roles of hot and cold reservoirs, respectively. 3. Electricity costs 8 cents per kW·h.

ANALYSIS: The coefficient of performance of a heat pump is less than, or equal to, the coefficient of performance of a reversible heat pump operating between reservoirs at T_C, T_H . That is, $\gamma \leq \gamma_{\max}$

$$\frac{\dot{Q}_H}{\dot{W}_{\text{cycle}}} \leq \frac{T_H}{T_H - T_C} \Rightarrow \dot{Q}_H \left[1 - \frac{T_C}{T_H} \right] \leq \dot{W}_{\text{cycle}}$$

Inserting values

$$\dot{W}_{\text{cycle}} \geq (21,100 \frac{\text{kJ}}{\text{h}}) \left[1 - \frac{T_C}{294 \text{ K}} \right] \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

(a) Air at $T_C = 268 \text{ K}$

$$\dot{W}_{\text{cycle}} \geq (21,100) \left[1 - \frac{268}{294} \right] \left| \frac{1}{3600} \right| = 0.52 \text{ kW}$$

$$\text{Costing: } \$ \geq (0.52 \text{ kW}) \left| \frac{24 \text{ h}}{1 \text{ day}} \right| \left(\$ \frac{0.08}{\text{kW} \cdot \text{h}} \right) = \$1.00/\text{day} \leftarrow \text{minimum daily cost (a)}$$

(b) Well water at $T_C = 281 \text{ K}$

$$\dot{W}_{\text{cycle}} = (21,100) \left[1 - \frac{281}{294} \right] \left| \frac{1}{3600} \right| = 0.26 \text{ kW}$$

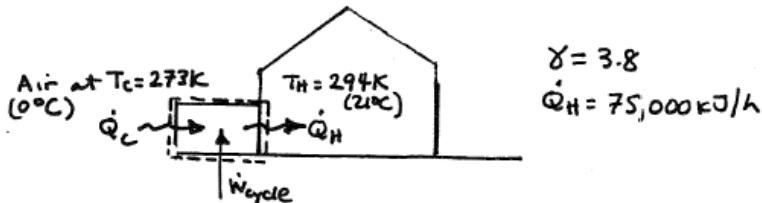
$$\text{Costing: } \$ \geq (0.26 \text{ kW}) \left| \frac{24 \text{ h}}{\text{day}} \right| \left(\$ \frac{0.08}{\text{kW} \cdot \text{h}} \right) = \$0.50/\text{day} \leftarrow \text{minimum daily cost (b)}$$

PROBLEM 5.66

KNOWN: Operating data are provided for a heat pump that maintains the temperature within a building at a specified value.

FIND: Determine the actual and minimum theoretical daily operating costs, and compare with the daily cost for electrical resistance heating.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system shown in the schematic undergoes a heat pump cycle. 2. Electricity costs 8 cents/kW·h. 3. The dwelling and outside air play the roles of the hot and cold reservoirs.

ANALYSIS: (a) Using Eq. 5.6, $\gamma = \dot{Q}_H / \dot{W}_{\text{cycle}}$

$$\dot{W}_{\text{cycle}} = \frac{\dot{Q}_H}{\gamma} = \frac{75,000 \text{ kJ/h}}{3.8} = 19,737 \frac{\text{kJ}}{\text{h}}$$

$$\text{Costing: } \$ = \left(19,737 \frac{\text{kJ}}{\text{h}} \right) \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \left(\frac{24 \text{ h}}{\text{day}} \right) \left(\frac{\$0.08}{\text{kW}\cdot\text{h}} \right) = \$10.53/\text{day} \quad \leftarrow \text{(actual)}$$

For a reversible heat pump operating between reservoirs at T_c , T_H , Eq. 5.11 gives

$$\gamma_{\text{MAX}} = \frac{T_H}{T_H - T_c} = \frac{294}{294 - 273} = 14$$

Then

$$(\dot{W}_{\text{cycle}})_{\text{MIN}} = \frac{75,000 \text{ kJ/h}}{14} = 5357 \frac{\text{kJ}}{\text{h}}$$

$$\text{Costing: } (\$)_{\text{MIN}} = (5357) \left| \frac{1}{3600} \right| (24)(0.08) = \$2.86/\text{day} \quad \leftarrow \text{(minimum)}$$

(b) For electrical-resistance heating, electricity would provide the full 75,000 kJ/h needed. Thus

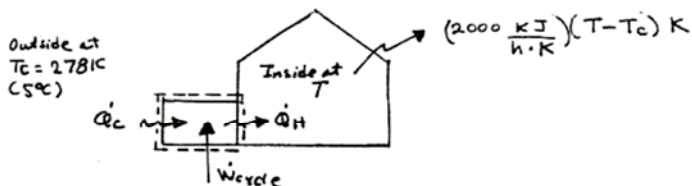
$$\text{Costing: } (\$)_{\text{electrical resistance}} = (75,000) \left| \frac{1}{3600} \right| (24)(0.08) = \$40/\text{day} \quad \leftarrow \text{(resistance heating)}$$

PROBLEM 5.67

KNOWN: Operating data are provided for a heat pump that maintains a dwelling at temperature T .

FIND: (a) When $T = 20^\circ\text{C}$, determine the minimum theoretical daily operating cost. (b) Plot the minimum theoretical daily operating cost versus T .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system shown in the schematic undergoes a heat pump cycle. 2. The inside air and outside air play the roles of hot and cold reservoirs, respectively. 3. Electricity costs 8cents per kW·h.

ANALYSIS: To maintain the dwelling at temperature T , the heat pump must provide energy to the dwelling at the rate

$$\dot{Q}_H = \left(\frac{2000 \text{ KJ}}{\text{h} \cdot \text{K}} \right) (T - T_c) \quad , \quad T_c = 278 \text{ K} \quad (1)$$

The coefficient of performance must satisfy the following relationship, written using Eqs. 5.6 and 5.11

$$\frac{\dot{Q}_H}{\dot{W}_{\text{cycle}}} \leq \frac{T}{T - T_c} \Rightarrow \dot{W}_{\text{cycle}} \geq \left[\frac{T - T_c}{T} \right] \dot{Q}_H$$

Introducing Eq. (1)

$$\dot{W}_{\text{cycle}} \geq \left[\frac{T - T_c}{T} \right] \left[\left(\frac{2000 \text{ KJ}}{\text{h} \cdot \text{K}} \right) (T - T_c) \right] = \left(\frac{2000 \text{ KJ}}{\text{h} \cdot \text{K}} \right) \frac{(T - T_c)^2}{T} \quad (2)$$

where T, T_c are in K.

(a) When $T = 293 \text{ K} (20^\circ\text{C})$, Eq. (2) gives

$$\dot{W}_{\text{cycle}} \geq \left(\frac{2000 \text{ KJ}}{\text{h} \cdot \text{K}} \right) \frac{(293 - 278)^2}{293} \text{ K} = 1536 \frac{\text{KJ}}{\text{h}}$$

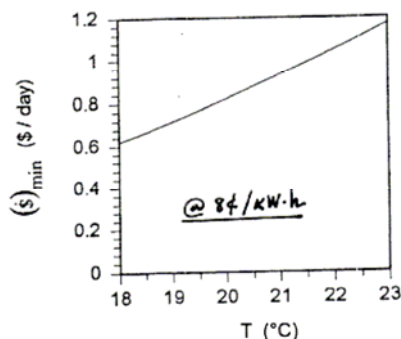
Thus, the minimum theoretical power required is 1536 KJ/h. The corresponding cost on a daily basis is

$$(\$/\text{day})_{\text{min}} = \left(1536 \frac{\text{KJ}}{\text{h}} \right) \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ KJ/s}} \right| \left| \frac{24 \text{ h}}{\text{day}} \right| \left(\frac{\$ 0.08}{\text{kW} \cdot \text{h}} \right) = \$ 0.82/\text{day} \leftarrow$$

(b) For T ranging from 18 to 23°C , the cost on a daily basis is

$$(\$/\text{day})_{\text{min}} = \left[2000 \left(\frac{T - T_c}{T} \right)^2 \frac{\text{KJ}}{\text{h}} \right] \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ KJ/s}} \right| \left| \frac{24 \text{ h}}{\text{day}} \right| \left(\frac{\$ 0.08}{\text{kW} \cdot \text{h}} \right) = 1.067 \left[\frac{(T - T_c)^2}{T} \right] (\$/\text{day})$$

PLOT:



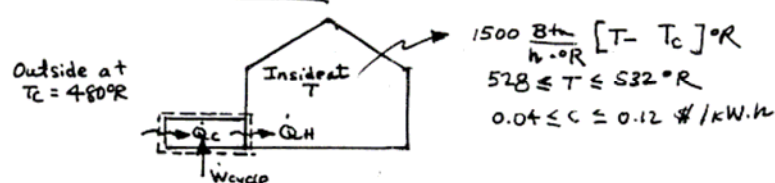
The minimum daily cost increases significantly as the inside temperature is required to be higher.

PROBLEM 5.68

KNOWN: Operating data are provided for a heat pump that maintains a dwelling at temperature T .

FIND: For a given electricity cost, plot the minimum theoretical operating cost versus T . For a given value of T , plot the minimum theoretical operating cost versus the cost of electricity in cents per kW·h.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system shown in the schematic undergoes a heat pump cycle. 2. The inside air and outside air play the roles of hot and cold reservoirs, respectively.

ANALYSIS: To maintain the dwelling at temperature T , the heat pump must provide energy to the dwelling at the rate

$$\dot{Q}_H = \left(1500 \frac{\text{Btu}}{\text{h} \cdot ^\circ\text{R}} \right) [T - T_c], \quad T_c = 480^\circ\text{R} \quad (1)$$

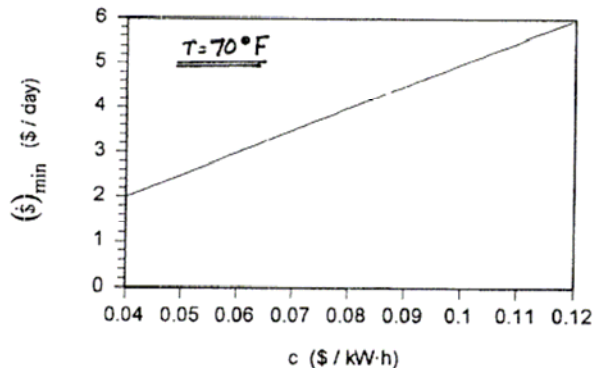
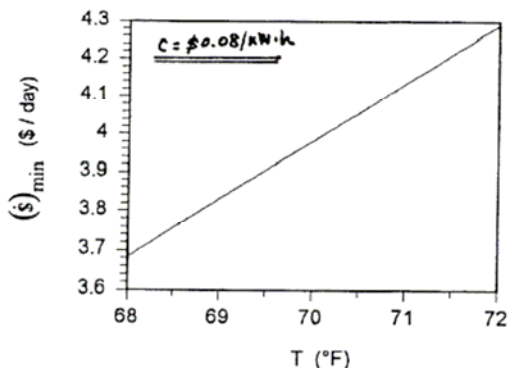
The coefficient of performance must satisfy the following relationship, written using Eqs. 5.6 and 5.11

$$\frac{\dot{Q}_H}{W_{\text{cycle}}} \leq \frac{T}{T - T_c} \Rightarrow W_{\text{cycle}} \geq \left[\frac{T - T_c}{T} \right] \dot{Q}_H \Rightarrow (W_{\text{cycle}})_{\text{MIN}} = \left[\frac{T - T_c}{T} \right] \dot{Q}_H$$

Introducing Eq. (1) and the unit cost of electricity, c , the minimum theoretical daily operating cost is

$$\begin{aligned} (\dot{\$})_{\text{MIN}} &= \left[\frac{T - T_c}{T} \right] \left[\left(1500 \frac{\text{Btu}}{\text{h} \cdot ^\circ\text{R}} \right) [T - T_c]^\circ\text{R} \right] \left(\frac{24 \text{ h}}{\text{day}} \right) \left(\frac{1 \text{ kW} \cdot \text{h}}{3413 \text{ Btu}} \right) c \left(\frac{\$}{\text{kW} \cdot \text{h}} \right) \\ &= 10.55 \left[\frac{(T - T_c)^2}{T} \right] c \left(\frac{\$}{\text{day}} \right) \end{aligned} \quad (2)$$

Sample calculation: $T = 530^\circ\text{R}$ (70°F), $c = \$0.08/\text{kW} \cdot \text{h}$. Then, $(\dot{\$})_{\text{MIN}} = 10.55 \left[\frac{(530 - 480)^2}{530} \right] (0.08) = \$3.98/\text{day}$.

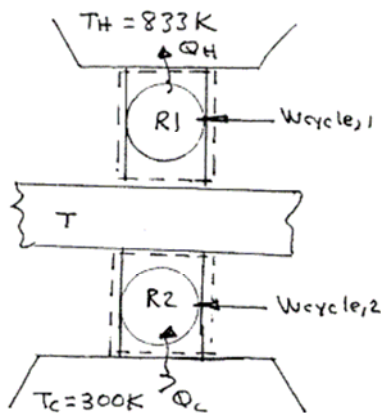


The minimum daily cost increases significantly as the inside temperature is required to be higher and/or as the unit cost of electricity increases.

PROBLEM 5.69

Two reversible refrigeration cycles operate in series. The first cycle receives energy by heat transfer from a cold reservoir at 300 K and rejects energy by heat transfer to a reservoir at an intermediate temperature T greater than 300 K. The second cycle receives energy by heat transfer from the reservoir at temperature T and rejects energy by heat transfer to a higher-temperature reservoir at 883 K. If the refrigeration cycles have the same coefficient of performance, determine (a) T , in K, and (b) the value of each coefficient of performance.

SCHEMATIC & GIVEN DATA



$$(b) \quad \beta_2 = \frac{300}{500-300} = 1.5$$

$$\beta_1 = \frac{500}{883-500} = 1.5$$

ANALYSIS:

- (a) Each cycle is reversible and they each have the same coefficient of performance:

$$\Rightarrow \beta_2 = \beta_1$$

$$\text{Eq. 5.10: } \frac{T_c}{T - T_c} = \frac{T}{T_H - T}$$

$$\Rightarrow T_c [T_H - T] = T [T - T_c]$$

$$T_c T_H - T T_c = T^2 - T T_c$$

$$\Rightarrow T = \sqrt{T_c T_H}$$

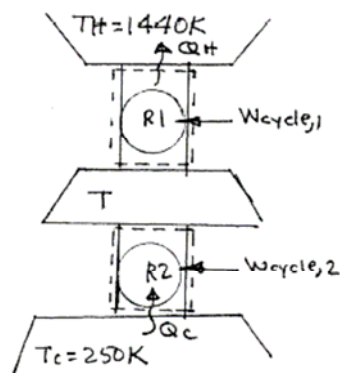
$$= \sqrt{(300\text{K})(883\text{K})}$$

$$= 500\text{K}$$

PROBLEM 5.70

Two reversible heat pump cycles operate in series. The first cycle receives energy by heat transfer from a cold reservoir at 250 K and rejects energy by heat transfer to a reservoir at an intermediate temperature T greater than 250 K. The second cycle receives energy by heat transfer from the reservoir at temperature T and rejects energy by heat transfer to a higher-temperature reservoir at 1440 K. If the heat pump cycles have the same coefficient of performance, determine (a) T , in K, and (b) the value of each coefficient of performance.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

- (a) Each heat pump cycle is reversible and they each have the same coefficient of performance.

$$\Rightarrow \gamma_2 = \gamma_1$$

Eq. 5.11:

$$\frac{T}{T - T_c} = \frac{T_H}{T_H - T}$$

$$\Rightarrow T(T_H - T) = T_H(T - T_c)$$

$$T T_H - T^2 = T_H T - T_c T_H$$

$$\Rightarrow T = \sqrt{T_c T_H}$$

$$= \sqrt{(250\text{ K})(1440\text{ K})}$$

$$= 600\text{ K} \quad \leftarrow$$

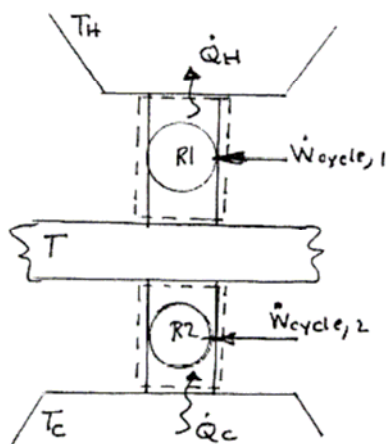
(b) $\gamma_2 = \frac{600}{600 - 250} = 1.71$

$$\gamma_1 = \frac{1440}{1440 - 600} = 1.71 \quad \leftarrow$$

PROBLEM 5.71

Two reversible refrigeration cycles are arranged in series. The first cycle receives energy by heat transfer from a cold reservoir at temperature T_C and rejects energy by heat transfer to a reservoir at an intermediate temperature T greater than T_C . The second cycle receives energy by heat transfer from the reservoir at temperature T and rejects energy by heat transfer to a higher-temperature reservoir at T_H . Obtain an expression for the coefficient of performance of a single reversible refrigeration cycle operating directly between cold and hot reservoirs at T_C and T_H , respectively, in terms of the coefficients of performance of the two cycles.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

$$\beta_1 = \frac{T}{T_H - T} \Rightarrow \frac{T_H}{T} = 1 + \frac{1}{\beta_1}$$

$$\beta_2 = \frac{T_C}{T - T_C} \Rightarrow \frac{T}{T_C} = 1 + \frac{1}{\beta_2}$$

$$\beta = \frac{T_C}{T_H - T_C} \Rightarrow \frac{T_H}{T_C} = 1 + \frac{1}{\beta}$$

Then

$$\frac{T_H}{T_C} = \left(\frac{T_H}{T} \right) \left(\frac{T}{T_C} \right)$$

$$1 + \frac{1}{\beta} = \left(1 + \frac{1}{\beta_1} \right) \left(1 + \frac{1}{\beta_2} \right)$$

$$\Rightarrow \frac{1}{\beta} = \frac{(\beta_1 + 1)(\beta_2 + 1)}{\beta_1 \beta_2} - 1$$

$$\Rightarrow \frac{1}{\beta} = \frac{(\beta_1 \beta_2 + \beta_1 + \beta_2 + 1) - \beta_1 \beta_2}{\beta_1 \beta_2}$$

$$\Rightarrow \beta = \frac{\beta_1 \beta_2}{\beta_1 + \beta_2 + 1}$$

PROBLEM 5.72

Repeat Problem 5.71 for the case of two reversible heat pump cycles.

SCHEMATIC & GIVEN DATA: See figure in the solution to Problem 5.71.

ANALYSIS:

$$\gamma_1 = \frac{T_H}{T_H - T} \Rightarrow \frac{T}{T_H} = 1 - \frac{1}{\gamma_1}$$

$$\gamma_2 = \frac{T}{T - T_C} \Rightarrow \frac{T_C}{T} = 1 - \frac{1}{\gamma_2}$$

$$\gamma = \frac{T_H}{T_H - T_C} \Rightarrow \frac{T_C}{T_H} = 1 - \frac{1}{\gamma}$$

Then

$$\frac{T_C}{T_H} = \left(\frac{T}{T_H} \right) \left(\frac{T_C}{T} \right)$$

$$1 - \frac{1}{\gamma} = \left(1 - \frac{1}{\gamma_1} \right) \left(1 - \frac{1}{\gamma_2} \right)$$

$$\Rightarrow \frac{1}{\gamma} = 1 - \left(1 - \frac{1}{\gamma_1} \right) \left(1 - \frac{1}{\gamma_2} \right)$$

$$= \frac{\gamma_1 \gamma_2 - (\gamma_1 - 1)(\gamma_2 - 1)}{\gamma_1 \gamma_2}$$

$$= \frac{\gamma_1 \gamma_2 - \gamma_1 \gamma_2 + \gamma_1 + \gamma_2 - 1}{\gamma_1 \gamma_2}$$

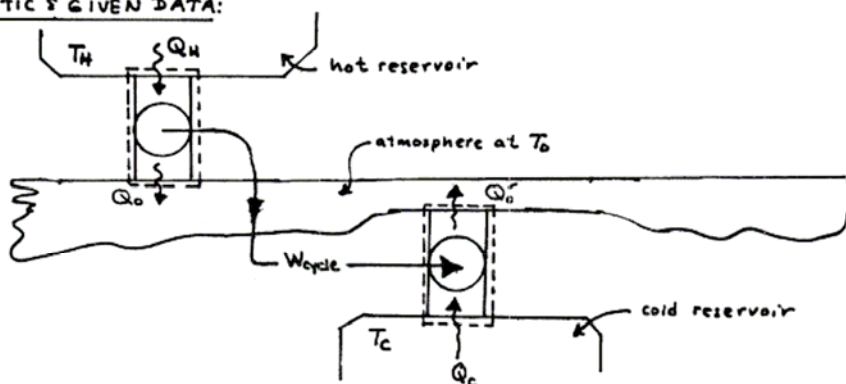
$$\Rightarrow \gamma = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 - 1} \quad \leftarrow$$

PROBLEM 5.73

KNOWN: A reversible power cycle receiving Q_H from a hot reservoir drives a refrigeration cycle that removes Q_C from a cold reservoir.

FIND: Develop an expression for Q_C/Q_H in terms of T_H/T_0 , T_C/T_0 .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) Each of the systems shown in the accompanying figure undergo reversible cycles. (2) The atmosphere plays the role of a thermal reservoir.

ANALYSIS: (a) An energy balance for the power cycle gives

$$W_{cycle} = Q_H - Q_0$$

An energy balance for the refrigeration cycle gives

$$W_{cycle} = Q_0' - Q_C$$

Combining the last two expressions

$$Q_H - Q_0 = Q_0' - Q_C \quad (1)$$

As each of the systems undergo reversible cycles between thermal reservoirs, Eq. 5.7 is applicable for each. That is

$$\frac{Q_0}{Q_H} = \frac{T_0}{T_H} \Rightarrow Q_0 = \frac{T_0}{T_H} Q_H \quad (2)$$

$$\frac{Q_C}{Q_0'} = \frac{T_C}{T_0} \Rightarrow Q_0' = \frac{T_0}{T_C} Q_C \quad (3)$$

Substituting Eqs. (2) and (3) into Eq. (1)

$$Q_H \left[1 - \frac{T_0}{T_H} \right] = Q_C \left[\frac{T_0}{T_C} - 1 \right]$$

Solving for Q_C/Q_H

$$\begin{aligned} \frac{Q_C}{Q_H} &= \frac{\left[1 - \frac{T_0}{T_H} \right]}{\left[\frac{T_0}{T_C} - 1 \right]} = \frac{T_C}{T_H} \left[\frac{T_H - T_0}{T_0 - T_C} \right] \\ &= \frac{(T_C/T_0) \left[\frac{T_H}{T_0} - 1 \right]}{(T_H/T_0) \left[1 - \frac{T_C}{T_0} \right]} \end{aligned}$$

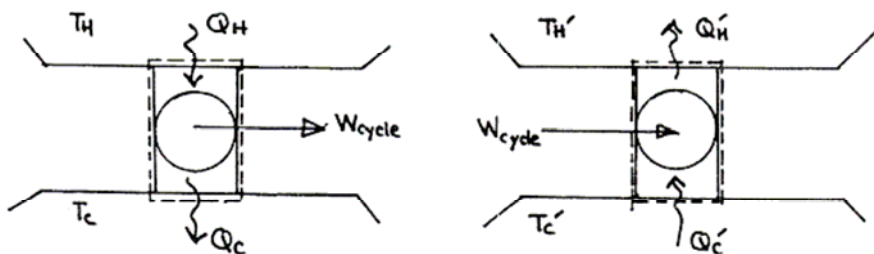


PROBLEM 5.74

KNOWN: A reversible power cycle receiving Q_H from a hot reservoir at T_H drives a heat pump that discharges Q_H' to a reservoir at T_H' .

FIND: Develop an expression for Q_H'/Q_H in terms of T_H, T_C, T_H', T_C' , and determine the relationship among the temperatures for the ratio to exceed unity.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: Each of the systems shown in the accompanying figure undergo reversible cycles.

ANALYSIS: (a) An energy balance on the power cycle gives

$$W_{cycle} = Q_H - Q_C$$

An energy balance for the heat pump cycle gives

$$W_{cycle} = Q_H' - Q_C'$$

Combining

$$Q_H - Q_C = Q_H' - Q_C' \quad (1)$$

As each of the systems undergo reversible cycles between thermal reservoirs, Eq. 5.7 is applicable for each. That is

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} \Rightarrow Q_C = \frac{T_C}{T_H} Q_H \quad (2)$$

$$\frac{Q_C'}{Q_H'} = \frac{T_C'}{T_H'} \Rightarrow Q_C' = \frac{T_C'}{T_H'} Q_H' \quad (3)$$

Substituting Eqs. (2) and (3) into Eq. (1)

$$Q_H \left[1 - \frac{T_C}{T_H} \right] = Q_H' \left[1 - \frac{T_C'}{T_H'} \right]$$

Solving for Q_H'/Q_H

$$\frac{Q_H'}{Q_H} = \frac{\left[1 - \frac{T_C}{T_H} \right]}{\left[1 - \frac{T_C'}{T_H'} \right]} = \frac{T_H' [T_H - T_C]}{T_H [T_H' - T_C']} \quad \leftarrow \frac{Q_H'}{Q_H}$$

(b) $Q_H'/Q_H > 1$ when

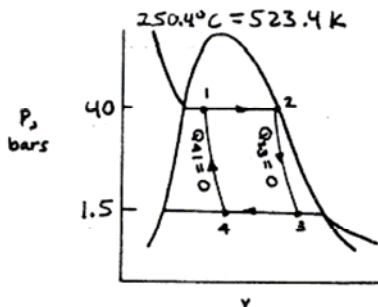
$$\frac{T_C}{T_C'} < \frac{T_H}{T_H'} \quad \leftarrow$$

PROBLEM 5.75

KNOWN: 2 kg of water executes a Carnot cycle for which property data are provided.

FIND: Sketch the cycle on p-v coordinates, evaluate the heat and work for each process, and evaluate the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



$$x_1 = 15\%$$

$$\frac{W_{23}}{m} = 491.5 \frac{\text{kJ}}{\text{kg}}$$

$$111.4^\circ\text{C} = 384.4 \text{ K}$$

Summary:

Process	Q	W
1-2	2914.0	330.0
2-3	0	983.0
3-4	-2140.2	-167.1
4-1	0	-373.0
Overall	773.8	772.9

agreement to within roundoff

ENGR. MODEL: The system shown in the figure undergoes a Carnot cycle.

Process 1-2: $\frac{W_{12}}{m} = \int p dv = p(v_2 - v_1)$. An energy balance gives $\frac{Q_{12}}{m} = u_2 - u_1 + \frac{W_{12}}{m}$. Thus

$$\frac{Q_{12}}{m} = (u_2 - u_1) + p(v_2 - v_1) = h_2 - h_1$$

With data from Table A-3, $v_2 = 0.04978 \text{ m}^3/\text{kg}$, $h_2 = 2801.4 \text{ kJ/kg}$, $u_2 = 2602.3 \text{ kJ/kg}$

$$v_1 = v_f + x(v_g - v_f) = 1.2522 \times 10^{-3} + 0.15(0.04978 - 1.2522 \times 10^{-3}) = 8.531 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$h_1 = h_f + x(h_g - h_f) = 1082.3 + 0.15(1714.1) = 1344.42 \text{ kJ/kg}$$

With these values, $Q_{12} = (2 \text{ kg})(2801.4 - 1344.42) \text{ kJ/kg} = 2914.0 \text{ kJ}$, and $\frac{Q_{12}}{m}$

$$W_{12} = (2.0 \text{ kg})(40 \text{ bar}) \left(\frac{10^5 \text{ N/m}^2}{\text{bar}} \right) (49.78 - 8.531) \times 10^{-3} \frac{\text{m}^3}{\text{kg}} \left(\frac{\text{kJ}}{10^3 \text{ N}\cdot\text{m}} \right) = 330.0 \text{ kJ}$$

Process 2-3: $Q_{23} = 0$, $W_{23}/m = 491.5 \text{ kJ/kg}$. Thus $W_{23} = 983.0 \text{ kJ}$. $\frac{Q_{23}, W_{23}}{m}$

Using an energy balance and data from Table A-3, $u_3 = 2110.8 \text{ kJ/kg}$,

giving $x_3 = 0.801$, $h_3 = 2250.5 \text{ kJ/kg}$, $v_3 = 0.929 \text{ m}^3/\text{kg}$.

Process 3-4: As for process 1-2, $W_{34}/m = p(v_4 - v_3)$, $Q_{34}/m = (h_4 - h_3)$. Also, since the system undergoes a reversible cycle while communicating with reservoirs at $T_H = 523.4 \text{ K}$, $T_C = 384.4 \text{ K}$, Eq. 5.7 is applicable

$$\frac{1}{Q_{12}} = \frac{384.4}{523.4} \Rightarrow h_3 - h_4 = \frac{384.4}{523.4} (1457.0) = 1070.1 \text{ kJ/kg}$$

Thus $h_4 = h_3 - 1070.1 = 2250.5 - 1070.1 = 1180.4 \text{ kJ/kg}$. Using this value,

$$x_4 = (1180.4 - 467.11) / 2226.5 = 0.320 \quad \text{Then } v_4 = 0.001528 + (0.320)(1.159 - 0.001528) = 0.372 \text{ m}^3/\text{kg}$$

$$u_4 = 466.94 + (0.320)(2519.7 - 466.94) = 1123.8 \text{ kJ/kg}$$

Finally, $Q_{34} = -2140.2 \text{ kJ}$, $W_{34} = -167.1 \text{ kJ}$. $\frac{Q_{34}, W_{34}}{m}$

Process 4-1: $Q_{41} = 0$. $W_{41}/m = u_4 - u_1 = 1123.8 - (1082.3 + 0.15(1520.0)) = -186.5 \text{ kJ/kg}$

So, $W_{41} = -373.0 \text{ kJ}$. $\frac{Q_{41}, W_{41}}{m}$

Thermal Efficiency.

$$\text{Method \#1: } \eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{330.0 + 983.0 - 167.1 - 373.0}{2914.0} = 0.265 \text{ (26.5\%)} \quad \eta$$

Method \#2: Equation 5.8

$$\eta_{\text{MAX}} = 1 - \frac{T_C}{T_H} = 1 - \frac{384.4}{523.4} = 0.266 \text{ (26.6\%)} \quad \eta_{\text{MAX}}$$

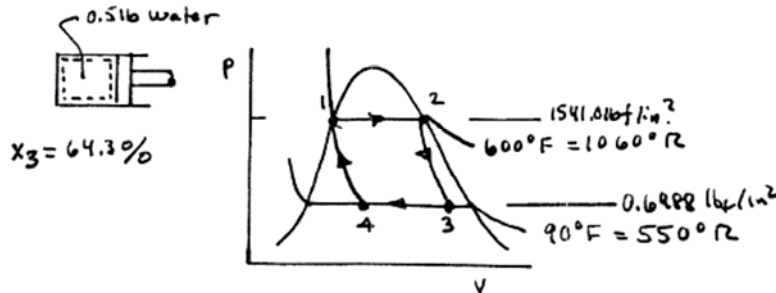
which agrees within roundoff with the result of Method \#1.

PROBLEM 5.76

KNOWN: 0.5 lb of water executes a Carnot cycle for which property data are provided.

FIND: Sketch the cycle in p-v coordinates, evaluate the heat and work for each process, and evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:



SUMMARY:

Process	Q	W
1-2	274.85	34.81
2-3	0	200.21
3-4	-142.61	-8.29
4-1	0	-94.71
Cycle	132.24	132.02

(agrees to within roundoff)

ENGR. MODEL: 1. The system is shown in the schematic. 2. Volume change is the only work mode.

ANALYSIS: (b) **Process 1-2:** $W_{12}/m = \int p dv = p(v_2 - v_1)$. Energy balance: $Q_{12}/m = (u_2 - u_1) + W_{12}/m$. Thus, $Q_{12}/m = (u_2 - u_1) + p(v_2 - v_1) = h_2 - h_1$. With data from Table A-3E, $Q_{12} = 0.5 \text{ lb}(549.7 \text{ Btu/lb}) = 274.85 \text{ Btu}$. $W_{12} = (0.5 \text{ lb})(1541.0 \text{ lbf/ft}^2)(0.2677 - 0.02363) \text{ ft}^3/\text{lb} | 1 \text{ Btu}/778 \text{ ft} \cdot \text{lbf}| = 34.81 \text{ Btu}$.

Process 2-3: $Q_{23} = 0$. An energy balance reduces to $W_{23} = m(u_2 - u_3)$. From Table A-2E $u_2 = 1090.0 \text{ Btu/lb}$, $u_3 = 58.07 + 0.643(1040.2 - 58.07) = 689.58 \text{ Btu/lb}$. Then $W_{23} = 0.5(1090.0 - 689.58) = 200.21 \text{ Btu}$.

Process 3-4: As for process 1-2: $W_{34} = m(p(v_4 - v_3))$, $Q_{34} = m(h_4 - h_3)$. Since the system undergoes a reversible cycle while communicating with reservoirs at $T_H = 1060^\circ\text{R}$ and $T_C = 550^\circ\text{R}$, Eq. 5.7 is applicable: $\frac{Q_{34}}{550} = \frac{Q_{12}}{1060} \Rightarrow Q_{34} = -142.61 \text{ Btu}$. Thus, $h_4 = \frac{Q_{34}}{m} + h_3 = -285.22 + (58.07 + 0.643(1040.2)) = 443.31 \text{ Btu/lb}$. Using this, the quality at state 4 can be determined: $x_4 = (h_4 - h_f)/h_{fg} = (443.31 - 58.07)/1040.2 = 0.369$.

Accordingly, $(v_4 - v_3) = (v_f + x_4 v_{fg}) - (v_f + x_3 v_{fg}) = v_{fg}(x_4 - x_3) = [467.7 - 0.01610](0.369 - 0.643) = -128.15 \text{ ft}^3/\text{lb}$. Finally, $W_{34} = (0.5 \text{ lb})(0.6988 \times 144 \text{ lbf/ft}^2)(-128.15 \text{ ft}^3/\text{lb}) | 1 \text{ Btu}/778 \text{ ft} \cdot \text{lbf}| = -8.29 \text{ Btu}$.

Process 4-1: $Q_{41} = 0$. Energy balance, $W_{41} = m(u_4 - u_1)$, $u_4 = 58.07 + 0.369(1040.2 - 58.07) = 420.48 \text{ Btu/lb}$, $u_1 = 609.90 \text{ Btu/lb}$. Then, $W_{41} = 0.5(420.48 - 609.9) = -94.71 \text{ Btu}$.

(c) The thermal efficiency is found using

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{550}{1060} = 0.481 \quad (48.1\%)$$

$$\textcircled{1} \quad \eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{132.02}{274.85} = 0.480 \quad (48.0\%)$$

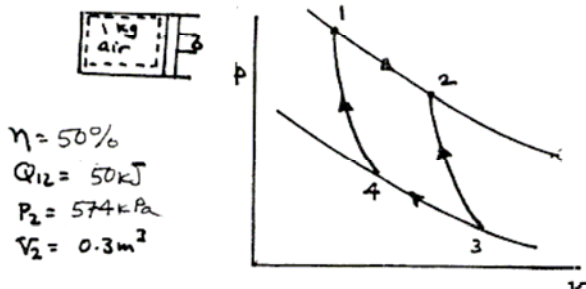
1. For any cycle, $W_{\text{cycle}} = Q_{\text{cycle}}$. From the process summary, $132.24 \approx 132.02$ (difference due to round-off)

PROBLEM 5.77

KNOWN: One kg of air undergoes a Carnot cycle for which $\eta = 50\%$.

FIND: Determine the minimum and maximum temperatures, the pressure and volume at the beginning of the isothermal expansion, the work and heat transfer for each process, and sketch the cycle on $p-v$ coordinates.

SCHEMATIC & GIVEN DATA:



Summary:

process	Q	W
1-2	50	50
2-3	0	220.7
3-4	-25	-25
4-1	0	-220.7
cycle	25	25

ENGR. MODEL: 1. The system shown in the schematic consists of air modeled as an ideal gas. 2. Volume change is the only work mode.

ANALYSIS: (a) Using the ideal gas model equation of state, $T_2 = P_2 V_2 / m R$. Then

$$T_2 = \frac{(574 \text{ kPa}) (0.3 \text{ m}^3)}{(1 \text{ kg}) \left(\frac{8.314 \text{ N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right)} = 600 \text{ K} \quad \text{Then, since } \eta = 1 - \frac{T_C}{T_H} \Rightarrow T_C = T_H (1 - \eta) \quad \text{With } T_H = T_2, \quad (a)$$

$$T_C = 600 (1 - 0.5) = 300 \text{ K} \quad \text{where } T_C = T_3 = T_4.$$

(b) For process 1-2, $Q_{12} = 50 \text{ kJ}$ (given). An energy balance reads $m(u_2 - u_1) = Q_{12} - W_{12}$, but since internal energy of an ideal gas depends on temperature and $T_1 = T_2$, $W_{12} = Q_{12}$. Further, $W_{12} = \int_1^2 p dV = \int_1^2 \frac{m R T_H}{V} dV = m R T_H \ln V_2 / V_1$. Solving and inserting values

$$\ln \frac{V_2}{V_1} = \frac{W_{12}}{m R T_H} \Rightarrow \ln \frac{V_2}{V_1} = \frac{50 \text{ kJ}}{(1 \text{ kg}) \left(\frac{8.314 \text{ N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (600 \text{ K})} = 0.2904 \Rightarrow V_1 = 0.224 \text{ m}^3 \quad (b)$$

$$\text{Since } T_1 = T_2, \quad P_1 V_1 = m R T, \quad P_2 V_2 = m R T \Rightarrow P_2 V_2 = P_1 V_1, \quad P_1 = P_2 (V_2 / V_1) = 574 \text{ kPa} (0.3 / 0.224) = 769 \text{ kPa}.$$

(c) For process 2-3: $Q_{23} = 0$. An energy balance reduces to give $W_{23} = m(u_2 - u_3)$ with data from Table A-22, $W_{23} = (1 \text{ kg}) (434.78 - 214.07) = 220.7 \text{ kJ}$. For process 3-4, $W_{34} = Q_{34}$ (as for process 1-2). Also, Eq. 5.7 is applicable: (c)

$$\frac{|Q_{34}|}{T_C} = \frac{Q_{12}}{T_H} \Rightarrow |Q_{34}| = \left(\frac{300}{600} \right) (50 \text{ kJ}) = 25 \text{ kJ} \Rightarrow Q_{34} = -25 \text{ kJ}, \quad W_{34} = -25 \text{ kJ}.$$

① Process 4-1, $Q_{41} = 0$. An energy balance reduces to give, $W_{41} = m(u_4 - u_1)$. Since $u_1 = u_2$, $u_4 = u_3$, $W_{41} = -220.7 \text{ kJ}$. The work and heat transfers are summarized in the table.

1. As checks on the calculations, note that (1) $W_{\text{cycle}} = Q_{\text{cycle}}$ (see summary above), and (2)

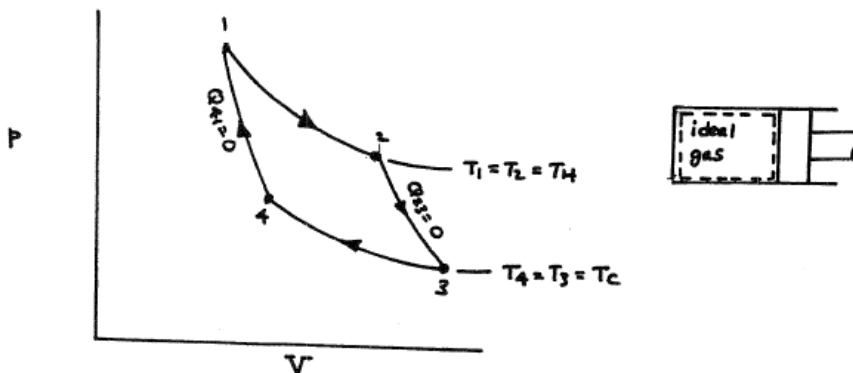
$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{25}{50} = 0.5$$

PROBLEM 5.78

KNOWN: A Carnot cycle is executed by an ideal gas with constant specific heat ratio k .

FIND: Show that (a) $V_4 V_2 = V_1 V_3$, (b) $T_2/T_3 = (P_2/P_3)^{(k-1)/k}$, (c) $T_2/T_3 = (V_3/V_2)^{k-1}$

SCHEMATIC & GIVEN DATA.



ENGR. MODEL: (1) The system shown in the figure consists of an ideal gas. (2) The specific heat ratio k is constant (required in part (b) only). (3) The system undergoes a Carnot cycle.

ANALYSIS: (a) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{W_{12} + W_{23}}{Q_{12}}$$

Since internal energy of an ideal gas depends on temperature only, an energy balance for process 1-2 reduces to $u_2 - u_1 = Q_{12} - W_{12}$, where $u_2 = u_1$. Thus, $Q_{12} = W_{12}$. Furthermore

$$W_{12} = \int_1^2 p dV = \int_1^2 \frac{m R T_H}{V} dV = m R T_H \ln \frac{V_2}{V_1}$$

Similarly, $W_{23} = m R T_C \ln V_4/V_3$. Collecting results

$$\eta = 1 - \frac{m R T_C \ln V_4/V_3}{m R T_H \ln V_2/V_1} = 1 - \left(\frac{\ln V_2/V_4}{\ln V_2/V_1} \right) \frac{T_C}{T_H}$$

However, for the Carnot cycle $\eta = 1 - T_C/T_H$, so it is necessary that

$$\frac{\ln(V_2/V_4)}{\ln(V_2/V_1)} = 1 \Rightarrow \ln\left(\frac{V_2}{V_4}\right) = \ln\left(\frac{V_2}{V_1}\right) \Rightarrow V_4 V_2 = V_3 V_1 \quad \text{(a)}$$

(b) As process 2-3 is adiabatic, a energy balance in differential form reads

$$dU = \delta Q - \delta W$$

where $\delta W = p dV$ and with assumption 1 $dU = m c_v dT$. Collecting results and using $pV = mRT$ and $c_v = R/(k-1)$ (Eq. 3.47b)

$$\frac{1}{k-1} d \ln T = - d \ln V$$

Integration for constant k (assumption 2) gives

$$\ln \frac{T_3}{T_2} = - \ln \left(\frac{V_3}{V_2} \right)^{k-1} \Rightarrow \frac{T_3}{T_2} = \left(\frac{V_3}{V_2} \right)^{k-1} \quad \text{(c)}$$

Finally, using $V = mRT/p$

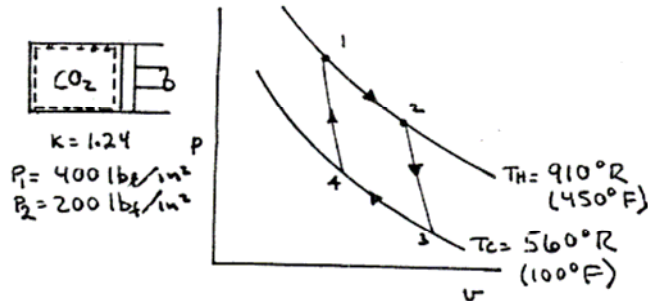
$$\frac{T_3}{T_2} = \left[\frac{T_3}{T_2} \frac{P_2}{P_3} \right]^{k-1} \Rightarrow \frac{T_3}{T_2} = \left(\frac{P_2}{P_3} \right)^{\frac{k-1}{k}} \quad \text{(b)}$$

PROBLEM 5.79

KNOWN: A quantity of CO_2 undergoes a Carnot cycle.

FIND: Determine the work and heat transfer for each process, the thermal efficiency, and the pressures at the initial and final states of the isothermal compression.

SCHEMATIC & GIVEN DATA:



Summary:

Process	Q/m	W/m
1-2	28.46	28.46
2-3	0	65.80
3-4	-17.52	-17.52
4-1	0	-65.80
Cycle	10.94	10.94

ENGR. MODEL: 1. The system consists of a quantity of CO_2 modeled as an ideal gas with $k=1.24$. 2. Volume change is the only work mode.

ANALYSIS: (a) **Process 1-2:** Since $T_2 = T_1$, for an ideal gas at fixed temperature, an energy balance reduces to give $Q_{12} = W_{12}$, where $W_{12} = \int_1^2 p \, dV = \int_1^2 \frac{mRT}{V} dV$, or $W_{12}/m = RT_1 \ln(V_2/V_1)$. With $P_1 V_1 = mRT_1$, $P_2 V_2 = mRT_1$, we have

$$\frac{W_{12}}{m} = RT_1 \ln \frac{P_1}{P_2} = \left(\frac{1545}{44.01} \right) \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \left(\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right) (910^\circ\text{R}) \ln \left(\frac{400}{200} \right) = 28.46 \text{ Btu/lb}$$

Process 2-3: $Q_{23} = 0$. An energy balance gives $W_{23} = m(u_2 - u_3)$. For an ideal gas with constant k , Eq. 3.47b gives $c_v = \frac{R}{k-1} = \left(\frac{1545}{44.01} \right) \left(\frac{1}{1.24-1} \right) = 0.188 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$

$$\frac{W_{23}}{m} = c_v (T_2 - T_3) = \left(0.188 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (910 - 560)^\circ\text{R} = 65.8 \text{ Btu/lb}$$

Process 3-4: Since $T_3 = T_4$, an energy balance gives $Q_{34} = W_{34}$, where

$$\frac{W_{34}}{m} = \int_3^4 \frac{RT_C}{V} dV = RT_C \ln \frac{V_4}{V_3} = RT_C \ln \frac{P_3}{P_4} = \left(\frac{1545}{44.01} \right) \left(\frac{1}{778} \right) (560) \ln \left(\frac{16.28}{32.56} \right) = -17.52 \frac{\text{Btu}}{\text{lb}}$$

where to find P_3, P_4 , use part (b) of Problem 5.78

$$\left. \begin{aligned} \frac{P_3}{P_2} &= \left(\frac{T_2}{T_3} \right)^{\frac{k}{k-1}} \Rightarrow P_3 = 200 \frac{\text{lbf}}{\text{in}^2} \left[\frac{560}{910} \right]^{\frac{1.24}{0.24}} = 16.28 \frac{\text{lbf}}{\text{in}^2} \\ \frac{P_4}{P_1} &= \left(\frac{T_1}{T_4} \right)^{\frac{k}{k-1}} \Rightarrow \frac{P_4}{P_1} = \frac{P_3}{P_2} \Rightarrow P_4 = \frac{P_1}{P_2} P_3 = 32.56 \frac{\text{lbf}}{\text{in}^2} \end{aligned} \right\} \leftarrow (c)$$

Process 4-1: $Q_{41} = 0$, and an energy balance gives $W_{41} = m(u_4 - u_1)$, or

$$\frac{W_{41}}{m} = c_v [T_4 - T_1] = \left(0.188 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (560 - 910)^\circ\text{R} = -65.8 \text{ Btu/lb}$$

(b) The thermal efficiency is given with this case by

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{10.94}{28.46} = 0.384 \text{ (38.4\%)}$$

or using Eq. 5.9

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{560}{910} = 0.384 \text{ (38.4\%)}$$

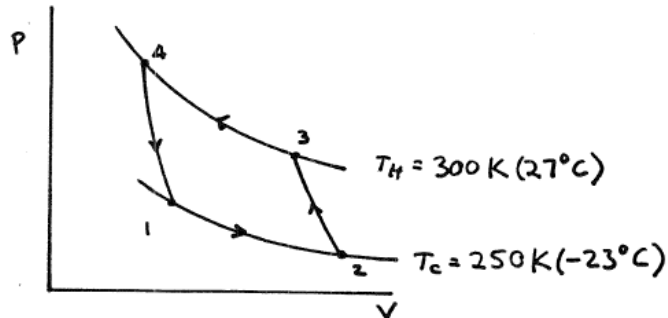
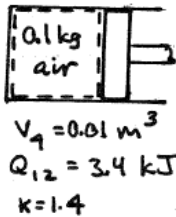
1. For any cycle, $Q_{\text{cycle}} = W_{\text{cycle}}$. See summary table above. This provides a check on the calculations.

PROBLEM 5.80

KNOWN: One-tenth kilogram of air executes a Carnot refrigeration cycle for which data are provided.

FIND: Determine (a) the pressure at each of the four principal states, (b) the work for each of the four processes, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system consists of air modeled as an ideal gas with $k = 1.4$. (2) Volume change is the only work mode.

ANALYSIS: From the solution to Problem 5.78, $T_4/T_1 = (P_4/P_1)^{1/k}$ and $V_4 V_2 = V_3 V_1$.

(a) Using the ideal gas equation

$$P_4 = \frac{mRT_4}{V_4} = \frac{(0.1 \text{ kg}) \left(\frac{8.314}{28.97} \right) \frac{\text{kJ}}{\text{kgK}} (300 \text{ K}) \left| \frac{1 \text{ kNm}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right|}{(0.01 \text{ m}^3)} = 861.0 \text{ kPa} \leftarrow P_4$$

Now, for process 4-1; $T_4/T_1 = (P_4/P_1)^{k-1/k}$, or

$$P_1 = \left(\frac{T_1}{T_4} \right)^{k/k-1} P_4 = \left(\frac{250}{300} \right)^{1.4/1.4-1} (861 \frac{\text{lb}_f}{\text{in}^2}) = 454.9 \text{ kPa} \leftarrow P_1$$

The volume at 1 is

$$V_1 = \frac{mRT_1}{P_1} = \frac{(0.1) \left(\frac{8.314}{28.97} \right) (250)}{(454.9)} = 0.01577 \text{ m}^3$$

For process 1-2: $m(u_2 - u_1) = Q_{12} - W_{12}$. Since $T_1 = T_2 = T_c$, $u_2 = u_1$, and

$$Q_{12} = W_{12} = \int_1^2 P dV = \int_1^2 \frac{mRT}{V} dV = mRT_c \ln \frac{V_2}{V_1}$$

Solving for V_2/V_1 and inserting values

$$\frac{V_2}{V_1} = \exp \left[\frac{W_{12}}{mRT_c} \right] = \exp \left[\frac{(3.4 \text{ kJ})}{(0.1 \text{ kg}) \left(\frac{8.314}{28.97} \right) \frac{\text{kJ}}{\text{kgK}} (250 \text{ K})} \right] = 1.6062$$

$$\therefore V_2 = (1.6062)(0.01577 \text{ m}^3) = 0.02533 \text{ m}^3$$

Thus

$$P_2 = \frac{(0.1) \left(\frac{8.314}{28.97} \right) (250)}{(0.02533)} = 283.2 \text{ kPa} \leftarrow P_2$$

Next, from above

$$V_4 V_2 = V_3 V_1 \Rightarrow V_3 = \left(\frac{V_4}{V_1} \right) V_2 = \left(\frac{0.01}{0.01577} \right) (0.02533 \text{ m}^3) = 0.01606 \text{ m}^3$$

Continued on next slide

Problem 5-80 continued

Finally

$$P_3 = \frac{(0.1) \left(\frac{8.314}{28.97} \right) (300)}{(0.01606)} = 536.1 \text{ kPa} \leftarrow P_3$$

(b) Process 1-2: $Q_{12} = W_{12} = 3.4 \text{ kJ} \leftarrow W_{12}$

process 2-3: $Q_{23} = 0 \Rightarrow W_{23} = -m(u_3 - u_2) = -m c_v (T_3 - T_2)$

with $c_v = R/(k-1) = \left(\frac{8.314}{28.97} \right) \left(\frac{1}{1.4-1} \right) = 0.717 \text{ kJ/kg K}$

$$W_{23} = -(0.1 \text{ kg}) (0.717 \frac{\text{kJ}}{\text{kg K}}) (300 - 250) \text{ K} = -3.585 \text{ kJ} \leftarrow W_{23}$$

Process 3-4: As for process 1-2 in part (a)

$$W_{34} = \int_3^4 p dV = m R T_H \ln \frac{V_4}{V_3} = (0.1 \text{ kg}) \left(\frac{8.314}{28.97} \right) \frac{\text{kJ}}{\text{kg K}} (300 \text{ K}) \ln \left(\frac{0.01}{0.01606} \right) = -4.079 \text{ kJ} \leftarrow W_{34}$$

Process 4-1: $Q_{41} = 0 \Rightarrow W_{41} = -m(u_1 - u_4) = -m c_v (T_1 - T_4)$. Again

$$W_{41} = -(0.1 \text{ kg}) (0.717 \frac{\text{kJ}}{\text{kg K}}) (250 - 300) \text{ K} = 3.585 \text{ kJ} \leftarrow W_{41}$$

(c) For the Carnot cycle

$$\textcircled{1} \quad \beta = \beta_{\max} = \frac{T_c}{T_H - T_c} = \frac{250}{300 - 250} = 5.00 \leftarrow \beta$$

1. Alternatively,

$$|W_{\text{cycle}}| = |W_{12} + W_{23} + W_{34} + W_{41}| = |-0.679 \text{ kJ}|$$

and, with $Q_{\text{in}} = Q_{12}$ for the cycle

$$\beta = \frac{Q_{\text{in}}}{|W_{\text{cycle}}|} = \frac{3.4}{0.679} = 5.01$$

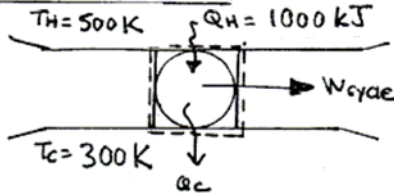
The slight difference is due to round-off.

PROBLEM 5.81

KNOWN: A system undergoes a power cycle for which data are provided.

FIND: Using Eq. 5.13, evaluate σ_{cycle} if η is 60, 40, and 20%.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The system shown in the accompanying figure undergoes a power cycle while receiving Q_H at T_H and discharging Q_C at T_C .

ENGR. MODEL: For any cycle, $\eta = 1 - Q_C/Q_H \Rightarrow Q_C = (1 - \eta)Q_H$. Then, Eq. 5.13 gives

$$\sigma_{\text{cycle}} = -\oint \left(\frac{\delta Q}{T} \right)_b = - \left[\frac{Q_H}{T_H} - \frac{Q_C}{T_C} \right] = -Q_H \left[\frac{1}{T_H} - \frac{(1-\eta)}{T_C} \right]$$

(a) $\eta = 60\%$

$$\sigma_{\text{cycle}} = -(1000 \text{ kJ}) \left[\frac{1}{500 \text{ K}} - \frac{(1-0.6)}{300 \text{ K}} \right] = -0.667 \frac{\text{kJ}}{\text{K}}$$

① Since σ_{cycle} must be positive or zero in value, this case is impossible.

(b) $\eta = 40\%$

$$\sigma_{\text{cycle}} = -(1000 \text{ kJ}) \left[\frac{1}{500 \text{ K}} - \frac{(1-0.4)}{300 \text{ K}} \right] = 0$$

Since $\sigma_{\text{cycle}} = 0$, this case corresponds to internally reversible operation.

(c) $\eta = 20\%$

$$\sigma_{\text{cycle}} = -(1000 \text{ kJ}) \left[\frac{1}{500 \text{ K}} - \frac{(1-0.2)}{300 \text{ K}} \right] = +0.667 \frac{\text{kJ}}{\text{K}}$$

In this case, irreversibilities are present within the system.

1. Using Eq. 5.13, the maximum thermal efficiency any cycle can have while operating between reservoirs at T_H and T_C is

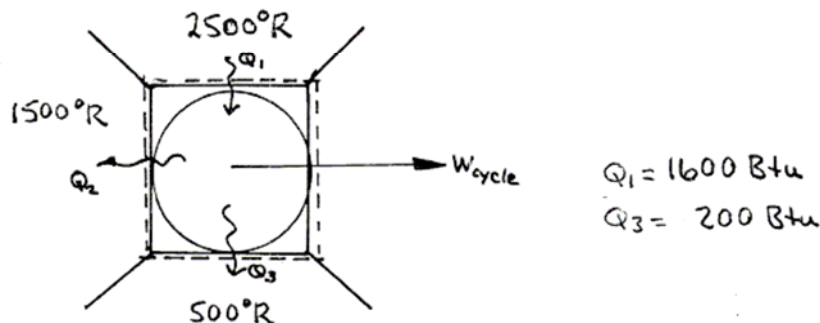
$$\eta_{\text{max}} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{500} = 0.4 \text{ (40\%)}$$

PROBLEM 5.82

KNOWN: A system undergoes a reversible power cycle while communicating thermally with three reservoirs at specified temperatures. The values for the energy transfer by heat between two of the reservoirs and the system are also specified.

FIND: Determine the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The system shown in the accompanying figure undergoes a power cycle with no internal irreversibilities. 2. All heat transfers take place at the indicated temperatures and are positive in the direction of the accompanying arrow.

ANALYSIS: The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{W_{\text{cycle}}}{Q_1}$$

An energy balance gives

$$\begin{aligned} W_{\text{cycle}} &= Q_1 - Q_2 - Q_3 = 1600 - Q_2 - 200 \\ &= 1400 - Q_2 \end{aligned}$$

Since the cycle involves no irreversibilities, and heat transfers are at T_1, T_2, T_3 , Eq. 5.13 reads

$$\begin{aligned} \frac{Q_1}{T_1} - \frac{Q_2}{T_2} - \frac{Q_3}{T_3} &= -\oint \frac{\delta Q}{T} = 0 \\ \Rightarrow 0 &= \frac{1600}{2500} - \frac{Q_2}{1500} - \frac{200}{500} \Rightarrow Q_2 = 360 \text{ Btu} \end{aligned}$$

Thus

$$W_{\text{cycle}} = 1400 - 360 = 1040 \text{ Btu}$$

and

$$\eta = \frac{1040}{1600} = 0.65 \text{ (65\%)}$$

PROBLEM 5.83

As shown in Fig. P5.83, a system executes a power cycle while receiving 750 kJ by heat transfer at a temperature of 1500 K and discharging 100 kJ by heat transfer at a temperature of 500 K. Another heat transfer from the system occurs at a temperature of 1000 K. Using Eq. 5.13, determine the thermal efficiency if σ_{cycle} is (a) 0.1 kJ/K, (b) 0.2 kJ/K, (c) 0.35 kJ/K.

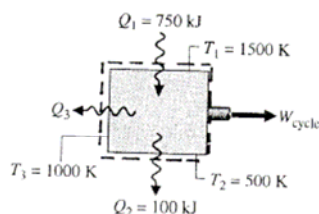


Fig. P5.83

ENGR. MODEL:

1. The system shown in Fig. P5.83 executes a power cycle.
2. All heat transfers are at the indicated temperatures and in the direction of the accompanying arrow.

ANALYSIS: For any cycle, $\eta = W_{\text{cycle}}/Q_{\text{in}}$.

Also, $W_{\text{cycle}} = Q_{\text{cycle}} = Q_1 - Q_2 - Q_3$. Thus,

$$\eta = \frac{Q_1 - Q_2 - Q_3}{Q_1} = \frac{750 - 100 - Q_3}{750} \quad (1)$$

Using Eq. 5.13

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = -\sigma_{\text{cycle}}$$

$$\begin{aligned} \Rightarrow Q_3 &= T_3 \left[\frac{Q_1}{T_1} - \frac{Q_2}{T_2} + \sigma_{\text{cycle}} \right] \\ &= 1000 \text{ K} \left[\frac{750 \text{ kJ}}{1500 \text{ K}} - \frac{100 \text{ kJ}}{500 \text{ K}} + \sigma_{\text{cycle}} \right] \\ &= 1000 \text{ K} [0.3 + \sigma_{\text{cycle}}] \frac{\text{kJ}}{\text{K}} \quad (2) \end{aligned}$$

1) (a) $\sigma_{\text{cycle}} = 0.1$. Eq. (2) gives 400 kJ. Then, Eq. (1) gives

$$\eta = \frac{750 - 100 - 400}{750} = 0.333 \quad (33.3\%)$$

← (a)

(b) $\sigma_{\text{cycle}} = 0.2$. Eq. (2) gives 500 kJ. Then, Eq. (1) gives

$$\eta = \frac{750 - 100 - 500}{750} = 0.2 \quad (20\%)$$

← (b)

(c) $\sigma_{\text{cycle}} = 0.35$. Eq. (2) gives 650 kJ. Then, Eq. (1) gives

$$\eta = \frac{750 - 100 - 650}{750} = 0 \quad (0\%)$$

← (c)

1. If $\sigma_{\text{cycle}} = 0$, Eq. (2) gives 300 kJ. Then, Eq. (1) gives

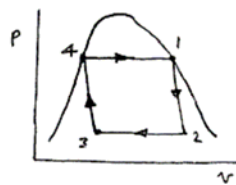
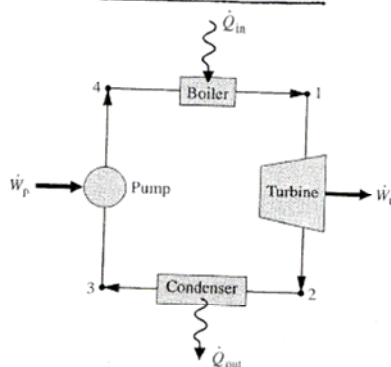
$$\eta = \frac{750 - 100 - 300}{750} = 0.467 \quad (46.7\%) \quad \leftarrow \text{MAX } \eta$$

PROBLEM 5.84

A simple vapor power plant operating at steady state is shown in Fig. P5.84. In each of the following three cases, use Eq. 5.13 to determine whether the cycle is *possible*, *internally reversible*, or *impossible*. Processes 1-2 and 3-4 are adiabatic. Kinetic and potential energy changes can be ignored.

- Process 4-1: constant-pressure at 1 MPa from saturated liquid to saturated vapor. Process 2-3: constant-pressure at 20 kPa from $x_2 = 88\%$ to $x_3 = 18\%$.
- Process 4-1: constant-pressure at 8 MPa from saturated liquid to saturated vapor. Process 2-3: constant-pressure at 8 kPa from $x_2 = 67.5\%$ to $x_3 = 34.2\%$.
- Process 4-1: constant-pressure at 0.15 MPa from saturated liquid to saturated vapor. Process 2-3: constant-pressure at 20 kPa from $x_2 = 90\%$ to $x_3 = 10\%$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

- A control volume at steady state encloses each of the four components.
- Power and heat transfer rates are in the directions of the arrows.
- Stray heat transfer can be ignored.
- Kinetic and potential energy changes can be ignored.

ANALYSIS: Property data from Table A-3,

	P_{41} (MPa)	T_{41} (°C)	$(h_g - h_f)$ (kJ/kg)	P_{23} (kPa)	T_{23} (°C)	$(h_g - h_f)$ (kJ/kg)
a	1	179.9	2015.3	20	60.06	2358.3
b	8	295.1	1441.3	8	41.51	2403.1
c	0.15	111.4	2226.5	20	60.06	2358.3

Expressed on a rate basis, Eq. 5.13 takes the form,

$$\frac{\dot{Q}_{41}}{T_{41}} - \frac{\dot{Q}_{23}}{T_{23}} = -\dot{\sigma}_{\text{cycle}} \quad \text{where } \dot{Q}_{41} = \dot{m}(h_1 - h_4) = \dot{m}(h_{fg})_{41} \text{ and } \dot{Q}_{23} = \dot{m}(h_2 - h_3) = \dot{m}[(h_f + x h_{fg})_2 - (h_f + x h_{fg})_3] = \dot{m}(x_2 - x_3)(h_{fg})_{23}$$

Accordingly,

$$\frac{\dot{\sigma}_{\text{cycle}}}{\dot{m}} = \frac{(x_2 - x_3)(h_{fg})_{23}}{T_{23}} - \frac{(h_{fg})_{41}}{T_{41}} \quad (1)$$

(a) $x_2 = 0.88, x_3 = 0.18$

$$\frac{\dot{\sigma}_{\text{cycle}}}{\dot{m}} = \frac{(0.88 - 0.18)(2358.3)}{333.21} - \frac{2015.3}{453.01} = 0.5 \frac{\text{KJ}}{\text{K}} \quad \text{possible} \quad \leftarrow (a)$$

(b) $x_2 = 0.675, x_3 = 0.342$

$$\frac{\dot{\sigma}_{\text{cycle}}}{\dot{m}} = \frac{(0.675 - 0.342)(2403.1)}{314.66} - \frac{1441.3}{568.25} \approx 0 \frac{\text{KJ}}{\text{K}} \quad \text{internally reversible} \quad \leftarrow (b)$$

(c) $x_2 = 0.9, x_3 = 0.10$

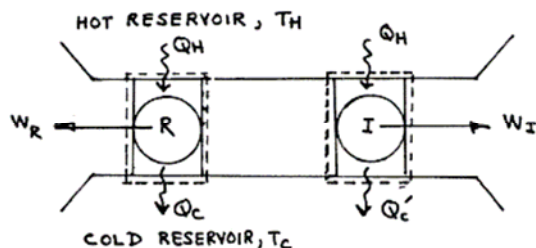
$$\frac{\dot{\sigma}_{\text{cycle}}}{\dot{m}} = \frac{(0.9 - 0.10)(2358.3)}{333.21} - \frac{2226.5}{354.55} = -0.13 \frac{\text{KJ}}{\text{K}} \quad \text{impossible} \quad \leftarrow (c)$$

PROBLEM 5.85

KNOWN: A reversible power cycle R and an irreversible cycle I operate between the same two reservoirs.

FIND: (a) Evaluate σ_{cycle} for I in terms of W_R , W_I , and the temperature T_c .
(b) Demonstrate that $W_I < W_R$, $Q'_c > Q_c$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The systems shown in the accompanying figure undergo power cycles. R is reversible and I is irreversible. (2) Each system receives Q_H at T_H from the hot reservoir and discharges energy at T_c to the cold reservoir.

ANALYSIS: (a) For cycle I, Eq. 5.13 takes the form

$$\sigma_{\text{cycle}} = - \oint \left(\frac{\delta Q}{T} \right)_b = - \left[\frac{Q_H}{T_H} - \frac{Q'_c}{T_c} \right]$$

Energy balances for cycles R and I give, respectively

$$\begin{aligned} Q_H &= W_R + Q_c \\ Q_H &= W_I + Q'_c \end{aligned} \quad \Rightarrow \quad Q'_c = Q_c + W_R - W_I$$

Inserting this result into the expression for σ_{cycle}

$$\sigma_{\text{cycle}} = - \left[\left(\frac{Q_H}{T_H} - \frac{Q_c}{T_c} \right) - \left(\frac{W_R - W_I}{T_c} \right) \right]$$

Since R is reversible, Eq. 5.7 applies and the first term on the right vanishes, leaving

$$\sigma_{\text{cycle}} = \frac{W_R - W_I}{T_c} \quad \leftarrow \quad (a)$$

(b) When irreversibilities are present, $\sigma_{\text{cycle}} > 0$. Thus, Eq. (a) reads

$$\frac{W_R - W_I}{T_c} > 0 \Rightarrow W_R > W_I$$

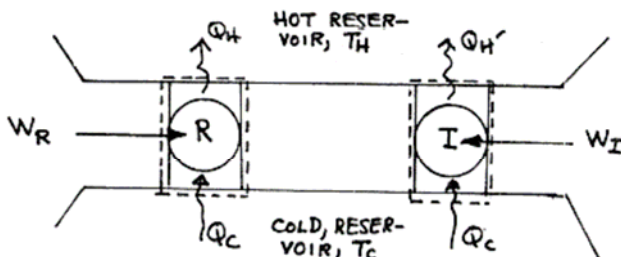
The energy balance result: $Q'_c = Q_c + W_R - W_I$ then indicates that $Q'_c > Q_c$. Accordingly, the effect of irreversibilities is to reduce the desired outcome: W_{cycle} and increase the heat rejected, which for actual power cycles increases the thermal pollution effect.

PROBLEM 5.86

KNOWN: A reversible refrigeration cycle R and an irreversible refrigeration cycle I operate between the same two reservoirs.

FIND: Show that $W_I > W_R$ and $Q_H' > Q_H$

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The systems shown in the accompanying figure undergo refrigeration cycles. R is reversible and I is irreversible. (2) Each system receives Q_C at T_C from the cold reservoir and discharges energy at T_H to the hot reservoir.

ANALYSIS: For cycle I, Eq. 5.13 takes the form

$$\sigma_{\text{cycle}} = - \oint \left(\frac{\delta Q}{T} \right)_b = - \left[\frac{Q_C}{T_C} - \frac{Q_H'}{T_H} \right] \quad (1)$$

Energy balances for cycles R and I give, respectively

$$\begin{aligned} Q_H &= W_R + Q_C \\ Q_H' &= W_I + Q_C \end{aligned} \quad \Rightarrow \quad Q_H' = Q_H + W_I - W_R \quad (2)$$

Inserting this result into the expression for σ_{cycle} , Eq. (1),

$$\sigma_{\text{cycle}} = - \left[\left(\frac{Q_C}{T_C} - \frac{Q_H}{T_H} \right) - \left(\frac{W_I - W_R}{T_H} \right) \right]$$

Since R is reversible, Eq. 5.7 applies and the first term on the right vanishes leaving

$$\sigma_{\text{cycle}} = \frac{W_I - W_R}{T_H}$$

Since cycle I is irreversible, $\sigma_{\text{cycle}} > 0$. Thus, $W_I - W_R > 0$, and $W_I > W_R$. Equation (2) then gives $Q_H' > Q_H$.

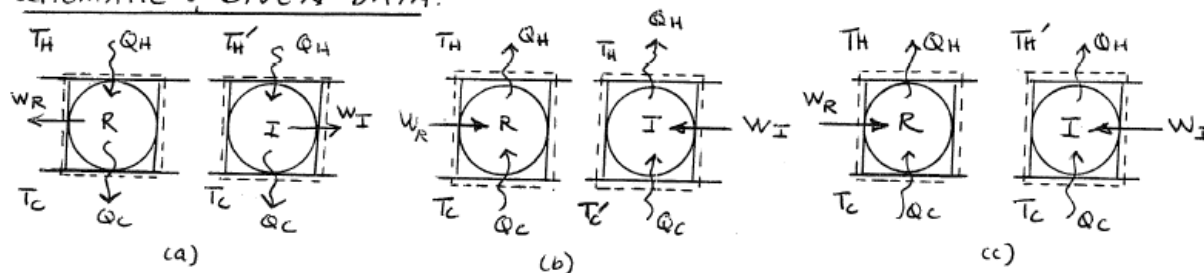
PROBLEM 5.87

KNOWN: Operating details are provided for reversible and irreversible cycles.

FIND: (a) For power cycles, show $T_H' > T_H$, (b) For refrigeration cycles, show $T_C' > T_C$,

(c) For heat pump cycles, show $T_H' < T_H$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The systems shown in the figure undergo cycles. R denotes a reversible cycle and I denotes an irreversible cycle, (2) The systems receive and discharge energy by heat transfer at the indicated temperatures, (3) The only energy transfers taking place are shown on the figures.

ANALYSIS: (a) Applying Eq. 5.13 to R and I, respectively

$$\begin{aligned}
 R: \quad \frac{Q_H}{T_H} - \frac{Q_C}{T_C} &= -\sigma_{\text{cycle}} \Rightarrow Q_C = \frac{T_C}{T_H} Q_H \\
 I: \quad \frac{Q_H}{T_H'} - \frac{Q_C}{T_C} &= -\sigma_{\text{cycle}}
 \end{aligned}
 \Rightarrow \frac{Q_H}{T_H'} - \left(\frac{T_C}{T_H} \frac{Q_H}{T_H} \right) \frac{1}{T_C} = -\sigma_{\text{cycle}}$$

$$Q_H \left[\frac{1}{T_H'} - \frac{1}{T_H} \right] = -\sigma_{\text{cycle}} \quad \leftarrow (a)$$

$$\Rightarrow T_H' > T_H$$

(b) Similarly

$$\begin{aligned}
 R: \quad \frac{Q_C}{T_C} - \frac{Q_H}{T_H} &= 0 \Rightarrow Q_H = \frac{T_H}{T_C} Q_C \\
 I: \quad \frac{Q_C}{T_C'} - \frac{Q_H}{T_H} &= -\sigma_{\text{cycle}}
 \end{aligned}
 \Rightarrow \frac{Q_C}{T_C'} - \left(\frac{T_H}{T_C} Q_C \right) \frac{1}{T_H} = -\sigma_{\text{cycle}}$$

$$Q_C \left[\frac{1}{T_C'} - \frac{1}{T_C} \right] = -\sigma_{\text{cycle}} \quad \leftarrow (b)$$

$$\Rightarrow T_C' > T_C$$

(c) Similarly

$$\begin{aligned}
 R: \quad \frac{Q_C}{T_C} - \frac{Q_H}{T_H} &= 0 \Rightarrow Q_C = \frac{T_C}{T_H} Q_H \\
 I: \quad \frac{Q_C}{T_C} - \frac{Q_H}{T_H'} &= -\sigma_{\text{cycle}}
 \end{aligned}
 \Rightarrow \left(\frac{T_C}{T_H} Q_H \right) \frac{1}{T_C} - \frac{Q_H}{T_H'} = -\sigma_{\text{cycle}}$$

$$Q_H \left[\frac{1}{T_H} - \frac{1}{T_H'} \right] = -\sigma_{\text{cycle}} \quad \leftarrow (c)$$

$$\Rightarrow T_H' < T_H$$

PROBLEM 5.88

Figure P5.88 shows a system consisting of a power cycle driving a heat pump. At steady state, the power cycle receives $\dot{Q}_s$ by heat transfer at T_s from the high-temperature source and delivers $\dot{Q}_1$ to a dwelling at T_d . The heat pump receives $\dot{Q}_0$ from the outdoors at T_0 , and delivers $\dot{Q}_2$ to the dwelling. Using Eq. 5.13, obtain an expression for the maximum theoretical value of the performance parameter $(\dot{Q}_1 + \dot{Q}_2)/\dot{Q}_s$ in terms of the temperature ratios T_s/T_d and T_0/T_d .

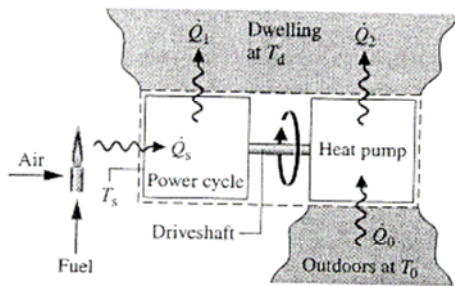


Fig. P5.88

ENGR. MODEL:

1. The system in the figure is at steady state.
2. Each heat transfer rate is in the direction of the accompanying arrow and occurs at the specified temperature.

ANALYSIS: An energy rate balance gives, $\dot{Q}_0 + \dot{Q}_s = \dot{Q}_1 + \dot{Q}_2$, or

$$\dot{Q}_0 = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_s \quad (1)$$

Applying Eq. 5.13,

$$0 = \frac{\dot{Q}_s}{T_s} + \frac{\dot{Q}_0}{T_0} - \frac{\dot{Q}_1}{T_d} - \frac{\dot{Q}_2}{T_d} = -\dot{\sigma}_{\text{cycle}}$$

$$\Rightarrow \frac{\dot{Q}_1 + \dot{Q}_2}{T_d} = \frac{\dot{Q}_s}{T_s} + \frac{\dot{Q}_0}{T_0} + \dot{\sigma}_{\text{cycle}} \quad (2)$$

Inserting Eq. (1), Eq. (2) becomes

$$\frac{\dot{Q}_1 + \dot{Q}_2}{T_d} = \frac{\dot{Q}_s}{T_s} + \frac{(\dot{Q}_1 + \dot{Q}_2 - \dot{Q}_s)}{T_0} + \dot{\sigma}_{\text{cycle}}$$

$$\text{or} \quad (\dot{Q}_1 + \dot{Q}_2) \left[\frac{1}{T_d} - \frac{1}{T_0} \right] = \dot{Q}_s \left[\frac{1}{T_s} - \frac{1}{T_0} \right] + \dot{\sigma}_{\text{cycle}}$$

$$\Rightarrow \left(\frac{\dot{Q}_1 + \dot{Q}_2}{\dot{Q}_s} \right) = \underbrace{\frac{T_d}{T_s} \left[\frac{T_s - T_0}{T_d - T_0} \right]}_{(a)} - \underbrace{\frac{(T_0 T_d) \dot{\sigma}_{\text{cycle}}}{\dot{Q}_s [T_d - T_0]}}_{(b)}$$

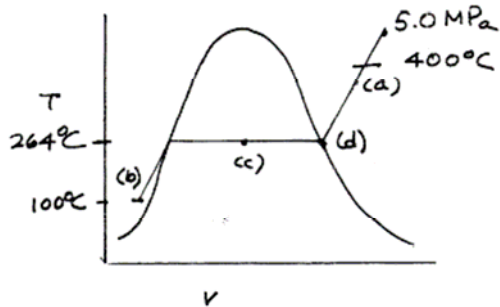
Term (a) is positive since $T_s > T_0$, $T_d > T_0$. Term (b) is positive when internal irreversibilities are present and vanishes when the system operates internally reversibly. Thus,

$$\left(\frac{\dot{Q}_1 + \dot{Q}_2}{\dot{Q}_s} \right)_{\text{MAX}} = \frac{T_d}{T_s} \left[\frac{T_s - T_0}{T_d - T_0} \right]$$



PROBLEM 6.1

FIND: Determine the specific entropy, in $\text{kJ/kg} \cdot \text{K}$, of water at the following states:



(a) 5.0 MPa, 400°C. Interpolation in Table A-4: $s = 6.6549 \text{ kJ/kg} \cdot \text{K}$. ← (a)

(b) 5.0 MPa, 100°C. Table A-5: $s = 1.303 \text{ kJ/kg} \cdot \text{K}$. ← (b)

(c) 5.0 MPa, $u = 1872.5 \text{ kJ/kg}$. Table A-3 gives $u_f = 1147.8 \text{ kJ/kg}$, $u_g = 2597.1 \text{ kJ/kg}$.

Thus

$$x = \frac{1872.5 - 1147.8}{2597.1 - 1147.8} = 0.5$$

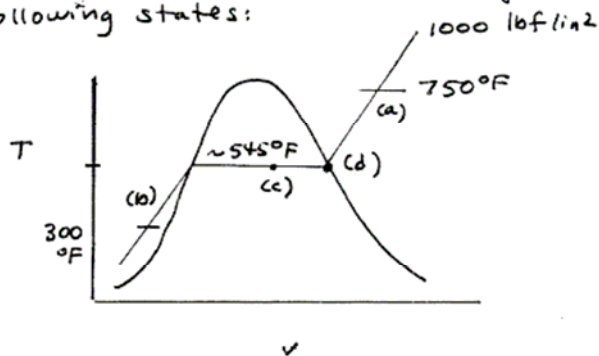
Then

$$s = s_f + x(s_g - s_f) = 2.9202 + 0.5[5.9734 - 2.9202] = 4.4468 \text{ kJ/kg} \cdot \text{K} \leftarrow (c)$$

(d) 5.0 MPa, Saturated vapor. Table A-3: $s_g = 5.9734 \text{ kJ/kg} \cdot \text{K}$ ← (d)

PROBLEM 6.2

FIND: Determine the specific entropy, in Btu/lb·°R, of water at the following states:



(a) 1000 lbf/in², 750°F. Interpolation in Table A-4E: $s = 1.54$ Btu/lb·°R. ← (a)

(b) 1000 lbf/in², 300°F. Table A-5E: $s = 0.43552$ Btu/lb·°R. ← (b)

(c) 1000 lbf/in², $h = 932.4$ Btu/lb. Table A-3E gives $h_f = 542.4$ Btu/lb, $h_g = 650.0$, $h_g = 1142.4$ Btu/lb. Thus

$$x = \frac{932.4 - 542.4}{650} = 0.6$$

Then,

$$s = s_f + x(s_g - s_f)$$

$$= 0.7432 + 0.6(0.6471) = 1.1315 \text{ Btu/lb·°R.} \quad \leftarrow (c)$$

(d) 1000 lbf/in², saturated vapor. Table A-3E: $s_g = 1.3903$ Btu/lb·°R ← (d)

PROBLEM 6.3

FIND: Determine the change in specific entropy, in kJ/kg·K, between the specified states.

- (a) Water. $P_1 = 10 \text{ MPa}$, $T_1 = 400^\circ\text{C}$, $P_2 = 10 \text{ MPa}$, $T_2 = 100^\circ\text{C}$.

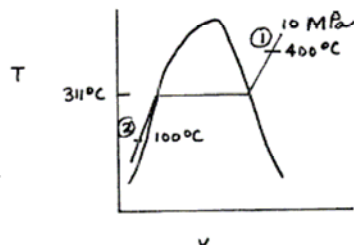


Table A-4, $s_1 = 6.2120 \text{ kJ/kg}\cdot\text{K}$

Table A-5, $s_2 = 1.2992 \text{ kJ/kg}\cdot\text{K}$

$$s_2 - s_1 = 1.2992 - 6.2120 = -4.9128 \text{ kJ/kg}\cdot\text{K} \quad (a)$$

- (b) R134a. $h_1 = 111.44 \text{ kJ/kg}$, $T_1 = -40^\circ\text{C}$, saturated vapor at $P_2 = 5 \text{ bar}$.

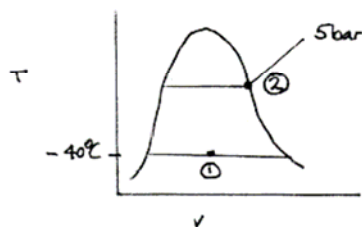


Table A-10 at -40°C

$$h_f = 0, h_{fg} = 222.88, h_g = 222.88 \text{ kJ/kg}$$

Thus,

$$x_1 = \frac{111.44 - 0}{222.88} = 0.5$$

Then

$$s_1 = s_f + x_1(s_g - s_f) = 0 + 0.5[0.9560 - 0] = 0.478 \text{ kJ/kg}\cdot\text{K}$$

Then, with s_2 from Table A-11

$$s_2 - s_1 = 0.9117 - 0.478 = 0.4337 \text{ kJ/kg}\cdot\text{K} \quad (b)$$

- (c) Air as an ideal gas. $T_1 = 280 \text{ K}$ (7°C), $P_1 = 2 \text{ bar}$, $T_2 = 600 \text{ K}$ (327°C), $P_2 = 1 \text{ bar}$.

With Eq. 6.20a and s° data from Table A-22

$$\begin{aligned} s_2 - s_1 &= s^\circ(T_2) - s^\circ(T_1) - \frac{\bar{R}}{M} \ln \frac{P_2}{P_1} \\ &= (2.40902 - 1.63279) \frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \left(\frac{8.314}{29.97} \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \right) \ln \frac{1}{2} \\ &= 0.97515 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad (c) \end{aligned}$$

- (d) Hydrogen (H_2) as an ideal gas. $T_1 = 1000 \text{ K}$ (727°C), $P_1 = 1 \text{ bar}$, $T_2 = 298 \text{ K}$ (25°C), $P_2 = 3 \text{ bar}$

Table A-21 provides the following expression for $\bar{c}_p(T)$

$$\frac{\bar{c}_p}{\bar{R}} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4$$

Inserting this into Eq. 6.18 written on a molar basis, and integrating

$$\begin{aligned} \frac{\bar{s}_2 - \bar{s}_1}{\bar{R}} &= \int_{T_1}^{T_2} \left(\frac{\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4}{T} \right) dT - \ln \frac{P_2}{P_1} \\ &= \alpha \ln \frac{T_2}{T_1} + \beta(T_2 - T_1) + \frac{\gamma}{2}(T_2^2 - T_1^2) + \frac{\delta}{3}(T_2^3 - T_1^3) + \frac{\epsilon}{4}(T_2^4 - T_1^4) - \ln \frac{P_2}{P_1} \end{aligned}$$

The values for the constants α , β , γ , δ , and ϵ for H_2 are obtained from Table A-21. Hence

Continued on next slide

Problem 6-3 continued

$$\begin{aligned} \frac{\bar{s}_2 - \bar{s}_1}{R} = & 3.057 \ln \frac{298}{1000} + 2.677 \frac{(298 - 1000)}{10^3} - \frac{5.810}{2} \left[\frac{298^2 - 1000^2}{10^6} \right] \\ & + \frac{5.521}{3} \left[\frac{298^3 - 1000^3}{10^9} \right] - \frac{1.812}{4} \left[\frac{298^4 - 1000^4}{10^{12}} \right] - \ln \frac{3}{1} \end{aligned}$$

$$\begin{aligned} \bar{s}_2 - \bar{s}_1 &= 8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \left[-3.701 - 1.8793 + 2.647 - 1.7916 + 0.4494 - 1.0986 \right] \\ &= -44.68 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \end{aligned}$$

With the molecular weight for H_2 from Table A-1, $M = 2.016$

$$s_2 - s_1 = \frac{\bar{s}_2 - \bar{s}_1}{M} = \frac{-44.68 \text{ kJ/kmol} \cdot \text{K}}{2.016 \text{ kg/kmol}} = -22.163 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

PROBLEM 6.4

FIND: Determine the change in specific entropy, in Btu/lb·°R.

(a) Water. $P_1 = 1000 \text{ lbf/in}^2$, $T_1 = 800^\circ\text{F}$, $P_2 = 1000 \text{ lbf/in}^2$, $T_2 = 100^\circ\text{F}$.

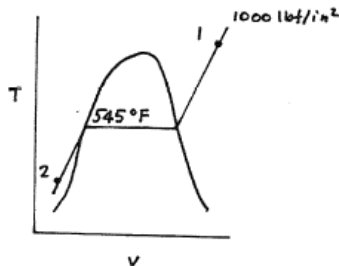


Table A-4E. $s_1 = 1.5665 \text{ Btu/lb} \cdot ^\circ\text{R}$

Table A-5E. $s_2 = 0.12901 \text{ Btu/lb} \cdot ^\circ\text{R}$

$$s_2 - s_1 = -1.43749 \text{ Btu/lb} \cdot ^\circ\text{R} \quad (a)$$

(b) R-134a. $h_1 = 47.91 \text{ Btu/lb}$, $T_1 = -40^\circ\text{F}$, saturated vapor at $P_2 = 40 \text{ lbf/in}^2$.

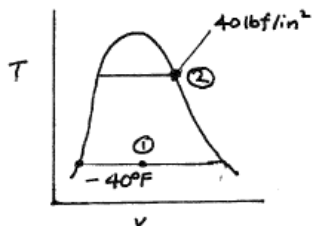


Table A-10E at -40°F : $h_f = 0$, $h_{fg} = 95.82 \text{ Btu/lb}$

$$\therefore x_1 = \frac{47.91 - 0}{95.82} = 0.5$$

$$\text{And } s_1 = s_f + x_1(s_g - s_f) = 0 + 0.5(0.2283 - 0) = 0.11415 \text{ Btu/lb} \cdot ^\circ\text{R}.$$

Table A-11E, $s_2 = 0.2197 \text{ Btu/lb} \cdot ^\circ\text{R}$. Then

$$s_2 - s_1 = 0.2197 - 0.11415 = 0.1056 \text{ Btu/lb} \cdot ^\circ\text{R} \quad (b)$$

(c) Air as an ideal gas. $T_1 = 40^\circ\text{F}$, $P_1 = 2 \text{ atm}$, $T_2 = 420^\circ\text{F}$, $P_2 = 1 \text{ atm}$.

With Eq. 6.20a and s° data from Table A-22E at 500°R and 880°R

$$\begin{aligned} s_2 - s_1 &= s^\circ(T_2) - s^\circ(T_1) - \bar{R} \ln \frac{P_2}{P_1} \\ &= (0.71886 - 0.58233) \left(\frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) - \left(\frac{1.986}{28.97} \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) \ln \frac{1}{2} \\ &= 0.18405 \text{ Btu/lb} \cdot ^\circ\text{R} \quad (c) \end{aligned}$$

(d) Carbon dioxide as an ideal gas. $T_1 = 820^\circ\text{F}$, $P_1 = 1 \text{ atm}$, $T_2 = 77^\circ\text{F}$, $P_2 = 3 \text{ atm}$.

With Eq. 6.20b and $\bar{s}^\circ$ data from Table A-23E at 1280°R and 537°R

$$\begin{aligned} \bar{s}_2 - \bar{s}_1 &= \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln \frac{P_2}{P_1} \\ &= (51.032 - 60.044) \left(\frac{\text{Btu}}{\text{lbmol} \cdot ^\circ\text{R}} \right) - \left(\frac{1.986}{44.01} \frac{\text{Btu}}{\text{lbmol} \cdot ^\circ\text{R}} \right) \ln \frac{3}{1} \\ &= -11.1938 \frac{\text{Btu}}{\text{lbmol} \cdot ^\circ\text{R}} \end{aligned}$$

Then, with the molecular weight of CO_2 from Table A-1E

$$\begin{aligned} s_2 - s_1 &= \frac{\bar{s}_2 - \bar{s}_1}{M} \\ &= \frac{-11.1938 \frac{\text{Btu}}{\text{lbmol} \cdot ^\circ\text{R}}}{44.01 \text{ lb/lbmol}} \\ &= -0.25435 \text{ Btu/lb} \cdot ^\circ\text{R} \quad (d) \end{aligned}$$

PROBLEM 6.5

(a) Re: Problem 6.1.

IT Code

```
p = 50 // bar
Ta = 400 // °C
sa = s_PT("Water/Steam", p, Ta)
Tb = 100
sb = s_PT("Water/Steam", p, Tb)
uc = 1872.5 // kJ/kg
uc = usat_Px("Water/Steam", p, xc)
sc = ssat_Px("Water/Steam", p, xc)
xd = 1
sd = ssat_Px("Water/Steam", p, xd)
```

IT Results

```
(a) sa = 6.645 kJ/kg·K
(b) sb = 1.308 kJ/kg·K
(c) xc = 0.5001
    sc = 4.447 kJ/kg·K
(d) sd = 5.973 kJ/kg·K
```

At liquid states,
IT returns specific
entropy via Eq. 6.5

(b) Re: Problem 6.2.

IT Code

```
p = 1000 // lbf/in.2
Ta = 750 // °F
sa = s_PT("Water/Steam", p, Ta)
Tb = 300
sb = s_PT("Water/Steam", p, Tb)
hc = 932.4 // Btu/lb
hc = hsat_Px("Water/Steam", p, xc)
sc = ssat_Px("Water/Steam", p, xc)
xd = 1
sd = ssat_Px("Water/Steam", p, xd)
```

IT Results

```
(a) sa = 1.541 Btu/lb·°R
(b) sb = 0.4369 Btu/lb·°R
(c) xc = 0.6001
    sc = 1.131 Btu/lb·°R
(d) sd = 1.39 Btu/lb·°R
```

At liquid states,
IT returns
specific entropy
via Eq. 6.5.

PROBLEM 6.6

(a) Re: Problem 6.3.

IT Code

```
Ta1 = 400 // °C
pa1 = 100 // bar
Ta2 = 100
pa2 = 100
sa1 = s_PT("Water/Steam", pa1, Ta1)
sa2 = s_PT("Water/Steam", pa2, Ta2)
dels_a = sa2 - sa1
```

```
hb1 = 111.44 // kJ/kg
Tb1 = -40
xb1 = 1
pb1 = Psat_T("R134A", Tb1)
hb1 = hsat_Px("R134A", pb1, xb1)
sb1 = ssat_Px("R134A", pb1, xb1)
sb2 = ssat_Px("R134A", pb2, xb2)
dels_b = sb2 - sb1
```

```
Tc1 = 7
pc1 = 2
Tc2 = 327
pc2 = 1
dels_c = s_TP("Air", Tc2, pc2) - s_TP("Air", Tc1, pc1)
```

```
Td1 = 727
pd1 = 1
Td2 = 25
pd2 = 3
dels_d = s_TP("H2", Td2, pd2) - s_TP("H2", Td1, pd1)
```

IT Results

(a) $\Delta s = -4.903 \text{ kJ/kg} \cdot \text{K}$
 (b) $\Delta s = 0.4338 \text{ kJ/kg} \cdot \text{K}$
 (c) $\Delta s = 0.9749 \text{ kJ/kg} \cdot \text{K}$
 (d) $\Delta s = -22.16 \text{ kJ/kg} \cdot \text{K}$

(b) Re: Problem 6.4.

IT Code

```
p = 1000 // lbf/in.2
Ta1 = 800 // °F
Ta2 = 100
sa1 = s_PT("Water/Steam", p, Ta1)
sa2 = s_PT("Water/Steam", p, Ta2)
dels_a = sa2 - sa1
```

```
hb1 = 47.91 // Btu/lb
Tb1 = -40
xb1 = 1
pb1 = Psat_T("R134A", Tb1)
hb1 = hsat_Px("R134A", pb1, xb1)
sb1 = ssat_Px("R134A", pb1, xb1)
sb2 = ssa_Px("R134A", pb2, xb2)
dels_b = sb2 - sb1
```

```
Tc1 = 40
pc1 = 2 * 14.696 // lbf/in.2
Tc2 = 420
pc2 = 1 * 14.696
sc1 = s_TP("Air", Tc1, pc1)
sc2 = s_TP("Air", Tc2, pc2)
dels_c = sc2 - sc1
```

```
Td1 = 820
pd1 = 1 * 14.696
Td2 = 77
pd2 = 3 * 14.696
sd1 = s_TP("CO2", Td1, pd1)
sd2 = s_TP("CO2", Td2, pd2)
dels_d = sd2 - sd1
```

IT Results

(a) $\Delta s = -1.437 \text{ Btu/lb} \cdot ^\circ\text{R}$
 (b) $\Delta s = 0.1056 \text{ Btu/lb} \cdot ^\circ\text{R}$
 (c) $\Delta s = 0.184 \text{ Btu/lb} \cdot ^\circ\text{R}$
 (d) $\Delta s = -0.2545 \text{ Btu/lb} \cdot ^\circ\text{R}$

PROBLEM 6.7

Using steam table data, determine the indicated property data for a process in which there is no change in specific entropy between state 1 and state 2. In each case, locate the states on a sketch of the T - s diagram.

- (a) $T_1 = 40^\circ\text{C}$, $x_1 = 100\%$, $p_2 = 150 \text{ kPa}$. Find T_2 , in $^\circ\text{C}$, and Δh , in kJ/kg .
 (b) $T_1 = 10^\circ\text{C}$, $x_1 = 75\%$, $p_2 = 1 \text{ MPa}$. Find T_2 , in $^\circ\text{C}$, and Δu , in kJ/kg .

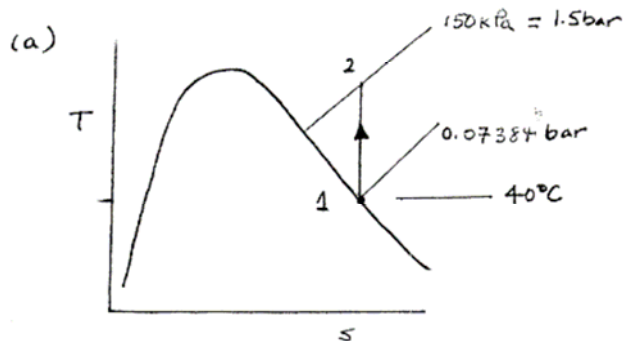


Table A-2,

$$s_1 = 8.2570 \text{ kJ/kg} \cdot \text{K}$$

$$h_1 = 2574.3 \text{ kJ/kg}$$

Interpolating in Table A-4,

$$T_2 = 368.8^\circ\text{C}$$

$$h_2 = 3213.1 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta h = 638.8 \frac{\text{kJ}}{\text{kg}}$$

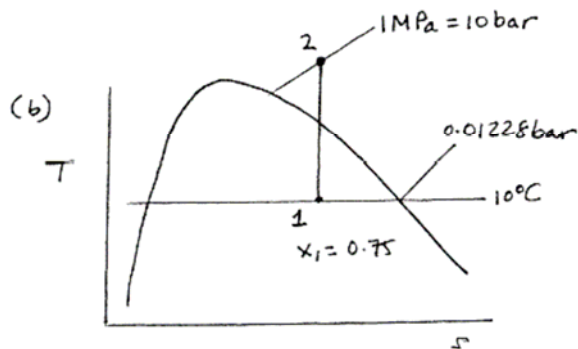


Table A-2,

$$s_1 = s_f + x_1 (s_g - s_f)$$

$$= 0.1510 + 0.75(8.9008 - 0.1510)$$

$$= 6.71335 \text{ kJ/kg} \cdot \text{K}$$

$$u_1 = u_f + x_1 (u_g - u_f)$$

$$= 42.0 + 0.75(2389.2 - 42.0)$$

$$= 1802.4 \text{ kJ/kg}$$

Interpolating in Table A-4,

$$T_2 = 204.1^\circ\text{C}$$

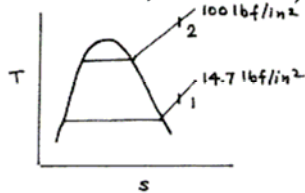
$$u_2 = 2629.2 \text{ kJ/kg}$$

$$\Delta u = 826.8 \text{ kJ/kg}$$

PROBLEM 6.8

FIND: Determine the indicated property for a process where $s_2 = s_1$.

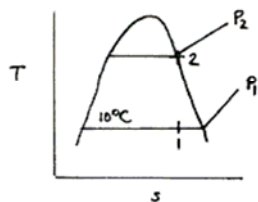
- (a) Water. $P_1 = 14.7 \text{ lbf/in}^2$, $T_1 = 500^\circ\text{F}$, $P_2 = 100 \text{ lbf/in}^2$. Find T_2 .



From Table A-4E, $s_1 = 1.9263 \text{ Btu/lb}\cdot^\circ\text{R}$.
Interpolating with $s_2 = s_1$ in Table A-4E at 100 lbf/in^2 ,
 $T_2 = 101.7^\circ\text{F}$.

(a)

- (b) Water. $T_1 = 10^\circ\text{C}$, $x_1 = 0.75$. Saturated vapor at state 2. Find P_2 .



Using data from Table A-2

$$s_1 = s_f + x_1(s_g - s_f) = 0.1510 + 0.75(8.9008 - 0.1510) = 6.71335 \text{ kJ/kg}\cdot\text{K}$$

Then, interpolating with $s_2 = s_1$ in Table A-3, $P_2 = 6.897 \text{ bar}$.

(b)

- (c) Air as an ideal gas. $T_1 = 300 \text{ K}$ (27°C), $P_1 = 1.5 \text{ bar}$, $T_2 = 400 \text{ K}$ (127°C). Find P_2 .

Since $s_2 = s_1$, Eq. 6.20a gives

$$\ln \frac{P_2}{P_1} = \frac{s^\circ(T_2) - s^\circ(T_1)}{R}$$

with s° data from Table A-22

$$\ln \frac{P_2}{P_1} = \frac{1.99194 - 1.70203}{(8.314/28.97)}$$

$$\Rightarrow \frac{P_2}{P_1} = 2.746 \Rightarrow P_2 = 4.119 \text{ bar}$$

(c)

- (d) Air as an ideal gas. $T_1 = 560^\circ\text{R}$ (100°F), $P_1 = 3 \text{ atm}$, $P_2 = 2 \text{ atm}$. Find T_2 .

Since $s_2 = s_1$, Eq. 6.20a gives

$$s^\circ(T_2) = s^\circ(T_1) + R \ln \frac{P_2}{P_1}$$

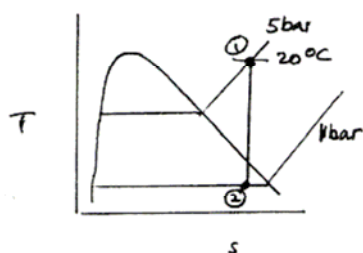
With s° at T_1 from Table A-22E

$$s^\circ(T_2) = 0.60950 + \frac{1.986}{28.97} \ln \frac{2}{3} = 0.5817 \text{ Btu/lb}\cdot^\circ\text{R}$$

Interpolating with $s^\circ(T_2)$ in Table A-22E, $T_2 = 498.7^\circ\text{R}$ (39°F).

(d)

- (e) R134a. $T_1 = 20^\circ\text{C}$, $P_1 = 5 \text{ bar}$, $P_2 = 1 \text{ bar}$. Find v_2 .



From Table A-12, $s_1 = 0.9264 \text{ kJ/kg}\cdot\text{K}$.
Then, with data from Table A-11 and $s_2 = s_1$,

$$x_2 = \frac{0.9264 - 0.0678}{0.9395 - 0.0678} = 0.985$$

And

$$\begin{aligned} v_2 &= v_f + x_2(v_g - v_f) \\ &= \frac{0.7258}{103} + 0.985 \left[\frac{0.1917 - 0.7258}{103} \right] \\ &= 0.188 \text{ m}^3/\text{kg} \end{aligned}$$

(e)

PROBLEM 6.9

Using IT, obtain the property data requested in

(a) Problem 6.7.

(b) Problem 6.8

Problem 6.7 Using IT

Problem 6.7(a)

```
T1=40
x1=1
p2=150
s2=s1
deltah=h2-h1
p1 = Psat_T("Water/Steam", T1)
s1 = ssat_Px("Water/Steam", p1, x1)
s2 = s_PT("Water/Steam", p2, T2)
h1 = hsat_Px("Water/Steam", p1, x1)
h2 = h_PT("Water/Steam", p2, T2)
```

```
Solving...
T2=368.6
deltah=638.4
```

Problem 6.7(b)

```
T1=10
x1=0.75
p2=10 // bar
s2=s1
deltau=u2-u1
p1 = Psat_T("Water/Steam", T1)
s1 = ssat_Px("Water/Steam", p1, x1)
u1 = usat_Px("Water/Steam", p1, x1)
s2 = s_PT("Water/Steam", p2, T2)
u2 = u_PT("Water/Steam", p2, T2)
```

```
Solving...
T2=203.7
deltau=826.5
```

Continued on next slide

PROBLEM 6.8 using IT:

IT Code

```
/* (a) Water
p1 = 14.7 // lbf/in.2
T1 = 500 // °F
p2 = 100
s1 = s_PT("Water/Steam", p1, T1)
s2 = s_PT("Water/Steam", p2, T2)
s2 = s1
```

```
(b) Water
T1 = 10 // °C
x1 = 0.75
x2 = 1
p1 = Psat_T("Water/Steam", T1)
s1 = ssat_Px("Water/Steam", p1, x1)
s2 = ssat_Px("Water/Steam", p2, x2)
s2 = s1
```

```
(c) Air
T1 = 27 // °C
p1 = 1.5 // bar
T2 = 127
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)
s2 = s1
```

```
(d) Air
T1 = 100 // °F
p1 = 3 // atm
p2 = 2
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)
s2 = s1
```

```
(e) R-134a
*/
T1 = 20 // °C
p1 = 5 // bar
p2 = 1
s1 = s_PT("R134A", p1, T1)
x2 = x_sP("R134A", s2, p2)
v2 = vsat_Px("R134A", p2, x2)
s2 = s1
```

IT Results

- (a) $s = 1.926 \text{ Btu/lb} \cdot ^\circ\text{R}$; $T_2 = 1016^\circ\text{F}$
- (b) $s = 6.712 \text{ kJ/kg} \cdot \text{K}$; $p_2 = 6.903 \text{ bar}$
- (c) $p_2 = 4.117 \text{ bar}$
- (d) $T_2 = 38.75^\circ\text{F}$
- (e) $s = 0.9264 \text{ kJ/kg} \cdot \text{K}$; $x_2 = 0.985$, $v_2 = 0.1888 \text{ m}^3/\text{kg}$

Comment: As expected, good agreement exists between data obtained from the Appendix tables and data from IT.

PROBLEM 6.10

Propane undergoes a process from state 1, where $p_1 = 1.4 \text{ MPa}$, $T_1 = 60^\circ\text{C}$, to state 2, where $p_2 = 1.0 \text{ MPa}$, during which the change in specific entropy is $s_2 - s_1 = -0.035 \text{ kJ/kg}\cdot\text{K}$.

At state 2, determine the temperature, in $^\circ\text{C}$, and the specific enthalpy, in kJ/kg .

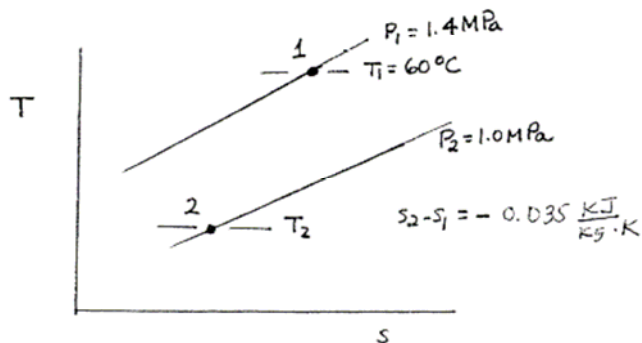


Table A-18:

$$s_1 = 1.845 \text{ kJ/kg}\cdot\text{K}$$

$$\text{Given: } s_2 - s_1 = -0.035 \text{ kJ/kg}\cdot\text{K}$$

$$\Rightarrow s_2 = (1.845 - 0.035) \text{ kJ/kg}\cdot\text{K} \\ = 1.81 \text{ kJ/kg}\cdot\text{K}$$

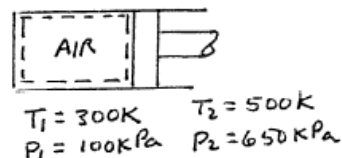
Then, at $p_2 = 1.0 \text{ MPa}$,

$$T_2 = 70^\circ\text{C}$$

$$h_2 = 524.6 \text{ kJ/kg}$$

PROBLEM 6.11

Air in a piston-cylinder assembly undergoes a process from state 1, where $T_1 = 300 \text{ K}$, $p_1 = 100 \text{ kPa}$, to state 2, where $T_2 = 500 \text{ K}$, $p_2 = 650 \text{ kPa}$. Using the ideal gas model for air, determine the change in specific entropy between these states, in $\text{kJ/kg} \cdot \text{K}$, if the process occurs (a) without internal irreversibilities, (b) with internal irreversibilities.



Entropy is a property. Thus, the change in specific entropy has the same value for fixed end states whether or not irreversibilities are present during a process between the end states.

With Eq. 6.20a and data from Table A-22,

$$\begin{aligned}
 \Delta s &= s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1} \\
 &= \left[2.21952 - 1.70203 - \frac{8.314}{28.97} \ln \left(\frac{650}{100} \right) \right] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\
 &= -0.0197 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}
 \end{aligned}$$

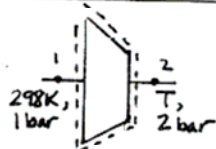


PROBLEM 6.12

KNOWN: CH₄ is compressed from 298K, 1 bar to temperature T₂, 2 bar in a process for which $s_2 = s_1$.

FIND: Determine T₂.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: CH₄ is modeled as an ideal gas.

① ANALYSIS Using IT

IT code

T1 = 298 // K

p1 = 1 // bar

p2 = 2 // bar

s1 = s_TP("CH4", T1, p1)

s2 = s_TP("CH4", T2, p2)

s2 = s1

IT Result

T₂ = 348.7 K

1. Alternatively, using Eq. 6.18 with $\bar{c}_p(T)$ from Table A-21

$$\bar{s}_2 - \bar{s}_1 = \int_{T_1}^{T_2} \left(\frac{\bar{c}_p}{T} \right) dT - \bar{R} \ln \left(\frac{p_2}{p_1} \right)$$

$$0 = \bar{R} \left[\alpha \ln T + \beta T + \frac{\gamma}{2} T^2 + \frac{\delta}{3} T^3 + \frac{\epsilon}{4} T^4 \right]_{298}^{T_2} - \bar{R} \ln \left(\frac{p_2}{p_1} \right)$$

Using an equation solver, such as IT, this equation can be solved for T₂.

PROBLEM 6.13

One-quarter lbmol of nitrogen gas (N_2) undergoes a process from $p_1 = 20 \text{ lbf/in.}^2$, $T_1 = 500^\circ\text{R}$ to $p_2 = 150 \text{ lbf/in.}^2$. For the process $W = -500 \text{ Btu}$ and $Q = -125.9 \text{ Btu}$. Employing the ideal gas model, determine

- T_2 , in $^\circ\text{R}$.
 - the change in entropy, in $\text{Btu}/^\circ\text{R}$.
- Show the initial and final states on a T - s diagram.

ANALYSIS:

$$(a) \Delta U + \Delta KE + \Delta PE = Q - W$$

$$\Rightarrow n[\bar{u}(T_2) - \bar{u}(T_1)] = Q - W$$

$$\Rightarrow \bar{u}(T_2) = \bar{u}(T_1) + \frac{Q - W}{n}$$

With $\bar{u}(500^\circ\text{R}) = 2479.3 \text{ Btu/lbmol}$ from Table A-23E,

$$\bar{u}(T_2) = 2479.3 + \frac{[-125.9 - (-500)] \text{ Btu}}{0.25 \text{ lbmol}} = 3975.7 \frac{\text{Btu}}{\text{lbmol}}$$

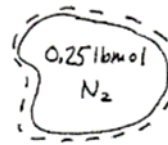
From Table A-23E, $T_2 = 800^\circ\text{R}$.

$$(b) \Delta S = n[\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln p_2/p_1]$$

$$= 0.25 \text{ lbmol} \left[48.522 - 45.246 - 8.314 \ln \left(\frac{150}{20} \right) \right] \frac{\text{Btu}}{\text{lbmol} \cdot ^\circ\text{R}}$$

$$= -3.369 \frac{\text{Btu}}{^\circ\text{R}}$$

SCHEMATIC & GIVEN DATA:



$$W = -500 \text{ Btu}$$

$$Q = -125.9 \text{ Btu}$$

ENGR. MODEL: (1) The ideal gas model applies to the N_2 . (2) Ignore changes in kinetic & potential energy.

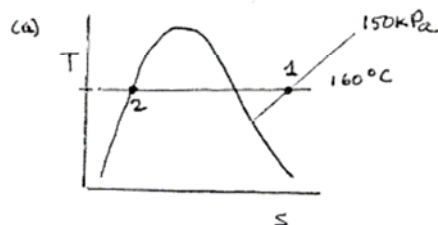


PROBLEM 6.14

One kilogram of water contained in a piston-cylinder assembly, initially at 160°C , 150 kPa , undergoes an isothermal compression process to saturated liquid. For the process, $W = -471.5\text{ kJ}$. Determine for the process,

- the heat transfer, in kJ.
 - the change in entropy, in kJ/K.
- Show the process on a sketch of the T - s diagram.

ANALYSIS:



(b) $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow Q = W + \Delta U$$

$$\Rightarrow Q = W + m(u_2 - u_1)$$

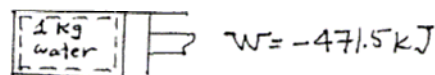
Table A-4, $u_1 = 2595.2\text{ kJ/kg}$

Table A-2, $u_2 = 674.86\text{ kJ/kg}$

$$\therefore Q = -471.5 + (1\text{ kg})(674.86 - 2595.2)\frac{\text{kJ}}{\text{kg}}$$

$$= -2391.84\text{ kJ}$$

SCHEMATIC & GIVEN DATA:



$$T = 160^\circ\text{C}$$

$$P_1 = 150\text{ kPa}$$

State 2: Saturated liquid at 160°C

ENGR. MODEL: Ignore changes in kinetic and potential energy.

(c) Table A-4, $s_1 = 7.4665\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Table A-2, $s_2 = 1.9427\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

$$\Delta S = 1\text{ kg}(1.9427 - 7.4665)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$= -5.5238\frac{\text{kJ}}{\text{K}} \quad \leftarrow \Delta S$$

$$\leftarrow Q$$

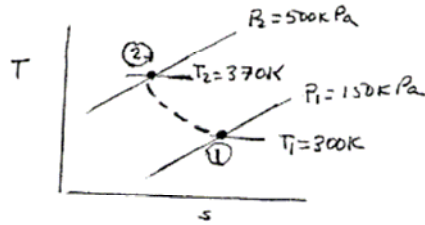
PROBLEM 6.15

One-tenth kmol of carbon monoxide (CO) in a piston-cylinder assembly undergoes a process from $p_1 = 150 \text{ kPa}$, $T_1 = 300 \text{ K}$ to $p_2 = 500 \text{ kPa}$, $T_2 = 370 \text{ K}$. For the process, $W = -300 \text{ kJ}$. Employing the ideal gas model, determine

- the heat transfer, in kJ.
- the change in entropy, in kJ/K.

Show the process on a sketch of the T - s diagram.

ANALYSIS:



- Energy balance:

$$\Delta U + \Delta KE + \Delta PE = Q - W$$

$$\Rightarrow Q = \Delta U + W$$

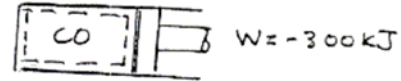
$$= n[\bar{u}(T_2) - \bar{u}(T_1)] + W$$

With data from Table A-23,

$$Q = 0.1 \text{ kmol} \left[7689 - 6229 \right] \frac{\text{kJ}}{\text{kmol}} + (-300 \text{ kJ})$$

$$= -154 \text{ kJ}$$

SCHEMATIC & GIVEN DATA:



$$n = 0.1 \text{ kmol}$$

$$p_1 = 150 \text{ kPa} \quad p_2 = 500 \text{ kPa}$$

$$T_1 = 300 \text{ K} \quad T_2 = 370 \text{ K}$$

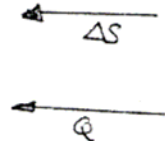
ENGR. MODEL 1. The CO is modeled as an ideal gas. 2. Kinetic and potential energy effects can be ignored.

- With data from Table A-23,

$$\Delta S = n \left[\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - R \ln \frac{p_2}{p_1} \right]$$

$$\Delta S = (0.1 \text{ kmol}) \left[203.842 - 197.723 - 8.314 \ln \left(\frac{500}{150} \right) \right] \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

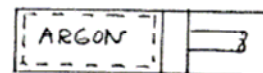
$$= -0.3891 \frac{\text{kJ}}{\text{K}}$$



PROBLEM 6.16

Argon in a piston-cylinder assembly is compressed from state 1, where $T_1 = 300 \text{ K}$, $V_1 = 1 \text{ m}^3$, to state 2, where $T_2 = 200 \text{ K}$. If the change in specific entropy is $s_2 - s_1 = -0.27 \text{ kJ/kg} \cdot \text{K}$, determine the final volume, in m^3 . Assume the ideal gas model with $k = 1.67$.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} T_1 &= 300 \text{ K} & T_2 &= 200 \text{ K} \\ V_1 &= 1 \text{ m}^3 & V_2 &=? \\ s_2 - s_1 &= -0.27 \text{ kJ/kg} \cdot \text{K} \end{aligned}$$

ENGR. MODEL: The argon is modeled as an ideal gas with $k = 1.67$.

ANALYSIS: If k is constant, then $C_v = \frac{R}{k-1}$ is also constant (Eq. 3.476). With Eq. 6.21,

$$s_2 - s_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

where

$$C_v = \frac{R}{k-1} = \frac{(8.314/39.94) \text{ kJ/kg} \cdot \text{K}}{1.67-1} = 0.3107 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

That is

$$-0.27 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.3107 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln \left(\frac{200 \text{ K}}{300 \text{ K}} \right) + \frac{8.314}{39.94} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln \left(\frac{V_2}{1 \text{ m}^3} \right)$$

Solving,

$$V_2 = 0.5 \text{ m}^3$$

 V_2

PROBLEM 6.17

Steam enters a turbine operating at steady state at 1 MPa, 200°C and exits at 40°C with a quality of 83%. Stray heat transfer and kinetic and potential energy effects are negligible. Determine (a) the power developed by the turbine, in kJ per kg of steam flowing, (b) the change in specific entropy from inlet to exit, in kJ/K per kg of steam flowing.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and kinetic and potential energy effects are negligible.

ANALYSIS:

(a) An energy rate balance reduces to read, $\dot{W}_{cv}/\dot{m} = h_1 - h_2$, where

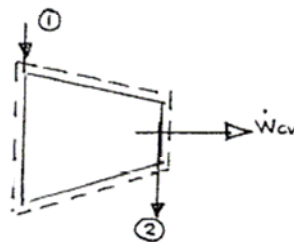
$$h_1 = 2827.9 \text{ kJ/kg (Table A-4) and } h_2 = h_f + x_2 h_{fg} = 167.57 + 0.83(2406.7) = 2165.1 \frac{\text{kJ}}{\text{kg}} \text{ (Table A-2). Thus,}$$

$$\frac{\dot{W}_{cv}}{\dot{m}} = (2827.9 - 2165.1) \frac{\text{kJ}}{\text{kg}} = 662.8 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow (a)$$

(b) $s_1 = 6.6940 \text{ kJ/kg} \cdot \text{K}$, $s_2 = s_f + x_2 s_{fg} = 0.5720 + 0.83(8.2570 - 0.5720) = 6.9506 \text{ kJ/kg} \cdot \text{K}$.

$$\therefore s_2 - s_1 = (6.9506 - 6.6940) = 0.2566 \text{ kJ/kg} \cdot \text{K} \quad \leftarrow (b)$$

SCHEMATIC & GIVEN DATA:



PROBLEM 6.18

(a) True. Entropy is a property. Thus, the change in entropy between two states is the same for any process between these states.

(b) False. When T is constant, Eq. 6.17 reduces to read

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v}{T} dT + R \ln \frac{v_2}{v_1} \Rightarrow s_2 - s_1 = R \ln \frac{v_2}{v_1}$$

Thus, when $v_2 < v_1$, $(s_2 - s_1) < 0$.

(c) True. See, for example, Eqs. 6.17, 6.18.

(d) False. See Eq. 6.10a: $Tds = du + pdv$

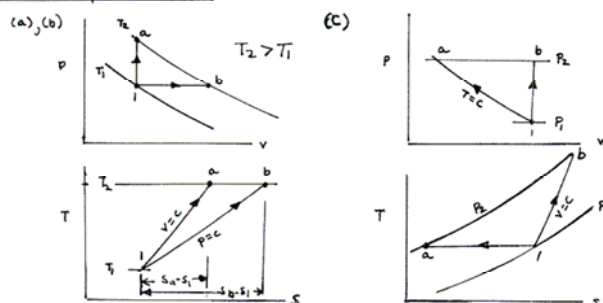
(e) False. See Eq. 6.13: $s_2 - s_1 = c \ln \frac{T_2}{T_1}$

PROBLEM 6.19

KNOWN: A system consisting of an ideal gas with constant specific heat ratio k undergoes constant volume, constant pressure, and constant temperature processes from the same initial state.

FIND: (a) Show that the entropy change is greater for the constant pressure process than for the constant volume process. Sketch the processes on $p-v$, $T-s$ coordinates. (b) Using the $T-s$ diagram, show that a line of constant v has a greater slope than a line of constant p . (c) Show that the ratio of the entropy change for a constant T process to the entropy change for a constant v process is $(1-k)$. Sketch the processes on $p-v$ and $T-s$ coordinates.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The system consists of an ideal gas with constant k .

ANALYSIS: (a) For process 1-2, specific volume is constant. For process 1-3, pressure is constant. Thus, Eqs. 6.21 and 6.22 reduce, respectively, to give

$$\begin{aligned} s_2 - s_1 &= c_v \ln \frac{T_2}{T_1} \\ s_3 - s_1 &= c_p \ln \frac{T_3}{T_1} \end{aligned} \Rightarrow \frac{s_3 - s_1}{s_2 - s_1} = \frac{c_p \ln T_3/T_1}{c_v \ln T_2/T_1} = \frac{c_p}{c_v} = k$$

As $k > 1$, $(s_3 - s_1) > (s_2 - s_1)$.

(b) Here, we compare at state 1 $(\partial T/\partial s)_v$ to $(\partial T/\partial s)_p$. Since $(\partial T/\partial s)$ at fixed v (or fixed p) = $\lim_{\Delta s \rightarrow 0} \frac{\Delta T}{\Delta s}$, it is evident from the $T-s$ diagram that a constant specific volume line passing through state 1 has a greater slope than a constant pressure line.

(c) For process 1-2, temperature is constant. For process 2-3, volume is constant. Thus, Eqs. 6.22 and 6.21 reduce, respectively, to give

$$\begin{aligned} s_2 - s_1 &= -R \ln \frac{p_2}{p_1} \quad (\text{entropy decreases}) \\ s_3 - s_2 &= c_v \ln \frac{T_3}{T_2} \quad (\text{entropy increases}) \end{aligned}$$

Using Eq. 3.44, the first of these can be written as

$$s_2 - s_1 = -(c_p - c_v) \ln \frac{p_2}{p_1}$$

Using the ideal gas equation of state with $v_2 = v_1$, the second can be written as

$$s_3 - s_2 = c_v \ln \frac{p_2}{p_1}$$

Forming the ratio

$$\frac{s_2 - s_1}{s_3 - s_2} = \frac{-(c_p - c_v) \ln p_2/p_1}{c_v \ln p_2/p_1} = 1 - k$$

PROBLEM 6.20

One kilogram of water in a piston-cylinder assembly undergoes the two internally reversible processes in series shown in Fig. P6.20. For each process, determine, in kJ, the heat transfer and the work.

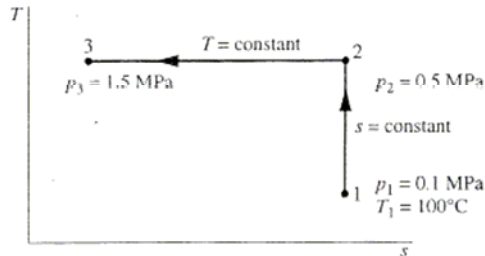
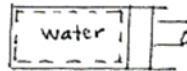


Fig. P6.20

SCHEMATIC & GIVEN DATA



$$m = 1 \text{ kg}$$

ENGR. MODEL:

1. The water is the closed system.
2. For the system, kinetic and potential energy changes can be ignored.
3. The processes are internally reversible.

ANALYSIS:

Process 1-2. With Eq. 6.23, $Q = 0$. An energy balance reduces to give $\Delta U = -W$

$$\Rightarrow W = -\Delta U = -m(u_2 - u_1)$$

Table A-4, $u_1 = 2506.7 \frac{\text{kJ}}{\text{kg}}$, $s_1 = 7.3614 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$. Interpolating with $s_2 = s_1$ at 0.5 MPa, $u_2 = 2760.95 \text{ kJ/kg}$, $T_2 = 273.56^\circ\text{C}$.

$$W = -1 \text{ kg} (2760.95 - 2506.7) = -254.25 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow$$

Process 2-3. With Eq. 6.23, $Q = mT(s_3 - s_2)$. From Table A-4 at 1.5 MPa, $T = 273.56^\circ\text{C}$, $s_3 = 6.8099 \text{ kJ/kg} \cdot \text{K}$, $u_3 = 2737.05 \text{ kJ/kg}$. Thus,

$$Q = (1 \text{ kg})(546.71 \text{ K})(6.8099 - 7.3614) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = -301.51 \text{ kJ} \quad \leftarrow$$

An energy balance reduces to read, $\Delta U = Q - W$, or

$$\begin{aligned} W &= Q - m(u_3 - u_2) \\ &= -301.51 \text{ kJ} - (1 \text{ kg})(2737.05 - 2760.95) \frac{\text{kJ}}{\text{kg}} \\ &= -277.61 \text{ kJ} \quad \leftarrow \end{aligned}$$

PROBLEM 6.21

One kilogram of water in a piston-cylinder assembly undergoes the two internally reversible processes in series shown in Fig. P6.21. For each process, determine, in kJ, the heat transfer and the work.

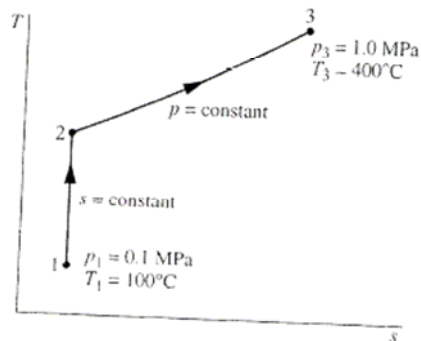
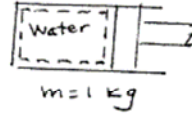


Fig. P6.21

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The water is the closed system.
2. For the system, kinetic and potential energy changes can be ignored.
3. The processes are internally reversible.

ANALYSIS:

Process 1-2. With Eq. 6.23, $Q = 0$. An energy balance reduces to give $\Delta U = -W$

$$\Rightarrow W = -\Delta U = -m(u_2 - u_1)$$

From Table A-4, $s_1 = 7.3614 \text{ kJ/kg} \cdot \text{K}$, $u_1 = 2506.7 \text{ kJ/kg}$. Interpolating at 1.0 MPa with $s_2 = s_1$, $u_2 = 2904.97 \text{ kJ/kg}$, $v_2 = 0.2912 \text{ m}^3/\text{kg}$

$$\therefore W = -(1 \text{ kg})(2904.97 - 2506.7) \frac{\text{kJ}}{\text{kg}} = -398.27 \text{ kJ}$$

Process 2-3. $W = \int_2^3 p dV = mp(v_3 - v_2)$. From Table A-4, $v_3 = 0.3066 \text{ m}^3/\text{kg}$.

$$\therefore W = (1 \text{ kg}) \left(10^6 \frac{\text{N}}{\text{m}^2} \right) (0.3066 - 0.2912) \frac{\text{m}^3}{\text{kg}} \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 15.4 \text{ kJ}$$

An energy balance reduces to read, $\Delta U = Q - W \Rightarrow Q = \Delta U + W$

Thus, with $u_3 = 2957.3 \text{ kJ/kg}$

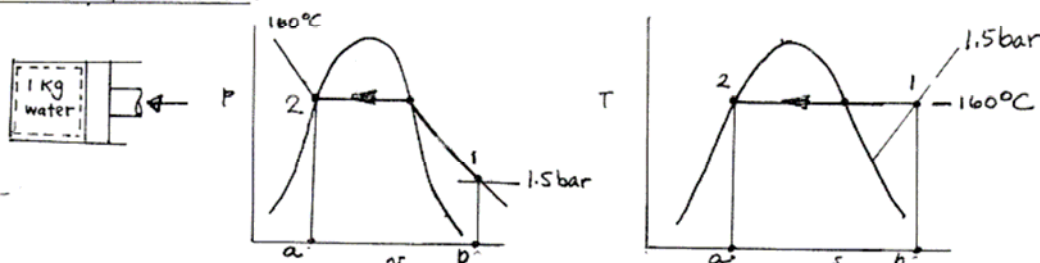
$$\begin{aligned} Q &= m(u_3 - u_2) + W \\ &= (1 \text{ kg})(2957.3 - 2904.97) + 15.4 \text{ kJ} \\ &= 67.73 \text{ kJ} \end{aligned}$$

PROBLEM 6.22

KNOWN: One kg of water undergoes an isothermal process between two specified states.

FIND: Determine the heat transfer and the work. Sketch the process on p-v and T-s coordinates

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) As shown in the accompanying figure, the system consists of one kg of water. (2) The compression takes place isothermally and without internal irreversibilities. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: Using assumption 2, Eq. 6.23 gives

$$Q = \int_1^2 T ds = m T [s_2 - s_1]$$

Then, with data from Tables A-2 and A-4

$$Q = (1 \text{ kg})(433 \text{ K})(1.9427 - 7.4665) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = -2391.8 \text{ kJ} \leftarrow Q$$

The magnitude of the heat transfer is represented by area 1-2-a-b-1 on the T-s diagram.

The energy balance reduces to give $W = Q - m(u_2 - u_1)$. With data from Tables A-2 and A-4

$$W = -2391.8 - (1 \text{ kg})[674.86 - 2545.2] \frac{\text{kJ}}{\text{kg}} = -471.5 \text{ kJ} \leftarrow$$

Alternatively, $W = \int_1^2 p dv$. The magnitude of the work is represented by area 1-2-a-b-1 on the p-v diagram.

PROBLEM 6.23

One pound mass of water initially a saturated liquid at 1 atm undergoes a constant-pressure, internally reversible expansion to $x = 90\%$. Determine the work and heat transfer, each in Btu. Sketch the process on p - v and T - s coordinates. Associate the work and heat transfer with areas on these diagrams.

ENGR. MODEL:

1. The water is the closed system.
2. The process is internally reversible.

ANALYSIS:

$$W = \int_1^2 p \, dv = m p (v_2 - v_1) = m p x_2 (v_g - v_f)$$

$$v_f + x_2 (v_g - v_f) \quad v_g$$

$$= (1 \text{ lb}) (14.7 \times 144 \frac{\text{lbf}}{\text{ft}^2}) (0.90) (26.80 - 0.01672) \frac{\text{ft}^3}{\text{lb}} \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 65.6 \text{ Btu}$$

Using Eq. 6.23,

$$Q = \int_1^2 T \, ds = m T (s_2 - s_1) = m T x_2 (s_g - s_f)$$

$$s_f + x_2 (s_g - s_f) \quad s_g$$

$$\textcircled{1} = (1 \text{ lb}) (671.66^\circ \text{R}) (0.90) (1.4446 \frac{\text{Btu}}{\text{lb} \cdot ^\circ \text{R}}) = 873.3 \text{ Btu}$$

$$211.99 + 459.67$$

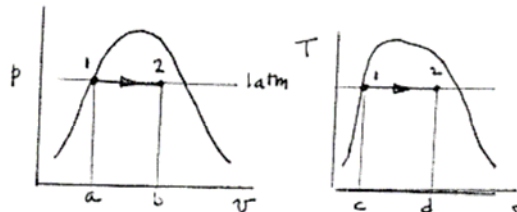
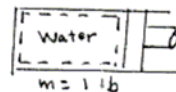
1. Alternatively, an energy balance reduces to read $\Delta U = Q - W$ or

$$m(u_2 - u_1) = Q - W \Rightarrow m x_2 (u_g - u_f) = Q - W$$

$$\Rightarrow Q - W = (1 \text{ lb}) (0.90) (1077.6 - 180.10) = 807.8 \text{ Btu}$$

Thus, knowing Q or W allows the other quantity to be evaluated from an energy balance.

SCHEMATIC & GIVEN DATA:



$$\text{Area 1-2-b-a-1} = W/m$$

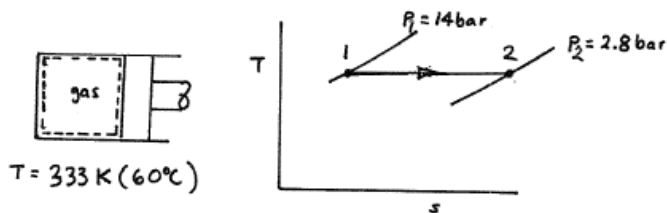
$$\text{Area 1-2-d-c-1} = Q/m$$

PROBLEM 6.24

KNOWN: A gas expands from a specified initial state to a specified final pressure isothermally and without internal irreversibilities.

FIND: Determine the heat transfer and work, per unit of mass, for (a) R134a (b) Air as an ideal gas.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) As shown in the accompanying figure, the system is the gas. (2) The expansion takes place isothermally and without internal irreversibilities. (3) There is no change in kinetic or potential energy between the end states. (4) Air is modeled as an ideal gas.

ANALYSIS: Using assumption 2, Eq. 6.23 becomes

$$Q = \int_1^2 T ds = mT(s_2 - s_1) \Rightarrow Q/m = T(s_2 - s_1)$$

With assumption 2, an energy balance gives

$$W = Q - \Delta U \Rightarrow W/m = Q/m - (u_2 - u_1)$$

(a) **R134a.** Table A-12: $u_1 = 262.17 \text{ kJ/kg}$, $s_1 = 0.9297 \text{ kJ/kg} \cdot \text{K}$, $u_2 = 277.23$, $s_2 = 1.1079 \text{ kJ/kg} \cdot \text{K}$.

$$\begin{aligned} \Rightarrow Q/m &= 333 \text{ K} (1.1079 - 0.9297) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 59.34 \text{ kJ/kg} \quad \leftarrow Q/m \\ W/m &= 59.34 - (277.23 - 262.17) = 44.28 \text{ kJ/kg} \quad \leftarrow W/m \end{aligned}$$

(b) **AIR.** Since $T_1 = T_2$, Eq. 6.20a reduces to read

$$s_2 - s_1 = -R \ln P_2/P_1 = -\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}}\right) \ln \frac{2.8}{14} = +0.46188 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

and

$$Q/m = (333)(0.46188) = 153.81 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow Q/m$$

Since internal energy depends on temperature alone for an ideal gas, $\Delta U = 0$ because $T_1 = T_2$. The energy balance then gives

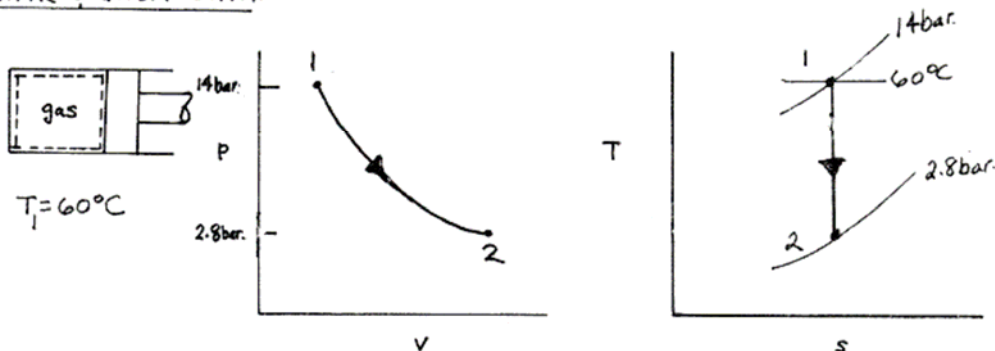
$$\begin{aligned} W/m &= Q/m - (u_2 - u_1) \\ &= Q/m = 153.81 \text{ kJ/kg} \quad \leftarrow W/m \end{aligned}$$

PROBLEM 6.25

KNOWN: A gas expands from a specified initial state to a specified final pressure adiabatically and without internal irreversibilities.

FIND: Determine the work, per unit of mass, for (a) R134a, (b) air as an ideal gas.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) As shown in the accompanying figure, the system is the gas. (2) The expansion takes place adiabatically and without internal irreversibilities. (3) There is no change in kinetic or potential energy between the end states. (4) Air is modeled as an ideal gas.

ANALYSIS: Since the process is both adiabatic and internally reversible, it is isentropic (Sec. 6.6). This together with the specified final pressure fixes state 2. With assumption 3, an energy balance for the adiabatic process reduces to give $W = -m(u_2 - u_1)$, or $W/m = -(u_2 - u_1)$.

(a) **R134a.** Table A-12: $u_1 = 262.17 \text{ kJ/kg}$, $s_1 = 0.9297 \text{ kJ/kg}\cdot\text{K}$. Then, with $s_2 = s_1$ and data from Table A-12 at 2.8 bar , interpolation gives $u_2 = 228.82 \text{ kJ/kg}$. Finally,

$$\frac{W}{m} = -(228.82 - 262.17) \frac{\text{kJ}}{\text{kg}} = 33.25 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow \text{(a)}$$

(b) **AIR.** With $s_2 = s_1$, Eq. 6.20a reduces to

$$s^\circ(T_2) = s^\circ(T_1) + R \ln P_2/P_1$$

With $s^\circ(T_1)$ from Table A-22

$$s^\circ(T_2) = 1.80685 + \frac{8.314}{28.97} \ln \frac{2.8}{14} = 1.345$$

Using this value, interpolation in Table A-22 gives $u_2 = 149.8 \text{ kJ/kg}$.

Also, $u_1 = 237.8 \text{ kJ/kg}$. Finally

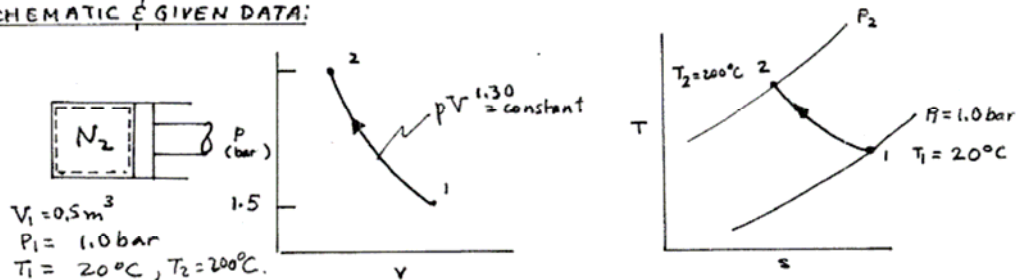
$$\frac{W}{m} = -(149.8 - 237.8) = 88 \text{ kJ/kg} \quad \leftarrow \text{(b)}$$

PROBLEM 6.26

KNOWN: N_2 undergoes an internally reversible compression between specified states during which $pV^n = \text{constant}$, where $n = 1.30$.

FIND: Determine (a) P_2 , (b) the work and heat transfer, (c) the entropy change.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) As shown in the accompanying figure, the system consists of the N_2 which is modeled as an ideal gas. (2) The compression is internally reversible and described by $pV^n = \text{constant}$. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: (a) With Eq. 3.56,

$$P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{\frac{n}{1-n}} = (1.0 \text{ bar}) \left(\frac{473}{293} \right)^{\frac{1.30}{-0.30}} = 7.97 \text{ bar} \leftarrow P_2$$

(b) The mass of N_2 present can be found using the ideal gas equation of state

$$m = \frac{P_1 V_1}{RT_1} = \frac{(1.0 \times 10^5 \text{ N/m}^2)(0.5 \text{ m}^3)}{\left(\frac{8.314}{28.01} \right) \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} (293 \text{ K})} = 0.5749 \text{ kg}$$

The work is

$$W = \int_1^2 p dV = \int_1^2 \frac{C}{V^n} dV = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n} = \frac{(0.5749 \text{ kg}) \left(\frac{8.314 \text{ kJ}}{28.01 \text{ kg}\cdot\text{K}} \right) (180 \text{ K})}{1-1.3} = -102.4 \text{ kJ} \leftarrow W$$

With assumption 3, an energy balance reduces to $Q = m(u_2 - u_1) + W$. Evaluating

$$Q = (0.5749 \text{ kg}) (9849 - 6083) \frac{\text{kJ}}{\text{kmol}} \left(\frac{1}{28.01} \right) \frac{\text{kmol}}{\text{kg}} + (-102.4 \text{ kJ}) = -25.1 \text{ kJ} \leftarrow Q$$

(c) The entropy change can be found with s° data from Table A-23 and Eq. 6.20a

$$\Delta S = m[s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1}] = (0.5749) \left[(204.989 - 190.991) / 28.01 - \left(\frac{8.314}{28.01} \right) \ln \left(\frac{7.97}{1.0} \right) \right] = -0.0670 \text{ kJ/K} \leftarrow \Delta S$$

PROBLEM 6.27

Air in a piston-cylinder assembly and modeled as an ideal gas undergoes two internally reversible processes in series from state 1, where $T_1 = 290 \text{ K}$, $p_1 = 1 \text{ bar}$.

Process 1-2: Compression to $p_2 = 5 \text{ bar}$ during which $pV^{1.19} = \text{constant}$

Process 2-3: Isentropic expansion to $p_3 = 1 \text{ bar}$.

- Sketch the two processes in series on T - s coordinates.
- Determine the temperature at state 2, in K.
- Determine the net work, in kJ/kg.

ENGR. MODEL:

- The air is the closed system.
- The processes are internally reversible.
- Kinetic and potential energy play no role.
- The air is modeled as an ideal gas.

ANALYSIS:

(b) With Eq. 3.56, $T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(n-1)/n} = (290 \text{ K}) (5)^{(1.19-1)/1.19} = 375 \text{ K} \leftarrow T_2$

$u_2 = 268.08 \frac{\text{kJ}}{\text{kg}}$ (Table A-22).

(c) Following Example 2.1,

$$\begin{aligned} \frac{W_{12}}{m} &= \frac{p_2 u_2 - p_1 u_1}{1-n} \stackrel{\text{Ideal gas model: } pV = RT}{=} \frac{R(T_2 - T_1)}{1-n} \\ &= \frac{(8.314/28.97) \left(\frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (375 - 290) \text{ K}}{1-1.19} = -128.39 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

With Eq. 6.23, $Q_{23} = 0$. Thus, an energy balance reduces to read

$$\Delta U + \Delta KE + \Delta PE = Q - W \Rightarrow W = -\Delta U = -m(u_3 - u_2).$$

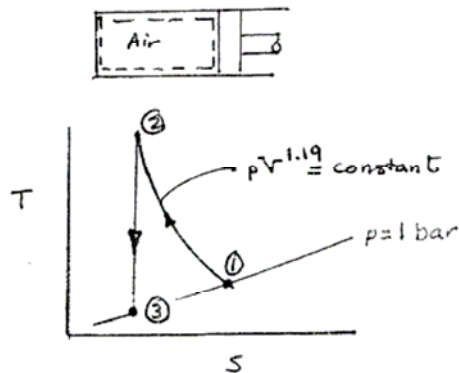
To find u_3 , apply Eq. 6.20a: $\underbrace{s_3 - s_2}_{=0} = s^\circ(T_3) - s^\circ(T_2) - R \ln(p_3/p_2)$

$$\begin{aligned} \Rightarrow s^\circ(T_3) &= s^\circ(T_2) + R \ln(p_3/p_2) = 1.92657 \underset{\text{(Table A-22)}}{+} \frac{8.314}{28.97} \ln\left(\frac{1}{5}\right) = 1.46468 \text{ kJ/kg} \cdot \text{K} \\ \Rightarrow u_3 &= 168.86 \text{ kJ/kg} \end{aligned}$$

$$\therefore \frac{W_{23}}{m} = -(168.86 - 268.08) = 99.22 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{W_{\text{net}}}{m} = \frac{W_{12}}{m} + \frac{W_{23}}{m} = -128.39 \frac{\text{kJ}}{\text{kg}} + 99.22 \frac{\text{kJ}}{\text{kg}} = -29.17 \frac{\text{kJ}}{\text{kg}} \leftarrow \frac{W_{\text{net}}}{m}$$

SCHEMATIC & GIVEN DATA:

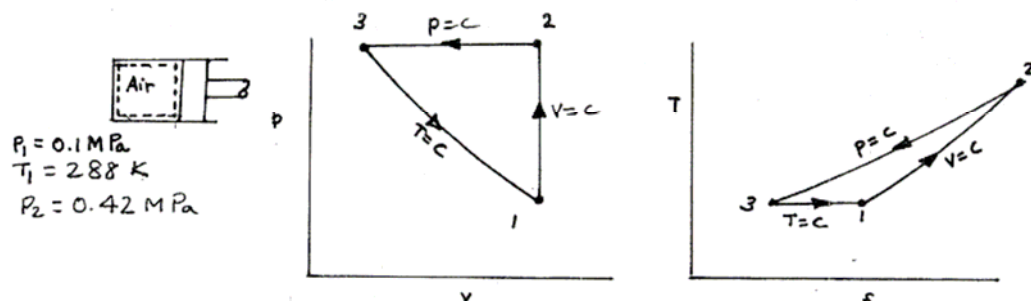


PROBLEM 6.28

KNOWN: A quantity of air undergoes a thermodynamic cycle consisting of three processes.

FIND: Evaluate the change in entropy for each process, and sketch the cycle on p-v coordinates.

SCHEMATIC & GIVEN DATA:



ENGINEER MODEL: (1) As shown in the accompanying figure, the system consists of the quantity of air. (2) The air behaves as an ideal gas with $c_p = 1 \text{ kJ/kg} \cdot \text{K}$.

ANALYSIS: Since $V_2 = V_1$,

$$\left. \begin{aligned} P_1 V_1 &= R T_1 \\ P_2 V_2 &= R T_2 \end{aligned} \right\} T_2 = \frac{P_2}{P_1} T_1 = \left(\frac{0.42}{0.10} \right) (288 \text{ K}) = 1210 \text{ K}$$

Also, with Eq. 3.44,

$$c_v = c_p - R = 1 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - \frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.713 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Process 1-2: With $V_2 = V_1$, Eq. 6.21 reduces to

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} = 0.713 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln \left(\frac{1210}{288} \right) = 1.0235 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad \leftarrow (s_2 - s_1)$$

Process 2-3: With $P_3 = P_2$, Eq. 6.22 reduces to

$$s_3 - s_2 = c_p \ln \frac{T_3}{T_2} \quad (= T_1) = 1 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln \left(\frac{288}{1210} \right) = -1.4354 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad \leftarrow (s_3 - s_2)$$

Process 3-1: With $T_3 = T_1$, Eq. 6.22 reduces to

$$s_1 - s_3 = -R \ln \frac{P_1}{P_3} \quad (= P_2) = -\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln \left(\frac{0.10}{0.42} \right) = 0.4118 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad \leftarrow (s_1 - s_3)$$

As a check, note that

$$(s_2 - s_1) + (s_3 - s_2) + (s_1 - s_3) = 0 \quad (\text{to within roundoff})$$

PROBLEM 6.29

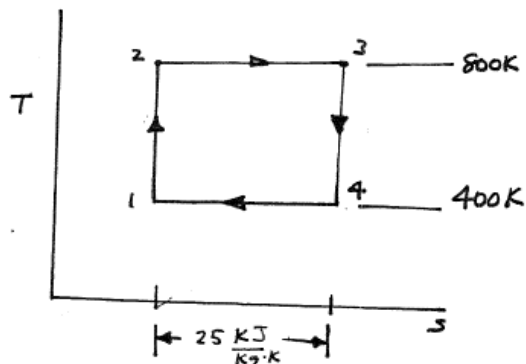
One-tenth kilogram of a gas in a piston-cylinder assembly undergoes a Carnot power cycle for which the isothermal expansion occurs at 800 K. The change in specific entropy of the gas during the isothermal compression, which occurs at 400 K, is $-25 \text{ kJ/kg} \cdot \text{K}$. Determine (a) the net work developed per cycle, in kJ, and (b) the thermal efficiency.

ENGR. MODEL:

1. The gas is the closed system.
2. The gas undergoes a Carnot power cycle.

ANALYSIS:

On T - s coordinates, the Carnot cycle is



The enclosed area is the net work developed per unit of mass. That is

$$W_{\text{net}}/m = \text{Area } 1-2-3-4-1 \\ = (800\text{K} - 400\text{K}) \left(25 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) = 10,000 \frac{\text{kJ}}{\text{kg}} / \text{cycle}$$

With $m = 0.1 \text{ kg}$,

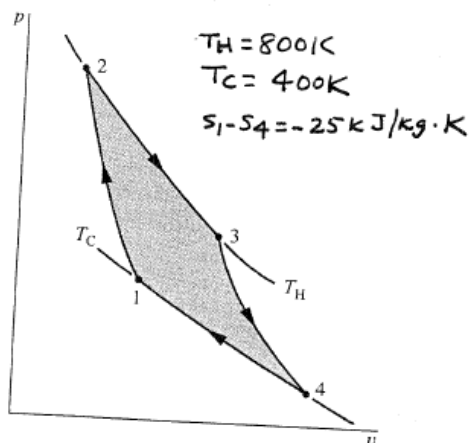
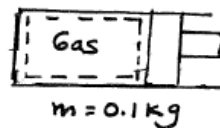
$$W_{\text{net}} = 1000 \text{ kJ} / \text{cycle}$$

With Eq. 5.9,

$$\textcircled{1} \quad \eta = 1 - \frac{T_C}{T_H} = 1 - \frac{400}{800} = 0.5 \text{ (50\%)}$$

1. Alternatively, η can be found in terms of areas on the T - s diagram. See Sec. 6.6.2 for discussion.

SCHEMATIC & GIVEN DATA:

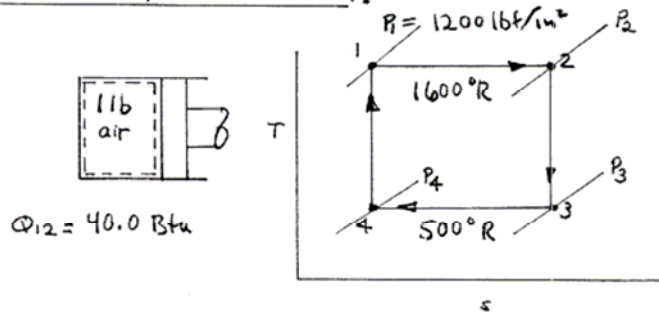


PROBLEM 6.30

KNOWN: Data is provided for 1 lb of air undergoing a Carnot power cycle.

FIND: Determine (a) the pressures at the end of the isothermal expansion, adiabatic expansion, and isothermal compression, (b) the net work, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system is the 1 lb of air. (2) The air is modeled as an ideal gas. (3) The system undergoes a Carnot power cycle.

ANALYSIS: As discussed in Sec. 6.6.2, for process 1-2 $Q_{12} = m T_H (s_2 - s_1)$. And for an ideal gas at fixed temperature, Eq. 6.18 reduces to read $s_2 - s_1 = -R \ln P_2/P_1$. Combining these $Q_{12} = -m R T_H \ln P_2/P_1$. Solving

$$\ln \frac{P_2}{P_1} = -\frac{Q_{12}}{m R T_H} = \frac{-40.0 \text{ Btu}}{(1.0 \text{ lb}) \left(\frac{1.986 \text{ Btu}}{28.97 \text{ lb} \cdot \text{R}} \right) (1600^\circ \text{R})}$$

giving $P_2/P_1 = 0.6944$, $P_2 = 833.3 \text{ lbf/in}^2$ ← P_2

For process 2-3 and data from Table A-22E

$$0 = s^\circ(T_3) - s^\circ(T_2) - R \ln \frac{P_3}{P_2} \Rightarrow \ln \frac{P_3}{P_2} = \frac{s^\circ(T_3) - s^\circ(T_2)}{R} = \frac{0.58233 - 0.87130}{(1.986/28.97)}$$

giving $\ln P_3/P_2 = -4.2152$, $P_3/P_2 = 0.01477$, $P_3 = 12.31 \text{ lbf/in}^2$ ← P_3

Similarly, for process 4-1

$$\ln \frac{P_1}{P_4} = \frac{s^\circ(T_1) - s^\circ(T_4)}{R} = +4.2152, \frac{P_1}{P_4} = 67.708, P_4 = 17.72 \text{ lbf/in}^2 \leftarrow P_4$$

(b) For any cycle, $W_{\text{net}} = \eta Q_{\text{in}}$. Here, η corresponds to η_{MAX} and $Q_{\text{in}} = Q_{12}$.

$$\text{Thus } W_{\text{net}} = \left[1 - \frac{500}{1600} \right] 40.0 \text{ Btu} = 27.5 \text{ Btu} \leftarrow W_{\text{net}}$$

$$\underline{0.6875 \text{ (68.75\%)} \leftarrow \eta}$$

Alternatively $W_{\text{net}} = Q_{\text{net}} = Q_{12} + Q_{34}$. From an energy balance, since T is constant, $Q_{34} = W_{34}$, where

$$W_{34} = m \int_3^4 p dv = m \int_3^4 \frac{RT}{v} dv = m R T \ln \frac{v_4}{v_3} = m R T \ln \frac{P_3}{P_4}$$

$$= (1.0) \left(\frac{1.986}{28.97} \right) (500) \ln \left(\frac{12.31}{17.72} \right) = -12.5 \text{ Btu}$$

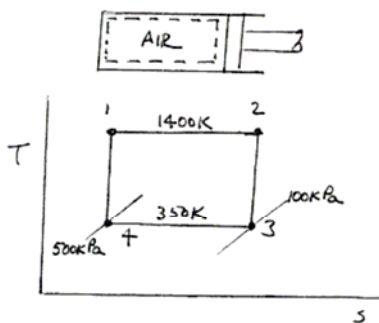
So $Q_{\text{net}} = 40.0 - 12.5 = 27.5 \text{ Btu}$, and $W_{\text{net}} = 27.5 \text{ Btu}$.

PROBLEM 6.31

Air in a piston-cylinder assembly undergoes a Carnot power cycle. The isothermal expansion and compression processes occur at 1400 K and 350 K, respectively. The pressures at the beginning and end of the isothermal compression are 100 kPa and 500 kPa, respectively. Assuming the ideal gas model with $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$, determine

- the pressures at the beginning and end of the isothermal expansion, each in kPa.
- the heat transfer and work, in kJ/kg, for each process.
- the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

- The air is the closed system.
- The air undergoes a Carnot power cycle.
- Kinetic and potential energy play no role.
- The air is modeled as an ideal gas with $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$

ANALYSIS: (a) Use the relations $s_2 - s_3 = 0$ and $s_1 - s_4 = 0$ together with Eq. 6.22. Thus

$$s_2 - s_3 = 0 = c_p \ln \frac{T_2}{T_3} - R \ln \frac{P_2}{P_3} \Rightarrow \ln \frac{P_2}{P_3} = \frac{c_p}{R} \ln \frac{T_2}{T_3}$$

$$= \left(\frac{1.005}{8.314/28.97} \right) \ln 4 \Rightarrow P_2 = 128.20 \text{ kPa} \leftarrow P_2$$

$$s_1 - s_4 = 0 = c_p \ln \frac{T_1}{T_4} - R \ln \frac{P_1}{P_4} \Rightarrow \ln \frac{P_1}{P_4} = \frac{c_p}{R} \ln \frac{T_1}{T_4} \Rightarrow P_1 = 64,150 \text{ kPa} \leftarrow P_1$$

(b) Process 1-2:

$$\frac{Q_{12}}{m} = \int_1^2 T ds = T(s_2 - s_1) \Rightarrow \frac{Q_{12}}{m} = T \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right] = -1400 \text{ K} \left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) \ln(0.2)$$

$$\Rightarrow \frac{Q_{12}}{m} = 646.64 \frac{\text{kJ}}{\text{kg}} \leftarrow \text{Process 1-2}$$

An energy balance reads, $\Delta U = Q_{12} - W_{12} \Rightarrow$
(T=c)

$$\frac{W_{12}}{m} = 646.64 \frac{\text{kJ}}{\text{kg}}$$

Process 2-3: $Q_{23} = \int_2^3 T ds = 0$. An energy balance reduces to $\Delta U = W_{23}$

$$\Rightarrow W_{23} = -m(u_3 - u_2) = -m c_v (T_3 - T_2) \Rightarrow \frac{W_{23}}{m} = -0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (350 - 1400) \text{ K}$$

$$= 753.9 \frac{\text{kJ}}{\text{kg}} \leftarrow \text{Process 2-3}$$

Eg. 3.44:
 $c_v = c_p - R = 0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

Process 3-4: $\frac{Q_{34}}{m} = T(s_4 - s_3) = T \left[c_p \ln \frac{T_4}{T_3} - R \ln \frac{P_4}{P_3} \right]$

$$= -(350 \text{ K}) \left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) \ln \left(\frac{500}{100} \right) = -161.66 \frac{\text{kJ}}{\text{kg}}$$

Energy balance: $\Delta U = Q_{34} - W_{34} \Rightarrow$
(T=c)

$$\frac{W_{34}}{m} = -161.66 \frac{\text{kJ}}{\text{kg}} \leftarrow \text{Process 3-4}$$

Continued on next slide

Problem 6-31 continued

Process 4-1: $Q_{41} = \int_4^1 T ds = 0$. Energy balance: $\Delta U = \cancel{Q}_{41} - W_{41}$

$$\Rightarrow W_{41} = -m(u_1 - u_4) \Rightarrow \frac{W_{41}}{m} = -c_v(T_1 - T_4)$$

①

$$= -0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (1400 - 350) = -753.9 \frac{\text{kJ}}{\text{kg}}$$

Process
4-1

$$(c) \quad \eta = \frac{W_{\text{net}}/m}{Q_{\text{in}}/m} = \frac{646.64 + 753.9 - 161.66 - 753.9}{646.64} = 0.75 \quad (75\%) \quad \eta$$

Alternatively, apply Eq. 5.9

$$\eta = 1 - \frac{T_c}{T_H} = 1 - \frac{350}{1400} = 0.75 \quad (75\%)$$

1. For any cycle $W_{\text{net}} = Q_{\text{net}}$. Thus, to check the current calculations,

$$\frac{W_{\text{net}}}{m} = 646.64 + 753.9 - 161.66 - 753.9 = 484.98 \frac{\text{kJ}}{\text{kg}}$$

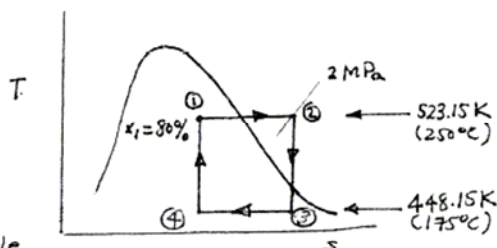
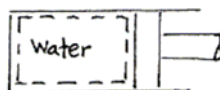
$$\frac{Q_{\text{net}}}{m} = 646.64 + 0 - 161.66 + 0 = 484.98 \frac{\text{kJ}}{\text{kg}}$$

PROBLEM 6.32

Water in a piston-cylinder assembly undergoes a Carnot power cycle. At the beginning of the isothermal expansion, the temperature is 250°C and the quality is 80%. The isothermal expansion continues until the pressure is 2 MPa. The adiabatic expansion then occurs to a final temperature of 175°C.

- Sketch the cycle on $T-s$ coordinates.
- Determine the heat transfer and work, in kJ/kg, for each process.
- Evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

- The water is the closed system.
- The water undergoes a Carnot cycle.
- Kinetic and potential energy play no role.

ANALYSIS:

(b) Process 1-2

$$\frac{Q_{12}}{m} = \int_1^2 T ds = T(s_2 - s_1) = 523.15 \text{ K} (6.5421 - 5.417) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 588.6 \text{ kJ/kg} \quad \leftarrow \frac{Q_{12}}{m}$$

Table A-2 at 250°C:

$$s_1 = s_f + x_1(s_g - s_f) = 2.7927 + 0.8(6.0730 - 2.7927) = 5.417 \text{ kJ/kg} \cdot \text{K}$$

$$u_1 = u_f + x_1(u_g - u_f) = 1080.4 + 0.8(2602.4 - 1080.4) = 2298 \text{ kJ/kg}$$

Table A-4 at 2 MPa, 250°C: $s_2 = 6.5421 \text{ kJ/kg} \cdot \text{K}$, $u_2 = 2678.8 \text{ kJ/kg}$

An energy balance reduces to read, $\Delta U = Q_{12} - W_{12} \Rightarrow$

$$\frac{W_{12}}{m} = \frac{Q_{12}}{m} - (u_2 - u_1) = 588.6 - (2678.8 - 2298) = 207.8 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow \frac{W_{12}}{m}$$

Process 2-3. $\frac{Q_{23}}{m} = \int_2^3 T ds = 0$. An energy balance reads, $\leftarrow \frac{Q_{23}}{m}$

$$\Delta U = Q_{23} - W_{23} \Rightarrow \frac{W_{23}}{m} = -(u_3 - u_2) = -(2546.1 - 2678.8) = 132.7 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow \frac{W_{23}}{m}$$

To find u_3 , use $s_3 = s_2$. Thus $x_3 = \frac{s_3 - s_f}{s_g - s_f} = \frac{6.5421 - 2.09075}{6.626 - 2.09075} = 0.9815$

$$\text{So, } u_3 = u_f + x_3 u_{fg} = 740.21 + 0.9815(2580.1 - 740.21) = 2546.1 \text{ kJ/kg}.$$

Process 3-4.

$$\frac{Q_{34}}{m} = \int_3^4 T ds = T(s_4 - s_3) = -448.15 \text{ K} (6.5421 - 5.417) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = -504.21 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow \frac{Q_{34}}{m}$$

An energy balance gives $\Delta U = Q_{34} - W_{34} \Rightarrow \frac{W_{34}}{m} = \frac{Q_{34}}{m} - (u_4 - u_3)$

To get u_4 , use $s_4 = s_1 = 5.417 \text{ kJ/kg} \cdot \text{K}$. Then

$$x_4 = \frac{5.417 - 2.09075}{6.626 - 2.09075} = 0.7334 \Rightarrow u_4 = 740.21 + (0.7334)(2580.1 - 740.21) = 2089.6 \frac{\text{kJ}}{\text{kg}}$$

Finally

$$\frac{W_{34}}{m} = (-504.21) - (2089.6 - 2546.1) = -47.7 \frac{\text{kJ}}{\text{kg}}$$

Continued on next slide

Problem 6-32 continued

Process 4-1. $Q_{41}/m = \int_4^1 T ds = 0.$

$\leftarrow Q_{41}/m$

An energy balance reads, $\Delta U = Q_{41} - W_{41}$. So

① $\frac{W_{41}}{m} = -(u_1 - u_4) = -(2298 - 2089.6) = -208.4 \frac{kJ}{kg}$ $\leftarrow \frac{W_{41}}{m}$

② $\eta = \frac{W_{net}/m}{Q_{in}/m} = \frac{207.8 + 132.7 - 47.7 - 208.4}{588.6} = 0.143 \quad (14.3\%)$

Alternatively, apply Eq. 5.9

$\eta = 1 - \frac{T_c}{T_H} = 1 - \frac{448.15}{523.15} = 0.143 \quad (14.3\%)$

1. For any cycle $W_{net} = Q_{net}$. Thus, to check the calculations,

$\frac{W_{net}}{m} = 207.8 + 132.7 - 47.7 - 208.4 = 84.4 \frac{kJ}{kg}$

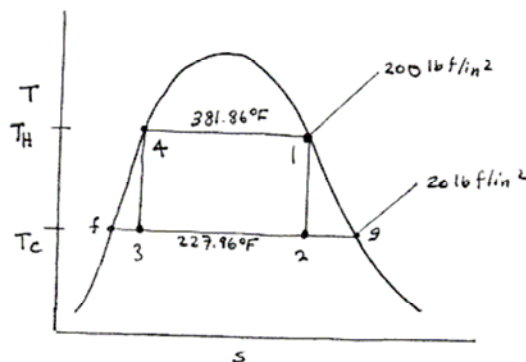
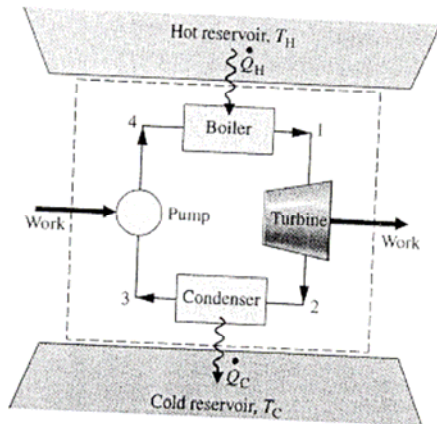
$\frac{Q_{net}}{m} = 588.6 + 0 - 504.21 + 0 = 84.4 \frac{kJ}{kg}$

PROBLEM 6.33

KNOWN: Steady-state operating data are provided for a Carnot power cycle.

FIND: Sketch the cycle on T - s coordinates. Also, evaluate $\dot{Q}/\dot{m}$ and $\dot{W}/\dot{m}$ for each process and the thermal efficiency of the overall cycle

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. A control volume at steady state encloses each of the four components.
2. For the turbine and pump, $\dot{Q}_{cv} = 0$.
3. Kinetic and potential energy effects are ignored.

ANALYSIS: From Table A-4E,

$$h_1 = 1199.3 \text{ Btu/lb}, \quad s_1 = 1.5465 \text{ Btu/lb} \cdot ^\circ\text{R}$$

$$h_4 = 355.6 \text{ Btu/lb}, \quad s_4 = 0.544 \text{ Btu/lb} \cdot ^\circ\text{R}$$

$$s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_f}{s_g - s_f} = \frac{1.5465 - 0.3358}{1.3962} = 0.867$$

$$s_4 = s_3 \Rightarrow x_3 = \frac{s_3 - s_f}{s_g - s_f} = \frac{0.544 - 0.3358}{1.3962} = 0.149$$

① **Process 1-2:** $\dot{Q}_{12}/\dot{m} = 0$.

$$\begin{aligned} \frac{\dot{W}_t}{\dot{m}} &= h_1 - h_2, \quad h_2 = h_f + x_2(h_g - h_f) \\ &= 196.26 + 0.867(960.1) \\ &= 1028.67 \text{ Btu/lb} \end{aligned}$$

$$\therefore \frac{\dot{W}_t}{\dot{m}} = (1199.3 - 1028.67) = 170.63 \frac{\text{Btu}}{\text{lb}}$$

① **Process 3-4:** $\dot{Q}_{34}/\dot{m} = 0$

$$\begin{aligned} \frac{\dot{W}_p}{\dot{m}} &= h_3 - h_4, \quad h_3 = h_f + x_3(h_g - h_f) \\ &= 196.26 + 0.149(960.1) \\ &= 339.31 \text{ Btu/lb} \end{aligned}$$

$$\therefore \frac{\dot{W}_p}{\dot{m}} = (339.31 - 355.6) = -16.29 \frac{\text{Btu}}{\text{lb}}$$

① **Process 2-3:** $\dot{W}_{23}/\dot{m} = 0$.

$$\begin{aligned} \frac{\dot{Q}_{23}}{\dot{m}} &= h_3 - h_2 = 339.31 - 1028.67 \\ &= -689.36 \text{ Btu/lb} \end{aligned}$$

$$\text{or } \frac{\dot{Q}_{23}}{\dot{m}} = T_C(s_3 - s_2) = 687.63^\circ\text{R} (0.544 - 1.5465) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = -689.35 \text{ Btu/lb}$$

① **Process 4-1:** $\dot{W}_{41}/\dot{m} = 0$

$$\begin{aligned} \frac{\dot{Q}_{41}}{\dot{m}} &= h_1 - h_4 = (1199.3 - 355.6) \\ &= 843.7 \text{ Btu/lb} \end{aligned}$$

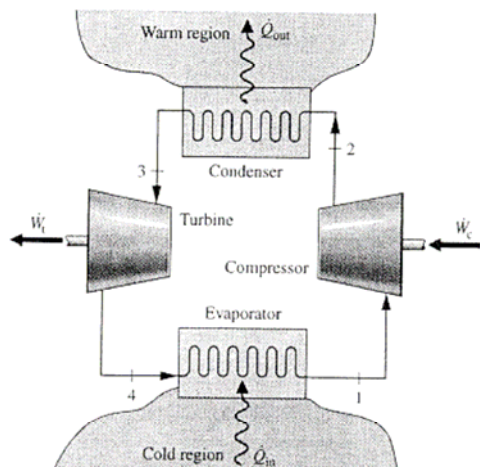
$$\text{or } \frac{\dot{Q}_{41}}{\dot{m}} = T_H(s_1 - s_4) = 841.53^\circ\text{R} (1.5465 - 0.544) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 843.63 \text{ Btu/lb}$$

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{170.63 - 16.29}{843.7} = 0.183$$

$$\text{or } \eta = \frac{T_H - T_C}{T_H} = \frac{841.53 - 687.63}{841.53} = 0.183$$

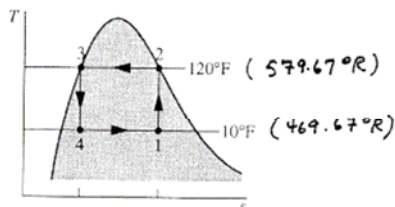
PROBLEM 6.34

Figure P6.34 shows a Carnot heat pump cycle operating at steady state with ammonia as the working fluid. The



condenser temperature is 120°F, with saturated vapor entering and saturated liquid exiting. The evaporator temperature is 10°F.

- Determine the heat transfer and work for each process, in Btu per lb of ammonia flowing.
- Evaluate the coefficient of performance for the heat pump.
- Evaluate the coefficient of performance for a Carnot refrigeration cycle operating as shown in the figure.



ENGR. MODEL:

- Each of the four components is enclosed by a control volume at steady state.
- Ammonia circulating through the components executes a Carnot heat pump cycle.
- Energy transfers are positive in the directions of the arrows.

ANALYSIS:

(a) Process 1-2,

$$\frac{\dot{Q}_{12}}{\dot{m}} = \int_1^2 T ds = 0$$

An energy rate balance reduces to give $0 = \dot{Q}_{12} - \dot{W}_{12} + \dot{m}(h_1 - h_2) \Rightarrow \frac{\dot{W}_{12}}{\dot{m}} = h_1 - h_2$.

$h_2 = 632.95 \text{ Btu/lb}$, $s_2 = 1.1405 \text{ Btu/lb} \cdot ^\circ\text{R}$ (Table A-13E). To find h_1 , use $s_1 = s_2$.

$$\text{Then } x_1 = \frac{1.1405 - 0.1196}{1.3141 - 0.1196} = 0.8547 \Rightarrow h_1 = h_f + x_1 h_{fg} = 53.27 + 0.8547(561) = 532.76 \frac{\text{Btu}}{\text{lb}}$$

$$\text{So, } \frac{\dot{W}_{12}}{\dot{m}} = h_1 - h_2 = 532.76 - 632.95 = -100.19 \text{ Btu/lb}$$

Process 2-3, $\dot{W}_{23}/\dot{m} = 0$

$$\textcircled{1} \quad \frac{\dot{Q}_{23}}{\dot{m}} = \int_2^3 T ds = T(s_3 - s_2) = 579.67^\circ\text{R}(0.3570 - 1.1405) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = -454.17 \frac{\text{Btu}}{\text{lb}}$$

Process 3-4,

$$\frac{\dot{Q}_{34}}{\dot{m}} = \int_3^4 T ds = 0$$

$$\frac{\dot{W}_{34}}{\dot{m}} = h_3 - h_4$$

To find h_4 use $s_4 = s_3 = 0.3570$. Then, $x_4 = \frac{0.3570 - 0.1196}{1.3141 - 0.1196} = 0.1987$ and

$$h_4 = h_f + x_4 h_{fg} = 53.27 + 0.1987(561) = 164.74 \text{ Btu/lb. Finally}$$

$$\frac{\dot{W}_{34}}{\dot{m}} = 178.79 - 164.74 = 14.05 \text{ Btu/lb}$$

Continued on next slide

Problem 6-34 continued

Process 4-1. $\dot{W}_{41}/\dot{m} = 0$

$$\textcircled{2} \quad \frac{\dot{Q}_{41}}{\dot{m}} = \int_4^1 T ds = T(s_1 - s_4) = 469.67^\circ R (1.1405 - 0.3570) \frac{\text{Btu}}{\text{lb} \cdot ^\circ R} = 367.99 \frac{\text{Btu}}{\text{lb}} \leftarrow$$

$= (s_2 - s_3)$

(b) The coefficient of performance for the heat pump shown in the figure is

$$\gamma = \frac{\dot{Q}_{\text{out}}}{\dot{W}_{\text{c}} - \dot{W}_{\text{t}}} = \frac{|\dot{Q}_{23}/\dot{m}|}{|\frac{\dot{W}_{12}}{\dot{m}}| - \frac{\dot{W}_{34}}{\dot{m}}} = \frac{454.17}{100.19 - 14.05} = 5.27 \leftarrow$$

All quantities are positive

Alternatively, Eq. 5.11 can be used

$$\gamma = \frac{T_H}{T_H - T_C} = \frac{579.67^\circ R}{110^\circ R} = 5.27$$

(c) The coefficient of performance for a Carnot refrigeration cycle obtained using Eq. 5.10 is

$$\textcircled{3} \quad \beta = \frac{T_C}{T_H - T_C} = \frac{469.67^\circ R}{110^\circ R} = 4.27 \leftarrow$$

1. Applying an energy rate balance, $\frac{\dot{Q}_{23}}{\dot{m}} = h_3 - h_2 = h_f - h_g = -454.16 \text{ Btu/lb}$,

2. For any cycle $\dot{Q}_{\text{net}}/\dot{m} = \dot{W}_{\text{net}}/\dot{m}$. Thus, to check the calculations

$$\dot{Q}_{\text{net}}/\dot{m} = 0 + (-454.17) + 0 + (367.99) = -86.18 \text{ Btu/lb}$$

$$\dot{W}_{\text{net}}/\dot{m} = (-100.19) + 0 + (14.05) + 0 = -86.14 \text{ Btu/lb}$$

3. Alternatively

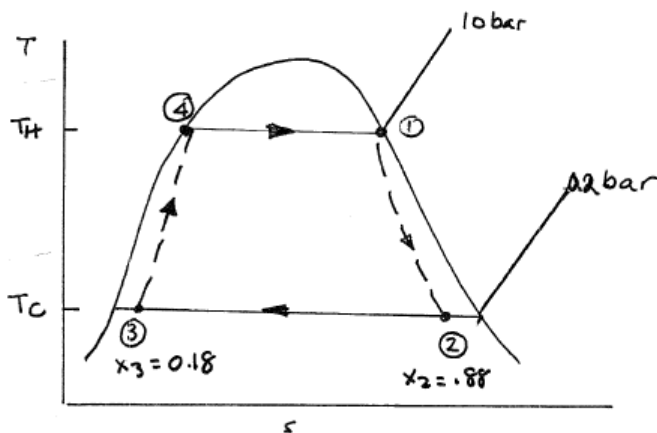
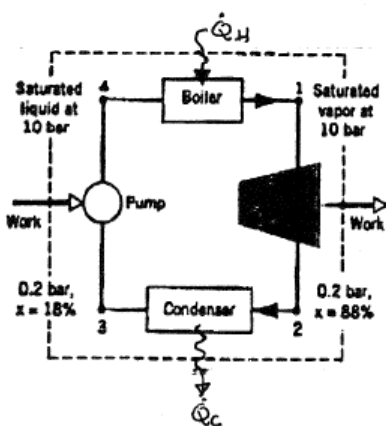
$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{c}} - \dot{W}_{\text{t}}} = \frac{\dot{Q}_{41}/\dot{m}}{|\frac{\dot{W}_{12}}{\dot{m}}| - \frac{\dot{W}_{34}}{\dot{m}}} = \frac{367.99}{100.19 - 14.05} = 4.27$$

PROBLEM 6.35

KNOWN: The schematic and steady state operating data are provided for a vapor power plant.

FIND: (a) sketch the cycle on T-s coordinates, (b) determine the thermal efficiency and compare with the thermal efficiency of a Carnot cycle operating between the same maximum and minimum temperatures.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The overall system shown in the figure and thus each component is at steady state. (2) The only significant heat transfers occur with the two reservoirs. (3) Kinetic and potential energy effects can be ignored. (3) The maximum and minimum temperatures correspond, respectively, to the saturation temperatures at 10 bar and 0.2 bar.

ANALYSIS: (b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = 1 - \frac{\dot{Q}_C}{\dot{Q}_H}$$

The heat transfer rates are found by reducing mass and energy balances for the boiler and condenser: $\dot{Q}_H = \dot{m}(h_1 - h_4)$, $\dot{Q}_C = \dot{m}(h_2 - h_3)$. Thus

$$\eta = 1 - \left[\frac{h_2 - h_3}{h_1 - h_4} \right].$$

With data from Table A-3, $h_1 - h_4 = 2015.3 \text{ kJ/kg}$. Also, $h_2 = h_f + x_2(h_g - h_f)$ and $h_3 = h_f + x_3(h_g - h_f)$. So, $h_2 - h_3 = (x_2 - x_3)(h_g - h_f)$. That is,

$$h_2 - h_3 = (0.88 - 0.18)(2358.3) = 1650.8 \text{ kJ/kg}$$

Accordingly,

$$\eta = 1 - \left[\frac{1650.8}{2015.3} \right] = 0.181 \quad (18.1\%)$$

For a Carnot Cycle

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{(60.06 + 273.15)}{(179.9 + 273.15)} = 0.265 \quad (26.5\%) \quad \leftarrow \eta_{MAX}$$

PROBLEM 6.36

KNOWN: A closed system undergoes a process in which Q occurs at T_b .

FIND: For each of several cases, determine whether the entropy change is positive, negative, zero, or indeterminate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The system shown in the figure interacts thermally with its surroundings only at a place on its boundary where temperature is T_b .

ANALYSIS: In this case, Eq. 6.24 takes the form $\Delta S = \frac{Q}{T_b} + \sigma$.

(a) $\sigma = 0, Q > 0 \Rightarrow \Delta S = \frac{Q}{T_b} + \cancel{\sigma}^0 > 0$

(b) $\sigma = 0, Q = 0 \Rightarrow \Delta S = \frac{\cancel{Q}^0}{T_b} + \cancel{\sigma}^0 = 0$

(c) $\sigma = 0, Q < 0 \Rightarrow \Delta S = \frac{Q}{T_b} + \cancel{\sigma}^0 < 0$

(d) $\sigma > 0, Q > 0 \Rightarrow \Delta S = \frac{Q}{T_b} + \sigma > 0$

(e) $\sigma > 0, Q = 0 \Rightarrow \Delta S = \frac{\cancel{Q}^0}{T_b} + \sigma > 0$

(f) $\sigma > 0, Q < 0 \Rightarrow \Delta S = \underbrace{\frac{Q}{T_b}}_{(-)} + \underbrace{\sigma}_{(+)}$

Indeterminate: ΔS may be positive or negative depending on the relative magnitudes of the two terms on the right side.

PROBLEM 6.36

KNOWN: A closed system undergoes a process in which Q occurs at T_b .

FIND: For each of several cases, determine whether the entropy change is positive, negative, zero, or indeterminate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The system shown in the figure interacts thermally with its surroundings only at a place on its boundary where temperature is T_b .

ANALYSIS: In this case, Eq. 6.24 takes the form $\Delta S = \frac{Q}{T_b} + \sigma$.

(a) $\sigma = 0, Q > 0 \Rightarrow \Delta S = \frac{Q}{T_b} + \cancel{\sigma}^0 > 0$

(b) $\sigma = 0, Q = 0 \Rightarrow \Delta S = \frac{\cancel{Q}^0}{T_b} + \cancel{\sigma}^0 = 0$

(c) $\sigma = 0, Q < 0 \Rightarrow \Delta S = \frac{Q}{T_b} + \cancel{\sigma}^0 < 0$

(d) $\sigma > 0, Q > 0 \Rightarrow \Delta S = \frac{Q}{T_b} + \sigma > 0$

(e) $\sigma > 0, Q = 0 \Rightarrow \Delta S = \frac{\cancel{Q}^0}{T_b} + \sigma > 0$

(f) $\sigma > 0, Q < 0 \Rightarrow \Delta S = \underbrace{\frac{Q}{T_b}}_{(-)} + \underbrace{\sigma}_{(+)}$

Indeterminate: ΔS may be positive or negative depending on the relative magnitudes of the two terms on the right side.

PROBLEM 6.37

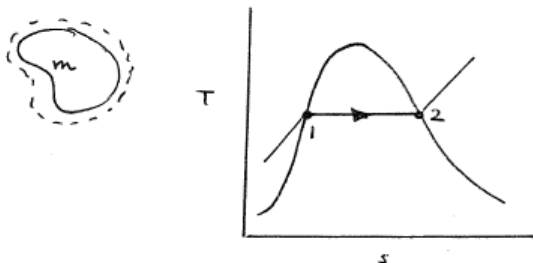
- (a) False. See discussion of Sec. 5.1
- (b) False. For any process of a closed system, internally reversible or otherwise, $\Delta E = Q - W$. Accordingly, the magnitude and direction of Q is not determined by W alone -- unless $\Delta E = 0$.
- (c) False. See discussion of Sec. 6.7.1
- (d) False. See discussion of Sec. 6.7.1
- (e) False. See discussion of Sec. 6.7.1
- (f) True. See Eq. 6.24
- (g) False. See Sec. 6.8.

PROBLEM 6.38

KNOWN: A fixed mass of water, initially a saturated liquid, undergoes a constant temperature, constant pressure process to a saturated vapor condition.

FIND: (a) Derive expressions for W, Q in terms of m and steam table properties. (b) Show that the process is internally reversible.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system is a fixed mass of water. (2) Kinetic and potential energy effects are absent. (3) In the process, $p = \text{constant}$, $T = \text{constant}$. (4) Volume change is the only work mode.

ANALYSIS: (a) With assumption (4) and since the pressure remains constant,

$$W = m \int_1^2 p \, dv = m p (v_2 - v_1) = m p [v_g - v_f] \quad \leftarrow W$$

With assumption (2) and the above expression for W , an energy balance gives

$$\begin{aligned} m(u_2 - u_1) &= Q - W \Rightarrow Q = m(u_g - u_f) + m p (v_g - v_f) \\ &= m[(u_g + p v_g) - (u_f + p v_f)] \\ &= m(h_g - h_f) \quad \leftarrow Q \end{aligned}$$

(b) An entropy balance reduces to

$$m(s_2 - s_1) = \frac{Q}{T} + \sigma$$

Introducing the expression for Q

$$m(s_g - s_f) = \frac{m(h_g - h_f)}{T} + \sigma$$

From the discussion of Sec. 6.3, $(s_g - s_f) = \frac{(h_g - h_f)}{T}$ (Eq. 6.12). Thus,

$$\sigma = 0 \Rightarrow \text{the process is internally reversible.}$$

PROBLEM 6.39

Five kg of water contained in a piston-cylinder assembly expand from an initial state where $T_1 = 400^\circ\text{C}$, $p_1 = 700 \text{ kPa}$ to a final state where $T_2 = 200^\circ\text{C}$, $p_2 = 300 \text{ kPa}$, with no significant effects of kinetic and potential energy. The accompanying table provides additional data at the two states. It is claimed that the water undergoes an adiabatic process between these states, while developing work. Evaluate this claim.

State	$T(^{\circ}\text{C})$	$p(\text{kPa})$	$v(\text{m}^3/\text{kg})$	$u(\text{kJ/kg})$	$h(\text{kJ/kg})$	$s(\text{kJ/kg} \cdot \text{K})$
1	400	700	0.4397	2960.9	3268.7	7.6350
2	200	300	0.7160	2650.7	2865.5	7.3115

ANALYSIS:

Every process between the given end states must satisfy the second law.

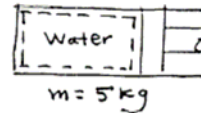
That is,

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma \Rightarrow \sigma = \Delta S = m(s_2 - s_1)$$

$$\begin{aligned} \sigma &= 5 \text{ kg} (7.3115 - 7.6350) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ &= -1.6175 \frac{\text{kJ}}{\text{K}} \end{aligned}$$

Since $\sigma < 0$, this process cannot occur as claimed.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The water is the closed system.
2. For the system, $Q = 0$ and there are no significant effects of kinetic and potential energy.

PROBLEM 6.40

Two m^3 of air in a rigid, insulated container fitted with a paddle wheel is initially at 293 K, 200 kPa. The air receives 710 kJ by work from the paddle wheel. Assuming the ideal gas model with $c_v = 0.72 \text{ kJ/kg} \cdot \text{K}$, determine for the air (a) the mass, in kg, (b) final temperature, in K, and (c) the amount of entropy produced, in kJ/K.

ENGR. MODEL:

1. The air is the closed system.
2. For the air, $Q=0$ and there are no overall changes in KE or PE.
3. The air is modeled as an ideal gas with $c_v = 0.72 \text{ kJ/kg} \cdot \text{K}$.

ANALYSIS: Using the ideal gas equation of state, the mass, m , of the air is

$$m = \frac{P_1 V}{R T_1} = \frac{(200 \times 10^3 \text{ N/m}^2)(2 \text{ m}^3)}{\left(\frac{8314}{28.97} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}\right)(293 \text{ K})} = 4.76 \text{ kg}$$

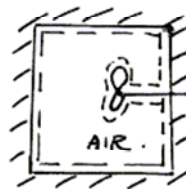
An energy balance reduces as, $\Delta U + \Delta KE + \Delta PE = Q - W$. Thus

$$W = -\Delta U = -m c_v (T_2 - T_1) \Rightarrow T_2 = T_1 - \frac{W}{m c_v} = 293 - \frac{(-710 \text{ kJ})}{(4.76 \text{ kg})(0.72 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})} = 500 \text{ K}$$

An entropy balance reduces as, $\Delta S = \int_1^2 \frac{\delta Q}{T} + \sigma \Rightarrow \sigma = \Delta S = m(s_2 - s_1)$
with Eq. 6.21 and $v_2 = v_1$,

$$\sigma = m c_v \ln \frac{T_2}{T_1} = (4.76 \text{ kg})(0.72 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \ln \frac{500}{293} = 1.832 \frac{\text{kJ}}{\text{K}}$$

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} W &= -710 \text{ kJ} \\ V &= 2 \text{ m}^3 \\ T_1 &= 293 \text{ K} \\ P_1 &= 200 \text{ kPa} \end{aligned}$$

PROBLEM 6.41

A rigid, insulated container fitted with a paddle wheel contains 5 lb of water, initially at 260°F and a quality of 60%. The water is stirred until the temperature is 350°F. For the water, determine (a) the work, in Btu, and (b) the amount of entropy produced, in Btu/°R.

ENGR. MODEL:

1. The water is the closed system.
2. For the system, $Q = 0$ and there are no overall changes in KE or PE.

ANALYSIS:

With data from Table A-2E,

$$v_1 = (1-x_1)v_f + x_1 v_g$$

$$= (0.4)(0.0178) + 0.6(11.77) = 7.07 \frac{\text{ft}^3}{\text{lb}}$$

$$u_1 = (0.4)(228.6) + 0.6(1090.5) = 745.74 \frac{\text{Btu}}{\text{lb}}$$

$$s_1 = (0.4)(0.3819) + 0.6(1.6864) = 1.1646 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

Since $v_2 = v_1$ and $T_2 = 350^\circ\text{F}$, Table A-4E gives

$$u_2 = 1120.32 \frac{\text{Btu}}{\text{lb}}, \quad s_2 = 1.6699 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

(a) An energy balance reads,

$$\Delta U + \Delta KE + \Delta PE = \cancel{Q} - W$$

$$\Rightarrow W = -m(u_2 - u_1) = -5 \text{ lb} (1120.32 - 745.74) \frac{\text{Btu}}{\text{lb}}$$

$$= -1872.9 \text{ Btu}$$

(b) An entropy balance reads,

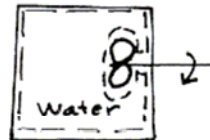
$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma$$

$$\Rightarrow \sigma = \Delta S = m(s_2 - s_1)$$

$$= 5 \text{ lb} (1.6699 - 1.1646) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

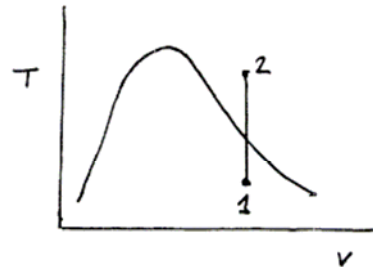
$$= 2.5265 \frac{\text{Btu}}{^\circ\text{R}}$$

SCHEMATIC & GIVEN DATA:



$$m = 5 \text{ lb} \quad T_1 = 260^\circ\text{F}, \quad x_1 = 0.6$$

$$T_2 = 350^\circ\text{F}$$



← W

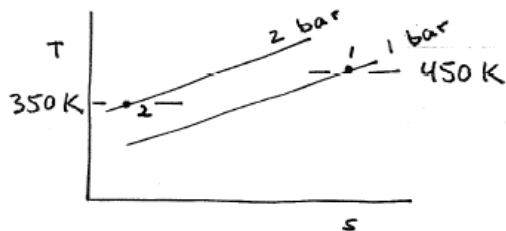
← σ

PROBLEM 6.42

KNOWN: One kg of air undergoes a process between two specified states.

FIND: Determine if the process can occur adiabatically.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system is the one kg of air. 2. Air is modeled as an ideal gas.

ANALYSIS: Whether the process can occur adiabatically can be determined using an entropy balance together with s° data from Table A-22 and Eq. 6.20a. The entropy balance reads

$$\Delta S = \int_1^2 \left(\frac{\dot{S}_Q}{T} \right)_b + \sigma$$

If the process takes place adiabatically, the underlined term vanishes, giving $\Delta S = \sigma$. Since σ cannot be negative, ΔS cannot be negative. Using Eq. 6.20a and s° from Table A-22

$$\Delta S = m \left[s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1} \right] = (1 \text{ kg}) \left[1.85708 - 2.11161 - \frac{8.314}{28.97} \ln \frac{2}{1} \right] \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$= -0.4535 \text{ kJ/K}$$

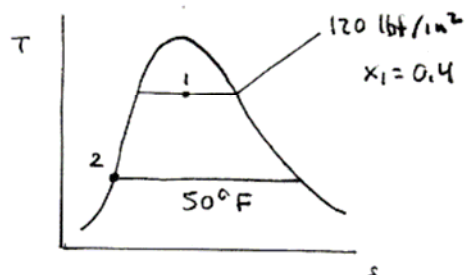
Since ΔS is negative, an adiabatic process between these states cannot occur.

PROBLEM 6.43

KNOWN: One lb of R134a undergoes a process between specified states.

FIND: Determine the entropy change for the process. Can the process occur with $\Phi=0$?

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The system is the one lb of R134a.

ANALYSIS: The entropy change can be determined using data from Tables A-10E, 11E

$$s_1 = s_f + x_1(s_g - s_f) = 0.0839 + 0.4(0.2165 - 0.0839) = 0.13694 \text{ Btu/lb}^\circ\text{R}$$

$$s_2 = 0.0585 \text{ Btu/lb}^\circ\text{R}$$

$$\text{Thus, } \Delta S = (1 \text{ lb})[0.0585 - 0.13694] \text{ Btu/lb}^\circ\text{R} = -0.07844 \text{ Btu/lb}^\circ\text{R} \leftarrow$$

To consider the possibility of an adiabatic process, begin with an entropy balance:

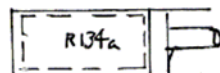
$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma$$

For an adiabatic process, the underlined term vanishes, giving $\Delta S = \sigma$. Since σ cannot be negative, ΔS also cannot be negative. Accordingly, the indicated process cannot occur adiabatically. $\leftarrow$

PROBLEM 6.44

Refrigerant 134a contained in a piston-cylinder assembly rapidly expands from an initial state where $T_1 = 140^\circ\text{F}$, $p_1 = 200 \text{ lbf/in}^2$ to a final state where $p_2 = 5 \text{ lbf/in}^2$ and the quality, x_2 , is (a) 99%, (b) 95%. In each case, determine if the process can occur adiabatically. If yes, determine the work, in Btu/lb, for an adiabatic expansion between these states. If no, determine the direction of the heat transfer.

SCHEMATIC & GIVEN DATA



$$T_1 = 140^\circ\text{F}, p_1 = 200 \text{ lbf/in}^2$$

$$p_2 = 5 \text{ lbf/in}^2$$

$$x_2: (a) 99\% \quad (b) 95\%$$

ENGR. MODEL:

1. The R134a is the closed system.
2. There are no overall changes in KE or PE.

ANALYSIS: An entropy balance reads

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma \Rightarrow m(s_2 - s_1) = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma \quad (1)$$

+ or 0, never -

+ , 0, -

From Table A-12E, $u_1 = 112.87 \text{ Btu/lb}$, $s_1 = 0.2226 \text{ Btu/lb} \cdot ^\circ\text{R}$.

(a) $x_2 = 0.99$. Data from Table A-11E

$$s_2 = s_f + x_2(s_g - s_f) = -0.0090 + 0.99(0.2311 - (-0.0090)) = 0.2287 \text{ Btu/lb} \cdot ^\circ\text{R}$$

So, Eq. (1) reads

$$m \left(\frac{0.2287 - 0.2226}{16.012} \right) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma$$

+0.0061

If the process occurs adiabatically, we get $\sigma = m(0.0061 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}})$, which is positive. Thus, such a process is allowed.

For an adiabatic process between these states, an energy balance reduces to, $\Delta U + \Delta KE + \Delta PE = Q - W$, or

$$W = -m(u_2 - u_1) \Rightarrow \frac{W}{m} = -(u_2 - u_1) = -(85.17 - 112.87) \frac{\text{Btu}}{\text{lb}}$$

$$= +27.7 \text{ Btu/lb}$$

$$\text{where } u_2 = u_f + x_2(u_g - u_f) = -3.74 + 0.99(86.07 - (-3.74)) = 85.17 \text{ Btu/lb}.$$

(b) $x = 0.95$. Data from Table A-11E.

$$s_2 = s_f + x_2(s_g - s_f) = -0.0090 + 0.95(0.2311 - (-0.0090)) = 0.2191 \text{ Btu/lb} \cdot ^\circ\text{R}.$$

So, Eq. (1) reads

$$m \left(\frac{0.2191 - 0.2226}{16.012} \right) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma$$

-0.0035

In this case, if the process occurred adiabatically, $\sigma < 0$.

Such a process is not allowed. Entropy transfer accompanying heat transfer from the system must occur if $\Delta S < 0$.

PROBLEM 6.45

One kg of air contained in a piston-cylinder assembly undergoes a process from an initial state where $T_1 = 300 \text{ K}$, $v_1 = 0.8 \text{ m}^3/\text{kg}$ to a final state where $T_2 = 420 \text{ K}$, $v_2 = 0.2 \text{ m}^3/\text{kg}$. Can this process occur adiabatically? If yes, determine the work, in kJ, for an adiabatic process between these states. If no, determine the direction of the heat transfer. Assume the ideal gas model for air.

SCHEMATIC & GIVEN DATA:



$$m = 1 \text{ kg}$$

ENGR. MODEL:

1. The air is the closed system.
2. The air is modeled as an ideal gas.

ANALYSIS:

An entropy balance reads,

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma \quad (1)$$

where

$$\Delta S = (s_2 - s_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1}$$

For an ideal gas, $P = RT/v$, so $\frac{P_2}{P_1} = \frac{T_2}{T_1} \frac{v_1}{v_2}$

$$\Rightarrow \Delta S = s^\circ(T_2) - s^\circ(T_1) - R \ln \left(\frac{T_2}{T_1} \frac{v_1}{v_2} \right)$$

With data from Table A-22

$$\Delta S = \left(2.04142 - 1.70203 - \frac{8.314}{28.97} \ln \left(\frac{420}{300} \times \frac{0.8}{0.2} \right) \right) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$= -0.155 \text{ kJ/kg} \cdot \text{K}$$

Thus, in Eq. (1), $\Delta S < 0$. Accordingly, since $\sigma \geq 0$, this process cannot occur adiabatically. Entropy transfer accompanying heat transfer from the system must occur if $\Delta S < 0$.

PROBLEM 6.46

Air as an ideal gas contained within a piston-cylinder assembly is compressed between two specified states. In each of the following cases, can the process occur adiabatically? If yes, determine the work in appropriate units for an adiabatic process between these states. If no, determine the direction of the heat transfer.

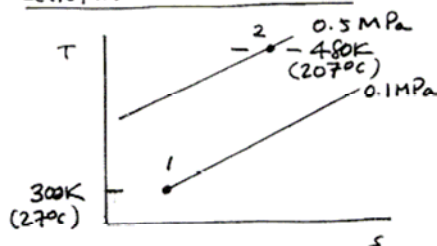
- (a) State 1: $p_1 = 0.1 \text{ MPa}$, $T_1 = 27^\circ\text{C}$. State 2: $p_2 = 0.5 \text{ MPa}$, $T_2 = 207^\circ\text{C}$. Use Table A-22 data.
 (b) State 1: $p_1 = 3 \text{ atm}$, $T_1 = 80^\circ\text{F}$. State 2: $p_2 = 10 \text{ atm}$, $T_2 = 240^\circ\text{F}$. Assume $c_p = 0.241 \text{ Btu/lb}^\circ\text{R}$.

PART (a)

KNOWN: Air is compressed between specified states.

FIND: Determine if the process can occur adiabatically. If so, evaluate the work. If not, determine the direction of the heat transfer.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system is the quantity of air under consideration. 2. Air is modeled as an ideal gas. 3. Ignore kinetic and potential energy.

ANALYSIS: Whether the process can occur adiabatically can be determined using an entropy balance together with Eq. 6.20a and s° data from Table A-22:

The entropy balance reads, $\Delta S = \int \left(\frac{\delta Q}{T} \right)_b + \sigma$. In an adiabatic process, the underlined term vanishes, giving $\Delta S = \sigma$. Since σ cannot be negative, ΔS cannot be negative. Then, with Eq. 6.20a and s° data,

$$\Delta S = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1} = 2.1776 - 1.70203 - \frac{8.314}{28.97} \ln 5 = +0.0137 \text{ kJ/kg} \cdot \text{K}$$

Accordingly, an adiabatic process between these states is allowed.

For any such process, the energy balance reads, $\Delta U = Q - W$, or $W = -m(u_2 - u_1)$, $W/m = u_1 - u_2 = 214.07 - 344.7 = -130.6 \text{ kJ/kg}$. $\leftarrow$

Part(b).... Continued...

Continued on next slide

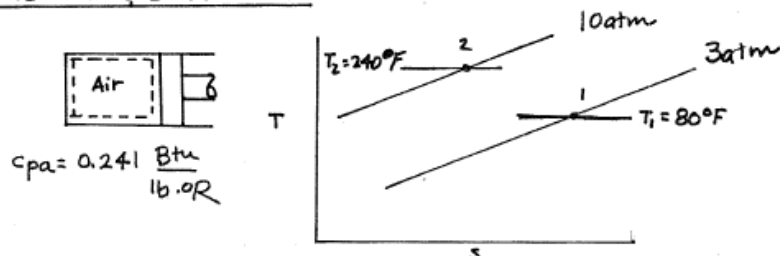
Problem 6-46 continued

Part (b)

KNOWN: Air is compressed in a process between two specified states.

FIND: Determine if the process can occur adiabatically. If adiabatic, evaluate the work. If not adiabatic, determine the direction of heat transfer.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) As shown in the accompanying figure, the system is the air.

(2) Air is modeled as an ideal gas with $c_p = 0.241 \text{ Btu/lb} \cdot ^\circ\text{R}$.

ANALYSIS: Whether the process can occur adiabatically can be determined using an entropy balance together with Eq. 6.22 which gives

$$\Delta S = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 0.241 \ln \frac{700}{540} - \frac{1.986}{28.97} \ln \frac{10}{3} = -0.02 \text{ Btu/lb} \cdot ^\circ\text{R}$$

An entropy balance then reads

$$m[-0.02] = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma$$

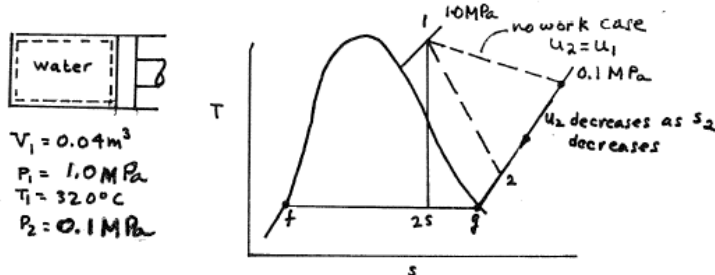
Since $\sigma \geq 0$ for all processes, the entropy transfer term must be negative which implies that energy is removed by heat transfer during the process. $\longleftarrow$

PROBLEM 6.47

KNOWN: A quantity of water expands adiabatically from a specified state to a specified pressure.

FIND: Plot the work done versus the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) As shown in the accompanying figure, the system is the water. (2) The expansion occurs adiabatically. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: With assumptions 2 and 3 an energy balance reduces to

$$W = Q - \Delta U = m(u_1 - u_2)$$

With data from Table A-4, $u_1 = 2826.1 \text{ kJ/kg}$, $v_1 = 0.2678 \text{ m}^3/\text{kg}$, $s_1 = 7.1962 \text{ kJ/kg}\cdot\text{K}$.

So,

$$W = \left(\frac{0.04 \text{ m}^3}{0.2678 \text{ m}^3/\text{kg}} \right) (2826.1 - u_2) \frac{\text{kJ}}{\text{kg}} = 0.1494 (2826.1 - u_2) \text{ kJ} \quad (1)$$

An entropy balance reduces to read, $m(s_2 - s_1) = \sigma$. Thus

$$\sigma = (0.1494)(s_2 - 7.1962) \frac{\text{kJ}}{\text{K}} \quad (2)$$

The T-s diagram shows that u_2 decreases as s_2 decreases. Thus W is a maximum and $\sigma = 0$, when $s_2 = s_1$.

Case $s_2 = s_1$: The specific internal energy at this state is the smallest allowed value: u_{2s} . To determine u_{2s} , first evaluate the quality with data from Tables A-3 and A-4

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{s_1 - s_f}{s_g - s_f} = \frac{7.1962 - 1.3026}{7.3594 - 1.3026} = 0.9731$$

$$\Rightarrow u_{2s} = u_f + x_{2s}(u_g - u_f) = 417.36 + 0.9731(2506.1 - 417.36) = 2449.9 \text{ kJ/kg}$$

So

$$W_{\text{MAX}} = 0.1494(2826.1 - 2449.9) = 56.20 \text{ kJ} \quad \leftarrow W_{\text{MAX}}$$

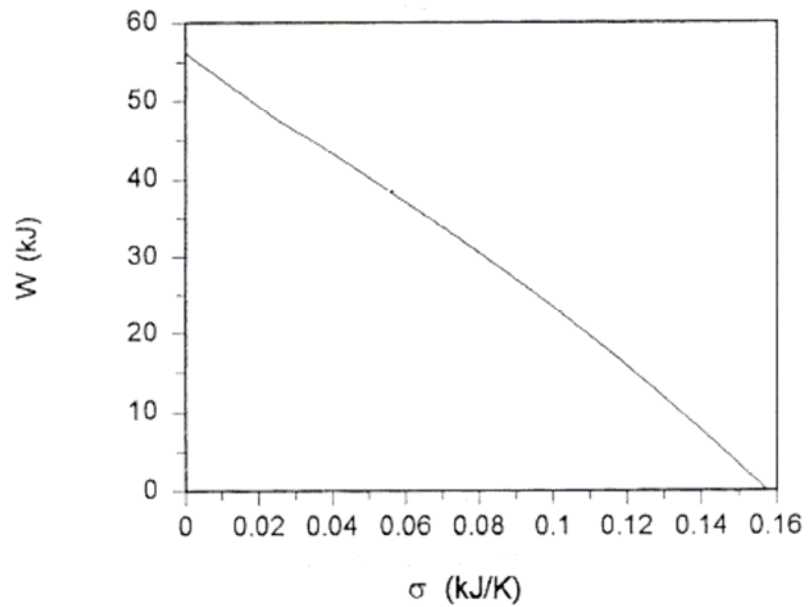
Case $u_2 = u_1$: In the limit as $u_2 \rightarrow u_1$, $W \rightarrow 0$, and the entropy production approaches a maximum. Interpolating in Table A-4 at 0.1 MPa with $u_2 = u_1$ gives $s_2 = 8.25 \text{ kJ/kg}\cdot\text{K}$. Then

$$\sigma_{\text{MAX}} = (0.1494)(8.25 - 7.1962) = 0.1574 \frac{\text{kJ}}{\text{K}} \quad \leftarrow \sigma_{\text{MAX}}$$

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Problem 6-47 continued

PLOT: W vs. σ



PROBLEM 6.48

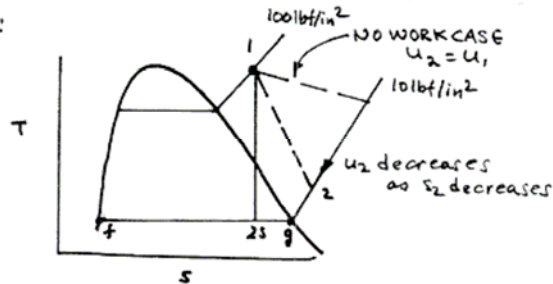
KNOWN: Two lb of steam expand adiabatically from a specified state to a specified pressure.

FIND: Plot the work done versus the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} P_1 &= 100 \text{ lbf/in}^2 \\ T_1 &= 500^\circ\text{F} \\ P_2 &= 10 \text{ lbf/in}^2 \end{aligned}$$



ENGR. MODEL: (1) As shown in the accompanying figure, the system is the water. (2) The expansion occurs adiabatically. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: (a) The work can be found from an energy balance using assumptions 2 and 3

$$W = \cancel{Q} - \Delta U = m(u_1 - u_2)$$

From Table A-4E at 100 lbf/in², 500°F, $u_1 = 1175.7 \text{ Btu/lb}$, $s_1 = 1.7085 \text{ Btu/lb} \cdot ^\circ\text{R}$. Thus

$$W = (2 \text{ lb}) (1175.7 - u_2) \frac{\text{Btu}}{\text{lb}} \quad (1)$$

An entropy balance reduces to read $m(s_2 - s_1) = \sigma$. Thus

$$\sigma = (2 \text{ lb}) (s_2 - 1.7085) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \quad (2)$$

The T-s diagram shows that u_2 decreases as s_2 decreases. Thus W is a maximum and $\sigma = 0$ when $s_2 = s_1$.

Continued on next slide

Problem 6-48 continued

Case $s_2 = s_1$. To determine u_{2s} , first evaluate the quality x_{2s} with data from Tables A-3E and A-4E

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{s_1 - s_f}{s_g - s_f} = \frac{1.7085 - 0.2836}{1.5041} = 0.9473$$

Then

$$u_{2s} = u_f + x_{2s}(u_g - u_f) = 161.2 + 0.9473(1072.2 - 161.2) = 1024.19 \text{ Btu/lb}$$

Finally

$$W_{\max} = 2(1175.7 - 1024.19) = 303.02 \text{ Btu} \quad \leftarrow W_{\max}$$

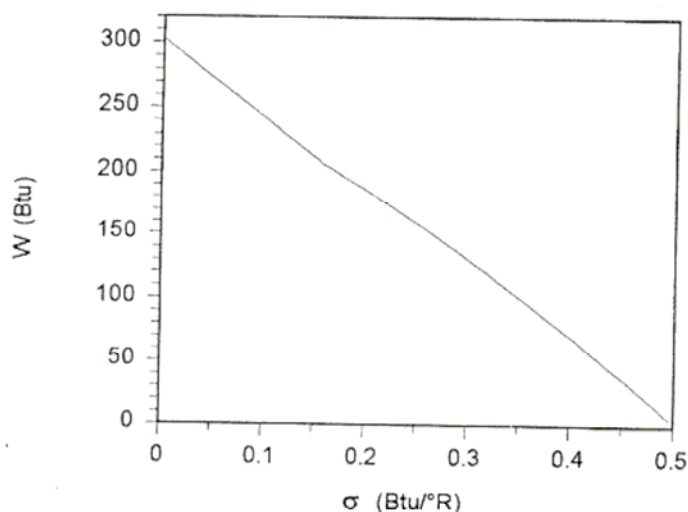
Case $u_2 = u_1$. In the limit as $u_2 \rightarrow u_1$, $W \rightarrow 0$, and the entropy production approaches a maximum.

Interpolation in Table A-4E at 1016f/in² with $u_2 = u_1$, gives

$s_2 = 1.9597 \text{ Btu/lb} \cdot ^\circ\text{R}$. Then

$$T_{\max} = 2(1.9597 - 1.7085) = 0.5024 \text{ Btu/}^\circ\text{R} \quad \leftarrow T_{\max}$$

PLOT: W vs σ :



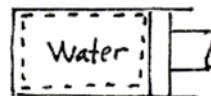
PROBLEM 6.49

One-tenth kilogram of water, initially at 300 kPa, 200°C, is compressed in a piston-cylinder assembly to 1500 kPa, 210°C. If heat transfer from the water occurs at an average temperature of 205°C, determine the minimum theoretical work of compression, in kJ.

ENGR. MODEL:

1. The water is the closed system.
2. Heat transfer occurs at $T_b = 205^\circ\text{C}$.
3. There are no overall changes in KE or PE.

SCHEMATIC & GIVEN DATA:



$$m = 0.1 \text{ kg}$$

$$P_1 = 300 \text{ kPa}, T_1 = 200^\circ\text{C}$$

$$P_2 = 1500 \text{ kPa}, T_2 = 210^\circ\text{C}$$

ANALYSIS: An energy balance reads, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow W = Q - m(u_2 - u_1) \quad (1)$$

An entropy balance reads, $\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma$. With assumption 2,

$$\Delta S = \frac{Q}{T_b} + \sigma \Rightarrow Q = T_b \Delta S - T_0 \sigma \quad (2)$$

Combining Eqs. (1), (2)

$$\begin{aligned} W &= T_b m(s_2 - s_1) - m(u_2 - u_1) - T_0 \sigma \\ &= m [T_b(s_2 - s_1) - (u_2 - u_1)] - T_0 \sigma \end{aligned}$$

With data from Table A-4 and $T_b = 205 + 273.15 = 478.15 \text{ K}$

$$\begin{aligned} W &= (0.1 \text{ kg}) \left[478.15 \text{ K} (6.5067 - 7.3115) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - (2617.8 - 2650.7) \frac{\text{kJ}}{\text{kg}} \right] - T_0 \sigma \\ &= -35.19 \text{ kJ} - T_0 \sigma \end{aligned}$$

Since $\sigma \geq 0$,

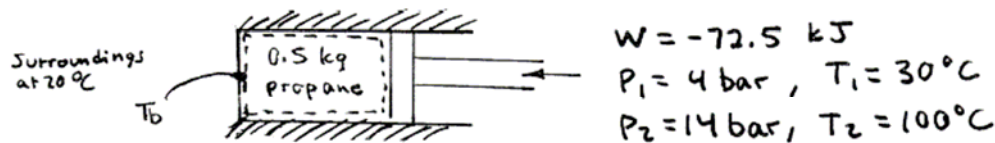
$$W_{\min} = -35.19 \text{ kJ}$$

PROBLEM 6.50

KNOWN: Data is provided for a quantity of propane rapidly compressed.

FIND: Determine whether the reported value for the work is feasible.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The propane is the closed system. 2. Heat transfer with the surroundings occurs at T_b , which cannot be less than 293 K (20°C) and is most likely in the interval from 303 K (30°C) to 373 K (100°C). 3. Kinetic and potential energy effects can be ignored.

ANALYSIS: An energy balance reduces to $\Delta U = Q - W$, or with data from Table A-15

$$Q = m(u_2 - u_1) + W = (0.5 \text{ kg})[1480.79 - 1377.49] \frac{\text{kJ}}{\text{kg}} + (-72.5 \text{ kJ}) = -20.85 \text{ kJ}$$

An entropy balance takes the form,

$$\Delta S = \frac{Q}{T_b} + \sigma \Rightarrow \sigma = m[s_2 - s_1] - \frac{Q}{T_b}$$

With Eq. 6.20a and data from Table A-15

$$\sigma = (0.5 \text{ kg})[(5.4433 - 5.6328) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}] - \frac{(-20.85 \text{ kJ})}{T_b}$$

$$\sigma = -0.09475 \frac{\text{kJ}}{\text{kg}} + \frac{20.85 \text{ kJ}}{T_b}$$

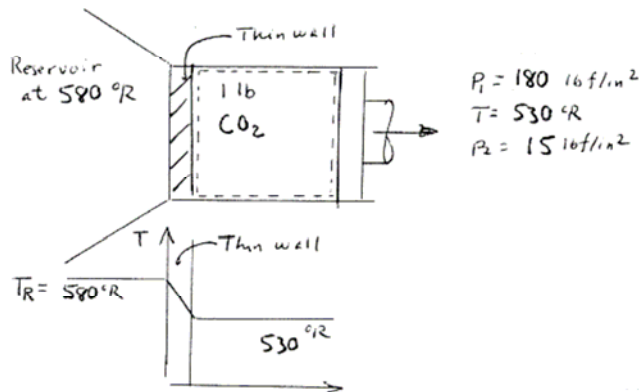
For $T_b \geq 293 \text{ K}$, $\sigma < 0$. Thus, the value for W cannot be correct.

PROBLEM 6.51

KNOWN: One lb of CO_2 expands isothermally between specified states while receiving energy through a thin intervening wall from a reservoir.

FIND: (a) For the CO_2 as the system, evaluate W, Q, σ . (b) For the CO_2 plus the thin wall as the system, evaluate σ and compare with the result of part (a).

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) In part (a) the system is the CO_2 only. In part (b) the system is the CO_2 plus the thin wall. (2) CO_2 is modeled as an ideal gas. (3) There is no change in kinetic or potential energy. (4) The state of the thin wall does not change.

ANALYSIS: With assumptions (2), (3) an energy balance reduces to $\Delta U = Q - W$, where $\Delta U = 0$ because $u(T)$ for an ideal gas. Thus $Q = W$. To find W , use the ideal gas equation of state to write

$$W = m \int_1^2 p dv = m \int_1^2 \frac{RT}{v} dv = mRT \ln \frac{v_2}{v_1} = mRT \ln \frac{P_1}{P_2}$$

$$= (1 \text{ lb}) \left(\frac{1.986}{44.01} \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (530 ^\circ\text{R}) \ln \left(\frac{180}{15} \right) = 59.43 \text{ Btu}$$

W, Q

An entropy balance reads

$$\Delta S = \frac{Q}{T} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T}$$

From Eq. 6.20a, $\Delta S = -mR \ln P_2/P_1$. Also, from above $Q = mRT \ln P_1/P_2$. Thus

$$\sigma = -mR \ln \frac{P_2}{P_1} - \frac{mRT \ln \frac{P_1}{P_2}}{T} \equiv 0 \quad (\text{the process is internally reversible})$$

(b) For the enlarged system of O_2 plus thin wall an entropy balance reads

$$\Delta S = \frac{Q}{T_R} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T_R}$$

where by assumption (4) ΔS is the same as in part (a): $-mR \ln \frac{P_2}{P_1}$. Introducing $Q = mRT \ln \frac{P_1}{P_2}$ and simplifying

$$\sigma = -mR \ln \frac{P_2}{P_1} - \frac{mRT \ln \frac{P_1}{P_2}}{T_R} = mR \ln \frac{P_1}{P_2} \left[1 - \frac{T}{T_R} \right]$$

$$= (1 \text{ lb}) \left(\frac{1.986}{44.01} \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) \ln \left(\frac{180}{15} \right) \left[1 - \frac{530}{580} \right] = 0.009667 \text{ Btu}/^\circ\text{R}$$

σ

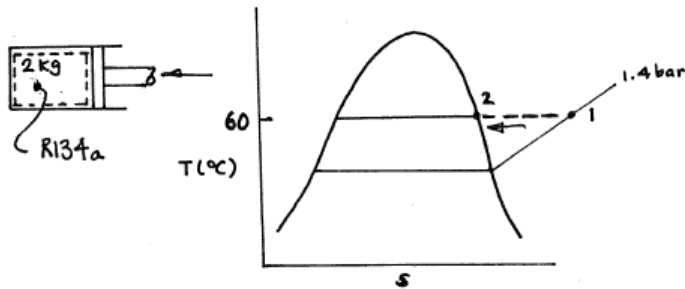
Discussion: The enlarged system includes an irreversibility not present in the CO_2 alone: heat transfer through a finite temperature difference, and so a nonzero value of σ is determined for the enlarged system.

PROBLEM 6.52

KNOWN: Two kg of R12 is compressed between two specified states, during which the temperature of the R12 is $60^\circ\text{C} \pm 0.1^\circ\text{C}$.

FIND: Determine the minimum theoretical heat transfer from the refrigerant.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The R12 is the closed system. (2) The temperature at which heat transfer occurs is $T_b = 60^\circ\text{C}$.

ANALYSIS: An entropy balance for the process reads

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma = \frac{Q}{T_b} + \sigma$$

Thus

$$Q = T_b \Delta S - T_b \sigma$$

With data from Tables A-10 and A-12, $s_1 = 1.1690 \text{ kJ/kg} \cdot \text{K}$, $s_2 = 0.8973 \text{ kJ/kg} \cdot \text{K}$.

Then

$$\begin{aligned} Q &= (333\text{K})(2\text{kg})(0.8973 - 1.1690) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - T_b \sigma \\ &= -180.95 \text{ kJ} - \underline{T_b \sigma} \end{aligned}$$

As $\sigma \geq 0$, the underlined term is positive (or zero). Thus, $Q_{\min} = -180.95 \text{ kJ}$

PROBLEM 6.53

An inventor claims that the electricity-generating unit shown in Fig. P6.53 receives a heat transfer at the rate of 250 Btu/s at a temperature of 500°R, a second heat transfer at the rate of 350 Btu/s at 700°R, and a third at the rate of 500 Btu/s at 1000°R. For operation at steady state, evaluate this claim.

SCHEMATIC & GIVEN DATA:

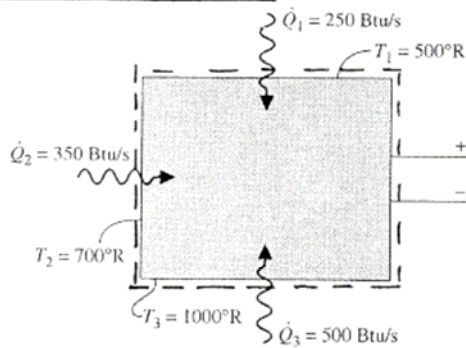


Fig. P6.53

ENGR. MODEL:

1. The system is shown by the dashed line on the sketch.
2. The system is at steady state.
3. Heat transfer rates are each positive in the direction of the accompanying arrow.

ANALYSIS: Applying Eq. 6.28,

$$\begin{aligned}
 \frac{dS}{dt} &= \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} + \frac{\dot{Q}_3}{T_3} + \dot{\sigma} \\
 \Rightarrow \dot{\sigma} &= - \left[\frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} + \frac{\dot{Q}_3}{T_3} \right] \\
 &= - \left[\frac{250 \text{ Btu/s}}{500^\circ\text{R}} + \frac{350 \text{ Btu/s}}{700^\circ\text{R}} + \frac{500 \text{ Btu/s}}{1000^\circ\text{R}} \right] \\
 &= - [0.5 + 0.5 + 0.5] \frac{\text{Btu/s}}{^\circ\text{R}} \\
 &= -1.5 \frac{\text{Btu/s}}{^\circ\text{R}}
 \end{aligned}$$

Since $\dot{\sigma} \geq 0$, the claim cannot be valid.

PROBLEM 6.54

KNOWN: Data are provided for a silicon chip at steady state.

FIND: Determine the rate of entropy production and identify the principal source of irreversibility.

SCHEMATIC & GIVEN DATA: See Fig. E 2.5.

ENGR. MODEL: 1. The chip is a closed system at steady state. 2. There is no heat transfer between the chip and the substrate. All heat transfer is by convection from the top surface at T_b .

ANALYSIS: An entropy rate balance at steady state reads

$$\frac{dS^0}{dt} = \frac{\dot{Q}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b} \quad \text{where } \dot{Q} = -0.225 \text{ W (from Example 2.5).}$$

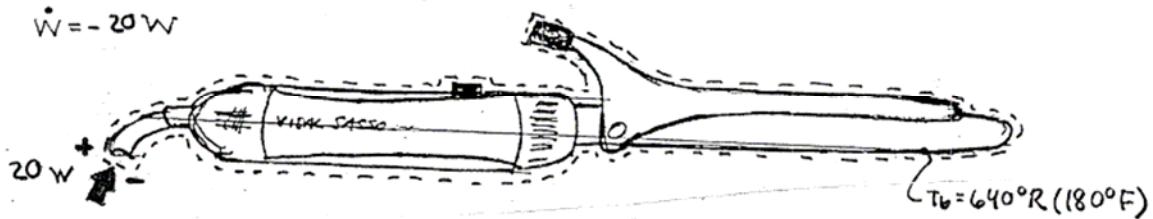
Thus

$$\dot{\sigma} = \frac{-(-0.225)}{353 \text{ K}} = 6.37 \times 10^{-4} \frac{\text{Watt}}{\text{K}} \left| \frac{1 \text{ kW}}{10^3 \text{ W}} \right| = 6.37 \times 10^{-7} \frac{\text{KW}}{\text{K}} \quad \longleftarrow$$

The principal source of internal irreversibility is electric current flow through a resistance.

PROBLEM 6.55

KNOWN: At steady state the outer surface of a 20-W curling iron is at 180°F
FIND: Determine the rate of heat transfer and the rate of entropy production.
SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the figure is at steady state.
 (2) Heat transfer occurs at $T_b = 640^\circ\text{R} (180^\circ\text{F})$ only.

ANALYSIS: An energy rate balance reduces to read

$$\frac{dE}{dt} \overset{0}{=} \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W} = (-20\text{ W}) \left(3.413 \frac{\text{Btu}}{\text{W}\cdot\text{h}} \right) = -68.3 \text{ Btu/h} \leftarrow \dot{Q}$$

An entropy rate balance reduces to read

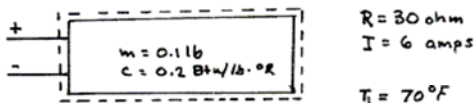
$$\frac{dS}{dt} \overset{0}{=} \frac{\dot{Q}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b} = \frac{68.3 \text{ Btu/h}}{640^\circ\text{R}} = 0.107 \frac{\text{Btu}}{\text{h}\cdot^\circ\text{R}} \leftarrow \dot{\sigma}$$

PROBLEM 6.56

KNOWN: Data we provided for a thermally insulated resistor.

FIND: Plot the temperature and amount of entropy produced versus t for $0 \leq t \leq 3$ s.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the accompanying figure is modeled as an incompressible substance. (2) $Q = 0$.

ANALYSIS: An energy balance reduces to give

$$\Delta U = \cancel{Q} - W \Rightarrow m(u_2 - u_1) = -W$$

With assumption 1

$$mc(T_2 - T_1) = -W \Rightarrow T_2 = T_1 - W/mc$$

Electrical work is done on the system. Using given data

$$W = -I^2 R \Delta t = -(6 \text{ amps})^2 (30 \text{ ohm}) (\Delta t) \left(\frac{W}{(\text{amps})^2 \cdot \text{ohm}} \right)$$

$$= -1080 \Delta t \text{ (W} \cdot \text{s)}$$

Accordingly

$$T_2 = 530^\circ\text{R} - \frac{(-1080 \Delta t \text{ (W} \cdot \text{s)})}{(0.116)(0.2 \text{ Btu/lb} \cdot ^\circ\text{R})} \left(\frac{3.413 \text{ Btu/h}}{1 \text{ W}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right)$$

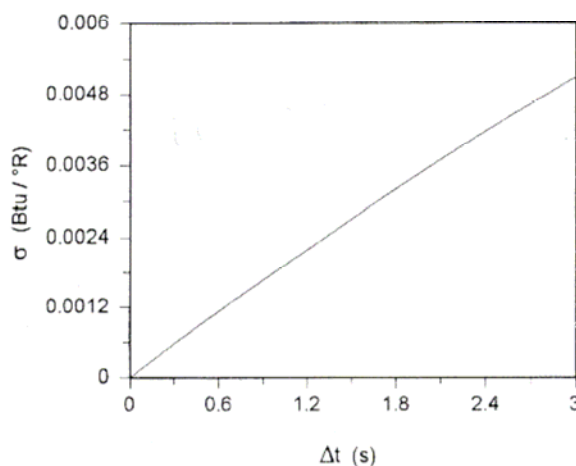
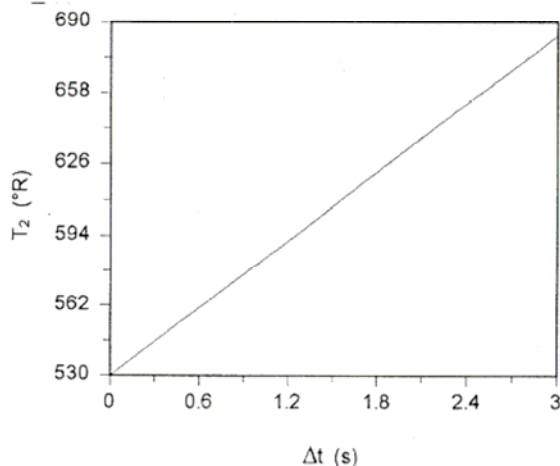
$$= (530 + 51.2 \Delta t) ^\circ\text{R} \quad (1)$$

An entropy balance reduces to give

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma \Rightarrow \sigma = \Delta S = mc \ln \frac{T_2}{T_1} \quad (\text{Eq. 6.13})$$

Introducing Eq.(1) the expression for σ becomes

$$\sigma = (0.116)(0.2 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}) \ln \left(1 + \frac{51.2 \Delta t}{530} \right) = 0.02 \frac{\text{Btu}}{^\circ\text{R}} \ln \left(1 + \frac{51.2 \Delta t}{530} \right) \quad (2)$$

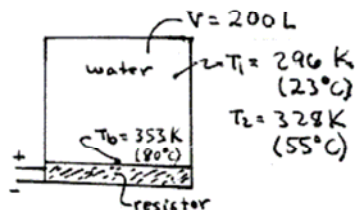


PROBLEM 6.57

KNOWN: Data are provided for an electric water heater.

FIND: For each of two systems determine the amount of entropy produced. Compare.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. In part (a) the system is the water alone. In part (b) the system is the water plus the resistor. 2. The water is modeled as incompressible. 3. Heat transfer from the outer surface of the water heater can be ignored. 4. The states of the tank wall and the resistor are constant.

(a) For the water as the system, the entropy balance reduces to $\Delta S = \frac{Q}{T_b} + \sigma$. The heat transfer to the water from the resistor is found from an energy balance on the water: $\Delta U = Q - W$, which with Eq. 3.20a takes the form

$$Q = \Delta U \Rightarrow Q = m[c\Delta T]. \text{ With Eq. 6.13 } \Delta S = mc \ln T_2/T_1. \text{ Collecting results}$$

$$\sigma = \Delta S - \frac{Q}{T_b} \Rightarrow \sigma = mc \left[\ln \frac{T_2}{T_1} - \frac{(T_2 - T_1)}{T_b} \right]$$

Using the known volume and $v = v_f(T_{ave}) = 1.008 \times 10^{-3} \text{ m}^3/\text{kg}$ from Table A-2 and $c = 4.18 \text{ kJ/kg}\cdot\text{K}$ from Table A-19

$$m = \frac{V}{v} = \frac{(200 \text{ L}) (10^{-3} \text{ m}^3/\text{L})}{1.008 \times 10^{-3} \text{ m}^3/\text{kg}} = 198.4 \text{ kg}, \quad \sigma = (198.4 \text{ kg})(4.18 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}) \left[\ln \frac{328}{296} - \frac{(328 - 296)}{353} \right] = 9.95 \frac{\text{kJ}}{\text{K}} \quad (a)$$

(b) For the water plus the resistor as the system, the entropy balance reduces to $\Delta S = \frac{Q}{T_b} + \sigma \Rightarrow \sigma = \Delta S$. With assumption 4, ΔS can be identified with the water only: $\Delta S = mc \ln \frac{T_2}{T_1}$. Thus

$$\sigma = mc \ln \frac{T_2}{T_1} = (198.4 \text{ kg})(4.18 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}) \ln \frac{328}{296} = 85.13 \frac{\text{kJ}}{\text{K}} \quad (b)$$

The entropy production in the enlarged system is greater because there is an additional source of irreversibility — namely, electric current flow through the resistor. For the water alone, the irreversibility is associated with internal heat transfer from the resistor through the water as it is heated.

PROBLEM 6.58

See solution to Problem 2.59. For a closed system consisting of the motor, $\dot{Q} = -0.03 \text{ kW}$.

ANALYSIS: An entropy rate balance reads, $\frac{dS^0}{dt} = \frac{\dot{Q}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b}$

(a) $T_b = 315 \text{ K} (42^\circ\text{C})$:

$$\dot{\sigma} = \frac{-(0.03 \text{ kW})}{315 \text{ K}} = 9.5 \times 10^{-5} \frac{\text{kW}}{\text{K}}$$

(b) $T_b = 295 \text{ K} (22^\circ\text{C})$:

$$\dot{\sigma} = \frac{-(0.03 \text{ kW})}{295 \text{ K}} = 10.2 \times 10^{-5} \frac{\text{kW}}{\text{K}}$$

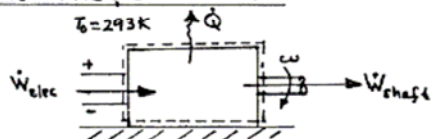
COMMENT: The rate of entropy production is greater for the enlarged system because it includes an additional source of irreversibility associated with heat transfer from the motor to the surroundings.

PROBLEM 6.59

KNOWN: Operating data are provided for an electric motor at steady state.

FIND: (a) Determine the motor outer surface temperature. Evaluate the rate of entropy production for (b) the motor as the system, (c) the motor plus the nearby surroundings as the system.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned}\dot{Q} &= hA(T_b - T_o) \\ h &= 100 \text{ W/m}^2 \cdot \text{K} \\ A &= 0.195 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\omega &= 1000 \text{ RPM} \\ \mathcal{T} &= 16 \text{ N}\cdot\text{m} \\ i &= 10 \text{ amp} \\ \mathcal{E} &= 220 \text{ Volts}\end{aligned}$$

ENGR. MODEL: (1) Two systems at steady state are considered: The motor. The motor plus the nearby surroundings. (2) The temperature at the outer surface of the motor, T_b , is uniform with position. (3) Energy transfers are in the directions of the arrows.

ANALYSIS: (a) The temperature T_b can be evaluated from an energy rate balance applied to the motor. At steady-state, the rate energy enters equals the rate energy exits. Thus, for energy transfer in the directions indicated by the arrows

$$\dot{W}_{\text{elec}} = \dot{W}_{\text{shaft}} + \dot{Q}$$

where $\dot{Q}$ is given by the expression above and the other two terms are evaluated using Eq. 2.20 and 2.21:

$$\dot{W}_{\text{shaft}} = \mathcal{T}\omega = (16 \text{ N}\cdot\text{m})(1000 \frac{\text{rev}}{\text{min}})(\frac{2\pi \text{ rad}}{\text{rev}})(\frac{\text{min}}{60 \text{ s}})(\frac{1 \text{ kW}}{10^3 \text{ N}\cdot\text{m}}) = 1.676 \text{ kW}$$

$$\dot{W}_{\text{elec}} = \mathcal{E}i = (220 \text{ volts})(10 \text{ amp})[\frac{1 \text{ watt/amp}}{1 \text{ volt}}](\frac{\text{kW}}{10^3 \text{ W}}) = 2.2 \text{ kW}$$

Thus

$$2.2 \text{ kW} = 1.676 \text{ kW} + (100 \frac{\text{W}}{\text{m}^2 \cdot \text{K}})(0.195 \text{ m}^2)(\frac{\text{kW}}{10^3}) (T_b - 293 \text{ K}) \Rightarrow T_b = 319.9 \text{ K} \leftarrow T_b$$

(b) With assumption 2, and taking in account that $\dot{Q}$ is positive in the direction of the arrow on the sketch above, an entropy rate balance for the motor as the system reduces at steady state to give

$$\frac{dS}{dt} = \frac{(-\dot{Q})}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = \frac{\dot{Q}}{T_b}$$

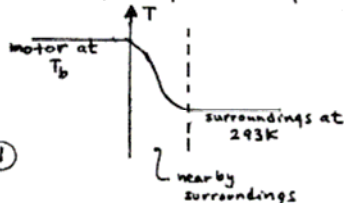
From part (a), $\dot{Q} = \dot{W}_{\text{elec}} - \dot{W}_{\text{shaft}} = 0.524 \text{ kW}$, so

$$\dot{\sigma} = \frac{0.524 \text{ kW}}{319.9 \text{ K}} = 1.64 \times 10^{-3} \frac{\text{kW}}{\text{K}} \leftarrow \dot{\sigma}$$

(c) For a system consisting of the motor plus the nearby surroundings, the heat transfer takes place at $T_o = 293 \text{ K}$. At steady state an entropy rate balance reduces to give for this enlarged system

$$\begin{aligned}\dot{\sigma} &= \frac{\dot{Q}}{T_o} \\ &= \frac{0.524 \text{ kW}}{293 \text{ K}} = 1.79 \times 10^{-3} \frac{\text{kW}}{\text{K}} \leftarrow \dot{\sigma}\end{aligned}$$

①



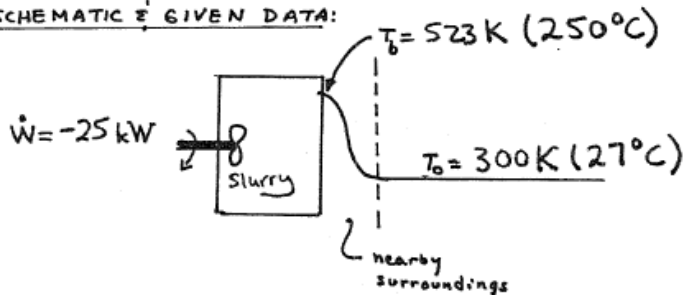
- The rate of entropy production is greater for the enlarged system because it includes an additional source of irreversibility associated with heat transfer from the motor at T_b to the surroundings at T_o .

PROBLEM 6.60

KNOWN: Operating data are provided for a closed tank fitted with a paddle wheel and containing a slurry.

FIND: Determine the rate of entropy production for systems consisting of (a) the tank and its contents, (b) the tank and its nearby surroundings.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) Two systems are under consideration. One comprises the tank and the slurry within the tank. The other includes the tank, the slurry, and as shown in the sketch the nearby surroundings. (2) Each system operates at steady state, while receiving a power input from a paddle wheel.

ANALYSIS: An energy rate balance reduces at steady state to give

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W} = -25 \text{ kW}$$

(a) Tank and slurry as the system. An entropy rate balance reduces at steady state to give

$$\frac{dS}{dt} = \frac{\dot{Q}}{T_b} + \dot{\sigma}$$

$$\Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b} = -\frac{(-25 \text{ kW})}{523 \text{ K}} = 0.0478 \text{ kW/K}$$

(b) Tank, slurry, and nearby surroundings as the system. An entropy rate balance reduces at steady state to give

$$\textcircled{1} \quad \frac{dS}{dt} = \frac{\dot{Q}}{T_o} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{(-25 \text{ kW})}{300 \text{ K}} = 0.0833 \frac{\text{kW}}{\text{K}}$$

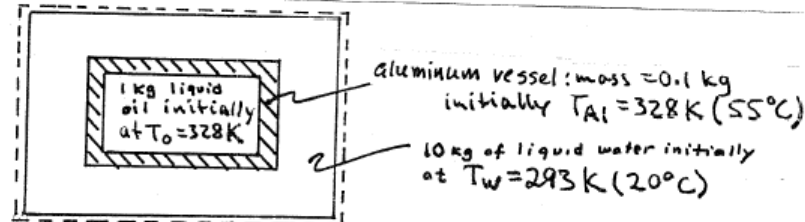
1. The entropy production rate is greater in part (b) than in part (a) because the enlarged system of part (b) has an additional source of irreversibility: the irreversibility associated with heat transfer from the tank to the surroundings.

PROBLEM 6.61

KNOWN: Data are provided on an isolated system consisting of an aluminum vessel, oil, and liquid water.

FIND: Determine (a) the final temperature when the system has come to equilibrium, (b) the entropy change for the aluminum vessel, the oil, and the water, (c) the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the accompanying figure is isolated. (2) The aluminum vessel and each of the liquid masses can be modeled as incompressible with constant specific heat.

ANALYSIS: From Table A-19, the specific heat of aluminum is $c_{Al} = 0.9 \text{ kJ/kg}\cdot\text{K}$ and of liquid water $c_w = 4.18 \text{ kJ/kg}\cdot\text{K}$, and of engine oil $c_o = 1.91 \text{ kJ/kg}\cdot\text{K}$.

(a) With assumption 1, $Q = W = 0$ and an energy balance reduces to

$$\Delta U = 0$$

$$(\Delta U)_w + (\Delta U)_o + (\Delta U)_{Al} = 0$$

With assumption 2

$$m_w c_w [T_f - T_w] + m_o c_o [T_f - T_o] + m_{Al} c_{Al} [T_f - T_{Al}] = 0$$

where m_w and T_w are the mass and initial temperature of liquid water, m_o and T_o are the mass and initial temperature of the oil, and m_{Al} and T_{Al} are the mass and initial temperature of the vessel.

Solving for the final temperature, T_f

$$T_f = \frac{m_w c_w T_w + m_o c_o T_o + m_{Al} c_{Al} T_{Al}}{m_w c_w + m_o c_o + m_{Al} c_{Al}} = \frac{(10)(4.18)(293) + (1)(1.91)(328) + (0.1)(0.9)(328)}{(10)(4.18) + (1)(1.91) + (0.1)(0.9)} = 294.6 \text{ K} (21.6^\circ\text{C}) \leftarrow T_f$$

(b) Entropy changes. Using Eq. 6.13 (assumption 2),

$$(\Delta S)_w = m_w c_w \ln \frac{T_f}{T_w} = (10)(4.18) \ln \frac{294.6}{293} = 0.22764 \text{ kJ/K} \leftarrow \Delta S$$

$$(\Delta S)_o = m_o c_o \ln \frac{T_f}{T_o} = (1)(1.91) \ln \frac{294.6}{328} = -0.20512 \text{ kJ/K}$$

$$(\Delta S)_{Al} = m_{Al} c_{Al} \ln \frac{T_f}{T_{Al}} = (0.1)(0.9) \ln \frac{294.6}{328} = -0.00967 \text{ kJ/K}$$

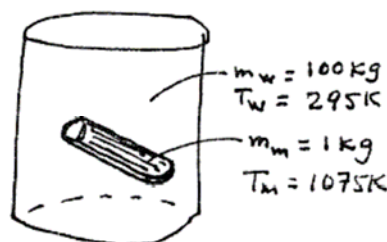
(c) Since there is no heat transfer to or from the system an entropy balance gives

$$\Delta S = \int_1^2 \frac{\delta Q}{T_b} + \sigma \Rightarrow \sigma = (\Delta S)_w + (\Delta S)_o + (\Delta S)_{Al} = 0.01285 \frac{\text{kJ}}{\text{K}} \leftarrow \sigma$$

PROBLEM 6.62

In a heat-treating process, a 1-kg metal part, initially at 1075 K, is quenched in a tank containing 100 kg of water, initially at 295 K. There is negligible heat transfer between the contents of the tank and their surroundings. Taking the specific heat of the metal part and water as constant at 0.5 kJ/kg · K and 4.2 kJ/kg · K, respectively, determine (a) the final equilibrium temperature after quenching, in K, and (b) the amount of entropy produced within the tank, in kJ/K.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The closed system is the water plus metal part.
2. For the system, $Q=0$, $W=0$. Kinetic and potential energy play no role.
3. The water and metal part are each modeled as incompressible with specific heats $c_m = 0.5 \text{ kJ/kg} \cdot \text{K}$, $c_w = 4.2 \text{ kJ/kg} \cdot \text{K}$, respectively.

ANALYSIS: (a) The energy balance reduces as follows: $\Delta U + \Delta K + \Delta P = 0 - W$, or $\Delta U_{\text{water}} + \Delta U_{\text{metal}} = 0$. Using Eq. 3.20a we get

$$m_m c_m [T_f - T_m] + m_w c_w [T_f - T_w] = 0$$

Solving for T_f ,

$$\begin{aligned}
 T_f &= \frac{m_m c_m T_m + m_w c_w T_w}{m_m c_m + m_w c_w} \\
 &= \frac{(1 \text{ kg})(0.5 \text{ kJ/kg} \cdot \text{K})(1075 \text{ K}) + (100 \text{ kg})(4.2 \text{ kJ/kg} \cdot \text{K})(295 \text{ K})}{(1 \text{ kg})(0.5 \text{ kJ/kg} \cdot \text{K}) + (100 \text{ kg})(4.2 \text{ kJ/kg} \cdot \text{K})} \\
 &= 295.9 \text{ K}
 \end{aligned}$$

(b) An entropy balance reduces as follows: $\Delta S = \int \frac{\delta Q}{T} + \sigma$. Or $\sigma = (\Delta S)_m + (\Delta S)_w$. With Eq. 6.13,

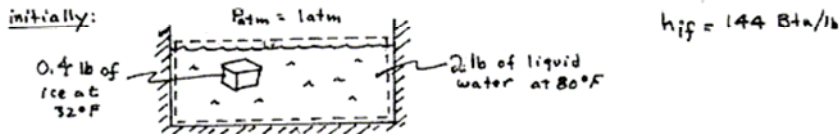
$$\begin{aligned}
 \sigma &= m_w c_w \ln \frac{T_f}{T_w} + m_m c_m \ln \frac{T_f}{T_m} \\
 &= (100 \text{ kg})(4.2 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \ln \left(\frac{295.9}{295} \right) + (1 \text{ kg})(0.5 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \ln \left(\frac{295.9}{1075} \right) \\
 &= 0.615 \frac{\text{kJ}}{\text{K}}
 \end{aligned}$$

PROBLEM 6.63

KNOWN: A system initially consisting of 0.4 lb of ice at 32°F and 2 lb of liquid water at 80°F attains an equilibrium state adiabatically at a constant pressure of 1 atm.

FIND: Determine the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGINEER MODEL: (1) As shown in the accompanying figure, the system consists of the ice and liquid. (2) $Q = 0$ and pressure remains constant. (3) The liquid is incompressible with constant specific heat.

ANALYSIS: (a) The final temperature can be determined using an energy balance. Thus, $\Delta U = \cancel{Q} - W$, where $W = \int p dV = p \Delta V$ since pressure is constant. So

$$\Delta U = -p \Delta V \Rightarrow \Delta H = 0 \text{ for the constant pressure process}$$

If all the ice does not melt, the final temperature would be 32°F. To investigate this, let T_f be the final temperature when all of the ice is regarded to have melted. Then with $c = 1.0 \text{ Btu/lb} \cdot ^\circ\text{R}$ for liquid water from Table A-19E

$$\begin{aligned} \{ \Delta H \}_{ice} + \Delta H \}_{melt_{32^\circ\text{F} \rightarrow T_f}} + \Delta H \}_{liquid_{80^\circ\text{F} \rightarrow T_f}} &= 0 \\ \{ (0.4 \text{ lb}) (144 \frac{\text{Btu}}{\text{lb}}) + (0.4 \text{ lb}) (1 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}) (T_f - 492)^\circ\text{R} \} + (2 \text{ lb}) (1 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}) (T_f - 540)^\circ\text{R} &= 0 \end{aligned}$$

$\Rightarrow T_f = 508^\circ\text{R} \text{ (48°F)} \leftarrow T_f$

As $T_f > 32^\circ\text{F}$, it can be concluded that all the ice melts.

(b) An entropy balance reduces to give

$$\Delta S = \cancel{\frac{Q}{T_b}} + \sigma \Rightarrow \sigma = \Delta S$$

Thus

$$\sigma = \{ \Delta S \}_{ice_{at 32^\circ\text{F}}} + \Delta S \}_{melt_{32^\circ\text{F} \rightarrow T_f}} + \Delta S \}_{liquid_{80^\circ\text{F} \rightarrow T_f}}$$

The two liquid terms can be evaluated using Eq. 6.13. The entropy change for the melting ice can be found using the Tds equation, Eq. 6.10b, which at constant pressure and temperature reduces to give

$$ds = \frac{dh}{T} \Rightarrow s_g - s_f = \frac{h_g - h_f}{T} = \frac{144 \text{ Btu/lb}}{492^\circ\text{R}} = 0.2927 \text{ Btu/lb} \cdot ^\circ\text{R}$$

Thus, with $c = 1 \text{ Btu/lb} \cdot ^\circ\text{R}$ for liquid water (Table A-19E),

$$\begin{aligned} \sigma &= (0.4 \text{ lb}) (0.2927 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}) + (0.4 \text{ lb}) (1 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}) \ln \frac{508}{492} + (2 \text{ lb}) (1 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}) \ln \frac{508}{540} \\ &= (0.1171) + (0.0128) + (-0.1222) = 0.0077 \text{ Btu/}^\circ\text{R} \end{aligned}$$

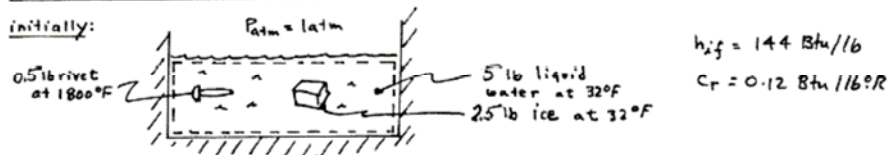
$\leftarrow \sigma$

PROBLEM 6.64

KNOWN: A system initially consisting of a rivet and a two-phase solid-liquid mixture of water attains an equilibrium state adiabatically at constant pressure.

FIND: Determine the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) As shown in the accompanying figure, the system consists of the ice, liquid, and rivet. (2) $Q = 0$ and pressure remains constant. (3) The rivet and liquid can be regarded as incompressible with constant specific heats.

ANALYSIS: (a) The final temperature can be determined using an energy balance. Thus, $\Delta U = Q - W$, where $W = \int p dV = p \Delta V$ since pressure is constant. So

$$\Delta U = -p \Delta V \Rightarrow \Delta H = 0 \text{ for the constant pressure process.}$$

If all the ice does not melt, the final temperature would be 32°F. To investigate this, let T_f be the final temperature when all of the ice is regarded to have melted. Then, with $c = 1.0 \text{ Btu/lb} \cdot ^\circ\text{R}$ for liquid water from Table A-19 E

$$\left\{ \Delta H \right\}_{\text{ice}} + \left\{ \Delta H \right\}_{\text{melt}} + \Delta H_{\text{liquid}} + \Delta H_{\text{rivet}} = 0$$

$$0 = \left\{ (2.5 \text{ lb}) \left(144 \frac{\text{Btu}}{\text{lb}} \right) + (2.5 \text{ lb}) \left(1 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (T_f - 492) \right\} + (5 \text{ lb}) \left(1 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (T_f - 492) + (0.5 \text{ lb}) \left(0.12 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (T_f - 2260)$$

$$\Rightarrow T_f = 458.7^\circ\text{R}$$

Since $T_f < 32^\circ\text{F}$, it can be concluded that not all of the ice melts and the final temperature is 32°F. $\xrightarrow{\quad\quad\quad} T_f$

(b) An entropy balance reduces to give $\Delta S = \int \frac{\delta Q}{T_b} + \sigma$ or $\sigma = \Delta S$. Since not all of the ice melts it is necessary to determine the amount that does melt. Let x denote the amount of ice that melts. Then since the liquid remains at 32°F, the energy balance gives

$$\Delta H_{\text{ice}} + \Delta H_{\text{rivet}} = 0$$

or

$$(x \text{ lb}) \left(144 \frac{\text{Btu}}{\text{lb}} \right) + (0.5 \text{ lb}) \left(0.12 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (492 - 2260)^\circ\text{R} = 0 \Rightarrow x = 0.737 \text{ lb}$$

Returning to the calculation of the entropy production.

$$\sigma = \Delta S_{\text{ice}} + \Delta S_{\text{rivet}} + \Delta S_{\text{liq}}$$

The entropy change of the rivet can be found using Eq. 6.13. Similarly, $\Delta S = 0$ for the liquid by Eq. 6.13 since the temperature remains constant. The entropy change for the melting ice can be found using the Tds equation, Eq. 6.10b, which at constant pressure and temperature reduces to give

$$ds = \frac{dh}{T} \Rightarrow s_f - s_i = \frac{h_f - h_i}{T} = \frac{144 \text{ Btu/lb}}{492^\circ\text{R}} = 0.2927 \text{ Btu/lb} \cdot ^\circ\text{R}$$

Thus

$$\sigma = (0.737 \text{ lb}) \left(0.2927 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) + (0.5 \text{ lb}) \left(0.12 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) \ln \left(\frac{492}{2260} \right)$$

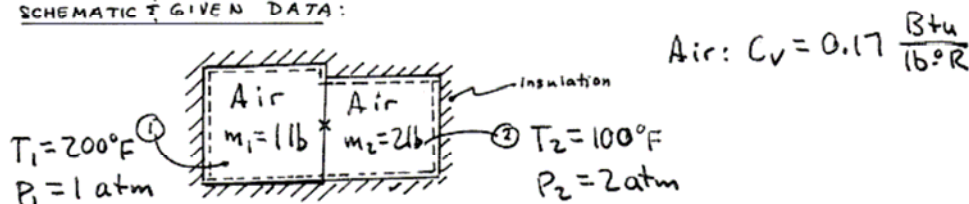
$$= (0.2157) + (-0.0915) = 0.1242 \text{ Btu/}^\circ\text{R} \quad \leftarrow \sigma$$

PROBLEM 6.65

KNOWN: Two insulated tanks containing air at known states are connected by a valve. The valve is opened and the two quantities of air mix.

FIND: Determine (a) the final temperature, (b) the final pressure, and (c) the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) As shown in the accompanying figure, the system is the total quantity of air. (2) Air is modeled as an ideal gas. (3) $Q = W = 0$ and there is no change in kinetic or potential energy between the initial and final states.

ANALYSIS: (a) An energy balance reduces with assumption 3 to give $\Delta U = 0$ or $\Delta U = 0$. That is

$$\Delta U = (m_1 + m_2)u(T_f) - [m_1 u(T_1) + m_2 u(T_2)] = 0$$

or

$$0 = m_1[u(T_f) - u(T_1)] + m_2[u(T_f) - u(T_2)] \Rightarrow 0 = m_1 c_v [T_f - T_1] + m_2 c_v [T_f - T_2]$$

$$\Rightarrow T_f = \frac{m_1 T_1 + m_2 T_2}{(m_1 + m_2)} = \frac{(1 \text{ lb})(660^\circ \text{R}) + (2 \text{ lb})(560^\circ \text{R})}{(3 \text{ lb})} = 593^\circ \text{R} (133^\circ \text{F}) \leftarrow (a)$$

(b) Using the ideal gas equation of state

$$P_f = \frac{(m_1 + m_2) R T_f}{(V_1 + V_2)}$$

where

$$V_1 = \frac{m_1 R T_1}{P_1} = \frac{(1) \left(\frac{1545}{28.97} \right) (660)}{(14.7)(144)} = 16.63 \text{ ft}^3$$

$$V_2 = \frac{m_2 R T_2}{P_2} = \frac{(2) \left(\frac{1545}{28.97} \right) (560)}{(2)(14.7)(144)} = 14.11 \text{ ft}^3$$

Thus

$$P_f = \frac{(3) \left(\frac{1545}{28.97} \right) (593)}{(16.63 + 14.11)(14.7)(144)} = 1.458 \text{ atm} \leftarrow (b)$$

(c) An entropy balance reduces to give $\Delta S = \int_1^2 \frac{\delta Q}{T} + \sigma$, or

$$\sigma = \Delta S = \{(m_1 + m_2) s_f - (m_1 s_1 + m_2 s_2)\} = m_1 (s_f - s_1) + m_2 (s_f - s_2)$$

$$= m_1 \left[c_p \ln \frac{T_f}{T_1} - R \ln \frac{P_f}{P_1} \right] + m_2 \left[c_p \ln \frac{T_f}{T_2} - R \ln \frac{P_f}{P_2} \right]$$

Using Eq. 3.44, $c_p = c_v + R = 0.17 + \left(\frac{1.986}{28.97} \right) = 0.24 \text{ Btu/lb} \cdot \text{R}$. Then

$$\sigma = (1) \left[(0.24) \ln \frac{593}{660} - \left(\frac{1.986}{28.97} \right) \ln \frac{1.458}{1} \right] + (2) \left[(0.24) \ln \frac{593}{560} - \left(\frac{1.986}{28.97} \right) \ln \frac{1.458}{2} \right]$$

$$= [-0.05154 + 0.07082] \text{ Btu/}^\circ \text{R}$$

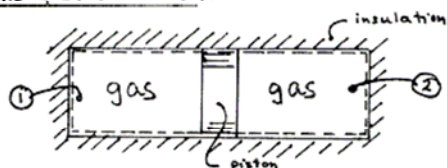
$$= 0.01928 \text{ Btu/}^\circ \text{R} \leftarrow (c)$$

PROBLEM 6.66

KNOWN: An insulated cylinder is initially divided into halves by a piston. On either side of the piston is a gas at a known state. The piston is released and equilibrium is attained.

FIND: Determine the final pressure, final temperature, and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



initially:
 $T_1 = 300 \text{ K}$ $T_2 = 300 \text{ K}$
 $P_1 = 2 \text{ bar}$ $P_2 = 1 \text{ bar}$
 $V_1 = 1 \text{ m}^3$ $V_2 = 1 \text{ m}^3$

ENGR. MODEL: (1) The system consists of both quantities of gas and the piston. But the piston experiences no net change in state. (2) The piston moves freely in the cylinder and is thermally conducting. (3) The gas is modeled as an ideal gas. (4) $Q = 0$.

ANALYSIS: (a) An energy balance reduces to give $\Delta U = \int_1^2 \delta Q - \int_1^2 \delta W \Rightarrow \Delta U = 0 \Rightarrow$

$$\Delta U_1 + \Delta U_2 + \Delta U_{\text{piston}} = 0 \Rightarrow m_1 [u(T_f) - u(300 \text{ K})] + m_2 [u(T_f) - u(300 \text{ K})] = 0$$

$$\Rightarrow u(T_f) = u(300 \text{ K}) \Rightarrow T_f = 300 \text{ K}$$

(b) Using the ideal gas equation of state, and noting that the total mass of gas occupies the same total volume at the final state as initially: $V = 2 \text{ m}^3$,

$$P_f = \frac{(m_1 + m_2) R T_f}{V}$$

where

$$m_1 = \frac{P_1 (V/2)}{R T_1}, \quad m_2 = \frac{P_2 (V/2)}{R T_2}$$

Thus, since $T_1 = T_2 = T_f$

$$P_f = \frac{\left(\frac{P_1 (V/2)}{R T} + \frac{P_2 (V/2)}{R T} \right) R T}{V} = \frac{P_1 + P_2}{2} = 1.5 \text{ bar}$$

(c) An entropy balance reduces to give $\Delta S = \int_1^2 \frac{\delta Q}{T} + \sigma$ or

$$\sigma = \Delta S \Rightarrow \sigma = \Delta S_1 + \Delta S_2 + \Delta S_{\text{piston}}$$

where

$$\Delta S_1 = m_1 \left[\underbrace{s^\circ(T_f) - s^\circ(T_1)}_{=0 \text{ (} T_f = T_1 \text{)}} - R \ln \frac{P_f}{P_1} \right] = -R m_1 \ln \frac{P_f}{P_1}$$

$$\Delta S_2 = m_2 \left[\underbrace{s^\circ(T_f) - s^\circ(T_2)}_{=0 \text{ (} T_f = T_2 \text{)}} - R \ln \frac{P_f}{P_2} \right] = -R m_2 \ln \frac{P_f}{P_2}$$

Thus

$$\begin{aligned} \sigma &= -R \left[m_1 \ln \frac{P_f}{P_1} + m_2 \ln \frac{P_f}{P_2} \right] = -R \left[\frac{P_1 (V/2)}{R T} \ln \frac{P_f}{P_1} + \frac{P_2 (V/2)}{R T} \ln \frac{P_f}{P_2} \right] \\ &= -\frac{V/2}{T} \left[P_1 \ln \frac{P_f}{P_1} + P_2 \ln \frac{P_f}{P_2} \right] \\ &= -\frac{1 \text{ m}^3}{300 \text{ K}} \left[(2 \text{ bar}) \ln \frac{1.5}{2} + (1 \text{ bar}) \ln \frac{1.5}{1} \right] \left| \frac{100 \text{ kN/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{1 \text{ kN}\cdot\text{m}} \right| \end{aligned}$$

①

$$= 0.0566 \text{ kJ/K}$$

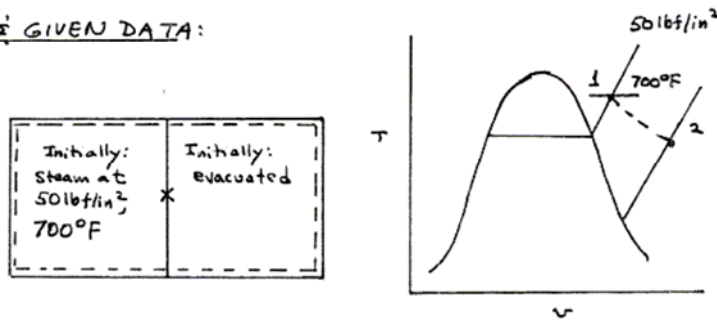
2. Provided the ideal gas model is assumed, the same numerical results are obtained for any gas.

PROBLEM 6.67

KNOWN: An insulated vessel is divided into equal-sized compartments connected by a valve. Initially one compartment contains steam at a known state and the other is evacuated. The valve is opened and the steam fills the entire volume.

FIND: Determine (a) the final temperature and (b) the amount of entropy produced per unit mass of steam.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system is shown by the dashed line in the figure above. (2) For the system, $Q=0$, $W=0$, and kinetic/potential energy effects can be ignored.

ANALYSIS: To fix the final state requires the values of two independent intensive properties: u_2 and v_2 . Thus, since the steam expands to fill twice the initial volume occupied by the steam, $v_2 = 2v_1$. And from an energy balance

$$\Delta U = Q - W \Rightarrow u_2 = u_1$$

An entropy balance reduces to

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right) + \sigma \Rightarrow \frac{\sigma}{m} = (s_2 - s_1) \quad (1)$$

As the use of the pair of independent properties u_2, v_2 to fix the state with tabular steam table data is cumbersome, IT is employed to evaluate σ/m using Eq. (1):

IT Code

p1 = 50 // lbf/in.²
T1 = 700 // °F

u2 = u1
v2 = 2 * v1
u1 = u_PT("Water/Steam", p1, T1)
v1 = v_PT("Water/Steam", p1, T1)
u2 = u_PT("Water/Steam", p2, T2)
v2 = v_PT("Water/Steam", p2, T2)

sigma / m = s2 - s1
m = 1 // lb
s1 = s_PT("Water/Steam", p1, T1)
s2 = s_PT("Water/Steam", p2, T2)

IT Results

T₂ = 697.4 °F
p₂ = 25.01 lbf/in.²
s₁ = 1.881 Btu/lb·°R
s₂ = 1.957 Btu/lb·°R
u₂ = 1255 Btu/lb
v₂ = 27.48 ft³/lb
σ/m = 0.07611 Btu/lb·°R

← T₂

← σ/m

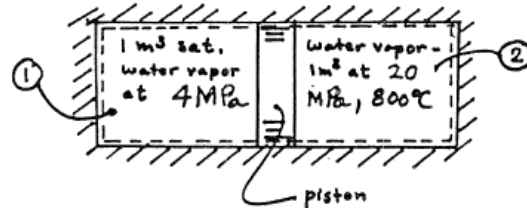
PROBLEM 6.68

KNOWN: An insulated tank is divided into two compartments by a piston. On either side of the piston is water vapor at known states. The piston is released and equilibrium is attained.

FIND: Determine (a) the final pressure, (b) the final temperature, and (c) the amount of entropy produced.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The closed system consists of both quantities of water vapor and the piston, but the piston experiences no change of state. (2) The piston moves freely and is thermally conducting. (3) For the system, $Q=0$, $W=0$, $\Delta KE = \Delta PE = 0$.



ANALYSIS: (a), (b) To fix the final state of the water vapor, we begin with an energy balance. That is, $\Delta U + \Delta KE + \Delta PE = Q - W \Rightarrow \Delta U = 0$. Thus

$$\Delta U_1 + \Delta U_2 + \Delta U_{\text{piston}} = 0 \Rightarrow m_1(\Delta u)_1 + m_2(\Delta u)_2 = 0 \quad (1)$$

For each side, $m = V/v$. With data from Table A-3 at the initial state

$$m_1 = \frac{V_1}{v_1} = \frac{1 \text{ m}^3}{0.04978 \text{ m}^3/\text{kg}} = 20.09 \text{ kg}$$

Similarly, for side 2 initially Table A-4 gives $v_2 = 0.02385 \text{ m}^3/\text{kg}$. Thus

$$m_2 = \frac{V_2}{v_2} = \frac{1 \text{ m}^3}{0.02385 \text{ m}^3/\text{kg}} = 41.93 \text{ kg}$$

It follows from assumption 2 that at equilibrium, the states of water vapor on either side of the piston are the same. Thus, from (1)

$$m_1(u - u_1) + m_2(u - u_2) = 0$$

From Table A-3, $u_1 = 2602.3 \text{ kJ/kg}$ and from Table A-4, $u_2 = 3592.7 \text{ kJ/kg}$. Inserting values and solving for u we get

$$20.09(u - 2602.3) + 41.93(u - 3592.7) = 0 \Rightarrow u = 3271.9 \text{ kJ/kg}$$

The overall system volume is constant: 2 m^3 , and the total mass of water vapor is fixed. Thus, at equilibrium $v = (2 \text{ m}^3)/(20.09 + 41.93) \text{ kg} = 0.03225 \text{ m}^3/\text{kg}$. The equilibrium state on both sides of the piston is fixed by $u = 3271.9 \text{ kJ/kg}$ and $v = 0.03225 \text{ m}^3/\text{kg}$. Interpolation in Table A-4 with u and v is very inconvenient. Instead, we use IT to get

$$P = 121.5 \text{ bar}$$

$$T = 622.6^\circ\text{C}$$

$$s = 6.861 \text{ kJ/kg}\cdot\text{K}$$

(a) (b)

Continued on next slide

Problem 6-68 continued

(c) The entropy produced can be determined from an entropy balance which reduces with assumptions (1) and (3) to give

$$\sigma = \Delta S \Rightarrow \sigma = \Delta S_1 + \Delta S_2 + \Delta S_{\text{piston}}^{\circ} \Rightarrow \sigma = m_1 (\Delta s)_1 + m_2 (\Delta s)_2$$

From Table A-3, $s_1 = 6.0701 \text{ kJ/kg} \cdot \text{K}$, and from Table A-4, $s_2 = 7.0544 \text{ kJ/kg} \cdot \text{K}$. Thus

$$\sigma = 20.09 [6.861 - 6.0701] + 41.93 [6.861 - 7.0544] = 7.78 \text{ kJ/kg} \cdot \text{K}$$

ALTERNATIVE IT SOLUTION

ITCode

V1 = 1 // m³

x1 = 1

p1 = 40 // bar

V2 = 1 // m³

p2 = 200 // bar

T2 = 800 // °C

m1 = V1 / v1

v1 = vsat_Px("Water/Steam", p1, x1)

m2 = V2 / v2

v2 = v_PT("Water/Steam", p2, T2)

m1 * (u - u1) + m2 * (u - u2) = 0

u1 = usat_Px("Water/Steam", p1, x1)

u2 = u_PT("Water/Steam", p2, T2)

v = (V1 + V2) / (m1 + m2)

u = u_PT("Water/Steam", p, T)

v = v_PT("Water/Steam", p, T)

sigma = m1 * (s - s1) + m2 * (s - s2)

s1 = ssat_Px("Water/Steam", p1, x1)

s2 = s_PT("Water/Steam", p2, T2)

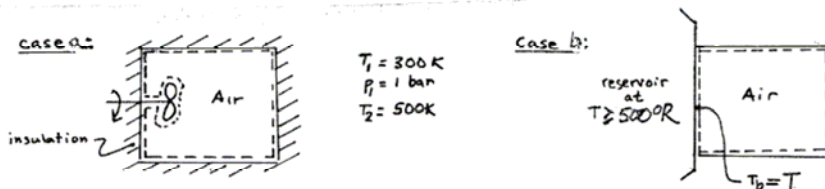
s = s_PT("Water/Steam", p, T)

PROBLEM 6.69

KNOWN: A system consisting of air at a specified state undergoes constant volume processes in which the temperature increases in each of two ways.

FIND: (a) When the air is stirred adiabatically, determine the entropy produced. (b) When the air is heated by a reservoir at temperature T , plot the entropy produced versus T . Compare and discuss.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system consists of the air. (2) Air is modeled as an ideal gas. (3) In case (a), $Q = 0$, $W \neq 0$. In case (b), $Q \neq 0$, $W = 0$.

ANALYSIS: The change in entropy of the air is required in the evaluation of σ in each case. Thus, with data from Table A-22 and the ideal gas equation which gives $P_2/P_1 = T_2/T_1$,

$$s_2 - s_1 = s^0(T_2) - s^0(T_1) - R \ln \frac{P_2}{P_1} = 2.21952 - 1.70203 - \frac{8.314}{28.97} \ln \frac{500}{300} = 0.3709 \text{ kJ/kg} \cdot \text{K}$$

CASE (a): An entropy balance reduces to give

$$\Delta S = \int \frac{\delta Q}{T_b} + \sigma_a \Rightarrow \frac{\sigma_a}{m} = s_2 - s_1 = 0.3709 \text{ kJ/kg} \cdot \text{K} \quad (a)$$

CASE (b): An entropy balance reduces to give

$$\Delta S = \frac{Q}{T_b} + \sigma_b \Rightarrow \frac{\sigma_b}{m} = (s_2 - s_1) - \frac{Q/m}{T_b}$$

To find Q , write an energy balance: $\Delta U = Q - W$. Thus, with data from Table A-22

$$\frac{Q}{m} = u(T_2) - u(T_1) = 359.49 - 214.07 = 145.42 \text{ kJ/kg}$$

Thus

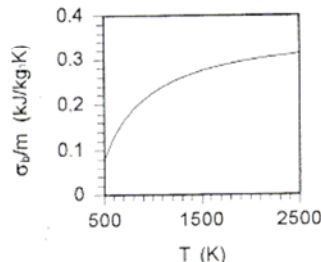
$$\frac{\sigma_b}{m} = 0.3709 - \frac{145.42}{T} \quad (b)$$

Sample calculation: When $T = 500\text{ K}$, $\sigma_b/m = 0.08 \text{ kJ/kg} \cdot \text{K}$.

Equation (b) is plotted below. Comparing the two cases, we get

$$(\sigma_b/m) = (\sigma_a/m) - (145.42/T).$$

Thus, the entropy produced by heating is always less than the entropy produced by stirring. The entropy produced by heating approaches the entropy produced stirring at higher reservoir temperatures (mathematically, as $T \rightarrow \infty$). See the accompanying plot.

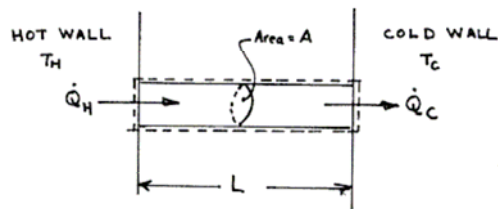


PROBLEM 6.70

KNOWN: Energy is conducted steadily through a copper rod from a hot wall to a cold wall. The rate of heat transfer, temperatures, and geometrical parameters are specified.

FIND: (a) Determine an expression for the rate of entropy production within the rod in terms of specified quantities, and (b) plot $\dot{Q}_H$ and $\dot{\sigma}$ versus L for a given set of numerical values for these quantities.

SCHEMATIC & GIVEN DATA:



$$\dot{Q}_H = kA(T_H - T_C)/L \quad (1)$$

$$T_H = 600 \text{ K } (327^\circ\text{C})$$

$$T_C = 350 \text{ K } (77^\circ\text{C})$$

$$k = 0.4 \text{ kW/m}\cdot\text{K}$$

$$A = 0.1 \text{ m}^2$$

ENGR. MODEL: (1) As shown in the accompanying figure, the system is the copper rod. (2) The system is at steady state. (3) The rod is insulated on its lateral surface. (4) An expression for evaluating $\dot{Q}_H$ is provided.

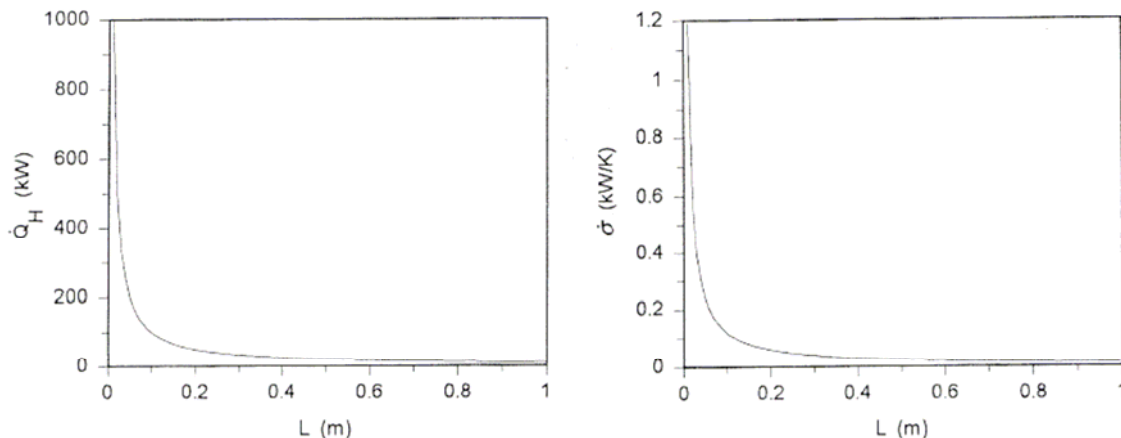
ANALYSIS: (a) At steady state an entropy rate balance reduces to give

$$\frac{d\dot{\sigma}}{dt} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{\sigma} \Rightarrow \dot{\sigma} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H}$$

Noting that the energy transfers are positive in the directions of the arrows, an energy rate balance gives $\dot{Q}_C = \dot{Q}_H$. Collecting results

$$\dot{\sigma} = \dot{Q}_H \left[\frac{1}{T_C} - \frac{1}{T_H} \right] = \frac{kA(T_H - T_C)^2}{L T_H T_C} \quad (2)$$

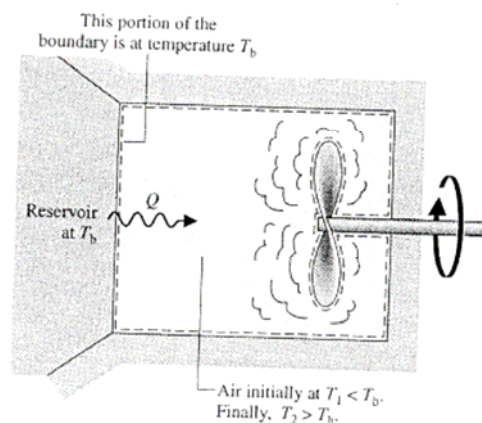
(b) If $T_H = 600 \text{ K } (327^\circ\text{C})$, $T_C = 350 \text{ K } (77^\circ\text{C})$, $k = 0.4 \text{ kW/m}\cdot\text{K}$, $A = 0.1 \text{ m}^2$, the variations of $\dot{Q}$ and $\dot{\sigma}$ with L are



The plots show that both heat transfer and entropy production rate decrease rapidly with increasing L . $\dot{Q}_H$ decreases due to greater resistance to heat transfer as the length of the rod increases. The variation of $\dot{\sigma}$ is associated with the changing temperature gradient, dt/dx , in the rod. Higher temperature gradient ($L \rightarrow 0$; $dt/dx \rightarrow \infty$) corresponds to high rates of entropy production, and conversely. As a final point, Eq. (2) above indicates that $\dot{\sigma}$ is positive, illustrating the irreversible nature of heat transfer.

PROBLEM 6.71

Figure P6.71 shows a system consisting of air in a rigid container fitted with a paddle wheel and in contact with a thermal energy reservoir. By heating and/or stirring, the air can achieve a specified increase in temperature from T_1 to T_2 in alternative ways. Discuss how the temperature increase of the air might be achieved with (a) minimum entropy production, and (b) maximum entropy production. Assume that the temperature on the boundary where heat transfer to the air occurs, T_b , is the same as the reservoir temperature. Let $T_1 < T_b < T_2$. The ideal gas model applies to the air.



ENGR. MODEL:

1. The closed system consists of the air only.
2. The temperature of the air at the air-reservoir interface is T_b .
3. The air is modeled as an ideal gas.
4. The volume of the air is constant and there are no overall changes in KE and PE.

ANALYSIS: The change of state can be brought about by a combination of heat and work. Q and W are related by an energy balance:

$$\Delta U = Q - W \Rightarrow m \int_{T_1}^{T_2} c_v(T) dT = Q - W, \quad W < 0,$$

where assumptions 3 and 4 have been used.

Since Q is received at T_b , the closed system entropy balance reads

$$\Delta S = \frac{Q}{T_b} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T_b}$$

Then, with Eq. 6.21 and assumption 4

$$\Delta S = m \left[\int_{T_1}^{T_2} \frac{c_v(T)}{T} dT + R \ln \frac{v_2}{v_1} \right]$$

Collecting results

$$\sigma = \overbrace{m \int_{T_1}^{T_2} \frac{c_v(T)}{T} dT}^{(\text{strictly positive})} - \overbrace{\frac{Q}{T_b}}^{(\text{positive or zero})}$$

Accordingly, σ increases as Q decreases, and conversely. This suggests,

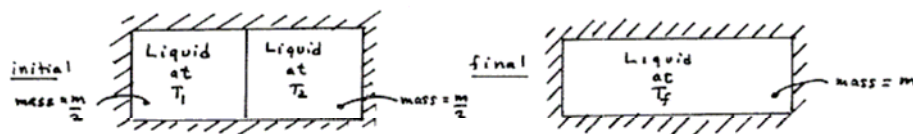
- Maximum σ corresponds to $Q = 0$: the process takes place by stirring only.
- Minimum σ corresponds to heating to the fullest extent, followed by stirring, as required. Observe that some stirring would be required because $T_2 > T_b$.

PROBLEM 6.72

KNOWN: An isolated system of total mass m is formed by mixing equal masses of the same liquid initially at temperatures T_1 and T_2 .

FIND: (a) Show that the amount of entropy produced is $\sigma = mc \ln \left[\frac{(T_1 + T_2)/2}{(T_1 T_2)^{1/2}} \right]$ and (b) that σ must be positive.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system consists of the total mass of liquid. (2) The system is isolated. (3) The liquid is incompressible with constant specific heat c .

ANALYSIS: (a) The final temperature T_f can be evaluated from an energy balance: $\Delta U = \cancel{Q} - \cancel{W}$. Thus $\Delta U = 0$, or

$$m u(T_f) - \left[\frac{m}{2} u(T_1) + \frac{m}{2} u(T_2) \right] = 0$$

$$\Rightarrow \frac{m}{2} [u(T_f) - u(T_1)] + \frac{m}{2} [u(T_f) - u(T_2)] = 0$$

Since the liquid is incompressible with constant specific heat c

$$\frac{m}{2} c [T_f - T_1] + \frac{m}{2} c [T_f - T_2] = 0$$

$$\Rightarrow T_f = \frac{T_1 + T_2}{2}$$

An entropy balance gives

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma$$

or

$$\sigma = m s_f - \left[\frac{m}{2} s_1 + \frac{m}{2} s_2 \right]$$

$$= \frac{m}{2} [(s_f - s_1) + (s_f - s_2)]$$

Using Eq. 6.13

$$\sigma = \frac{m}{2} c \left[\ln \frac{T_f}{T_1} + \ln \frac{T_f}{T_2} \right] = \frac{m}{2} c \ln \left[\frac{T_f^2}{T_1 T_2} \right]$$

$$= mc \ln \left[\frac{T_f}{(T_1 T_2)^{1/2}} \right]$$

$$= mc \ln \left[\frac{T_1 + T_2}{2 (T_1 T_2)^{1/2}} \right] \quad \leftarrow (a)$$

(b) $\sigma \geq 0$ when $\ln \left[\frac{T_1 + T_2}{2 (T_1 T_2)^{1/2}} \right] \geq 0$

$$\Rightarrow \frac{T_1 + T_2}{2 (T_1 T_2)^{1/2}} \geq 1 \quad \text{or} \quad T_1 + T_2 \geq 2 (T_1 T_2)^{1/2}$$

Squaring both sides

$$(T_1 + T_2)^2 \geq 4 (T_1 T_2) \quad \text{or} \quad T_1^2 + 2 T_1 T_2 + T_2^2 \geq 4 T_1 T_2$$

$$\Rightarrow T_1^2 - 2 T_1 T_2 + T_2^2 \geq 0$$

$$\Rightarrow (T_1 - T_2)^2 \geq 0$$

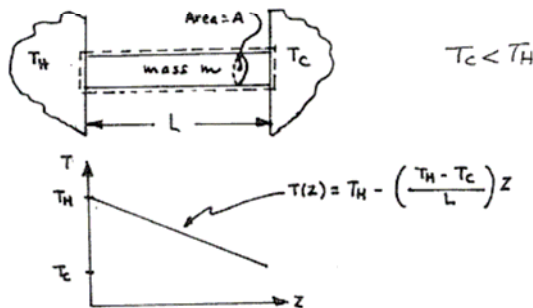
The inequality is satisfied for either $T_1 > T_2$ or $T_2 > T_1$. The equality applies only $\leftarrow (b)$ when $T_1 = T_2$.

PROBLEM 6.73

KNOWN: The temperature within a rod initially in contact with hot and cold walls at its ends is linear with position. The rod is insulated overall and eventually comes to a final equilibrium state where the temperature is T_f .

FIND: Evaluate the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system is the rod which is insulated on its lateral surface. (2) The rod is modeled as incompressible with constant specific heat c . (3) Initially, the temperature within the rod varies linearly from T_H to T_C .

ANALYSIS: The final temperature can be determined using an energy balance which reduces to give $\Delta U = 0$ or $\Delta U = 0$. Each element of rod dz changes temperature from $T(z)$ to the final temperature T_f , and thus contributes to the change in internal energy

$$dU = dm c (T_f - T(z)) = (\rho A dz) c (T_f - T(z))$$

Accordingly

$$\begin{aligned} \Delta U &= \int_0^L (\rho A dz) c (T_f - T(z)) \\ &= \rho A c \int_0^L \left[T_f - T_H + \left(\frac{T_H - T_C}{L}\right)z \right] dz \\ &= \rho A c \left[(T_f - T_H)z + \left(\frac{T_H - T_C}{L}\right)\frac{z^2}{2} \right]_0^L \\ &= \rho A c L \left[(T_f - T_H) + \frac{(T_H - T_C)}{2} \right] \end{aligned}$$

Since $\Delta U = 0$, $T_f = (T_H + T_C)/2$.

To find the entropy production, an entropy balance reduces to give $\Delta S = \int_1^2 \frac{\delta Q}{T} + \sigma$ or $\sigma = \Delta S$. With Eq. 6.13, the entropy change of an element of rod dz is

$$\begin{aligned} dS &= dm c \ln \frac{T_f}{T(z)} \\ &= (\rho A dz) c \ln \frac{T_f}{T(z)} \end{aligned}$$

Accordingly

$$\sigma = \rho A c \int_0^L (\ln T_f - \ln T(z)) dz = \rho A c \left[(\ln T_f)L - \int_0^L (\ln T(z)) dz \right]$$

Using the given temperature distribution, the variable of integration can be changed from z to T :

$$dT = -\left(\frac{T_H - T_C}{L}\right)dz \Rightarrow dz = -\left(\frac{L}{T_H - T_C}\right)dT$$

With this, the integral can be expressed as

Continued on next slide

Problem 6-73 continued

$$\begin{aligned}
 \int_0^L (\ln T(z)) dz &= \int_{T_H}^{T_C} (\ln T) \left(\frac{-L}{T_H - T_C} \right) dT \\
 &= \frac{L}{(T_H - T_C)} \int_{T_C}^{T_H} \ln T dT \\
 &= \frac{L}{(T_H - T_C)} \left[T \ln T - T \right]_{T_C}^{T_H} \\
 &= \frac{L}{T_H - T_C} \left[(T_H \ln T_H - T_H) - (T_C \ln T_C - T_C) \right] \\
 &= L \left[\frac{T_H \ln T_H}{T_H - T_C} - \frac{T_C \ln T_C}{T_H - T_C} - 1 \right]
 \end{aligned}$$

Collecting results

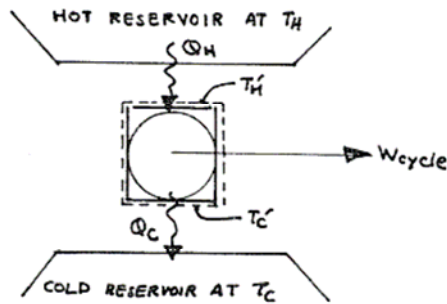
$$\begin{aligned}
 \sigma &= \rho A C \left[(\ln T_f) L - L \left[\frac{T_H \ln T_H}{T_H - T_C} - \frac{T_C \ln T_C}{T_H - T_C} - 1 \right] \right] \\
 &= m c \left[1 + \ln T_f + \frac{T_C \ln T_C}{T_H - T_C} - \frac{T_H \ln T_H}{T_H - T_C} \right] \longleftarrow \sigma
 \end{aligned}$$

PROBLEM 6.74

KNOWN: A system undergoes a cycle while receiving Q_H at T_H' and discharging Q_C at T_C' . Q_H and Q_C are with hot and cold reservoirs at T_H and T_C , respectively.

FIND: (a) Determine an expression for W_{cycle} in terms of Q_H , T_C' , T_H' , and σ . (b) State the relationship of T_H' to T_H and T_C' to T_C . (c) Obtain an expression for W_{cycle} when there are (i) no internal irreversibilities, (ii) no irreversibilities.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The system shown in the accompanying figure undergoes a power cycle while receiving Q_H at T_H' and discharging Q_C at T_C' . There are no other heat transfers.

ANALYSIS: (a) An energy balance gives

$$W_{\text{cycle}} = Q_H - Q_C \quad (1)$$

An entropy balance gives

$$\Delta S_{\text{cycle}}^0 = \frac{Q_H}{T_H'} - \frac{Q_C}{T_C'} + \sigma_{\text{cycle}} \quad (2)$$

where σ is the amount of entropy produced within the system per cycle and $\Delta S = 0$ because the system undergoes a cycle. Solving Eq. (2) for Q_C and substituting the result into Eq. (1)

$$W_{\text{cycle}} = Q_H \left[1 - \frac{T_C'}{T_H'} \right] - T_C' \sigma_{\text{cycle}} \quad (3) \quad \leftarrow (a)$$

(b) For heat transfer to occur from the hot reservoir to the system $T_H \geq T_H'$. For heat transfer to occur from the system to the cold reservoir $T_C' \geq T_C$. $\leftarrow (b)$

(c) If there were no irreversibilities within the system during the cycle, the term σ_{cycle} in Eq. (3) would vanish leaving

$$W_{\text{cycle}} = Q_H \left[1 - \frac{T_C'}{T_H'} \right] \quad (4) \quad \leftarrow (c-i)$$

External irreversibilities are associated with heat transfer between the reservoirs and the system. If these are also absent, $T_H' = T_H$ and $T_C' = T_C$, and Eq. (4) becomes

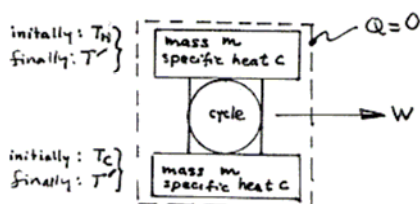
$$W_{\text{cycle}} = Q_H \left[1 - \frac{T_C}{T_H} \right] \quad (5) \quad \leftarrow (c-ii)$$

which is the maximum theoretical work that can be obtained.

PROBLEM 6.75

A thermodynamic power cycle receives energy by heat transfer from an incompressible body of mass m and specific heat c initially at temperature T_H . The cycle discharges energy by heat transfer to another incompressible body of mass m and specific heat c initially at a lower temperature T_C . There are no other heat transfers. Work is developed by the cycle until the temperature of each of the two bodies is the same. Develop an expression for the maximum theoretical amount of work that can be developed, $W_{\max}$, in terms of m , c , T_H , and T_C .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the accompanying figure is composed of three subsystems: An incompressible body of mass m and specific heat c initially at T_H . An incompressible body of mass m and specific heat c initially at T_C . A system that undergoes a power cycle. (2) For the overall system, $Q = 0$.

ANALYSIS: An energy balance gives $\Delta U = \cancel{Q} - W \Rightarrow$

$$W = -[\Delta U]_{\text{HOT}} + \cancel{\Delta U}_{\text{cycle}} + \Delta U_{\text{COLD}} = -[mc(T' - T_H) + mc(T' - T_C)]$$

or

$$W = mc[T_H + T_C - 2T'] \quad (1)$$

An entropy balance gives $\Delta S = \cancel{\int \frac{\delta Q}{T}}_0 + \sigma$. Then, with Eq. 6.13

$$\sigma = [\Delta S]_{\text{HOT}} + \cancel{\Delta S}_{\text{cycle}} + \Delta S_{\text{COLD}} = [mc \ln \frac{T'}{T_H} + mc \ln \frac{T'}{T_C}]$$

or

$$\sigma = mc \ln \left[\frac{(T')^2}{T_H T_C} \right]$$

Solving for T'

$$T' = [T_H T_C \exp(\sigma/mc)]^{1/2} \quad (2)$$

Substituting Eq. (2) into Eq. (1)

$$W = mc[T_H + T_C - 2(T_H T_C \exp(\sigma/mc))^{1/2}]$$

Since $\sigma \geq 0$, it follows that W is maximum when $\sigma = 0$, giving

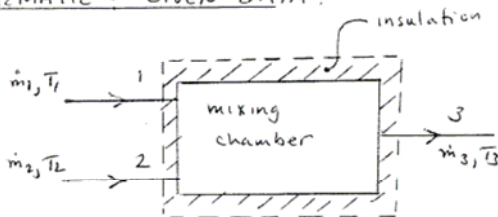
$$W_{\max} = mc[T_H + T_C - 2(T_H T_C)^{1/2}]$$

PROBLEM 6.76

KNOWN: An insulated mixing chamber at steady state receives liquid streams of the same substance at $\dot{m}_1, T_1$ and $\dot{m}_2, T_2$. A single stream exits at $\dot{m}_3, T_3$.

FIND: (a) Evaluate T_3 in terms of $T_1, T_2, \dot{m}_1/\dot{m}_3$. (b) Evaluate $\dot{\sigma}/\dot{m}_3$ in terms of the specific heat c , T_1/T_2 , $\dot{m}_1/\dot{m}_3$. (c) For fixed c and T_1/T_2 , determine the value of $\dot{m}_1/\dot{m}_3$ for which $\dot{\sigma}/\dot{m}_3$ is a maximum.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and all kinetic and potential energy effects are negligible. (3) The liquid streams can be modeled as incompressible with constant specific heat c and negligible effects of pressure.

ANALYSIS: (a) At steady state, a mass balance reads $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$. An energy balance becomes, with assumptions (2), $0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$. Combining these equations gives

$$0 = \dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_1) h_2 - \dot{m}_3 h_3$$

$$0 = \dot{m}_1 (h_1 - h_2) + \dot{m}_3 (h_2 - h_3)$$

① Or with assumption (3) and Eq. 3.20b

$$0 = \dot{m}_1 c [T_1 - T_2] + \dot{m}_3 c [T_2 - T_3] \Rightarrow T_3 = T_2 + \left[\frac{\dot{m}_1}{\dot{m}_3} \right] (T_1 - T_2) \quad \leftarrow (a)$$

(b) An entropy balance reduces at steady state to give

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma} \quad \text{or} \quad 0 = \dot{m}_1 s_1 + (\dot{m}_3 - \dot{m}_1) s_2 - \dot{m}_3 s_3 + \dot{\sigma}$$

Rearranging, and using Eq. 6.13,

$$0 = \dot{m}_1 (s_1 - s_2) + \dot{m}_3 (s_2 - s_3) + \dot{\sigma} \quad \text{or} \quad 0 = \dot{m}_1 c \ln \frac{T_1}{T_2} + \dot{m}_3 c \ln \frac{T_2}{T_3} + \dot{\sigma}$$

Solving

$$\textcircled{2} \quad \frac{\dot{\sigma}}{\dot{m}_3} = c \left[\frac{\dot{m}_1}{\dot{m}_3} \ln \frac{T_2}{T_1} + \ln \frac{T_2}{T_3} \right]$$

Using the result of part (a) this becomes

$$\frac{\dot{\sigma}}{\dot{m}_3} = c \left[\frac{\dot{m}_1}{\dot{m}_3} \ln \frac{T_2}{T_1} + \ln \left[\frac{T_2 + \left(\frac{\dot{m}_1}{\dot{m}_3} \right) (T_1 - T_2)}{T_2} \right] \right] = c \left[\frac{\dot{m}_1}{\dot{m}_3} \ln \frac{T_2}{T_1} + \ln \left[1 + \left(\frac{\dot{m}_1}{\dot{m}_3} \right) \left(\frac{T_1}{T_2} - 1 \right) \right] \right]$$

Letting $x = \dot{m}_1/\dot{m}_3$, this can be expressed as

$$\frac{\dot{\sigma}}{\dot{m}_3} = c \ln \left[\left(\frac{T_2}{T_1} \right)^x \left[1 + x \left(\frac{T_1}{T_2} - 1 \right) \right] \right] = c \ln \left[\left(\frac{T_1}{T_2} \right)^{-x} \left[1 + x \left[\left(\frac{T_1}{T_2} \right) - 1 \right] \right] \right] \quad \leftarrow (b)$$

(c) Taking the derivative of $\dot{\sigma}/\dot{m}_3$ with respect to x while holding c and T_2/T_1 constant,

$$\frac{\partial (\dot{\sigma}/\dot{m}_3)}{\partial x} \bigg|_{c, T_1/T_2} = \frac{- \left(\frac{T_1}{T_2} \right)^{-x} \ln \left(\frac{T_1}{T_2} \right) \left[1 + x \left[\left(\frac{T_1}{T_2} \right) - 1 \right] \right] + \left(\frac{T_1}{T_2} \right)^{-x} \left[\left(\frac{T_1}{T_2} \right) - 1 \right]}{\left(\frac{T_1}{T_2} \right)^{-x} \left[1 + x \left[\left(\frac{T_1}{T_2} \right) - 1 \right] \right]}$$

Continued on next slide

Problem 6-76 continued

Setting this to zero

$$-\ln\left(\frac{T_1}{T_2}\right) \left[1 + x \left[\left(\frac{T_1}{T_2}\right) - 1 \right] \right] + \left[\left(\frac{T_1}{T_2}\right) - 1 \right] = 0$$

Solving

$$\textcircled{3} \quad x = \frac{\ln\left(\frac{T_1}{T_2}\right) + \left[1 - \left(\frac{T_1}{T_2}\right) \right]}{\ln\left(\frac{T_1}{T_2}\right) \left[1 - \left(\frac{T_1}{T_2}\right) \right]} \quad \leftarrow (c)$$

1. In applying Eq. 3.20b the underlined term below has been omitted

$$h_2 - h_1 = c(T_2 - T_1) + \underline{v(P_2 - P_1)}$$

Since the specific volume of liquids are small, a substantial pressure difference ($P_2 - P_1$) would be required before the underlined term becomes significant. Effects of pressure are neglected in assumption 3.

2. When $\dot{m}_1/\dot{m}_3 = 0$, Eq.(a) gives $T_3 = T_2$, then

$$\frac{\dot{Q}}{\dot{m}_3} = c \left[\left(\frac{\dot{m}_1}{\dot{m}_3} \right) \ln \frac{T_2}{T_1} + \ln \frac{T_3}{T_2} \right] = 0$$

And when $\dot{m}_1/\dot{m}_3 = 1$, Eq.(a) gives $T_3 = T_1$, then

$$\frac{\dot{Q}}{\dot{m}_3} = c \left[\left(\frac{\dot{m}_1}{\dot{m}_3} \right) \ln \frac{T_2}{T_1} + \ln \frac{T_3}{T_2} \right] = c \left[\ln \frac{T_2}{T_1} + \ln \frac{T_1}{T_2} \right] = 0$$

In these cases there is no mixing, and thus no entropy production.

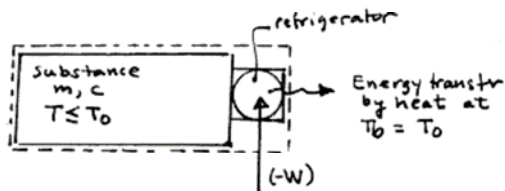
3. To verify that this locates a maximum, form the derivative $\partial^2(\dot{Q}/\dot{m}_3)/\partial x^2$ and evaluate the resulting expression with the expression for x here to show the second derivative is negative.

PROBLEM 6.77

KNOWN: The temperature of an incompressible substance of mass m and specific heat c is reduced from T_0 to T ($T < T_0$) by a refrigeration cycle.

FIND: Plot $(W_{\min}/m c T_0)$ versus T/T_0 , where $W_{\min}$ is the minimum theoretical work input required.

SCHEMATIC & GIVEN DATA:



ENER. MODEL: 1. The system shown in the accompanying figure is composed of an incompressible substance and a system that undergoes a refrigeration cycle. 2. The cycle discharges energy to the surroundings at T_0 .

ANALYSIS: An energy balance gives $(-W) = \Delta U - Q$, where ΔU is the change in internal energy of the substance because $(\Delta U)_{\text{cycle}} = 0$. With Eq. 3.20a

$$(-W) = mc[T - T_0] - Q \quad (1)$$

An entropy balance reads $\Delta S = (Q/T_0) + \sigma$, where ΔS is the change in entropy of the substance because $(\Delta S)_{\text{cycle}} = 0$. With Eq. 6.13

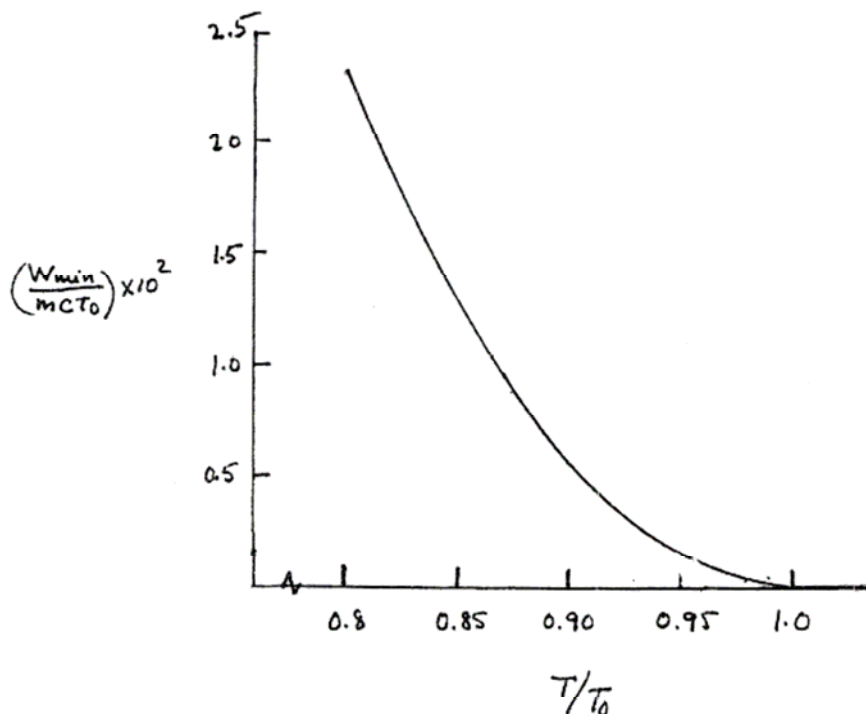
$$mc \ln\left(\frac{T}{T_0}\right) = \frac{Q}{T_0} + \sigma \Rightarrow Q = T_0 mc \ln\left(\frac{T}{T_0}\right) - T_0 \sigma \quad (2)$$

Collecting Eqs. (1), (2)

$$(-W) = mc[T - T_0] - T_0 mc \ln\left(\frac{T}{T_0}\right) + T_0 \sigma$$

Since $\sigma \geq 0$, the minimum work input corresponds to $\sigma = 0$, giving

$$W_{\min} = mc[T - T_0 - T_0 \ln\left(\frac{T}{T_0}\right)] \Rightarrow \left(\frac{W_{\min}}{m c T_0}\right) = \frac{T}{T_0} - 1 - \ln\left(\frac{T}{T_0}\right) \quad (3)$$

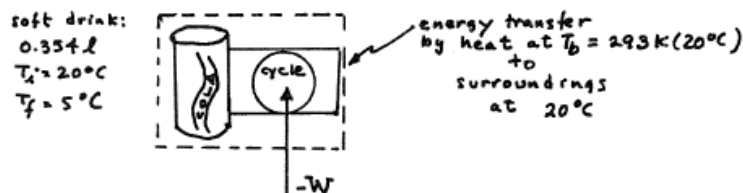


PROBLEM 6.78

KNOWN: A can of soft drink is cooled by a system undergoing a refrigeration cycle that receives energy by heat transfer from the soft drink and discharges energy by heat transfer to the surroundings.

FIND: Determine the minimum theoretical work required.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the accompanying figure is composed of two subsystems: The can of soft drink and a system that undergoes a refrigeration cycle. (2) The cycle discharges energy by heat transfer at $T_b = 293 \text{ K}$. (3) The soft drink can be modeled as an incompressible liquid with the properties of water. The aluminum can itself can be ignored.

ANALYSIS: An energy balance gives $(-W) = -Q + \Delta U$ where ΔU is the change in internal energy of the liquid because $\Delta U]_{\text{cycle}} = 0$ and the can is ignored. Thus, with Eq. 3.20a the work input is

$$(-W) = -Q + mc(T_f - T_i) \quad (1)$$

An entropy balance takes the form

$$\Delta S = \frac{Q}{T_b} + \sigma$$

where ΔS is the entropy change of the liquid because $\Delta S]_{\text{cycle}} = 0$ and the can is ignored. Thus, on solving for Q and using Eq. 6.13

$$Q = T_b \Delta S - T_b \sigma = T_b \left(mc \ln \frac{T_f}{T_i} \right) - T_b \sigma \quad (2)$$

Substituting Eq. (2) into Eq. (1)

$$(-W) = mc \left[T_b \ln \frac{T_i}{T_f} + T_f - T_i \right] + T_b \sigma$$

Since $\sigma \geq 0$, the work input $(-W)$ is a minimum when $\sigma = 0$, giving

$$(-W)_{\min} = mc \left[T_b \ln \frac{T_i}{T_f} + T_f - T_i \right]$$

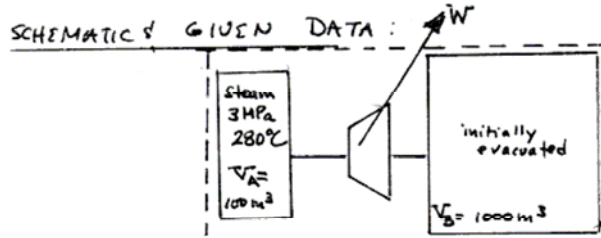
With $c \approx 4.2 \text{ kJ/kg} \cdot \text{K}$ from Table A-19 and $v \approx 1 \text{ cm}^3/\text{g}$ from Table A-2

$$\begin{aligned} (-W)_{\min} &= \left[(0.354 \text{ l}) \left(\frac{10^{-3} \text{ m}^3}{\text{l}} \right) \left(\frac{1 \text{ g}}{\text{cm}^3} \right) \left(\frac{10^6 \text{ cm}^3}{\text{m}^3} \right) \left(\frac{\text{kg}}{10^3 \text{ g}} \right) \right] (4.2 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \left[293 \ln \frac{293}{278} + 278 - 293 \right] \text{ K} \\ &= 0.591 \text{ kJ} \end{aligned}$$

PROBLEM 6.79

KNOWN: A turbine is located between two tanks, initially one tank is filled with steam and the other is evacuated. Steam is allowed to flow through the turbine until equilibrium is established.

FIND: Determine the maximum theoretical work that can be developed.



ENGR. MODEL: (1) The system is shown by the dashed line above. (2) Heat transfer can be ignored. (3) The initial and final states are equilibrium states. At the final state the mass of steam contained within the turbine and interconnecting piping can be ignored.

ANALYSIS: An energy balance reduces to $\Delta U = -W$, or $W = m(u_1 - u_2)$, where $m = \frac{V_A}{v_1}$. Thus, with steam table data, $u_1 = 2709.9 \text{ kJ/kg}$, $v_1 = 0.0771 \text{ m}^3/\text{kg}$, $s_1 = 6.4462 \text{ kJ/kg}\cdot\text{K}$,

$$W = \left(\frac{100 \text{ m}^3}{0.0771 \text{ m}^3/\text{kg}} \right) (2709.9 - u_2) \frac{\text{kJ}}{\text{kg}} = 1297 (2709.9 - u_2) \text{ kJ} \quad (1)$$

The final specific volume is

$$v_2 = \frac{V_A + V_B}{m} = v_1 \left[\frac{V_A + V_B}{V_A} \right] = (0.0771) \left(\frac{1100}{100} \right) = 0.8481 \frac{\text{m}^3}{\text{kg}} \quad (2)$$

An entropy balance reduces to $\Delta S = \int_1^2 \frac{\delta Q}{T} + \sigma \Rightarrow m(s_2 - s_1) = \sigma$. (3)

- By inspection of Eq. (1), the maximum value of W is attained when u_2 assumes the smallest allowed value at the value of v_2 given by Eq. (2). Since u and s change in the same direction at fixed v , the smallest allowed value for u_2 corresponds to the smallest allowed value for s_2 . Since $\sigma \geq 0$, Eq. (3) indicates that the smallest value of s_2 corresponds to $\sigma = 0$: $s_2 = s_1$. The final state of the steam is then fixed by: $v_2 = 0.8481 \text{ m}^3/\text{kg}$, $s_2 = 6.4462 \text{ kJ/kg}\cdot\text{K}$.

As this pair of independent properties is cumbersome to use with tabular steam table data, IT is employed to obtain $u_2 = 2270 \text{ kJ/kg}$. Returning to Eq. (1)

$$W_{\max} = 1297 (2709.9 - 2270) = 5.71 \times 10^5 \text{ kJ}$$

The final state is a two-phase liquid-vapor mixture with $T_2 = 117^\circ\text{C}$, $p_2 = 1.815 \text{ bar}$, $x_2 = 0.8741$.

When the entire problem is solved using IT, we get

Continued on next slide

Problem 6-79 continued

IT Solution:

p1 = 30 bar
T1 = 280 °C
VA = 100 m³
VB = 1000 m³

v1 = v_PT("Water/Steam", p1, T1)
u1 = u_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
m = VA / v1

W = m * (u1 - u2)
v2 = (VA + VB) / m
s2 = s1

v2 = vsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
u2 = usat_Px("Water/Steam", p2, x2)
T2 = Tsat_P("Water/Steam", p2)

Note: These are the IT results when the entire problem is solved using IT

T2	117.2
W	5.707E5
s2	6.445
u2	2269
v2	0.8483

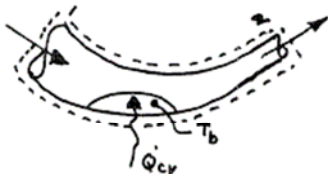
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1. When v is fixed, the first TDS equation — Eq. 6.10a — reads $Tds = du$. Thus, changes in u and s occur in the same direction.

PROBLEM 6.80

KNOWN: A gas flows through a one inlet, one exit control volume operating at steady state. Heat transfer $\dot{Q}_{cv}$ takes place only at temperature T_b .

FIND: For each of several cases, determine if the change in specific entropy from inlet to exit is positive, negative, or indeterminate

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state. (2) Heat transfer occurs only where the temperature is T_b .

ANALYSIS: At steady-state Eq. 6.34 reduces to Eq. 6.36. Then, since heat transfer takes place only at T_b and there is a single inlet and single exit, the entropy rate balance becomes

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

Solving for the change in specific entropy from inlet to exit

$$s_2 - s_1 = \frac{\dot{Q}_{cv}}{\dot{m} T_b} + \frac{\dot{\sigma}_{cv}}{\dot{m}}$$

(a) $\dot{\sigma}_{cv} = 0, \dot{Q}_{cv} = 0$

$$s_2 - s_1 = \frac{\cancel{\dot{Q}_{cv}}}{\dot{m} T_b} + \frac{\cancel{\dot{\sigma}_{cv}}}{\dot{m}} = 0$$

$$s_2 = s_1$$

(b) $\dot{\sigma}_{cv} = 0, \dot{Q}_{cv} < 0$:

$$s_2 - s_1 = \frac{\dot{Q}_{cv}}{\dot{m} T_b} + \frac{\cancel{\dot{\sigma}_{cv}}}{\dot{m}} < 0$$

negative

s decreases

(c) $\dot{\sigma}_{cv} = 0, \dot{Q}_{cv} > 0$:

$$s_2 - s_1 = \frac{\dot{Q}_{cv}}{\dot{m} T_b} + \frac{\cancel{\dot{\sigma}_{cv}}}{\dot{m}} > 0$$

positive

s increases

(d) $\dot{\sigma}_{cv} > 0, \dot{Q}_{cv} > 0$

$$s_2 - s_1 = \frac{\dot{Q}_{cv}}{\dot{m} T_b} + \frac{\dot{\sigma}_{cv}}{\dot{m}} > 0$$

positive or zero positive

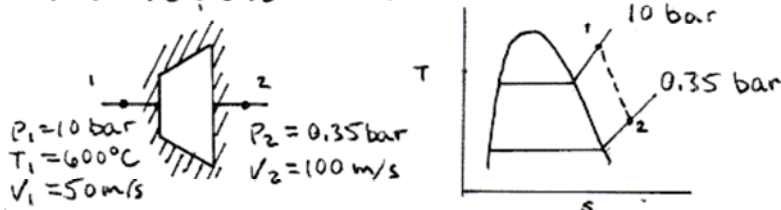
s increases

PROBLEM 6.81

KNOWN: An insulated turbine at steady state has steam entering at 10 bar, 600°C , 50 m/s and exiting at 0.35 bar, 100 m/s. The work developed is claimed to be (a) 1000 kJ/kg, (b) 500 kJ/kg.

FIND: Determine if the claim can be correct.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The turbine operates at steady-state and is well-insulated.
 (2) Potential energy effects can be ignored.

ANALYSIS: At steady-state the entropy rate balance reduces with the mass rate balance: $\dot{m}_1 = \dot{m}_2 = \dot{m}$ to give

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

Thus,

$$s_2 - s_1 = \frac{\dot{\sigma}_{cv}}{\dot{m}} \geq 0$$

Accordingly, the exiting specific entropy must be greater than, or equal to, the entering specific entropy.

From Table A-4 at 10 bar, 600°C , $s_1 = 8.0290 \text{ kJ/kg} \cdot \text{K}$. The exit specific entropy that corresponds to the claimed work value can be determined using an energy rate balance to fix state 2. Thus, at steady-state

$$0 = \dot{\sigma}_{cv} - \dot{W}_{cv} + \dot{m} \left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right)$$

Solving for h_2 , and introducing h_1 from Table A-4: $h_1 = 3697.9 \text{ kJ/kg}$

$$h_2 = -\frac{\dot{W}_{cv}}{\dot{m}} + h_1 + \frac{V_1^2 - V_2^2}{2} = -\frac{\dot{W}_{cv}}{\dot{m}} + 3697.9 + \left[\frac{(50)^2 - (100)^2}{2} \left(\frac{\text{m}^2}{\text{s}^2} \right) \right] \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= -\frac{\dot{W}_{cv}}{\dot{m}} + 3694.2 \frac{\text{kJ}}{\text{kg}} \quad (1)$$

(a) $\dot{W}_{cv}/\dot{m} = 1000 \text{ kJ/kg}$. Eq. (1) gives $h_2 = 2694.2 \text{ kJ/kg}$. Interpolating in Table A-4 at 0.35 bar with h_2 gives $s_2 = 7.8868 \text{ kJ/kg} \cdot \text{K}$. Since this value is less than s_1 , the claimed value cannot be correct.

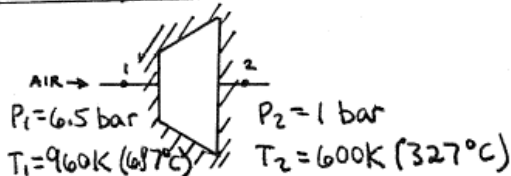
(b) $\dot{W}_{cv}/\dot{m} = 500 \text{ kJ/kg}$. Eq. (1) gives $h_2 = 3194.2 \text{ kJ/kg}$. Interpolating in Table A-4 at 0.35 bar with h_2 gives $s_2 = 8.8987 \text{ kJ/kg} \cdot \text{K}$. Since this value is greater than s_1 , the claimed value could be correct.

PROBLEM 6.82

KNOWN: Air enters and exits an insulated turbine at specified pressures and temperatures.

FIND: Determine the work developed per kg of air flowing, and whether the expansion is internally reversible, irreversible, or impossible.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The turbine operates at steady state and is well-insulated. (2) Changes in kinetic and potential energy from inlet to exit can be neglected. (3) Air is modeled as an ideal gas.

① **ANALYSIS:** An energy rate balance at steady state for a control volume enclosing the turbine reduces with a mass rate balance: $\dot{m}_1 = \dot{m}_2 = \dot{m}$ to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} (h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)})$$

or

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2$$

Then, with data from Table A-22

$$\frac{\dot{W}_{cv}}{\dot{m}} = 1000.55 - 607.02 = 393.53 \frac{\text{kJ}}{\text{kg}} \leftarrow$$

An entropy rate balance at steady state gives

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{Q}_{cv}$$

or

$$\frac{\dot{Q}_{cv}}{\dot{m}} = s_2 - s_1$$

With Eq. 6.26a and data from Table A-22

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}} &= s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1 \\ &= 2.40902 - 2.92128 - \frac{8.314}{28.97} \ln \frac{1}{6.5} = 0.0249 \text{ kJ/kg} \cdot \text{K} \end{aligned}$$

Since $\dot{Q}_{cv}/\dot{m}$ is positive, the expansion is irreversible.

1. From Table A-1, the critical pressure of air is $P_c = 37.7 \text{ bar}$ and the critical temperature is $T_c = 133 \text{ K}$. Thus, at the inlet and exit, respectively

$$\begin{aligned} P_{r1} &= \frac{6.5}{37.7} = 0.17 & P_{r2} &= \frac{1}{37.7} = 0.027 \\ T_{r1} &= \frac{960}{133} = 7.22 & T_{r2} &= \frac{600}{133} = 4.51 \end{aligned}$$

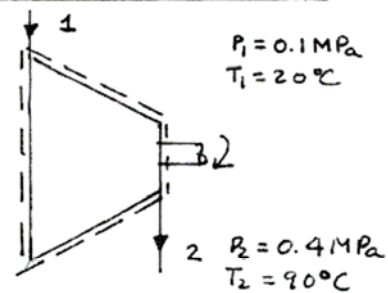
Referring to Fig A-1, these states fall into the region where the ideal gas model is appropriate.

PROBLEM 6.83

Propane at 0.1 MPa, 20°C enters an insulated compressor operating at steady state and exits at 0.4 MPa, 90°C. Neglecting kinetic and potential energy effects, determine

- the power required by the compressor, in kJ per kg of propane flowing.
- the rate of entropy production within the compressor, in kJ/K per kg of propane flowing.

SCHEMATIC & GIVEN DATA:



ENG. MODEL:

- The control volume shown in the sketch is at steady state.
- For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be ignored.

ANALYSIS: Table A-18, $h_1 = 517.6 \text{ kJ/kg}$, $s_1 = 2.194 \text{ kJ/kg} \cdot \text{K}$, $h_2 = 639.2 \text{ kJ/kg}$, $s_2 = 2.311 \text{ kJ/kg} \cdot \text{K}$.

An energy rate balance reduces as follows: $0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$

$$\Rightarrow \frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 = (517.6 - 639.2) \frac{\text{kJ}}{\text{kg}} = -121.6 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow (a)$$

An entropy rate balance reduces as follows:

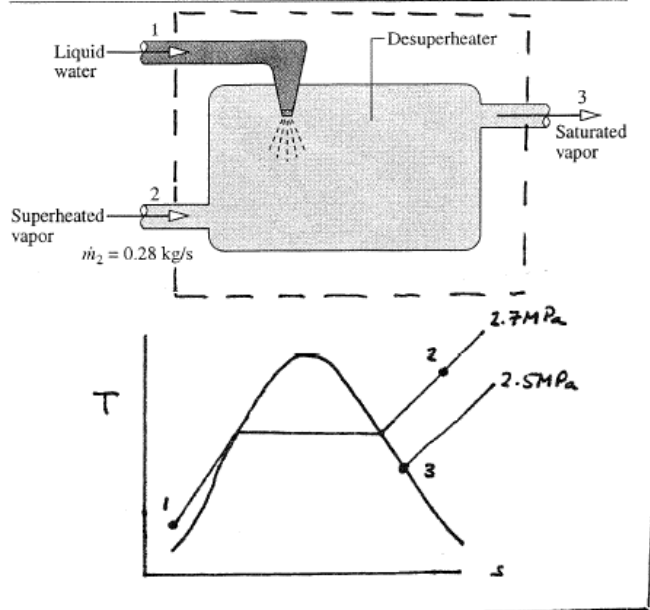
$$0 = \sum_j \cancel{\frac{\dot{Q}_j}{T_j}} + \dot{m}(s_1 - s_2) + \dot{S}$$

$$\Rightarrow \frac{\dot{S}}{\dot{m}} = s_2 - s_1 = (2.311 - 2.194) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.117 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad \leftarrow (b)$$

PROBLEM 6.84

By injecting liquid water into superheated steam, the desuperheater shown in Fig. P6.84 has a saturated vapor stream at its exit. Steady-state operating data are provided in the accompanying table. Stray heat transfer and all kinetic and potential energy effects are negligible. (a) Locate states 1, 2, and 3 on a sketch of the T - s diagram. (b) Determine the rate of entropy production within the desuperheater, in kW/K.

State	p (MPa)	T ($^{\circ}\text{C}$)	$v \times 10^3$ (m^3/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg $\cdot$ K)
1	2.7	40	1.0066	167.2	169.9	0.5714
2	2.7	300	91.01	2757.0	3002.8	6.6001
3	2.5	sat. vap.	79.98	2603.1	2803.1	6.2575



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. $\dot{W}_{cv} = 0$. $\dot{Q}_{cv}$ and the effects of kinetic and potential energy can be ignored.

ANALYSIS: An entropy rate balance reduces as follows:

$$0 = \sum_j \dot{Q}_j / T_j + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{S}_{cv}$$

$$\Rightarrow \dot{S}_{cv} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$\quad \quad \quad \uparrow$$

$$\quad \quad \quad L = (\dot{m}_1 + \dot{m}_2)$$

$$\Rightarrow \dot{S}_{cv} = (\dot{m}_1 + \dot{m}_2) s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 \quad (1)$$

To find $\dot{m}_1$, apply an energy rate balance:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left[h_1 + \frac{V_1^2}{2} + gz_1 \right] + \dot{m}_2 \left[h_2 + \frac{V_2^2}{2} + gz_2 \right] - \dot{m}_3 \left[h_3 + \frac{V_3^2}{2} + gz_3 \right]$$

$$\Rightarrow 0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

$$\quad \quad \quad \uparrow$$

$$\quad \quad \quad L = (\dot{m}_1 + \dot{m}_2)$$

$$\Rightarrow \dot{m}_1 = \dot{m}_2 \left[\frac{h_2 - h_3}{h_3 - h_1} \right]$$

$$= 0.28 \frac{\text{kg}}{\text{s}} \left[\frac{3002.8 - 2803.1}{2803.1 - 169.9} \right]$$

$$= 0.021 \text{ kg/s}$$

Inserting values, Eq. (1) gives

$$\dot{S}_{cv} = (0.301 \frac{\text{kg}}{\text{s}}) (6.2575 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) - (0.021) (0.5714) - (0.28) (6.6001)$$

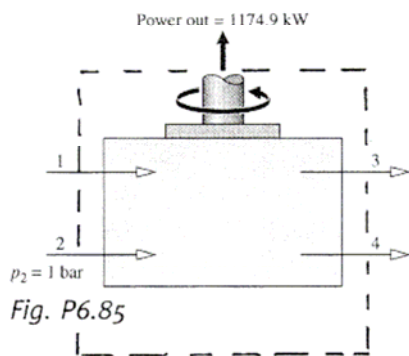
$$= 0.0235 \frac{\text{kJ/s}}{\text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 0.0235 \frac{\text{kW}}{\text{K}}$$

PROBLEM 6.85

An inventor claims that at steady state the device shown in Fig. P6.85 develops power from entering and exiting streams of water at a rate of 1174.9 kW. The accompanying table provides data for inlet 1 and exits 3 and 4. The pressure at inlet 2 is 1 bar. Stray heat transfer and kinetic and potential energy effects are negligible. Evaluate the inventor's claim.

State	$\dot{m}$ (kg/s)	p (bar)	T (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg·K)
1	4	1	450	3.334	3049.0	3382.4	8.6926
3	5	2	200	1.080	2654.4	2870.5	7.5066
4	3	4	400	0.773	2964.4	3273.4	7.8985



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects are negligible.

ANALYSIS: Begin by fixing the state at 2. This requires another property value to complement $p_2 = 1$ bar. To do this adopt the claimed power output and apply an energy rate balance.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 - \dot{m}_4 h_4$$

$$\text{mass rate balance: } \dot{m}_2 = \dot{m}_3 + \dot{m}_4 - \dot{m}_1 = 4 \text{ kg/s}$$

Solving the energy balance,

$$\begin{aligned} h_2 &= \frac{\dot{W}_{cv} - \dot{m}_1 h_1 + \dot{m}_3 h_3 + \dot{m}_4 h_4}{\dot{m}_2} \\ &= \frac{(1174.9 - 4(3382.4) + 5(2870.5) + 3(3273.4))}{4} \\ &= 2954.5 \text{ kJ/kg} \end{aligned}$$

With p_2 and h_2 , Table A-4 gives $s_2 = 7.9949 \text{ kJ/kg·K}$.

Now, checking the 2nd law via an entropy rate balance:

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 - \dot{m}_4 s_4 + \dot{Q}_{cv}$$

$$\begin{aligned} \Rightarrow \dot{Q}_{cv} &= \dot{m}_3 s_3 + \dot{m}_4 s_4 - \dot{m}_1 s_1 - \dot{m}_2 s_2 \\ &= (5)(7.5066) + (3)(7.8985) - (4)(8.6926) - 4(7.9949) \\ &= -5.52 \frac{\text{kJ/s}}{\text{K}} \end{aligned}$$

Since $\dot{Q}_{cv}$ must be positive for an actual device, the claimed performance is not allowed by the second law. The claimed power output and/or the given data must be in error.

PROBLEM 6.86

Figure P6.86 provides steady-state operating data for a well-insulated device having steam entering at one location and exiting at another. Neglecting kinetic and potential energy effects, determine (a) the direction of flow and (b) the power output or input, as appropriate, in kJ per kg of steam flowing.

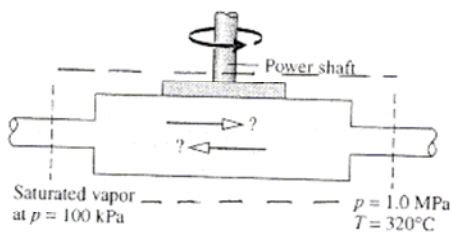


Fig. P6.86

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be neglected.

ANALYSIS: As discussed in Secs. 5.1 and 6.8, directionality normally can be established using the 2nd law. Here, a direction is assumed and the associated entropy production is evaluated. To begin, property data are found:

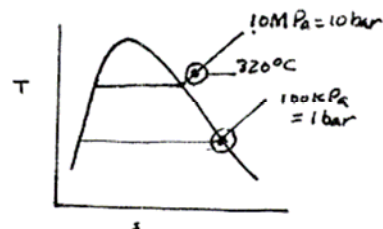
Table A-4: Saturated vapor at 100 kPa : $h = 2675.5 \text{ kJ/kg}$, $s = 7.3594 \text{ kJ/kg}\cdot\text{K}$
 $p = 1.0 \text{ MPa}$, 320°C : $h = 3093.9 \text{ kJ/kg}$, $s = 7.1962 \text{ kJ/kg}\cdot\text{K}$

Taking the inlet as the saturated vapor at 100 kPa , an entropy rate balance gives

$$\frac{dS^o}{dt} = \sum \frac{\dot{Q}_j^o}{T_j} + \dot{m}(s_{in} - s_{out}) + \dot{S}_{cv}$$

$$\Rightarrow \frac{\dot{S}_{cv}}{\dot{m}} = s_{out} - s_{in}$$

$$= (7.1962 - 7.3594) \frac{\text{kJ}}{\text{kg}\cdot\text{K}} = -0.1632 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$



As $\dot{S}_{cv}/\dot{m}$ must be ≥ 0 , the direction of flow must be opposite to that assumed: from 1.0 MPa , 320°C to saturated vapor at 100 kPa . (a)

Then, an energy rate balance reads,

$$\frac{dE^o}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_{in} - h_{out})$$

$$\Rightarrow \frac{\dot{W}_{cv}}{\dot{m}} = h_{in} - h_{out}$$

$$= (3093.9 - 2675.5) \frac{\text{kJ}}{\text{kg}}$$

$$= 418.4 \text{ kJ/kg}$$

(b)

PROBLEM 6.87

Steam enters a well-insulated nozzle operating at steady state at 1000°F , 500 lbf/in.^2 and a velocity of 10 ft/s . At the nozzle exit, the pressure is 14.7 lbf/in.^2 and the velocity is 4055 ft/s . Determine the rate of entropy production, in $\text{Btu}/^\circ\text{R}$ per lb of steam flowing.

ENGINEER MODEL:

1. A control volume at steady state encloses the nozzle.
2. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and potential energy effects play no role.

ANALYSIS: Reducing an entropy rate balance, $0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$

$$\Rightarrow \dot{\sigma}_{cv}/\dot{m} = s_2 - s_1 \quad (1)$$

From Table A-4E, $s_1 = 1.7371 \text{ Btu/lb} \cdot ^\circ\text{R}$, $h = 1520.7 \text{ Btu/lb}$.

To fix state 2, another property is needed to complement the pressure. This is specific enthalpy, h_2 found from an energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow h_2 = h_1 + \frac{V_1^2 - V_2^2}{2} = 1520.7 \frac{\text{Btu}}{\text{lb}} + \left[\frac{(10 \text{ ft/s})^2 - (4055 \text{ ft/s})^2}{2} \right] \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= 1192.5 \frac{\text{Btu}}{\text{lb}}$$

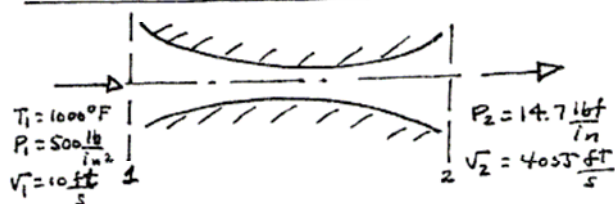
From Table A-4E at $P_2 = 14.7 \text{ lbf/in.}^2$, $s_2 = 1.8156 \text{ Btu/lb} \cdot ^\circ\text{R}$

Eq. (1) then gives

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = (1.8156 - 1.7371) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

$$= 0.079 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

SCHEMATIC & GIVEN DATA:



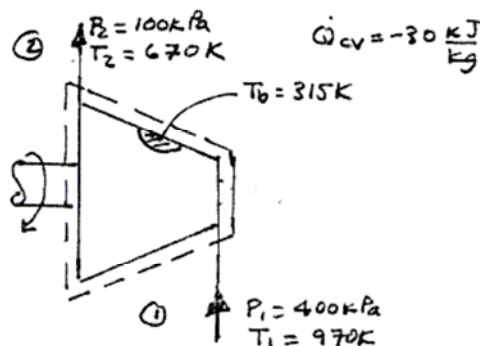
PROBLEM 6.88

Air at 400 kPa, 970 K enters a turbine operating at steady state and exits at 100 kPa, 670 K. Heat transfer from the turbine occurs at an average outer surface temperature of 315 K at the rate of 30 kJ per kg of air flowing. Kinetic and potential energy effects are negligible. For air as an ideal gas with $c_p = 1.1 \text{ kJ/kg} \cdot \text{K}$, determine (a) the rate power is developed, in kJ per kg of air flowing, and (b) the rate of entropy production within the turbine, in kJ/K per kg of air flowing.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Heat transfer occurs at $T_b = 315 \text{ K}$.
3. Kinetic and potential energy effects are negligible.
4. The air is modeled as an ideal gas with $c_p = 1.1 \text{ kJ/kg} \cdot \text{K}$.

SCHEMATIC & GIVEN DATA:



ANALYSIS: (a) An energy rate balance reads, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$.

or

$$\begin{aligned} \frac{\dot{W}_{cv}}{\dot{m}} &= \frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) \\ &= \frac{\dot{Q}_{cv}}{\dot{m}} + c_p(T_1 - T_2) = -\frac{30 \text{ kJ}}{\text{kg}} + 1.1 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}(970 - 670) \text{ K} \\ &= 300 \text{ kJ/kg} \end{aligned} \quad \leftarrow (a)$$

(b) An entropy rate balance reads,

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{\dot{Q}_{cv}/\dot{m}}{T_b} + (s_2 - s_1)$$

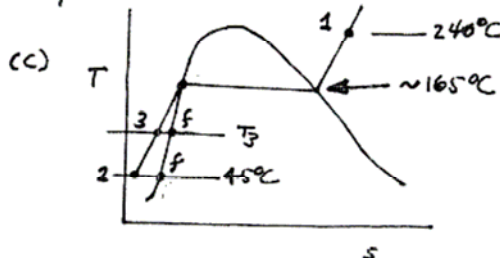
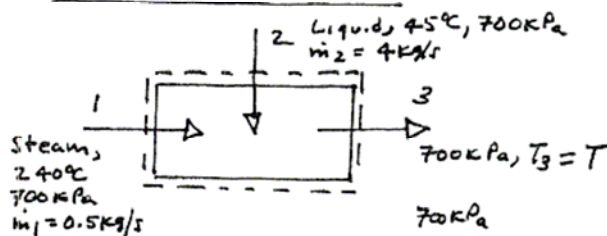
With Eq. 6.22

$$\begin{aligned} \frac{\dot{\sigma}_{cv}}{\dot{m}} &= -\frac{\dot{Q}_{cv}/\dot{m}}{T_b} + \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right] \\ &= -\frac{(-30 \text{ kJ/kg})}{315 \text{ K}} + \left[1.1 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln \frac{670}{970} - \frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \ln \left(\frac{100}{400} \right) \right] \\ &= 0.086 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \end{aligned} \quad \leftarrow (b)$$

PROBLEM 6.89

Steam at 240°C, 700 kPa enters an open feedwater heater operating at steady state with a mass flow rate of 0.5 kg/s. A separate stream of liquid water enters at 45°C, 700 kPa with a mass flow rate of 4 kg/s. A single mixed stream exits at 700 kPa and temperature T . Stray heat transfer and kinetic and potential energy effects can be ignored. Determine (a) T , in °C, and (b) the rate of entropy production within the feedwater heater, in kW/K. (c) Locate the three principal states on a sketch of the T - s diagram.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. Kinetic and potential energy effects can be ignored.
3. For liquid water at 700 kPa, $h \approx h_f(T)$ and $s \approx s_f(T)$.

ANALYSIS:

(a) To find T_3 , apply an energy rate balance: $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$.
Then with $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$

$$h_3 = \frac{\dot{m}_1 h_1 + \dot{m}_2 h_2}{\dot{m}_1 + \dot{m}_2} = \frac{(0.5 \text{ kg/s})(2932.2 \frac{\text{kJ}}{\text{kg}}) + (4)(188.45)}{4.5 \text{ kg/s}} \quad \leftarrow h_f(45^\circ\text{C})$$

$$= 493.3 \text{ kJ/kg}$$

Since this specific enthalpy is less than the saturated liquid value at 700 kPa (Table A-3), state 3 is in the liquid region. Thus, with $h_3 \approx h_f(T_3)$, we get $T_3 \approx 118^\circ\text{C}$, $s_3 \approx 1.501 \text{ kJ/kg}\cdot\text{K}$. (a)

(b) An entropy rate balance reduces to read,

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{Q}_{cv}$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$= (4.5 \frac{\text{kg}}{\text{s}})(1.501 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}) - (0.5)(7.0641) - (4)(0.6387) \quad \leftarrow s_f(45^\circ\text{C})$$

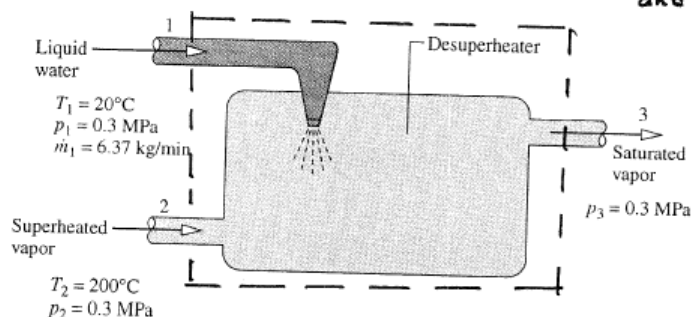
$$= 0.6676 \frac{\text{kJ/s}}{\text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 0.6676 \frac{\text{kW}}{\text{K}} \quad \leftarrow (b)$$

PROBLEM 6.90

By injecting liquid water into superheated vapor, the desuperheater shown in Fig. P6.90 has a saturated vapor stream at its exit. Steady-state operating data are shown on the figure. Ignoring stray heat transfer and kinetic and potential energy effects, determine (a) the mass flow rate of the superheated vapor stream, in kg/min, and (b) the rate of entropy production within the desuperheater, in kW/K.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$. Also, stray heat transfer and kinetic and potential energy effects are ignored.
3. At state 1, $h \approx h_f(T_1)$, $s \approx s_f(T_1)$

ANALYSIS: Property data:

Table A-2: $h_1 \approx h_f(T_1) = 83.96 \text{ kJ/kg}$ $s_1 \approx s_f(T_1) = 0.2966 \text{ kJ/kg} \cdot \text{K}$	Table A-4: $h_2 = 2865.5 \text{ kJ/kg}$ $s_2 = 7.3115 \text{ kJ/kg} \cdot \text{K}$	Table A-3: $h_3 = 2725.3 \text{ kJ/kg}$ $s_3 = 6.9919 \text{ kJ/kg} \cdot \text{K}$
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(a) To obtain the value of $\dot{m}_2$, write a mass rate balance: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ and an energy rate balance: $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$, obtaining

$$\dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3} \right] = 6.37 \frac{\text{kg}}{\text{min}} \left[\frac{2725.3 - 83.96}{2865.5 - 2725.3} \right] = 120.01 \frac{\text{kg}}{\text{min}} \quad \leftarrow (a)$$

(b) To obtain the rate of entropy production, write an entropy rate balance:

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{\sigma}_{cv} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$= (120.01 + 6.37) \frac{\text{kg}}{\text{min}} (6.9919 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) - (6.37)(0.2966) - (120.01)(7.3115)$$

$$= 4.3 \frac{\text{kJ/min}}{\text{K}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 0.072 \frac{\text{kW}}{\text{K}} \quad \leftarrow (b)$$

PROBLEM 6.91

Air at 600 kPa, 330 K enters a well-insulated, horizontal pipe having a diameter of 1.2 cm and exits at 120 kPa, 300 K. Applying the ideal gas model for air, determine at steady state (a) the inlet and exit velocities, each in m/s, (b) the mass flow rate, in kg/s, and (c) the rate of entropy production, in kW/K.

See solution to Problem 4.27.

(b) $\dot{m} = 0.04 \text{ kg/s}$.

(c) Reducing an entropy rate balance, $0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$
 $\Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) = \dot{m} \left(s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1} \right)$

$$= 0.04 \frac{\text{kg}}{\text{s}} \left[1.70203 - 1.79783 - \frac{8.314}{28.97} \ln \left(\frac{120}{600} \right) \right] \frac{\text{kJ}}{\text{K} \cdot \text{s}}$$

$$= 0.0146 \frac{\text{kJ/s}}{\text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 0.0146 \frac{\text{kW}}{\text{K}}$$

← (c)

PROBLEM 6.92

At steady state, air at 200 kPa, 52°C and a mass flow rate of 0.5 kg/s enters an insulated duct having differing inlet and exit cross-sectional areas. At the duct exit, the pressure of the air is 100 kPa, the velocity is 255 m/s, and the cross-sectional area is $2 \times 10^{-3} \text{ m}^2$. Assuming the ideal gas model, determine

- the temperature of the air at the exit, in °C.
- the velocity of the air at the inlet, in m/s.
- the inlet cross-sectional area, in m^2 .
- the rate of entropy production within the duct, in kW/K.

See solution to Problem 4.28.

(a) $T_2 = 353.4 \text{ K}$

(d) Reducing an entropy rate balance, $0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$

$$\Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) = \dot{m} \left(s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1} \right)$$

$$= 0.5 \frac{\text{kg}}{\text{s}} \left(1.87239 - 1.78249 - \frac{8.314}{28.97} \ln \frac{100}{200} \right) \frac{\text{kJ}}{\text{K} \cdot \text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 0.144 \frac{\text{kW}}{\text{K}}$$

← (d)

PROBLEM 6.93

KNOWN: Operating data are provided for an electronics enclosure at steady state.

FIND: For the control volume of Example 4.8 determine the rate of entropy production when air exits at 32°C.

SCHEMATIC & GIVEN DATA: See Fig. E 4.8

ENGINEERING MODEL: See Example 4.8. 2. Ignore the change in pressure from inlet to exit.

ANALYSIS: At steady state, mass and entropy rate balances reduce to give

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

Introducing $\dot{m}$ from Example 4.8 and evaluating $(s_2 - s_1)$ with Eq. 6.22

$$\dot{\sigma}_{cv} = \left[\frac{(-\dot{W}_{cv})}{c_p(T_2 - T_1)} \right] \left(c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right) \Rightarrow \dot{\sigma}_{cv} = \frac{(-\dot{W}_{cv})}{(T_2 - T_1)} \ln \frac{T_2}{T_1}$$

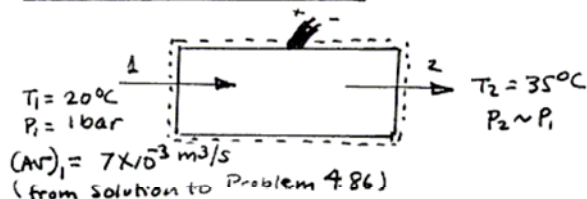
$$\Rightarrow \dot{\sigma}_{cv} = \frac{(98 \text{ W})}{(305 - 293) \text{ K}} \ln \frac{305}{293} = 0.328 \frac{\text{W}}{\text{K}} \quad \leftarrow \dot{\sigma}_{cv}$$

PROBLEM 6.94

KNOWN: Operating data are provided for an electronics enclosure at steady state.

FIND: For the control volume shown in the schematic determine the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$ and $P_2 \approx P_1$. 3. Air is modeled as an ideal gas.

ANALYSIS: Mass and entropy rate balances reduce at steady state to give

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

or with Eq. 6.22a and data from Table A-22

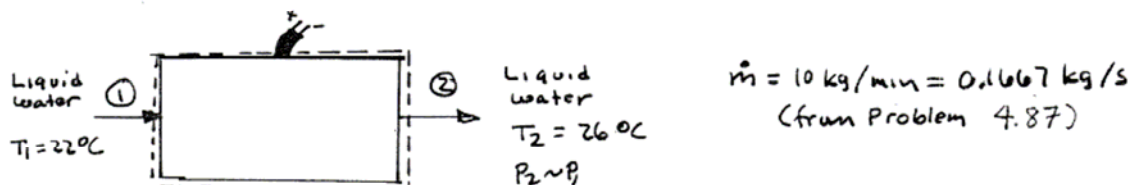
$$\begin{aligned} \dot{\sigma}_{cv} &= \frac{(\dot{AV})_1}{v_1} \left[s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1} \right] \\ &= \frac{(\dot{AV})_1}{(RT_1/P_1)} \left[s^\circ(T_2) - s^\circ(T_1) \right] = \left[\frac{(7 \times 10^{-3} \text{ m}^3/\text{s})(10^5 \text{ N/m}^2)}{(\frac{8314 \text{ N} \cdot \text{m}}{8.314 \text{ kg} \cdot \text{K}})(293 \text{ K})} \right] [1.7284 - 1.6785] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 4.15 \times 10^{-4} \frac{\text{kW}}{\text{K}} \quad \leftarrow \dot{\sigma}_{cv} \end{aligned}$$

PROBLEM 6.95

KNOWN: Steady-state operating data are provided for a water-jacketed electronics enclosure.

FIND: Determine the rate of entropy production when water exits at 26 °C.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$ 3. For the liquid water, $s \approx s_f(T)$.

ANALYSIS: Mass and entropy rate balances reduce at steady state to give

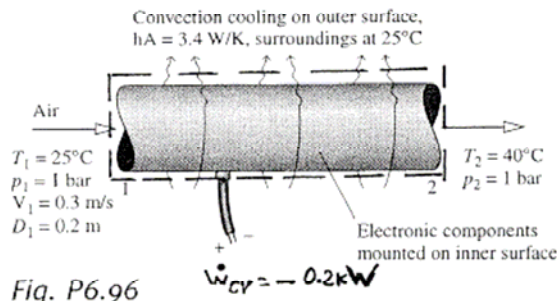
$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}[s_f(T_2) - s_f(T_1)]$$

With data from Table A-2

$$\dot{\sigma}_{cv} = \left(0.1667 \frac{\text{kg}}{\text{s}}\right) [0.3814 - 0.3251] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 9.4 \times 10^{-3} \frac{\text{kW}}{\text{K}} \quad \leftarrow \dot{\sigma}_{cv}$$

PROBLEM 6.96

Electronic components are mounted on the inner surface of a horizontal cylindrical duct whose inner diameter is 0.2 m, as shown in Fig. P6.96. To prevent overheating of the electronics, the cylinder is cooled by a stream of air flowing through it and by convection from its outer surface. Air enters the duct at 25°C, 1 bar and a velocity of 0.3 m/s and exits at 40°C with negligible changes in kinetic energy and



pressure. Convective cooling occurs on the outer surface to the surroundings, which are at 25°C, in accord with $hA = 3.4 \text{ W/K}$, where h is the heat transfer coefficient and A is the surface area. The electronic components require 0.20 kW of electric power. For a control volume enclosing the cylinder, determine at steady state (a) the mass flow rate of the air, in kg/s, (b) the temperature on the outer surface of the duct, in °C, and (c) the rate of entropy production, in W/K. Assume the ideal gas model for air.

ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. For the control volume, kinetic and potential energy effects can be ignored.
3. The air is modeled as an ideal gas.

ANALYSIS: (a) At steady state, $\dot{m}_1 = \dot{m}_2 = \dot{m}$, where

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{(\pi D_1^2/4) V_1}{RT_1/p_1}$$

$$= \frac{\pi (0.2 \text{ m})^2 (0.3 \text{ m/s}) (10^5 \text{ N/m}^2)}{4 \left(\frac{8314 \text{ N}\cdot\text{m}}{28.97 \text{ kg}\cdot\text{K}} \right) (298 \text{ K})} = 0.011 \frac{\text{kg}}{\text{s}} \quad \leftarrow (a)$$

(b) An energy rate balance is

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

or

$$\begin{aligned} \dot{Q}_{cv} &= \dot{W}_{cv} + \dot{m} (h_2 - h_1) \\ &= \dot{W}_{cv} + \dot{m} (h(T_2) - h(T_1)) \end{aligned}$$

Continued on next slide

Problem 6-96 continued

$$\begin{aligned}\dot{Q}_{cv} &= (-0.2 \text{ kW}) + (0.011 \frac{\text{kg}}{\text{s}}) \left[h(313 \text{ K}) - h(298 \text{ K}) \right] \left(\frac{\text{kJ}}{\text{kg}} \right) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= (-0.2 \text{ kW}) + (0.011 \frac{\text{kg}}{\text{s}}) \left[(313.3 - 298.2) \frac{\text{kJ}}{\text{kg}} \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= -0.034 \text{ kW} = -34 \text{ W}\end{aligned}$$

Invoking Eq. 2.34, $|\dot{Q}_{cv}| = hA(T_b - T_f)$ where $hA = 3.4 \frac{\text{W}}{\text{K}}$ and $T_f = 298 \text{ K}$.

$$\Rightarrow T_b = \frac{|\dot{Q}_{cv}|}{hA} + T_f = \frac{34 \text{ W}}{3.4 \text{ W/K}} + 298 \text{ K} = 308 \text{ K} \quad (35^\circ \text{C}) \quad \leftarrow (b)$$

(c) Mass and entropy rate balances reduce to read,

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_2 - s_1)$$

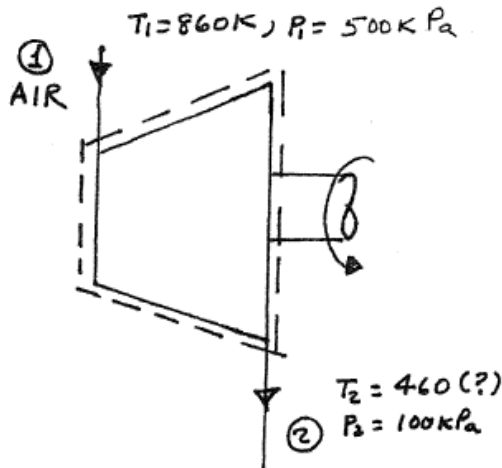
$$= -\frac{\dot{Q}_{cv}}{T_b} + \dot{m} \left(s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1} \right) \quad \text{with } P_2 = P_1$$

$$\begin{aligned}&= -\frac{(-34 \text{ W})}{308 \text{ K}} + 0.011 \frac{\text{kg}}{\text{s}} (1.7446 - 1.6953) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{10^3 \text{ J}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ W}}{1 \text{ J/s}} \right| \\ &= 0.653 \frac{\text{W}}{\text{K}}\end{aligned}$$

PROBLEM 6.97

Air enters a turbine operating at steady state at 500 kPa, 860 K and exits at 100 kPa. A temperature sensor indicates that the exit air temperature is 460 K. Stray heat transfer and kinetic and potential energy effects are negligible, and the air can be modeled as an ideal gas. Determine if the exit temperature reading can be correct. If yes, determine the power developed by the turbine for an expansion between these states, in kJ per kg of air flowing. If no, provide an explanation with supporting calculations.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential effects are negligible.
3. The air can be modeled as an ideal gas.

ANALYSIS: Apply an entropy rate balance:

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) \\ = \dot{m}(s_2^o - s_1^o - R \ln \frac{P_2}{P_1})$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2^o - s_1^o - R \ln \frac{P_2}{P_1} \\ = (2.13407 - 2.79783 - \frac{8.314}{28.97} \ln \frac{100}{500}) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ = -0.202 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Since $\dot{\sigma}_{cv}/\dot{m}$ cannot be negative, the exit temperature reading cannot be correct.

PROBLEM 6.98

Figure P6.98 provides steady-state test data for a control volume in which two entering streams of air mixed to form a single exiting stream. Stray heat transfer and kinetic and potential energy effects are negligible. A hard-to-read photocopy of the data sheet indicates that the pressure of the exiting stream is either 1.0 MPa or 1.8 MPa. Assuming the ideal gas model for air with $c_p = 1.02 \text{ kJ/kg} \cdot \text{K}$, determine if either or both of these pressure values can be correct.

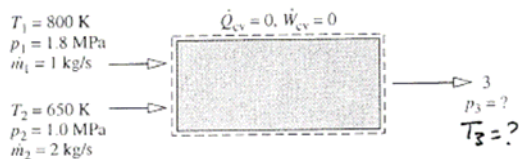


Fig. P6.98

ANALYSIS: Mass rate balance: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$. To find T_3 , apply an energy rate balance:

$$0 = \cancel{w_{cv}} + m_1 h_1 + m_2 h_2 - m_3 h_3 \Rightarrow 0 = m_1 \underbrace{(h_1 - h_3)}_{c_p(T_1 - T_3)} + m_2 \underbrace{(h_2 - h_3)}_{c_p(T_2 - T_3)}$$

$$\Rightarrow T_3 = \frac{m_1 T_1 + m_2 T_2}{(m_1 + m_2)} = \frac{(1 \text{ kg/s})(800\text{K}) + (2 \text{ kg/s})(650\text{K})}{3 \text{ kg/s}} = 700\text{K}$$

An entropy rate balance reduces to read, $0 = \sum \dot{Q}_i / T_i + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{S}_{cv}$

$$\Rightarrow Q_{cv} = (m_1 + m_2)S_3 - m_1 S_1 - m_2 S_2 = m_1(S_3 - S_1) + m_2(S_3 - S_2)$$

W. zu Eq. 6.22

$$\dot{Q}_{cv} = \dot{m}_1 \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right] + \dot{m}_2 \left[c_p \ln \frac{T_2}{T_2} - R \ln \frac{P_2}{P_2} \right]$$

$$\dot{\sigma}_{cv} = c_p \left[m_1 \ln \frac{T_3}{T_1} + m_2 \ln \frac{T_2}{T_2} \right] - R \left[m_1 \ln \frac{P_3}{P_1} + m_2 \ln \frac{P_2}{P_2} \right]$$

Inserting values

$$\begin{aligned} \sigma_{cv} &= 1.02 \frac{\text{KJ}}{\text{KJ} \cdot \text{K}} \left[\frac{1 \text{ KJ}}{5} \ln \frac{700}{800} + \frac{2 \text{ KJ}}{5} \ln \frac{700}{600} \right] - \frac{8.314}{28.97} \frac{\text{KJ}}{\text{KJ} \cdot \text{K}} \left[1 \frac{\text{KJ}}{5} \ln \frac{P_3}{1.8} + 2 \frac{\text{KJ}}{5} \ln \frac{P_3}{1.0} \right] \\ &= 0.01499 \frac{\text{KJ/s}}{\text{K}} - 0.287 \frac{\text{KJ}}{\text{KJ} \cdot \text{K}} \left[\frac{1 \text{ KJ}}{5} \ln \frac{P_3}{1.8} + 2 \frac{\text{KJ}}{5} \ln \frac{P_3}{1.0} \right] \quad (1) \end{aligned}$$

check $P_3 = 1.0 \text{ MPa}$. Eq. (1) gives

$$\dot{\sigma}_{cv} = 0.01499 \frac{\text{KJ/s}}{\text{K}} - 0.287 \frac{\text{KJ}}{\text{kg} \cdot \text{K}} \left[1 \frac{\text{kg}}{\text{s}} \ln \frac{1.0}{1.8} \right] = 0.1837 \frac{\text{KJ/s}}{\text{K}}$$

Yes. This value for B_2 can be correct.

Check $P_3 = 1.8 \text{ MPa}$. Eq. (1) gives

$$\dot{Q}_{CV} = 0.01499 \frac{\text{KJ}}{\text{s}} - 0.287 \frac{\text{KJ}}{\text{Ks}} \left[\frac{2 \text{ kg}}{3} \ln \frac{1.8}{1.0} \right] = -0.3224 \frac{\text{KJ}}{\text{K}}$$

NO. This value for P_3 cannot be correct.

ENGR. MODEL:

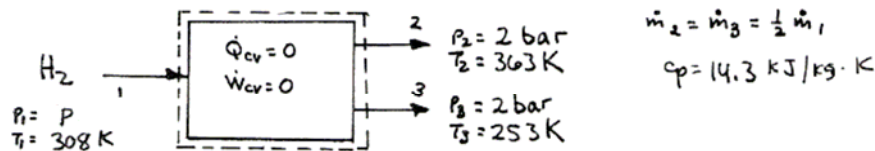
1. The control volume shown in the figure is at steady state.
2. Stray heat transfer and kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas with $c_p = 1.02 \text{ kJ/kg} \cdot \text{K}$.

PROBLEM 6.99

KNOWN: Hydrogen (H_2) gas enters an insulated control volume operating at steady state at $35^\circ C$ and pressure p . Half exits at $90^\circ C$, 2 bar and half exits at $-20^\circ C$, 2 bar. For the control volume, $\dot{W}_{cv} = 0$.

FIND: Determine the minimum allowed value for p .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state. (2) The effects of kinetic and potential energy are negligible. (3) Hydrogen is modeled as an ideal gas with constant c_p .

ANALYSIS: The given information must satisfy the conservation of energy principle. Thus, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and assumptions 1 and 2 an energy rate balance reduces to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

or with $\dot{m}_2 = \dot{m}_3 = \dot{m}_1/2$

$$0 = h_1 - \frac{1}{2} h_2 - \frac{1}{2} h_3$$

As c_p is constant, $h = c_p (T - T_{ref})$, where T_{ref} is an arbitrary reference temperature. With this for h_1 , h_2 , and h_3 the energy rate balance gives

$$0 = T_1 - \frac{1}{2} T_2 - \frac{1}{2} T_3$$

Introducing the given temperatures

$$0 = 308 - \frac{363}{2} - \frac{253}{2} \Rightarrow 0 = 308 - 181.5 - 126.5$$

Thus, the conservation of energy principle is satisfied.

Turning next to the second law, an entropy rate balance reduces to give

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

With $\dot{m}_2 = \dot{m}_3 = \dot{m}_1/2$

$$0 = \frac{1}{2} (s_1 - s_2) + \frac{1}{2} (s_1 - s_3) + \frac{\dot{\sigma}_{cv}}{\dot{m}_1}$$

Introducing Eq. 6.22 with $P_2 = P_3$ and $P_1 = P$

$$0 = \frac{1}{2} \left[c_p \ln \frac{T_1}{T_2} - R \ln \frac{P}{P_2} \right] + \frac{1}{2} \left[c_p \ln \frac{T_1}{T_3} - R \ln \frac{P}{P_2} \right] + \frac{\dot{\sigma}_{cv}}{\dot{m}_1}$$

$$\Rightarrow \ln \frac{P}{P_2} = \frac{c_p}{2R} \ln \left(\frac{T_1^2}{T_2 T_3} \right) + \frac{1}{R} \left(\frac{\dot{\sigma}_{cv}}{\dot{m}_1} \right) = \left[\frac{14.3 \text{ kJ/kg} \cdot \text{K}}{2(8.314/2.016) \text{ kJ/kg} \cdot \text{K}} \right] \ln \left(\frac{(308)^2}{363 \times 253} \right) + \frac{1}{R} \left(\frac{\dot{\sigma}_{cv}}{\dot{m}_1} \right)$$

$$= 0.0562 + \frac{1}{R} \left(\frac{\dot{\sigma}_{cv}}{\dot{m}_1} \right)$$

Since $P_2 = 2 \text{ bar}$

$$\Rightarrow p = (2.12 \text{ bar}) \exp \left(\frac{\dot{\sigma}_{cv}/\dot{m}_1}{R} \right)$$

where $\dot{\sigma}_{cv}/\dot{m}_1 \geq 0$. Thus, in the limit as $\dot{\sigma}_{cv}/\dot{m}_1 \rightarrow 0$, $\rightarrow 2.12 \text{ bar}$

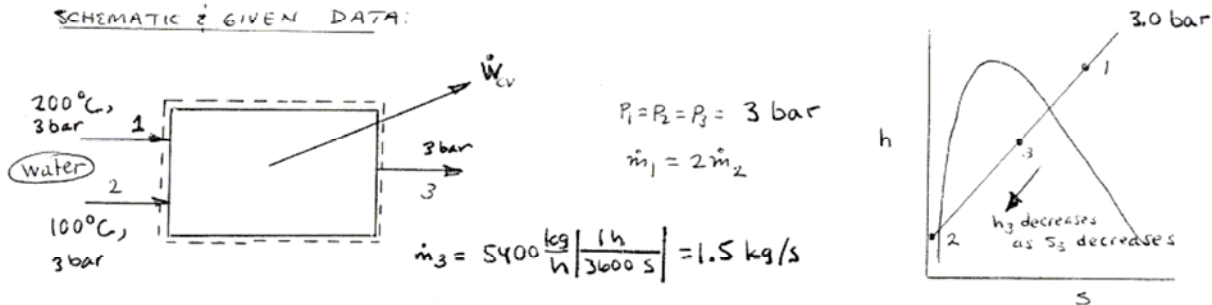
min. pressure $\leftarrow$

PROBLEM 6.100

KNOWN: Test data are provided for a new type of engine operating at steady state.

FIND: Determine the maximum theoretical rate power can be developed.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown above is at steady state.

(2) For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects are negligible. (3) For liquid water at 2, $h_2 \approx h_f(T_2)$, $s_2 \approx s_f(T_2)$.

ANALYSIS: At steady state, a mass rate balance gives $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$. Then, since $\dot{m}_3 = 1.5 \text{ kg/s}$ and $\dot{m}_1 = 2\dot{m}_2$, $\dot{m}_1 = 1 \text{ kg/s}$ and $\dot{m}_2 = 0.5 \text{ kg/s}$.

Using given assumptions, an energy rate balance reduces at steady state to give

$$\dot{W}_{cv} = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

With data from Tables A-2, 4, using $h_2 \approx h_f(T_2)$

$$\begin{aligned} \dot{W}_{cv} &= \left(1 \frac{\text{kg}}{\text{s}}\right) (2865.5 \frac{\text{kJ}}{\text{kg}}) + \left(0.5 \frac{\text{kg}}{\text{s}}\right) (419.04 \frac{\text{kJ}}{\text{kg}}) - \left(1.5 \frac{\text{kg}}{\text{s}}\right) (h_3) \\ &= (3075.0 - 1.5 h_3) \text{ kW} \quad (1) \end{aligned}$$

From Eq. (1) it can be concluded that $\dot{W}_{cv}$ increases as h_3 decreases.

The above Mollier diagram shows that the smallest allowed value for h_3 corresponds to the smallest allowed value for s_3 .

To determine the smallest allowed value for s_3 , apply an entropy rate balance at steady state: $0 = \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$

$$\Rightarrow s_3 = \frac{\dot{m}_1}{\dot{m}_3} s_1 + \frac{\dot{m}_2}{\dot{m}_3} s_2 + \frac{\dot{\sigma}_{cv}}{\dot{m}_3} \Rightarrow s_3 = \left(\frac{2}{3}\right) s_1 + \left(\frac{1}{3}\right) s_2 + \frac{\dot{\sigma}_{cv}}{\dot{m}_3}$$

Since $\dot{\sigma}_{cv} \geq 0$, the smallest allowed value for s_3 , and thus h_3 , corresponds to $\dot{\sigma}_{cv} = 0$. With data from Tables A-2, A-4, and using $s_2 = s_f(T_2)$

$$(s_3)_{\min} = \left(\frac{2}{3}\right) s_1 + \left(\frac{1}{3}\right) s_2 = \left(\frac{2}{3}\right) (7.3115) + \left(\frac{1}{3}\right) (1.3069) = 5.3100 \text{ kJ/kg} \cdot \text{K}$$

Using this value, state 3 is located in the two-phase region with quality

$$(x_3)_{\min} = \frac{5.3100 - 1.6718}{5.3201} = 0.684 \Rightarrow (h_3)_{\min} = 561.47 + 0.684(2163.8) = 2041.5 \text{ kJ/kg}$$

$$\textcircled{1} \text{ Finally, with Eq. (1), } (\dot{W}_{cv})_{\max} = [3075.0 - 1.5(2041.5)] = 12.75 \text{ kW} \leftarrow$$

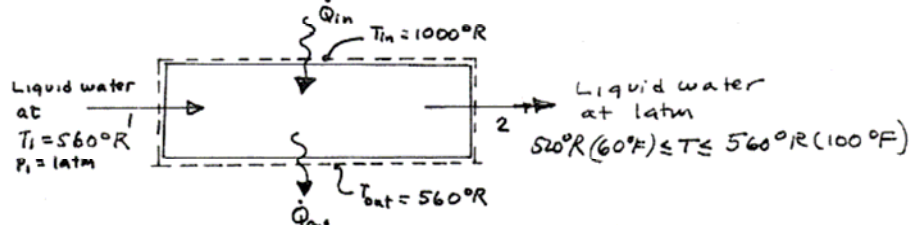
1. From Eq. (1), the case $\dot{W}_{cv} = 0$ corresponds to $h_3 = 2050 \text{ kJ/kg}$, or $x_3 = 0.688$, which differs only slightly from $(x_3)_{\min}$. Accordingly, with such a small interval between maximum power and no power output, the effect of irreversibilities within the engine is expected to erode significantly the power that would be obtained.

PROBLEM 6.101

KNOWN: A thermally activated device for chilling water requires no power input to operate. The chilled water exits at temperature T .

FIND: Plot the minimum theoretical heat addition required per lb of chilled water versus T .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$. (2) Changes in kinetic and potential energy from inlet to exit can be neglected. (3) The water can be modeled as incompressible with constant specific heat c .

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and an energy rate balance reduces with listed assumptions to give

$$0 = \dot{Q}_{in} - \dot{Q}_{out} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \Rightarrow \frac{\dot{Q}_{in}}{\dot{m}} = \frac{\dot{Q}_{out}}{\dot{m}} + h_2 - h_1 \quad (1)$$

At steady state an entropy rate balance reduces to give

$$0 = \frac{\dot{Q}_{in}}{T_{in}} - \frac{\dot{Q}_{out}}{T_{out}} + \dot{m}(s_1 - s_2) + \dot{S}_{cv}$$

Solving for $\dot{Q}_{out}$

$$\frac{\dot{Q}_{out}}{\dot{m}} = \left(\frac{T_{out}}{T_{in}} \right) \left(\frac{\dot{Q}_{in}}{\dot{m}} \right) + T_{out}(s_1 - s_2) + T_{out} \frac{\dot{S}_{cv}}{\dot{m}}$$

and substituting into Eq. (1)

$$\left[1 - \frac{T_{out}}{T_{in}} \right] \frac{\dot{Q}_{in}}{\dot{m}} = T_{out}(s_1 - s_2) + (h_2 - h_1) + T_{out} \frac{\dot{S}_{cv}}{\dot{m}}$$

Or, upon rearrangement

$$\frac{\dot{Q}_{in}}{\dot{m}} = \frac{T_{out}(s_1 - s_2) + (h_2 - h_1)}{\left[1 - \frac{T_{out}}{T_{in}} \right]} + \frac{T_{out} \frac{\dot{S}_{cv}}{\dot{m}}}{\left[1 - \frac{T_{out}}{T_{in}} \right]} \quad (1)$$

Evaluating $(h_2 - h_1)$ using Eq. 3.206 and $(s_2 - s_1)$ using Eq. 6.13, the minimum heat addition corresponds to $\dot{S}_{cv} = 0$ in Eq. (1), giving

$$\left(\frac{\dot{Q}_{in}}{\dot{m}} \right)_{min} = \frac{c \left[T_{out} \ln \left(\frac{T_1}{T} \right) + (T - T_1) \right]}{\left[1 - \frac{T_{out}}{T_{in}} \right]}, \text{ where } c = 1 \text{ Btu/lb} \cdot ^\circ\text{R (Table A-19E)},$$

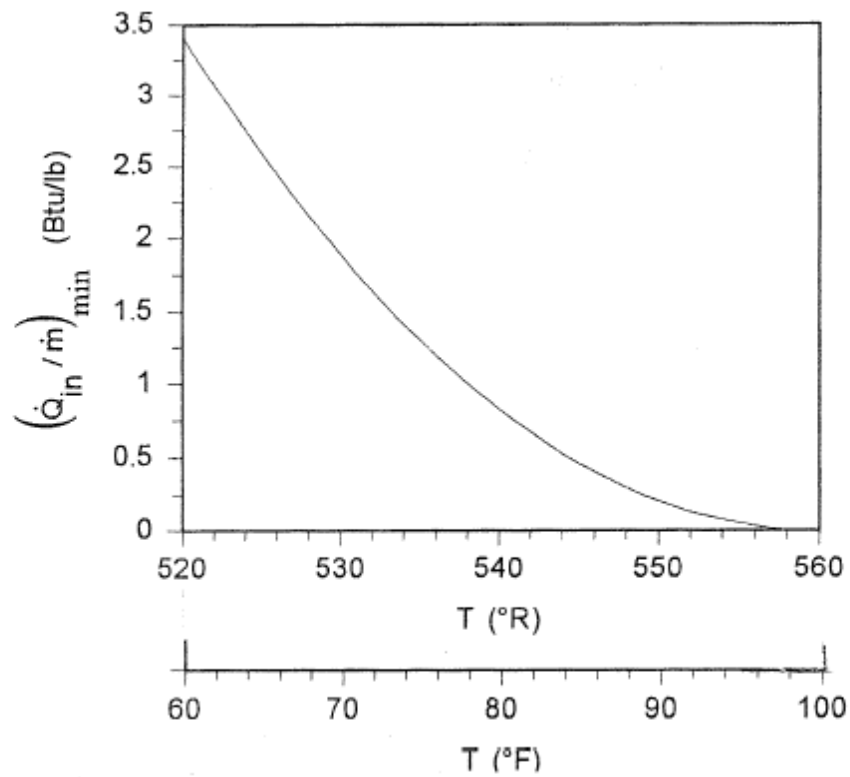
$T_{in} = 1000^\circ\text{R}, T_{out} = 560^\circ\text{R}, T_1 = 560^\circ\text{R}, \text{ and } 520 \leq T \leq 560^\circ\text{R}.$

Sample calculation: $T = 520^\circ\text{R}, (\dot{Q}_{in}/\dot{m})_{min} = 3.41 \text{ Btu/lb}.$

Plot on next page

Continued on next page

Problem 6-101 continued

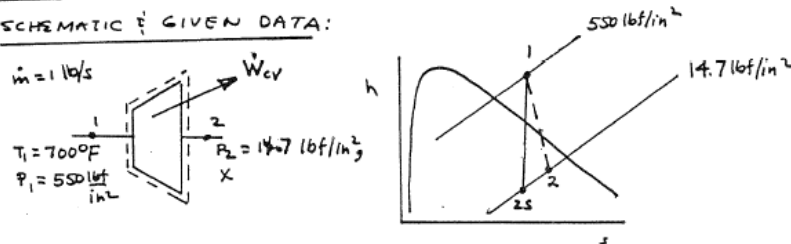


PROBLEM 6.102

KNOWN: Steady state operating data are provided for an insulated steam turbine. A two-phase liquid-vapor mixture with quality x exits the turbine.

FIND: Plot $\dot{W}_{cv}$ and $\dot{\sigma}_{cv}$ versus x

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the figure is at steady state. (2) At the exit the steam is a two-phase liquid-vapor mixture with quality x . (3) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy changes can be ignored.

ANALYSIS: Reducing mass, energy, and entropy rate balances

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2), \quad \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

From steam table data, $h_1 = 1353.7 \text{ Btu/lb}$, $s_1 = 1.5987 \text{ Btu/lb} \cdot ^\circ\text{R}$, $h_2 = h_f + x h_{fg} = 180.2 + x(970.4)$, $s_2 = s_f + x s_{fg} = 0.3121 + x(1.4446)$. Then

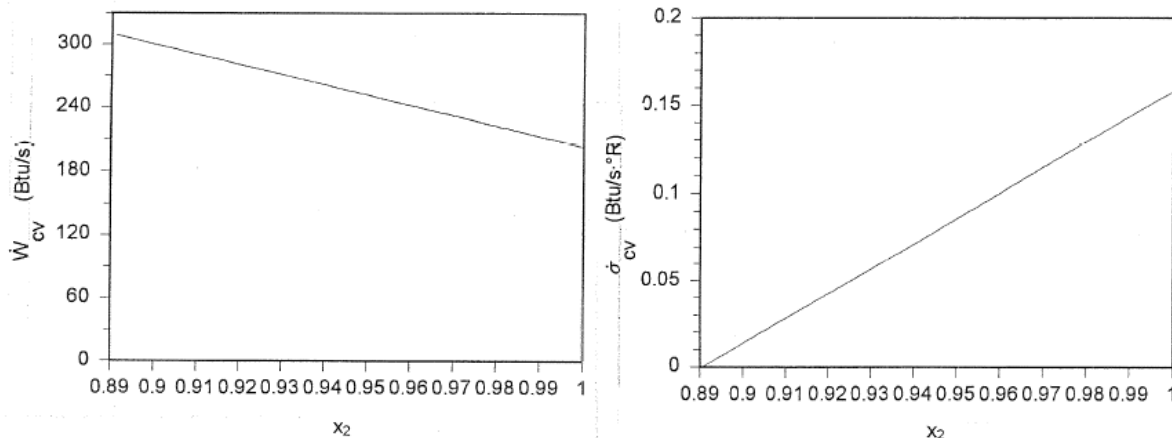
$$\dot{W}_{cv} = \left(1 \frac{\text{lb}}{\text{s}}\right)(1353.7 - 180.2 - 970.4x) \left(\frac{\text{Btu}}{\text{lb}}\right) = [1173.5 - 970.4x] \frac{\text{Btu}}{\text{s}} \quad (3)$$

$$\dot{\sigma}_{cv} = \left(1 \frac{\text{lb}}{\text{s}}\right)(0.3121 + 1.4446x - 1.5987) \left(\frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}\right) = [1.4446x - 1.2866] \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}} \quad (4)$$

Sample calculation: Since $\dot{\sigma}_{cv} \geq 0$, the smallest allowed value for x is x_{2s} corresponding to $\dot{\sigma}_{cv} = 0$. Setting Eq. (4) to zero and solving; $x_{2s} = 0.8906$. The corresponding value of $\dot{W}_{cv}$ from Eq. (3) is 309.3 Btu/s .

The following plots of Eq. (3) and (4) are made using IT, although other plotting software such as a spreadsheet could be used.

Plots:



The plots show that greater entropy production (irreversibility) corresponds to less power output for the given operating conditions, as expected.

Continued on next slide

Problem 6-102 continued

$\dot{m} = 1 \text{ // lb/s}$
 $T_1 = 700 \text{ // } ^\circ\text{F}$
 $p_1 = 550 \text{ // lbf/in.}^2$
 $p_2 = 14.7$
 $x_2 = 0.8906$

$\dot{W} = \dot{m} * (h_1 - h_2)$
 $\dot{\sigma} = \dot{m} * (s_2 - s_1)$

$h_1 = h_{\text{PT}}(\text{"Water/Steam"}, p_1, T_1)$
 $s_1 = s_{\text{PT}}(\text{"Water/Steam"}, p_1, T_1)$
 $h_2 = h_{\text{sat_Px}}(\text{"Water/Steam"}, p_2, x_2)$
 $s_2 = s_{\text{sat_Px}}(\text{"Water/Steam"}, p_2, x_2)$

IT Results for $x_2 = 1$

$h_1 = 1353 \text{ Btu/lb}$
 $h_2 = 1150 \text{ Btu/lb}$
 $s_1 = 1.599 \text{ Btu/lb} \cdot ^\circ\text{R}$
 $s_2 = 1.756 \text{ Btu/lb} \cdot ^\circ\text{R}$
 $\dot{W}_{\text{cv}} = 203.2 \text{ Btu/s}$
 $\dot{\sigma}_{\text{cv}} = 0.1579 \text{ Btu/s} \cdot ^\circ\text{R}$

IT Results for $x_2 = 0.8906$

$h_1 = 1353 \text{ Btu/lb}$
 $h_2 = 1044 \text{ Btu/lb}$
 $s_1 = 1.599 \text{ Btu/lb} \cdot ^\circ\text{R}$
 $s_2 = 1.598 \text{ Btu/lb} \cdot ^\circ\text{R}$
 $\dot{W}_{\text{cv}} = 309.3 \text{ Btu/s}$
 $\dot{\sigma}_{\text{cv}} = 0$

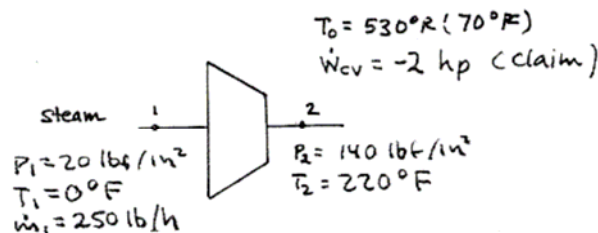
PROBLEM 6.103

KNOWN: R134a enters and exits a compressor operating at steady state at specified temperatures and pressures. The ambient temperature is also specified.

The required power input is claimed to be 2 hp.

FIND: Determine whether this claim can be correct.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) Two control volumes at steady state are considered. One encloses the compressor. The other encloses the compressor plus a portion of the nearby surroundings so heat transfer occurs at the ambient temperature T_0 . (2) Changes in kinetic and potential energy from inlet to exit can be ignored.

ANALYSIS: An entropy rate balance is used to determine if the claimed value for the power input can be correct. But first consider a control volume enclosing the compressor, and apply an energy rate balance to obtain the heat transfer. Thus with $\dot{m}_1 = \dot{m}_2 = \dot{m}$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2))$$

With assumption 2 and data from Table A-1E

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 - h_1 \Rightarrow \frac{\dot{Q}_{cv}}{\dot{m}} = \frac{(-2 \text{ hp}) \left(\frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right)}{(250 \text{ lb/h})} + (146.18 - 101.88) = 23.94 \text{ Btu/lb}$$

As the variation of temperature over the surface of this control volume is not specified, the entropy transfer accompanying heat transfer cannot be evaluated without further information. Accordingly, consider an entropy rate balance for an enlarged control volume enclosing the compressor plus a portion of the nearby surroundings so heat transfer occurs at the ambient temperature T_0 . That is

$$0 = \frac{\dot{Q}_{cv}}{T_0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

where $\dot{\sigma}_{cv}$ represents the rate of entropy production within the enlarged control volume. With data from Table A-4E

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{\dot{Q}_{cv}/\dot{m}}{T_0} + s_2 - s_1 = \frac{-(23.94 \text{ Btu/lb})}{530^\circ\text{R}} + 0.2667 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} - 0.228 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

①

$$= -0.00227 \text{ Btu/lb} \cdot ^\circ\text{R}$$

As $\dot{\sigma}_{cv} \geq 0$, the claimed value for the work input cannot be correct. $\leftarrow$

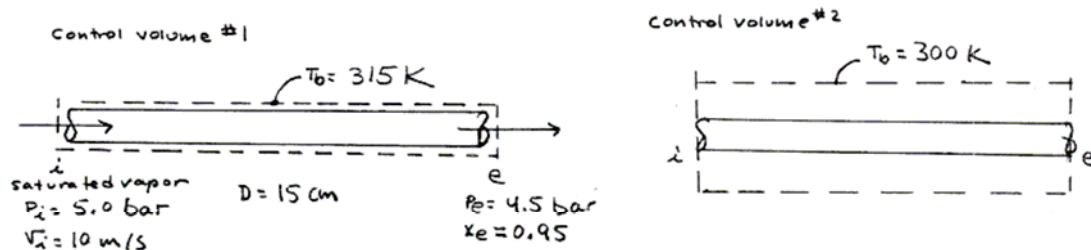
1. Note that the calculated value for $\dot{\sigma}_{cv}/\dot{m}$ is not for the compressor alone, but for an enlarged control volume of compressor plus nearby surroundings.

PROBLEM 6.104

KNOWN: Steady-state operating data are provided for steam flowing through a pipe.

FIND: Determine (a) the velocity of the steam at the pipe exit, (b) the rate of heat transfer from the pipe, (c) the rate of entropy production for each of two specifications of the control volume.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The two control volumes shown above are at steady state. (2) For the control volumes, heat transfer with the surroundings occurs at temperature T_0 only; $\dot{W}_{cv} = 0$; and potential energy effects are absent.

ANALYSIS: Consider control volume #1. At steady state a mass rate balance reads $\dot{m}_i = \dot{m}_e$. Thus

$$\frac{A_i V_i}{v_i} = \frac{A_e V_e}{v_e} \Rightarrow V_e = \frac{v_e}{v_i} V_i = \left(\frac{0.3935 \text{ m}^3/\text{kg}}{0.3749 \text{ m}^3/\text{kg}} \right) (10 \text{ m/s}) = 10.50 \text{ m/s} \leftarrow V_e$$

Steam table data: $v_i = 0.3749 \text{ m}^3/\text{kg}$, $v_e = v_f + x_e v_{fg} = 0.0010882 + 0.95(0.4140 - 0.0010882) = 0.3935 \text{ m}^3/\text{kg}$
With these data the mass flow rate is

$$\dot{m} = \frac{A_i V_i}{v_i} = \left(\frac{\pi D^2}{4} \right) \left(\frac{V_i}{v_i} \right) = \frac{\pi}{4} (0.15 \text{ m})^2 \left(\frac{10 \text{ m/s}}{0.3749 \text{ m}^3/\text{kg}} \right) = 0.471 \text{ kg/s}$$

Mass and energy balances reduce to give

$$\dot{Q}_{cv} = \dot{m} (h_e - h_i + \frac{V_e^2 - V_i^2}{2}) = (0.471 \text{ kg/s}) \left[(2637.9 - 2748.7) \frac{\text{kJ}}{\text{kg}} + \frac{(10.50^2 - 10^2) \text{ m}^2/\text{s}^2}{2} \left| \frac{1 \text{ kJ}}{1000 \text{ kg m}^2/\text{s}^2} \right| \right] = -52.18 \text{ kW} \leftarrow \dot{Q}_{cv}$$

Steam table data: $h_i = 2748.7 \text{ kJ/kg}$, $h_e = h_f + x_e h_{fg} = 623.5 + 0.95(2120.7) = 2637.9 \text{ kJ/kg}$
Mass and entropy balances reduce to give

$$\dot{\sigma}_{cv} = - \frac{\dot{Q}_{cv}}{T_0} + \dot{m} (s_e - s_i)$$

Steam table data: $s_i = 6.8212 \text{ kJ/kg K}$, $s_e = s_f + x_e s_{fg} = 1.8207 + 0.95(6.8565 - 1.8207) = 6.6047 \text{ kJ/kg K}$

Control volume #1, $T_0 = 315 \text{ K}$

$$\dot{\sigma}_{cv} = - \frac{(-52.18 \text{ kW})}{315 \text{ K}} + (0.471 \text{ kg/s}) (6.6047 - 6.8212) \frac{\text{kJ}}{\text{kg K}} = 0.064 \frac{\text{kW}}{\text{K}} \leftarrow (\dot{\sigma}_{cv})_1$$

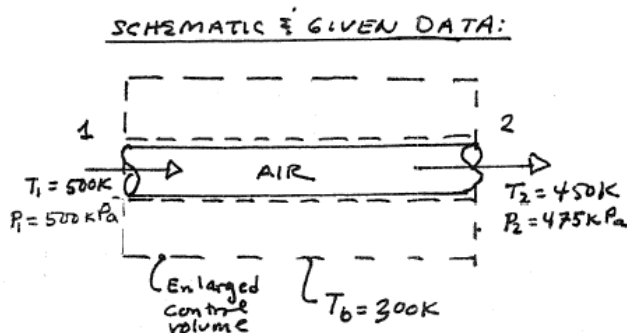
Control volume #2, $T_0 = 300 \text{ K}$:

$$\dot{\sigma}_{cv} = - \frac{(-52.18 \text{ kW})}{300 \text{ K}} + (0.471 \text{ kg/s}) (6.6047 - 6.8212) \frac{\text{kJ}}{\text{kg K}} = 0.072 \frac{\text{kW}}{\text{K}} \leftarrow (\dot{\sigma}_{cv})_2$$

COMMENT: For control volume #1 there is a pressure decrease as the steam flows through the pipe. For this control volume the pressure drop is the most significant source of irreversibility. For control volume #2, in addition to the pressure drop irreversibility, there is the heat transfer from the outer surface of the pipe to the surroundings. Accordingly, the entropy production is greater for C.V. #2 than for C.V. #1. For control volume #1, the heat transfer effect is an external irreversibility.

PROBLEM 6.105

Air at 500 kPa, 500 K and a mass flow of 600 kg/h enters a pipe passing overhead in a factory space. At the pipe exit, the pressure and temperature of the air are 475 kPa and 450 K, respectively. Air can be modeled as an ideal gas with $k = 1.39$. Kinetic and potential energy effects can be ignored. Determine at steady state, (a) the rate of heat transfer, in kW, and (b) the rate of entropy production, in kW/K, for an enlarged control volume that includes the pipe and enough of its surroundings that heat transfer occurs at the ambient temperature, 300 K.



ENGR. MODEL:

1. The control volumes shown in the sketch are at steady state.
2. For the control volumes, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be ignored.
3. Heat transfer from the enlarged control volume occurs at $T_b = 300 \text{ K}$.
4. The air is modeled as an ideal gas with $k = 1.39$.

ANALYSIS: (a) For the control volume enclosing the pipe an energy rate balance reads, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$. Thus,

$$\dot{Q}_{cv} = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1)$$

$$\Rightarrow \frac{kR}{k-1} = \frac{1.39(8.314/28.97)}{1.39-1} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 1.023 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

(Eq. 3.47a)

$$\therefore \dot{Q}_{cv} = 600 \frac{\text{kg}}{\text{h}} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| (1.023 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(450 - 500) \text{ K} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= -8.53 \text{ kW}$$

← (a)

(b) An entropy rate balance for the enlarged control volume reads

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_2 - s_1)$$

Or, with Eq. 6.22,

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m} \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right]$$

$$= \frac{8.53 \text{ kW}}{300 \text{ K}} - \left(\frac{600}{3600} \frac{\text{kg}}{\text{s}} \right) \left[1.023 \ln \frac{450}{500} - \frac{8.314}{28.97} \ln \frac{475}{500} \right] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= (0.0284 - 0.0155) \frac{\text{kW}}{\text{K}}$$

$$= 0.0129 \frac{\text{kW}}{\text{K}}$$

← (b)

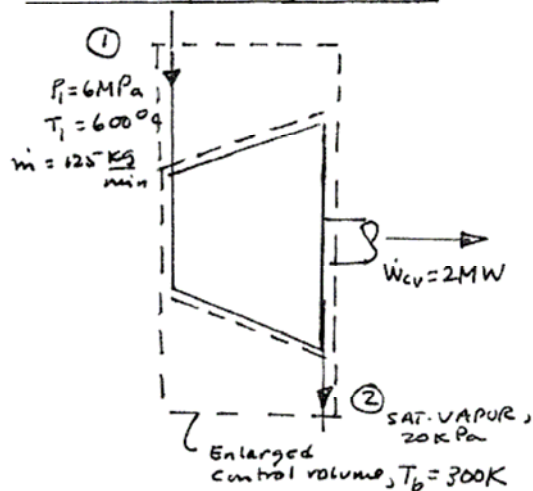
PROBLEM 6.106

Steam enters a turbine operating at steady state at 6 MPa, 600°C with a mass flow rate of 125 kg/min and exits as saturated vapor at 20 kPa, producing power at a rate of 2 MW. Kinetic and potential energy effects can be ignored. Determine (a) the rate of heat transfer, in kW, for a control volume including the turbine and its contents, and (b) the rate of entropy production, in kW/K, for an enlarged control volume that includes the turbine and enough of its surroundings that heat transfer occurs at the ambient temperature, 27°C.

ENGR. MODEL:

1. The control volumes shown in the sketch are at steady state.
2. Kinetic and potential energy effects can be ignored.
3. For the enlarged control volume heat transfer occurs at $T_b = 300\text{ K}$.

SCHEMATIC & GIVEN DATA



ANALYSIS: At 6 MPa, 600°C, Table A-4 gives $h_1 = 3658.4\text{ kJ/kg}$, $s_1 = 7.1677\text{ kJ/kg}\cdot\text{K}$.

At 20 kPa, saturated vapor Table A-3 gives $h_2 = 2609.7\text{ kJ/kg}$, $s_2 = 7.9085\text{ kJ/kg}\cdot\text{K}$.

- (a) An energy balance for the control volume enclosing just the turbine reads,
- $$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \quad \text{This gives}$$

$$\begin{aligned} \dot{Q}_{cv} &= \dot{W}_{cv} - \dot{m}(h_1 - h_2) \\ &= 2\text{ MW} \left| \frac{10^3\text{ kW}}{1\text{ MW}} \right| - 125 \frac{\text{kg}}{\text{min}} \left| \frac{1\text{ min}}{60\text{ s}} \right| (3658.4 - 2609.7) \frac{\text{kJ}}{\text{kg}} \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right| \\ &= -184.8\text{ kW} \end{aligned} \quad \leftarrow (a)$$

- (b) An entropy rate balance for the enlarged control volume reads,

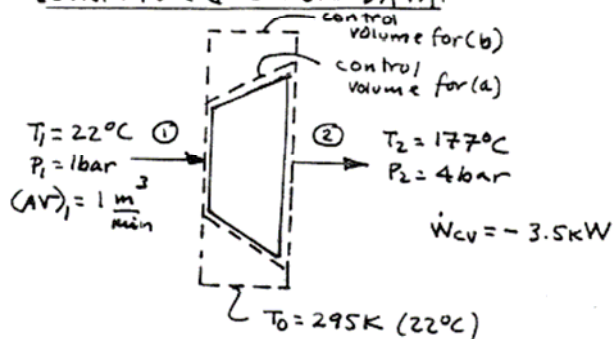
$$\begin{aligned} 0 &= \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \\ \Rightarrow \dot{\sigma}_{cv} &= -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_2 - s_1) \\ &= -\left(\frac{-184.8\text{ kW}}{300\text{ K}} \right) + \frac{125}{60} \frac{\text{kg}}{\text{s}} (7.9085 - 7.1677) \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \\ &= 2.16 \frac{\text{KW}}{\text{K}} \end{aligned} \quad \leftarrow (b)$$

PROBLEM 6.107

KNOWN: Steady-state operating data are provided for an air compressor.

FIND: (a) Determine the heat transfer rate and change in entropy from inlet to exit for a control volume enclosing the compressor only. Discuss. (b) For an enlarged control volume, determine the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volumes shown in the accompanying figure are at steady state.
2. Kinetic and potential energy effects can be ignored.
3. Air is modeled as an ideal gas.

ANALYSIS: (a) At steady state, the mass rate balance reads, $\dot{m}_2 = \dot{m}_1 = \dot{m}$, where

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1}{(RT_1/P_1)} = \frac{(1 \text{ m}^3/\text{min})(10^5 \text{ N/m}^2)}{\left(\frac{8314 \text{ N}\cdot\text{m}}{28.97 \text{ kg}\cdot\text{K}}\right)(295 \text{ K})} = 1.18 \text{ kg/min}$$

The energy rate balance at steady state reduces to give on rearrangement

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1) = -3.5 \text{ kW} + \left(1.18 \frac{\text{kg}}{\text{min}}\right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| [451.8 - 295.17] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= -0.42 \text{ kW} \quad \leftarrow \dot{Q}_{cv}$$

where enthalpy data is from Table A-22. The entropy change is obtained with Eq. 6.20a and s° data from Table A-22:

$$s_2 - s_1 = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1} = [2.11161 - 1.68515 - \frac{8.314}{28.97} \ln 4] = 0.0286 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad \leftarrow s_2 - s_1$$

To evaluate the entropy production for the control volume of part (a) would require information about the temperature at which heat transfer occurs on the boundary of the control volume.

(b) For the enlarged control volume of part (b), heat transfer takes place at $T_0 = 295 \text{ K}$. An entropy rate balance at steady state reads

$$0 = \frac{\dot{Q}_{cv}}{T_0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_0} + \dot{m}(s_2 - s_1)$$

with results from part (a)

$$\dot{\sigma}_{cv} = -\left(\frac{-0.42 \text{ kW}}{295 \text{ K}}\right) + \left(\frac{1.18 \text{ kg}}{60 \text{ s}}\right) \left[0.0286 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

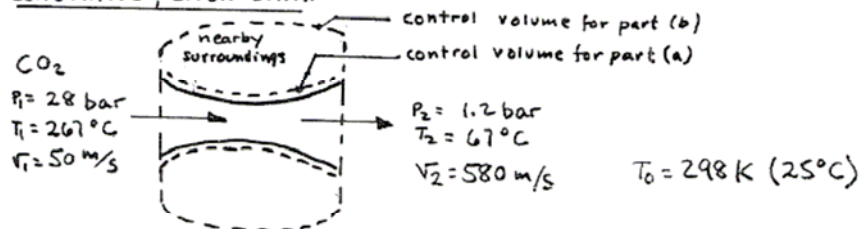
$$= 1.98 \times 10^{-3} \frac{\text{kW}}{\text{K}} \quad \leftarrow \dot{\sigma}_{cv}$$

PROBLEM 6.108

KNOWN: Operating data are provided for a nozzle at steady state through which CO_2 is passing.

FIND: (a) For a control volume enclosing the nozzle only, evaluate $\dot{Q}_{cv}/\dot{m}$, and the change in specific entropy from inlet to exit. Indicate the additional data required to evaluate $\dot{Q}_{cv}/\dot{m}$. (b) For an enlarged control volume including enough of the nearby surroundings that heat transfer occurs at the ambient temperature T_0 , evaluate $\dot{Q}_{cv}/\dot{m}$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) Each of the control volumes shown in the accompanying figure operates at steady state. (2) The change in potential energy from inlet to exit is negligible. (3) CO_2 can be modeled as an ideal gas.

ANALYSIS: (a) At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right)$$

or

$$\frac{\dot{Q}_{cv}}{\dot{m}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

With $M = 44.01$ from Table A-1 and data from Table A-23, $\bar{h}_1 = 19,485 \text{ kJ/kmol}$, $\bar{s}_1^\circ = 238.292 \text{ kJ/kmol} \cdot \text{K}$, $\bar{h}_2 = 10,459 \text{ kJ/kmol}$, $\bar{s}_2^\circ = 218.694 \text{ kJ/kmol} \cdot \text{K}$. Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{(10,459 - 19,485) \text{ kJ/kmol}}{44.01 \text{ kg/kmol}} + \left(\frac{580^2 - 50^2}{2} \right) \left(\frac{\text{m}^2}{\text{s}^2} \right) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left(\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right)$$

$$= -193.73 \text{ kJ/kg} + 166.95 \text{ kJ/kg} = -26.78 \text{ kJ/kg} \quad \left(\frac{\dot{Q}_{cv}}{\dot{m}} \right)$$

With Eq. 6.20b

$$s_2 - s_1 = \frac{1}{M} \left[\bar{s}_2^\circ - \bar{s}_1^\circ - \bar{R} \ln \frac{P_2}{P_1} \right] = \frac{[218.694 - 238.292 - 8.314 \ln \frac{1.2}{28}]}{44.01}$$

$$= +0.1497 \text{ kJ/kg} \cdot \text{K} \quad (s_2 - s_1)$$

To evaluate $\dot{Q}_{cv}/\dot{m}$ would require information about the temperature on the boundary of the control volume and the rate of heat transfer at each temperature.

(b) For the enlarged control volume the heat transfer takes place only at $T_0 = 298 \text{ K}$, and the entropy rate balance reduces to

$$0 = \frac{\dot{Q}_{cv}}{T_0} + \dot{m} (s_1 - s_2) + \dot{Q}_{cv}/\dot{m} \Rightarrow \dot{Q}_{cv}/\dot{m} = -\frac{\dot{Q}_{cv}/\dot{m}}{T_0} + s_2 - s_1$$

Using previously calculated values

$$\dot{Q}_{cv}/\dot{m} = -\frac{(-26.78 \text{ kJ/kg})}{298 \text{ K}} + 0.1497 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.2396 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad \left(\frac{\dot{Q}_{cv}}{\dot{m}} \right)$$

1. From Table A-1, for CO_2 $P_c = 23.9 \text{ bar}$, $T_c = 304 \text{ K}$. Then,

$$P_{R1} = \frac{28}{23.9} = 1.172 \quad T_{R1} = \frac{267}{304} = 0.878$$

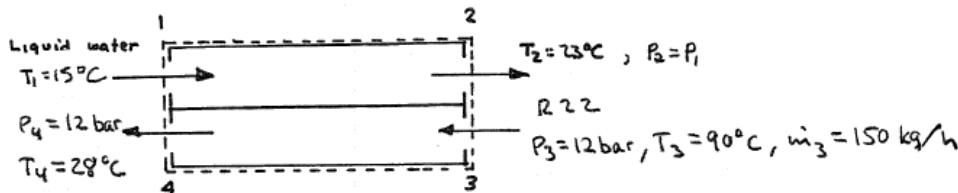
Inspection of Fig. A-1 indicates that the ideal gas model is appropriate at this state. Similarly, the ideal gas model is appropriate at state 2, as can be verified.

PROBLEM 6.109

KNOWN: Operating data are provided for a counterflow heat exchanger at steady state with liquid water flowing on one side and R22 flowing on the other side.

FIND: Determine the mass flow rate of the water stream and the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure is at steady state. (2) $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. (3) Liquid water is modeled as incompressible with constant specific heat c and negligible pressure drop.

ANALYSIS: At steady state the inlet and exit mass flow rates of each side of the heat exchanger are equal: $\dot{m}_1 = \dot{m}_2 = \dot{m}$, $\dot{m}_3 = \dot{m}_4 = \dot{m}_R$, and the energy rate balance reduces with assumption 2 to

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}(h_1 - h_2) + \dot{m}_R(h_3 - h_4)$$

Thus

$$\dot{m} = \frac{\dot{m}_R(h_3 - h_4)}{(h_2 - h_1)}$$

With assumption 3, Eq. 3.206 gives $(h_2 - h_1) = c(T_2 - T_1) + v(P_2 - P_1)$. With $c = 4.2 \text{ kJ/kg} \cdot \text{K}$ from Table A-19, $h_2 - h_1 = (4.2 \text{ kJ/kg} \cdot \text{K})(8 \text{ K}) = 33.6 \text{ kJ/kg}$. Then, approximating $h_4 \approx h_f(T_4)$ and using data from Tables A-7, 9

$$\dot{m} = \frac{(150 \text{ kg/h})(308.70 - 79.05) \text{ kJ/kg}}{33.6 \text{ kJ/kg}} = 1025.2 \text{ kg/h} \quad \leftarrow \dot{m}$$

An entropy rate balance at steady state reduces to give

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{m}_R(s_3 - s_4) + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) + \dot{m}_R(s_4 - s_3)$$

Introducing Eq. 6.13 and approximating $s_4 = s_f(T_4)$ and using data from Tables A-7, 9

$$\begin{aligned} \dot{\sigma}_{cv} &= \dot{m} c \ln \frac{T_2}{T_1} + \dot{m}_R(s_4 - s_3) \\ &= (1025.2 \frac{\text{kg}}{\text{h}})(4.2 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \ln \frac{296}{288} + (150 \frac{\text{kg}}{\text{h}})(0.2936 - 1.0363) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ &= (117.98 - 111.41) \frac{\text{kJ}}{\text{K}} \\ &= (6.57 \frac{\text{kJ}}{\text{K}}) \left| \frac{1 \text{ min}}{3600 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 1.83 \times 10^{-3} \text{ kW/K} \quad \leftarrow \dot{\sigma}_{cv} \end{aligned}$$

1. Alternatively, for the liquid water stream saturated liquid data can be used: $h_{fh}(T)$, $s_{fh}(T)$.

PROBLEM 6.110

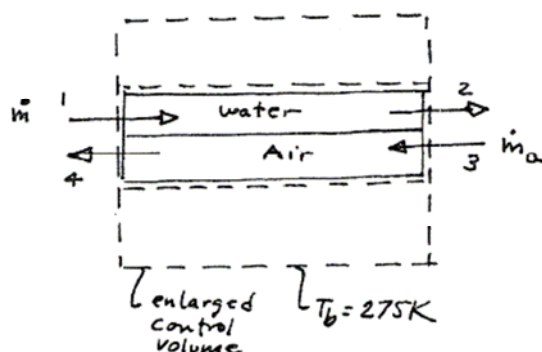
Saturated water vapor at 100 kPa enters a counterflow heat exchanger operating at steady state and exits at 20°C with a negligible change in pressure. Ambient air at 275 K, 1 atm enters in a separate stream and exits at 290 K, 1 atm. The air mass flow rate is 170 times that of the water. The air can be modeled as an ideal gas with $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$. Kinetic and potential energy effects can be ignored.

- For a control volume enclosing the heat exchanger, evaluate the rate of heat transfer, in kJ per kg of water flowing.
- For an enlarged control volume that includes the heat exchanger and enough of its immediate surroundings that heat transfer from the control volume occurs at the ambient temperature, 275 K, determine the rate of entropy production, in kJ/K per kg of water flowing.

ENGR. MODEL:

- The control volumes shown in the sketch are at steady state.
- Kinetic and potential energy effects are ignored. $\dot{W}_{cv} = 0$.
- For the enlarged control volume heat transfer occurs at $T_b = 275 \text{ K}$.
- The air is modeled as an ideal gas with $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$.
- For liquid water $h \sim h_f(T)$, $s \sim s_f(T)$.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

From Table A-3, $h_1 = h_g(\text{at } 100 \text{ kPa}) = 2778.1 \text{ kJ/kg}$, and $s_1 = 6.5863 \text{ kJ/kg} \cdot \text{K}$. From Table A-2, $h_2 \sim h_f(20^\circ\text{C}) = 83.96 \frac{\text{kJ}}{\text{kg}}$, $s_2 \sim s_f(20^\circ\text{C}) = 0.2966 \text{ kJ/kg} \cdot \text{K}$.



- An energy rate balance for a control volume enclosing just the heat exchanger reads,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) + \dot{m}_a(h_3 - h_4) \Rightarrow \dot{Q}_{cv} = \dot{m}(h_2 - h_1) + \dot{m}_a(h_4 - h_3)$$

Since $\dot{m}_a/\dot{m} = 170$, and using assumption 4,

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}} &= (h_2 - h_1) + 170(c_p(T_4 - T_3)) = [83.96 - 2778.1] + 170(1.005)(290 - 275) \frac{\text{kJ}}{\text{kg}(\text{w})} \\ &= -131.4 \text{ kJ/kg}(\text{w}) \end{aligned}$$

(a) ←

- An entropy rate balance for the enlarged control volume reads,

$$\begin{aligned} 0 &= \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{m}_a(s_3 - s_4) + \dot{S}_{cv} \\ \Rightarrow \dot{S}_{cv}/\dot{m} &= \left(-\frac{\dot{Q}_{cv}/\dot{m}}{T_b} \right) + (s_2 - s_1) + 170 \left[c_p \ln \frac{T_4}{T_3} - R \ln \frac{P_4}{P_3} \right] \\ &= \left[\frac{131.4}{275} + (0.2966 - 6.5863) + 170(1.005) \ln \frac{290}{275} \right] \frac{\text{kJ/kg}(\text{w})}{\text{K}} \\ &= 3.26 \frac{\text{kJ/kg}(\text{w})}{\text{K}} \end{aligned}$$

(b) ←

PROBLEM 6.111

Figure P6.111 shows data for a portion of the ducting in a ventilation system operating at steady state. The ducts are well insulated and the pressure is very nearly 1 atm throughout. Assuming the ideal gas model for air with $c_p = 0.24 \text{ Btu/lb} \cdot ^\circ\text{R}$, and ignoring kinetic and potential energy effects, determine (a) the temperature of the air at the exit, in $^\circ\text{F}$, (b) the exit diameter, in ft, and (c) the rate of entropy production within the duct, in $\text{Btu/min} \cdot ^\circ\text{R}$.

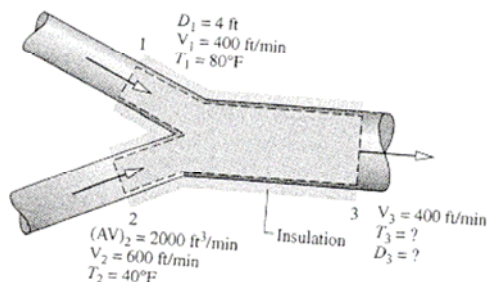


Fig. P6.111

ENGR. MODEL

1. The control volume shown in the figure is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and kinetic and potential energy effects can be ignored.
3. The air is modeled as an ideal gas with $c_p = 0.24 \text{ Btu/lb} \cdot ^\circ\text{R}$.

ANALYSIS: To find T_3 , begin with steady-state mass and energy balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left(h_1 + \frac{V_1^2}{2} + g z_1 \right) + \dot{m}_2 \left(h_2 + \frac{V_2^2}{2} + g z_2 \right) - \dot{m}_3 \left(h_3 + \frac{V_3^2}{2} + g z_3 \right)$$

and $\dot{m}_1 + \dot{m}_2 - \dot{m}_3 = 0 \Rightarrow \dot{m}_3 = \dot{m}_1 + \dot{m}_2$

Combining and incorporating assumption (2)

$$0 = \dot{m}_1 \left[(h_1 - h_3) + \frac{V_1^2 - V_3^2}{2} \right] + \dot{m}_2 \left[(h_2 - h_3) + \frac{V_2^2 - V_3^2}{2} \right]$$

Using $\Delta h = c_p \Delta T$, this becomes

$$0 = \dot{m}_1 [c_p(T_1 - T_3)] + \dot{m}_2 [c_p(T_2 - T_3)] \quad (*)$$

The mass flow rates are evaluated using Eq. 4-4b and the ideal gas equation of state

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{(\frac{\pi D_1^2}{4}) V_1 P_1}{R T_1} = \frac{(\frac{\pi (4^2)}{4}) (400 \text{ ft/min}) (14.7 \text{ lbf/in}^2) |144 \text{ in}^2|}{(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}) (540^\circ\text{R})} = 369.5 \text{ lb/min}$$

$$\dot{m}_2 = \frac{(AV)_2 P_2}{R T_2} = \frac{(2000) (14.7) |144|}{(\frac{1545}{28.97}) (500)} = 158.8 \text{ lb/min}$$

Returning to (*) and solving for T_3

$$T_3 = \frac{\dot{m}_1 c_p T_1 + \dot{m}_2 c_p T_2}{(\dot{m}_1 + \dot{m}_2) c_p} = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{(\dot{m}_1 + \dot{m}_2)}$$

Continued on next slide

Problem 6-111 continued

Inserting values

$$T_3 = \frac{(369.5 \text{ lb/min})(540^\circ\text{R}) + (158.8)(500)}{(369.5 + 158.8) \text{ lb/min}}$$

$$= 528^\circ\text{R} = 68^\circ\text{F} \leftarrow T_3$$

To get D_3 , note that

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 528.3 \text{ lb/min}$$

Thus

$$A_3 = \frac{v_3 \dot{m}_3}{V_3} = \frac{RT_3 \dot{m}_3}{P_3 V_3} = \frac{\left(\frac{1545}{28.97}\right)(528)(528.3)}{(14.7)(144)(400)} = 17.57 \text{ ft}^2$$

and

$$D_3 = \sqrt{\frac{4 A_3}{\pi}} = 4.73 \text{ ft} \leftarrow D_3$$

An entropy rate balance at steady state reduces to read

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

Since $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ at steady state

$$\dot{\sigma}_{cv} = (\dot{m}_1 + \dot{m}_2) s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$= \dot{m}_1 (s_3 - s_1) + \dot{m}_2 (s_3 - s_2)$$

Introducing Eq. 6.22 and noting that pressure is nearly 1 atm throughout,

$$\dot{\sigma}_{cv} = \dot{m}_1 \left[c_p \ln \frac{T_3}{T_1} - R \ln \frac{P_3}{P_1} \right] + \dot{m}_2 \left[c_p \ln \frac{T_3}{T_2} - R \ln \frac{P_3}{P_2} \right]$$

$$= c_p \left[\dot{m}_1 \ln \frac{T_3}{T_1} + \dot{m}_2 \ln \frac{T_3}{T_2} \right]$$

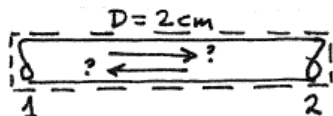
$$= 0.24 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \left[(369.5 \frac{\text{lb}}{\text{min}}) \ln \frac{528}{540} + (158.8 \frac{\text{lb}}{\text{min}}) \ln \frac{528}{500} \right]$$

$$= 0.084 \frac{\text{Btu}}{\text{min} \cdot ^\circ\text{R}} \leftarrow \dot{\sigma}_{cv}$$

PROBLEM 6.112

Air flows through an insulated circular duct having a diameter of 2 cm. Steady-state pressure and temperature data obtained by measurements at two locations, denoted as 1 and 2, are given in the accompanying table. Modeling air as an ideal gas with $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$, determine (a) the direction of the flow, (b) the velocity of the air, in m/s, at each of the two locations, and (c) the mass flow rate of the air, in kg/s.

Measurement location	1	2
Pressure (kPa)	100	500
Temperature ($^{\circ}\text{C}$)	20	50



ENGR. MODEL:

- The control volume shown in the sketch is at steady state.
- For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$ and potential energy effects are negligible.
- The air is modeled as an ideal gas with $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$.

ANALYSIS: As discussed in Secs. 5.1 and 6.8, directionality normally can be established using the 2nd law. Here, a direction is assumed and the associated entropy production is evaluated. Taking the inlet at 1 and the exit at 2, an entropy rate balance reads, $0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$

$$\Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 1.005 \ln \frac{323}{293} - \frac{8.314}{28.97} \ln \frac{500}{100} = -0.364 \frac{\text{KJ}}{\text{kg} \cdot \text{K}}$$

Since $\dot{\sigma}_{cv}/\dot{m}$ cannot be negative, the flow direction can only be from 2 to 1. $\leftarrow (a)$

A mass rate balance reads, $\dot{m}_1 = \dot{m}_2 \Rightarrow \frac{A \bar{V}_1}{\sqrt{T_1/P_1}} = \frac{A \bar{V}_2}{\sqrt{T_2/P_2}} \Rightarrow \frac{\bar{V}_2}{\bar{V}_1} = \frac{P_1}{P_2} \frac{T_2}{T_1}$

Inserting values, $\bar{V}_2 = \left(\frac{100}{500}\right) \left(\frac{323}{293}\right) \bar{V}_1 = 0.2205 \bar{V}_1$

An energy rate balance reads, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_2 - h_1 + \frac{\bar{V}_2^2 - \bar{V}_1^2}{2} \right]$. Or

$$0 = c_p(T_2 - T_1) + \frac{(0.2205 \bar{V}_1)^2 - \bar{V}_1^2}{2} \Rightarrow 0 = c_p(T_2 - T_1) - 0.9514 \bar{V}_1^2. \text{ Solving, we get}$$

$$\bar{V}_1 = \left[\frac{2(1.005 \frac{\text{KJ}}{\text{kg} \cdot \text{K}})(30 \text{ K})}{0.9514} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ KJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \right]^{\frac{1}{2}} = 251.75 \text{ m/s}$$

And

$$\bar{V}_2 = 0.2205 \bar{V}_1 = 55.51 \text{ m/s}$$

$\rightarrow (b)$

$$(c) \quad \dot{m} = \frac{A \bar{V}_1}{\sqrt{T_1/P_1}} = \frac{P_1 (\pi D^2/4) \bar{V}_1}{RT_1} = \frac{(105 \text{ N/m}^2) \left(\frac{\pi}{4} (0.02 \text{ m})^2 \right) 251.75 \text{ m/s}}{\left(\frac{8.314}{28.97} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (293 \text{ K})} = 0.094 \frac{\text{kg}}{\text{s}} \leftarrow (c)$$

PROBLEM 6.113

KNOWN: Steady state operating data are provided for two waste-heat recovery systems: (a) Example 4.10, (b) Problem 4.101.

FIND: For each system, determine the rates of entropy production for the steam generator and the turbine. Discuss.

SCHEMATIC & GIVEN DATA:

(a) See Fig. E 4.10

(b) See Fig. P 4.101. From the solution, $\dot{m}_A = 145.2 \text{ lb/min}$, $\dot{m}_1 = 2.8 \text{ lb/min}$.

ENGR. MODEL: 1. Control volumes enclosing the steam generator and the turbine are at steady state. 2. For each control volume, $\dot{Q}_{cv} = 0$. 3. The combustion products of part (a) can be modeled as air as an ideal gas. The ideal gas model also applies to the air of part (b).

ANALYSIS: At steady state the entropy rate balance for the control volume enclosing the turbine reduces to give

$$(a) \dot{S}_t = \dot{m}(s_5 - s_4) \quad (b) \dot{S}_t = \dot{m}(s_3 - s_2)$$

An entropy rate balance for the steam generator gives

$$(a) \dot{Q}_{HX} = \dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3) \quad (b) \dot{Q}_{HX} = \dot{m}_A(s_B - s_A) + \dot{m}_1(s_2 - s_1)$$

(a) From the solution to Example 4.10, $h_4 = 1213.6 \text{ Btu/lb}$, $p_4 = 40 \text{ lbf/in}^2$; thus Table A-4E gives $s_4 = 1.7334 \text{ Btu/lb} \cdot ^\circ\text{R}$. With data from Table A-3E

$s_5 = s_f + x_5(s_g - s_f) = 0.1327 + 0.93(1.8453) = 1.8488 \text{ Btu/lb} \cdot ^\circ\text{R}$. Thus,

$$\dot{S}_t = \left(275 \frac{\text{lb}}{\text{min}} \right) (1.8488 - 1.7334) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 0.529 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}} \quad \leftarrow$$

With s° data from Table A-22E and $s_3 = s_f(T_3)$ from Table A-2E

$$\begin{aligned} \dot{Q}_{HX} &= \left[(9230.6 \frac{\text{lb}}{\text{min}}) \left[0.67002 - 0.71323 - R \ln \frac{p_2}{p_1} \right] \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} + 275 (1.7334 - 0.1327) \right] \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \\ &= 0.687 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}} \quad \text{The steam generator contributes most to inefficient operation.} \quad \leftarrow \end{aligned}$$

(b) With data from Table A-4E and A-3E, $s_2 = 1.7121 \text{ Btu/lb} \cdot ^\circ\text{R}$, $s_3 = s_f + x_3 s_{fg}$, $s_3 = 0.1327 + 0.9(1.8453) = 1.7435 \text{ Btu/lb} \cdot ^\circ\text{R}$. Thus,

$$\dot{S}_t = \left(2.8 \frac{\text{lb}}{\text{min}} \right) (1.7435 - 1.7121) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 3.8 \times 10^{-3} \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}} \quad \leftarrow$$

With s° data from Table A-22E, noting that $p_B = p_A$, and $s_1 = s_f(T_1)$ from Table A-2E

$$\begin{aligned} \dot{Q}_{HX} &= \left[\left(145.2 \frac{\text{lb}}{\text{min}} \right) \left[0.67665 - 0.70160 \right] \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} + 2.8 [1.7121 - 0.3241] \right] \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \\ &= 4.4 \times 10^{-3} \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}} \quad \leftarrow \end{aligned}$$

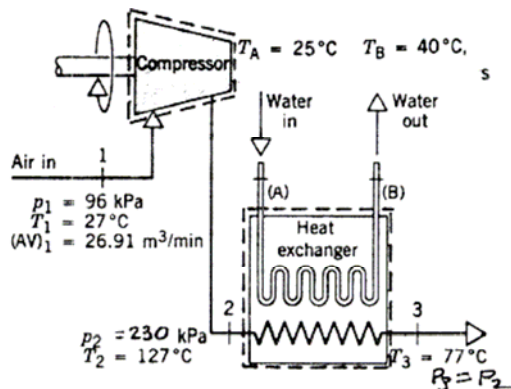
The steam generator contributes most to inefficient operation.

PROBLEM 6.114

KNOWN: Air passes through a compressor and heat exchanger. Data for the various flow streams are known.

FIND: Determine the compressor power and the mass flow rate of the cooling water. Also, evaluate the rates of entropy production in the compressor and heat exchanger.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) Both control volumes shown are at steady state. (2) Heat transfer is negligible, and $\dot{W}_{cv} = 0$ for the heat exchanger. (3) kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas.

ANALYSIS: (a) The compressor power is found from the steady-state energy balance for the compressor control volume

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_{air} [(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{air}$. With assumption (3)

$$\dot{W}_{cv} = \dot{m}_{air} (h_1 - h_2)$$

The mass flow rate of air is

$$\dot{m}_{air} = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1} = \frac{(96 \text{ kPa})(26.91 \text{ m}^3/\text{min})}{(0.314 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})} \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 30 \frac{\text{kg}}{\text{min}}$$

Thus

$$\dot{W}_{cv} = (30 \frac{\text{kg}}{\text{min}}) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) (300.19 - 400.98) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -50.4 \text{ kW} \leftarrow \dot{W}_{cv}$$

where the specific enthalpies are from Table A-22.

To find the cooling water flow rate, note that the air and water pass through the heat exchanger as separate streams. Thus

$$\dot{m}_2 = \dot{m}_3 \equiv \dot{m}_{air} \quad \text{and} \quad \dot{m}_A = \dot{m}_B \equiv \dot{m}_{cw}$$

With assumptions (2) and (3), the steady-state energy balance for the heat exchanger control volume reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_{air} (h_2 - h_3) + \dot{m}_{cw} (h_A - h_B)$$

or

$$\dot{m}_{cw} = \left(\frac{h_2 - h_3}{h_B - h_A} \right) \dot{m}_{air}$$

From Table A-22, $h_3 = 350.49 \text{ kJ/kg}$. For the cooling water, $h = h_f(T)$:

$$h_A = 104.89 \text{ kJ/kg}, \quad h_B = 167.57 \text{ kJ/kg}. \quad \text{Thus, the cooling water mass flow rate is}$$

$$\dot{m}_{cw} = \left(\frac{400.98 - 350.49}{167.57 - 104.89} \right) \left(30 \frac{\text{kg}}{\text{min}} \right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 0.403 \text{ kg/s} \leftarrow \dot{m}_{cw}$$

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Problem 6-114 continued

(b) Reducing mass and entropy rate balances for the compressor, and using Eq. 6.20a

$$\begin{aligned}
 \dot{\sigma}_C &= \dot{m}_{Air} (s_2 - s_1) \\
 &= \dot{m}_{Air} (s^0(T_2) - s^0(T_1) - R \ln P_2/P_1) \\
 &= \left(30 \frac{\text{kg}}{\text{min}} \right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left(1.99194 - 1.70203 - \frac{8.314}{28.97} \ln \left(\frac{230}{96} \right) \right) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\
 &= 0.0196 \frac{\text{KW}}{\text{K}} \quad \leftarrow \dot{\sigma}_C
 \end{aligned}$$

where s^0 values are from Table A-22.

Reducing mass and entropy rate balances for the heat exchanger

$$\begin{aligned}
 \dot{\sigma}_{HX} &= \dot{m}_{Air} [s_3 - s_2] + \dot{m}_{cw} [s_B - s_A] \\
 &= \dot{m}_{Air} \left[s^0(T_3) - s^0(T_2) - R \ln \frac{P_3}{P_2} \right] + \dot{m}_{cw} [s_f(T_B) - s_f(T_A)]
 \end{aligned}$$

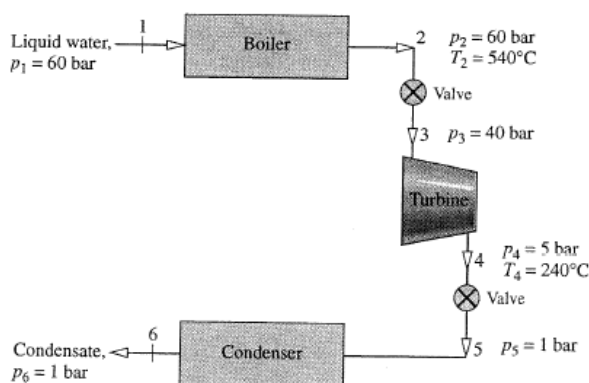
With data from Tables A-2, 22

$$\begin{aligned}
 \dot{\sigma}_{HX} &= \left[\left(0.5 \frac{\text{kg}}{\text{s}} \right) \left[1.85708 - 1.99194 \right] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} + 0.403 (0.5725 - 0.3674) \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\
 &= 0.0152 \frac{\text{KW}}{\text{K}} \quad \leftarrow \dot{\sigma}_{HX}
 \end{aligned}$$

PROBLEM 6.115

Figure P6.115 shows several components in series, all operating at steady state. Liquid water enters the boiler at 60 bar. Steam exits the boiler at 60 bar, 540°C and undergoes a throttling process to 40 bar before entering the turbine. Steam expands adiabatically through the turbine to 5 bar, 240°C, and then undergoes a throttling process to 1 bar before entering the condenser. Kinetic and potential energy effects can be ignored.

- Locate each of the states 2–5 on a sketch of the T - s diagram.
- Determine the power developed by the turbine, in kJ per kg of steam flowing.
- For the valves and the turbine, evaluate the rate of entropy production, each in kJ/K per kg of steam flowing.
- Using the result of part (c), place the components in rank order, beginning with the component contributing the most to inefficient operation of the overall system. Comment.

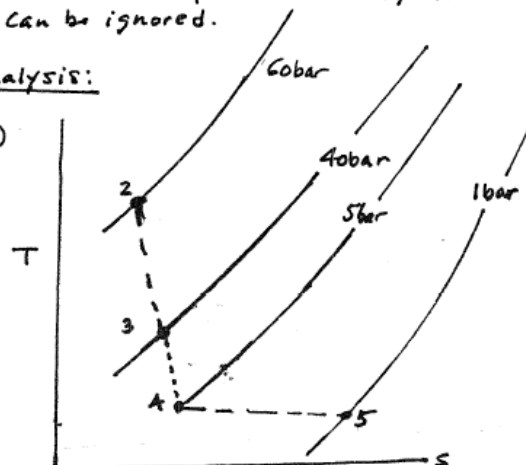


ENGR. MODEL:

- A control volume at steady state encloses each valve and the turbine.
- For the valves, the expansion is a throttling process: $h_3 = h_2$, $h_5 = h_4$.
- For the turbine, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be ignored.

Analysis:

(a)



- Mass and energy rate balances reduce for the turbine to give $\dot{W}_t/\dot{m} = h_3 - h_4$. But since $h_3 = h_2$, we have $\dot{W}_t/\dot{m} = h_2 - h_4 = (3517 - 2939.9) \frac{\text{kJ}}{\text{kg}}$ or $\dot{W}_t/\dot{m} = 577.1 \text{ kJ/kg}$, where h_2 and h_4 are from Table A-4.
- Since each of the three control volumes is at steady state, experiences no heat transfer, and has a single inlet and single exit, the entropy rate balance reduces to give for each one, $\dot{\sigma}_{cv}/\dot{m} = s_e - s_i$.

Valve #1: $\dot{\sigma}_{cv}/\dot{m} = (7.1805 - 6.9999) \text{ kJ/kg} \cdot \text{K} = 0.1806 \text{ kJ/kg} \cdot \text{K}$

Turbine: $\dot{\sigma}_{cv}/\dot{m} = (7.2307 - 7.1805) \text{ kJ/kg} \cdot \text{K} = 0.0502 \text{ kJ/kg} \cdot \text{K}$

Valve #2: $\dot{\sigma}_{cv}/\dot{m} = (7.9653 - 7.2307) \text{ kJ/kg} \cdot \text{K} = 0.7346 \text{ kJ/kg} \cdot \text{K}$

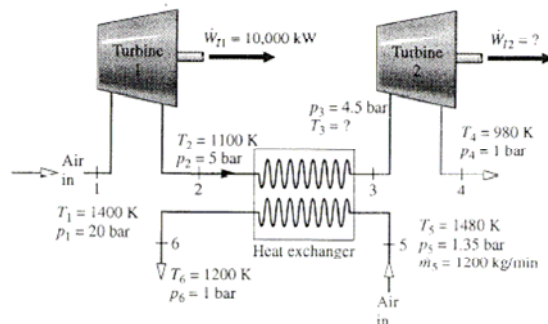
- Valve #2
Valve #1
Turbine

Valve #2 is a significant contributor to thermodynamic inefficiency. The proposed design should be reviewed to see if this component might be replaced or eliminated.

PROBLEM 6.116

Air as an ideal gas flows through the turbine and heat exchanger arrangement shown in Fig. P6.116. Steady-state data are given on the figure. Stray heat transfer and kinetic and potential energy effects can be ignored. Determine

- temperature T_3 , in K.
- the power output of the second turbine, in kW.
- the rates of entropy production, each in kW/K, for the turbines and heat exchanger.
- Using the result of part (c), place the components in rank order, beginning with the component contributing most to inefficient operation of the overall system.



ENGR. MODEL:

- A control volume at steady state encloses each turbine and the heat exchanger.
- Stray heat transfer and kinetic and potential energy can be ignored.
- The air is modeled as an ideal gas.

ANALYSIS: (a) First, find the air flow rate at 1. Begin with steady-state energy and mass balances for turbine 1

$$0 = \dot{Q}_{cv}^0 - \dot{W}_{t1} + \dot{m}_1 [(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2}] + g(z_1 - z_2)$$

and $0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2$

Solving for $\dot{m}_1$,

$$\dot{m}_1 = \frac{\dot{W}_{t1}}{(h_1 - h_2)}$$

From Table A-22; $h_1 = 1515.42 \text{ kJ/kg}$ and $h_2 = 1161.07 \text{ kJ/kg}$. Thus

$$\dot{m}_1 = \frac{(10,000 \text{ kW})}{(1515.42 - 1161.07) \text{ kJ/kg}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 28.22 \text{ kg/s}$$

Turning next to the heat exchanger

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4$$

$$\dot{m}_5 = \dot{m}_6$$

$$0 = \dot{Q}_{cv}^0 - \dot{W}_{cv}^0 + \dot{m}_2 [(h_2 - h_3) + \frac{V_2^2 - V_3^2}{2}] + g(z_2 - z_3) + \dot{m}_5 [(h_5 - h_6) + \frac{V_5^2 - V_6^2}{2}] + g(z_5 - z_6)$$

$$0 = \dot{m}_2 (h_2 - h_3) + \dot{m}_5 (h_5 - h_6)$$

or $h_3 = h_2 + \frac{\dot{m}_5}{\dot{m}_2} (h_5 - h_6)$

Again, from Table A-22; $h_5 = 1611.79 \text{ kJ/kg}$ and $h_6 = 1277.79 \text{ kJ/kg}$. Thus

$$h_3 = 1161.07 + \left[\frac{(1200 \text{ kg/min})}{(28.22 \text{ kg/s}) | 60 \text{ s/min} |} \right] (1611.79 - 1277.79)$$

$$= 1397.8 \text{ kJ/kg}$$

Interpolating in Table A-22

$$T_3 = 1301.5 \text{ K} \leftarrow T_3$$

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Problem 6-116 continued

(b) Now, writing the steady-state energy balance for turbine 2

$$0 = \dot{Q}_{cv} - \dot{W}_{t2} + \dot{m}_3 \left[(h_3 - h_4) + \frac{V_3^2 - V_4^2}{2} + g(z_3 - z_4) \right]$$

or

$$\dot{W}_{t2} = \dot{m}_3 (h_3 - h_4)$$

From Table A-22; $h_4 = 1023.25 \text{ kJ/kg}$, and

$$\dot{W}_{t2} = (28.22 \text{ kg/s})(1397.8 - 1023.25) \text{ kJ/kg} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 10,570 \text{ kW} \leftarrow \dot{W}_{t2}$$

(c) Mass and entropy rate balances for the first turbine reduce to give

$$\dot{\sigma}_{t1} = \dot{m}_1 (s_2 - s_1) = \dot{m}_1 [s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1]$$

$$= (28.22 \frac{\text{kg}}{\text{s}}) \left[3.07732 - 3.3620 - \frac{8.314}{28.97} \ln \frac{5}{20} \right] \frac{\text{kJ}}{\text{K} \cdot \text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 3.1936 \text{ kW/K} \leftarrow \dot{\sigma}_{t1}$$

where Eq. 6.20a and data from Table A-22 have been used. Likewise for turbine 2

$$\dot{\sigma}_{t2} = \dot{m}_1 (s_4 - s_3) = \dot{m}_1 [s^\circ(T_4) - s^\circ(T_3) - R \ln P_4/P_3]$$

$$= (28.22) \left[2.94468 - 3.27481 - \frac{8.314}{28.97} \ln \frac{1}{4.5} \right] = 2.8649 \text{ kW/K} \leftarrow \dot{\sigma}_{t2}$$

Mass and entropy rate balances for the interconnecting heat exchanger give

$$\dot{Q}_{HX} = \dot{m}_1 [s_3 - s_2] + \dot{m}_5 [s_6 - s_5]$$

$$= \dot{m}_1 \left[s^\circ(T_3) - s^\circ(T_2) - R \ln \frac{P_3}{P_2} \right] + \dot{m}_5 \left[s^\circ(T_6) - s^\circ(T_5) - R \ln \frac{P_6}{P_5} \right]$$

$$= 28.22 \left[3.27481 - 3.07732 - \frac{8.314}{28.97} \ln \frac{4.5}{5} \right] +$$

$$\left(\frac{1200}{60} \right) \left[3.17888 - 3.42892 - \frac{8.314}{28.97} \ln \frac{1}{1.35} \right]$$

$$= 6.4265 - 3.2783 = 3.1482 \text{ kW/K} \leftarrow \dot{\sigma}_{HX}$$

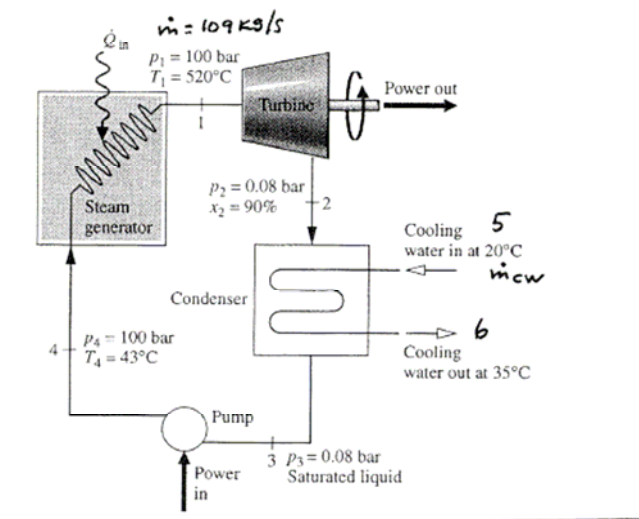
(d)

In rank order: Turbine 1, heat exchanger, turbine 2.

PROBLEM 6.117

Figure P6.117 shows a simple vapor power plant operating at steady state with water as the working fluid. Data at key locations are given on the figure. The mass flow rate of the water circulating through the components is 109 kg/s. Stray heat transfer and kinetic and potential energy effects can be ignored. Determine

- the mass flow rate of the cooling water, in kg/s.
- the thermal efficiency.
- the rates of entropy production, each in kW/K, for the turbine, condenser, and pump.
- Using the results of part (c), place the components in rank order, beginning with the component contributing most to inefficient operation of the overall system.



ENGR. MODEL

- For each of four principal components, a control volume at steady state encloses the component.
- For the turbine, pump, and condenser, $\dot{Q}_{cv} = 0$.
- Effects of kinetic and potential energy can be ignored.
- For liquid water, $h \sim h_f(T)$, $s \sim s_f(T)$.
- Energy transfers are positive in the direction of the arrows.

ANALYSIS:

State	h (kJ/kg)	s (kJ/kg·K)	Comment
1	3425.1	6.6622	Table A-4
2	2336.7	7.4651	Table A-3
3	173.9	0.5926	Table A-3
4	188.9	0.6061	Table A-5
5	83.96	0.2966	Table A-2
6	146.68	0.5053	Table A-2

- (a) Mass and energy rate balances for the condenser reduce to give

$$\dot{m}_{cw} = \dot{m} \left[\frac{h_2 - h_3}{h_6 - h_5} \right] = 109 \frac{\text{kg}}{\text{s}} \left[\frac{2336.7 - 173.9}{146.68 - 83.96} \right] = 3758.7 \text{ kg/s} \quad \leftarrow (a)$$

$$\begin{aligned} (b) \quad \eta &= \frac{\dot{W}_t - \dot{W}_p}{\dot{Q}_{in}}, \quad \dot{W}_t = \dot{m} [h_1 - h_2] = (109 \text{ kg/s}) (3425.1 - 2336.7) \frac{\text{kJ}}{\text{kg}} = 118,636 \frac{\text{kJ}}{\text{s}} \\ \dot{W}_p &= \dot{m} [h_4 - h_3] = (109) (188.9 - 173.9) = 1635 \text{ kJ/s} \\ \dot{Q}_{in} &= \dot{m} [h_1 - h_4] = (109) (3425.1 - 188.9) = 352,746 \text{ kJ/s} \\ &= \frac{117,001}{352,746} \\ &= 0.332 \quad (33.2\%) \end{aligned} \quad \leftarrow (b)$$

- (c) Mass and energy rate balances,

$$\begin{aligned} \text{turbine: } \dot{\sigma}_t &= \dot{m} (s_2 - s_1) = (109 \text{ kg/s}) (7.4651 - 6.6622) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 87.52 \frac{\text{kW}}{\text{K}} \\ \text{pump: } \dot{\sigma}_p &= \dot{m} (s_4 - s_3) = (109) (0.6061 - 0.5926) \left| \frac{1}{1} \right| = 1.47 \frac{\text{kW}}{\text{K}} \\ \text{condenser: } \dot{\sigma}_c &= \dot{m} (s_3 - s_2) + \dot{m}_{cw} (s_6 - s_5) \\ &= (109) (0.5926 - 7.4651) + 3758.7 (0.5053 - 0.2966) = 35.34 \frac{\text{kW}}{\text{K}} \end{aligned} \quad \leftarrow (c)$$

- (d) ✓ turbine
✓ condenser
✓ pump

PROBLEM 6.118

KNOWN: Steam contained in a large tank at a known state passes from the tank through a turbine into a small vessel until a specified final condition is attained in the vessel.

FIND: Determine the amount of entropy produced. Repeat for the case where no work is developed by the turbine.

SCHEMATIC & GIVEN DATA: See Figure E4.12.

ENGR. MODEL: See model listed in Example 4.12.

ANALYSIS: The mass rate balance reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

An entropy rate balance takes the form

$$\frac{dS_{cv}}{dt} = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}_i s_i + \dot{\sigma}_{cv}$$

Combining the mass and entropy rate balances

$$\frac{dS_{cv}}{dt} = s_i \frac{dm_{cv}}{dt} + \dot{\sigma}_{cv}$$

Integrating

$$\Delta S_{cv} = \int_1^2 s_i dm_{cv} + \sigma_{cv} \Rightarrow \Delta S_{cv} = s_i \Delta m_{cv} + \sigma_{cv}$$

In accordance with assumption 3, the specific entropy of the steam entering the control volume is constant at the value corresponding to the state in the large tank.

Since the small vessel is initially evacuated, the terms ΔS_{cv} and Δm_{cv} reduce to the entropy and mass within the vessel at the end of the process. That is

$$\Delta S_{cv} = m_2 s_2 - m_1 s_1^0, \quad \Delta m_{cv} = m_2 - m_1^0$$

Collecting results and solving for the amount of entropy produced

$$\sigma_{cv} = m_2 (s_2 - s_i) \quad (1)$$

At 15 bar, 320°C, Table A-4 gives $s_i = 6.9938 \text{ kJ/kg} \cdot \text{K}$. At 15 bar, 400°C, $s_2 = 7.2690 \text{ kJ/kg} \cdot \text{K}$. The solution to Example 4.12 provides m_2 . Thus, Eq. (1) gives

$$\sigma_{cv} = (2.96 \text{ kg})(7.2690 - 6.9938) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.8146 \frac{\text{kJ}}{\text{K}} \quad \sigma_{cv}$$

In the case where no work is developed, the solution to Example 4.12 gives the final temperature of the steam in the vessel as 477°C. Thus, interpolation in Table A-4 at 15 bar gives $s_2 = 7.5024 \text{ kJ/kg} \cdot \text{K}$, $v_2 = 0.22784 \text{ m}^3/\text{kg}$. Finally, substituting into Eq. (1)

$$\begin{aligned} \sigma_{cv} &= \left(\frac{0.6 \text{ m}^3}{0.22784 \text{ m}^3/\text{kg}} \right) (7.5024 - 6.9938) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ &= 1.3394 \text{ kJ/K} \quad \sigma_{cv} \end{aligned}$$

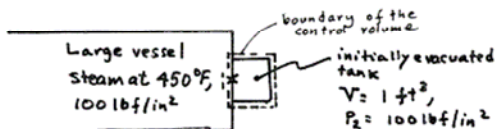
As there is less control of the entering stream in this case, the amount of entropy produced is greater.

PROBLEM 6.119

KNOWN: Steam contained in a large vessel at a specified state is allowed to flow into an initially evacuated tank until a specified pressure p is attained in the tank.

FIND: (a) For $p = 100 \text{ lbf/in}^2$, determine the final temperature of the steam in the tank and the amount of entropy produced. (b) Plot the quantities of part (a) versus p ranging from 10 to 100 lbf/in².

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) For the control volume shown in the accompanying figure, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. (2) The state of the steam in the large vessel remains constant.

ANALYSIS: The mass rate balance and the energy rate balance reduce, respectively, to

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

$$\frac{dU_{cv}}{dt} = \dot{m}_i h_i$$

Combining these expressions and integrating

$$\frac{dU_{cv}}{dt} = h_i \frac{dm_{cv}}{dt} \Rightarrow \Delta U_{cv} = \int_1^2 h_i dm_{cv} \Rightarrow \Delta U_{cv} = h_i \Delta m_{cv}$$

In accordance with assumption 2, the specific enthalpy of the steam entering the control volume is constant at the value corresponding to the state in the large vessel. Since the tank is initially evacuated, the terms ΔU_{cv} and Δm_{cv} reduce to the internal energy and mass within the tank at the end of the process: $\Delta U_{cv} = m_2 u_2$, $\Delta m_{cv} = m_2$. Accordingly, the energy balance reduces simply to

$$m_2 u_2 = m_2 h_i \Rightarrow u_2 = h_i \quad (1)$$

Thus, the final state of the steam in the tank is fixed by pressure p and $u_2 (= h_i)$.

An entropy rate balance for the control volume reduces to

$$\frac{dS_{cv}}{dt} = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}_i s_i + \dot{\sigma}_{cv}$$

Introducing the above mass rate balance and integrating

$$\frac{dS_{cv}}{dt} = s_i \frac{dm_{cv}}{dt} + \dot{\sigma}_{cv} \Rightarrow \Delta S_{cv} = \int_1^2 s_i dm_{cv} + \sigma_{cv} \Rightarrow \Delta S_{cv} = s_i \Delta m_{cv} + \sigma_{cv}$$

As for h_i , s_i remains constant during the process. Then, since the tank is initially evacuated, $\Delta S_{cv} = m_2 s_2$, $\Delta m_{cv} = m_2$. Inserting these expressions and solving for the amount of entropy produced is

$$\sigma_{cv} = m_2 (s_2 - s_i) = \left(\frac{V}{v_2} \right) (s_2 - s_i) \quad (2)$$

(a) $p = 100 \text{ lbf/in}^2$. From Table A-4E at 100 lbf/in², 450°F, $h_i = 1253.6 \text{ Btu/lb}$.

Then, with $u_2 = 1253.6 \text{ Btu/lb}$, interpolation in Table A-4E at 100 lbf/in² gives $T_2 = 702^\circ\text{F}$, $s_2 = 1.8042 \text{ Btu/lb} \cdot \text{R}$, $v_2 = 6.847 \text{ ft}^3/\text{lb}$ $\xrightarrow{T_2}$

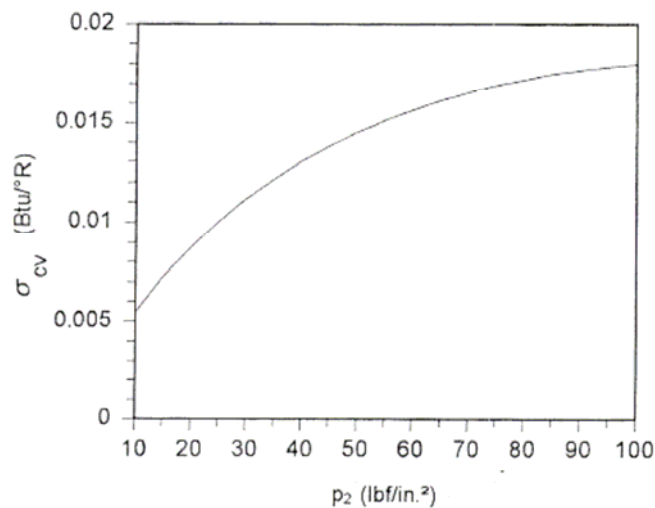
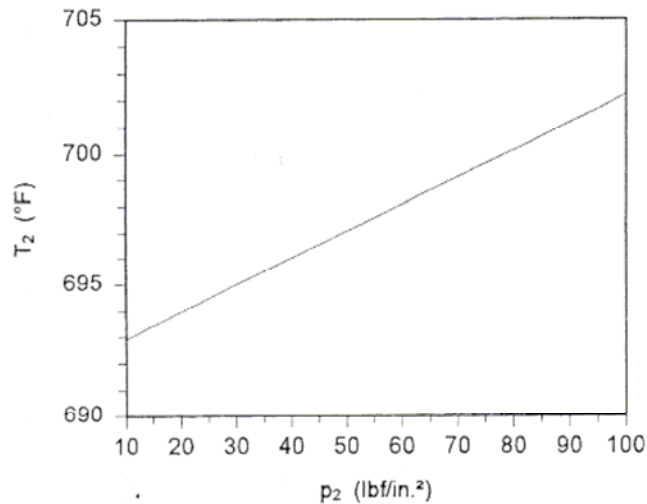
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Problem 6-119 continued

With Eq. (2) and s_i from Table A-4E at 100 lbf/in.², 450°F

$$q_{cv} = \frac{V}{U_2} (s_2 - s_i) = \left(\frac{1 \text{ ft}^3}{6.847 \text{ ft}^3/\text{lb}} \right) (1.8042 - 1.6812) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 0.018 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \leftarrow q_{cv}$$

(b) PLOTS

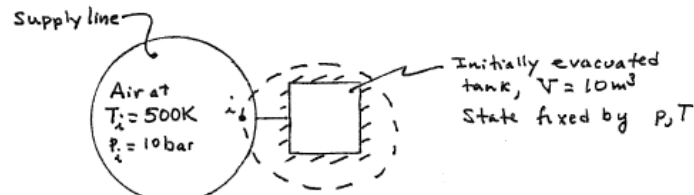


PROBLEM 6.120

KNOWN: Air flows from a large supply line into an initially evacuated, insulated tank until the tank pressure is p .

FIND: Plot the tank temperature, mass of air within the tank, and the amount of entropy produced versus p for $p < 10 \text{ bar}$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is shown in the above figure. (2) For the control volume, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects are negligible. (3) The state of the air within the supply line remains constant. (4) The mass of air within the piping connecting the supply line and tank can be ignored. (5) Air is modeled as an ideal gas with constant specific heat ratio k .

ANALYSIS: Paralleling the application of mass and energy balances in Example 4.12 but with $W_{cv} = 0$, the result is $u(T) = h(T_i)$, where T_i is the temperature of the air in the supply line and T is the temperature within the tank. For an ideal gas $h(T_i) = u(T_i) + RT_i$, so $u(T) = u(T_i) + RT_i$. Using ideal gas relations (assumption 5): $u(T) - u(T_i) = c_v [T - T_i]$, $R = c_p - c_v$, this gives

$$T = k T_i \quad (1)$$

That is, the temperature of the air within the tank is a constant independent of the pressure p .

Using the ideal gas equation of state, the mass of the air within the tank is

$$m = \frac{pV}{RT} = \frac{p}{P_i} \left[\frac{P_i V}{R(kT_i)} \right] = r \left[\frac{(10 \times 10^5 \text{ N/m}^2)(10 \text{ m}^3)}{\left(\frac{8.314 \text{ N} \cdot \text{m}}{28.97 \text{ kg} \cdot \text{K}} \right) (1.39 \times 500 \text{ K})} \right] = (50.14 r) \text{ kg} \quad (2)$$

where $r = p/P_i$ and $k = 1.39$ from Table A-20.

An entropy balance reads

$$\frac{dS_{cv}}{dt} = \dot{m}_i s_i + \dot{\sigma}_{cv} \quad (\text{integration}) \quad [m s(T, p) - 0] = \int \dot{m}_i s_i + \dot{\sigma}_{cv}$$

That is, with ideal gas relations

$$\dot{\sigma}_{cv} = m [s(T, p) - s(T_i, P_i)] = m \left[c_p \ln \frac{T}{T_i} - R \ln \frac{p}{P_i} \right]$$

(Eq. 2)
(Eq. 1)
(r)

$$\Rightarrow \dot{\sigma}_{cv} = (50.14 r) R \left[\frac{k}{k-1} \ln k - \ln r \right] = (50.14 r) \left(\frac{8.314}{28.97} \right) \left[\frac{1.39}{0.39} \ln 1.39 - \ln r \right] \frac{\text{kJ}}{\text{K}}$$

$$\Rightarrow \dot{\sigma}_{cv} = 14.39 r [1.174 - \ln r] \frac{\text{kJ}}{\text{K}} \quad (3)$$

SAMPLE CALCULATION: If $p = 10 \text{ bar}$, $r = 1$. Eq. (1), $T = 1.39(500 \text{ K}) = 695 \text{ K}$.

Eq. (2) gives, $m = 50.14 \text{ kg}$. Eq. (3) gives $\dot{\sigma}_{cv} = 16.89 \frac{\text{kJ}}{\text{K}}$.

Continued on next slide

To generate data for the required plots, we use IT, as follows:

IT Code

```
p_i = 10 // bar
Ti = 500 // K
V = 10 // m³
```

```
cp = cp_T("Air", Ti)
cv = cv_T("Air", Ti)
k = cp / cv
```

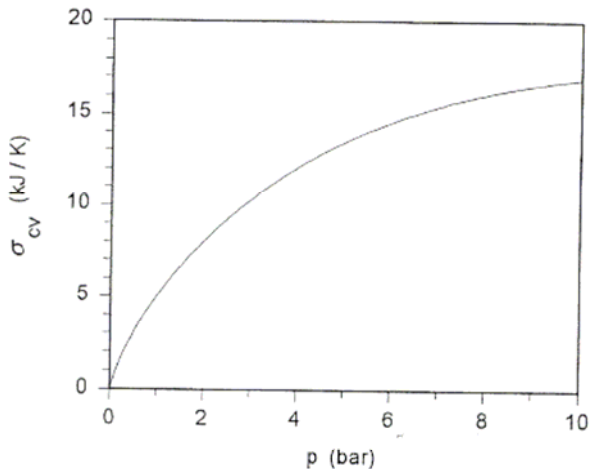
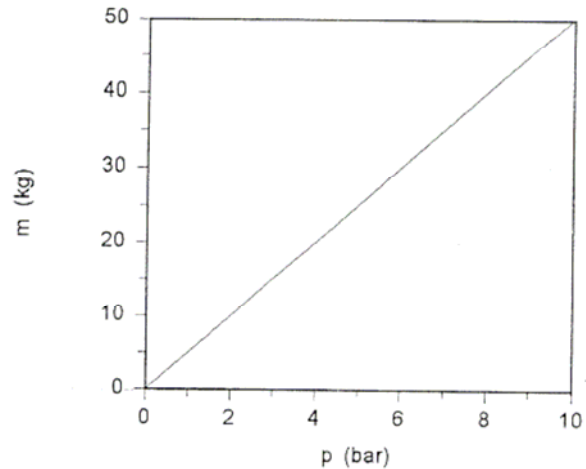
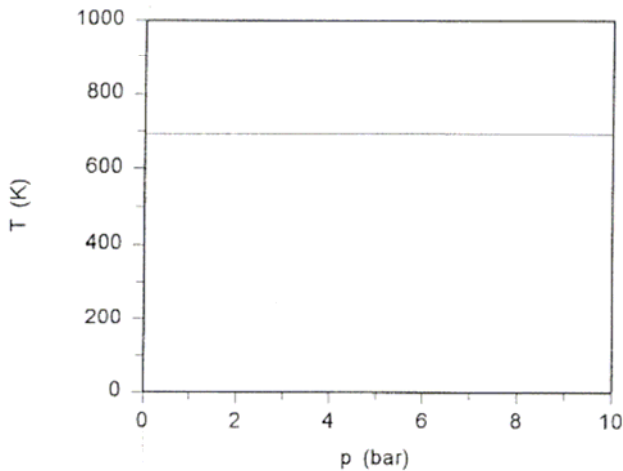
```
T = k * Ti
```

```
r = p / p_i
r = 1
v = v_TP("Air", T, p)
m = V / v
```

```
sigma_cv = m * (cp * ln(T / Ti) - R * ln(p / p_i))
R = 8.314 / 28.97
```

IT Results for p = 10 bar (r = 1)

```
cp = 1.029 kJ/kg·K
k = 1.387
v = 0.199 m³/kg
m = 50.26 kg
T = 693.3 K
σcv = 16.91 kJ/kg·K
```



Observe:

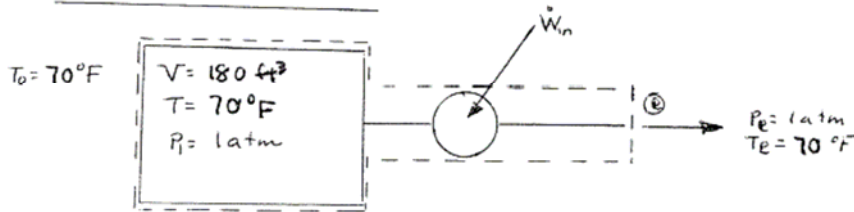
- The temperature in the tank is constant.
- The mass increases directly with tank pressure.
- There is more entropy produced as the pressure increases, as expected.

PROBLEM 6.121

KNOWN: A tank initially filled with air is evacuated by pumping out the air.

FIND: Determine the minimum theoretical work required.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is shown in the figure. (2) Heat transfer with the surroundings maintains the temperature of the air in the tank and at the discharge of the pump at 70°F. (3) Kinetic and potential energy effects can be ignored. (3) The air is modeled as an ideal gas.

ANALYSIS: A mass balance reduces to $dm/dt = -\dot{m}_e$. An energy balance reads $dU/dt = \dot{Q} + \dot{W}_{in} - \dot{m}_e h_e$. Combining these expressions

$$\frac{dU}{dt} = \dot{Q} + \dot{W}_{in} + h_e \frac{dm}{dt} \Rightarrow \Delta U = Q + W_{in} + h_e \Delta m \quad (1)$$

where h_e is determined by T_e and thus is constant.

An entropy balance reads $dS/dt = \dot{Q}/T_0 - \dot{m}_e s_e + \dot{\sigma}$, with $-\dot{m}_e = dm/dt$

$$\frac{dS}{dt} = \frac{\dot{Q}}{T_0} + s_e \frac{dm}{dt} + \dot{\sigma} \Rightarrow \Delta S = \frac{Q}{T_0} + s_e \Delta m + \sigma \quad (2)$$

where s_e is determined by T_e, P_e and thus is constant.

Eliminating Q between Eqs. (1), (2)

$$\begin{aligned} W_{in} &= \Delta U - T_0 \Delta S + T_0 s_e \Delta m - h_e \Delta m + T_0 \sigma \\ &= [m_2 u_2 - m_1 u_1] - T_0 [m_2 s_2 - m_1 s_1] + T_0 s_e [m_2 - m_1] - h_e (m_2 - m_1) + T_0 \sigma \\ &= m_1 T_0 (s_1 - s_e) + m_1 (h_e - u_1) + T_0 \sigma \end{aligned} \quad (3)$$

Since the states are the same initially and at the exit, $s_1 = s_e$. Also, with $h_e = u_e + P_e v_e$ ($h_e - u_1 = (u_e - u_1) + P_e v_e = P_e v_e$, since $T_1 = T_e$). With these, Eq. (3) reduces to

$$W_{in} = m_1 (P_e v_e) + T_0 \sigma = P_e V + T_0 \sigma \quad (4)$$

The specific volume at the exit is the same as the initial specific volume in the tank, and so $V = m_1 v_e$.

Finally, since $\sigma \geq 0$, the minimum theoretical value corresponds to $\sigma = 0$:

$$(W_{in})_{min} = P_e V$$

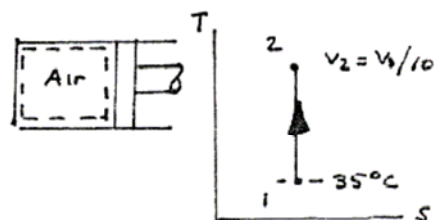
$$\begin{aligned} &= (14.7 \times 44 \frac{\text{lb}}{\text{ft}^2}) (180 \text{ ft}^3) \left(\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lb}} \right) \\ &= 489.7 \text{ Btu} \end{aligned}$$

$(W_{in})_{min}$

PROBLEM 6.122

Air in a piston-cylinder assembly is compressed isentropically from state 1, where $T_1 = 35^\circ\text{C}$, to state 2, where the specific volume is one-tenth of the specific volume at state 1. Applying the ideal gas model with $k = 1.4$, determine (a) T_2 , in $^\circ\text{C}$ and (b) the work, in kJ/kg .

SCHEMATIC & GIVEN DATA



ENGR. MODEL:

1. The air is the closed system.
2. The air undergoes an isentropic process.
3. The air is modeled as an ideal gas with $k = 1.4$.
4. Kinetic and potential energy play no role.

ANALYSIS: (a) With Eq. 6.44,

$$T_2 = T_1 \left(\frac{v_1}{v_2} \right)^{k-1} = (308.15\text{K}) (10)^{1.4-1} = 774.07\text{K} \quad (500.9^\circ\text{C}) \leftarrow$$

(b) Reducing an energy balance for the adiabatic process,

$$\Delta U + \Delta KE + \Delta PE = 0 - W \Rightarrow W = -\Delta U = -m c_v (T_2 - T_1).$$

From Eq. 3.47b, $c_v = R/(k-1)$. So

$$\frac{W}{m} = -\frac{R}{(k-1)} (T_2 - T_1) = -\frac{8.314}{28.97} \left(\frac{308.15 - 774.07}{1.4 - 1} \right) \frac{\text{kJ}}{\text{kg}} = -334.3 \frac{\text{kJ}}{\text{kg}} \leftarrow$$

PROBLEM 6.123

Air in a piston-cylinder assembly is compressed isentropically from $T_1 = 60^\circ\text{F}$, $p_1 = 20 \text{ lbf/in.}^2$ to $p_2 = 2000 \text{ lbf/in.}^2$. Assuming the ideal gas model, determine the temperature at state 2, in $^\circ\text{R}$, using (a) data from Table A-22E, and (b) a constant specific heat ratio, $k = 1.4$. Compare the values obtained in parts (a) and (b) and comment.

ENGR. MODEL:

1. The air is the closed system.
2. The air undergoes an isentropic process
3. The air is modeled as an ideal gas

(a) $\frac{P_2}{P_1} = \frac{P_r(2)}{P_r(1)} \Rightarrow P_r(2) = P_r(1) \left(\frac{2000}{20} \right) = 1.2147 (100) = 121.47$

Interpolating in Table A-22E gives, $T_2 = 1828^\circ\text{R}$

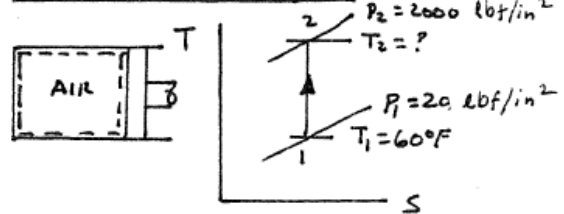
(b) With Eq. 6.43

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = 520^\circ\text{R} (100)^{\frac{0.4}{1.4}} = 1938^\circ\text{R}$$

The value obtained in part (b) is about 6% greater than in part (a).

The approach of part (a) accounts for the variation of specific heats with temperature whereas the approach of part (b) does not.

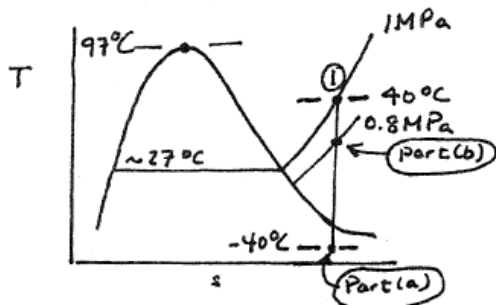
SCHEMATIC & GIVEN DATA:



PROBLEM 6.124

Propane undergoes an isentropic expansion from an initial state where $T_1 = 40^\circ\text{C}$, $p_1 = 1\text{ MPa}$ to a final state where the temperature and pressure are T_2, p_2 , respectively. Determine

- p_2 , in kPa, when $T_2 = -40^\circ\text{C}$
- T_2 , in $^\circ\text{C}$, when $p_2 = 0.8\text{ MPa}$



(a) From Table A-18, $s_1 = 1.810\text{ kJ/kg}\cdot\text{K}$.

From Table A-16 at -40°C ,

$$s_f = 0.000\text{ kJ/kg}\cdot\text{K}$$

$$s_g = 1.815\text{ kJ/kg}\cdot\text{K}$$

So, state 2 is in the two-phase region

$$\text{and } p_2 = p_{\text{sat}}(-40^\circ\text{C}) = 1.11\text{ bar} = 111\text{ kPa} \quad \leftarrow (a)$$

(b) From Table A-18 at 0.8 MPa , interpolation gives $T_2 = 30.6^\circ\text{C}$

$\leftarrow (b)$

PROBLEM 6.125

Argon in a piston-cylinder assembly is compressed isentropically from state 1, where $p_1 = 150 \text{ kPa}$, $T_1 = 35^\circ\text{C}$, to state 2, where $p_2 = 300 \text{ kPa}$. Assuming the ideal gas model with $k = 1.67$, determine (a) T_2 , in $^\circ\text{C}$, and (b) the work, in kJ per kg of argon.

ENGR. MODEL:

1. The argon is the closed system.
2. The argon undergoes an isentropic process.
3. The argon is modeled as an ideal gas with $k = 1.67$.
4. Kinetic and potential energy play no role.

ANALYSIS:

(a) w. th Eq. 6.43, $T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(k-1)/k} = (308 \text{ K}) \left(\frac{300}{150} \right)^{\frac{1.67-1}{1.67}} = 407 \text{ K} (134^\circ\text{C}) \leftarrow (a)$

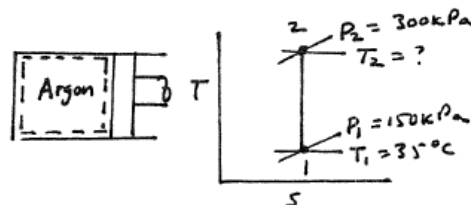
(b) Reducing an energy balance for the adiabatic process,
 $\Delta U + \cancel{Q} + \cancel{PE} = \cancel{0} - W \Rightarrow W = -m \Delta u = -m c_v (T_2 - T_1).$

From Eq. 3.47b, $c_v = R/(k-1)$. So

$$\begin{aligned} \frac{W}{m} &= -\frac{R}{(k-1)} (T_2 - T_1) = -\left(\frac{8.314}{39.94} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) \left(\frac{407 - 308}{1.67 - 1} \right) \text{K} \\ &= -30.8 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

$\leftarrow (b)$

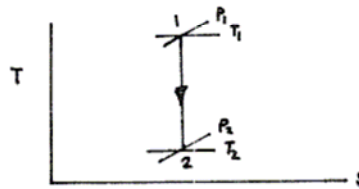
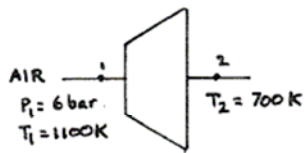
SCHEMATIC & GIVEN DATA:



PROBLEM 6.126

Air enters a turbine operating at steady state at 6 bar and 1100 K and expands isentropically to a state where the temperature is 700 K. Employing the ideal gas model with data from Table A-22, and ignoring kinetic and potential energy changes, determine the pressure at the exit, in bar, and the work, in kJ per kg of air flowing.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The turbine is at steady state and the expansion is isentropic. (2) Kinetic and potential energy changes from inlet to exit can be neglected. (3) Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$ and an energy rate balance reduces to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad (1)$$

Table A-22: With h_1 and h_2 from the table, Eq. (1) gives

$$\frac{\dot{W}_{cv}}{\dot{m}} = (1161.07 - 713.27) \frac{\text{kJ}}{\text{kg}} = 447.8 \text{ kJ/kg} \quad \left(\frac{\text{kJ}}{\text{kg}} \right)$$

The pressures and temperatures are related by Eq. 6.41, which with data from the table gives

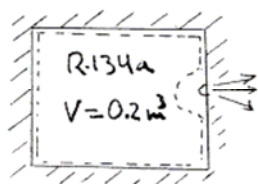
$$P_2 = P_1 \left(\frac{P_{r2}}{P_{r1}} \right) = 6 \text{ bar} \left(\frac{29.8}{167.1} \right) = 1.034 \text{ bar} \quad P_2$$

PROBLEM 6.127

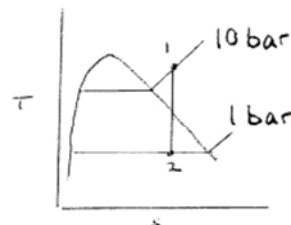
KNOWN: Data are provided for a rigid, insulated tank initially filled with R-134a vapor. A leak develops and R-134a slowly escapes until the pressure becomes 1 bar.

FIND: Determine (a) the final temperature of the R-134a within the tank and (b) the amount of mass that exits.

SCHEMATIC & GIVEN DATA:



Initially $P_1 = 10 \text{ bar}$, $T_1 = 40^\circ\text{C}$
 $P_2 = 1 \text{ bar}$



ENGR. MODEL: (1) The system consists of the mass initially in the tank that remains in the tank. (2) For the system, $\dot{Q} = 0$ and the effects of kinetic and potential energy can be ignored. (3) Irreversibilities within the tank can be ignored as the air slowly escapes.

ANALYSIS: The solution closely follows that of Example 6.10, leading to the result that the mass within the system undergoes an isentropic expansion: $s_2 = s_1$.

With values from Table A-12, $s_1 = 0.9066 \text{ kJ/kg}\cdot\text{K}$, $v_1 = 0.02029 \text{ m}^3/\text{kg}$.

Since $s_2 = s_1$, state 2 is located in the two phase liquid-vapor region with quality

$$x_2 = \frac{s_2 - s_f}{s_g - s_f} = \frac{0.9066 - 0.0678}{0.9395 - 0.0678} = 0.9623$$

(a) The final temperature is the saturation temperature corresponding to $P_2 = 1 \text{ bar}$: $T_2 = -26.43^\circ\text{C}$ (a)

(b) The mass that exits $= m_1 - m_2$, or

$$\Delta m = m_1 - m_2 = V \left[\frac{1}{v_1} - \frac{1}{v_2} \right]$$

where

$$v_2 = (0.7258 \times 10^{-3}) + 0.9623 [0.1917 - 0.7258 \times 10^{-3}] = 0.1845 \frac{\text{m}^3}{\text{kg}}$$

So

$$\Delta m = 0.2 \text{ m}^3 \left[\frac{1}{0.02029} - \frac{1}{0.1845} \right] \frac{\text{kg}}{\text{m}^3} = 8.77 \text{ kg} \quad \leftarrow (b)$$

PROBLEM 6.128

Air in a piston-cylinder assembly is compressed isentropically from an initial state where $T_1 = 340 \text{ K}$ to a final state where the pressure is 90% greater than at state 1. Assuming the ideal gas model, determine (a) T_2 , in K, and (b) the work, in kJ/kg.

ANALYSIS: (a) With $P_2/P_1 = 1.9$, Eq. 6.41 gives

$$P_2 = P_1 (P_2/P_1) = 2.149(1.9) = 4.083$$

Interpolating in Table A-22, $T_2 = 408 \text{ K}$,

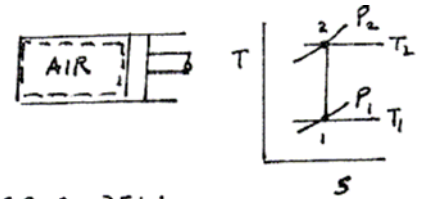
$$u_2 = 291.96 \text{ kJ/kg}$$

(b) An energy balance reduces to read,

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W \Rightarrow W = -m(u_2 - u_1).$$

$$\Rightarrow \frac{W}{m} = u_1 - u_2 = 292.82 - 291.96 = -49.14 \text{ kJ/kg}$$

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

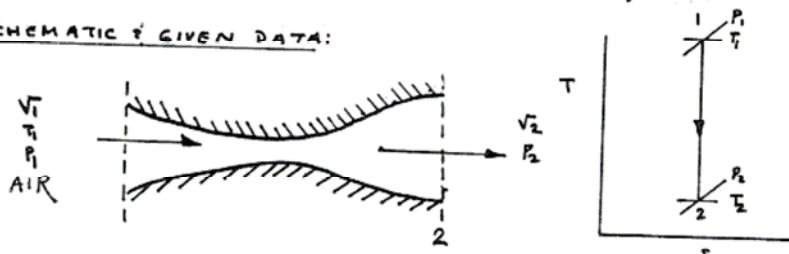
1. The air is the closed system.
2. The air undergoes an isentropic process.
3. The air is modeled as an ideal gas.
4. Kinetic and potential energy play no role.

PROBLEM 6.129

KNOWN: Air as an ideal gas with constant specific heat ratio k expands isentropically from P_1, T_1, V_1 to a pressure P_2 .

FIND: (a) Develop an expression for the exit velocity V_2 in terms of n, R, V_1, T_1, P_1, P_2 .
(b) Plot V_2 versus P_2/P_1 for selected values of k, V_1 , and T_1 .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The expansion through the nozzle is isentropic. (2) The nozzle operates at steady state. (3) The gas is modeled as an ideal gas with constant specific heat ratio k . (4) Potential energy effects are negligible.

ANALYSIS: (a) At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and an energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m} \left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right)$$

or

$$0 = h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}$$

With Eq. 3.47a, $c_p = kR/(k-1)$

$$\begin{aligned} \frac{V_2^2}{2} &= \frac{V_1^2}{2} + \frac{kR}{k-1} (T_1 - T_2) \\ &= \frac{V_1^2}{2} + \frac{kRT_1}{k-1} \left(1 - \frac{T_2}{T_1} \right) \end{aligned}$$

For constant k and no change in specific entropy, Eq. 6.43 relates the temperatures and pressures, giving

$$V_2 = \sqrt{V_1^2 + 2 \frac{kRT_1}{k-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \right]} \quad (a)$$

(b) Calculating with $V_1 = 0$, $T_1 = 1000\text{ K}$, $k = 1.4$, $(P_2/P_1) = 0.1$, we get

$$V_2 = \sqrt{\frac{(2)(1.4)}{1.4-1} \frac{8314 \text{ (N}\cdot\text{m)}}{28.97 \text{ (kg}\cdot\text{K)}} (1000 \text{ K}) \left[\frac{1 \text{ (kg}\cdot\text{m/s}^2)}{1 \text{ N}} \right] \left[1 - (0.1)^{0.4/1.4} \right]} = 984 \text{ m/s} \quad (b)$$

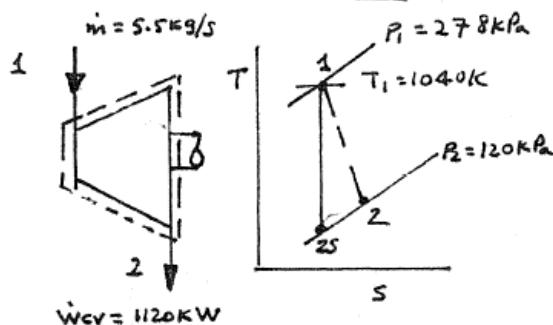
PROBLEM 6.130

Air modeled as an ideal gas enters a turbine operating at steady state at 1040 K, 278 kPa and exits at 120 kPa. The mass flow rate is 5.5 kg/s, and the power developed is 1120 kW. Stray heat transfer and kinetic and potential energy effects are negligible. Determine (a) the temperature of the air at the turbine exit, in K, and (b) the isentropic turbine efficiency.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas.

SCHEMATIC & GIVEN DATA:



ANALYSIS: (a) An energy rate balance reduces to give $0 = -\dot{W}_{cv} + \dot{m}(h_1 - h_2)$. Solving,

$$h_2 = h_1 - \frac{\dot{W}_{cv}}{\dot{m}} \Rightarrow h_2 = 1091.85 - \frac{1120 \text{ kJ/s}}{5.5 \text{ kg/s}} = 888.21 \text{ kJ/kg. Interpolating in}$$

Table A-22, $T_2 = 860 \text{ K.}$ ← (a)

(b) The isentropic turbine efficiency is $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$. To find h_{2s} use

$$\text{Eq. 6.41, } P_{r,2s} = \frac{P_2}{P_1} P_{r,1} = \frac{120}{278} (133.3) = 57.54 \Rightarrow h_{2s} = 865.82 \text{ kJ/kg}$$

$$\Rightarrow \eta_t = \frac{1091.85 - 888.21}{1091.85 - 865.82} = 0.901 \quad (90.1\%)$$
← (b)

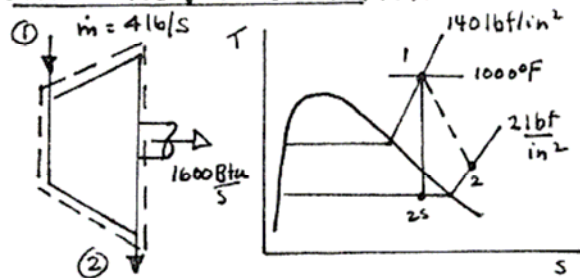
PROBLEM 6.131

Water vapor at 1000°F , 140 lbf/in^2 enters a turbine operating at steady state and expands to 2 lbf/in^2 . The mass flow rate is 4 lb/s and the power developed is 1600 Btu/s . Stray heat transfer and kinetic and potential energy effects are negligible. Determine the isentropic turbine efficiency.

ENGR. MODEL:

1. The control volume shown with the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects are negligible.

SCHEMATIC & GIVEN DATA:



ANALYSIS: The isentropic turbine efficiency is $\eta_t = \frac{\dot{W}_t}{\dot{m}(h_1 - h_{2s})}$. From Table A-4E, $h_1 = 1531.0\text{ Btu/lb}$, $s_1 = 1.8827\text{ Btu/lb}\cdot^\circ\text{R}$. With $s_{2s} = s_1$ and data from Table A-3E,

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{1.8827 - 0.1750}{1.7448} = 0.9787 \Rightarrow h_{2s} = h_f + x_{2s} h_{fg} = 94.02 + 0.9787(1022.1) = 1094.35 \frac{\text{Btu}}{\text{lb}}$$

Finally,

$$\eta_t = \frac{1600\text{ Btu/s}}{(4\text{ lb/s})(1531.0 - 1094.35) \frac{\text{Btu}}{\text{lb}}} = 0.916 \quad (91.6\%)$$

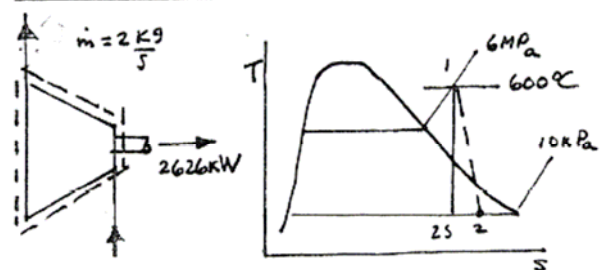
PROBLEM 6.132

Water vapor at 6 MPa, 600°C enters a turbine operating at steady state and expands to 10 kPa. The mass flow rate is 2 kg/s, and the power developed is 2626 kW. Stray heat transfer and kinetic and potential energy effects are negligible. Determine (a) the isentropic turbine efficiency and (b) the rate of entropy production within the turbine, in kW/K.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects are negligible.

SCHEMATIC & GIVEN DATA



Analysis: (a) To fix the exit state requires two independent intensive properties. One is the pressure. The other is h_2 found from applying the mass and energy rate balances:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)] \Rightarrow h_2 = h_1 - (\dot{W}_{cv}/\dot{m})$$

Table A-4, $h_1 = 3658.4 \text{ kJ/kg}$, $s_1 = 7.1677 \text{ kJ/kg} \cdot \text{K}$

$$\therefore h_2 = 3658.4 - \left[\frac{2626 \text{ kJ/s}}{2 \text{ kg/s}} \right] = 2345.4 \text{ kJ/kg} \quad \text{Since } h_f < h_2 < h_g, \text{ state 2 is a 2-phase liquid-vapor mixture.}$$

$$\therefore x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{2345.4 - 191.83}{2392.8} = 0.9$$

(data from Table A-2)
@ 0.1 bar

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \quad \text{To find } h_{2s}, \text{ use } s_{2s} = s_1 \Rightarrow x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{7.1677 - 0.6493}{8.1502 - 0.6493} = 0.869$$

$$\therefore h_{2s} = h_f + x_{2s}(h_g - h_f) = 191.83 + 0.869(2392.8) = 2271.2 \frac{\text{kJ}}{\text{kg}}$$

$$\therefore \eta_t = \frac{3658.4 - 2345.4}{3658.4 - 2271.2} = \frac{1313}{1387.2} = 0.947 \quad (94.7\%)$$

← (a)

(b) An entropy rate balance reduces to, $0 = \sum \dot{Q}_j/T_j + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$

$$\Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1), \quad \text{where } s_2 = s_f + x_2(s_g - s_f) = 0.6493 + 0.9(8.5009) = 7.4 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

So,

$$\dot{\sigma}_{cv} = (2 \frac{\text{kg}}{\text{s}})(7.4 - 7.1677) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 0.465 \frac{\text{kW}}{\text{K}}$$

← (b)

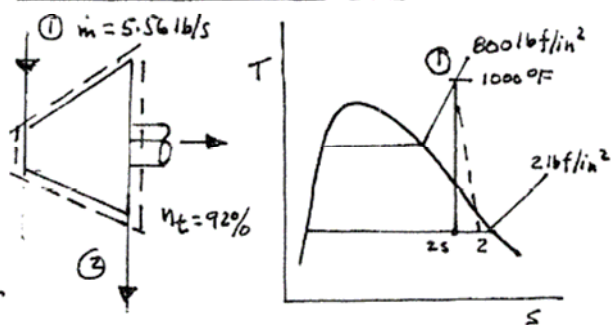
PROBLEM 6.133

Water vapor at 800 lbf/in^2 , 1000°F enters a turbine operating at steady state and expands to 2 lbf/in^2 . The mass flow rate is 5.56 lb/s , and the isentropic turbine efficiency is 92% . Stray heat transfer and kinetic and potential energy effects are negligible. Determine the power developed by the turbine, in hp.

ENGR MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects can be ignored.

SCHEMATIC & GIVEN DATA:



ANALYSIS: Reducing an energy rate balance, $\dot{W}_{cv} = \dot{m}(h_1 - h_2)$. Also, the isentropic turbine efficiency is $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$. Collecting these expressions,

$$\dot{W}_{cv} = \dot{m} \eta_t (h_1 - h_{2s}) \quad (1)$$

From Table A-4E at 800 lbf/in^2 , 1000°F , $h_1 = 1511.9 \text{ Btu/lb}$, $s_1 = 1.6807 \text{ Btu/lb} \cdot ^\circ\text{R}$.

With data from Table A-3E

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{1.6807 - 0.1753}{1.7448} = 0.863$$

$$\therefore h_{2s} = h_f + x_{2s} h_{fg} = 94.02 + 0.863(1022.1) = 976.09 \text{ Btu/lb}$$

Substituting values into Eq. (1)

$$\begin{aligned} \dot{W}_{cv} &= (5.56 \frac{\text{lb}}{\text{s}})(0.92)(1511.9 - 976.09) \frac{\text{Btu}}{\text{lb}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| \\ &= 3877 \text{ hp} \end{aligned}$$



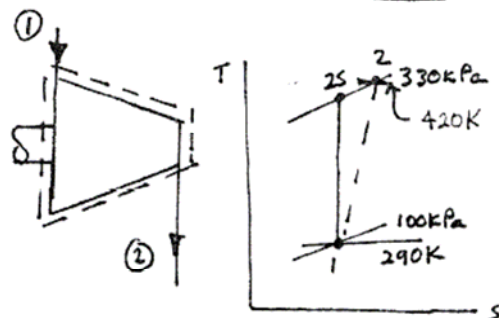
PROBLEM 6.134

Air enters the compressor of a gas turbine power plant operating at steady state at 290 K, 100 kPa and exits at 420 K, 330 kPa. Stray heat transfer and kinetic and potential energy effects are negligible. Using the ideal gas model for air, determine the isentropic compressor efficiency.

ENGINEER MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas.

SCHEMATIC & GIVEN DATA:



ANALYSIS: The isentropic compressor efficiency is $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$.

To find h_{2s} , use Eq. 6.41: $P_{r2} = P_{r1} (P_2/P_1) = 1.2311 (330/100) = 4.0626 \Rightarrow h_{2s} = 408.5 \text{ kJ/kg}$ (Table A-22)

So, with $h_1 = 290.16 \text{ kJ/kg}$, $h_2 = 421.26 \text{ kJ/kg}$ from Table A-22,

$$\eta_c = \frac{408.5 - 290.16}{421.26 - 290.16} = 0.903 \text{ (90.3\%)} \quad \leftarrow$$

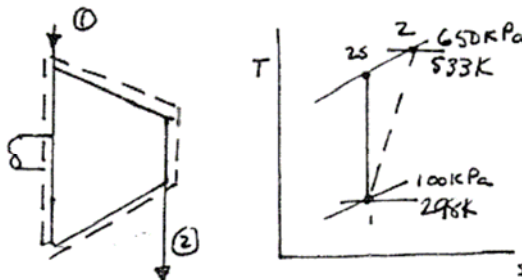
PROBLEM 6.135

Air at 25°C, 100 kPa enters a compressor operating at steady state and exits at 260°C, 650 kPa. Stray heat transfer and kinetic and potential energy effects are negligible. Modeling air as an ideal gas with $k = 1.4$, determine the isentropic compressor efficiency.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas with $k = 1.4$.

SCHEMATIC & GIVEN DATA:



ANALYSIS: By Eq. 3.47a, $c_p = \frac{kR}{k-1}$. So, if k is constant, c_p is constant. Accordingly, the isentropic compressor efficiency is $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)} = \frac{T_{2s} - T_1}{T_2 - T_1}$ (1)

With Eq. 6.43, $T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 298K \left(\frac{650}{100} \right)^{\frac{1.4-1}{1.4}} = 509K$

Substituting values into Eq. (1),

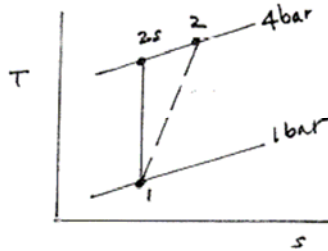
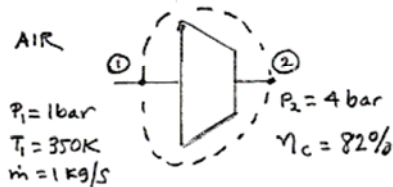
$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{(509 - 298)K}{235K} = 0.898 \text{ (89.8\%)} \quad \leftarrow$$

PROBLEM 6.136

KNOWN: Steady state operating data are provided for an air compressor.

FIND: Determine the power input and rate of entropy production using Table A-22.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects can be ignored. (4) Air is modeled as an ideal gas.

ANALYSIS: Reduction of mass, energy, and entropy balances with the given assumptions yields

$$\dot{W}_{cv} = \dot{m} (h_1 - h_2) \quad (1)$$

$$\dot{Q}_{cv} = \dot{m} (s_2 - s_1) \quad (2)$$

Air Table Analysis. Consider process 1-2s. With data from Table A-22

$h_1 = 350.49 \text{ kJ/kg}$, $s^\circ(T_1) = 1.85708 \text{ kJ/kg} \cdot \text{K}$, and

$$P_{r,2s} = \frac{P_2}{P_1} P_{r,1} = \left(\frac{4 \text{ bars}}{1 \text{ bar}} \right) (2.379) = 9.516$$

Interpolation in Table A-22 gives $h_{2s} = 520.98 \text{ kJ/kg}$. Then, using the isentropic compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 350.49 + \frac{520.98 - 350.49}{0.82} = 558.4 \text{ kJ/kg}$$

Interpolating in Table A-22 with h_2 gives $s^\circ(T_2) = 2.3247 \text{ kJ/kg} \cdot \text{K}$.

Substituting values in Eqs. (1), (2), using Eq. 6.20a,

$$\dot{W}_{cv} = \dot{m} (h_1 - h_2) = \left(1 \frac{\text{kg}}{\text{s}} \right) (350.49 - 558.4) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -207.9 \text{ kW} \leftarrow$$

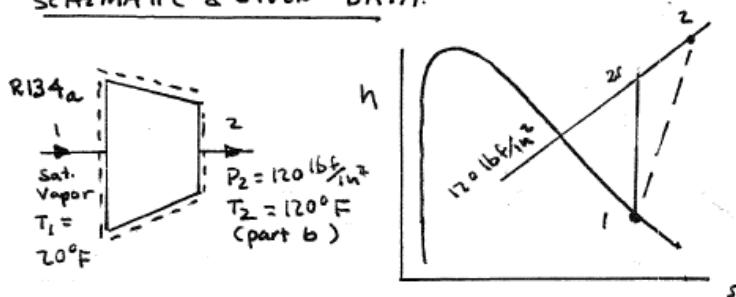
$$\begin{aligned} \dot{Q}_{cv} &= \dot{m} (s_2 - s_1) = \dot{m} (s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1) = \left(1 \frac{\text{kg}}{\text{s}} \right) \left(2.3247 - 1.8571 - \frac{8.314}{28.97} \ln 4 \right) \left(\frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) \\ &= 0.0698 \frac{\text{kJ/s}}{\text{K}} = 0.0698 \frac{\text{kW}}{\text{K}} \leftarrow \end{aligned}$$

PROBLEM 6.137

KNOWN: R134a enters a compressor operating adiabatically at steady state as a saturated vapor and exits at a known pressure.

FIND: (a) Determine the minimum theoretical work input per unit mass flowing and the corresponding exit temperature. (b) For a specified exit temperature, determine the isentropic compressor efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic operates at steady state with no appreciable heat transfer with the surroundings.
2. Kinetic and potential energy effects can be ignored.

ANALYSIS: (a) The mass, energy, and entropy rate balances reduce to give

$$\left(\frac{-\dot{W}_{cv}}{\dot{m}} \right) = h_2 - h_1, \quad \frac{\dot{Q}_{cv}}{\dot{m}} = s_2 - s_1$$

The work input decreases as h_2 decreases. The entropy balance and h - s diagram show that the smallest allowed value for h_2 corresponds to state 2s, where $s_{2s} = s_1$. From Table A-10E, at 20°F, $h_1 = 104.61$ Btu/lb, $s_1 = 0.2205$ Btu/lb·°R.

Then, interpolation in Table A-12E at 120 lbf/in² with $s_{2s} = s_1$ gives, $T_{2s} = 98.9^\circ\text{F}$ and $h_{2s} = 116.04$ Btu/lb. Accordingly,

$$\left(\frac{-\dot{W}_{cv}}{\dot{m}} \right)_{\min} = h_{2s} - h_1 = (116.04 - 104.61) = 11.43 \text{ Btu/lb} \quad \leftarrow$$

(b) The isentropic compressor efficiency is given by Eq. 6.48, which reduces to

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{11.43}{121.52 - 104.61} = \frac{11.43}{16.91} = 0.676 \text{ (67.6\%)} \quad \leftarrow$$

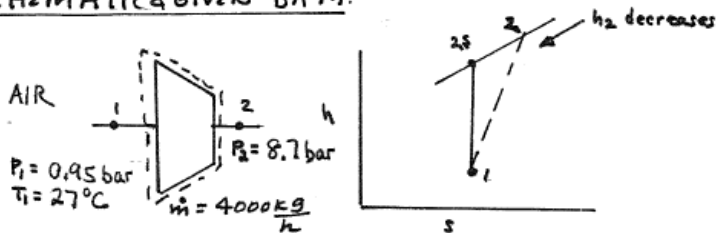
where h_2 is from Table A-12E at 120 lbf/in², 120°F.

PROBLEM 6.138

KNOWN: Air enters an insulated compressor operating at steady state at a specified state and mass flow rate and exits at a known pressure.

FIND: (a) Determine the minimum theoretical power input and the corresponding exit temperature. (b) For a specified exit temperature, determine the power input and the isentropic compressor efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The compressor operates at steady state and $\dot{Q}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) Air is modeled as an ideal gas.

①

ANALYSIS: (a) At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and an energy rate balance reduces to

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}} \right) = h_2 - h_1 \quad (1)$$

An entropy rate balance at steady state gives, $s_2 - s_1 = \dot{Q}_{cv}/\dot{m} \geq 0$. The work input decreases as h_2 decreases. The h - s diagram and the entropy balance shows that the smallest allowed value for h_2 corresponds to state 2s, where $s_1 = s_{2s}$. That is

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}} \right)_{min} = h_{2s} - h_1$$

To find h_{2s} use P_r data from Table A-22 and

$$\frac{P_r(T_{2s})}{P_r(T_1)} = \frac{P_2}{P_1} \Rightarrow P_r(T_{2s}) = P_r(T_1) \left(\frac{P_2}{P_1} \right) = 1.3860 \left(\frac{8.7}{0.95} \right) = 12.69$$

Interpolating in Table A-22, $T_{2s} = 560.4 \text{ K } (287.4^\circ\text{C})$, $h_{2s} = 565.54 \text{ kJ/kg}$. Thus, with h_1 from Table A-22

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}} \right)_{min} = 565.54 - 300.19 = 265.35 \text{ kJ/kg}$$

$$\Rightarrow (-\dot{W}_{cv})_{min} = 294.8 \text{ kW} \quad \leftarrow (-\dot{W}_{cv})_{min}$$

(b) $T_2 = 620 \text{ K } (347^\circ\text{C})$. With $h_2 = 628.07 \text{ kJ/kg}$ from Table A-22, Eq. (1)

gives

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}} \right) = h_2 - h_1 = 628.07 - 300.19 = 327.88 \text{ kJ/kg}$$

$$\Rightarrow (-\dot{W}_{cv}) = 364.3 \text{ kW} \quad \leftarrow (-\dot{W}_{cv})$$

With Eq. 6.48

$$\eta_c = \frac{294.8}{364.3} = 0.809 \text{ (80.9\%)} \quad \leftarrow \eta_c$$

1. Checking the compressibility chart with

$$P_{r1} = \frac{0.95 \text{ bar}}{37.7 \text{ bar}} = 0.025, \quad P_{r2} = \frac{8.7}{37.7} = 0.231$$

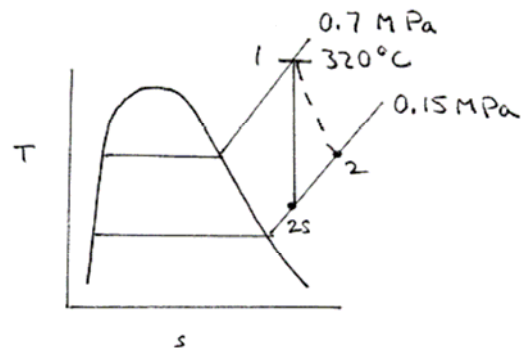
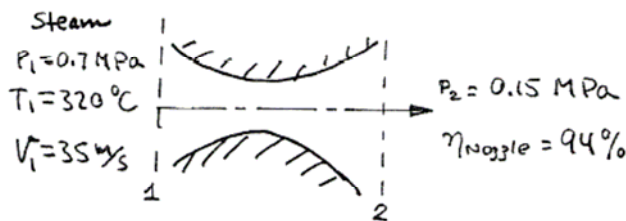
indicates that the ideal gas model is valid over a wide range of temperature including those here.

PROBLEM 6.139

KNOWN: Steady state operating data are provided for an insulated nozzle.

FIND: Determine the exit velocity.

SCHMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, and potential energy effects are negligible.

ANALYSIS: The isentropic nozzle efficiency, Eq. 6.47, yields

$$V_2 = \sqrt{\eta_{\text{nozzle}}} V_{2s} \quad (1)$$

where V_{2s} is the exit velocity for an isentropic expansion. Considering the isentropic expansion, reduction of mass and energy balances gives

$$\frac{V_{2s}^2}{2} = \frac{V_1^2}{2} + h_1 - h_{2s} \quad (2)$$

With data from Table A-4, $h_1 = 3100.9 \text{ kJ/kg}$, $s_1 = 7.3697 \text{ kJ/kg}\cdot\text{K}$. Then, interpolating at 0.15 MPa with $s_{2s} = s_1$, $h_{2s} = 2752.8 \text{ kJ/kg}$.

Substituting values in Eq. (2)

$$\frac{V_{2s}^2}{2} = \frac{(35 \text{ m/s})^2}{2} + (3100.9 - 2752.8) \frac{\text{kJ}}{\text{kg}} \left| \frac{1000 \text{ N}\cdot\text{m}}{\text{kJ}} \right| \left| \frac{1 \text{ kg}\cdot\text{m}}{1 \text{ N}\cdot\text{s}^2} \right|$$

gives

$$V_{2s} = 835.1 \text{ m/s} \Rightarrow V_2 = \sqrt{0.94} (835.1) = 809.7 \text{ m/s} \quad \leftarrow V_2$$

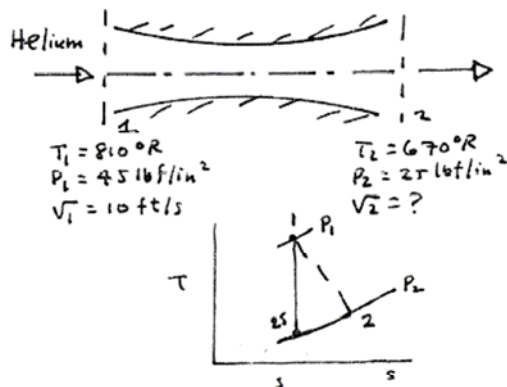
PROBLEM 6.140

Helium gas at 810°R , 45 lbf/in^2 and a velocity of 10 ft/s enters an insulated nozzle operating at steady state and exits at 670°R , 25 lbf/in^2 . Modeling helium as an ideal gas with $k = 1.67$, determine (a) the velocity at the nozzle exit, in ft/s , (b) the isentropic nozzle efficiency, and (c) the rate of entropy production within the nozzle, in $\text{Btu}/^\circ\text{R}$ per lb of helium flowing.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ and potential energy effects can be ignored.
3. The helium is modeled as an ideal gas with $k = 1.67$.

SCHEMATIC & GIVEN DATA:



ANALYSIS: (a) Mass and energy rate balances reduce to give,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right)$$

$$\Rightarrow V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

$$= \sqrt{V_1^2 + 2c_p(T_1 - T_2)} \quad (1)$$

With Eq. 3.47a, $c_p = \frac{kR}{k-1} = \left(\frac{1.67}{1.67-1} \right) \left(\frac{0.986 \text{ Btu}}{4.003 \text{ lb} \cdot ^\circ\text{R}} \right) = 1.24 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$. So,

$$V_2 = \sqrt{\left(\frac{10 \text{ ft}}{\text{s}} \right)^2 + 2 \left(1.24 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (810^\circ\text{R}) \left| \frac{778 \text{ ft} \cdot \text{bf}}{1 \text{ Btu}} \right| \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right|}$$

$$= 2949 \text{ ft/s} \quad \leftarrow (a)$$

(b) The isentropic nozzle efficiency is given by Eq. 6.47: $\eta_{\text{nozzle}} = \frac{V_2^2/2}{V_{2s}^2/2}$. To find V_{2s} , T_{2s} is required. Using Eq. 6.43,

$$T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = 810^\circ\text{R} \left(\frac{25}{45} \right)^{0.67/1.67} = 639.8^\circ\text{R}$$

Then, Eq. (1) gives

$$V_{2s} = \sqrt{V_1^2 + 2c_p(T_1 - T_{2s})} = \sqrt{\left(10 \text{ ft/s} \right)^2 + 2 \left(1.24 \right) (810 - 639.8) \left| \frac{778}{1} \right| \left| \frac{32.2}{1} \right|} = 3252 \text{ ft/s}$$

$$\Rightarrow \eta_{\text{nozzle}} = \frac{(2949)^2/2}{(3252)^2/2} = 0.822 \quad (82.2\%) \quad \leftarrow (b)$$

(c) Mass and entropy rate balances reduce to give $\dot{Q}_{cv}/\dot{m} = s_2 - s_1$. or

$$\dot{Q}_{cv}/\dot{m} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 1.24 \ln \frac{670}{810} - \frac{0.986}{4.003} \ln \frac{25}{45}$$

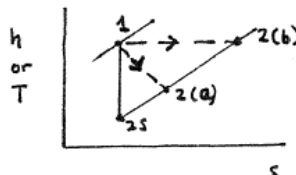
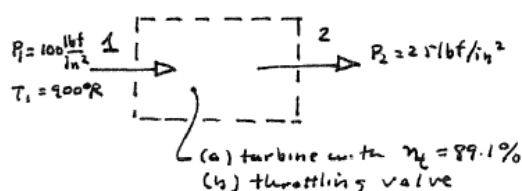
$$= 0.056 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \quad \leftarrow (c)$$

PROBLEM 6.141

Air modeled as an ideal gas enters a one-inlet, one-exit control volume operating at steady state at 100 lbf/in^2 , 900°R and expands adiabatically to 25 lbf/in^2 . Kinetic and potential energy effects are negligible. Determine the rate of entropy production, in $\text{Btu/}^\circ\text{R}$ per lb of air flowing,

- if the control volume encloses a turbine having an isentropic turbine efficiency of 89.1% .
- if the control volume encloses a throttling valve.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

- The control volume shown in the sketch is at steady state.
- In each case, the expansion occurs adiabatically, and kinetic and potential energy effects are negligible.
- The air is modeled as an ideal gas.

ANALYSIS: Mass and entropy rate balances reduce to give, with Eq. 6.20a,

$$\frac{\dot{Q}_{cv}}{\dot{m}} = s_2 - s_1 = s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1 \quad (1)$$

- (a) Turbine with $\eta_t = 0.891$. To find $s^\circ(T_2)$, use $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow$

$$h_2 = h_1 - \eta_t (h_1 - h_{2s}). \quad \text{To find } h_{2s}, \text{ use } P_{r(2s)} = P_{r(1)} \frac{P_2}{P_1} = 8.411 \left(\frac{25}{100} \right) = 2.103.$$

↑
TABLE A-22E

Interpolation in Table A-22E gives, $h_{2s} = 145.4 \text{ Btu/lb}$. Then,

$$h_2 = 216.26 - 0.891(216.26 - 145.4) = 153.12 \text{ Btu/lb} \Rightarrow s^\circ(T_2) \approx 0.64159.$$

Then, Eq. (1) gives

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}} &= 0.64159 - 0.72438 - \left[\frac{1.987}{28.97} \ln \left(\frac{25}{100} \right) \right] \\ &= 0.0122 \text{ Btu/lb} \cdot ^\circ\text{R} \end{aligned} \quad \leftarrow (a)$$

- (b) Throttling valve. Since $h_2 = h_1$, it follows that for an ideal gas $T_2 = T_1$.

So, Eq. (1) reads

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}} &= \underbrace{s^\circ(T_2) - s^\circ(T_1)}_{=0} - R \ln \frac{P_2}{P_1} \\ \textcircled{1} \quad &= -R \ln \frac{P_2}{P_1} = 0.095 \text{ Btu/lb} \cdot ^\circ\text{R} \end{aligned} \quad \leftarrow (b)$$

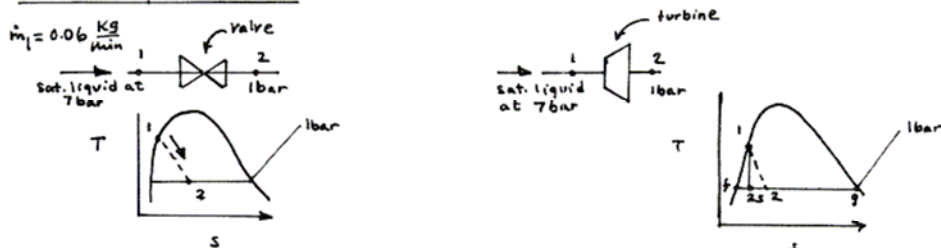
(See (a) for calculation)

- Entropy production associated with expansion through the throttling valve is significantly greater than in the more controlled expansion through the turbine.

PROBLEM 6.142

KNOWN: Operating data are provided for ammonia throttled across a valve.
FIND: Determine the rate of entropy production. If the valve were replaced by a power-recovery turbine, determine the maximum theoretical power that could be developed. Would you recommend the use of such a turbine?

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) Control volumes enclosing the valve and enclosing the turbine operate at steady state. (2) $\dot{Q}_{cv} = 0$, and kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: Considering the valve first, the expansion is a throttling process for which $h_2 = h_1$ (Sec. 4.10). Thus, state 2 is fixed by $P_2 = 1 \text{ bar}$ and $h_2 = h_1$. The rate of entropy production can be evaluated from an entropy rate balance which at steady state reduces with $\dot{m}_1 = \dot{m}_2 = \dot{m}$ to

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{Q}_{cv} \Rightarrow \dot{Q}_{cv} = \dot{m}(s_2 - s_1) \quad (1)$$

From Table A-14 at 7 bar, $s_1 = 0.9394 \text{ kJ/kg} \cdot \text{K}$. At state 2, $h_2 = h_1 = 244.69 \text{ kJ/kg}$. Then

$$x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{244.69 - 28.18}{1370.23} = 0.158$$

Then

$$s_2 = s_f + x_2(s_g - s_f) = 0.1191 + 0.158(5.8391 - 0.1191) = 1.0229 \text{ kJ/kg} \cdot \text{K}$$

Thus, with Eq. (1)

$$\dot{Q}_{cv} = \left(0.06 \frac{\text{kg}}{\text{min}}\right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (1.0229 - 0.9394) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 8.35 \times 10^{-5} \frac{\text{kW}}{\text{K}} \leftarrow \dot{Q}_{cv}$$

If the valve is replaced by a turbine operating at steady state, an energy rate balance reduces with assumption 2 to give

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

An entropy rate balance gives Eq. (1) above. As $h_1 = 244.69 \text{ kJ/kg}$, the work developed is

$$\dot{W}_{cv} = \left(\frac{0.06 \text{ kg}}{60 \text{ s}}\right) (244.69 - h_2) \frac{\text{kJ}}{\text{kg}} \quad (2)$$

Accordingly, the power developed increases as h_2 decreases. Since $s_2 \geq s_1$, the smallest allowed value is h_{2s} , corresponding to the state where $s_{2s} = s_1$, as shown on the sketch above. The quality at state 2s is

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{0.9394 - 0.1191}{5.8391 - 0.1191} = 0.1434 \Rightarrow h_{2s} = h_f + x_{2s}(h_g - h_f) = 28.18 + (0.1434)(1370.23) = 224.67 \frac{\text{kJ}}{\text{kg}}$$

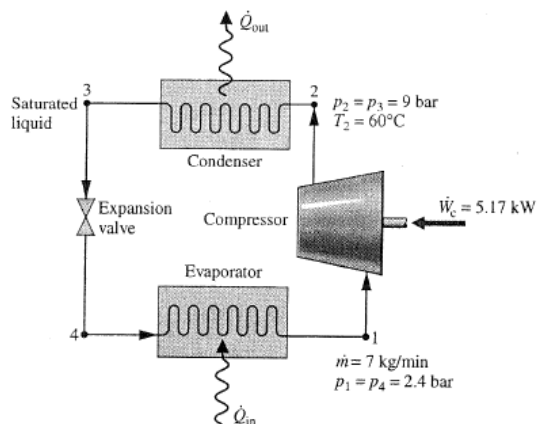
Then Eq. (2) gives

$$(\dot{W}_{cv})_{\text{max}} = \left(\frac{0.06 \text{ kg}}{60 \text{ s}}\right) (244.69 - 224.67) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 0.02 \text{ kW} \leftarrow (\dot{W}_{cv})_{\text{max}}$$

This is a small potential to develop power. Moreover, the effect of irreversibilities in an actual power-recovery turbine would be to reduce the actual power developed below the calculated value. Generally, for this type of application a power-recovery turbine would not be recommended.

PROBLEM 6.143

Figure P6.143 provides the schematic of a heat pump using Refrigerant 134a as the working fluid, together with steady-state data at key points. The mass flow rate of the refrigerant is 7 kg/min, and the power input to the compressor is 5.17 kW. (a) Determine the coefficient of performance for the heat pump. (b) If the valve were replaced by a turbine, power could be produced, reducing thereby the power requirement of the heat pump system. Would you recommend this power-saving measure? Explain.



ENGR. MODEL:

1. The heat pump system shown in the figure operates at steady state.
2. Stray heat transfer and kinetic and potential energy effects can be ignored.
3. $\dot{W}_c$ and $\dot{Q}_{out}$ are positive in the directions of the arrows.

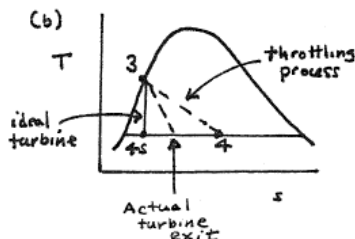
ANALYSIS: (a) Using Eq. 2.47 expressed on a rate basis,

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c}$$

An energy rate balance for the condenser gives, $\dot{Q}_{out} = \dot{m}(h_2 - h_3)$.

So,

$$\gamma = \frac{\dot{m}(h_2 - h_3)}{\dot{W}_c} = \frac{7 \frac{\text{kg}}{\text{min}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (293.21 - 99.56) \frac{\text{kJ}}{\text{kg}}}{5.17 \text{ kJ/s}} = 4.37 \quad (a)$$



To assess the merit of a power-recovery turbine, consider an ideal turbine: one whose isentropic turbine efficiency is 100%. Then,

$$\dot{W}_t = \dot{m}(h_3 - h_{4s}), \text{ where } h_3 = 99.56 \frac{\text{kJ}}{\text{kg}}, s_3 = 0.3656 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}.$$

Then, since $s_4s = s_3$,

$$x_{4s} = \frac{s_{4s} - s_f}{s_g - s_f} = \frac{0.3656 - 0.1710}{0.9222 - 0.1710} = 0.259$$

$$\Rightarrow h_{4s} = h_f + x_{4s} h_{fg} = 42.95 + 0.259(201.14) = 95.05 \frac{\text{kJ}}{\text{kg}}$$

Then

$$\dot{W}_t = \left(7 \frac{\text{kg}}{\text{min}} \right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left(99.56 \frac{\text{kJ}}{\text{kg}} - 95.05 \frac{\text{kJ}}{\text{kg}} \right) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 0.53 \text{ kW}$$

DISCUSSION:

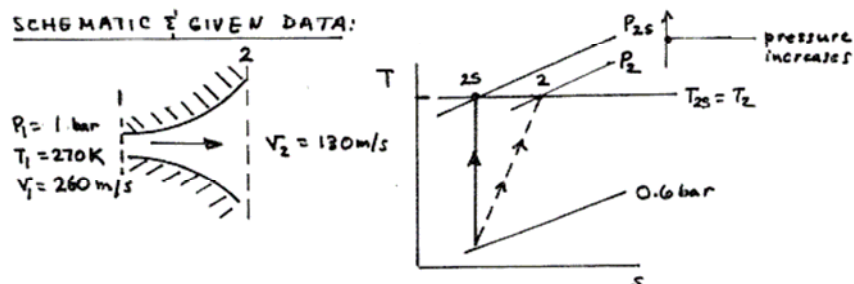
This is the maximum theoretical power that any power-recovery turbine would be able to develop. At best, the turbine could offset about 10% of the compressor power requirement. In most heat pump applications, such a turbine is not implemented owing to the turbine cost and operating difficulties related to the low-quality refrigerant that would be expanding through such a turbine.

PROBLEM 6.144

KNOWN: Air enters an insulated diffuser operating at steady state at 1 bar, -30°C , 260 m/s and exits at 130 m/s .

FIND: Determine (a) the temperature of the air at the exit, (b) the maximum attainable exit pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The diffuser operates at steady state and is insulated. (2) The change in potential energy from inlet to exit can be neglected. (3) Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$ and with stated assumptions an energy rate balance reduces to

$$0 = \dot{Q}_{cv} + \dot{W}_{cv} + \dot{m} \left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right)$$

or

$$h_2 = h_1 + \frac{V_1^2 - V_2^2}{2}$$

With h_1 from Table A-22 at 270 K

$$h_2 = 270.11 + \left(\frac{260^2 - 130^2}{2} \right) \left(\frac{\text{N}}{\text{kg}} \right) \left(\frac{1\text{ N}}{1\text{ kg} \cdot \text{m/s}^2} \right) \left(\frac{\text{kJ}}{10^3\text{ N} \cdot \text{m}} \right)$$

$$= 295.46\text{ kJ/kg}$$

Table A-22 then gives $T_2 = 295\text{ K}$ (22°C).

An entropy rate balance reduces to read

$$\frac{\dot{Q}_{cv}}{\dot{m}} = s_2 - s_1$$

Since $\dot{Q}_{cv} \geq 0$, $s_2 \geq s_1$. For fixed exit temperature T_2 (and thus fixed exit velocity V_2), the accompanying T - s diagram shows that the maximum allowed exit pressure corresponds to the case $s_2 = s_1$. To determine this pressure, P_{2s} , use Eq. 6.20a to write

$$\textcircled{1} \quad 0 = s^\circ(T_{2s}) - s^\circ(T_1) - R \ln \frac{P_{2s}}{P_1} \Rightarrow \text{with } T_{2s} = T_2 \quad \ln \frac{P_{2s}}{P_1} = \frac{s^\circ(T_2) - s^\circ(T_1)}{R}$$

With data from Table A-22 at $T_1 = 270\text{ K}$, $T_2 = 295\text{ K}$

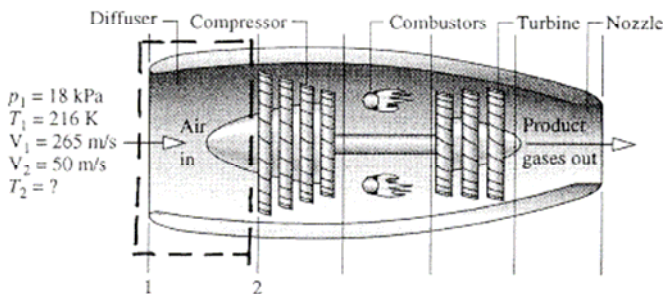
$$\ln \frac{P_{2s}}{P_1} = \frac{1.68515 - 1.59634}{(8.314/28.97)} \Rightarrow P_{2s} = 1.363 P_1 = 1.363\text{ bar} \quad \leftarrow (P_2)_{\text{MAX}}$$

1. Alternatively, Eq. 6.41 can be used.

PROBLEM 6.145

As shown in Fig. P6.145, air enters the diffuser of a jet engine at 18 kPa, 216 K with a velocity of 265 m/s, all data corresponding to high-altitude flight. The air flows adiabatically through the diffuser, decelerating to a velocity of 50 m/s at the diffuser exit. Assume steady-state operation, the ideal gas model for air, and negligible potential energy effects.

- Determine the temperature of the air at the exit of the diffuser, in K.
- If the air would undergo an isentropic process as it flows through the diffuser, determine the pressure of the air at the diffuser exit, in kPa.
- If friction were present, would the pressure of the air at the diffuser exit be greater than, less than, or equal to the value found in part (b)? Explain.



ENGR. MODEL:

- The control volume shown in the figure is at steady state.
- For the control volume, $\dot{Q}_{cv}$ and potential energy effects can be ignored. $\dot{W}_{cv} = 0$.
- The air is modeled as an ideal gas.

ANALYSIS: (a) Mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] \Rightarrow h_2 = h_1 + \frac{V_1^2 - V_2^2}{2}$$

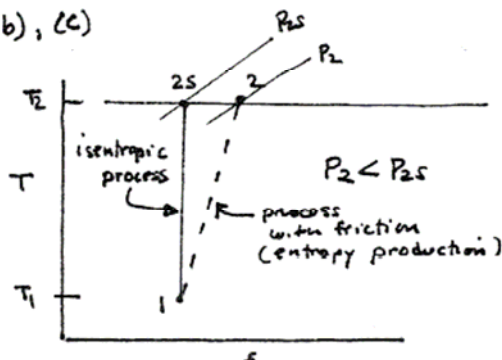
With data from Table A-22,

$$h_2 = 215.97 \frac{\text{kJ}}{\text{kg}} + \frac{(265 \text{ m/s})^2 - (50 \text{ m/s})^2}{2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 249.8 \frac{\text{kJ}}{\text{kg}}$$

Interpolation in Table A-22 gives $T_2 = 249.8 \text{ K}$

(a)

(b), (c)



To find P_{2s} , use Eq. 6.41:

$$\frac{P_{2s}}{P_1} = \frac{Pr(2s)}{Pr(1)}$$

$$P_{2s} = P_1 \frac{Pr(2s)}{Pr(1)}$$

$$= (18 \text{ kPa}) \left(\frac{0.7810}{0.4409} \right)$$

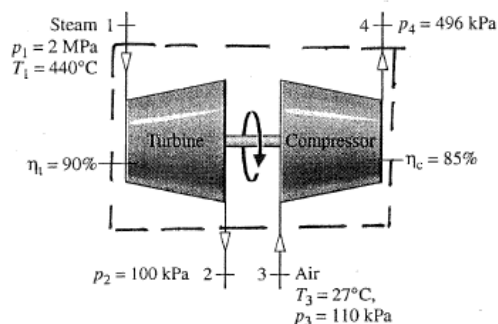
$$= 30 \text{ kPa}$$

(b)

PROBLEM 6.146

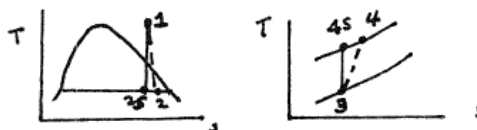
As shown in Fig. P6.146, a steam turbine having an isentropic turbine efficiency of 90% drives an air compressor having an isentropic compressor efficiency of 85%. Steady-state operating data are provided on the figure. Assume the ideal gas model for air, and ignore stray heat transfer and kinetic and potential energy effects.

- Determine the mass flow rate of the steam entering the turbine, in kg of steam per kg of air exiting the compressor.
- Repeat part (a) if $\eta_t = \eta_c = 100\%$



ENGR. MODEL:

- The control volume shown in the figure is at steady state.
- For the control volume, $\dot{W}_{cv} = 0$. Also, stray heat transfer and kinetic and potential energy effects can be ignored.
- The air is modeled as an ideal gas.



ANALYSIS: Mass balances give, $\dot{m}_1 = \dot{m}_2$ and $\dot{m}_3 = \dot{m}_4$. An energy rate balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 [h_1 - h_2] + \dot{m}_3 [h_3 - h_4] \Rightarrow \dot{m}_1 = \dot{m}_4 \left[\frac{h_4 - h_3}{h_1 - h_2} \right] \quad (1)$$

The respective isentropic efficiencies are, $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$, $\eta_c = \frac{h_{4s} - h_3}{h_4 - h_3}$. With these Eq. (1) becomes

$$\textcircled{1} \quad \frac{\dot{m}_1}{\dot{m}_4} = \frac{1}{\eta_t \eta_c} \left[\frac{h_{4s} - h_3}{h_1 - h_{2s}} \right] \quad (2)$$

Property data: Table A-4 $h_1 = 3335.5 \text{ kJ/kg}$, $s_1 = 7.2540 \text{ kJ/kg} \cdot \text{K}$

$$s_{2s} = s_1 \Rightarrow x_{2s} = \frac{7.2540 - 1.3026}{7.3594 - 1.3026} = 0.9826 \Rightarrow h_{2s} = 417.46 + 0.9826(2258) = 2636.2 \text{ kJ/kg}$$

$$\text{Table A-22 } h_3 = 300.19 \text{ kJ/kg}, P_{r3} = 1.3860, P_{r4} = P_{r3} \left(\frac{P_4}{P_3} \right) = 1.3860 \left(\frac{4.96}{1.1} \right) = 6.2496 \\ \Rightarrow h_{4s} = 462.1 \text{ kJ/kg}$$

$$(a) \quad \eta_t = 90\%, \eta_c = 85\%$$

$$\frac{\dot{m}_1}{\dot{m}_4} = \frac{1}{(0.9)(0.85)} \left[\frac{462.1 - 300.19}{3335.5 - 2636.2} \right] = 0.303$$

(a)

$$(b) \quad \eta_t = 100\%, \eta_c = 100\%$$

$$\frac{\dot{m}_1}{\dot{m}_4} = 0.232$$

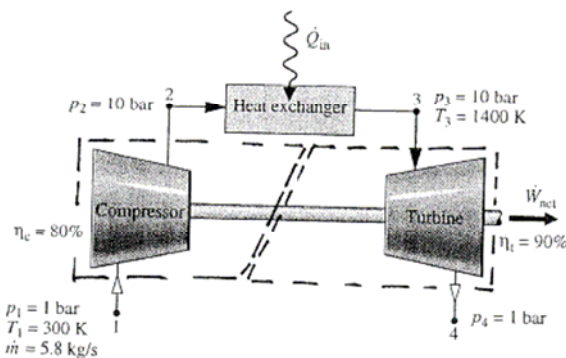
(b)

1. Note that

$$\frac{\dot{m}_1}{\dot{m}_4} = \frac{1}{\eta_t \eta_c} \left(\frac{\dot{m}_1}{\dot{m}_4} \right)_{\text{int rev}}$$

PROBLEM 6.147

Figure P6.147 shows a power system operating at steady state consisting of three components in series: an air compressor having an isentropic compressor efficiency of 80%, a heat exchanger, and a turbine having an isentropic turbine efficiency of 90%. Air enters the compressor at 1 bar, 300 K with a mass flow rate of 5.8 kg/s and exits at a pressure of 10 bar. Air enters the turbine at 10 bar, 1400 K and exits at a pressure of 1 bar. Air can be modeled as an ideal gas. Stray heat transfer and kinetic and potential energy effects are negligible. Determine, in kW, (a) the power required by the compressor, (b) the power developed by the turbine, and (c) the net power output of the overall power system.



ENGR. MODEL:

1. Control volumes at steady state enclose the turbine and compressor.
2. For the control volumes, stray heat transfer and kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS: Mass and energy rate balances reduce to give,

$$\text{turbine: } \dot{W}_t = \dot{m}(h_3 - h_4) \quad \text{compressor: } \dot{W}_c = \dot{m}(h_1 - h_2)$$

Isentropic efficiencies,

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Collecting results

$$\dot{W}_t = \dot{m} \eta_t (h_3 - h_{4s}) \quad (1)$$

$$\dot{W}_c = -\frac{\dot{m}(h_{2s} - h_1)}{\eta_c} \quad (2)$$

From Table A-22, $h_1 = 300.19 \text{ kJ/kg}$, $h_3 = 1515.4 \text{ kJ/kg}$; to find h_{2s} and h_{4s} , use

$$Pr(4s) = Pr(3) \left[\frac{P_4}{P_3} \right] = 450.5 \left[\frac{1}{10} \right] = 45.05 \Rightarrow h_{4s} = 808.5 \frac{\text{kJ}}{\text{kg}}$$

$$Pr(2s) = Pr(1) \left[\frac{P_2}{P_1} \right] = 1.386 \left[\frac{10}{1} \right] = 13.86 \Rightarrow h_{2s} = 579.9 \frac{\text{kJ}}{\text{kg}}$$

Inserting values into Eqs. (1), (2)

$$\dot{W}_c = -\left(5.8 \frac{\text{kg}}{\text{s}}\right) \left(\frac{579.9 - 300.19}{0.80} \right) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -2028 \quad \leftarrow (a)$$

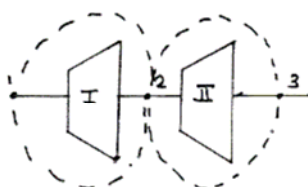
$$\dot{W}_t = (5.8)(0.9)(1515.4 - 808.5) |1| = 3690 \text{ kW} \quad \leftarrow (b)$$

$$\dot{W}_{\text{net}} = 3690 \text{ kW} - 2028 \text{ kW} = 1662 \text{ kW} \quad \leftarrow (c)$$

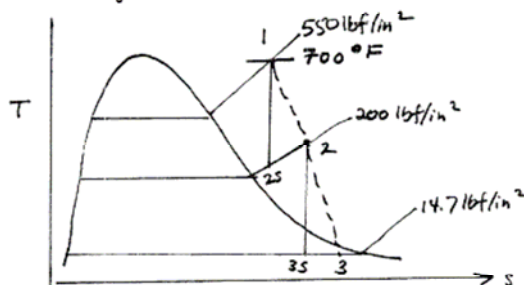
PROBLEM 6.148

KNOWN: Steady state operating data are provided for a well-insulated steam turbine having two stages in series.

FIND: Show the principal states on a T-s diagram. Determine the state at the exit of the second stage. Determine the work developed by each stage, per unit of mass of steam flowing.



$$\begin{aligned} T_1 &= 700^\circ\text{F} \\ P_1 &= 550 \text{ lbf/in}^2 \\ P_2 &= 200 \text{ lbf/in}^2 \\ P_3 &= 14.7 \text{ lbf/in}^2 \\ \eta_{II} &= 88\% \\ \eta_{III} &= 92\% \end{aligned}$$



ENGR. MODEL: (1) Each control volume shown in the figure is at steady state.

(2) For each control volume, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects can be ignored.

ANALYSIS: Steam table data give $h_1 = 1353.7 \text{ Btu/lb}$, $s_1 = 1.5987 \text{ Btu/lb} \cdot ^\circ\text{R}$. Interpolating at 200 lbf/in^2 with $s_{2s} = s_1$, $h_{2s} = 1245.2 \text{ Btu/lb}$. Then, using the isentropic turbine efficiency of the first stage

$$\eta_{II} = 0.88 = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - 0.88(h_1 - h_{2s}) = 1353.7 - 0.88(1353.7 - 1245.2) = 1258.2 \frac{\text{Btu}}{\text{lb}}$$

Interpolating at 200 lbf/in^2 with h_2 gives $s_2 = 1.6127 \text{ Btu/lb} \cdot ^\circ\text{R}$. Using $s_{3s} = s_2$,

$$x_{3s} = \frac{s_{3s} - s_f}{s_g - s_f} = \frac{1.6127 - 0.31212}{1.4446} = 0.9, \quad h_{3s} = h_f + x_{3s} h_{fg} = 180.15 + 0.9(970.4) = 1053.5 \text{ Btu/lb}.$$

Then, using the isentropic turbine efficiency of the second stage

$$\eta_{III} = 0.92 = \frac{h_2 - h_3}{h_2 - h_{3s}} \Rightarrow h_3 = h_2 - 0.92(h_2 - h_{3s}) = 1258.2 - 0.92(1258.2 - 1053.5) = 1069.9 \frac{\text{Btu}}{\text{lb}}$$

Using h_3

$$x_3 = \frac{h_3 - h_f}{h_{fg}} = \frac{1069.9 - 180.15}{970.4} = 0.917 \quad (91.7\%)$$

The state at the exit of stage I is fixed by two of P_2 , h_2 , s_2 . The state at the exit of stage II is fixed by two of P_3 , h_3 , x_3 .

The work developed for each of the stages is obtained by reducing mass and energy rate balances to give

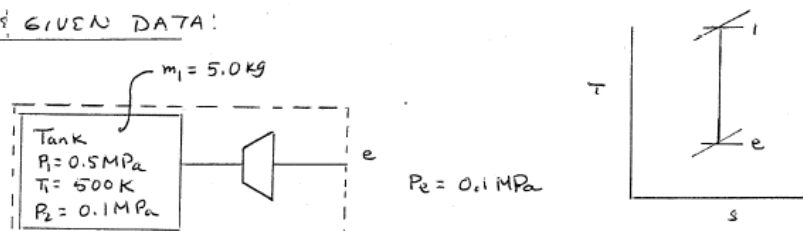
$$\begin{aligned} (\dot{w}_{cv}/\dot{m})_I &= h_1 - h_2 = 1353.7 - 1258.2 = 95.5 \text{ Btu/lb} \\ (\dot{w}_{cv}/\dot{m})_{II} &= h_2 - h_3 = 1258.2 - 1069.9 = 188.3 \text{ Btu/lb} \end{aligned}$$

PROBLEM 6.149

KNOWN: A tank initially filled with air is allowed to discharge through a turbine until the pressure in the tank becomes atmospheric.

FIND: Determine the maximum theoretical work that could be developed.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is shown above. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) The control volume is free of irreversibilities.

ANALYSIS: A mass rate balance reduces to $dm/dt = -\dot{m}_e$. Using this, an energy rate balance reads

$$\frac{dU}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} - \dot{m}_e h_e \quad , \quad \frac{dU}{dt} = -\dot{W}_{cv} + h_e \frac{dm}{dt}$$

or

$$\dot{W}_{cv} = -\frac{dU}{dt} + h_e \frac{dm}{dt} \Rightarrow W_{cv} = -\Delta U + \int h_e dm \quad (1)$$

We expect that the maximum theoretical work would be developed in the absence of irreversibilities within the tank and the turbine (assumption (4)). The data given are recognized as corresponding to those for Example 6.10. The solution to Example 6.10 shows that a typical unit of mass remaining in the tank would undergo an isentropic expansion from T_1, P_1 to T_2, P_2 . Moreover, each unit of mass passing through the turbine expands isentropically. Accordingly, if P, T denote the pressure and temperature within the tank at a particular instant

$$\frac{P_r(T)}{P_r(T_1)} = \frac{P}{P_1} \quad (2)$$

But P, T would also correspond to the condition of the mass entering the turbine. So

$$\frac{P_r(T_e)}{P_r(T)} = \frac{P_e}{P} \quad (3)$$

Combining Eqs. (2), (3)

$$\frac{P_r(T_e)}{P_r(T_1)} = \frac{P_e}{P_1} \Rightarrow \frac{P_r(T_e)}{P_r(T_1)} = \frac{P_2}{P_1} \quad \text{since } P_e = P_2$$

Then, with the result of Example 6.10, $T_e = T_2 = 317 \text{ K}$.

Returning to Eq. (1), h_e is fixed by T_e , and thus a constant:

$$\begin{aligned} (W_{cv})_{\max} &= -\Delta U + h_e \Delta m \\ &= (m_1 u_1 - m_2 u_2) + h_e (m_1 - m_2) \end{aligned}$$

From Example 6.10, $m_2 = 1.58 \text{ kg}$. With data from Table A-22

$$\begin{aligned} (W_{cv})_{\max} &= (5 \text{ kg}) \left(359.49 \frac{\text{kJ}}{\text{kg}} \right) - (1.58 \text{ kg}) \left(226.28 \frac{\text{kJ}}{\text{kg}} \right) + (317.28 \frac{\text{kJ}}{\text{kg}}) (1.58 - 5) \text{ kg} \\ &= 1797.45 \text{ kJ} - 357.52 \text{ kJ} - 1085.1 \text{ kJ} \\ &= 354.83 \text{ kJ} \end{aligned}$$

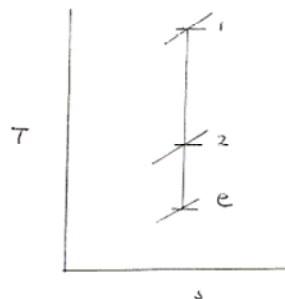
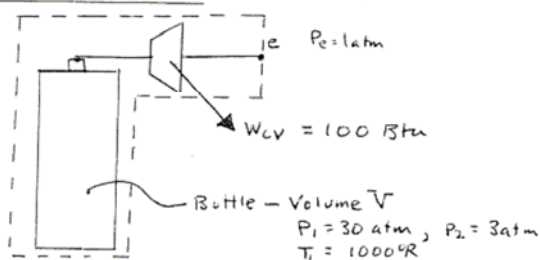
← MAX

PROBLEM 6.15D

KNOWN: A bottle initially filled with air is allowed to discharge through a turbine, developing work, until the pressure in the bottle becomes 3 atm.

FIND: Determine the volume of the bottle when irreversibilities are absent.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is shown in the figure. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) Irreversibilities are absent within the control volume.

ANALYSIS: A mass rate balance reduces to $dm/dt = -\dot{m}_e$. Using this an energy rate balance reads

$$\frac{dU}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} - \dot{m}_e h_e, \quad \frac{dU}{dt} = -\dot{W}_{cv} + h_e \frac{dm}{dt} \Rightarrow W_{cv} = -\Delta U + \int h_e dm \quad (1)$$

The plan is to evaluate the bottle volume in the absence of irreversibilities (assumption (4)). Following the reasoning of Example 6.10, a typical unit of mass remaining in the bottle would undergo an isentropic expansion from P_1, T_1 to P_e, T_2 . Moreover, each unit of mass passing through the turbine expands isentropically. Accordingly, if P, T denote the pressure and temperature within the tank at a particular instant,

$$\frac{P_r(T)}{P_r(T_1)} = \frac{P}{P_1} \quad (2)$$

But P, T would also correspond to the condition of the mass entering the turbine, so

$$\frac{P_r(T_e)}{P_r(T)} = \frac{P_e}{P} \quad (3)$$

Combining Eqs. (2), (3), and using data from Table A-22E

$$\frac{P_r(T_e)}{P_r(T_1)} = \frac{P_e}{P_1} \Rightarrow P_r(T_e) = \left(\frac{1 \text{ atm}}{30 \text{ atm}} \right) (12.3) = 0.41 \Rightarrow h_e = 90.98 \text{ Btu/lb.}$$

Since h_e is fixed, Eq. (1) becomes, with the ideal gas equation of state ($m = PV/RT$),

$$\begin{aligned} W_{cv} &= -\Delta U + h_e (\Delta m) \Rightarrow W_{cv} = (m_1 u_1 - m_2 u_2) + h_e (m_2 - m_1) \\ &= \left(\frac{P_1 V}{RT_1} \right) u_1 - \left(\frac{P_2 V}{RT_2} \right) u_2 + h_e \left[\frac{P_2 V}{RT_2} - \frac{P_1 V}{RT_1} \right] \\ &= \frac{P_1 V}{R} \left[\frac{u(T_1)}{T_1} - \left(\frac{P_2}{P_1} \right) \frac{u(T_2)}{T_2} + h_e \left[\left(\frac{P_2}{P_1} \right) \frac{1}{T_2} - \frac{1}{T_1} \right] \right] \quad (4) \end{aligned}$$

With data from Table A-22E, $u(T_1) = 172.43 \text{ Btu/lb.}$, $P_r(T_2) = P_2/P_1$, $P_r(T_1) = (3/30)(12.3) = 1.23$, giving $T_2 = 522^\circ\text{R}$, $u(T_2) = 88.43 \text{ Btu/lb.}$. Evaluating Eq. (4)

$$100 \text{ Btu} = \frac{V(30)(14.7)(144)(10^4/\text{ft}^2)}{(1545/28.97)(1 \text{ lb}/\text{lb} \cdot \text{mol} \cdot \text{R})} \left[\frac{172.43}{1000} - \left(\frac{3}{30} \right) \frac{88.43}{522} + 90.98 \left[\left(\frac{3}{30} \right) \frac{1}{522} - \frac{1}{1000} \right] \right] \frac{\text{Btu}}{\text{lb.}} \quad (5)$$

Solving gives $V = 1.03 \text{ ft}^3$.

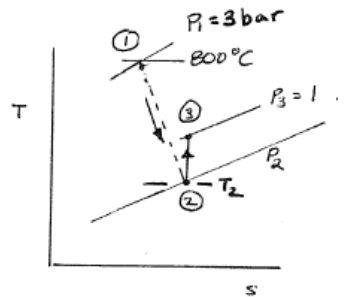
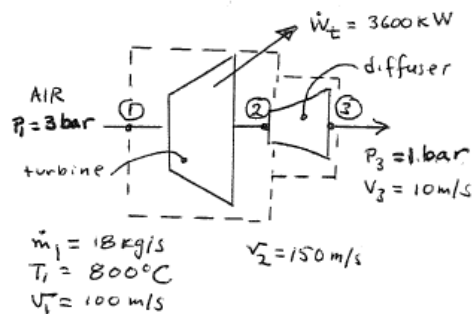
1. Checking the compressibility chart with $TR_1 = 1000/239 = 4.18$, $Pr_2 = 3/37.2 = 0.081$, it can be concluded that the ideal gas model is appropriate at these states.

PROBLEM 6.151

KNOWN: Steady state operating data are provided for an air turbine fitted with a diffuser at its exit.

FIND: Determine the pressure and temperature at the turbine exit and the rate of entropy production in the turbine. Show the processes on a T-s diagram.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The overall control volume consists of two sub control volumes, each at steady state. (2) The turbine operates adiabatically. (3) Flow through the diffuser is isentropic. (4) Potential energy effects can be ignored. (5) Air is modeled as an ideal gas.

ANALYSIS: (a) For the isentropic process of air from 2 to 3, $P_2/P_3 = P_r(T_2)/P_r(T_3)$. Accordingly, to find P_2 requires $P_r(T_2)$ and $P_r(T_3)$.

Mass and energy rate balances for the turbine reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_t + \dot{m} (h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}) \Rightarrow h_2 = -\frac{\dot{W}_t}{\dot{m}} + h_1 + \frac{V_1^2 - V_2^2}{2}$$

With h_1 from Table A-22

$$h_2 = -\frac{3600 \text{ kJ/s}}{18 \text{ kg/s}} + 1129.83 \text{ kJ/kg} + \frac{(100)^2 - (150)^2}{2} \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left(\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right) = 923.6 \text{ kJ/kg}$$

Then, interpolation in Table A-22 gives $P_{r2} = 72.66$, $T_2 = 892 \text{ K}$ (619°C). ← T_2

Mass and energy rate balances for the diffuser reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} (h_2 - h_3 + \frac{V_2^2 - V_3^2}{2}) \Rightarrow h_3 = h_2 + \frac{V_2^2 - V_3^2}{2}$$

$$\text{or } h_3 = 923.6 + \frac{(150)^2 - (10)^2}{2000} = 934.8 \text{ kJ/kg}$$

Interpolating in Table A-22, $P_{r3} = 75.85$. Then

$$P_2 = P_3 \frac{P_{r2}}{P_{r3}} = (1 \text{ bar}) \left(\frac{72.66}{75.85} \right) = 0.958 \text{ bar} \quad \leftarrow P_2$$

(b) Mass and entropy rate balances reduce to give for the turbine

$$\dot{\sigma}_t = \dot{m} (s_2 - s_1)$$

With Eq. 6.20a this becomes

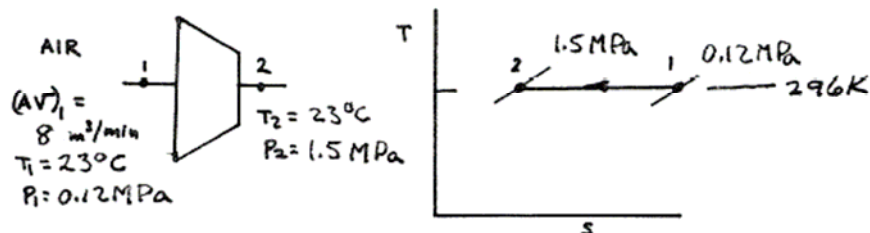
$$\begin{aligned} \dot{\sigma}_t &= \dot{m} [s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1] \\ &= (18 \text{ kg/s}) \left[2.8381 - 3.0485 - \frac{8.314}{28.97} \ln \left(\frac{0.958}{3} \right) \right] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 2.11 \frac{\text{kW}}{\text{K}} \quad \leftarrow \dot{\sigma}_t \end{aligned}$$

PROBLEM 6.152

KNOWN: Operating data are provided for an air compressor at steady state.

FIND: Determine the power required and the rate of heat transfer.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The compressor is at steady state. (2) The process is isothermal and internally reversible. (3) Kinetic and potential energy effects are negligible. (4) Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and an energy rate balance reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}$$

As the enthalpy of an ideal gas depends on temperature only, and temperature remains constant, $h_2 = h_1$. Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}}$$

As the process is internally reversible, there are two solution approaches that can be taken: (1) Equation 6.49 can be integrated giving

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}} &= T(s_2 - s_1) \\ &= T(s^0(T_2) - s^0(T_1) - R \ln P_2/P_1) = -RT \ln P_2/P_1 \end{aligned}$$

$= 0 \text{ since } T_1 = T_2$

(2) Equation 6.51b can be integrated, giving

$$\frac{\dot{W}_{cv}}{\dot{m}} = - \int_1^2 v dp = - \int_1^2 \frac{RT}{p} dp = -RT \ln \frac{P_2}{P_1}$$

Alternative Approaches

The mass flow rate is

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1} = \frac{(1.2 \times 10^5 \text{ N/m}^2)(8 \frac{\text{m}^3}{\text{min}}) \left| \frac{\text{min}}{60 \text{ s}} \right|}{\left(\frac{8314 \text{ N} \cdot \text{m}}{28.97 \text{ kg} \cdot \text{K}} \right)(296 \text{ K})} = 0.1884 \frac{\text{kg}}{\text{s}}$$

Thus

$$\begin{aligned} \dot{Q}_{cv} = \dot{W}_{cv} &= - \dot{m} RT \ln \frac{P_2}{P_1} = -(0.1884 \frac{\text{kg}}{\text{s}}) \left(\frac{8314 \text{ J}}{28.97 \text{ kg} \cdot \text{K}} \right) (296 \text{ K}) \left(\ln \frac{1.5}{0.12} \right) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= -40.4 \text{ kW} \end{aligned}$$

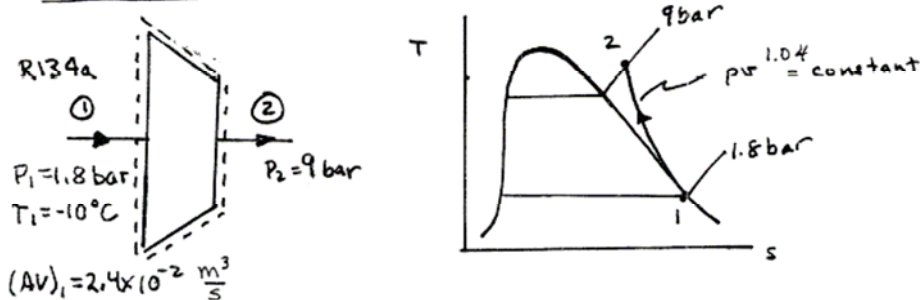
$\frac{\dot{Q}_{cv}}{\dot{W}_{cv}}$

PROBLEM 6.153

KNOWN: Steady state operating data are provided for a R134a compressor.

FIND: Determine the power required and rate of heat transfer.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the accompanying sketch is at steady state. 2. For the control volume, kinetic and potential energy effects can be ignored. 3. The compression is described by $p v^{1.04} = \text{constant}$. The process is internally reversible.

ANALYSIS: (a) Using Eq. 6.53, the power is

$$(\dot{W}_{cv})_{int, rev} = -\dot{m} \left(\frac{n}{n-1} \right) [P_2 v_2 - P_1 v_1]$$

To find v_2 , write $P_2 v_2^{1.04} = P_1 v_1^{1.04} \Rightarrow v_2 = \left(\frac{P_1}{P_2} \right)^{\frac{1}{1.04}} v_1$, where $v_1 = 0.11135 \text{ m}^3/\text{kg}$ from Table A-12. Thus

$$v_2 = \left(\frac{1.8}{9} \right)^{\frac{1}{1.04}} (0.11135) = 0.02369 \text{ m}^3/\text{kg}.$$

Also,

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{2.4 \times 10^{-2} \text{ m}^3/\text{s}}{0.11135 \text{ m}^3/\text{kg}} = 0.2155 \text{ kg/s}$$

Finally

$$\begin{aligned} (\dot{W}_{cv})_{int, rev} &= -(0.2155 \frac{\text{kg}}{\text{s}}) \left(\frac{1.04}{1.04-1} \right) \left[(9 \times 10^5 \frac{\text{N}}{\text{m}^2}) (0.02369 \frac{\text{m}^3}{\text{kg}}) - (1.8 \times 10^5) (0.11135) \right] \\ &= -7.161 \text{ kW} \end{aligned}$$

(b) Mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{v_1^2 - v_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m} [h_2 - h_1]$$

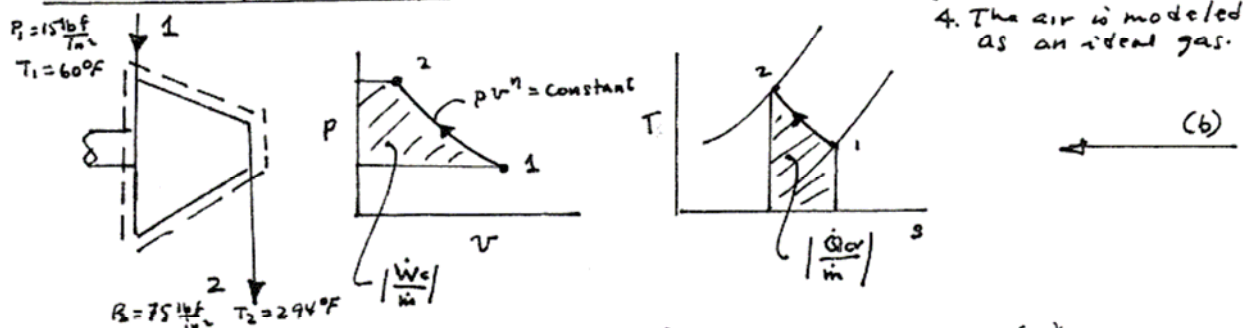
State 2 is fixed by 9 bar, $v_2 = 0.02369 \text{ m}^3/\text{kg}$. Thus, from Table A-12, $h_2 = 274.57 \text{ kJ/kg}$ and at state 1, $h_1 = 242.06 \text{ kJ/kg}$.

$$\begin{aligned} \dot{Q}_{cv} &= -7.161 \text{ kW} + (0.2155 \frac{\text{kg}}{\text{s}}) [274.57 - 242.06] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= -0.16 \text{ kW} \end{aligned}$$

PROBLEM 6.154

An air compressor operates at steady state with air entering at $p_1 = 15 \text{ lbf/in.}^2$, $T_1 = 60^\circ\text{F}$. The air undergoes a polytropic process, and exits at $p_2 = 75 \text{ lbf/in.}^2$, $T_2 = 294^\circ\text{F}$. (a) Evaluate the work and heat transfer, each in Btu per lb of air flowing. (b) Sketch the process on p - v and T - s diagrams and associate areas on the diagrams with work and heat transfer, respectively. Assume the ideal gas model for air and neglect changes in kinetic and potential energy.

SCHEMATIC & GIVEN DATA:



ENGINEER MODEL:

1. The control volume shown in the sketch is at steady state.
2. The air undergoes a process described by $p v^n = \text{constant}$.
3. Kinetic and potential energy effects can be neglected.
4. The air is modeled as an ideal gas.

ANALYSIS: With Eq. 3.56, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} \Rightarrow \frac{754^\circ\text{R}}{520^\circ\text{R}} = \left(\frac{75 \text{ lbf/in.}^2}{15 \text{ lbf/in.}^2}\right)^{(n-1)/n} \Rightarrow n = 1.3$

(a) With Eq. 6.55a

$$\begin{aligned} \frac{\dot{W}_{cv}}{\dot{m}} &= \frac{-nR}{(n-1)} (T_2 - T_1) \\ &= \frac{-1.3}{(1.3-1)} \left(\frac{1.986 \text{ Btu}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right) (754 - 520)^\circ\text{R} = -69.5 \frac{\text{Btu}}{\text{lb}} \end{aligned}$$

Reducing an energy balance, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$

$$\Rightarrow \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$$

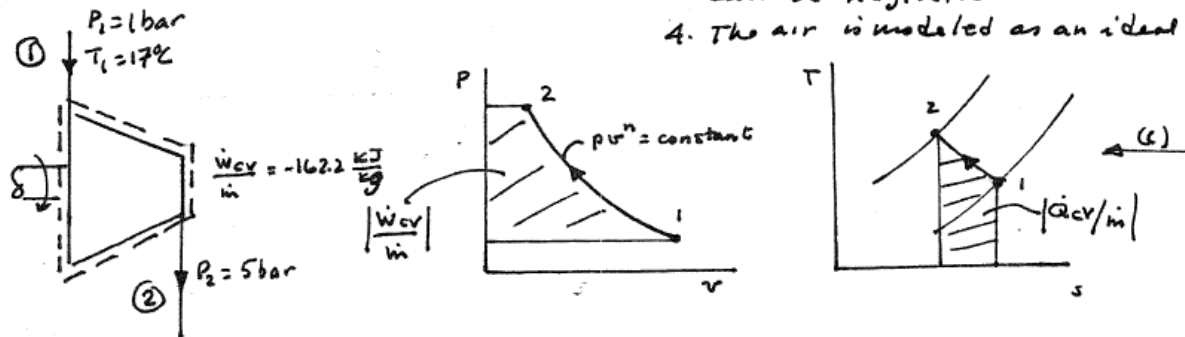
$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}} &= \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1) \\ &= -69.5 \frac{\text{Btu}}{\text{lb}} + (180.63 - 124.27) \frac{\text{Btu}}{\text{lb}} \\ &= -13.14 \text{ Btu/lb} \end{aligned}$$

where h_1 and h_2 are from Table A-22E.



PROBLEM 6.155

An air compressor operates at steady state with air entering at $p_1 = 1 \text{ bar}$, $T_1 = 17^\circ\text{C}$ and exiting at $p_2 = 5 \text{ bar}$. The air undergoes a polytropic process for which the compressor work input is $162.2 \text{ kJ per kg of air flowing}$. Determine (a) the temperature of the air at the compressor exit, in $^\circ\text{C}$, and (b) the heat transfer, in $\text{kJ per kg of air flowing}$. (c) Sketch the process on p - v and T - s diagrams and associate areas on the diagrams with work and heat transfer, respectively. Assume the ideal gas model for air and neglect changes in kinetic and potential energy.



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. The air undergoes a process described by $p v^n = \text{constant}$.
3. Kinetic and potential energy effects can be neglected.
4. The air is modeled as an ideal gas.

ANALYSIS: Using Eq. 6.55b

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\frac{nRT_1}{(n-1)} \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \Rightarrow -162.2 \frac{\text{kJ}}{\text{kg}} = -\frac{n}{(n-1)} \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (290 \text{ K}) \left[\left(\frac{5}{1} \right)^{\frac{(n-1)}{n}} - 1 \right]$$

$$\Rightarrow n = 1.3$$

(a) With Eq. 3.56,

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{(n-1)}{n}} = 290 \text{ K} \left(\frac{5}{1} \right)^{0.3/1.3} = 420.4 \text{ K} \quad \leftarrow (a)$$

(b) Reducing an energy rate balance, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$

$$\begin{aligned} \Rightarrow \frac{\dot{Q}_{cv}}{\dot{m}} &= \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1) \\ &= -162.2 \frac{\text{kJ}}{\text{kg}} + (421.67 - 290.16) \\ &= -30.69 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

$\leftarrow (b)$

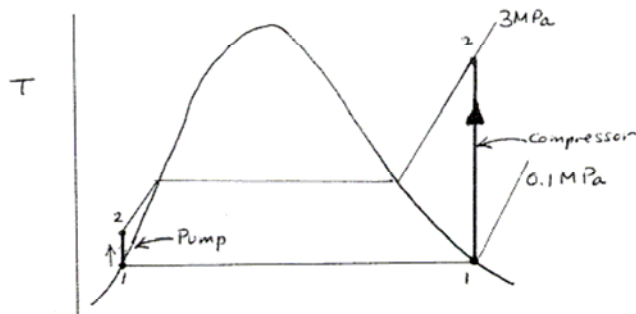
where h_1 and h_2 are from Table A-22.

PROBLEM 6.156

KNOWN: At steady state, water vapor is compressed isentropically to 3 MPa from the saturated vapor state at 0.1 MPa and liquid water is pumped isentropically to 3 MPa from the saturated liquid state at 0.1 MPa.

① FIND: Compare the work required in each case.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) In each case, a control volume enclosing the device is under consideration. (2) For each control volume, $\dot{Q}_{cv} = 0$, kinetic/potential energy effects are negligible, and operation is at steady state.

ANALYSIS: Reducing mass and energy rate balances

$$-\frac{\dot{W}_{cv}}{\dot{m}} = h_2 - h_1$$

Compressor: From Table A-4, $h_1 = 2675.5 \text{ kJ/kg}$, $s_1 = 7.3594 \text{ kJ/kg} \cdot \text{K}$. Since $s_2 = s_1$, $h_2 = 3556.71 \text{ kJ/kg}$. Thus

$$-\frac{\dot{W}_{cv}}{\dot{m}} = 3556.71 - 2675.5 = 881.21 \frac{\text{kJ}}{\text{kg}}$$

PUMP: From Table A-3, $h_1 = 417.46 \text{ kJ/kg}$, $s_1 = 1.3026 \text{ kJ/kg} \cdot \text{K}$. Then, interpolating in Table A-5 (double interpolation), $h_2 = 420.49 \text{ kJ/kg}$. Thus

$$-\frac{\dot{W}_{cv}}{\dot{m}} = 420.49 - 417.46 = 3.03 \frac{\text{kJ}}{\text{kg}}$$

Comparison:

$$\% = \left(\frac{\text{Pump Work}}{\text{Compressor Work}} \right) 100 = \frac{(3.03)(100)}{881.21} = 0.34\%$$

- In chapters 8 and 9 the significance of pump work and compressor work and their relation to each other will be apparent in the study of Rankine and Brayton power cycles. Also, see the discussion of Eq. 6.51b in Sec. 6.13.

PROBLEM 6.157

A pump operating at steady state receives saturated liquid water at 50°C with a mass flow rate of 20 kg/s . The pressure of the water at the pump exit is 1 MPa . If the pump operates with negligible internal irreversibilities and negligible changes in kinetic and potential energy, determine the power required in kW .

ENGR. MODEL:

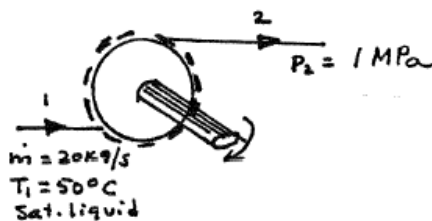
1. The control volume shown in the sketch operates at steady state.
2. Internal irreversibilities are negligible.
3. Kinetic and potential energy effects can be ignored.
4. $v \approx v_f(T_1)$.

ANALYSIS: Invoking Eq. 6.51c

$$\begin{aligned} \left(\frac{\dot{W}_{cv}}{\dot{m}} \right)_{\text{int rev}} &= -v(P_2 - P_1) \quad \begin{array}{l} \downarrow P_{\text{sat at } 50^\circ\text{C}} \\ L \approx v_f(50^\circ\text{C}) \end{array} = - \left(\frac{1.0121 \text{ m}^3}{10^3 \text{ kg}} \right) \left(10^6 \frac{\text{N}}{\text{m}^2} - 0.1235 \times 10^5 \frac{\text{N}}{\text{m}^2} \right) \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= -1 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

$$\therefore (\dot{W}_{cv})_{\text{int rev}} = 20 \frac{\text{kg}}{\text{s}} \left(-1 \frac{\text{kJ}}{\text{kg}} \right) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -20 \text{ kW}$$

SCHEMATIC & GIVEN DATA:



PROBLEM 6.158

A pump operating at steady state receives liquid water at 20°C , 100 kPa with a mass flow rate of 53 kg/min . The pressure of the water at the pump exit is 5 MPa . The isentropic pump efficiency is 70% . Stray heat transfer and changes in kinetic and potential energy are negligible. Determine the power required by the pump, in kW .

ENGINEER MODEL

1. The control volume shown in the sketch operates at steady state.
2. Kinetic and potential energy effects can be ignored.
3. $v \approx v_f(T_1)$

ANALYSIS: Involving Eq. 6.51c,

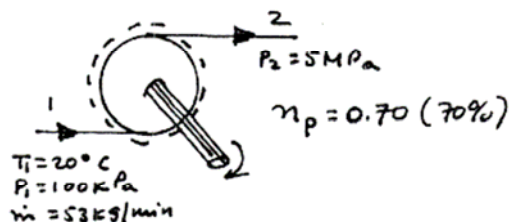
$$(\dot{W}_{cv})_{int} = -\dot{m} v (P_2 - P_1) = -53 \frac{\text{kg}}{\text{min}} \left(\frac{1.0018 \text{ m}^3}{103 \text{ kg}} \right) (5 \times 10^6 \frac{\text{N}}{\text{m}^2} - 10^5 \frac{\text{N}}{\text{m}^2}) \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right|$$

$$= -4.34 \text{ kW}$$

Using the pump efficiency,

$$-\dot{W} = \frac{(\dot{W}_{cv})_{int}}{\eta_p} = \frac{4.34 \text{ kW}}{0.70} = 6.19 \text{ kW}$$

SCHEMATIC & GIVEN DATA:



PROBLEM 6.159

A pump operating at steady state receives liquid water at 50°C, 1.5 MPa. The pressure of the water at the pump exit is 15 MPa. The magnitude of the work required by the pump is 18 kJ per kg of water flowing. Stray heat transfer and changes in kinetic and potential energy are negligible. Determine the isentropic pump efficiency.

ENGR. MODEL

1. The control volume shown in the sketch operates at steady state.
2. Kinetic and potential energy effects can be ignored.
3. $v \approx v_f(T_1)$.

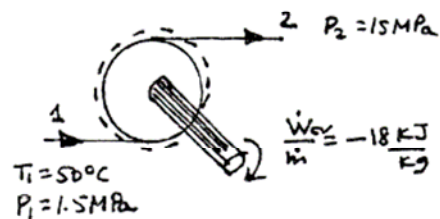
ANALYSIS: Invoking Eq. 6.51c,

$$\begin{aligned} \left(\frac{\dot{W}_{cv}}{\dot{m}} \right)_{int rev} &= -v(P_2 - P_1) \\ &= - \left(\frac{1.0121 \text{ m}^3}{10^3 \text{ kg}} \right) \left(15 \times 10^6 \frac{\text{N}}{\text{m}^2} - 1.5 \times 10^6 \frac{\text{N}}{\text{m}^2} \right) \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= -13.66 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

Then,

$$\eta_{\text{pump}} = \frac{\left(\frac{\dot{W}_{cv}}{\dot{m}} \right)_{int rev}}{\left(\frac{\dot{W}_{cv}}{\dot{m}} \right)} = \frac{13.66}{18} = 0.759 \text{ (75.9\%)} \quad \leftarrow$$

SCHMATIC & GIVEN DATA:

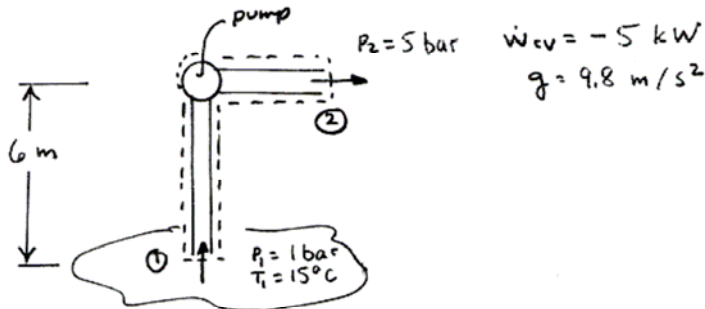


PROBLEM 6.160

KNOWN: Steady state operating data are provided for a 5-kW pump.

FIND: Determine if it would be possible to pump 7.5 m^3 in 10 min or less.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The control volume shown with the schematic is at steady state. 2. For the control volume, kinetic energy effects can be ignored. 3. Liquid water is modeled as incompressible with $v = v_f(T_1) = 1.0009 \times 10^{-3} \text{ m}^3/\text{kg}$ (Table A-3).

ANALYSIS: To pump 7.5 m^3 in 10 minutes, or less, would require a volumetric flow rate (AV)

$$(AV) \geq \left(\frac{7.5 \text{ m}^3}{10 \text{ min}} \right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 0.0125 \text{ m}^3/\text{s}$$

which corresponds to a mass flow rate

$$\dot{m} \geq \frac{0.0125 \text{ m}^3/\text{s}}{1.0009 \times 10^{-3} \text{ m}^3/\text{kg}} = 12.49 \frac{\text{kg}}{\text{s}} \quad (1)$$

In the absence of internal irreversibilities, the power input required would be given by the following expression obtained from Eq. 6.51a using assumptions 2, 3

$$(-\dot{W}_{cv})_{\text{int rev}} = \dot{m} \left[v(P_2 - P_1) + g(z_2 - z_1) + \frac{V_2^2 - V_1^2}{2} \right]$$

Then, with (1)

$$\begin{aligned} (-\dot{W}_{cv})_{\text{int rev}} &\geq (12.49 \frac{\text{kg}}{\text{s}}) \left[(1.0009 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}) (4 \times 10^2 \frac{\text{kN}}{\text{m}^2}) \left| \frac{1 \text{ kJ}}{1 \text{ kN m}} \right| \right. \\ &\quad \left. + (9.8 \frac{\text{m}}{\text{s}^2}) (6 \text{ m}) \left| \frac{1 \text{ N s}^2}{1 \text{ kg m}} \right| \left| \frac{1 \text{ kJ}}{1000 \text{ N m}} \right| \right] \\ &\geq (12.49 \frac{\text{kg}}{\text{s}}) \left[0.400 \frac{\text{kJ}}{\text{kg}} + 0.059 \frac{\text{kJ}}{\text{kg}} \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &\geq 5.73 \text{ kW} \end{aligned}$$

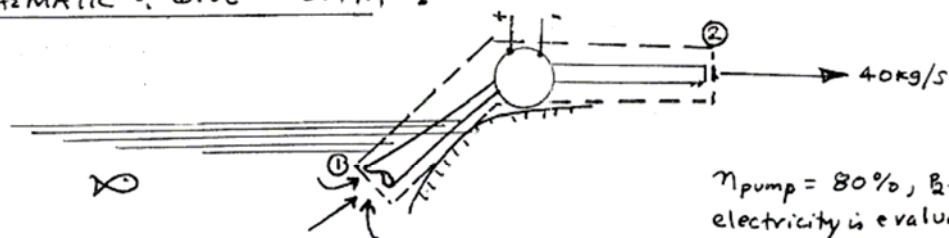
To overcome the effect of internal irreversibilities, such as friction between the flowing liquid and pipe wall, we expect the actual power input to the pump to be greater than calculated here. Since, only a 5-kW pump is available, the desired flow rate cannot be accommodated.

PROBLEM 6.161

KNOWN: Steady state operating data are provided for an electrically-driven pump that draws water from a pond.

FIND: Estimate the hourly cost of operating the pump.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, the changes in kinetic energy and potential energy from the inlet to the exit are ignored. (3) Liquid water is modeled as incompressible.

ANALYSIS: Using the pump efficiency, the power required to operate the pump is

$$\dot{W}_{\text{pump}} = \frac{(\dot{W}_{\text{int rev}})}{\eta_p}$$

where the numerator is provided by Eq. 6.51c

$$\dot{W}_{\text{pump}} = -\frac{\dot{m} v (P_2 - P_1)}{\eta_p}$$

With v from Table A-2, $v \approx v_f \approx 10^{-3} \text{ m}^3/\text{kg}$.

$$\dot{W}_{\text{pump}} = -\frac{(40 \frac{\text{kg}}{\text{s}})(10^{-3} \frac{\text{m}^3}{\text{kg}})(3 \times 10^5 \frac{\text{N}}{\text{m}^2})}{0.80} \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= -15 \text{ kW} \quad (\text{minus denotes input})$$

Then, the hourly cost of operating the pump is

$$(\text{hourly cost}) = (15 \text{ kW})(1 \text{ h}) \left(\frac{0.08 \text{ ¢}}{\text{kW}\cdot\text{h}} \right)$$

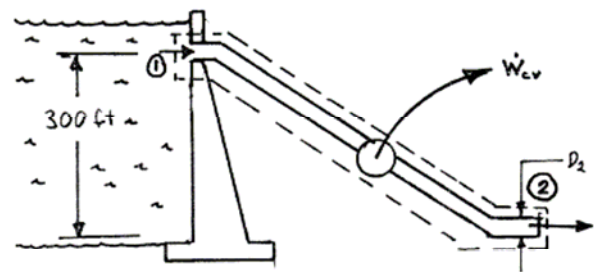
$$= \$1.20/\text{h}$$

PROBLEM 6.162

KNOWN: Steady-state operating data are provided for a hydraulic turbine-generator.

FIND: In the absence of internal irreversibilities, determine the value of the power produced on a daily basis.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned}
 P_1 &= 16 \text{ lbf/in}^2, V_1 = 4 \text{ ft/s} \\
 P_2 &= 14.7 \text{ lbf/in}^2, V_2 = 35 \text{ ft/s}, T_2 = 50^\circ \text{F}, D_2 = 4 \text{ ft} \\
 g &= 32.2 \text{ ft/s}^2 \\
 \text{electricity is evaluated at } &8 \text{¢/kW}\cdot\text{h}
 \end{aligned}$$

ENGR. MODEL: 1. The control volume shown in the figure is at steady state. 2. Internal irreversibilities are ignored. 3. Liquid water is modeled as incompressible with $v \approx v_f(50^\circ \text{F})$.

ANALYSIS: Using assumption 2 indicating that internal irreversibilities are ignored, Eq. 6.51a can be used to evaluate the power developed.

Since water is modeled as incompressible, we get

$$\left(\frac{\dot{W}_{cv}}{\dot{m}} \right)_{\text{int rev}} = -v(P_2 - P_1) + \frac{V_1^2 - V_2^2}{2} + g(Z_1 - Z_2)$$

With $v = v_f(50^\circ \text{F}) = 0.01602 \text{ ft}^3/\text{lb}$ and given data

$$\begin{aligned}
 \left(\frac{\dot{W}_{cv}}{\dot{m}} \right)_{\text{int rev}} &= -(0.01602 \frac{\text{ft}^3}{\text{lb}}) [(14.7 - 16) \frac{\text{lbf}}{\text{ft}^2}] + \left[\left(\frac{4^2 - 35^2}{2} \right) \frac{\text{ft}^2}{\text{s}^2} \right] \left| \frac{1 \text{ lbf} \cdot \text{s}^2}{32.2 \text{ lb} \cdot \text{ft}} \right| + (32.2 \frac{\text{ft}}{\text{s}^2}) (300 \text{ ft}) \left| \frac{1 \text{ lbf} \cdot \text{s}^2}{32.2 \text{ lb} \cdot \text{ft}} \right| \\
 &= (2.999 \frac{\text{ft} \cdot \text{lbf}}{\text{lb}}) + (-18.773 \frac{\text{ft} \cdot \text{lbf}}{\text{lb}}) + (300 \frac{\text{ft} \cdot \text{lbf}}{\text{lb}}) \\
 &= 284.23 \frac{\text{ft} \cdot \text{lbf}}{\text{lb}} \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 0.365 \text{ Btu/lb}
 \end{aligned}$$

The mass flow rate is

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{\pi D_2^2 V_2}{4 v_2} = \frac{\pi (4 \text{ ft})^2 (35 \text{ ft/s})}{4 (0.01602 \text{ ft}^3/\text{lb})} = 27,455 \frac{\text{lb}}{\text{s}}$$

$$\text{So, } (\dot{W}_{cv})_{\text{int rev}} = (27,455 \frac{\text{lb}}{\text{s}}) (0.365 \frac{\text{Btu}}{\text{lb}}) \left| \frac{1 \text{ kW} \cdot \text{h}}{3413 \text{ Btu}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = 10,570 \text{ kW}$$

$$\textcircled{1} \text{ Daily Value} = (10,570 \text{ kW}) \left(\frac{24 \text{ h}}{\text{day}} \right) \left(\frac{\$0.08}{\text{kW} \cdot \text{h}} \right) = \$20,294 / \text{day}$$

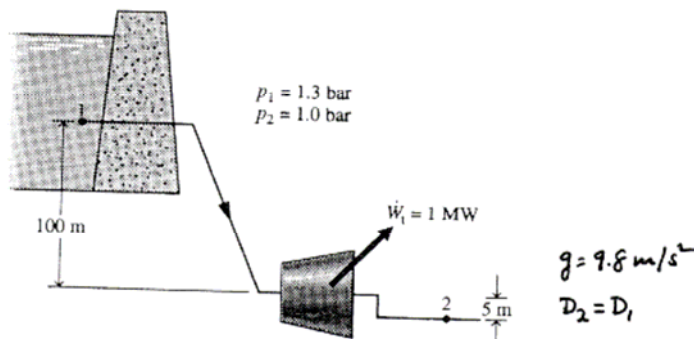
- Owing to the effect of irreversibilities, the actual daily revenue would be less than the calculated value. Also, the daily \$ value of the power developed is not the profit. The plant owners would have a considerable capital investment to recoup, and a variety of operating and maintenance costs to pay.

PROBLEM 6.163

KNOWN: Steady-state operating data are provided for a hydraulic turbine.

FIND: For a turbine power output of 1 MW, estimate the minimum required mass flow rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. A one-inlet, one-exit control volume with inlet at 1 and exit at 2, and enclosing the turbine, is at steady state.

2. Liquid water is modeled as incompressible, with $v \approx v_f \approx 10^{-3} \frac{\text{m}^3}{\text{kg}}$.

ANALYSIS: We expect that the effect of internal irreversibilities would be a lower power output in the actual case than in the ideal case — that is, we expect

$$(\dot{W}_t)_{\text{act}} \leq (\dot{W})_{\text{int rev}}$$

Introducing Eq. 6.51a with assumption 2

$$(\dot{W}_t)_{\text{act}} \leq \dot{m} \left[-v(p_2 - p_1) + g(z_1 - z_2) + \cancel{\frac{V_1^2}{2} - \frac{V_2^2}{2}} \right] \quad (1)$$

where the kinetic energy term drops out because $D_1 = D_2$, v is constant, and $\dot{m}_1 = \dot{m}_2$. From (1), we have

$$\dot{m} \geq \left[\frac{(\dot{W}_t)_{\text{act}}}{-v(p_2 - p_1) + g(z_1 - z_2)} \right] = \frac{(1 \text{ MW}) \left| \frac{10^6 \text{ N} \cdot \text{m/s}}{1 \text{ MW}} \right|}{\left[-\left(10^{-3} \frac{\text{m}^3}{\text{kg}}\right) \left(-0.3 \times 10^5 \frac{\text{N}}{\text{m}^2}\right) \right] + \left[\left(9.8 \frac{\text{m}}{\text{s}^2}\right) (105 \text{ m}) \right] \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right|}$$

$$\dot{m} \geq 944.3 \text{ kg/s}$$

The minimum required mass flow rate is 944.3 kg/s.

PROBLEM 6.164

Nitrogen (N_2) enters a nozzle operating at steady state at 0.2 MPa, 550 K with a velocity of 1 m/s and undergoes a polytropic expansion with $n = 1.3$ to 0.15 MPa. Using the ideal gas model with $k = 1.4$, and ignoring potential energy effects, determine (a) the exit velocity, in m/s, and (b) the rate of heat transfer, in kJ per kg of gas flowing.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. The N_2 undergoes an expansion with $pv^{1.3} = \text{constant}$.
3. Potential energy effects can be ignored. $\dot{W}_{cv} = 0$.
4. The N_2 is modeled as an ideal gas with $k = 1.4$.

ANALYSIS: (a) Invoking Eq. 6.51a with $(\dot{W}_{cv}/\dot{m})_{int} = 0$ and dropping the potential energy term, we get

$$\int_1^2 v dp + \frac{V_2^2 - V_1^2}{2} = 0$$

where, with $pv^n = \text{constant}$, $\int_1^2 v dp = \frac{n}{n-1} (P_2 V_2 - P_1 V_1)$, Eq. 6.53. Using the ideal gas equation of state, and collecting results,

$$\frac{nR}{n-1} [T_2 - T_1] + \frac{V_2^2 - V_1^2}{2} = 0$$

Solving for V_2 ,

$$V_2 = \sqrt{V_1^2 + \frac{2nR}{(n-1)} (T_1 - T_2)} \quad (1)$$

With Eq. 3.56

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(n-1)/n} = 550 \text{ K} \left(\frac{0.15 \text{ MPa}}{0.2 \text{ MPa}} \right)^{(1.3-1)/1.3} = 514.7 \text{ K}$$

Inserting values in Eq. (1),

$$V_2 = \sqrt{\left(\frac{1 \text{ m}}{\text{s}} \right)^2 + 2 \left(\frac{1.3}{0.3} \right) \left(\frac{8.314 \text{ kJ}}{28.01 \text{ kg} \cdot \text{K}} \right) (550 - 514.7) \text{ K} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|} \quad (a)$$

$$= 301.3 \text{ m/s}$$

(b) Reducing an energy rate balance,

$$\dot{Q}_{cv} = \dot{h}_2 - \dot{h}_1 + \frac{V_2^2 - V_1^2}{2} = c_p (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

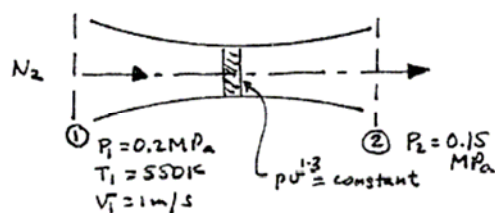
($= \frac{kR}{k-1} (T_2 - T_1)$)

$$\Rightarrow \dot{Q}_{cv} = \frac{kR}{k-1} (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$= \left(\frac{1.4}{0.4} \right) \left(\frac{8.314}{28.01} \right) (514.7 - 550) \frac{\text{kJ}}{\text{kg}} + \left[\frac{(301.3)^2 - (1)^2}{2} \right] \left(\frac{\text{m}}{\text{s}} \right)^2 \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= +8.72 \frac{\text{kJ}}{\text{kg}} \quad (b)$$

SCHEMATIC & GIVEN DATA:

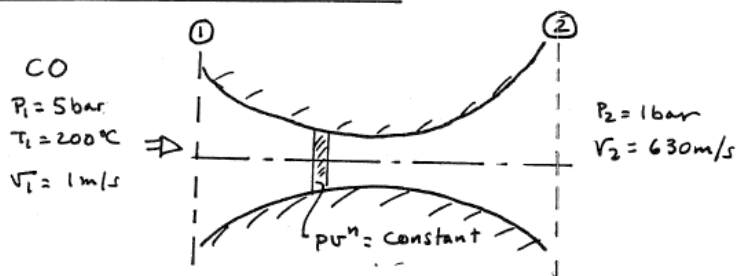


PROBLEM 6.165

Carbon monoxide enters a nozzle operating at steady state at 5 bar, 200°C with a velocity of 1 m/s and undergoes a polytropic expansion to 1 bar and an exit velocity of 630 m/s. Using the ideal gas model and ignoring potential energy effects, determine

- the exit temperature, in °C.
- the rate of heat transfer, in kJ per kg of gas flowing.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The nozzle is at steady state. (2) The expansion is described by $pv^n = \text{constant}$. (3) CO is modeled as an ideal gas. (4) Potential energy effects are ignored.

ANALYSIS: (a) As a polytropic process is internally reversible, Eq. 6.51a is applicable. Then, with $\dot{W}_{cv} = 0$, the Bernoulli equation, Eq. 6.52 results:

$$\int_1^2 v dp + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0 \quad (1)$$

Using the relationship $pv^n = \text{constant}$, the integral can be performed (Sec. 6.13), giving

$$\begin{aligned} \int_1^2 v dp &= \frac{n}{n-1} (P_2 V_2 - P_1 V_1) \\ &= \frac{nR}{n-1} (T_2 - T_1) \end{aligned} \quad (2)$$

where the ideal gas equation of state has been used to obtain the last expression. For a polytropic process of an ideal gas (Eq. 3.56)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(n-1)/n} \quad (3)$$

With Eqs. (1)–(3),

$$\frac{nR}{(n-1)} (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} = 0 \Rightarrow \frac{nRT_1}{n-1} \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right] + \frac{V_2^2 - V_1^2}{2} = 0.$$

Inserting values,

$$\left(\frac{n}{n-1} \right) \left(\frac{8.314 \text{ kJ}}{28.01 \text{ kg} \cdot \text{K}} \right) (473 \text{ K}) \left[\left(\frac{1}{5} \right)^{(n-1)/n} - 1 \right] + \left[\frac{(630)^2 - (1)^2}{2} \right] \left(\frac{\text{m}^2}{\text{s}^2} \right) \left(\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 0$$

$$\text{Solving, } n = 1.2. \text{ Then, with Eq. (3), } T_2 = 473 \text{ K} \left(\frac{1}{5} \right)^{1.2/1.2} = 362 \text{ K} (89^\circ \text{C}) \quad (c)$$

(b) An energy balance reduces to give $\dot{Q}_{cv}/\dot{m} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$

With data from Table A-23

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \left(\frac{10531 - 13797}{28.01} \right) \frac{\text{kJ}}{\text{kg}} + \left[\frac{(630)^2 - (1)^2}{2} \right] \left(\frac{\text{m}^2}{\text{s}^2} \right) \left(\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = +81.8 \frac{\text{kJ}}{\text{kg}}$$

PROBLEM 7.1

By inspection of Fig. P7.1 giving a $T-v$ diagram for water, indicate whether exergy would increase, decrease, or remain the same in (a) Process 1-2, (b) Process 3-4, (c) Process 5-6. Explain.

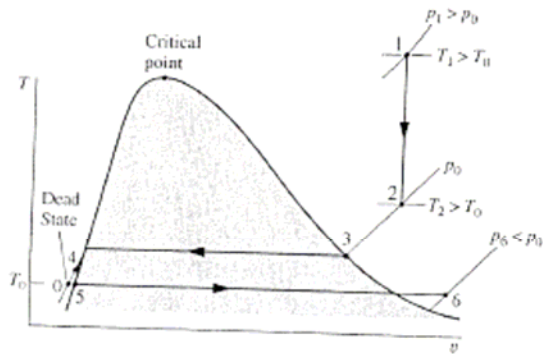


Fig. P7.1

See discussion of Fig. 7.4.

Process 1-2:

Exergy decreases
 $p \rightarrow p_0$ while
 T moves closer to T_0

Process 3-4:

Exergy decreases
 T moves closer to T_0
 while $p = p_0$

Process 5-6:

Exergy increases
 p moves away from p_0
 while $T = T_0$

PROBLEM 7.2

KNOWN: An ideal gas is stored in a closed vessel at p, T .

FIND: (a) If $T = T_0$, derive $e = f(p, p_0, T_0, R)$

(b) If $p = p_0$, derive $e = f(T, T_0, c_p)$

ENGR. MODEL: (1) The gas obeys the ideal gas model. In part (b), c_p is constant as well. (2) The effects of motion and gravity can be ignored.

ANALYSIS:

(a) With assumptions (1), (2), Eq. 7.2 becomes

$$e = \cancel{(u - u_0)} + p_0(v - v_0) - T_0(s - s_0)$$

The first term vanishes because u of an ideal gas depends only on temperature, and $T = T_0$. Also,

$$s - s_0 = \cancel{(s^0(T) - s^0(T_0))} - R \ln \frac{p}{p_0}$$

Collecting results

$$e = p_0(v - v_0) + R T_0 \ln \frac{p}{p_0}$$

With $v = R T_0 / p$ and $v_0 = R T_0 / p_0$

$$e = R T_0 \left[\frac{p_0}{p} - \frac{R T_0}{p_0} \right] + R T_0 \ln \frac{p}{p_0}$$

$$= R T_0 \left[\frac{p_0}{p} - 1 + \ln \frac{p}{p_0} \right] \leftarrow$$

(b) With assumption (2), Eq. 7.2 becomes

$$e = u - u_0 + p_0(v - v_0) - T_0(s - s_0)$$

Introducing ideal gas relations, with $p = p_0$

$$e = u(T) - u(T_0) + p_0 \left[\frac{R T}{p_0} - \frac{R T_0}{p_0} \right] - T_0 \left[\int_{T_0}^T \frac{c_p}{T} dT - \cancel{R \ln \frac{p_0}{p_0}} \right]$$

If c_p is constant, so is c_v and the two differ by the gas constant R . Thus

$$e = c_v [T - T_0] + R [T - T_0] - T_0 c_p \ln \frac{T}{T_0}$$

$$= \underbrace{(c_v + R)}_{c_p} [T - T_0] - T_0 c_p \ln \frac{T}{T_0}$$

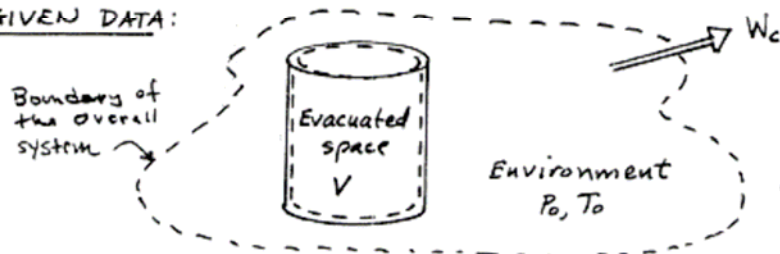
$$= c_p T_0 \left[\frac{T}{T_0} - 1 - \ln \frac{T}{T_0} \right] \leftarrow$$

PROBLEM 7.3

KNOWN: An evacuated container of known volume is under consideration.

FIND: For the space inside the tank, determine the exergy.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) An overall system is considered consisting of the evacuated space (a closed system) and the environment. (2) The volume of the overall system is constant. (3) For the overall system, $Q=0$. (4) For the environment, P_0 and T_0 are constant. (5) The effects of gravity and motion are ignored.

ANALYSIS: Consider a process where the evacuated space collapses and work is developed by the overall system. An energy balance for the overall system reduces with assumption 3 to give

$$W_c = -\Delta E_c$$

Since $\Delta U=0$ for the evacuated space, $\Delta E_c = \Delta U_c$. With Eq. (c) of p. 334, we get

$$W_c = -[T_0 \Delta S_c - P_0 \Delta V_c]$$

By assumption 2, $\Delta V_c = -\Delta V = -(V_0 - V) = V$. Further, an entropy balance for the overall system gives

$$\Delta S_c = \sigma_c \Rightarrow \Delta S + \Delta S_c = \sigma_c$$

Collecting results

$$W_c = P_0 V - T_0 \sigma_c$$

Since $\sigma_c \geq 0$, the maximum theoretical work, or exergy, is obtained when $\sigma_c = 0$. Thus

$$E = P_0 V \quad (1)$$

Alternatively, Eq. (1) can be regarded as the minimum theoretical work required to produce the evacuated space.

PROBLEM 7.4

KNOWN: Equal molar amounts of CO_2 and He are at the same T, P .

FIND: Determine which gas has the greater exergy value, $\bar{e}$.

ENGR. MODEL: (1) Each gas obeys the ideal gas model with constant $\bar{c}_v$.
(2) There are no significant effects of motion and gravity.

ANALYSIS: With assumption (2), Eq. 7.2 reduces to give on a molar basis

$$\textcircled{1} \quad \bar{e} = \bar{u} - \bar{u}_0 + P_0(\bar{v} - \bar{v}_0) - T_0(\bar{s} - \bar{s}_0)$$

Then, with assumption (1)

$$\begin{aligned} \bar{e} &= \bar{c}_v(T - T_0) + \bar{R}T_0 \left[\frac{T}{T_0} \cdot \frac{P_0}{P} - 1 \right] - T_0 \left[\bar{c}_p \ln \frac{T}{T_0} - \bar{R} \ln \frac{P}{P_0} \right] \\ &= \bar{c}_v(T - T_0) - T_0 \bar{c}_p \ln \frac{T}{T_0} + \bar{R}T_0 \left\{ \frac{T}{T_0} \cdot \frac{P_0}{P} - 1 \right\} + \bar{R}T_0 \ln \frac{P}{P_0} \end{aligned}$$

Applying this to each of CO_2 and He , and subtracting the resulting equations gives

$$\begin{aligned} \bar{e}_{\text{CO}_2} - \bar{e}_{\text{He}} &= \underbrace{[\bar{c}_{v,\text{CO}_2} - \bar{c}_{v,\text{He}}]}_{\equiv (\bar{c}_{p,\text{CO}_2} - \bar{c}_{p,\text{He}})} (T - T_0) - [\bar{c}_{p,\text{CO}_2} - \bar{c}_{p,\text{He}}] T_0 \ln \frac{T}{T_0} \\ &\quad \text{(see Eq. 3.45: } \bar{c}_p = \bar{c}_v + \bar{R}) \\ &= [\bar{c}_{p,\text{CO}_2} - \bar{c}_{p,\text{He}}] \left[T - T_0 - T_0 \ln \frac{T}{T_0} \right] \end{aligned}$$

By inspection of Figure 3.13, $\bar{c}_{p,\text{CO}_2} > \bar{c}_{p,\text{He}}$, giving

$$\textcircled{2} \quad \bar{e}_{\text{CO}_2} > \bar{e}_{\text{He}} \quad \longleftarrow$$

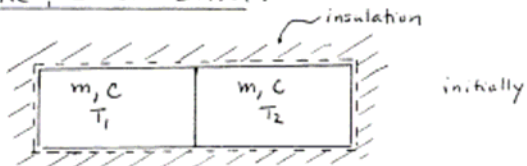
-
1. Here only the thermomechanical component is considered. The chemical contribution of Section 13.6 is not included in the present discussion.
 2. But note that the molecular weights are much different: $M_{\text{CO}_2} = 44.01$, $M_{\text{He}} = 4.003$.

PROBLEM 7.5

KNOWN: Two solid blocks of mass m and specific heat C are brought into contact and attain thermal equilibrium.

FIND: Derive an expression for the energy destruction; demonstrate that the energy destruction cannot be negative, and discuss.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) As shown in the figure, the two solid blocks form a closed system. (2) The blocks are modeled as incompressible with constant specific heat C . (3) For the system, $Q = W = 0$ and there are no kinetic and potential energy effects.

ANALYSIS: (a) Using assumptions (3), the energy balance reduces to

$$\Delta U = \cancel{Q} - \cancel{W} \Rightarrow \Delta U = 0 \Rightarrow [(2m)u(T_f) - [mu(T_1) + mu(T_2)]] = 0$$

where T_f is the final temperature at equilibrium. Accordingly

$$[2u(T_f) - u(T_1) - u(T_2)] = 0 \Rightarrow [u(T_f) - u(T_1)] + [u(T_f) - u(T_2)] = 0$$

$$\Rightarrow c[T_f - T_1] + c[T_f - T_2] = 0 \Rightarrow T_f = \frac{T_1 + T_2}{2}$$

The amount of exergy destroyed is $E_d = T_0 \sigma$, where σ is the amount of entropy produced, obtained from an entropy balance:

$$\Delta S = \cancel{\int \frac{\delta Q}{T}}_b + \sigma \Rightarrow [(2m)s(T_f) - [ms(T_1) + ms(T_2)]] = \sigma$$

or

$$\begin{aligned} \sigma &= m[s(T_f) - s(T_1)] + m[s(T_f) - s(T_2)] \\ &= mc \ln \frac{T_f}{T_1} + mc \ln \frac{T_f}{T_2} \\ &= mc \ln \left[\frac{T_f^2}{T_1 T_2} \right] = mc \ln \left[\frac{(T_1 + T_2)^2}{4 T_1 T_2} \right] \end{aligned}$$

Finally

$$E_d = mc T_0 \ln \left[\frac{(T_1 + T_2)^2}{4 T_1 T_2} \right] \quad \leftarrow E_d$$

(b) The value of E_d would be negative only if $\left[\frac{(T_1 + T_2)^2}{4 T_1 T_2} \right]$ were less than unity. Considering this possibility...

$$\frac{(T_1 + T_2)^2}{4 T_1 T_2} < 1 \Rightarrow (T_1 + T_2)^2 < 4 T_1 T_2 \Rightarrow T_1^2 + 2 T_1 T_2 + T_2^2 < 4 T_1 T_2$$

$$\text{or } T_1^2 - 2 T_1 T_2 + T_2^2 < 0 \Rightarrow (T_1 - T_2)^2 < 0$$

Since this inequality cannot be satisfied, we can conclude that

$E_d \geq 0$.

(c) The exergy destruction in this case can be traced to the spontaneous heat transfer that takes place within the two blocks as they come to thermal equilibrium.

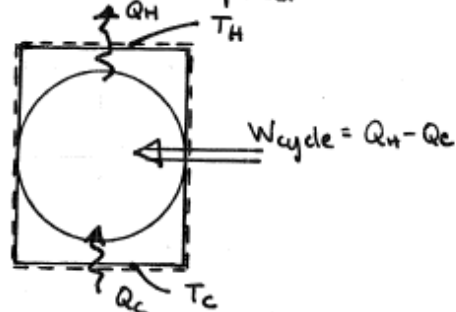
PROBLEM 7-6

KNOWN: A system undergoes a refrigeration cycle while receiving Q_c at temperature T_c and discharging Q_H at T_H , where $T_H > T_c$. Q_c and Q_H are the only heat transfers.

FIND: (a) Show that W_{cycle} cannot be zero. (b) Obtain a specified expression for coefficient of performance β . (c) Determine β_{max} .

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The system shown undergoes a refrigeration cycle. (2) Q_c and Q_H are the only heat transfers and are in the directions of the arrows. (3) T_c and T_H are constant and $T_H > T_c$. (4) The environment temperature is T_o .



ANALYSIS: (a) An exergy balance for the cycle reads

$$\Delta E^o_{\text{cycle}} = \left[1 - \frac{T_o}{T_c}\right] Q_c - \left[1 - \frac{T_o}{T_H}\right] Q_H - [W_{\text{cycle}} - P_o \Delta V]_{\text{cycle}} - E_d$$

where $\Delta E = \Delta V = 0$ for a cycle. Introducing the energy balance, $Q_H = W_{\text{cycle}} + Q_c$ we get

$$\begin{aligned} 0 &= \left[1 - \frac{T_o}{T_c}\right] Q_c - \left[1 - \frac{T_o}{T_H}\right] (W_{\text{cycle}} + Q_c) - W_{\text{cycle}} - E_d \\ &= \left[\left(1 - \frac{T_o}{T_c}\right) - \left(1 - \frac{T_o}{T_H}\right)\right] Q_c + \left[\frac{T_o}{T_H} - 1\right] W_{\text{cycle}} - E_d \\ &= T_o \left[\frac{1}{T_H} - \frac{1}{T_c}\right] Q_c + \frac{T_o}{T_H} W_{\text{cycle}} - E_d \end{aligned} \quad (1)$$

Solving for E_d , and setting W_{cycle} to zero

$$E_d = T_o \left[\underbrace{\frac{1}{T_H}}_{\text{pos.}} - \underbrace{\frac{1}{T_c}}_{\text{neg.}} \right] Q_c + \frac{T_o}{T_H} W_{\text{cycle}} \Rightarrow E_d < 0 \text{ impossible!}$$

Thus W_{cycle} cannot be zero. (a)

(b) Solving (1) for $\beta = Q_c / W_{\text{cycle}}$

$$\left[\frac{T_H - T_c}{T_H T_c} \right] Q_c = \frac{W_{\text{cycle}}}{T_H} - E_d / T_o$$

$$\beta = \frac{Q_c}{W_{\text{cycle}}} = \left[\frac{T_c}{T_H - T_c} \right] \left[1 - \frac{T_H E_d}{T_o W_{\text{cycle}}} \right] = \left[\frac{T_c}{T_H - T_c} \right] \left[1 - \frac{T_H E_d}{T_o (Q_H - Q_c)} \right] \quad (b)$$

(c) From the result of part (b), β increases as $E_d \rightarrow 0$. Thus, when $E_d = 0$

$$\beta_{\text{max}} = \frac{T_c}{T_H - T_c}$$

as expected.

PROBLEM 7.7

When matter flows across the boundary of a control volume, an energy transfer by work, called *flow work*, occurs. The rate is $\dot{m}(pv)$ where $\dot{m}$, p , and v denote the mass flow rate, pressure, and specific volume, respectively, of the matter crossing the boundary (see Sec. 4.4.2). Show that the *exergy transfer accompanying flow work* is given by $\dot{m}(pv - p_0v)$, where p_0 is the pressure at the dead state.

ANALYSIS: The objective is to demonstrate the following expression:

$$\left[\begin{array}{l} \text{time rate of exergy transfer} \\ \text{accompanying flow work} \end{array} \right] = \dot{m}(pv - p_0v) \quad (1)$$

Let us develop Eq. (1) for the case shown in the figure. The figure shows a closed system that occupies different regions at time t and a later time $t + \Delta t$. The fixed quantity of matter under consideration is shown shaded. During the time interval Δt , some of the mass initially within the region labeled *control volume* exits to fill the small region e adjacent to the control volume, as shown in Fig. (b). We assume that the increase in the volume of the closed system in the time interval Δt is equal to the volume of region e and, for further simplicity, that the only work is associated with this volume change. With Eq. 7.6, the exergy transfer accompanying work is

$$\left[\begin{array}{l} \text{exergy transfer} \\ \text{accompanying work} \end{array} \right] = W - p_0 \Delta V \quad (2)$$

where ΔV is the volume change of the system. The volume change of the system equals the volume of region e . Thus, we may write $\Delta V = m_e v_e$, where m_e is the mass within region e and v_e is the specific volume, which is regarded as uniform throughout region e . With this expression for ΔV , Eq. (2) becomes

$$\left[\begin{array}{l} \text{exergy transfer} \\ \text{accompanying work} \end{array} \right] = W - m_e(p_0 v_e) \quad (3)$$

Equation (3) can be placed on a time rate basis by dividing each term by the time interval Δt and taking the limit as Δt approaches zero. That is

$$\left[\begin{array}{l} \text{time rate of exergy} \\ \text{transfer accompanying work} \end{array} \right] = \lim_{\Delta t \rightarrow 0} \left(\frac{W}{\Delta t} \right) - \lim_{\Delta t \rightarrow 0} \left[\frac{m_e}{\Delta t} (p_0 v_e) \right] \quad (4)$$

In the limit as Δt approaches zero, the boundaries of the closed system and control volume coincide. Accordingly, in this limit the rate of energy transfer by work from the closed system is also the rate of energy transfer by work from the control volume. For the present case, this is just the flow work. Thus, the first term on the right side of Eq. (4) becomes

$$\lim_{\Delta t \rightarrow 0} \left(\frac{W}{\Delta t} \right) = \dot{m}_e(p_e v_e) \quad (5)$$

where $\dot{m}_e$ is the mass flow rate at the exit of the control volume. In the limit as Δt approaches zero, the second term on the right side of Eq. (4) becomes

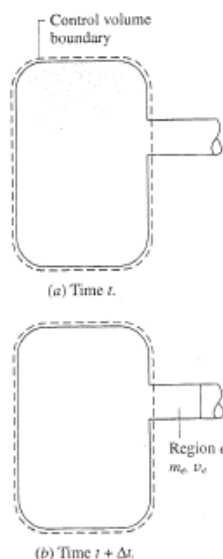
$$\lim_{\Delta t \rightarrow 0} \left[\frac{m_e}{\Delta t} (p_0 v_e) \right] = \dot{m}_e(p_0 v_e) \quad (6)$$

In this limit, the assumption of uniform specific volume throughout region e corresponds to the assumption of uniform specific volume across the exit (one-dimensional flow).

Substituting Eqs. (5) and (6) into Eq. (4) gives

$$\begin{aligned} \left[\begin{array}{l} \text{time rate of exergy transfer} \\ \text{accompanying flow work} \end{array} \right] &= \dot{m}_e(p_e v_e) - \dot{m}_e(p_0 v_e) \\ &= \dot{m}_e(p_e v_e - p_0 v_e) \end{aligned} \quad (7)$$

Extending the reasoning given here, it can be shown that an expression having the same form as Eq. (7) accounts for the transfer of exergy accompanying flow work at inlets to control volumes as well. The general result applying at both inlets and exits is given by Eq. (1).



PROBLEM 7.8

When matter flows across the boundary of a control volume, an exergy transfer accompanying mass flow occurs, which is given by $\dot{m}e$, where e is the specific exergy (Eq. 7.2) and $\dot{m}$ is the mass flow rate. An exergy transfer accompanying flow work, which is given by the result of Problem 7.7, also occurs at the boundary. Show that the sum of these exergy transfers is given by $\dot{m}e_f$, where e_f is the specific flow exergy (Eq. 7.14).

ANALYSIS:

When mass enters or exits a control volume, there is an accompanying exergy transfer given by

$$\left[\begin{array}{l} \text{time rate of exergy transfer} \\ \text{accompanying mass flow} \end{array} \right] = \dot{m}e \quad \begin{array}{l} \text{specific exergy} \\ \text{specific energy} \end{array}$$

$$= \dot{m}[(e - u_0) + p_0(v - v_0) - T_0(s - s_0)] \quad (1)$$

where Eq. 7.2 for specific exergy is written compactly in terms of specific energy: $e = u + V^2/2 + gz$. In addition to an exergy transfer accompanying mass flow, an exergy transfer accompanying flow work takes place at locations where mass enters or exits a control volume. Transfers of exergy accompanying flow work are accounted for by $\dot{m}(pv - p_0v)$, which is developed in Problem 7.7.

Since transfers of exergy accompanying mass flow and flow work occur at locations where mass enters or exits a control volume, a single expression giving the sum of these effects is convenient. Thus,

$$\left[\begin{array}{l} \text{time rate of exergy transfer} \\ \text{accompanying mass flow and flow work} \end{array} \right] = \dot{m}[e + (pv - p_0v)]$$

$$= \dot{m}[\underbrace{(e - u_0) + p_0(v - v_0) - T_0(s - s_0)}_{\text{specific flow exergy } e_f} + \underbrace{(pv - p_0v)}_{\text{flow work contribution}}] \quad (2)$$

The underlined terms in Eq. (2) represent, per unit of mass, the exergy transfer accompanying mass flow and flow work, respectively. The sum identified by underlining is the specific flow exergy e_f . That is

$$\textcircled{1} \quad e_f = (e - u_0) + p_0(v - v_0) - T_0(s - s_0) + (pv - p_0v) \quad (3)$$

The specific flow exergy can be placed in a more convenient form for calculation by introducing $e = u + V^2/2 + gz$ in Eq. (3) and simplifying to obtain

$$e_f = \left(u + \frac{V^2}{2} + gz - u_0 \right) + (pv - p_0v_0) - T_0(s - s_0)$$

$$= \underbrace{(u + pv)}_h - \underbrace{(u_0 + p_0v_0)}_{h_0} - T_0(s - s_0) + \frac{V^2}{2} + gz \quad (4)$$

Finally, with $h = u + pv$ and $h_0 = u_0 + p_0v_0$, Eq. (4) gives Eq. 7.14, which is the objective.

1. The specific flow exergy, e_f , and specific exergy, e , are related by

$$e_f = e + v(p - p_0) \quad (5)$$

specific flow exergy ← specific exergy ← flow work contribution — negative when $p < p_0$.

Although the specific exergy cannot be negative, the specific flow exergy can take on negative values when the local pressure is less than the pressure at the dead state: $p < p_0$. In such cases, the net transfer of exergy where matter crosses the boundary is opposite to the direction of mass flow.

PROBLEM 7.9

KNOWN: An ideal gas with constant specific heat ratio k .

FIND: Show that in the absence of motion and gravity effects

$$\frac{e_f}{c_p T_0} = \frac{T}{T_0} - 1 - \ln \frac{T}{T_0} + \ln \left(\frac{P}{P_0} \right)^{(k-1)/k}$$

and develop plots for $k = 1.2, 1.3, 1.4$ of $e_f/c_p T_0$ vs. T/T_0 for $P/P_0 = 0.25, 0.5, 1, 2, 4$. Discuss the significance of $e_f < 0$.

ENG. MODEL: (1) The system consists of an ideal gas with constant specific heat ratio. (2) The effects of motion and gravity are not significant.

ANALYSIS: With assumption (2) and ideal gas relationships, Eq. 7.14 gives

$$\begin{aligned} e_f &= h - h_0 - T_0 (s - s_0) \\ &= c_p [T - T_0] - T_0 \left[c_p \ln \frac{T}{T_0} - R \ln \frac{P}{P_0} \right] \\ &\quad \uparrow = c_p \left[\frac{k-1}{k} \right] \quad (\text{Eq. 3.47a}) \end{aligned}$$

Thus

$$\frac{e_f}{c_p T_0} = \frac{T}{T_0} - 1 - \ln \frac{T}{T_0} + \ln \left[\frac{P}{P_0} \right]^{(k-1)/k}$$

(a) See plots on the next page.

(b) Discussion.

The specific exergy is

$$e = (u - u_0) + P_0 (v - v_0) - T_0 (s - s_0)$$

and the specific flow exergy is

$$\begin{aligned} e_f &= h - h_0 - T_0 (s - s_0) \\ &= (u + pv) - (u_0 + P_0 v_0) - T_0 (s - s_0) = (u - u_0) + (Pv - P_0 v_0) - T_0 (s - s_0) \end{aligned}$$

Subtracting e from e_f gives

$$e_f = e + v(P - P_0)$$

Thus, although the specific exergy e cannot be negative, e_f can take on negative values when $P < P_0$, as shown in the plots below.

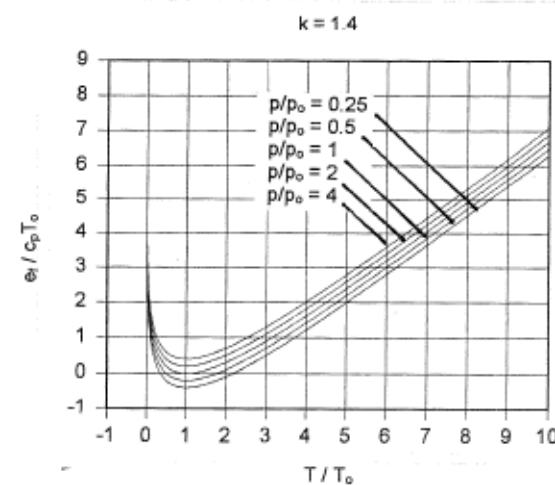
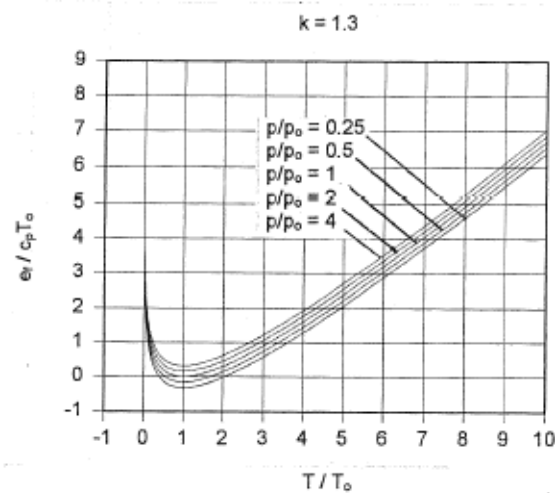
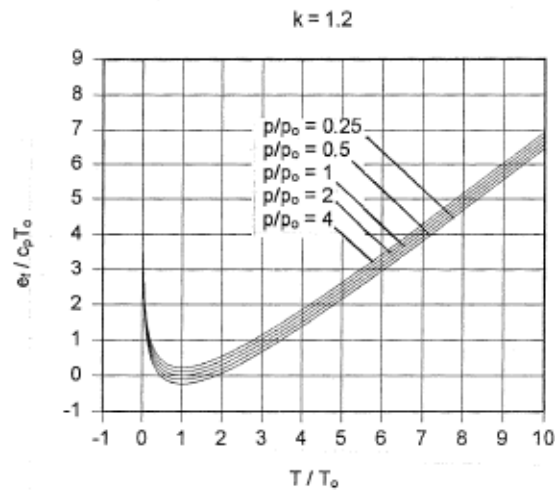
From the developments of Problems 7.7 and 7.8, the specific flow exergy can be viewed as the sum of exergy transfers accompanying mass flow and flow work.

The value of e_f can be negative, then, when these contributions have opposite signs: the flow work contribution is opposite to the direction of flow.

Continued on next slide

Problem 7-9 continued

PLOTS:

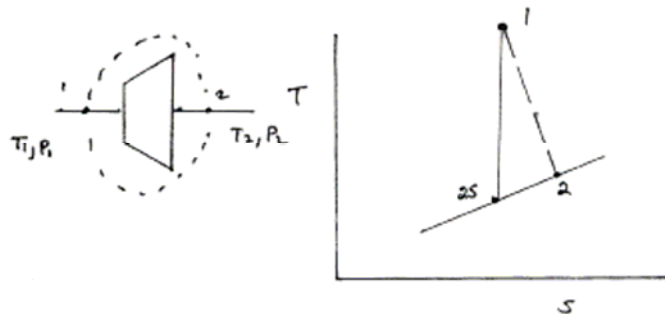


PROBLEM 7.10

KNOWN: An ideal gas with constant specific heat ratio k enters a turbine operating at steady state at T_1, P_1 and expands adiabatically to T_2, P_2 .

FIND: Determine when the value of the exergetic turbine efficiency exceeds the value of the isentropic efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown with the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and the effects of motion and gravity can be ignored. (3) The gas is modeled as an ideal gas with constant specific heat ratio k . (4) the exergy reference environment is at T_0 .

ANALYSIS: The requirement is $\epsilon > \eta_t$. That is, with Eqs. 6.46 and 7.24

$$\frac{h_1 - h_2}{h_1 - h_2 - T_0(s_1 - s_2)} > \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow (h_1 - h_{2s}) > (h_1 - h_2) - T_0(s_1 - s_2)$$

$$\Rightarrow (h_2 - h_{2s}) > T_0(s_2 - s_1)$$

$\uparrow$
 $= s_{2s}$

Thus, $\epsilon > \eta_t$ when

$$h_2 - h_{2s} > T_0(s_2 - s_{2s})$$

Since k is constant

$$c_p[T_2 - T_{2s}] > T_0 \left[c_p \ln \frac{T_2}{T_{2s}} - R \ln \frac{P_2}{P_{2s}} \right]$$

$$\Rightarrow (T_2 - T_{2s}) > T_0 \ln \frac{T_2}{T_{2s}}$$

or

$$\frac{T_2 - T_{2s}}{\ln(T_2/T_{2s})} > T_0 \quad (1)$$

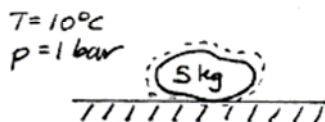
The left side is recognized as the logarithmic average of the temperature difference $T_2 - T_{2s}$.

PROBLEM 7.11

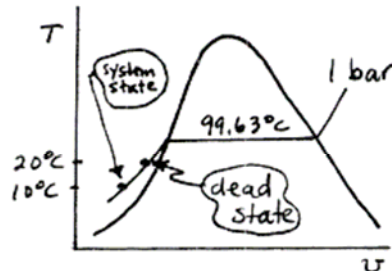
KNOWN: A system consists of 5 kg of water at 10°C, 1 bar. The system is at rest and zero elevation relative to the exergy reference environment.

FIND: Determine the exergy.

SCHEMATIC & GIVEN DATA:



$T_0 = 20^\circ\text{C}$
 $p_0 = 1 \text{ bar}$



ENGR. MODEL: The closed system shown in the figure is at rest and zero elevation relative to the environment for which $T_0 = 20^\circ\text{C}$, $p_0 = 1 \text{ bar}$.

ANALYSIS: Equation 7.2 gives

$$\Xi = m \left[(u - u_0) + p_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gZ \right]$$

From Table A-2, using the approximations of Eqs. 3.11, 3.12: $v \approx v_f(T)$ and $u \approx u_f(T)$ with the approximation of Eq. 6.5: $s \approx s_f(T)$

$$v_0 = 1.0018 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$v = 1.0004 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$u_0 = 83.95 \text{ kJ/kg}$$

$$u = 42.00 \text{ kJ/kg}$$

$$s_0 = 0.2966 \text{ kJ/kg}\cdot\text{K}$$

$$s = 0.1510 \text{ kJ/kg}\cdot\text{K}$$

Thus

$$\begin{aligned} \Xi &= (5 \text{ kg}) \left[(42.00 - 83.95) \frac{\text{kJ}}{\text{kg}} + (1 \text{ bar}) \left(\frac{1.0004 - 1.0018}{10^3} \right) \frac{\text{m}^3}{\text{kg}} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \right. \\ &\quad \left. - (293 \text{ K})(0.1510 - 0.2966) \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \right] \\ &= (5 \text{ kg}) [-41.95 - 1.4 \times 10^{-4} + 42.66] \frac{\text{kJ}}{\text{kg}} \\ &= 3.55 \text{ kJ} \end{aligned}$$

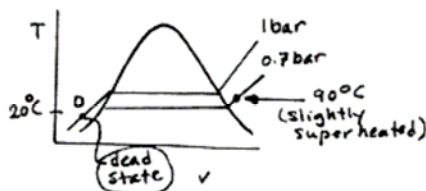
PROBLEM 7.12

KNOWN: A state is specified for each of three substances.

FIND: Determine the exergy.

ENGR. MODEL: 1. Ignore the effects of motion and gravity. 2. $T_0 = 20^\circ\text{C}$, $P_0 = 1\text{ bar}$.

ANALYSIS: (a) Water at 0.7 bar, 90°C.



Tables A-2,3: $T_0 = 20^\circ\text{C}$, $P_0 = 1\text{ bar}$

$$u_0 \approx u_f(T_0) = 83.95 \text{ kJ/kg}$$

$$v_0 \approx v_f(T_0) = \frac{1.0018 \text{ m}^3/\text{kg}}{10^3}$$

$$s_0 \approx s_f(T_0) = 0.2966 \text{ kJ/kg}\cdot\text{K}$$

$T = 90^\circ\text{C}$, $p = 0.7\text{ bar}$

$$u \approx u_g(p) = 2494.5$$

$$v \approx v_g(p) = 2.365$$

$$s \approx s_g(p) = 7.4797$$

Then, with Eq. 7.2, dropping $V^2/2$, gz terms

$$e = (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$$

$$= (2494.5 - 83.95) \frac{\text{kJ}}{\text{kg}} + (10^5 \frac{\text{N}}{\text{m}^2}) \left(2.365 - \frac{1.0018}{10^3} \right) \frac{\text{m}^3}{\text{kg}} \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| - (293 \text{ K}) (7.4797 - 0.2966) \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$= [(2410.55) + (236.4) - (2104.65)] \text{ kJ/kg}$$

$$= 542.27 \frac{\text{kJ}}{\text{kg}} \Rightarrow E = (1 \text{ kg}) (542.27 \frac{\text{kJ}}{\text{kg}}) = 542.27 \text{ kJ} \leftarrow E$$

(b) R134a at 0.7 bar, 90°C

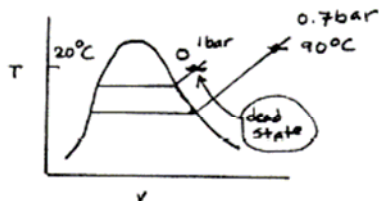


Table A-12:

$$u_0 = 246.67 \text{ kJ/kg}$$

$$v_0 = 0.23349 \text{ m}^3/\text{kg}$$

$$s_0 = 1.0829 \text{ kJ/kg}\cdot\text{K}$$

$$u = 305.5 \text{ kJ/kg}$$

$$v = 0.441 \text{ m}^3/\text{kg}$$

$$s = 1.3118 \text{ kJ/kg}\cdot\text{K}$$

$$e = (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$$

$$= (305.5 - 246.67) + 10^5 (0.441 - 0.23349) \left| \frac{1}{10^3} \right| - 293 (1.3118 - 1.0829)$$

$$= (58.83) + (20.75) - (67.07) = 12.51 \text{ kJ/kg} \Rightarrow E = 12.51 \text{ kJ} \leftarrow E$$

(c) Air as an ideal gas with c_p constant.

Constant $c_p \Rightarrow$ constant c_v . Evaluating c_v at T_{ave} from Table A-20: $c_v = 0.72 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

$$e = (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$$

$$c_p = 1.007 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$= c_v [T - T_0] + P_0 \left[\frac{RT}{P} - \frac{RT_0}{P_0} \right] - T_0 \left[c_p \ln \frac{T}{T_0} - R \ln \frac{P}{P_0} \right]$$

$$= c_v [T - T_0] + R \left[T \left(\frac{P_0}{P} \right) - T_0 \right] - T_0 \left[c_p \ln \frac{T}{T_0} - R \ln \frac{P}{P_0} \right]$$

$$= \left(0.72 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \right) [70 \text{ K}] + \frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \left[363 \text{ K} \left(\frac{1}{0.7} \right) - 293 \text{ K} \right] - 293 \text{ K} \left[1.007 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \ln \frac{363}{293} - \frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \ln \left(\frac{0.7}{1} \right) \right]$$

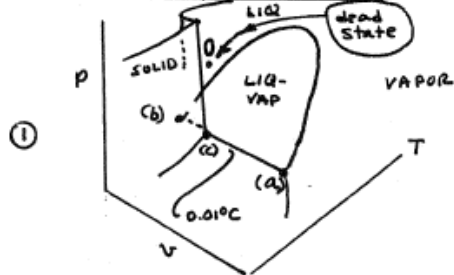
$$= [(50.4) + (64.74) - (93.2)] \frac{\text{kJ}}{\text{kg}} = 21.94 \frac{\text{kJ}}{\text{kg}} \Rightarrow E = 21.94 \text{ kJ} \leftarrow E$$

PROBLEM 7.13

KNOWN: Water at each of three alternative states.

FIND: Determine the specific exergy.

SCHEMATIC & GIVEN DATA:



dead state: $T_0 = 20^\circ\text{C}$, $P_0 = 1 \text{ bar}$

$$u_0 \sim u_f(20^\circ\text{C}) = 83.95 \text{ kJ/kg}$$

$$v_0 \sim v_f(20^\circ\text{C}) = (1.0018/10^3) \text{ m}^3/\text{kg}$$

$$s_0 \sim s_f(20^\circ\text{C}) = 0.2966 \text{ kJ/kg}\cdot\text{K}$$

States (a), (b), (c) fall on the triple line — see Figure 3.1.

ENGR. MODEL: 1. At the dead state $v \sim v_f(T_0)$, $u \sim u_f(T_0)$, $s \sim s_f(T_0)$. $T_0 = 20^\circ\text{C}$, $P_0 = 1 \text{ bar}$.
2. Ignore the effects of motion and gravity.

(a) Saturated vapor at 0.01°C . With Eq. 7.2, dropping $V^2/2$, gz terms, we get

$$e = (u - u_0) + P_0(v - v_0) - T_0(s - s_0), \text{ with data from Table A-2:}$$

$$\begin{aligned} e &= (2375.3 - 83.95) \frac{\text{kJ}}{\text{kg}} + (10^5 \frac{\text{N}}{\text{m}^2}) \left[206.136 - \frac{1.0018}{10^3} \right] \frac{\text{m}^3}{\text{kg}} \left| \frac{1 \text{ kJ}}{10^3 \frac{\text{N}\cdot\text{m}}{\text{kg}}} \right| - 293 [9.1562 - 0.2966] \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \\ &= [(2291.35) + 20,613.5 - (2595.86)] \frac{\text{kJ}}{\text{kg}} = 20,309 \text{ kJ/kg} \quad \leftarrow e \end{aligned}$$

(b) Saturated liquid at 0.01°C . With data from Table A-2:

$$\begin{aligned} e &= (0 - 83.95) + (10^5) \left[\frac{1.002}{10^3} - \frac{1.0018}{10^3} \right] \left| \frac{1}{10^3} \right| - 293 (0 - 0.2966) \\ &= [(-83.95) + (0) + (86.90)] \frac{\text{kJ}}{\text{kg}} = 2.95 \text{ kJ/kg} \quad \leftarrow e \end{aligned}$$

(c) Saturated solid at 0.01°C . With data from Table A-5:

$$\begin{aligned} e &= (-333.4 - 83.95) + (10^5) \left[\frac{1.0908}{10^3} - \frac{1.0018}{10^3} \right] \left| \frac{1}{10^3} \right| - 293 ((-1.221) - 0.2966) \\ &= [(-417.35) + (0.01) + (444.66)] \frac{\text{kJ}}{\text{kg}} = 27.32 \text{ kJ/kg} \quad \leftarrow e \end{aligned}$$

1. The discussion of Figure 7.4 indicates that the more distant a state of a system is from the dead state, the greater is its value for exergy. Thus, state (a) is most distant, state (c) is closer, and state (b) is closest. The calculated exergy values confirm this rule.

PROBLEM 7.14

KNOWN: Systems at specified states are defined.

FIND: Determine the specific exergy for 1 lb of each substance.

ENGR. MODEL: The closed systems are at rest and zero elevation relative to the environment for which $T_0 = 70^\circ\text{F} = 530^\circ\text{R}$, $P_0 = 14.7 \text{ lbf/in}^2$.

ANALYSIS: Equation 7.2 gives

$$E = m[(u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \cancel{\frac{V^2}{2}} + \cancel{gZ}]$$

(a) Saturated water vapor at 212°F . Using $u_0 = u_f(T_0)$, $v_0 = v_f(T_0)$, $s_0 = s_f(T_0)$, Table A-2E gives $u_0 = 38.09 \text{ Btu/lb}$, $v_0 = 0.01605 \text{ ft}^3/\text{lb}$, $s_0 = 0.07463 \text{ Btu/lb}\cdot^\circ\text{R}$. Table A-2E also gives $u = 1077.6 \text{ Btu/lb}$, $v = 26.80 \text{ ft}^3/\text{lb}$, $s = 1.7567 \text{ Btu/lb}\cdot^\circ\text{R}$.

$$\begin{aligned} \text{Thus } E &= (1 \text{ lb}) \left[(1077.6 - 38.09) \frac{\text{Btu}}{\text{lb}} + (14.7)(144) \frac{\text{lbf}}{\text{ft}^2} (26.80 - 0.01605) \frac{\text{ft}^3}{\text{lb}} \left| \frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}} \right| \right. \\ &\quad \left. - (530^\circ\text{R})(1.7567 - 0.07463) \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}} \right] \\ &= (1 \text{ lb}) [1039.51 + 72.87 - 891.50] \frac{\text{Btu}}{\text{lb}} = 220.88 \text{ Btu} \quad \leftarrow (a) \end{aligned}$$

(b) Saturated liquid water at 40°F . The values of u_0 , v_0 , s_0 are the same as in part (a). From Table A-2E, $u = 8.02 \text{ Btu/lb}$, $v = 0.01602 \text{ ft}^3/\text{lb}$, $s = 0.01617 \text{ Btu/lb}\cdot^\circ\text{R}$. Thus

$$\begin{aligned} E &= (1) [(8.02 - 38.09) + (14.7)(144)(0.01602 - 0.01605) \left| \frac{1}{778} \right| - (530)(0.01617 - 0.07463)] \\ \textcircled{1} \quad &= (1 \text{ lb}) [-30.07 - 8.16 \times 10^{-5} + 30.98] \frac{\text{Btu}}{\text{lb}} = 0.914 \text{ Btu} \quad \leftarrow (b) \end{aligned}$$

(c) Ammonia at -40°F , 6 lbf/in^2 .

at 70°F , 14.7 lbf/in^2
 $v_0 = 22.55 \text{ ft}^3/\text{lb}$
 $u_0 = 592.00 \text{ Btu/lb}$
 $s_0 = 1.5020 \text{ Btu/lb}\cdot^\circ\text{R}$

From Table A-15
 at -40°F , 6 lbf/in^2
 $v = 43.533 \text{ ft}^3/\text{lb}$
 $u = 551.03 \text{ Btu/lb}$
 $s = 1.4915 \text{ Btu/lb}\cdot^\circ\text{R}$

$$\begin{aligned} \textcircled{2} \quad E &= (1) [(551.03 - 592.00) + (14.7)(144)(43.533 - 22.55) \left| \frac{1}{778} \right| - (530)(1.4915 - 1.5020)] \\ &= (1 \text{ lb}) [-40.97 + 57.09 + 5.57] \frac{\text{Btu}}{\text{lb}} = 21.69 \text{ Btu} \quad \leftarrow (c) \end{aligned}$$

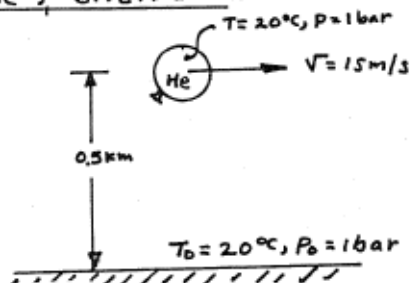
1. Even though $T < T_0$, the value of exergy is positive, as expected. See discussion of Figure 7.4.
2. Even though $T < T_0$ and $P < P_0$, the value of exergy is positive, as expected. See discussion of Figure 7.4.

PROBLEM 7.15

KNOWN: A balloon filled with helium is at a specified state.

FIND: Determine the specific exergy.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The closed system is the helium within the balloon. (2) Helium is modeled as an ideal gas. (3) $g = 9.807 \text{ m/s}^2$.

ANALYSIS: The specific exergy is given by

$$e = u - u_0 + P_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

For a monatomic gas such as Helium, the specific heats c_p and c_v are constant*, so with ideal gas relations

$$e = \underline{c_v[T - T_0] + P_0\left[\frac{RT}{P} - \frac{RT_0}{P_0}\right] - T_0\left[c_p \ln \frac{T}{T_0} - R \ln \frac{P}{P_0}\right]} + \frac{V^2}{2} + gz$$

Since $T = T_0$ and $P = P_0$, the underlined term vanishes, leaving

$$e = \frac{V^2}{2} + gz$$

$$= \left[\frac{(15 \text{ m/s})^2}{2} + (9.807 \text{ m/s}^2)(500 \text{ m}) \right] \left[\frac{1 \text{ N}}{1 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2}} \right] \left[\frac{\text{kJ}}{10^3 \text{ N} \cdot \text{m}} \right]$$

$$= 5.016 \frac{\text{kJ}}{\text{kg}}$$

e

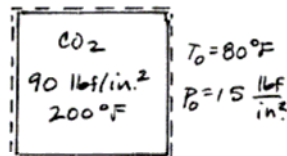
* See Tables A-21 or Fig. 3.13.

PROBLEM 7.16

KNOWN: A vessel contains carbon dioxide (CO_2).

FIND: (a) Determine specific exergy e at $p = 90 \text{ lbf/in.}^2$, $T = 200^\circ\text{F}$. (b) Plot e versus pressure for $T = 80^\circ\text{F}$. (c) Plot e versus T for $p = 15 \text{ lbf/in.}^2$.

ENGR. MODEL: (1) The CO_2 is a closed system at rest and zero elevation relative to the reference environment at $T_0 = 80^\circ\text{F}$, $p_0 = 15 \text{ lbf/in.}^2$. (2) The CO_2 is modeled as an ideal gas.



ANALYSIS: Equation 7.2 reduces to give

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) \quad (1)$$

(a) When using gas table data, $u - u_0 = \frac{\bar{u}(T) - \bar{u}(T_0)}{M}$, $s - s_0 = \frac{\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - R \ln p/p_0}{M}$

Thus, with ideal gas relations, (1) becomes

$$e = \frac{1}{M} \left\{ [\bar{u}(T) - \bar{u}(T_0)] + p_0 \left[\frac{RT}{p} - \frac{RT_0}{p_0} \right] - T_0 \left[\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - R \ln \frac{p}{p_0} \right] \right\}$$

$$= \frac{1}{M} \left\{ [\bar{u}(T) - \bar{u}(T_0)] + RT \left[\frac{p_0}{p} - \frac{T_0}{T} \right] - T_0 \left[\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - R \ln \frac{p}{p_0} \right] \right\}$$

Using data from Table A-23E

$$\bar{u}(T) - \bar{u}(T_0) = 3854.6 - 2984.4 = 870.2 \text{ Btu/lbmol}$$

$$RT \left[\frac{p_0}{p} - \frac{T_0}{T} \right] = (1.986 \frac{\text{Btu}}{\text{lbmol} \cdot ^\circ\text{R}})(660^\circ\text{R}) \left[\frac{15}{90} - \frac{540}{660} \right] = -853.98 \text{ Btu/lbmol}$$

$$T_0 [\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - R \ln p/p_0] = (540^\circ\text{R}) [52.934 - 51.082 - (1.986) \ln \frac{90}{15}] \frac{\text{Btu}}{\text{lbmol} \cdot ^\circ\text{R}}$$

$$= -921.47 \text{ Btu/lbmol}$$

Finally

$$e = \left(\frac{1}{44.01 \frac{\text{lb}}{\text{lbmol}}} \right) \{ (870.2 - 853.98 + 921.47) \} \frac{\text{Btu}}{\text{lbmol}} = 21.31 \frac{\text{Btu}}{\text{lb}} \leftarrow e$$

(b)(c) The following IT code is used to generate data for the required plots. The evaluations are based on Eq.(1) above and internal functions in IT for u , v , and s of CO_2 as an ideal gas.

IT Code

$T = 200 \text{ // } ^\circ\text{F}$

$p = 90 \text{ // lbf/in.}^2$

$T_0 = 80 \text{ // } ^\circ\text{F}$

$p_0 = 15 \text{ // lbf/in.}^2$

$e = (u - u_0) + (p_0 * (144 / 778)) * (v - v_0) - (T_0 + 460) * (s - s_0)$

$u = u_T(\text{"CO2"}, T)$

$u_0 = u_T(\text{"CO2"}, T_0)$

$v = v_TP(\text{"CO2"}, T, p)$

$v_0 = v_TP(\text{"CO2"}, T_0, p_0)$

$s = s_TP(\text{"CO2"}, T, p)$

$s_0 = s_TP(\text{"CO2"}, T_0, p_0)$

IT Result for $p = 90 \text{ lbf/in.}^2$, $T = 200^\circ\text{F}$

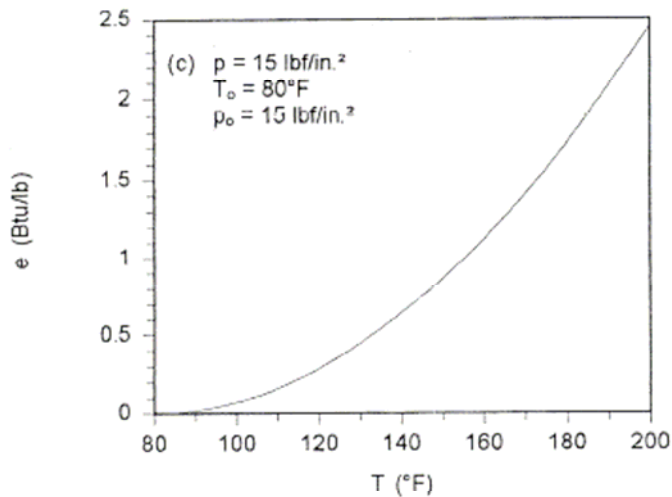
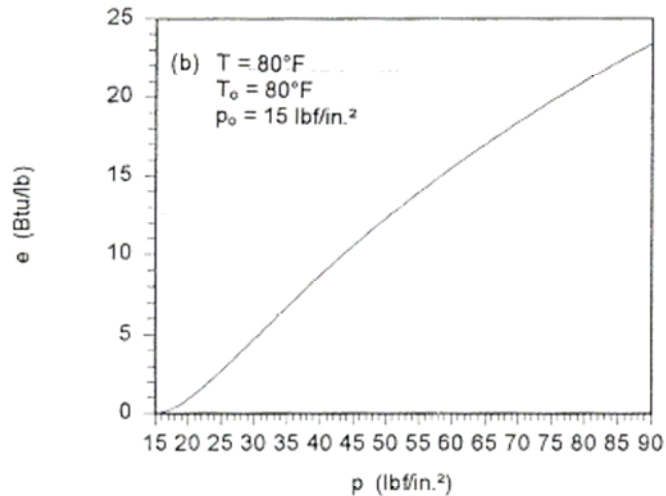
$e = 21.32 \text{ Btu/lb}$

①

Continued on next slide

Problem 7-16 continued

PLOTS:



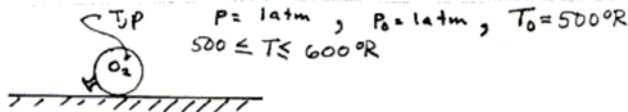
1. The IT result compares very favorably with the ideal gas table result in part (a).

PROBLEM 7.17

KNOWN: Oxygen (O_2) at T_1 latm fills a balloon at rest on the surface of the Earth where the ambient temperature and pressure are known.

END: Plot specific exergy versus T .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The O_2 is at rest and zero elevation relative to the environment for which $T_0 = 500^\circ\text{R}$, $P_0 = 1 \text{ atm}$. (2) O_2 is modeled as an ideal gas with constant C_p .

ANALYSIS: Eq. 7.2 reduces to

$$e = u - u_0 + P_0(V - V_0) - T_0(s - s_0)$$

With ideal gas relations, and noting that $P = P_0$

$$\begin{aligned}
 e &= u(T) - u(T_0) + P_0 \left(\frac{RT}{P} - \frac{RT_0}{P_0} \right) - T_0 \left(s^\circ(T) - s^\circ(T_0) - R \ln \frac{P}{P_0} \right) \\
 &= u(T) - u(T_0) + R(T - T_0) - T_0 (s^\circ(T) - s^\circ(T_0)) \quad (1)
 \end{aligned}$$

Since C_p is constant: $C_p = 0.22 \text{ Btu/lb} \cdot ^\circ\text{R}$, Eq. (1) can be expressed as

$$e = C_v [T - T_0] + R [T - T_0] - T_0 C_p \ln \frac{T}{T_0}$$

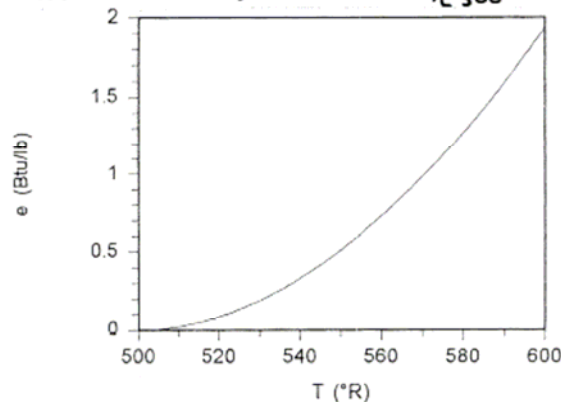
$$= \underbrace{(C_v + R)}_{C_p \text{ (Eq. 3.44)}} [T - T_0] - T_0 C_p \ln \frac{T}{T_0}$$

or

$$e = T_0 C_p \left[\frac{T}{T_0} - 1 - \ln \frac{T}{T_0} \right]$$

$$= \underbrace{\left(0.22 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (500^\circ\text{R})}_{110 \text{ Btu/lb}} \left[\frac{T}{T_0} - 1 - \ln \frac{T}{T_0} \right] = \left(110 \frac{\text{Btu}}{\text{lb}} \right) \left[\frac{T}{T_0} - 1 - \ln \frac{T}{T_0} \right] \quad (2)$$

SAMPLE CALCULATION: $T = 600^\circ\text{R}$, $e = (110 \text{ Btu/lb}) \left[\frac{600}{500} - 1 - \ln \frac{600}{500} \right] = 1.945 \text{ Btu/lb}$



1. Alternatively, $(u - u_0)$ and $(s - s_0)$ can be evaluated using data from Table A-23E.

PROBLEM 7.18

KNOWN: A vessel contains a known amount of air at pressure p and 200°F .

FIND: Plot the specific exergy versus p ranging from 0.5 to 2 atm.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: (1) The air is a closed system, at rest and zero elevation relative to the reference environment which is at $T_0 = 60^\circ\text{F}$, $p_0 = 1 \text{ atm}$. (2) The air is modeled as an ideal gas.



$T = 200^\circ\text{F}$
 $0.5 \leq p \leq 2 \text{ atm}$
 $T_0 = 60^\circ\text{F}$
 $p_0 = 1 \text{ atm}$

ANALYSIS: Equation 7.2 reduces to $e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0)$ (1)

SAMPLE CALCULATION: This calculation will be done using gas table data. That is, $u - u_0 = u(T) - u(T_0)$, $s - s_0 = s^\circ(T) - s^\circ(T_0) - R \ln p/p_0$. With the ideal gas equation of state

$$e = u(T) - u(T_0) + p_0 \left(\frac{RT}{p} - \frac{RT_0}{p_0} \right) - T_0 \left[s^\circ(T) - s^\circ(T_0) - R \ln p/p_0 \right]$$

Using data from Table A-22E, for the case $T = 200^\circ\text{F}$, $p = 0.5 \text{ atm}$

$$u(T) - u(T_0) = 112.67 - 88.62 = 24.05 \text{ Btu/lb}$$

$$p_0 R \left(\frac{T}{p} - \frac{T_0}{p_0} \right) = (1 \text{ atm}) \left(\frac{1.986 \text{ Btu}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right) \left(\frac{660^\circ\text{R}}{0.5 \text{ atm}} - \frac{520^\circ\text{R}}{1 \text{ atm}} \right) = 54.843 \text{ Btu/lb}$$

$$T_0 [s^\circ(T) - s^\circ(T_0) - R \ln p/p_0] = (520^\circ\text{R}) \left[(0.64902 - 0.59172) - \left(\frac{1.986}{28.97} \right) \ln \left(\frac{0.5}{1} \right) \right] \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

$$= 54.505 \text{ Btu/lb}$$

Finally

$$e = 24.05 + 54.843 - 54.505 = 24.388 \text{ Btu/lb}$$

Using IT to generate the data required, the evaluations are based on Eq. (1) above and internal IT functions for u , v , and s , as follows:

IT Code

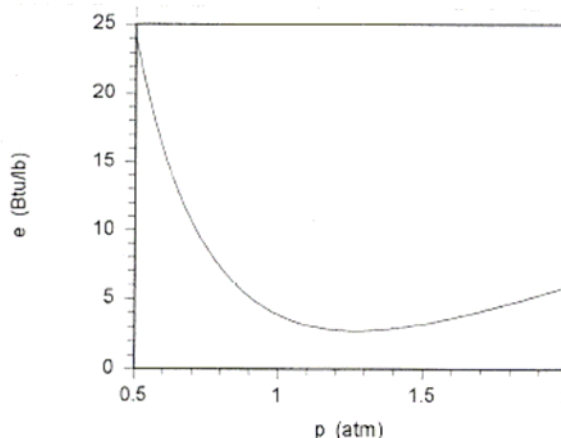
$T = 200 \text{ // } ^\circ\text{F}$
 $p = 0.5 \text{ // atm}$
 $T_0 = 60 \text{ // } ^\circ\text{F}$
 $p_0 = 1 \text{ // atm}$

$e = (u - u_0) + p_0 * (v - v_0) * (14.696 * 144 / 778) - (T_0 + 460) * (s - s_0)$
 $u = u_T(\text{"Air"}, T)$
 $u_0 = u_T(\text{"Air"}, T_0)$
 $v = v_TP(\text{"Air"}, T, p)$
 $v_0 = v_TP(\text{"Air"}, T_0, p_0)$
 $s = s_TP(\text{"Air"}, T, p)$
 $s_0 = s_TP(\text{"Air"}, T_0, p_0)$

IT Result for $p = 0.5 \text{ atm}$, $T = 200^\circ\text{F}$

① $e = 24.37 \text{ Btu/lb}$

1. The IT result is in close agreement with the sample calculation based on Table A-22E, as expected.



PROBLEM 7.19

KNOWN: A state is specified for each of three substances.

FIND: Determine the specific exergy

ENGR. MODEL: 1. Ignore the effects of motion and gravity. 2. $T_0 = 0^\circ\text{C}$, $P_0 = 1\text{ bar}$.

ANALYSIS: (a) Ammonia at 0.6 bar, -10°C .

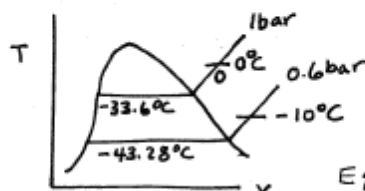


Table A-15

$u_0 = 1341\text{ kJ/kg}$	$u = 1327.37$
$v_0 = 1.3136\text{ m}^3/\text{kg}$	$v = 2.1188$
$s_0 = 6.1281\text{ kJ/kg}\cdot\text{K}$	$s = 6.3077$

Equation 7.2 reduces with assumption 1 to give

$$\begin{aligned}
 e &= (u - u_0) + P_0(v - v_0) - T_0(s - s_0) \\
 &= (1327.37 - 1341)\frac{\text{kJ}}{\text{kg}} + (10^5\frac{\text{N}}{\text{m}^2})\left[2.1188 - 1.3136\right]\frac{\text{m}^3}{\text{kg}}\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| - 273\text{K}[6.3077 - 6.1281]\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \\
 &= [(-13.63) + (80.52) - (49.03)]\frac{\text{kJ}}{\text{kg}} = 17.86\text{ kJ/kg} \quad \leftarrow (a)
 \end{aligned}$$

(b) R22 at 0.6 bar, -10°C

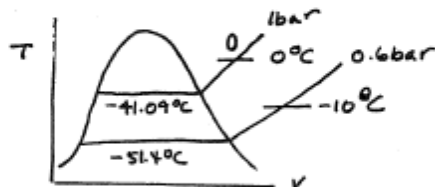


Table A-9

$u_0 = 232.36\text{ kJ/kg}$	$u = 227.65$
$v_0 = 0.25747\text{ m}^3/\text{kg}$	$v = 0.41608$
$s_0 = 1.1035\text{ kJ/kg}\cdot\text{K}$	$s = 1.1314$

$$\begin{aligned}
 e &= (u - u_0) + P_0(v - v_0) - T_0(s - s_0) \\
 &= (227.65 - 232.36) + (10^5)[0.41608 - 0.25747]\left|\frac{1}{10^3}\right| - 273(1.1314 - 1.1035) \\
 &= (-4.71) + (15.86) - (7.62) = 3.53\text{ kJ/kg} \quad \leftarrow (b)
 \end{aligned}$$

(c) R134a at 0.6 bar, -10°C

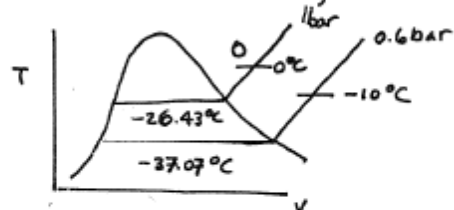


Table A-12

$u_0 = 231.41\text{ kJ/kg}$	$u = 224.97$
$v_0 = 0.21587\text{ m}^3/\text{kg}$	$v = 0.34992$
$s_0 = 1.0227\text{ kJ/kg}\cdot\text{K}$	$s = 1.0371$

$$\begin{aligned}
 e &= (u - u_0) + P_0(v - v_0) - T_0(s - s_0) \\
 &= [224.97 - 231.41] + 10^5[0.34992 - 0.21587]\left|\frac{1}{10^3}\right| - 273(1.0371 - 1.0227) \\
 &= (-6.44) + (13.41) - (3.93) = 3.04\text{ kJ/kg} \quad \leftarrow (c)
 \end{aligned}$$

PROBLEM 7.22

Refrigerant 134a vapor initially at 1 bar and 20°C fills a rigid vessel. The vapor is cooled until the temperature becomes -32°C. There is no work during the process. For the refrigerant, determine the heat transfer per unit mass and the change in specific exergy, each in kJ/kg. Comment. Let $T_0 = 20^\circ\text{C}$, $p_0 = 0.1\text{ MPa}$ and ignore the effects of motion and gravity.

ENGR. MODEL:

1. The R134a is the closed system.
2. For the system, $W=0$ and the effects of motion and gravity can be ignored.
3. The environment is at $T_0 = 20^\circ\text{C}$, $p_0 = 1\text{ bar}$

ANALYSIS: Property data: Note for the process, $v_2 = v_1$.

Table A-12: At 0.1 MPa, 20°C

$$\begin{aligned} v_1 &= 0.23349 \text{ m}^3/\text{kg} \\ u_1 &= 246.67 \text{ kJ/kg} \\ s_1 &= 1.0829 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

Table A-10:

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.23349 - (0.7172/10^3)}{0.2451 - (0.7172/10^3)} = 0.9525$$

$$u_2 = u_f + x_2(u_g - u_f) = 94.7 + 0.9525(209.01 - 94.7) = 199.53 \text{ kJ/kg}$$

$$s_2 = s_f + x_2(s_g - s_f) = 0.0401 + 0.9525(0.9456 - 0.0401) = 0.9026 \text{ kJ/kg}\cdot\text{K}$$

Reducing an energy balance, $\dot{Q} + \dot{m}e + \dot{W} = 0$

$$\Rightarrow \frac{\dot{Q}}{\dot{m}} = u_2 - u_1 = 199.53 - 246.67 = -47.14 \text{ kJ/kg} \quad \leftarrow \frac{\dot{Q}}{\dot{m}}$$

With Eq. 7.2,

$$\Delta e = (u_2 - u_1) + p_0(v_2 - v_1) - T_0(s_2 - s_1)$$

$$= -47.14 \frac{\text{kJ}}{\text{kg}} - 293 \text{ K} (0.9026 - 1.0829) \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

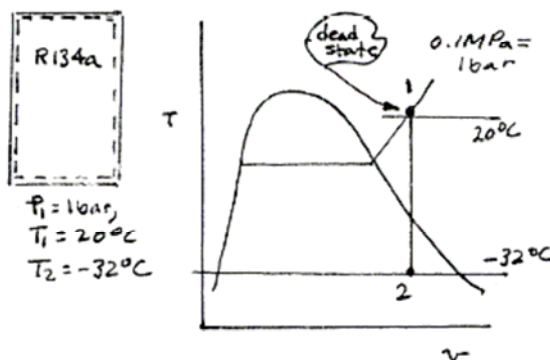
①

$$= +5.69 \frac{\text{kJ}}{\text{kg}}$$

$\leftarrow \Delta e$

1. In keeping with the discussion of Fig. 7.4, the exergy increases because the system is brought from the dead state to a state where p and T differ from p_0 and T_0 , respectively.

SCHEMATIC & GIVEN DATA:

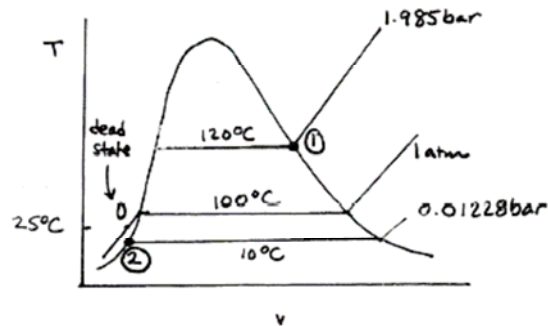
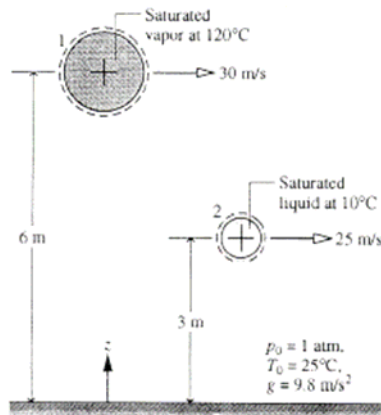


PROBLEM 7.23

KNOWN: Two kg of water undergo a process between specified states.

FIND: Determine the exergy at the initial and final states and the change in exergy.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The water is a closed system at equilibrium states initially and finally. 2. The velocities and elevations are measured relative to the environment. 3. $T_0 = 25^\circ\text{C}$, $P_0 = 1 \text{ atm}$, $v_0 \sim v_f(T_0)$, $u_0 \sim u_f(T_0)$, $s_0 \sim s_f(T_0)$.

ANALYSIS: The exergy at the initial and final states can be calculated using

Eq. 7.2:

$$\textcircled{1} \quad IE = m \left[(u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \right]$$

(a) For saturated vapor at 120°C, Table A-2 gives $v = 0.8919 \text{ m}^3/\text{kg}$, $u = 2529.3 \text{ kJ/kg}$, $s = 7.1296 \text{ kJ/kg} \cdot \text{K}$. Table A-2 gives $v_0 = 1.0029 \times 10^{-3} \text{ m}^3/\text{kg}$, $u_0 = 104.88 \text{ kJ/kg}$, $s_0 = 0.3674 \text{ kJ/kg} \cdot \text{K}$. Thus,

$$IE_1 = (2 \text{ kg}) \left\{ (2529.3 - 104.88) \frac{\text{kJ}}{\text{kg}} + (1.013 \times 10^5 \frac{\text{N}}{\text{m}^2}) \left[0.8919 - \frac{1.0029}{10^3} \frac{\text{m}^3}{\text{kg}} \right] \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \right. \\ \left. - (298 \text{ K}) [7.1296 - 0.3674] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} + \left[\frac{(30 \text{ m/s})^2}{2} + (9.8 \text{ m/s}^2)(6 \text{ m}) \right] \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \right\} \\ = (2) [2424.42 + 90.25 - 205.14 + 0.45 + 0.06] \text{ kJ} = 1000.1 \text{ kJ} \quad \leftarrow IE_1$$

(b) For saturated liquid at 10°C, $v = 1.0004 \times 10^{-3} \text{ m}^3/\text{kg}$, $u = 42 \text{ kJ/kg}$, $s = 0.151 \text{ kJ/kg} \cdot \text{K}$. We get $IE_2 = 3.9 \text{ kJ}$. $\leftarrow IE_2$

(c) The change in exergy is $\Delta IE = IE_2 - IE_1 = 3.9 - 1000.1 = -996.2 \text{ kJ} \quad \leftarrow \Delta IE$

- The kinetic and potential energies measured relative to the environment contribute their full magnitudes to the values of exergy, for in principle each could be completely converted to work were the system brought to rest at zero elevation relative to the environment.
- Exergy is a measure of the departure of the state of the system from that of the environment. At all states, $IE \geq 0$. This applies when $T > T_0$, $P > P_0$, as in part (a), and when $T < T_0$, $P < P_0$, as in part (b).
- Alternatively, Eq. 7.3 can be used. This requires dead state property values only for T_0 , P_0 . In parts (a), (b) u_0 , v_0 and s_0 are also required; so more computation is needed with that approach.

PROBLEM 7.24

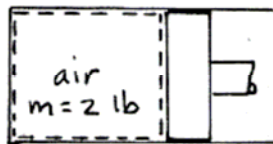
KNOWN: Air undergoes two specified processes in series.

FIND: (a) Represent each process on a p-v diagram and indicate the dead state. (b) Determine the change in exergy for each process.

SCHEMATIC & GIVEN DATA:

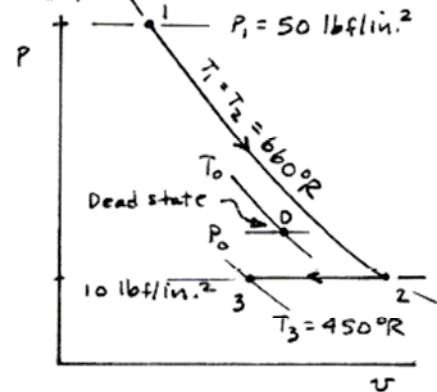
Process 1-2: isothermal to $P_2 = 10 \text{ lbf/in.}^2$

Process 2-3: constant pressure to $T_3 = -10^\circ\text{F} = 450^\circ\text{R}$



$$T_0 = 77^\circ\text{F} = 537^\circ\text{R}$$

$$P_0 = 14.7 \text{ lbf/in.}^2$$



ENGR. MODEL: (1) The air is the closed system. (2) The effects of motion and gravity can be ignored. (3) The air is modeled as an ideal gas. (4) For the environment, $T_0 = 77^\circ\text{F}$, $P_0 = 14.7 \text{ lbf/in.}^2$

ANALYSIS: (a) From the given information, $P_1 = 50 \text{ lbf/in.}^2$, $T_1 = 660^\circ\text{R}$. Thus

$$v_1 = \frac{RT_1}{P_1} = \frac{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}})(660^\circ\text{R})}{(50 \text{ lbf/in.}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in.}^2} \right| = 4.889 \text{ ft}^3/\text{lb}$$

Similarly, $P_2 = 10 \text{ lbf/in.}^2$, $T_2 = T_1 = 660^\circ\text{R} \Rightarrow v_2 = 24.44 \text{ ft}^3/\text{lb}$ and $P_3 = P_2 = 10 \text{ lbf/in.}^2$ and $T_3 = 450^\circ\text{R} \Rightarrow v_3 = 16.67 \text{ ft}^3/\text{lb}$.

(b) Using Eq. 7.3 and ideal gas relations

$$E_2 - E_1 = m [u(T_2) - u(T_1) + P_0(v_2 - v_1) - T_0(S^\circ(T_2) - S^\circ(T_1)) - R \ln(P_2/P_1)]$$

$$E_3 - E_2 = m [u(T_3) - u(T_2) + P_0(v_3 - v_2) - T_0(S^\circ(T_3) - S^\circ(T_2)) - R \ln(P_3/P_2)]$$

With data from Table A-22E as needed

$$E_2 - E_1 = (2 \text{ lb}) \left[(14.7 \frac{\text{lbf}}{\text{in.}^2})(24.44 - 4.889) \frac{\text{ft}^3}{\text{lb}} \left| \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| - (537^\circ\text{R}) \left(- \frac{1.986 \text{ Btu}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \ln \frac{10}{50} \right) \right]$$

$$\textcircled{1} \quad = -12.11 \text{ Btu} \quad \leftarrow \quad E_2 - E_1$$

$$E_3 - E_2 = (2) \left[(76.645 - 112.67) + (14.7)(16.67 - 24.44) \left| \frac{144}{778} \right| - (537)(.55704 - .64902) \right]$$

$$= -15.55 \text{ Btu} \quad \leftarrow \quad E_3 - E_2$$

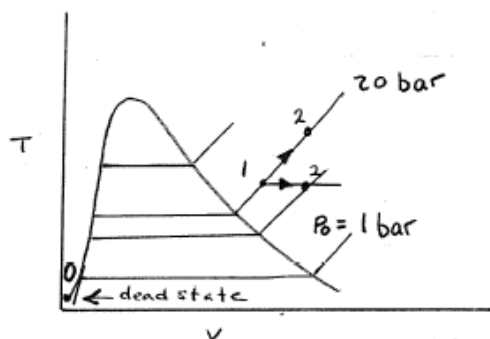
1 Although exergy is positive at states 1, 2, and 3, changes in exergy can be, and in this instance are, negative.

PROBLEM 7.2b

KNOWN: One kg of steam initially at 20 bar and 240°C undergoes two different processes.

FIND: For each process determine the change in exergy.

SCHEMATIC & GIVEN DATA:



$$T_1 = 240^\circ\text{C}$$

$$1-2: \text{ constant pressure, } v_2 = 2v_1$$

$$1-2': \text{ isothermal, } v_2 = 2v_1$$

ENGR. MODEL: (1) The system is the 1 kg of steam. (2) For the environment, $T_0 = 20^\circ\text{C}$, $P_0 = 1 \text{ bar}$. (3) Ignore the effects of motion and gravity.

ANALYSIS: Equation 7.3 reduces to give $E_2 - E_1 = m[u_2 - u_1 + P_0(v_2 - v_1) - T_0(s_2 - s_1)]$. From Table A-4, $u_1 = 2659.6 \text{ kJ/kg}$, $v_1 = 0.1085 \text{ m}^3/\text{kg}$, $s_1 = 6.4952 \text{ kJ/kg}\cdot\text{K}$. For each process $v_2 = 2v_1 = 0.217 \text{ m}^3/\text{kg}$.

(a) **Constant pressure process:** Interpolating in Table A-4 at 20 bar with $v_2 = 0.217 \text{ m}^3/\text{kg}$, $u_2 = 3423.1 \text{ kJ/kg}$, $s_2 = 7.8849 \text{ kJ/kg}\cdot\text{K}$

$$E_2 - E_1 = (1 \text{ kg}) \left[(3423.1 - 2659.6) \frac{\text{kJ}}{\text{kg}} + (100 \frac{\text{kN}}{\text{m}^2}) (0.217 - 0.1085) \frac{\text{m}^3}{\text{kg}} \left| \frac{1 \text{ kJ}}{1 \text{ kN}\cdot\text{m}} \right| - 293 \text{ K} (7.8849 - 6.4952) \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \right]$$

$$\textcircled{1} = (1 \text{ kg}) [763.5 + 10.9 - 407.2] \frac{\text{kJ}}{\text{kg}} = 367.2 \text{ kJ} \leftarrow (a)$$

(b) **Isothermal Process:** Interpolating in Table A-4 at 240°C with $v_2 = 0.217 \text{ m}^3/\text{kg}$, $u_2 = 2690.8 \text{ kJ/kg}$, $s_2 = 6.8527 \text{ kJ/kg}\cdot\text{K}$

$$E_2 - E_1 = (1) [(2690.8 - 2659.6) + (100)(0.1085) - 293(6.8527 - 6.4952)]$$

$$\textcircled{2} = (1 \text{ kg}) [31.2 + 10.9 - 104.7] \frac{\text{kJ}}{\text{kg}} = -62.6 \text{ kJ} \leftarrow (b)$$

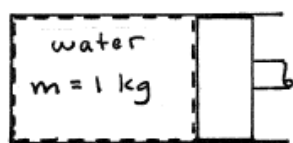
- Here, pressure is kept constant and temperature increases relative to T_0 . Exergy increases in the process.
- Here, temperature is kept constant and pressure decreases relative to P_0 . Exergy decreases in the process.

PROBLEM 7.27

KNOWN: One kilogram of water cools at constant pressure from an initial state to saturated liquid.

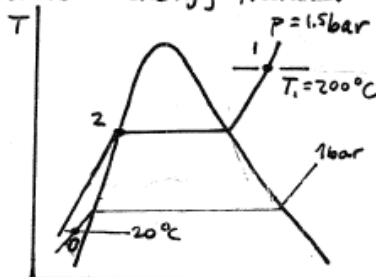
FIND: Determine the work, heat transfer, and amounts of exergy transfer accompanying heat and work.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} P_1 &= 1.5 \text{ bar} \\ T_1 &= 200^\circ\text{C} \\ P &= \text{const.} \\ x_2 &= 0 \end{aligned}$$

$$\begin{aligned} T_0 &= 20^\circ\text{C} \\ P_0 &= 1 \text{ bar} \end{aligned}$$



ENGR. MODEL: (1) The water is the closed system. (2) The process occurs with no internal irreversibilities. (3) The effects of motion and gravity can be ignored. (4) $T_0 = 20^\circ\text{C}$, $P_0 = 1 \text{ bar}$.

ANALYSIS: To determine the work, we use

$$W = \int_1^2 p dv = m p (v_2 - v_1)$$

With data from Tables A-4 and A-3, respectively, $v_1 = 1.444 \text{ m}^3/\text{kg}$ and $v_2 = 1.0528 \times 10^{-3} \text{ m}^3/\text{kg}$

$$W = (1 \text{ kg})(1.5 \text{ bar})(1.0528 \times 10^{-3} - 1.444) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = -216.4 \text{ kJ} \quad \leftarrow W$$

The heat transfer is determined by using the energy balance, which reduces to

$$Q = m(u_2 - u_1) + W$$

From Table A-4, $u_1 = 2656.2 \text{ kJ/kg}$, and From Table A-3, $u_2 = 466.94 \text{ kJ/kg}$. Thus

$$Q = (1 \text{ kg})(466.94 - 2656.2) \text{ kJ/kg} + (-216.4 \text{ kJ}) = -2405.7 \text{ kJ} \quad \leftarrow Q$$

Now, the exergy transfer accompanying work, E_w , is

$$\begin{aligned} E_w &= W - P_0 \Delta V = W - m P_0 (v_2 - v_1) \\ &= (-216.4 \text{ kJ}) - (1)(1)(1.0528 \times 10^{-3} - 1.444) \left| \frac{10^5}{10^3} \right| = -72.1 \text{ kJ} \quad \leftarrow E_w \end{aligned}$$

The exergy transfer accompanying heat transfer, E_Q , is

$$E_Q = \int_1^2 \left(1 - \frac{T_0}{T_b}\right) \delta Q = Q - T_0 \int_1^2 \frac{\delta Q}{T_b} \quad \text{assumption 2}$$

From an entropy balance: $m(s_2 - s_1) = \int_1^2 \frac{\delta Q}{T_b} + \delta \sigma$. Thus

$$E_Q = Q - m T_0 (s_2 - s_1)$$

With data from the appropriate tables

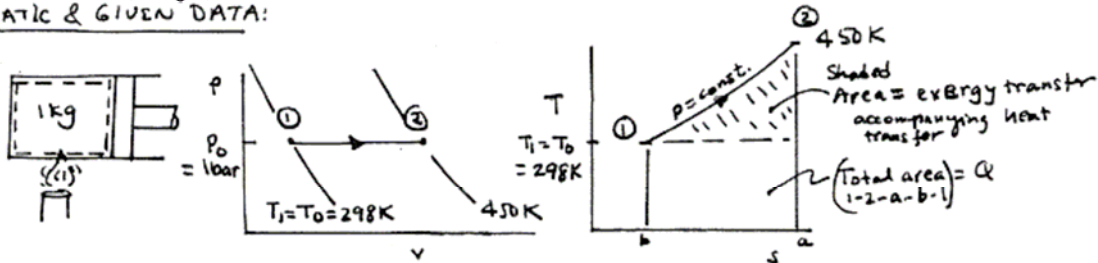
$$\begin{aligned} E_Q &= (-2405.7 \text{ kJ}) - (1 \text{ kg})(293 \text{ K})(1.4336 - 7.6433) \left| \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \right| \\ &= -586.3 \text{ kJ} \quad \leftarrow E_Q \end{aligned}$$

PROBLEM 7.28

KNOWN: 1 kg of air is heated at constant pressure with no internal irreversibilities.

FIND: Determine the work, heat transfer, and the amounts of exergy transfer accompanying work and heat transfer.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The closed system is 1 kg of air, as shown in the schematic.

2. The process occurs at constant pressure without internal irreversibilities.

3. Air is modeled as an ideal gas. 4. $T_0 = 298\text{ K}$ (25°C), $P_0 = 1\text{ bar}$. 4. The effects of motion and gravity can be ignored.

ANALYSIS: Using the ideal gas equation of state, the work is

$$W = \int_1^2 p dV = m p (v_2 - v_1) = m R (T_2 - T_1) = (1\text{ kg}) \left(\frac{8.314\text{ kJ}}{28.97\text{ kg} \cdot \text{K}} \right) (450 - 298)\text{ K} = 43.62\text{ kJ} \leftarrow$$

Using Eq. 7.6, the accompanying exergy transfer is

$$\begin{aligned} E_W &= \left[\text{Exergy transfer accompanying work} \right] = W - P_0 \Delta V = m p (v_2 - v_1) - m P_0 (v_2 - v_1) \\ &= m (p - P_0) (v_2 - v_1) = 0, \text{ since } p = P_0. \end{aligned} \leftarrow$$

Reducing an energy balance for the process: $\Delta U = Q - W \Rightarrow Q = \Delta U + W$.

Or, with data from Table A-22

$$\begin{aligned} \textcircled{1} \quad Q &= m(u_2 - u_1) + W = (1\text{ kg}) [322.62 - 212.64] \frac{\text{kJ}}{\text{kg}} + 43.62 \\ &= 153.6\text{ kJ} \end{aligned} \leftarrow$$

Using Eq. 7.5, the accompanying exergy transfer is

$$\begin{aligned} E_Q &= \left[\text{Exergy transfer accompanying heat} \right] = \int_1^2 \left[1 - \frac{T_0}{T_b} \right] \delta Q = Q - T_0 \int_1^2 \left(\frac{\delta Q}{T} \right) \\ &= Q - T_0 [S_2 - S_1] \quad \text{since the process is without internal irrevers. (assumption 2)} \\ &= Q - m T_0 [s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1] \\ &= 153.6\text{ kJ} - (1\text{ kg})(298\text{ K}) [2.11161 - 1.69528] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ &= 29.53\text{ kJ} \end{aligned} \leftarrow$$

1. Alternatively, for the constant pressure process

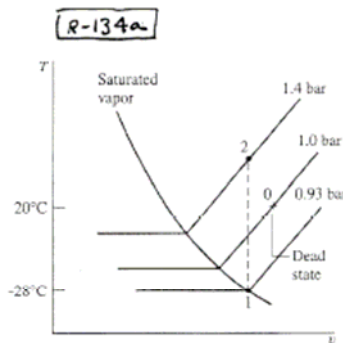
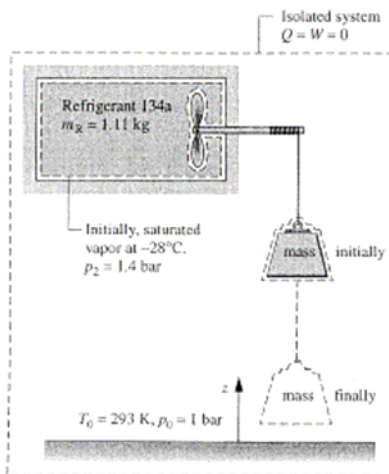
$$\begin{aligned} Q &= m(u_2 - u_1) + m p (v_2 - v_1) \\ &= m [h_2 - h_1] = 1\text{ kg} (451.8 - 298.2) \frac{\text{kJ}}{\text{kg}} = 153.6\text{ kJ} \end{aligned}$$

PROBLEM 7.29

As shown in Fig. P7.29, 1.11 kg of Refrigerant 134a is contained in a rigid, insulated vessel. The refrigerant is initially saturated vapor at -28°C . The vessel is fitted with a paddle wheel from which a mass is suspended. As the mass descends a certain distance, the refrigerant is stirred until it attains a final equilibrium state at a pressure of 1.4 bar. The only significant changes in state are experienced by the refrigerant and the suspended mass. Determine, in kJ,

- the change in exergy of the refrigerant.
- the change in exergy of the suspended mass.
- the change in exergy of an isolated system of the vessel and pulley-mass assembly.
- the destruction of exergy within the isolated system.

Let $T_0 = 293\text{ K}$ (20°C), $p_0 = 1\text{ bar}$.



ENGR. MODEL:

- As shown in the schematic, three systems are under consideration: the refrigerant, the suspended mass, and an isolated system consisting of the vessel and pulley-mass assembly. For the isolated system $Q = 0$, $W = 0$.
- The only significant changes of state are experienced by the refrigerant and the suspended mass. For the refrigerant, there is no change in kinetic or potential energy. For the suspended mass, there is no change in kinetic or internal energy. Elevation is the only intensive property of the suspended mass that changes.
- For the environment, $T_0 = 293\text{ K}$ (20°C), $p_0 = 1\text{ bar}$.

Analysis:

(a) The initial and final exergies of the refrigerant can be evaluated using Eq. 7.2. From assumption 2, it follows that for the refrigerant there are no significant effects of motion or gravity, and thus the exergy at the initial state is

$$E_1 = m_R[(u_1 - u_0) + p_0(v_1 - v_0) - T_0(s_1 - s_0)]$$

The initial and final states of the refrigerant are shown on the accompanying T - v diagram. From Table A-10, $u_1 = u_g(-28^\circ\text{C}) = 211.29\text{ kJ/kg}$, $v_1 = v_g(-28^\circ\text{C}) = 0.2052\text{ m}^3/\text{kg}$, $s_1 = s_g(-28^\circ\text{C}) = 0.9411\text{ kJ/kg} \cdot \text{K}$. From Table A-12 at 1 bar, 20°C , $u_0 = 246.67\text{ kJ/kg}$, $v_0 = 0.23349\text{ m}^3/\text{kg}$, $s_0 = 1.0829\text{ kJ/kg} \cdot \text{K}$. Then

$$\begin{aligned} E_1 &= 1.11\text{ kg} \left[(211.29 - 246.67) \frac{\text{kJ}}{\text{kg}} + \left(10^5 \frac{\text{N}}{\text{m}^2} \right) (0.2052 - 0.23349) \frac{\text{m}^3}{\text{kg}} \left| \frac{1\text{ kJ}}{10^3\text{ N} \cdot \text{m}} \right| - 293\text{ K} (0.9411 - 1.0829) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right] \\ &= 1.11\text{ kg} [(-35.38) + (-2.83) + (41.55)] \frac{\text{kJ}}{\text{kg}} = 3.7\text{ kJ} \end{aligned}$$

The final state of the refrigerant is fixed by $p_2 = 1.4\text{ bar}$ and $v_2 = v_1$. Interpolation in Table A-12 gives $u_2 = 300.16\text{ kJ/kg}$, $s_2 = 1.2369\text{ kJ/kg} \cdot \text{K}$. Then

$$E_2 = 1.11\text{ kg} [(53.49) + (-2.83) + (-45.12)] \frac{\text{kJ}}{\text{kg}} = 6.1\text{ kJ}$$

For the refrigerant, the change in exergy is

$$(\Delta E)_{\text{refrigerant}} = E_2 - E_1 = 6.1\text{ kJ} - 3.7\text{ kJ} = 2.4\text{ kJ}$$

The exergy of the refrigerant increases as it is stirred.

← (a)

Continued on next slide

Problem 7-29 continued

(b) With assumption 2, Eq. 7.3 reduces to give the exergy change for the suspended mass

$$(\Delta E)_{\text{mass}} = (\Delta U + p_0 \Delta V - T_0 \Delta S + \Delta KE + \Delta PE)_{\text{mass}} \\ = (\Delta PE)_{\text{mass}}$$

Thus, the exergy change for the suspended mass equals its change in potential energy.

- ③ The change in potential energy of the suspended mass is obtained from an energy balance for the isolated system as follows: The change in energy of the isolated system is the sum of the energy changes of the refrigerant and suspended mass. There is no heat transfer or work, and with assumption 2 we have

$$(\Delta KE + \Delta PE + \Delta U)_{\text{refrigerant}} + (\Delta KE + \Delta PE + \Delta U)_{\text{mass}} = Q - W$$

Solving for $(\Delta PE)_{\text{mass}}$ and using previously determined values for the specific internal energy of the refrigerant

$$(\Delta PE)_{\text{mass}} = -(\Delta U)_{\text{refrigerant}} \\ = -(1.11 \text{ kg})(300.16 - 211.29) \left(\frac{\text{kJ}}{\text{kg}} \right) \\ = -98.6 \text{ kJ}$$

Collecting results, $(\Delta E)_{\text{mass}} = -98.6 \text{ kJ}$. The exergy of the mass decreases because its elevation decreases. (b)

(c) The change in exergy of the isolated system is the sum of the exergy changes of the refrigerant and suspended mass. With the results of parts (a) and (b)

$$(\Delta E)_{\text{isol}} = (\Delta E)_{\text{refrigerant}} + (\Delta E)_{\text{mass}} \\ = (2.4 \text{ kJ}) + (-98.6 \text{ kJ}) \\ = -96.2 \text{ kJ}$$

The exergy of the isolated system decreases. (c)
(d)

To summarize

	Energy Change	Exergy Change
Refrigerant	+98.6 kJ	+ 2.4 kJ
Suspended mass	-98.6 kJ	-98.6 kJ
Isolated system	0.0 kJ	-96.2 kJ

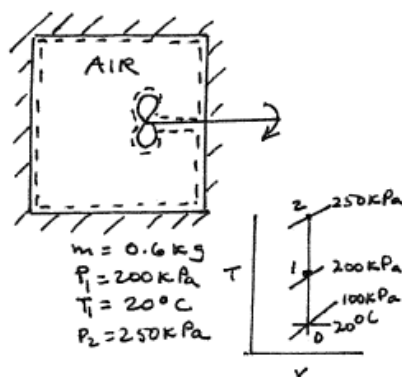
For the isolated system there is no net change in energy. The increase in the internal energy of the refrigerant equals the decrease in potential energy of the suspended mass. However, the *increase* in exergy of the refrigerant is much less than the *decrease* in exergy of the mass. For the isolated system, exergy decreases because stirring destroys exergy.

- Exergy is a measure of the departure of the state of the system from that of the environment. At all states, $E \geq 0$. This applies when $T > T_0$ and $p > p_0$, as at state 2, and when $T < T_0$ and $p < p_0$, as at state 1.
- The exergy change of the refrigerant can be determined more simply with Eq. 7.3 which requires dead state property values only for T_0 and p_0 . With the approach used in part (a), values for u_0 , v_0 , and s_0 are also required.
- The change in potential energy of the suspended mass, $(\Delta PE)_{\text{mass}}$, cannot be determined from Eq. 2.10 (Sec. 2.1) since the mass and change in elevation are unknown. Moreover, for the suspended mass as the system, $(\Delta PE)_{\text{mass}}$ cannot be obtained from an energy balance without first evaluating the work. Thus, we resort here to an energy balance for the isolated system, which does not require such information.

PROBLEM 7.30

A rigid, insulated tank contains 0.6 kg of air, initially at 200 kPa, 20°C. The air is stirred by a paddle wheel until its pressure is 250 kPa. Using the ideal gas model with $c_v = 0.72 \text{ kJ/kg} \cdot \text{K}$, determine, in kJ, (a) the work, (b) the change in exergy of the air, and (c) the amount of exergy destroyed. Ignore the effects of motion and gravity, and let $T_0 = 20^\circ\text{C}$, $p_0 = 100 \text{ kPa}$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The air is the closed system.
2. For the system, $Q = 0$ and the effects of motion and gravity can be ignored.
3. For the environment, $T_0 = 293 \text{ K}$ (20°C),
 $p_0 = 100 \text{ kPa}$

4. Air is modeled as an ideal gas with $c_v = 0.72 \text{ kJ/kg} \cdot \text{K}$

ANALYSIS: Use the ideal gas model equation of state to get T_2 :

$$\frac{p_1 V = m R T_1}{p_2 V = m R T_2} \Rightarrow T_2 = \frac{p_2}{p_1} T_1 = \frac{250}{200} (293 \text{ K}) = 366 \text{ K}$$

- (a) An energy balance reduces to give, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$

$$\Rightarrow W = -m(u_2 - u_1) = -m c_v (T_2 - T_1) = (0.6 \text{ kg}) (0.72 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) (366 - 293) \text{ K}$$

$$= -31.54 \text{ kJ} \quad \leftarrow (a)$$

- (b) With Eq. 7.3

$$\Delta E = m [(u_2 - u_1) + p_0 (v_2 - v_1) - T_0 (s_2 - s_1)]$$

Then, with Eq. 6.21

$$\Delta E = m \left[c_v (T_2 - T_1) - T_0 \left[c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \right] \right]$$

$$= +31.54 \text{ kJ} - (0.6 \text{ kg}) (293 \text{ K}) \left(0.72 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln \frac{366}{293} \right)$$

$$= +31.54 \text{ kJ} - 28.16 \text{ kJ}$$

$$= +3.38 \text{ kJ} \quad \leftarrow (b)$$

①

- (c) $E_d = T_0 \sigma$, where σ is obtained from an entropy balance:

$$\sigma = m (s_2 - s_1) = m \left[c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \right]$$

$$\therefore E_d = m c_v \ln \frac{T_2}{T_1} = 28.16 \text{ kJ} \quad (\text{from Part (b)}) \quad \leftarrow (c)$$

Alternatively, an exergy balance reduces to give $\Delta E = E_q - E_w - E_d$

$$\Rightarrow E_d = -E_w - \Delta E = -[W - p_0 \Delta V] - \Delta E \Rightarrow E_d = -W - \Delta E$$

$$= 31.54 - 3.38 = 28.16 \text{ kJ}$$

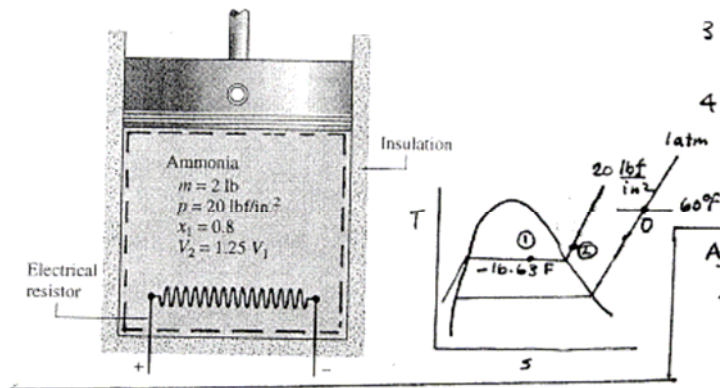
1. As shown by the T-v diagram, state 2 is farther from the dead state than state 1, so, an increase in exergy is expected.

PROBLEM 7.31

As shown in Fig. P7.31, two lb of ammonia is contained in a well-insulated piston-cylinder assembly fitted with an electrical resistor of negligible mass. The ammonia is initially at 20 lbf/in.² and a quality of 80%. The resistor is activated until the volume of the ammonia increases by 25%, while its pressure varies negligibly. Determine, in Btu,

- the amount of energy transfer by electrical work and the accompanying exergy transfer.
- the amount of energy transfer by work to the piston and the accompanying exergy transfer.
- the change in exergy of the ammonia.
- the amount of exergy destruction.

Ignore the effects of motion and gravity and let $T_0 = 60^\circ\text{F}$, $p_0 = 1 \text{ atm}$.



ENGR. MODEL

- The ammonia is the closed system.
- For the system, $Q=0$ and the effects of motion and gravity can be ignored.
- The electrical resistor has negligible mass.
- For the environment, $T_0 = 520^\circ\text{R}$ (60°F), $p_0 = 14.7 \text{ lbf/in}^2$.

ANALYSIS: Property data:

$$v_1 = v_f + x_1 (v_g - v_f)$$

$$= 0.02377 + 0.8(13.497 - 0.02377)$$

$$= 10.8 \text{ ft}^3/\text{lb}$$

$$u_1 = u_f + x_1 (u_g - u_f)$$

$$= 24.58 + 0.8(555.78 - 24.58) = 449.54 \frac{\text{Btu}}{\text{lb}}$$

$$s_1 = s_f + x_1 (s_g - s_f)$$

$$= 0.0571 + 0.8(1.3687 - 0.0571)$$

$$= 1.1064 \text{ Btu/lb} \cdot ^\circ\text{R}$$

$$v_2 = 1.25 v_1 = 1.25(10.8 \text{ ft}^3/\text{lb})$$

$$= 13.5 \text{ ft}^3/\text{lb}$$

Note: State 2 is closely saturated vapor at 20 lbf/in.². So,

$$u_2 \approx u_g = 555.78 \frac{\text{Btu}}{\text{lb}}, s_2 \approx s_g = 1.3687 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

$$(a) W_{\text{piston}} = m \int_1^2 p \, dv = m p (v_2 - v_1)$$

$$= (2 \text{ lb}) \left(20 \frac{\text{lbf}}{\text{in}^2} \right) \left(13.5 - 10.8 \right) \frac{\text{ft}^3}{\text{lb}} \left| \frac{144 \text{ in}^2}{\text{ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= 19.998 \text{ Btu}$$

$$[E_w]_{\text{piston}} = W_{\text{piston}} - p_0 \Delta V$$

$$= W_{\text{piston}} - p_0 m (v_2 - v_1)$$

$$= 19.99 \text{ Btu} - (14.7)(2)(13.5 - 10.8) \left| \frac{144}{778} \right|$$

$$= 5.3 \text{ Btu}$$

(b) To get W_{elec} , apply an energy balance: $\Delta U + \cancel{KE} + \cancel{PE} = Q - (W_{\text{piston}} + W_{\text{elec}})$

$$\Rightarrow W_{\text{elec}} = -W_{\text{piston}} - \Delta U$$

$$= -19.99 \text{ Btu} - 2 \text{ lb} (555.78 - 449.54) \frac{\text{Btu}}{\text{lb}}$$

$$= -232.47 \text{ Btu}$$

$$\Rightarrow [E_w]_{\text{elec}} = -232.47 \text{ Btu} \quad (\text{no accompanying volume change})$$

Continued on next slide

Problem 7-31 continued

$$\begin{aligned}
 (c) \quad \Delta E &= m \left((u_2 - u_1) + P_0 (v_2 - v_1) - T_0 (s_2 - s_1) \right) \\
 &= 2 \left[(555.78 - 449.54) + 14.7 \left(\frac{144}{778} \right) (13.5 - 10.8) - 520 (1.3687 - 1.1064) \right] \text{ Btu} \\
 \textcircled{1} \quad &= 2 [106.24 + 7.35 - 136.40] = -45.62 \text{ Btu}
 \end{aligned}$$

(d) An exergy balance reads

$$\begin{aligned}
 \Delta E &= \cancel{E_q}^0 - E_w - E_d \\
 \Rightarrow E_d &= -E_w - \Delta E \\
 &= -[5.3 - 232.47] \text{ Btu} - [-45.62 \text{ Btu}] \\
 &= 272.8 \text{ Btu}
 \end{aligned}$$

Alternatively, $E_d = T_0 \sigma$, where σ is obtained from an entropy balance as $\sigma = m(s_2 - s_1)$. So

$$\begin{aligned}
 E_d &= T_0 m (s_2 - s_1) = (520^\circ \text{R})(216) (1.3687 - 1.1064) \frac{\text{Btu}}{16.012} \\
 &= 272.8 \text{ Btu}
 \end{aligned}$$

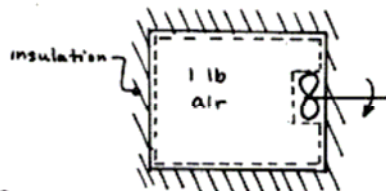
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1. As suggested by the location of state 2 on the T-s diagram, and gauged by the values of v , u , and s at states 1 and 2 relative to the respective values at the dead state, state 2 is closer to the dead state than state 1. Accordingly, the change in exergy from state 1 to state 2 is negative, as calculated here.

PROBLEM 7.32

KNOWN: Air contained in a closed, rigid, insulated tank is stirred by a paddle wheel.

FIND: Determine the change in exergy, the transfer of exergy accompanying work, and the exergy destruction.

SCHEMATIC & GIVEN DATA:



$$T_0 = 500^\circ\text{R}, T_1 = 500^\circ\text{R}, T_2 = 700^\circ\text{R}$$

$$P_0 = 1\text{atm}, P_1 = 1\text{atm}$$

ENGR. MODEL: (1) For the system shown in the accompanying figure, $Q=0$, and kinetic and potential energy effects can be ignored. (2) Air is modeled as an ideal gas. (3) For the environment $T_0 = 500^\circ\text{R}$, $P_0 = 1\text{atm}$.

ANALYSIS: With Eq. 7.3 the change in exergy is

$$E_2 - E_1 = m \left[(u_2 - u_1) + P_0 \left(\frac{v_2}{v_1} - 1 \right) - T_0 (s_2 - s_1) \right]$$

$$= m \left[u(T_2) - u(T_1) - T_0 (s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1) \right] \quad (1)$$

This requires P_2 . Using the ideal gas equation of state: $P_2/P_1 = T_2/T_1$. Then, with data from Table A-22E

$$E_2 - E_1 = (1\text{ lb}) \left[119.58 - 85.2 - 500 \left(0.66321 - 0.58222 - \frac{1.916}{28.97} \ln \frac{700}{500} \right) \right] = 5.47\text{ Btu} \quad \leftarrow$$

28.91 Btu/lb

To find W , reduce the energy balance to read $W = m(u_1 - u_2)$, or

$$W = (1\text{ lb}) [85.2 - 119.58] = -34.38\text{ Btu}$$

Then, since $\Delta V = 0$ in the process,

$$E_W = \left[\begin{array}{c} \text{exergy transfer} \\ \text{accompanying work} \end{array} \right] = W - P_0 \Delta V = -34.38\text{ Btu} \quad \leftarrow$$

① Using the exergy balance,

$$E_d = \int_1^2 \left(1 - \frac{T_0}{T_b} \right) \delta Q - [W - P_0 \Delta V] - [E_2 - E_1]$$

$$= -W - [E_2 - E_1]$$

$$\textcircled{2} \quad = -(-34.38\text{ Btu}) - [5.47\text{ Btu}] = 28.91\text{ Btu} \quad \leftarrow$$

1. Alternatively, $E_d = T_0 \sigma$, where σ is the amount of entropy produced, which is obtained from an entropy balance. Since the tank involves no heat transfer, the entropy balance reduces to give $\sigma = s_2 - s_1$. Thus, in this case, $E_d = T_0 (s_2 - s_1) = 28.91\text{ Btu}$.

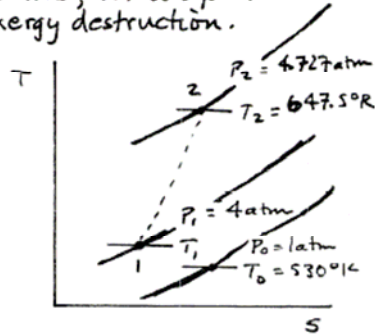
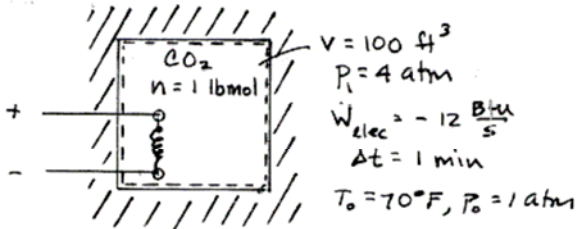
2. In this case, the destruction of exergy can be traced to the shearing action of the paddle wheel on the air in the tank.

PROBLEM 7.34

KNOWN: A known amount of carbon dioxide gas is contained in a rigid, insulated vessel of known volume and at a specified initial pressure. An electric resistor in the vessel transfers energy to the gas at a known rate for a specified period of time.

FIND: Determine the exergy change of the gas and, for a system consisting of the resistor and gas, the work and exergy destruction.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system consists of the gas and the resistor. (2) The volume is constant and kinetic and potential energy effects are neglected. (3) The resistor is of negligible mass and thus undergoes no change of state. (4) The carbon dioxide is modeled as an ideal gas. (5) For the system, $Q = 0$.

ANALYSIS: First, fix both states, as follows:

$$T_1 = \frac{P_1 V}{n \bar{R}} = \frac{(4 \text{ atm})(100 \text{ ft}^3)}{(1 \text{ lbmol})(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lbmol} \cdot \text{R}})} \left| \frac{14.696 \text{ lbf/in}^2}{1 \text{ atm}} \right| \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 548^\circ \text{R} \quad (88^\circ \text{F})$$

To determine T_2 , begin with an energy balance: $\Delta U = Q - W$. With $\Delta U_{\text{res}} = 0$

$$n(\bar{u}_2 - \bar{u}_1) = Q - W$$

where $\bar{u}_2$ and $\bar{u}_1$ are obtained from Table A-23E. To get W ,

$$W = \int_1^2 \dot{W} dt = \dot{W}_{\text{elec}} \Delta t = (-12 \frac{\text{Btu}}{\text{s}})(1 \text{ min}) \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = -720 \text{ Btu}$$

Thus, solving for $\bar{u}_2$ and inserting data

$$\bar{u}_2 = -\frac{W}{n} + \bar{u}_1 = -\frac{(-720 \text{ Btu})}{(1 \text{ lbmol})} + 3040.1 \text{ Btu/lbmol} = 3760.1 \text{ Btu/lbmol}$$

Interpolating in Table A-23E, $T_2 = 647.5^\circ \text{R}$ (187.5°F). Thus

$$P_2 = \frac{n \bar{R} T_2}{V} = \frac{(1)(1545)(647.5)}{(100)} \left| \frac{1}{144} \right| \left| \frac{1}{14.696} \right| = 4.727 \text{ atm}$$

Now, the change in exergy of the gas is obtained using Eq. 7.3 and ideal gas relations

$$E_2 - E_1 = n \left[\bar{u}(T_2) - \bar{u}(T_1) + P_0 (\bar{v}_2 - \bar{v}_1) - T_0 (3^\circ(T_2) - 3^\circ(T_1)) - \bar{R} \ln P_2/P_1 \right]$$

Inserting data interpolated from Table A-23E

$$\begin{aligned} E_2 - E_1 &= (1 \text{ lbmol}) \left[(3760.1 - 3040.1) \frac{\text{Btu}}{\text{lbmol}} - (530^\circ \text{R}) (52.750 - 51.2124 - 1.986 \ln \frac{4.727}{4}) \frac{\text{Btu}}{\text{lbmol} \cdot \text{R}} \right] \\ &= 80.85 \text{ Btu} \end{aligned}$$

As state 2 is further from the dead state than state 1, see the T-s diagram, an increase in exergy occurs during this process.

Continued on next slide

Problem 7-34 continued

The exergy transfer accompanying work, E_w , is

$$E_w = W - p_0 \Delta V^0 = -720 \text{ Btu} \leftarrow E_w$$

The exergy destruction, E_d , is determined by solving the exergy balance

$$E_2 - E_1 = \int_1^2 \left(1 - \frac{T_0}{T_b}\right) \delta Q^0 - (W - p_0 \Delta V^0) - E_d$$

or

$$E_d = -(E_2 - E_1) - (W) = -(80.85 \text{ Btu}) - (-720 \text{ Btu})$$

$$\textcircled{1} \textcircled{2} \quad = 639.15 \text{ Btu} \leftarrow E_d$$

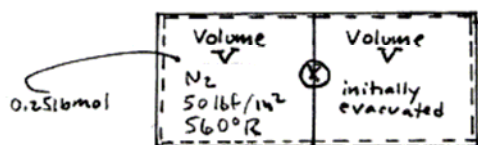
1. Note that the exergy input by work (720 Btu) results in only 80.85 Btu being stored as increased exergy of the gas. The rest, 639.15 Btu, is destroyed by irreversibilities.

2. Alternatively, $E_d = T_0 \sigma$, where $\sigma = \eta(\bar{s}_2 - \bar{s}_1)$, giving $E_d = 639.15 \text{ Btu}$.

PROBLEM 7.35

KNOWN: A rigid insulated tank has two equal-volume compartments separated by a valve. Initially, one is evacuated and the other holds N_2 gas at a known condition. The valve is opened and the gas fills the entire volume.
FIND: Determine the final temperature and pressure, and evaluate the exergy destruction. Discuss.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The closed system consists of both volumes, as shown in the schematic. 2. For the system, $Q = W = 0$ and effects of motion and gravity can be ignored. 3. N_2 is modeled as an ideal gas. 4. For the environment, $T_0 = 530^\circ R$, $p_0 = 1 \text{ atm}$.

ENGR. MODEL: With assumptions 1, 2 the energy balance reduces to $\Delta U = 0 \Rightarrow \Delta U = 0$. Since the specific internal energy of an ideal gas depends on temperature alone: $T_2 = T_1 = 560^\circ R$

Using the ideal gas equation of state

$$T_2 = 100^\circ F$$

$$\text{initially: } p_1 V = m R T_1$$

$$\text{finally: } p_2 (2V) = m R T_2$$

but $T_2 = T_1$, giving

$$p_2 (2V) = p_1 V \Rightarrow p_2 = \frac{1}{2} p_1 = 25 \text{ lbf/in}^2$$

The exergy destruction can be found from an exergy balance or using $E_d = T_0 \sigma$, where σ is obtained from an entropy balance. That is,

$$\Delta S = \int_1^2 \frac{\delta Q}{T} + \sigma$$

$$\Rightarrow \sigma = \Delta S = n \left[\bar{s}^0(T_2) - \bar{s}^0(T_1) - \bar{R} \ln p_2/p_1 \right]$$

$= 0 \text{ since } T_2 = T_1$

$$\sigma = -n \bar{R} \ln \frac{p_2}{p_1} = -(0.25 \text{ lbmol}) \left(\frac{1.986 \text{ Btu}}{\text{lbmol} \cdot ^\circ R} \right) \ln \left(\frac{1}{2} \right)$$

$$= 0.344 \text{ Btu/}^\circ R$$

Finally

$$E_d = T_0 \sigma$$

$$= (530^\circ R) (0.344 \frac{\text{Btu}}{^\circ R})$$

$$= 182.3 \text{ Btu}$$

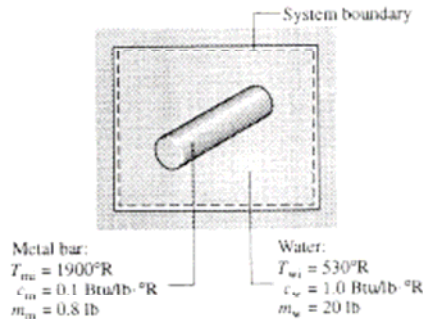
Exergy is destroyed in this case because the N_2 undergoes an unrestrained and thus irreversible expansion to a lower pressure.

PROBLEM 7.36

KNOWN: A hot metal bar is quenched by immersing it in a tank of water.

FIND: Determine the exergy destruction.

SCHEMATIC & GIVEN DATA:



- ENGR. MODEL** 1. As shown in the schematic, the metal bar and water form a closed system. 2. For the system, $Q = W = 0$ and there are no effects of motion or gravity. 3. The metal bar and water are each modeled as incompressible. 4. $T_0 = 537^\circ\text{R}$ (77°F).

ANALYSIS: An energy balance for the system reduces to give

$$\Delta U)_{\text{metal}} + \Delta U)_{\text{water}} = 0 \quad (1)$$

An exergy balance reduces to give

$$\Delta E = \int_1^2 \left[1 - \frac{T_0}{T_b} \right] \delta Q - \left[W - P_0 \Delta V \right] - E_d \Rightarrow E_d = -\Delta E$$

Since exergy is an extensive property, $\Delta E = \Delta E)_{\text{metal}} + \Delta E)_{\text{water}}$. Then, with Eq. 7.3

$$E_d = - \left[\Delta U + P_0 \Delta V - T_0 \Delta S \right]_{\text{metal}} + \left[\Delta U + P_0 \Delta V - T_0 \Delta S \right]_{\text{water}}$$

Using Eq. (1) this becomes

$$E_d = T_0 [\Delta S)_{\text{metal}} + \Delta S)_{\text{water}}] \quad (2)$$

The term in square brackets is the amount of entropy produced, which is evaluated in Example 6.5 as $0.0864 \text{ Btu/}^\circ\text{R}$. Thus

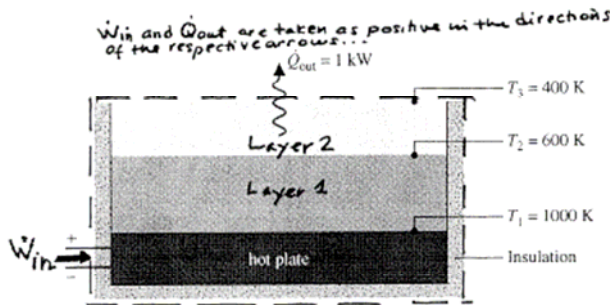
$$E_d = 537^\circ\text{R} (0.0864 \text{ Btu/}^\circ\text{R}) = 46.4 \text{ Btu}$$

1. With the indicated assumptions, the system is isolated. Eq. (1) indicates that the total energy of the system remains constant. Eq. (2) indicates that the exergy of the system does not remain constant because exergy is destroyed.

2. Alternatively, Eq. (2) can be expressed as $E_d = T_0 \sigma$, where σ is the amount of entropy produced within the system.

PROBLEM 7.37

Figure P7.37 provides steady-state data for a composite of a hot plate and two solid layers. Perform a full exergy accounting, in kW, of the electrical power provided to the composite, including the exergy transfer accompanying heat transfer from the composite and the destruction of exergy in the hot plate and each of the two layers. Let $T_0 = 300$ K.



ENGR. MODEL:

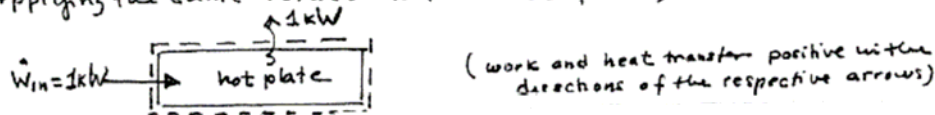
1. Closed systems consist of the hot plate, layer 1, layer 2, and the overall composite.
2. Each system is at steady state.
3. For the environment, $T_0 = 300$ K

ANALYSIS:

For any closed system at steady state, $\frac{dE}{dt} = \dot{Q} - \dot{W} \Rightarrow \dot{W} = \dot{Q}$. (1)

When Eq. (1) is applied to the overall composite, we get $\dot{W}_{in} = \dot{Q}_{out} = 1$ kW.

Applying the same result to the hot plate,



For each layer, $\dot{W} = 0$; so $\dot{Q} = 0$: the rate of heat transfer into the layer equals the rate of heat transfer out of the layer:



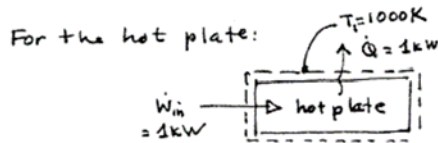
With these preliminaries, we can account for the exergy carried in electrically, $\dot{W}_{in}$, in terms of the exergy carried out accompanying heat transfer and the exergy destroyed in each of the three subsystems.

$$\begin{aligned} \left[\begin{array}{l} \text{Exergy Carried} \\ \text{Out accompanying} \\ \text{heat transfer} \end{array} \right] &= \left[1 - \frac{T_0}{T_3} \right] \dot{Q}_{out} \\ &= \left[1 - \frac{300}{400} \right] (1 \text{ kW}) = 0.25 \text{ kW} \end{aligned}$$

The exergy destroyed in each of the three subsystems is conveniently evaluated as $\dot{E}_d = T_0 \dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production.

Continued on next slide

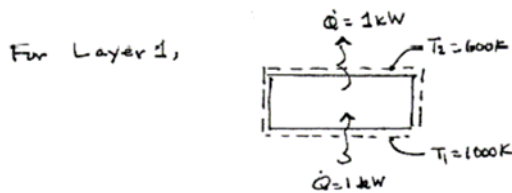
Problem 7-37 continued



where $\dot{Q}$ is positive in the direction of the arrow. An entropy rate balance reads,

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{\sigma} \Rightarrow 0 = -\frac{\dot{Q}}{T_f} + \dot{\sigma} \Rightarrow \dot{\sigma} = \frac{\dot{Q}}{T_f}$$

$$\therefore \dot{E}_d = T_0 \dot{\sigma} = \frac{T_0}{T_f} \dot{Q} = \left(\frac{300}{1000}\right)(1 \text{ kW}) = 0.3 \text{ kW}$$



$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{\sigma} \Rightarrow 0 = \frac{\dot{Q}}{T_f} - \frac{\dot{Q}}{T_2} + \dot{\sigma} \Rightarrow \dot{\sigma} = \left[\frac{1}{T_2} - \frac{1}{T_f}\right] \dot{Q}$$

$$\therefore \dot{E}_d = T_0 \dot{\sigma} = T_0 \left[\frac{1}{T_2} - \frac{1}{T_f}\right] \dot{Q} = 300 \left[\frac{1}{600} - \frac{1}{1000}\right](1 \text{ kW}) = 0.2 \text{ kW}$$

With the same approach for Layer 2,

$$\dot{E}_d = T_0 \dot{\sigma} = T_0 \left[\frac{1}{T_3} - \frac{1}{T_2}\right] \dot{Q} = 300 \left[\frac{1}{400} - \frac{1}{600}\right](1 \text{ kW}) = 0.25 \text{ kW}$$

① Thus, an exergy accounting for the overall composite reads:

Exergy supplied:

$$\left[\begin{array}{l} \text{Rate exergy} \\ \text{is carried in} \\ \text{Electrically} \end{array} \right] = 1.0 \text{ kW}$$

Disposition of the exergy carried in:

$$\textcircled{a} \left[\begin{array}{l} \text{Exergy carried} \\ \text{out accompanying} \\ \text{heat transfer} \end{array} \right] = 0.25 \text{ kW}$$

② Exergy destroyed:

✓ Hot plate	= 0.30 kW	}	0.75 kW
✓ Layer 1	= 0.20 kW		
✓ Layer 2	= 0.25 kW		
1.0 kW			

1. On an exergy basis, the efficiency of the composite can be evaluated as

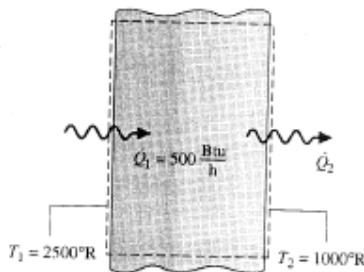
$$\frac{\left[\begin{array}{l} \text{Exergy carried out} \\ \text{accompanying heat transfer} \end{array} \right]}{\left[\begin{array}{l} \text{Exergy carried in} \\ \text{electrically} \end{array} \right]} = 0.25 \quad (25\%)$$

PROBLEM 7.38

KNOWN: There is heat transfer through a wall for which the temperatures at the inner and outer surfaces are known.

FIND: Determine the rates of exergy transfer accompanying heat transfer into and out of the wall and the rate of exergy destruction. Discuss.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the schematic is at steady state with $\dot{W} = 0$. (2) $T_0 = 500^\circ\text{R}$

ANALYSIS: At steady state an, energy rate balance reduces to

$\frac{dE}{dt} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = 0 \Rightarrow \dot{Q}_2 = \dot{Q}_1 = 500 \text{ Btu/h}$. The directions of $\dot{Q}_1, \dot{Q}_2$ are indicated by the arrows on the schematic.

Since T_1 and T_2 are uniform over the inner and outer surfaces, the rates of exergy transfer accompanying heat transfer are, respectively

$$\begin{aligned} \left[\begin{array}{l} \text{rate of exergy transfer} \\ \text{in accompanying} \\ \text{heat transfer} \end{array} \right] &= \left[1 - \frac{T_0}{T_1} \right] \dot{Q}_1 \\ &= \left[1 - \frac{500}{2500} \right] (500 \frac{\text{Btu}}{\text{h}}) = 400 \frac{\text{Btu}}{\text{h}} \quad \leftarrow \end{aligned}$$

$$\begin{aligned} \left[\begin{array}{l} \text{rate of exergy transfer} \\ \text{out accompanying} \\ \text{heat transfer} \end{array} \right] &= \left[1 - \frac{T_0}{T_2} \right] \dot{Q}_2 \\ &= \left[1 - \frac{500}{1000} \right] (500 \frac{\text{Btu}}{\text{h}}) = 250 \frac{\text{Btu}}{\text{h}} \quad \leftarrow \end{aligned}$$

Reducing the exergy rate balance for closed systems at steady state, Eq. 7.11a,

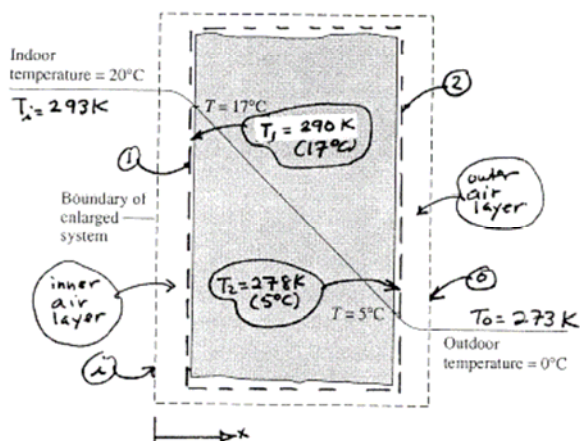
$$\begin{aligned} 0 &= \left[1 - \frac{T_0}{T_1} \right] \dot{Q}_1 - \left[1 - \frac{T_0}{T_2} \right] \dot{Q}_2 - \dot{W} - \dot{E}_d \\ \Rightarrow \dot{E}_d &= \left[1 - \frac{T_0}{T_1} \right] \dot{Q}_1 - \left[1 - \frac{T_0}{T_2} \right] \dot{Q}_2 = 400 \frac{\text{Btu}}{\text{h}} - 250 \frac{\text{Btu}}{\text{h}} = 150 \frac{\text{Btu}}{\text{h}} \quad \leftarrow \end{aligned}$$

- ① Exergy is destroyed because heat transfer occurs spontaneously through a finite temperature difference.

1. The term $\left[1 - \frac{T_0}{T_1} \right] \dot{Q}_1$ is the power that could be obtained in principle by supplying $\dot{Q}_1$ to a reversible power cycle operating between T_1 and T_0 . Likewise, $\left[1 - \frac{T_0}{T_2} \right] \dot{Q}_2$ is the power that could be obtained in principle by supplying $\dot{Q}_2$ to a reversible power cycle operating between T_2 and T_0 . The difference between these — $\dot{E}_d$ — represents the opportunity to develop power that is irrevocably lost in the spontaneous conduction process.

PROBLEM 7.39

Figure P7.39 provides steady-state data for the outer wall of a dwelling on a day when the indoor temperature is maintained at 20°C and the outdoor temperature is 0°C . The heat transfer rate through the wall is 1100 W . Determine, in W , the rate of exergy destruction (a) within the wall, and (b) within the enlarged system shown on the figure by the dashed line. Comment. Let $T_0 = 0^\circ\text{C}$.



ENGR. MODEL:

1. As shown in the figure, two closed systems are considered: One is a section of the wall. The other is an enlarged system.
2. The systems are at steady state.
3. Heat transfer at a rate of 1100 W takes place in the positive x direction.
4. For the environment, $T_0 = 273\text{ K}$ (0°C)

- (a) An energy rate balance for the wall reads, $\frac{dE}{dt} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = 0$. That is, $\dot{Q}_1 = \dot{Q}_2 = 1100\text{ W}$, where the heat transfer rates at the inner and outer surfaces of the wall are positive in the positive x direction. An exergy rate balance at steady state reads,

$$0 = \left[1 - \frac{T_0}{T_1}\right] \dot{Q}_1 - \left[1 - \frac{T_0}{T_2}\right] \dot{Q}_2 - \dot{E}_d$$

$$\Rightarrow \dot{E}_d = \left[1 - \frac{T_0}{T_1}\right] \dot{Q}_1 - \left[1 - \frac{T_0}{T_2}\right] \dot{Q}_2$$

$$= \left[1 - \frac{273}{290}\right] (1100\text{ W}) - \left[1 - \frac{273}{278}\right] (1100\text{ W}) = 44.7\text{ W} \quad \leftarrow (a)$$

- (b) An energy rate balance for the enlarged system reads, $\frac{dE}{dt} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = 0$. That is, $\dot{Q}_1 = \dot{Q}_0 = 1100\text{ W}$, where the heat transfer rates at the inner and outer surfaces of the enlarged system are positive in the positive x direction. An exergy rate balance at steady state reads,

$$0 = \left[1 - \frac{T_0}{T_1}\right] \dot{Q}_1 - \left[1 - \frac{T_0}{T_0}\right] \dot{Q}_0 - \dot{E}_d$$

$$\Rightarrow \dot{E}_d = \left[1 - \frac{T_0}{T_1}\right] \dot{Q}_1 = \left[1 - \frac{273}{290}\right] (1100\text{ W}) = 75.1\text{ W} \quad \leftarrow$$

The exergy destruction in the enlarged system is greater than for the wall alone because of exergy destruction associated with heat transfer through the air layers at each of the wall surfaces.

1. All of the exergy exiting the indoor space: $\left[1 - \frac{T_0}{T_1}\right] \dot{Q}_1$ is destroyed owing to spontaneous heat transfer through the wall and its adjacent air layers. Typically, such exergy has its origin in fossil fuels.

PROBLEM 7.40

A gearbox operating at steady state receives 2 hp along the input shaft and delivers 1.89 hp along the output shaft. The outer surface of the gearbox is at 110°F. For the gearbox, (a) determine, in Btu/s, the rate of heat transfer and (b) perform a full exergy accounting, in Btu/s, of the input power. Let $T_0 = 70^\circ\text{F}$.

ENGR. MODEL:

1. The system shown in the schematic is at steady state.
2. Heat transfer occurs only at T_b .
3. For the environment, $T_0 = 520^\circ\text{R}$ (70°F)

ANALYSIS:

(a) At steady state, an energy rate balance reduces to read,

$$\begin{aligned}\frac{dE^0}{dt} &= \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W} \\ &= (1.89 \text{ hp} - 2 \text{ hp}) \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \\ &= -0.078 \text{ Btu/s}\end{aligned}$$

(b) The rate exergy enters the system equals the input power: 2 hp. Expressing this in units of Btu/s,

$$\dot{W}_{in} = 2 \text{ hp} \left| \frac{2545}{3600} \right| = 1.414 \frac{\text{Btu}}{\text{s}}$$

Exergy exits the system via output power: 1.89 hp

$$\dot{W}_{out} = 1.89 \left| \frac{2545}{3600} \right| = 1.336 \frac{\text{Btu}}{\text{s}}$$

Exergy also exits accompanying the heat transfer: $\left[1 - \frac{T_0}{T_b}\right] |\dot{Q}|$

That is

$$\left[1 - \frac{T_0}{T_b}\right] |\dot{Q}| = \left[1 - \frac{520}{570}\right] (0.078 \frac{\text{Btu}}{\text{s}}) = 0.005 \frac{\text{Btu}}{\text{s}}$$

Finally, exergy is destroyed within the gearbox. This is conveniently evaluated in terms of the rate of entropy production: $\dot{E}_d = T_0 \dot{\sigma}$, where

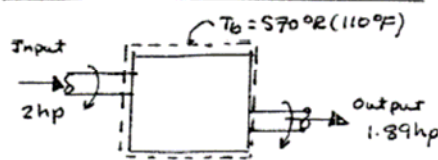
$$\dot{\sigma} = -\frac{\dot{Q}}{T_b} \rightarrow \text{giving}$$

$$\dot{E}_d = \frac{T_0}{T_b} \dot{Q} = \frac{520}{570} (0.078 \frac{\text{Btu}}{\text{s}}) = 0.073 \frac{\text{Btu}}{\text{s}}$$

Exergy Accounting Summary:

① Rate exergy is carried in:	1.414 Btu/s	
② Disposition of exergy:		
✓ Output power	1.336	(94.48%)
✓ accompanying heat transfer	0.005	(0.35%)
✓ destroyed	0.073	(5.16%)
	1.414	

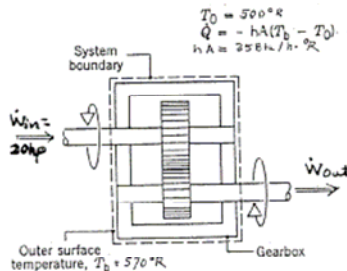
SCHEMATIC & GIVEN DATA:



PROBLEM 7.41

KNOWN: A gearbox operates at steady state. Performance data are provided.
FIND: Determine a full exergy accounting of the input power.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the figure is at steady state. (2) The temperature of the outer surface of the gearbox and the temperature of the surroundings are each uniform. (3) For the environment $T_0 = 500^\circ\text{R}$.

ANALYSIS: An energy rate balance reduces at steady state to give:
 $\dot{W} = \dot{Q}$. Then, inserting given information

$$\begin{aligned}\dot{W}_{\text{out}} = \dot{Q} + \dot{W}_{\text{in}} &= -hA[T_0 - T_b] + \dot{W}_{\text{in}} \\ &= -(2545 \frac{\text{Btu}}{\text{h} \cdot ^\circ\text{R}})(570 - 500)^\circ\text{R} + (20 \text{ hp}) \left(\frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right) \\ &= \underbrace{-2450 \frac{\text{Btu}}{\text{h}}}_{\dot{Q}} + 50,900 \frac{\text{Btu}}{\text{h}} = 48,450 \frac{\text{Btu}}{\text{h}} \quad \leftarrow\end{aligned}$$

The magnitude of the rate of exergy transfer accompanying heat is

$$\left[1 - \frac{T_0}{T_b}\right] |\dot{Q}| = \left[1 - \frac{500}{570}\right] (2450 \frac{\text{Btu}}{\text{h}}) = 300.9 \frac{\text{Btu}}{\text{h}} \quad \leftarrow$$

The rate of exergy destruction can be found from $\dot{E}_d = T_0 \dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production, or by reducing the exergy rate balance at steady state to obtain

$$\begin{aligned}\dot{E}_d &= \left[1 - \frac{T_0}{T_b}\right] \dot{Q} - \dot{W} \\ &= -300.9 \frac{\text{Btu}}{\text{h}} - [48,450 - 50,900] \frac{\text{Btu}}{\text{h}} = 2149.1 \frac{\text{Btu}}{\text{h}} \quad \leftarrow\end{aligned}$$

Exergy Accounting: Expressing values as percentages of the input power,

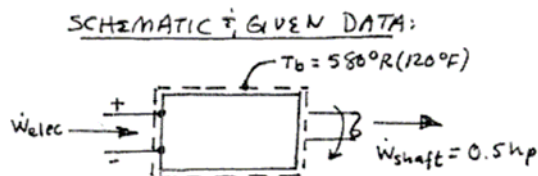
$$\begin{aligned}\frac{\dot{W}_{\text{out}}}{\dot{W}_{\text{in}}} &= \left(\frac{48,450}{50,900} \right) (100) = 95.19\% \\ \textcircled{1} \quad \frac{\left[1 - \frac{T_0}{T_b}\right] \dot{Q}}{\dot{W}_{\text{in}}} &= \left(\frac{300.9}{50,900} \right) (100) = 0.59\% \\ \frac{\dot{E}_d}{\dot{W}_{\text{in}}} &= \left(\frac{2149.1}{50,900} \right) (100) = 4.22\%\end{aligned} \quad \left. \vphantom{\begin{aligned} \frac{\dot{W}_{\text{out}}}{\dot{W}_{\text{in}}} \\ \frac{\left[1 - \frac{T_0}{T_b}\right] \dot{Q}}{\dot{W}_{\text{in}}} \\ \frac{\dot{E}_d}{\dot{W}_{\text{in}}} \end{aligned}} \right\} \text{Exergy accounting}$$

$$\underline{\underline{= 100\%}}$$

1. This represents the true thermodynamic value of the heat loss. An energy analysis overstates its significance: $\frac{|\dot{Q}|}{\dot{W}_{\text{in}}} = \left(\frac{2450}{50,900} \right) (100) = 5\%$

PROBLEM 7.42

At steady state, an electric motor develops power along its output shaft of 0.5 hp while drawing 4 amps at 120 V. The outer surface of the motor is at 120°F. For the motor, (a) determine, in Btu/h, the rate of heat transfer and (b) perform a full exergy accounting, in Btu/h, of the electrical power input. Let $T_0 = 60^\circ\text{F}$.



ENGR. MODEL

1. The system shown in the schematic operates at steady state.
2. Heat transfer occurs at $T_b = 580^\circ\text{R}$ and the power quantities are positive in the directions of the arrows.
3. For the environment, $T_0 = 520^\circ\text{R}$ (60°F).

ANALYSIS: (a) At steady state, the energy rate balance reduces to

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W}$$

or,

$$\begin{aligned} \dot{Q} &= \dot{W}_{\text{shaft}} - \dot{W}_{\text{elec}} \\ &= 0.5 \text{ hp} \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right| - \left[(4 \text{ amps})(120 \text{ volts}) \left| \frac{1 \text{ W}}{1 \text{ amp} \cdot \text{volt}} \right| \left| \frac{3413 \text{ Btu/h}}{1 \text{ W}} \right| \right] \\ &= (1272.5 \frac{\text{Btu}}{\text{h}}) - (1638.2 \frac{\text{Btu}}{\text{h}}) = -365.7 \frac{\text{Btu}}{\text{h}} \quad (a) \end{aligned}$$

(b) Exergy is carried into the motor by electric power: $1638.2 \frac{\text{Btu}}{\text{h}}$.

Exergy is carried out of the motor by shaft power: $1272.5 \frac{\text{Btu}}{\text{h}}$

Exergy is also carried out accompanying heat transfer; the magnitude is

$$\left[1 - \frac{T_0}{T_b} \right] |\dot{Q}| = \left[1 - \frac{520}{580} \right] (365.7 \frac{\text{Btu}}{\text{h}}) = 37.8 \frac{\text{Btu}}{\text{h}}$$

Finally, exergy is destroyed within the motor. This is conveniently evaluated in terms of the entropy production: $\dot{E}_d = T_0 \dot{\sigma}$, where

$$\dot{\sigma} = -\frac{\dot{Q}}{T_b}, \text{ giving}$$

$$\dot{E}_d = \frac{T_0}{T_b} (-\dot{Q}) = \left(\frac{520^\circ\text{R}}{580^\circ\text{R}} \right) (365.7 \frac{\text{Btu}}{\text{h}}) = 327.9 \frac{\text{Btu}}{\text{h}}$$

EXERGY ACCOUNTING %

① Rate exergy is carried in: $1638.2 \frac{\text{Btu}}{\text{h}}$

② Disposition of exergy:

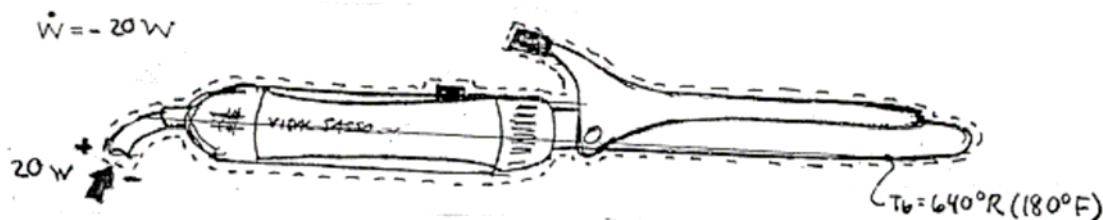
✓ output power:	1272.5 Btu/h	(77.68%)
✓ accompanying heat transfer	37.8 Btu/h	(2.31%)
✓ destroyed	327.9 Btu/h	(20.02%)
	<u>1638.2 Btu/h</u>	

Electric motors typically develop a greater fraction of output power from the input power than seen in this case.

PROBLEM 7.43

For the curling iron of Problem 6.55, perform a full exergy accounting, in Btu/h, of the electrical power supplied. Let $T_0 = 70^\circ\text{F}$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the figure is at steady state. (2) Heat transfer occurs at $T_b = 640^\circ\text{R}$ (180°F) only. (3) $T_0 = 530^\circ\text{R}$ (70°F).

ANALYSIS: An energy rate balance reduces to read

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W} = (-20\text{ W}) \left(3.413 \frac{\text{Btu}}{\text{W}\cdot\text{h}} \right) = -68.3 \text{ Btu/h} \leftarrow \dot{Q}$$

An entropy rate balance reduces to read

$$\frac{dS}{dt} = \frac{\dot{Q}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b} = \frac{68.3 \text{ Btu/h}}{640^\circ\text{R}} = 0.107 \frac{\text{Btu}}{\text{h}\cdot^\circ\text{R}} \leftarrow \dot{\sigma}$$

Using the rate of entropy production, the rate of exergy destruction is

$$\dot{E}_d = T_0 \dot{\sigma} = 530^\circ\text{R} (0.107 \frac{\text{Btu}}{\text{h}\cdot^\circ\text{R}}) = 56.7 \text{ Btu/h}$$

Using the heat transfer rate, the rate exergy is carried out of the curling iron with heat transfer is

$$\left[1 - \frac{T_0}{T_b} \right] |\dot{Q}| = \left[1 - \frac{530}{640} \right] (68.3 \frac{\text{Btu}}{\text{h}}) = 11.7 \text{ Btu/h}$$

EXERGY ACCOUNTING:

① Rate exergy is carried in electrically: 20 W.

68.3 Btu/h

② Disposition:

✓ exergy rate out accompanying heat transfer

11.7 Btu/h (17 %)

①

✓ destroyed

56.7 Btu/h (83 %)

- The significant exergy destruction in this case indicates that good use is not being made of the valuable power input.

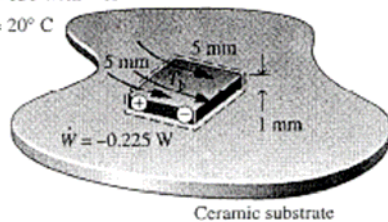
PROBLEM 7.44

As shown in Fig. P7.44, a silicon chip measuring 5 mm on a side and 1 mm in thickness is embedded in a ceramic substrate. At steady state, the chip has an electrical power input of 0.225 W. The top surface of the chip is exposed to a coolant whose temperature is 20°C. The heat transfer coefficient for convection between the chip and the coolant is 150 W/m² · K. Heat transfer by conduction between the chip and the substrate is negligible. Determine (a) the surface temperature of the chip, in °C, and (b) the rate of exergy destruction within the chip, in W. What causes the exergy destruction in this case? Let $T_0 = 293$ K.

Coolant

$$h = 150 \text{ W/m}^2 \cdot \text{K}$$

$$T_f = 20^\circ \text{C}$$



Note: This application is the subject of Example 2.5 and Problem 6.54.

$$(a) \text{ From Example 2.5, } T_b = 353 \text{ K } (80^\circ \text{C})$$

← (a)

$$(b) \text{ From Problem 6.54, } \dot{\sigma} = 6.37 \times 10^{-7} \text{ kW/K}$$

The principal source of entropy production — and exergy destruction — is electric current flow through a resistance.

$$\begin{aligned} \dot{E}_d &= T_0 \dot{\sigma} = 293 \text{ K} \left(6.37 \times 10^{-7} \frac{\text{kW}}{\text{K}} \right) \left| \frac{10^3 \text{ W}}{1 \text{ kW}} \right| \\ &= 0.187 \text{ W} \end{aligned}$$

← (b)

PROBLEM 7.45

An electric water heater having a 200 liter capacity heats water from 23 to 55°C. Heat transfer from the outside of the water heater is negligible, and the states of the electrical heating element and the tank holding the water do not change significantly. Perform a full exergy accounting, in kJ, of the electricity supplied to the water heater. Model the water as incompressible with a specific heat $c = 4.18 \text{ kJ/kg} \cdot \text{K}$. Let $T_0 = 23^\circ\text{C}$.

Note: This application is the subject of Problem 6.57 where the mass of water is found as 198.4 kg and the entropy production for the overall water heater is found as $\sigma = 85.13 \text{ kJ/K}$.

An energy balance for the overall water heater reads, $\Delta U = Q - W \Rightarrow W = -\Delta U$, where ΔU is for the water alone because the states of the tank wall and resistor are constant. Thus,

$$W = -m(u_2 - u_1) = -m c (T_2 - T_1) \\ = (198.4 \text{ kg})(4.18 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(32 \text{ K}) = -26,538 \text{ kJ}$$

This accounts for the exergy carried into the water heater.

Of this, exergy is destroyed in an amount

$$E_d = T_0 \sigma = (296 \text{ K})(85.13 \frac{\text{kJ}}{\text{K}}) = 25,198 \text{ kJ}$$

Further, exergy is stored in the water, whose temperature is increased, with Eq. 7.3 and using previously found values

$$E_2 - E_1 = (u_2 - u_1) + P_0(v_2 - v_1) - T_0(s_2 - s_1) \\ = 26,538 - 25,198 = 1,340 \text{ kJ}$$

EXERGY ACCOUNTING

① Exergy carried in electrically	26,538 kJ
② Destroying the exergy	
✓ Exergy stored	1,340 kJ (5%)
✓ Exergy destroyed	25,198 kJ (95%)
	<u>26,538 kJ</u>

The significant fraction of the supplied exergy that is destroyed in this application illustrates the wasteful nature of such water heaters.

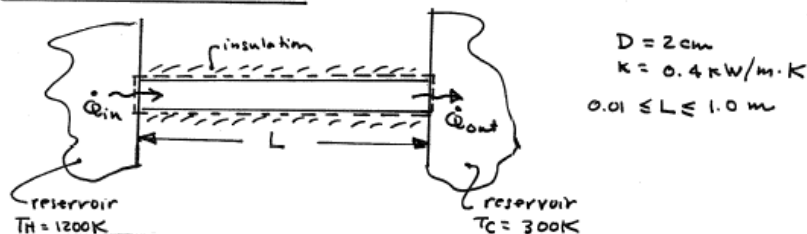
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PROBLEM 7.46

KNOWN: Energy is conducted from a thermal reservoir through a cylindrical rod at steady state to another thermal reservoir at a lower temperature.

FIND: Plot the rate of conduction through the rod, the rates of exergy transfer accompanying heat transfer into and out of the rod, and the rate of exergy destruction, each versus the rod length, L .

SCHEMATIC & GIVEN DATA:



ENER. MODEL: 1. The system shown in the schematic is at steady state. 2. Energy transfer is in the directions of the arrows only. 3. $T_0 = 300 \text{ K}$.

ANALYSIS: An energy rate balance reduces to give $\frac{dE}{dt} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = 0$, or $\dot{Q}_{in} = \dot{Q}_{out}$. Using Eq. 2.31, the common heat transfer rate by conduction is

$$\dot{Q}_{in} = \dot{Q}_{out} = kA \left[\frac{T_H - T_C}{L} \right] = \left(\frac{0.4 \text{ kW}}{\text{m.K}} \right) \left(\frac{\pi}{4} (0.02 \text{ m})^2 \right) \left(\frac{1200 - 300}{L} \right) \text{ K}$$

$$= \left(\frac{0.1131}{L} \right) \text{ kW} \quad (1)$$

The rates of exergy transfer accompanying heat transfer are, respectively

$$\left[\text{rate of exergy transfer in} \right] = \left[1 - \frac{T_0}{T_H} \right] \dot{Q}_{in} = \left[1 - \frac{300}{1200} \right] \dot{Q}_{in} = 0.75 \dot{Q}_{in} \quad (2)$$

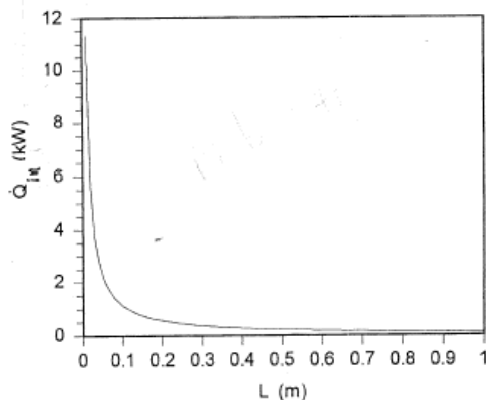
$$\left[\text{rate of exergy transfer out} \right] = \left[1 - \frac{T_0}{T_C} \right] \dot{Q}_{out} = \left[1 - \frac{300}{300} \right] \dot{Q}_{out} = 0 \quad (3)$$

The rate of exergy destruction can be obtained by reducing an exergy rate balance, which in this case corresponds to finding the difference between the exergy entering and exiting the rod. Thus,

$$\dot{E}_d = \left[\text{rate of exergy transfer in} \right] - \left[\text{rate of exergy transfer out} \right] = 0.75 \dot{Q}_{in} \quad (4)$$

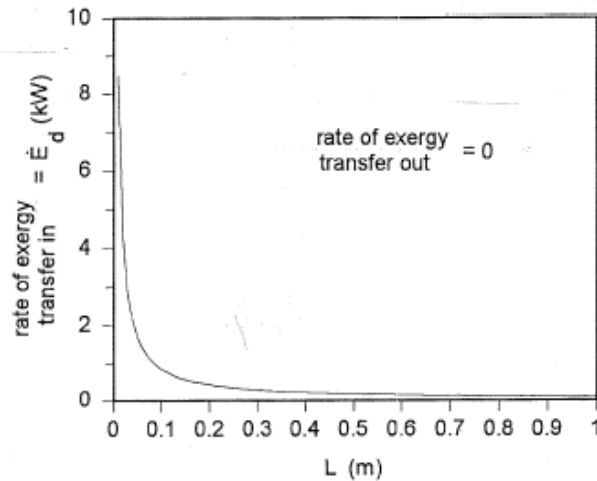
Sample Calculation: $L = 1 \text{ m}$, $\dot{Q}_{in} = 0.1131 \text{ kW}$, $\left[\text{Rate of exergy transfer in} \right] = \dot{E}_d = 0.0848 \text{ kW}$.

PLOTS:



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Problem 7-46 continued



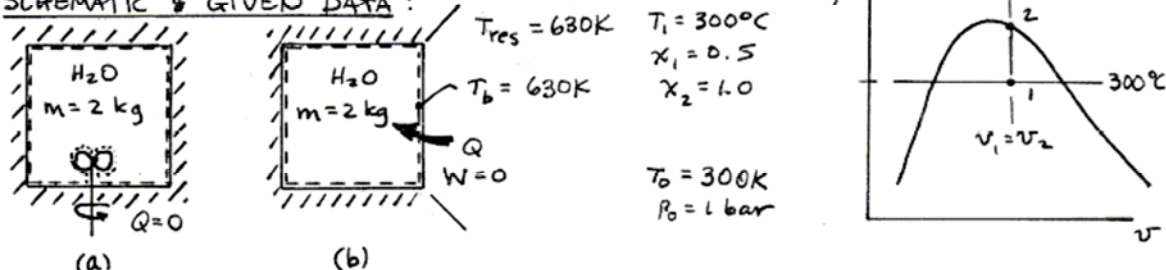
Discussion: As the length of the rod increases, the heat transfer rate and the exergy destruction rate decrease rapidly. Since exergy can be viewed as having economic value (see Sec. 7.7), a reduction in exergy destruction can be viewed as a cost saving. However, the cost of the rod increases with length, so an economic tradeoff is inherent in selecting a value for L.

PROBLEM 7.47

KNOWN: Two kilograms of a two-phase liquid-vapor mixture of water undergo two different processes at constant volume between the same end states: (a) adiabatic process, stirring with a paddle wheel, (b) heat transfer from a thermal reservoir at 630K.

FIND: For each case, determine the change in exergy, the net amounts of exergy transfer by work and heat, and the amount of exergy destruction.

SCHEMATIC & GIVEN DATA:



ENGR MODEL: (1) The water is a closed system. (2) The volume is constant. (3) In case (a); $Q=0$, in case (b); $W=0$. (4) Kinetic and potential energy effects are negligible. (5) For the environment, $T_0 = 300\text{K}$, $p_0 = 1\text{bar}$.

ANALYSIS: For both cases, the initial and final states are the same. Let us begin by obtaining data at each state. From Table A-2 at 300°C

$$v_1 = v_f + x_1(v_g - v_f) = 1.4036 \times 10^{-3} + (0.5)(0.02167 - 1.4036 \times 10^{-3}) = 0.01154 \text{ m}^3/\text{kg}$$

$$u_1 = u_f + x_1(u_g - u_f) = 1332.0 + (0.5)(2563.0 - 1332.0) = 1947.5 \text{ kJ/kg}$$

$$s_1 = s_f + x_1(s_g - s_f) = 3.2534 + (0.5)(5.7045 - 3.2534) = 4.479 \text{ kJ/kg}\cdot\text{K}$$

By assumptions (1) and (2), $v_2 = v_1 = v_g(T_2)$. From Table A-2, $T_2 = 336.8^\circ\text{C}$ and $u_2 = 2474.2 \text{ kJ/kg}$, $s_2 = 5.3673 \text{ kJ/kg}\cdot\text{K}$

(a) In this case, the volume is constant. Thus

$$\Delta E = m[(u_2 - u_1) + p_0(v_2 - v_1) - T_0(s_2 - s_1)] = (2 \text{ kg})[(2474.2 - 1947.5) - (300)(5.3673 - 4.479)] \frac{\text{kJ}}{\text{kg}} = 520.4 \text{ kJ} \leftarrow \Delta E$$

$E_W = (W - p_0 \Delta v) = W$. From an energy balance, $m(u_2 - u_1) = Q - W$. Thus

$$W = -m(u_2 - u_1) = -(2 \text{ kg})(2474.2 - 1947.5) \frac{\text{kJ}}{\text{kg}} = -1053.4 \text{ kJ} \leftarrow E_W$$

The exergy transfer accompanying heat transfer, denoted E_Q , is

$$E_Q = \int_1^2 (1 - \frac{T_0}{T_b}) \delta Q = 0 \leftarrow E_Q$$

Now, from an exergy balance

$$\Delta E = E_Q - E_W - E_d$$

$$E_d = -E_W - \Delta E = -(-1053.4 \text{ kJ}) - (520.4 \text{ kJ})$$

$$= 533 \text{ kJ} \leftarrow E_d$$

Continued on next slide

Problem 7-47 continued

(b) Since the change of state is the same, the change in energy is the same as in part (a). That is

$$\Delta E = 520.4 \text{ kJ} \quad \xleftarrow{\hspace{10em}} \Delta E$$

Since the work is zero, $E_w = 0$ $\xleftarrow{\hspace{10em}} E_w$

The exergy transfer accompanying heat transfer is evaluated at the boundary temperature, $T_b = 630\text{K}$. Thus

$$E_Q = \left(1 - \frac{T_o}{T_b}\right) Q$$

In this case, $m(u_2 - u_1) = Q - W^o \Rightarrow Q = 1053.4 \text{ kJ}$ and

$$E_Q = \left(1 - \frac{300}{630}\right) (1053.4 \text{ kJ}) = 551.8 \text{ kJ} \quad \xleftarrow{\hspace{10em}} E_Q$$

Now, from an exergy balance

$$\Delta E = E_Q - E_w^o - E_d$$

$$E_d = E_Q - \Delta E = 551.8 - 520.4 = 31.4 \text{ kJ} \quad \xleftarrow{\hspace{10em}} E_d$$

Discussion

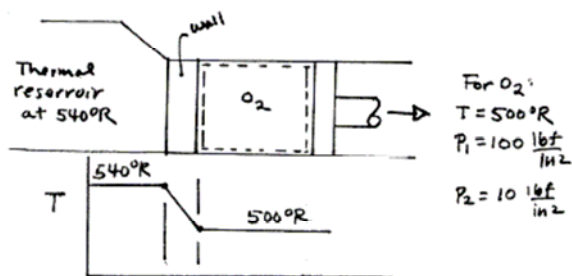
The exergy destruction for case (b) is significantly less than for case (a). Since the exergy change is the same in both cases, case (b) allows the exergy change to be brought about by less exergy transfer from the surroundings.

PROBLEM 7.48

KNOWN: One lb of O_2 expands isothermally while receiving energy by heat transfer through a wall separating the O_2 from a thermal reservoir.

FIND: (a) For the O_2 as the system, evaluate W , Q , the exergy transfers accompanying work and heat transfer, and the exergy destruction. (b) Evaluate the exergy destruction for a system consisting of the wall and O_2 .
Discuss:

SCHEMATIC & GIVEN DATA:



ENGINEER MODEL: 1. In part (a), the system is the O_2 only. In part (b), the system is the O_2 plus the wall. 2. O_2 is modeled as an ideal gas. 3. The state of the wall does not change. 4. Effects of motion and gravity are ignored. 5. $T_0 = 540^\circ R$, $P_0 = 15 \frac{\text{lbf}}{\text{in}^2}$.

ANALYSIS: (a) An energy balance reduces to $\Delta U = Q - W$. But $\Delta U = 0$ for an ideal gas at fixed temperature, giving $Q = W$. (1)

To find W , use the ideal gas model with

$$W = \int_1^2 p dV = \int_1^2 \frac{mRT}{V} dV = mRT \ln \frac{V_2}{V_1}. \quad \text{Since } P_1 V_1 = mRT, P_2 V_2 = mRT,$$

$$V_2/V_1 = \frac{P_1}{P_2} \Rightarrow W = mRT \ln \frac{P_1}{P_2}. \quad (2)$$

An exergy balance takes the form

$$\Delta E = \underbrace{\left[1 - \frac{T_0}{T}\right] Q}_{\text{Exergy transfer with heat}} - \underbrace{[W - P_0 \Delta V]}_{\text{Exergy transfer with work}} - \underbrace{E_d}_{\text{Exergy destruction}}$$

Alternatively

$$(\Delta U + P_0 \Delta V - T_0 \Delta S) = \left[1 - \frac{T_0}{T}\right] Q - [W - P_0 \Delta V] - E_d$$

With $\Delta U = 0$, $Q = W$, and cancellation of $P_0 \Delta V$ on both sides, we get

$$\begin{aligned} \textcircled{1} \quad E_d &= T_0 \left[\Delta S - \frac{Q}{T} \right] \\ &= T_0 \left[m \left[s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1} \right] - \frac{Q}{T} \right] \quad \left(\begin{array}{l} Q \text{ from Eqs. (1), (2)} \\ = 0 \text{ since } T = \text{constant} \end{array} \right) \end{aligned}$$

Accordingly, the O_2 undergoes an internally reversible process.

With Eqs. (1), (2)

$$Q = W = (1 \text{ lb}) \left(\frac{1.986 \text{ Btu}}{32 \text{ lb} \cdot \text{mol}} \right) (500^\circ R) \ln \left(\frac{100}{10} \right) = 71.452 \text{ Btu}$$

Using the ideal gas equation of state and introducing Eq. (2)

$$\begin{aligned} \left[\text{Exergy transfer associated with work} \right] &= W - P_0 \Delta V \\ &= mRT \ln \frac{P_1}{P_2} - P_0 \left[\frac{mRT}{P_2} - \frac{mRT}{P_1} \right] = mRT \left[\ln \frac{P_1}{P_2} - \frac{P_0}{P_2} + \frac{P_0}{P_1} \right] \\ &= (1 \text{ lb}) \left(\frac{1.986 \text{ Btu}}{32 \text{ lb} \cdot \text{mol}} \right) (500^\circ R) \left[\ln 10 - \frac{15}{10} + \frac{15}{100} \right] = 29.56 \text{ Btu} \end{aligned}$$

Continued on next slide

Problem 7-48 continued

$$\left[\begin{array}{l} \text{Exergy transfer} \\ \text{associated with heat} \end{array} \right] = \left[1 - \frac{T_0}{T} \right] Q$$

$$\textcircled{2} \textcircled{3} \quad = \left[1 - \frac{540}{500} \right] (71.452 \text{ Btu}) = -5.716 \text{ Btu} \quad \leftarrow$$

(h) For an enlarged system of O_2 plus wall, $E_d = T_0 \sigma$, where σ is the amount of entropy produced within the enlarged system. Using assumption 3,

$$\Delta S = \frac{Q}{T_{res}} + \sigma \quad \text{or} \quad (\Delta S)_{wall} + (\Delta S)_{O_2} = \frac{Q}{T_{res}} + \sigma \Rightarrow \quad \text{with Eq. (1), (2) and Assumption 3}$$

$$\sigma = (\Delta S)_{O_2} - \frac{Q}{T_{res}} = m \left[\frac{s^o(T_2) - s^o(T_1)}{=0} - R \ln \frac{P_2}{P_1} \right] - \frac{m R T \ln(P_1/P_2)}{T_{res}}$$

or

$$\textcircled{4} \textcircled{5} \quad \sigma = m R \ln(P_1/P_2) \left[1 - \frac{T}{T_{res}} \right] = (1 \text{ lb}) \left(\frac{1.986 \text{ Btu}}{32 \cdot 10 \cdot 0.2} \right) \ln 10 \left[1 - \frac{500}{540} \right]$$

$$= 1.059 \times 10^{-2} \text{ Btu/or}$$

$$\text{Finally, } E_d = T_0 \sigma = 540^\circ R (1.059 \times 10^{-2} \text{ Btu/or}) = 5.72 \text{ Btu} \quad \leftarrow$$

Exergy is destroyed within the enlarged system owing to spontaneous heat transfer through a finite temperature difference through the wall.

1. The term in brackets is the entropy production within the O_2 , as can be seen by applying the entropy balance to the system of part (a). For this system, we get $\sigma = 0$.

2. Since heat transfer to the system occurs at $T < T_0$, the accompanying exergy transfer is oppositely directed.

3. Inserting calculated values into exergy balance

$$\Delta E = (-5.716) - (29.56) - (0) = -35.28 \text{ Btu}$$

This also can be determined using $\Delta E = \Delta U + P_0 \Delta V - T_0 \Delta S$, as can be readily checked.

4. $\sigma \rightarrow 0$ so $T_{res} \rightarrow T$.

5. Since the O_2 undergoes an internally reversible process, the entropy production (and exergy destruction) can be evaluated by study of the wall. That is, an entropy balance for the wall reduces with assumption 3 to give

$$(\Delta S)_{wall}^0 = \frac{Q}{T_{res}} - \frac{Q}{T} + \sigma$$

$$\Rightarrow \sigma = Q \left[\frac{1}{T} - \frac{1}{T_{res}} \right] = 71.452 \text{ Btu} \left[\frac{1}{500} - \frac{1}{540} \right] \left(\frac{1}{0.2} \right)$$

$$= 1.059 \times 10^{-2} \text{ Btu/or}$$

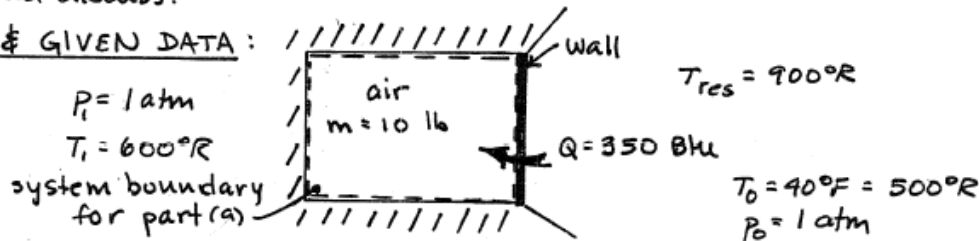
which agrees with the result reported here.

PROBLEM 7.49

KNOWN: Air at a known initial state is contained in a closed, rigid tank. The air receives a specified energy transfer by heat through a wall separating the gas from a thermal reservoir at 900°R .

FIND: (a) For the air as the system, determine the exergy change, the exergy transfer accompanying heat transfer, and the exergy destruction. (b) Evaluate the exergy destruction for an enlarged system that includes the wall and discuss.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) Closed systems are under consideration for which the effects of motion and gravity can be ignored. (2) The volume is constant and there is no work. (3) The air behaves as an ideal gas. (4) The state of the wall remains unchanged. (5) The air undergoes an internally reversible process.

ANALYSIS: (a) First, we apply the energy balance in order to fix state 2. That is

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - \cancel{W} \Rightarrow m(u_2 - u_1) = Q$$

Solving for u_2 and inserting data from Table A-22E

$$u_2 = \frac{Q}{m} + u_1 = \frac{350 \text{ Btu}}{10 \text{ lb}} + 102.34 \text{ Btu/lb} = 137.34 \text{ Btu/lb}$$

Interpolating in Table A-22E; $T_2 = 802.1^\circ\text{R}$, $s^\circ(T_2) = 0.69622 \text{ Btu/lb} \cdot ^\circ\text{R}$.

The exergy change of the air is

$$\Delta E_{\text{air}} = m [u(T_2) - u(T_1) + p_0(v_2 - v_1) - T_0(s^\circ(T_2) - s^\circ(T_1)) - R \ln p_2/p_1]$$

Noting that $p_2/p_1 = T_2/T_1$, and with $s^\circ(T_1)$ from Table A-22E

$$\Delta E_{\text{air}} = (10 \text{ lb}) \left[(137.34 - 102.34) \frac{\text{Btu}}{\text{lb}} - (500^\circ\text{R})(0.69622 - 0.62607) - \left(\frac{1.986}{28.97} \right) \ln \left(\frac{802.1}{600} \right) \right] \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

ΔE_{air}

Denoting the exergy transfer accompanying heat transfer as $E_{Q,\text{air}}$, the exergy balance gives

$$\Delta E_{\text{air}} = E_{Q,\text{air}} - (W - p_0 \Delta V) - E_{d,\text{air}}$$

With assumption 5,

$$E_{d,\text{air}} = 0$$

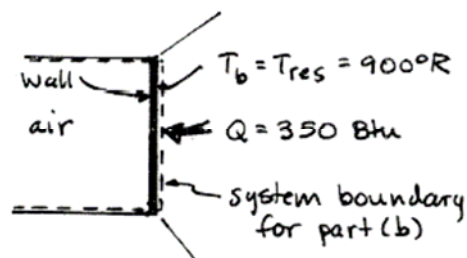
Thus,

$$E_{Q,\text{air}} = 98.76$$

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PROBLEM 7.49 (cont'd.)

(b) Applying the exergy balance to the enlarged system that includes the air and the wall



$$\cancel{\Delta E_{wall}^o} + \Delta E_{air} = \left(1 - \frac{T_o}{T_b}\right) Q - \cancel{(W - p_o \Delta V)^o} - (E_d)_{\text{enlarged system}}$$

Thus

$$(E_d)_{\text{enlarged system}} = \left(1 - \frac{T_o}{T_b}\right) Q - \Delta E_{air}$$

$$= \left(1 - \frac{500}{900}\right) (350 \text{ Btu}) - (98.76 \text{ Btu})$$

$$= 155.56 - 98.76 = 56.80 \text{ Btu} \quad \leftarrow (b)$$

Discussion

From the analysis of the enlarged system, the exergy transfer accompanying heat transfer to the wall is 155.56 Btu. This occurs at the reservoir temperature of 900°R. The exergy transfer from the wall to the air, $E_{Q,air}$, occurs at a much lower temperature, and amounts to only 98.76 Btu. The difference is accounted for by the exergy destruction associated with the temperature gradient across the wall.

PROBLEM 7.50

KNOWN: Various substances are at specified states at the inlet to a control volume.

FIND: In each case, determine the specific exergy and specific flow exergy.

ENGR. MODEL: (1) The velocity is relative to an exergy reference environment for which $T_0 = 20^\circ\text{C}$, $p_0 = 1\text{ bar}$. (2) The effects of gravity are negligible.

ANALYSIS: With assumption (2), the specific exergy is

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} \quad (1)$$

and the specific flow exergy is

$$e_f = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} \quad (2)$$

Using $h = u + pv$ and $h_0 = u_0 + p_0 v_0$, the relation between the two is

$$e_f = e + v(p - p_0) \quad (3)$$

(a) Water vapor at 100 bar, 520°C , 100 m/s . With $h_0 = h_f(20^\circ\text{C}) = 83.96\text{ kJ/kg}$ and $s_0 = s_f(25^\circ\text{C}) = 0.2966\text{ kJ/kg}\cdot\text{K}$ from Table A-2; and $h = 3425.1\text{ kJ/kg}$, $s = 6.6622\text{ kJ/kg}\cdot\text{K}$, and $v = 0.03394\text{ m}^3/\text{kg}$ from Table A-4, we get

$$e_f = (3425.1 - 83.96) \frac{\text{kJ}}{\text{kg}} - (293\text{ K})(6.6622 - 0.2966) \frac{\text{kJ}}{\text{kg}\cdot\text{K}} + \left(\frac{100^2}{2}\right) \frac{\text{m}^2}{\text{s}^2} \left| \frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right|$$

$$= 1481\text{ kJ/kg} \quad \leftarrow e_f$$

From (3) above

$$e = e_f - v(p - p_0) = 1481 \frac{\text{kJ}}{\text{kg}} - (0.03394 \frac{\text{m}^3}{\text{kg}})(100 - 1)\text{ bar} \left| \frac{10^5\text{ N/m}^2}{1\text{ bar}} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right|$$

$$= 1145\text{ kJ/kg} \quad \leftarrow e$$

(b) Ammonia at 3 bar, 0°C , 5 m/s . With data from Table A-15, $h_0 = 1515.8\text{ kJ/kg}$, $s_0 = 6.2816\text{ kJ/kg}\cdot\text{K}$, $h = 1454.43\text{ kJ/kg}$, $s = 5.5409\text{ kJ/kg}\cdot\text{K}$, $v = 0.42382\text{ m}^3/\text{kg}$. Thus

$$e_f = (1454.43 - 1515.8) - (293)(5.5409 - 6.2816) + \left(\frac{5^2}{2}\right) \left| \frac{1}{10^3} \right| = 155.67\text{ kJ/kg} \quad \leftarrow e_f$$

and

$$e = e_f - v(p - p_0) = 155.67 - (0.42382)(3 - 1) \left| \frac{10^5}{10^3} \right| = 70.91\text{ kJ/kg} \quad \leftarrow e$$

(c) Nitrogen as an ideal gas at 527°C (800 K), 50 bar , 200 m/s . Using ideal gas relations, Eq. (2) becomes

$$e_f = \left(\frac{1}{M}\right) \left[\bar{h}(T) - \bar{h}(T_0) - T_0 \{ \bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R} \ln \frac{p}{p_0} \} \right] + \frac{V^2}{2}$$

With data from Table A-23

$$e_f = \frac{1}{28.01} \left[(23,714 - 8521) - (293) \{ (220.907 - 190.805) - (8.314) \ln \left(\frac{50}{1} \right) \} \right] + \left(\frac{200^2}{2} \right) \left| \frac{1}{10^3} \right|$$

$$= 587.8\text{ kJ/kg} \quad \leftarrow e_f$$

Now, from (3) above

$$e = e_f - v(p - p_0) = e_f - RT \left[1 - \frac{p_0}{p} \right]$$

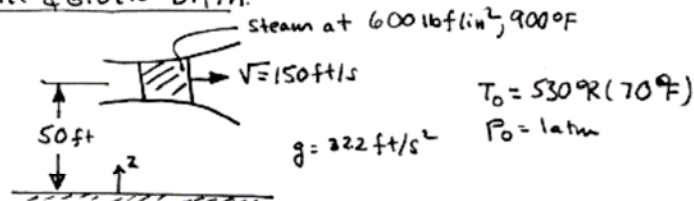
$$= 587.8 \frac{\text{kJ}}{\text{kg}} - \left(\frac{8.314\text{ kJ}}{28.01\text{ kg}\cdot\text{K}} \right) (800\text{ K}) \left[1 - \frac{1}{50} \right] = 355.1\text{ kJ/kg} \quad \leftarrow e$$

PROBLEM 7.51

KNOWN: Steam at a specified state is under consideration.

FIND: Determine the specific exergy and the specific flow exergy.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The velocity and elevation are each relative to the exergy reference environment for which $T_0 = 530^\circ\text{R}$ (70°F), $P_0 = 1\text{ atm}$, $v_0 \approx v_f(T_0)$, $u_0 \approx u_f(T_0)$, $h_0 \approx h_f(T_0)$, $s_0 \approx s_f(T_0)$.

ANALYSIS: The specific exergy is found from Eq. 7.2

$$e = (u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

With data from Tables A-2E, 4E

$$\begin{aligned} e &= (1318.4 - 38.09) \frac{\text{Btu}}{\text{lb}} + (14.7 \times 144 \frac{\text{lbf}}{\text{ft}^2})(1.302 - 0.01605) \frac{\text{ft}^3}{\text{lb}} \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\ &\quad - 530^\circ\text{R} (1.6766 - 0.07463) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} + \left[\frac{(150 \text{ ft/s})^2}{2} + (32.2 \text{ ft/s}^2)(50 \text{ ft}) \right] \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\ &= 1280.31 + 3.50 - 849.04 + 0.51 = 435.28 \text{ Btu/lb} \end{aligned}$$

① The specific flow exergy is found from Eq. 7.14 and data from Tables A-2E, 4E:

$$\begin{aligned} e_f &= (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \\ &= (1462.9 - 38.09) - 849.04 + 0.51 = 576.3 \text{ Btu/lb} \end{aligned}$$

1. Alternatively, using Eqs. 7.2 and 7.14, e_f and e can be related: $e_f = e + v(P - P_0)$, giving

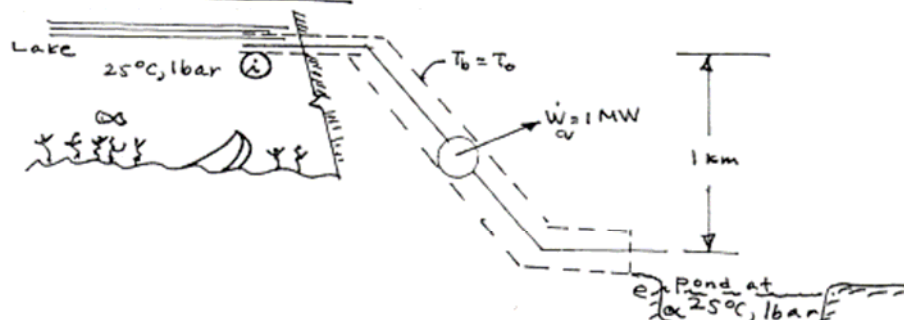
$$\begin{aligned} e_f &= e + v(P - P_0) = 435.28 \frac{\text{Btu}}{\text{lb}} + (1.302 \frac{\text{ft}^3}{\text{lb}})(600 - 14.7) \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{\text{ft}^2} \right| \left| \frac{\text{Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\ &= (435.28 + 141.05) \frac{\text{Btu}}{\text{lb}} = 576.3 \frac{\text{Btu}}{\text{lb}} \end{aligned}$$

PROBLEM 7.52

KNOWN: Water drawn from a lake flows through a hydraulic turbine-generator to a pond located 1 km below the lake.

FIND: Determine the minimum mass flow rate required to generate electricity at a rate of 1 MW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. Heat transfer (if any) occurs only at $T_b = T_0$. 2. At the exit e , the state corresponds to the dead state: $T_0, P_0, V_0 \sim 0, z_0 \sim, h_e = h_0, s_e = s_0$. 3. At the inlet i , kinetic energy can be ignored. 4. For the environment, $T_0 = 25^\circ\text{C}, P_0 = 1 \text{ bar}, g = 9.81 \text{ m/s}^2$.

ANALYSIS: Applying the steady-state control volume exergy rate balance, 7.13a,

$$0 = \sum_j \left[1 - \frac{T_0}{T_j} \right] \dot{Q}_j - \dot{W}_{cv} + \dot{m}(e_{fi} - e_{fe}) - \dot{E}_d$$

$= 0$ since $T_b = T_0$

$= 0$ since the stream is reduced to the dead state

$$\Rightarrow \frac{\dot{W}_{cv}}{\dot{m}} = e_{fi} - \frac{\dot{E}_d}{\dot{m}}$$

$$\Rightarrow \dot{m} = \frac{\dot{W}_{cv}}{e_{fi} - (\dot{E}_d/\dot{m})}$$

By inspection, for fixed $\dot{W}_{cv}$ and inlet state the minimum mass flow rate corresponds to $\dot{E}_d/\dot{m} = 0$. That is,

$$\Rightarrow \dot{m}_{\min} = \frac{\dot{W}_{cv}}{e_{fi}}$$

$$\text{where } e_{fi} = h_i - h_0 - T_0(s_i - s_0) + \frac{V_i^2}{2} + gz$$

$\Rightarrow 0$ because $P_i = P_0, T_i = T_0$
and $V_i^2/2$ is assumed negligible.

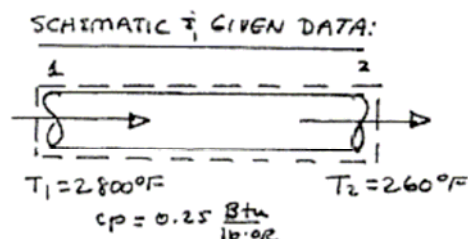
Then,

$$\dot{m}_{\min} = \frac{\dot{W}_{cv}}{gz} = \frac{(10^6 \text{ J/s}) \left| \frac{1 \text{ N} \cdot \text{m}}{1 \text{ J}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|}{(9.81 \text{ m/s}^2)(1000 \text{ m})}$$

$$= 101.9 \text{ kg/s}$$

PROBLEM 7.53

At steady state, hot gaseous products of combustion cool from 2800°F to 260°F as they flow through a pipe. Owing to negligible fluid friction, the flow occurs at nearly constant pressure. Applying the ideal gas model with $c_p = 0.25 \text{ Btu/lb} \cdot ^\circ\text{R}$, determine the exergy transfer accompanying heat transfer from the gas, in Btu per lb of gas flowing. Let $T_0 = 60^\circ\text{F}$ and ignore the effects of motion and gravity.



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. The flow is internally reversible.
Fluid friction is negligible and the gas flows at constant pressure.
3. The gas is modeled as an ideal gas with $c_p = 0.25 \text{ Btu/lb} \cdot ^\circ\text{R}$.
4. Ignore the effects of motion and gravity.
5. $T_0 = 520^\circ\text{R} (60^\circ\text{F})$.

ANALYSIS: Applying the steady-state control volume exergy rate balance, Eq. 7.13

$$0 = \dot{E}_q - \dot{E}_{cv} + \dot{m}[e_{f1} - e_{f2}] - \dot{E}_d$$

$$\Rightarrow \frac{\dot{E}_q}{\dot{m}} = e_{f2} - e_{f1}$$

$$= (h_2 - h_1) - T_0(s_2 - s_1)$$

$$= c_p[T_2 - T_1] - T_0 \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right]$$

$$\textcircled{1} = c_p \left[(T_2 - T_1) - T_0 \ln \frac{T_2}{T_1} \right]$$

$$= 0.25 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \left[(720 - 3260)^\circ\text{R} - 520^\circ\text{R} \ln \left(\frac{720}{3260} \right) \right]$$

$$= -438.7 \text{ Btu/lb}$$

1. Alternatively

$$\frac{\dot{E}_q}{\dot{m}} = \int_{T_1}^{T_2} \left[1 - \frac{T_0}{T} \right] \delta q_{rev}^{int}$$

From Eq. 6.49, $\delta q_{rev}^{int} = T ds$

From Eq. 6.10b, $T ds = dh - v dp$ (assumption 2)

$\Rightarrow \delta q_{rev}^{int} = dh \xrightarrow{\text{ideal gas}} c_p dT$

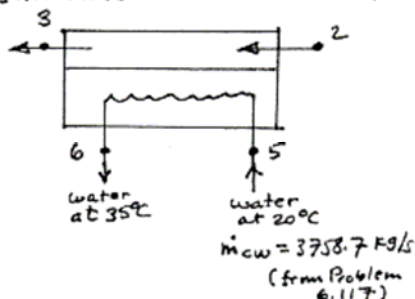
$$\Rightarrow \dot{E}_q = \int_{T_1}^{T_2} \left[1 - \frac{T_0}{T} \right] c_p dT = c_p \left[(T_2 - T_1) - T_0 \ln \frac{T_2}{T_1} \right]$$

PROBLEM 7.54

For the simple vapor power plant of Problem 6.117, evaluate, in MW, (a) the net rate energy exits the plant with the cooling water and (b) the net rate exergy exits the plant with the cooling water. Comment. Let $T_0 = 20^\circ\text{C}$, $p_0 = 1\text{ atm}$ and ignore the effects of motion and gravity.

SCHEMATIC & GIVEN DATA:

States numbered as in Problem 6.117.



For the power plant, the net power output
 $= 117\text{ MW}$ (Problem 6.117)

ENGR. MODEL:

- See model of Problem 6.117
- For the environment, $T_0 = 20^\circ\text{C}$,
 $p_0 = 1\text{ atm}$.

ANALYSIS:

- (a) The net rate energy exits with the cooling water

$$\begin{aligned}
 &= \dot{m} (h_6 - h_5) \\
 &= 3758.7 \frac{\text{kg}}{\text{s}} (146.68 - 83.96) \frac{\text{kJ}}{\text{kg}} \left| \frac{1\text{ MW}}{10^3 \text{ kJ/s}} \right| = 235.7\text{ MW} \quad \leftarrow
 \end{aligned}$$

$h \approx h(T)$

- (b) The net rate exergy exits with the cooling water

$$\begin{aligned}
 &= \dot{m} [(h_6 - h_5) - T_0 (s_6 - s_5)] \\
 &= 3758.7 \frac{\text{kg}}{\text{s}} [(146.68 - 83.96) - 293 (0.5053 - 0.2966)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1\text{ MW}}{10^3 \text{ kJ/s}} \right| \\
 &= 5.9\text{ MW} \quad \leftarrow
 \end{aligned}$$

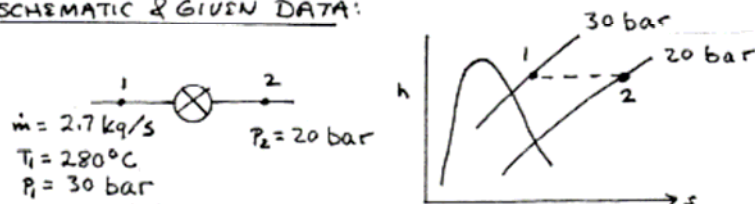
$s \approx s(T)$

COMMENT: Part (a) shows that considerable energy is carried out with the cooling water. However, since the exiting water temperature is only 35°C , it has little thermodynamic utility. Part (b) confirms this conclusion by quantifying the net exergy carried out. In keeping with expectations, the net exergy carried out with the cooling water is much less than the net power developed by the plant.

PROBLEM 7.55

KNOWN: Data are provided for the throttling of steam across a valve.
FIND: (a) Determine the flow exergy rates at the valve inlet and exit and the rate of exergy destruction. (b) Evaluate the annual cost associated with exergy destruction.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The expansion across the valve is a throttling process: $h_2 \approx h_1$. 2. For the environment, $T_0 = 298 \text{ K}$ (25°C), $P_0 = 1 \text{ atm}$. 3. Exergy is valued at $\$0.08/\text{kW}\cdot\text{h}$. 4. 8000 operating hours annually. 5. Ignore the effects of motion and gravity.

ANALYSIS: (a) With assumption 1, $h_2 \approx h_1$. Thus, state 2 is fixed by P_2, h_2 .

From Table A-4, $h_1 = 2941.3 \text{ kJ/kg}$, $s_1 = 6.4462 \text{ kJ/kg}\cdot\text{K}$, $s_2 = 6.6169 \text{ kJ/kg}\cdot\text{K}$.

Table A-2 gives $h_0 \approx h_f(T_0) = 104.89 \text{ kJ/kg}$, $s_0 \approx s_f(T_0) = 0.3674 \text{ kJ/kg}\cdot\text{K}$.

Then, with assumption 5, Eq. 7.14 gives

$$e_{f1} = h_1 - h_0 - T_0(s_1 - s_0) = (2941.3 - 104.89) - 298(6.4462 - 0.3674) = 1024.9 \text{ kJ/kg}$$

$$e_{f2} = h_2 - h_0 - T_0(s_2 - s_0) = (2941.3 - 104.89) - 298(6.6169 - 0.3674) = 974.1 \text{ kJ/kg}$$

The associated rates are

$$\dot{E}_{f1} = \dot{m} e_{f1} = (2.7 \text{ kg/s})(1024.9 \text{ kJ/kg}) = 2767.2 \text{ kW}$$

$$\dot{E}_{f2} = \dot{m} e_{f2} = (2.7)(974.1) = 2630.1 \text{ kW}$$

An exergy rate balance reduces to give

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W}^0 + (\dot{E}_{f1} - \dot{E}_{f2}) - \dot{E}_d$$

$$\Rightarrow \dot{E}_d = \dot{E}_{f1} - \dot{E}_{f2} = (2767.2 - 2630.1) = 132.1 \text{ kW}$$

(b) Evaluating exergy at $\$0.08/\text{kW}\cdot\text{h}$, the cost associated with the exergy destroyed for 8000 hours of operation annually is

$$\dot{\$} = (132.1 \text{ kW}) \left(\frac{8000 \text{ h}}{\text{year}} \right) \left(\frac{\$0.08}{\text{kW}\cdot\text{h}} \right)$$

$$\textcircled{1} \quad = \$84,540/\text{year}$$

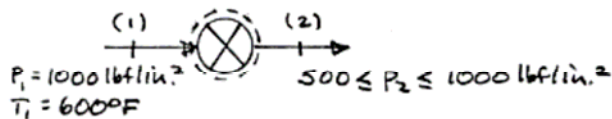
1. The irreversibility associated with the expansion across the valve can be reduced by replacing the valve with a power-recovery turbine. However, the costs of the turbine and its operation would have to be considered carefully before such a replacement would be approved.

PROBLEM 7.56

KNOWN: Steam at a known state enters a valve operating at steady state and undergoes a throttling process to pressure P_2 .

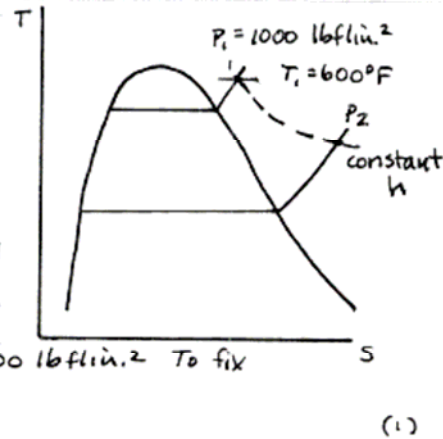
FIND: (a) For $P_2 = 500 \text{ lbf/in}^2$, determine the exit temperature and the exergy destruction per unit of steam flowing. (b) Plot these quantities versus P_2 ranging from 500 to 1000 lbf/in^2 .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown is at steady state. (2) The steam undergoes a throttling process in passing through the valve. (3) For the reference environment, $T_0 = 70^\circ\text{F}$, $P_0 = 14.7 \text{ lbf/in}^2$.

ANALYSIS: State 1 is fixed by $T_1 = 600^\circ\text{F}$, $P_1 = 1000 \text{ lbf/in}^2$. To fix state 2, we use assumption (2) to get $h_1 = h_2$



Thus, with P_2 and h_2 , state 2 is fixed as well and T_2 can be determined.

To get $\dot{E}_d/\dot{m}$, we begin with an exergy balance at steady state

$$0 = \sum_j (1 - \frac{T_0}{T_j}) \dot{Q}_j - \dot{W} + \dot{m} [e_{f1} - e_{f2}] - \dot{E}_d$$

or
$$\dot{E}_d/\dot{m} = e_{f1} - e_{f2} = (h_1 - h_2) - T_0 (s_1 - s_2) \quad (2)$$

- ① (a) From Table A-4E; $h_1 = 1248.8 \text{ Btu/lb}$ and $s_1 = 1.4450 \text{ Btu/lb} \cdot ^\circ\text{R}$. Interpolating in Table A-4E at $P_2 = 500 \text{ lbf/in}^2$, $h_2 = h_1 = 1248.8 \text{ Btu/lb}$ we get

$$T_2 = 524.6^\circ\text{F} \quad (a) T_2$$

$$s_2 = 1.5098 \text{ Btu/lb} \cdot ^\circ\text{R}$$

Now, from Eq. (2) above

$$\begin{aligned} \dot{E}_d/\dot{m} &= -T_0 (s_1 - s_2) \\ &= -(530^\circ\text{R})(1.4450 - 1.5098) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \\ &= 34.34 \text{ Btu/lb} \quad (a) \dot{E}_d/\dot{m} \end{aligned}$$

(b) The data to construct the required plots are obtained using IT, as follows:

Continued on next slide

Problem 7-56 continued

IT Code

```
p1 = 1000 // lbf/in.2
T1 = 600 // °F
p2 = 500 // lbf/in.2
To = 530 // °R
```

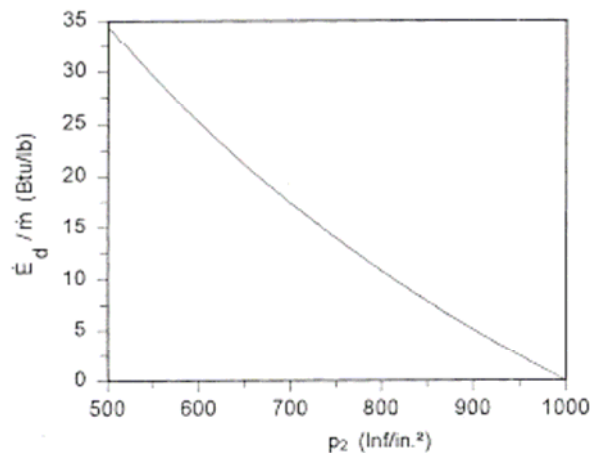
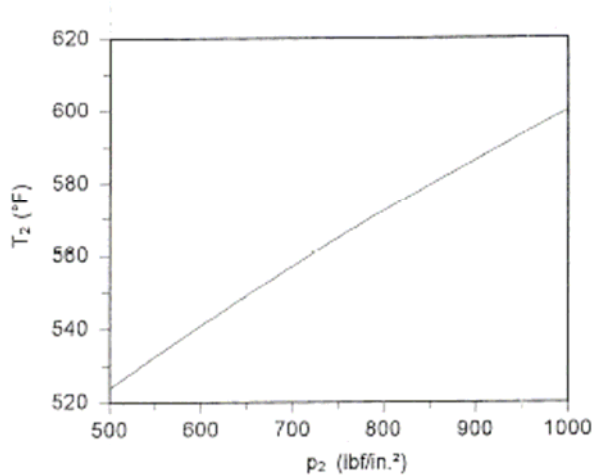
```
h1 = h_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
h1 = h2
s1 = s_PT("Water/Steam", p1, T1)
s2 = s_PT("Water/Steam", p2, T2)
```

$$\dot{E}_d = -T_o (s_1 - s_2)$$

IT Results for $p_2 = 500$ lbf/in.²

```
h1 = 1249 Btu/lb
s1 = 1.445 Btu/lb·°R
s2 = 1.51 Btu/lb·°R
 $\dot{E}_d / \dot{m} = 34.43$  Btu/lb
T2 = 523.8 °F
```

Plots:



Discussion

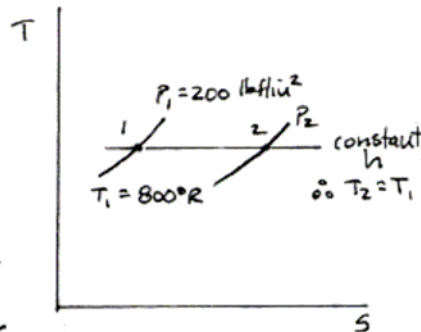
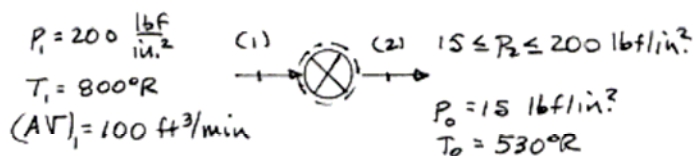
- As seen on the accompanying T-s diagram, the exit temperature decreases as p_2 decreases for fixed h . Thus, the plot of exit temperature exhibits expected behavior.
- The rate of exergy destruction per unit mass of steam flowing increases with decreasing p_2 . This is also expected based on greater entropy production and hence increasing s_2 (See the T-s diagram.)

PROBLEM 7.57

KNOWN: Air at a given state enters a valve operating at steady state with a known volumetric flow rate. The air undergoes a throttling process to exit pressure p_2 .

FIND: (a) Determine the rate of exergy destruction for $p_2 = 15 \text{ lbf/in}^2$. (b) Plot the exergy destruction rate versus p_2 ranging from 15 to 200 lbf/in^2 .

SCHEMATIC & GIVEN DATA:



ENG. MODEL: (1) The control volume shown is at steady state. (2) The air undergoes a throttling process in passing through the valve. (3) The air is modeled as an ideal gas. (4) For the reference environment, $T_0 = 530^\circ\text{R}$, $P_0 = 15 \text{ lbf/in}^2$.

ANALYSIS: State 1 is fixed by $T_1 = 800^\circ\text{R}$, $P_1 = 200 \text{ lbf/in}^2$. To fix state 2, we use assumption (2) to get $h_1 = h_2$. By assumption (3)

$$T_2 = T_1 \quad (1)$$

To get $\dot{E}_d$, begin with an exergy balance at steady state. With $\dot{m}_1 = \dot{m}_2 = \dot{m}$

$$0 = \sum_j \left[1 - \frac{T_0}{T_j} \right] \dot{Q}_j - \dot{W} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$$

$$\text{or} \quad \dot{E}_d = \dot{m}(e_{f1} - e_{f2}) = \dot{m}[(h_1 - h_2) - T_0(s_1 - s_2)] \quad (2)$$

To get $\dot{m}$, we use $\dot{m} = (AV)_1/v_1$ and the ideal gas equation for v_1 .

(a) Evaluating $\dot{m}$

$$\dot{m} = \frac{P_1 (AV)_1}{RT_1} = \frac{(200 \text{ lbf/in}^2)(100 \text{ ft}^3/\text{min})}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)(800^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 67.5 \text{ lb/min}$$

Using ideal gas relations with Eq. (2) and inserting values

$$\begin{aligned} \dot{E}_d &= -\dot{m} T_0 [s^\circ(T_1) - s^\circ(T_2) - R \ln P_1/P_2] \\ &= (67.5 \frac{\text{lb}}{\text{min}})(530^\circ\text{R}) \left[-\frac{1.986 \text{ Btu}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \ln \left(\frac{200}{15} \right) \right] \\ &= 6353 \text{ Btu/min} \quad (a) \dot{E}_d \end{aligned}$$

(b) The data to construct the required plot are obtained using IT, as follows. For the IT solution, $v_1(T_1, P_1)$ and the entropy values $s_1(T_1, P_1)$ and $s_2(T_2, P_2)$ are evaluated directly using internal IT property functions.

Continued on next slide

Problem 7-57 continued

IT Code

```
p1 = 200 // lbf/in.2
T1 = 800 // °R
p2 = 15 // lbf/in.2
AV1 = 100 // ft3/min
To = 530 // °R
```

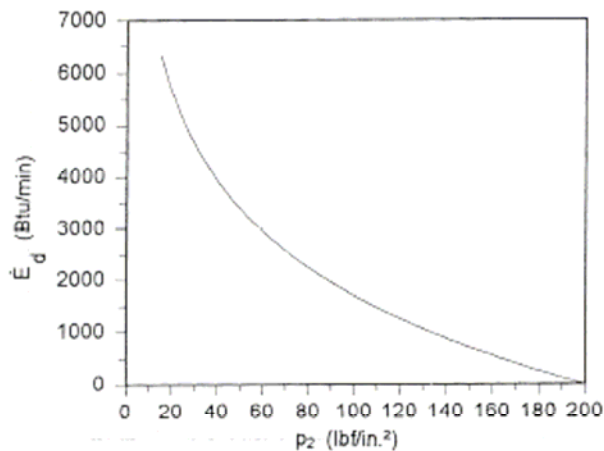
```
T2 = T1
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)

mdot = AV1 / v1
Edot = - mdot * To * (s1 - s2)
```

IT Results for $p_2 = 15$ lbf/in.²

```
v1 = 1.481 ft3/lb
s1 = 0.5167 Btu/lb·°R
s2 = 0.6941 Btu/lb·°R
ṁ = 67.5 lb/min
Ėd = 6347 Btu/min
```

PLOT:



From the plot, we see that the rate of exergy destruction increases with decreasing p_2 . This is expected based on greater entropy production and hence increasing s_2 . (See the T-s diagram.)

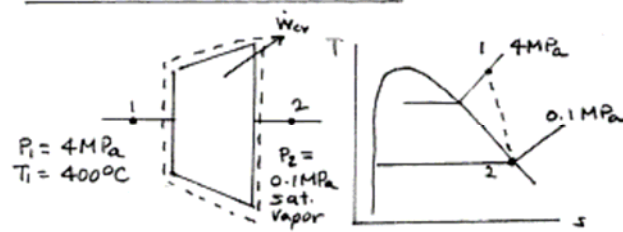
PROBLEM 7.58

Water vapor at 4.0 MPa and 400°C enters an insulated turbine operating at steady state and expands to saturated vapor at 0.1 MPa. The effects of motion and gravity can be neglected. Determine the work developed and the exergy destruction, each in kJ per kg of water vapor passing through the turbine. Let $T_0 = 27^\circ\text{C}$, $p_0 = 0.1\text{ MPa}$.

ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ and the effects of motion and gravity can be ignored.
3. For the environment, $T_0 = 300\text{ K}$ (27°C), $p_0 = 0.1\text{ MPa}$.

SCHEMATIC & GIVEN DATA:



ANALYSIS: (a) Reducing mass and energy rate balances, we get

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 = (3213.6 - 2675.5) \frac{\text{kJ}}{\text{kg}} = 538.1 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow$$

Reducing mass and entropy rate balances, the rate of entropy production is

$$\frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1 = (7.3594 - 6.9690) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.3904 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

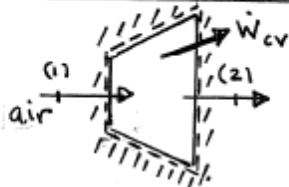
The rate of exergy destruction is then

$$\frac{\dot{E}_d}{\dot{m}} = T_0 \left(\frac{\dot{\sigma}}{\dot{m}} \right) = 300\text{ K} (0.3904) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 117.12 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow$$

PROBLEM 7.59

Air enters an insulated turbine operating at steady state at 8 bar, 500 K, and 150 m/s. At the exit the conditions are 1 bar, 320 K, and 10 m/s. There is no significant change in elevation. Determine the work developed and the exergy destruction, each in kJ per kg of air flowing. Let $T_0 = 300 \text{ K}$, $p_0 = 1 \text{ bar}$.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} P_1 &= 8 \text{ bar} & P_2 &= 1 \text{ bar} \\ T_1 &= 500 \text{ K} & T_2 &= 320 \text{ K} \\ V_1 &= 150 \text{ m/s} & V_2 &= 10 \text{ m/s} \\ T_0 &= 300 \text{ K}, & p_0 &= 1 \text{ bar} \end{aligned}$$

ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ and the effect of gravity can be ignored.
3. For the environment, $T_0 = 300 \text{ K}$, $p_0 = 1 \text{ bar}$.

ANALYSIS: The mass and energy rate balances reduce with assumptions (1)–(3) to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \Rightarrow \frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 + \frac{(V_1^2 - V_2^2)}{2} \quad (1)$$

With data from Table A-22

$$\frac{\dot{W}_{cv}}{\dot{m}} = (503.02 - 320.29) \frac{\text{kJ}}{\text{kg}} + \left(\frac{150^2 - 10^2}{2} \right) \frac{\text{m}^2}{\text{s}^2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 193.93 \frac{\text{kJ}}{\text{kg}} \leftarrow (\dot{W}_{cv}/\dot{m})$$

The exergy destruction rate is found using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ and an entropy balance.

$$0 = \sum \left(\frac{\dot{Q}_j}{T_j} \right) + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \frac{\dot{E}_d}{\dot{m}} = T_0 (s_2 - s_1) \quad (2)$$

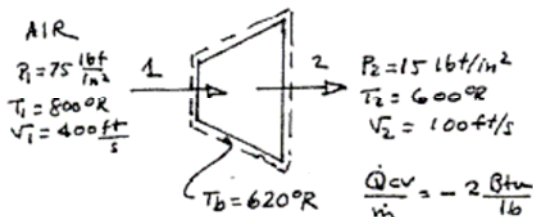
With ideal gas relations and data from Table A-22

$$\begin{aligned} \frac{\dot{E}_d}{\dot{m}} &= T_0 [s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1] \\ &= (300 \text{ K}) [(1.76690 - 2.21952) - \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) \ln \left(\frac{1}{8} \right)] = 43.25 \text{ kJ/kg} \leftarrow \frac{\dot{E}_d}{\dot{m}} \end{aligned}$$

PROBLEM 7.60

Air enters a turbine operating at steady state with a pressure of 75 lbf/in.², a temperature of 800°R, and a velocity of 400 ft/s. At the turbine exit, the conditions are 15 lbf/in.², 600°R, and 100 ft/s. Heat transfer from the turbine to its surroundings takes place at an average surface temperature of 620°R. The rate of heat transfer is 2 Btu per lb of air passing through the turbine. For the turbine, determine the work developed and the exergy destruction, each in Btu per lb of air flowing. Let $T_0 = 40^\circ\text{F}$, $p_0 = 15 \text{ lbf/in.}^2$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. Heat transfer takes place at temperature T_b .
3. The effect of gravity can be ignored.
4. Air is modeled as an ideal gas. (This can be checked using the generalized compressibility chart.)

ANALYSIS: At steady state an energy rate balance reduces to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$

or

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} \quad (1)$$

With data from Table A-22E

$$\begin{aligned} \frac{\dot{W}_{cv}}{\dot{m}} &= -2 \frac{\text{Btu}}{\text{lb}} + (91.81 - 143.47) \frac{\text{Btu}}{\text{lb}} + \frac{[(400)^2 - (100)^2] \left(\frac{\text{ft}^2}{\text{s}^2} \right)}{(2) 32.2 \frac{\text{lb} \cdot \text{ft}}{\text{lb} \cdot \text{s}^2} \cdot 778 \frac{\text{ft} \cdot \text{lbf}}{\text{Btu}}} \\ &= -2 \frac{\text{Btu}}{\text{lb}} + 48.34 \frac{\text{Btu}}{\text{lb}} + 2.99 \frac{\text{Btu}}{\text{lb}} = 49.33 \frac{\text{Btu}}{\text{lb}} \leftarrow \frac{\dot{W}_{cv}}{\dot{m}} \end{aligned}$$

The exergy destruction can be evaluated by reducing an exergy rate balance. Thus

$$\frac{\dot{E}_d}{\dot{m}} = \left[1 - \frac{T_0}{T_b} \right] \left(\frac{\dot{Q}_{cv}}{\dot{m}} \right) - \frac{\dot{W}_{cv}}{\dot{m}} + \left[(h_1 - h_2) - T_0 (s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} \right]$$

Introducing Eq. (1) and simplifying

$$\textcircled{1} \quad \frac{\dot{E}_d}{\dot{m}} = T_0 \left[(s_2 - s_1) - \frac{\dot{Q}_{cv}/\dot{m}}{T_b} \right] \quad (2)$$

Or, for an ideal gas (assumption 5)

$$\frac{\dot{E}_d}{\dot{m}} = T_0 \left[(s^\circ(T_2) - s^\circ(T_1)) - \frac{\bar{R}}{M} \ln \frac{P_2}{P_1} \right] - \frac{\dot{Q}_{cv}/\dot{m}}{T_b} \quad (3)$$

With data from Table A-22E

$$\begin{aligned} \frac{\dot{E}_d}{\dot{m}} &= 500^\circ\text{R} \left[(0.62607 - 0.69558) - \frac{1.986}{28.97} \ln \frac{15}{75} \right] \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} - \frac{(-2 \text{ Btu/lb})}{620} \\ &= 500^\circ\text{R} [0.04082 + 0.0032] \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 22.01 \frac{\text{Btu}}{\text{lb}} \leftarrow \frac{\dot{E}_d}{\dot{m}} \end{aligned}$$

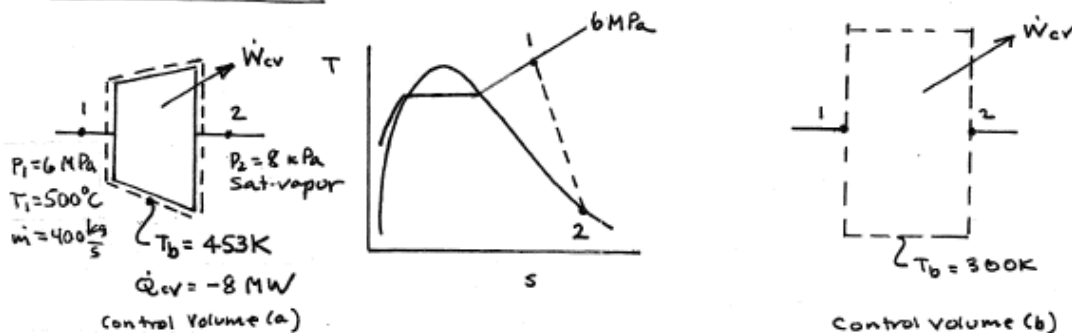
1. This corresponds to $\frac{\dot{E}_d}{\dot{m}} = T_0 \frac{\dot{Q}}{\dot{m}}$

PROBLEM 7.61

KNOWN: Steady-state operating data are provided for a steam turbine.

FIND: Determine the power developed. For each of two choices for the control volume, evaluate the rate of exergy destruction. Discuss.

SCHEMATIC & GIVEN DATA:



Engr. Model: 1. The control volumes shown in the schematic are at steady state. 2. In each case, heat transfer occurs at T_b . 3. Effects of motion and gravity can be ignored. 4. For the environment, $T_0 = 300\text{ K}$, $P_0 = 100\text{ kPa}$.

ANALYSIS: (a) Mass and energy rate balances reduce at steady state to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}(h_1 - h_2).$$

Then, with data from Tables A-3, 4

$$\dot{W}_{cv} = (-8\text{ MW}) + (400\text{ kg/s})[3422.2 - 2577.0]\text{ kJ/kg} \left| \frac{1\text{ MW}}{10^3\text{ kJ/s}} \right| = 330.1\text{ MW} \leftarrow$$

The rate of exergy destruction can be obtained by reducing an exergy rate balance or using $\dot{E}_d = T_0 \dot{\sigma}$. Selecting the second, an entropy rate balance reads

$$0 = \frac{\dot{Q}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b} + \dot{m}(s_2 - s_1)$$

Then,

$$\begin{aligned} \dot{\sigma} &= -\left(\frac{-8\text{ MW}}{453\text{ K}}\right) + (400\text{ kg/s})[8.2287 - 6.8803]\text{ kJ/kg} \left| \frac{1\text{ MW}}{10^3\text{ kJ/s}} \right| \\ &= 0.55702\text{ MW/K} \Rightarrow \dot{E}_d = (300\text{ K})(0.55702\text{ MW/K}) = 167.1\text{ MW} \end{aligned}$$

(b) For the enlarged control volume, the entropy rate balance reads

$$\begin{aligned} \dot{\sigma} &= -\frac{\dot{Q}}{T_b} + \dot{m}(s_2 - s_1) \\ &= -\left(\frac{-8\text{ MW}}{300\text{ K}}\right) + (400\text{ kg/s})[8.2287 - 6.8803]\text{ kJ/kg} \left| \frac{1}{10^3} \right| = 0.56603\text{ MW/K} \end{aligned}$$

$$\Rightarrow \dot{E}_d = (300\text{ K})(0.56603\text{ MW/K}) = 169.8\text{ MW}$$

The exergy destruction in control volume (b) is greater because there is an additional source of irreversibility — namely, the irreversibility related to heat transfer from the outer surface of the turbine to the ambient. For control volume (a), the heat transfer effect is an external irreversibility.

1. Using values obtained in the analysis, the net rate exergy is supplied to the turbine by the steam, $(\dot{E}_{f1} - \dot{E}_{f2})$, is 499.9 MW. Since $[\dot{E}_d / (\dot{E}_{f1} - \dot{E}_{f2})] = [169.8 / 499.9] = 0.34$ (34%), there is considerable scope in this case for improving thermodynamic performance.

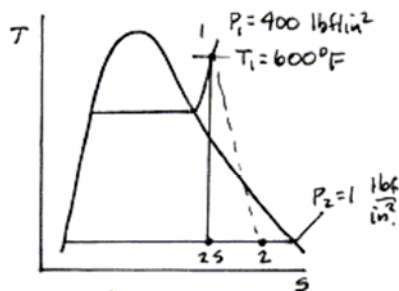
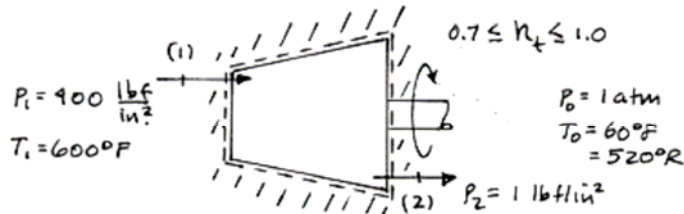
→ 1.

PROBLEM 7.62

KNOWN: An insulated steam turbine operating at steady state receives steam at a known state and exhausts at a given pressure.

FIND: Plot the exergy destruction rate per unit of steam flowing versus isentropic turbine efficiency ranging from 70 to 100%.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown is at steady state. (2) For the turbine, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be neglected. (3) For the exergy reference environment, $T_0 = 520^\circ\text{R}$, $P_0 = 1\text{ atm}$.

ANALYSIS: To determine the exergy destruction rate, we begin with mass and entropy rate balances which reduce to give

$$0 = \sum_j \left(\frac{\dot{Q}_j}{T_j} \right) + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv}/\dot{m} = (s_2 - s_1)$$

With $\dot{E}_d = T_0 \dot{\sigma}_{cv}$

$$\dot{E}_d/\dot{m} = T_0(s_2 - s_1) \quad (1)$$

Since $p_1 = 400\text{ lbf/in}^2$, $T_1 = 600^\circ\text{F}$, state 1 is in the superheated vapor region and the state is fixed. To fix state 2, we use the isentropic turbine efficiency

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) \quad (2)$$

where h_{2s} is determined using $P_2 = 1\text{ lbf/in}^2$ and $s_{2s} = s_1$. Then, with h_2 known from Eq. (2) and P_2 , s_2 can be determined.

Sample calculation: **CASE: $\eta_t = 0.7$** From Table A-4E, $h_1 = 1306.6\text{ Btu/lb}$ and $s_1 = 1.5892\text{ Btu/lb} \cdot ^\circ\text{R}$. With $s_{2s} = s_1 = 1.5892$

$$x_{2s} = \frac{s_{2s} - s_{f2}}{s_{fg2}} = \frac{1.5892 - 0.1327}{1.8453} = 0.7893$$

$$\text{and } h_{2s} = h_{f2} + x_{2s} h_{fg2} = 69.74 + (0.7893)(1036.0) = 887.45\text{ Btu/lb}$$

From (2)

$$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1306.6 - (0.7)(1306.6 - 887.45) = 1013.2\text{ Btu/lb}$$

$$\text{Finally, with } P_2 = 1\text{ lbf/in}^2; \quad x_2 = \frac{h_2 - h_{f2}}{h_{fg2}} = 0.9107 \text{ and } s_2 = s_{f2} + x_2 s_{fg2} = 1.8132 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

Thus

$$\dot{E}_d/\dot{m} = T_0(s_2 - s_1) = (520^\circ\text{R})(1.8132 - 1.5892) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

$$= 116.5\text{ Btu/lb} \quad \leftarrow \quad \dot{E}_d/\dot{m} \quad (\eta_t = 70\%)$$

Continued on next slide

Problem 7-62 continued

The data to construct the required plot are obtained using IT, as follows:

IT Code

$$p1 = 400 \text{ // lbf/in.}^2$$

$$T1 = 600 \text{ // } ^\circ\text{F}$$

$$p2 = 1 \text{ // lbf/in.}^2$$

$$\text{eff} = 0.7$$

$$T0 = 60 + 460 \text{ // } ^\circ\text{R}$$

$$h1 = h_PT(\text{"Water/Steam"}, p1, T1)$$

$$s1 = s_PT(\text{"Water/Steam"}, p1, T1)$$

$$s2s = s1$$

$$h2s = h_Ps(\text{"Water/Steam"}, p2, s2s)$$

$$h2 = h1 - \text{eff} * (h1 - h2s)$$

$$Ed = T0 * (s2 - s1)$$

$$h2 = h_Ps(\text{"Water/Steam"}, p2, s2)$$

IT Results for $\eta_t = 70\%$

$$h1 = 1306 \text{ Btu/lb}$$

$$h2 = 1013 \text{ Btu/lb}$$

$$h2s = 887.3 \text{ Btu/lb}$$

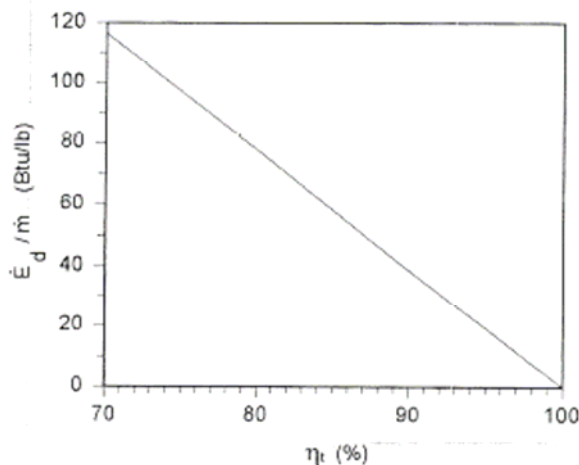
$$s1 = 1.589 \text{ Btu/lb} \cdot ^\circ\text{R}$$

$$s2 = 1.813 \text{ Btu/lb} \cdot ^\circ\text{R}$$

$$s2s = 1.589 \text{ Btu/lb} \cdot ^\circ\text{R}$$

$$\dot{E}_d / \dot{m} = 116.5 \text{ Btu/lb}$$

PLOT:

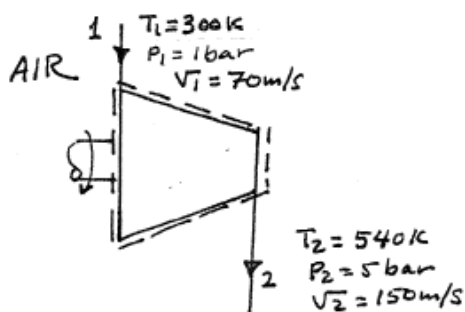


From the plot we see that lower values of isentropic turbine efficiency correspond to increased exergy destruction, as expected.

PROBLEM 7.63

Air enters a compressor operating at steady state at $T_1 = 300 \text{ K}$, $p_1 = 1 \text{ bar}$ with a velocity of 70 m/s . At the exit, $T_2 = 540 \text{ K}$, $p_2 = 5 \text{ bar}$ and the velocity is 150 m/s . The air can be modeled as an ideal gas with $c_p = 1.01 \text{ kJ/kg} \cdot \text{K}$. Stray heat transfer can be ignored. Determine, in $\text{kJ per kg of air flowing}$, (a) the power required by the compressor and (b) the rate of exergy destruction within the compressor. Let $T_0 = 300 \text{ K}$, $p_0 = 1 \text{ bar}$. Ignore the effects of motion and gravity.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ and the effects of gravity can be ignored.
3. The air can be modeled as an ideal gas with $c_p = 1.01 \text{ kJ/kg} \cdot \text{K}$.
4. For the environment, $T_0 = 300 \text{ K}$, $p_0 = 1 \text{ bar}$.

ANALYSIS: (a) Reducing an energy rate balance: $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left((h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right)$

$$\begin{aligned} \Rightarrow \frac{\dot{W}_{cv}}{\dot{m}} &= (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} \\ &= c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2} \\ &= 1.01 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (-240 \text{ K}) + \left[\frac{(70)^2 - (150)^2}{2} \right] \left(\frac{\text{m}}{\text{s}} \right)^2 \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= -[242.4 + 8.8] \frac{\text{kJ}}{\text{kg}} = -251.2 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow (a) \end{aligned}$$

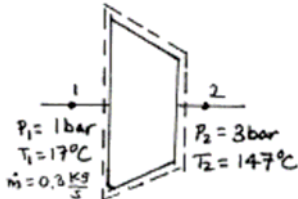
- (b) The rate of exergy destruction is $\dot{E}_d = T_0 \dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production. From an entropy rate balance, $\dot{\sigma} = \dot{m}(s_2 - s_1)$. Thus,

$$\begin{aligned} \frac{\dot{E}_d}{\dot{m}} &= T_0(s_2 - s_1) = T_0 \left(c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right) \\ &= 300 \text{ K} \left[1.01 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln \frac{540 \text{ K}}{300 \text{ K}} - \frac{8.314}{28.97} \ln \frac{5 \text{ bar}}{1 \text{ bar}} \right] \\ &= 39.5 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow (b) \end{aligned}$$

PROBLEM 7.64

KNOWN: Steady-state operating data are provided for an air compressor.
FIND: Determine the power required by the compressor and the rate of energy destruction.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state, $\dot{Q}_{cv} = 0$.
2. Effects of motion and gravity can be ignored.
- ① 3. Air is modeled as an ideal gas.
4. $T_0 = 290\text{ K}$, $P_0 = 1\text{ bar}$

ANALYSIS: Mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} [(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)]$$

with data from Table A-22.

$$\dot{W}_{cv} = \dot{m} (h_1 - h_2) = (0.3 \frac{\text{kg}}{\text{s}}) [290.16 - 421.26] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -39.33 \text{ kW} \leftarrow \dot{W}_{cv}$$

The rate of energy destruction can be obtained by reducing an energy rate balance or using $\dot{E}_d = T_0 \dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production from an entropy balance. Using the entropy approach,

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m} (s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\begin{aligned} \Rightarrow \dot{\sigma}_{cv} &= \dot{m} (s_2 - s_1) = \dot{m} [s^0(T_2) - s^0(T_1) - R \ln P_2/P_1] \\ &= (0.3 \frac{\text{kg}}{\text{s}}) \left[2.04142 - 1.66802 - \frac{8.314}{28.97} \ln \frac{3}{1} \right] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 0.0174 \frac{\text{KW}}{\text{K}} \end{aligned}$$

Then,

$$\dot{E}_d = (290 \text{ K}) (0.0174 \frac{\text{KW}}{\text{K}}) = 5.05 \text{ kW} \leftarrow \dot{E}_d$$

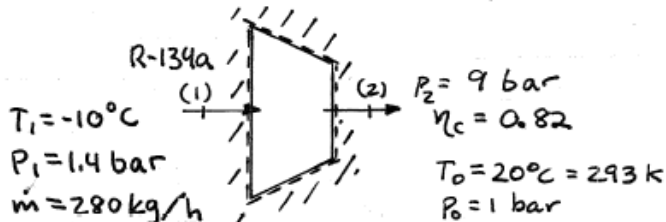
1. Can be checked using the generalized compressibility chart.

PROBLEM 7.65

KNOWN: Operating data are provided for an insulated compressor at steady state.

FIND: Determine (a) the exit temperature, (b) the power input, and (c) the exergy destruction rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be neglected. (3) For the exergy reference environment, $T_0 = 293 \text{ K}$, $P_0 = 1 \text{ bar}$.

ANALYSIS: (a) To find the exit temperature, we begin with the isentropic compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c}$$

From Table A-12, $h_1 = 243.40 \text{ kJ/kg}$, $s_1 = 0.9606 \text{ kJ/kg}\cdot\text{K}$. Interpolating in Table A-12 with $P_2 = 9 \text{ bar}$ and $s_{2s} = s_1 = 0.9606$; $h_{2s} = 283.65 \text{ kJ/kg}$. Thus

$$h_2 = 243.40 + \frac{(283.65 - 243.40)}{0.82} = 292.49 \text{ kJ/kg}$$

Interpolating again in Table A-12 with $P_2 = 9 \text{ bar}$, $h_2 = 292.49 \text{ kJ/kg}$

$$T_2 = 59.3^\circ\text{C} \quad T_2$$

$$s_2 = 0.9875 \text{ kJ/kg}\cdot\text{K}$$

(b) Applying energy and mass rate balances with assumptions (1) and (2)

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$

Solving for the power input, $(-\dot{W}_{cv})$, and inserting values

$$-(\dot{W}_{cv}) = \dot{m}(h_2 - h_1)$$

$$= (280 \frac{\text{kg}}{\text{h}}) \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| (292.49 - 243.40) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 3.818 \text{ kW} \quad (-\dot{W}_{cv})$$

(c) The exergy destruction rate is conveniently evaluated using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy balance

$$0 = \sum_j \left(\frac{\dot{Q}_j}{T_j} \right) + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{E}_d = T_0 \dot{m}(s_2 - s_1)$$

Thus

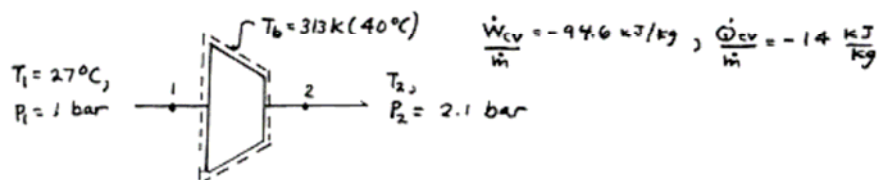
$$\dot{E}_d = (293 \text{ K}) (280 \frac{\text{kg}}{\text{h}}) (0.9875 - 0.9606) \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 0.613 \text{ kW}$$

PROBLEM 7.66

KNOWN: Operating data are provided for an air compressor at steady state.

FIND: Determine the temperature of the air exiting the compressor and the rate of energy destruction per kg of air flowing

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure is at steady state. (2) Heat transfer takes place at T_b only. (3) Kinetic and potential energy changes from inlet to exit can be ignored. (4) Air is modeled as an ideal gas. (5) For the environment, $T_0 = 293 \text{ K } (20^\circ\text{C})$, $P_0 = 1 \text{ atm}$.

ANALYSIS: (a) At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and an energy rate balance reduces with assumption 3

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right)$$

or

$$h_2 = h_1 + \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} = (300.19 - 14 + 94.6) \frac{\text{kJ}}{\text{kg}} = 380.79 \frac{\text{kJ}}{\text{kg}}$$

where h_1 is from Table A-22. From the same table with $h_2 = 380.79 \frac{\text{kJ}}{\text{kg}}$, $T_2 = 380 \text{ K}$.

(b) With assumption 2, an entropy rate balance at steady state takes the form

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m} (s_1 - s_2) + \dot{Q}_{cv}$$

or

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{(-\dot{Q}_{cv}/\dot{m})}{T_b} + s_2 - s_1$$

$$= \left(-\frac{\dot{Q}_{cv}/\dot{m}}{T_b} \right) + (s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1})$$

$$= \left(\frac{14 \text{ kJ/kg}}{313 \text{ K}} \right) + (1.94001 - 1.70203 - \frac{8.314}{28.97} \ln \frac{2.1}{1}) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$= (0.0447 + 0.0251) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$= 0.0698 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Then

$$\frac{\dot{E}_d}{\dot{m}} = T_0 \frac{\dot{Q}_{cv}}{\dot{m}} = 293 \text{ K} (0.0698 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) = 20.45 \frac{\text{KJ}}{\text{kg}}$$

$$\frac{\dot{E}_d}{\dot{m}}$$

PROBLEM 7.67

Figure P7.67 shows a device to develop power using a heat transfer from a high-temperature industrial process together with a steam input. The figure provides data for steady-state operation. All surfaces are well insulated, except for the one at 527°C , across which heat transfer occurs at a rate of 4.21 kW . The device develops power at a rate of 6 kW . Determine, in kW ,

- the rate exergy enters accompanying heat transfer.
- the net rate exergy is carried in by the steam, $(\dot{E}_{f1} - \dot{E}_{f2})$.
- the rate of exergy destruction within the device.

Ignore the effects of motion and gravity and let $T_0 = 293\text{ K}$, $p_0 = 1\text{ bar}$.

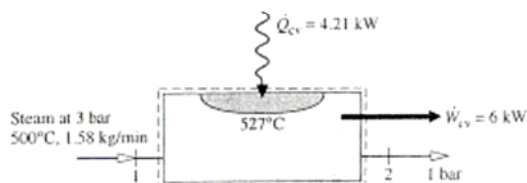


Fig. P7.67

ENGR. MODEL:

- The control volume shown in the figure is at steady state.
- Heat transfer occurs only at $T_b = 800\text{ K}$ (527°C).
- The effects of motion and gravity can be ignored.
- Let $T_0 = 293\text{ K}$, $p_0 = 1\text{ bar}$.

ANALYSIS: (a) The rate exergy enters accompanying heat transfer is obtained with Eq. 7.15

$$\dot{E}_q = \left[1 - \frac{T_0}{T_b}\right] \dot{Q}_{cv} = \left[1 - \frac{293}{800}\right] (4.21\text{ kW}) = 2.67\text{ kW}$$

(b) To find $(\dot{E}_{f1} - \dot{E}_{f2})$ requires state 2 to be fixed. Begin with a mass and energy rate balance to obtain with $h_1 = 3486\text{ kJ/kg}$ from Table A-4

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}[h_1 - h_2] \Rightarrow h_2 = h_1 + \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{\dot{m}} = 3486 \frac{\text{kJ}}{\text{kg}} + \frac{(4.21 - 6) \text{ kJ/s}}{(1.58/60) \text{ kg/s}} = 3418 \frac{\text{kJ}}{\text{kg}}$$

Then, from Table A-4, with $p_2 = 1\text{ bar}$ and $h_2 = 3418\text{ kJ/kg}$, $s_2 = 8.7398\text{ kJ/kg} \cdot \text{K}$. Also, Table A-4 gives $s_1 = 8.3251\text{ kJ/kg} \cdot \text{K}$. With Eq. 7.18

$$(\dot{E}_{f1} - \dot{E}_{f2}) = \dot{m}[(h_1 - h_2) - T_0(s_1 - s_2)] = \left(\frac{1.58}{60} \frac{\text{kg}}{\text{s}}\right) [(3486 - 3418) - 293(8.3251 - 8.7398)] = 4.99\text{ kW}$$

① (c) The rate of exergy destruction can be obtained from the exergy rate balance

$$0 = \dot{E}_q - \dot{W}_{cv} + (\dot{E}_{f1} - \dot{E}_{f2}) - \dot{E}_d$$

$$\Rightarrow \dot{E}_d = \dot{E}_q - \dot{W}_{cv} + (\dot{E}_{f1} - \dot{E}_{f2}) = (2.67 - 6 + 4.99)\text{ kW} = 1.66\text{ kW}$$

1. Alternatively, $\dot{E}_d = T_0 \dot{\sigma}$, where

$$\begin{aligned} \dot{\sigma} &= \left(-\frac{\dot{Q}_{cv}}{T_b}\right) + \dot{m}(s_2 - s_1) = -\frac{4.21\text{ kW}}{800\text{ K}} + \left(\frac{1.58}{60} \frac{\text{kg}}{\text{s}}\right) (8.7398 - 8.3251) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right| \\ &\Rightarrow \dot{\sigma} = -0.00526 + 0.01092 = 0.00566\text{ kW/K} \\ &\Rightarrow \dot{E}_d = T_0 \dot{\sigma} = (293\text{ K})(0.00566\text{ kW/K}) = 1.66\text{ kW} \end{aligned}$$

Continued on next slide

Problem 6-67 continued

2. Consider an exergy accounting:

⊙ Exergy supplied to the device:

✓ via heat transfer	2.67 kW	(34.9%)
✓ via steam	<u>4.99 kW</u>	(65.1%)
	7.66 kW	

⊙ Disposition of the exergy supplied:

✓ power developed	6.00 kW	(78.3%)
✓ exergy destroyed	<u>1.66 kW</u>	(21.7%)
	7.66 kW	

PROBLEM 7.68

Determine the rate of exergy destruction, in Btu/min, for the duct system of Problem 6.111. Let $T_0 = 500^\circ\text{R}$, $p_0 = 1 \text{ atm}$.

ANALYSIS: From the solution to Problem 6.111, $\dot{\sigma}_{cv} = 0.084 \frac{\text{Btu}}{\text{min} \cdot ^\circ\text{R}}$.

Then, with $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ we get

$$\dot{E}_d = 500^\circ\text{R} (0.084 \frac{\text{Btu}}{\text{min} \cdot ^\circ\text{R}}) = 42 \frac{\text{Btu}}{\text{min}}$$

4

PROBLEM 7.69

For the vortex tube of Example 6.7, determine the rate of exergy destruction, in Btu per lb of air entering. Referring to this value for exergy destruction, comment on the inventor's claim. Let $T_0 = 530^\circ\text{R}$, $p_0 = 1 \text{ atm}$.

KNOWN: Steady-state operating data are provided for a vortex tube.

FIND: Evaluate the rate of exergy destruction per unit mass of air entering.

SCHEMATIC & GIVEN DATA: See Figure E6.7

ENGR. MODEL: 1. The assumptions of Example 6.7 apply. 2. $T_0 = 530^\circ\text{R}$, $p_0 = 1 \text{ atm}$.

ANALYSIS: Using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ and $\dot{\sigma}_{cv}/\dot{m}_1 = 0.1086 \text{ Btu/lb}\cdot^\circ\text{R}$ from the solution to Example 6.7, we get

$$\begin{aligned}\frac{\dot{E}_d}{\dot{m}_1} &= T_0 \frac{\dot{\sigma}_{cv}}{\dot{m}_1} = (530^\circ\text{R})(0.1086) \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}} \\ &= 57.6 \frac{\text{Btu}}{\text{lb}}\end{aligned}$$

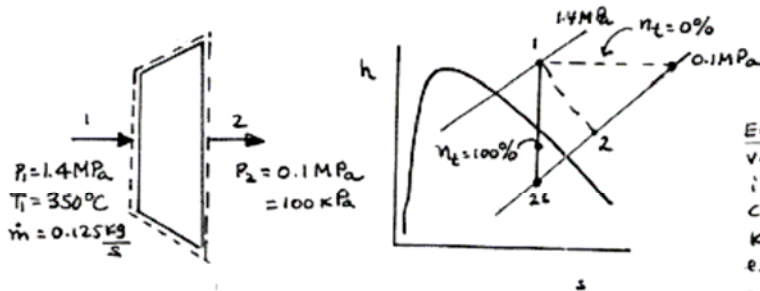
The inventor claims the device operates without work or heat transfer. However, the exergy destroyed within the vortex tube can be traced to a device, or devices, located upstream of the vortex tube that would require some combination of work, heat transfer, and fuel input.

PROBLEM 7.70

KNOWN: Steady state operating data are provided for a steam turbine.

FIND: Plot versus the isentropic turbine efficiency ranging from 0 to 100%, the turbine exit temperature, power developed, and the rate of exergy destruction within the turbine.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. 3. $T_0 = 293 \text{ K} (20^\circ\text{C})$, $p_0 = 1 \text{ atm}$.

ANALYSIS: At steady state, mass and energy rate balances reduce to give $\dot{W}_{cv} = \dot{m} (h_1 - h_2)$. Introducing the isentropic turbine efficiency

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \quad (1) \Rightarrow \dot{W}_{cv} = \dot{m} \eta_t (h_1 - h_{2s}) \quad (2)$$

Note that Eq. (1) gives h_2 , which together with p_2 fixes T_2 . Mass and entropy rate balances reduce to give

$$\dot{E}_d = \dot{m} T_0 (s_2 - s_1) \quad (3)$$

Sample Calculation: $\eta_t = 80\%$. From Table A-4, $h_1 = 3149.5 \frac{\text{kJ}}{\text{kg}}$, $s_1 = 7.1406 \text{ kJ/kg} \cdot \text{K}$ (double interpolation). Then, with Eq. (1) and h_{2s} determined with $s_{2s} = s_1$,

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{7.1406 - 1.3026}{7.3594 - 1.3026} = 0.964, \quad h_{2s} = 417.6 + (0.964)(2258) = 2594.3 \frac{\text{kJ}}{\text{kg}}$$

$$h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 3149.5 - 0.8(3149.5 - 2594.3) = 2705.3 \frac{\text{kJ}}{\text{kg}} \Rightarrow T_2 = 114.4^\circ\text{C},$$

$$s_2 = 7.4373 \text{ kJ/kg} \cdot \text{K}. \text{ So, with Eqs. (2), (3)}$$

$$\dot{W}_{cv} = (0.125 \frac{\text{kg}}{\text{s}})(0.8)(3149.5 - 2594.3) \frac{\text{kJ}}{\text{kg}} \left| \frac{\text{KJ}}{\text{kg}} \right| \frac{1 \text{ kW}}{1 \frac{\text{kJ}}{\text{s}}} = 55.52 \text{ kW}, \quad \dot{E}_d = (0.125)(293)(7.4373 - 7.1406) = 10.87 \text{ kW}$$

IT Code

```
p1 = 14 // bar
T1 = 350 // °C
p2 = 1 // bar
To = 293 // K
mdot = 0.125 // kg/s
eta = 0.8
```

```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s1)
h2 = h1 - eta * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)
```

$T_2 = T_{Ph}(\text{"Water/Steam", } p_2, h_2)$

$\dot{W}_{dotcv} = \text{mdot} * (h_1 - h_2)$

$\dot{E}_{dotcv} = \text{mdot} * T_0 * (s_2 - s_1)$

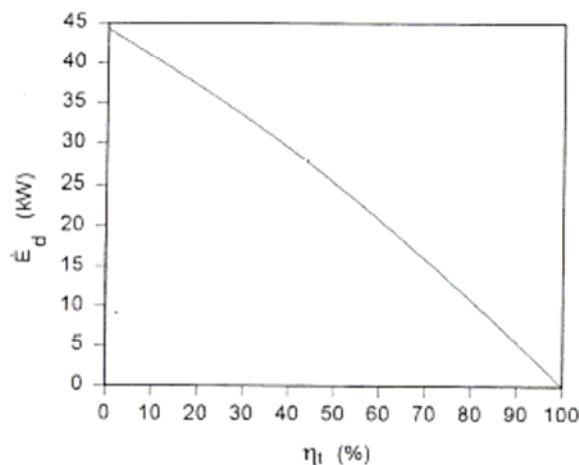
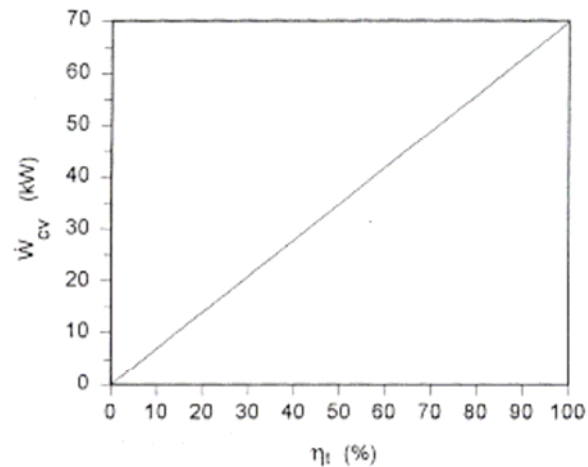
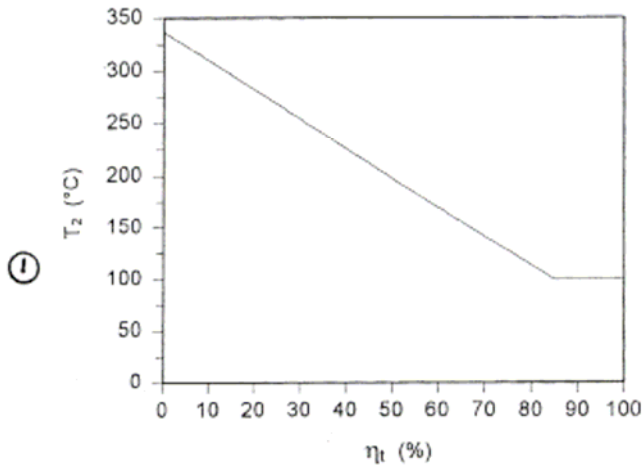
IT Results for $\eta_t = 80\%$

```
h1 = 3149 kJ/kg
s1 = 7.135 kJ/kg·K
h2s = 2592 kJ/kg
h2 = 2703 kJ/kg
s2 = 7.433 kJ/kg·K
T2 = 113.6 °C
W_cv = 55.73 kW
E_d = 10.9 kW
```

Continued on next slide

Problem 6-70 continued

PLOTS:



Discussion:

1. The case of $\eta_t = 100\%$ corresponds to maximum power and zero exergy destruction. The case of $\eta_t = 0$ corresponds to zero power and maximum exergy destruction.
2. In the case of $\eta_t = 0$, there is no work, and the turbine acts as a throttling process, with $h_2 = h_1$.

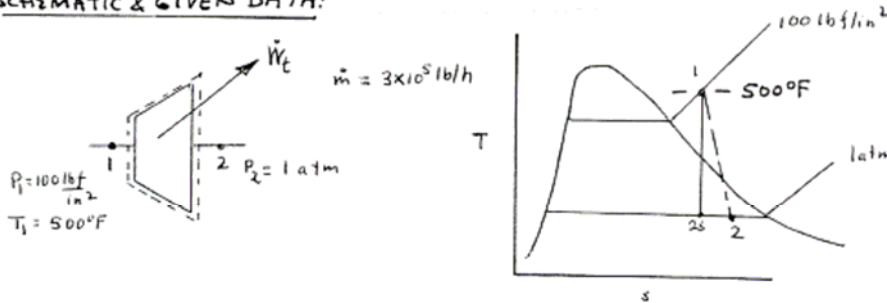
-
1. The flat portion of the plot corresponds to $x_2 \leq 1.0$. At these states the temperature corresponds to the saturation temperature at $p = 0.1 \text{ MPa}$.

PROBLEM 7.71

KNOWN: Steady-state operating data are provided for a steam turbine.

FIND: (a), (b) If the isentropic efficiency is 80% and exergy is valued at 8 cents per kW·h, determine the value of the power produced and the cost of the exergy destroyed. (c) Plot the quantities of (a), (b) versus isentropic efficiency ranging from 80 to 100%.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume above is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects can be ignored. (3) For the environment $T_0 = 53.0^\circ\text{R}$, $P_0 = 1 \text{ atm}$. (4) Exergy is valued at 8 cents per kW·h.

ANALYSIS: Reducing mass and energy rate balances and using the definition of isentropic turbine efficiency (Eq. 6.48)

$$\dot{W}_t = \dot{m}(h_1 - h_2) = \eta_t \dot{m}(h_1 - h_{2s}) \quad (1)$$

Using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ and $\dot{\sigma}_{cv}$ from the mass and entropy rate balances

$$\dot{E}_d = T_0 \dot{m}(s_2 - s_1) \quad (2)$$

(a) $\eta_t = 80\%$. From Table A-4E, $h_1 = 1279.1 \text{ Btu/lb}$, $s_1 = 1.7085 \text{ Btu/lb} \cdot ^\circ\text{R}$. State 2s is fixed by $P_2 = 1 \text{ atm}$ and x_{2s} :

$$x_{2s} = \frac{1.7085 - 0.7121}{1.4446} = 0.687$$

Then

$$h_{2s} = h_f + x_{2s}(h_g - h_f) = 180.15 + 0.687(970.4) = 850.5 \text{ Btu/lb}$$

Substituting into Eq. (1)

$$\dot{W}_t = (0.8)(3 \times 10^5 \frac{\text{lb}}{\text{h}})(1279.1 - 850.5) \frac{\text{Btu}}{\text{lb}} = 385.4 \times 10^5 \frac{\text{Btu}}{\text{h}}$$

The value is

$$\dot{\Phi} = \left(385.4 \times 10^5 \frac{\text{Btu}}{\text{h}} \right) \left| \frac{1 \text{ kW} \cdot \text{h}}{3413 \text{ Btu}} \right| \left(0.08 \frac{\$}{\text{kW} \cdot \text{h}} \right) = 903.4 \frac{\$}{\text{h}}$$

(b) Continuing the discussion of the case $\eta_t = 80\%$, to fix state 2 use Eq. (1) to write

$$h_2 = h_1 - (\dot{W}_t / \dot{m}) = 1279.1 - (385.4 \times 10^5 / 3 \times 10^5) = 1150.6 \text{ Btu/lb. Then,}$$

at $P_2 = 1 \text{ atm}$, state 2 is closely a saturated vapor for which $s_2 = 1.7567 \text{ Btu/lb} \cdot ^\circ\text{R}$.

Substituting into Eq. (2)

$$\dot{E}_d = (53.0^\circ\text{R})(3 \times 10^5 \frac{\text{lb}}{\text{h}})(1.7567 - 1.7085) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 76.6 \times 10^5 \frac{\text{Btu}}{\text{h}}$$

The cost rate is

$$\dot{\Phi} = \left(76.6 \times 10^5 \frac{\text{Btu}}{\text{h}} \right) \left| \frac{1 \text{ kW} \cdot \text{h}}{3413 \text{ Btu}} \right| \left(0.08 \frac{\$}{\text{kW} \cdot \text{h}} \right) = 179.5 \frac{\$}{\text{h}}$$

Continued on next slide

Problem 6-68 continued

The data required for the plot are generated using IT, as follows:

IT Code

```
p1 = 100 // lbf/in.2
T1 = 500 // °F
p2 = 1 * 14.696 // lbf/in.2
mdot = 300000 // lb/h
eta = 80 // %
To = 70 + 460 // °R

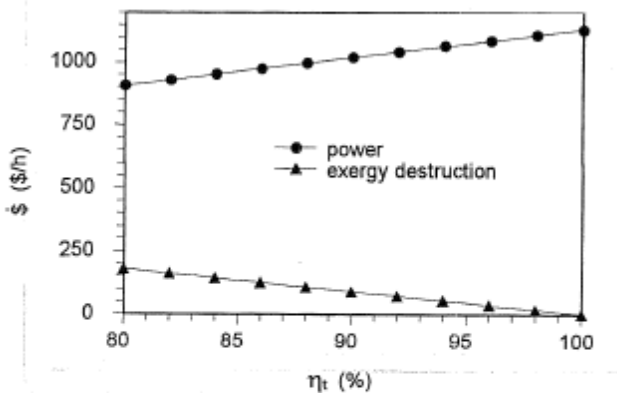
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1
h2 = h1 - (eta / 100) * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)

Wdot = mdot * (h1 - h2)
CostW = Wdot * (0.08 / 3413)
Edot = mdot * To * (s2 - s1)
CostE = Edot * (0.08 / 3413)
```

IT Results for $\eta_t = 80\%$

```
h1 = 1279 Btu/lb
s1 = 1.708 Btu/lb·°R
h2s = 1118 Btu/lb
h2 = 1150 Btu/lb
s2 = 1.756 Btu/lb
Wdot_cv = 3.864 x 107 Btu/h
$power = 905.7 $/h
Edot_d = 7.623 x 106 Btu/h
$exergy = 178.7 $/h
```

Plot:



At higher values of η_t , there is less exergy destruction and greater power output, as expected.

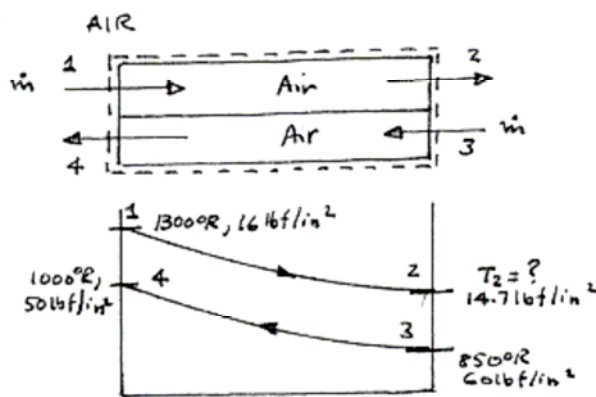
PROBLEM 7.72

Air at $T_1 = 1300^\circ\text{R}$, $p_1 = 16 \text{ lbf/in}^2$ enters a counter-flow heat exchanger operating at steady state and exits at $p_2 = 14.7 \text{ lbf/in}^2$. A separate stream of air enters at $T_3 = 850^\circ\text{R}$, $p_3 = 60 \text{ lbf/in}^2$ and exits at $T_4 = 1000^\circ\text{R}$, $p_4 = 50 \text{ lbf/in}^2$. The mass flow rates of the streams are equal. Stray heat transfer and the effects of motion and gravity can be ignored. Assuming the ideal gas model with $c_p = 0.24 \text{ Btu/lb}\cdot^\circ\text{R}$, determine (a) T_2 , in $^\circ\text{R}$, and (b) the rate of exergy destruction within the heat exchanger, in Btu per lb of air flowing. Let $T_0 = 520^\circ\text{R}$, $p_0 = 1 \text{ atm}$.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and the effects of motion and gravity can be ignored.
3. The air is modeled as an ideal gas with $c_p = 0.24 \text{ Btu/lb}\cdot^\circ\text{R}$.
4. For the environment, $T_0 = 520^\circ\text{R}$, $p_0 = 1 \text{ atm}$.

SCHEMATIC & GIVEN DATA:



ANALYSIS: (a) An energy rate balance reduces to read, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) + \dot{m}(h_3 - h_4)$

$$\Rightarrow 0 = \dot{m}(h_1 - h_2) + \dot{m}(h_3 - h_4) = \dot{m}c_p(T_1 - T_2) + \dot{m}c_p(T_3 - T_4)$$

$$\Rightarrow T_2 = T_1 + T_3 - T_4 = 1300^\circ\text{R} + 850^\circ\text{R} - 1000^\circ\text{R} = 1150^\circ\text{R} \quad (a)$$

(b) To determine $\dot{E}_d$ it is convenient to use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy rate balance:

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{m}(s_3 - s_4) + \dot{\sigma}_{cv} \Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}} = (s_2 - s_1) + (s_4 - s_3)$$

$$\therefore \frac{\dot{E}_d}{\dot{m}} = T_0 [(s_2 - s_1) + (s_4 - s_3)] = T_0 \left[\left(c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right) + \left(c_p \ln \frac{T_4}{T_3} - R \ln \frac{p_4}{p_3} \right) \right]$$

$$= 520^\circ\text{R} \left[\left((0.24 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}) \ln \frac{1150}{1300} - \frac{1.986}{28.97} \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}} \ln \frac{14.7}{16} \right) + \left(0.24 \ln \frac{1000}{850} - \frac{1.986}{28.97} \ln \frac{50}{60} \right) \right]$$

$$= 14.5 \text{ Btu/lb} \quad (b)$$

PROBLEM 7.74

Water at $T_1 = 100^\circ\text{F}$, $p_1 = 30 \text{ lbf/in}^2$ enters a counter-flow heat exchanger operating at steady state with a mass flow rate of 100 lb/s and exits at $T_2 = 200^\circ\text{F}$ with closely the same pressure. Air enters in a separate stream at $T_3 = 540^\circ\text{F}$ and exits at $T_4 = 140^\circ\text{F}$ with no significant change in pressure. Air can be modeled as an ideal gas and stray heat transfer can be ignored. Determine (a) the mass flow rate of the air, in lb/s , and (b) the rate of exergy destruction within the heat exchanger, in Btu/s . Ignore the effects of motion and gravity and let $T_0 = 60^\circ\text{F}$, $p_0 = 1 \text{ atm}$.

ENGR MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and the effects of motion and gravity can be ignored.
3. The air is modeled as an ideal gas.
4. For the liquid water at 1 and 2, $h \sim h_f(T)$, $s \sim s_f(T)$.
5. For the environment, $T_0 = 520^\circ\text{R}$, $p_0 = 1 \text{ atm}$.

ANALYSIS: At 200 lbf/in^2 , $p_{\text{sat}} = 381.86^\circ\text{F}$. Thus, the water is a liquid at 1 and 2.

(a) An energy rate balance reduces to, $0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$

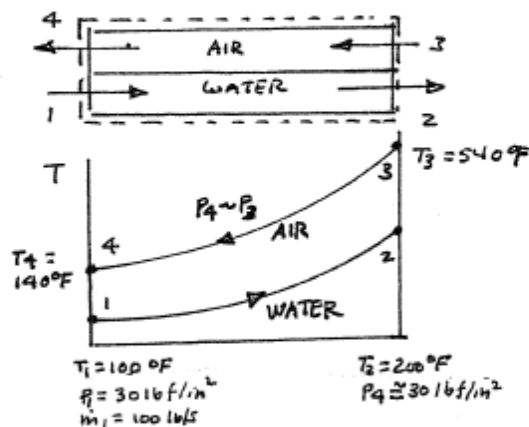
$$\Rightarrow \dot{m}_3 = \dot{m}_1 \left[\frac{h_2 - h_1}{h_3 - h_4} \right] = 100 \frac{\text{lb}}{\text{s}} \left[\frac{168.07 - 68.05}{240.98 - 143.47} \right] = 102.6 \frac{\text{lb}}{\text{s}} \quad \leftarrow \text{ca)}$$

where data are from Tables A-2E and A-22E.

(b) To find $\dot{E}_d$, it is convenient to use $\dot{E}_d = T_0 \dot{\sigma}_{\text{cv}}$, where $\dot{\sigma}_{\text{cv}}$ is from an entropy rate balance:

$$\begin{aligned} \dot{E}_d &= T_0 \dot{\sigma} = T_0 [\dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3)] \\ &= T_0 [\dot{m}_1(s_f(T_2) - s_f(T_1)) + \dot{m}_3(s_4 - s_3 - R \ln p_4/p_3)] \\ &= 520^\circ\text{R} \left[100 \frac{\text{lb}}{\text{s}} (0.2940 - 0.1296) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} + 102.6 (0.62607 - 0.75042) \right] \\ &= 1914.5 \frac{\text{Btu}}{\text{s}} \quad \leftarrow \text{(b)} \end{aligned}$$

SCHEMATIC & GIVEN DATA:

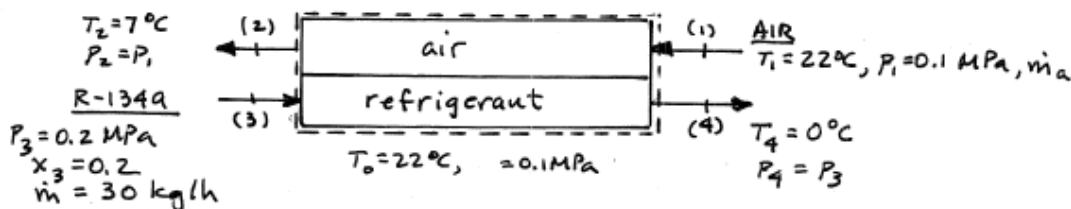


PROBLEM 7.75

KNOWN: Operating data are provided for a counterflow heat exchanger at steady state. One stream is R-134a and the other is air.

FIND: (a) For the R-134a, determine the rate of heat transfer, and (b) For each of the two streams, evaluate the change in flow exergy rate and compare.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown is at steady state, with $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and negligible effects of motion and gravity. (2) For a control volume enclosing only the refrigerant stream, the foregoing applies, except $\dot{Q}_{cv} \neq 0$. (3) For each stream, pressure change is negligible. (4) The air is modeled as an ideal gas. (5) For the environment, $T_0 = 22^\circ\text{C} = 295\text{K}$, $P_0 = 0.1\text{ MPa}$.

(a) For a control volume enclosing only the refrigerant stream, the mass and energy rate balances reduce to give

$$0 = \dot{Q} - \dot{W}_{cv} + \dot{m}(h_3 - h_4) \Rightarrow \dot{Q} = \dot{m}(h_4 - h_3) \quad (1)$$

From Table A-11 at 2 bar, $x_3 = 0.2$; $h_3 = h_f + x_3 h_{fg} = 36.84 + (0.2) 204.46 = 77.732 \frac{\text{kJ}}{\text{kg}}$
From Table A-12; $h_4 = 250.10 \text{ kJ/kg}$. Thus

$$\dot{Q} = (30 \frac{\text{kg}}{\text{h}})(250.10 - 77.732) \text{ kJ/kg} = 5171 \text{ kJ/h} \quad \dot{Q}$$

(b) The change in flow exergy rate for the refrigerant stream is, with data from Tables A-11 and A-12

$$(\dot{\Delta E}_f)_{\text{ref}} = \dot{m}[(h_4 - h_3) - T_0(s_4 - s_3)] = (30)[(250.1 - 77.732) - (295)(0.9502 - 0.30354)]$$

$$\textcircled{1} \quad = -622.7 \text{ kJ/h} \quad (\dot{\Delta E}_f)_{\text{ref}}$$

To determine the change in flow exergy rate for the air, we first evaluate $\dot{m}_a$ using mass and energy rate balances for the overall control volume.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_a[h_1 - h_2] + \dot{m}[h_3 - h_4] \quad (1)$$

That is

$$\dot{m}_a = \dot{m} \left[\frac{h_4 - h_3}{h_1 - h_2} \right] = (30 \frac{\text{kg}}{\text{h}}) \left[\frac{250.1 - 77.732}{295.17 - 280.13} \right] \frac{\text{kJ}}{\text{kg}} = 343.8 \text{ kg/h}$$

where the specific enthalpies for air are from Table A-22.

Now, the change in exergy rate of the air is

$$(\dot{\Delta E}_f)_{\text{air}} = \dot{m}_a(e_{f2} - e_{f1}) = \dot{m}_a[(h_2 - h_1) - T_0(s_2 - s_1)]$$

$$= \dot{m}_a[(h_2 - h_1) - T_0(s_0(T_2) - s_0(T_1) - R \ln(T_2/P_1))] \quad \dot{m}_a$$

Continued on next slide

Problem 6-75 continued

With $s^\circ(T)$ data from Table A-22

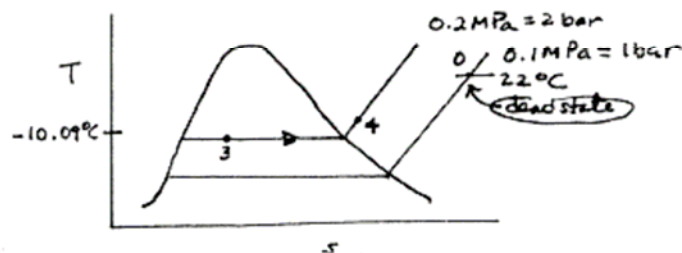
$$(\dot{\Delta E}_f)_{\text{air}} = (343.8) [280.13 - 295.17] - (2.95) (1.63275 - 1.68515)$$

② ③

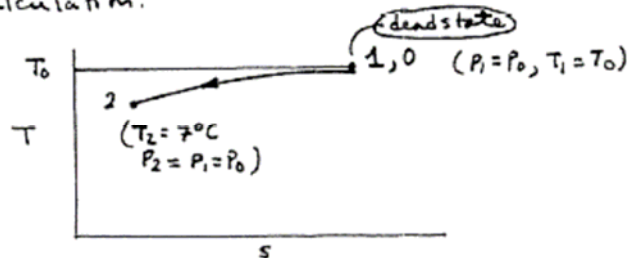
$$= 139.7 \text{ kJ/h}$$

$$\leftarrow (\dot{\Delta E}_f)_{\text{air}}$$

- As the refrigerant flows through the heat exchanger, its state is brought closer to the dead state and its exergy decreases, as confirmed by this calculation.



- As the air is brought at constant pressure from the dead state to a lower temperature, its exergy increases, as confirmed by this calculation.

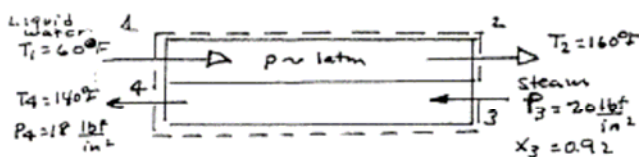


- When heat transfer occurs at temperatures below T_0 , as in the present case, the accompanying exergy transfer is oppositely directed. This can be seen by study of Eq. 7.12. In the present case, heat transfer occurs from the warmer air to the cooler refrigerant. Still, exergy transfer is from the refrigerant. Further, the exergy transferred from the refrigerant is accounted for by the exergy increase of the air and by the exergy destroyed within the heat exchanger owing to spontaneous heat transfer.

PROBLEM 7.76

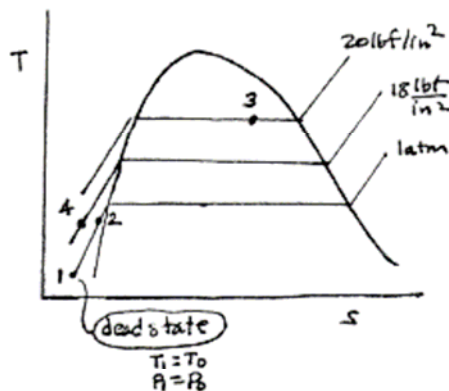
Liquid water enters a heat exchanger operating at steady state at $T_1 = 60^\circ\text{F}$, $p_1 = 1 \text{ atm}$ and exits at $T_2 = 160^\circ\text{F}$ with a negligible change in pressure. In a separate stream, steam enters at $T_3 = 20 \text{ lbf/in}^2$, $x_3 = 92\%$ and exits at $T_4 = 140^\circ\text{F}$, $p_4 = 18 \text{ lbf/in}^2$. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 60^\circ\text{F}$, $p_0 = 1 \text{ atm}$. Determine (a) the ratio of the mass flow rates of the two streams and (b) the rate of exergy destruction, in Btu per lb of steam entering the heat exchanger.

SCHEMATIC & GIVEN DATA:



ENGGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. Stray heat transfer and the effects of motion and gravity are negligible.
3. At states 1, 2, and 4, $h \sim h_f(T)$, $s \sim s_f(T)$.
4. Let $T_0 = 520^\circ\text{R}$ (60°F), $p_0 = 1 \text{ atm}$.



ANALYSIS:

(a) An energy rate balance reads: $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$

$$\Rightarrow \frac{\dot{m}_1}{\dot{m}_3} = \frac{h_3 - h_4}{h_2 - h_1} = \frac{1079.55 - 107.96}{127.96 - 28.08} = 9.73 \frac{\text{lb(liquid)}}{\text{lb(steam)}} \quad (a)$$

$$\text{where } h_3 = h_f + x_3(h_g - h_f) = 196.26 + 0.92(960.1) = 1079.55 \text{ Btu/lb}$$

(b) To find $\dot{E}_d$ it is convenient to use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy balance. That is,

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = T_0 [\dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3)]$$

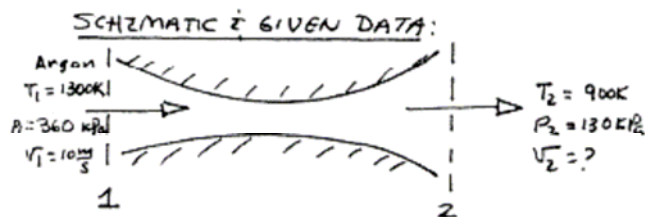
$$\Rightarrow \frac{\dot{E}_d}{\dot{m}_3} = T_0 \left[\frac{\dot{m}_1}{\dot{m}_3} (s_2 - s_1) + (s_4 - s_3) \right] \quad (b)$$

$$= 520^\circ\text{R} \left[9.73 (0.2313 - 0.05555) + (0.1985 - 1.6203) \right] \frac{\text{Btu/}^\circ\text{R}}{\text{lb(steam)}} = 149.9 \frac{\text{Btu}}{\text{lb(steam)}}$$

$$\text{where } s_3 = s_f + x_3(s_g - s_f) = 0.3358 + 0.92(1.3962) = 1.6203 \text{ Btu/lb} \cdot ^\circ\text{R}$$

PROBLEM 7.77

Argon enters a nozzle operating at steady state at 1300 K, 360 kPa with a velocity of 10 m/s and exits the nozzle at 900 K, 130 kPa. Stray heat transfer can be ignored. Modeling argon as an ideal gas with $k = 1.67$, determine (a) the velocity at the exit, in m/s, and (b) the rate of exergy destruction, in kJ per kg of argon flowing. Let $T_0 = 293$ K, $p_0 = 1$ bar.



ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. Stray heat transfer and the effects of gravity can be ignored.
3. The argon is modeled as an ideal gas with $k = 1.67$.
4. For the environment, $T_0 = 293 \text{ K}$, $P_0 = 1 \text{ bar}$.

ANALYSIS: With Eq. 3.47a, $c_p = \frac{kR}{k-1} = \left(\frac{1.67}{0.67} \right) \frac{8.314 \text{ kJ}}{39.94 \text{ kg} \cdot \text{K}} = 0.52 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

(a) Reducing an energy rate balance,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

$$\Rightarrow V_2 = \sqrt{V_1^2 + 2 \underbrace{(h_1 - h_2)}_{c_p(T_1 - T_2)}} = \sqrt{V_1^2 + 2 c_p (T_1 - T_2)}$$

$$\therefore V_2 = \sqrt{\left(10 \frac{\text{m}}{\text{s}} \right)^2 + 2 \left(0.52 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (1300 - 900) \text{ K}} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| = 645 \frac{\text{m}}{\text{s}} \quad (a)$$

(b) To find $\dot{E}_d$ it is convenient to use $\dot{E}_d = T_0 \dot{S}_{cv}$, where $\dot{S}_{cv}$ is obtained from an entropy rate balance. Thus, with Eq. 6.22,

$$\frac{\dot{E}_d}{\dot{m}} = T_0 [s_2 - s_1] = T_0 \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right]$$

$$= 293 \text{ K} \left[0.52 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln \frac{900}{1300} - \frac{8.314 \text{ kJ}}{39.94 \text{ kg} \cdot \text{K}} \ln \left(\frac{130}{360} \right) \right]$$

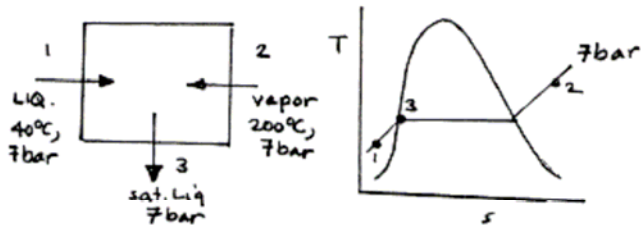
$$= 6.10 \frac{\text{kJ}}{\text{kg}} \quad (b)$$

PROBLEM 7.79

KNOWN: Steady-state operating data are provided for an open feedwater heater.

FIND: Determine the ratio of the incoming mass flow rates, $\dot{m}_1/\dot{m}_2$, and the exergy destruction per unit mass exiting.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$. Kinetic and potential energy effects can be ignored.
3. $T_0 = 25^\circ\text{C}$, $P_0 = 1 \text{ atm}$

ANALYSIS: (a) A mass rate balance reads, $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$. An energy rate balance reduces with assumptions 2 to give

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 \Rightarrow 0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$\Rightarrow \frac{\dot{m}_1}{\dot{m}_2} = \frac{h_3 - h_2}{h_1 - h_3} = \frac{2844.8 - 697.22}{697.22 - 167.57} = 4.05$$

where data is from Tables A-2, 3, 4. At state 1, $h_1 \approx h_f(T_1)$.

(b) The exergy destruction can be found using $(\dot{E}_d/\dot{m}_3) = T_0 (\dot{\sigma}_{cv}/\dot{m}_3)$, where $\dot{\sigma}_{cv}/\dot{m}_3$ is obtained from an entropy balance:

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

$$\Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}_3} = s_3 - \frac{\dot{m}_1}{\dot{m}_3} s_1 - \frac{\dot{m}_2}{\dot{m}_3} s_2$$

From part (a) $\dot{m}_1 = 4.05 \dot{m}_2$, $\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 5.05 \dot{m}_2$. Thus, $\dot{m}_2/\dot{m}_3 = 0.198$, $\dot{m}_1/\dot{m}_3 = 0.802$. With data from Tables A-2, 3, 4, At state 1, $s_1 \approx s_f(T_1)$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = [1.9922 - (0.802)(0.5725) - (0.198)(6.8865)] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.1696 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

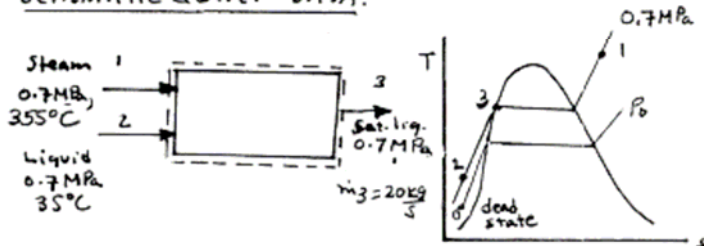
$$\Rightarrow \dot{E}_d/\dot{m}_3 = T_0 (\dot{\sigma}_{cv}/\dot{m}_3) = (298 \text{ K})(0.1696 \text{ kJ/kg} \cdot \text{K}) = 50.5 \text{ kJ/kg}$$

PROBLEM 7.80

Steam at 0.7 MPa, 355°C enters an open feedwater heater operating at steady state. A separate stream of liquid water enters at 0.7 MPa, 35°C. A single mixed stream exits as saturated liquid at 0.7 MPa with a mass flow rate of 20 kg/s. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 20^\circ\text{C}$, $p_0 = 1\text{ atm}$. Determine

(a) the mass flow rates of the entering streams, each in kg/s, (b) the rate of exergy destruction, in kW, and (c) the cost of the exergy destroyed, in \$/year, for 8000 hours of operation annually. Evaluate exergy at 8 cents per kW·h.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. Kinetic and potential energy effects can be ignored.
3. Evaluate electricity at 8 cents per kW·h. 4. Take $T_0 = 20^\circ\text{C}$.

ANALYSIS: (a) At steady state, the mass rate balance gives $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$, and the energy rate balance reduces as follows

$$0 = \dot{Q}_{cv} + \dot{W}_{cv} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3 \Rightarrow \frac{\dot{m}_1}{\dot{m}_2} = \frac{h_3 - h_2}{h_1 - h_3} \quad (1)$$

With $h_2 \approx h_f(T_2)$ and data from Tables A-2, 3, 4,

$$\frac{\dot{m}_1}{\dot{m}_2} = \frac{(697.22 - 146.68)}{(3174.2 - 697.22)} = 0.2223$$

$$\therefore \dot{m}_1 + \dot{m}_2 = \dot{m}_3 \Rightarrow 0.2223 \dot{m}_2 + \dot{m}_2 = 20 \text{ kg/s} \Rightarrow \begin{aligned} \dot{m}_2 &= 16.3626 \text{ kg/s} \\ \dot{m}_1 &= 3.6374 \text{ kg/s} \end{aligned} \quad (a)$$

(b) To find $\dot{E}_d$ it is convenient to use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance:

$$0 = \sum_j \dot{Q}_j / T_j + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{\sigma}_{cv} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 = (20 \text{ kg/s})(1.9922 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) - 3.6374(7.4892) - 16.3626(6.5033) = 4.37 \frac{\text{kJ/s}}{\text{K}}$$

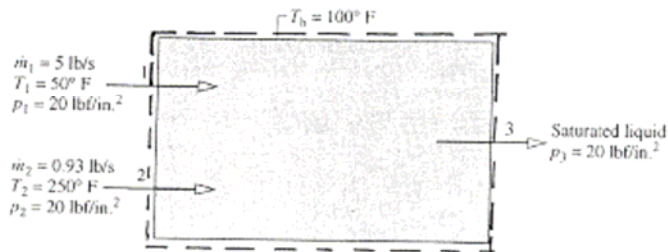
$$\text{Then, } \dot{E}_d = 293 \text{ K} \left(4.37 \frac{\text{kJ/s}}{\text{K}} \right) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 1280.4 \text{ kW} \quad (b)$$

(c) Costing:

$$\dot{\$} = (1280.4 \text{ kW}) \left(\frac{8000 \text{ h}}{\text{year}} \right) \left(\frac{\$0.08}{\text{kW} \cdot \text{h}} \right) = \$819,456/\text{year} \quad (c)$$

PROBLEM 7.81

Figure P7.81 provides steady-state operating data for a mixing chamber in which entering liquid and vapor streams of water mix to form an exiting saturated liquid stream. Heat transfer from the mixing chamber to its surroundings occurs at an average surface temperature of 100°F . The effects of motion and gravity are negligible. Let $T_0 = 70^\circ\text{F}$, $p_0 = 1 \text{ atm}$. For the mixing chamber, determine, each in Btu/s, (a) the rate of heat transfer and the accompanying rate of exergy transfer and (b) the rate of exergy destruction.



ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. $\dot{W}_{cv} = 0$ and the effects of gravity and motion can be ignored.
3. All heat transfer from the control volume occurs at T_b .
4. For the environment, $T_0 = 530^\circ\text{R}$ (70°F), $p_0 = 1 \text{ atm}$.

ANALYSIS: From a mass rate balance, $\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 5.93 \text{ lb/s}$.
Also, using Tables A-2E, 3E, 4E:

	$h \text{ (Btu/lb)}$	$s \text{ (Btu/lb}\cdot^\circ\text{R)}$
1	18.06 (whf)	0.03607 (~sf)
2	1167.2	1.7475
3	196.26	0.3358

(a) An energy rate balance reduces to
read, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$
 $\Rightarrow \dot{Q}_{cv} = \dot{m}_3 h_3 - \dot{m}_1 h_1 - \dot{m}_2 h_2$
 $= (5.93 \frac{\text{lb}}{\text{s}})(196.26 \frac{\text{Btu}}{\text{lb}}) - 5(18.06) - 0.93(1167.2)$
 $= -12 \text{ Btu/s}$ (a)

$\dot{E}_q = [1 - \frac{T_0}{T_b}] \dot{Q}_{cv}$
① $= [1 - \frac{530}{560}](-12 \frac{\text{Btu}}{\text{s}}) = -0.64 \frac{\text{Btu}}{\text{s}}$

(b) To get $\dot{E}_d$ it is convenient to use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance:

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{E}_d = T_0 \left[-\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 \right]$$

$$= 530^\circ\text{R} \left[-\frac{(-12 \text{ Btu/s})}{560^\circ\text{R}} + (5.93 \frac{\text{lb}}{\text{s}})(0.3358 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}) - 5(0.03607) - 0.93(1.7475) \right]$$

$$= 109.8 \text{ Btu/s}$$
 (b)

1. In comparison to the value of $\dot{E}_d$ found in part (b), the exergy transfer accompanying heat transfer is a minor effect.

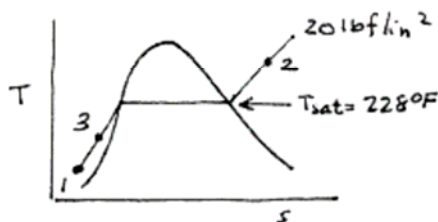
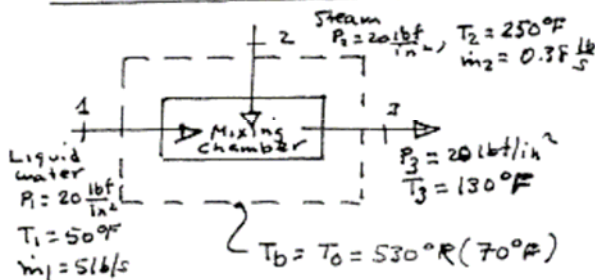
PROBLEM 7.82

Liquid water at 20 lbf/in^2 , 50°F enters a mixing chamber operating at steady state with a mass flow rate of 5 lb/s and mixes with a separate stream of steam entering at 20 lbf/in^2 , 250°F with a mass flow rate of 0.38 lb/s . A single mixed stream exits at 20 lbf/in^2 , 130°F . Heat transfer from the mixing chamber occurs to its surroundings. Neglect the effects of motion and gravity and let $T_0 = 70^\circ\text{F}$, $p_0 = 1 \text{ atm}$. Determine the rate of exergy destruction, in Btu/s , for a control volume including the mixing chamber and enough of its immediate surroundings that heat transfer occurs at 70°F .

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Heat transfer occurs at $T_b = T_0 = 530^\circ\text{R}$ only. $\dot{W}_{cv} = 0$.
3. The effects of motion and gravity can be ignored.
4. For the environment, $T_0 = 530^\circ\text{R}$, $p_0 = 1 \text{ atm}$.

SCHEMATIC & GIVEN DATA:



ANALYSIS: An energy rate balance reduces to, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$

or $\dot{Q}_{cv} = \dot{m}_3 h_3 - \dot{m}_1 h_1 - \dot{m}_2 h_2 = 5.38 \frac{\text{lb}}{\text{s}} \left(97.98 \frac{\text{Btu}}{\text{lb}} \right) - (5)(18.06) - 0.38(1167.2)$

$L (= \dot{m}_1 + \dot{m}_2)$

$= -6.71 \text{ Btu/s}$

To find $\dot{E}_d$, it is convenient to use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance:

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{E}_d = T_0 \left[-\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 \right]$$

$$= 530^\circ\text{R} \left[-\frac{(-6.71 \text{ Btu/s})}{530^\circ\text{R}} + (5.38 \frac{\text{lb}}{\text{s}}) \left(0.1817 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) - (5)(0.03607) - 0.38(1.7475) \right]$$

① $= 77.3 \frac{\text{Btu}}{\text{s}}$

1. This value accounts for exergy destruction within the mixing chamber and exergy destruction in the immediate surroundings of the mixing chamber owing to spontaneous heat transfer there.

PROBLEM 7.83

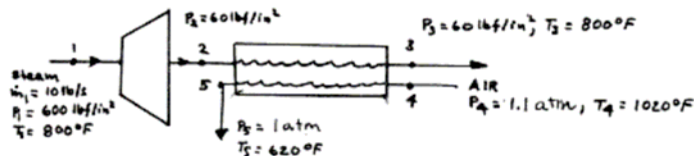
At steady state, steam with a mass flow rate of 10 lb/s enters a turbine at 800°F and 600 lbf/in.² and expands to 60 lbf/in.². The power developed by the turbine is 2852 horsepower. The steam then passes through a counterflow heat exchanger with a negligible change in pressure, exiting at 800°F. Air enters the heat exchanger in a separate stream at 1.1 atm, 1020°F and exits at 1 atm, 620°F. The effects of motion and gravity can be ignored and there is no significant heat transfer between either component and its surroundings. Determine

- the mass flow rate of air, in lb/s.
- the rates of exergy destruction in the turbine and heat exchanger, each in Btu/s.

Evaluating exergy at 8 cents per kW · h, determine the hourly cost of each of the exergy destructions found in part (b). Let $T_0 = 40^\circ\text{F}$, $p_0 = 1 \text{ atm}$.

SCHEMATIC & GIVEN DATA:

$$\dot{W}_{\text{cv}} = 2852 \text{ hp}$$



ASSUMPTIONS: (1) The turbine and heat exchanger operate at steady state. (2) There is no significant heat transfer between either component and its surroundings. (3) Kinetic and potential energy effects can be ignored. (4) Air is modeled as an ideal gas. (5) $T_0 = 500^\circ\text{R}$, $p_0 = 1 \text{ atm}$. Evaluate exergy at 8 cents per kW · h.

ANALYSIS: (a) At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4$ and $\dot{m}_5 = \dot{m}_6 = \dot{m}_a$. Taking a control volume about the turbine and heat exchanger, an energy rate balance at steady state reduces to give with assumption 3

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}(h_1 - h_2) + \dot{m}_a(h_4 - h_5)$$

Thus

$$\dot{m}_a = \frac{\dot{W}_{\text{cv}} + \dot{m}(h_2 - h_1)}{h_4 - h_5} = \frac{2852 \text{ hp} \left| \frac{2545 \text{ Btu/h}}{\text{hp}} \right| \left| \frac{1}{3600 \text{ s}} \right| + (10 \text{ lb/s})(1434.2 - 1407.6)}{363.89 - 260.97} = 21.88 \text{ lb/s}$$

where enthalpies are from Tables A-4E and A-22E.

(b) Taking a control volume about the turbine only, an entropy rate balance at steady state reduces to $\dot{\sigma}_{\text{cv}} = \dot{m}(s_2 - s_1)$. To determine $\dot{\sigma}_{\text{cv}}$ requires state 2 to be fixed. This can be accomplished using an energy rate balance for the same control volume:

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}(h_1 - h_2) \Rightarrow h_2 = h_1 - \dot{W}_{\text{cv}}/\dot{m}$$

Thus

$$h_2 = 1407.6 \text{ Btu/lb} - \frac{(2852) \left| \frac{2545 \text{ Btu/h}}{\text{hp}} \right| \left| \frac{1}{3600 \text{ s}} \right|}{10 \text{ lb/s}} = 1205.98 \text{ Btu/lb}$$

State 2 is fixed by $p_2 = 60 \text{ lbf/in}^2$, $h_2 = 1205.98 \text{ Btu/lb}$. Interpolating in Table A-4E, $s_2 = 1.6802 \text{ Btu/lb} \cdot ^\circ\text{R}$. Thus, for the turbine

$$\dot{\sigma}_{\text{cv}} = \dot{m}(s_2 - s_1) = 10 \text{ lb/s} (1.6802 - 1.6343) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 0.459 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}}$$

Taking a control volume around the heat exchanger only, an entropy rate balance reduces at steady state to

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{m}_a(s_4 - s_5) + \dot{\sigma}_{\text{cv}}$$

or with Eq. 6.20a and data from Tables A-4E and A-22E

$$\begin{aligned} \dot{\sigma}_{\text{cv}} &= \dot{m}(s_2 - s_1) + \dot{m}_a(s_5 - s_4 - R \ln p_4/p_5) \\ &= (10 \text{ lb/s}) \left(1.9021 - 1.6802 \right) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} + (21.88 \text{ lb/s}) \left(0.76964 - 0.85062 - \frac{1.986}{28.97} \ln \frac{1}{1.1} \right) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \\ &= 2.22 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}} - 1.629 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}} = 0.591 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}} \end{aligned}$$

Continued on next slide

Problem 6-83 continued

The rates of exergy destruction can be determined using
 $\dot{E}_d = T_0 \dot{\sigma}_{cv}$. Thus

$$\text{Turbine: } \dot{E}_d = 500^\circ\text{R} \left(0.459 \frac{\text{Btu/s}}{^\circ\text{R}} \right) = 229.5 \text{ Btu/s}$$

$\dot{E}_d$

$$\text{Heat Exchanger: } \dot{E}_d = 500^\circ\text{R} \left(0.591 \frac{\text{Btu/s}}{^\circ\text{R}} \right) = 295.5 \text{ Btu/s}$$

The corresponding cost rates are,

Cost Rates

$$\text{Turbine: } \$ = \left(229.5 \frac{\text{Btu}}{\text{s}} \right) \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left| \frac{1 \text{ kW} \cdot \text{h}}{3413 \text{ Btu}} \right| (0.08 \frac{\$}{\text{kW} \cdot \text{h}}) = 19.37 \text{ \$/h}$$

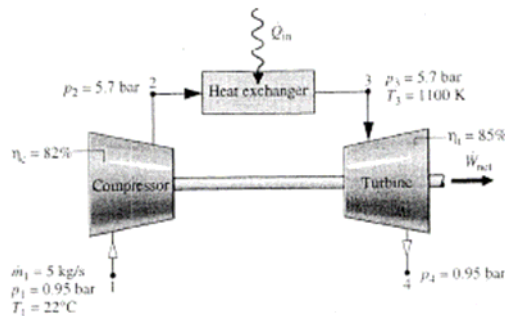
$$\text{Heat Exchanger: } \$ = (295.5) \left| \frac{3600}{1} \right| \left| \frac{1}{3413} \right| (0.08) = 24.94 \text{ \$/h}$$

PROBLEM 7.84

A gas turbine operating at steady state is shown in Fig. P7.84. Air enters the compressor with a mass flow rate of 5 kg/s at 0.95 bar and 22°C and exits at 5.7 bar. The air then passes through a heat exchanger before entering the turbine at 1100 K, 5.7 bar. Air exits the turbine at 0.95 bar. The compressor and turbine operate adiabatically and the effects of motion and gravity can be ignored. The compressor and turbine isentropic efficiencies are 82 and 85%, respectively. Using the ideal gas model for air, determine, each in kW,

- the net power developed.
- the rates of exergy destruction for the compressor and turbine.
- the net rate exergy is carried out of the plant at the turbine exit, $(\dot{E}_{44} - \dot{E}_{11})$.

Let $T_0 = 22^\circ\text{C}$, $p_0 = 0.95$ bar.



ENGR. MODEL:

- The gas turbine operates at steady state.
- The compressor and turbine operate adiabatically.
- The effects of motion and gravity can be ignored.
- The air is modeled as an ideal gas.
- For the environment, $T_0 = 295\text{ K}$, $p_0 = 0.95$ bar.

ANALYSIS: Property evaluation: From Table A-22, $h_1 = 295.17\text{ kJ/kg}$, $s_1^0 = 1.68515\text{ kJ/kg}\cdot\text{K}$,

$$Pr_1 = 1.3068, h_3 = 1161.07\text{ kJ/kg}, s_3^0 = 3.07732\text{ kJ/kg}\cdot\text{K}, Pr_3 = 167.1.$$

$$Pr_2 = Pr_1 \left[\frac{p_2}{p_1} \right] = 1.3068 \left[\frac{5.7}{0.95} \right] = 7.8408 \Rightarrow h_{2s} = 493.03\text{ kJ/kg}$$

$$Pr_4 = Pr_3 \left[\frac{p_4}{p_3} \right] = 167.1 \left[\frac{0.95}{5.7} \right] = 27.85 \Rightarrow h_{4s} = 706.51\text{ kJ/kg}$$

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 774.69\text{ kJ/kg}, s_4^0 = 2.6571\text{ kJ/kg}\cdot\text{K}$$

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 536.46\text{ kJ/kg}, s_2^0 = 2.2843\text{ kJ/kg}\cdot\text{K}$$

$$(a) \text{ The net power developed} = \dot{W}_t - \dot{W}_c = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$$

$$\dot{W}_{\text{net}} = 5\text{ kg/s} \left[(1161.07 - 774.69) - (536.46 - 295.17) \right] \frac{\text{kJ}}{\text{kg}} \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right| = 725.5\text{ kW} \quad (a)$$

(b) $\dot{E}_d$ is conveniently determined as $\dot{E}_d = T_0 \dot{Q}_{\text{rev}}$. For the compressor, $\dot{E}_d = T_0 \dot{m}(s_2 - s_1)$. For the turbine $\dot{E}_d = T_0 \dot{m}(s_4 - s_3)$. That is,

$$\text{Compressor: } \dot{E}_d = T_0 \dot{m}(s_2 - s_1 - R \ln p_2/p_1) = 295\text{ K}(5\text{ kg/s}) \left(2.2843 - 1.68515 - \frac{8.314}{28.97} \ln \frac{5.7}{0.95} \right) \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$= 125.3\text{ kW}$$

$$\text{Turbine: } \dot{E}_d = T_0 \dot{m}(s_4 - s_3 - R \ln p_4/p_3) = 295\text{ K}(5) \left(2.6571 - 3.07732 - \frac{8.314}{28.97} \ln \left(\frac{0.95}{5.7} \right) \right)$$

$$= 138.6\text{ kW}$$

$$(c) \dot{E}_{f4} - \dot{E}_{f1} = \dot{m}[(h_4 - h_1) - T_0(s_4 - s_1 - R \ln p_4/p_1)]$$

$$= \dot{m}[(h_4 - h_1) - T_0(s_4 - s_1^0)] = 5\text{ kg/s}[(774.69 - 295.17) - 295(2.6571 - 1.68515)] \frac{\text{kJ}}{\text{kg}}$$

$$= 964\text{ kW}$$

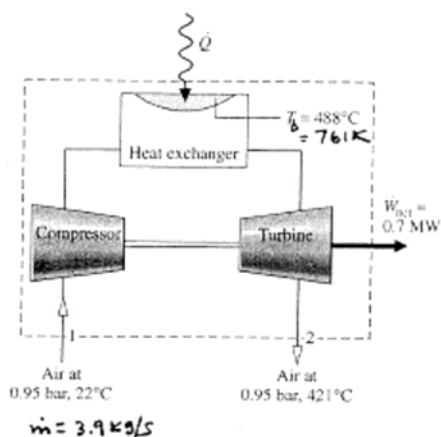
1. Consideration should be given to means for utilizing the considerable exergy of the high-temperature gas exiting the turbine at 4. See Secs. 9.7, 9.10 for options.

PROBLEM 7.85

Figure P7.85 shows a gas turbine power plant operating at steady state consisting of a compressor, a heat exchanger, and a turbine. Air enters the compressor with a mass flow rate of 3.9 kg/s at 0.95 bar, 22°C and exits the turbine at 0.95 bar, 421°C. Heat transfer to the air as it flows through the heat exchanger occurs at an average temperature of 488°C. The compressor and turbine operate adiabatically. Using the ideal gas model for the air, and neglecting the effects of motion and gravity, determine, in MW,

- the rate of exergy transfer accompanying heat transfer to the air flowing through the heat exchanger.
- the net rate exergy is carried out of the plant at the turbine exit, $(\dot{E}_{t2} - \dot{E}_{t1})$.
- the rate of exergy destruction within the power plant.

Compare the results of (a)–(c), and comment. Let $T_0 = 295 \text{ K}$ (22°C), $p_0 = 0.95 \text{ bar}$.



ENGR. MODEL:

- As shown in the sketch, consider a control volume enclosing the gas turbine power plant.
- The control volume is at steady state.
- Heat transfer occurs only at $T_b = 761 \text{ K}$.
- The effects of motion and gravity can be ignored.
- The air is modeled as an ideal gas.
- For the environment, $T_0 = 295 \text{ K}$, $p_0 = 0.95 \text{ bar}$.

ANALYSIS: (a) Mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \Rightarrow \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1). \text{ Then with data from Table A-22}$$

$$\dot{Q}_{cv} = 0.7 \text{ MW} + \left(3.9 \frac{\text{kg}}{\text{s}}\right) \left[706.8 - 295.17\right] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| = 2.305 \text{ MW}$$

Using Eq. 7.15

$$\dot{E}_q = \left[1 - \frac{T_0}{T_b}\right] \dot{Q}_{cv} = \left[1 - \frac{295}{761}\right] (2.305 \text{ MW}) = 1.411 \text{ MW} \quad \leftarrow (a)$$

(b) with Eqs. 7.16 and 7.18

$$\begin{aligned} \dot{E}_{t2} - \dot{E}_{t1} &= \dot{m}[(h_2 - h_1) - T_0(s_2 - s_1)] = \dot{m}[(h_2 - h_1) - T_0(s_2 - s_1) - R \ln \frac{p_2}{p_1}] \\ &= \left(3.9 \frac{\text{kg}}{\text{s}}\right) \left[(706.8 - 295.17) - 295(2.5635 - 1.68515)\right] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| \\ &= 0.595 \text{ MW} \quad \leftarrow (b) \end{aligned}$$

Continued on next slide

Problem 6-85 continued

(c) The total rate of exergy destruction can be obtained by use of the exergy rate balance, 7.13b

$$\begin{aligned}\dot{I}E_d &= \dot{I}E_q - \dot{W}_{cv} + (\dot{I}E_{f1} - \dot{I}E_{f2}) \\ &= (1.411 - 0.7 - 0.595) \text{ MW} \\ &= 0.116 \text{ MW}\end{aligned}$$

← (c)

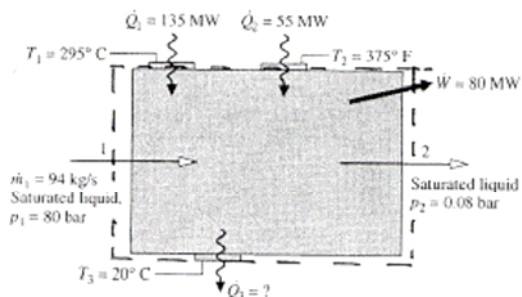
Discussion: The foregoing results provide the basis for an exergy accounting of the gas turbine power plant:

⊙ Rate exergy is supplied accompanying heat transfer.	1.411 MW
⊙ Disposition of the exergy supplied.	
✓ Power developed	0.700 MW (49.6%)
✓ Exergy destroyed	0.116 MW (8.2%)
✓ <u>Net</u> exergy carried out the stream exiting the turbine	0.595 MW (42.2%)
	<u>1.411 MW</u>

The exergy carried out by the exiting stream is significant. Consideration should be given to means for cost-effectively using it. For options, see Secs. 9.7 and 9.10.

PROBLEM 7.86

Figure P7.86 shows a power-generating system at steady state. Saturated liquid water enters at 80 bar with a mass flow rate of 94 kg/s. Saturated liquid exits at 0.08 bar with the same mass flow rate. As indicated by arrows, three heat transfers occur, each at a specified temperature in the direction of the arrow: The first adds 135 MW at 295°C, the second adds 55 MW at 375°C, and the third removes energy at 20°C. The system generates power at the rate of 80 MW. The effects of motion and gravity can be ignored. Let $T_0 = 20^\circ\text{C}$, $p_0 = 1\text{ atm}$. Determine, in MW, (a) the rate of heat transfer $\dot{Q}_3$ and the accompanying rate of exergy transfer and (b) a full exergy accounting of the total exergy supplied to the system with the two heat additions and with the net exergy, $(\dot{E}_{f1} - \dot{E}_{f2})$, carried in by the water stream as it passes from inlet to exit.



ENGR. MODEL:

1. The control volume shown in the figure is at steady state
2. Heat transfer occurs only at the specified temperatures and are positive in the direction of the arrows
3. The effects of motion and gravity can be ignored.
4. For the environment, $T_0 = 293\text{ K}$, $p_0 = 1\text{ atm}$

ANALYSIS: At state 1, $h_1 = 1316.6\text{ kJ/kg}$, $s_1 = 3.2068\text{ kJ/kg}\cdot\text{K}$. At state 2,
 $h_2 = 173.9\text{ kJ/kg}$, $s_2 = 0.5926\text{ kJ/kg}\cdot\text{K}$.

(a) Energy rate balance: $0 = (\dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3) - \dot{W} + \dot{m}(h_1 - h_2)$

$$\Rightarrow \dot{Q}_3 = \dot{Q}_1 + \dot{Q}_2 - \dot{W} + \dot{m}(h_1 - h_2) = 135\text{ MW} + 55\text{ MW} - 80\text{ MW} + 94\frac{\text{kg}}{\text{s}} \left(\frac{1316.6 - 173.9}{10^3} \right) \frac{\text{kJ}}{\text{kg}} \left| \frac{1\text{ MW}}{10^3\text{ kJ/s}} \right|$$

$$= 217\text{ MW}$$

$$\dot{E}_g = \left[1 - \frac{T_0}{T_3} \right] \dot{Q}_3 = 0 \quad \text{since } T_3 = T_0$$

Comment: There is considerable energy exiting the control volume by heat transfer at 3, but the exergetic value is zero because heat transfer occurs at $T_0 (= T_3)$.

(b) Exergy supplied = $65.4\text{ MW} + 30.1\text{ MW} + 35.4\text{ MW} = 130.9\text{ MW}$

$$\checkmark \quad \left[1 - \frac{T_0}{T_1} \right] \dot{Q}_1 = \left[1 - \frac{293}{568} \right] (135\text{ MW}) = 65.4\text{ MW}$$

$$\checkmark \quad \left[1 - \frac{T_0}{T_2} \right] \dot{Q}_2 = \left[1 - \frac{293}{648} \right] (55\text{ MW}) = 30.1\text{ MW}$$

$$\checkmark \quad \dot{E}_{f1} - \dot{E}_{f2} = \dot{m} [h_1 - h_2 - T_0(s_1 - s_2)] = \left(94\frac{\text{kg}}{\text{s}} \right) \left[(1316.6 - 173.9) - 293(3.2068 - 0.5926) \right] \frac{\text{kJ}}{\text{kg}} \left| \frac{1\text{ MW}}{10^3\text{ kJ/s}} \right|$$

$$= 35.4\text{ MW}$$

The exergy destruction is obtained from an exergy rate balance, Eq. 7.13a,

$$\dot{E}_d = [\text{Rate exergy is supplied}] - \dot{W} = 130.9\text{ MW} - 80\text{ MW} = 50.9\text{ MW}.$$

Summary:

- Exergy Supplied: 130.9 MW
 - Disposition of Exergy:
 - ✓ Power developed: 80 MW (61.1%)
 - ✓ Exergy destroyed: 50.9 MW (38.9%)
- 130.9 MW

PROBLEM 7.87

Figure P7.87 shows a gas turbine power plant using air as the working fluid. The accompanying table gives steady-state operating data. Air can be modeled as an ideal gas. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 290 \text{ K}$, $p_0 = 100 \text{ kPa}$. Determine, each in kJ per kg of air flowing, (a) the net power developed, (b) the net exergy increase of the air passing through the heat exchanger, $(\dot{e}_{13} - \dot{e}_{12})$, and (c) a full exergy accounting based on the exergy supplied to the plant found in part (b). Comment.

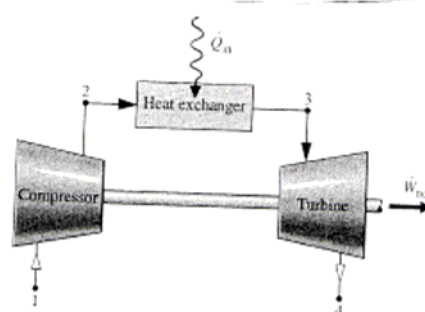


Fig. P7.87

State	p (kPa)	T (K)	h (kJ/kg)	s^0 (kJ/kg K)
1	100	290	290.16	1.6680
2	500	505	508.17	2.2297
3	500	875	904.99	2.8170
4	100	635	643.93	2.4688

s^0 is the variable appearing in Eq. 6.20a and Table A-22.

ENGR. MODEL:

1. The gas turbine power plant operates at steady state. Control volumes at steady state enclose each of the three components and the overall power plant.
2. Effects of motion and gravity can be ignored.
3. Air is modeled as an ideal gas.
4. $T_0 = 290 \text{ K}$, $p_0 = 100 \text{ kPa}$

(a) The net power developed equals the power developed by the turbine less the power required by the compressor: $\frac{\dot{W}_{\text{net}}}{\dot{m}} = (h_3 - h_4) - (h_2 - h_1) = (904.99 - 643.93) - (508.17 - 290.16) = 43.05 \text{ kJ/kg}$ (a.)

(b) $\frac{\dot{e}_{13} - \dot{e}_{12}}{\dot{m}} = (h_3 - h_2) - T_0(s_3 - s_2) = (h_3 - h_2) - T_0(s_3^0 - s_2^0 - R \ln p_3/p_2)$
 $= (904.99 - 508.17) - 290(2.8170 - 2.2297) = 226.5 \text{ kJ/kg}$ (b.)

(c) The net exergy carried out with the gas exiting the turbine:

$$\frac{\dot{e}_{14} - \dot{e}_{13}}{\dot{m}} = (h_4 - h_1) - T_0(s_4 - s_1) = (h_4 - h_1) - T_0(s_4^0 - s_1^0 - R \ln p_4/p_1)$$

$$= (643.93 - 290.16) - 290(2.4688 - 1.6680) = 121.54 \text{ kJ/kg}$$

Rate exergy is destroyed within the turbine and compressor:

$$\left(\frac{\dot{E}_d}{\dot{m}}\right)_{\text{comp}} = T_0 \frac{\dot{S}_{\text{dev}}}{\dot{m}} = T_0(s_2 - s_1) = T_0[s_2^0 - s_1^0 - R \ln p_2/p_1]$$

$$= 290 \text{ K} [2.2297 - 1.6680 - \frac{8.314}{29.97} \ln(5/1)] = 28.95 \frac{\text{kJ}}{\text{kg}}$$

$$\left(\frac{\dot{E}_d}{\dot{m}}\right)_{\text{turb}} = T_0[s_4^0 - s_3^0 - R \ln p_4/p_3] = 290 [2.4688 - 2.8170 - \frac{8.314}{29.97} \ln(1/5)]$$

$$= 32.97 \text{ kJ/kg}$$

Summary:

① Exergy supplied:	226.5 kJ/kg	
② Disposition of exergy:		
✓ power developed:	43.05 kJ/kg	(19%) ← Relatively little power is developed.
✓ exergy destroyed:	28.95 kJ/kg	(12.8%)
compressor:	32.97 kJ/kg	(14.6%)
turbine:		
✓ Net exergy carried out at turbine exit:	121.54 kJ/kg	(53.7%) ← Considerable exergy is carried out of the power plant.
	226.5 kJ/kg	

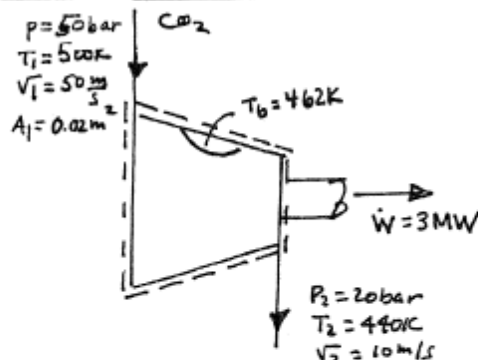
Comment: Steps should be taken to cost-effectively utilize the exergy carried out at the turbine exit. For options, see Sects. 9.7 and 9.10.

PROBLEM 7.88

Carbon dioxide (CO_2) gas enters a turbine operating at steady state at 50 bar, 500 K with a velocity of 50 m/s. The inlet area is 0.02 m^2 . At the exit, the pressure is 20 bar, the temperature is 462 K, and the velocity is 10 m/s. The power developed by the turbine is 3 MW, and heat transfer occurs across a portion of the surface where the average temperature is 462 K. Assume ideal gas behavior for the carbon dioxide and neglect the effect of gravity. Let $T_0 = 298 \text{ K}$, $p_0 = 1 \text{ bar}$.

- Determine the rate of heat transfer, in kW.
- Perform a full exergy accounting, in kW, based on the net rate exergy is carried into the turbine by the carbon dioxide.

SCHEMATIC & GIVEN DATA



ENGINE MODEL:

- The control volume shown in the sketch is at steady state.
- Heat transfer occurs only at temperature $T_b = 462 \text{ K}$.
- The effect of gravity can be ignored.
- CO_2 is modeled as an ideal gas.
- For the environment, $T_0 = 298 \text{ K}$, $p_0 = 1 \text{ bar}$.

ANALYSIS: (a) An energy rate balance reduces to give

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m} \left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} \right] \quad (1)$$

where

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{p_1 A_1 V_1}{RT_1} = \frac{(5 \times 10^6 \text{ N/m}^2)(0.02 \text{ m}^2)(50 \text{ m/s})}{\left(\frac{8.314}{44.01} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (500 \text{ K})} = 52.93 \frac{\text{kg}}{\text{s}}$$

From Table A-23, $\bar{h}_1 = 17,678 \text{ kJ/kmol}$, $\bar{h}_2 = 15,054 \text{ kJ/kmol}$. Thus, Eq. (1) gives

$$\begin{aligned} \dot{Q}_{cv} &= 3 \times 10^3 \text{ kW} + 52.93 \frac{\text{kg}}{\text{s}} \left[\frac{15,054 - 17,678}{44.01} \frac{\text{kJ}}{\text{kg}} + \left(\frac{(10)^2 - (50)^2}{2} \right) \left(\frac{\text{m}^2}{\text{s}^2} \right) \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \right] \\ &= -219 \text{ kW} \end{aligned} \quad (a)$$

(b) The net rate exergy is carried into the turbine $= \dot{E}_{f1} - \dot{E}_{f2}$, or

$$\begin{aligned} (\dot{E}_{f1} - \dot{E}_{f2}) &= \dot{m} \left[(h_1 - h_2) - T_0 (s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} \right] \\ &= \dot{m} \left[(h_1 - h_2) - T_0 [s_1^0 - s_2^0 - R \ln p_1/p_2] + \frac{V_1^2 - V_2^2}{2} \right] \end{aligned}$$

From Table A-23, $\bar{s}_1^0 = 234.814 \text{ kJ/kmol} \cdot \text{K}$, $\bar{s}_2^0 = 229.230 \text{ kJ/kmol} \cdot \text{K}$. Thus

$$\begin{aligned} \dot{E}_{f1} - \dot{E}_{f2} &= 52.93 \frac{\text{kg}}{\text{s}} \left[\frac{17,678 - 15,054}{44.01} \frac{\text{kJ}}{\text{kg}} - 298 \text{ K} \left(\frac{234.814 - 229.230}{44.01} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - \frac{8.314}{44.01} \ln \frac{50}{20} \right) \right. \\ &\quad \left. + \left[\frac{(50)^2 - (10)^2}{2} \right] \left(\frac{\text{m}^2}{\text{s}^2} \right) \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \right] \\ &= 3948 \text{ kW} \end{aligned}$$

Continued on next slide

Problem 6-88 continued

$$\dot{E}_q = \left[1 - \frac{T_0}{T_b}\right] \dot{Q}_{cv} = \left[1 - \frac{298}{462}\right] (-219 \text{ kW}) = -78 \text{ kW}$$

The rate of exergy destruction can be found from an exergy rate balance or using $\dot{E}_d = T_0 \dot{S}_{cv}$. Selecting the former,

$$\begin{aligned} \dot{E}_d &= \dot{E}_q - \dot{W} + (\dot{E}_{f1} - \dot{E}_{f2}) \\ &= (-78) + (-3000) + 3948 = 870 \text{ kW} \end{aligned}$$

Summary:

← (b)

⊙ Net exergy supplied to the turbine:

3948 kW

⊙ Disposition of exergy supplied:

✓ Power developed:	3000 kW	(76%)
✓ Exergy carried out via heat trans fer	78 kW	(2%)
✓ Exergy destroyed	870 kW	(22%)
	3948 kW	

PROBLEM 7.89

For the air compressor of Problem 7.66 perform a full exergy accounting, in kJ per kg, of air flowing, based on the work input.

ANALYSIS:

From Problem 7.66,

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right) = 94.6 \frac{\text{kJ}}{\text{kg}}$$

$$\left(-\frac{\dot{Q}_{cv}}{\dot{m}}\right) = 14.0 \frac{\text{kJ}}{\text{kg}} \Rightarrow \frac{\dot{E}_q}{\dot{m}} = \left[1 - \frac{T_0}{T_b}\right] \frac{\dot{Q}_{cv}}{\dot{m}} = \left[1 - \frac{293}{313}\right] (-14 \frac{\text{kJ}}{\text{kg}}) = -0.89 \frac{\text{kJ}}{\text{kg}}$$

$$\dot{E}_d = 20.45 \frac{\text{kJ}}{\text{kg}}$$

The net change in exergy of the air stream is

$$\begin{aligned} \left(\frac{\dot{E}_{f2} - \dot{E}_{f1}}{\dot{m}}\right) &= (h_2 - h_1) - T_0(S_2 - S_1) \\ &= (380.79 - 300.19) - 293 \text{K} (0.025 \text{J}) \\ &\quad \xrightarrow{\text{from Problem 7.66 solution.}} \\ &= 73.25 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

Summary:

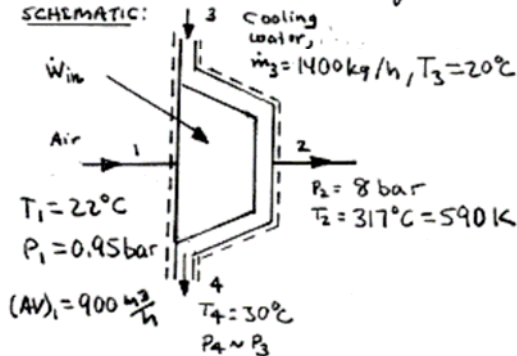
① Exergy supplied to compressor = work:	94.6 kJ/kg	
② Disposition of exergy:		
✓ Exergy increase of air stream:	73.25 kJ/kg	(77%)
✓ Exergy carried out via heat transfer:	0.89 kJ/kg	(1%)
✓ Exergy destroyed:	20.45 kJ/kg	(22%)
	94.6 kJ/kg	

PROBLEM 7.90

KNOWN: Steady-state operating data are provided for a water-jacketed air compressor.

FIND: Perform an exergy accounting of the power input

SCHEMATIC:



EXGR. MODEL:

1. The control volume shown in the schematic operates at steady state with negligible kinetic and potential energy effects and $\dot{Q}_{cv} = 0$.
2. Air is modeled as an ideal gas.
3. Water is modeled as incompressible (1) with $c = 4.19 \text{ kJ/kg} \cdot \text{K}$ (Table A-19).
4. $T_0 = 293 \text{ K}$ (20°C), $P_0 = 1 \text{ atm}$.

ANALYSIS: The exergy entering the control volume with the power input has the following disposition: (i) the exergy of the air stream is increased, (ii) the exergy of the water stream is increased, and (iii) exergy is destroyed by irreversibilities within the control volume. These quantities are now evaluated:

At steady state $\dot{m}_1 = \dot{m}_2$, $\dot{m}_3 = \dot{m}_4$. The energy rate balance reduces to give

$$\dot{W}_{cv} = \dot{m}_1 (h_1 - h_2) + \dot{m}_3 (h_3 - h_4)$$

With the ideal gas model equation of state

$$\dot{m}_1 = \frac{(\dot{A}V)_1}{v_1} = \frac{(\dot{A}V)_1}{(RT_1/P_1)} = \frac{(95,000 \text{ m}^3/\text{h})(900 \text{ m}^3/\text{h})}{\left(\frac{8.314 \text{ N} \cdot \text{m}}{28.97 \text{ kg} \cdot \text{K}}\right)(295 \text{ K})} = 1009.9 \frac{\text{kg}}{\text{h}}$$

Then, with h_1, h_2 from Table A-22 and Eq. 3.20b ($P_4 \approx P_3$)

$$\begin{aligned} \dot{W}_{cv} &= (1009.9 \frac{\text{kg}}{\text{h}})[295.17 - 596.52] \frac{\text{kJ}}{\text{kg}} + (1400 \frac{\text{kg}}{\text{h}})(4.19 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(-10 \text{ K}) \\ &= [-304,333 + (-58,660)] \frac{\text{kJ}}{\text{h}} \left| \frac{1 \text{ kW}}{3600 \text{ kJ/h}} \right| = -100.83 \text{ kW} \end{aligned}$$

The rate of exergy destruction can be determined using an exergy rate balance or $\dot{E}_d = T_0 \dot{\sigma}$ and an entropy rate balance. At steady state the entropy rate balance reduces to give

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}_1 (s_1 - s_2) + \dot{m}_3 (s_3 - s_4) + \dot{\sigma}_{cv}$$

Or with s° data from Table A-22 and Eq. 6.13

$$\begin{aligned} \dot{\sigma}_{cv} &= \dot{m}_1 [s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1] + \dot{m}_3 c \ln \frac{T_4}{T_3} \\ &= (1009.9 \frac{\text{kg}}{\text{h}})[2.39140 - 1.68515 - \frac{8.314}{28.97} \ln \frac{8}{1}] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} + (1400)(4.19) \ln \frac{303}{293} \\ &= [110.56 + 196.86] \frac{\text{kJ}}{\text{h} \cdot \text{K}} \left| \frac{1 \text{ kW}}{3600 \text{ kJ/h}} \right| = 0.05839 \frac{\text{kW}}{\text{K}} \end{aligned}$$

$$\therefore \dot{E}_d = T_0 \dot{\sigma}_{cv} = (293 \text{ K})(0.05839 \frac{\text{kW}}{\text{K}}) = 17.12 \text{ kW}$$

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Problem 6-90 continued

For the air, the increase in flow exergy rate from inlet to exit is

$$\begin{aligned}\dot{E}_{f2} - \dot{E}_{f1} &= \dot{m}_1(e_{f2} - e_{f1}) = \dot{m}_1[(h_2 - h_1) - T_0(s_2 - s_1)] \\ &= \dot{m}_1(h_2 - h_1) - T_0 \dot{m}_1(s_2 - s_1) \\ &= \left(\frac{304,333}{3600}\right) \text{ kW} - (293 \text{ K}) \left[\left(\frac{110.56}{3600}\right)\right] \frac{\text{ kW}}{\text{ K}} = 75.54 \text{ kW}\end{aligned}$$

For the water, the increase in flow exergy rate from inlet to exit is

$$\begin{aligned}\dot{E}_{f4} - \dot{E}_{f3} &= \dot{m}_3(e_{f4} - e_{f3}) = \dot{m}_3[(h_4 - h_3) - T_0(s_4 - s_3)] \\ &= \dot{m}_3(h_4 - h_3) - T_0 \dot{m}_3(s_4 - s_3) \\ &= \left(\frac{58,660}{3600}\right) \text{ kW} - (293 \text{ K}) \left(\frac{196.86}{3600}\right) \frac{\text{ kW}}{\text{ K}} = 0.27 \text{ kW}\end{aligned}$$

Exergy Accounting:

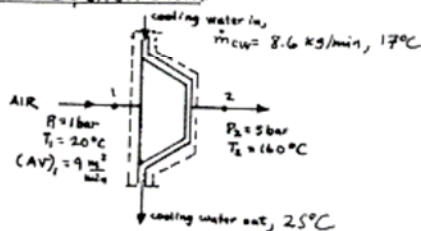
• Exergy input: power input	100.83 kW
• Disposition:	
- Increase in exergy of the air stream	75.54 kW (74.9 %)
- Increase in exergy of the water stream	0.27 kW (0.3 %)
- Exergy destruction	25.02 kW (24.9 %)
	<u>100.83 kW</u>

1. Alternatively, for the liquid water $h \sim h_f(T)$, $s \sim s_f(T)$.

PROBLEM 7.91

According to a laboratory data sheet, air enters a compressor operating at steady state at 1 bar, 20°C with a volumetric flow rate of 9 m³/min and exits at 5 bar, 160°C. Cooling water is circulated through a water jacket enclosing the compressor at a rate of 8.6 kg/min, entering at 17°C, and exiting at 25°C with a negligible change in pressure. There is no significant heat transfer from the outer surface of the water jacket, and the effects of motion on gravity are negligible. Let $T_0 = 17^\circ\text{C}$, $p_0 = 1$ bar. Perform a full exergy accounting in kW, based on the compressor power input and comment.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the figure operates at steady state with negligible kinetic and potential energy effects and $\dot{Q}_{cv} = 0$. (2) Air is modeled as an ideal gas. (3) The cooling water is modeled as incompressible with constant specific heat c and a negligible change in pressure. (4) $T_0 = 290\text{K}$, $p_0 = 1$ bar.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$ and the rates of cooling water entering and exiting are equal. An energy rate balance reduces with listed assumptions to give

$$\dot{W}_{cv} = \dot{m} [h_1 - h_2] + \dot{m}_{cw} [h_{cw,in} - h_{cw,out}]$$

With the ideal gas equation of state

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{R T_1} = \frac{(10^5 \text{ N/m}^2) (9 \frac{\text{m}^3}{\text{min}})}{(28.97 \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}) (293 \text{ K})} = 10.703 \frac{\text{kg}}{\text{min}}$$

Then, with specific enthalpies h_1 and h_2 from Table A-22, and a specific heat value for liquid water from Table A-19, $c = 4.19 \text{ kJ/kg} \cdot \text{K}$, together with Eq. 3.20b: $\Delta h = c \Delta T + v \Delta p$, where Δp is dropped by assumption 3, the power is

$$\begin{aligned} \dot{W}_{cv} &= (10.703 \frac{\text{kg}}{\text{min}}) (293.2 - 434.5) \frac{\text{kJ}}{\text{kg}} + (8.6 \frac{\text{kg}}{\text{min}}) (4.19 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) (290 - 298) \text{ K} \\ &= (-1512.3 - 288.3) \frac{\text{kJ}}{\text{min}} \left| \frac{\text{min}}{60 \text{ s}} \right| \left| \frac{\text{KW}}{\text{kJ/s}} \right| = -30.01 \text{ KW} \leftarrow \dot{W}_{cv} \end{aligned}$$

At steady state an entropy rate balance reduces to give

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m} (s_1 - s_2) + \dot{m}_{cw} (s_{cw,in} - s_{cw,out}) + \dot{\sigma}_{cv}$$

Or, with Eq. 6.20a and Eq. 6.13

$$\begin{aligned} \dot{\sigma}_{cv} &= \dot{m} (s_2 - s_1 - R \ln \frac{p_2}{p_1}) + \dot{m}_{cw} c \ln \frac{T_{cw,out}}{T_{cw,in}} \\ &= (10.703 \frac{\text{kg}}{\text{min}}) (2.07234 - 1.6783 - \frac{8.314}{28.97} \ln 5) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} + \\ &\quad (8.6 \frac{\text{kg}}{\text{min}}) (4.19 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \ln \frac{298}{290} \\ &= (-0.72617 + 0.9806) \frac{\text{kJ}}{\text{min}} \left| \frac{\text{min}}{60 \text{ s}} \right| \left| \frac{\text{KW}}{\text{kJ/s}} \right| \\ &= 4.24 \times 10^{-3} \text{ KW/K} \leftarrow \end{aligned}$$

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Problem 6-91 continued

$$\text{Then, } \dot{E}_d = T_0 \dot{S}_{cv} = (290 \text{ K}) (4.24 \times 10^{-3} \frac{\text{K W}}{\text{K}}) = 1.23 \text{ kW}$$

The exergy increase of the air stream is

$$\begin{aligned} \dot{E}_{f2} - \dot{E}_{f1} &= \dot{m}_a [h_2 - h_1 - T_0 (S_2 - S_1)] = \dot{m}_a [h_2 - h_1 - T_0 (S_2^0 - S_1^0 - R \ln P_2/P_1)] \\ &= \left(\frac{10.703}{60} \frac{\text{kg}}{\text{s}} \right) \left[(434.5 - 293.2) - 290 \text{ K} \left[(2.07234 - 1.6783) - \frac{8.314}{28.97} \ln 5 \right] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 28.72 \text{ kW} \end{aligned}$$

The exergy increase of the water stream is

$$\begin{aligned} (\dot{E}_{f,e} - \dot{E}_{f,i}) &= \dot{m}_{cw} [h_e - h_i - T_0 (s_e - s_i)] = \dot{m}_{cw} [c(T_e - T_i) - c T_0 \ln(T_e/T_i)] \\ &= \left(\frac{8.6}{60} \right) \frac{\text{kg}}{\text{s}} \left[4.18 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right] \left[8 \text{ K} - 290 \ln \frac{298}{290} \right] = 0.07 \text{ kW} \end{aligned}$$

Summary:

⊙ Exergy input: power input	30.01 kW
⊙ Disposition of the exergy input:	
- Increase in exergy of the air stream:	28.72 kW (95.7%)
- Increase in exergy of the water stream:	0.07 kW (0.2%)
- Exergy destroyed:	1.23 kW (4.1%)
	30.02 kW

Comment: Based on experience with compressors, the exergy increase of the air stream relative to the work input: 95.7% is too high. Values in the 80-percent range would be typical. On this basis there is reason to suspect an error in the laboratory data set.

1. Alternatively for the liquid water $h \approx h_f(T)$, $s \approx s_f(T)$.

PROBLEM 7.92

Refrigerant 134a enters a water-jacketed compressor operating at steady state at 10°C , 3.2 bar and exits at 70°C , 10 bar. Cooling water enters as a separate stream at 20°C and exits at 32°C with no significant change in pressure. The refrigerant mass flow rate is 1.63 kg/s , and the power input to the compressor is $55.2 \text{ kJ per kg of refrigerant flowing}$. Assuming no heat transfer between the outer surface of the water jacket and the surroundings, and neglecting the effects of motion and gravity

- determine the mass flow rate of cooling water, in kg/s , and
- perform a full exergy accounting, in kW , based on the compressor power input and comment.

Let $T_0 = 20^\circ\text{C}$, $p_0 = 1 \text{ bar}$.

ENGR. MODEL:

- The control volume shown in the sketch is at steady state.
- For the control volume $\dot{Q}_{cv}$ and the effects of motion and gravity can be neglected.
- For the cooling water, $h \sim h_f(T)$, $s \sim s_f(T)$.
- $T_0 = 293 \text{ K}$, $p_0 = 1 \text{ bar}$.

ANALYSIS: The compressor power is $\dot{W}_{cv} = -(1.63 \frac{\text{kg}}{\text{s}})(55.2 \frac{\text{kJ}}{\text{kg}}) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -89.98 \text{ kW}$

(a) An energy rate balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) + \dot{m}_{cw}(h_3 - h_4)$$

$$\Rightarrow \dot{m}_{cw} = \frac{\dot{W}_{cv} + \dot{m}(h_2 - h_1)}{h_3 - h_4} = \frac{-89.98 \frac{\text{kJ}}{\text{s}} + (1.63 \frac{\text{kg}}{\text{s}})(302.34 - 255.65) \frac{\text{kJ}}{\text{kg}}}{(83.96 - 134.15)} \frac{\text{kJ}}{\text{kg}} = 0.276 \text{ kg/s}$$

← (a)

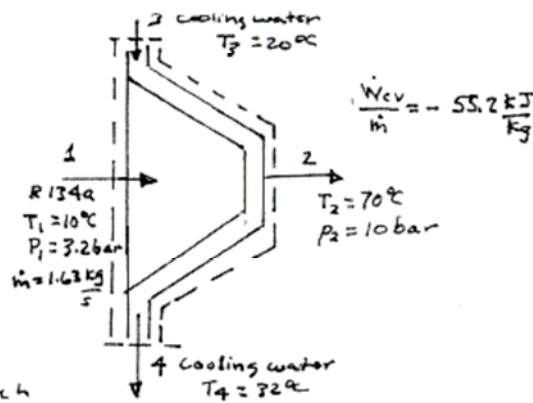
(b) The increase in exergy of the refrigerant stream is

$$\begin{aligned} \dot{E}_{f,2} - \dot{E}_{f,1} &= \dot{m} [(h_2 - h_1) - T_0(s_2 - s_1)] \\ &= 1.63 \frac{\text{kg}}{\text{s}} \left[(302.34 - 255.65) \frac{\text{kJ}}{\text{kg}} - 293 \text{ K} (1.0093 - 0.9427) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 44.3 \text{ kW} \end{aligned}$$

The increase in exergy of the cooling water stream is

$$\begin{aligned} \dot{E}_{f,4} - \dot{E}_{f,3} &= \dot{m}_{cw} [(h_4 - h_3) - T_0(s_4 - s_3)] \\ &= 0.276 \frac{\text{kg}}{\text{s}} \left[(134.15 - 83.96) \frac{\text{kJ}}{\text{kg}} - 293 \text{ K} (0.4649 - 0.2766) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 0.28 \text{ kW} \end{aligned}$$

SCHEMATIC & GIVEN DATA:



$$\frac{\dot{W}_{cv}}{\dot{m}} = -55.2 \frac{\text{kJ}}{\text{kg}}$$

$$T_2 = 70^\circ\text{C}$$

$$P_2 = 10 \text{ bar}$$

$$T_4 = 32^\circ\text{C}$$

← (a)

Continued on next slide

Problem 6-92 continued

The rate of exergy destruction can be found from an exergy rate balance:

$$\begin{aligned}\dot{E}_d &= \dot{E}_q^o - \dot{W} + (\dot{E}_{f,1} - \dot{E}_{f,2}) + (\dot{E}_{f,3} - \dot{E}_{f,4}) \\ &= -(-89.98) + (-44.3) + (-0.28) \\ &= 45.4 \text{ kW}\end{aligned}$$

Summary:

(b)

⊙ Exergy input: power input	89.98 kW
⊙ Disposition of the exergy input:	
✓ Increase in exergy of the refrigerant stream:	44.3 kW (49.2%)
✓ Increase in exergy of the cooling water stream:	0.28 kW (0.3%)
✓ Exergy destruction:	$\frac{45.4 \text{ kW}}{89.98 \text{ kW}}$ (50.5%)

Comment: Based on experience with compressors, the exergy increase of the R134a stream relative to the power input: 49.2% is too low. Values in the 80% range would be typical. The exergy destruction rate is also too high, suggesting that an opportunity exists for improving the thermodynamic performance of this compressor.

1. Alternatively, the liquid water can be modeled as incompressible.

PROBLEM 7.93

For the compressor and heat exchanger of Problem 6.114, perform a full exergy accounting, in kW, based on the compressor power input. Let $T_0 = 300 \text{ K}$, $p_0 = 96 \text{ kPa}$.

KNOWN: Steady-state operating data are provided for a compressor and heat exchanger in series.

FIND: Develop a full exergy accounting of the compressor power input.

SCHEMATIC & GIVEN DATA

- See the schematic in the solution to Problem 6.114
- From the solution

$$(-\dot{W}_{\text{cv}}) = 50.4 \text{ kW}, \dot{m}_{\text{air}} = 0.5 \text{ kg/s}, \dot{m}_{\text{cw}} = 0.403 \text{ kg/s}$$

$$\dot{\sigma}_{\text{comp}} = 0.0196 \frac{\text{KW}}{\text{K}}, \dot{\sigma}_{\text{HX}} = 0.0152 \frac{\text{KW}}{\text{K}}$$

State	h	s^0	State	h	s
1	300.19	1.70203	A	104.89	0.3674
3	350.49	1.85708	B	167.57	0.5725

ENGR. MODEL: 1. See assumptions listed for Problem 6.114. 2. $T_0 = 300 \text{ K}$, $p_0 = 96 \text{ kPa}$

ANALYSIS: The rates of exergy destruction can be found using $\dot{E}_d = T_0 \dot{\sigma}_{\text{cv}}$.

$$\text{compressor: } (\dot{E}_d)_c = 300 \text{ K} \left(0.0196 \frac{\text{KW}}{\text{K}} \right) = 5.88 \text{ kW}$$

$$\text{heat exchanger: } (\dot{E}_d)_{\text{HX}} = 300 (0.0152) = 4.56 \text{ kW}$$

The increase in exergy of the air stream is

$$\begin{aligned} \dot{E}_f - \dot{E}_i &= \dot{m}_{\text{air}} (h_3 - h_1 - T_0 (s_3 - s_1) - R \ln p_3/p_1) \\ &= \left(0.5 \frac{\text{kg}}{\text{s}} \right) \left[(350.49 - 300.19) - 300 (1.85708 - 1.70203) - \frac{8.314}{28.97} \ln \frac{230}{96} \right] \frac{\text{KJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ KJ/s}} \right| \\ &= 39.5 \text{ kW} \end{aligned}$$

The increase in exergy of the water stream is

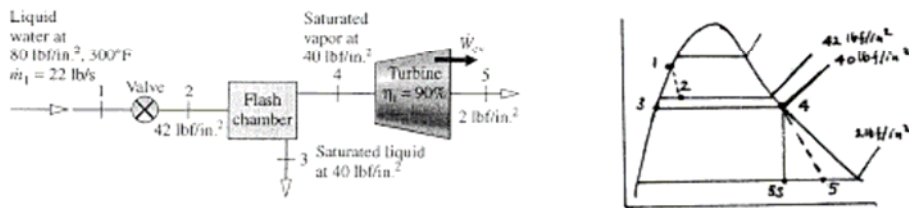
$$\begin{aligned} \dot{E}_f - \dot{E}_i &= \dot{m}_{\text{cw}} [(h_B - h_A) - T_0 (s_B - s_A)] = \left(0.403 \frac{\text{kg}}{\text{s}} \right) [(167.57 - 104.89) - 300 (0.5725 - 0.3674)] \frac{\text{KJ}}{\text{kg}} \\ &= 0.46 \text{ kW} \end{aligned}$$

Exergy Balance Sheet (rate basis)

① rate exergy is supplied by compressor power input (= power input).	50.4 kW	
② disposition of the exergy		
- net exergy carried out by the air	39.5 kW	(78.4%)
- net exergy carried out by the water	0.46 kW	(0.9%)
- exergy destruction		
• compressor	5.88 kW	(11.7%)
• heat exchanger	4.56 kW	(9.0%)
	50.4 kW	

PROBLEM 7.94

Figure P7.94 shows liquid water at 80 lbf/in.², 300°F entering a flash chamber through a valve at the rate of 22 lb/s. At the valve exit, the pressure is 42 lbf/in.². Saturated liquid at 40 lbf/in.² exits from the bottom of the flash chamber and saturated vapor at 40 lbf/in.² exits from near the top. The vapor stream is fed to a steam turbine having an isentropic efficiency of 90% and an exit pressure of 2 lbf/in.². For steady-state operation, negligible heat transfer with the surroundings, and no significant effects of motion and gravity, perform a full exergy accounting, in Btu/s, of the net rate at which exergy is supplied: $(\dot{E}_{11} - \dot{E}_{13} - \dot{E}_{15})$. Let $T_0 = 500^\circ\text{R}$, $p_0 = 1\text{ atm}$.



ENGR. MODEL: (1) Each component operates at steady state with negligible heat transfer between each component and its surroundings. (2) Kinetic and potential energy effects can be ignored. (3) The expansion across the valve is a throttling process. (4) $T_0 = 500^\circ\text{R}$, $p_0 = 1\text{ atm}$.

ANALYSIS: At steady state, mass rate balances reduce to give $\dot{m}_2 = \dot{m}_1$, $\dot{m}_3 + \dot{m}_4 = \dot{m}_2$.

$\dot{m}_3 = \dot{m}_5$ An entropy rate balance for a control volume enclosing the flash chamber takes the form

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}_2 s_2 - \dot{m}_3 s_3 - \dot{m}_4 s_4 + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}_4 s_4 + \dot{m}_3 s_3 - \dot{m}_2 s_2 \quad [\text{flash chamber}] \quad (1)$$

Similarly, for the valve and turbine, the rates of entropy production are

$$\dot{\sigma}_{cv} = \dot{m}_1 (s_2 - s_1) \quad [\text{valve}] \quad (2)$$

$$\dot{\sigma}_{cv} = \dot{m}_4 (s_5 - s_4) \quad [\text{turbine}] \quad (3)$$

To evaluate Eqs. (1)–(3) requires $\dot{m}_3$, $\dot{m}_4$, s_1 , s_2 , s_3 , s_4 , and s_5 . These will now be evaluated in turn.

Using the mass rate balance together with listed assumptions, an energy rate balance for a control volume enclosing the flash chamber reduces to

$$0 = \dot{m}_2 h_2 - \dot{m}_3 h_3 - \dot{m}_4 h_4$$

$$0 = \dot{m}_1 h_2 - \dot{m}_3 h_3 - (\dot{m}_1 - \dot{m}_3) h_4$$

Across the valve $h_2 = h_1$, so on solving the above equation

$$\dot{m}_3 = \dot{m}_1 \left[\frac{h_1 - h_4}{h_3 - h_4} \right] = \left(\frac{22\text{ lb}}{\text{s}} \right) \left[\frac{269.7 - 1170}{236.16 - 1170} \right] = 21.21\text{ lb/s}$$

where h_1 , h_3 , h_4 from Table A-2E and h_2 and h_4 are from Table A-3E. Then, $\dot{m}_4 = \dot{m}_2 - \dot{m}_3 = 22 - 21.21 = 0.79\text{ lb/s}$.

From Table A-2E, $s_1 = s_f(T_1) = 0.4772\text{ Btu/lb}\cdot^\circ\text{R}$. State 2 is fixed by 42 lbf/in.² and $h_2 = h_1 = 269.7\text{ Btu/lb}$. Thus, with data from Table A-3E

$$x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{269.7 - 239.1}{921.8} = 0.0328$$

Thus

$$s_2 = s_f + x_2 s_{fg} = 0.5961 + 0.0328(1.2767) = 0.438\text{ Btu/lb}\cdot^\circ\text{R}$$

From Table A-3E at 40 lbf/in.², $s_3 = 0.2921\text{ Btu/lb}\cdot^\circ\text{R}$, $s_4 = 1.6767\text{ Btu/lb}\cdot^\circ\text{R}$. State 5 is fixed by 2 lbf/in.² and h_5 . Using the turbine isentropic efficiency,

$$\eta_t = \frac{h_4 - h_5}{h_4 - h_{5s}} \quad (4)$$

where h_{5s} is the specific enthalpy at the turbine exit for an isentropic expansion from state 4 to 2 lbf/in.². The value of h_{5s} is determined by 2 lbf/in.² and $s_{5s} = s_4$.

Continued on next slide

Problem 6-94 continued

Thus, with data from Table A-3E at $2.16 \text{ ft}^3/\text{in}^3$

$$x_{ss} = \frac{s_{ss} - s_f}{s_{fg}} = \frac{1.6767 - 0.175}{1.7448} = 0.861 \Rightarrow h_{ss} = h_f + x_{ss} h_{fg} = 94.02 + 0.861(1022.1) = 974.05 \frac{\text{Btu}}{\text{lb}}$$

Then, with data from Table A-3E at $2.16 \text{ ft}^3/\text{in}^3$

$$x_{ss} = \frac{s_{ss} - s_f}{s_{fg}} = \frac{1.6767 - 0.175}{1.7448} = 0.861, h_{ss} = h_f + x_{ss} h_{fg} = 94.02 + 0.861(1022.1) = 974.05 \frac{\text{Btu}}{\text{lb}}$$

Solving Eq. (4) for h_5 and inserting values

$$h_5 = h_4 - \eta_t (h_4 - h_{ss}) = 1170 - 0.9(1170 - 974.05) = 993.65 \frac{\text{Btu}}{\text{lb}}$$

Using this together with data from Table A-3E at $2.16 \text{ ft}^3/\text{in}^3$

$$x_s = \frac{h_s - h_f}{h_{fg}} = \frac{993.65 - 94.02}{1022.1} = 0.88$$

$$s_s = s_f + x_s s_{fg} = 0.175 + 0.88(1.7448) = 1.7104 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

Finally, substituting values into Eq. (1) - (3)

- flash chamber: $\dot{\sigma}_{cv} = (0.79)[1.6767] + (2.12)[0.392] - (2.2)[0.438] = 0.0054 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}}$
- valve: $\dot{\sigma}_{cv} = (2.2)[0.438 - 0.4372] = 0.0176 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}}$
- turbine: $\dot{\sigma}_{cv} = (0.79)[1.7104 - 1.6767] = 0.0266 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}}$

Mass and energy rate balances reduce for the turbine to give

$$\dot{W}_t = \dot{m}_4 [h_4 - h_5] = (0.79 \frac{\text{lb}}{\text{s}})[1170 - 993.65] = 139.3 \frac{\text{Btu}}{\text{s}}$$

Using the entropy production rates, exergy destruction rates are found from $\dot{E}_d = T_0 \dot{\sigma}_{cv}$:

- flash chamber: $\dot{E}_d = 500^\circ\text{R}(0.0054 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}}) = 2.7 \frac{\text{Btu}}{\text{s}}$
- valve: $\dot{E}_d = 500^\circ\text{R}(0.0176 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}}) = 8.8 \frac{\text{Btu}}{\text{s}}$
- turbine: $\dot{E}_d = 500^\circ\text{R}(0.0266 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}}) = 13.3 \frac{\text{Btu}}{\text{s}}$

The net rate at which exergy is supplied is $(\dot{E}_{f1} - \dot{E}_{f3} - \dot{E}_{f5}) = \dot{m}_1 e_{f1} - \dot{m}_3 e_{f3} - \dot{m}_5 e_{f5}$

$$\begin{aligned} \Rightarrow (\dot{E}_{f1} - \dot{E}_{f3} - \dot{E}_{f5}) &= \dot{m}_3 (e_{f1} - e_{f3}) + \dot{m}_5 (e_{f1} - e_{f5}) \\ &= \dot{m}_3 [(h_1 - h_3) - T_0(s_1 - s_3)] + \dot{m}_5 [(h_1 - h_5) - T_0(s_1 - s_5)] \\ &= 2.12 \frac{\text{lb}}{\text{s}} [(269.7 - 236.16) - 500(0.4372 - 0.3921)] \frac{\text{Btu}}{\text{lb}} + 0.79 [(269.7 - 993.65) - 500(0.4372 - 1.7104)] \\ &= 164.1 \frac{\text{Btu}}{\text{s}} \end{aligned}$$

Summary:

① Net rate exergy is supplied:

$$164.1 \frac{\text{Btu}}{\text{s}}$$

② Disposition:

- ✓ Power developed:
- ✓ Exergy destruction
 - turbine
 - valve
 - flash chamber

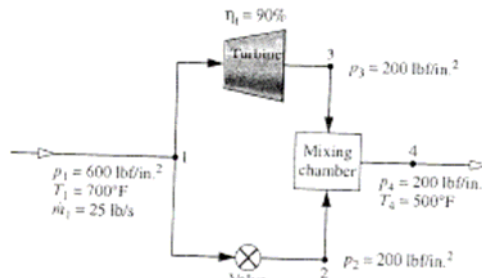
$$\begin{array}{r} 139.3 \frac{\text{Btu}}{\text{s}} \quad (84.9\%) \\ 13.3 \frac{\text{Btu}}{\text{s}} \quad (8.1\%) \\ 8.8 \frac{\text{Btu}}{\text{s}} \quad (5.4\%) \\ 2.7 \frac{\text{Btu}}{\text{s}} \quad (1.6\%) \\ \hline 164.1 \frac{\text{Btu}}{\text{s}} \end{array}$$

PROBLEM 7.95

Figure P7.95 provides steady-state operating data for a throttling valve in parallel with a steam turbine having an isentropic turbine efficiency of 90%. The streams exiting the valve and the turbine mix in a mixing chamber. Heat transfer with the surroundings and the effects of motion and gravity can be neglected. Determine

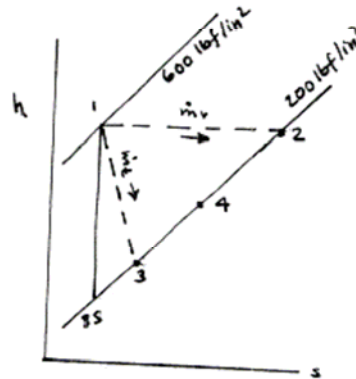
- the power developed by the turbine, in Btu/s.
- the mass flow rates through the turbine and valve, each in lb/s.
- a full exergy accounting, in Btu/s, of the net rate at which exergy is supplied: $(E_{11} - E_{14})$.

Let $T_0 = 500^\circ\text{R}$, $p_0 = 1 \text{ atm}$.



ENGR. MODEL:

- Control volumes enclosing the turbine, valve, mixing chamber, and overall configuration are at steady state.
- Heat transfer with the surroundings and the effects of motion and gravity can be neglected.
- The expansion across the valve is a throttling process.
- $T_0 = 500^\circ\text{R}$, $p_0 = 1 \text{ atm}$.



ANALYSIS: (a) Considering an overall control volume at steady state including the turbine, valve, and mixing chamber, mass and energy rate balances reduce to give $\dot{W}_t = \dot{m} [h_1 - h_4]$. With data from Table A-4E

$$\dot{W}_t = (25 \frac{\text{lb}}{\text{s}}) [1350.6 - 1268.8] \frac{\text{Btu}}{\text{lb}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 2893 \text{ hp} \quad \leftarrow (a)$$

(b) Then mass and energy rate balances for a control volume enclosing only the turbine give $\dot{W}_t = \dot{m}_t [h_1 - h_3]$. Using the isentropic turbine efficiency: $\eta_t = (h_1 - h_3) / (h_1 - h_{3s}) \Rightarrow (h_1 - h_3) = \eta_t (h_1 - h_{3s})$. So

$$\dot{W}_t = \dot{m}_t \eta_t (h_1 - h_{3s}) \Rightarrow \dot{m}_t = \frac{\dot{W}_t}{\eta_t (h_1 - h_{3s})} \quad \text{With } s_1 = s_{3s} = 1.5872 \text{ Btu/lb} \cdot ^\circ\text{R},$$

Table A-4E gives $h_{3s} = 1234.86 \text{ Btu/lb}$. Then

$$\dot{m}_t = \frac{(25) [1350.6 - 1268.8]}{(0.9) [1350.6 - 1234.86]} = 19.63 \text{ lb/s}$$

The mass flow rate through the valve is then $\dot{m} = \dot{m}_t + \dot{m}_v$ or $\dot{m}_v = \dot{m} - \dot{m}_t = 25 - 19.63 = 5.37 \text{ lb/s}$ (b)

(c) Mass and entropy rate balances for the valve give

$$\dot{Q}_v = \dot{m}_v [s_2 - s_1]$$

Since $h_2 \approx h_1$, state 2 is fixed by $h_2 = 1350.6 \text{ Btu/lb}$, $p_2 = 200 \text{ lbf/in}^2$, giving $s_2 = 1.7024 \text{ Btu/lb} \cdot ^\circ\text{R}$. Also, $s_1 = 1.5872 \text{ Btu/lb} \cdot ^\circ\text{R}$. So

$$\dot{Q}_v = (5.37 \frac{\text{lb}}{\text{s}}) [1.7024 - 1.5872] \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 0.6186 \frac{\text{Btu}}{\text{s} \cdot ^\circ\text{R}}$$

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Problem 6-95 continued

Mass and entropy rate balances for the turbine give

$$\dot{Q}_t = \dot{m}_t [s_3 - s_1]$$

To find s_3 , use the isentropic turbine efficiency to get

$$\eta_t = \frac{h_1 - h_3}{h_1 - h_{3s}} \Rightarrow h_3 = h_1 - \eta_t (h_1 - h_{3s}) = 1350.6 - .9(1350.6 - 1234.86) = 1246.4 \text{ Btu/lb}$$

Interpolating at 200 lbf/in² in Table A-4E, $s_3 = 1.6 \text{ Btu/lb} \cdot \text{OR}$. Thus

$$\dot{Q}_t = 19.63 [1.6 - 1.5872] = 0.2513 \frac{\text{Btu}}{\text{s} \cdot \text{OR}}$$

Mass and entropy rate balances for the mixing chamber give

$$\dot{Q}_{\text{mix}} = \dot{m}_4 s_4 - \dot{m}_t s_3 - \dot{m}_v s_2, \text{ where } s_4 = 1.6239 \frac{\text{Btu}}{\text{lb} \cdot \text{OR}} \text{ from Table A-4E.}$$

Thus

$$\begin{aligned} \dot{Q}_{\text{mix}} &= (25)(1.6239) - (19.63)(1.6) - (5.37)(1.7024) \\ &= 0.0476 \frac{\text{Btu}}{\text{s} \cdot \text{OR}} \end{aligned}$$

The rates of exergy destruction in the three components can be evaluated using $\dot{E}_d = T_0 \dot{Q}_{\text{ev}}$. Thus

$$\text{valve: } (\dot{E}_d)_v = (500^\circ\text{R})(0.6186 \frac{\text{Btu/s}}{\text{OR}}) = 309.3 \text{ Btu/s}$$

$$\text{turbine: } (\dot{E}_d)_t = (500^\circ\text{R})(0.2513 \frac{\text{Btu/s}}{\text{OR}}) = 125.65 \text{ Btu/s}$$

$$\text{mixer: } (\dot{E}_d)_m = (500^\circ\text{R})(0.0476 \frac{\text{Btu/s}}{\text{OR}}) = 23.8 \text{ Btu/s}$$

The net exergy carried in the water stream is

$$\begin{aligned} \dot{E}_{f1} - \dot{E}_{f4} &= \dot{m} [(h_1 - h_4) - T_0(s_1 - s_4)] = 25 \frac{\text{lb}}{\text{s}} [(1350.6 - 1268.8) - 500(1.5872 - 1.6239)] \frac{\text{Btu}}{\text{lb} \cdot \text{OR}} \\ &= 2503.75 \text{ Btu/s} \end{aligned}$$

Summary:

- net rate exergy is carried in by the water stream

$$2503.75 \text{ Btu/s}$$

- Disposition of the exergy carried in

- power developed:
- exergy destroyed:
 - valve
 - turbine
 - mixer

2045 Btu/s	(82 %)
309.3 Btu/s	(12 %)
125.65 Btu/s	(5 %)
23.8 Btu/s	(1 %)
<u>2503.75 Btu/s</u>	

← (c)

PROBLEM 7.96

For the water heater of Problem 7.45, devise and evaluate an exergetic efficiency.

See solution to Problem 7.45 for data. Then,

$$\textcircled{1} \quad \epsilon = \frac{(\text{Exergy Stored})}{(\text{Exergy Carried In Electrically})} = \frac{1340 \text{ kJ}}{26538 \text{ kJ}} = 0.05 \text{ (5\%)}$$

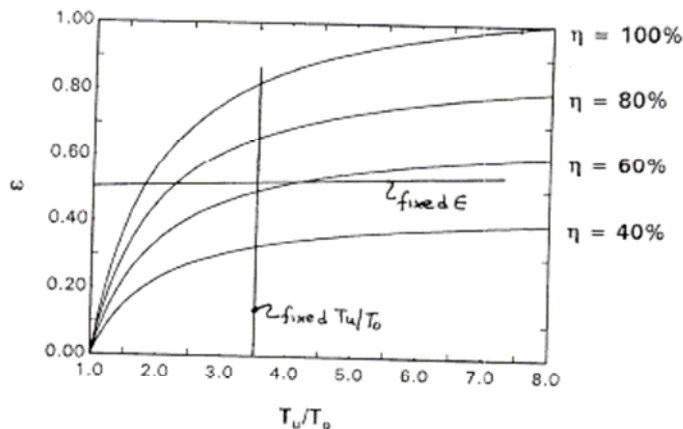
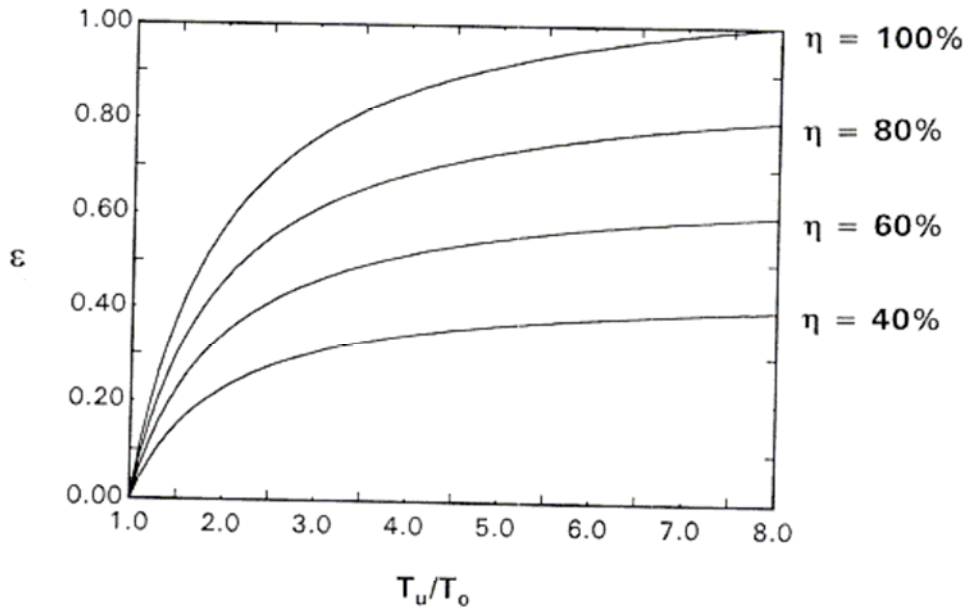
1. This value is typical for the exergetic efficiencies exhibited by domestic water heaters operating from an input of electricity or natural gas.

PROBLEM 7.97

Plot the exergetic efficiency given by Eq. 7.21b versus T_u/T_0 for $T_u/T_0 = 8.0$ and $\eta = 0.4, 0.6, 0.8, 1.0$. What can be learned from the plot when T_u/T_0 is fixed? When ϵ is fixed? Discuss.

Eq. 7.21b $\epsilon = \eta \left[\frac{1 - T_0/T_u}{1 - T_0/T_s} \right] = \eta \left[\frac{1 - T_0/T_u}{1 - \frac{1}{8}} \right] = \frac{8}{7} \eta \left[1 - \frac{T_0}{T_u} \right]$

$\left(= 1/8 \right)$



- When ϵ is fixed, as η increases, lower use temperatures are allowed.
- When T_u/T_0 is fixed, as η increases, ϵ increases.

In sum, a high first-law efficiency, η , has practical benefit. So, both first-law and second-law considerations are important in proper energy resource utilization.

PROBLEM 7.9B

KNOWN: From an input of electricity, a furnace delivers energy by heat transfer at a rate of $\dot{Q}_u$ at a use temperature of T_u . The furnace operates at steady state.

FIND: (a) Devise an exergetic efficiency. (b) Plot this efficiency versus T_u ranging from 300 to 900K.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The closed system is at steady state. (2) There are no significant energy transfers other than those shown. (3) For the environment, $T_o = 20^\circ\text{C} = 293\text{K}$.

ANALYSIS: (a) At steady state, the exergy balance with assumption 2 reads

$$0 = \dot{W}_e - \left[1 - \frac{T_o}{T_u}\right] \dot{Q}_u - \dot{E}_d$$

Where the quantities $\dot{W}_e$ and $\dot{Q}_u$ have been taken as positive in the direction of the arrows. Rearranging

$$\dot{W}_e = \left[1 - \frac{T_o}{T_u}\right] \dot{Q}_u + \dot{E}_d$$

(input) (transfer out) (destroyed)

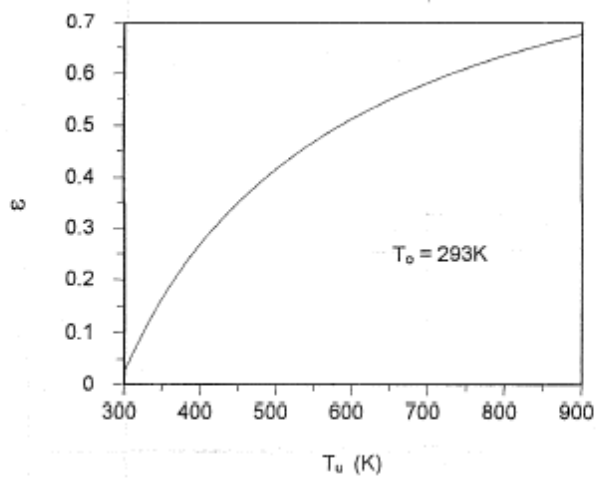
An exergetic efficiency in the form output/input is

$$\epsilon = \frac{\left[1 - \frac{T_o}{T_u}\right] \dot{Q}_u}{\dot{W}_e}$$

With $\dot{Q}_u = \dot{W}_e$ from an energy balance at steady state

$$\epsilon = \left[1 - \frac{T_o}{T_u}\right] \longleftarrow \epsilon$$

(b) The following plot is generated using IT with $T_o = 293$:



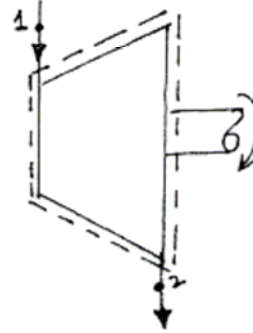
From the plot, we observe that as $T_u \rightarrow T_o$, $\epsilon \rightarrow 0$. Also, ϵ increases rapidly with T_u .

PROBLEM 7.99

Steam enters a turbine operating at steady state at $p_1 = 12 \text{ MPa}$, $T_1 = 700^\circ\text{C}$ and exits at $p_2 = 0.6 \text{ MPa}$. The isentropic turbine efficiency is 88%. Property data are provided in the accompanying table. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 300 \text{ K}$, $p_0 = 100 \text{ kPa}$. Determine (a) the power developed and the rate of exergy destruction, each in kJ per kg of steam flowing, and (b) the exergetic turbine efficiency.

State	$p \text{ (MPa)}$	$T \text{ (}^\circ\text{C)}$	$h \text{ (kJ/kg)}$	$s \text{ (kJ/kg} \cdot \text{K)}$
1 Turbine inlet	12	700	3858.4	7.0749
2 Turbine exit	0.6	($\eta_r = 88\%$)	3017.5	7.2938

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and the effects of motion and gravity are negligible.
3. $T_0 = 300 \text{ K}$, $p_0 = 100 \text{ kPa}$

ANALYSIS: (a) An energy rate balance reduces to

$$\dot{W}_{cv}/\dot{m} = h_1 - h_2 = (3858.4 - 3017.5) \frac{\text{kJ}}{\text{kg}} = 840.9 \text{ kJ/kg} \quad (a)$$

- (b) The rate of exergy destruction can be found from $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production found from an entropy rate balance. Thus,

$$\frac{\dot{E}_d}{\dot{m}} = T_0 (s_2 - s_1) = 300 \text{ K} (7.2938 - 7.0749) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 65.67 \text{ kJ/kg}$$

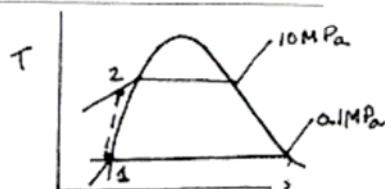
- (c) The exergetic turbine efficiency is

$$\begin{aligned} \epsilon &= \frac{\dot{W}_{cv}}{\dot{m} [e_{f1} - e_{f2}]} = \frac{\dot{W}_{cv}/\dot{m}}{(h_1 - h_2) - T_0 (s_1 - s_2)} \\ &= \frac{(h_1 - h_2)}{(h_1 - h_2) + \frac{\dot{E}_d}{\dot{m}}} = \frac{840.9}{840.9 + 65.67} \\ &= 0.928 \quad (92.8\%) \end{aligned}$$

PROBLEM 7.100

Saturated liquid water at 0.01 MPa enters a power plant pump operating at a steady state. Liquid water exits the pump at 10 MPa. The isentropic pump efficiency is 90%. Property data are provided in the accompanying table. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 300 \text{ K}$, $p_0 = 100 \text{ kPa}$. Determine (a) the power required by the pump and the rate of exergy destruction, each in kJ per kg of water flowing, and (b) the exergetic pump efficiency.

State	p (MPa)	h (kJ/kg)	s (kJ/kg·K)
1 Pump inlet	0.01	191.8	0.6493
2 Pump exit	10	204.5	0.6531



ANALYSIS:

(a) An energy rate balance reduces to

$$\begin{aligned} \frac{\dot{W}_{\text{ev}}}{\dot{m}} &= h_1 - h_2 = (191.8 - 204.5) \frac{\text{kJ}}{\text{kg}} \\ &= -12.7 \text{ kJ/kg} \end{aligned}$$

← (a)

(b) The rate of exergy destruction can be found from $\dot{E}_d = T_0 \dot{\sigma}_{\text{ev}}$, where $\dot{\sigma}_{\text{ev}}$ is the rate of entropy production found from an entropy rate balance.

Thus

$$\begin{aligned} \frac{\dot{E}_d}{\dot{m}} &= T_0 (s_2 - s_1) = 300 \text{ K} (0.6531 - 0.6493) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ &= 1.14 \text{ kJ/kg} \end{aligned}$$

← (b)

(c) The exergetic pump efficiency is

$$\epsilon = \frac{e_{f2} - e_{f1}}{(-\dot{W}_{\text{ev}}/\dot{m})} = \frac{(h_2 - h_1) - T_0 (s_2 - s_1)}{(-\dot{W}_{\text{ev}}/\dot{m})}$$

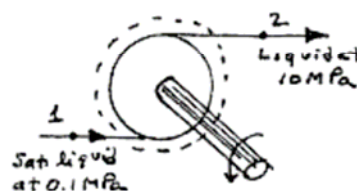
← $\dot{E}_d/\dot{m}$ from (b)

← $(h_2 - h_1)$ from (a)

$$= \frac{(h_2 - h_1) - T_0 (s_2 - s_1)}{(h_2 - h_1)}$$

$$= \frac{12.7 - 1.14}{12.7} = 0.91 \text{ (91\%)} \quad \leftarrow (c)$$

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

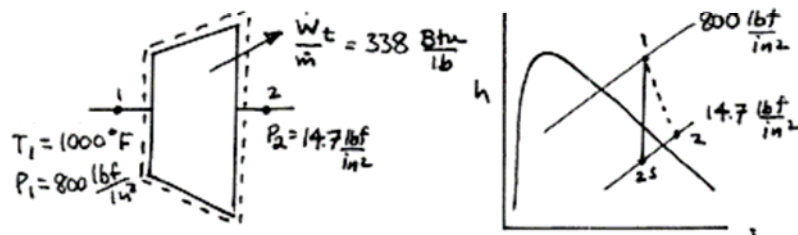
1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and the effects of motion and gravity are negligible.
3. $T_0 = 300 \text{ K}$, $p_0 = 100 \text{ kPa}$

PROBLEM 7.101

KNOWN: Steady-state operating data are provided for a steam turbine.

FIND: Evaluate the isentropic turbine efficiency and the exergetic turbine efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects are negligible. 3. $T_0 = 520^\circ\text{R}$, $P_0 = 14.7 \text{ lbf/in}^2$.

ANALYSIS: Using Eq. 6.46, the isentropic turbine efficiency is

$$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{h_1 - h_{2s}}$$

From Table A-4E $h_1 = 1511.9 \text{ Btu/lb}$, $s_1 = 1.6807 \text{ Btu/lb} \cdot ^\circ\text{R}$. With $s_{2s} = s_1$ at 14.7 lbf/in^2 , Table A-3E gives

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{1.6807 - 0.3121}{1.4446} = 0.947$$

$$\Rightarrow h_{2s} = h_f + x_{2s} h_{fg} = 180.15 + 0.947(970.4) = 1099.5 \text{ Btu/lb}$$

Thus,

$$\eta_t = \frac{338 \text{ Btu/lb}}{[1511.9 - 1099.5] \text{ Btu/lb}} = \frac{338}{412.4} = 0.82 \text{ (82\%)}$$

Using Eq. 7.24 the exergetic turbine efficiency is

$$\epsilon = \frac{\dot{W}_{cv}/\dot{m}}{(h_1 - h_2) - T_0(s_1 - s_2)}$$

To fix the exit state, reduce the mass and energy rate balance to get
 $0 = \dot{Q}_{cv}^0 - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \Rightarrow h_2 = h_1 - \dot{W}_{cv}/\dot{m} = 1511.9 - 338 = 1173.9 \text{ Btu/lb}$

Then, at 14.7 lbf/in^2 , with h_2 known, Table A-4E gives $s_2 = 1.7902 \text{ Btu/lb} \cdot ^\circ\text{R}$.

Accordingly,

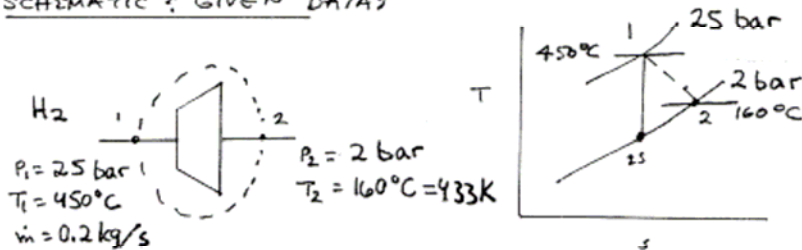
$$\epsilon = \frac{338 \text{ Btu/lb}}{\left[\frac{(1511.9 - 1173.9)}{338} - 520(1.6807 - 1.7902) \right] \text{ Btu/lb}} = \frac{338}{394.9} = 0.856 \text{ (85.6\%)}$$

PROBLEM 7.102

KNOWN: Steady state operating data are provided for a hydrogen turbine.

FIND: Determine (a) the isentropic turbine efficiency, (b) the exergetic turbine efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$, and kinetic/potential energy changes can be ignored. (3) H_2 is modeled as an ideal gas with $k = 1.37$. (4) The exergy reference environment is at $T_0 = 298\text{ K}$, $P_0 = 1\text{ atm}$.

ANALYSIS: (a) The isentropic turbine efficiency is given by Eq. 6.46

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{c_p(T_1 - T_2)}{c_p(T_1 - T_{2s})} = \frac{T_1 - T_2}{T_1 - T_{2s}} \quad \text{With Eq. 6.43, } T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = (723\text{ K}) \left(\frac{2}{25} \right)^{\frac{0.37}{1.37}} = 365.5\text{ K}$$

$$\therefore \eta_t = \frac{723 - 433}{723 - 365.5} = 0.81 \text{ (81\%)} \leftarrow \eta_t$$

(b) The exergetic turbine efficiency is given by Eq. 7.24, which becomes

$$\epsilon = \frac{h_1 - h_2}{(h_1 - h_2) - T_0(s_1 - s_2)} = \frac{c_p(T_1 - T_2)}{c_p(T_1 - T_2) - T_0 \left[c_p \ln \frac{T_1}{T_2} - R \ln \frac{P_1}{P_2} \right]} \quad \text{Eq. 3.47k gives } R = c_p \left(\frac{k-1}{k} \right)$$

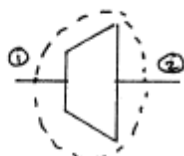
$$\Rightarrow \epsilon = \frac{T_1 - T_2}{T_1 - T_2 - T_0 \left[\ln \frac{T_1}{T_2} - \left(\frac{k-1}{k} \right) \ln \frac{P_1}{P_2} \right]}$$

$$\epsilon = \frac{723 - 433}{723 - 433 - 298 \left[\ln \left(\frac{723}{433} \right) - \left(\frac{1.37-1}{1.37} \right) \ln \left(\frac{25}{2} \right) \right]} = 0.852 \text{ (85.2\%)} \leftarrow \epsilon$$

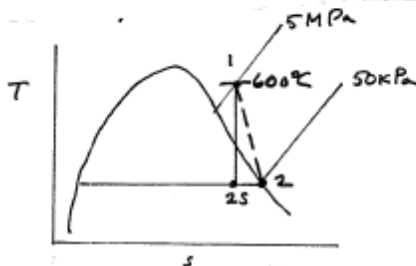
PROBLEM 7.103

Steam enters a turbine operating at steady state at 5 MPa, 600°C and exits at 50 kPa. Stray heat transfer and the effects of motion and gravity can be ignored. If the rate of exergy destruction is 95.4 kJ per kg of steam flowing, determine (a) the isentropic turbine efficiency and (b) the exergetic turbine efficiency. Let $T_0 = 293 \text{ K}$, $p_0 = 1 \text{ bar}$.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} P_1 &= 5 \text{ MPa} \\ T_1 &= 600^\circ\text{C} \\ P_2 &= 50 \text{ kPa} \\ \frac{\dot{E}_d}{\dot{m}} &= 95.4 \text{ kJ/kg} \end{aligned}$$



ENGR. MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, and the effects of motion and gravity can be ignored. (3). $T_0 = 293 \text{ K}$, $p_0 = 1 \text{ bar}$.

ANALYSIS: $\frac{\dot{E}_d}{\dot{m}} = T_0 \frac{\dot{Q}_{cv}}{\dot{m}} = T_0 (s_2 - s_1)$. Thus, with data from Table A-4

$$s_2 = \frac{\dot{E}_d/\dot{m}}{T_0} + s_1 = \frac{95.4 \text{ kJ/kg}}{293 \text{ K}} + 7.2683 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 7.5939 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Thus, state 2 is a saturated vapor state at 50 kPa, as shown on the T-s diagram.

$$\text{Also, } x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{7.2683 - 1.0910}{7.5939 - 1.0910} = 0.95 \Rightarrow h_{2s} = 340.48 + 0.95(2205.4) = 2530.6 \frac{\text{kJ}}{\text{kg}}$$

(a) Using Eq. 6.46

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{3666.4 - 2645.9}{3666.4 - 2530.6} = 0.9 \text{ (90\%)} \quad \leftarrow \eta_t$$

(b) Using Eq. 7.24

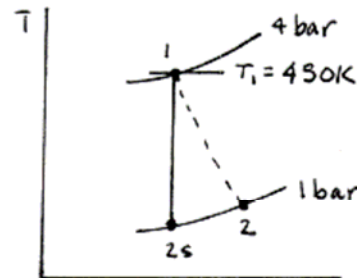
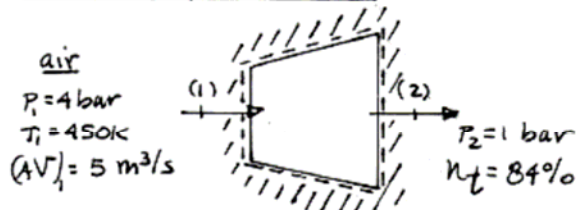
$$\begin{aligned} \epsilon &= \frac{\dot{W}_{cv}/\dot{m}}{e_{f1} - e_{f2}} = \frac{(h_1 - h_2)}{(h_1 - h_2) - T_0(s_1 - s_2)} = \frac{(h_1 - h_2)}{(h_1 - h_2) + \dot{E}_d/\dot{m}} = \frac{1020.5}{1020.5 + 95.4} \\ &= 0.915 \text{ (91.5\%)} \quad \leftarrow \epsilon \end{aligned}$$

PROBLEM 7.104

KNOWN: Air enters an insulated turbine operating at steady state at a given state and with a known volumetric flow rate. The exit pressure and the isentropic turbine efficiency are specified.

FIND: Determine (a) the power developed and the exergy destruction rate. (b) the exergetic turbine efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and the effects of motion and gravity are neglected. (3) The air is modeled as an ideal gas. (4) For the environment, $T_0 = 20^\circ\text{C} = 293\text{ K}$, $P_0 = 1\text{ bar}$.

ANALYSIS: (a) We begin by using the isentropic turbine efficiency to fix state 2. With data from Table A-22

$$Pr(T_{2s}) = \left(\frac{P_2}{P_1}\right) Pr(T_1) = \left(\frac{1}{4}\right) \cdot 775 = 1.44375 \Rightarrow \begin{cases} T_{2s} = 303.5\text{ K} \\ h_{2s} = 303.71 \frac{\text{kJ}}{\text{kg}} \end{cases}$$

Now

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 451.8 - (0.84)(451.8 - 303.71) = 327.4 \text{ kJ/kg}$$

The mass flow rate of air is

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1} = \frac{(4\text{ bar})(5\text{ m}^3/\text{s})}{\left(\frac{8.314\text{ kJ}}{28.97\text{ kg}\cdot\text{K}}\right)(450\text{ K})} \left| \frac{10^5\text{ N/m}^2}{1\text{ bar}} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right| = 15.49 \text{ kg/s}$$

From mass and energy rate balances: $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)]$

$$\dot{W}_{cv} = \dot{m} [h_1 - h_2] = (15.49 \frac{\text{kg}}{\text{s}}) [451.8 - 327.4] \frac{\text{kJ}}{\text{kg}} \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right| = 1927 \text{ kW} \leftarrow \dot{W}_{cv}$$

From mass and entropy balances: $0 = \dot{Q}_{cv}/T_0 + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$

Thus

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = T_0 \dot{m}(s_2 - s_1) = T_0 \dot{m} [s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1]$$

Interpolating in Table A-22 with $h_2 = 327.4$; $s^\circ(T_2) = 1.78886$.

$$\dot{E}_d = (293\text{ K})(15.49 \frac{\text{kg}}{\text{s}}) [(1.78886 - 2.11161) - \frac{8.314}{28.97} \ln(\frac{1}{4})] \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right| = 340.8 \text{ kW} \leftarrow \dot{E}_d$$

(b) The exergetic turbine efficiency is

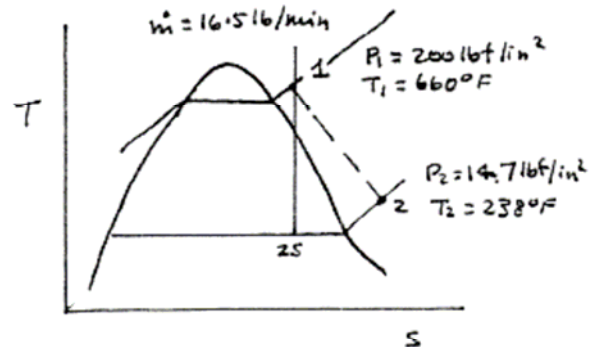
$$\epsilon = \frac{\dot{W}_{cv}}{[\dot{m}(h_1 - h_2) - T_0 \dot{m}(s_1 - s_2)]} = \frac{\dot{W}_{cv}}{\dot{W}_{cv} + \dot{E}_d} = \frac{1927}{1927 + 340.8} = 0.8497 \text{ (85\%)} \leftarrow \epsilon$$

$\underbrace{[\dot{m}(h_1 - h_2) - T_0 \dot{m}(s_1 - s_2)]}_{= -\dot{E}_d/\dot{m}}$

PROBLEM 7.105

Steam at 200 lbf/in^2 , 660°F enters a turbine operating at steady state with a mass flow rate of 16.5 lb/min and exits at 14.7 lbf/in^2 , 238°F . Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 537^\circ\text{R}$, $p_0 = 14.7 \text{ lbf/in}^2$. Determine for the turbine (a) the power developed and the rate of exergy destruction, each in Btu/min , and (b) the isentropic and exergetic turbine efficiencies.

SCHEMATIC & GIVEN DATA



ENGR. MODEL:

1. A control volume at steady state encloses the turbine.
 2. Stray heat transfer and the effects of motion and gravity can be ignored.
- $T_0 = 537^\circ\text{R}$, $p_0 = 14.7 \text{ lbf/in}^2$

ANALYSIS: From Table A-4E, $h_1 = 1353.12 \text{ Btu/lb}$, $s_1 = 1.7047 \text{ Btu/lb} \cdot ^\circ\text{R}$.

$$h_2 = 1163.02 \frac{\text{Btu}}{\text{lb}}, \quad s_2 = 1.7748 \text{ Btu/lb} \cdot ^\circ\text{R}.$$

$$x_{2s} = \frac{s_2 - s_f}{s_g - s_f} = \frac{1.7047 - 0.3121}{1.4446} = 0.964 \Rightarrow h_{2s} = 180.15 + 0.964(970.4) = 1115.62 \frac{\text{Btu}}{\text{lb}}$$

$$(a) \quad \dot{W}_t = \dot{m}(h_1 - h_2) = 16.5 \frac{\text{lb}}{\text{min}} (1353.12 - 1163.02) \frac{\text{Btu}}{\text{lb}} = 3136.65 \frac{\text{Btu}}{\text{min}} \quad \leftarrow \dot{W}_t$$

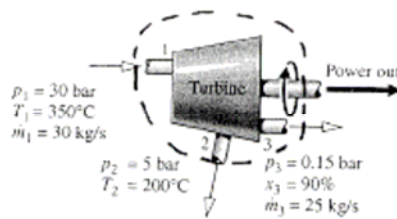
$$\begin{aligned} \dot{E}_d &= T_0 \dot{\sigma}_{cv} = \dot{m} T_0 (s_2 - s_1) = (16.5 \frac{\text{lb}}{\text{min}})(537^\circ\text{R})(1.7748 - 1.7047) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \\ &= 621.12 \frac{\text{Btu}}{\text{min}} \quad \leftarrow \dot{E}_d \end{aligned}$$

$$(b) \quad \eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{1353.12 - 1163.02}{1353.12 - 1115.62} = \frac{190.1}{237.5} = 0.8 \quad (80\%) \quad \leftarrow \eta_t$$

$$\epsilon = \frac{h_1 - h_2}{h_1 - h_2 - T_0(s_1 - s_2)} = \frac{190.1}{190.1 - 537(1.7047 - 1.7748)} = 0.835 \quad (83.5\%) \quad \leftarrow \epsilon$$

PROBLEM 7.106

Figure P7.106 shows a turbine operating at steady state with steam entering at $p_1 = 30$ bar, $T_1 = 350^\circ\text{C}$ and a mass flow rate of 30 kg/s. Process steam is extracted at $p_2 = 5$ bar, $T_2 = 200^\circ\text{C}$. The remaining steam exits at $p_3 = 0.15$ bar, $x_3 = 90\%$, and a mass flow rate of 25 kg/s. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 25^\circ\text{C}$, $p_0 = 1$ bar. The accompanying table provides property data at key states. For the turbine, determine the power developed and rate of exergy destruction, each in MW. Also devise and evaluate an exergetic efficiency for the turbine.



State	p (bar)	T ($^\circ\text{C}$)	h (kJ/kg)	s (kJ/kg $\cdot$ K)
1	30	350	3115.3	6.7428
2	5	200	2855.4	7.0592
3	0.15	($x = 90\%$)	2361.7	7.2831

ENGR MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and the effects of motion and gravity are negligible.
3. $T_0 = 298\text{K}$, $p_0 = 1$ bar

ANALYSIS: $\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 5$ kg/s.

An energy rate balance reduces to give

$$\dot{W}_{cv} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = \left[(30 \frac{\text{kg}}{\text{s}}) (3115.3 \frac{\text{kJ}}{\text{kg}}) - (5) (2855.4) - (25) (2361.7) \right] \left[\frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right]$$

$$= 20.14 \text{ MW}$$

$\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy rate balance. Thus

$$\dot{E}_d = T_0 [\dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1] = 298 \text{ K} \left[(5 \frac{\text{kg}}{\text{s}}) (7.0592 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) + (25) (7.2831) - (30) (6.7428) \right] \left[\frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right]$$

$$= 4.5 \text{ MW}$$

Writing an exergy rate balance,

$$0 = \dot{E}_q - \dot{W}_{cv} + \dot{m}_1 e_{f1} - \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3} - \dot{E}_d$$

$$\Rightarrow \underbrace{\dot{m}_1 e_{f1} - \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3}}_{\text{net rate exergy is supplied}} = \dot{W}_{cv} + \dot{E}_d$$

$$\textcircled{1} \quad \Rightarrow \epsilon = \frac{\dot{W}_{cv}}{(\dot{m}_1 e_{f1} - \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3})} = \frac{\dot{W}_{cv}}{\dot{W}_{cv} + \dot{E}_d} = \frac{20.14}{20.14 + 4.5} = 0.817 \quad (81.7\%)$$

1. Alternatively, note that

$$\begin{aligned} \dot{m}_1 e_{f1} - \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3} &= \dot{m}_2 (e_{f1} - e_{f2}) + \dot{m}_3 (e_{f1} - e_{f3}) \\ \uparrow (\dot{m}_2 + \dot{m}_3) & \\ &= \dot{m}_2 [(h_1 - h_2) - T_0 (s_1 - s_2)] + \\ &\quad \dot{m}_3 [(h_1 - h_3) - T_0 (s_1 - s_3)] \end{aligned}$$

PROBLEM 7.107

For the turbine and heat exchanger arrangement of Problem 6.116, evaluate an exergetic efficiency for (a) each turbine, (b) the heat exchanger, and (c) an overall control volume enclosing the turbines and heat exchanger. Comment. Let $T_0 = 300 \text{ K}$, $p_0 = 1 \text{ bar}$.

FROM Problem 6.116:

State	$h \text{ (kJ/kg)}$	$s^0 \text{ (kJ/Ks}\cdot\text{K)}$
1	1515.42	3.3620
2	1161.07	3.07732
3	1397.78	3.27481
4	1023.25	2.94468
5	1611.79	3.42892
6	1277.79	3.17888

$$\dot{W}_{t1} = 10,000 \text{ kW}$$

$$\dot{m}_1 = 28.22 \text{ kg/s}, \dot{W}_{t2} = 10,570 \text{ kW}$$

$$\dot{\sigma}_1^{\text{turbine}} = 3.1936 \text{ kW/K}$$

$$\dot{\sigma}_2^{\text{turbine}} = 2.8649 \text{ kW/K}$$

$$\dot{\sigma}_{\text{HX}} = 3.1482 \text{ kW/K}$$

ANALYSIS: Using the known entropy production values, the net exergy destruction can be found using $\dot{E}_d = T_0 \dot{\sigma}$:

$$(\dot{E}_d)_{\text{turbine 1}} = (300 \text{ K})(3.1936 \text{ kW/K}) = 958 \text{ kW}$$

$$(\dot{E}_d)_{\text{turbine 2}} = (300 \text{ K})(2.8649 \text{ kW/K}) = 859 \text{ kW}$$

$$(\dot{E}_d)_{\text{HX}} = (300 \text{ K})(3.1482 \text{ kW/K}) = 944 \text{ kW}$$

(a) With Eqs. 7.23 and 7.24, $\epsilon = \frac{\dot{W}_{\text{ev}}}{\dot{W}_{\text{ev}} + \dot{E}_d}$

$$\text{Turbine 1: } \epsilon_1 = \frac{10,000 \text{ kW}}{(10,000 + 958) \text{ kW}} = 0.913 \text{ (91.3\%)}$$

$$\text{Turbine 2: } \epsilon_2 = \frac{10,570 \text{ kW}}{(10,570 + 859) \text{ kW}} = 0.925 \text{ (92.5\%)}$$

(a)

(b) With Eqs. 7.26 and 7.27

$$\epsilon_{\text{HX}} = \frac{\dot{m}_1 (e_{f3} - e_{f2})}{\dot{m}_5 (e_{f5} - e_{f6})} = \frac{\dot{m}_1 (e_{f3} - e_{f2})}{\dot{m}_1 (e_{f3} - e_{f2}) + \dot{E}_d} \quad (1)$$

where

$$\begin{aligned} \dot{m}_1 (e_{f3} - e_{f2}) &= \dot{m}_1 [h_3 - h_2 - T_0 (s_3 - s_2)] \\ &= \dot{m}_1 [h_3 - h_2 - T_0 (s_3^0 - s_2^0 - R \ln p_3/p_2)] \\ &= (28.22 \frac{\text{kg}}{\text{s}}) \left[1397.78 - 1161.07 - 300 (3.27481 - 3.07732 - \frac{8.314}{28.97} \ln \frac{4.5}{5.0}) \right] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 4752 \text{ kW} \end{aligned}$$

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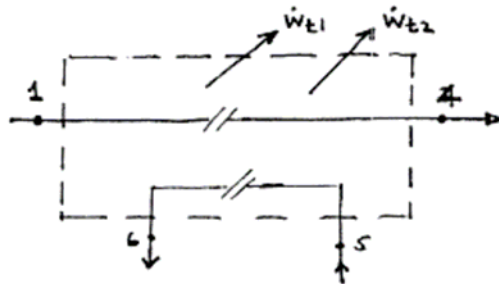
Problem 7-107 continued

Then, Eq. 1 gives

$$\epsilon = \frac{4752 \text{ kW}}{(4752 + 944)} = 0.834 \text{ (83.4\%)}$$

(b)

(c) Taking an overall control volume,



An exergy rate balance reads

$$0 = \dot{E}_q - (\dot{W}_{t1} + \dot{W}_{t2}) + \dot{m}_1(e_{f1} - e_{f4}) + \dot{m}_5(e_{f5} - e_{f6}) - (\sum \dot{E}_d)$$

$$\Rightarrow \underbrace{\dot{m}_1(e_{f1} - e_{f4}) + \dot{m}_5(e_{f5} - e_{f6})}_{\text{net exergy supplied to the control volume}} = (\dot{W}_{t1} + \dot{W}_{t2}) + (\sum \dot{E}_d)$$

$$\Rightarrow \epsilon = \frac{(\dot{W}_{t1} + \dot{W}_{t2})}{\dot{m}_1(e_{f1} - e_{f4}) + \dot{m}_5(e_{f5} - e_{f6})} = \frac{(\dot{W}_{t1} + \dot{W}_{t2})}{(\dot{W}_{t1} + \dot{W}_{t2}) + (\sum \dot{E}_d)}$$

$$= \frac{(10,000 + 10,570) \text{ kW}}{(20,570 + (958 + 859 + 944)) \text{ kW}} = 0.882 \text{ (88.2\%)}$$

When an overall control volume is under consideration, its exergetic efficiency is impacted by all sources of exergy destruction present within the control volume.

PROBLEM 7.108

Determine the exergetic efficiency for the turbine of

- (a) Problem 6.130. Let $T_0 = 300 \text{ K}$.
- (b) Problem 6.131. Let $T_0 = 500^\circ \text{R}$.
- (c) Problem 6.132. Let $T_0 = 295 \text{ K}$.
- (d) Problem 6.133. Let $T_0 = 530^\circ \text{R}$.

ANALYSIS: From Eq. 7.24,

$$\epsilon = \frac{\dot{W}_{cv}}{\dot{m}(e_{f1} - e_{f2})} = \frac{\dot{W}_{cv}}{\dot{m}[(h_1 - h_2) - T_0(s_1 - s_2)]} = \frac{\dot{W}_{cv}}{\dot{W}_{cv} + \dot{m}T_0(s_2 - s_1)} \quad (1)$$

In these cases, $\dot{W}_{cv} = h_1 - h_2$. Also note that $\dot{m}T_0(s_2 - s_1) = \dot{E}_d$.

(a) Problem 6.130. $\dot{W}_{cv} = 1120 \text{ kW}$, $\dot{m} = 5.5 \text{ kg/s}$.

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} = [2.79776 - 3.0126 - \frac{8.314}{28.97} \ln \frac{120}{278}] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.0263 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Then, Eq. (1) gives

$$\epsilon = \frac{1120 \text{ kJ/s}}{1120 \frac{\text{kJ}}{\text{s}} + (5.5 \frac{\text{kg}}{\text{s}})(300 \text{ K})(0.0263 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})} = 0.963 \text{ (96.3\%)} \quad \leftarrow (a)$$

(b) Problem 6.131. $\dot{W}_{cv} = 1600 \text{ Btu/s}$, $\dot{m} = 4 \text{ lb/s}$

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) \Rightarrow h_2 = h_1 - \dot{W}_{cv}/\dot{m} = 1531 - \frac{1600 \text{ Btu/s}}{4 \text{ lb/s}} = 1131 \frac{\text{Btu}}{\text{lb}} \Rightarrow s_2 = 1.945 \frac{\text{Btu}}{\text{lb} \cdot ^\circ \text{R}} \quad (\text{IT value})$$

Then Eq. (1) gives

$$\epsilon = \frac{1600 \text{ Btu/s}}{1600 \frac{\text{Btu}}{\text{s}} + (4 \frac{\text{lb}}{\text{s}})(500^\circ \text{R})(1.945 - 1.8827) \frac{\text{Btu}}{\text{lb} \cdot ^\circ \text{R}}} = 0.928 \text{ (92.8\%)} \quad \leftarrow (b)$$

(c) Problem 6.132. Eq. (1) gives

$$\epsilon = \frac{2626 \text{ kJ/s}}{2626 \frac{\text{kJ}}{\text{s}} + (2 \frac{\text{kg}}{\text{s}})(295 \text{ K})(7.4 - 7.1677) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}} = 0.95 \text{ (95\%)} \quad \leftarrow (c)$$

(d) Problem 6.133 $\dot{m} = 5.56 \text{ lb/s}$, $\dot{W}_{cv} = 2740.8 \text{ Btu/s}$

With $\dot{W}_{cv} = \dot{m}(h_1 - h_2)$, $h_2 = h_1 - (\dot{W}_{cv}/\dot{m})$. Thus,

$$h_2 = 1511.9 \frac{\text{Btu}}{\text{lb}} - \left[\frac{2740.8 \text{ Btu/s}}{5.56 \text{ lb/s}} \right] = 1018.95 \frac{\text{Btu}}{\text{lb}}$$

$$\Rightarrow x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{1018.95 - 94.02}{1022.1} = 0.9049$$

$$\Rightarrow s_2 = s_f + x_2(s_g - s_f) = 0.1750 + 0.9049(1.7448) = 1.7539 \text{ Btu/lb} \cdot ^\circ \text{R}$$

Then, with Eq. (1)

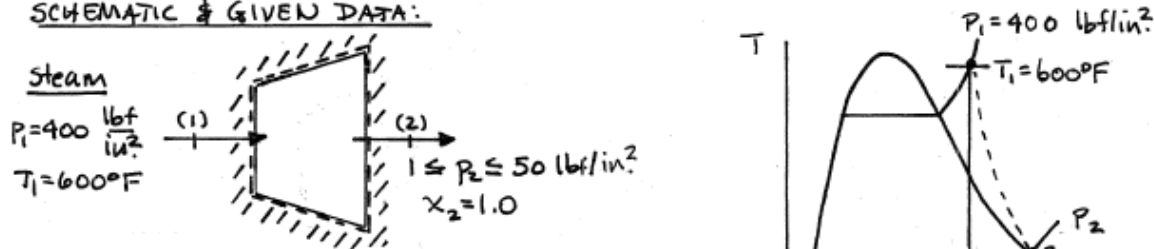
$$\epsilon = \frac{2740.8 \text{ Btu/s}}{2740.8 \frac{\text{Btu}}{\text{s}} + (5.56 \frac{\text{lb}}{\text{s}})(530^\circ \text{R})(1.7539 - 1.6867) \frac{\text{Btu}}{\text{lb} \cdot ^\circ \text{R}}} = 0.927 \text{ (92.7\%)} \quad \leftarrow$$

PROBLEM 7.109

KNOWN: Steam enters a well-insulated turbine operating at steady state at a specified state and exits at pressure p .

FIND: (a) For $p = 50 \text{ lbf/in}^2$, determine the exergy destruction rate per unit mass of steam flowing and the isentropic and exergetic turbine efficiencies. (b) Plot each of these quantities versus p ranging from 1 to 50 lbf/in^2 .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown operates at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are ignored. (3) For the environment, $T_0 = 60^\circ\text{F} = 520^\circ\text{R}$, $P_0 = 1 \text{ atm}$.

ANALYSIS: With assumptions 1 and 2, the mass and energy rate balances reduce to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad (1)$$

Further, mass and entropy rate balances reduce to give $\dot{Q}_{cv}/\dot{m} = s_2 - s_1$. The exergy destruction rate is $\dot{E}d/\dot{m} = T_0(\dot{Q}_{cv}/\dot{m})$, or

$$\dot{E}d/\dot{m} = T_0(s_2 - s_1) \quad (2)$$

Furthermore, the isentropic and exergetic turbine efficiencies are, respectively

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \quad (3)$$

$$\epsilon = \frac{\dot{W}_{cv}/\dot{m}}{e_{f1} - e_{f2}} = \frac{h_1 - h_2}{(h_1 - h_2) - T_0(s_1 - s_2)} \quad (4)$$

(a) From Table A-4E, $h_1 = 1306.6 \text{ Btu/lb}$, $s_1 = 1.5892 \text{ Btu/lb} \cdot ^\circ\text{R}$. Further, from Table A-3E at 50 lbf/in^2 , $h_2 = h_g = 1174.4 \text{ Btu/lb}$, $s_2 = s_g = 1.6589 \text{ Btu/lb} \cdot ^\circ\text{R}$. Accordingly

$$\dot{E}d/\dot{m} = (520^\circ\text{R})(1.6589 - 1.5892) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 36.24 \text{ Btu/lb} \leftarrow \dot{E}d/\dot{m}$$

State 2s is fixed by p_2 and $s_{2s} = s_1$: $x_{2s} = (s_{2s} - s_f)/s_{fg} = (1.5892 - 0.4113)/1.2476 = 0.9441$. Therefore $h_{2s} = h_f + x_{2s}h_{fg} = 250.24 + (0.9441)(924.2) = 1122.8 \text{ Btu/lb}$.

Thus

$$\eta_t = \frac{1306.6 - 1174.4}{1306.6 - 1122.8} = 0.719 \text{ (71.9 \%)} \leftarrow \eta_t$$

Now, from Eq. (4)

$$\epsilon = \frac{(1306.6 - 1174.4)}{(1306.6 - 1174.4) - (520)(1.5892 - 1.6589)} = 0.785 \text{ (78.5 \%)} \leftarrow \epsilon$$

Continued on next slide

Problem 7-109 continued

(b) The data for the required plots are obtained using IT, as follows:

IT Code

```
p1 = 400 // lbf/in.2
T1 = 600 // °F
x2 = 1.0
p2 = 50 // lbf/in.2
To = 60 + 460 // °R

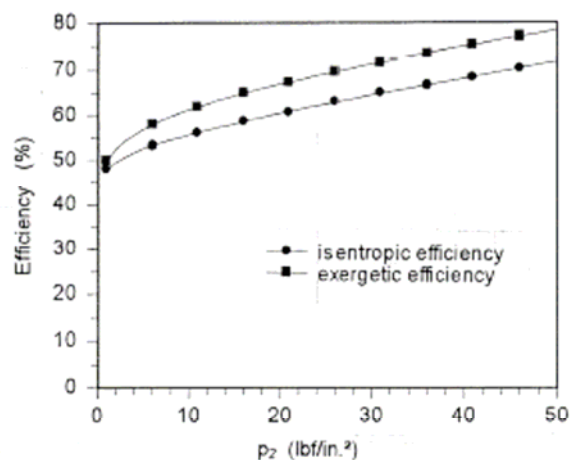
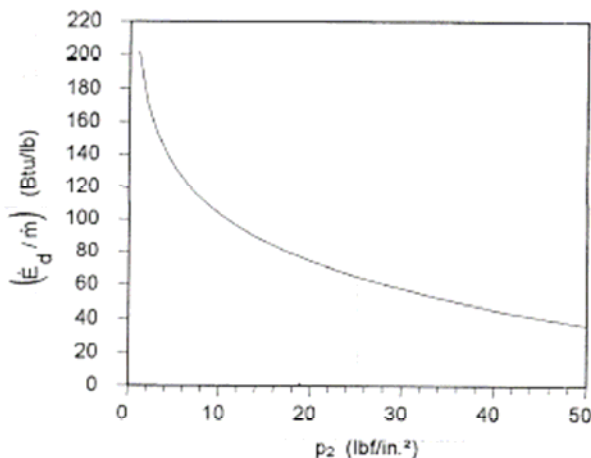
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = hsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
s2s = s_Ph("Water/Steam", p2, h2s)
s2s = s1

Ed = To * (s2 - s1)
eta = (h1 - h2) / (h1 - h2s)
eff = (h1 - h2) / ((h1 - h2) - To * (s1 - s2s))
```

IT Results for $p_2 = 50 \text{ lbf/in.}^2$

```
h1 = 1306 Btu/lb
h2 = 1174 Btu/lb
h2s = 1123 Btu/lb
s1 = 1.589 Btu/lb·°R
s2 = 1.659 Btu/lb·°R
s2s = 1.589 Btu/lb·°R
Ed / m = 36.26
ηt = 0.719 (71.9%)
ε = 0.7847 (78.47%)
```

PLOTS:



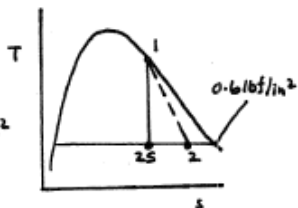
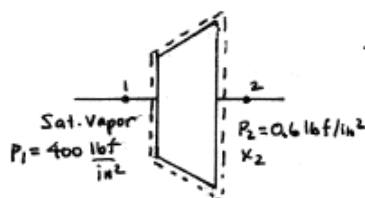
As expected, increased efficiency corresponds with decreased exergy destruction.

PROBLEM 7.110

KNOWN: Steady-state operating data are provided for a steam turbine.

FIND: Plot versus the steam quality at the turbine exit (a) the power developed and rate of exergy destruction, each per unit mass of steam flowing, (b) isentropic turbine efficiency, (c) exergetic turbine efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects are negligible.
3. $T_0 = 520^\circ R$, $P_0 = 1 \text{ atm}$.

ANALYSIS: Reducing mass, energy, and entropy balances gives

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad (1) \quad \text{and} \quad \frac{\dot{E}_d}{\dot{m}} = T_0 \frac{\dot{Q}_{cv}}{\dot{m}} = T_0 (s_2 - s_1) \quad (2)$$

h_1 and s_1 are fixed by P_1 , saturated vapor. h_2 and s_2 are fixed by P_2 , x_2 . Using Eq. 6.46,

$$(3) \quad \eta_t = \frac{(h_1 - h_2)}{(h_1 - h_{2s})} \quad \text{where } h_{2s} \text{ is fixed by } P_2, s_2 = s_1. \text{ Then, with Eq. 7.24}$$

$$(4) \quad \epsilon = \frac{\dot{W}_{cv}/\dot{m}}{\dot{e}_{f1} - \dot{e}_{f2}} = \frac{(h_1 - h_2)}{(h_1 - h_2) - T_0 (s_1 - s_2)}$$

Sample calculation: $x_2 = 0.81$. From Table A-3E, $h_1 = 1205.5 \text{ Btu/lb}$, $s_1 = 1.4856 \text{ Btu/lb} \cdot ^\circ R$,

$$h_2 = h_f + x_2 h_{fg} = 53.27 + 0.81(1045.4) = 900.8 \text{ Btu/lb}, \quad s_2 = s_f + x_2 s_{fg} = 0.1029 + 0.81(1.9184) = 1.6568 \text{ Btu/lb} \cdot ^\circ R. \quad \text{With } s_{2s} = s_1,$$

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{1.4856 - 0.1029}{1.9184} = 0.721 \Rightarrow h_{2s} = 53.27 + (0.721)(1045.4) = 807.8 \text{ Btu/lb}$$

Then, we get from Eq. (1), (2), (3), and (4), respectively

$$\frac{\dot{W}_{cv}}{\dot{m}} = (1205.5 - 900.8) = 305.5 \text{ Btu/lb}, \quad \frac{\dot{E}_d}{\dot{m}} = 520(1.6568 - 1.4856) = 89.8 \text{ Btu/lb}$$

$$\eta_t = \left(\frac{305.5}{1205.5 - 807.8} \right) = \frac{305.5}{397.7} = 0.767 \text{ (76.7\%)}, \quad \epsilon = \frac{305.5}{305.5 + 89.8} = \frac{305.5}{395.3} = 0.773 \text{ (77.3\%)}$$

Data required for the plots on the next page are obtained with the following IT code:

IT Code

```
p1 = 400 // lbf/in.^2
x1 = 1
p2 = 0.6 // lbf/in.^2
x2 = 0.81
To = 60 + 460 // °R
```

```
h1 = hsat_Px("Water/Steam", p1, x1)
s1 = ssat_Px("Water/Steam", p1, x1)
h2 = hsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
s2s = s_Ph("Water/Steam", p2, h2s)
s2s = s1
```

```
Wcv = h1 - h2
Ed = To * (s2 - s1)
eta = Wcv / (h1 - h2s)
eff = Wcv / ((h1 - h2) - To * (s1 - s2))
```

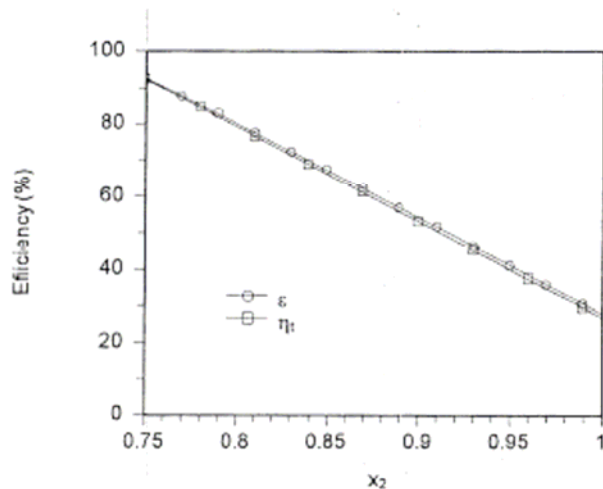
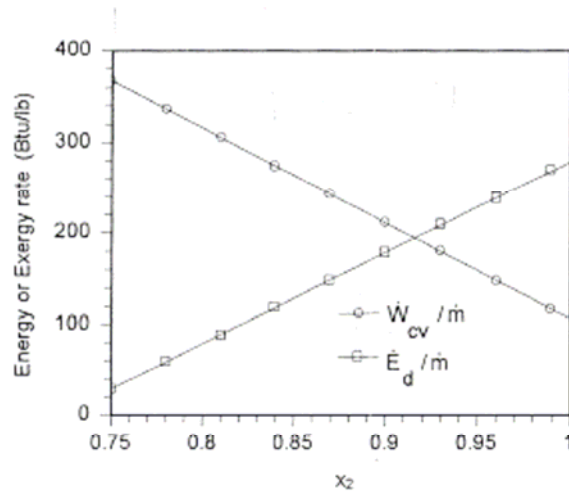
IT Results for $x_2 = 0.81$

```
h1 = 1205 Btu/lb
h2 = 899.8 Btu/lb
h2s = 806.6 Btu/lb
s1 = 1.485 Btu/lb.°R
s2 = 1.656 Btu/lb.°R
Wcv / m = 305.5 Btu/lb
Ed / m = 88.97 Btu/lb
epsilon = 0.7744 (77.44%)
eta = 0.7662 (76.62%)
```

Continued on next slide

Problem 7-110 continued

PLOTS:



As expected, increased exergy destruction corresponds with decreased power output and efficiency.

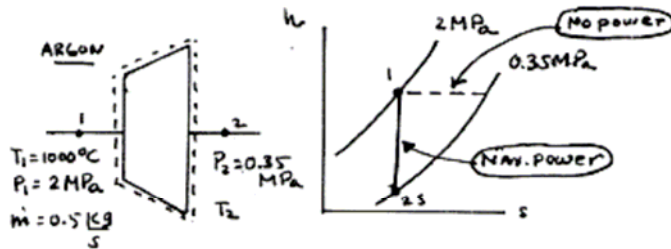
1. In this case the numerical values of ϵ and η_t for each X_2 are nearly the same, but this is not observed generally.

PROBLEM 7.111

KNOWN: Steady-state operating data are provided for a turbine through which argon is expanding.

FIND: Plot versus the turbine exit temperature, the power developed, the rate of exergy destruction, the exergetic turbine efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects are negligible.
3. Argon is modeled as an ideal gas with $c_p = \frac{5}{2}R$ (Table A-21). ①
4. $T_0 = 293\text{ K}$, $P_0 = 1\text{ bar}$.

ANALYSIS: Reducing mass, energy, and entropy balances together with ideal gas model relations for c_p constant

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) = \dot{m}c_p(T_2 - T_1)$$

$$= \dot{m} \left(\frac{5}{2}R \right) (T_2 - T_1) \quad (1)$$

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = \dot{m} T_0 (s_2 - s_1) = \dot{m} T_0 \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right]$$

$$= \dot{m} T_0 R \left[2.5 \ln \frac{T_2}{T_1} - \ln \frac{P_2}{P_1} \right] \quad (2)$$

$$\epsilon = \frac{\dot{W}_{cv}}{\dot{m}(e_{f1} - e_{f2})} = \frac{\dot{W}_{cv}}{\dot{m}(h_1 - h_2 - T_0(s_1 - s_2))} = \frac{\dot{W}_{cv}}{\dot{W}_{cv} + \dot{E}_d} \quad (3)$$

Observe that T_2 can range from T_{2s} , corresponding to an isentropic expansion, to $T_2 = T_1$, corresponding to no power developed. With Eq. 6.43, $T_{2s} = T_1 (P_2/P_1)^{(k-1)/k}$. Using Eq. 3.47a, $\frac{k-1}{k} = \frac{R}{c_p} = \frac{R}{2.5R} = 0.4$. Thus, $T_{2s} = (1273\text{ K}) \left(\frac{0.35}{2} \right)^{0.4} = 634\text{ K}$. So, $634\text{ K} \leq T_2 \leq 1273\text{ K}$.

The data for the plots on the next page are obtained using IT as follows:

IT Code

```
p1 = 2 // MPa
T1 = 1000 + 273 // K
p2 = 0.35 // MPa
T2 = 1273 // K
T0 = 20 + 273 // K
mdot = 0.5 // kg/s
cp = 2.5 * R
R = 8.314 / 39.94 // kJ/kg.K
```

```
Wdot = mdot * cp * (T1 - T2)
delta_s = cp * ln(T2 / T1) - R * ln(p2 / p1)
Edot = T0 * mdot * delta_s
eff = Wdot / (Wdot + Edot)
```

IT Results

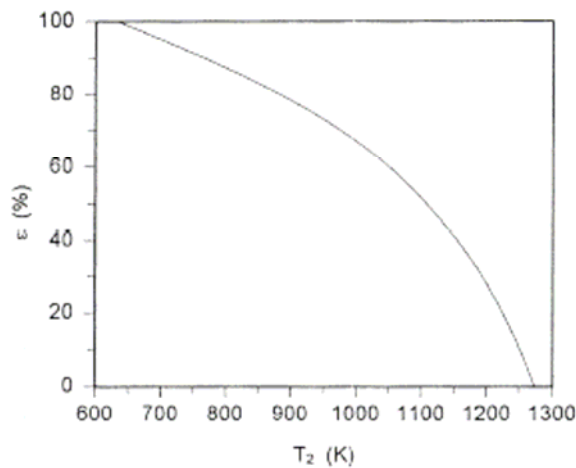
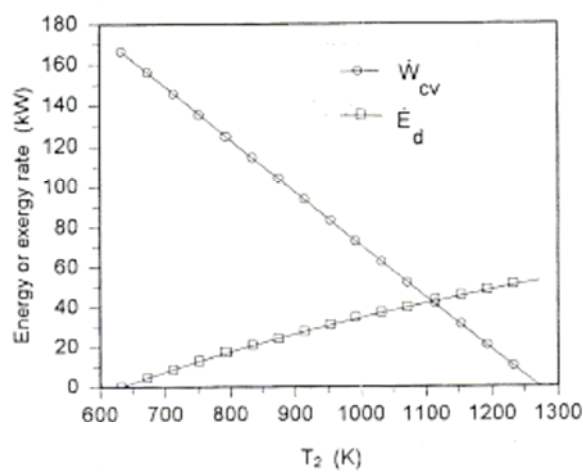
```
for T2 = 634 K:
Wdot_cv = 166.3 kW
Edot_d = 0
epsilon = 1 (100%)
```

```
For T2 = 1273 K:
Wdot_cv = 0
Edot_d = 53.15 kW
epsilon = 0
```

Continued on next slide

Problem 7-111 continued

PLOTS:



1. Since $c_p = \frac{5}{2} R$ and $c_p = c_v + R$ for an ideal gas, we get $c_v = \frac{3}{2} R$ and

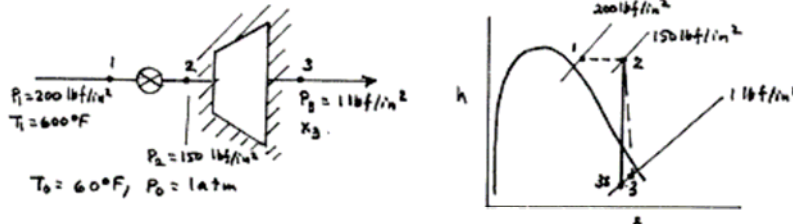
$$k = \frac{c_p}{c_v} = \frac{5/2 R}{3/2 R} = \frac{5}{3} = 1.67.$$

PROBLEM 7.112

KNOWN: Steady-state operating data are provided for a throttling valve and turbine in series.

FIND: For the turbine plot the rate of energy destruction and exergetic efficiency versus quality ranging from 90 to 100%.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The turbine is insulated and at steady state. $\dot{Q}_{cv} = 0$. (2) The expansion across the valve is a throttling process. (3) For the environment, $T_0 = 60^\circ\text{F}$, $P_0 = 1 \text{ atm}$.

ANALYSIS: For a 1-inlet, 2-exit control volume at steady state for which there is no heat transfer, an entropy rate balance reduces to give

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{Q}_{cv} \Rightarrow \dot{Q}_{cv} = \dot{m}(s_2 - s_1)$$

Then, with $\dot{E}_d = T_0 \dot{Q}_{cv}$, we get

$$\text{Valve: } \frac{\dot{E}_d}{\dot{m}} = T_0(s_2 - s_1) \quad (1) \quad \text{Turbine: } \frac{\dot{E}_d}{\dot{m}} = T_0(s_3 - s_2) \quad (2)$$

Sample calculation: $x_3 = 0.98$. From Table A-4E, $h_1 = 1322.1 \text{ Btu/lb}$, $s_1 = 1.6767 \text{ Btu/lb} \cdot ^\circ\text{R}$. With assumption 2, $h_2 \sim h_1$. Interpolating in Table A-4E at 150 lbf/in^2 with $h_2 = 1322.1 \text{ Btu/lb}$, $s_2 = 1.7078 \text{ Btu/lb} \cdot ^\circ\text{R}$. With data from Table A-3E, $s_3 = s_f + x_3 s_{fg}$ or $s_3 = 0.1327 + 0.98(1.8453) = 1.9411 \text{ Btu/lb} \cdot ^\circ\text{R}$. Inserting values into Eqs. (1), (2)

$$(\dot{E}_d/\dot{m})_{\text{valve}} = 520^\circ\text{R}(1.7078 - 1.6767) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 16.17 \frac{\text{Btu}}{\text{lb}} \quad \leftarrow \text{valve}$$

$$(\dot{E}_d/\dot{m})_{\text{turbine}} = 520^\circ\text{R}(1.9411 - 1.7078) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 121.32 \frac{\text{Btu}}{\text{lb}} \quad \leftarrow \text{turbine}$$

Note that the valve is considered only for comparison.

IT Code

```
p1 = 200 // lbf/in.^2
T1 = 600 // °F
p2 = 150 // lbf/in.^2
p3 = 1 // lbf/in.^2
T0 = 60 + 460 // °R
x3 = 0.98
```

```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = h_Ps("Water/Steam", p2, s1)
h2 = h1
s3 = ssat_Px("Water/Steam", p3, x3)
Edvalve = T0 * (s2 - s1)
Edturb = T0 * (s3 - s2)
```

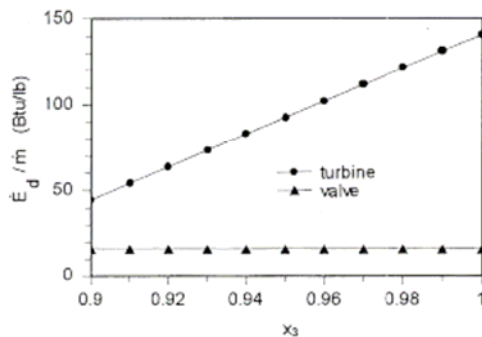
IT Results for $x_3 = 0.98$

```
h1 = 1322 Btu/lb
s1 = 1.676 Btu/lb.°R
s2 = 1.707 Btu/lb.°R
s3 = 1.941 Btu/lb.°R
(Ed/m)_valve = 121.4 Btu/lb
(Ed/m)_turbine = 16.04 Btu/lb
```

Continued on next slide

Problem 7-112 continued

PLOT:



- Note that the exergy destruction in the valve is independent of x_3 .
- The exergy destruction in the turbine increases as x_3 increases, corresponding to a decrease in isentropic turbine efficiency, as expected.

The turbine exergetic efficiency is given by Eq. 7.24. With the indicated assumptions, this can be expressed as

$$\epsilon = \frac{h_2 - h_3}{(h_2 - h_3) - T_0(s_2 - s_3)} \quad \text{Since } h_2 = h_1 \text{ for the throttling process, we get}$$

$$\epsilon = \frac{h_1 - h_3}{(h_1 - h_3) - T_0(s_2 - s_3)} \quad (1) \quad \text{In Eq. (1) } h_1 \text{ is fixed by } T_1, p_1, s_2 \text{ is fixed by } h_2 = h_1, p_2, \text{ and } h_3 \text{ and } s_3 \text{ are fixed by } p_3, x_3.$$

Sample Calculation: $x_3 = 0.98$. From Table A-4E, $h_1 = 1322.1$ Btu/lb. Interpolating in Table A-4E with $h_2 = h_1$ at 150 lbf/in.², $s_2 = 1.7078$ Btu/lb·°R. With data from Table A-9E at 1 lbf/in.², $h_3 = 69.74 + 0.98(1086) = 1085$ Btu/lb, $s_3 = 0.1327 + 0.98(1.8453) = 1.9411$ Btu/lb·°R. Inserting values, Eq. (1) gives

$$\epsilon = \frac{(1322.1 - 1085)}{(1322.1 - 1085) - 520(1.7078 - 1.9411)} = \frac{237.1}{358.4} = 0.662 \quad (66.2\%)$$

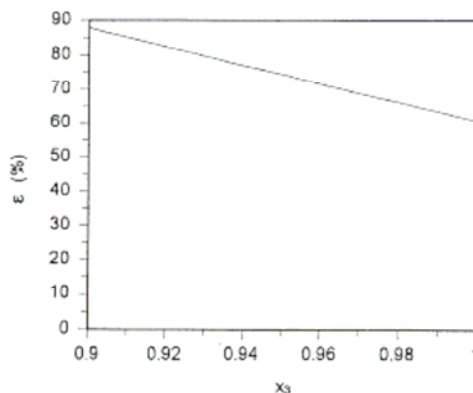
IT Code

```
p1 = 200 // lbf/in.2
T1 = 600 // °F
p2 = 150 // lbf/in.2
p3 = 1 // lbf/in.2
x3 = 0.98
To = 60 + 460 // °R
```

```
h1 = h_PT("Water/Steam", p1, T1)
s2 = s_Ph("Water/Steam", p2, h1)
s3 = ssat_Px("Water/Steam", p3, x3)
h3 = hsat_Px("Water/Steam", p3, x3)
eff = (h1 - h3) / ((h1 - h3) - To * (s2 - s3))
```

IT Results for $x_3 = 0.98$

```
eff 0.6613
h1 1322
h3 1085
s2 1.707
s3 1.941
```

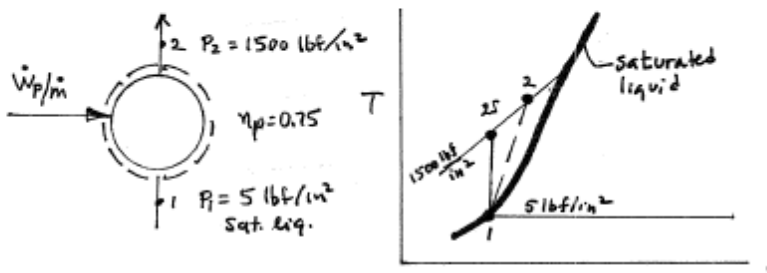


PROBLEM 7.113

KNOWN: Steady state operating data are provided for a pump.

FIND: Determine (a) exergy destruction per unit of mass flowing, (b) the exergetic pump efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects are negligible. (3) $T_0 = 530^\circ R$

ANALYSIS: (a) The exergy destruction per unit of mass flowing is conveniently found with

$$\frac{\dot{E}_d}{\dot{m}} = T_0 \frac{\dot{Q}_{cv}}{\dot{m}} \Rightarrow \frac{\dot{E}_d}{\dot{m}} = T_0 (s_2 - s_1)$$

State 1 is fixed. State 2 is fixed by P_2 and h_2 obtained as follows: using Eqs. 6.48 and 6.51c together with an average value for the specific volume,

$$\eta_p = \frac{v_{ave}(P_2 - P_1)}{(-\dot{W}_{cv}/\dot{m})} = \frac{v_{ave}(P_2 - P_1)}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{v_{ave}(P_2 - P_1)}{\eta_p} \quad (1)$$

Taking $v_{ave} \approx v_f(T_1)$, $h_1 = h_f(T_1)$ and data from Table A-3E, Eq. (1) gives

$$h_2 = 130.17 \frac{\text{Btu}}{\text{lb}} + \frac{(0.01641 \text{ ft}^3/\text{lb})(1500 - 5)(144) \frac{\text{lbf}}{\text{ft}^2} \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|}{0.75} = 136.22 \frac{\text{Btu}}{\text{lb}}$$

- ① Interpolating in Table A-5E, $s_2 = 0.2367 \text{ Btu/lb} \cdot ^\circ R$, $v_2 = 0.01635 \text{ ft}^3/\text{lb}$. Also, $s_1 = s_f(T_1) = 0.2349 \text{ Btu/lb} \cdot ^\circ R$. Then

$$\frac{\dot{E}_d}{\dot{m}} = 530^\circ R (0.2367 - 0.2349) \frac{\text{Btu}}{\text{lb} \cdot ^\circ R} = 0.954 \frac{\text{Btu}}{\text{lb}}$$

(b) With Eq. 7.25

$$\epsilon = \frac{e_{f2} - e_{f1}}{(-\dot{W}_{cv}/\dot{m})} = \frac{(h_2 - h_1) - T_0(s_2 - s_1)}{(h_2 - h_1)} = \frac{(136.22 - 130.17) - 0.954}{136.22 - 130.17} = 0.842 \quad (84.2\%)$$

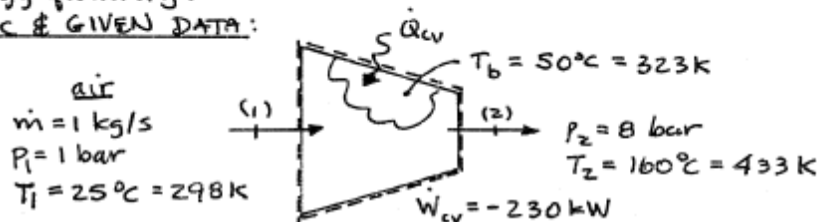
1. Comparing v_2 with $v_f(T_1)$, it is clear that specific volume does not vary appreciably in this case.

PROBLEM 7.114

KNOWN: The states at the inlet and exit of an air compressor operating at steady state are known. The compressor power and the average surface temperature for heat transfer from the compressor to its surrounding are also specified.

FIND: (a) Perform a full exergy accounting of the power input. (2) Devise and evaluate an exergetic efficiency. (3) Determine the hourly costs of each energy quantity.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown is at steady state. (2) For the control volume, kinetic and potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) For the environment, $T_0 = 25^\circ\text{C} = 298\text{ K}$, $P_0 = 1\text{ bar}$.

ANALYSIS: (a) First, we determine the heat transfer rate using mass and energy rate balances: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$ and $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1) = -230\text{ kW} + (1\text{ kg/s})(434.48 - 298.18)\text{ kJ/kg} \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right|$$

$$= -93.69\text{ kW}$$

Exergy transfer out with heat

$$\left(\text{exergy out} \right)_{\text{with heat}} = \left(1 - \frac{T_0}{T_b} \right) |\dot{Q}_{cv}| = \left(1 - \frac{298}{323} \right) (93.69) = \underline{7.252\text{ kW}}$$

Net flow exergy increase of air

$$\dot{E}_{f2} - \dot{E}_{f1} = \dot{m} [(h_2 - h_1) - T_0 (s_2 - s_1)]$$

$$= \dot{m} [(h_2 - h_1) - T_0 (s^\circ(T_2) - s^\circ(T_1)) - R \ln P_2/P_1]$$

$$= (1\text{ kg/s}) [(434.48 - 298.18) - (298\text{ K})(2.07234 - 1.69528) - \frac{8.314\text{ kJ}}{28.97\text{ kg}\cdot\text{K}} \ln \frac{8}{1}] = \underline{201.8\text{ kW}}$$

Exergy destroyed

Using an exergy rate balance

$$0 = \left[1 - \frac{T_0}{T_b} \right] \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$$

$$\dot{E}_d = \left[1 - \frac{T_0}{T_b} \right] \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(e_{f1} - e_{f2})$$

$$= (-7.252) - (-230) + (-201.8) = \underline{20.95\text{ kW}}$$

Continued on next slide

Problem 7-114 continued

Exergy Accounting Summary

① Input:

$$\text{power} = 230 \text{ kW} \quad (100\%)$$

② Disposition of Exergy Input:

$$\checkmark \text{ flow exergy} = 201.8 \text{ kW} \quad (87.74\%)$$

$$\checkmark \text{ heat transfer} = 7.252 \text{ kW} \quad (3.15\%)$$

$$\checkmark \text{ Exergy destroyed} = \frac{20.95 \text{ kW}}{230 \text{ kW}} \quad (9.11\%)$$

exergy
accounting

(b) Defining an exergetic efficiency in terms of (desired output/input)

$$\textcircled{1} \quad \epsilon = \frac{\text{net flow exergy increase of air}}{\text{power input}} = \frac{201.8}{230} = 0.8774 \quad (87.74\%)$$

② (c) If the exergy cost rate is \$0.08/kW·h

$$\left(\text{hourly cost of power input} \right) = (230 \text{ kW})(1 \text{ h}) \left(0.08 \frac{\$}{\text{kW}\cdot\text{h}} \right) = \$18.40/\text{h}$$

$$\left(\text{hourly cost of exergy loss due to heat transfer} \right) = (7.252 \text{ kW})(1 \text{ h}) \left(0.08 \frac{\$}{\text{kW}\cdot\text{h}} \right) = \$0.58/\text{h}$$

$$\left(\text{cost of exergy destruction} \right) = (20.95 \text{ kW})(1 \text{ h}) \left(0.08 \frac{\$}{\text{kW}\cdot\text{h}} \right) = \$1.66/\text{h}$$

\$

1. The exergy transfer with heat is regarded as a loss.

2. As discussed in Sec. 7.72, a more complete costing evaluation must take into account the cost of owning and operating the compressor.

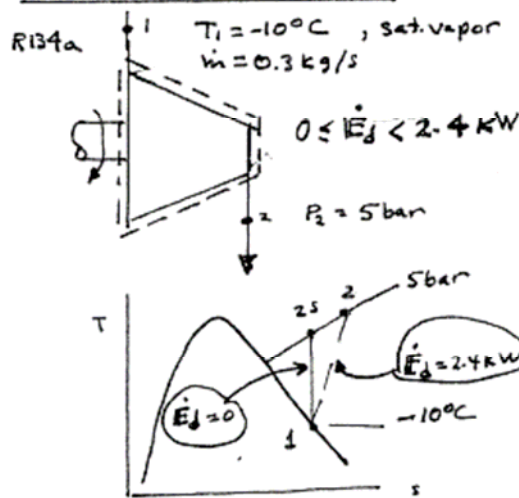
PROBLEM 7.115

Refrigerant 134a as saturated vapor at -10°C enters a compressor operating at steady state with a mass flow rate of 0.3 kg/s . At the compressor exit the pressure of the refrigerant is 5 bar. Stray heat transfer and the effects of motion and gravity can be ignored. If the rate of exergy destruction within the compressor must be kept less than 2.4 kW , determine the allowed ranges for (a) the power required by the compressor, in kW, and (b) the exergetic compressor efficiency. Let $T_0 = 298 \text{ K}$, $p_0 = 1 \text{ bar}$.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and the effects of motion and gravity can be ignored.
3. $T_0 = 298 \text{ K}$, $p_0 = 1 \text{ bar}$

SCHEMATIC & GIVEN DATA:



ANALYSIS: $\dot{E}_d = T_0 \dot{Q}_{cv} = T_0 \dot{m} (s_2 - s_1)$. So, $T_0 \dot{m} (s_2 - s_1) < 2.4 \text{ kW}$, or

$$s_2 < s_1 + \frac{2.4 \text{ kW}}{\dot{m} T_0} \Rightarrow s_2 < 0.9253 + \frac{2.4 \text{ kJ/s}}{(0.3 \text{ kg/s})(298 \text{ K})} = 0.9521 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Thus, from Table A-12 at 5 bar, $h_2 < 268.01 \text{ kJ/kg}$

(a) Reducing mass and energy rate balances, $(-\dot{W}_c) = h_2 - h_1$. Thus,

$$\dot{m} [h_{2s} - h_1] \leq (-\dot{W}_c) < \dot{m} (h_2 - h_1) \quad (1)$$

$\uparrow \dot{E}_d = 0$
 $\uparrow h_2 < 268.01 \text{ kJ/kg}$

From Table A-10, $h_1 = 241.35 \text{ kJ/kg}$, $s_1 = 0.9253 \text{ kJ/kg} \cdot \text{K}$. Then, with $s_{2s} = s_1$, $h_{2s} = 260.02 \text{ kJ/kg}$. Eq. (1) gives

$$(0.3 \frac{\text{kg}}{\text{s}})(260.02 - 241.35) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \leq (-\dot{W}_c) < 0.3(268.01 - 241.35) |1|$$

or $5.6 \text{ kW} \leq (-\dot{W}_c) < 8 \text{ kW} \quad \leftarrow (a)$

(b) Invoking Eq. 7.25,

$$\epsilon = \frac{e_{f2} - e_{f1}}{-\dot{W}_c} = \frac{(h_2 - h_1) - T_0(s_2 - s_1)}{(h_2 - h_1)}$$

When $\dot{E}_d = 0$, $\epsilon = 1$ (100%). Otherwise,

$$\epsilon > \frac{(268.01 - 241.35) - 298(0.9521 - 0.9253)}{(268.01 - 241.35)} = 0.7 \text{ (70\%)}$$

$\Rightarrow 70\% < \epsilon \leq 100\% \quad \leftarrow (b)$

PROBLEM 7.116

Determine the exergetic efficiency for the compressor of

- (a) Problem 6.134 Let $T_0 = 290\text{K}$.
 (b) Problem 6.135. Let $T_0 = 298\text{K}$.

(a) From Eq. 7.25

$$\epsilon = \frac{(h_2 - h_1) - T_0(s_2 - s_1)}{h_2 - h_1} = \frac{(h_2 - h_1) - T_0(s_2^0 - s_1^0 - R \ln P_2/P_1)}{h_2 - h_1}$$

From Table A-22, $h_1 = 290.16 \text{ kJ/kg}$, $h_2 = 421.26 \text{ kJ/kg}$

$$s_1^0 = 1.66802, \quad s_2^0 = 2.04142 \text{ kJ/kg} \cdot \text{K}$$

$$\therefore \epsilon = \frac{131.1 - 290(2.04142 - 1.66802 - \frac{8.314}{28.97} \ln(\frac{320}{100}))}{131.1} = \frac{122.18}{131.1} = 0.932 (93.2\%) \quad \leftarrow (a)$$

(b) From Eq. 7.25

$$\epsilon = \frac{(h_2 - h_1) - T_0(s_2 - s_1)}{h_2 - h_1} = \frac{c_p(T_2 - T_1) - T_0 \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right]}{c_p(T_2 - T_1)}$$

$$= \frac{(T_2 - T_1) - T_0 \left[\ln \frac{T_2}{T_1} - \frac{R}{c_p} \ln \frac{P_2}{P_1} \right]}{T_2 - T_1}, \quad \text{Eq. 3.47a: } c_p = \frac{KR}{K-1} \Rightarrow \frac{R}{c_p} = \frac{K-1}{K}$$

$$= \frac{235\text{K} - 298\text{K} \left[\ln \frac{533}{298} - \left(\frac{1.4-1}{1.4} \right) \ln \left(\frac{650}{100} \right) \right]}{235\text{K}} = 0.941 (94.1\%) \quad \leftarrow (b)$$

PROBLEM 7.117

For the turbine-compressor arrangement of Problem 6.146, determine the exergetic efficiency for (a) the turbine, (b) the compressor, (c) an overall control volume enclosing the turbine and compressor. Let $T_0 = 300 \text{ K}$.

ANALYSIS:

$$(a) \quad \epsilon_t = \frac{\dot{W}_t}{\dot{m}_1(e_{f1} - e_{f2})} = \frac{\dot{m}_1(h_1 - h_2)}{\dot{m}_1[h_1 - h_2 - T_0(s_1 - s_2)]} = \frac{629.37}{629.37 - 300(7.2540 - 7.4395)}$$

$$h_1 = 3335.5 \text{ kJ/kg}, \quad s_1 = 7.2540 \text{ kJ/kg} \cdot \text{K} \quad = \frac{629.37}{685.02} = 0.919 \quad (91.9\%)$$

$$h_2 = h_1 - \eta_t(h_1 - h_{2s}) \\ = 3335.5 - 0.9(3335.5 - 2636.2) \\ = 2706.13 \text{ kJ/kg}$$

$$\Rightarrow s_2 = 7.4395 \text{ kJ/kg} \cdot \text{K} \quad (\text{Interpolation in Table A-4 at 1 bar})$$

$$(b) \quad \epsilon_c = \frac{\dot{m}_4(e_{f4} - e_{f3})}{(-\dot{W}_c)} = \frac{h_4 - h_3 - T_0(s_4 - s_3)}{(h_4 - h_3)} = \frac{(h_4 - h_3) - T_0(s_4 - s_3) - R \ln P_4/P_3}{(h_4 - h_3)}$$

From Problem 6.146,

$$h_3 = 300.19 \text{ kJ/kg}, \quad s_3 = 1.70203 \text{ kJ/kg} \cdot \text{K}, \quad h_{4s} = 462.1 \text{ kJ/kg}$$

$$h_4 = h_3 + \frac{(h_{4s} - h_3)}{\eta_c} = 300.19 + \frac{(462.1 - 300.19)}{0.85} = 490.67 \text{ kJ/kg} \Rightarrow s_4 = 2.19449$$

$$\epsilon_c = \frac{190.48 - 300(2.19449 - 1.70203 - (8.314/28.97) \ln(446/110))}{190.48} \\ = \frac{172.41}{190.48} = 0.905 \quad (90.5\%)$$

(c) For an overall control volume enclosing the turbine and condenser, an exergy rate balance reads

$$0 = \dot{E}_q^o - \dot{W}^o + \dot{m}_1[e_{f1} - e_{f2}] + \dot{m}_3[e_{f3} - e_{f4}] - \dot{E}_d$$

$$\Rightarrow \underbrace{\dot{m}_1[e_{f1} - e_{f2}]}_{\text{net rate exergy is supplied}} = \dot{m}_4(e_{f4} - e_{f3}) + \dot{E}_d$$

$$\textcircled{1} \quad \Rightarrow \quad \epsilon = \frac{\dot{m}_4(e_{f4} - e_{f3})}{\dot{m}_1(e_{f1} - e_{f2})} = \left[\frac{\dot{m}_4}{\dot{m}_1} \right] \frac{(e_{f4} - e_{f3})}{(e_{f1} - e_{f2})} = \left[\frac{1}{0.303} \right] \frac{172.41}{685.02} = 0.831 \quad (83.1\%)$$

from Problem 6.146

part (c)

part (a)

1. Note that since $\dot{W}_t = -\dot{W}_c$

$$\epsilon_t \epsilon_c = \left[\frac{\dot{W}_t}{\dot{m}_1(e_{f1} - e_{f2})} \right] \left[\frac{\dot{m}_4(e_{f4} - e_{f3})}{(-\dot{W}_c)} \right] = \frac{\dot{m}_4(e_{f4} - e_{f3})}{\dot{m}_1(e_{f1} - e_{f2})} = \epsilon \Rightarrow \underline{\underline{\epsilon = \epsilon_t \epsilon_c}}$$

Using the values determined in parts (a), (b),

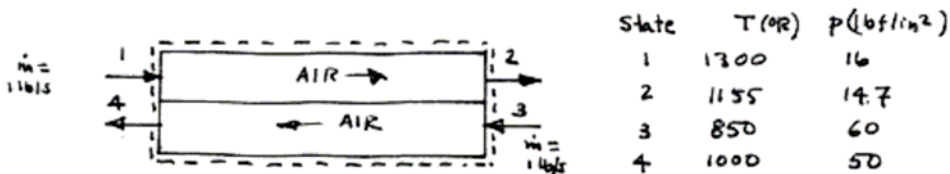
$$\epsilon = (0.919)(0.905) = 0.832, \text{ which agrees to within round-off.}$$

PROBLEM 7.118

KNOWN: Steady-state operating data is provided for a counterflow heat exchanger having air flowing on both sides.

FIND: Determine (a) the rate of exergy destruction, (b) the exergetic efficiency given by Eq. 7.27.

SCHEMATIC & GIVEN DATA



ENGR. MODEL: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and kinetic and potential energy effects are negligible. 3. Air is modeled as an ideal gas. 4. $T_0 = 520^\circ\text{R}$, $p_0 = 14.7 \text{ lbf/in}^2$.

ANALYSIS: Mass and energy rate balances reduce to give $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$.
 $\Rightarrow 0 = (h_1 - h_2) + (h_3 - h_4)$. With data from Table A-22E, $h_1 = 316.95 \text{ Btu/lb}$, $h_2 = 279.88 \text{ Btu/lb}$, $h_3 = 204.01 \text{ Btu/lb}$, $h_4 = 240.98 \text{ Btu/lb}$. These values check the energy balance, indicating that the given state data are consistent with the conservation of mass and energy principles.

Reducing an exergy rate balance at steady state

$$\begin{aligned}
 0 &= \sum_j \left[1 - \frac{T_0}{T_j} \right] \dot{Q}_j - \dot{W}_{cv} + \dot{m}_1(e_{f1} - e_{f2}) + \dot{m}_3(e_{f3} - e_{f4}) - \dot{E}_d \\
 \Rightarrow \frac{\dot{E}_d}{\dot{m}} &= [h_1 - h_2 - T_0(s_1 - s_2)] + [h_3 - h_4 - T_0(s_3 - s_4)] \\
 &= [(h_1 - h_2) + (h_3 - h_4)] + T_0[(s_2 - s_1) + (s_4 - s_3)] \quad (= 0 \text{ from the energy balance}) \\
 \textcircled{1} \quad \frac{\dot{E}_d}{\dot{m}} &= T_0 \left[(s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1}) + (s^\circ(T_4) - s^\circ(T_3) - R \ln \frac{p_4}{p_3}) \right] \\
 &= 520^\circ\text{R} \left[\left[0.78665 - 0.81678 - \frac{1.986}{28.97} \ln \frac{14.7}{16} \right] \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} + \right. \\
 &\quad \left. \left[0.75042 - 0.71035 - \frac{1.986}{28.97} \ln \frac{50}{60} \right] \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right] \\
 &= 520[-0.02432 + 0.05257] = 14.69 \text{ Btu/lb} \Rightarrow \dot{E}_d = 14.69 \frac{\text{Btu}}{\text{s}}
 \end{aligned}$$

(b) The exergetic efficiency given by Eq. 7.27

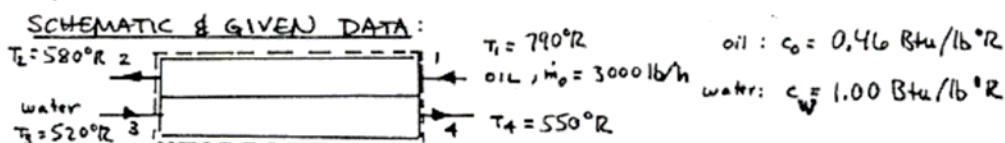
$$\begin{aligned}
 \textcircled{2} \quad \epsilon &= \frac{\dot{m}(e_{f4} - e_{f3})}{\dot{m}(e_{f1} - e_{f2})} = \frac{(h_4 - h_3) - T_0(s_4 - s_3)}{(h_1 - h_2) - T_0(s_1 - s_2)} = \frac{(240.98 - 204.01) - 520[0.05257]}{(316.95 - 279.88) - 520[0.02432]} \\
 &= \frac{9.634}{24.424} = 0.394 \quad (39.4\%)
 \end{aligned}$$

1. This corresponds to $\dot{E}_d = T_0 \dot{\sigma}$.

2. In accord with the discussion of Eq. 7.27, both streams are at temperatures above T_0 .

PROBLEM 7.119

KNOWN: Steady state operating data are provided for a counter flow heat exchanger through which oil and water flow in separate streams.
FIND: Determine (a) the mass flow rate of the water, (b) the exergetic efficiency given by Eq. 7.27, (c) the hourly cost of exergy destruction.



ENGR. MODEL: (1) The control volume shown in the accompanying figure is at steady state. (2) $\dot{Q}_{\text{cv}} = 0$ and the effects of kinetic and potential energy can be ignored. (3) The oil and water can each be regarded as incompressible with constant specific heats c_o and c_w , respectively, and negligible change in pressure in flowing through the heat exchanger. (4) For the environment $T_o = 60^\circ\text{F}$, $P_o = 1 \text{ atm}$. (5) Exergy is valued at $\$$ per kW·h.

ANALYSIS: (a) At steady state, mass rate balances reduce to give $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_o$ and $\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_w$. With assumptions 1 and 2, an energy balance gives

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}_o(h_1 - h_2) + \dot{m}_w(h_3 - h_4) \Rightarrow 0 = \dot{m}_o c_o (T_1 - T_2) + \dot{m}_w c_w (T_3 - T_4)$$

In writing the last expression, Eq. 3.20b was used together with assumption 3 to evaluate the enthalpy changes. Solving for $\dot{m}_w$

$$\dot{m}_w = \dot{m}_o \frac{c_o (T_1 - T_2)}{c_w (T_4 - T_3)} = 3000 \frac{\text{lb}}{\text{h}} \frac{(0.46)(790 - 580)}{(1.00)(550 - 520)} = 9660 \text{ lb/h} \quad \leftarrow \dot{m}_w$$

(b) The exergetic efficiency is, with Eq. 6.13

$$\begin{aligned} \textcircled{1} \quad \epsilon &= \frac{\dot{m}_w (e_{f4} - e_{f3})}{\dot{m}_o (e_{f1} - e_{f2})} = \frac{\dot{m}_w c_w [(T_4 - T_3) - T_o \ln(T_4/T_3)]}{\dot{m}_o c_o [(T_1 - T_2) - T_o \ln(T_1/T_2)]} \\ &= \frac{(9660)(1.00)[(550 - 520) - (520) \ln(550/520)]}{(3000)(0.46)[(790 - 580) - (520) \ln(790/580)]} \\ \textcircled{2} \quad &= \frac{8,051.4 \text{ Btu/h}}{68,058.1 \text{ Btu/h}} = 0.1183 \quad (11.83\%) \quad \leftarrow \epsilon \end{aligned}$$

(c) The rate of exergy destruction is obtained using an exergy rate balance, as follows:

$$0 = \sum_j \left[1 - \frac{T_o}{T_j} \right] \dot{Q}_j - \dot{W}_{\text{cv}} + \dot{m}_o (e_{f1} - e_{f2}) + \dot{m}_w (e_{f3} - e_{f4}) - \dot{E}_d$$

$$\text{or} \quad \dot{E}_d = \dot{m}_o (e_{f1} - e_{f2}) - \dot{m}_w (e_{f4} - e_{f3})$$

With values from above, $\dot{E}_d = 68,058.1 - 8,051.4 = 60,006.7 \frac{\text{Btu}}{\text{h}}$. Thus

$$\left(\text{cost} \right)_{\text{hourly}} = \left(60,006.7 \frac{\text{Btu}}{\text{h}} \right) \left| \frac{1 \text{ kW}}{3413 \text{ Btu/h}} \right| \left(0.08 \frac{\$}{\text{kW} \cdot \text{h}} \right) = \$1.41/\text{h} \quad \leftarrow \$$$

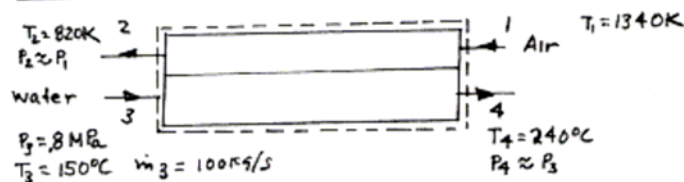
- In accord with the discussion of Eq. 7.27, both streams are at temperatures $\geq T_o$.
- For a counterflow heat exchanger this is very low exergetic efficiency. The considerable exergy yielded by the cooling oil is not well utilized.

PROBLEM 7.120

KNOWN: Steady-state operating data are provided for boiler tubes carrying water over which combustion gases flow.

FIND: Determine (a) the mass flow rate of the combustion gases, (b) the rate of exergy destruction, (c) the exergetic efficiency Eq. 7.27.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The combustion gases are modeled as air as an ideal gas. (3) There is no significant change in pressure for either stream as it passes from inlet to exit. (4) For the environment, $T_0 = 298K$, $P_0 = 1 \text{ atm}$.

ANALYSIS: At steady state mass rate balances give $\dot{m}_1 = \dot{m}_2$ and $\dot{m}_3 = \dot{m}_4$. An energy rate balance reduces with assumption 1 to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4) \Rightarrow \frac{\dot{m}_1}{\dot{m}_3} = \frac{h_4 - h_3}{h_1 - h_2}$$

With data from Tables A-2 and A-4, $h_4 = 2928.3 \text{ kJ/kg}$ and $h_3 \approx h_f(T_3) = 632.2 \text{ kJ/kg}$

Table A-22 gives $h_1 = 1443.6 \text{ kJ/kg}$, $h_2 = 843.98 \text{ kJ/kg}$.

$$\Rightarrow \dot{m}_1 = \left(\frac{100 \text{ kg/s}}{5} \right) \left[\frac{2928.3 - 632.2}{1443.6 - 843.98} \right] = (100) \left[\frac{2296.1}{599.62} \right] = 382.93 \text{ kg/s} \quad \leftarrow$$

The exergy rate balance reduces at steady state to give

$$\dot{E}_d = \dot{m}_1(e_{f1} - e_{f2}) + \dot{m}_3(e_{f3} - e_{f4}) = \dot{m}_1[(h_1 - h_2) - T_0(s_1 - s_2)] + \dot{m}_3[(h_3 - h_4) - T_0(s_3 - s_4)]$$

$$\dot{E}_d / \dot{m}_3 = \frac{\dot{m}_1}{\dot{m}_3} \left[h_1 - h_2 - T_0(s_1 - s_2) - R \ln \frac{P_1}{P_2} \right] + [(h_3 - h_4) - T_0(s_3 - s_4)]$$

With s° data from Table A-22, $s_3 \approx s_f(T_3)$ from Table A-2 and s_4 from Table A-4

$$\begin{aligned} \frac{\dot{E}_d}{\dot{m}_3} &= 3.8293 [699.62 - 298(3.30959 - 2.74504)] + [-2296.1 - 298(1.8418 - 7.0033)] \\ &= 3.8293 [431.38] + [-757.97] = 893.91 \frac{\text{kJ}}{\text{kg}} \Rightarrow \dot{E}_d = 8.939 \times 10^4 \frac{\text{kJ}}{\text{s}} \quad \leftarrow \end{aligned}$$

Alternatively, the exergy destruction can be found using $\dot{E}_d = T_0 \dot{\sigma}$.

① With Eq. 7.27 and previously calculated values

$$\epsilon = \frac{\dot{m}_3(e_{f4} - e_{f3})}{\dot{m}_1(e_{f1} - e_{f2})} = \frac{757.97}{3.8293(431.38)} = 0.459 \quad (45.9\%) \quad \leftarrow$$

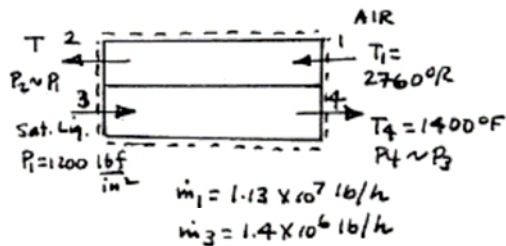
1. In accord with the discussion of Eq. 7.27, both streams are at temperatures above T_0 .

PROBLEM 7.121

KNOWN: Steady-state operating data are provided for boiler tubes carrying water over which combustion gases flow.

FIND: Determine (a) the exit temperature of the combustion gases, (b) the rate of exergy destruction, (c) the exergetic efficiency given by Eq. 7.27.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects are negligible.
3. The combustion gases are modeled as air as an ideal gas.
4. There is no significant change in pressure for either stream.
5. $T_0 = 530^\circ R$, $P_0 = 1 \text{ atm}$.

ANALYSIS: (a) Reducing mass and energy rate balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4) \Rightarrow h_2 = h_1 + \frac{\dot{m}_3}{\dot{m}_1}(h_3 - h_4)$$

With data from Table A-22E and Tables A-35 & 4E

$$h_2 = 720.73 + \left(\frac{1.4 \times 10^6}{1.13 \times 10^7} \right) (571.7 - 1730.7) = 577.14 \text{ Btu/lb} \cdot \text{Interpolating in Table A-22E}$$

$$T_2 = 2259^\circ R.$$

(b) The rate of exergy destruction is obtained using an exergy rate balance, which reduces to give

$$\begin{aligned} \dot{E}_d &= \dot{m}_1[e_{f1} - e_{f2}] + \dot{m}_3[e_{f3} - e_{f4}] \\ &= \dot{m}_1[(h_1 - h_2) - T_0(s_1 - s_2)] + \dot{m}_3[(h_3 - h_4) - T_0(s_3 - s_4)] \\ &= \dot{m}_1[(h_1 - h_2) - T_0(s_1^o - s_2^o - R \ln \frac{P_1}{P_2})] + \dot{m}_3[(h_3 - h_4) - T_0(s_3 - s_4)] \end{aligned}$$

Inserting table data

$$\begin{aligned} \dot{E}_d &= (1.13 \times 10^7 \frac{\text{lb}}{\text{h}}) [(720.73 - 577.14) - 530(1.02349 - 0.96609)] \frac{\text{Btu}}{\text{lb}} + \\ &\quad (1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) [(571.7 - 1730.7) - 530(0.7412 - 1.7696)] \frac{\text{Btu}}{\text{lb}} \\ &= (1.13 \times 10^7) [\frac{143.59 - 30.42}{113.17}] + (1.4 \times 10^6) [\frac{-1159 - (-529.15)}{-629.85}] \\ &= 1.28 \times 10^9 + (-0.88 \times 10^9) = 4 \times 10^8 \frac{\text{Btu}}{\text{h}} \end{aligned}$$

Alternatively, the exergy destruction can be evaluated using $\dot{E}_d = T_0 \dot{S}$.

① (c) With Eq. 7.27 and previously calculated values

$$\epsilon = \frac{\dot{m}_3(e_{f4} - e_{f3})}{\dot{m}_1(e_{f1} - e_{f2})} = \frac{0.88 \times 10^9}{1.28 \times 10^9} = 0.688 \text{ (68.8\%)} \leftarrow$$

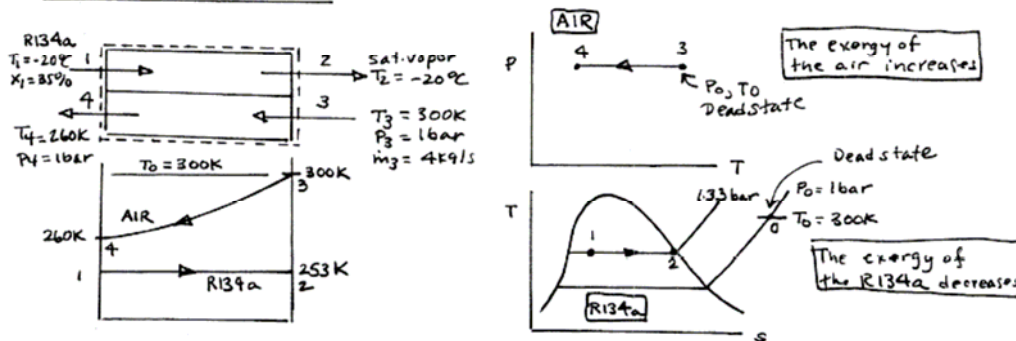
1. In accord with the discussion of Eq. 7.27, both streams are at temperatures above T_0 .

PROBLEM 7.122

KNOWN: Steady-state operating data are provided for a counterflow heat exchanger.

FIND: (a) Sketch the variation of the temperature of each stream with position. Locate T_o . (b) Determine the rate of exergy destruction. (c) Devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The control volume shown in the accompanying sketch is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects can be ignored. 3. Air is modeled as an ideal gas. (Can be verified by reference to the compressibility chart.) 4. $T_o = 300\text{K}$, $P_o = 1\text{bar}$.

ANALYSIS: The refrigerant mass flow rate is required for parts (b), (c). Reducing mass and energy rate balances,

$$0 = \dot{Q}_{cv} + \dot{m}_1[h_1 - h_2] + \dot{m}_3[h_3 - h_4] \Rightarrow \text{with data from Tables A-10, A-22}$$

$$\dot{m}_1 = \frac{\dot{m}_3[h_3 - h_4]}{[h_2 - h_1]} = \frac{(4\text{kg/s})[300.19 - 260.09]\text{kJ/kg}}{[255.31 - 98.13]\text{kJ/kg}} = 1.17\text{ kg/s}$$

$\leftarrow [4.26 + 0.35(2105)]$

(b) The rate of exergy destruction can be determined by reducing an exergy rate balance:

$$0 = \sum \left[\dot{Q}_j \left(1 - \frac{T_o}{T_j} \right) \right] - \dot{Q}_{cv} + \dot{m}_1[e_{f1} - e_{f2}] + \dot{m}_3[e_{f3} - e_{f4}] - \dot{E}_d$$

$$\Rightarrow \dot{E}_d = \dot{m}_1[(h_1 - h_2) - T_o(s_1 - s_2)] + \dot{m}_3[h_3 - h_4 - T_o(s_3 - s_4)]$$

$$= \dot{m}_1[(h_1 - h_2) - T_o(s_1 - s_2)] + \dot{m}_3[h_3 - h_4 - T_o(s_3 - s_4 - R \ln(P_3/P_4))]$$

$$= 1.17[(98.13 - 255.31) - 300(0.3914 - 0.9332)] + 4[300.19 - 260.09 - 300(1.70203 - 1.55848)]$$

$$= [(+29.67) + (-11.86)] \left(\frac{\text{kJ}}{\text{s}} \right) \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right| = 17.8\text{ kW}$$

(c) Since the air is brought at constant pressure from the dead state to a lower temperature, its exergy increases in the process. The state of the R134a is brought closer to the dead state, so its exergy decreases. These points are confirmed by the calculations of part (b) that show

$$\dot{m}_3(e_{f4} - e_{f3}) = +11.86\text{ kW}, \quad \dot{m}_1(e_{f1} - e_{f2}) = 29.67\text{ kW}$$

Thus, the colder refrigerant provides the exergy that is either transferred to the air or destroyed by irreversibilities within the control volume.

$$\Rightarrow \epsilon = \frac{\dot{m}_3(e_{f4} - e_{f3})}{\dot{m}_1(e_{f1} - e_{f2})} = \frac{11.86}{29.67} = 0.403 \quad (40.3\%)$$

4. Since the streams are each below T_o , this expression is formulated differently than Eq. 7.27, which regards the cold stream as receiving exergy from the hot stream. When heat transfer occurs below T_o , as in the present case, the accompanying exergy transfer is oppositely directed. This can be seen from study of Eq. 7.12. In the present case, heat transfer occurs from the warmer air to the cooler refrigerant. Still, exergy transfer is from the refrigerant. Further, the exergy transferred from the refrigerant is accounted for by the exergy increase of the air and by the exergy destroyed within the heat exchanger owing to spontaneous heat transfer.

PROBLEM 7.123

Ammonia enters a counterflow heat exchanger at -20°C , with a quality of 35%, and exits as saturated vapor at -20°C . Air at 300 K , 1 bar enters the heat exchanger in a separate stream with a mass flow rate of 240 kg/min and exits at 285 K , 1 bar . The heat exchanger is at steady state. There is no appreciable heat transfer between the heat exchanger and its surroundings. Neglect the effects of motion and gravity and let $T_0 = 300\text{ K}$, $p_0 = 1\text{ bar}$.

- As in Fig. E7.6, sketch the variation with position of the temperature of each stream. Locate T_0 on the sketch.
- Determine the mass flow rate of the ammonia, in kg/s .
- Devise and evaluate an exergetic efficiency for the heat exchanger.

ENGR MODEL:

- The control volume shown in the sketch is at steady state.
- Stray heat transfer and the effects of motion and gravity can be ignored.
- The air is modeled as an ideal gas. (Can be verified by reference to the compressibility chart.)
- $T_0 = 300\text{ K}$, $p_0 = 1\text{ bar}$.

ANALYSIS: (b) Reducing an energy rate balance,

$$0 = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} + \dot{m}_r(h_1 - h_2) + \dot{m}_a(h_3 - h_4)$$

$$\Rightarrow \dot{m}_r = \dot{m}_a \left[\frac{h_3 - h_4}{h_2 - h_1} \right]$$

$$= 240 \frac{\text{kg}}{\text{min}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left[\frac{300.19 - 285.14}{863.9} \right]$$

$$= 0.0697 \frac{\text{kg}}{\text{s}} \quad \leftarrow (b)$$

(c) An exergy rate balance reads,

$$0 = \dot{E}_q - \dot{E}_d + \dot{m}_r(e_{f1} - e_{f2}) + \dot{m}_a(e_{f3} - e_{f4}) - \dot{E}_d$$

$$\Rightarrow \dot{m}_r(e_{f1} - e_{f2}) = \dot{m}_a(e_{f4} - e_{f3}) + \dot{E}_d$$

$$\textcircled{1} \Rightarrow e = \frac{\dot{m}_a(e_{f4} - e_{f3})}{\dot{m}_r(e_{f1} - e_{f2})} = \frac{\dot{m}_a[(h_4 - h_3) - T_0(s_4 - s_3)]}{\dot{m}_r[(h_1 - h_2) - T_0(s_1 - s_2)]} = \frac{\dot{m}_a[(h_4 - h_3) - T_0(s_4 - s_3 - R \ln p_4/p_3)]}{\dot{m}_r[(h_1 - h_2) - T_0(s_1 - s_2)]}$$

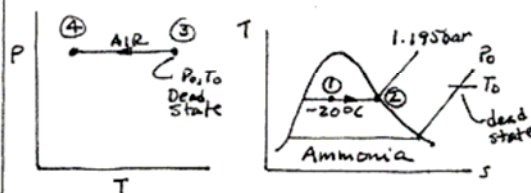
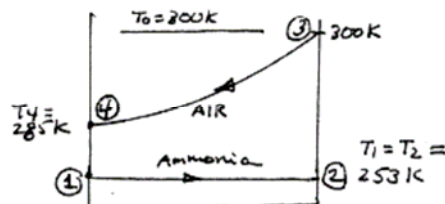
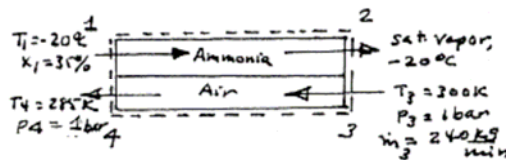
$$\textcircled{2} = \frac{(4 \text{ kg/s})[285.14 - 300.19 - 300(1.65055 - 1.70203)] \text{ kJ/kg}}{(0.0697 \frac{\text{kg}}{\text{s}})[-863.9 - 300(-3.4126)] \text{ kJ/kg}} = \frac{1.576 \text{ kJ/s}}{11.144 \text{ kJ/s}} = 0.141 \quad (14.1\%)$$

- Since the streams are each below T_0 , this expression is formulated differently than Eq. 7.27, which regards the cold stream as receiving exergy from the hot stream.

When heat transfer occurs at temperatures below T_0 , as in the present case, the accompanying exergy transfer is oppositely directed. This can be seen by study of Eq. 7.12. In the present case, heat transfer occurs from the warmer air to the cooler refrigerant. Still, exergy transfer is from the refrigerant. Further, the exergy transferred from the refrigerant is accounted for by the exergy increase of the air and by the exergy destroyed within the heat exchanger owing to spontaneous heat transfer.

- For a counterflow heat exchanger this is a very low exergetic efficiency. The considerable exergy yielded by the ammonia is not well utilized.

SCHEMATIC & GIVEN DATA:



Since the air is brought from the dead state to a lower temperature, its exergy increases in the process. The state of the ammonia is brought closer to the dead state, so its exergy decreases.

From Table A-22,

$$h_3 = 300.19 \frac{\text{kJ}}{\text{kg}}, \quad s_3^0 = 1.70203 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$h_4 = 285.14 \frac{\text{kJ}}{\text{kg}}, \quad s_4^0 = 1.65055 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

From Table A-13,

$$h_2 - h_1 = h_g - [h_f + x_1(h_g - h_f)] = (1 - x_1)(h_g - h_f)$$

$$= 0.65(1329.10) = 863.9 \text{ kJ/kg}$$

$$s_2 - s_1 = s_g - [s_f + x_1(s_g - s_f)] = (1 - x_1)(s_g - s_f)$$

$$= 0.65(5.2502) = 3.4126 \text{ kJ/kg} \cdot \text{K}$$

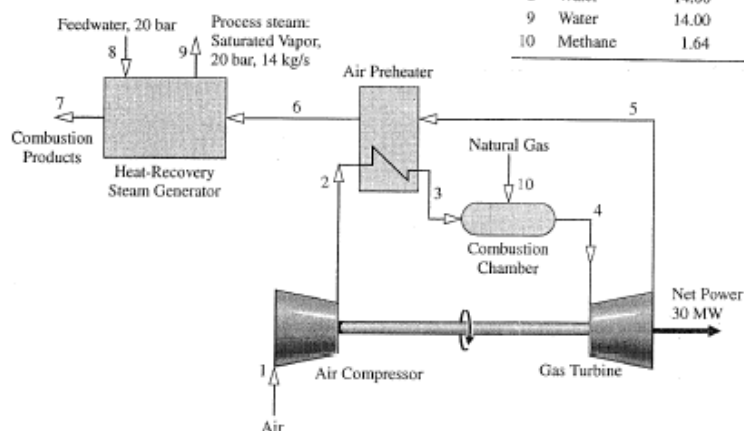
PROBLEM 7.125

Figure P7.125 shows a cogeneration system producing two useful products: net power and process steam. The accompanying table provides steady-state mass flow rate, temperature, pressure, and flow exergy data at the 10 numbered states on the figure. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 298.15 \text{ K}$, $p_0 = 1.013 \text{ bar}$. Determine, in MW,

- the net rate exergy is carried out with the process steam, $(\dot{E}_{f9} - \dot{E}_{f8})$.
- the net rate exergy is carried out with the combustion products, $(\dot{E}_{f7} - \dot{E}_{f1})$.
- the rates of exergy destruction in the air preheater, heat-recovery steam generator, and the combustion chamber.

Devise and evaluate an exergetic efficiency for the overall cogeneration system.

State	Substance	Mass Flow Rate (kg/s)	Temperature (K)	Pressure (bar)	Flow Exergy Rate, $\dot{E}_f$ (MW)
1	Air	91.28	298.15	1.013	0.00
2	Air	91.28	603.74	10.130	27.54
3	Air	91.28	850.00	9.623	41.94
4	Combustion products	92.92	1520.00	9.142	101.45
5	Combustion products	92.92	1006.16	1.099	38.78
6	Combustion products	92.92	779.78	1.066	21.75
7	Combustion products	92.92	426.90	1.013	2.77
8	Water	14.00	298.15	20.000	0.06
9	Water	14.00	485.57	20.000	12.81
10	Methane	1.64	298.15	12.000	84.99



ENGR. MODEL:

- Consider control volumes enclosing the overall system and, individually, around the preheater, HRSG, and combustion chamber.
- All control volumes are at steady state.
- Ignore stray heat transfer and the effects of motion and gravity.
- $T_0 = 298.15 \text{ K}$, $p_0 = 1.013 \text{ bar}$.
- The net exergy carried out with the combustion products is a loss.

ANALYSIS:

$$(a) (\dot{E}_{f9} - \dot{E}_{f8}) = 12.81 - 0.06 = 12.75 \text{ MW.}$$

$$(b) (\dot{E}_{f7} - \dot{E}_{f1}) = 2.77 - 0.00 = 2.77 \text{ MW}$$

(c) Exergy rate balances with assumption 3:

$$\text{Preheater: } 0 = \sum \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W}_{cv} + \dot{E}_{f2} - \dot{E}_{f3} + \dot{E}_{f5} - \dot{E}_{f6} - \dot{E}_d$$

$$\Rightarrow \dot{E}_d = \dot{E}_{f2} - \dot{E}_{f3} + \dot{E}_{f5} - \dot{E}_{f6}$$

$$= 27.54 - 41.94 + 38.78 - 21.75 = \underline{\underline{2.63 \text{ MW}}}$$

(1) HRSG:

$$\Rightarrow \dot{E}_d = \dot{E}_{f6} - \dot{E}_{f7} + \dot{E}_{f8} - \dot{E}_{f9}$$

$$= 21.75 - 2.77 + 0.06 - 12.81 = \underline{\underline{6.23 \text{ MW}}}$$

Comb. Chamber:

$$\Rightarrow \dot{E}_d = \dot{E}_{f3} + \dot{E}_{f10} - \dot{E}_{f4}$$

$$= 41.94 + 84.99 - 101.45 = \underline{\underline{25.48 \text{ MW}}}$$

Continued on next slide

Problem 7-125 continued

- (c) This is a cogeneration system having two products: the net power developed and the net exergy carried out with the process steam.

In accordance with assumption 5, the net exergy carried out with the combustion products — calculated in part (b) — is regarded here as a loss.

The exergy supplied to the overall system is the exergy provided by the methane.

Thus, in summary

$$\begin{aligned} \textcircled{2} \quad \epsilon &= \frac{\text{exergy products}}{\text{exergy supplied}} = \frac{W_{\text{net}} + (\dot{E}_{f9} - \dot{E}_{f8})}{\dot{E}_{f,10}} \\ &= \frac{30 \text{ MW} + 12.75 \text{ MW}}{84.99 \text{ MW}} = 0.503 \quad (50.3\%) \end{aligned}$$

1. An exergy rate balance for the compressor/turbine gives the rate of exergy destruction in these components:

$$\begin{aligned} \dot{E}_d &= (\dot{E}_{f1} - \dot{E}_{f2}) + (\dot{E}_{f4} - \dot{E}_{f5}) - W_{\text{net}} \\ &= (0 - 27.54) + (101.45 - 38.78) - 30 = 5.13 \text{ MW} \end{aligned}$$

2. An exergy accounting for the cogeneration system reads,

① Exergy supplied by the fuel: 84.99 MW

② Disposition of the exergy supplied:

✓ Products			
Power Developed	30.00 MW	35.3%	} 50.3%
Steam	12.75 MW	15.0%	
✓ Loss associated with combustion products	2.77 MW	3.3%	
✓ Exergy Destruction			
— Preheater	2.63 MW	3.1%	
— HRSG	6.23 MW	7.3%	
— Combustion Chamber	25.48 MW	30.0%	←
— Compressor/turbine	5.13 MW	6.0%	
	84.99 MW		

Note that the most significant source of exergy destruction is combustion

PROBLEM 7.126

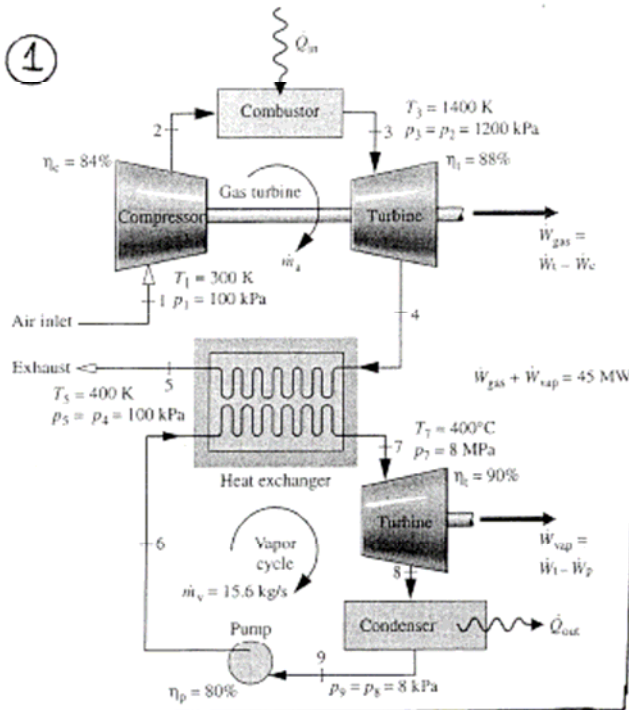
Figure P7.126 shows a combined gas turbine-vapor power plant operating at steady state. The gas turbine is numbered 1-5. The vapor power plant is numbered 6-9. The accompanying table gives data at these numbered states. The total net power output is 45 MW and the mass flow rate of the water flowing through the vapor power plant is 15.6 kg/s. The ideal gas model applies to the air. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 300$ K, $p_0 = 100$ kPa. Determine

- the mass flow rate of the air flowing through the gas turbine, in kg/s.
- the net rate exergy is carried out with the exhaust air stream, $(\dot{E}_5 - \dot{E}_{f1})$, in MW.
- the rate of exergy destruction in the compressor and pump, each in MW.
- the net rate of exergy increase of the air flowing through the combustor, $(\dot{E}_{13} - \dot{E}_{12})$, in MW.

Devise and evaluate an exergetic efficiency for the overall combined power plant.

Gas Turbine ^a			Vapor Cycle		
State	h (kJ/kg)	s^0 (kJ/kg · K)	State	h (kJ/kg)	s (kJ/kg · K)
1	300.19	1.7020	6	183.96	0.5975
2	669.79	2.5088	7	3138.30	6.3634
3	1515.42	3.3620	8	2104.74	6.7282
4	858.02	2.7620	9	173.88	0.5926
5	400.98	1.9919			

^a s^0 is the variable appearing in Eq. 6.20a and Table A-22.



ENGR. MODEL:

- Control volumes at steady state enclose principal components and the overall system.
- Stray heat transfer and the effects of motion and gravity can be ignored.
- $T_0 = 300$ K, $p_0 = 100$ kPa.
- The air is modeled as an ideal gas.

ANALYSIS:

- An energy rate balance for the heat exchanger reduces to

$$0 = \dot{Q}_{ev} \cdot \dot{m}_v + \dot{m}_a (h_4 - h_7) + \dot{m}_w (h_6 - h_7)$$

$$\Rightarrow \dot{m}_a = \dot{m}_v \left[\frac{h_7 - h_6}{h_4 - h_5} \right]$$

$$= (15.6 \frac{\text{kg}}{\text{s}}) \left[\frac{3138.3 - 183.96}{858.02 - 400.98} \right]$$

$$= 100.84 \frac{\text{kg}}{\text{s}} \quad \leftarrow \text{ca)}$$

$$\begin{aligned}
 (b) \quad (\dot{E}_5 - \dot{E}_{f1}) &= \dot{m}_a [h_5 - h_1 - T_0 (s_5 - s_1)] \\
 &= \dot{m}_a [h_5 - h_1 - T_0 (s_5^0 - s_1^0 - R \ln \frac{p_5}{p_1})] \\
 &= (100.84 \frac{\text{kg}}{\text{s}}) [400.98 - 300.19 - 300 (1.9919 - 1.7020)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| \\
 &= 1.39 \text{ MW}
 \end{aligned}$$

Continued on next slide

Problem 7-126 continued

(c) Rate of exergy destruction.

Compressor:

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \quad , \quad \dot{E}_d = T_0 \dot{\sigma}_{cv}$$

$$\dot{E}_d = \dot{m}_a T_0 (s_2 - s_1) = \dot{m}_a T_0 \left(s^0(T_2) - s^0(T_1) - R \ln \frac{P_2}{P_1} \right)$$

$$= (100.84 \frac{\text{kg}}{\text{s}}) (300 \text{ K}) \left[2.5887 - 1.7020 - \frac{8.314}{28.97} \ln \frac{1200}{100} \right] \frac{\text{kJ}}{\text{K}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right|$$

$$= \underline{\underline{2.83 \text{ MW}}}$$

Pump:

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = T_0 \dot{m}_r (s_6 - s_9)$$

$$= (300 \text{ K}) (15.6 \text{ kg/s}) (0.5975 - 0.5926) \frac{\text{kJ}}{\text{K}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right|$$

$$= \underline{\underline{0.02 \text{ MW}}}$$

(d)

$$\dot{E}_{f3} - \dot{E}_{f2} = \dot{m}_a \left[h_3 - h_2 - T_0 (s_3 - s_2) \right]$$

$$= (100.84 \frac{\text{kg}}{\text{s}}) \left[(1515.42 - 669.79) - 300 (7.7620 - 2.3285) \right] \frac{\text{kJ}}{\text{K}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right|$$

$$= \underline{\underline{59.46 \text{ MW}}}$$

(e) Although exergy is carried from the overall Combined system via the exhaust stream (part (b)) and via the cooling water, they are assumed to be losses here. The product of the overall system is then the net power developed.

The exergy provided to the overall system is obtained in part (d). Thus, the exergetic efficiency is

$$\epsilon = \frac{\dot{W}_{net}}{\dot{E}_{f3} - \dot{E}_{f2}} = \frac{45 \text{ MW}}{59.46 \text{ MW}} = 0.757 \text{ (75.7\%)}$$

1. See Sec. 9.10 for further discussion of the combined gas turbine-vapor cycle.

PROBLEM 7.127

KNOWN: An expression is provided for the total cost rate $\dot{C}$ of a device as a function of pressure drop, $P_1 - P_2$.

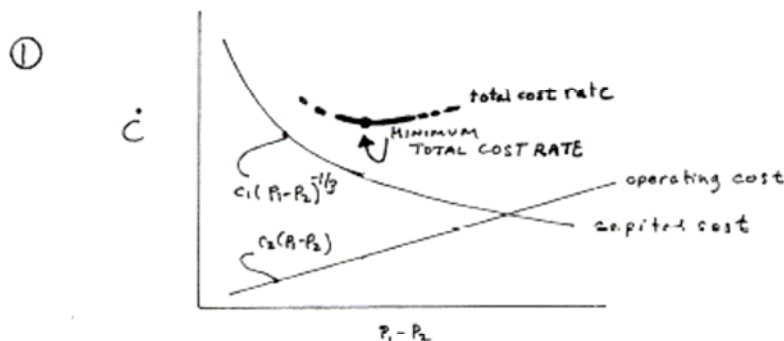
FIND: (a) sketch $\dot{C}$ versus $(P_1 - P_2)$. (b) At the point of minimum total cost, determine the percent contributions of the capital and operating cost rates.

ENGR. MODEL: The total cost rate is given by

$$\dot{C} = \underbrace{C_1 (P_1 - P_2)^{-1/3}}_{\text{Capital Cost}} + \underbrace{C_2 (P_1 - P_2)}_{\text{operating Cost}} \quad (1)$$

where C_1 and C_2 are constants.

ANALYSIS: (a) sketch of Eq. (1)



$$(b) \frac{\partial \dot{C}}{\partial (P_1 - P_2)} = -\frac{1}{3} C_1 (P_1 - P_2)^{-4/3} + C_2 = 0 \Rightarrow \frac{1}{3} C_1 (P_1 - P_2)^{-4/3} = C_2$$

or, on rearrangement, at the point of minimum total cost

$$\underbrace{C_1 (P_1 - P_2)^{-1/3}}_{\text{Cap. cost}} = \underbrace{3 C_2 (P_1 - P_2)}_{\text{op. cost}}$$

In other words, at this point the capital cost is three times the operating cost. Thus, at the point of minimum total cost, the percent contributions to the total cost rate are

$$\% \text{ operating cost} = 25\%$$

$$\% \text{ capital cost} = 75\%$$

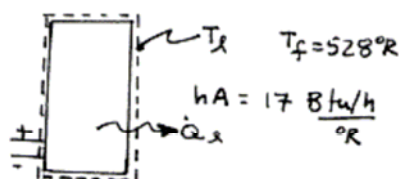
1. Compare with Figure 7.13.

PROBLEM 7.128

KNOWN: Heat transfer and cost data are provided for an electric water heater.

FIND: (a) Determine the cost of the heat loss when the outer surface temperature is 535°R . (b) Plot cost of heat loss versus outer surface temperature ranging from 535 to 570°R .

SCHEMATIC & GIVEN DATA:



ENER. MODEL: 1. For the system shown in the accompanying schematic, heat transfer occurs only at T_s .
2. Electricity is evaluated at 8 cents/kW·h.
3. $T_0 = 528^\circ\text{R}$.

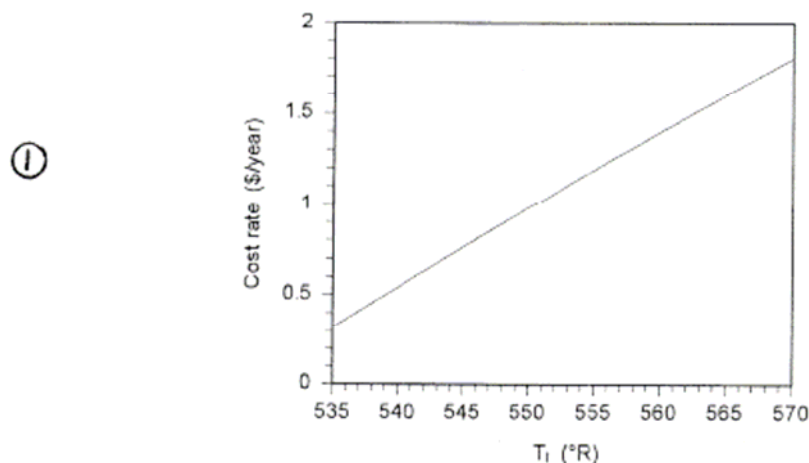
ANALYSIS: Letting $\dot{Q}_s$ denote the rate of heat loss from the water heater,
 $\dot{Q}_s = hA[T_s - T_f] \frac{\text{Btu}}{\text{h}}$. Following the discussion of Sec. 7.6.1 concerning costing of heat loss, and letting electricity play the role of the fuel, we get

$$\dot{\$}_s = c_e \left[1 - \frac{T_0}{T_s} \right] \dot{Q}_s = c_e \left[1 - \frac{T_0}{T_s} \right] (hA)(T_s - T_f) \quad (1)$$

(a) When $T_s = 535^\circ\text{R}$, the annual cost ($24 \times 365 = 8760 \text{ h}$) is

$$\dot{\$}_s = \left(\frac{\$.08}{\text{kW}\cdot\text{h}} \right) \left[1 - \frac{528}{535} \right] \left(17 \frac{\text{Btu}}{\text{h}\cdot^\circ\text{R}} \right) (7^\circ\text{R}) \left| \frac{8760 \text{ h}}{\text{year}} \right| \left| \frac{1 \text{ kW}}{3413 \text{ Btu/h}} \right| = \$.32/\text{year}$$

(b) Plot



1. When considered over the several years life of such a water heater, the use of an insulating blanket to reduce the outer surface temperature can be cost effective for the owner.

PROBLEM 7.129

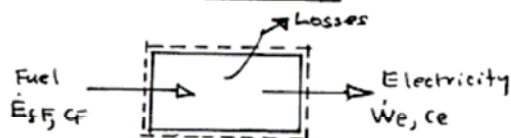
KNOWN: Cost rate data are provided for a system operating at steady state. The system generates electricity at the rate $\dot{W}_e$. The total cost rate is

$$\dot{C} = \underbrace{c_F \dot{E}_{ff}}_{\text{fuel cost}} + \underbrace{c \left(\frac{\epsilon}{1-\epsilon} \right) \dot{W}_e}_{\text{owning and operating cost}} \quad (1)$$

where c_F is the unit cost based on exergy, $\epsilon = \dot{W}_e / \dot{E}_{ff}$, and c is a constant.

FIND: (a) Derive an expression for c_e , the unit cost of electricity based on $\dot{W}_e$, in terms of ϵ , c_e/c_F , and c/c_F . (b) For fixed c/c_F , derive an expression for ϵ corresponding to minimum c_e/c_F . (c) Plot c_e/c_F versus ϵ for selected values of c/c_F . For each c/c_F , evaluate the minimum value for c_e/c_F and the corresponding ϵ .

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system shown in the figure is at steady state.
2. In Eq. (1), c and c_F are constants.
3. Eq. (1) accounts for all significant contributors to the total cost rate.

ANALYSIS: (a) Beginning with Eq. (1), use $\dot{E}_{ff} = \dot{W}_e / \epsilon$ to obtain

$$\dot{C} = c_F \left[\frac{\dot{W}_e}{\epsilon} \right] + c \left[\frac{\epsilon}{1-\epsilon} \right] \dot{W}_e = \left(\frac{c_F}{\epsilon} + c \left[\frac{\epsilon}{1-\epsilon} \right] \right) \dot{W}_e$$

Thus, expressing $\dot{C}$ in terms of the unit cost of electricity, c_e , as $\dot{C} = c_e \dot{W}_e$, we get

$$c_e = \frac{c_F}{\epsilon} + c \left(\frac{\epsilon}{1-\epsilon} \right)$$

or

$$\frac{c_e}{c_F} = \frac{1}{\epsilon} + \underbrace{\left(\frac{c}{c_F} \right)}_{\epsilon = \gamma} \left(\frac{\epsilon}{1-\epsilon} \right) = \frac{1}{\epsilon} + \gamma \left(\frac{\epsilon}{1-\epsilon} \right) \quad (a)$$

(b) For fixed $\gamma (= c/c_F)$, differentiation of Eq. (a) with respect to ϵ gives

$$\begin{aligned} \frac{\partial (c_e/c_F)}{\partial \epsilon} &= -\frac{1}{\epsilon^2} + \gamma \left[\frac{1(1-\epsilon) - \epsilon(-1)}{(1-\epsilon)^2} \right] \\ &= -\frac{1}{\epsilon^2} + \gamma \left[\frac{1}{(1-\epsilon)^2} \right] \end{aligned}$$

Setting $\frac{\partial (c_e/c_F)}{\partial \epsilon} = 0$ and solving

$$\frac{1}{\epsilon^2} = \frac{\gamma}{(1-\epsilon)^2} \Rightarrow \left(\frac{1-\epsilon}{\epsilon} \right)^2 = \gamma \Rightarrow \frac{1-\epsilon}{\epsilon} = \sqrt{\gamma} \Rightarrow \epsilon = \frac{1}{1+\sqrt{\gamma}} \quad (b)$$

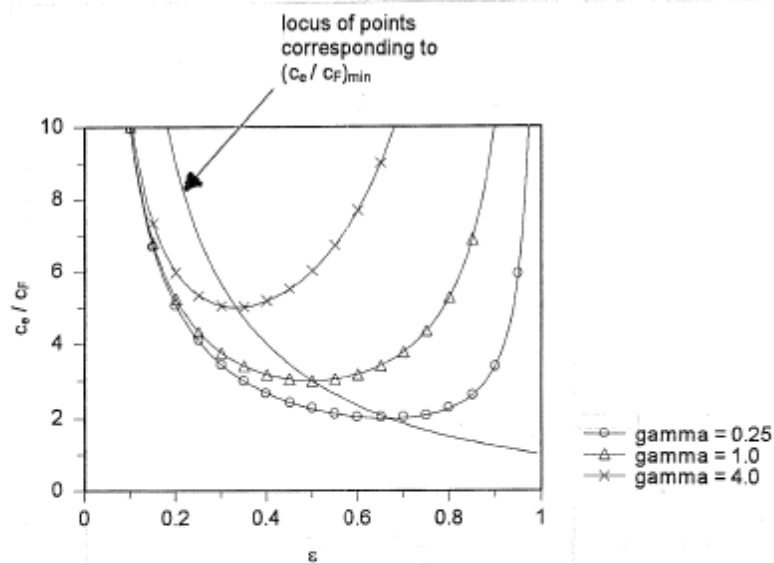
which is the value of ϵ corresponding to a minimum in c_e/c_F when γ is fixed. Using this in the result of part (a)

$$\left(\frac{c_e}{c_F} \right)_{\min} = 1 + 2\sqrt{\gamma}$$

Continued on next slide

Problem 7-129 continued

PLOT :



The following chart lists values of $(C_e / C_f)_{\min}$ and the corresponding values of ϵ for $\gamma = 0.25, 1$, and 4 :

γ	ϵ	$(C_e / C_f)_{\min}$
.25	2/3	2
1	.5	3
4	1/3	5

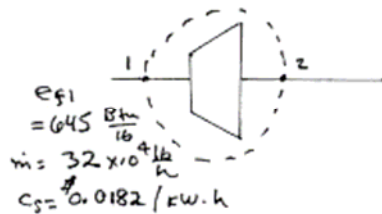
PROBLEM 7.130

KNOWN: Operating and cost data are provided for a turbine operating at steady state.

FIND: (a) Evaluate the unit cost of the power developed.

(b) Evaluate the unit cost based on exergy of the steam entering and exiting the turbine, each in cents per lb of steam flowing.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned}\dot{W}_t &= 18 \times 10^7 \text{ kW.h/yr} \\ \dot{Z} &= 5.0 \times 10^5 \text{ $/yr} \\ \epsilon &= 85\% \\ 8000 \text{ h of operation/yr}\end{aligned}$$

ENGR. MODEL: The control volume shown in the figure is at steady state.

ANALYSIS: (a) In this case, Eq. 7.34c is applicable:

$$\begin{aligned}C_e &= \frac{c_s}{\epsilon} + \frac{\dot{Z}}{\dot{W}_t} = \left(\frac{0.0182 \text{ $/kW.h}}{0.85} \right) + \left(\frac{5.0 \times 10^5 \text{ $/yr}}{18 \times 10^7 \text{ kW.h/yr}} \right) \\ &= (0.02141 + 0.00278) \text{ $/kW.h} = 0.0242 \text{ $/kW.h} = 2.42 \frac{\text{cents}}{\text{kW.h}} \quad (a)\end{aligned}$$

(b) By assumption leading to Eq. 7.34c, the same unit cost based on exergy applies to the steam at the inlet and at the exit: 1.82 cents/kW.h. Thus the unit cost of steam expressed on a unit of mass basis is

$$\left(\text{unit cost of steam, per lb, entering the turbine} \right) = \left(1.82 \frac{\text{cents}}{\text{kW.h}} \right) \left[\left(645 \frac{\text{Btu}}{\text{lb}} \right) \left| \frac{\text{kW.h}}{3413 \text{ Btu}} \right| \right] = 0.344 \frac{\text{cents}}{\text{lb}}$$

The same approach is used at the exit, but the value of e_{f2} is required.

$$\text{using } \epsilon = \frac{\dot{W}_t}{\dot{m}(e_{f2} - e_{f1})} \Rightarrow e_{f2} = e_{f1} - \frac{\dot{W}_t}{\dot{m}\epsilon}$$

$$\text{or } e_{f2} = 645 \frac{\text{Btu}}{\text{lb}} - \frac{(18 \times 10^7 \text{ kW.h/yr}) \left| \frac{3413 \text{ Btu}}{\text{kW.h}} \right|}{(32 \times 10^4 \frac{\text{lb}}{\text{h}}) \left(\frac{8000 \text{ h}}{\text{yr}} \right) (0.85)} = 363 \frac{\text{Btu}}{\text{lb}}$$

Then

$$\left(\text{unit cost of steam, per lb, exiting the turbine} \right) = \left(1.82 \frac{\text{cents}}{\text{kW.h}} \right) \left[\left(363 \frac{\text{Btu}}{\text{lb}} \right) \left| \frac{\text{kW.h}}{3413 \text{ Btu}} \right| \right] = 0.194 \frac{\text{cents}}{\text{lb}}$$

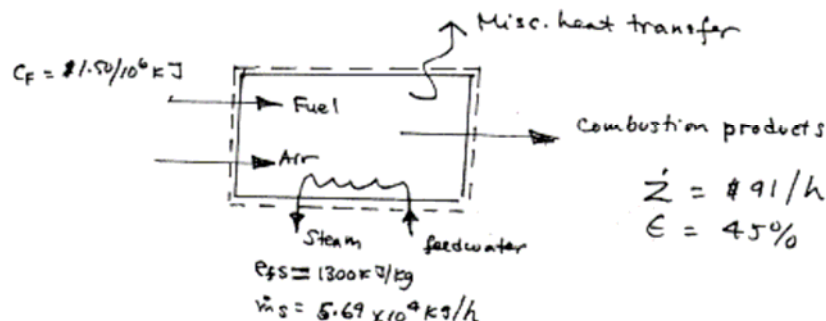
PROBLEM 7.131

KNOWN: Operating and cost data are provided for the boiler of Fig. P7.131.

FIND: (a) Develop an expression for the unit cost, based on exergy, of the steam exiting the boiler.

(b) Using the result of (a), determine the unit cost of the exiting steam in cents per kg of steam exiting the boiler.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the figure is at steady state.

(2) The cost rates of the feedwater, combustion products, and heat transfer are ignored.

ANALYSIS: (a) With assumption (2), Equation 7.32b is applicable:

$$c_s = c_F \left(\frac{\dot{E}_{FF}}{\dot{E}_{fs}} \right) + \frac{\dot{Z}}{\dot{E}_{fs}}. \text{ With } E = \frac{\dot{E}_{fs}}{\dot{E}_{FF}}, \quad c_s = \frac{c_F}{E} + \frac{\dot{Z}}{\dot{E}_{fs}} \quad (a)$$

(b) Inserting values

$$c_s = \frac{(\$1.50/10^6 \text{ kJ})}{0.45} + \frac{91 (\$/h)}{(5.69 \times 10^4 \text{ kg/h}) (1300 \text{ kJ/kg})} = \left[\frac{(3.33 + 1.23) (\$/\text{kJ})}{10^6} \right] \left| \frac{3600 \text{ kJ}}{\text{kw} \cdot \text{h}} \right| = 0.0164 \text{ \$/kw} \cdot \text{h}$$

This unit cost is on a per kw.h of exergy basis. It can be expressed on a per unit mass of steam basis as follows:

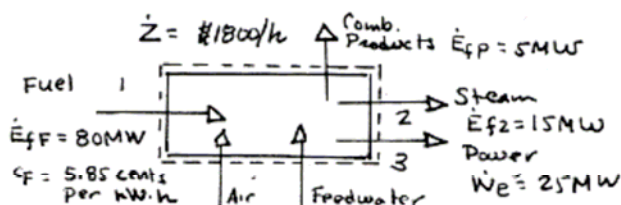
$$\left(\text{unit cost of steam per kg} \right) = \left(0.0164 \frac{\$}{\text{kw} \cdot \text{h}} \right) \left(1300 \frac{\text{kJ}}{\text{kg}} \right) \left| \frac{\text{kw} \cdot \text{h}}{3600 \text{ kJ}} \right| = 0.0059 \text{ \$/kg} = 0.59 \frac{\text{cents}}{\text{kg}} \quad (b)$$

PROBLEM 7.132

known: Steady-state operating and cost data are provided for a cogeneration system.

FIND: (a) Determine the rate of exergy destruction. (b) Devise and evaluate an exergetic efficiency. (c) Evaluate the unit cost, and the cost rates of the power and steam produced, each on an exergy basis.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. Feedwater and combustion air enter with negligible exergy and cost.
3. Combustion products exit with negligible cost.
4. Heat transfer with the surroundings can be ignored.
5. The exiting steam and power each have the same unit cost based on exergy.

ANALYSIS:

(a) At steady state, the exergy rate balance reduces with assumptions 2 and 4 to give

$$\dot{E}_d = \dot{E}_{fF} - \dot{E}_{fP} - \dot{E}_{f2} - \dot{W}_e$$

$$= 80 - 5 - 15 - 25 = 35 \text{ MW}$$

(b) Regarding the exiting steam and power as the product of the system and the combustion products as a loss

$$\epsilon = \frac{\dot{E}_{f2} + \dot{W}_e}{\dot{E}_{fF}} = \frac{15 + 25}{80} = 0.5 \text{ (50\%)}$$

(c) With assumptions 2-4, a cost rate balance reads

$$\dot{C}_2 + \dot{C}_3 = \dot{C}_1 + \dot{Z} \quad \text{Then with assumption 5, } C \dot{E}_{f2} + C \dot{W}_e = C_F \dot{E}_{fF} + \dot{Z}$$

Solving for the unit cost of the steam and the power

$$C = \frac{C_F \dot{E}_{fF} + \dot{Z}}{\dot{E}_{f2} + \dot{W}_e} = C_F \left[\frac{\dot{E}_{fF}}{\dot{E}_{f2} + \dot{W}_e} \right] + \frac{\dot{Z}}{[\dot{E}_{f2} + \dot{W}_e]}$$

$$= \frac{C_F}{\epsilon} + \frac{\dot{Z}}{[\dot{E}_{f2} + \dot{W}_e]}$$

$$= \left[\frac{5.85 \text{ ¢/kW}\cdot\text{h}}{0.5} \right] + \frac{\$1800/\text{h}}{[15 + 25] \text{ MW}} \left| \frac{1 \text{ MW}}{10^3 \text{ kW}} \right| \left| \frac{10^2 \text{ ¢}}{1 \$} \right|$$

$$= [11.7 + 4.5] \frac{\text{cents}}{\text{kW}\cdot\text{h}} = 16.2 \frac{\text{cents}}{\text{kW}\cdot\text{h}}$$

The cost rates are

$$\dot{C}_2 = C \dot{E}_{f2} = (16.2 \frac{\text{cents}}{\text{kW}\cdot\text{h}}) (15 \text{ MW}) \left| \frac{10^3 \text{ kW}}{1 \text{ MW}} \right| \left| \frac{1 \$}{10^2 \text{ ¢}} \right| = \$2430/\text{h}$$

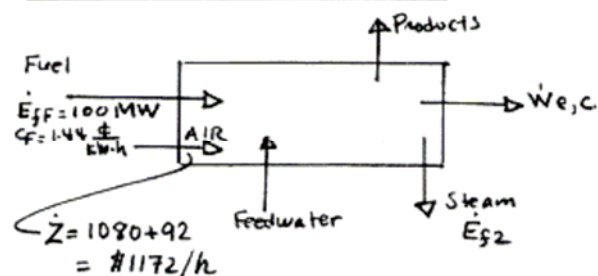
$$\dot{C}_3 = C \dot{W}_e = (16.2 \frac{\text{cents}}{\text{kW}\cdot\text{h}}) (25 \text{ MW}) \left| \frac{10^3 \text{ kW}}{1 \text{ MW}} \right| \left| \frac{1 \$}{10^2 \text{ ¢}} \right| = \$4050/\text{h}$$

PROBLEM 7.133

KNOWN: Steady-state operating and cost data are provided in Example 7.10 for a cogeneration system.

FIND: Taking an overall control volume and regarding the unit cost based on exergy is the same for the power and steam produced, evaluate the unit cost and compare with the values obtained in Example 7.10

SCHEMATIC & GIVEN DATA:



ANALYSIS: A cost rate balance reads

$$C \dot{W}_e + C \dot{E}_{f2} = C_F \dot{E}_{fF} + \dot{Z}$$

$$\Rightarrow C = \frac{C_F \dot{E}_{fF} + \dot{Z}}{\dot{W}_e + \dot{E}_{f2}}$$

From the solution to Example 7.10, $\dot{W}_e = 12.75 \text{ MW}$, $\dot{E}_{f2} = 20.67 \text{ MW}$

$$C = \frac{(1.44 \text{ $/kW-h})(100 \text{ MW})}{(12.75 + 20.67) \text{ MW}} + \frac{\$1172/\text{h}}{(12.75 + 20.67) \text{ MW}} \left| \frac{1 \text{ MW}}{10^3 \text{ kW}} \right| \left| \frac{10^2 \text{ $}}{\text{h}} \right|$$

$$= (4.309 + 3.507) = 7.82 \text{ cents/kW-h}$$

From the solution to Example 7.10, the unit cost for power is 8.81 cents/kW-h and for process steam 7.2 cents/kW-h. When the overall control volume is considered in the present case, the unit cost of the steam is higher because it now bears a part of the costs associated with the turbine in the analysis of Example 7.10. On the other hand, the unit cost of the power is lower because it is no longer required to bear the full burden of these costs. The approach of Example 7.10 is preferred because it takes into account important information related to the actual production processes.

PROBLEM 7.134

KNOWN: Steady-state operating and cost data are provided for the cogeneration system of Example 7.10.

FIND: Plot versus the pressure of the process steam (a) the power developed, (b) the unit costs of the power and process steam, per kW·h, (c) the unit cost of the process steam, per kg of process steam flowing.

SCHEMATIC & GIVEN DATA: See Figure E 7.10.

$\dot{Z}_t = 7.2 \dot{W}_e \frac{\$}{h}$ (in MW)	$\dot{P}_2$ (bar)	40	30	20	9	5	2	1
	T_2 (°C)	436	398	349	262	205	128	sat.

ENGR. MODEL: See Example 7.10

Case considered in Example 7.10.

ANALYSIS: Following the analysis of Example 7.10

$$\dot{W}_e = \dot{m} (h_1 - h_2) \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| \quad (1)$$

where $\dot{m} = 26.15 \text{ kg/s}$, h_1 is fixed by $P_1 = 50 \text{ bar}$, $T_1 = 466^\circ\text{C}$, h_2 is fixed by P_2, T_2 from the table above.

The unit cost of the process steam is 7.2 cents/kW·h, as in Example 7.10. This value is determined by the boiler analysis together with the assumption underlying Eq. 7.34b. The unit cost of the power is determined by

$$c_e = c_2 \left[\frac{\dot{E}_{f1} - \dot{E}_{f2}}{\dot{W}_e} \right] + \frac{\dot{Z}_t}{\dot{W}_e} \quad (2)$$

where $c_2 = 7.2 \text{ cents/kW·h}$, $\dot{W}_e$ is given by Eq. (1), $\dot{Z}_t = 7.2 \dot{W}_e$, where $\dot{W}_e$ is in MW, and

$$\dot{E}_{f1} - \dot{E}_{f2} = \dot{m} [h_1 - h_2 - T_0 (s_1 - s_2)] \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| \quad (3)$$

To determine the unit cost of the process steam on a unit mass basis

$$c_m = c_2 \left[\frac{\dot{E}_{f2}}{\dot{m}} \right] \left| \frac{10^3 \text{ kW}}{1 \text{ MW}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \quad (4)$$

where $\dot{E}_{f2}$ is found using Eq. (3) and $\dot{E}_{f1} = 35 \text{ MW}$.

The following IT code is used to generate data for each case in the above table. The data were imported to a spreadsheet program to obtain the plots on the next page.

IT Code

```
p1 = 50 // bar
T1 = 466 // °C
p2 = 40 // bar
T2 = 436 // °C
mdot = 26.15 // kg/s
To = 298 // K
```

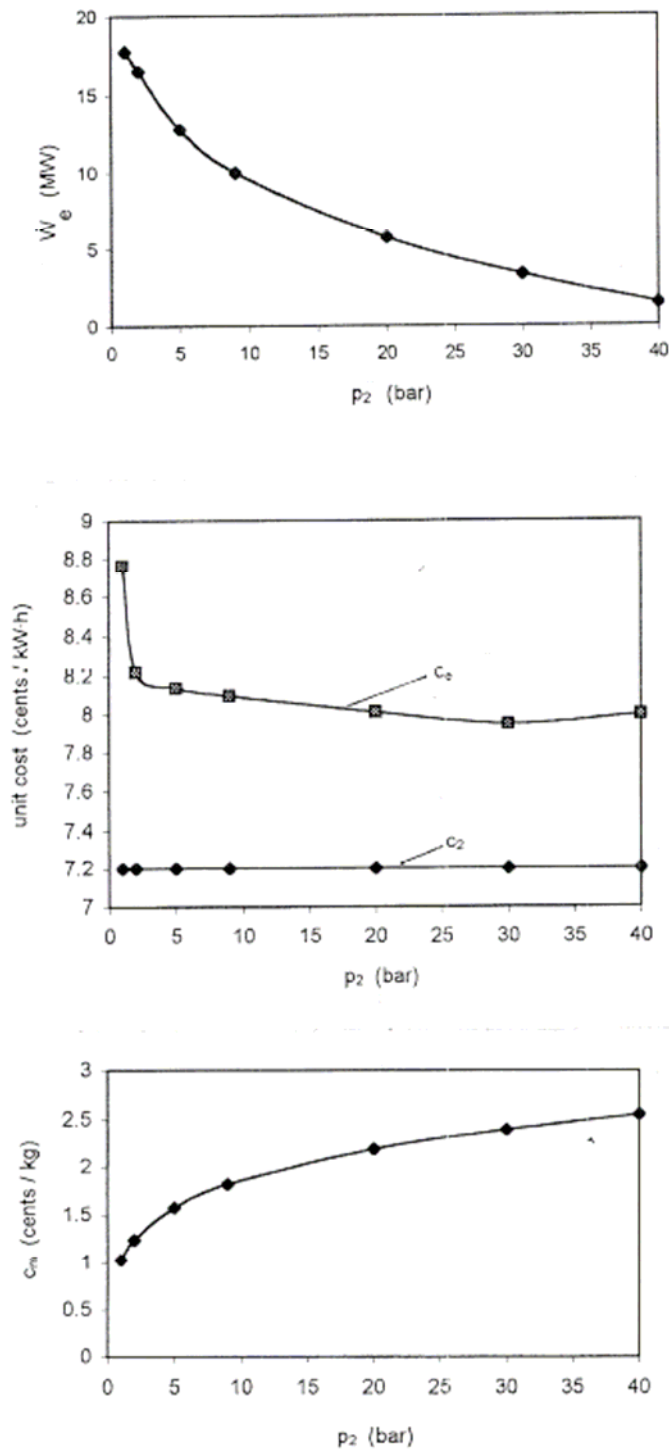
```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
s2 = s_PT("Water/Steam", p2, T2)
s2s = s_Ph("Water/Steam", p2, h2s)
s2s = s1
```

```
Wdote = mdot * (h1 - h2) / 1000
Edotf1 - Edotf2 = mdot * ((h1 - h2) - To * (s1 - s2)) / 1000
Edotf1 = 35
ce = c2 * ((Edotf1 - Edotf2) / Wdote) + (Zdott / Wdote) / 1000
Zdott = 7.2 * Wdote
c2 = 7.2
cm = c2 * (Edotf2 / mdot) * (1000 / 3600)
```

Continued on next slide

Problem 7-134 continued

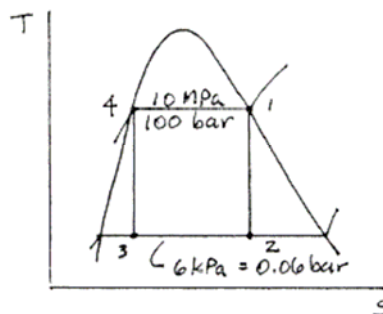
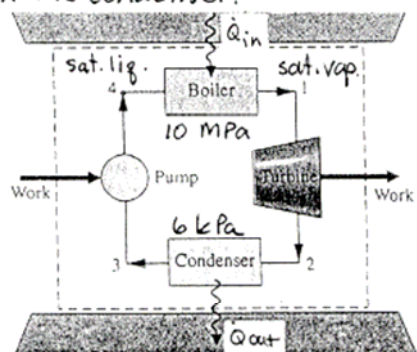
PLOTS:



PROBLEM 8.3

KNOWN: Water is the working fluid in a Carnot vapor power cycle. The states at the boiler and turbine inlets are specified and the condenser pressure is known.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the heat transfer to the working fluid per unit mass passing through the boiler, and (d) the heat transfer from the working fluid passing through the condenser.



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible. (3) The turbine and the pump operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 100 \text{ bar}$, sat. vapor $\Rightarrow h_1 = 2724.7 \text{ kJ/kg}$, $s_1 = 5.6141 \text{ kJ/kg}\cdot\text{K}$

State 2: $p_2 = 0.06 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.6522$, $h_2 = 1727.2 \text{ kJ/kg}$

State 3: $p_3 = 0.06 \text{ bar}$, $s_3 = s_4 = 3.3546 \text{ kJ/kg}\cdot\text{K} \Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.3635$
 $h_3 = 1029.7 \text{ kJ/kg}$

State 4: $p_4 = 100 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 1407.6 \text{ kJ/kg}$

(a) The thermal efficiency of the Carnot cycle is given by Eq. 5.9. With data from Table A-3; $T_H = T_{\text{sat}} @ 100 \text{ bar} = 584.23 \text{ K}$ and $T_C = T_{\text{sat}} @ 0.06 \text{ bar} = 309.31 \text{ K}$. Thus

$$\textcircled{1} \quad \eta_{\text{max}} = 1 - \frac{T_C}{T_H} = 1 - \frac{309.31 \text{ K}}{584.23 \text{ K}} = 0.4706 \text{ (47.06\%)} \leftarrow \eta$$

(b) The back work ratio is

$$\text{bwr} = \frac{(\dot{W}_P/\dot{m})}{(\dot{W}_T/\dot{m})} = \frac{h_4 - h_3}{h_1 - h_2} = \frac{1407.6 - 1029.7}{2724.7 - 1727.2} = 0.3788 \text{ (37.88\%)} \leftarrow \text{bwr}$$

$$\textcircled{2} \text{ (c) For the boiler: } \dot{Q}_{\text{in}}/\dot{m} = h_1 - h_4 = 2724.7 - 1407.6 = 1317.1 \frac{\text{kJ}}{\text{kg}} \leftarrow \frac{\dot{Q}_{\text{in}}}{\dot{m}}$$

$$\text{(d) For the condenser: } \dot{Q}_{\text{out}}/\dot{m} = h_2 - h_3 = 1727.2 - 1029.7 = 697.5 \frac{\text{kJ}}{\text{kg}} \leftarrow \frac{\dot{Q}_{\text{out}}}{\dot{m}}$$

$$1. \text{ Alternatively, } \eta = 1 - \frac{\dot{Q}_{\text{out}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = 1 - \frac{697.5}{1317.1} = 0.4704$$

2. These results allow comparison of the Carnot cycle with the corresponding ideal Rankine cycle of Problem 8.1.

PROBLEM 8.4

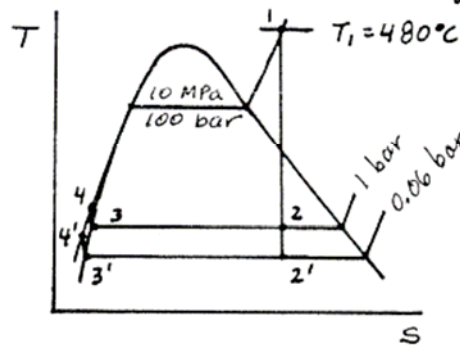
KNOWN: Water is the working fluid in an ideal Rankine cycle with superheated vapor entering the turbine. Data are given in Problem 8.2.

FIND: Plot each of the quantities calculated in Problem 8.2 versus condenser pressure ranging from 6 kPa to 0.1 MPa.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.2.

ENGINEERING MODEL: See solution to Problem 8.2.

ANALYSIS: The procedure for fixing each of the principal states is the same as in Problem 8.2. State 1 is fixed, and states 2, 3, and 4 all vary with condenser pressure, as illustrated on the T-s diagram below



For a sample calculation, see the solution to Problem 8.2.

IT Code:

```

p1 = 100 // bar
T1 = 480 // °C
p2 = 0.06 // bar
mdot = 1
// Turbine
h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2 = s1
h2 = h_Ps("Water", p2, s2)
Wdott = mdot * (h1 - h2)
// Condenser
p3 = p2
x3 = 0
h3 = hsat_Px("Water", p3, x3)
Qdotout = mdot * (h2 - h3)
// Pump
v3 = vsat_Px("Water", p3, x3)
p4 = p1
h4 = h3 + v3 * (p4 - p3) * (10^5 / 10^3)
Wdotp = mdot * (h4 - h3)
// Cycle parameters
Wdotnet = Wdott - Wdotp
eta = Wdotnet / Qdotin
Qdotin = mdot * (h1 - h4)

```

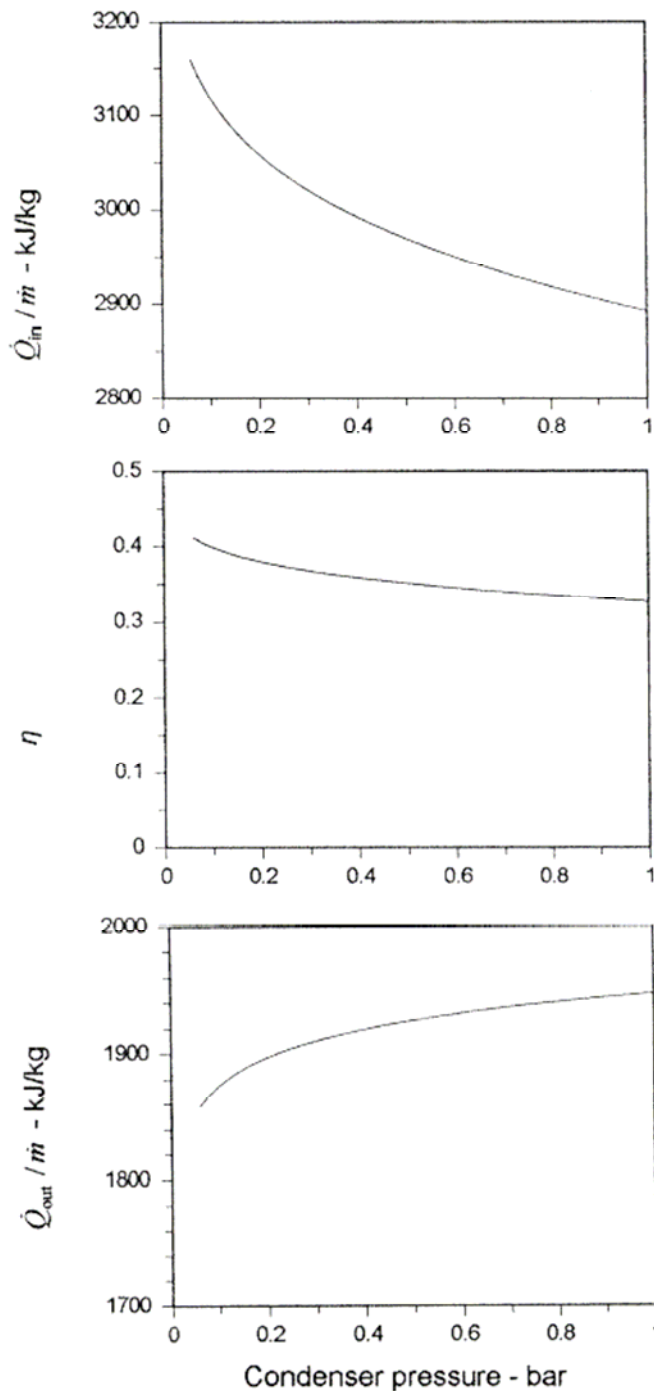
IT Results:

p_2 (bar)	$\dot{Q}_{in} / \dot{m}$	$\dot{Q}_{out} / \dot{m}$	$\dot{W}_t / \dot{m}$	$\dot{W}_p / \dot{m}$	$\dot{W}_{net} / \dot{m}$	η	h_1	h_2	h_3	h_4
0.06	3160	1858	1311	10.06	1301	0.4119	3321	2009	151	161.1
1	2893	1947	955.8	10.33	945.5	0.3269	3321	2365	417.9	428.3

Continued on next slide

Problem 8-4 continued

Plotting for the entire range of condenser pressures



The thermal efficiency decreases as condenser pressure increases. Although less heat input is needed per unit mass of steam flowing, the net work per unit mass decreases as well.

PROBLEM 8.5

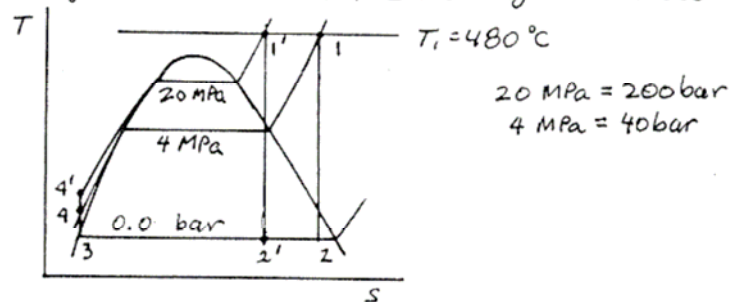
KNOWN: Water is the working fluid in an ideal Rankine cycle with superheated vapor at the turbine inlet. Data are given in Problem 8.2.

FIND: Plot each of the quantities calculated in Problem 8.2 versus steam generator pressure ranging from 4 to 20 MPa while maintaining constant turbine inlet temperature.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.2.

ENGINEERING MODEL: See solution to Problem 8.2.

ANALYSIS: The procedure for fixing each of the principal states is the same as in Problem 8.2. State 3 is fixed, and states 1, 2, and 4 all vary with steam generator pressure, as illustrated on the T-s diagram below



For a sample calculation, see the solution to problem 8.2.

IT Code

```

p1 = 100 // bar
T1 = 480 // deg C
p2 = 0.06 // bar
mdot = 1
// Turbine
h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2 = s1
h2 = h_Ps("Water", p2, s2)
Wdott = mdot * (h1 - h2)
// Condenser
p3 = p2
x3 = 0
h3 = hsat_Px("Water", p3, x3)
Qdotout = mdot * (h2 - h3)
// Pump
v3 = vsat_Px("Water", p3, x3)
p4 = p1
h4 = h3 + v3 * (p4 - p3) * (10^5 / 10^3)
Qdotin = mdot * (h1 - h4)
Wdotp = mdot * (h4 - h3)
Wdotnet = Wdott - Wdotp
eta = Wdotnet / Qdotin
    
```

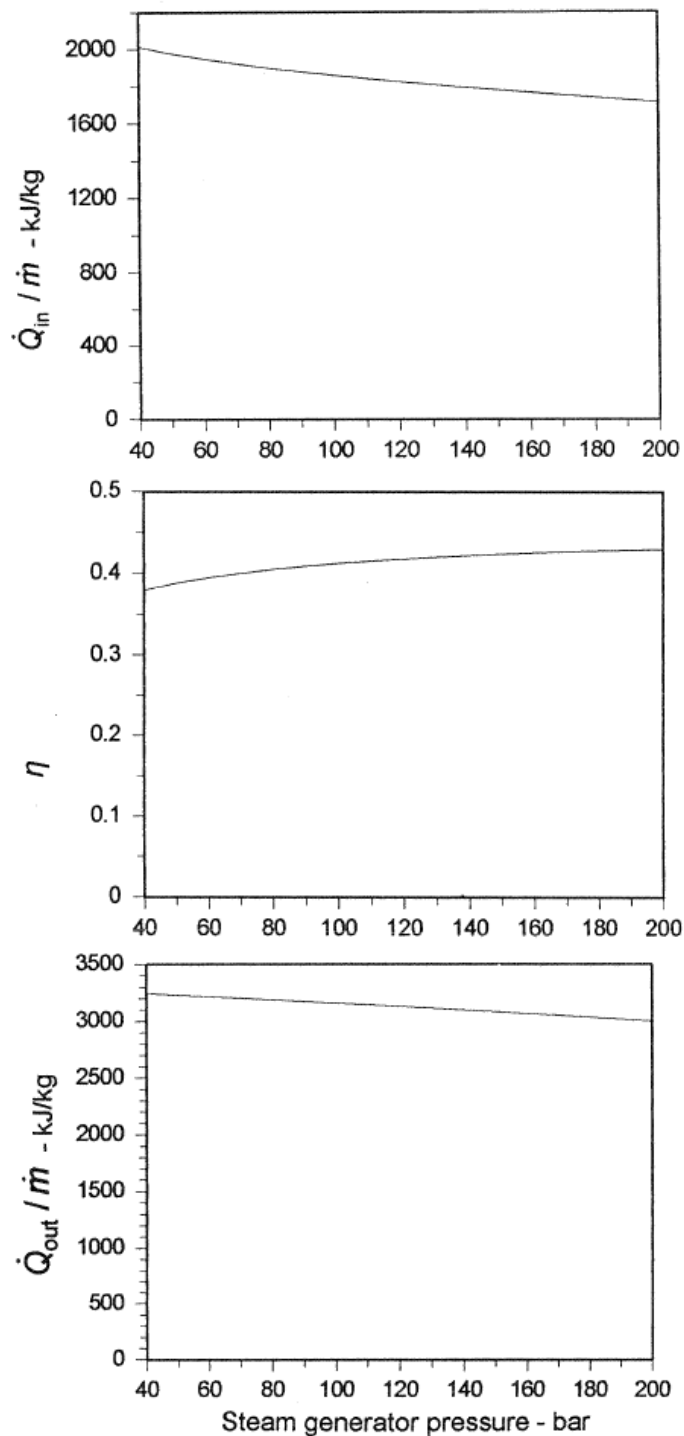
IT Results:

p_1 (bar)	$\dot{Q}_{in}$ / $\dot{m}$	$\dot{Q}_{out}$ / $\dot{m}$	$\dot{W}_t$ / $\dot{m}$	$\dot{W}_p$ / $\dot{m}$	$\dot{W}_{net}$ / $\dot{m}$	η	h_1	h_2	h_3	h_4
40	3244	2014	1234	4.02	1230	0.3792	3399	2165	151	155.1
200	3160	1858	1311	10.06	1301	0.4119	3321	2009	151	161.1

Continued on next slide

Problem 8-5 continued

Plotting for the entire range of steam generator pressures



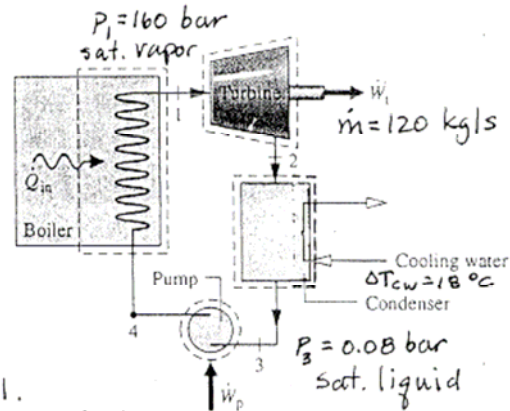
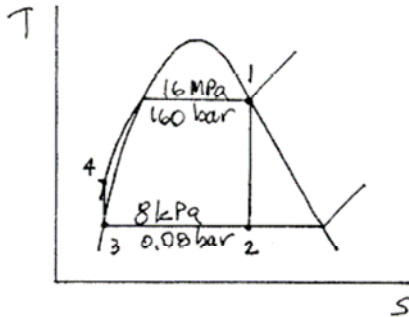
The thermal efficiency increases with increasing steam generator pressure. Although less heat input is needed per unit mass of steam flowing, the net work per unit mass decreases as well.

PROBLEM 8.6

KNOWN: Water is the working fluid in an ideal Rankine cycle. The condenser pressure and the turbine inlet state are specified. The mass flow of steam is given.

FIND: Determine (a) the net power, (b) the rate of heat transfer to the steam passing through the boiler, (c) the thermal efficiency, and (d) the mass flow rate of cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.1.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 160 \text{ bar}$, sat. vapor $\Rightarrow h_1 = 2580.6 \text{ kJ/kg}$, $s_1 = 5.2455 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

State 2: $P_2 = 0.08 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.6091$, $h_2 = 1637.6 \text{ kJ/kg}$

State 3: $P_3 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 173.88 \text{ kJ/kg}$

State 4: $P_4 = 160 \text{ bar}$, $h_4 \approx h_3 + v_3(P_4 - P_3)$
 $= 173.88 \frac{\text{kJ}}{\text{kg}} + (1.0084 \times 10^{-3} \frac{\text{m}^3}{\text{kg}})(160 - 0.08) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$
 $= 173.88 + 16.13 = 190.01 \text{ kJ/kg}$

(a) The net power developed is

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

$$= 120 \frac{\text{kg}}{\text{s}} [(2580.6 - 1637.6) - (190.01 - 173.88)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 1.112 \times 10^5 \text{ kW} \leftarrow \dot{W}_{\text{cycle}}$$

(b) For the steam passing through the boiler

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = (120)(2580.6 - 190.01) = 2.869 \times 10^5 \text{ kW} \leftarrow \dot{Q}_{\text{in}}$$

$$(c) \eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = 0.388 \text{ (38.8\%)} \leftarrow \eta$$

(d) For the control volume enclosing the condenser (assuming $\Delta h_{\text{cw}} = c_w \Delta T_{\text{cw}}$)

$$\dot{m}_{\text{cw}} = \frac{\dot{m}(h_2 - h_3)}{c_w \Delta T_{\text{cw}}}$$

With $c_w = 4.179 \text{ kJ/kg} \cdot \text{K}$ from Table A-19

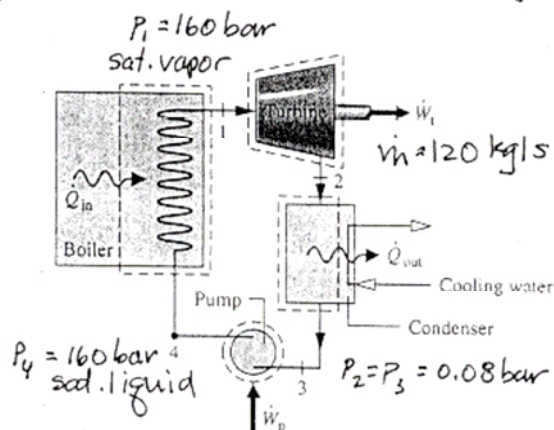
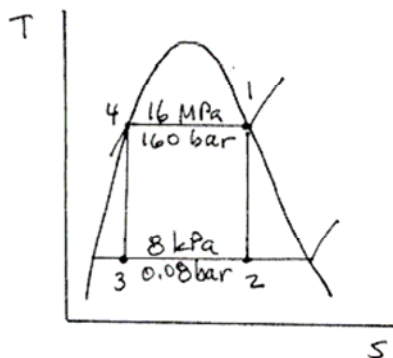
$$\dot{m}_{\text{cw}} = \frac{(120)(1637.6 - 173.88)}{(4.179)(18)} = 2335 \frac{\text{kg}}{\text{s}} \leftarrow \dot{m}_{\text{cw}}$$

PROBLEM 8.7

KNOWN: Water is the working fluid in a Carnot vapor power cycle. Saturated liquid enters the boiler and saturated vapor enters the turbine, and the boiler pressure is specified. The condenser pressure is specified, and the mass flow rate of steam is given.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power, and (d) the rate of heat transfer from the working fluid passing through the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The turbine and the pump operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 160 \text{ bar}$, sat. vapor $\Rightarrow h_1 = 2580.6 \text{ kJ/kg}$, $s_1 = 5.2455 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 0.08 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.6091$, $h_2 = 1637.6 \text{ kJ/kg}$

State 3: $P_3 = 0.08 \text{ bar}$, $s_3 = s_4 = s_f @ 160 \text{ bar} = 3.7461 \text{ kJ/kg}\cdot\text{K}$
 $\Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.4130$, $h_3 = 1166.4 \text{ kJ/kg}$

State 4: $P_4 = 160 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 1650.1 \text{ kJ/kg}$

(a) The thermal efficiency of the Carnot cycle is given by Eq. 5.9. With data from Table A-3; $T_H = T_{\text{sat}} @ 160 \text{ bar} = 620.55 \text{ K}$ and $T_C = T_{\text{sat}} @ 0.08 \text{ bar} = 314.66 \text{ K}$. Thus

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{314.66}{620.55} = 0.493 \text{ (49.3\%)} \quad \leftarrow \eta$$

(b) The back work ratio is

$$\text{bwr} = \frac{\dot{W}_p}{\dot{W}_t} = \frac{\dot{m}(h_4 - h_3)}{\dot{m}(h_1 - h_2)} = \frac{1650.1 - 1166.4}{2580.6 - 1637.6} = 0.513 \quad \leftarrow \text{bwr}$$

(c) The net power developed is

$$\dot{W}_{\text{cycle}} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)] = (120 \frac{\text{kg}}{\text{s}})[(2580.6 - 1637.6) - (1650.1 - 1166.4)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 55,120 \text{ kW} \quad \leftarrow \dot{W}_{\text{cycle}}$$

(d) For the condenser: $\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3) = (120)(1637.6 - 1166.4) = 56,540 \text{ kW} \quad \leftarrow \dot{Q}_{\text{out}}$

1. Alternatively, $\eta = \dot{W}_{\text{cycle}} / \dot{Q}_{\text{in}} = \dot{W}_{\text{cycle}} / (\dot{W}_{\text{cycle}} + \dot{Q}_{\text{out}}) = 0.4936$.

2. These results can be compared with the Rankine cycle of Problem 8.6.

PROBLEM 8.8

KNOWN: Water is the working fluid in an ideal Rankine cycle. The condenser and steam generator pressures are specified as in Problem 8.6.

FIND: Plot each of the quantities calculated in Problem 8.6 versus turbine inlet temperature ranging from saturation to superheated vapor.

SCHEMATIC & GIVEN DATA: See Problem 8.6.

ENGINEERING MODEL: See Problem 8.6.

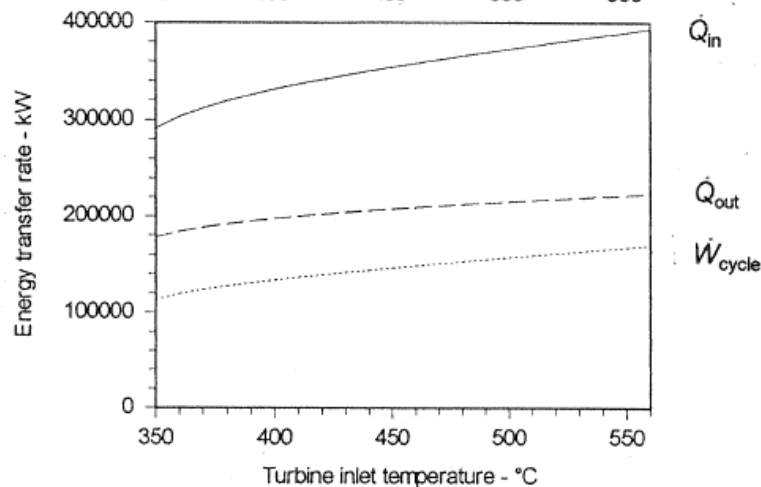
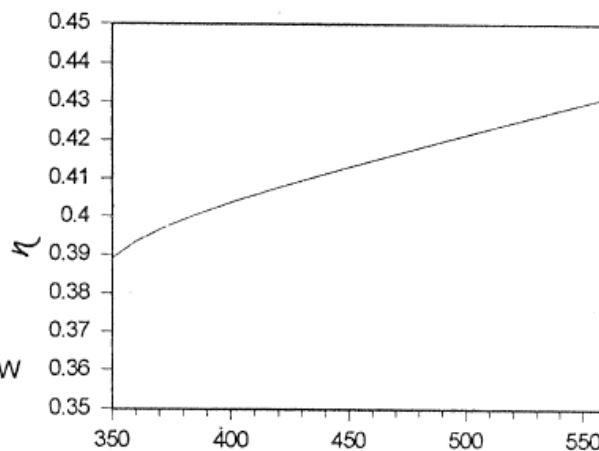
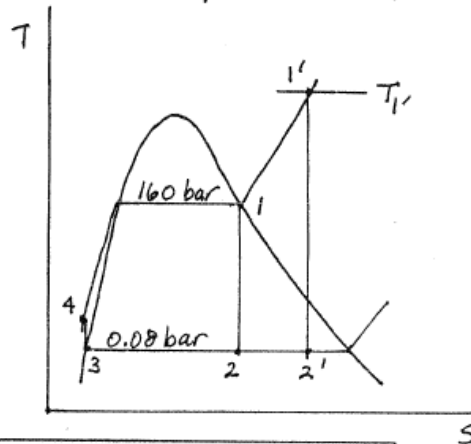
ANALYSIS: The turbine inlet and exit states change as in the T-s diagram:

IT Code

```
p1 = 160 // bar
p2 = 0.08 // bar
mdot = 120 // kg/s
// Turbine
T1sat = Tsat_P("Steam", p1)
T1 = 350
h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2 = s1
h2 = h_Ps("Water", p2, s2)
// Condenser
p3 = p2
x3 = 0
h3 = hsat_Px("Water", p3, x3)
// Pump
v3 = vsat_Px("Water", p3, x3)
p4 = p1
h4 = h3 + v3 * (p4 - p3) * (10^5 / 10^3)
// Cycle parameters
Wdotnet = mdot * ((h1 - h2) - (h4 - h3)) // kW
Qdotin = mdot * (h1 - h4) // kW
eta = Wdotnet / Qdotin
Qdotout = mdot * (h2 - h3) // kW
```

IT Results ($T_1 = T_{\text{sat @ 160 bar}}$)

$\dot{Q}_{\text{in}} = 2.868\text{E5 kW}$
 $\dot{Q}_{\text{out}} = 1.757\text{E5 kW}$
 $\dot{W}_{\text{cycle}} = 1.111\text{E5 kW}$
 $\eta = 0.3875$
 $h_1 = 2580 \text{ kJ/kg}$
 $h_2 = 1638 \text{ kJ/kg}$
 $h_3 = 173.6 \text{ kJ/kg}$
 $h_4 = 189.8 \text{ kJ/kg}$



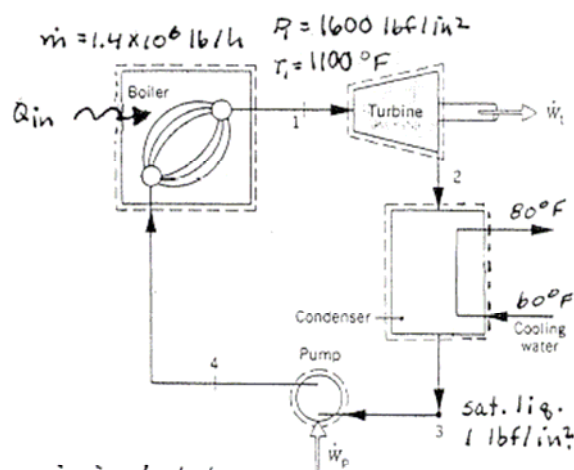
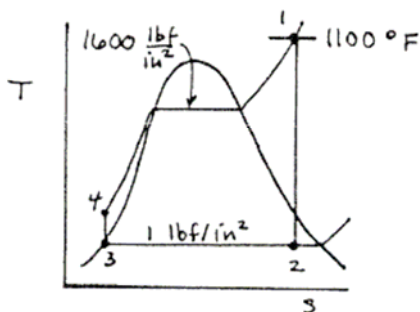
The cycle with superheating (1'-2'-3-4-1') has a higher average temperature of heat addition than the cycle without superheating (1-2-3-4-1), so the thermal efficiency is higher. The energy transfer rates vary accordingly.

PROBLEM 8.9

KNOWN: Water is the working fluid in an ideal Rankine cycle. The turbine inlet state and the condenser pressure are known. Also, the mass flow rate of steam entering the turbine is given.

FIND: Determine (a) the net power developed, (b) the thermal efficiency, and (c) the mass flow rate of cooling water.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: See Example 8.1, assumptions 1-4.

ANALYSIS: First, fix each of the principal states.

STATE 1: $P_1 = 1600 \text{ lbf/in}^2$, $T_1 = 1100^\circ\text{F} \Rightarrow h_1 = 1547.7 \frac{\text{Btu}}{\text{lb}}$, $s_1 = 1.6315 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}$

STATE 2: $P_2 = 1 \text{ lbf/in}^2$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = 0.81223$, $h_2 = 911.2 \frac{\text{Btu}}{\text{lb}}$

STATE 3: $P_3 = 1 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_3 = 69.74 \frac{\text{Btu}}{\text{lb}}$

STATE 4: $h_4 \approx h_3 + v_3 (P_4 - P_3)$

$$= 69.74 \frac{\text{Btu}}{\text{lb}} + (0.01614 \frac{\text{ft}^3}{\text{lb}}) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}} \right| (1600 - 1) \frac{\text{lbf}}{\text{in}^2}$$

$$= 69.74 + 4.78 = 74.52 \frac{\text{Btu}}{\text{lb}}$$

(a) The net power developed by the cycle is

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = \dot{m} [(h_1 - h_2) - (h_4 - h_3)]$$

$$= (1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) [(1547.7 - 911.2) - (74.52 - 69.74)] \frac{\text{Btu}}{\text{lb}}$$

$$= 8.84 \times 10^8 \frac{\text{Btu}}{\text{h}} \quad \dot{W}_{\text{cycle}}$$

(b) To find the cycle thermal efficiency, first find the heat transfer rate to the working fluid passing through the steam generator. Thus

$$\dot{Q}_{\text{in}} = \dot{m} [h_1 - h_4]$$

$$= (1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) (1547.7 - 74.52) \frac{\text{Btu}}{\text{lb}}$$

$$= 2.06 \times 10^9 \frac{\text{Btu}}{\text{h}}$$

Continued on next slide

Problem 8-9 continued

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{8.84 \times 10^8}{2.06 \times 10^9} = 0.429 (42.9\%) \quad \eta$$

(c) For the control volume enclosing the condenser

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}(h_2 - h_3) + \dot{m}_{\text{cw}}(h_{\text{cw},\text{in}} - h_{\text{cw},\text{out}})$$

where $\dot{m}_{\text{cw}}$ is the mass flow rate of the cooling water. Thus

$$\dot{m}_{\text{cw}} = \frac{\dot{m}(h_2 - h_3)}{(h_{\text{cw},\text{out}} - h_{\text{cw},\text{in}})}$$

For the cooling water, $h_{\text{cw}} \approx h_f(T)$. Thus, with data from Table A-2E

$$\begin{aligned} \dot{m}_{\text{cw}} &= \frac{(1.4 \times 10^6 \text{ lb/h})(911.2 - 69.74) \text{ Btu/lb}}{(48.09 - 28.08) \text{ Btu/lb}} \\ &= 5.89 \times 10^7 \text{ lb/h} \quad \dot{m}_{\text{cw}} \end{aligned}$$

PROBLEM 8.10

KNOWN: Water is working fluid in an ideal Rankine cycle. Data are given in Prob. 8.9.

FIND: Plot each of the quantities calculated in Prob. 8.9 versus condenser pressure ranging from 0.4 to 14.7 lbf/in.². Maintain constant steam mass flow rate.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.9.

ENGINEERING MODEL: See Example 8.1, assumptions 1–4.

ANALYSIS: The following IT code is used to develop the data for the required plots. Results are presented for the two limiting cases.

IT Code

```
p1 = 1600 // lbf/in.2
T1 = 1100 // °F
p2 = 1
p3 = p2
x3 = 0
p4 = p1
mdot = 1.4E06
Twin = 60
Twout = 80

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1
h3 = hsat_Px("Water/Steam", p3, x3)
v3 = vsat_Px("Water/Steam", p3, x3)
h4 = h3 + v3 * (p4 - p3) * (144 / 778)
pw1 = Psat_T("Water/Steam", Twin)
hwin = hsat_Px("Water/Steam", pw1, 0)
pw2 = Psat_T("Water/Steam", Twout)
hwout = hsat_Px("Water/Steam", pw2, 0)

Wcycle = mdot * ((h1 - h2s) - (h4 - h3))
Qin = mdot * (h1 - h4)
eta = Wcycle / Qin
Q = mdot * (h2s - h3) + mdotw * (hwin - hwout)
```

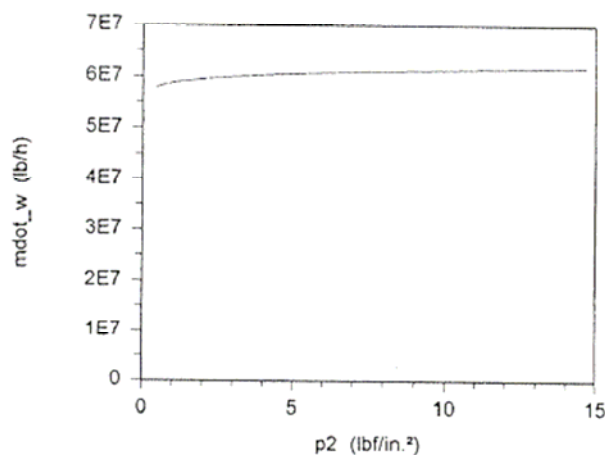
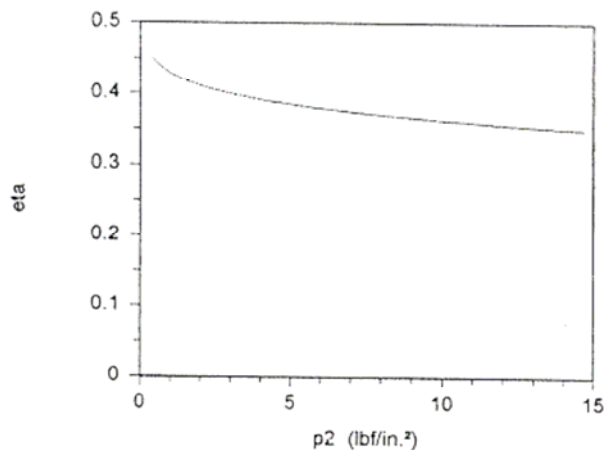
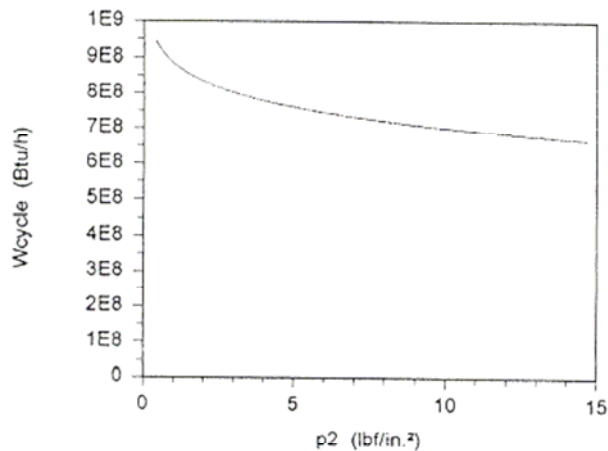
IT Results for p2 = 0.4 and 14.7 lbf/in.²

p2 (lbf/in. ²)	0.4	14.7
h1 (Btu/lb)	1547	1547
s1 (Btu/lb.R)	1.631	1.631
h2s	867.1	1066
h3	40.49	180.4
h4	45.25	185.3
hwin	27.63	27.63
hwout	47.7	47.7
v3 (ft ³ /lb)	0.01606	0.01671
Qin (Btu/h)	2.103 × 10 ⁹	1.907 × 10 ⁹
Wcycle (Btu/h)	9.46 × 10 ⁸	6.669 × 10 ⁸
eta	0.4498	0.3497
mdotw (lb/h)	5.765 × 10 ⁷	6.178 × 10 ⁷

Continued on next slide

Problem 8-10 continued

PLOTS :



We see from the plots that as condenser pressure decreases, the power output increases as does the cycle thermal efficiency. Further, less energy is rejected in the conder, so the cooling water flow rate decreases slightly. These results are consistent with the discussion of Fig. 8.4(b) in Sec. 8.2.3.

PROBLEM 8.11

KNOWN: Water is the working fluid in an ideal Rankine cycle. Data are given in Prob. 8.9.

FIND: Plot each of the quantities calculated in Prob. 8.9 versus steam generator pressure ranging from 250 to 4000 lbf/in.². Maintain constant steam mass flow rate and turbine inlet temperature.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.9.

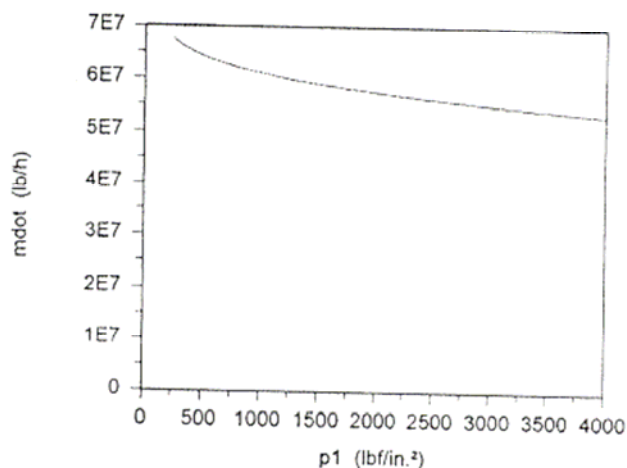
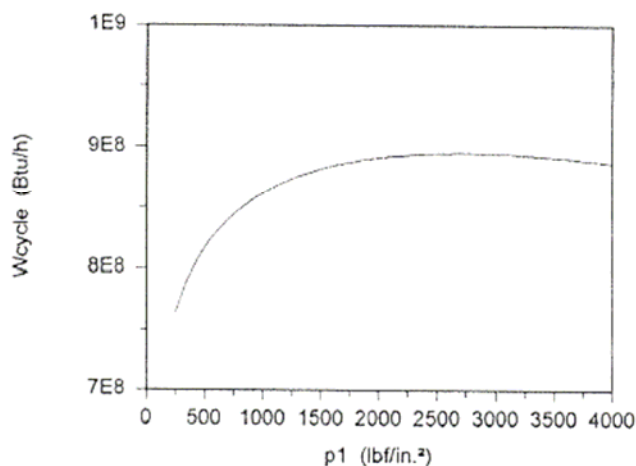
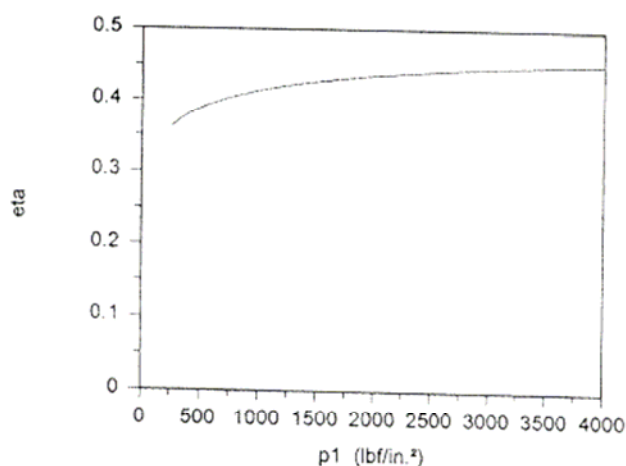
ENGINEERING MODEL: See Example 8.1, items 1-4.

ANALYSIS: See solution to Prob. 8.10 for the IT code. Results are presented here for the two limiting cases.

IT Results for $p_1 = 250$ and 4000 lbf/in.²

p_1 (lbf/in. ²)	250	4000
h_1 (Btu/lb)	1581	1481
s_1 (Btu/lb.R)	1.852	1.497
h_{2s}	1035	835.8
h_3	69.58	69.58
h_4	70.32	81.53
h_{win}	27.63	27.63
h_{wout}	47.7	47.7
v_3 (ft ³ /lb)	0.01614	0.01614
Q_{in} (Btu/h)	2.115×10^9	1.96×10^9
W_{cycle} (Btu/h)	7.629×10^8	8.872×10^8
η	0.3608	0.4527
$\dot{m}_{dotw}$ (lb/h)	6.734×10^7	5.343×10^7

PLOTS:



We see from the plots that the power output exhibits a maximum, but the thermal efficiency increases with increasing steam generator pressure. Further, less energy is rejected in the condenser, so the cooling water flow rate decreases slightly. For further discussion, see Sec. 8.2.3.

PROBLEM 8.12

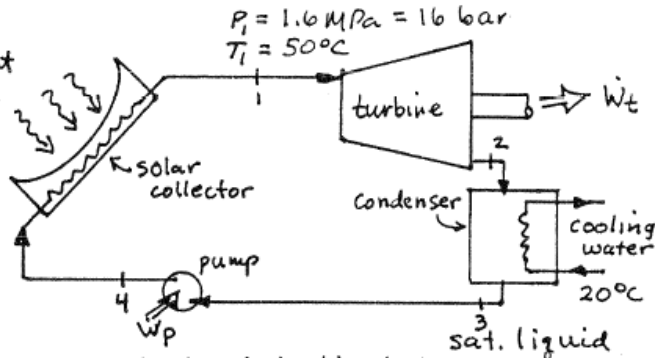
KNOWN: A power plant using R-22 as the working fluid is under development based on the Rankine cycle. The desired net power output, turbine inlet state, and available cooling water temperature are specified.

FIND: Specify the preliminary design of the cycle, estimate the thermal efficiency and steam and cooling water flow rates.

SCHEMATIC & GIVEN DATA:

ENGINEERING

MODEL: (1) Each component operates at steady state. (2) There are no stray heat transfers from any component. (3) Kinetic and potential energy effects are negligible. (4) Saturated liquid exits the condenser. (Other assumptions discussed below in developing the preliminary design.)



ANALYSIS: To maximize performance, it is desirable that the condenser pressure be as low as possible. For $P_{\text{cond}} = 12 \text{ bar}$; $T_{\text{sat}} = 30.25^\circ\text{C}$. With this choice, it is likely that the temperature rise of the cooling water passing through the condenser would be about 15°C .

Further, a characteristic value of $\eta_t \approx 85\%$ is reasonable for the preliminary design. Also, let $\eta_p \approx 70\%$. With these choices, each principal state can be fixed.

State 1: $P_1 = 16 \text{ bar}$, $T_1 = 50^\circ\text{C} \Rightarrow h_1 = 269.18 \text{ kJ/kg}$, $s_1 = 0.8971 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 12 \text{ bar}$, $s_{2s} = s_1 \Rightarrow h_{2s} = 262.2 \text{ kJ/kg}$. Thus, with the turbine efficiency

$$h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 263.2 \text{ kJ/kg}$$

State 3: $P_3 = 12 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 81.90 \text{ kJ/kg}$

State 4: $h_{4s} \approx h_3 + v_3 (P_4 - P_3)$

$$= 81.90 \frac{\text{kJ}}{\text{kg}} + (0.8546 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (16 - 12) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 82.24 \frac{\text{kJ}}{\text{kg}}$$

Thus, with the pump efficiency

$$h_4 = h_3 + (h_{4s} - h_3) / \eta_p = 82.39 \text{ kJ/kg}$$

Using this data

$$\left. \begin{aligned} \dot{W}_t / \dot{m} &= h_1 - h_2 = 5.98 \text{ kJ/kg} \\ \dot{W}_p / \dot{m} &= h_4 - h_3 = 0.49 \text{ kJ/kg} \\ \dot{Q}_{\text{in}} / \dot{m} &= h_1 - h_4 = 186.8 \text{ kJ/kg} \end{aligned} \right\} \eta = \frac{\dot{W}_t / \dot{m} - \dot{W}_p / \dot{m}}{\dot{Q}_{\text{in}} / \dot{m}} = 0.0294 \text{ (2.94\%)} \leftarrow \eta$$

For a net power output of $10 \text{ MW} = 10,000 \text{ kJ/s}$

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{\dot{W}_t / \dot{m} - \dot{W}_p / \dot{m}} = \frac{10,000 \text{ kJ/s}}{5.98 - 0.49 \text{ kJ/kg}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = 6.56 \times 10^6 \text{ kg/h} \leftarrow \dot{m}$$

With $h_{\text{cw}} \approx h_f(T_{\text{cw}})$, the cooling water flow rate $\dot{m}_{\text{cw}}$ is

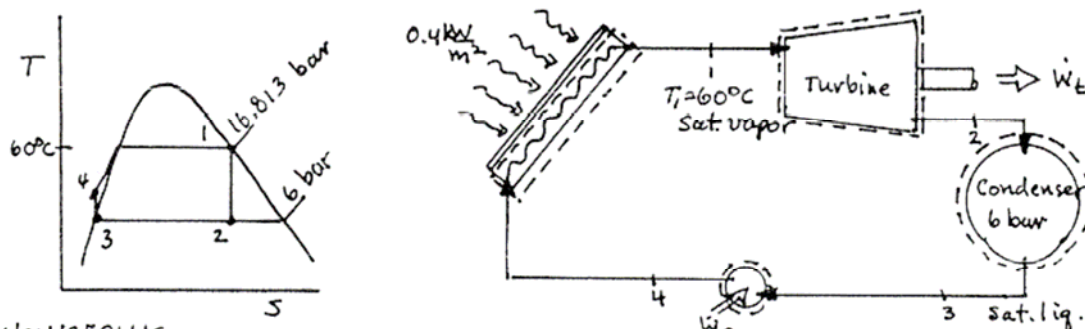
$$\dot{m}_{\text{cw}} = \frac{\dot{m} (h_2 - h_3)}{(h_{\text{cw,out}} - h_{\text{cw,in}})} = \frac{(6.56 \times 10^6) (263.2 - 81.90)}{(117.43 - 83.96)} = 35.53 \times 10^6 \text{ kg/h} \leftarrow \dot{m}_{\text{cw}}$$

PROBLEM 8.13

KNOWN: Refrigerant 134a is the working fluid in a solar power plant operating on a Rankine cycle. The turbine inlet state and the condenser pressure are known.

FIND: Determine the minimum possible solar collector surface area per kW developed by the plant.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each control volume is at steady state. (2) All processes of the working fluid are internally reversible. (3) There are no stray heat transfers, and all solar radiation incident on the collector is absorbed by the working fluid. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the condenser as saturated liquid at 6 bars.

ANALYSIS: Fix each of the principal states.

State 1: $T_1 = 60^\circ\text{C}$, sat. vapor $\Rightarrow h_1 = 275.99 \text{ kJ/kg}$, $s_1 = 0.8973 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 6 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = 0.9797$, $h_2 = 255.54 \text{ kJ/kg}$

State 3: $P_3 = 6 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 79.48 \text{ kJ/kg}$

State 4: $h_4 \approx h_3 + v_3(P_4 - P_3)$
 $= 79.48 \frac{\text{kJ}}{\text{kg}} + (0.196 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (16.813 - 6) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$
 $= 79.48 + 0.89 = 80.37 \text{ kJ/kg}$

The ideal Rankine cycle described here requires the minimum $\dot{Q}_{\text{in}}$ to develop 1 kW under the prescribed constraints for turbine inlet state and condenser pressure. The mass flow rate of refrigerant is obtained from

$$\dot{W}_{\text{cycle}} = \dot{m} [\dot{W}_{t/\dot{m}} - \dot{W}_{p/\dot{m}}] = \dot{m} [(h_1 - h_2) - (h_4 - h_3)]$$

or

$$\dot{m} = \frac{(1 \text{ kJ/s})}{[(275.99 - 255.54) - (80.37 - 79.48)] \frac{\text{kJ}}{\text{kg}}} = 0.05112 \text{ kg/s}$$

Thus, the minimum value of $\dot{Q}_{\text{in}}$ is

$$\dot{Q}_{\text{in}} = \dot{m} (h_1 - h_4) = (0.05112 \frac{\text{kg}}{\text{s}}) (275.99 - 80.37) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 10.00 \text{ kW}$$

$$\left(\frac{\text{minimum collector area}}{\text{area}} \right) = \frac{10.00 \text{ kW}}{0.4 \text{ kW/m}^2} = 25 \text{ m}^2/\text{kW power developed} \leftarrow \text{area}$$

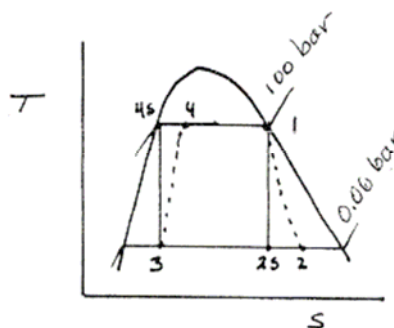
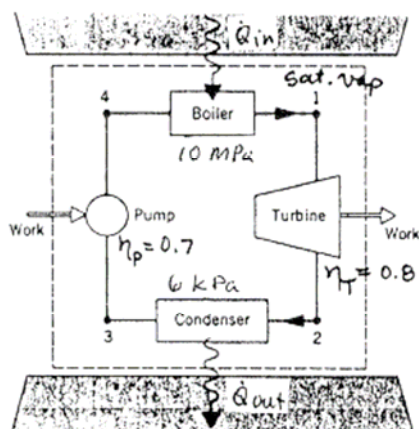
Irreversibilities or stray heat transfer from the collector will result in greater collector area being required.

PROBLEM 8.15

KNOWN: A Carnot vapor power cycle is modified to include the effects of irreversibilities in the adiabatic expansion and compression processes.

FIND: Repeat the analysis of Problem 8.3 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The processes of the working fluid in the boiler and condenser are internally reversible. (3) The turbine and the pump operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: From Problem 8.3, $h_1 = 2724.7 \text{ kJ/kg}$ and $h_3 = 1029.7 \text{ kJ/kg}$. Further, $h_{2s} = 1727.2 \text{ kJ/kg}$ and $h_{4s} = 1407.6 \text{ kJ/kg}$.

State 2: $h_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - h_t(h_1 - h_{2s}) = 1926.7 \text{ kJ/kg}$

State 4: $\eta_p = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + (h_{4s} - h_3)/\eta_p = 1569.6 \text{ kJ/kg}$

(a) The thermal efficiency is

① $\eta = 1 - \frac{(\dot{Q}_{out}/\dot{m})}{(\dot{Q}_{in}/\dot{m})} = 1 - \frac{h_2 - h_3}{h_1 - h_4} = 0.223 (22.3\%) \leftarrow \eta$

② (b) The back work ratio is $bwr = \frac{h_4 - h_3}{h_1 - h_2} = 0.677 (67.7\%) \leftarrow bwr$

(c) For the boiler: $\dot{Q}_{in}/\dot{m} = h_1 - h_4 = 1155.1 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}$

(d) For the condenser: $\dot{Q}_{out}/\dot{m} = h_2 - h_3 = 897 \text{ kJ/kg} \leftarrow \dot{Q}_{out}/\dot{m}$

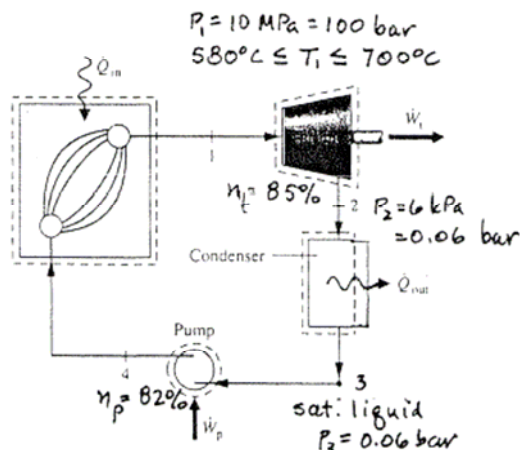
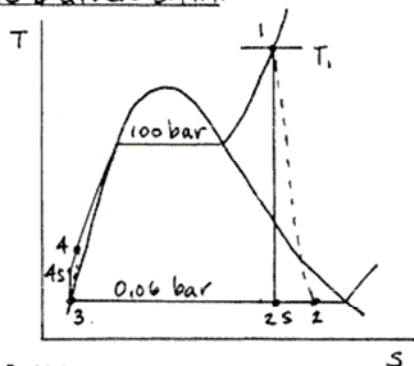
1. Comparing this result to the thermal efficiency calculated in Problem 8.3, we see that irreversibilities in the turbine and pump greatly reduce the thermal efficiency.
2. The back work ratio is much greater than the result of Problem 8.3, indicating another significant effect of internal irreversibilities.

PROBLEM 8.16

KNOWN: Water is the working fluid in a simple vapor power plant. Data are given at various states in the cycle. The turbine inlet temperature is T_1 .

FIND: (a) For $T_1 = 580^\circ\text{C}$, determine the turbine exit quality and the cycle thermal efficiency. (a) Plot the quantities of part (a) for T_1 ranging from 580 to 700°C .

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: See Example 8.1, assumptions 1-4.

ANALYSIS: First, fix each of the principal states.

State 1 $P_1 = 100 \text{ bar}$, T_1 fix the state in the superheat region $\Rightarrow h_1, s_1$

State 2 P_2 and $s_{2s} = s_1$ gives x_{2s}, h_{2s} . Now, $h_2 = h_1 - (h_1 - h_{2s})/\eta_t \Rightarrow x_2$ (1)

State 3 $P_3 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_3, v_3$

State 4 $h_4 = h_3 + v_3(P_4 - P_3)/\eta_p$

Now, the net work of the cycle is

$$\frac{W_{\text{cycle}}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = (h_1 - h_2) - (h_4 - h_3) \quad (2)$$

And, for the steam generator, $\dot{Q}_{\text{in}}/\dot{m} = h_1 - h_4$ (3)

$$\text{Thus } \eta = \frac{W_{\text{cycle}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)} \quad (4)$$

(a) For $T_1 = 580^\circ\text{C}$, Table A-4 gives $h_1 = 3575.7 \text{ kJ/kg}$, $s_1 = 6.8447 \text{ kJ/kg}\cdot\text{K}$. Thus $s_{2s} = s_1 = 6.8447 \Rightarrow x_{2s} = 0.80975 \Rightarrow h_{2s} = 2107.8 \text{ kJ/kg}$. With $\eta_t = 0.85$

$$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3575.7 - (0.85)(3575.7 - 2107.8) = 2328.0 \text{ kJ/kg} \Rightarrow x_2 = 0.901 \leftarrow x_2$$

Further, from Table A-3, $h_3 = 151.53 \text{ kJ/kg}$, $v_3 = 1.0064 \times 10^{-3} \text{ m}^3/\text{kg}$. Thus

$$h_4 = 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (100 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| / (0.82) \\ = 163.8 \text{ kJ/kg}$$

Now

$$\eta = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)} = \frac{(3575.7 - 2328) - (163.8 - 151.53)}{(3575.7 - 163.8)} \\ = \frac{1235.4}{3411.9} = 0.362 \leftarrow \eta$$

Continued on next slide

Problem 8-16 continued

(b) The data for the required plots are obtain using IT, as follows:

IT Code

```
p1 = 100 // bar
T1 = 580 // °C
p2 = 0.06 // bar
eff_t = 0.85
eff_p = 0.82

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1

h2 = h1 - eff_t * (h1 - h2s)
h2 = hsat_Px("Water/Steam", p2, x2)
x2pct = x2 * 100
p3 = p2
h3 = hsat_Px("Water/Steam", p3, 0)
v3 = vsat_Px("Water/Steam", p3, 0)
p4 = p1
h4 = h3 + (v3 * (p4 - p3) * 100) / eff_p

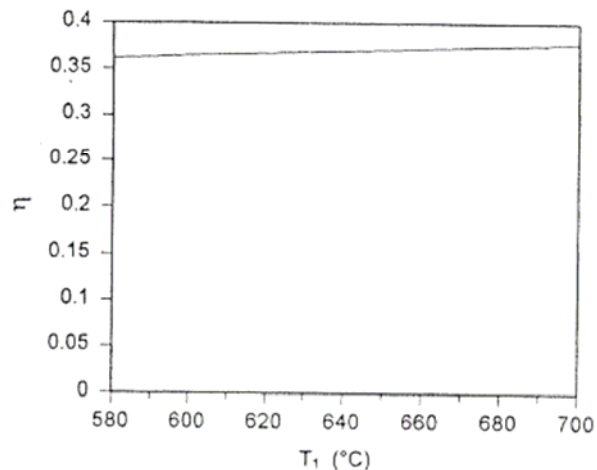
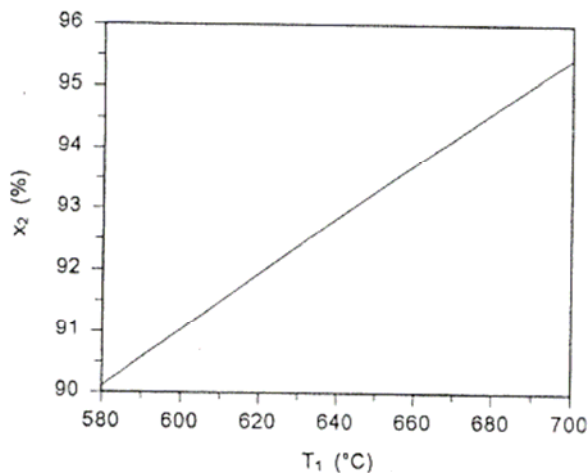
Wcycle = (h1 - h2) - (h4 - h3)

Qin = h1 - h4
eta = Wcycle/Qin
```

IT Results for $T_1 = 580^\circ\text{C}$

```
h1 = 3575 kJ/kg
s1 = 6.845 kJ/kg·K
h2 = 2328 kJ/kg
h2s = 2108 kJ/kg
h3 = 151 kJ/kg
v3 = 0.001007 m³/kg
h4 = 163.3 kJ/kg
Q_in / m = 3412 kJ/kg
W_cycle / m = 1235 kJ/kg
x2 = 90.1 %
eta = 0.362
```

PLOTS:



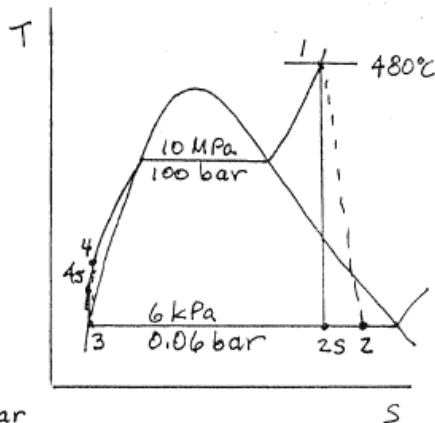
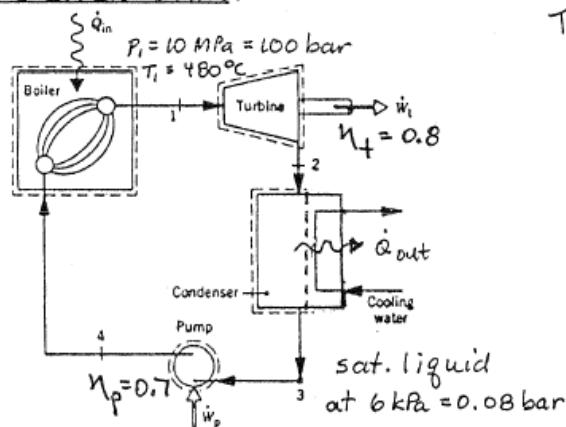
From the T-s diagram, we see that as T_1 increases, points 2s and 2 move to the right. Thus, x_2 increases. Further, as T_1 increases, the average temperature of heat addition increases. Thus, η increases slightly, as indicated in the above plot.

PROBLEM 8.17

KNOWN: Water is the working fluid in a Rankine cycle with superheated vapor entering the turbine. The states at the turbine inlet and condenser exit are specified and turbine and pump isentropic efficiencies are given.

FIND: Determine (a) the heat transfer rate for the steam generator, per kg of steam flowing, (b) the thermal efficiency, and (c) the rate of heat transfer for the condenser, per kg of steam condensing.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as Example 8.1, items 1-4, except $\eta_t = 0.8$ and $\eta_p = 0.7$.

ANALYSIS: From Problem 8.2, $h_1 = 3321.4 \text{ kJ/kg}$ and $h_3 = 151.53 \text{ kJ/kg}$. The specific enthalpy at state 2 is found using the turbine efficiency

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s})$$

From Problem 8.2, $h_{2s} = 2009.8 \text{ kJ/kg}$. Thus, $h_2 = 2272.1 \text{ kJ/kg}$. Similarly,

$$\eta_p = \frac{h_4 - h_3}{h_4 - h_{3s}} \Rightarrow h_4 = h_3 + (h_4 - h_{3s})/\eta_p$$

From Problem 8.2, $h_{4s} = 161.59$. Thus, $h_4 = 165.9 \text{ kJ/kg}$.

(a) For the control enclosing the steam generator

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) \Rightarrow \frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4 = 3321.4 - 165.9 = 3155.5 \text{ kJ/kg} \leftarrow \frac{\dot{Q}_{in}}{\dot{m}}$$

(b) The thermal efficiency is $\eta = \frac{\dot{W}_{net}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$

$$\dot{W}_{net}/\dot{m} = (h_1 - h_2) - (h_4 - h_3) = 1049.3 - 14.37 = 1034.9 \text{ kJ/kg}$$

$$\eta = \frac{1034.9}{3155.5} = 0.328 (32.8\%) \leftarrow \eta$$

(c) For the condenser

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) \Rightarrow \frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 = 2272.1 - 151.53 = 2120.6 \frac{\text{kJ}}{\text{kg}} \leftarrow \frac{\dot{Q}_{out}}{\dot{m}}$$

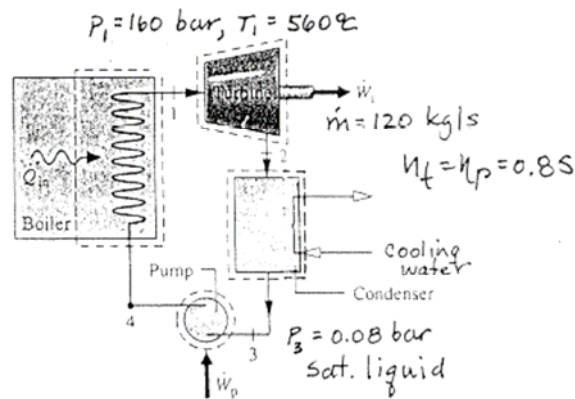
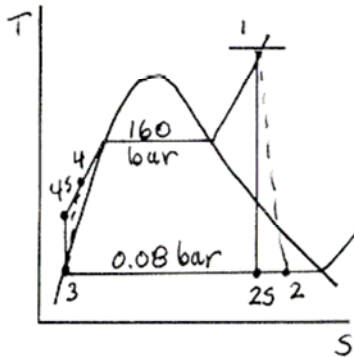
1. These results can be compared with those of Problem 8.2 to see some of the effects of irreversibilities in the turbine and pump on the ideal Rankine cycle.

PROBLEM 8.18

KNOWN: Water is the working fluid in a Rankine cycle. The condenser pressure and the turbine inlet state are specified. The mass flow of steam is given, and the turbine and pump efficiencies are specified.

FIND: Determine (a) the net power, (b) the rate of heat transfer to the steam passing through the boiler, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.2.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 160 \text{ bar}$, $T_1 = 560^\circ\text{C} \Rightarrow h_1 = 3465.4 \text{ kJ/kg}$, $s_1 = 6.5132 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 0.08 \text{ bar}$, $s_{2s} = s_1 \Rightarrow x_{2s} = \frac{s_{2s} - s_{f2}}{s_{g2} - s_{f2}} = 0.7753$, $h_{2s} = 2037.0 \frac{\text{kJ}}{\text{kg}}$

The specific enthalpy h_2 is found using the isentropic turbine efficiency.

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 2251.3 \text{ kJ/kg}$$

State 3: $P_3 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 173.88 \text{ kJ/kg}$

$$\begin{aligned} \text{State 4: } P_4 = 160 \text{ bar}, h_{4s} &\approx h_3 + v_3(P_4 - P_3) \\ &= 173.88 \frac{\text{kJ}}{\text{kg}} + (1.0084 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (160 - 0.08 \text{ bar}) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \\ &= 190.01 \text{ kJ/kg} \end{aligned}$$

With the pump efficiency: $\eta_p = (h_{4s} - h_3) / (h_4 - h_3)$

$$h_4 = h_3 + (h_{4s} - h_3) / \eta_p = 192.86 \text{ kJ/kg}$$

(a) The net power is $\dot{W}_{\text{cycle}} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$

$$\begin{aligned} &= (120 \frac{\text{kg}}{\text{s}}) [(3465.4 - 2251.3) - (192.86 - 173.88)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 1.434 \times 10^5 \text{ kW} \end{aligned}$$

(b) For the steam passing through the steam generator

$$\dot{Q}_{\text{in}} = \dot{m}[h_1 - h_4] = (120) [3465.4 - 192.86] \left| \frac{1}{1} \right| = 3.927 \times 10^5 \text{ kW}$$

(c) The thermal efficiency is $\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = 0.365 (36.5\%)$

Continued on next slide

Problem 8-18 continued

The plots are developed using IT, as follows:

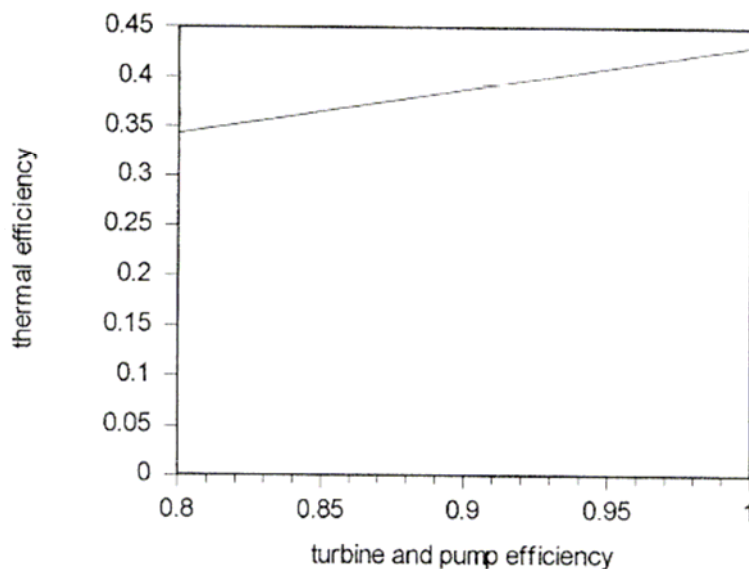
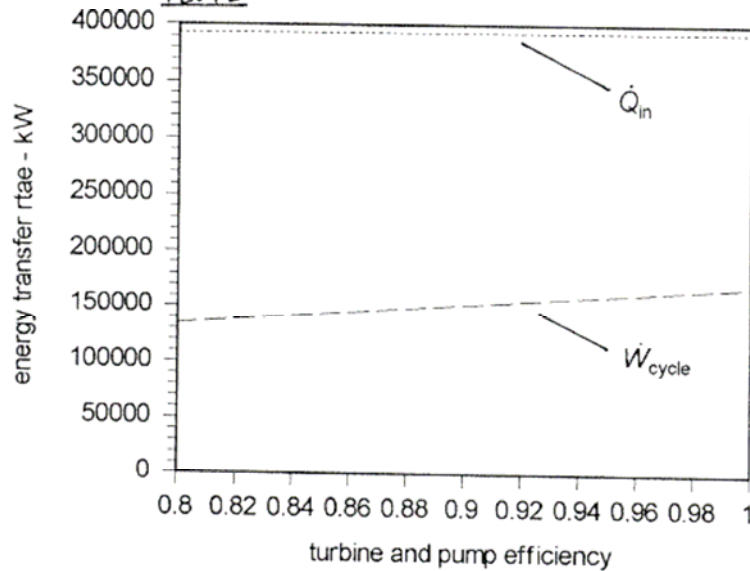
IT Code

```
p1 = 160 // bar
p2 = 0.08 // bar
mdot = 120 // kg/s
T1 = 560 // °C
etat = 0.85
etap = etat
// Turbine
T1sat = Tsat_P("Steam", p1)
h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water", p2, s2s)
h2 = h1 - etat * (h1 - h2s)
// Condenser
p3 = p2
x3 = 0
h3 = hsat_Px("Water", p3, x3)
// Pump
v3 = vsat_Px("Water", p3, x3)
p4 = p1
h4s = h3 + v3 * (p4 - p3) * (10^5 / 10^3)
h4 = h3 + (h4s - h3) / etap
// Cycle parameters
Wdotnet = mdot * ((h1 - h2) - (h4 - h3)) // kW
Qdotin = mdot * (h1 - h4) // kW
eta = Wdotnet / Qdotin
```

IT Results

```
Q_in = 3.927 x 10^5 kW
W_cycle = 1.434 x 10^5 kW
eta = 0.3652
h1 = 3465 kJ/kg
h2 = 2251 kJ/kg
h3 = 173.6 kJ/kg
h4 = 192.6 kJ/kg
```

Plots:



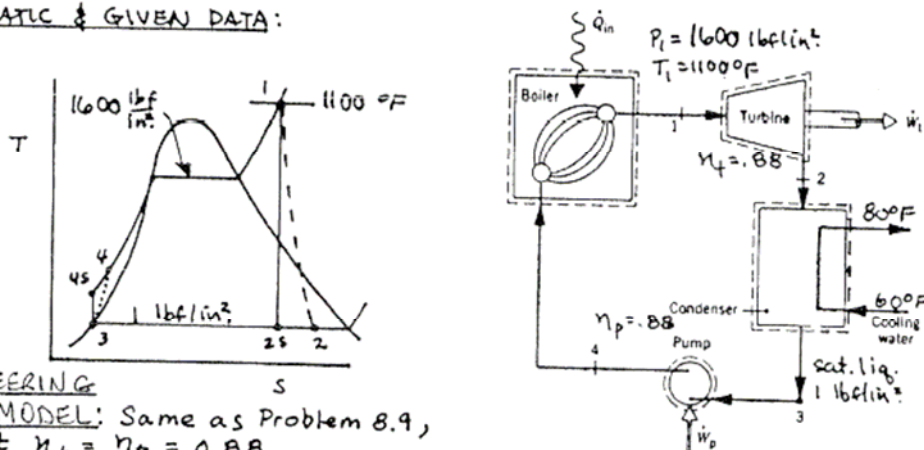
As turbine and pump isentropic efficiencies decrease, the net power developed decreases significantly, resulting in reduced thermal efficiency. The heat input changes only slightly.

PROBLEM 8.19

KNOWN: The ideal Rankine cycle of Problem 8.9 is modified to include that the turbine and pump each have an isentropic efficiency of 88%.

FIND: Determine (a) the net power developed, (b) the rate of heat transfer to the working fluid passing through the steam generator, (c) the thermal efficiency, and (d) the volumetric flow rate of the cooling water entering the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Problem 8.9, except $\eta_t = \eta_p = 0.88$.

ANALYSIS: From Problem 8.9, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R and $h_3 = 69.74$ Btu/lb. The specific enthalpy at state 2 is found using the turbine efficiency

$$\eta_t = \frac{(W_t/\dot{m})}{(W_t/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t (h_1 - h_{2s})$$

From Problem 8.9, $h_{2s} = 911.2$ Btu/lb. Thus, $h_2 = 987.6$ Btu/lb. Similarly, the specific enthalpy at state 4 is found using the pump efficiency

$$\eta_p = \frac{(W_p/\dot{m})}{(W_p/\dot{m})_s} = \frac{h_4 - h_3}{h_{4s} - h_3} \Rightarrow h_4 = h_3 + (h_{4s} - h_3)/\eta_p$$

From Problem 8.9, $h_{4s} = 74.52$ Btu/lb. Thus, $h_4 = 75.17$ Btu/lb.

(a) The net power developed is

$$\dot{W}_{\text{cycle}} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)] = (1.4 \times 10^6 \frac{\text{lb}}{\text{h}})[(1547.7 - 987.6) - (75.17 - 69.74)] \frac{\text{Btu}}{\text{lb}} = 7.77 \times 10^8 \frac{\text{Btu}}{\text{h}} \leftarrow \dot{W}_{\text{cycle}}$$

(b) The heat transfer rate for the steam generator is

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = (1.4 \times 10^6 \frac{\text{lb}}{\text{h}})(1547.7 - 75.17) \frac{\text{Btu}}{\text{lb}} = 2.062 \times 10^9 \frac{\text{Btu}}{\text{h}} \leftarrow \dot{Q}_{\text{in}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{7.77 \times 10^8}{2.062 \times 10^9} = 0.377 \text{ (37.7\%)} \leftarrow \eta$$

(d) The mass flow rate of cooling water through the condenser is

$$\dot{m}_{\text{cw}} = \frac{\dot{m}(h_2 - h_3)}{h_{\text{cw,out}} - h_{\text{cw,in}}} = \frac{(1.4 \times 10^6 \frac{\text{lb}}{\text{h}})(987.6 - 69.74) \frac{\text{Btu}}{\text{lb}}}{(48.09 - 28.08) \frac{\text{Btu}}{\text{lb}} [60 \text{ min/h}]} = 1.071 \times 10^6 \frac{\text{lb}}{\text{min}}$$

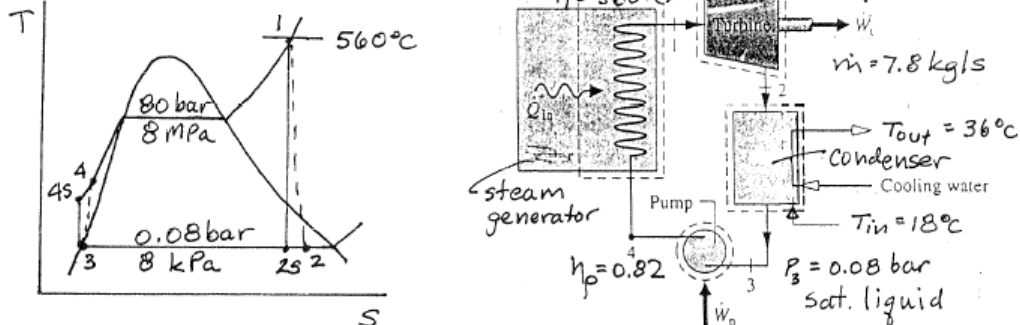
With $v_{\text{cw}} = v_f(60^\circ\text{F}) = 0.01604 \text{ ft}^3/\text{lb}$, $(\dot{V})_{\text{cw}} = v_{\text{cw}} \dot{m}_{\text{cw}} = 17,176 \text{ ft}^3/\text{min} \leftarrow (\dot{V})_{\text{cw}}$

PROBLEM 8.20

KNOWN: Water is the working fluid in a Rankine cycle. The condenser pressure and the turbine inlet state are specified. The mass flow rate of steam is given and the turbine and pump isentropic efficiencies are known. Temperature are given for cooling water entering and exiting the condenser.

FIND: Determine (a) the net power, (b) the thermal efficiency, and (c) the mass flow rate of cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as Problem 8.2, except $\eta_t = 0.88$ and $\eta_p = 0.82$. Also, we assume that the pressure drop is negligible for the cooling water passing through the condenser, and that $h \approx h_f(T)$.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 80 \text{ bar}$, $T_1 = 560^\circ\text{C} \Rightarrow h_1 = 3545.3 \text{ kJ/kg}$, $s_1 = 6.9072 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 0.08 \text{ bar}$, $s_{2s} = s_1 \Rightarrow x_{2s} = \frac{s_{2s} - s_{f2}}{s_{g2} - s_{f2}} = 0.8269$, $h_{2s} = 2161.0 \text{ kJ/kg}$

Using the isentropic turbine efficiency to get h_2

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 2327.1 \text{ kJ/kg}$$

State 3: $P_3 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 173.88 \text{ kJ/kg}$

State 4: $P_4 = 80 \text{ bar}$, $h_4 \approx h_3 + v_3(P_4 - P_3)$
 $= 173.88 \frac{\text{kJ}}{\text{kg}} + (1.0084 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (80 - 0.08) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$
 $= 181.94 \text{ kJ/kg}$

Using the isentropic pump efficiency to get h_4

$$\eta_p = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + (h_{4s} - h_3) / \eta_p = 183.58 \text{ kJ/kg}$$

(a) The net power developed is

$$\dot{W}_{\text{cycle}} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

$$= (7.8 \text{ kg/s})[(3545.3 - 2327.1) - (183.58 - 173.88)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 9426 \text{ kW} \leftarrow \dot{W}_{\text{cycle}}$$

(b) To find the thermal efficiency, first get $\dot{Q}_{\text{in}}$.

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = (7.8 \frac{\text{kg}}{\text{s}})(3545.3 - 183.58) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 26,221 \text{ kW}$$

$$\text{Thus } \eta = \dot{W}_{\text{cycle}} / \dot{Q}_{\text{in}} = 9426 / 26,221 = 0.359 (35.9\%) \leftarrow \eta$$

Continued on next slide

Problem 8-20 continued

(c) The mass flow rate of cooling water passing through the condenser is

$$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{h_{cw,in} - h_{cw,out}}$$

With $h_{cw} \approx h_f(T)$

$$\dot{m}_{cw} = \frac{(7.8 \text{ kg/s})(2327.1 - 173.88) \text{ kJ/kg}}{(150.86 - 75.58) \text{ kJ/kg}}$$

$$= 223.1 \text{ kg/s} \leftarrow$$

$\dot{m}_{cw}$

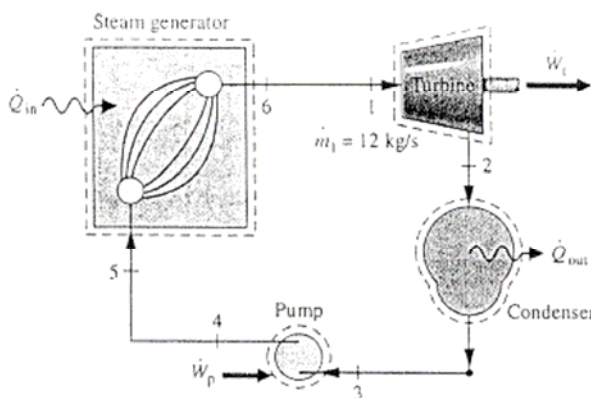
PROBLEM 8.21

KNOWN: A vapor power plant operates at steady state with water as the working fluid. Data are known at principal states in the cycle.

FIND: Determine the (a) thermal efficiency, and (b) the rates of heat transfer $\dot{Q}_{in}$ and $\dot{Q}_{out}$.

SCHEMATIC & GIVEN DATA:

State	p	$T(^{\circ}\text{C})$	h (kJ/kg)
1	6 MPa	500	3422.2
2	10 kPa	---	1633.3
3	10 kPa	Sat.	191.83
4	7.5 MPa	---	199.4
5	7 MPa	40	167.57
6	6 MPa	550	3545.3



ENGINEERING MODEL: (1) Each control volume operates at steady state, (2) The turbine and pump operate adiabatically, (3) Kinetic and potential energy effects can be neglected.

ANALYSIS: (a) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_6 - h_5)}$$

$$= \frac{(3422.2 - 1633.3) - (199.4 - 191.83)}{(3545.3 - 167.57)} = 0.527 (52.7\%) \leftarrow \eta$$

(b) For the steam generator

$$\textcircled{1} \quad \dot{Q}_{in} = \dot{m}(h_6 - h_5) = (12 \frac{\text{kg}}{\text{s}})(3545.3 - 167.57) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 4.054 \times 10^4 \text{ kW} \leftarrow \dot{Q}_{in}$$

For the condenser

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) = (12)(1633.3 - 191.83) = 1.73 \times 10^4 \text{ kW} \leftarrow \dot{Q}_{out}$$

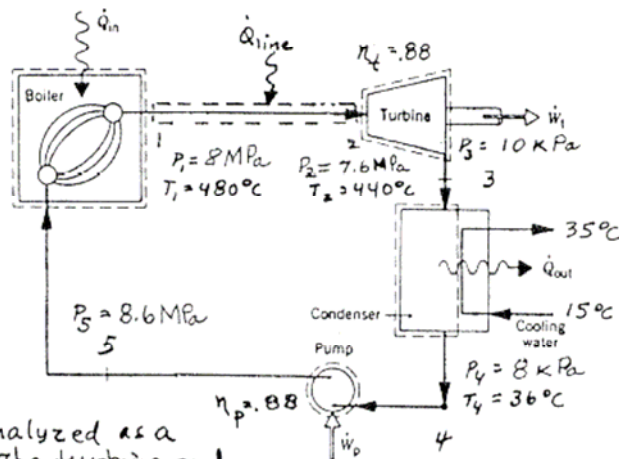
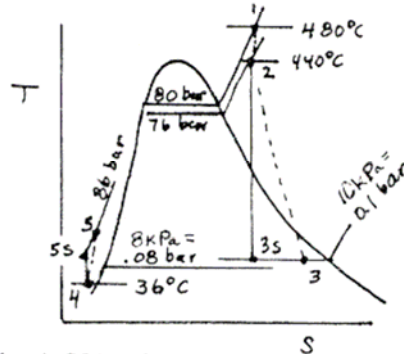
1. Note that stray heat loss from the piping between the pump and the steam generator and the steam generator and turbine result in more energy input ($\dot{Q}_{in}$) without additional power being developed.

PROBLEM 8.22

KNOWN: Water is the working fluid in a vapor power plant. Data are known at various locations, and the mass flow rate is given.

FIND: Determine (a) the net power output, (b) thermal efficiency, (c) the rate of heat transfer from the line connecting the steam generator and the turbine, and (d) the mass flow rate of condenser cooling water.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The turbine and pump operate adiabatically. (3) Kinetic & potential energy effects are negligible. (4) For the cooling water, assume $h_{cw} \approx h_f(T)$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 8 \text{ MPa}$, $T_1 = 480^\circ\text{C} \Rightarrow h_1 = 3348.4 \text{ kJ/kg}$, $s_1 = 6.6586 \text{ kJ/kg} \cdot \text{K}$

State 2: $p_2 = 7.6 \text{ MPa}$, $T_2 = 440^\circ\text{C} \Rightarrow$ interpolating in Table A-4;
 $h_2 = 3252.3 \text{ kJ/kg}$, $s_2 = 6.5526 \text{ kJ/kg} \cdot \text{K}$

State 3: State 3 is fixed using the turbine efficiency. First, at $p_3 = 10 \text{ kPa}$, $s_{3s} = s_2 = 6.5526 \Rightarrow x_{3s} = 0.787$; $h_{3s} = 2075.0 \text{ kJ/kg}$. Thus

$$\eta_t = \frac{(\dot{w}_t/\dot{m})}{(\dot{w}_t/\dot{m})_s} = \frac{h_2 - h_3}{h_2 - h_{3s}} \Rightarrow h_3 = h_2 - \eta_t(h_2 - h_{3s}) = 2216.3 \text{ kJ/kg}$$

Further, with $h_3 = 2216.3 \text{ kJ/kg}$; $x_3 = 0.8461 \Rightarrow s_3 = 6.9958 \text{ kJ/kg} \cdot \text{K}$

State 4: $p_4 = 8 \text{ kPa}$, $T_4 = 36^\circ\text{C} \Rightarrow h_4 \approx h_f(T_4) = 150.86 \text{ kJ/kg}$

State 5: $h_{5s} \approx h_4 + v_4(p_5 - p_4)$

$$= 150.86 + (1.0063 \times 10^{-3} \frac{\text{m}^3}{\text{kg}})(86 - 8 \text{ bars}) \left(\frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right) \left(\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right) = 150.86 + 8.646 = 159.51 \text{ kJ/kg}$$

Thus, using the pump efficiency

$$\eta_p = \frac{(\dot{w}_p/\dot{m})_s}{(\dot{w}_p/\dot{m})} = \frac{h_{5s} - h_4}{h_5 - h_4} \Rightarrow h_5 = h_4 + \frac{(h_{5s} - h_4)}{\eta_p} = 150.86 + \frac{(159.51 - 150.86)}{0.88} = 160.69 \text{ kJ/kg}$$

Continued on next slide

Problem 8-22 continued

(a) To determine the net power output, we use energy and mass balances for the control volumes surrounding the turbine and pump to get

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = \dot{m} [(h_2 - h_3) - (h_5 - h_4)]$$

Inserting values

$$\begin{aligned} \dot{W}_{\text{cycle}} &= (79.53 \frac{\text{kg}}{\text{s}}) [(3252.3 - 2216.3) - (160.69 - 150.86)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 8.161 \times 10^4 \text{ kW} \end{aligned} \quad \xleftarrow{\dot{W}_{\text{cycle}}}$$

(b) The thermal efficiency is

$$\begin{aligned} \eta &= \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{cycle}}}{\dot{m} (h_1 - h_5)} = \frac{8.161 \times 10^4}{(79.53)(3348.4 - 160.69)} \left| \frac{1}{1} \right| \\ &= 0.322 \text{ (32.2\%)} \end{aligned} \quad \xleftarrow{\eta}$$

(c) For a control volume enclosing the line connecting the steam generator and the turbine

$$0 = \dot{Q}_{\text{line}} + \dot{m} (h_1 - h_2)$$

$$\begin{aligned} \text{or } \dot{Q}_{\text{line}} &= \dot{m} (h_2 - h_1) = (79.53 \frac{\text{kg}}{\text{s}}) (3252.3 - 3348.4) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= -7643 \text{ kW} \end{aligned} \quad \xleftarrow{\dot{Q}_{\text{line}}}$$

(d) The mass flow rate of cooling water is found from

$$0 = \dot{Q}_{\text{cv}}^{\text{out}} - \dot{Q}_{\text{cv}}^{\text{in}} + \dot{m} (h_3 - h_4) + \dot{m}_{\text{cw}} (h_{\text{cw},\text{in}} - h_{\text{cw},\text{out}})$$

$$\text{or } \dot{m}_{\text{cw}} = \frac{\dot{m} (h_3 - h_4)}{(h_{\text{cw},\text{out}} - h_{\text{cw},\text{in}})}$$

From Table A-2, $h_{\text{cw},\text{out}} \approx h_f(35^\circ\text{C}) = 146.68 \text{ kJ/kg}$ and $h_{\text{cw},\text{in}} \approx h_f(15^\circ\text{C}) = 62.99 \text{ kJ/kg}$.
Inserting values

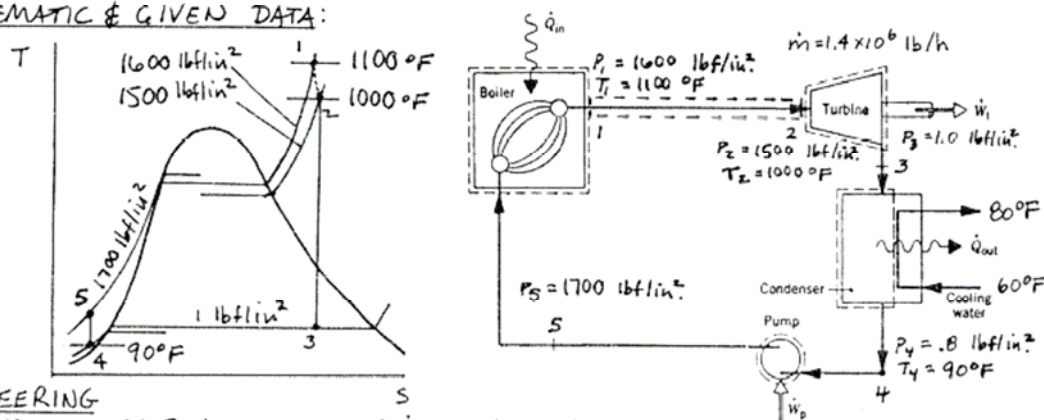
$$\dot{m}_{\text{cw}} = (79.53 \frac{\text{kg}}{\text{s}}) \frac{(2216.3 - 150.86)}{(146.68 - 62.99)} = 1963 \text{ kg/s} \quad \xleftarrow{\dot{m}_{\text{cw}}}$$

PROBLEM 8.23

KNOWN: The ideal Rankine cycle of Prob. 8.9 is modified to include the effects of friction in the line connecting the steam generator to the turbine, the condenser, and the steam generator.

FIND: Determine for the modified cycle (a) the net power developed, (b) the thermal efficiency, (c) the heat rate, and (d) the mass flow rate of cooling water in the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The turbine and pump operate adiabatically and reversibly. (3) Kinetic and potential energy effects are negligible. (4) For the cooling water, assume $h_{cw} \approx h_f(T)$.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 1600 \text{ lbf/in}^2$, $T_1 = 1100 \text{ °F} \Rightarrow h_1 = 1547.7 \text{ Btu/lb}$, $s_1 = 1.6315 \text{ Btu/lb} \cdot \text{°R}$

State 2: $P_2 = 1500 \text{ lbf/in}^2$, $T_2 = 1100 \text{ °F} \Rightarrow h_2 = 1490.3 \text{ Btu/lb}$, $s_2 = 1.6004 \text{ Btu/lb} \cdot \text{°R}$

State 3: $P_3 = 1.0 \text{ lbf/in}^2$, $s_3 = s_2 \Rightarrow x_3 = \frac{s_3 - s_f}{s_{fg}} = \frac{1.6004 - 0.1327}{1.8453} = 0.795$

and $h_3 = h_f + x_3 h_{fg} = 69.74 + (0.795)(1036.0) = 893.4 \text{ Btu/lb}$

State 4: $P_4 = 0.8 \text{ lbf/in}^2$, $T_4 = 90 \text{ °F} \Rightarrow h_4 \approx h_f(90 \text{ °F}) = 58.07 \text{ Btu/lb}$

$s_4 \approx s_f(90 \text{ °F}) = 0.1117 \text{ Btu/lb} \cdot \text{°R}$

$v_4 \approx v_f(90 \text{ °F}) = 0.01610 \text{ ft}^3/\text{lb}$

State 5: $h_5 \approx h_4 + v_4 (P_5 - P_4) = 58.07 \frac{\text{Btu}}{\text{lb}} + (0.01610 \frac{\text{ft}^3}{\text{lb}})(1699.2 \frac{\text{lbf}}{\text{in}^2}) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$
 $= 63.13 \text{ Btu/lb}$

(a) The net power developed is

$$\begin{aligned} \dot{W}_{\text{cycle}} &= \dot{W}_t - \dot{W}_p = \dot{m} [(h_2 - h_3) - (h_5 - h_4)] \\ &= (1.4 \times 10^6 \text{ lb/h}) [(1490.3 - 893.4) - (63.13 - 58.07)] \frac{\text{Btu}}{\text{lb}} \\ &= 8.286 \times 10^8 \text{ Btu/h} \end{aligned}$$

(b) The thermal efficiency is

$$\begin{aligned} \eta &= \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{cycle}}}{\dot{m} (h_1 - h_5)} = \frac{8.286 \times 10^8}{(1.4 \times 10^6)(1547.7 - 63.13)} \\ &= \frac{8.286 \times 10^8}{2.078 \times 10^9} = 0.3987 \text{ (39.87\%)} \end{aligned}$$

Continued on next slide

Problem 8-23 continued

(c) Using the definition on Pg. 376, the heat rate is

$$\left(\begin{array}{c} \text{heat} \\ \text{rate} \end{array} \right) = \frac{\dot{Q}_{in}}{W_{cycle}} = \frac{2.078 \times 10^9 \text{ Btu/h}}{8.286 \times 10^8 \text{ Btu/h}} \left| \frac{3413 \text{ Btu/h}}{1 \text{ kW}} \right|$$

$$= 8559 \text{ Btu/kW} \cdot \text{h} \quad \leftarrow \text{heat rate}$$

(d) Mass and energy balances for the control volume enclosing the condenser reduce as follows:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_3 - h_4) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

or

$$\dot{m}_{cw} = \dot{m} \left(\frac{h_3 - h_4}{h_{cw,out} - h_{cw,in}} \right)$$

From Table A-2E, $h_{cw,in} = h_f(60^\circ\text{F}) = 28.08 \text{ Btu/lb}$ and $h_{cw,out} = h_f(80^\circ\text{F}) = 48.09 \text{ Btu/lb}$.
Thus

$$\dot{m}_{cw} = (1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) \left(\frac{893.4 - 58.07}{48.09 - 28.08} \right)$$

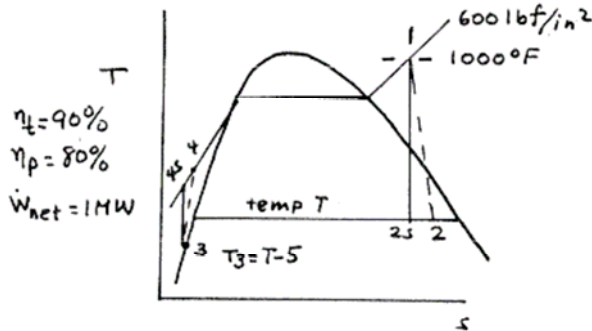
$$= 5.844 \times 10^7 \text{ lb/h} \quad \leftarrow \dot{m}_{cw}$$

PROBLEM 8.24

KNOWN: Steady-state operating data are provided for a vapor power plant in which steam exits the turbine as a two-phase liquid-vapor mixture at temperature T , and condensate exits the condenser at 5°F lower than T .

FIND: (a) For $T = 80^\circ\text{F}$, determine the steam quality at the turbine exit, the mass flow rate, and the cycle thermal efficiency. (b) Plot the quantities of (a) versus T ranging from 80 to 105°F .

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. Control volumes enclosing each of the principal components is at steady state.
2. The pump and turbine operate adiabatically.
3. Kinetic/potential energy can be ignored.
4. Condensate exits the condenser and enters the pump as sat. liquid.

ANALYSIS: First fix each of the principal states for $T = 80^\circ\text{F}$: From Table A-4E, $h_1 = 1517.8 \text{ Btu/lb}$, $s_1 = 1.7155 \text{ Btu/lb} \cdot \text{OR}$. Then, with $s_{2s} = s_1$ and data from Table A-2G

$$x_{2s} = \frac{s_1 - s_f}{s_{fg}} = \frac{1.7155 - 0.09332}{2.0356 - 0.09332} = 0.835 \Rightarrow h_{2s} = 48.09 + (0.835)(1048.3) = 923.4 \text{ Btu/lb}$$

Then, with

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1517.8 - 0.9(1517.8 - 923.4) = 982.8 \text{ Btu/lb}$$

$$\Rightarrow x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{982.8 - 48.09}{1048.3} = 0.892 \quad \leftarrow x_2$$

At state 3, $h_3 = h_f(75^\circ\text{F}) = 43.1 \text{ Btu/lb}$. Too, $h_{4s} \approx h_3 + v_3(P_4 - P_3)$, or

$$h_{4s} = 43.1 \frac{\text{Btu}}{\text{lb}} + (0.0161 \frac{\text{ft}^3}{\text{lb}})(600 - 0.43) \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 44.9 \text{ Btu/lb}$$

Then, with

$$\eta_p = \frac{h_4 - h_3}{h_{4s} - h_3} \Rightarrow h_4 = h_3 + \eta_p(h_{4s} - h_3) = 43.1 + (0.8)(44.9 - 43.1) = 45.4 \text{ Btu/lb}$$

The thermal efficiency is obtained as

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} = \frac{(1517.8 - 982.8) - (45.4 - 43.1)}{1517.8 - 45.4} = 0.362 \quad \leftarrow \eta$$

The mass flow rate is

$$\dot{m} = \frac{\dot{W}_{net}}{(\dot{W}_t/\dot{m}) - (\dot{W}_p/\dot{m})} = \frac{(103 \text{ kW}) \left| \frac{3413 \text{ Btu/h}}{1 \text{ kW}} \right|}{(532.7 \text{ Btu/lb})} = 6.41 \times 10^3 \frac{\text{lb}}{\text{h}} \quad \leftarrow \dot{m}$$

The data for the required plots are obtained using IT, as follows:

Continued on next slide

IT Code

```
p1 = 600 // lbf/in.2
T1 = 1000 // °F
T2 = 80
T3 = T2 - 5
p3 = p2
p4 = p1
efft = 0.9
effp = 0.8
Wdotcycle = 1 * 1000 * 3413 // Btu/h
```

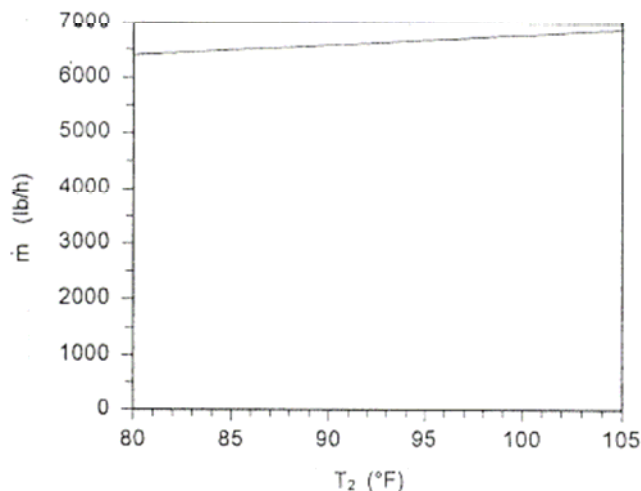
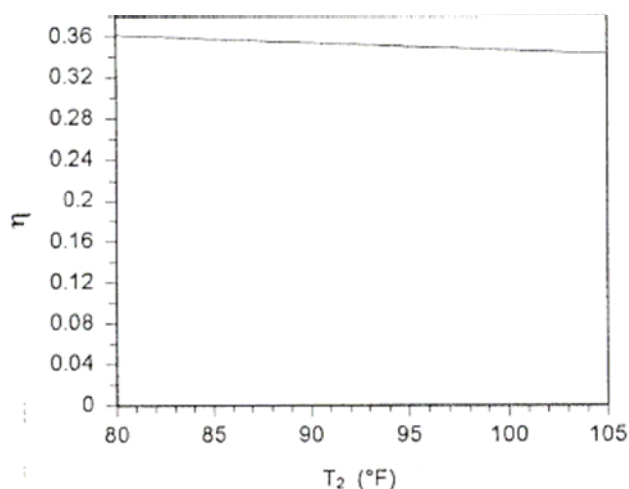
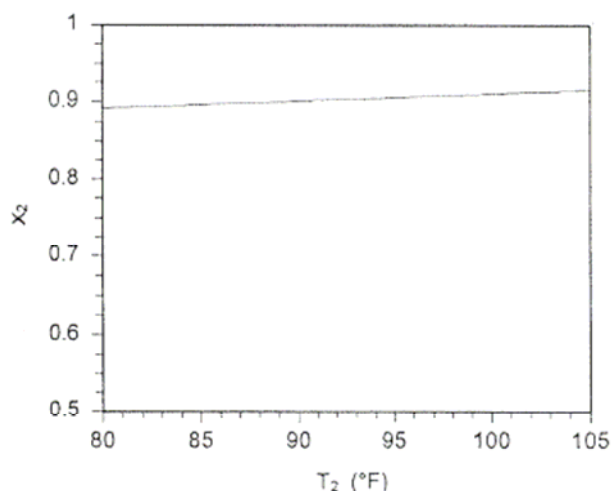
```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
p2 = Psat_T("Water/Steam", T2)
h2s = h_Ps("Water/Steam", p2, s1)
h2 = h1 - (h1 - h2s) * efft
x2 = x_hP("Water/Steam", h2, p2)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 0)
v3 = vsat_Px("Water/Steam", psat, 0)
h4 = h3 + v3 * (p4 - p3) * (144 / 778) /
effp
```

```
Wcycle = (h1 - h2) - (h4 - h3)
eta = Wcycle / (h1 - h4)
Wdotcycle = mdot * (Wcycle)
```

IT Results for $T_1 = 80^\circ\text{F}$

```
h1 = 1518 Btu/lb
h2 = 982.9 Btu/lb
h3 = 42.67 Btu/lb
h4 = 44.9 Btu/lb
x2 = 0.8919
 $\eta$  = 0.3616
 $\dot{m}$  = 6409 lb/h
```

PLOTS:



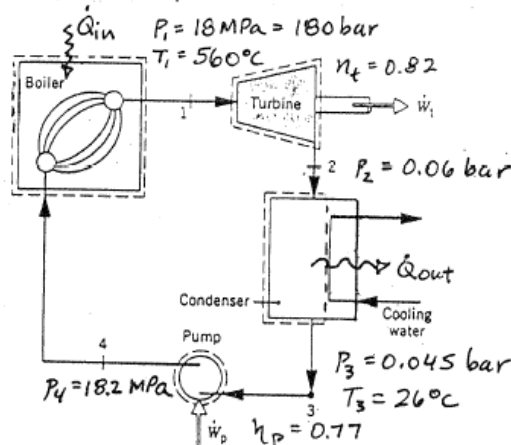
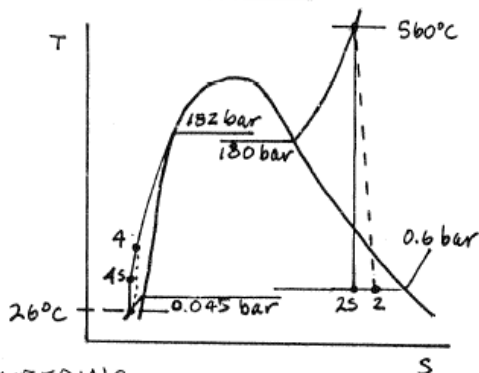
As the condenser temperature increases, the cycle thermal efficiency decreases, as expected. As a result, the turbine exit quality increases and the required mass flow rate increases.

PROBLEM 8.25

KNOWN: Water is the working fluid in a simple vapor power plant. Data are known at various locations.

FIND: Determine (a) the net work per unit mass of steam flow, (b) the heat transfer per unit mass of steam passing through the boiler, (c) the thermal efficiency, and (d) the heat transfer per unit mass of steam passing through the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each control volume shown is at steady state. (2) The turbine and the pump operate adiabatically. (3) Kinetic and potential energy effects can be neglected.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 180 \text{ bar}$, $T_1 = 560^\circ\text{C} \Rightarrow h_1 = 3444.4 \text{ kJ/kg}$, $s_1 = 6.4392 \text{ kJ/kg}\cdot\text{K}$

State 2: First, $P_2 = 0.06 \text{ bar}$, and $s_{2s} = s_1 \Rightarrow x_{2s} = 0.75783$, $h_{2s} = 1982.4 \text{ kJ/kg}$
Using the turbine efficiency, $\eta_t = (h_1 - h_2)/(h_1 - h_{2s})$. Thus
 $h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 2245.6 \text{ kJ/kg}$

State 3: From Table A-2, $P_3 > P_{\text{sat}} @ 26^\circ\text{C}$. Thus, state 3 is a sub-cooled liquid state. Since the pressure is low, $h_3 \approx h_f @ 26^\circ\text{C} = 109.07 \text{ kJ/kg}$ and $v_3 \approx v_f @ 26^\circ\text{C} = 1.0032 \times 10^{-3} \text{ m}^3/\text{kg}$.

State 4: First, $h_{4s} \approx h_3 + v_3(P_4 - P_3)$
 $= 109.07 \text{ kJ/kg} + (1.0032 \times 10^{-3} \frac{\text{m}^3}{\text{kg}})(18.2 - 0.045) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$
 $= 127.32 \text{ kJ/kg}$

Using the pump efficiency, $\eta_p = (h_{4s} - h_3)/(h_4 - h_3)$. Thus

$$h_4 = h_3 + (h_{4s} - h_3)/\eta_p = 132.78 \text{ kJ/kg}$$

(a) The net work per unit mass of steam flow is

$$\begin{aligned} \frac{\dot{W}_{\text{cycle}}}{\dot{m}} &= \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = (h_1 - h_2) - (h_4 - h_3) \\ &= (3444.4 - 2245.6) - (132.78 - 109.07) \\ &= 1175.1 \text{ kJ/kg} \leftarrow \frac{\dot{W}_{\text{cycle}}}{\dot{m}} \end{aligned}$$

(b) For control volume enclosing the steam passing through the boiler

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_1 - h_4$$

Continued on next slide

Problem 8-25 continued

Thus

$$\begin{aligned}\frac{\dot{Q}_{in}}{\dot{m}} &= (3444.4 - 132.78) \text{ kJ/kg} \\ &= 3311.6 \text{ kJ/kg} \leftarrow \frac{\dot{Q}_{in}}{\dot{m}}\end{aligned}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{1175.1}{3311.6} = 0.355 \text{ (35.5\%)} \leftarrow \eta$$

(d) The heat transfer to cooling water passing through the condenser per kg of steam condensed is

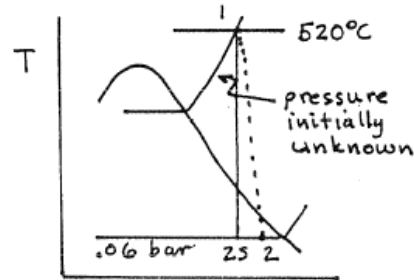
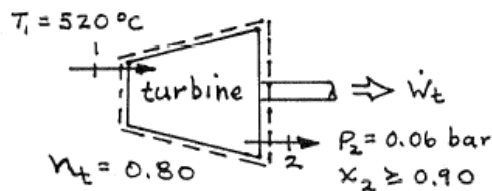
$$\begin{aligned}\frac{\dot{Q}_{out}}{\dot{m}} &= h_2 - h_3 = (2245.6 - 109.07) \text{ kJ/kg} \\ &= 2136.5 \text{ kJ/kg} \leftarrow \frac{\dot{Q}_{out}}{\dot{m}}\end{aligned}$$

PROBLEM 8.26

KNOWN: The turbine inlet temperature and condenser pressure are specified in the preliminary design of a vapor power plant with water as the working fluid. The turbine efficiency is also known.

FIND: Determine the steam generator pressure required if the quality at the turbine exit must be at least 90%.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state. (2) The turbine operates adiabatically. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: For fixed turbine inlet temperature, the value of turbine exit quality, x_2 , decreases as turbine inlet pressure p_1 is increased. A trial and error procedure can be used to find p_1 corresponding to $x_2 = 90\%$ using table data. Alternatively, the following IT code can be used to sweep p_1 from 60 bar to 120 bar and determine the corresponding values of x_2 .

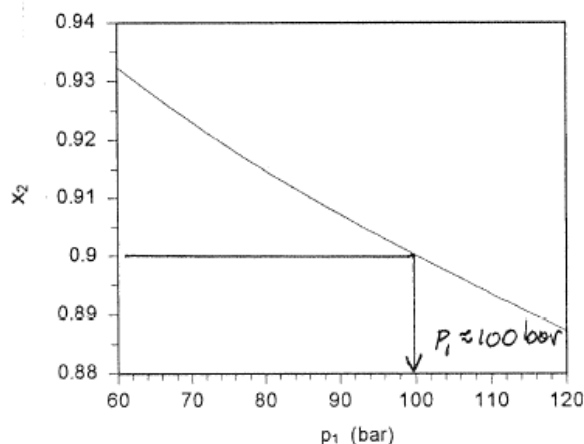
IT Code

```
T1 = 520 // °C
p2 = 0.06 // bar
p1 = 60
efft = 0.8
```

```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s1)
s2s = s1
h2 = h1 - efft * (h1 - h2s)
x2 = x_hP("Water/Steam", h2, p2)
```

IT Results

p_1 (bar)	60	80	100	120
h_1 (kJ/kg)	3469	3447	3425	3401
h_2 (kJ/kg)	2404	2361	2326	2295
h_{2s} (kJ/kg)	2137	2090	2051	2018
s_1 (kJ/kg·K)	6.94	6.786	6.661	6.555
s_{2s} (kJ/kg·K)	6.94	6.786	6.661	6.555
x_2	0.9324	0.9148	0.9001	0.8873



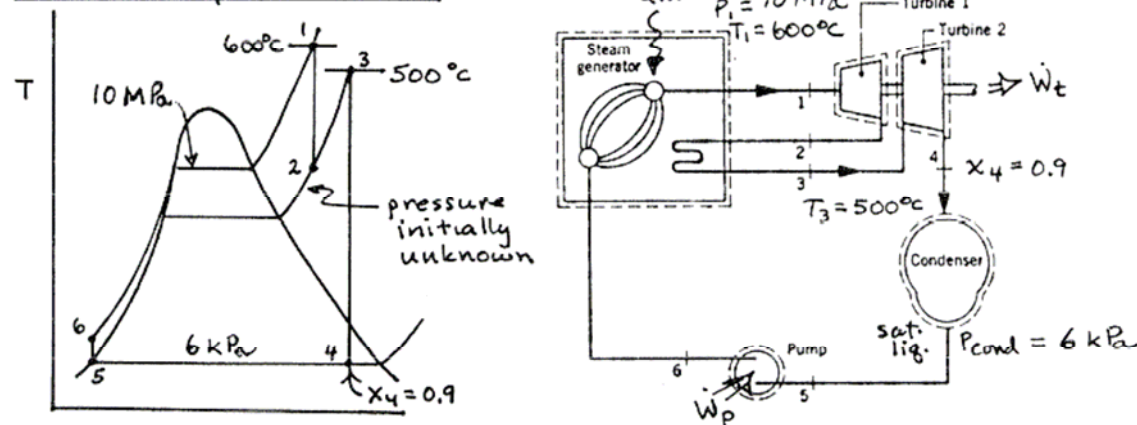
1. It is common practice to maintain at least 90% quality at the exit of a steam turbine to avoid undesirable effects.

PROBLEM 8.27

KNOWN: Water is the working fluid in an ideal Rankine cycle with reheat. Data are known at various locations.

FIND: Determine the thermal efficiency of the cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as in Example 8.3

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 10 \text{ MPa}$, $T_1 = 600^\circ\text{C} \Rightarrow h_1 = 3625.3 \text{ kJ/kg}$, $s_1 = 6.9029 \text{ kJ/kg}\cdot\text{K}$

Next, skip to **State 4:** $P_4 = 6 \text{ kPa}$, $x_4 = 0.9 \Rightarrow h_4 = 2325.8 \text{ kJ/kg}$
 $s_4 = 7.5495 \text{ kJ/kg}\cdot\text{K}$

State 3: $T_3 = 500^\circ\text{C}$, $s_3 = s_4 = 7.5495 \text{ kJ/kg}\cdot\text{K} \Rightarrow$ Interpolating in Table A-4; $P_3 \approx 15.735 \text{ bar}$, $h_3 \approx 3472.3 \text{ kJ/kg}$

Now, **State 2:** With $s_2 = s_1 = 6.9029 \text{ kJ/kg}$, $P_2 = P_3 = 15.735 \Rightarrow$ Double interpolation in Table A-4 gives $h_2 \approx 3040.5 \text{ kJ/kg}$

State 5: $P_5 = 6 \text{ kPa}$, sat. liquid $\Rightarrow h_5 = 151.53 \text{ kJ/kg}$

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5)$
 $= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (100 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$
 $= 151.53 + 10.06 = 161.59 \text{ kJ/kg}$

To determine the thermal efficiency, first evaluate the net work per unit mass of steam flow

$$\begin{aligned} \frac{\dot{W}_{\text{cycle}}}{\dot{m}} &= \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = (h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5) \\ &= (3625.3 - 3040.5) + (3472.3 - 2325.8) - (161.59 - 151.53) \\ &= 1721.2 \text{ kJ/kg} \end{aligned}$$

Also, the total heat transfer for the boiling/superheating, per unit mass of steam flow is

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = (h_1 - h_6) + (h_3 - h_2) = (3625.3 - 161.59) + (3472.3 - 3040.5) = 3895.5 \frac{\text{kJ}}{\text{kg}}$$

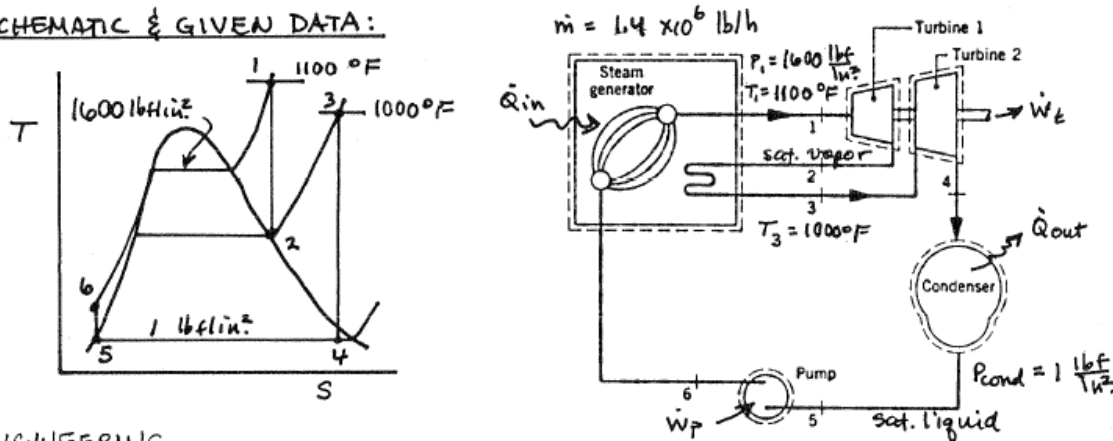
Thus $\eta = \frac{\dot{W}_{\text{cycle}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = \frac{1721.2}{3895.5} = 0.442 \text{ (44.2\%)} \leftarrow \eta$

PROBLEM 8.28

KNOWN: The ideal Rankine cycle of Problem 8.9 is modified to include reheat. The mass flow rate is the same as in Problem 8.9.

FIND: Determine for the modified cycle (a) the net power developed, (b) the rate of heat transfer to the working fluid in the reheat process, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Example 8.3.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 1600 \text{ lbf/in}^2$, $T_1 = 1100^\circ\text{F} \Rightarrow h_1 = 1547.7 \text{ Btu/lb}$, $s_1 = 1.6315 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2: $s_2 = s_1 = 1.6315 \text{ Btu/lb}\cdot^\circ\text{R}$, saturated vapor $\Rightarrow$ Interpolating in Table A-3E; $P_2 \approx 70.5 \text{ lbf/in}^2$, $h_2 \approx 1181.1 \text{ Btu/lb}$

State 3: $P_3 = 70.5 \text{ lbf/in}^2$, $T_3 = 1000^\circ\text{F} \Rightarrow h_3 = 1532.9 \text{ Btu/lb}$, $s_3 = 1.9605 \text{ Btu/lb}\cdot^\circ\text{R}$

State 4: $P_4 = 1 \text{ lbf/in}^2$, $s_4 = s_3 \Rightarrow x_4 = 0.9905$, $h_4 = 1095.9 \text{ Btu/lb}$

State 5: $P_5 = 1 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_5 = 69.74 \text{ Btu/lb}$

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5) = 74.52 \text{ Btu/lb}$ (see Problem 8.9)

(a) The net power developed is

$$\begin{aligned} \dot{W}_{\text{cycle}} &= \dot{m} [\dot{W}_t / \dot{m} - \dot{W}_p / \dot{m}] = \dot{m} [(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)] \\ &= (1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) [(1547.7 - 1181.1) + (1532.9 - 1095.9) - (74.52 - 69.74)] \frac{\text{Btu}}{\text{lb}} \\ &= 1.118 \times 10^9 \text{ Btu/h} \end{aligned}$$

(b) For the reheat process

$$\dot{Q}_{\text{reheat}} = \dot{m} (h_3 - h_2) = (1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) (1532.9 - 1181.1) \frac{\text{Btu}}{\text{lb}} = 4.93 \times 10^8 \frac{\text{Btu}}{\text{h}}$$

(c) The thermal efficiency is

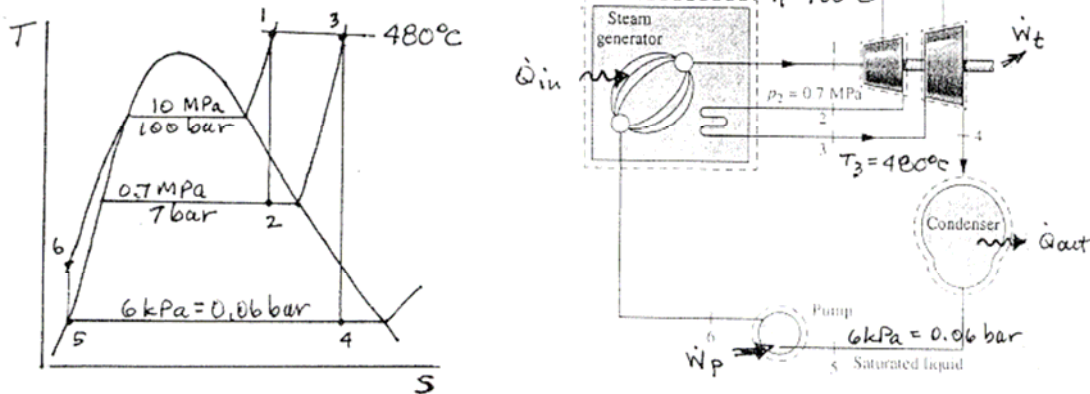
$$\begin{aligned} \eta &= \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{cycle}}}{\dot{m} [(h_1 - h_6) + (h_3 - h_2)]} = \frac{1.118 \times 10^9}{1.4 \times 10^6 [(1547.7 - 74.52) + (1532.9 - 1181.1)]} \\ &= 0.438 \text{ (43.8\%)} \end{aligned}$$

PROBLEM 8.29

KNOWN: Water is the working fluid in an ideal Rankine cycle with reheat. The states of the inlets to both turbine stages and the condenser exit are specified.

FIND: Determine (a) the rate of heat addition per kg of steam flowing, (b) the thermal efficiency, (c) the rate of heat transfer for the condenser per kg of steam condensing.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.3.

ANALYSIS: First, fix all of the principal states.

State 1: $p_1 = 100 \text{ bar}$, $T_1 = 480^\circ\text{C} \Rightarrow h_1 = 3321.4 \text{ kJ/kg}$, $s_1 = 6.5282 \text{ kJ/kg} \cdot \text{K}$

State 2: $p_2 = 7 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.9619$, $h_2 = 2684.8 \text{ kJ/kg}$

State 3: $p_3 = 7 \text{ bar}$, $T_3 = 480^\circ\text{C} \Rightarrow h_3 = 3438.9 \text{ kJ/kg}$, $s_3 = 7.8723 \text{ kJ/kg} \cdot \text{K}$

State 4: $p_4 = 0.06 \text{ bar}$, $s_4 = s_3 \Rightarrow x_4 = \frac{s_4 - s_{f4}}{s_{g4} - s_{f4}} = 0.9413$, $h_4 = 2425.6 \frac{\text{kJ}}{\text{kg}}$

State 5: $p_5 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_5 = 151.53 \text{ kJ/kg}$

State 6: $h_6 \approx h_5 + v_5(p_6 - p_5)$
 $= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.006 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (100 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$
 $= 151.53 + 10.06 = 161.59 \text{ kJ/kg}$

(a) For the control volume enclosing the steam generator

$$\dot{Q}_{\text{in}} = \dot{m} [(h_1 - h_6) + (h_3 - h_2)] \Rightarrow \dot{Q}_{\text{in}}/\dot{m} = (h_1 - h_6) + (h_3 - h_2)$$

$$\dot{Q}_{\text{in}}/\dot{m} = (3321.4 - 161.59) + (3438.9 - 2684.8)$$

$$= 3913.9 \text{ kJ/kg} \leftarrow \dot{Q}_{\text{in}}/\dot{m}$$

(b) The thermal efficiency is $\eta = \frac{\dot{W}_{\text{cycle}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}}$

$$\dot{W}_{\text{cycle}}/\dot{m} = (h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)$$

$$= 636.6 + 1013.3 - 10.06 = 1639.8 \text{ kJ/kg}$$

Thus

$$\eta = \frac{1639.8}{3913.9} = 0.419 \text{ (41.9 \%)} \leftarrow \eta$$

Continued on next slide

Problem 8-29 continued

(c) For the condenser

$$\dot{Q}_{\text{out}} = \dot{m} (h_4 - h_5) \Rightarrow \frac{\dot{Q}_{\text{out}}}{\dot{m}} = h_4 - h_5$$
$$= 2425.6 - 151.53 = 2274.1 \frac{\text{kJ}}{\text{kg}} \leftarrow \dot{Q}_{\text{out}}/\dot{m}$$

Alternatively

$$\textcircled{1} \quad \dot{Q}_{\text{out}}/\dot{m} = \dot{Q}_{\text{in}}/\dot{m} - \dot{w}_{\text{cycle}}/\dot{m}$$
$$= 3913.9 - 1639.8 = 2274.1 \text{ kJ/kg}$$

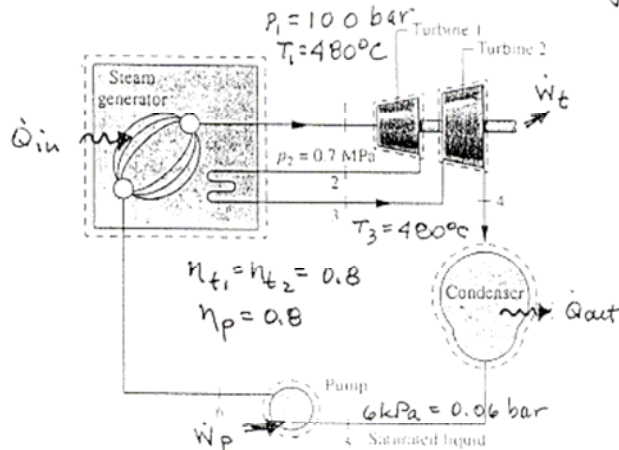
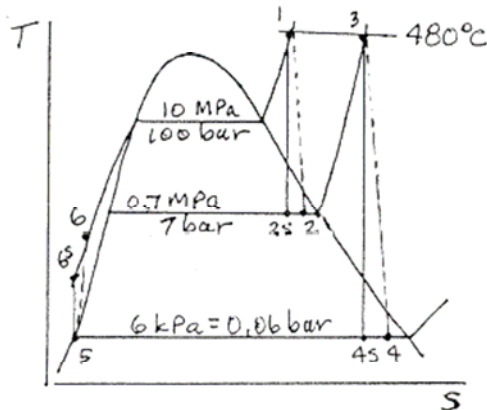
1. The results of this analysis can be compared to the results of Problem 8.2 to see some of the effects of incorporating reheat into the ideal Rankine cycle.

PROBLEM 8.30

KNOWN: The ideal Rankine cycle with reheat of Problem 8.29 is modified to have turbine stage and pump isentropic efficiencies of 80%.

FIND: Answer the same questions as in Problem 8.29 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.3, except $\eta_{t1} = \eta_{t2} = \eta_p = 0.8$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 10.0 \text{ bar}$, $T_1 = 480^\circ\text{C} \Rightarrow h_1 = 3321.4 \text{ kJ/kg}$, $s_1 = 6.5282 \text{ kJ/kg}\cdot\text{K}$

State 2: Using the isentropic efficiency of the first turbine stage

$$\eta_{t1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s})$$

With $h_{2s} = 2684.8 \text{ kJ/kg}$ from Problem 8.29, $h_2 = 2812.1 \text{ kJ/kg}$

State 3: $p_3 = 7 \text{ bar}$, $T_3 = 480^\circ\text{C} \Rightarrow h_3 = 3438.9 \text{ kJ/kg}$, $s_3 = 7.8723 \text{ kJ/kg}\cdot\text{K}$

State 4: For the second turbine stage, $h_4 = h_3 - \eta_{t2}(h_3 - h_{4s})$

With $h_{4s} = 2425.6 \text{ kJ/kg}$ from Problem 8.29, $h_4 = 2628.3 \text{ kJ/kg}$

State 5: $p_5 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_5 = 151.53 \text{ kJ/kg}$

State 6: Using the isentropic pump efficiency

$$\eta_p = \frac{h_6 - h_5}{h_{6s} - h_5} \Rightarrow h_6 = h_5 + (h_{6s} - h_5)/\eta_p$$

With $h_{6s} = 161.59 \text{ kJ/kg}$ from Problem 8.29, $h_6 = 164.11 \text{ kJ/kg}$

(a) The heat addition is

$$\dot{Q}_{in}/\dot{m} = (h_1 - h_6) + (h_3 - h_2) = 3786.6 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}$$

(b) The net power developed, per kg of steam flowing, is

$$W_{cycle}/\dot{m} = (h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5) = 1307.3 \text{ kJ/kg}$$

$$\text{The thermal efficiency is } \eta = \frac{W_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 0.345 (34.5\%) \leftarrow \eta$$

(c) For the condenser

$$\dot{Q}_{out}/\dot{m} = h_4 - h_5 = 2479.3 \text{ kJ/kg} \leftarrow \dot{Q}_{out}/\dot{m}$$

1. These results can be compared with those of Problem 8.29 to see some of the effects of turbine and pump irreversibilities on the performance of the Rankine cycle with reheat.

PROBLEM 8.31

(a) Refer to Problem 8.29.

IT Code

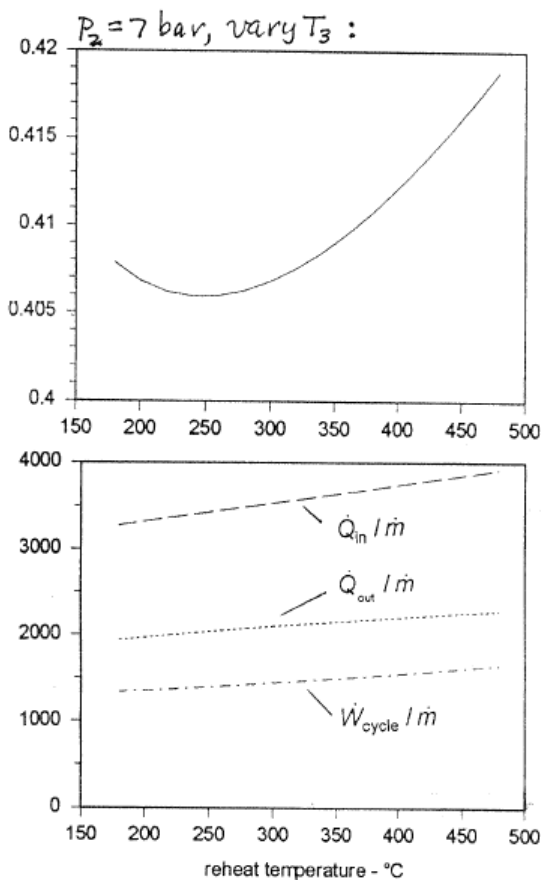
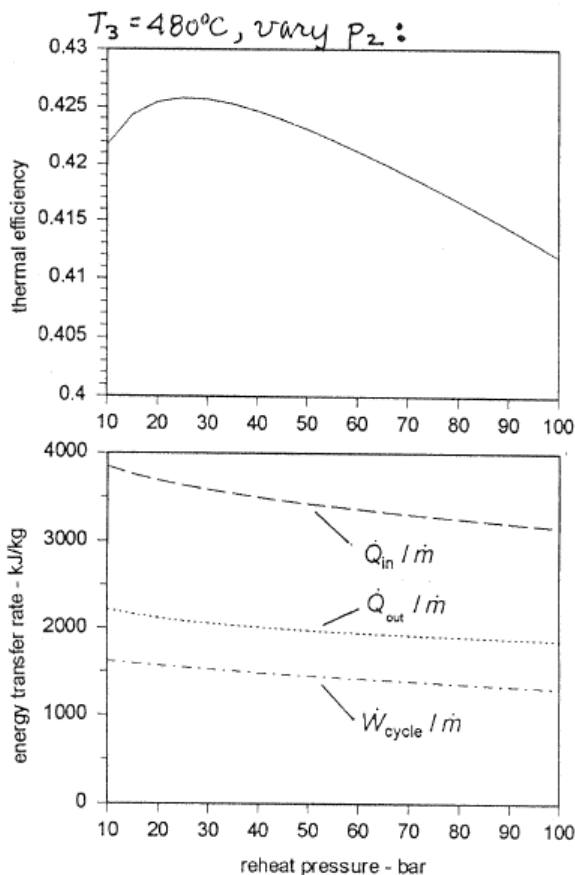
```
p1 = 100 // bar
T1 = 480 // °C
p2 = 7 // bar
p3 = p2
T3 = 480 // °C
p4 = 0.06 // bar
p5 = p4
p6 = p1
mdot = 1
```

```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = h_Ps("Water/Steam", p2, s1)
s2 = s1
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4 = h_Ps("Water/Steam", p4, s3)
s4 = s3
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6s = h5 + v5 * (p6 - p5) * 100
```

```
Wdott = mdot * ((h1 - h2) + (h3 - h4))
Wdotp = mdot * (h6 - h5)
Wdotcycle = Wdott - Wdotp
Qdotin = mdot * ((h1 - h6) + (h3 - h2))
eta = Wdotcycle / Qdotin
Qdotout = mdot * (h4 - h5)
```

IT Results ($p_2 = 7$ bar, $T_3 = 480$ °C)

```
Qin / ṁ = 3914 kJ/kg
Qout / ṁ = 2275 kJ/kg
Wcycle / ṁ = 1639 kJ/kg
η = 0.4188
h1 = 3321 kJ/kg
h2 = 2684 kJ/kg
h3 = 3438 kJ/kg
h4 = 2426 kJ/kg
h5 = 151 kJ/kg
h6 = 161.1 kJ/kg
```



Continued on next slide

Problem 8-31 continued

(b) Refer to Problem 8.30.

IT Code

```
p1 = 100 // bar
T1 = 480 // °C
p2 = 7 // bar
p3 = p2
T3 = 480 // °C
p4 = 0.06 // bar
p5 = p4
p6 = p1
etat1 = 0.8
etat2 = etat1
etap = etat1
mdot = 1
```

```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s1)
s2s = s1
h2 = h1 - etat1 * (h1 - h2s)
```

```
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4s = h_Ps("Water/Steam", p4, s3)
s4s = s3
h4 = h3 - etat2 * (h3 - h4s)
```

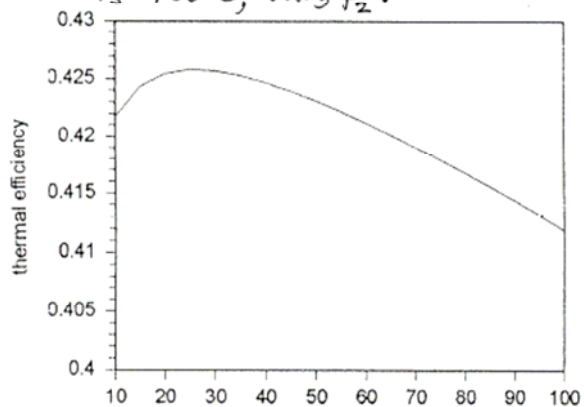
```
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6s = h5 + v5 * (p6 - p5) * 100
h6 = h5 + (h6s - h5) / etap
```

```
Wdott = mdot * ((h1 - h2) + (h3 - h4))
Wdotp = mdot * (h6 - h5)
Wdotcycle = Wdott - Wdotp
Qdotin = mdot * ((h1 - h6) + (h3 - h2))
eta = Wdotcycle / Qdotin
Qdotout = mdot * (h4 - h5)
```

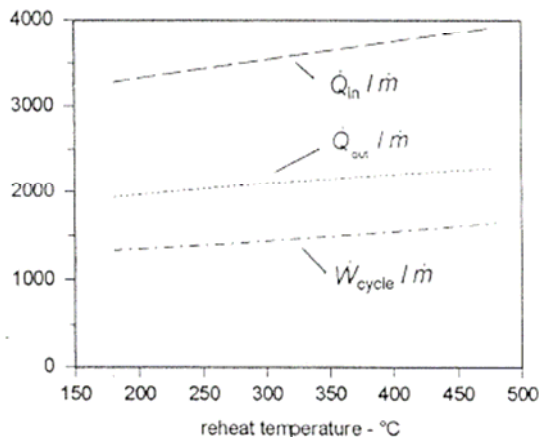
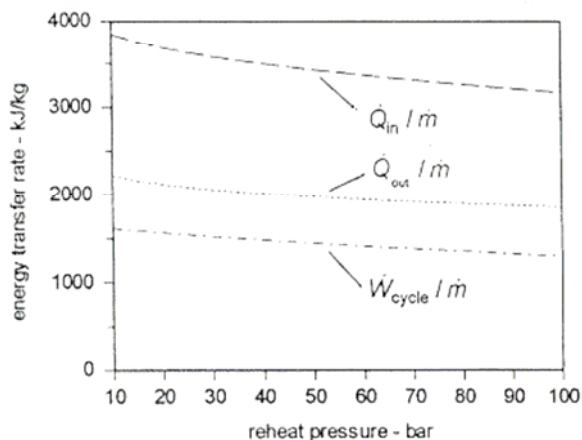
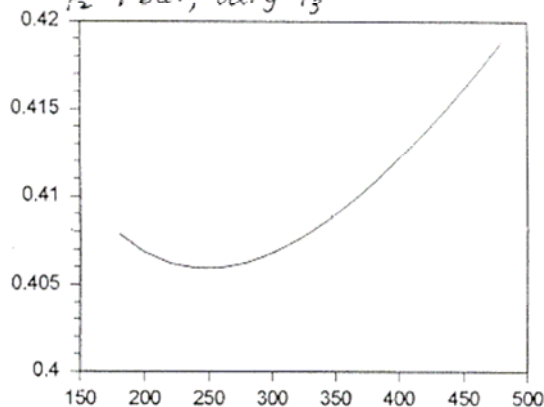
IT Results ($p_2 = 7$ bar, $T_3 = 480$ °C)

```
Qin / ṁ = 3784 kJ/kg
Qout / ṁ = 2477 kJ/kg
Wcycle / ṁ = 1307 kJ/kg
η = 0.3454
h1 = 3321 kJ/kg
h2 = 2812 kJ/kg
h3 = 3438 kJ/kg
h4 = 2628 kJ/kg
h5 = 151 kJ/kg
h6 = 163.6 kJ/kg
```

$T_3 = 480^\circ\text{C}$, vary p_2 :



$p_2 = 7$ bar, vary T_3 :

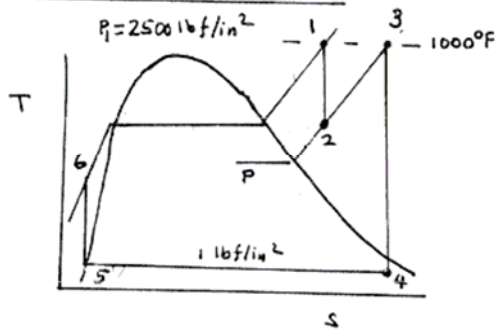


PROBLEM 8.32

KNOWN: Operating data are provided for an ideal Rankine cycle that has reheat at pressure p between the turbine stages.

FIND: (a) If $p/p_1 = 0.2$, where p_1 is the turbine inlet pressure, determine the cycle thermal efficiency and steam quality at the exit of the second turbine stage. (b) Plot the quantities of (a) versus p/p_1 ranging from 0.05 to 1.0.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. Control volumes enclosing each of the principal components are at steady state.
2. Each process is internally reversible, and the turbine stages and pump operate adiabatically.
3. Kinetic/potential energy can be ignored.

ANALYSIS: (a) For the case $p/p_1 = 0.2$, or $p = 500 \text{ lbf/in}^2$, the principal states are fixed as follows: From Table A-4E, $h_1 = 1457.2 \text{ Btu/lb}$, $s_1 = 1.5262 \text{ Btu/lb} \cdot \text{R}$. With $s_2 = s_1$, interpolation in Table A-4E at 500 lbf/in^2 gives $h_2 = 1264.9 \text{ Btu/lb}$.
 ① At state 3, $h_3 = 1520.7 \text{ Btu/lb}$, $s_3 = 1.7371 \text{ Btu/lb} \cdot \text{R}$. Then, with data from Table A-3E at 1 lbf/in^2 , and $s_4 = s_3$

$$x_4 = \frac{s_3 - s_f}{s_g - s_f} = \frac{1.7371 - 0.1327}{1.8453} = 0.8695 \Rightarrow h_4 = 69.74 + 0.8695(1036.0) = 970.5 \text{ Btu/lb}$$

From Table A-3E, $h_5 = 69.7 \text{ Btu/lb}$ and $h_6 \approx h_5 + v_5(p_6 - p_5)$, or

$$h_6 = 69.7 \frac{\text{Btu}}{\text{lb}} + 0.01614 \frac{\text{ft}^3}{\text{lb}} (2500 - 1) \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{\text{ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 77.2 \frac{\text{Btu}}{\text{lb}}$$

The thermal efficiency is given by

$$\eta = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)} = \frac{(1457.2 - 1264.9) + (1520.7 - 970.5) - (77.2 - 69.7)}{(1457.2 - 77.2) + (1520.7 - 1264.9)} = 0.449 \text{ (44.9\%)} \quad \leftarrow h$$

(b) The data for the required plots are obtained using IT, as follows:

IT Code

$p_1 = 2500 \text{ // lbf/in.}^2$

$T_1 = 1000 \text{ // } ^\circ\text{F}$

$\text{pratio} = p_2 / p_1$

$\text{pratio} = 0.2$

$p_3 = p_2$

$T_3 = 1000 \text{ // } ^\circ\text{F}$

$p_4 = 1 \text{ // lbf/in.}^2$

$p_5 = p_4$

$p_6 = p_1$

$h_1 = h_{\text{PT}}(\text{"Water/Steam"}, p_1, T_1)$

$s_1 = s_{\text{PT}}(\text{"Water/Steam"}, p_1, T_1)$

$h_2 = h_{\text{Ps}}(\text{"Water/Steam"}, p_2, s_1)$

$s_2 = s_1$

$h_3 = h_{\text{PT}}(\text{"Water/Steam"}, p_3, T_3)$

$s_3 = s_{\text{PT}}(\text{"Water/Steam"}, p_3, T_3)$

$h_4 = h_{\text{Ps}}(\text{"Water/Steam"}, p_4, s_3)$

$s_4 = s_3$

$h_5 = h_{\text{sat_Px}}(\text{"Water/Steam"}, p_5, 0)$

$v_5 = v_{\text{sat_Px}}(\text{"Water/Steam"}, p_5, 0)$

$h_6 = h_5 + v_5 \cdot (p_6 - p_5) \cdot (144 / 778)$

$x_4 = x_{\text{hP}}(\text{"Water/Steam"}, h_4, p_4)$

$\text{eta} = ((h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)) / ((h_1 - h_6) + (h_3 - h_2))$

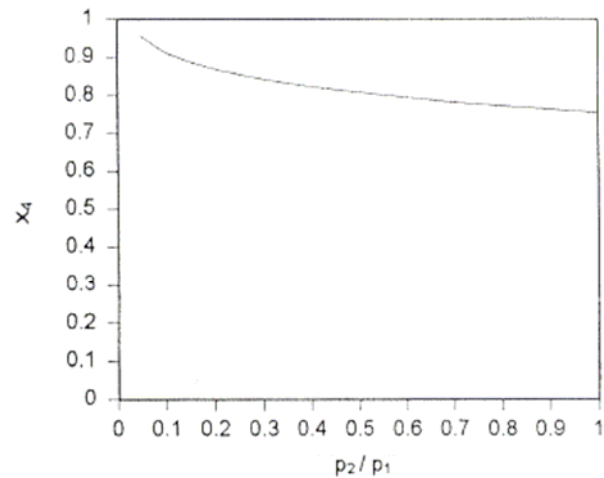
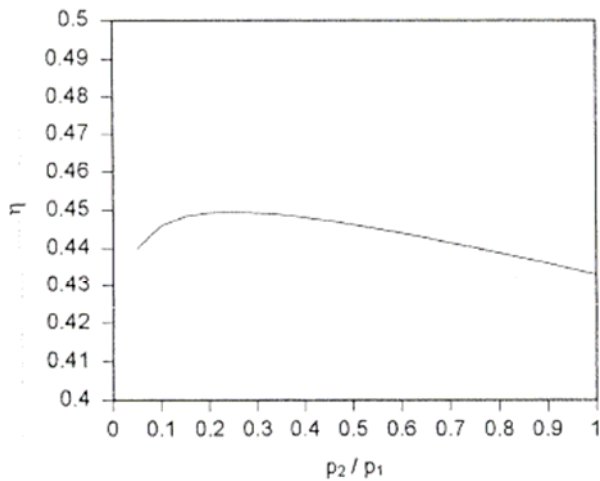
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Problem 8-32 continued

/T Results for $p_2/p_1 = 0.2$ ($p_2 = 500 \text{ lbf/in.}^2$)

$h_1 = 1457 \text{ Btu/lb}$
 $s_1 = 1.526 \text{ Btu/lb} \cdot ^\circ\text{R}$
 $h_2 = 1265 \text{ Btu/lb}$
 $h_3 = 1521 \text{ Btu/lb}$
 $s_3 = 1.737 \text{ Btu/lb} \cdot ^\circ\text{R}$
 $h_4 = 970.4 \text{ Btu/lb}$
 $h_5 = 69.58 \text{ Btu/lb}$
 $h_6 = 77.05 \text{ Btu/lb}$
 $v_5 = 0.01614 \text{ ft}^3/\text{lb}$
 $\eta = 0.4493$
 $x_4 = 0.8695$

Plots:



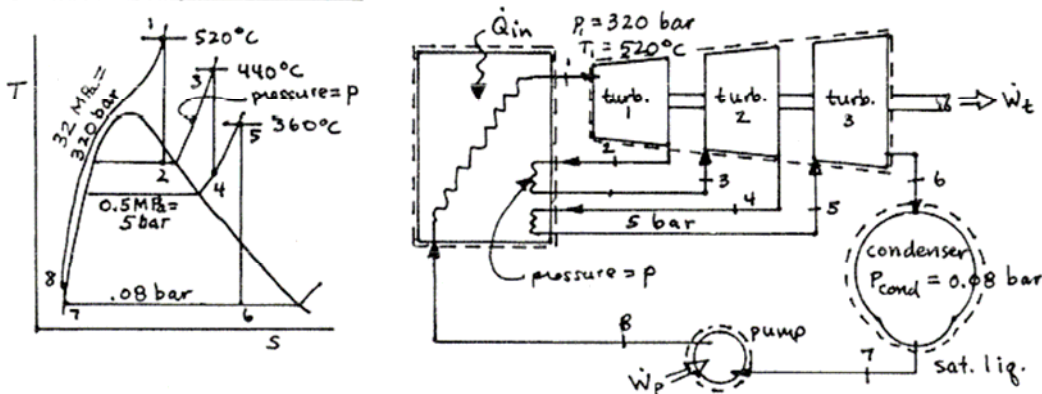
The plots show the thermal efficiency is affected by reheat pressure, with a maximum at $p_2/p \approx 0.2$ to 0.25 . Also, as the reheat pressure increases the quality at the turbine exit decreases, which is consistent with point 4 on the T-s diagram moving to the left.

PROBLEM 8.33

KNOWN: Water is the working fluid in an ideal Rankine cycle modified to include three turbine stages with reheat between the stages. Data are known at various locations. The pressure of the steam exiting the first-stage turbine is p .

FIND: (a) If $p = 4 \text{ MPa}$, determine the net work per unit of mass flowing and the thermal efficiency.
(b) Plot the quantities of part (a) versus p ranging from 0.5 to 1 MPa.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.3.

ANALYSIS: For $p = 4 \text{ MPa} = 40 \text{ bar}$. First, fix each of the principal states.

State 1: $P_1 = 320 \text{ bar}$, $T_1 = 520^\circ\text{C} \Rightarrow h_1 = 3133.7 \text{ kJ/kg}$, $s_1 = 5.8357 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 40 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = 0.9284$, $h_2 = 2678.7 \text{ kJ/kg}$

State 3: $P_3 = 40 \text{ bar}$, $T_3 = 440^\circ\text{C} \Rightarrow h_3 = 3307.1 \text{ kJ/kg}$, $s_3 = 6.9041 \text{ kJ/kg}\cdot\text{K}$

State 4: $P_4 = 5 \text{ bar}$, $s_4 = s_3 \Rightarrow h_4 = 2785.0 \text{ kJ/kg}$

State 5: $P_5 = 5 \text{ bar}$, $T_5 = 360^\circ\text{C} \Rightarrow h_5 = 3188.4 \text{ kJ/kg}$, $s_5 = 7.666 \text{ kJ/kg}\cdot\text{K}$

State 6: $P_6 = 0.08 \text{ bar}$, $s_6 = s_5 \Rightarrow x_6 = 0.92631$, $h_6 = 2399.9 \text{ kJ/kg}$

State 7: $P_7 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 173.88 \text{ kJ/kg}$

State 8: $h_8 \approx h_7 + v_7 (P_8 - P_7)$

$$= 173.88 \frac{\text{kJ}}{\text{kg}} + (1.0084 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (320 - 0.08) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$= 173.88 + 32.26 = 206.14 \text{ kJ/kg}$$

(a) To determine the net work per unit mass of steam flowing, begin with the turbine stages. That is

$$\frac{\dot{W}_t}{\dot{m}} = (h_1 - h_2) + (h_3 - h_4) + (h_5 - h_6)$$

$$= (3133.7 - 2678.7) + (3307.1 - 2785.0) + (3188.4 - 2399.9)$$

$$= 1765.6 \text{ kJ/kg}$$

And, for the pump

$$\frac{\dot{W}_p}{\dot{m}} = h_8 - h_7 = 32.26 \text{ kJ/kg}$$

Continued on next slide

Problem 8-33 continued

Thus

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = 1765.6 - 32.26 = 1733.3 \text{ kJ/kg} \leftarrow \frac{\dot{W}_{\text{cycle}}}{\dot{m}}$$

For the working fluid passing through the steam generator, the total heat transfer including the reheat processes is

$$\begin{aligned} \frac{\dot{Q}_{\text{in}}}{\dot{m}} &= (h_1 - h_8) + (h_3 - h_2) + (h_5 - h_4) \\ &= (3133.7 - 206.14) + (3307.1 - 2678.7) + (3188.4 - 2785.0) \\ &= 3959.4 \text{ kJ/kg} \end{aligned}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}} / \dot{m}}{\dot{Q}_{\text{in}} / \dot{m}} = \frac{1733.3}{3959.4} = 0.438 \text{ (43.8\%)} \leftarrow \eta$$

(b) The data for the required plots are obtained using IT, as follows:

IT Code

```
p1 = 320 // bar
T1 = 520 // °C
p2 = 40 // bar
p3 = p2
T3 = 440 // °C
p4 = 5 // bar
p5 = p4
T5 = 360 // °C
p6 = 0.08 // bar
p7 = p6
p8 = p1
```

// First turbine stage

```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s1)
s2s = s1
```

IT Results for $p_2 = 4 \text{ MPa (40 bar)}$

```
h1 = 3133 kJ/kg
s1 = 5.835 kJ/kg·K
h2s = 2678 kJ/kg
h3 = 3307 kJ/kg
s3 = 6.903 kJ/kg·K
h4s = 2784 kJ/kg
h5 = 3188 kJ/kg
s5 = 7.665 kJ/kg·K
h6s = 2399 kJ/kg
h7 = 173.6 kJ/kg
h8s = 205.9 kJ/kg
Wcycle / m = 1734 kJ/kg
Qin / m = 3960 kJ/kg
η = 0.4379 (43.79%)
```

// Second turbine stage

```
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4s = h_Ps("Water/Steam", p4, s3)
s4s = s3
```

// Third turbine stage

```
h5 = h_PT("Water/Steam", p5, T5)
s5 = s_PT("Water/Steam", p5, T5)
h6s = h_Ps("Water/Steam", p6, s5)
s6s = s5
```

// Pump

```
h7 = hsat_Px("Water/Steam", p7, 0)
v7 = vsat_Px("Water/Steam", p7, 0)
h8s = h7 + v7 * (p8 - p7) * 100
```

```
Wt = (h1 - h2s) + (h3 - h4s) + (h5 - h6s)
```

```
Wp = h8s - h7
```

```
Wcycle = Wt - Wp
```

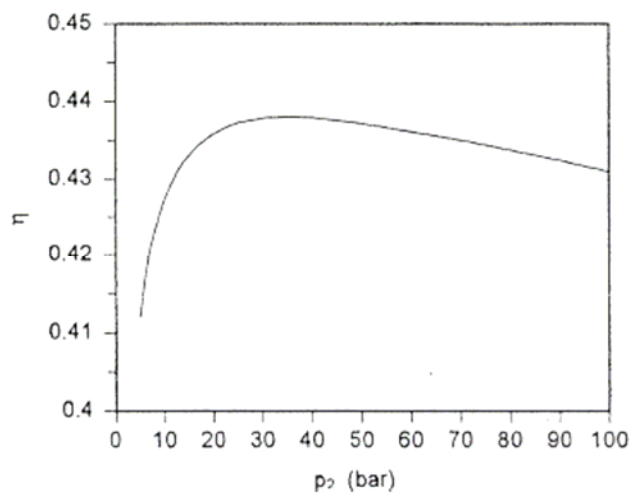
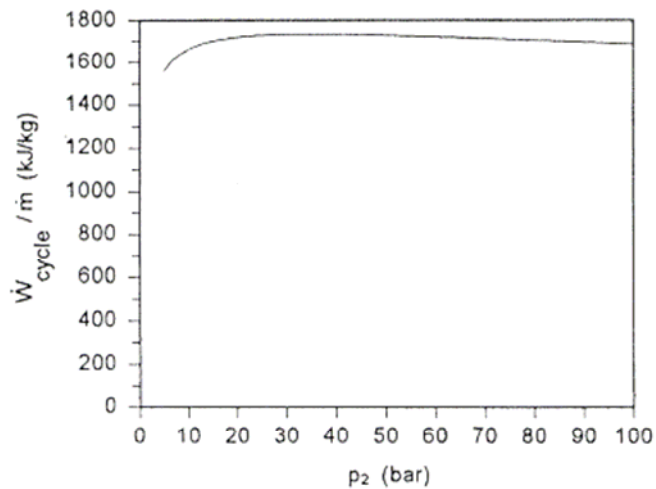
```
Qin = (h1 - h8s) + (h3 - h2s) + (h5 - h4s)
```

```
eta = Wcycle / Qin
```

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Problem 8-33 continued

PLOTS:



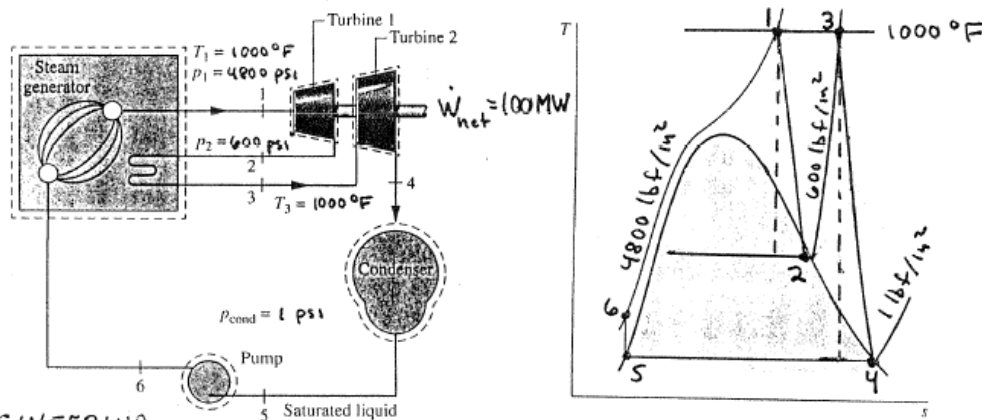
The plots show that the cycle net work and thermal efficiency are affected by the value of p_2 . Each plot exhibits a maximum.

PROBLEM 8.34

KNOWN: Water is the working fluid in an ideal Rankine cycle modified to include two turbine stages with reheat between the stages. The turbine and pump efficiencies are 85%.

FIND: (a) the rate of heat transfer to the working fluid passing through the steam generator, (b) the rate of heat transfer from the working fluid passing through the condenser, and (c) the cycle efficiency.

SCHEMATIC + GIVEN DATA:



ENGINEERING

MODEL: Same as in Example 8.3

ANALYSIS: First, fix each of the principle states.

State 1: $P_1 = 4800 \text{ lbf/in}^2$, $T_1 = 1000^\circ\text{F} \Rightarrow h_1 = 1317.4 \text{ Btu/lb}$, $s_1 = 1.4078 \text{ Btu/lb}^\circ\text{R}$

State 2: $P_2 = 600 \text{ lbf/in}^2$, $s_{2s} = s_1 \Rightarrow x_{2s} = 0.9500$, $h_{2s} = 1167.49 \text{ Btu/lb}$

$$\eta_t = 0.85 = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = 1189.98 \text{ Btu/lb}$$

State 3: $P_3 = 600 \text{ lbf/in}^2$, $T_3 = 1000^\circ\text{F} \Rightarrow h_3 = 1517.8 \text{ Btu/lb}$, $s_3 = 1.7155 \text{ Btu/lb}^\circ\text{R}$

State 4: $P_4 = 1 \text{ lbf/in}^2$, $s_{4s} = s_3 \Rightarrow x_{4s} = 0.8577$, $h_{4s} = 958.37 \text{ Btu/lb}$

$$\eta_t = 0.85 = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = 1042.28 \text{ Btu/lb}$$

State 5: $P_5 = 1 \text{ lbf/in}^2$, sat. liq. $\Rightarrow h_5 = 69.74 \text{ Btu/lb}$, $v_5 = 0.01614 \text{ ft}^3/\text{lb}$

State 6: $P_6 = 4800 \text{ lbf/in}^2$, $h_6 \approx h_5 + v_5(P_6 - P_5)/\eta_p$
 $h_6 = 69.74 \frac{\text{Btu}}{\text{lb}} + (0.01614) \frac{\text{ft}^3}{\text{lb}} \frac{(4800 - 1) \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{\text{ft}^2} \right| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}}}{1.0551} = 69.74 + 14.34 = 84.08 \frac{\text{Btu}}{\text{lb}}$

Next, determine the flow rate of the working fluid.

$$\dot{W}_{\text{net}} = \dot{W}_t - \dot{W}_p = \dot{m} [(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)]$$

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)} = \frac{(100 \text{ MW}) \left| \frac{1000 \text{ kJ/s}}{1 \text{ MW}} \right| \left| \frac{1 \text{ Btu}}{1.0551 \text{ kJ}} \right|}{[(1317.4 - 1189.98) + (1517.8 - 1042.28) - (84.08 - 69.74)] \frac{\text{Btu}}{\text{lb}}} = 161.02 \text{ lb/s}$$

(a) The rate of heat transfer to the working fluid in the steam generator is

$$\dot{Q}_{\text{in}} = \dot{m} (h_1 - h_6 + h_3 - h_2) = (161.02) \frac{\text{lb}}{\text{s}} (1317.4 - 84.08 + 1517.8 - 1189.98) \frac{\text{Btu}}{\text{lb}} = 2.514 \times 10^5 \frac{\text{Btu}}{\text{s}} \left| \frac{1.0551 \text{ kJ}}{1 \text{ Btu}} \right| \left| \frac{1 \text{ MW}}{1000 \text{ kJ/s}} \right| = 265.23 \text{ MW} \leftarrow \dot{Q}_{\text{in}}$$

Continued on next slide

Problem 8-34 continued

(b) The rate of heat transfer from the working fluid in the condenser is

$$\dot{Q}_{out} = \dot{m}(h_4 - h_5) = (161.02)(1042.28 - 69.74) \left| \frac{1.0551}{1000} \right|$$

① $\dot{Q}_{out} = 165.23 \text{ MW} \leftarrow \dot{Q}_{out}$

(c) The cycle thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{sg}} = \frac{100 \text{ MW}}{265.23 \text{ MW}} = 0.377 (37.7\%) \leftarrow \eta$$

1. The overall energy balance holds for the cycle:

$$\dot{Q}_{in} = \dot{W}_{net} + \dot{Q}_{out}$$

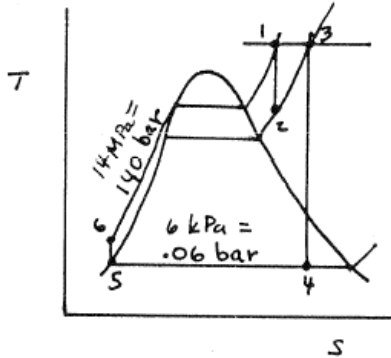
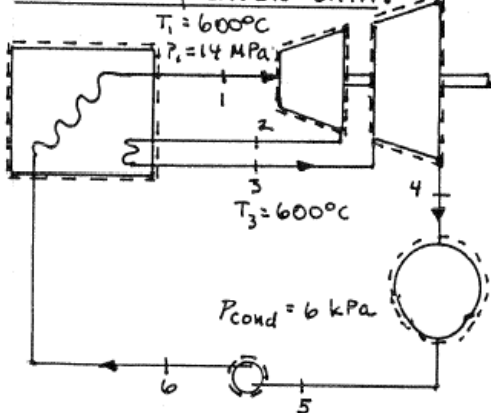
$$265.23 \text{ MW} = 100 \text{ MW} + 165.23 \text{ MW}$$

PROBLEM 8.35

KNOWN: An ideal reheat cycle operates with steam as the working fluid. Operating pressures and temperatures are given.

FIND: Plot thermal efficiency versus reheat pressure for pressures ranging from 20 bars to 120 bars.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.3

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 140 \text{ bar}$, $T_1 = 600^\circ\text{C} \Rightarrow h_1, s_1$

State 2: For a given P_2 , and with $s_2 = s_1$, we can get h_2 .

State 3: $P_3 = P_2$, $T_3 = 600^\circ\text{C} \Rightarrow h_3, s_3$

State 4: $P_4 = 0.06 \text{ bar}$, and with $s_4 = s_3$, we can get x_4 and h_4 .

State 5: $P_5 = 0.06 \text{ bar}$, sat. liq. $\Rightarrow h_5, v_5$

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5)$

Sample calculation: $P_2 = 2 \text{ MPa} = 20 \text{ bar}$. Using data from the steam tables, $h_1 = 3591.1 \text{ kJ/kg}$, $s_1 = 6.7172 \text{ kJ/kg}\cdot\text{K}$. Now, with $P_2 = 20 \text{ bar}$, $s_2 = s_1$, $h_2 = 2996.1 \text{ kJ/kg}$. At state 3, $h_3 = 3690.1 \text{ kJ/kg}$, $s_3 = 7.7024 \text{ kJ/kg}\cdot\text{K}$. Thus, $P_4 = 0.06 \text{ bar}$, $s_4 = s_3$ gives

$$x_4 = \frac{s_4 - s_f}{s_g - s_f} = \frac{7.7024 - 0.5210}{8.3304 - 0.5210} = 0.9196 \Rightarrow h_4 = 151.53 + (0.9196)(2415.9) = 2373.2 \text{ kJ/kg}$$

Now, $P_5 = 0.06 \text{ bar}$, sat. liq. $\Rightarrow h_5 = 151.53 \text{ kJ/kg}$, $v_5 = 1.0064 \times 10^{-3} \text{ m}^3/\text{kg}$. Finally

$$h_6 = h_5 + v_5(P_6 - P_5) = 151.53 + (1.0064 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (140 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 165.61 \text{ kJ/kg}$$

Cycle Calculations

The work per unit mass of steam flow is obtained as follows:

$$W_t/\dot{m} = (h_1 - h_2) + (h_3 - h_4) \quad (1)$$

$$W_p/\dot{m} = h_6 - h_5 \quad (2)$$

$$\text{and } W_{\text{cycle}}/\dot{m} = W_t/\dot{m} - W_p/\dot{m} \quad (3)$$

For the heat input in the steam generator and reheat processes

$$\dot{Q}_{\text{in}}/\dot{m} = (h_1 - h_6) + (h_3 - h_2) \quad (4)$$

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Problem 8-35 continued

The thermal efficiency is $\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}}$ (s)

Inserting data from the sample calculation into Eqs. (1)–(5) gives, for $p_2 = 20 \text{ bar}$

$$\dot{W}_t/\dot{m} = 1911.9 \text{ kJ/kg}, \dot{W}_p/\dot{m} = 14.08 \text{ kJ/kg} \Rightarrow \dot{W}_{\text{cycle}}/\dot{m} = 1897.8 \text{ kJ/kg}$$

$$\dot{Q}_{\text{in}}/\dot{m} = 4119.5 \text{ kJ/kg} \Rightarrow \eta = 0.4607 (46.07\%)$$

To obtain data for the required plot, we use IT, as follows

IT Code

```
p1 = 140 // bar
T1 = 600 // °C
p2 = 20 // bar
p3 = p2
T3 = 600 // °C
p4 = 0.06 // bar
p5 = p4
p6 = p1
```

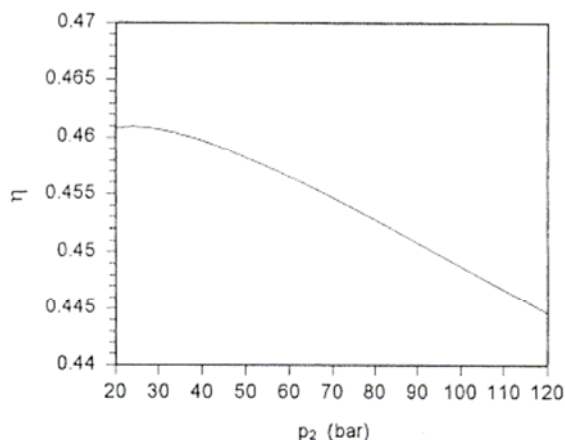
```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s1)
s2s = s1
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4s = h_Ps("Water/Steam", p4, s3)
s4s = s3
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6s = h5 + v5 * (p6 - p5) * 100
```

```
Wt = (h1 - h2s) + (h3 - h4s)
Wp = h6s - h5
Wcycle = Wt - Wp
Qin = (h1 - h6s) + (h3 - h2s)
eta = Wcycle / Qin
```

IT Results for $p_2 = 2 \text{ MPa (20 bar)}$

```
h1 = 3591 kJ/kg
s1 = 6.716 kJ/kg·K
h2s = 2995 kJ/kg
h3 = 3690 kJ/kg
s3 = 7.702 kJ/kg·K
h4s = 2373 kJ/kg
h5 = 151 kJ/kg
h6s = 165.1 kJ/kg
Wt / m = 1912 kJ/kg
Wp / m = 14.09 kJ/kg
Wcycle / m = 1898 kJ/kg
Qin / m = 4120 kJ/kg
eta = 0.4608
```

PLOT:



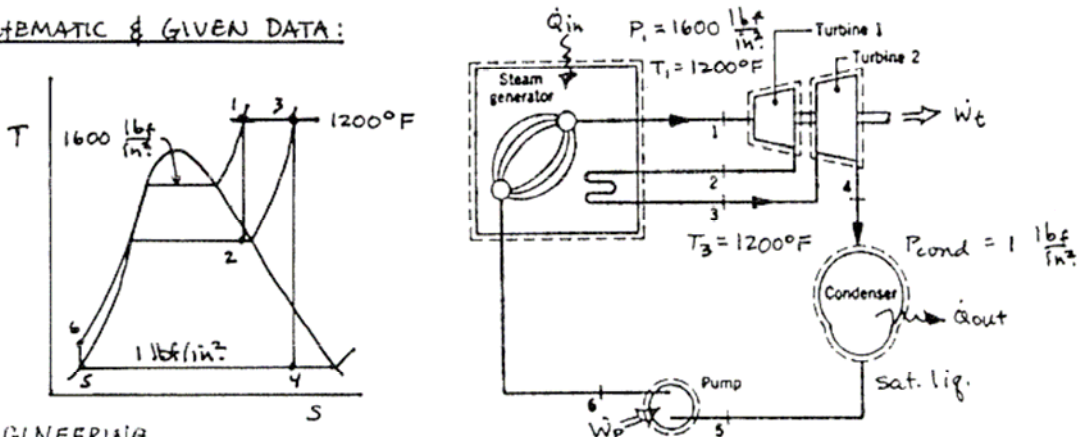
We see that the thermal efficiency exhibits a maximum value in the range of reheat pressures studied.

PROBLEM 8.36

KNOWN: An ideal reheat cycle uses steam as the working fluid. Operating pressures and temperatures are given.

FIND: Plot thermal efficiency versus reheat pressure for pressures ranging from 60 lbf/in² to 1200 lbf/in².

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: See Example 8.3

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 1600 \text{ lbf/in}^2$, $T_1 = 1200^\circ\text{F} \Rightarrow h_1, s_1$

State 2: For a given P_2 , and with $s_2 = s_1$, we can get h_2 .

State 3: $P_3 = P_2$, $T_3 = 1200^\circ\text{F} \Rightarrow h_3, s_3$

State 4: $P_4 = 1 \text{ lbf/in}^2$, and with $s_4 = s_3$, we can get x_4 and h_4 , or h_4 if superheated.

State 5: $P_5 = 1 \text{ lbf/in}^2$, sat. liq. $\Rightarrow h_5, v_5$

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5)$

Cycle calculations.

The work per unit mass of steam flow is

$$W_t/\dot{m} = (h_1 - h_2) + (h_3 - h_4) \quad (1)$$

$$W_p/\dot{m} = h_6 - h_5 \quad (2)$$

$$W_{\text{cycle}}/\dot{m} = W_t/\dot{m} - W_p/\dot{m} \quad (3)$$

The heat input to the steam generator and reheat process is

$$\dot{Q}_{\text{in}}/\dot{m} = (h_1 - h_6) + (h_3 - h_2) \quad (4)$$

$$\text{Finally, the thermal efficiency is } \eta = \frac{W_{\text{cycle}}}{\dot{Q}_{\text{in}}} \quad (5)$$

Sample calculation. $P_2 = 60 \text{ lbf/in}^2$ Using data from the steam tables,

$h_1 = 1607.1 \text{ Btu/lb}$, $s_1 = 1.6684 \text{ Btu/lb} \cdot ^\circ\text{R}$. Now, with $P_2 = 60 \text{ lbf/in}^2$, $s_2 = s_1$; $h_2 = 1196.7 \text{ Btu/lb}$. At State 3, $h_3 = 1638.5 \text{ Btu/lb}$, $s_3 = 2.0448 \text{ Btu/lb} \cdot ^\circ\text{R}$. Thus, $P_4 = 1 \text{ lbf/in}^2$, $s_4 = s_3$, gives $h_4 = 1146.3 \text{ Btu/lb}$. Now, $P_5 = 1 \text{ lbf/in}^2$, sat. liq. $\Rightarrow$

$h_5 = 69.74 \text{ Btu/lb}$, $v_5 = 0.01614 \text{ ft}^3/\text{lb}$. Finally

$$h_6 \approx h_5 + v_5(P_6 - P_5) = 69.74 + (0.01614) \left(\frac{\text{ft}^3}{\text{lb}} \right) (1600 - 1) \left(\frac{\text{lbf}}{\text{in}^2} \right) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= 74.52 \text{ lbf/in}^2$$

Inserting data into Eqs. (1)–(5) gives, for $P_2 = 60 \text{ lbf/in}^2$

Continued on next slide

Problem 8-36 continued

$$\dot{W}_t/\dot{m} = 902.6 \text{ Btu/lb}, \dot{W}_p/\dot{m} = 4.78 \text{ Btu/lb} \Rightarrow \dot{W}_{\text{cycle}}/\dot{m} = 897.8 \text{ Btu/lb}$$

$$\dot{Q}_{\text{in}}/\dot{m} = 1974.4 \text{ Btu/lb} \Rightarrow \eta = 0.4547 \text{ (45.47\%)}$$

To obtain data for the required plots, we use IT, as follows:

IT Code

```
p1 = 1600 // lbf/in.2
T1 = 1200 // °F
p2 = 60 // lbf/in.2
p3 = p2
T3 = 1200 // °F
p4 = 1 // lbf/in.2
p5 = p4
p6 = p1

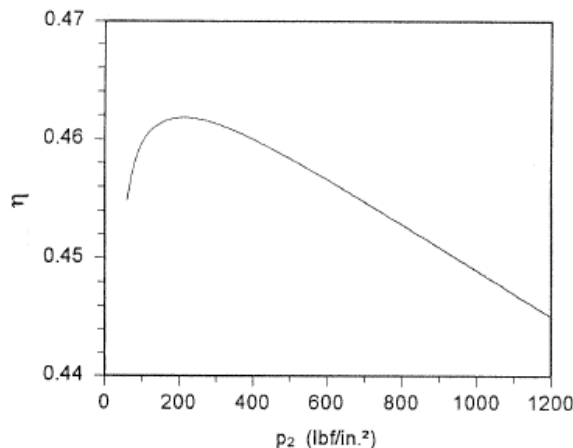
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s1)
s2s = s1
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4s = h_Ps("Water/Steam", p4, s3)
s4s = s3
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6s = h5 + v5 * (p6 - p5) * (144 / 778)

Wt = (h1 - h2s) + (h3 - h4s)
Wp = h6s - h5
Wcycle = Wt - Wp
Qin = (h1 - h6s) + (h3 - h2s)
eta = Wcycle / Qin
```

IT Results for $p_2 = 60 \text{ lbf/in.}^2$

```
h1 = 1607 Btu/lb
s1 = 1.668 Btu/lb·°R
h2s = 1196 Btu/lb
h3 = 1638 Btu/lb
s3 = 2.045 Btu/lb·°R
h4s = 1146 Btu/lb
h5 = 69.58 Btu/lb
h6s = 74.36 Btu/lb
Wt / ṁ = 902.8 Btu/lb
Wp / ṁ = 4.776 Btu/lb
Wcycle / ṁ = 898.1 Btu/lb
Qin / ṁ = 1975 Btu/lb
η = 0.4548
```

PLOT:



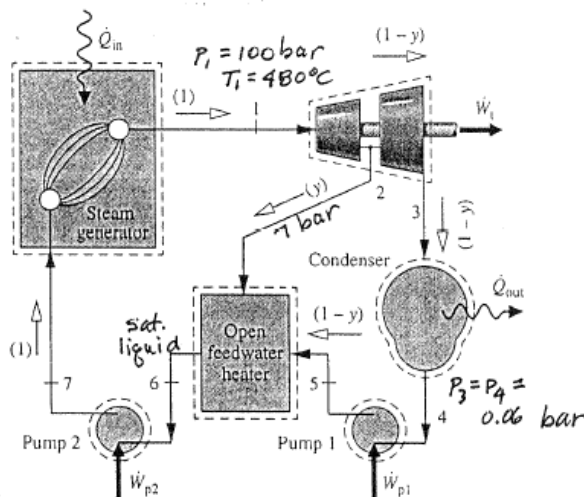
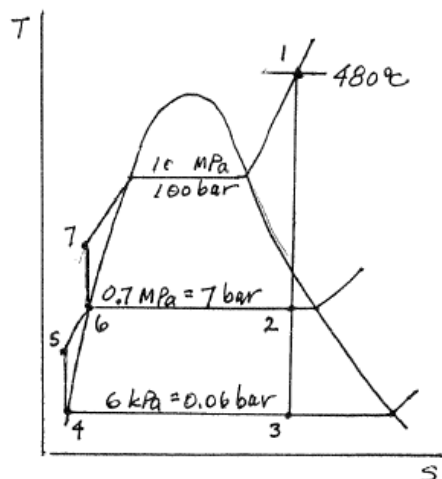
We see that the thermal efficiency exhibits a maximum in the range of reheat pressures studied.

PROBLEM 8.37

KNOWN: Water is the working fluid in an ideal regenerative Rankine cycle with one open feedwater heater. Data at various locations are known.

FIND: Determine (a) the rate of heat addition per kg of steam entering the first-stage turbine, (b) the thermal efficiency, and (c) the rate of heat transfer for the condenser per kg of steam entering the first-stage turbine.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as Example 8.5, except turbine stages and pumps operate in an internally reversible manner.

ANALYSIS: First, fix each principal state.

State 1: $p_1 = 100 \text{ bar}$, $T_1 = 480^\circ\text{C} \Rightarrow h_1 = 3321.4 \text{ kJ/kg}$, $s_1 = 6.5282 \text{ kJ/kg}\cdot\text{K}$

State 2: $p_2 = 7 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.9619$, $h_2 = 2684.8 \text{ kJ/kg}$

State 3: $p_3 = 0.06 \text{ bar}$, $s_3 = s_2 \Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.7692$, $h_3 = 2009.8 \text{ kJ/kg}$

State 4: $p_4 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 151.53 \text{ kJ/kg}$

State 5: $h_5 \approx h_4 + v_4(p_5 - p_4) = 151.53 + (1.0064 \times 10^{-3} \text{ m}^3/\text{kg})(7 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 151.53 + 0.698 = 152.23 \text{ kJ/kg}$

State 6: $p_6 = 7 \text{ bar}$, sat. liquid $\Rightarrow h_6 = 697.22 \text{ kJ/kg}$

State 7: $h_7 \approx h_6 + v_6(p_7 - p_6) = 697.22 + (1.1080 \times 10^{-3})(100 - 7) \left| \frac{10^5}{10^3} \right| = 707.52 \text{ kJ/kg}$

(a) For the steam generator

$$\dot{Q}_{in}/\dot{m}_1 = h_1 - h_7 = (3321.4 - 707.52) = 2613.9 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}$$

(b) Applying mass and energy balances to the control volume enclosing the open feedwater heater

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{697.22 - 152.23}{2684.8 - 152.23} = 0.2152$$

For the control volume enclosing the turbine stages

$$\begin{aligned} \frac{\dot{W}_t}{\dot{m}_1} &= (h_1 - h_2) + (1 - y)(h_2 - h_3) \\ &= (3321.4 - 2684.8) + (1 - 0.2152)(2684.8 - 2009.8) = 1166.3 \text{ kJ/kg} \end{aligned}$$

Continued on next slide

Problem 8-37 continued

For the pumps

$$\dot{W}_P = \dot{W}_{P1} + \dot{W}_{P2} = \dot{m}_1 [(1-y)(h_5 - h_4) + (h_7 - h_6)]$$

$$\begin{aligned} \text{or } \dot{W}_P / \dot{m}_1 &= (1-y)(h_5 - h_4) + (h_7 - h_6) \\ &= (1 - 0.2152)(152.23 - 151.53) + (707.52 - 697.22) \\ &= 10.85 \text{ kJ/kg} \end{aligned}$$

Thus, the net power developed, per unit mass entering the first-stage turbine is

$$\dot{W}_{\text{cycle}} / \dot{m}_1 = \dot{W}_t / \dot{m}_1 - \dot{W}_P / \dot{m}_1 = 1166.3 - 10.85 = 1155.3 \text{ kJ/kg}$$

And the thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}} / \dot{m}_1}{\dot{Q}_{\text{in}} / \dot{m}_1} = \frac{1155.3}{2613.9} = 0.442 \text{ (44.2\%)} \quad \leftarrow \eta$$

(c) For the condenser

$$\dot{Q}_{\text{out}} = \dot{m}_1 (1-y)(h_3 - h_4)$$

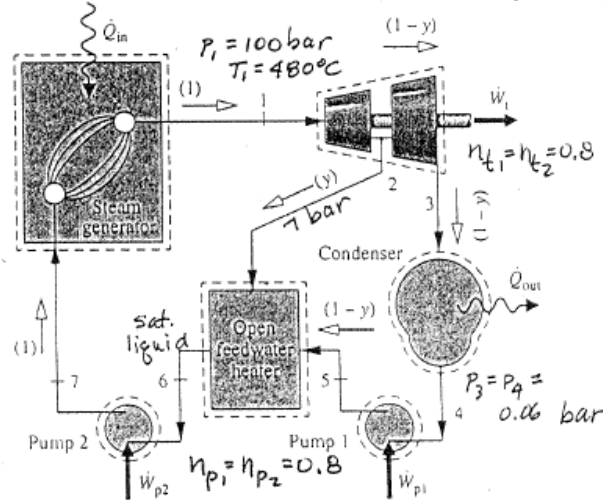
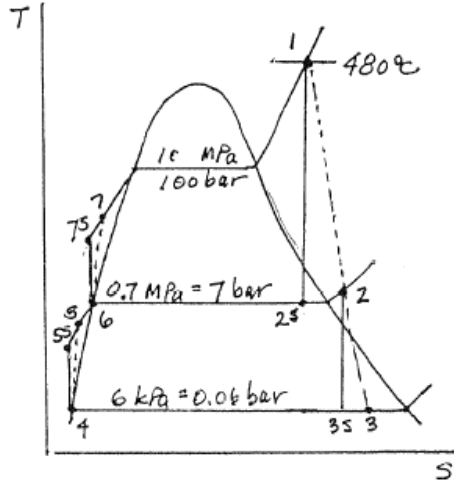
$$\begin{aligned} \text{or } \dot{Q}_{\text{out}} / \dot{m}_1 &= (1-y)(h_3 - h_4) = (1 - 0.2152)(2009.8 - 151.53) \\ &= 1458.4 \text{ kJ/kg} \quad \leftarrow \dot{Q}_{\text{out}} / \dot{m}_1 \end{aligned}$$

PROBLEM 8.38

KNOWN: The ideal regenerative Rankine cycle of Problem 8.37 is modified to include turbine stage and pump isentropic efficiencies of 0.8.

FIND: Answer the same questions as in Problem 8.37 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as Example 8.5.

ANALYSIS: First, fix each principal state.

State 1: $P_1 = 100 \text{ bar}$, $T_1 = 480^\circ\text{C} \Rightarrow h_1 = 3321.4 \text{ kJ/kg}$, $s_1 = 6.5282 \text{ kJ/kg}\cdot\text{K}$

State 2: using the isentropic efficiency of the first turbine stage

$$\eta_{t1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s})$$

With $h_{2s} = 2684.8 \text{ kJ/kg}$ from Problem 8.37, $h_2 = 2812.1 \text{ kJ/kg}$

State 3: $P_3 = 0.06 \text{ bar}$, $s_{3s} = s_2$. To get s_2 , interpolate in Table A-3: $s_2 = 6.810 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

$$\text{Thus } x_{3s} = \frac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.8061; h_{3s} = 2099.0 \text{ kJ/kg}$$

Using the second-stage turbine isentropic efficiency

$$h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 2241.6 \text{ kJ/kg}$$

State 4: $P_4 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 181.53 \text{ kJ/kg}$

State 5: using the isentropic pump efficiency; $\eta_p = (h_{5s} - h_4)/(h_5 - h_4)$

$$\text{Thus } h_5 = h_4 + (h_{5s} - h_4)/\eta_{p1} = 152.41 \text{ kJ/kg}$$

State 6: $P_6 = 7 \text{ bar}$, sat. liquid $\Rightarrow h_6 = 697.22 \text{ kJ/kg}$

State 7: with $\eta_{p2} = (h_{7s} - h_6)/(h_7 - h_6)$; $h_7 = h_6 + (h_{7s} - h_6)/\eta_{p2} = 710.1 \text{ kJ/kg}$

$$(a) \dot{Q}_{in}/\dot{m}_1 = h_1 - h_2 = 2611.3 \text{ kJ/kg} \leftarrow \frac{\dot{Q}_{in}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1}$$

$$(b) y = \frac{h_6 - h_5}{h_2 - h_5} = 0.2048 \text{ and } \dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1-y)(h_2 - h_3) = 962.96 \text{ kJ/kg}$$

$$\text{For the pumps, } \dot{W}_p/\dot{m}_1 = (1-y)(h_5 - h_4) + (h_7 - h_6) = 13.58 \text{ kJ/kg}$$

$$\text{And } \dot{W}_{cycle}/\dot{m}_1 = \dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1 = 949.38 \text{ kJ/kg}$$

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{949.3}{2611.3} = 0.364 \text{ (36.4 \%)} \leftarrow \eta$$

$$(c) \dot{Q}_{out}/\dot{m}_1 = (1-y)(h_3 - h_4) = 1662 \text{ kJ/kg} \leftarrow \frac{\dot{Q}_{out}/\dot{m}_1}{\dot{Q}_{out}/\dot{m}_1}$$

1. These results can be compared with those of Problem 8.37 to see some of the effects of turbine and pump irreversibilities on cycle performance.

PROBLEM 8.39

(a) Refer to Problem 8.37.

IT Code

```
p1 = 100 // bar
T1 = 480 // °C
p2 = 7 // bar
p3 = 0.06 // bar
p4 = p3
p5 = p2
p6 = p5
p7 = p1
mdot1 = 1
```

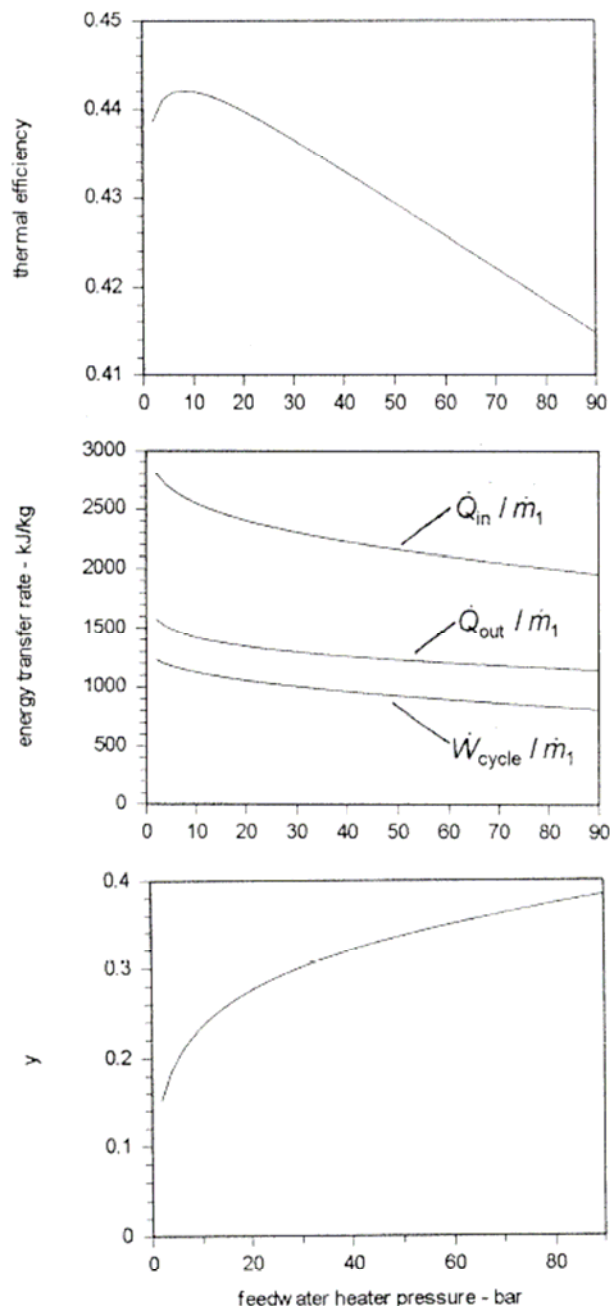
```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s1)
s2s = s1
s3s = s2s
h3s = h_Ps("Water/Steam", p3, s3s)
h4 = hsat_Px("Water/Steam", p4, 0)
v4 = vsat_Px("Water/Steam", p4, 0)
h5s = h4 + v4 * (p5 - p4) * 100
h6 = hsat_Px("Water/Steam", p6, 0)
v6 = vsat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * 100
```

```
Qdotin = mdot1 * (h1 - h7s)
y = (h6 - h5s) / (h2s - h5s)
Wdott = mdot1 * ((h1 - h2s) + (1 - y) * (h2s - h3s))
Wdotp = mdot1 * ((1 - y) * (h5s - h4) + (h7s - h6))
Wdotcycle = Wdott - Wdotp
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * (1 - y) * (h3s - h4)
```

IT Results ($p_2 = 7$ bar)

$\dot{Q}_{in} / \dot{m}_1 = 2614$ kJ/kg
 $\dot{Q}_{out} / \dot{m}_1 = 1459$ kJ/kg
 $\dot{W}_{cycle} / \dot{m}_1 = 1155$ kJ/kg
 $\eta = 0.442$
 $h_1 = 3321$ kJ/kg
 $h_{2s} = 2684$ kJ/kg
 $h_{3s} = 2009$ kJ/kg
 $h_4 = 151$ kJ/kg
 $h_{5s} = 151.8$ kJ/kg
 $h_6 = 696.8$ kJ/kg
 $h_{7s} = 707.1$ kJ/kg
 $y = 0.2152$

Plots:



From the plots, we see that the thermal efficiency exhibits a maximum in the range of feedwater heater pressures studied. Also, the fraction of steam extracted increases with increasing feedwater heater pressure.

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Problem 8-39 continued

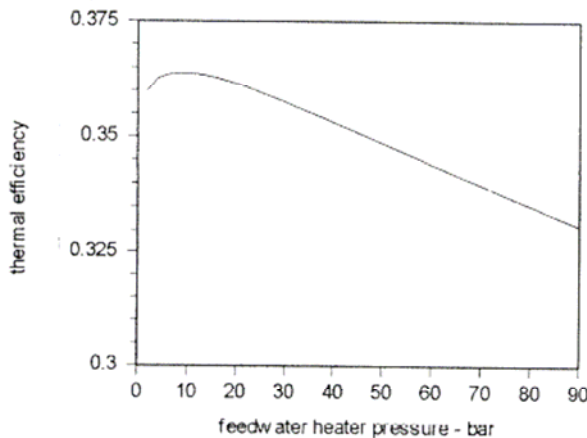
(b) Refer to Problem 8.38.

IT Code

```
p1 = 100 // bar
T1 = 480 // °C
p2 = 7 // bar
p3 = 0.06 // bar
p4 = p3
p5 = p2
p6 = p5
p7 = p1
etat = 0.8
etap = etat
mdot1 = 1
```

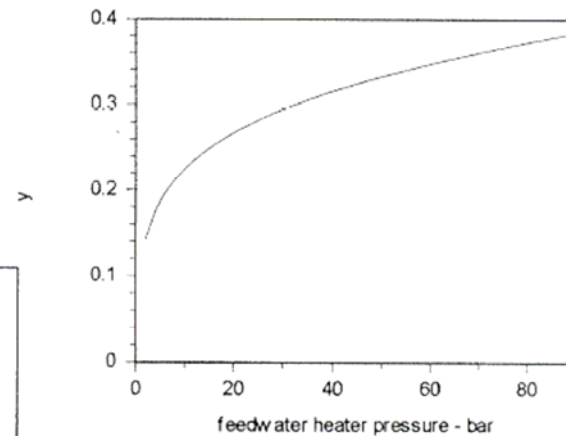
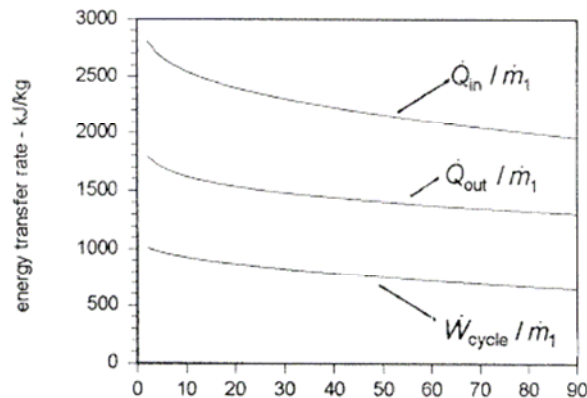
```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
h2 = h1 - etat * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)
s3s = s2
h3s = h_Ps("Water/Steam", p3, s3s)
h3 = h2 - etat * (h2 - h3s)
h4 = hsat_Px("Water/Steam", p4, 0)
v4 = vsat_Px("Water/Steam", p5, 0)
h5s = h4 + v4 * (p5 - p4) * 100
h5 = h4 + (h5s - h4) / etap
h6 = hsat_Px("Water/Steam", p6, 0)
v6 = vsat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * 100
h7 = h6 + (h7s - h6) / etap
```

```
Qdotin = mdot1 * (h1 - h7)
y = (h6 - h5s) / (h2 - h5s)
Wdott = mdot1 * ((h1 - h2) + (1 - y) * (h2 - h3))
Wdotp = mdot1 * ((1 - y) * (h5 - h4) + (h7 - h6))
Wdotcycle = Wdott - Wdotp
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * (1 - y) * (h3 - h4)
```



IT Results ($p_2 = 7$ bar)

```
Qin / m1 = 2611 kJ/kg
Qout / m1 = 1662 kJ/kg
Wcycle / m1 = 949.2 kJ/kg
η = 0.3635
h1 = 3321 kJ/kg
h2 = 2812 kJ/kg
h3 = 2241 kJ/kg
h4 = 151 kJ/kg
h5 = 152 kJ/kg
h6 = 696.8 kJ/kg
h7 = 709.6 kJ/kg
v = 0.2049
```



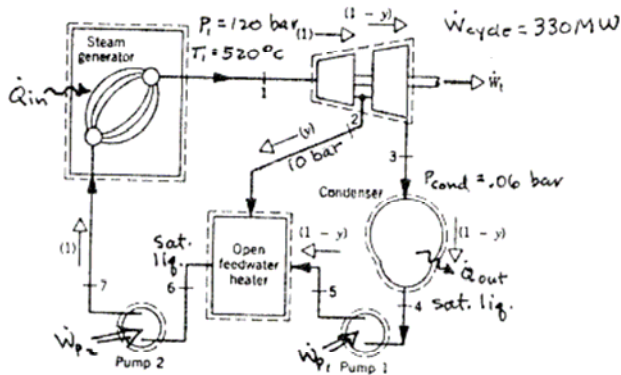
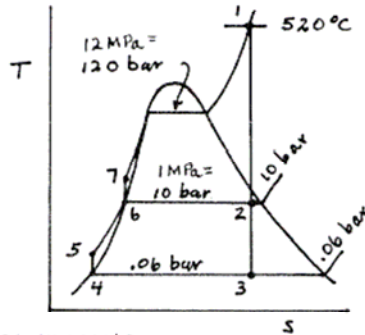
From the plots, we see that the thermal efficiency exhibits a maximum and the fraction of steam extracted increases with p_2 for the range of pressures studied. Further, these results can be compared to those of part (a) to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

PROBLEM 8.40

KNOWN: Water is the working fluid in a regenerative power cycle with one open feedwater heater. Data at various locations are known and the net power output is given.

FIND: Determine the thermal efficiency and the mass flow rate into the first turbine stage.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Example 8.5, except that the turbine stages operate in an internally reversible manner.

ANALYSIS: First, fix each principal state.

State 1: $P_1 = 120 \text{ bar}$, $T_1 = 520^\circ\text{C} \Rightarrow h_1 = 3401.8 \text{ kJ/kg}$, $s_1 = 6.5555 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 10 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = 0.9931$, $h_2 = 2764.2 \text{ kJ/kg}$

State 3: $P_3 = 0.06 \text{ bar}$, $s_3 = s_2 \Rightarrow x_3 = 0.7727$, $h_3 = 2018.3 \text{ kJ/kg}$

State 4: $P_4 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 151.53 \text{ kJ/kg}$

State 5: $h_5 \approx h_4 + v_4(P_5 - P_4)$
 $= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064 \times 10^{-3} \frac{\text{m}^3}{\text{kg}})(10 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$
 $= 151.53 + 1.00 = 152.53 \text{ kJ/kg}$

State 6: $P_6 = 10 \text{ bar}$, sat. liquid $\Rightarrow h_6 = 762.81 \text{ kJ/kg}$

State 7: $P_7 = 120 \text{ bar}$, $h_7 \approx h_6 + v_6(P_7 - P_6)$
 $= 762.81 + (1.1273 \times 10^{-3})(120 - 10) \left| \frac{10^5}{10^3} \right| = 775.21 \text{ kJ/kg}$

(a) Applying energy and mass balances to the control volume enclosing the open heater, the fraction of flow y extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{762.81 - 152.53}{2764.2 - 152.53} = 0.2337 \quad (1)$$

For the control volume surrounding the turbine stages

$$\begin{aligned} \frac{\dot{W}_t}{\dot{m}_1} &= (h_1 - h_2) + (1 - y)(h_2 - h_3) \\ &= (3401.8 - 2764.2) + (1 - 0.2337)(2764.2 - 2018.3) = 1209.2 \text{ kJ/kg} \end{aligned} \quad (2)$$

And, for the pumps

$$\begin{aligned} \frac{\dot{W}_p}{\dot{m}_1} &= (h_7 - h_6) + (1 - y)(h_5 - h_4) \\ &= (775.21 - 762.81) + (0.7663)(152.53 - 151.53) \\ &= 13.17 \text{ kJ/kg} \end{aligned} \quad (3)$$

Continued on next slide

Problem 6-40 continued

For the working fluid passing through the steam generator

$$\dot{Q}_{in}/\dot{m}_1 = h_1 - h_7 = 3401.8 - 775.21 = 2626.6 \text{ kJ/kg} \quad (4)$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.455 \text{ (45.5\%)} \quad \leftarrow \eta \quad (6)$$

(b) The net power developed is

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1] \quad (7)$$

$$\begin{aligned} \Rightarrow \dot{m}_1 &= \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]} \\ &= \frac{(330 \times 10^3 \text{ kJ/s})}{(1209.2 - 13.17) \text{ kJ/kg}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \\ &= 9.93 \times 10^5 \text{ kg/h} \quad \leftarrow \dot{m}_1 \end{aligned}$$

PROBLEM 8.41

KNOWN: Steady-state operating data are provided for the regenerative cycle of Problem 8.40

FIND: Plot the thermal efficiency of the cycle and the rate of exergy destruction within the feedwater heater versus feedwater heater pressure ranging from

- ① 0.5 to 10 MPa (5 to 100 bar).

SCHEMATIC & GIVEN DATA: See Problem 8.40.

ENGINEERING MODEL: See Problem 8.40. Also, let $T_o = 293\text{ K}$.

ANALYSIS: The states are fixed as in the solution to Problem 8.40. Further, Eqs. (1)–(7) from the solution to Problem 8.40 are used.

The rate of exergy destruction in the feedwater heater can be obtained from $\dot{E}_d = T_o \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production in the control volume surrounding the feedwater heater. That is

$$0 = \sum_j \left(\frac{\dot{Q}_j}{T_j} \right) + \dot{m}_1 [(1-y)s_5 + y s_2 - s_6] + \dot{\sigma}_{cv}$$

$$\text{or} \quad \dot{E}_d = T_o \dot{m}_1 [s_6 - y s_2 - (1-y)s_5] \quad (8)$$

Sample calculation: For $p_2 = 1.0\text{ MPa} = 10\text{ bar}$, see the solution to Problem 8.40. For the exergy destruction rate

$$\begin{aligned} \dot{E}_d &= (293\text{ K}) (1.93 \times 10^5 \frac{\text{kg}}{\text{h}}) [2.1387 - (0.2337)(6.5555) - (1.7663)(5.210)] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ &= 6.0354 \times 10^7 \text{ kJ/h} \left| \frac{1\text{ h}}{3600\text{ s}} \right| \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right| = 1.677 \times 10^4 \text{ kW} \end{aligned}$$

To obtain the data required for the plots, we use IT, as follows:

<u>IT Code</u> p1 = 120 // bar T1 = 520 // °C p2 = 10 // bar p3 = 0.06 // bar p4 = p3 p5 = p2 p6 = p2 p7 = p1 Wdotcycle = 330 // MW h1 = h_PT("Steam", p1, T1) s1 = s_PT("Steam", p1, T1) s2s = s1 h2s = h_Ps("Water", p2, s2s) s3s = s2s h3s = h_Ps("Water", p3, s3s) h4 = hsat_Px("Water", p4, 0) v4 = vsat_Px("Water", p4, 0)	s4 = ssat_Px("Water/Steam", p4, 0) h5s = h4 + v4 * (p5 - p4) * (100) s5s = s4 v6 = vsat_Px("Water", p6, 0) h6 = hsat_Px("Water", p6, 0) s6 = ssat_Px("Water/Steam", p6, 0) h7s = h6 + v6 * (p7 - p6) * (100) 0 = y * h2s + (1 - y) * h5s - h6 Wdott1 = mdot1 * (h1 - h2s) Wdott2 = mdot1 * (1 - y) * (h2s - h3s) Wdotp1 = mdot1 * (1 - y) * (h5s - h4) Wdotp2 = mdot1 * (h7s - h6) Wdotcycle * 10^3 = Wdott1 + Wdott2 - Wdotp1 - Wdotp2 mdot = mdot1 * 3600 // kg/h Qdotin = mdot1 * (h1 - h7s) Qdotout = mdot1 * (1 - y) * (h3s - h4) eta = Wdotcycle * 10^3 / Qdotin Edotd = To * mdot1 * (s6 - y * s2s - (1 - y) * s5s) To = 293 // K
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Problem 8-41 continued

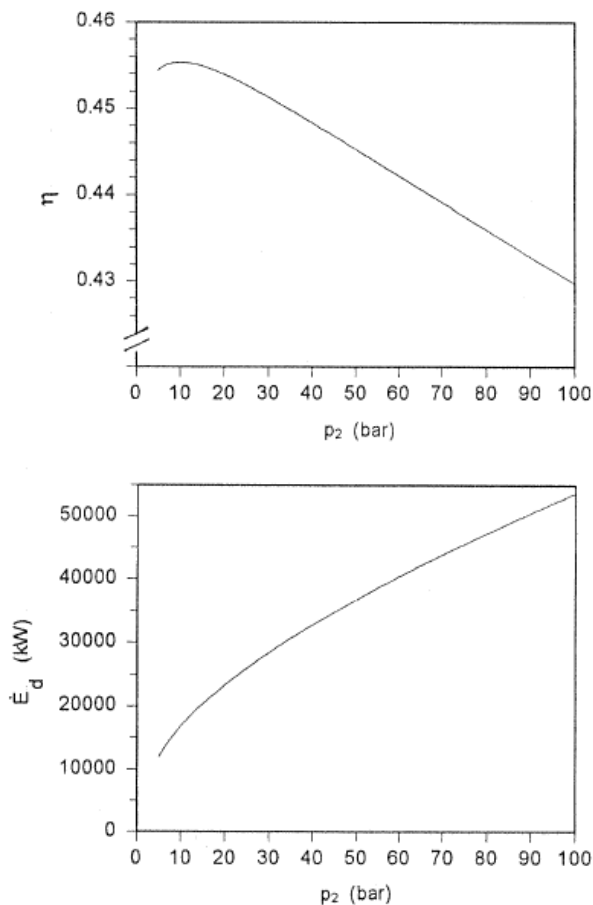
IT Results for $p_2 = 1 \text{ MPa}$ (10 bar)

$h_1 = 3401 \text{ kJ/kg}$
 $h_{2s} = 2764 \text{ kJ/kg}$
 $h_{3s} = 2018 \text{ kJ/kg}$
 $h_4 = 151 \text{ kJ/kg}$
 $h_{5s} = 152 \text{ kJ/kg}$
 $h_6 = 762.5 \text{ kJ/kg}$
 $h_{7s} = 774.9 \text{ kJ/kg}$
 $s_1 = 6.555 \text{ kJ/kg}\cdot\text{K}$
 $s_4 = 0.519 \text{ kJ/kg}\cdot\text{K}$
 $s_6 = 2.138 \text{ kJ/kg}\cdot\text{K}$

$y = 0.2338$

$\dot{W}_{t1} = 1.76 \times 10^5 \text{ kW}$
 $\dot{W}_{t2} = 1.577 \times 10^5 \text{ kW}$
 $\dot{W}_{p1} = 211.6 \text{ kW}$
 $\dot{W}_{t2} = 3422 \text{ kW}$
 $\dot{Q}_{in} = 7.247 \times 10^5 \text{ kW}$
 $\eta = 0.4553$
 $\dot{m}_1 = 9.934 \times 10^5 \text{ kg/h}$
 $\dot{E}_d = 1.681 \times 10^4 \text{ kW}$

PLOTS:



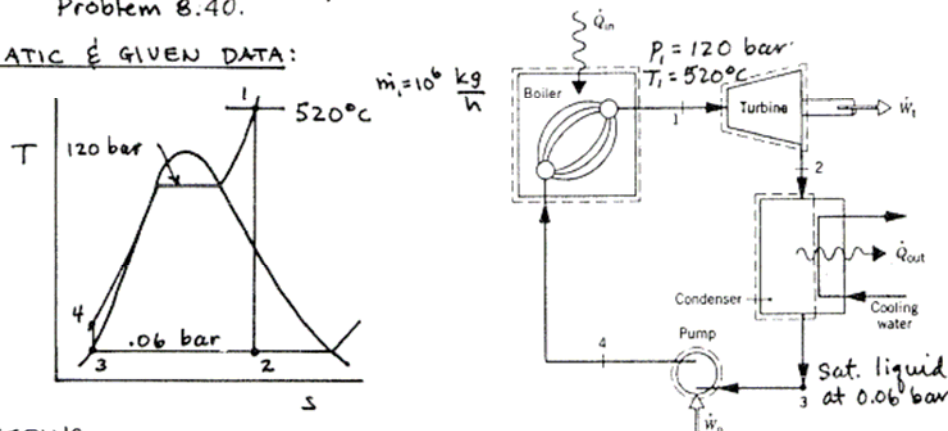
From the plots, we see that the thermal efficiency exhibits a maximum in the range of feedwater heaters studied. In addition, the exergy destruction rate increases with increasing p_2 , as the temperature difference between the extracted steam and the steam generator feedwater increases.

PROBLEM 8.42

KNOWN: An ideal Rankine cycle having the same turbine inlet conditions and condenser pressure, but no regenerator, as the cycle of Problem 8.40 is considered.

FIND: Answer the same questions about the simple cycle as those in Problem 8.40.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.1, assumptions 1-4

ANALYSIS: Using appropriate data from the solution to Problem 8.40.

$$h_1 = 3401.8 \text{ kJ/kg}, \quad s_1 = 6.5555 \text{ kJ/kg} \cdot \text{K}$$

$$h_2 = 2018.3 \text{ kJ/kg}, \quad h_3 = 151.53 \text{ kJ/kg}$$

State 4: $h_4 \approx h_3 + v_3 (P_4 - P_3)$

$$= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064 \times 10^{-3} \frac{\text{m}^3}{\text{kg}})(120 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 151.53 + 12.07 = 163.6 \text{ kJ/kg}$$

(a) For the turbine

$$\dot{W}_t / \dot{m} = h_1 - h_2 = 3401.8 - 2018.3 = 1383.5 \text{ kJ/kg}$$

And, for the pump

$$\dot{W}_p / \dot{m} = h_4 - h_3 = 12.07 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\dot{Q}_{in} / \dot{m} = h_1 - h_4 = 3401.8 - 163.6 = 3238.2 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t / \dot{m} - \dot{W}_p / \dot{m}}{\dot{Q}_{in} / \dot{m}} = 0.424 \quad (42.4\%) \quad \leftarrow \eta$$

(b) The net power developed is

$$\dot{W}_{cycle} = \dot{m} [\dot{W}_t / \dot{m} - \dot{W}_p / \dot{m}]$$

$$= (10^6 \frac{\text{kg}}{\text{h}}) \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| [1383.5 - 12.07] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| = 381 \text{ MW} \quad \leftarrow \dot{W}_{cycle}$$

The introduction of the open feedwater heater into the cycle of Problem 8.40 results in an increase in thermal efficiency and a decrease in the net power developed.

PROBLEM 8.43

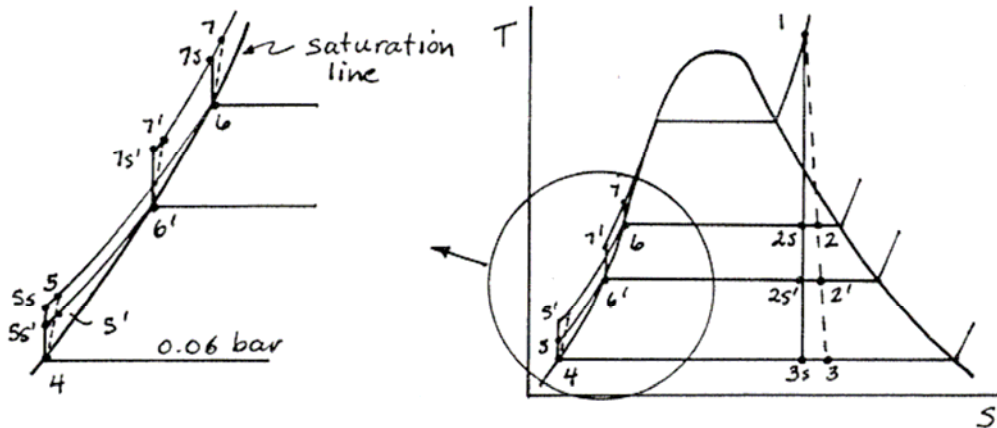
KNOWN: The regenerative vapor power cycle of Problem 8.40 is reconsidered.

FIND: Investigate the effects on cycle performance as the feedwater heater pressure takes on other values. Construct suitable plots and discuss. Consider the cycle for which each of the turbine stages and the pump have isentropic efficiencies less than unity.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.40.

ENGINEERING MODEL: See Example 8.5, and $\eta_{t1} = \eta_{t2} = \eta_p = \eta_{s2} = 80\%$.

ANALYSIS: The performance of the cycle is investigated by determining the effects on thermal efficiency of varying the feedwater heater pressure.



To obtain the data for the required plots, we use IT, as follows

IT Code

p1 = 120 // bar
T1 = 520 // °C
p2 = 10 // bar
p3 = 0.06 // bar

p4 = p3
p5 = p2
p6 = p2
p7 = p1
Wdotcycle = 330 // MW
etat1 = .8
etat2 = .8
etap1 = .8
etap2 = .8

h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water", p2, s2s)
h2 = h1 - etat1 * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)
s3s = s2
h3s = h_Ps("Water", p3, s3s)
h3 = h2 - etat2 * (h2 - h3s)

h4 = hsat_Px("Water", p4, 0)
v4 = vsat_Px("Water", p4, 0)
s4 = ssat_Px("Water/Steam", p4, 0)
h5s = h4 + v4 * (p5 - p4) * (100)
h5 = h4 + (h5s - h4) / etap1
s5s = s4
s5 = s_Ph("Water/Steam", p5, h5)
v6 = vsat_Px("Water", p6, 0)
h6 = hsat_Px("Water", p6, 0)
s6 = ssat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * (100)
h7 = h6 + (h7s - h6) / etap2

0 = y * h2 + (1 - y) * h5 - h6

Wdott1 = mdot1 * (h1 - h2)
Wdott2 = mdot1 * (1 - y) * (h2 - h3)
Wdotp1 = mdot1 * (1 - y) * (h5 - h4)
Wdotp2 = mdot1 * (h7 - h6)
Wdotcycle * 10^3 = Wdott1 + Wdott2 - Wdotp1 - Wdotp2
Qdotin = mdot1 * (h1 - h7)
Qdotout = mdot1 * (1 - y) * (h3 - h4)
eta = Wdotcycle * 10^3 / Qdotin
mdot = mdot1 * 3600

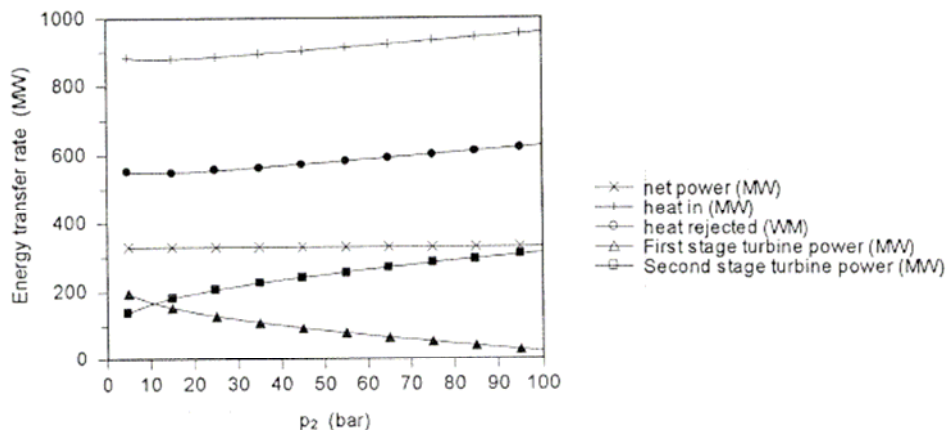
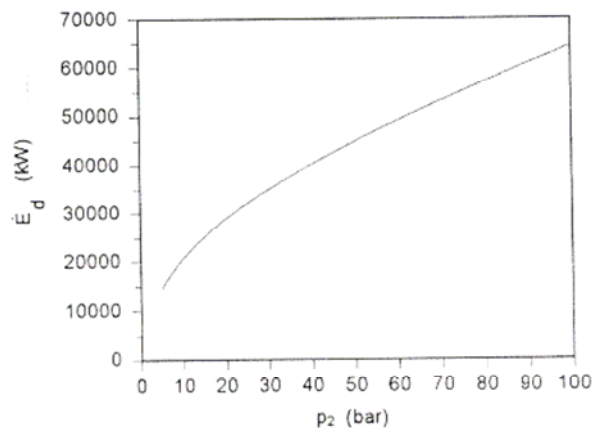
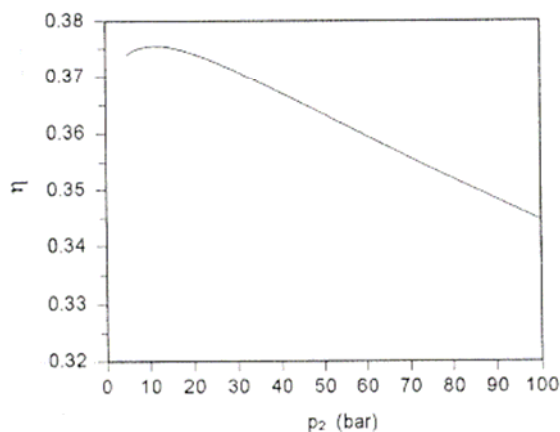
Edotd = To * mdot1 * (s6 - y * s2 - (1 - y) * s5)
To = 293

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Problem 8-43 continued

IT Results for $p_2 = 1.0$ MPa (10 bar)

$h_1 = 3401$ kJ/kg	$y = 0.2228$
$h_2 = 2891$ kJ/kg	$\dot{E}_d = 2.07 \times 10^4$ kW
$h_{2s} = 2764$ kJ/kg	$\dot{W}_{p1} = 325.7$ kW
$h_3 = 2259$ kJ/kg	$\dot{W}_{p2} = 5194$ kW
$h_{3s} = 2101$ kJ/kg	$\dot{W}_{t1} = 1.71 \times 10^5$ kW
$h_4 = 151$ kJ/kg	$\dot{W}_{t2} = 1.646 \times 10^5$ kW
$h_5 = 152.3$ kJ/kg	$\dot{m}_1 = 1.206 \times 10^6$ kg/h
$h_{5s} = 152$ kJ/kg	$\dot{Q}_{in} = 8.791 \times 10^5$ kW
$h_6 = 762.5$ kJ/kg	$\eta = 0.375$
$h_7 = 778$ kJ/kg	
$h_{7s} = 774.9$ kJ/kg	
$s_1 = 6.555$ kJ/kg·K	
$s_2 = 6.824$ kJ/kg·K	
$s_4 = 0.519$ kJ/kg·K	
$s_5 = 0.5231$ kJ/kg·K	
$s_6 = 2.138$ kJ/kg·K	



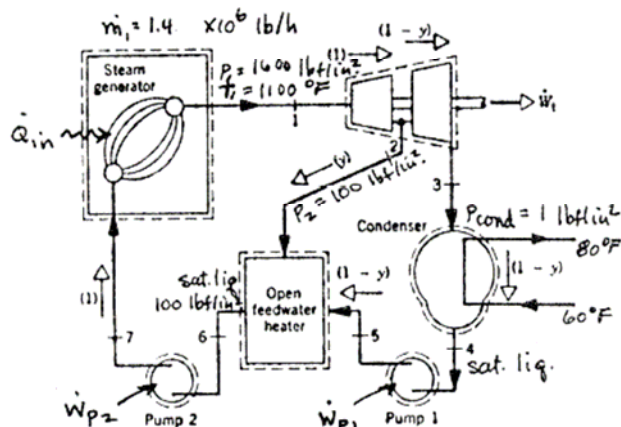
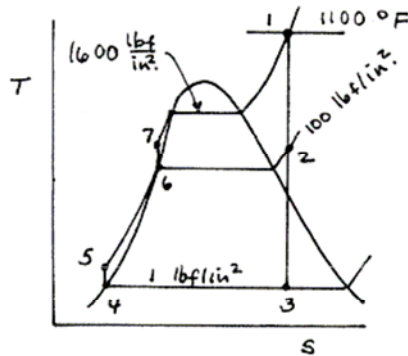
Even though turbine and pump isentropic efficiencies are less than unity, the trends for η and $\dot{E}_d$ are similar to those in Problem 8.41. Also, note that the power output of the first turbine stage decreases as $p_2 \rightarrow p_1$, and the output of stage two increases accordingly.

PROBLEM 8.44

KNOWN: The ideal Rankine cycle of Problem 8.9 is modified to include one open feedwater heater operating at 100 lbf/in².

FIND: Answer the same questions about the modified cycle as in Problem 8.9 and discuss the results.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Example 8.5, except that the turbine stages operate in an internally reversible manner.

ANALYSIS: First, fix each of the principal states. From the solution to Problem 8.9; $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R, $h_3 = 911.2$ Btu/lb, and $h_4 = 69.74$ Btu/lb.

State 2: $P_2 = 100$ lbf/in², $s_2 = s_1 \Rightarrow h_2 = 1210.7$ Btu/lb (Table A-4E)

State 5: $h_5 \approx h_4 + v_4(P_5 - P_4)$

$$= 69.74 \frac{\text{Btu}}{\text{lb}} + (0.01614 \frac{\text{ft}^3}{\text{lb}}) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{18 \text{ lbf}}{778 \text{ ft} \cdot \text{lbf}} \right| (100 - 1) \frac{\text{lbf}}{\text{in}^2}$$

$$= 69.74 + .296 = 70.04 \text{ Btu/lb}$$

State 6: $P_6 = 100$ lbf/in², sat. liquid $\Rightarrow h_6 = 298.6$ Btu/lb

State 7: $h_7 \approx h_6 + v_6(P_7 - P_6) = 298.6 + (0.01774) \left| \frac{144}{778} \right| (1600 - 100) = 303.5$ Btu/lb

Applying energy and mass rate balances to the control volume enclosing the open heater, the fraction y of flow extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{298.6 - 70.04}{1210.7 - 70.04} = 0.2004$$

(a) To determine the net power developed by the cycle, begin with a control volume around the turbine

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$

$$= (1547.7 - 1210.7) + (1 - 0.2004)(1210.7 - 911.2) = 576.48 \text{ Btu/lb}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_7 - h_6) + (1 - y)(h_5 - h_4)$$

$$= (303.5 - 298.6) + (1 - 0.2004)(70.04 - 69.74) = 5.14 \text{ Btu/lb}$$

Thus, with $\dot{m}_1 = 1.4 \times 10^6$ lb/h (Problem 8.9)

$$\dot{W}_{\text{cycle}} = \dot{m}_1 \left[\frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} \right] = (1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) [576.48 - 5.14] \frac{\text{Btu}}{\text{lb}} = 8.00 \times 10^8 \frac{\text{Btu}}{\text{h}}$$

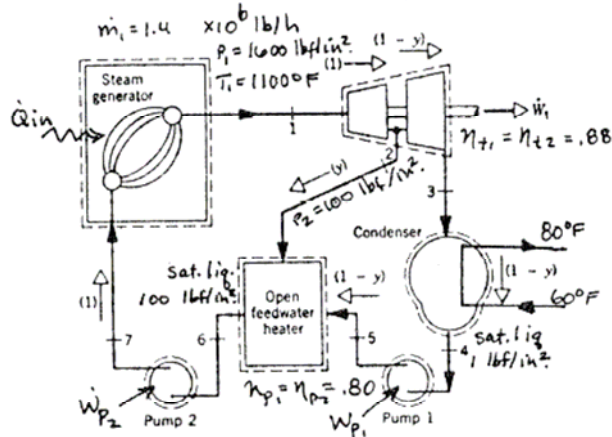
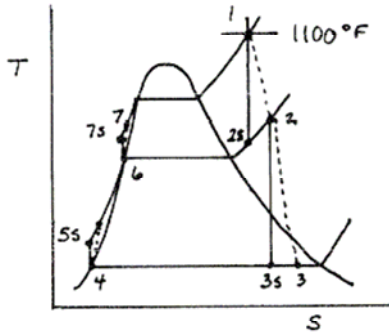
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Problem 8-44 continued

KNOWN: The regenerative power cycle of Problem 8.44 is modified to include that each turbine stage has an efficiency of 88% and each pump has an efficiency of 80%.

FIND: Answer the same questions for the modified cycle as in Problem 8.44.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Problem 8.41, except $\eta_{t1} = \eta_{t2} = 0.88$ and $\eta_{p1} = \eta_{p2} = 0.80$.

ANALYSIS: From Problem 8.41, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·R, $h_4 = 69.74$ Btu/lb, and $h_6 = 298.6$ Btu/lb.

State 2: Use the turbine stage efficiency. First, $P_2 = 100$ lbf/in² and $s_{2s} = s_1$;
 $h_{2s} = 1210.7$ Btu/lb (Problem 8.41) Thus

$$\eta_{t1} = \frac{w_{t1}}{(w_{t1}/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 1251.1 \text{ Btu/lb}$$

From Table A-4E, $s_2 = 1.6784$ Btu/lb·R.

State 3: Similarly, $P_3 = 1$ lbf/in², $s_{3s} = s_2 \Rightarrow x_{3s} = 0.8376$, $h_{3s} = 937.52$ Btu/lb.

Thus $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 975.15$ Btu/lb

State 5: State 5 is determined using the pump efficiency. From Problem 8.41

$h_{5s} = 70.04$ Btu/lb. Thus

$$\eta_{p1} = \frac{(w_{p1}/\dot{m})_s}{(w_{p1}/\dot{m})} = \frac{h_{5s} - h_4}{h_5 - h_4} \Rightarrow h_5 = h_4 + (h_{5s} - h_4)/\eta_{p1} = 70.115 \text{ Btu/lb}$$

State 7: Similarly, $h_{7s} = 303.5$ Btu/lb and $h_7 = h_6 + (h_{7s} - h_6)/\eta_{p2}$
 $= 304.7$ Btu/lb

The fraction y of the turbine flow extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{298.6 - 70.115}{1251.1 - 70.115} = 0.1935$$

(a) For the turbines

$$\begin{aligned} \dot{w}_t/\dot{m}_1 &= (h_1 - h_2) + (1-y)(h_2 - h_3) \\ &= (1547.7 - 1251.1) + (1 - 0.1935)(1251.1 - 975.15) = 519.15 \text{ Btu/lb} \end{aligned}$$

and the pumps

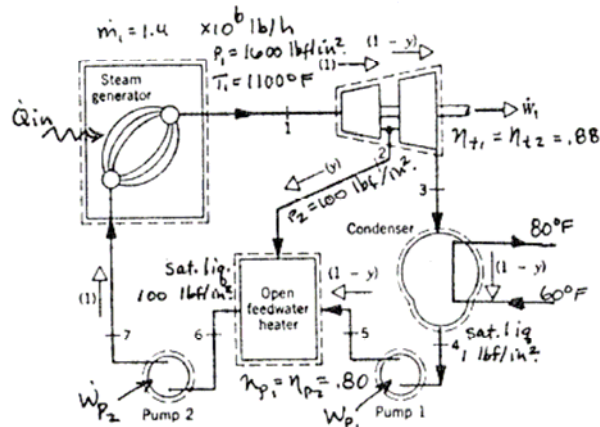
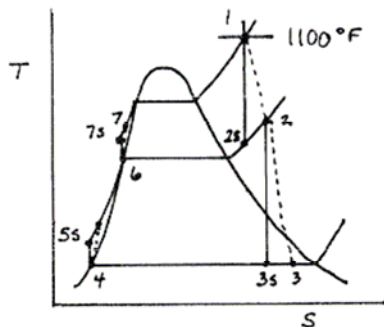
$$\begin{aligned} \dot{w}_p/\dot{m}_1 &= (h_7 - h_6) + (1-y)(h_5 - h_4) \\ &= (304.7 - 298.6) + (1 - 0.1935)(70.115 - 69.74) = 6.402 \text{ Btu/lb} \end{aligned}$$

PROBLEM 8.45

KNOWN: The regenerative power cycle of Problem 8.44 is modified to include that each turbine stage has an efficiency of 88% and each pump has an efficiency of 80%.

FIND: Answer the same questions for the modified cycle as in Problem 8.44.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Problem 8.41, except $\eta_{t1} = \eta_{t2} = 0.88$ and $\eta_{p1} = \eta_{p2} = 0.80$.

ANALYSIS: From Problem 8.41, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·R, $h_4 = 69.74$ Btu/lb, and $h_6 = 298.6$ Btu/lb.

State 2: Use the turbine stage efficiency. First, $P_2 = 100$ lbf/in² and $s_{2s} = s_1$; $h_{2s} = 1210.7$ Btu/lb (Problem 8.41). Thus

$$\eta_{t1} = \frac{\dot{W}_{t1}/\dot{m}_1}{(\dot{W}_{t1}/\dot{m}_1)_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 1251.1 \text{ Btu/lb}$$

From Table A-4E, $s_2 = 1.6784$ Btu/lb·R.

State 3: Similarly, $P_3 = 1$ lbf/in², $s_{3s} = s_2 \Rightarrow x_{3s} = 0.8376$, $h_{3s} = 937.52$ Btu/lb.

Thus $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 975.15$ Btu/lb

State 5: State 5 is determined using the pump efficiency. From Problem 8.41

$h_{5s} = 70.04$ Btu/lb. Thus

$$\eta_{p1} = \frac{(\dot{W}_{p1}/\dot{m}_1)_s}{(\dot{W}_{p1}/\dot{m}_1)} = \frac{h_{5s} - h_4}{h_5 - h_4} \Rightarrow h_5 = h_4 + (h_{5s} - h_4)/\eta_{p1} = 70.115 \text{ Btu/lb}$$

State 7: Similarly, $h_{7s} = 303.5$ Btu/lb and $h_7 = h_6 + (h_{7s} - h_6)/\eta_{p2} = 304.7$ Btu/lb

The fraction y of the turbine flow extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{298.6 - 70.115}{1251.1 - 70.115} = 0.1935$$

(a) For the turbines

$$\begin{aligned} \dot{W}_t/\dot{m}_1 &= (h_1 - h_2) + (1 - y)(h_2 - h_3) \\ &= (1547.7 - 1251.1) + (1 - 0.1935)(1251.1 - 975.15) = 519.15 \text{ Btu/lb} \end{aligned}$$

and the pumps

$$\begin{aligned} \dot{W}_p/\dot{m}_1 &= (h_7 - h_6) + (1 - y)(h_5 - h_4) \\ &= (304.7 - 298.6) + (1 - 0.1935)(70.115 - 69.74) = 6.402 \text{ Btu/lb} \end{aligned}$$

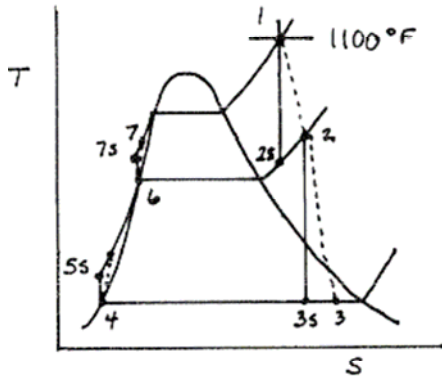
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Problem 8-45 continued

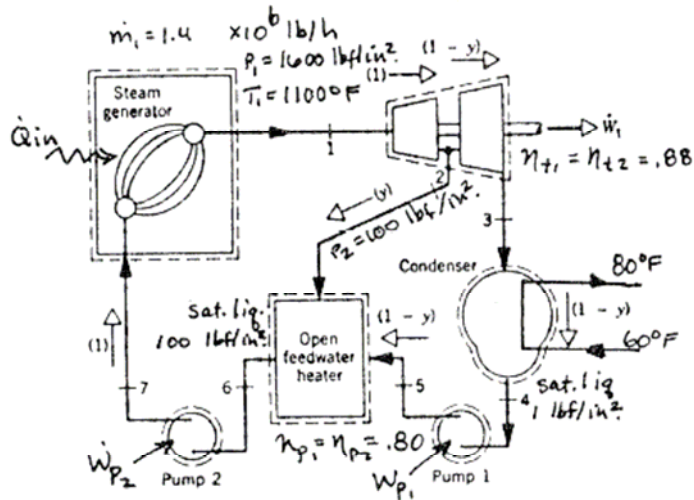
OWN: The regenerative power cycle of Problem 8.44 is modified to include that each turbine stage has an efficiency of 88% and each pump has an efficiency of 80%.

VD: Answer the same questions for the modified cycle as in Problem 8.44.

HEMATIC & GIVEN DATA:



ENGINEERING

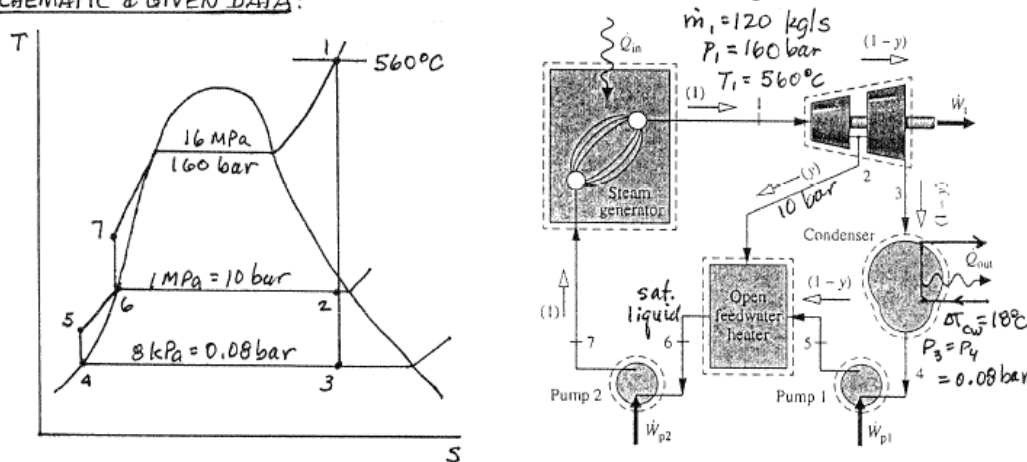


PROBLEM 8.46

KNOWN: Water is the working fluid in an ideal regenerative Rankine cycle with one open-feedwater heater. Data at various locations are known. The mass flow rate entering the first-stage turbine is given, and the temperature rise of cooling water passing through the condenser is specified.

FIND: Determine (a) the net power, (b) the rate of heat transfer $\dot{Q}_{in}$, (c) the thermal efficiency, and (d) the mass flow rate of cooling water.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as Example 8.5, except turbine stages and pumps operate in an internally reversible manner.

ANALYSIS: First, fix each principal state.

State 1: $P_1 = 160 \text{ bar}$, $T_1 = 560^\circ\text{C} \Rightarrow h_1 = 3465.4 \text{ kJ/kg}$, $s_1 = 6.5132 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 10 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.9836$, $h_2 = 2745.1 \text{ kJ/kg}$

State 3: $P_3 = 0.08 \text{ bar}$, $s_3 = s_2 \Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.7753$, $h_3 = 2037.0 \text{ kJ/kg}$

State 4: $P_4 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 173.88 \text{ kJ/kg}$

State 5: $h_5 \approx h_4 + v_4(P_5 - P_4)$
 $= 173.88 \frac{\text{kJ}}{\text{kg}} + (1.0084 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (10 - 0.08) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$
 $= 173.88 + 1.00 = 174.88 \text{ kJ/kg}$

State 6: $P_6 = 10 \text{ bar}$, sat. liquid $\Rightarrow h_6 = 762.81 \text{ kJ/kg}$

State 7: $h_7 \approx h_6 + v_6(P_7 - P_6) = 762.81 + (1.1273 \times 10^{-3}) (160 - 10) \left| \frac{10^5}{10^3} \right| = 779.72 \frac{\text{kJ}}{\text{kg}}$

(a) For the control volume enclosing the turbine stages

$$\dot{W}_t = \dot{m}_1 [(h_1 - h_2) + (1 - y)(h_2 - h_3)]$$

To get y , apply mass and energy balances to the control volume enclosing the feedwater heater to get

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{762.81 - 174.88}{2745.1 - 174.88} = 0.2287$$

Thus

$$\dot{W}_t = (120 \frac{\text{kg}}{\text{s}}) [(3465.4 - 2745.1) + (1 - 0.2287)(2745.1 - 2037.0)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 1.519 \times 10^5 \text{ kW}$$

Continued on next slide

Problem 8-46 continued

For the pumps

$$\begin{aligned}\dot{W}_p &= \dot{W}_{p1} + \dot{W}_{p2} = \dot{m}_1 [(1-y)(h_5 - h_4) + (h_7 - h_6)] \\ &= (120)[(1-0.2287)(174.88 - 173.88) + (779.72 - 762.81)] \left(\frac{1}{1} \right) \\ &= 2122 \text{ kW}\end{aligned}$$

Thus, the net power developed is

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = 1.498 \times 10^5 \text{ kW} \leftarrow \dot{W}_{\text{cycle}}$$

(b) For the steam generator

$$\dot{Q}_{\text{in}} = \dot{m}_1 (h_1 - h_7) = (120)(3465.4 - 779.72) \left(\frac{1}{1} \right) = 3.223 \times 10^5 \text{ kW} \leftarrow \dot{Q}_{\text{in}}$$

(c) The thermal efficiency is $\eta = \dot{W}_{\text{cycle}} / \dot{Q}_{\text{in}} = 0.465$ (46.5%) $\leftarrow \eta$

(d) The rate of heat transfer from the working fluid to the cooling water passing through the condenser is

$$\dot{Q}_{\text{out}} = \dot{m}_1 (1-y)(h_3 - h_4) = (120)(1-0.2287)(2037.0 - 173.88) \left(\frac{1}{1} \right) = 1.724 \times 10^5 \text{ kW}$$

Thus, the mass flow rate of cooling water is

$$\dot{Q}_{\text{out}} = \dot{m}_{\text{cw}} (h_{\text{out,cw}} - h_{\text{in,cw}})$$

Assuming $h_{\text{out,cw}} - h_{\text{in,cw}} = c_{\text{cw}} \Delta T_{\text{cw}}$, and using $c_{\text{cw}} = 4.179 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$, we get

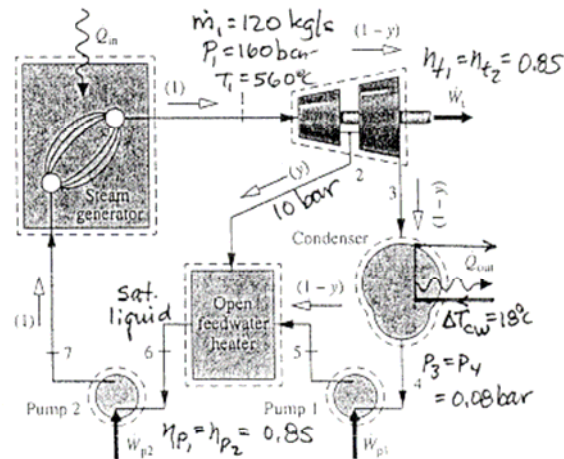
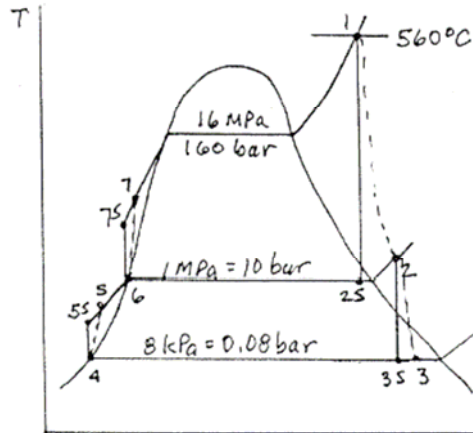
$$\dot{m}_{\text{cw}} = \frac{\dot{Q}_{\text{out}}}{c_{\text{cw}} \Delta T_{\text{cw}}} = \frac{1.724 \times 10^5 \text{ kW}}{(4.179)(18) \frac{\text{kJ}}{\text{kg}}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 2292 \frac{\text{kg}}{\text{s}} \leftarrow \dot{m}_{\text{cw}}$$

PROBLEM 8.47

KNOWN: The ideal regenerative Rankine cycle of Problem 8.46 is modified to include turbine stage and pump isentropic efficiencies of 0.85.

FIND: Answer the same questions as in Problem 8.46 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as Example 8.5.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 160 \text{ bar}$, $T_1 = 560^\circ\text{C} \Rightarrow h_1 = 3465.1 \text{ kJ/kg}$, $s_1 = 6.5132 \text{ kJ/kg}\cdot\text{K}$

State 2: Using the isentropic efficiency of the first-stage turbine

$$\eta_{t1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s})$$

With $h_{2s} = 2745.1 \text{ kJ/kg}$ from Problem 8.46, $h_2 = 2853.1 \text{ kJ/kg}$

State 3: $P_3 = 0.08 \text{ bar}$, $s_3 = s_2 \Rightarrow$ Interpolating in Table A-4; $s_2 = 6.7451 \text{ kJ/kg}\cdot\text{K}$

$$\text{Thus } x_{3s} = \frac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.8056; h_{3s} = 2109.8 \text{ kJ/kg}$$

Using the second-stage turbine isentropic efficiency

$$h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 2221.3 \text{ kJ/kg}$$

State 4: $P_4 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 173.88 \text{ kJ/kg}$

State 5: Using the isentropic pump efficiency; $\eta_{p1} = (h_{5s} - h_4)/(h_5 - h_4)$

$$\text{Thus } h_5 = h_4 + (h_{5s} - h_4)/\eta_{p1} = 175.06 \text{ kJ/kg}$$

State 6: $P_6 = 10 \text{ bar}$, sat. liquid $\Rightarrow h_6 = 762.81 \text{ kJ/kg}$

State 7: With $\eta_{p2} = (h_{7s} - h_6)/(h_7 - h_6)$; $h_7 = h_6 + (h_{7s} - h_6)/\eta_{p2} = 782.7 \text{ kJ/kg}$

$$(a) y = \frac{h_6 - h_5}{h_2 - h_5} = 0.2195 \text{ and } \dot{W}_t = \dot{m}_1[(h_1 - h_2) + (1 - y)(h_2 - h_3)] = 1.326 \times 10^5 \text{ kW}$$

$$\text{For the pumps, } \dot{W}_p = \dot{m}_1[(1 - y)(h_5 - h_4) + (h_7 - h_6)] = 2497 \text{ kW}$$

$$\text{Thus } \dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = 1.301 \times 10^5 \text{ kW}$$

$$(b) \dot{Q}_{\text{in}} = \dot{m}_1(h_1 - h_7) = 3.219 \times 10^5 \text{ kW}$$

$$(c) \eta = \dot{W}_{\text{cycle}}/\dot{Q}_{\text{in}} = 0.404 \text{ (40.4\%)}$$

$$(d) \dot{Q}_{\text{out}} = \dot{m}_1(1 - y)(h_3 - h_4) = 1.9176 \times 10^5 \text{ kW}$$

$$\dot{m}_{\text{cw}} = \dot{Q}_{\text{out}}/c_{\text{cw}}\Delta T_{\text{cw}} = 2549 \text{ kW}$$

1. These results can be compared with those of 8.46 to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

Continued on next slide

Problem 8-47 continued

PROBLEM 8.48

(a) Refer to Problem 8.47

IT Code

```
p1 = 160 // bar
T1 = 560 // °C
p2 = 10 // bar
p3 = 0.08 // bar
p4 = p3
p5 = p2
p6 = p5
p7 = p1
mdot1 = 120 // kg/s
Ccw = 4.179 // kJ/kg·K
delTcw = 18
```

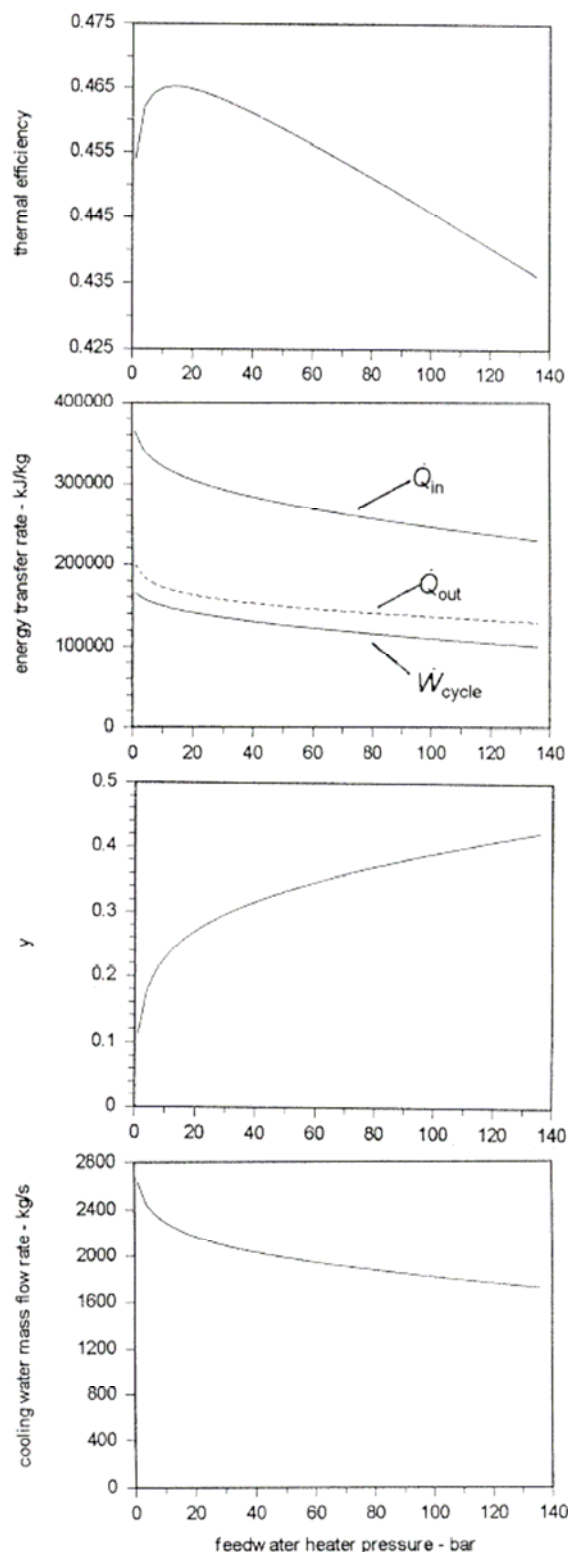
```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s1)
s2s = s1
s3s = s2s
h3s = h_Ps("Water/Steam", p3, s3s)
h4 = hsat_Px("Water/Steam", p4, 0)
v4 = vsat_Px("Water/Steam", p5, 0)
h5s = h4 + v4 * (p5 - p4) * 100
h6 = hsat_Px("Water/Steam", p6, 0)
v6 = vsat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * 100
```

```
Qdotin = mdot1 * (h1 - h7s)
y = (h6 - h5s) / (h2s - h5s)
Wdot = mdot1 * ((h1 - h2s) + (1 - y) * (h2s - h3s))
Wdotp = mdot1 * ((1 - y) * (h5s - h4) + (h7s - h6))
Wdotcycle = Wdot - Wdotp
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * (1 - y) * (h3s - h4)
mdotcw = Qdotout / (Ccw * delTcw)
```

IT Results ($p_2 = 10$ bar)

```
Qin = 3.223 × 105 kW
Qout = 1.724 × 105 kW
Wcycle = 1.498 × 105 kW
ṁcw = 2292 kg/s
η = 0.4649
h1 = 3465 kJ/kg
h2s = 2744 kJ/kg
h3s = 2037 kJ/kg
h4 = 173.6 kJ/kg
h5s = 174.8 kJ/kg
h6 = 762.5 kJ/kg
h7s = 779.4 kJ/kg
y = 0.2287
```

The thermal efficiency exhibits a maximum in the range of feedwater heater pressures studied.



Continued on next slide

Problem 9-48 continued

(b) Refer to Problem 8.47

IT Code

```
p1 = 180 // bar
T1 = 560 // °C
p2 = 10 // bar
p3 = 0.08 // bar
p4 = p3
p5 = p2
p6 = p5
p7 = p1
etat = 0.85
etap = etat
mdot1 = 120 // kg/s
Ccw = 4.179 // kJ/kg·K
delTcw = 18
```

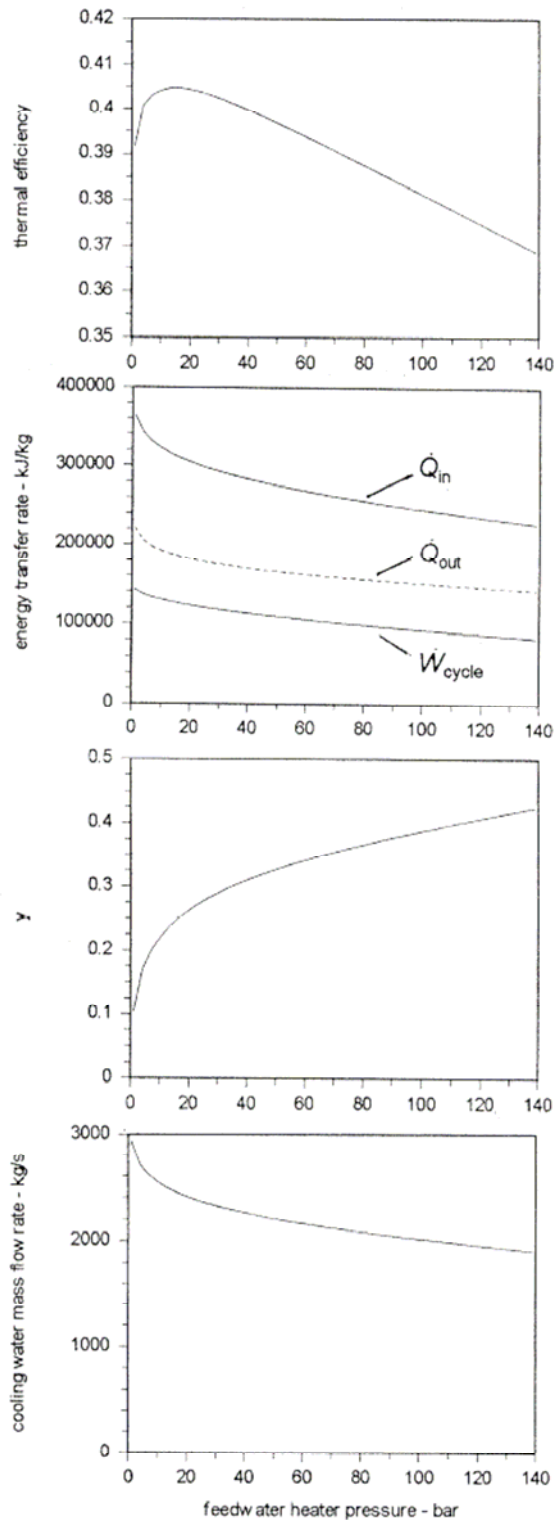
```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
h2 = h1 - etat * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)
s3s = s2
h3s = h_Ps("Water/Steam", p3, s3s)
h3 = h2 - etat * (h2 - h3s)
h4 = hsat_Px("Water/Steam", p4, 0)
v4 = vsat_Px("Water/Steam", p5, 0)
h5s = h4 + v4 * (p5 - p4) * 100
h5 = h4 + (h5s - h4) / etap
h6 = hsat_Px("Water/Steam", p6, 0)
v6 = vsat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * 100
h7 = h6 + (h7s - h6) / etap
```

```
Qdotin = mdot1 * (h1 - h7)
y = (h6 - h5s) / (h2 - h5s)
Wdott = mdot1 * ((h1 - h2) + (1 - y) * (h2 - h3))
Wdotp = mdot1 * ((1 - y) * (h5 - h4) + (h7 - h6))
Wdotcycle = Wdott - Wdotp
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * (1 - y) * (h3 - h4)
mdotcw = Qdotout / (Ccw * delTcw)
```

IT Results ($p_2 = 10$ bar)

```
Qin = 3.219 × 105 kW
Qout = 1.918 × 105 kW
Wcycle = 1.301 × 105 kW
mcw = 2550 kg/s
η = 0.4041
h1 = 3465 kJ/kg
h2 = 2853 kJ/kg
h3 = 2221 kJ/kg
h3s = 2110 kJ/kg
h4 = 173.6 kJ/kg
h5 = 175 kJ/kg
h6 = 762.5 kJ/kg
h7 = 782.4 kJ/kg
```

The thermal efficiency exhibits a maximum in the range of feedwater heater pressures studied.

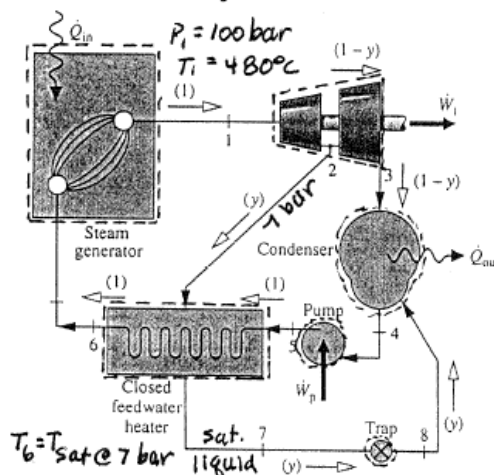
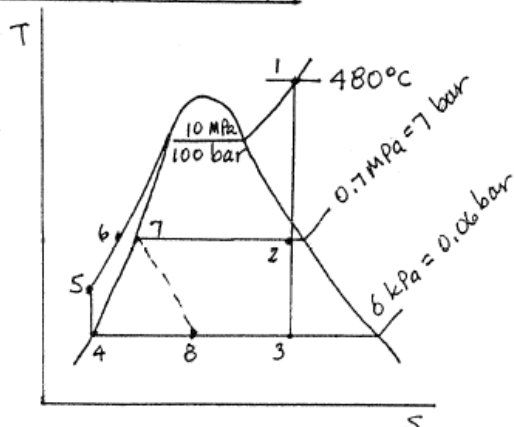


PROBLEM 8.49

KNOWN: Water is the working fluid in an ideal regenerative Rankine cycle with one closed feedwater heater. Data at various locations are known.

FIND: Determine (a) the rate of heat addition per kg of steam entering the first-stage turbine, (b) the thermal efficiency, and (c) the rate of heat transfer for the condenser per kg of steam entering the first-stage turbine.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is modeled as a control volume at steady state. (2) There are no stray heat transfers. (3) The working fluid undergoes an internally reversible process in passing through each component except the trap. (4) For the trap, $h_7 = h_8$ (throttling process). (5) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each principal state.

State 1: $P_1 = 100 \text{ bar}$, $T_1 = 480^\circ\text{C} \Rightarrow h_1 = 3321.4 \text{ kJ/kg}$, $s_1 = 6.5282 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 7 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.9619$, $h_2 = 2684.8 \text{ kJ/kg}$

State 3: $P_3 = 0.06 \text{ bar}$, $s_3 = s_2 \Rightarrow x_3 = 0.7692$, $h_3 = 2009.8 \text{ kJ/kg}$

State 4: $P_4 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 151.53 \text{ kJ/kg}$

State 5: $h_5 = h_4 + v_4(P_5 - P_4)$
 $= 151.53 + (1.0064 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (100 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$
 $= 151.53 + 10.06 = 161.59 \text{ kJ/kg}$

State 6: $T_{\text{sat}} @ 7 \text{ bar} = 165.0^\circ\text{C} \Rightarrow h_6 = h_f(T_7) = 697.22 \text{ kJ/kg}$

State 7: $P_7 = 7 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 697.22 \text{ kJ/kg}$

State 8: $h_8 = h_7 = 697.22 \text{ kJ/kg}$

(a) For the steam generator

$$\dot{Q}_{\text{in}}/\dot{m}_1 = h_1 - h_6 = (3321.4 - 697.22) = 2624 \text{ kJ/kg} \leftarrow \dot{Q}_{\text{in}}/\dot{m}_1$$

(b) Applying mass and energy balances to the control volume enclosing the closed feedwater heater

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{697.22 - 161.59}{2684.8 - 697.22} = 0.2695$$

For a control volume enclosing the turbine stages

$$\begin{aligned} \dot{W}_t/\dot{m}_1 &= (h_1 - h_2) + (1-y)(h_2 - h_3) \\ &= (3321.4 - 2684.8) + (1 - 0.2695)(2684.8 - 2009.8) = 1129.7 \text{ kJ/kg} \end{aligned}$$

For the pump

$$\dot{W}_p/\dot{m}_1 = h_5 - h_4 = 161.59 - 151.53 = 10.06 \text{ kJ/kg}$$

Continued on next slide

Problem 8-49 continued

The net power developed, per unit mass entering the first-stage turbine, is

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 1129.7 - 10.06 = 1119.6 \text{ kJ/kg}$$

And the thermal efficiency is

$$\eta = \dot{W}_{\text{cycle}} / \dot{Q}_{\text{in}} = 1119.6 / 2624 = 0.427 \text{ (42.7\%)} \leftarrow \eta$$

(c) For the condenser

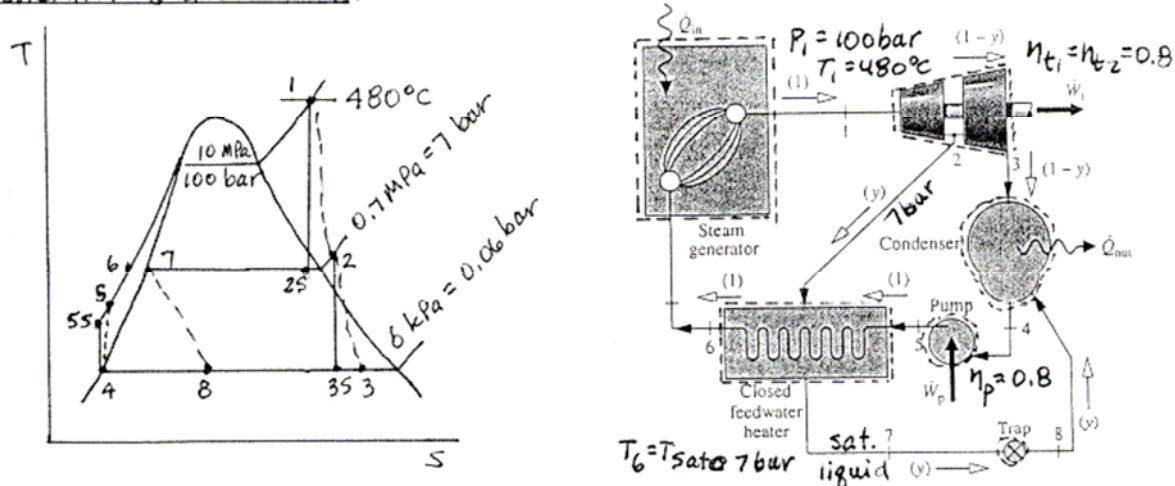
$$\begin{aligned} \frac{\dot{Q}_{\text{out}}}{\dot{m}_1} &= (1-y) h_3 + y h_8 - h_4 \\ &= (1 - 0.2695)(3321.4) + (0.2695)(697.22) - 151.53 \\ &= 1504.1 \text{ kJ/kg} \leftarrow \dot{Q}_{\text{out}}/\dot{m}_1 \end{aligned}$$

PROBLEM 8.50

KNOWN: The ideal regenerative Rankine cycle of Problem 8.49 is modified to include turbine stage and pump isentropic efficiencies of 0.8.

FIND: Answer the same questions as in Problem 8.49 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Problem 8.49, except $\eta_{t1} = \eta_{t2} = \eta_p = 0.8$.

ANALYSIS: First, fix each principal state.

State 1: $P_1 = 100 \text{ bar}$, $T_1 = 480^\circ\text{C} \Rightarrow h_1 = 3321.4 \text{ kJ/kg}$, $s_1 = 6.5282 \text{ kJ/kg}\cdot\text{K}$

State 2: Using $\eta_{t1} = (h_1 - h_2)/(h_1 - h_{2s}) \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s})$

With $h_{2s} = 2684.8 \text{ kJ/kg}$ from Problem 8.49, $h_2 = 2812.1 \text{ kJ/kg}$

State 3: $P_3 = 0.06 \text{ bar}$, $s_{3s} = s_2$. To get s_2 , interpolate in Table A-3: $s_2 = 6.810 \frac{\text{kJ}}{\text{kg}}$

Thus $x_{3s} = \frac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.8061$; $h_{3s} = 2099.0 \text{ kJ/kg}$

Using η_{t2} : $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 2241.6 \text{ kJ/kg}$

State 4: $P_4 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 151.53 \text{ kJ/kg}$

State 5: Using $\eta_p = (h_5 - h_4)/(h_5 - h_{4s})$ and $h_{4s} = 161.59 \frac{\text{kJ}}{\text{kg}}$ from Problem 8.49

$h_5 = h_4 + (h_{5s} - h_4)/\eta_p = 164.11 \text{ kJ/kg}$

State 6: $h_6 \approx h_f(T_7) = 697.22 \text{ kJ/kg}$

State 7: $P_7 = 7 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 697.22 \text{ kJ/kg}$

State 8: $h_8 = h_7 = 697.22 \text{ kJ/kg}$

(a) $\dot{Q}_{in}/\dot{m}_1 = h_1 - h_6 = 2624 \text{ kJ/kg} \leftarrow \frac{\dot{Q}_{in}/\dot{m}_1}{\text{kJ/kg}}$

(b) $y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{697.22 - 164.11}{2812.1 - 697.22} = 0.2521$

$\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1 - y)(h_2 - h_3) = 935.98 \text{ kJ/kg}$

$\dot{W}_p/\dot{m}_1 = h_5 - h_4 = 12.58 \text{ kJ/kg}$

$\dot{W}_{cycle}/\dot{m}_1 = \dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1 = 923.4 \text{ kJ/kg}$

$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{923.4}{2624} = 0.352 (35.2\%) \leftarrow \eta$

(c) $\dot{Q}_{out}/\dot{m}_1 = (1 - y)h_3 + yh_8 - h_4 = 1700.7 \text{ kJ/kg} \leftarrow \frac{\dot{Q}_{out}/\dot{m}_1}{\text{kJ/kg}}$

1. These results can be compared with those of Problem 8.49 to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

PROBLEM 8.51

See Problem 8.50.

IT Code

```
p1 = 100 // bar
T1 = 480 // °C
p2 = 7 // bar
p3 = 0.06 // bar
p4 = p3
p5 = p1
p6 = p1
p7 = p2
p8 = p3
etat = 0.8
etap = etat
mdot1 = 1
```

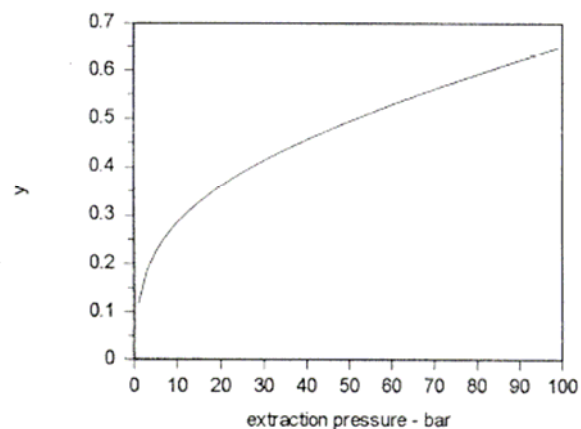
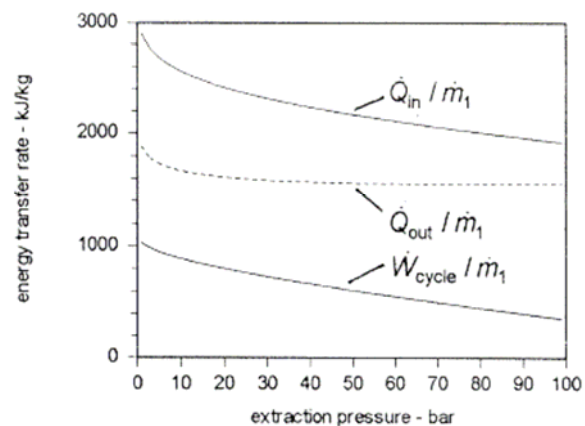
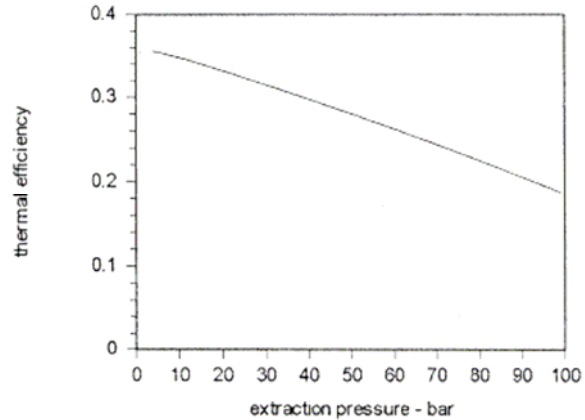
```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
h2 = h1 - etat * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)
s3s = s2
h3s = h_Ps("Water/Steam", p3, s3s)
h3 = h2 - etat * (h2 - h3s)
h4 = hsat_Px("Water/Steam", p4, 0)
v4 = vsat_Px("Water/Steam", p4, 0)
h5s = h4 + v4 * (p5 - p4) * 100
h5 = h4 + (h5s - h4) / etap
h6 = hsat_Px("Water/Steam", p2, 0)
h7 = hsat_Px("Water/Steam", p7, 0)
h8 = h7
```

```
Qdotin = mdot1 * (h1 - h6)
y = (h6 - h5) / (h2 - h7)
Wdott = mdot1 * ((h1 - h2) + (1 - y) * (h2 - h3))
Wdotp = mdot1 * (h5 - h4)
Wdotcycle = Wdott - Wdotp
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * ((1 - y) * h3 + y * h8 - h4)
```

IT Results ($p_2 = 7$ bar)

```
 $\dot{Q}_{in} / \dot{m}_1 = 2624$  kJ/kg
 $\dot{Q}_{out} / \dot{m}_1 = 1701$  kJ/kg
 $\dot{W}_{cycle} / \dot{m}_1 = 923.4$  kJ/kg
 $\eta = 0.3519$ 
 $h_1 = 3321$  kJ/kg
 $h_2 = 2812$  kJ/kg
 $h_3 = 2241$  kJ/kg
 $h_4 = 151$  kJ/kg
 $h_5 = 163.6$  kJ/kg
 $h_6 = 696.8$  kJ/kg
 $h_7 = 696.8$  kJ/kg
 $h_8 = 696.8$  kJ/kg
 $y = 0.2521$ 
```

PLOTS



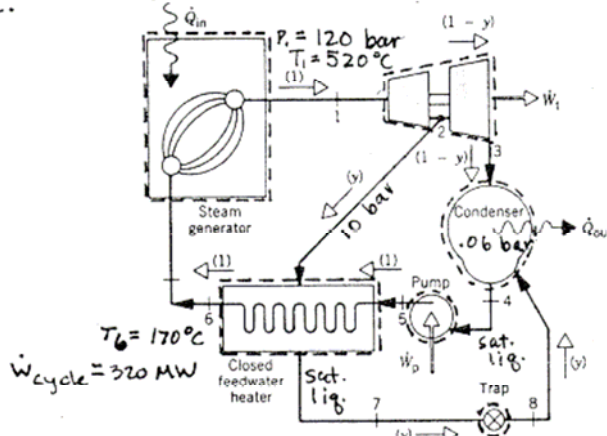
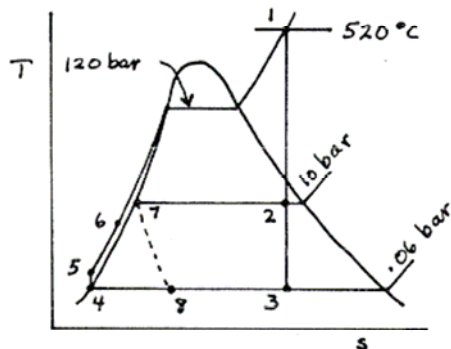
From the thermal efficiency plot, we see that η decreases considerably as the extraction pressure increases.

PROBLEM 8.52

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed feedwater heater. Data at various locations are known and the net power output is given.

FIND: Determine (a) the thermal efficiency and (b) the mass flow rate into the first turbine stage.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the expansion through the trap (a throttling process) and in the closed feedwater heater. (3) The turbines, pump, and feedwater heater operate adiabatically. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the closed heater and the condenser as saturated liquid at the respective pressures.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 120 \text{ bar}$, $T_1 = 520^\circ\text{C} \Rightarrow h_1 = 3401.8 \text{ kJ/kg}$, $s_1 = 6.5555 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 10 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = 0.9931$, $h_2 = 2764.2 \text{ kJ/kg}$

State 3: $P_3 = 0.06 \text{ bar}$, $s_3 = s_1 \Rightarrow x_3 = 0.7727$, $h_3 = 2018.3 \text{ kJ/kg}$

State 4: $P_4 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 151.53 \text{ kJ/kg}$

State 5: $h_5 \approx h_4 + v_4(P_5 - P_4)$

$$= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (120 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$= 151.53 + 12.07 = 163.6 \text{ kJ/kg}$$

State 6: $P_6 = 120 \text{ bar}$, $T_6 = 170^\circ\text{C} \Rightarrow$ Interpolating in Table A-5, $h_6 = 725.86 \frac{\text{kJ}}{\text{kg}}$

State 7: $P_7 = 10 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 762.81 \text{ kJ/kg}$

State 8: Throttling process $\Rightarrow h_8 = h_7 = 762.81 \text{ kJ/kg}$

(a) Applying mass and energy rate balances to the control volume surrounding the closed feedwater heater, the fraction of the flow y extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{725.86 - 163.6}{2764.2 - 762.81} = 0.2809$$

For the control volume surrounding the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$

Continued on next slide

Problem 8-52 continued

$$= (3401.8 - 2764.2) + (1 - 0.2809)(2764.2 - 2018.3) = 1174.0 \text{ kJ/kg}$$

And, for the pump

$$\frac{\dot{W}_p}{\dot{m}_1} = h_5 - h_4 = 163.6 - 151.53 = 12.07 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_6 = (3401.8 - 725.86) = 2675.9 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1174.0 - 12.07}{2675.9} = 0.434 \text{ (43.4\%)} \leftarrow \eta$$

(b) The mass flow rate into the first turbine stage is found from

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]$$

or

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]}$$

$$= \frac{(320 \text{ MW})}{[1174.0 - 12.07] \text{ kJ/kg}} \left| \frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right|$$

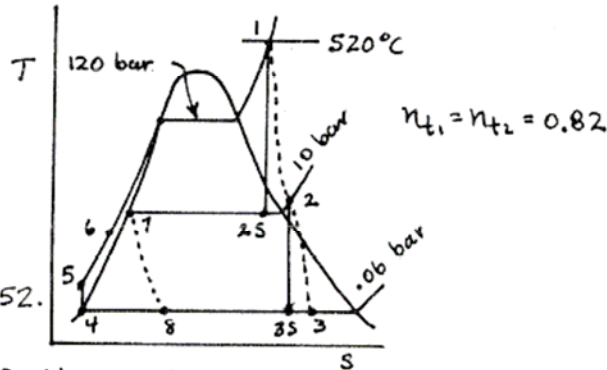
$$= 9.91 \times 10^5 \text{ kg/h} \leftarrow \dot{m}_1$$

PROBLEM 8.53

KNOWN: The regenerative vapor power cycle of Problem 8.52 is modified to include that each turbine stage has an efficiency of 82%.

FIND: Answer the same questions about the modified cycle as in Problem 8.52.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Problem 8.52, except $\eta_{t1} = \eta_{t2} = 0.82$.

ANALYSIS: From the solution of Problem 8.52,

$$h_1 = 3401.8 \text{ kJ/kg}, s_1 = 6.5555 \text{ kJ/kg} \cdot \text{K}$$

$$h_4 = 151.53 \text{ kJ/kg}$$

$$h_5 = 163.6 \text{ kJ/kg}$$

$$h_6 = 725.86 \text{ kJ/kg}$$

$$h_7 = h_8 = 762.81 \text{ kJ/kg}$$

State 2: Using the turbine efficiency, $\eta_{t1} = (h_1 - h_2) / (h_1 - h_{2s})$. With $h_{2s} = 2764.2 \text{ kJ/kg}$ (Problem 8.52)

$$h_2 = h_1 - \eta_{t1} (h_1 - h_{2s}) = 2879.0 \text{ kJ/kg}$$

From Table A-4, $s_2 = 6.7977 \text{ kJ/kg} \cdot \text{K}$

State 3: Similarly, with $p_3 = 0.06 \text{ bar}$, $s_{3s} = s_2 \Rightarrow x_{3s} = 0.8037$ and $h_{3s} = 2093.2 \text{ kJ/kg}$. Thus

$$h_3 = h_2 - \eta_{t2} (h_2 - h_{3s}) = 2234.6 \text{ kJ/kg}$$

(a) The fraction of flow y is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = 0.2657$$

For the turbine stages

$$\dot{W}_t / \dot{m}_1 = (h_1 - h_2) + (1 - y)(h_2 - h_3) = 996.0 \text{ kJ/kg}$$

And, for the pump

$$\dot{W}_p / \dot{m}_1 = h_5 - h_4 = 12.07 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\dot{Q}_{in} / \dot{m}_1 = h_1 - h_6 = 2675.9 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t / \dot{m}_1 - \dot{W}_p / \dot{m}_1}{\dot{Q}_{in} / \dot{m}_1} = 0.368 \text{ (36.8\%)} \quad \eta$$

(b) The mass flow rate into the first turbine stage is

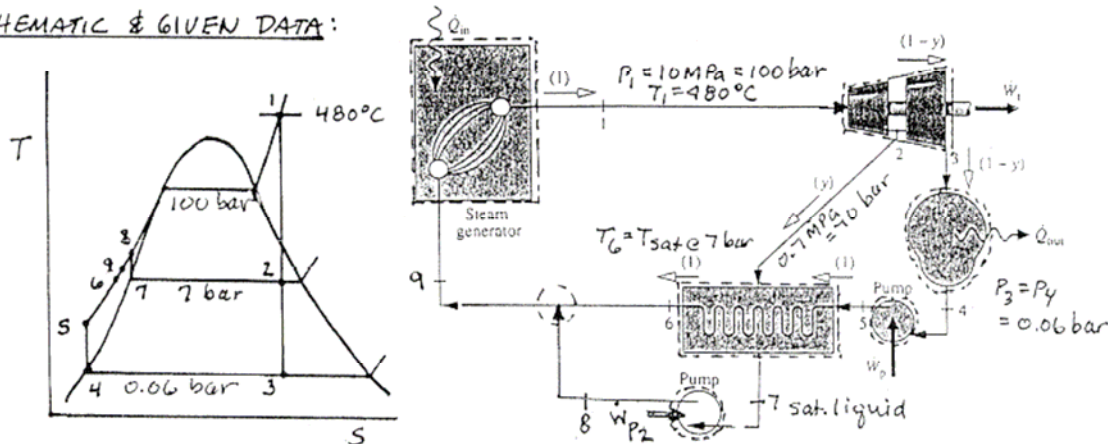
$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{[\dot{W}_t / \dot{m}_1 - \dot{W}_p / \dot{m}_1]} = \frac{(320 \text{ MW})}{[996.0 - 12.07] \text{ kJ/kg}} \left(\frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) = 1.17 \times 10^6 \text{ kg/h} \quad \dot{m}_1$$

PROBLEM 8.54

KNOWN: The modified regenerative cycle in Problem 8.49 is reconsidered. The condensate is pumped into the boiler feedwater line.

FIND: Answer the same questions about the cycle with this further modification as in Problem 8.49. Compare the two condensate options.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the mixing of streams 8 and 6 to form 9. (3) Other assumptions are as in Problem 8.49.

ANALYSIS: First, fix each of the principal states. States 1-7 are the same as in Problem 8.49. Summarizing

$$\begin{aligned} h_1 &= 3321.4 \text{ kJ/kg} & h_5 &= 161.59 \\ h_2 &= 2684.8 & h_6 &= 697.22 \\ h_3 &= 2009.8 & h_7 &= 697.22 \\ h_4 &= 151.53 \end{aligned}$$

State 8: $h_8 \approx h_7 + v_7 (p_8 - p_7)$

$$\begin{aligned} &= 697.22 \frac{\text{kJ}}{\text{kg}} + (1.108 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (100 - 7) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= 697.22 + 10.30 = 707.52 \text{ kJ/kg} \end{aligned}$$

State 9: First, applying mass and energy rate balances to the closed feedwater heater, the fraction of flow extracted at 2 is obtained as follows

$$0 = (1-y)(h_5 - h_6) + y(h_2 - h_7)$$

or

$$y = \frac{(h_6 - h_5)}{(h_2 - h_7) + (h_6 - h_5)} = \frac{(697.22 - 161.59)}{(2684.8 - 697.22) + (697.22 - 161.59)} = 0.2123$$

Now, for the control volume enclosing the mixing process

$$0 = (1-y)h_6 + yh_8 - h_9$$

or

$$\begin{aligned} h_9 &= (1-y)h_6 + yh_8 = (0.7877)(697.22) + (0.2123)(707.52) \\ &= 699.41 \text{ kJ/kg} \end{aligned}$$

(a) The heat transfer rate $\dot{Q}_{in}/\dot{m}_1$ is

$$\dot{Q}_{in}/\dot{m}_1 = h_1 - h_9 = (3321.4 - 699.41) \text{ kJ/kg} = 2622 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}_1$$

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Problem 8-54 continued

(b) For the turbines

$$\begin{aligned}\frac{\dot{W}_t}{\dot{m}_1} &= (h_1 - h_2) + (1-y)(h_2 - h_3) \\ &= (3321.4 - 2684.8) + (0.7877)(2684.8 - 2009.8) = 1168.3 \text{ kJ/kg}\end{aligned}$$

And, for the pumps

$$\begin{aligned}\frac{\dot{W}_p}{\dot{m}_1} &= (1-y)(h_5 - h_4) + y(h_8 - h_7) \\ &= (0.7877)(161.59 - 151.53) + (0.2123)(707.52 - 697.22) \\ &= 10.11 \text{ kJ/kg}\end{aligned}$$

$$\text{Thus } \frac{\dot{W}_{\text{cycle}}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 1168.3 - 10.11 = 1158.2 \text{ kJ/kg}$$

$$\text{And } \eta = \frac{\dot{W}_{\text{cycle}}/\dot{m}_1}{\dot{Q}_{\text{in}}/\dot{m}_1} = \frac{1158.2}{2622} = 0.442 \text{ (44.2\%)} \quad \leftarrow \eta$$

(c) For the condenser

$$\begin{aligned}\frac{\dot{Q}_{\text{out}}}{\dot{m}_1} &= (1-y)(h_3 - h_4) = (0.7877)(2009.8 - 151.53) \\ &= 1463.8 \text{ kJ/kg} \quad \leftarrow \frac{\dot{Q}_{\text{out}}}{\dot{m}_1}\end{aligned}$$

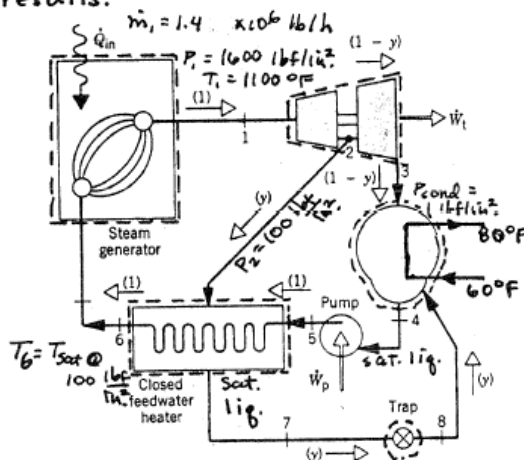
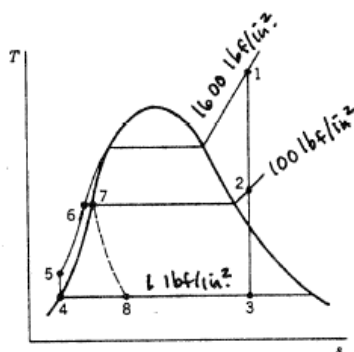
Discussion: Pumping the condensate into the boiler feedwater line has advantages over trapping it into the condenser. The fraction of steam y extracted at 2 is lower, resulting in greater work per unit mass flow through the turbine. Since the power is constant, less flow is required. This results in lower heat input and higher thermal efficiency. The pumping option also eliminates the irreversibility due to the trapping valve. Finally, the pumping option is somewhat more complex in terms of piping and requires an additional pump in the system with associated maintenance needs.

PROBLEM 8.55

KNOWN: The ideal Rankine cycle of Problem 8.9 is modified to include one closed feedwater heater operating at 100 lbf/in². The condensate from the heater is trapped into the condenser.

FIND: Answer the same questions about the modified cycle as in Problem 8.9, and discuss the results.

SHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the expansion through the trap (a throttling process) and in the closed feedwater heater. (3) The turbines, pump, and feedwater heater operate adiabatically. (4) kinetic and potential energy effects are negligible. (5) Condensate exits the closed heater as saturated liquid at 100 lbf/in² and the condenser as saturated liquid at 1 lbf/in².

ANALYSIS: First, fix each of the principal states. From the solution to Problem 8.9; $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R, $h_3 = 911.2$ Btu/lb, $h_4 = 69.74$ Btu/lb, and $h_5 = 74.52$ Btu/lb.

State 2: $P_2 = 100$ lbf/in², $s_2 = s_1 \Rightarrow h_2 = 1210.7$ Btu/lb (Table A-4E)

State 6: $P_6 = 1600$ lbf/in², $T_6 = T_{\text{sat @ } 1600 \text{ lbf/in}^2} = 327.86^\circ\text{F} \Rightarrow$ from Table A-5E $h_6 \approx 301.6$ Btu/lb (double interpolation)

State 7: $P_7 = 100$ lbf/in², sat. liquid $\Rightarrow h_7 = 298.6$ Btu/lb

State 8: Throttling process $\Rightarrow h_8 = h_7 = 298.6$ Btu/lb

Applying mass and energy rate balances to the closed feedwater heater, the fraction of flow y extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = 0.2490$$

(a) To determine the power developed by the cycle, begin with a control volume around the turbine

$$\begin{aligned} \frac{\dot{W}_t}{\dot{m}_1} &= (h_1 - h_2) + (1 - y)(h_2 - h_3) \\ &= [(1547.7 - 1210.7) + (1 - 0.2490)(1210.7 - 911.2)] \text{ Btu/lb} \\ &= 561.2 \text{ Btu/lb} \end{aligned}$$

Continued on next slide

Problem 8-55 continued

For the pump

$$\frac{\dot{W}_p}{\dot{m}_1} = h_5 - h_4 = (74.52 - 69.74) \text{ Btu/lb} = 4.78 \text{ Btu/lb}$$

Thus, with $\dot{m}_1 = 1.4 \times 10^6 \text{ lb/h}$ (Problem 8.9)

$$\begin{aligned} \dot{W}_{\text{cycle}} &= \dot{m}_1 [\dot{W}_{t/\dot{m}_1} - \dot{W}_{p/\dot{m}_1}] \\ &= (1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) [561.92 - 4.78] \frac{\text{Btu}}{\text{lb}} = 7.800 \times 10^8 \text{ Btu/h} \end{aligned} \quad \leftarrow \dot{W}_{\text{cycle}}$$

(b) The thermal efficiency is

$$\begin{aligned} \eta &= \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{cycle}}}{\dot{m}_1 (h_1 - h_6)} \\ &= \frac{(7.800 \times 10^8) \text{ Btu/h}}{(1.4 \times 10^6 \text{ lb/h}) (1547.7 - 301.6) \text{ Btu/lb}} = 0.447 \text{ (44.7\%)} \end{aligned} \quad \leftarrow \eta$$

(c) For the condenser, the mass and energy rate balances reduce to

$$0 = \dot{m}_1 [(1-y)h_3 + y h_8 - h_4] + \dot{m}_{\text{cw}} (h_{\text{cw},\text{in}} - h_{\text{cw},\text{out}})$$

where $\dot{m}_{\text{cw}}$ is the mass flow rate of cooling water. Solving for $\dot{m}_{\text{cw}}$

$$\dot{m}_{\text{cw}} = \frac{\dot{m}_1 [(1-y)h_3 + y h_8 - h_4]}{(h_{\text{cw},\text{out}} - h_{\text{cw},\text{in}})}$$

With $h_{\text{cw}} \approx h_f(T)$ from Table A-2E

$$\begin{aligned} \dot{m}_{\text{cw}} &= \frac{(1.4 \times 10^6 \text{ lb/h}) [(0.751)911.2 + (0.249)298.6 - 69.74] \text{ Btu/lb}}{(48.09 - 28.08) \text{ Btu/lb}} \\ &= 4.820 \times 10^7 \text{ lb/h} \end{aligned} \quad \leftarrow \dot{m}_{\text{cw}}$$

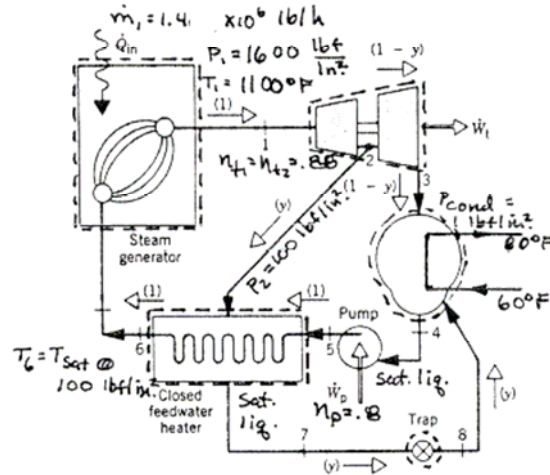
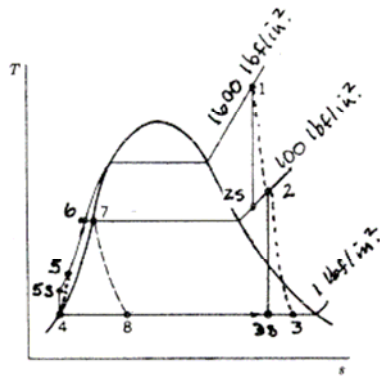
Discussion: (1) Since some mass is extracted from the turbine, the power developed in the modified cycle is less than for the cycle of Problem 8.9. (2) The rate of heat transfer to the working fluid passing through the steam generator is less for the modified cycle due to regeneration in the closed feed water heater. In this case, the thermal efficiency is increased due to regeneration.

PROBLEM 8.56

KNOWN: The regenerative power cycle of Problem 8.55 is modified to include that each turbine stage has an efficiency of 88% and the pump has an efficiency of 80%.

FIND: Answer the same questions for the modified cycle as in Problem 8.55.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Problem 8.55, except $\eta_{t1} = \eta_{t2} = 0.88$ and $\eta_p = 0.80$.

ANALYSIS: From Problem 8.55, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb $\cdot$ °R, $h_4 = 69.74$ Btu/lb, $h_6 = 301.6$ Btu/lb, $h_7 = 298.6$ Btu/lb, $h_8 = h_7 = 298.6$ Btu/lb.

State 2: Use the turbine stage efficiency. First, $p_2 = 100$ lbf/in² and $s_{2s} = s_1$;
 $h_{2s} = 1210.7$ Btu/lb (Problem 8.55). Thus

$$\eta_{t1} = \frac{\dot{W}_{t1}/\dot{m}_1}{(\dot{W}_{t1}/\dot{m}_1)_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 1251.1 \text{ Btu/lb}$$

From Table A-4E, $s_2 = 1.6784$ Btu/lb $\cdot$ °R.

State 3: Similarly, $p_3 = 1$ lbf/in², $s_{3s} = s_2 \Rightarrow x_{3s} = 0.8377$, $h_{3s} = 937.55$ Btu/lb

Thus, $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 975.2$ Btu/lb

State 5: State 5 is fixed using the pump efficiency. From Problem 8.55,
 $h_{5s} = 74.52$ Btu/lb. Thus

$$\eta_p = \frac{h_{5s} - h_4}{h_5 - h_4} \Rightarrow h_5 = h_4 + (h_{5s} - h_4)/\eta_p = 75.715 \text{ Btu/lb}$$

The fraction y of the turbine flow extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = 0.2371$$

(a) For the turbine stages

$$\begin{aligned} \frac{\dot{W}_t}{\dot{m}_1} &= (h_1 - h_2) + (1 - y)(h_2 - h_3) \\ &= (1547.7 - 1251.1) + (1 - 0.2371)(1251.1 - 975.2) \\ &= 507.08 \text{ Btu/lb} \end{aligned}$$

For the pump

Continued on next slide

Problem 8-56 continued

$$\frac{\dot{W}_P}{\dot{m}} = h_5 - h_4 = 75.715 - 69.74 = 5.975 \text{ Btu/lb}$$

And the net power developed is

$$\begin{aligned} \dot{W}_{\text{cycle}} &= \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_P/\dot{m}_1] \\ &= (1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) [507.08 - 5.975] \frac{\text{Btu}}{\text{lb}} = 7.016 \times 10^8 \text{ Btu/h} \leftarrow \dot{W}_{\text{cycle}} \end{aligned}$$

(b) The thermal efficiency is

$$\begin{aligned} \eta &= \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{cycle}}}{\dot{m}_1 (h_1 - h_6)} \\ &= \frac{(7.017 \times 10^8)}{(1.4 \times 10^6)(1547.7 - 301.6)} = 0.402 \text{ (40.2\%)} \leftarrow \eta \end{aligned}$$

(c) The mass flow rate of cooling water passing through the condenser is

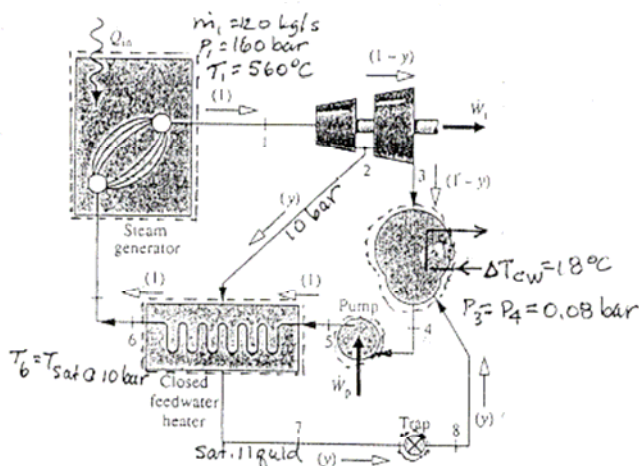
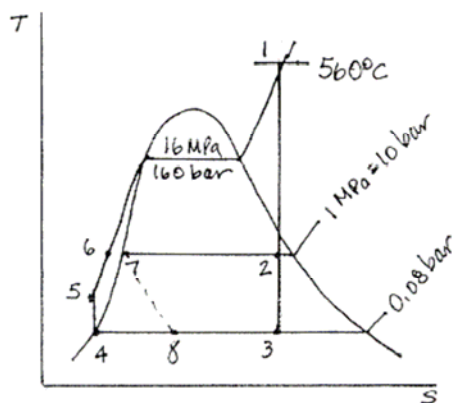
$$\begin{aligned} \dot{m}_{\text{cw}} &= \frac{\dot{m}_1 [(1-y)h_3 + y h_8 - h_4]}{(h_{\text{cw,out}} - h_{\text{cw,in}})} \\ &= \frac{(1.4 \times 10^6) [(0.7629)975.2 + (0.2371)298.6 - 69.74]}{(48.09 - 28.08)} \\ &= 5.213 \times 10^7 \text{ lb/h} \leftarrow \dot{m}_{\text{cw}} \end{aligned}$$

PROBLEM 8.57

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed feedwater heater. Data are specified at various locations, and the mass flow rate of steam entering the first-stage turbine is given. The temperature of the cooling water passing through the condenser is known.

FIND: Determine (a) the net power developed, (b) the rate of heat addition, (c) the thermal efficiency, and (d) the mass flow rate of cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the expansion through the trap (a throttling process) and in the closed feedwater heater. (3) The turbines, pump, and feedwater heater operate adiabatically. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the closed heater and the condenser as saturated liquid at the respective pressures.

ANALYSIS: First, fix each principal state.

State 1: $p_1 = 160 \text{ bar}$, $T_1 = 560^\circ\text{C} \Rightarrow h_1 = 3465.4 \text{ kJ/kg}$, $s_1 = 6.5132 \text{ kJ/kg}$

State 2: $p_2 = 10 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.9836$, $h_2 = 2745.1 \text{ kJ/kg}$

State 3: $p_3 = 0.08 \text{ bar}$, $s_3 = s_2 \Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.7753$, $h_3 = 2037.0 \text{ kJ/kg}$

State 4: $p_4 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 173.88 \text{ kJ/kg}$

State 5: $h_5 \approx h_4 + v_4(p_5 - p_4)$
 $= 173.88 + (1.0084 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (160 - 0.08) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$
 $= 173.88 + 16.13 = 190.01 \text{ kJ/kg}$

State 6: $p_6 = 160 \text{ bar}$, $T_6 = T_{\text{sat}} @ 160 \text{ bar} \approx 180^\circ\text{C}$. From Table A-5, $h_6 \approx 771 \text{ kJ/kg}$

State 7: $p_7 = 10 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 762.81 \text{ kJ/kg}$

State 8: Throttling process $\Rightarrow h_8 = h_7 = 762.81 \text{ kJ/kg}$

(a) For the control volume enclosing the closed feedwater heater

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{771 - 190.01}{2745.1 - 762.81} = 0.2931$$

Thus, for the control volume enclosing the turbine stages

$$\dot{W}_t = \dot{m}_1 [(h_1 - h_2) + (1 - y)(h_2 - h_3)]$$

$$= (120 \frac{\text{kg}}{\text{s}}) [(3465.4 - 2745.1) + (0.7069)(2745.1 - 2037.0)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 1.465 \times 10^5 \text{ kW}$$

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Problem 8-57 continued

For the pump

$$\dot{W}_p = \dot{m}_1 (h_5 - h_4) = (120 \frac{\text{kg}}{\text{s}})(190.01 - 173.88) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 1936 \text{ kW}$$

Thus $\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = 1.4456 \times 10^5 \text{ kW} \longleftarrow \dot{W}_{\text{cycle}}$

(b) For the steam generator

$$\dot{Q}_{\text{in}} = \dot{m}_1 (h_1 - h_6) = (120)(3465.4 - 771) \left| \frac{1}{1} \right| = 3.233 \times 10^5 \text{ kW} \longleftarrow \dot{Q}_{\text{in}}$$

(c) The thermal efficiency is

$$\eta = \dot{W}_{\text{cycle}} / \dot{Q}_{\text{in}} = (1.445 \times 10^5) / (3.233 \times 10^5) = 0.447 (44.7\%) \longleftarrow \eta$$

(d) For the control volume enclosing the condenser

$$0 = \dot{m}_1 [(1-y)h_3 + yh_8 - h_4] + \dot{m}_{\text{cw}} (h_{\text{in,cw}} - h_{\text{out,cw}})$$

or
$$\dot{m}_{\text{cw}} = \frac{\dot{m}_1 [(1-y)h_3 + yh_8 - h_4]}{(h_{\text{out,cw}} - h_{\text{in,cw}})}$$

With $(h_{\text{out,cw}} - h_{\text{in,cw}}) = c_{\text{cw}} \Delta T_{\text{cw}}$ and $c_{\text{cw}} = 4.179$ from Table A-19

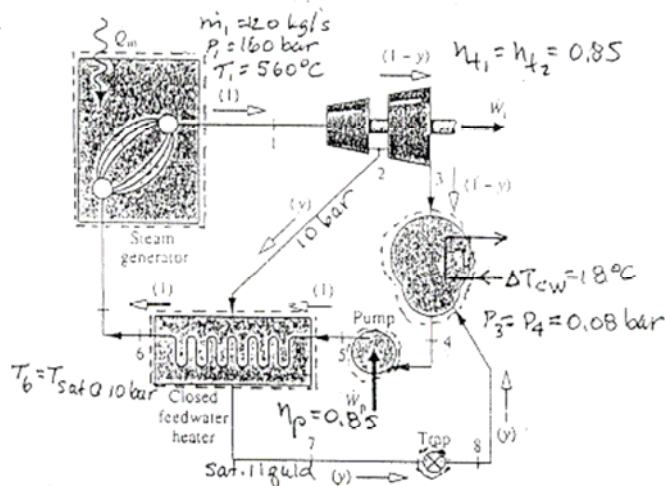
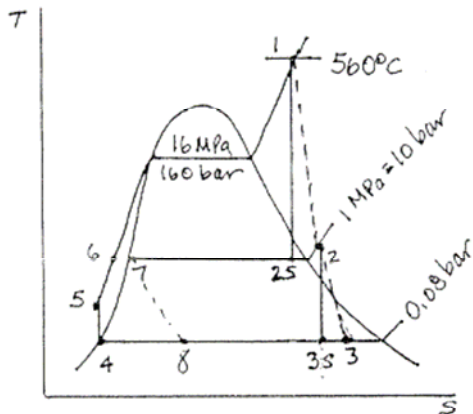
$$\begin{aligned} \dot{m}_{\text{cw}} &= \frac{(120 \text{ kg/s}) [(0.7069)(2037.0) + (0.2931)(762.81 - 173.88) \text{ kJ/kg}]}{(4.179)(18) \text{ kJ/kg}} \\ &= \frac{1.788 \times 10^5}{75.222} = 2376 \text{ kg/s} \longleftarrow \dot{m}_{\text{cw}} \end{aligned}$$

PROBLEM 8.58

KNOWN: The ideal regenerative Rankine cycle of Problem 8.57 is modified to include turbine stage and pump isentropic efficiencies of 0.85.

FIND: Answer the same questions as in Problem 8.57 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Problem 8.57, except $\eta_{t1} = \eta_{t2} = \eta_p = 0.85$.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 160 \text{ bar}$, $T_1 = 560^\circ\text{C} \Rightarrow h_1 = 3465.1 \text{ kJ/kg}$, $s_1 = 6.5132 \text{ kJ/kg}\cdot\text{K}$

State 2: Using the isentropic efficiency of the first-stage turbine

$$\eta_{t1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s})$$

With $h_{2s} = 2745.1 \text{ kJ/kg}$ from Problem 8.57 $h_2 = 2853.1 \text{ kJ/kg}$

State 3: $P_3 = 0.08 \text{ bar}$, $s_3 = s_2 \Rightarrow$ Interpolating in Table A-4; $s_2 = 6.7451 \text{ kJ/kg}\cdot\text{K}$

$$\text{Thus } x_{3s} = \frac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.8056; h_{3s} = 2109.8 \text{ kJ/kg}$$

Using the second-stage turbine isentropic efficiency

$$h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 2221.3 \text{ kJ/kg}$$

State 4: $P_4 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 173.88 \text{ kJ/kg}$

State 5: Using the isentropic pump efficiency; $\eta_p = (h_{5s} - h_4) / (h_5 - h_4)$

Thus, with $h_{5s} = 190.01 \text{ kJ/kg}$ from Problem 8.57

$$h_5 = h_4 + (h_{5s} - h_4) / \eta_p = 192.86 \text{ kJ/kg}$$

State 6: $P_6 = 160 \text{ bar}$. From Problem 8.57, $h_6 = 771 \text{ kJ/kg}$

State 7: $h_7 = 762.8 \text{ kJ/kg}$

State 8: $h_8 = h_7 = 762.8 \text{ kJ/kg}$

$$(a) y = (h_6 - h_5) / (h_2 - h_7) = (771 - 192.86) / (2853.1 - 762.8) = 0.2766$$

$$\dot{W}_t = \dot{m}_1 [(h_1 - h_2) + (1 - y)(h_2 - h_3)] = 1.2828 \text{ kW} \quad \dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = 1.26 \times 10^5 \text{ kW}$$

$$\dot{W}_p = \dot{m}_1 (h_5 - h_4) = 2278 \text{ kW}$$

$$(b) \dot{Q}_{\text{in}} = \dot{m}_1 (h_1 - h_6) = 3.233 \times 10^5 \text{ kW}$$

$$(c) \eta = \dot{W}_{\text{cycle}} / \dot{Q}_{\text{in}} = 0.390 \text{ (39.0\%)} \quad \eta$$

$$(d) \dot{m}_{\text{cw}} = \dot{m}_1 [(1 - y)h_3 + yh_8 - h_4] / (c_{\text{cw}} \Delta T) = 2623 \text{ kg/s} \quad \dot{m}_{\text{cw}}$$

1. These results can be compared to those of Problem 8.57 to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

PROBLEM 8.59

KNOWN: The power plant layout of Fig. 8.12 is under consideration.

FIND: Determine expressions for the fractions of the total flow at states 8, 11, 17 in terms of the fractions y_2, y_3, y_6 , and y_7 .

SCHEMATIC & GIVEN DATA: See Fig. 8.12

ENGINEERING MODEL: Control volumes at steady state enclose each of the principal components.

ANALYSIS: Letting $\dot{m}_1$ denote the total flow entering the first turbine stage, we have

$$\text{state 2: } y_2 = \frac{\dot{m}_2}{\dot{m}_1}, \text{ state 3: } y_3 = \frac{\dot{m}_3}{\dot{m}_1}, \text{ state 6: } y_6 = \frac{\dot{m}_6}{\dot{m}_1}, \text{ state 7: } y_7 = \frac{\dot{m}_7}{\dot{m}_1}$$

Applying a mass rate balance to a control volume enclosing the overall turbine,

$$\dot{m}_8 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 - \dot{m}_6 - \dot{m}_7 + \frac{\dot{m}_5 - \dot{m}_4}{= 0, \text{ since } \dot{m}_4 = \dot{m}_5}$$

$$\therefore \frac{\dot{m}_8}{\dot{m}_1} = 1 - \frac{\dot{m}_2}{\dot{m}_1} - \frac{\dot{m}_3}{\dot{m}_1} - \frac{\dot{m}_6}{\dot{m}_1} - \frac{\dot{m}_7}{\dot{m}_1} = 1 - y_2 - y_3 - y_6 - y_7 \quad \leftarrow y_8$$

A mass balance for a control volume enclosing the condenser reads

$$\dot{m}_9 = \dot{m}_8 + \dot{m}_{20}$$

For the closed feedwater heater and valve, $\dot{m}_7 = \dot{m}_9 = \dot{m}_{20}$. And for the closed feedwater heater and pump, $\dot{m}_{11} = \dot{m}_{10} = \dot{m}_9$. Collecting results

$$\dot{m}_{11} = \dot{m}_8 + \dot{m}_7$$

$$\Rightarrow y_{11} = y_8 + y_7 \quad \text{or} \quad y_{11} = (1 - y_2 - y_3 - y_6 - y_7) + y_7 \\ = 1 - y_2 - y_3 - y_6 \quad \leftarrow y_{11}$$

For the intermediate-pressure closed feedwater heater, $\dot{m}_{17} = \dot{m}_3 + \dot{m}_{16}$. And for the high-pressure closed feedwater heater and valve,

$\dot{m}_2 = \dot{m}_{15} = \dot{m}_{16}$. Collecting results

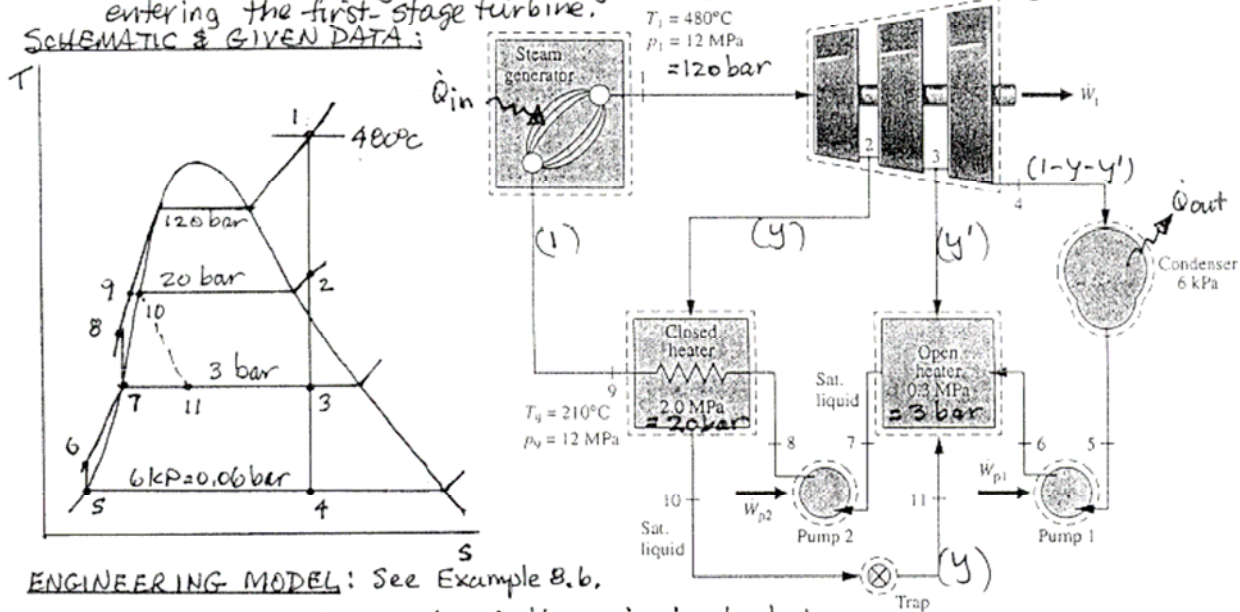
$$\dot{m}_{17} = \dot{m}_3 + \dot{m}_2 \Rightarrow y_{17} = y_3 + y_2 \quad \leftarrow y_{17}$$

PROBLEM 8.60

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed and one open feedwater heater. Data are known at various locations in the cycle.

FIND: Determine (a) the rate of heat addition per kg of steam entering the first-stage turbine, (b) the thermal efficiency, and (c) the rate of heat transfer to cooling water passing through the condenser, per kg of steam entering the first-stage turbine.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.6.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 120 \text{ bar}$, $T_1 = 480^\circ\text{C} \Rightarrow h_1 = 3293.5 \text{ kJ/kg}$, $s_1 = 6.4154 \text{ kJ/kg}\cdot\text{K}$

State 2: $p_2 = 20 \text{ bar}$, $s_2 = s_1 \Rightarrow$ Interpolating in Table A-4; $h_2 = 2836.7 \text{ kJ/kg}$

State 3: $p_3 = 3 \text{ bar}$, $s_3 = s_1 \Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.89164$, $h_3 = 2490.8 \text{ kJ/kg}$

State 4: $p_4 = 0.06 \text{ bar}$, $s_4 = s_1 \Rightarrow x_4 = \frac{s_4 - s_{f4}}{s_{g4} - s_{f4}} = 0.7548$, $h_4 = 1975.1 \text{ kJ/kg}$

State 5: $p_5 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_5 = 151.53 \text{ kJ/kg}$

State 6: $h_6 \approx h_5 + v_5(p_6 - p_5)$
 $= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (3 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$
 $= 151.53 + 0.296 = 151.83 \text{ kJ/kg}$

State 7: $p_7 = 3 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 561.47 \text{ kJ/kg}$

State 8: $h_8 \approx h_7 + v_7(p_8 - p_7) = 561.47 + (1.0732 \times 10^{-3})(120 - 3) \left| \frac{10^5}{10^3} \right|$
 $= 561.47 + 12.56 = 574.03 \text{ kJ/kg}$

State 9: Assume $h_9 \approx h_{f@210^\circ\text{C}} = 897.76 \text{ kJ/kg}$

State 10: $p_{10} = 20 \text{ bar}$, sat. liquid $\Rightarrow 908.79 \text{ kJ/kg}$

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 908.79 \text{ kJ/kg}$

Continued on next slide

Problem 8-60 continued

(a) For the control volume enclosing the steam generator

$$\dot{Q}_{in}/\dot{m}_1 = h_1 - h_9 = 3293.5 - 897.76 = 2395.7 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}_1$$

(b) First, get the fractions extracted at 2 and 3 from energy and mass balances on the closed and open heater control volumes, respectively.

$$0 = y(h_2 - h_{10}) + (h_8 - h_9)$$

$$\text{or } y = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{897.76 - 574.03}{2836.7 - 908.79} = 0.1679$$

$$\text{Further } 0 = y'h_3 + (1 - y - y')h_6 + yh_{11} - h_7$$

$$\text{or } y' = \frac{h_7 - h_6 + y(h_6 - h_{11})}{h_3 - h_6} = \frac{561.47 - 151.83 + (0.1679)(151.83 - 908.79)}{2490.8 - 151.83} = 0.1208$$

For the control volume enclosing the turbine stages

$$\dot{W}_t = \dot{m}_1 [h_1 - y h_2 - y' h_3 - (1 - y - y') h_4]$$

$$\text{or } \dot{W}_t/\dot{m}_1 = h_1 - y h_2 - y' h_3 - (1 - y - y') h_4$$

$$= 3293.5 - (0.1679)(2836.7) - (0.1208)(2490.8) - (0.7113)(1975.1) = 1111.44 \text{ kJ/kg}$$

For the pumps

$$\dot{W}_{P1}/\dot{m}_1 = (1 - y - y')(h_6 - h_5) = (0.7113)(151.83 - 151.53) = 0.2134 \text{ kJ/kg}$$

$$\dot{W}_{P2}/\dot{m}_1 = h_8 - h_7 = 574.03 - 561.47 = 12.56 \text{ kJ/kg}$$

$$\text{Thus } \dot{W}_{cycle}/\dot{m}_1 = \dot{W}_t/\dot{m}_1 - \dot{W}_{P1}/\dot{m}_1 - \dot{W}_{P2}/\dot{m}_1 = 1098.7 \text{ kJ/kg}$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1098.7}{2395.7} = 0.459 \text{ (45.9\%)} \leftarrow \eta$$

(c) For the condenser

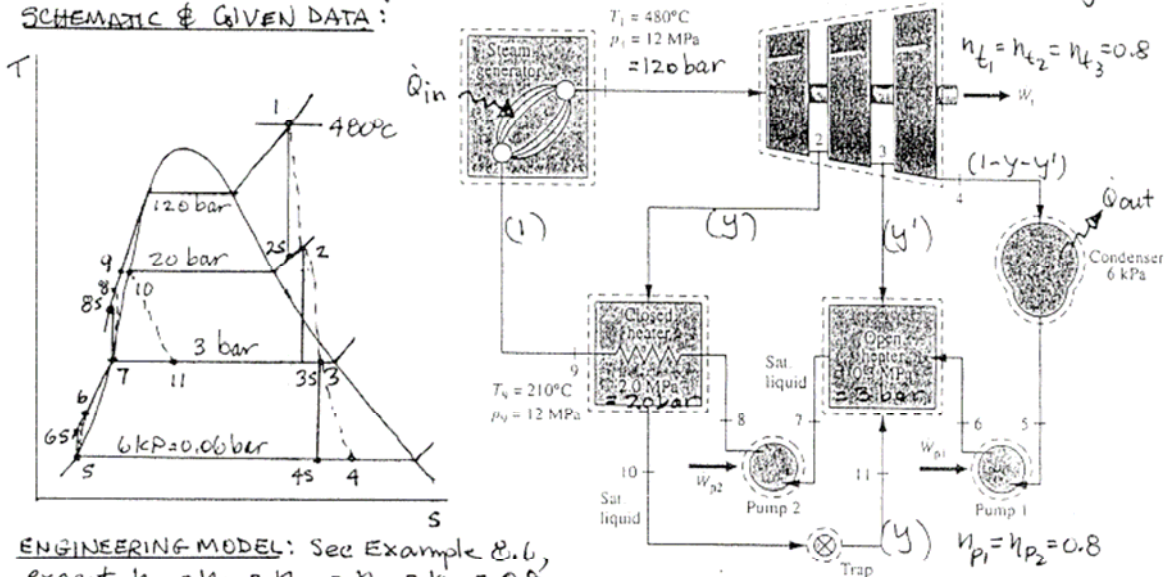
$$\dot{Q}_{out}/\dot{m}_1 = (1 - y - y')(h_4 - h_5) = (0.7113)(1975.1 - 151.53) = 1297.1 \text{ kJ/kg} \leftarrow \dot{Q}_{out}/\dot{m}_1$$

PROBLEM 8.61

KNOWN: The regenerative vapor power cycle of Problem 8.60 is modified to include turbine stage and pump isentropic efficiencies of 0.8.

FIND: Answer the same questions as in Problem 8.60 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.6, except $\eta_{t1} = \eta_{t2} = \eta_{t3} = \eta_{p1} = \eta_{p2} = 0.8$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 120\text{ bar}$, $T_1 = 480^\circ\text{C} \Rightarrow h_1 = 3292.5\text{ kJ/kg}$, $s_1 = 6.4154\text{ kJ/kg}\cdot\text{K}$

State 2: Using the first-stage turbine efficiency: $\eta_{t1} = (h_1 - h_2)/(h_1 - h_{2s})$
With $h_{2s} = 2836.7\text{ kJ/kg}$ from Problem 8.60

$h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 2928.1\text{ kJ/kg}$. From Table A-4; $s_2 = 6.5921\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

State 3: $p_3 = 3\text{ bar}$, $s_{3s} = s_2 \Rightarrow x_{3s} = \frac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.92485$; $h_{3s} = 2562.7\text{ kJ/kg}$

Using the turbine stage efficiency

$h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 2635.8\text{ kJ/kg}$; $s_3 = 6.7722\text{ kJ/kg}\cdot\text{K}$
($x_3 = 0.9587$)

State 4: $p_4 = 0.06\text{ bar}$, $s_{4s} = s_3 \Rightarrow x_{4s} = \frac{s_{4s} - s_{f4}}{s_{g4} - s_{f4}} = 0.8005$; $h_{4s} = 2085.5\text{ kJ/kg}$

Using the turbine stage efficiency

$h_4 = h_3 - \eta_{t3}(h_3 - h_{4s}) = 2195.6\text{ kJ/kg}$

State 5: $p_5 = 0.06\text{ bar}$, sat. liquid $\Rightarrow h_5 = 151.53\text{ kJ/kg}$

State 6: Using the pump efficiency with $h_{6s} = 151.83\text{ kJ/kg}$ from Problem 8.60

$h_6 = h_5 + (h_{6s} - h_5)/\eta_p = 151.91\text{ kJ/kg}$

State 7: $p_7 = 3\text{ bar}$, sat. liquid $\Rightarrow h_7 = 561.47\text{ kJ/kg}$

State 8: Using the pump efficiency with $h_{8s} = 574.03\text{ kJ/kg}$ from Problem 8.60

$h_8 = h_7 + (h_{8s} - h_7)/\eta_p = 577.17\text{ kJ/kg}$

State 9: Assume $h_9 \approx h_f @ 210^\circ\text{C} = 897.76\text{ kJ/kg}$

State 10: $p_{10} = 20\text{ bar}$, sat. liquid $\Rightarrow h_{10} = 908.79\text{ kJ/kg}$

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 908.79\text{ kJ/kg}$

Continued on next slide

Problem 8-61 continued

$$(a) \dot{Q}_{in}/\dot{m}_1 = h_1 - h_9 = 3293.5 - 897.76 = 2395.7 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}_1$$

$$(b) y = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{897.76 - 577.17}{2928.1 - 908.79} = 0.1588$$

$$y' = \frac{h_7 - h_6 + y(h_6 - h_{11})}{h_3 - h_6} = \frac{561.47 - 151.91 + (0.1588)(151.91 - 908.79)}{2635.8 - 151.91} = 0.1165$$

$$\begin{aligned} \dot{W}_t/\dot{m}_1 &= h_1 - y h_2 - y' h_3 - (1 - y - y') h_4 \\ &= 3293.5 - (0.1588)(2928.1) - (0.1165)(2635.8) - (0.7247)(2195.6) \\ &= 930.3 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \dot{W}_p/\dot{m}_1 &= \dot{W}_{p1}/\dot{m}_1 + \dot{W}_{p2}/\dot{m}_2 \\ &= (h_6 - h_5)(1 - y - y') + (h_8 - h_7) \\ &= (151.91 - 151.53)(0.7247) + (577.17 - 561.47) = 15.98 \text{ kJ/kg} \end{aligned}$$

$$\frac{\dot{W}_{cycle}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 914.3 \text{ kJ/kg}$$

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.382 (38.2\%) \leftarrow \eta$$

①

$$\begin{aligned} (c) \dot{Q}_{out}/\dot{m}_1 &= (1 - y - y')(h_4 - h_5) = (0.7247)(2195.6 - 151.53) \\ &= 1481.3 \text{ kJ/kg} \leftarrow \dot{Q}_{out}/\dot{m}_1 \end{aligned}$$

1. These results can be compared with those of Problem 8.60 to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

PROBLEM 8.62

Refer to Problem 8.60.

IT Code

```
p1 = 120 // bar
T1 = 480 // °C
p2 = 20 // bar
p3 = 3 // bar
p4 = 0.06 // bar
p5 = p4
p6 = p3
p7 = p3
p8 = p1
p9 = p1
p10 = p2
p11 = p3
T9 = 210 // °C
mdot1 = 1
```

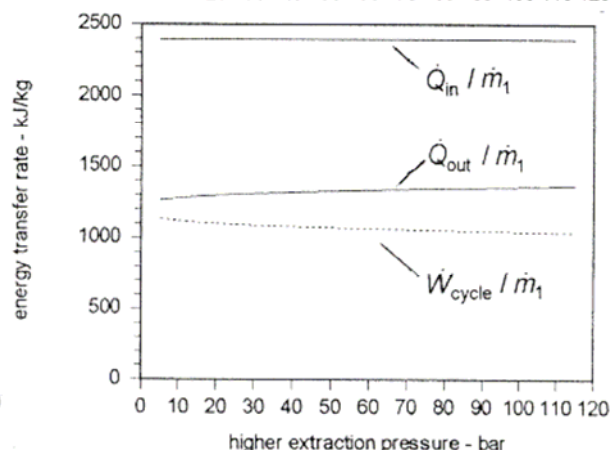
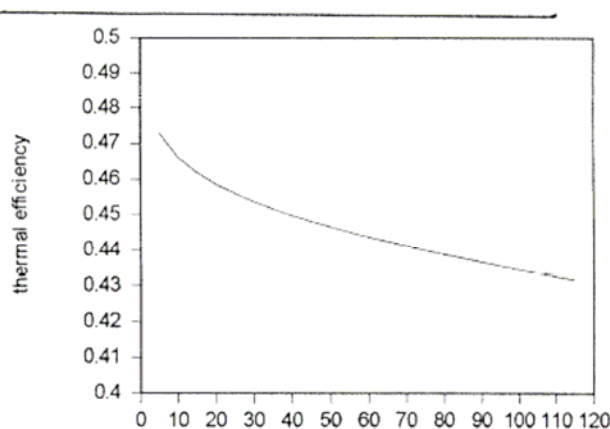
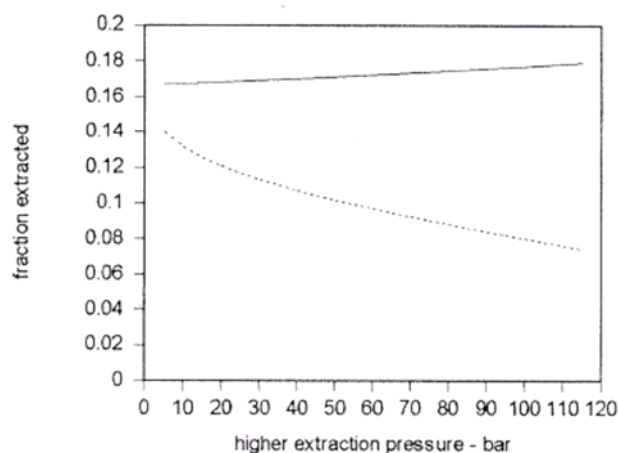
```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2 = s1
h2 = h_Ps("Water/Steam", p2, s2)
s3 = s2
h3 = h_Ps("Water/Steam", p3, s3)
s4 = s3
h4 = h_Ps("Water/Steam", p4, s4)
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6 = h5 + v5 * (p5 - p4) * 100
h7 = hsat_Px("Water/Steam", p7, 0)
v7 = vsat_Px("Water/Steam", p7, 0)
h8 = h7 + v7 * (p8 - p7) * 100
psat = Psat_T("Water/Steam", T9)
h9 = hsat_Px("Water/Steam", psat, 0)
h10 = hsat_Px("Water/Steam", p10, 0)
h11 = h10
```

```
Qdotin = mdot1 * (h1 - h9)
y = (h9 - h8) / (h2 - h10)
y' = (h7 - h6 + y * (h6 - h11)) / (h3 - h6)
Wdott = mdot1 * (h1 - y * h2 - y' * h3 - (1 - y - y') * h4)
Wdotp1 = mdot1 * ((1 - y - y') * (h6 - h5))
Wdotp2 = mdot1 * (h8 - h7)
Wdotcycle = Wdott - Wdotp1 - Wdotp2
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * ((1 - y - y') * (h4 - h5))
```

IT Results ($p_2 = 20$ bar)

$\dot{Q}_{in} / \dot{m}_1 = 2395$ kJ/kg	$h_4 = 1975$ kJ/kg
$\dot{Q}_{out} / \dot{m}_1 = 1296$ kJ/kg	$h_5 = 151$ kJ/kg
$\dot{W}_{cycle} / \dot{m}_1 = 1099$ kJ/kg	$h_6 = 151$ kJ/kg
$\eta = 0.4587$	$h_7 = 561.2$ kJ/kg
$h_1 = 3293$ kJ/kg	$h_8 = 573.7$ kJ/kg
$h_2 = 2836$ kJ/kg	$h_9 = 897.9$ kJ/kg
$h_3 = 2490$ kJ/kg	$h_{10} = 908.9$ kJ/kg
	$h_{11} = 908.9$ kJ/kg
	$y = 0.1682$
	$y' = 0.1208$

PLOTS:

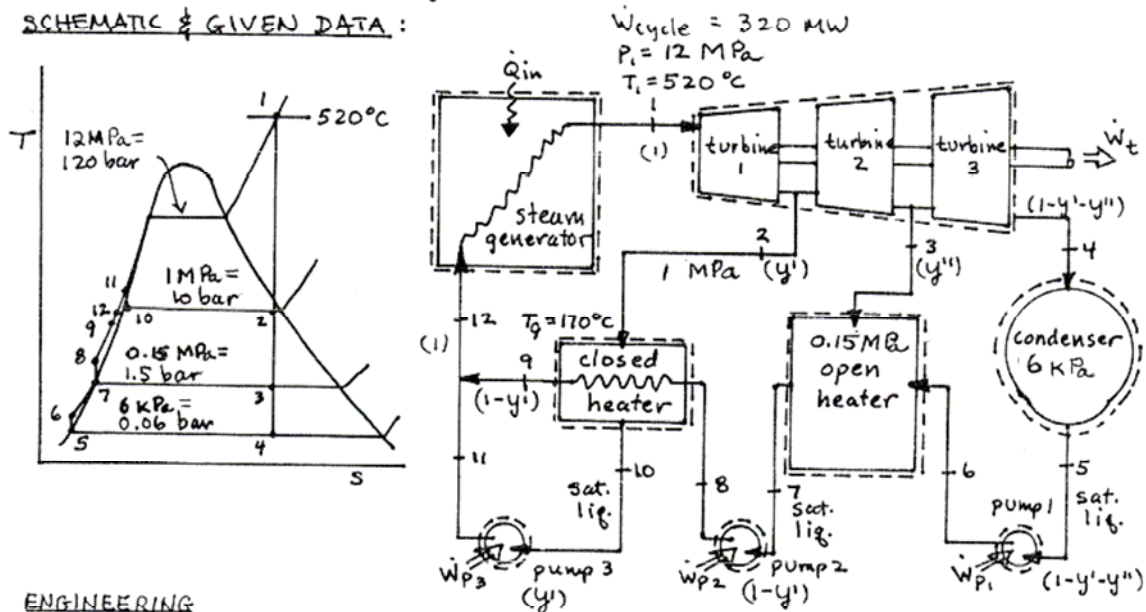


PROBLEM 8.63

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed and one open feedwater heater. Data are known at various locations, and the net power output is given.

FIND: Determine (a) the thermal efficiency, and (b) the mass flow rate into the first turbine stage.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) Each process of the working fluid is internally reversible, except in the feedwater heaters and the mixing of streams 9 and 11 to form stream 12. (3) The turbines, pumps, and feedwater heaters operate adiabatically. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the closed heater and condenser, and feedwater exits the open heater as saturated liquid the respective pressures.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 120 \text{ bar}$, $T_1 = 520^\circ\text{C} \Rightarrow h_1 = 3401.8 \text{ kJ/kg}$, $s_1 = 6.5555 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 10 \text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = 0.9931$, $h_2 = 2764.2 \text{ kJ/kg}$

State 3: $P_3 = 1.5 \text{ bar}$, $s_3 = s_1 \Rightarrow x_3 = 0.8847$, $h_3 = 2436.9 \text{ kJ/kg}$

State 4: $P_4 = 0.06 \text{ bar}$, $s_4 = s_1 \Rightarrow x_4 = 0.7727$, $h_4 = 2018.3 \text{ kJ/kg}$

State 5: $P_5 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_5 = 151.53 \text{ kJ/kg}$

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5)$

$$= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (1.5 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$= 151.53 + 0.14 = 151.67 \text{ kJ/kg}$$

State 7: $P_7 = 1.5 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 467.11 \text{ kJ/kg}$

State 8: $h_8 \approx h_7 + v_7(P_8 - P_7) = 467.11 + (1.0528 \times 10^{-3})(120 - 1.5) \left| \frac{10^5}{10^3} \right| = 479.59 \frac{\text{kJ}}{\text{kg}}$

State 9: $P_9 = 120 \text{ bar}$, $T_9 = 170^\circ\text{C} \Rightarrow$ Interpolating in Table A-5; $h_9 = 725.86 \text{ kJ/kg}$

State 10: $P_{10} = 10 \text{ bar}$, sat. liquid $\Rightarrow h_{10} = 762.81 \text{ kJ/kg}$

Continued on next slide

Problem 8-63 continued

State 11: $h_{11} \approx h_{10} + v_{10}(P_{11} - P_{10}) = 762.81 + (1.1273 \times 10^{-3})(120 - 10) \left| \frac{10^5}{10^5} \right| = 775.21 \text{ kJ/kg}$

(a) Applying mass and energy rate balances to the control volume enclosing the closed heater, the fraction of flow y' extracted at location 2 is

$$y' = \frac{h_9 - h_8}{h_2 - h_{10} + h_9 - h_8} = \frac{725.86 - 479.59}{2764.2 - 762.81 + 725.86 - 479.59} = 0.1096$$

This allows the specific enthalpy at 12 to be determined by analyzing the mixing of streams 9 and 11:

$$0 = (1 - y')h_9 + y'h_{11} - h_{12}$$

or

$$h_{12} = (1 - y')h_9 + y'h_{11} = (0.8904)(725.86) + (0.1096)(775.21) = 731.27 \text{ kJ/kg}$$

Next, energy and mass balances for the control volume enclosing the open heater give the fraction y'' extracted at 3:

$$0 = y''h_3 + (1 - y' - y'')h_6 - (1 - y')h_7$$

or

$$y'' = \frac{(1 - y')(h_7 - h_6)}{(h_3 - h_6)} = \frac{(0.8904)(467.11 - 151.67)}{(2436.9 - 151.67)} = 0.1229$$

For the control volume enclosing the turbine stages

$$\begin{aligned} \frac{\dot{W}_t}{\dot{m}_1} &= (h_1 - h_2) + (1 - y')(h_2 - h_3) + (1 - y' - y'')(h_3 - h_4) \\ &= (3401.8 - 2764.2) + (0.8904)(2764.2 - 2436.9) - (0.7675)(2436.9 - 2018.3) = 1250.3 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

For the pumps

$$\begin{aligned} \frac{\dot{W}_p}{\dot{m}_1} &= y'(h_{11} - h_{10}) + (1 - y')(h_8 - h_7) + (1 - y' - y'')(h_6 - h_5) \\ &= (0.1096)(775.21 - 762.81) + (0.8904)(479.59 - 467.11) + (0.7675)(151.67 - 151.23) \\ &= 12.58 \text{ kJ/kg} \end{aligned}$$

For the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_{12} = (3401.8) - (731.27) = 2670.5 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1250.3 - 12.58}{2670.5} = 0.463 \text{ (46.3\%)} \quad \leftarrow \eta$$

(b) The mass flow rate into the first turbine stage is

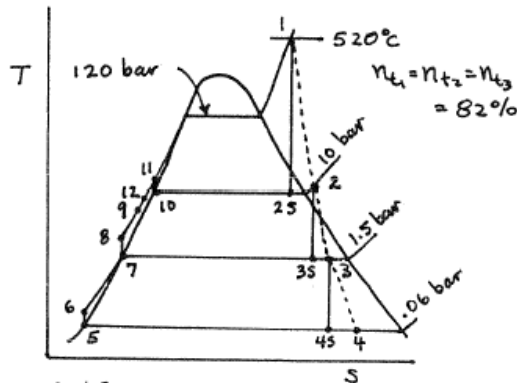
$$\begin{aligned} \dot{m}_1 &= \frac{\dot{W}_{cycle}}{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1} \\ &= \frac{(320 \text{ MW})}{(1250.3 - 12.58) \text{ kJ/kg}} \left| \frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \\ &= 9.31 \times 10^5 \text{ kg/h} \quad \leftarrow \dot{m}_1 \end{aligned}$$

PROBLEM 8.64

KNOWN: The regenerative vapor power cycle of Problem 8.63 is modified to include that each turbine stage has an efficiency of 82%.

FIND: Answer the same questions about the modified cycle as in Problem 8.57.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Problem 8.63, except $\eta_{t1} = \eta_{t2} = \eta_{t3} = 0.82$.

ANALYSIS: From the solution of Problem 8.63

$$\begin{aligned} h_1 &= 3401.8 \text{ kJ/kg}, s_1 = 6.5555 \text{ kJ/kg}\cdot\text{K} \\ h_5 &= 151.53 \text{ kJ/kg} & h_9 &= 725.86 \text{ kJ/kg} \\ h_6 &= 151.67 \text{ kJ/kg} & h_{10} &= 762.81 \text{ kJ/kg} \\ h_7 &= 467.11 \text{ kJ/kg} & h_{11} &= 775.21 \text{ kJ/kg} \\ h_8 &= 479.59 \text{ kJ/kg} \end{aligned}$$

State 2: Using the turbine efficiency, $\eta_{t1} = (h_1 - h_2) / (h_1 - h_{2s})$. With $h_{2s} = 2764.2$ (Problem 8.63)

$$h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 2879.0 \text{ kJ/kg}$$

From Table A-4, $s_2 = 6.7977 \text{ kJ/kg}\cdot\text{K}$.

State 3: Similarly, $p_3 = 1.5 \text{ bar}$, $s_{2s} = s_2 \Rightarrow x_{2s} = .9265$, $h_{2s} = 2530.0 \text{ kJ/kg}$.

$$\text{Thus } h_3 = h_2 - \eta_{t2}(h_2 - h_{2s}) = 2592.8 \text{ kJ/kg}$$

and, using data from Table A-3; $x_3 = .9547$, $s_3 = 6.9610 \text{ kJ/kg}\cdot\text{K}$.

State 4: Again, $p_4 = 0.06 \text{ bar}$, $s_{3s} = s_3 \Rightarrow x_{3s} = .82465$, $h_{3s} = 2143.8 \text{ kJ/kg}$.

$$\text{Thus } h_4 = h_3 - \eta_{t3}(h_3 - h_{3s}) = 2224.6 \text{ kJ/kg}$$

(a) The flow fraction y' is

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{725.86 - 479.59}{2879.0 - 762.81} = 0.1164$$

Thus, specific enthalpy h_{12} is

$$\begin{aligned} h_{12} &= (1 - y')h_9 + y'h_{11} = (.8836)(725.86) + (.1164)(775.21) \\ &= 731.6 \text{ kJ/kg} \end{aligned}$$

and the flow fraction y'' is

$$y'' = \frac{(1 - y')(h_7 - h_6)}{(h_3 - h_6)} = \frac{(.8836)(467.11 - 151.67)}{(2592.8 - 151.67)} = 0.1142$$

For the control volume enclosing the turbine stages

$$\begin{aligned} \frac{\dot{W}_t}{\dot{m}_1} &= (h_1 - h_2) + (1 - y')(h_2 - h_3) + (1 - y' - y'')(h_3 - h_4) \\ &= (3401.8 - 2879.0) + (.8836)(2879.0 - 2592.8) + (.7694)(2592.8 - 2224.6) \\ &= 1059.0 \text{ kJ/kg} \end{aligned}$$

Continued on next slide

Problem 8-64 continued

And, for the pumps

$$\begin{aligned}\frac{\dot{W}_p}{\dot{m}_1} &= y'(h_{11}-h_{10}) + (1-y')(h_8-h_7) + (1-y'-y'')(h_6-h_5) \\ &= (.1164)(775.21-762.81) + (.8836)(479.59-467.11) + (.7694)(151.67-151.53) \\ &= 12.58 \text{ kJ/kg}\end{aligned}$$

For the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_{12} = 3401.8 - 731.6 = 2670.2 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1059 - 12.58}{2670.2} = 0.392 \text{ (39.2\%)} \quad \leftarrow \eta$$

(b) The mass flow rate into the first turbine stage is

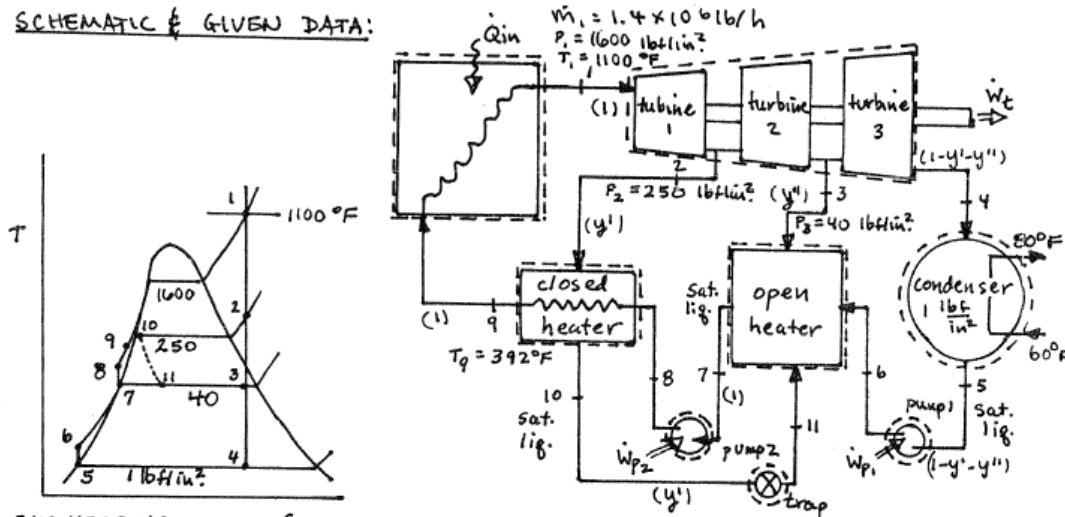
$$\begin{aligned}\dot{m}_1 &= \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]} \\ &= \frac{(320 \text{ MW})}{[1059.0 - 12.58] \text{ kJ/kg}} \left| \frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \\ &= 1.1 \times 10^6 \text{ kg/h} \quad \leftarrow \dot{m}_1\end{aligned}$$

PROBLEM 8.65

KNOWN: The ideal Rankine cycle of Problem 8.9 is modified to include one closed and one open feedwater heater. The condensate from the closed heater is trapped into the open heater.

FIND: Answer the same questions about the modified cycle as in Problem 8.9 and discuss the results.

SCHEMATIC & GIVEN DATA:



ENGINEERING

5

MODEL: Same as Example 8.6, except no reheater.

ANALYSIS: From the solution to Problem 8.9, $h_1 = 1547.7 \text{ Btu/lb}$, $s_1 = 1.6315 \text{ Btu/lb} \cdot ^\circ\text{R}$, $h_4 = 911.2 \text{ Btu/lb}$, and $h_5 = 69.74 \text{ Btu/lb}$.

State 2: $P_2 = 250 \text{ lbf/in}^2$, $s_2 = s_1 \Rightarrow h_2 = 1299.8 \text{ Btu/lb}$

State 3: $P_3 = 40 \text{ lbf/in}^2$, $s_3 = s_1 \Rightarrow x_3 = 0.9649$, $h_3 = 1137.2 \text{ Btu/lb}$

State 6: $h_6 \approx h_5 + v_5 (P_6 - P_5)$
 $= 69.74 \frac{\text{Btu}}{\text{lb}} + (0.01614 \frac{\text{ft}^3}{\text{lb}}) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| (40 - 1) \frac{\text{lbf}}{\text{in}^2}$
 $= 69.74 + 0.12 = 69.86 \text{ Btu/lb}$

State 7: $P_7 = 40 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_7 = 236.16 \text{ Btu/lb}$

State 8: $h_8 \approx h_7 + v_7 (P_8 - P_7) = 236.16 + (0.01715) \frac{144}{778} (1600 - 40)$
 $= 236.16 + 4.95 = 241.11 \text{ Btu/lb}$

State 9: $P_9 = 1600 \text{ lbf/in}^2$, $T_9 = 392^\circ\text{F} \Rightarrow$ (Interpolating in Table A-5)
 $h_9 \approx 368.38 \text{ Btu/lb}$

State 10: $P_{10} = 250 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_{10} = 376.2 \text{ Btu/lb}$

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 376.2 \text{ Btu/lb}$

(a) Begin by determining the flow rate ratios y' and y'' . Energy and mass rate balances for the control volume enclosing the closed feedwater heater reduce to

$$0 = y' (h_2 - h_{10}) + (h_8 - h_9)$$

or

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{368.38 - 241.11}{1299.8 - 376.2} = 0.1365$$

Continued on next slide

Problem 8-65 continued

For the open heater

$$0 = y'' h_3 + (1 - y' - y'') h_6 + y' h_{11} - h_7$$

or

$$y'' = \frac{y'(h_6 - h_{11}) + (h_7 - h_6)}{h_3 - h_6} = \frac{(0.1365)(69.86 - 376.2) + (236.16 - 69.86)}{(1137.2 - 69.86)} \\ = 0.1166$$

For the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y')(h_2 - h_3) + (1 - y' - y'')(h_3 - h_4) \\ = (1547.7 - 1299.8) + (0.8635)(1299.8 - 1137.2) + (0.7469)(1137.2 - 911.2) \\ = 557.10 \text{ Btu/lb}$$

and for the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_8 - h_7) + (1 - y' - y'')(h_6 - h_5) \\ = (241.11 - 236.16) + (0.7469)(69.86 - 69.74) = 5.04 \text{ Btu/lb}$$

With $\dot{m}_1 = 1.4 \times 10^6 \text{ lb/h}$ from Problem 8.9, the net power developed is

$$\dot{W}_{\text{cycle}} = \dot{m}_1 \left[\frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} \right] \\ = (1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) [557.10 - 5.04] \frac{\text{Btu}}{\text{lb}} = 7.729 \times 10^8 \text{ Btu/h} \leftarrow \dot{W}_{\text{cycle}}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{cycle}}}{\dot{m}_1 (h_1 - h_9)} = \frac{7.729 \times 10^8}{(1.4 \times 10^6)(1547.7 - 368.38)} \\ = 0.468 \text{ (46.8\%)} \leftarrow \eta$$

(c) The mass flow rate of cooling water passing through the condenser is found from

$$\dot{m}_{\text{cw}} = \frac{\dot{m}_1 (1 - y' - y'')(h_4 - h_5)}{h_{\text{cw,out}} - h_{\text{cw,in}}} \\ = \frac{(1.4 \times 10^6 \text{ lb/h})(0.7469)(911.2 - 69.74) \text{ Btu/lb}}{(48.09 - 28.08) \text{ Btu/lb}} \\ = 4.40 \times 10^7 \text{ lb/h} \leftarrow \dot{m}_{\text{cw}}$$

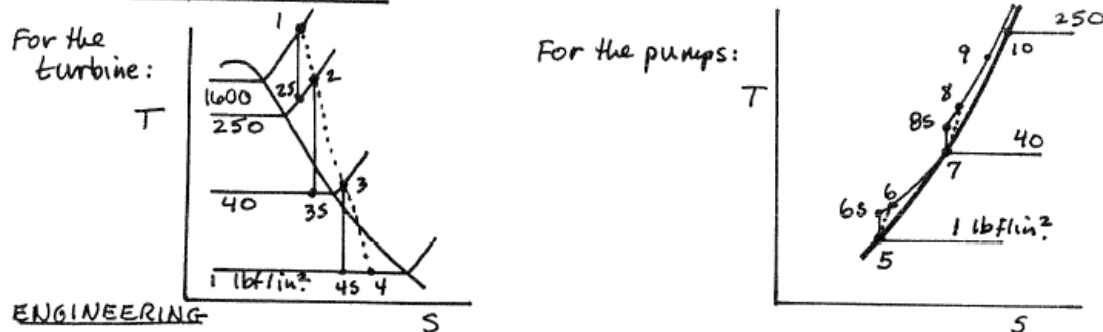
Discussion: (1) The net power developed in the modified cycle is less than in Problem 8.9 due to some mass being extracted. (2) The rate of heat transfer to the working fluid passing through the steam generator is less for the modified cycle due to regeneration in the feedwater heaters. In this case, the thermal efficiency is increased due to regeneration. (3) The cooling water flow rate is less in the modified cycle because of the reduced flow of working fluid through the condenser in this case.

PROBLEM 8.66

KNOWN: The regenerative vapor power cycle of Problem 8.65 is modified to include that each turbine stage has an efficiency of 88% and each pump has an efficiency of 80%.

FIND: Repeat the analysis of Problem 8.65 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

Same as Problem 8.65, except $\eta_{t1} = \eta_{t2} = \eta_{t3} = 0.88$ and $\eta_{p1} = \eta_{p2} = 0.80$.

ANALYSIS: From Problem 8.65, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb $\cdot$ °R, $h_5 = 69.74$ Btu/lb, $h_7 = 236.16$ Btu/lb, $h_9 = 368.38$ Btu/lb, and $h_{10} = h_{11} = 376.2$ Btu/lb.

State 2: Using the turbine efficiency, $\eta_{t1} = (h_1 - h_2)/(h_1 - h_{2s})$. Thus, with h_{2s} from Problem 8.65

$$h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 1329.5 \text{ Btu/lb}$$

and, interpolating in Table A-4E; $s_2 = 1.6595$ Btu/lb $\cdot$ °R.

State 3: First, $P_3 = 40$ lbf/in 2 , $s_{3s} = s_2 \Rightarrow x_{3s} = 0.9867$, $h_{3s} = 1157.5$ Btu/lb.

Then $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 1178.1$ Btu/lb

and, interpolating in Table A-4E; $s_3 = 1.6877$ Btu/lb $\cdot$ °R

State 4: With $p_4 = 1$ lbf/in 2 , $s_{4s} = s_3 \Rightarrow h_{4s} = 942.7$ Btu/lb

Thus $h_4 = h_3 - \eta_{t3}(h_3 - h_{4s}) = 971.0$ Btu/lb

State 6: Using the pump efficiency, $\eta_{p1} = (h_6 - h_5)/(h_6 - h_{5s})$. Thus, with h_{5s} from Problem 8.65

$$h_6 = h_5 + (h_{6s} - h_5)/\eta_{p1} = 69.89 \text{ Btu/lb}$$

State 8: Similarly, with h_{8s} from Problem 8.59

$$h_8 = h_7 + (h_{8s} - h_7)/\eta_{p2} = 242.35 \text{ Btu/lb}$$

The mass flow rate ratios y' and y'' are, respectively

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{368.38 - 242.35}{1329.5 - 376.2} = 0.1322$$

and

$$y'' = \frac{y'(h_6 - h_{11}) + (h_7 - h_6)}{(h_3 - h_6)} = \frac{(0.1322)(69.89 - 376.2) + (236.16 - 69.89)}{(1178.1 - 69.89)} = 0.1135$$

Continued on next slide

Problem 8-66 continued

(a) For the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y')(h_2 - h_3) + (1 - y' - y'')(h_3 - h_4) \\ = 505.80 \text{ Btu/lb}$$

And for the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_8 - h_7) + (1 - y' - y'')(h_6 - h_5) = 6.303 \text{ Btu/lb}$$

thus, with $\dot{m}_1 = 1.4 \times 10^6$ from Problem 8.59

$$\dot{W}_{\text{cycle}} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1] = 6.99 \times 10^8 \text{ Btu/h} \quad \leftarrow \dot{W}_{\text{cycle}}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{cycle}}}{\dot{m}_1(h_1 - h_9)} = 0.424 (42.4\%) \quad \leftarrow \eta$$

(c) The mass flow rate of cooling water through the condenser is

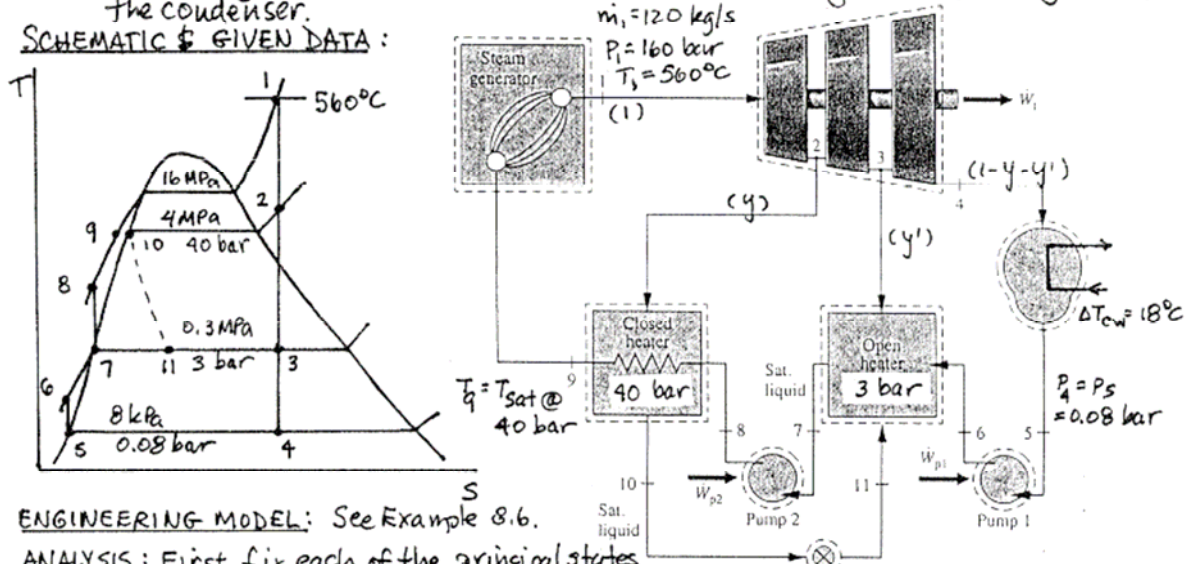
$$\dot{m}_{\text{cw}} = \frac{\dot{m}_1(1 - y' - y'')(h_4 - h_5)}{h_{\text{cw,out}} - h_{\text{cw,in}}} = 4.76 \times 10^7 \text{ lb/h} \quad \leftarrow \dot{m}_{\text{cw}}$$

PROBLEM 8.67

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed and one open feedwater heater. Data are specified at various locations for the cycle. The mass flow rate of steam entering the first-stage turbine is known.

FIND: Determine (a) the net power, (b) rate of heat addition, (c) the thermal efficiency, and (d) the mass flow rate of cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.6.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 160 \text{ bar}$, $T_1 = 560^\circ\text{C} \Rightarrow h_1 = 3465.4 \text{ kJ/kg}$, $s_1 = 6.5132 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 40 \text{ bar}$, $s_2 = s_1 \Rightarrow$ Interpolating in Table A-4: $h_2 = 3050.9 \text{ kJ/kg}$

State 3: $P_3 = 3 \text{ bar}$, $s_3 = s_2 \Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.9100$, $h_3 = 2530.5 \text{ kJ/kg}$

State 4: $P_4 = 0.08 \text{ bar}$, $s_4 = s_3 \Rightarrow x_4 = \frac{s_4 - s_{f4}}{s_{g4} - s_{f4}} = 0.7753$, $h_4 = 2037.0 \text{ kJ/kg}$

State 5: $P_5 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_5 = 173.88 \text{ kJ/kg}$

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5)$
 $= 173.88 \frac{\text{kJ}}{\text{kg}} + (1.0084 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (3 - 0.08) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$
 $= 173.88 + 0.294 = 174.17 \text{ kJ/kg}$

State 7: $P_7 = 3 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 561.47 \text{ kJ/kg}$

State 8: $h_8 \approx h_7 + v_7(P_8 - P_7) = 561.47 + (1.0732 \times 10^{-3})(160 - 3) \left| \frac{10^5}{10^3} \right|$
 $= 561.47 + 16.85 = 578.32 \text{ kJ/kg}$

State 9: Assume $h_9 \approx h_{f@T_{10}} = 1087.3 \text{ kJ/kg}$

State 10: $P_{10} = 40 \text{ bar}$, sat. liquid $\Rightarrow h_{10} = 1087.3 \text{ kJ/kg}$

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 1087.3 \text{ kJ/kg}$

(a) Apply mass and energy balances to the closed heater to get the fraction extracted y

$$0 = y(h_2 - h_{10}) + (h_8 - h_9)$$

Continued on next slide

Problem 8-67 continued

$$\text{or } y = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{1087.3 - 578.32}{3050.9 - 1087.3} = 0.2592$$

For the open feedwater heater

$$0 = y' h_3 + y h_{11} + (1 - y - y') h_6 - h_7$$

Solving for y'

$$y' = \frac{h_7 - h_6 + y(h_6 - h_{11})}{(h_3 - h_6)} = \frac{561.47 - 174.17 + (0.2592)(174.17 - 1087.3)}{(2530.5 - 174.17)}$$

$$= 0.0639$$

For the control volume enclosing the turbine stages

$$\begin{aligned} \dot{W}_t &= \dot{m}_1 [h_1 - y h_2 - y' h_3 - (1 - y - y') h_4] \\ &= (120 \frac{\text{kg}}{\text{s}}) [3465.4 - (0.2592)(3050.9) - (0.0639)(2530.5) - (0.6769)(2037.0)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 1.3609 \times 10^5 \text{ kW} \end{aligned}$$

For the pumps

$$\begin{aligned} \dot{W}_p &= \dot{W}_{p1} + \dot{W}_{p2} = \dot{m}_1 [(1 - y - y')(h_6 - h_5) + (h_8 - h_7)] \\ &= (120) [(0.6769)(174.17 - 173.88) + (578.32 - 561.47)] = 2045.6 \text{ kW} \end{aligned}$$

The net power developed is

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = 1.340 \times 10^5 \text{ kW} \leftarrow \dot{W}_{\text{cycle}}$$

(b) For the steam generator

$$\dot{Q}_{\text{in}} = \dot{m}_1 (h_1 - h_9) = (120)(3465.4 - 1087.3) = 2.854 \times 10^5 \text{ kW} \leftarrow \dot{Q}_{\text{in}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = 0.4695 (46.95\%) \leftarrow \eta$$

(d) For a control volume enclosing the condenser

$$0 = \dot{m}_1 (1 - y - y') (h_4 - h_5) + \dot{m}_{\text{cw}} (h_{\text{cw}, \text{in}} - h_{\text{cw}, \text{out}})$$

Solving for $\dot{m}_{\text{cw}}$

$$\dot{m}_{\text{cw}} = \frac{\dot{m}_1 (1 - y - y') (h_4 - h_5)}{(h_{\text{cw}, \text{out}} - h_{\text{cw}, \text{in}})}$$

Using $h_{\text{cw}, \text{out}} - h_{\text{cw}, \text{in}} = c_{\text{cw}} (T_{\text{cw}, \text{out}} - T_{\text{cw}, \text{in}})$ with $c_{\text{cw}} = 4.179$ from Table A-19

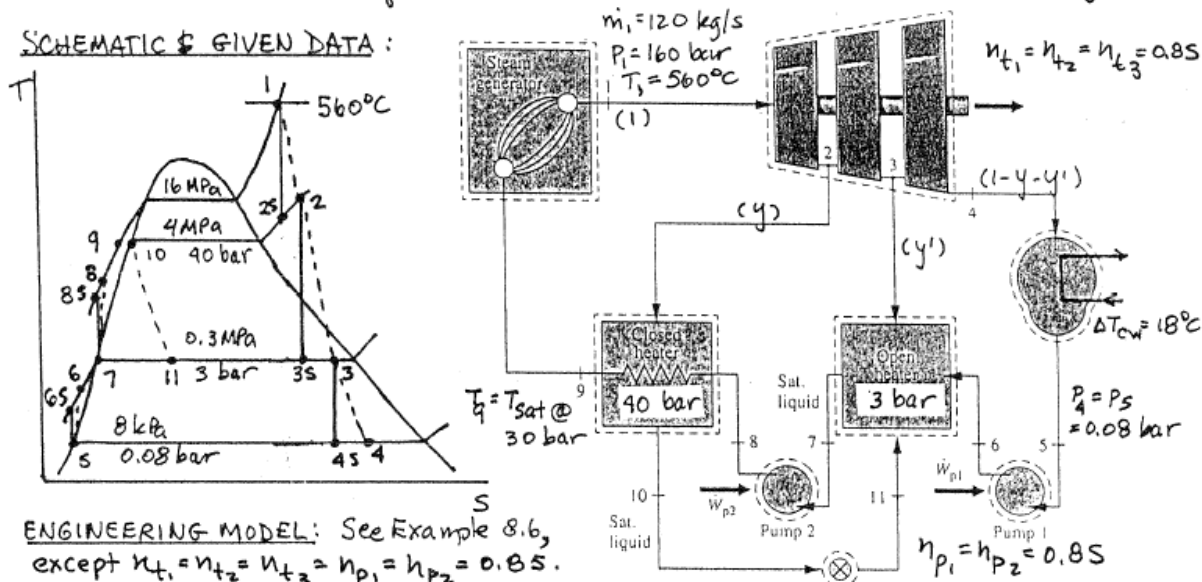
$$\begin{aligned} \dot{m}_{\text{cw}} &= \frac{(120 \text{ kg/s})(0.6769)(2037.0 - 173.88) \text{ kJ/kg}}{(4.179)(18) \text{ kJ/kg}} \\ &= \frac{1.5134 \times 10^5}{75.222} = 1354 \text{ kg/s} \leftarrow \dot{m}_{\text{cw}} \end{aligned}$$

PROBLEM 8.68

KNOWN: The regenerative vapor power cycle of Problem 8.67 is modified to include turbine stage and pump isentropic efficiencies of 0.85.

FIND: Answer the same questions as in Problem 8.67 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.6, except $\eta_{t1} = \eta_{t2} = \eta_{t3} = \eta_{p1} = \eta_{p2} = 0.85$.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 160 \text{ bar}$, $T_1 = 560^\circ\text{C} \Rightarrow h_1 = 3465.4 \text{ kJ/kg}$, $s_1 = 6.5132 \text{ kJ/kg}\cdot\text{K}$

State 2: Using the first-stage turbine efficiency: $\eta_{t1} = (h_1 - h_2)/(h_1 - h_{2s})$
With $h_{2s} = 3050.9 \text{ kJ/kg}$ from Problem 8.67

$h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 3113.1 \text{ kJ/kg}$. From Table A-4; $s_2 = 6.6148 \text{ kJ/kg}\cdot\text{K}$

State 3: $P_3 = 3 \text{ bar}$, $s_{3s} = s_2 \Rightarrow x_{3s} = \frac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.9291$, $h_{3s} = 2571.9 \text{ kJ/kg}$

Using the second-stage turbine efficiency

$h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 2653.1 \text{ kJ/kg}$; $s_3 = 6.8142$
($x_3 = 0.9666$)

State 4: $P_4 = 0.08 \text{ bar}$, $s_{4s} = s_3 \Rightarrow x_{4s} = 0.8148$; $h_{4s} = 2131.9 \text{ kJ/kg}$

Using the third-stage turbine efficiency

$h_4 = h_3 - \eta_{t3}(h_3 - h_{4s}) = 2210.1 \text{ kJ/kg}$

State 5: $P_5 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_5 = 173.88 \text{ kJ/kg}$

State 6: Using the pump efficiency with $h_{6s} = 174.17 \text{ kJ/kg}$ from Problem 8.67

$h_6 = h_5 + (h_{6s} - h_5)/\eta_{p1} = 174.22 \text{ kJ/kg}$

State 7: $P_7 = 3 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 561.47 \text{ kJ/kg}$

State 8: Using the pump efficiency with $h_{8s} = 578.32 \text{ kJ/kg}$ from Problem 8.67

$h_8 = h_7 + (h_{8s} - h_7)/\eta_{p2} = 581.29 \text{ kJ/kg}$

State 9: Assume $h_9 \approx h_f @ T_{10} = 1087.3 \text{ kJ/kg}$

State 10: $P_{10} = 40 \text{ bar}$, sat. liquid $\Rightarrow h_{10} = 1087.3 \text{ kJ/kg}$

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 1087.3 \text{ kJ/kg}$

Continued on next slide

Problem 8-68 continued

$$(a) \quad y = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{1087.3 - 581.29}{3113.1 - 1087.3} = 0.2498$$

$$y = \frac{h_7 - h_6 + y(h_6 - h_{11})}{(h_3 - h_6)} = \frac{561.47 - 174.22 + (0.2498)(174.22 - 1087.3)}{(2653.1 - 174.22)} = 0.0642$$

$$\begin{aligned} \dot{W}_t &= \dot{m}_1 [h_1 - y h_2 - y' h_3 - (1 - y - y') h_4] \\ &= (120 \frac{\text{kg}}{\text{s}}) [3465.4 - (0.2498)(3113.1) - (0.0642)(2653.1) - (0.6860)(2210.1)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 1.202 \times 10^5 \text{ kW} \end{aligned}$$

$$\begin{aligned} \dot{W}_p &= \dot{m}_1 [(1 - y - y')(h_6 - h_5) + (h_8 - h_7)] \\ &= (120) [(0.6860)(174.22 - 173.88) + (581.29 - 561.47)] = 2406 \text{ kW} \end{aligned}$$

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = 1.178 \times 10^5 \text{ kW} \leftarrow \dot{W}_{\text{cycle}}$$

$$(b) \quad \dot{Q}_{\text{in}} = \dot{m}_1 (h_1 - h_9) = (120)(3465.4 - 1087.3) = 2.854 \times 10^5 \text{ kW} \leftarrow \dot{Q}_{\text{in}}$$

$$(c) \quad \eta = \dot{W}_{\text{cycle}} / \dot{Q}_{\text{in}} = 0.413 \text{ (41.3 \%)} \leftarrow \eta$$

$$(d) \quad \dot{m}_{\text{cw}} = \frac{\dot{m}_1 (1 - y - y')(h_4 - h_5)}{c_{\text{cw}} \Delta T_{\text{cw}}}$$

①

$$= \frac{(120 \text{ kg/s})(0.6860)(2210.1 - 173.88) \text{ kJ/kg}}{(4.179)(18) \text{ kJ/kg}}$$

$$= 2228 \text{ kg/s} \leftarrow \dot{m}_{\text{cw}}$$

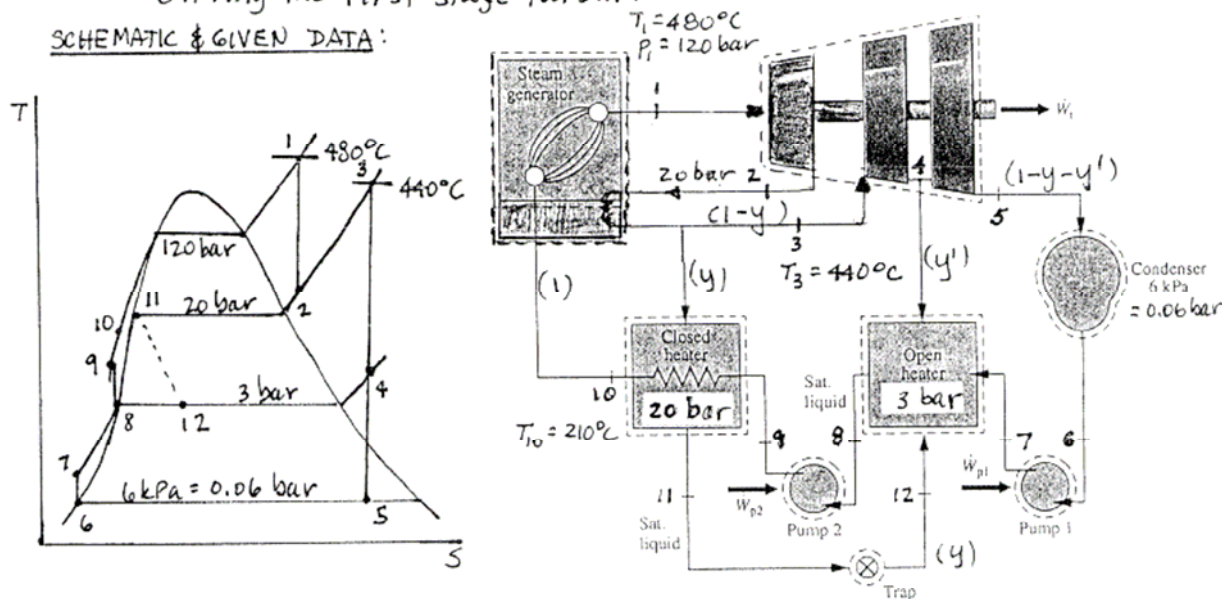
1. These results can be compared with those of Problem 8.27 to see some of the effects of turbine stage and pump isentropic efficiencies on cycle performance.

PROBLEM 8.69

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed and one open feedwater heater and reheat. Data are known at various locations in the cycle.

FIND: Determine (a) the rate of heat addition per kg of steam entering the first-stage turbine, (b) the thermal efficiency, and (c) the rate of heat transfer to cooling water passing through the condenser, per kg of steam entering the first-stage turbine.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.6.

ANALYSIS: First, fix each of the principal states.

From the solution to problem 8.60: (Note - some states are renumbered)

$$\begin{aligned} h_1 &= 3293.5 \text{ kJ/kg} & h_9 &= 574.03 \text{ kJ/kg} \\ h_2 &= 2836.7 \text{ kJ/kg} & h_{10} &= 897.76 \text{ kJ/kg} \\ h_b &= 151.53 \text{ kJ/kg} & h_{11} &= 908.79 \text{ kJ/kg} \\ h_7 &= 151.83 \text{ kJ/kg} & h_{12} &= 908.79 \text{ kJ/kg} \\ h_8 &= 561.47 \text{ kJ/kg} \end{aligned}$$

State 3: $P_3 = 20 \text{ bar}, T_3 = 440^\circ\text{C} \Rightarrow h_3 = 3335.5, s_3 = 7.2540 \text{ kJ/kg}\cdot\text{K}$

State 4: $P_4 = 3 \text{ bar}, s_4 = s_3 = 7.2540 \text{ kJ/kg}\cdot\text{K} \Rightarrow \text{From Table A-4; } h_4 = 2839.5 \text{ kJ/kg}$

State 5: $P_5 = 0.06 \text{ bar}, s_5 = s_4 \Rightarrow x_5 = \frac{s_5 - s_{g5}}{s_{g5} - s_{f5}} = 0.8622, h_5 = 2234.4 \text{ kJ/kg}$

(a) For the control volume enclosing the steam generator

$$\begin{aligned} \dot{Q}_{in}/\dot{m}_1 &= (h_1 - h_{10}) + (h_3 - h_2) \\ &= (3293.5 - 897.76) + (3335.5 - 2836.7) = 2894.5 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}_1 \end{aligned}$$

(b) First, get the fractions extracted at 3 and 4 from energy and mass balances on the closed and open heater control volumes, respectively.

$$0 = y(h_3 - h_{11}) + (h_9 - h_{10})$$

Continued on next slide

Problem 8-69 continued

$$\text{or } y = \frac{h_{10} - h_9}{h_3 - h_{11}} = \frac{897.76 - 574.03}{3335.5 - 908.79} = 0.1334$$

Further

$$0 = y' h_4 + (1 - y - y') h_7 + y h_{12} - h_8$$

or

$$y' = \frac{h_8 - h_7 + y(h_7 - h_{12})}{h_4 - h_7} = \frac{561.47 - 151.83 + (0.1334)(151.83 - 908.79)}{2839.5 - 151.83} = 0.1148$$

For the control volume enclosing the turbine stages

$$\begin{aligned} \dot{W}_t / \dot{m}_1 &= h_1 - h_2 + (1 - y) h_3 - y' h_4 - (1 - y - y') h_5 \\ &= 3243.5 - 2836.7 + (0.8666)(3335.5) - (0.1148)(2839.5) - (0.7518)(2234.4) \\ &= 1341.5 \text{ kJ/kg} \end{aligned}$$

For the pumps

$$\begin{aligned} \dot{W}_p / \dot{m}_1 &= \dot{W}_{p1} / \dot{m}_1 + \dot{W}_{p2} / \dot{m}_1 = (1 - y - y')(h_7 - h_6) + (h_9 - h_8) \\ &= (0.7518)(151.83 - 151.53) + (574.03 - 561.47) \\ &= 12.78 \text{ kJ/kg} \end{aligned}$$

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 1328.7 \text{ kJ/kg}$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}} / \dot{m}_1}{\dot{Q}_{\text{in}} / \dot{m}_1} = \frac{1328.7}{2894.5} = 0.459 \text{ (45.9\%)} \leftarrow \eta$$

(c) For the condenser

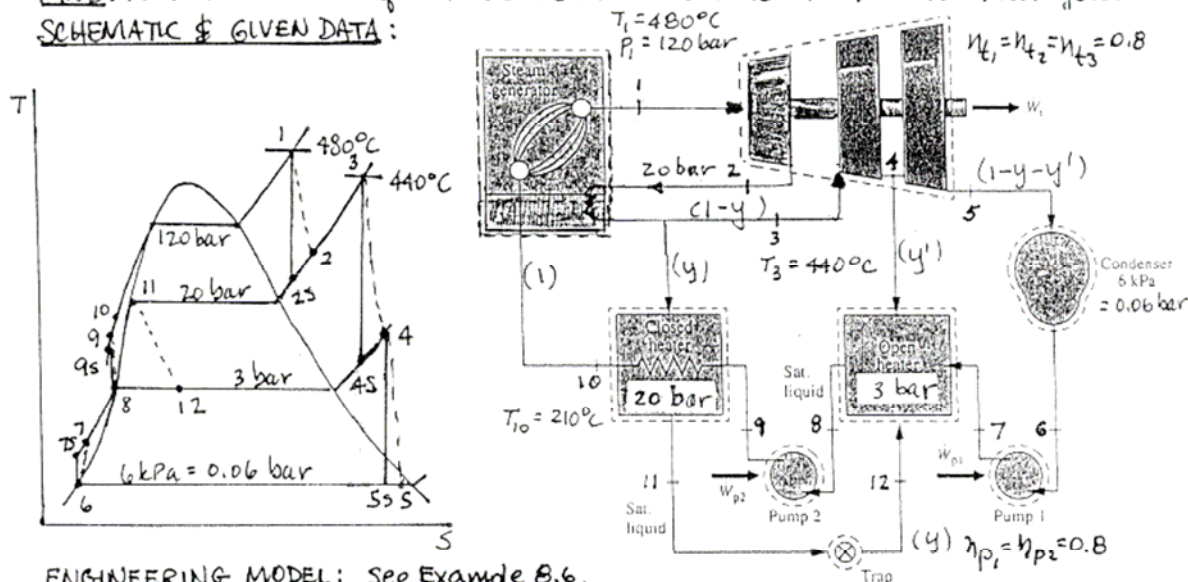
$$\begin{aligned} \dot{Q}_{\text{out}} / \dot{m}_1 &= (1 - y - y')(h_5 - h_6) \\ &= (0.7518)(2234.4 - 151.53) = 1565.9 \text{ kJ/kg} \leftarrow \dot{Q}_{\text{out}} / \dot{m}_1 \end{aligned}$$

PROBLEM 8.70

KNOWN: The regenerative vapor power cycle of Problem 8.69 is modified to include turbine stage and pump isentropic efficiencies of 0.8.

FIND: Answer the same questions as in Problem 8.69 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.6, except $\eta_{t1} = \eta_{t2} = \eta_{t3} = \eta_{p1} = \eta_{p2} = 0.8$.

ANALYSIS: First, fix each of the principal states. From the solution to Problem 8.69:

$$\begin{aligned} h_1 &= 3293.5 \text{ kJ/kg} & h_6 &= 151.53 \text{ kJ/kg}, h_{7s} = 151.83 \\ h_{2s} &= 2836.7 & h_8 &= 561.47, h_{9s} = 574.03 \\ h_3 &= 3335.5 & h_{10} &= 897.76 \\ h_{4s} &= 2837.5 & h_{11} &= h_{12} = 908.79 \end{aligned}$$

State 2: Using the first-stage turbine efficiency; $\eta_{t1} = (h_1 - h_2)/(h_1 - h_{2s})$

$$h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 2928.1 \text{ kJ/kg}$$

State 4: For the second-stage turbine

$$h_4 = h_3 - \eta_{t2}(h_3 - h_{4s}) = 2938.7 \text{ kJ/kg}; s_4 = 7.4600 \text{ kJ/kg} \cdot \text{K}$$

State 5: $P_5 = 0.06 \text{ bar}, s_{5s} = s_4 \Rightarrow x_{5s} = 0.885; h_{5s} = 2298.1 \text{ kJ/kg}$

$$\text{With } \eta_{t3} = 0.8; h_5 = h_4 - (h_4 - h_{5s}) = 2426.2 \text{ kJ/kg}$$

State 7: Using the pump efficiency; $\eta_{p1} = (h_{7s} - h_6)/(h_7 - h_6)$

$$h_7 = h_6 + (h_{7s} - h_6)/\eta_{p1} = 151.91 \text{ kJ/kg}$$

State 9: For pump 2

$$h_9 = h_8 + (h_{9s} - h_8)/\eta_{p2} = 577.17 \text{ kJ/kg}$$

(a) For the control volume enclosing the steam generator

$$\begin{aligned} \dot{Q}_{in}/\dot{m}_1 &= (h_1 - h_{10}) + (h_3 - h_2) \\ &= (3293.5 - 897.76) + (3335.5 - 2928.1) = 2803.1 \text{ kJ/kg} \end{aligned}$$

(b) For the closed feedwater heater

$$y = \frac{h_{10} - h_9}{h_3 - h_{11}} = \frac{897.76 - 577.17}{3335.5 - 908.79} = 0.1321$$

Continued on next slide

Problem 8-70 continued

For the open feedwater heater

$$y' = \frac{h_8 - h_7 + y(h_7 - h_{12})}{h_4 - h_7}$$

$$= \frac{561.47 - 151.91 + (0.1321)(151.91 - 908.79)}{2938.7 - 151.91} = 0.1111$$

For the control volume enclosing the turbine stages

$$\dot{W}_t/\dot{m}_1 = h_1 - h_2 + (1-y)h_3 - y'h_4 - (1-y-y')h_5$$

$$= 3293.5 - 2928.1 + (0.8679)(3335.5) - (0.1111)(2938.7) - (0.7568)(2426.2)$$

$$= 1097.6 \text{ kJ/kg}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = \frac{\dot{W}_{p1}}{\dot{m}_1} + \frac{\dot{W}_{p2}}{\dot{m}_1} = (1-y-y')(h_7 - h_6) + (h_9 - h_8)$$

$$= (0.7568)(151.91 - 151.53) + (577.17 - 561.47)$$

$$= 15.99 \text{ kJ/kg}$$

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 1081.6 \text{ kJ/kg} \quad \leftarrow \frac{\dot{W}_{\text{cycle}}}{\dot{m}_1}$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}/\dot{m}_1}{\dot{Q}_{\text{in}}/\dot{m}_1} = \frac{1081.6}{2803.1} = 0.386 \text{ (38.6\%)} \quad \leftarrow \eta$$

(c) For the condenser

$$\textcircled{1} \quad \dot{Q}_{\text{out}}/\dot{m}_1 = (1-y-y')(h_5 - h_6)$$

$$= (0.7568)(2426.2 - 151.53) = 1721.5 \text{ kJ/kg} \quad \leftarrow \dot{Q}_{\text{out}}/\dot{m}_1$$

1. These results can be compared with those of Problem 8.69 to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

PROBLEM 8.71
Refer to Problem 8.70

IT Code

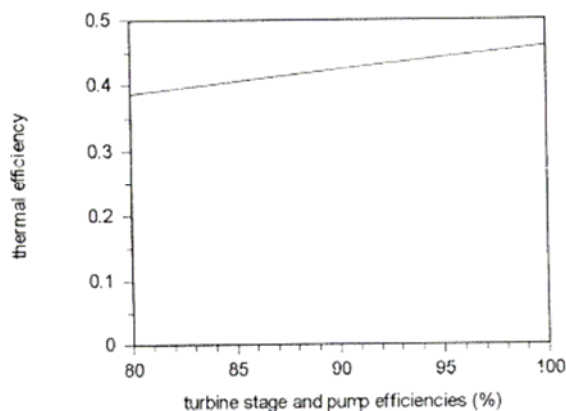
```
p1 = 120 // bar
T1 = 480 // °C
p2 = 20 // bar
p3 = p2
T3 = 440 // °C
p4 = 3 // bar
p5 = 0.06 // bar
p6 = p5
p7 = p4
p8 = p4
p9 = p1
p10 = p1
p11 = p2
p12 = p4
T10 = 210 // °C
etat = .8
etap = etat
mdot1 = 1
etatt = etat * 100
```

```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
h2 = h1 - etat * (h1 - h2s)
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
s4s = s3
h4s = h_Ps("Water/Steam", p4, s4s)
h4 = h3 - etat * (h3 - h4s)
s4 = s_Ph("Water/Steam", p4, h4)
s5s = s4
h5s = h_Ps("Water/Steam", p5, s5s)
h5 = h4 - etat * (h4 - h5s)
h6 = hsat_Px("Water/Steam", p6, 0)
v6 = vsat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p6 - p5) * 100
h7 = h6 + (h7s - h6) / etap
h8 = hsat_Px("Water/Steam", p8, 0)
v8 = vsat_Px("Water/Steam", p8, 0)
h9s = h8 + v8 * (p9 - p8) * 100
h9 = h8 + (h9s - h8) / etap
psat = Psat_T("Water/Steam", T10)
h10 = hsat_Px("Water/Steam", psat, 0)
h11 = hsat_Px("Water/Steam", p11, 0)
h12 = h11
```

```
Qdotin = mdot1 * ((h1 - h10) + (h3 - h2))
y = (h10 - h9) / (h3 - h11)
y' = (h8 - h7 + y * (h7 - h12)) / (h4 - h7)
Wdott = mdot1 * (h1 - h2 + (1 - y) * h3 - y' * h4 - (1 - y - y') * h5)
Wdotp1 = mdot1 * ((1 - y - y') * (h7 - h6))
Wdotp2 = mdot1 * (h9 - h8)
Wdotcycle = Wdott - Wdotp1 - Wdotp2
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * ((1 - y - y') * (h5 - h6))
```

IT Results ($\eta_{h1} = \eta_{h2} = \eta_{h3} = \eta_{p1} = \eta_{p2} = 0.8$)

```
Qdotin / m1 = 2803 kJ/kg
Qdotout / m1 = 1721 kJ/kg
Wcycle / m1 = 1082 kJ/kg
eta = 0.3862
h1 = 3293 kJ/kg
h2 = 2927 kJ/kg
h3 = 3335 kJ/kg
h4 = 2938 kJ/kg
h5 = 2426 kJ/kg
h6 = 151 kJ/kg
h7 = 151 kJ/kg
h8 = 561.2 kJ/kg
h9 = 576.9 kJ/kg
h10 = 897.9 kJ/kg
h11 = 908.9 kJ/kg
h12 = 908.9 kJ/kg
y = 0.1323
y' = 0.1112
```



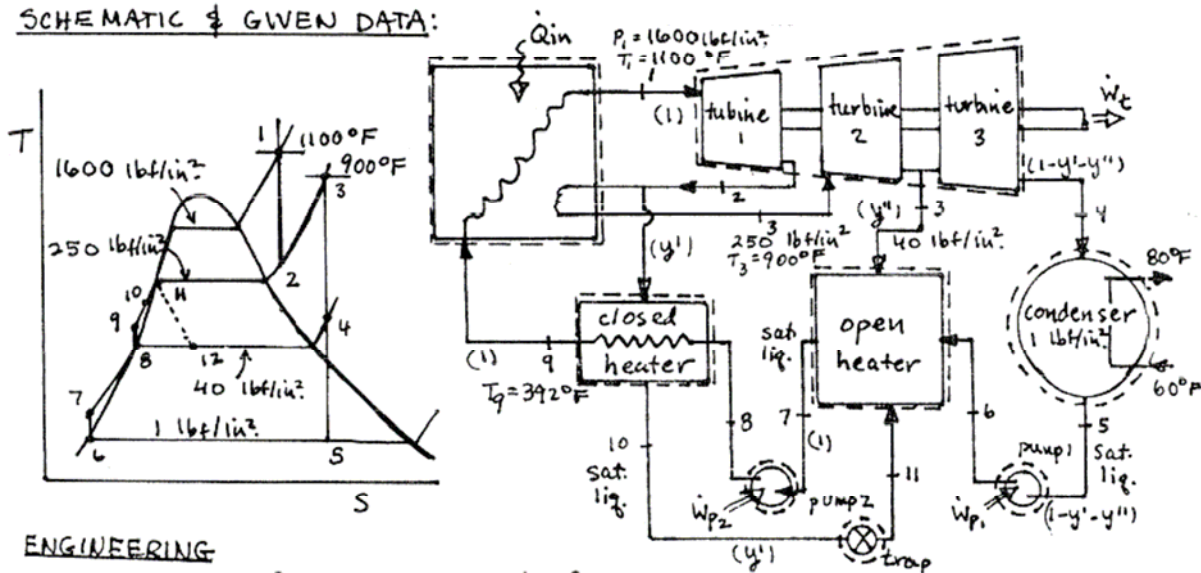
Increased irreversibilities in the turbine stages and pumps (lower isentropic efficiencies) result in lower thermal efficiency (as expected).

PROBLEM 8.72

KNOWN: The regenerative power cycle in Problem 8.65 is modified to include reheat at 250 lbf/in^2 .

FIND: Determine the thermal efficiency of the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Example 8.6

ANALYSIS: Using appropriate data from Problem 8.65

$$h_1 = 1547.7 \text{ Btu/lb}$$

$$h_9 = 241.11 \text{ Btu/lb}$$

$$h_2 = 1299.8 \text{ Btu/lb}$$

$$h_{10} = 368.38 \text{ Btu/lb}$$

$$h_6 = 69.74 \text{ Btu/lb}$$

$$h_{11} = 376.2 \text{ Btu/lb}$$

$$h_7 = 69.86 \text{ Btu/lb}$$

$$h_{12} = 376.2 \text{ Btu/lb}$$

$$h_8 = 236.16 \text{ Btu/lb}$$

$$\text{State 3: } p_3 = 250 \text{ lbf/in}^2, T_3 = 900^\circ\text{F} \Rightarrow h_3 = 1475.3 \text{ Btu/lb}, s_3 = 1.7799 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

$$\text{State 4: } p_4 = 40 \text{ lbf/in}^2, s_4 = s_3 \Rightarrow \text{sup. vapor}; h_4 = 1253.9 \text{ Btu/lb}$$

$$\text{State 5: } p_5 = 1 \text{ lbf/in}^2, s_5 = s_3 \Rightarrow x_5 = 0.8926, h_5 = 944.5 \text{ Btu/lb}$$

From the solution to Problem 8.65, $y' = 0.1365$. Regarding y''

$$y'' = \frac{y'(h_7 - h_{12}) + (h_8 - h_7)}{(h_4 - h_7)} = \frac{(0.1365)(69.86 - 376.2) + (236.16 - 69.86)}{(1253.9 - 69.86)} \\ = 0.1051$$

For the turbine stages

$$\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1 - y')(h_3 - h_4) + (1 - y' - y'')(h_4 - h_5) \\ = (1547.7 - 1299.8) + (0.8635)(1475.3 - 1253.9) + (0.7584)(1253.9 - 944.5) \\ = 635.8 \text{ Btu/lb}$$

Continued on next slide

Problem 8-72 continued

And, for the pumps

$$\begin{aligned}\dot{W}_P/\dot{m}_1 &= (h_9 - h_8) + (1 - y' - y'')(h_7 - h_6) \\ &= (241.11 - 236.16) + (.7584)(69.86 - 69.74) = 5.04 \text{ Btu/lb}\end{aligned}$$

The total heat added during boiling/superheating and reheating is expressed per unit mass entering the first turbine as

$$\begin{aligned}\dot{Q}_{in}/\dot{m}_1 &= (h_1 - h_{10}) + (1 - y')(h_3 - h_2) \\ &= (1547.7 - 368.38) + (.8635)(1475.3 - 1299.8) \\ &= 1330.9 \text{ Btu/lb}\end{aligned}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_P/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.474 (47.4\%) \quad \leftarrow \eta$$

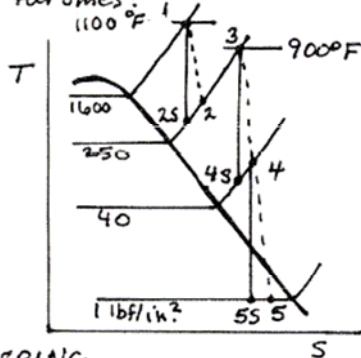
PROBLEM 8.73

KNOWN: The vapor power cycle of Problem 8.72 is modified to include that each turbine stage has an efficiency of 88% and each pump has an efficiency of 80%.

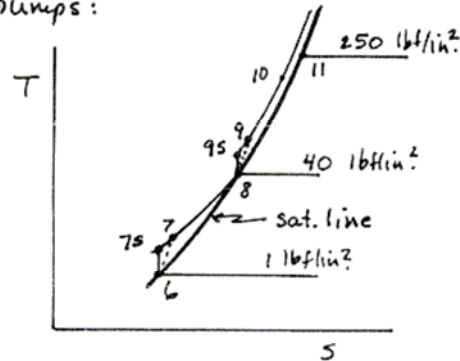
FIND: Repeat the analysis of Problem 8.65 for the modified cycle.

SCHEMATIC & GIVEN DATA:

For the turbines:



For the pumps:



ENGINEERING

MODEL: Same as Problem 8.72, except $\eta_{t1} = \eta_{t2} = \eta_{t3} = 0.88$ and $\eta_{p1} = \eta_{p2} = 0.80$.

ANALYSIS: From Problem 8.65, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R, $h_3 = 1475.3$ Btu/lb, $s_3 = 1.7799$ Btu/lb·°R, $h_6 = 69.74$ Btu/lb, $h_8 = 236.16$ Btu/lb, $h_{10} = 368.38$ Btu/lb, and $h_{11} = h_{12} = 376.2$ Btu/lb.

State 2: Using the turbine efficiency, $\eta_{t1} = (h_1 - h_{2s}) / (h_1 - h_{2s})$. Thus, with h_{2s} from Problem 8.65

$$h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 1329.5 \text{ Btu/lb}$$

State 4: $P_4 = 40$ lbf/in², $s_{4s} = s_3 \Rightarrow h_{4s} = 1253.9$ Btu/lb (Problem 8.65). Then, with $\eta_{t2} = 0.88$

$$h_4 = h_3 - \eta_{t2}(h_3 - h_{4s}) = 1280.5 \text{ Btu/lb}$$

Further, interpolating in Table A-4E; $s_4 = 1.8092$ Btu/lb·°R

State 5: $P_5 = 1$ lbf/in², $s_{5s} = s_4 \Rightarrow x_{5s} = 0.9085$, $h_{5s} = 1010.9$ Btu/lb. Then, with $\eta_{t3} = 0.88$

$$h_5 = h_4 - \eta_{t3}(h_4 - h_{5s}) = 1043.2 \text{ Btu/lb}$$

State 7: Using the pump efficiency, $\eta_{p1} = (h_7s - h_6) / (h_7 - h_6)$. Thus, with h_{7s} from Problem 8.72

$$h_7 = h_6 + (h_{7s} - h_6) / \eta_{p1} = 69.89 \text{ Btu/lb}$$

State 9: Similarly, with h_{9s} from Problem 8.65

$$h_9 = h_8 + (h_{9s} - h_8) / \eta_{p2} = 242.35 \text{ Btu/lb}$$

The mass flow rate ratios y' and y'' are, respectively

$$y' = \frac{h_{10} - h_9}{h_2 - h_{11}} = \frac{368.38 - 242.35}{1329.5 - 376.2} = 0.1322$$

and

Continued on next slide

Problem 8-73 continued

$$y'' = \frac{y'(h_7 - h_{12}) + (h_8 - h_7)}{(h_4 - h_7)}$$

$$= \frac{(.1322)(69.89 - 376.2) + (236.16 - 69.89)}{(1280.5 - 69.89)} = 0.1039$$

For the turbine stages

$$\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1 - y')(h_3 - h_4) + (1 - y' - y'')(h_4 - h_5)$$

$$= (1547.7 - 1329.5) + (.8678)(1475.3 - 1280.5) + (.7639)(1280.5 - 1043.2)$$

$$= 568.5 \text{ Btu/lb}$$

And, for the pumps

$$\dot{W}_p/\dot{m}_1 = (h_9 - h_8) + (1 - y' - y'')(h_7 - h_6)$$

$$= (242.35 - 236.16) + (.7639)(69.89 - 69.74) = 6.30 \text{ Btu/lb}$$

The total heat added per unit mass entering the first turbine stage is

$$\dot{Q}_{in}/\dot{m}_1 = (h_1 - h_{10}) + (1 - y')(h_3 - h_2)$$

$$= (1547.7 - 368.38) + (.8678)(1475.3 - 1329.5)$$

$$= 1305.8 \text{ Btu/lb}$$

Thus, the thermal efficiency is

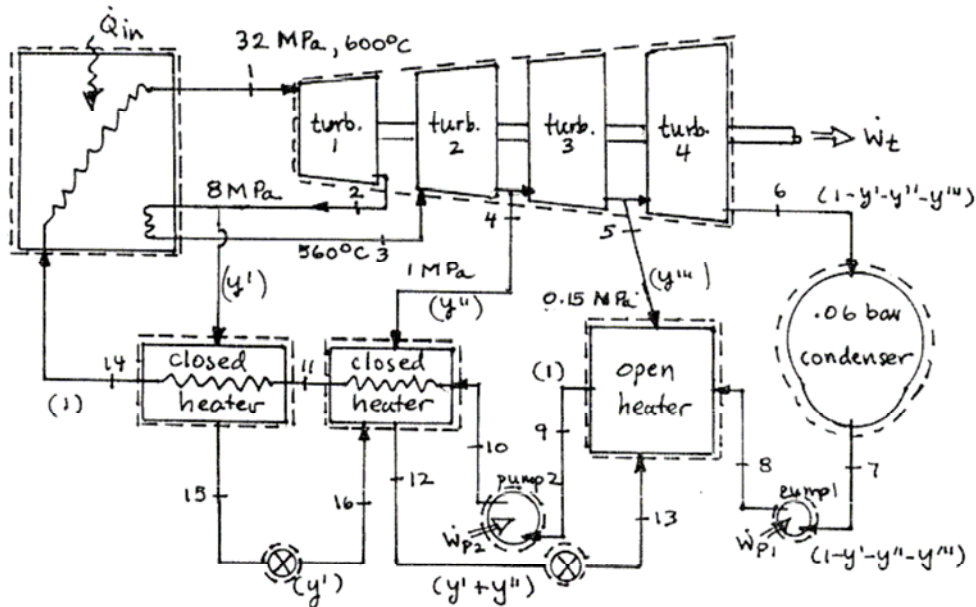
$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{568.5 - 6.30}{1305.8} = 0.431 (43.1\%) \quad \leftarrow \eta$$

PROBLEM 8.74

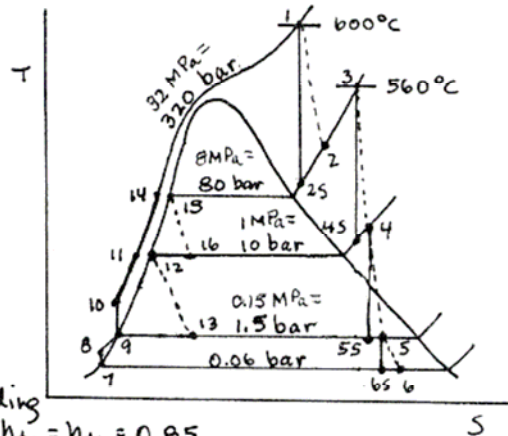
KNOWN: Water is the working fluid in a vapor power cycle with reheat and three regenerative feedwater heaters. Data are given at various locations and the net power output is known.

FIND: Sketch the layout of the cycle and determine the thermal efficiency and mass flow rate into the first turbine stage.

SCHEMATIC & GIVEN DATA:



$$\dot{W}_{cycle} = 500 \text{ MW}$$



ENGINEERING MODEL: Same as in Example 8.6, items 1–5, (6) the feedwater leaving each closed heater is at the saturation temperature corresponding to the extraction pressure. Also, $\eta_{t1} = \eta_{t2} = \eta_{t3} = \eta_{t4} = 0.85$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 320 \text{ bar}$, $T_1 = 600^\circ\text{C} \Rightarrow h_1 = 3424.6 \text{ kJ/kg}$, $s_1 = 6.1858 \text{ kJ/kg}\cdot\text{K}$

State 2: $p_2 = 80 \text{ bar}$, $s_{2s} = s_1 \Rightarrow h_{2s} = 3022.3 \text{ kJ/kg}$

Using the turbine efficiency

$$h_2 = h_1 - (h_1 - h_{2s})\eta_{t1} = 3082.6 \text{ kJ/kg} \text{ and } s_2 = 6.2781 \text{ kJ/kg}\cdot\text{K}$$

Continued on next slide

Problem 8-74 continued

State 3: $P_3 = 80 \text{ bar}$, $T_3 = 560^\circ\text{C} \Rightarrow h_3 = 3545.3 \text{ kJ/kg}$, $s_3 = 6.9072 \text{ kJ/kg}\cdot\text{K}$

State 4: $P_4 = 10 \text{ bar}$, $s_4 = s_3 \Rightarrow h_4 = 2934.0 \text{ kJ/kg}$

$$h_4 = h_3 - (h_3 - h_{4s}) \eta_{t2} = 3025.7 \text{ kJ/kg}, s_4 = 7.0771 \text{ kJ/kg}\cdot\text{K}$$

State 5: $P_5 = 1.5 \text{ bar}$, $s_5 = s_4 \Rightarrow x_{5s} = 0.9747$, $h_{5s} = 2637.3 \text{ kJ/kg}$

$$h_5 = h_4 - (h_4 - h_{5s}) \eta_{t3} = 2695.6 \text{ kJ/kg}, s_5 = 7.2284 \text{ kJ/kg}\cdot\text{K}$$

State 6: $P_6 = 0.06 \text{ bar}$, $s_6 = s_5 \Rightarrow x_{6s} = 0.0589$, $h_{6s} = 2226.5 \text{ kJ/kg}$

$$h_6 = h_5 - (h_5 - h_{6s}) \eta_{t4} = 2296.9 \text{ kJ/kg}$$

State 7: $P_7 = 0.06 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 151.53 \text{ kJ/kg}$

State 8: $h_8 \approx h_7 + v_7 (P_8 - P_7)$

$$= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (1.5 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$= 151.53 + 0.14 = 151.67 \text{ kJ/kg}$$

State 9: $P_9 = 1.5 \text{ bar}$, sat. liquid $\Rightarrow h_9 = 467.11 \text{ kJ/kg}$

State 10: $h_{10} \approx h_9 + v_9 (P_{10} - P_9) = 467.11 + (1.0528 \times 10^{-3}) (320 - 1.5) \left| \frac{10^5}{10^5} \right|$

$$= 467.11 + 33.53 = 500.64 \text{ kJ/kg}$$

State 11: $P_{11} = 320 \text{ bar}$, $T_{11} = T_{\text{sat}} @ 10 \text{ bar} = 180^\circ\text{C} \Rightarrow$ Extrapolating using data from Table A-5 gives $h_{11} \approx 781.7 \text{ kJ/kg}$

State 12: $P_{12} = 10 \text{ bar}$, sat. liquid $\Rightarrow h_{12} = 762.81 \text{ kJ/kg}$

State 13: Throttling process $\Rightarrow h_{13} = h_{12} = 762.81 \text{ kJ/kg}$

State 14: $P_{14} = 320 \text{ bar}$, $T_{14} = T_{\text{sat}} @ 80 \text{ bar} = 295^\circ\text{C} \Rightarrow$ Extrapolating using data from Table A-5 gives $h_{14} \approx 1303.8 \text{ kJ/kg}$

State 15: $P_{15} = 80 \text{ bar}$, sat. liquid $\Rightarrow h_{15} = 1316.6 \text{ kJ/kg}$

State 16: Throttling process $\Rightarrow h_{16} = h_{15} = 1316.6 \text{ kJ/kg}$

Applying mass and energy balances to the control volume enclosing the 80 bar closed heater, the fraction of the flow y' extracted at location 2 is

$$0 = y'(h_2 - h_{15}) + (1)(h_{11} - h_{14})$$

or
$$y' = \frac{h_{14} - h_{11}}{h_2 - h_{15}} = \frac{1303.8 - 781.7}{3025.7 - 1316.6} = 0.2956$$

Similarly, for the 10 bar closed heater

$$0 = y'' h_4 + y' h_{16} + (h_{10} - h_{11}) - (y' + y'') h_{12}$$

or
$$y'' = \frac{(h_{11} - h_{10}) + y'(h_{12} - h_{16})}{(h_4 - h_{12})}$$

$$= \frac{(781.7 - 500.64) + (0.2956)(762.81 - 1316.6)}{(3025.7 - 762.81)} = 0.0519$$

Continued on next slide

Problem 8-74 continued

And, for the open heater

$$0 = y''' h_5 + (y' + y'') h_{13} + (1 - y' - y'' - y''') h_8 - h_9$$

or

$$y''' = \frac{(h_9 - h_8) + (y' + y'')(h_8 - h_{13})}{(h_5 - h_8)}$$

$$= \frac{(467.11 - 151.67) + (0.3475)(151.67 - 762.81)}{(2695.6 - 151.67)} = 0.0405$$

For the control volume enclosing the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y')(h_3 - h_4) + (1 - y' - y'')(h_4 - h_5) + (1 - y' - y'' - y''')(h_5 - h_6)$$

$$= (3424.6 - 3082.6) + (0.7044)(3545.3 - 3025.7)$$

$$+ (0.6525)(3025.7 - 2695.6) + (0.6120)(2695.6 - 2296.9)$$

$$= 1167.4 \text{ kJ/kg}$$

And, for the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_{10} - h_9) + (1 - y' - y'' - y''')(h_8 - h_7)$$

$$= (500.64 - 467.11) + (0.6120)(151.67 - 151.53) = 33.62 \text{ kJ/kg}$$

The total rate of heat transfer for the boiler/superheater and reheater, per unit mass of steam entering the first turbine stage, is

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = (h_1 - h_{14}) + (1 - y')(h_3 - h_2)$$

$$= (3424.6 - 1303.8) + (0.7044)(3545.3 - 3082.6)$$

$$= 2446.7 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1167.4 - 33.62}{2446.7} = 0.463 (46.3\%) \leftarrow \eta$$

The net power developed by the cycle is

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]$$

or, with $\dot{W}_{cycle} = 500 \text{ MW}$, the flow rate $\dot{m}_1$ is

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]}$$

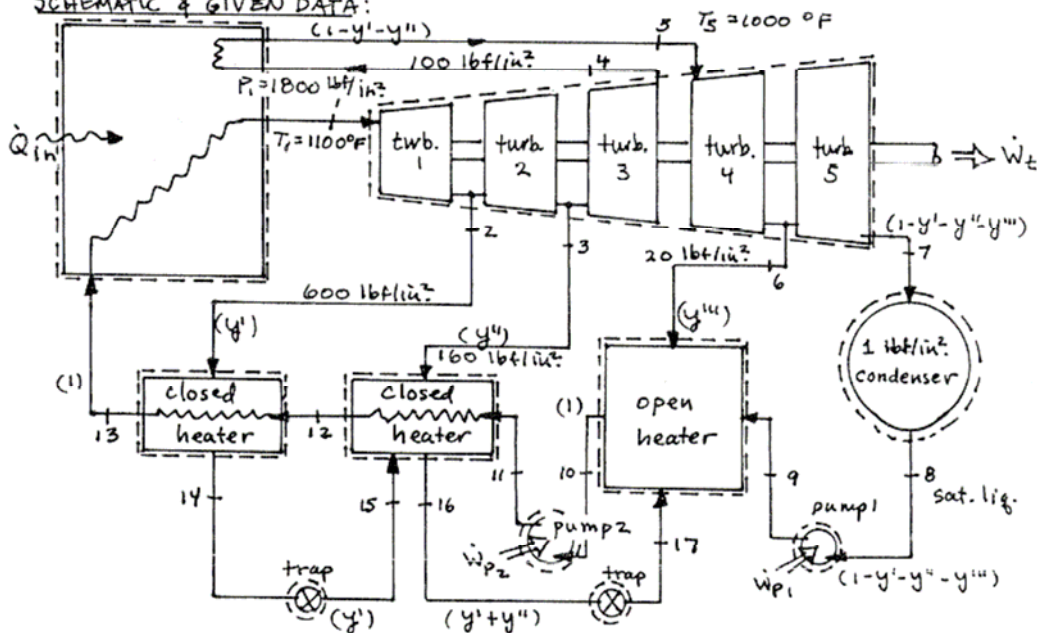
$$= \frac{(500 \text{ MW})}{(1167.4 - 33.62) \text{ kJ/kg}} \left| \frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = 1.588 \times 10^6 \text{ kg/h} \leftarrow \dot{m}_1$$

PROBLEM 8.75

KNOWN: Water is the working fluid in a vapor power cycle with reheat and three regenerative feedwater heaters. Data are given at various locations, and the net power output is known.

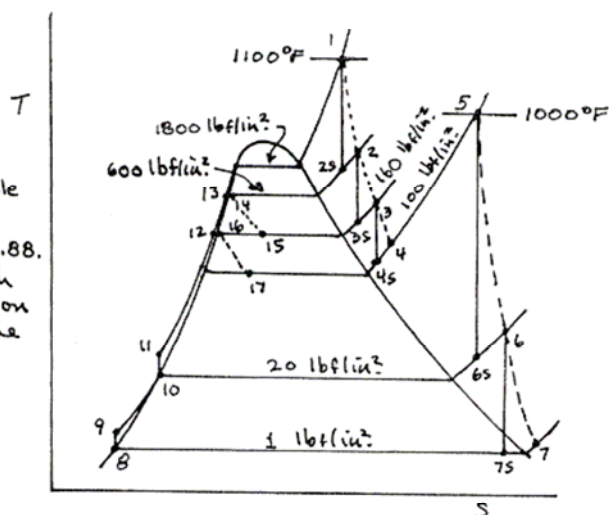
FIND: Sketch the layout of the cycle and determine the thermal efficiency, the mass flow rate into the first turbine stage, and the heat rate.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as in Example 8.6, assumptions 1-5, except $\eta_{t1} = \eta_{t2} = \eta_{t3} = \eta_{t4} = \eta_{t5} = .88$. Also, the feedwater leaving each closed heater is at the saturation temperature corresponding to the extraction pressure.



ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 1800 \text{ lbf/in}^2$, $T_1 = 1100^\circ\text{F} \Rightarrow h_1 = 1542.5 \text{ Btu/lb}$, $s_1 = 1.6159 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2: First, $P_2 = 600 \text{ lbf/in}^2$, $s_{2s} = s_1 \Rightarrow h_{2s} = 1385.3 \text{ Btu/lb}$. With the turbine efficiency
 $h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 1404.2 \text{ Btu/lb}$

And, interpolating in Table A-4; $s_2 = 1.6315 \text{ Btu/lb}\cdot^\circ\text{R}$

Continued on next slide

Problem 8-75 continued

State 3: Similarly, $h_{3s} = 1254.0 \text{ Btu/lb}$, and $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 1272.0 \text{ Btu/lb}$. From Table A-4; $s_3 = 1.6507 \text{ Btu/lb} \cdot ^\circ\text{R}$

State 4: Similarly, $h_{4s} = 1226.7 \text{ Btu/lb}$, and $h_4 = h_3 - \eta_{t3}(h_3 - h_{4s}) = 1232.1 \text{ Btu/lb}$. From Table A-4; $s_4 = 1.6569 \text{ Btu/lb} \cdot ^\circ\text{R}$

State 5: $p_5 = 100 \text{ lbf/in}^2$, $T_5 = 1000^\circ\text{F} \Rightarrow h_5 = 1532.1 \text{ Btu/lb}$, $s_5 = 1.9204 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$

State 6: $p_6 = 20 \text{ lbf/in}^2$, $s_{6s} = s_5 \Rightarrow h_{6s} = 1315.5 \text{ Btu/lb}$. With the turbine efficiency; $h_6 = h_5 - \eta_{t4}(h_5 - h_{6s}) = 1341.5 \text{ Btu/lb}$. From Table A-4; $s_6 = 1.9455 \text{ Btu/lb}$.

State 7: Similarly, $x_{7s} = .9824$, $h_{7s} = 1087.5$, and $h_7 = h_6 - \eta_{t5}(h_6 - h_{7s}) = 1118.0 \text{ Btu/lb}$

State 8: $p_8 = 1 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_8 = 69.74 \text{ Btu/lb}$

State 9: $h_9 \approx h_8 + v_8(p_9 - p_8)$
 $= 69.74 \frac{\text{Btu}}{\text{lb}} + (0.01614 \frac{\text{ft}^3}{\text{lb}}) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| (20 - 1) \frac{\text{lbf}}{\text{in}^2}$
 $= 69.80 \text{ Btu/lb}$

State 10: $p_{10} = 20 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_{10} = 196.26 \text{ Btu/lb}$

State 11: $p_{11} = 1800 \text{ lbf/in}^2$, $s_{11} = s_{10} = 0.3358 \text{ Btu/lb} \cdot ^\circ\text{R}$. Double interpolation in Table A-5 gives $h_{11} = 203.28 \text{ Btu/lb}$.

State 12: $p_{12} = 1800 \text{ lbf/in}^2$, $T_{12} = T_{\text{sat}} @ 1800 \text{ lbf/in}^2 = 363.6^\circ\text{F} \Rightarrow$ Double interpolation in Table A-5 gives $h_{12} = 339.10 \text{ Btu/lb}$

State 13: $p_{13} = 1800 \text{ lbf/in}^2$, $T_{13} = T_{\text{sat}} @ 600 \text{ lbf/in}^2 = 486.3^\circ\text{F} \Rightarrow$ Double interpolation in Table A-5 gives $h_{13} = 472.22 \text{ Btu/lb}$

State 14: $p_{14} = 600 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_{14} = 471.7 \text{ Btu/lb}$

State 15: Throttling process $\Rightarrow h_{15} = h_{14} = 471.7 \text{ Btu/lb}$

State 16: $p_{16} = 160 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_{16} = 336.2 \text{ Btu/lb}$

State 17: Throttling process $\Rightarrow h_{17} = h_{16} = 336.2 \text{ Btu/lb}$

Applying mass and energy rate balances to the control volume enclosing the 600 lbf/in^2 closed heater, the fraction of the flow y' extracted at location 2 is

$$0 = y'(h_2 - h_{14}) + (h_{12} - h_{13})$$

or

$$y' = \frac{h_{13} - h_{12}}{h_2 - h_{14}} = \frac{472.22 - 339.10}{1404.2 - 471.7} = 0.1428$$

Similarly, for the 160 lbf/in^2 heater

$$0 = y'' h_3 + y' h_{15} - (y' + y'') h_{16} + (h_{11} - h_{12})$$

or

$$y'' = \frac{(h_{12} - h_{11}) + y'(h_{16} - h_{15})}{(h_3 - h_{16})}$$

$$= \frac{(339.10 - 203.28) + (0.1428)(336.2 - 471.7)}{(1272.0 - 336.2)} = 0.1245$$

Continued on next slide

Problem 8-75 continued

Also, for the open feedwater heater

$$0 = y''' h_6 + (y' + y'') h_{17} + (1 - y' - y'' - y''') h_9 - h_{10}$$

or

$$y''' = \frac{(h_{10} - h_9) + (y' + y'')(h_9 - h_{17})}{(h_6 - h_9)}$$

$$= \frac{(196.26 - 69.80) + (0.2673)(69.80 - 336.2)}{(1341.5 - 69.80)} = 0.0434$$

For the control volume enclosing the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y')(h_2 - h_3) + (1 - y' - y'')[h_3 - h_4] + (h_5 - h_6) + (1 - y' - y'' - y''')(h_6 - h_7)$$

$$= (1542.5 - 1404.2) + (0.8572)(1404.2 - 1272.0) + (0.7327)[(1272.0 - 1232.1) + (1532.1 - 1341.5)] + (0.6893)(1341.5 - 1118.0) = 574.6 \text{ Btu/lb}$$

And, for the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_{11} - h_{10}) + (1 - y' - y'' - y''')(h_9 - h_8)$$

$$= (203.28 - 196.26) + (0.6893)(69.80 - 69.74) = 7.06 \text{ Btu/lb}$$

The total rate of heat transfer for the boiler/super heater and reheater, per unit mass of steam entering the first turbine stage, is

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = (h_1 - h_{13}) + (1 - y' - y'')(h_5 - h_4)$$

$$= (1542.5 - 472.22) + (0.7327)(1532.1 - 1232.1) = 1290.1 \text{ Btu/lb}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{574.6 - 7.06}{1290.1} = 0.440 \text{ (44.0\%)} \leftarrow \eta$$

The net power developed by the cycle is

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]$$

or, with $\dot{W}_{cycle} = 3 \times 10^9 \text{ Btu/h}$, the flow rate $\dot{m}_1$ is

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]} = \frac{3 \times 10^9 \text{ Btu/h}}{[574.6 - 7.06] \text{ Btu/lb}} = 5.286 \times 10^6 \text{ lb/h} \leftarrow \dot{m}_1$$

The heat rate — the heat added to the cycle to produce a unit of net work output — is given here by

$$\text{heat rate} = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{(5.286 \times 10^6 \frac{\text{lb}}{\text{h}})(1290.1 \text{ Btu/lb})}{3 \times 10^9 \text{ Btu/h}} \left| \frac{3413 \text{ Btu/h}}{1 \text{ kW}} \right|$$

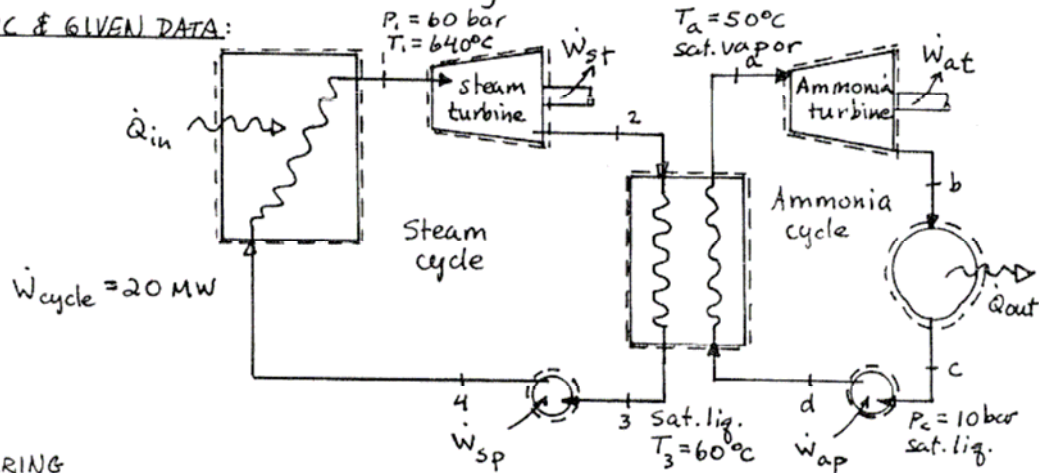
$$= 7758 \frac{\text{Btu}}{\text{kW} \cdot \text{h}} \leftarrow \text{heat rate}$$

PROBLEM 8.76

KNOWN: Steam and ammonia are the working fluids in a binary vapor power cycle consisting of two ideal Rankine cycles. The heat rejected from the steam cycle is provided to the ammonia cycle. Data are known at various locations.

FIND: Determine (a) the power output of the steam and ammonia turbines, respectively, (b) the rate of heat addition to the cycle, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluids are internally reversible, except in the interconnecting heat exchanger. (3) There are no stray heat transfers from the turbines or the heat exchanger. (4) Kinetic and potential energy effects can be neglected.

ANALYSIS: First, fix each of principal states. For steam, use Tables A-2 and A-4.

State 1: $P_1 = 60 \text{ bar}$, $T_1 = 640^\circ\text{C} \Rightarrow h_1 = 3752.6 \text{ kJ/kg}$, $s_1 = 7.2731 \text{ kJ/kg}$

State 2: $T_2 = T_3 = 60^\circ\text{C}$, $s_2 = s_1 \Rightarrow x_2 = 0.9101$, $h_2 = 2397.6 \text{ kJ/kg}$

State 3: $T_3 = 60^\circ\text{C}$, sat. liquid $\Rightarrow h_3 = 251.13 \text{ kJ/kg}$, $v_3 = 1.0172 \times 10^{-3} \text{ m}^3/\text{kg}$

State 4: $h_4 \approx h_3 + v_3(P_4 - P_3)$; $P_3 = 0.1994 \text{ bar}$

$$h_4 = 251.13 + (1.0172 \times 10^{-3} \frac{\text{m}^3}{\text{kg}})(60 - 0.1994) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$= 251.13 + 6.083 = 257.21 \text{ kJ/kg}$$

Now, for Ammonia, use Tables A-13 and A-14.

State a: $T_a = 50^\circ\text{C}$, sat. vapor $\Rightarrow h_a = 1471.26 \text{ kJ/kg}$, $s_a = 4.7604 \text{ kJ/kg}\cdot\text{K}$

State b: $P_b = P_c = 10 \text{ bar}$, $s_b = s_a \Rightarrow x_b = 0.9312$, $h_b = 1383.0 \text{ kJ/kg}$

State c: $P_c = 10 \text{ bar}$, sat. liquid $\Rightarrow h_c = 297.76 \text{ kJ/kg}$, $v_c = 1.6584 \times 10^{-3} \text{ m}^3/\text{kg}$

State d: $h_d \approx h_c + v_c(P_d - P_c)$; $P_d = P_a = 20.331 \text{ bar}$

$$h_d = 297.76 + (1.6584 \times 10^{-3})(20.331 - 10)(100) = 299.47 \text{ kJ/kg}$$

(a) To determine the power output of each turbine, we must find the mass flow rates $\dot{m}_s$ and $\dot{m}_a$ for the steam and ammonia cycles, respectively. Beginning with mass and energy rate balances for the inter-connecting heat exchanger

Continued on next slide

Problem 8-76 continued

$$0 = \dot{m}_s (h_2 - h_3) + \dot{m}_a (h_d - h_a)$$

or $\frac{\dot{m}_a}{\dot{m}_s} = \frac{h_2 - h_3}{h_a - h_d} = \frac{2397.6 - 251.13}{1471.26 - 299.47} = 1.832$

For the steam cycle

$$\dot{W}_{\text{cycle},s} = \dot{W}_{st} - \dot{W}_{sp} = \dot{m}_s [(h_1 - h_2) - (h_4 - h_3)]$$

And, for the ammonia cycle

$$\dot{W}_{\text{cycle},a} = \dot{W}_{at} - \dot{W}_{ap} = \dot{m}_a [(h_a - h_b) - (h_d - h_c)]$$

Noting that $\dot{W}_{\text{cycle}} = \dot{W}_{\text{cycle},s} + \dot{W}_{\text{cycle},a}$ and combining

$$\dot{W}_{\text{cycle}} = \dot{m}_s \left\{ [(h_1 - h_2) - (h_4 - h_3)] + \left(\frac{\dot{m}_a}{\dot{m}_s} \right) [(h_a - h_b) - (h_d - h_c)] \right\}$$

Solving

$$\begin{aligned} \dot{m}_s &= \frac{\dot{W}_{\text{cycle}}}{[(h_1 - h_2) - (h_4 - h_3)] + \left(\frac{\dot{m}_a}{\dot{m}_s} \right) [(h_a - h_b) - (h_d - h_c)]} \\ &= \frac{(20 \text{ MW})}{[(1355) - (6.08)] + (1.832)[(88.26) - (1.71)] \text{ kJ/kg}} \left| \frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right| \\ &= 13.27 \text{ kg/s} \end{aligned}$$

And $\dot{m}_a = 1.832 \dot{m}_s = 24.31 \text{ kg/s}$

Finally

$$\begin{aligned} \dot{W}_{st} &= \dot{m}_s (h_1 - h_2) = 17.98 \text{ MW} \longleftarrow \dot{W}_{st} \\ \dot{W}_{at} &= \dot{m}_a (h_a - h_b) = 2.15 \text{ MW} \longleftarrow \dot{W}_{at} \end{aligned}$$

(b) The rate of heat addition is

$$\begin{aligned} \dot{Q}_{in} &= \dot{m}_s (h_1 - h_4) \\ &= (13.27 \frac{\text{kg}}{\text{s}})(3752.6 - 257.21) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| \\ &= 46.38 \text{ MW} \longleftarrow \dot{Q}_{in} \end{aligned}$$

(c) The thermal efficiency is

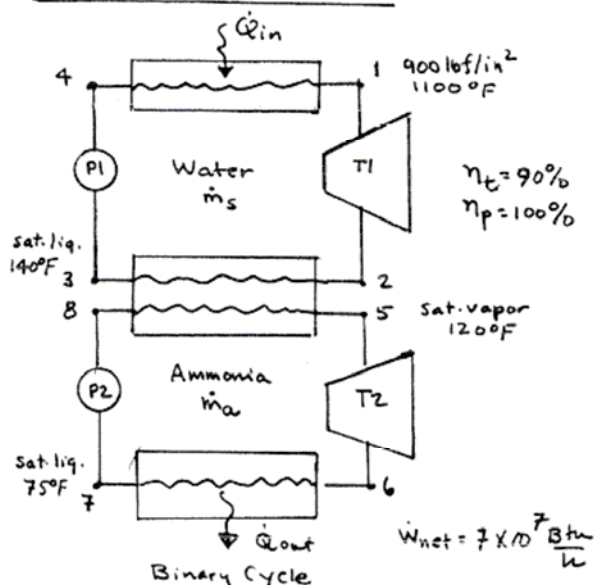
$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{in}} = \frac{20}{46.38} = 0.431 \text{ (43.1\%)} \longleftarrow \eta$$

PROBLEM 8.77

KNOWN: Operating data are provided for a binary vapor cycle consisting of two Rankine cycles with steam and ammonia as the working fluids.

FIND: (a) For the binary cycle, determine the quality at the exit of each turbine, the mass flow rate of each working fluid, and the overall thermal efficiency. (b) Compare the performance to a single Rankine cycle using water as the working fluid, condensing at 75°F.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

- Control volumes enclosing the principal components are at steady state.
- The turbines and pumps operate adiabatically. There are no stray heat transfers, and kinetic and potential energy effects can be ignored.
- For the heat exchangers, the pressure of the working fluid remains constant.

ANALYSIS: (a) Begin by fixing the numbered states on the schematic.

From Table A-4E, $h_1 = 1565.4 \text{ Btu/lb}$,

$s_1 = 1.7036 \text{ Btu/lb} \cdot \text{R}$. With $s_{2s} = s_1$, and data from Table A-2E at 140°F

$$x_{2s} = \frac{1.7036 - 0.1985}{1.8892 - 0.1985} = 0.89$$

$$\Rightarrow h_{2s} = 107.96 + 0.89(1014) = 1010.42 \text{ Btu/lb}. \text{ Then, with } \eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}},$$

$$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1565.4 - 0.9(1565.4 - 1010.42) = 1065.92 \text{ Btu/lb}. \text{ And}$$

$$x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{1065.92 - 107.96}{1014} = 0.945$$

$$\text{For pump P1, Eq. 8.76 gives, } \frac{\dot{W}_{P1}}{\dot{m}_s} = v_3 \Delta p = (0.01629 \frac{\text{ft}^3}{\text{lb}}) (900 - 2.892) \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 2.7 \text{ Btu/lb}$$

$$\Rightarrow h_4 = h_3 + \frac{\dot{W}_{P1}}{\dot{m}_s} = 107.96 + 2.7 = 110.66 \text{ Btu/lb}.$$

Considering the ammonia cycle, Table A-13E gives $h_5 = 632.95 \text{ Btu/lb}$, $s_1 = 1.1058 \text{ Btu/lb} \cdot \text{R}$.

With $s_{6s} = s_5$ and data from Table A-13E at 75°F

$$x_{6s} = \frac{1.1405 - 0.2636}{1.2048 - 0.2636} = 0.932 \Rightarrow h_{6s} = 126.02 + 0.932(503.18) = 594.98 \text{ Btu/lb}. \text{ And}$$

$$h_6 = h_5 - \eta_t(h_5 - h_{6s}) = 632.95 - 0.9(632.95 - 594.98) = 599.78 \text{ Btu/lb}$$

$$\Rightarrow x_6 = \frac{599.78 - 126.02}{503.18} = 0.94$$

$$\text{For pump P2, } \frac{\dot{W}_{P2}}{\dot{m}_a} = v_7 \Delta p = (0.02650) (286.4 - 140.6) \left| \frac{144}{778} \right| = 0.72 \text{ Btu/lb}$$

$$\Rightarrow h_8 = h_7 + \frac{\dot{W}_{P2}}{\dot{m}_a} = 126.02 + 0.72 = 126.74 \text{ Btu/lb}$$

Continued on next slide

Problem 8-77 continued

Next, an energy rate balance on the interconnecting heat exchanger gives,

$$\frac{\dot{m}_s}{\dot{m}_a} = \frac{h_5 - h_8}{h_2 - h_3} = \frac{632.95 - 126.74}{1065.92 - 107.96} = 0.528$$

Then,

$$\dot{W}_{net} = \dot{m}_s \left[h_1 - h_2 - \frac{\dot{W}_{P1}}{\dot{m}_s} \right] + \dot{m}_a \left[h_5 - h_6 - \frac{\dot{W}_{P2}}{\dot{m}_a} \right]$$

$$(7 \times 10^7 \frac{\text{Btu}}{\text{h}}) = 0.528 \dot{m}_a [1565.4 - 1065.92 - 2.7] + \dot{m}_a [632.95 - 595.78 - 0.72]$$

$$\Rightarrow \dot{m}_a = 236,686 \frac{\text{lb}}{\text{h}}, \quad \dot{m}_s = 124,970 \frac{\text{lb}}{\text{h}}$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{m}_s (h_1 - h_4)} = \frac{7 \times 10^7 \text{ Btu/h}}{(124,970 \frac{\text{lb}}{\text{h}})(1565.4 - 110.66) \frac{\text{Btu}}{\text{lb}}} = 0.385 \quad (38.5\%)$$

(a) For a single steam cycle condensing at 75°F, the exit condition for the steam turbine is fixed using $s_{2s} = s_1$. Then, with data from Table A-2E

$$x_{2s} = \frac{1.7036 - 0.08402}{2.04975 - 0.08402} = 0.824 \Rightarrow h_{2s} = 43.09 + 0.824(1051.15) = 909.24 \text{ Btu/lb}$$

$$\Rightarrow h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 1565.4 - 0.9(1565.4 - 909.24) = 974.86 \text{ Btu/lb}$$

$$\textcircled{1} \Rightarrow x_2 = \frac{974.86 - 43.09}{1051.15} = 0.886$$

The pump work per unit of steam flowing is

$$\frac{\dot{W}_P}{\dot{m}_s} = v_3 \Delta p = 0.01606 (900 - 0.43) \left| \frac{14.7}{778} \right| = 2.67 \text{ Btu/lb}$$

$$\Rightarrow h_4 = h_3 + \frac{\dot{W}_P}{\dot{m}_s} = 43.09 + 2.67 = 45.76 \text{ Btu/lb}$$

The mass flow rate of the steam is

$$\textcircled{2} \quad \dot{m}_s = \frac{\dot{W}_{net}}{(h_1 - h_2) - \dot{W}_P/\dot{m}_s} = \frac{7 \times 10^7 \text{ Btu/h}}{[(1565.4 - 974.86) - 2.67] \text{ Btu/lb}} = 119,074 \frac{\text{lb}}{\text{h}}$$

The thermal efficiency is

$$\textcircled{3} \quad \eta = \frac{\dot{W}_{net}}{\dot{m}_s (h_1 - h_4)} = \frac{7 \times 10^7}{119,074 (1565.4 - 45.76)} = 0.387 \quad (38.7\%)$$

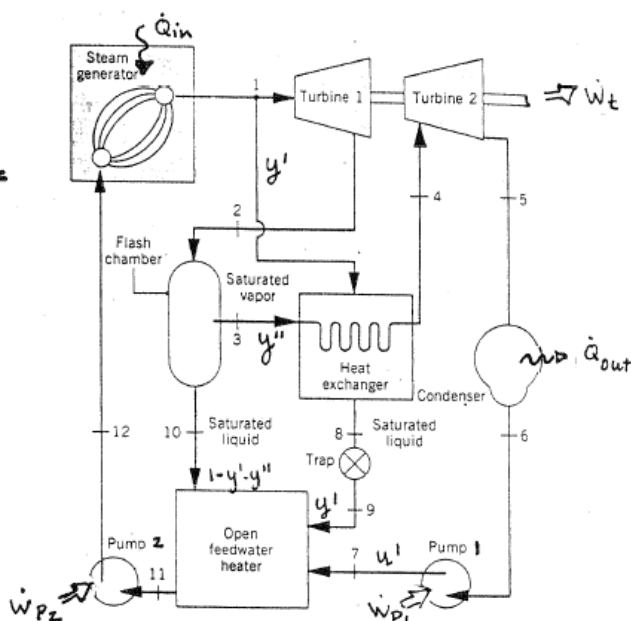
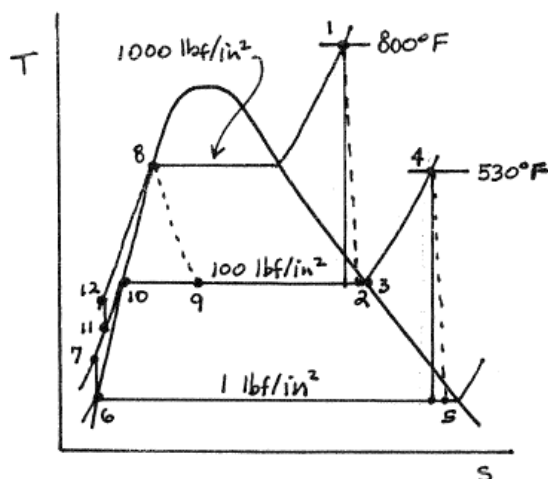
1. Since the quality at the exit of a steam turbine should be at least 90%, the performance of the single Rankine cycle is less satisfactory than the binary cycle in this respect.
2. The mass flow rate of the steam in the single Rankine cycle is about 5% less than in the binary cycle.
3. Since the average temperatures of heat addition and heat rejection are the same in the two cycles, the thermal efficiencies are the same when roundoff is taken into account.

PROBLEM 8.78

KNOWN: Water is the working fluid in a vapor power cycle with reheat and regeneration. Data are known at various locations and the net power is given.

FIND: Determine (a) the mass flow rate through the steam generator, (b) the thermal efficiency, and (c) the rate of heat transfer to cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) There are no stray heat transfers. (3) The working fluid undergoes internally reversible processes in passing through the pumps, steam generator, flash chamber, and condenser. (4) The expansion through the trap is a throttling process. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 1000 \text{ lbf/in}^2$, $T_1 = 800^\circ\text{F} \Rightarrow h_1 = 1388.5 \text{ Btu/lb}$, $s_1 = 1.5665 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2: $p_2 = 100 \text{ lbf/in}^2$, $s_{2s} = s_1 \Rightarrow x_{2s} = 0.9673$, $h_{2s} = 1158.7 \text{ Btu/lb}$

Then, with $\eta_{t1} = (h_1 - h_2)/(h_1 - h_{2s})$

$$h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 1186.3 \text{ Btu/lb}$$

State 3: $p_3 = 100 \text{ lbf/in}^2$, sat. vapor $\Rightarrow h_3 = 1187.8 \text{ Btu/lb}$

State 4: $p_4 = 100 \text{ lbf/in}^2$, $T_4 = 530^\circ\text{F} \Rightarrow h_4 = 1294.2 \text{ Btu/lb}$, $s_4 = 1.7234 \text{ Btu/lb}\cdot^\circ\text{R}$

State 5: First, $p_5 = 1 \text{ lbf/in}^2$, $s_{5s} = s_4 \Rightarrow x_{5s} = 0.8620$, $h_{5s} = 962.77 \text{ Btu/lb}$

Then, with $\eta_{t2} = 0.88$; $h_5 = h_4 - \eta_{t2}(h_4 - h_{5s}) = 1002.5 \text{ Btu/lb}$

State 6: $p_6 = 1 \text{ lbf/in}^2$, sat. vapor $\Rightarrow h_6 = 69.74 \text{ Btu/lb}$

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Problem 8-78 continued

$$\begin{aligned}\text{State 7: } h_7 &\approx h_6 + v_6 (p_7 - p_6) \\ &= 69.74 \frac{\text{Btu}}{\text{lb}} + (0.01614 \frac{\text{ft}^3}{\text{lb}})(100-1) \frac{\text{lb}_f}{\text{in}^2} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lb}_f} \right| \\ &= 69.74 + 2.96 = 70.04 \text{ Btu/lb}\end{aligned}$$

$$\text{State 8: } p_8 = 1000 \text{ lb}_f/\text{in}^2, \text{ sat. liquid} \Rightarrow h_8 = 542.4 \text{ Btu/lb}$$

$$\text{State 9: throttling process} \Rightarrow h_9 = h_8 = 542.4 \text{ Btu/lb}$$

$$\text{State 10: } p_{10} = 100 \text{ lb}_f/\text{in}^2, \text{ sat. liquid} \Rightarrow h_{10} = 298.6 \text{ Btu/lb}$$

State 11: Must be determined in the analysis.

The mass flow rate ratios y' and y'' are determined from energy and mass rate balances on the flash chamber and heat exchanger, respectively.

$$(1-y')h_2 = y'h_3 + (1-y'-y'')h_{10} \quad (*)$$

$$y'(h_1 - h_8) + y''(h_3 - h_4) = 0 \quad (**)$$

From (**)

$$y'' = \left(\frac{h_1 - h_8}{h_4 - h_3} \right) y' = \left(\frac{1388.5 - 542.4}{1294.2 - 1187.8} \right) y' = 7.952 y'$$

Solving (*) and incorporating this result gives

$$y' = \frac{(h_2 - h_{10})}{(7.952 h_3 - 8.952 h_{10} + h_2)} = 0.1115$$

$$\text{and } y'' = 7.952 y' = 0.8866$$

(a) For the turbine stages

$$\frac{\dot{W}_{t1}}{\dot{m}_1} = (1-y')(h_1 - h_2)$$

$$\frac{\dot{W}_{t2}}{\dot{m}_2} = y''(h_4 - h_5)$$

and for the pumps

$$\frac{\dot{W}_{p1}}{\dot{m}_1} = y''(h_7 - h_6)$$

$$\frac{\dot{W}_{p2}}{\dot{m}_1} = (h_{12} - h_{11})$$

To find h_{11} , use an energy balance for the open feedwater heater

$$(1-y'-y'')h_{10} + y'h_9 + y''h_7 = h_{11} = 123.1 \text{ Btu/lb.}$$

From Table A-2E, $T_{11} \approx 155^\circ\text{F}$ and $v_{11} \approx 0.01637 \text{ ft}^3/\text{lb}$. Thus, for state 12

$$h_{12} \approx h_{11} + v_{11}(p_{12} - p_{11}) = 123.1 + 2.73 = 125.83 \text{ Btu/lb}$$

Finally

$$\begin{aligned}\dot{m}_1 &= \frac{\dot{W}_{\text{cycle}}}{(1-y')(h_1 - h_2) + y''(h_4 - h_5) + y''(h_7 - h_6) + (h_{12} - h_{11})} \\ &= \frac{5 \times 10^9 \text{ Btu/h}}{[179.6 + 258.6 - 0.03 - 2.73] \text{ Btu/lb}} \\ &= 1.148 \times 10^7 \text{ lb/h} \leftarrow \dot{m}_1\end{aligned}$$

Continued on next slide

Problem 8-78 continued

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{cycle}}}{\dot{m}_1 (h_1 - h_{12})} = \frac{(5 \times 10^9)}{1.148 \times 10^7 (1388.5 - 125.83)} \\ = 0.345 (34.5\%) \quad \leftarrow \eta$$

(c) The rate of heat transfer to cooling water passing through the condenser is

$$\dot{Q}_{\text{out}} = \dot{m}_1 y'' (h_5 - h_6) \\ = (1.148 \times 10^7 \text{ lb/h}) (0.8866) (1002.5 - 69.74) \text{ Btu/lb} \\ = 9.494 \times 10^9 \text{ Btu/h} \quad \leftarrow \dot{Q}_{\text{out}}$$

COMMENT: As a check on the calculations, note that

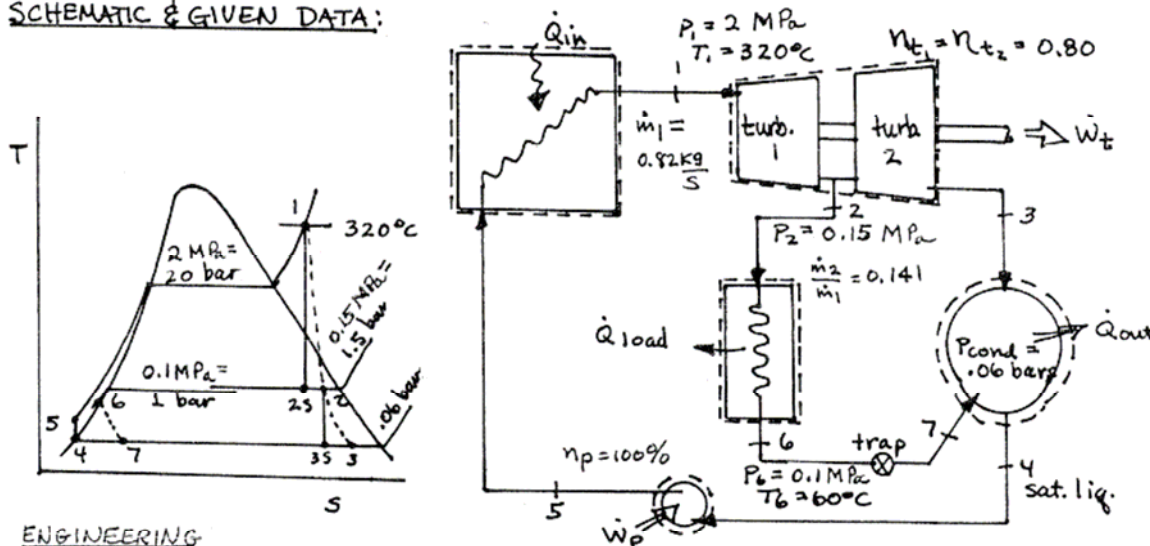
$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{cycle}} = 9.495 \times 10^9 \text{ Btu/h}$$

which is the same as the result determined in part (c), except for round-off errors.

PROBLEM 8.79

KNOWN: Water is the working fluid in a cogeneration cycle that generates electricity and provides heating for buildings. Data are provided at several locations. The fraction of the total flow extracted is also known. **Determine:** (a) the heat transfer rate to the working fluid passing through the steam generator, (b) the net power developed, (c) the rate of heat transfer for building heating, (d) the rate of heat transfer to cooling water.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each control volume shown operates at steady state. (2) The turbine stages operate adiabatically, with $\eta_{t1} = \eta_{t2} = 0.8$, and the pump operates isentropically. (3) The working fluid undergoes a throttling process in passing through the trap. (4) Kinetic and potential energy effects can be neglected. (5) Condensate leaves the condenser as saturated liquid at 0.06 bars.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 2.0 \text{ bar}$, $T_1 = 320^\circ\text{C} \Rightarrow h_1 = 3069.5 \text{ kJ/kg}$, $s_1 = 6.8452 \text{ kJ/kg}\cdot\text{K}$

State 2: First, $P_2 = 1.5 \text{ bar}$, $s_{2s} = s_1 \Rightarrow h_{2s} = 2548.2$. Using the turbine efficiency

$$\eta_{t1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 2652.5 \text{ kJ/kg}$$

And, with data from Table A-3; $s_2 = 7.1163 \text{ kJ/kg}\cdot\text{K}$

State 3: $P_3 = 0.06 \text{ bar}$, $s_{3s} = s_2 \Rightarrow h_{3s} = 2191.8$. Thus, $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 2283.9 \text{ kJ/kg}$. And, from Table A-3, $s_3 = 7.4139 \text{ kJ/kg}\cdot\text{K}$.

State 4: $P_4 = 0.06 \text{ bar}$, Sat. liquid $\Rightarrow h_4 = 151.53 \text{ kJ/kg}$

$$\begin{aligned} \text{State 5: } h_5 &\approx h_4 + v_4(P_5 - P_4) \\ &= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (20 - 0.06) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \\ &= 153.5 \text{ kJ/kg} \end{aligned}$$

State 6: $P_6 = 1 \text{ bar}$, $T_6 = 60^\circ\text{C}$, $h_6 \approx h_f(T_6) = 251.13 \text{ kJ/kg}$

State 7: Throttling process $\Rightarrow h_7 = h_6 = 251.13 \text{ kJ/kg}$

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Problem 8-79 continued

(a) For a control volume enclosing the tubes carrying the working fluid through the steam generator

$$\dot{Q}_{in} = \dot{m}_1 [h_1 - h_5] = (0.82 \frac{kg}{s}) \left| \frac{3600s}{1h} \right| [3069.5 - 153.5] \frac{kJ}{kg} = 8.61 \times 10^6 \frac{kJ}{h} \quad \leftarrow \dot{Q}_{in}$$

(b) The net power developed is obtained as follows:

$$\text{Turbines: } \dot{W}_t = \dot{m}_1 (h_1 - h_2) + (\dot{m}_1 - \dot{m}_2) (h_2 - h_3)$$

$$\text{Pump: } \dot{W}_p = \dot{m}_1 [h_5 - h_4]$$

$$\begin{aligned} \therefore \dot{W}_{net} &= \dot{m}_1 (h_1 - h_2) + (\dot{m}_1 - \dot{m}_2) (h_2 - h_3) - \dot{m}_1 (h_5 - h_4) \\ &= \dot{m}_1 [(h_1 - h_2) + (h_2 - h_3) - (h_5 - h_4)] - \dot{m}_2 (h_2 - h_3) \\ &= \dot{m}_1 [h_1 - h_3 - h_5 + h_4] - \dot{m}_2 [h_2 - h_3] \\ &= (0.82 \times 3600) \frac{kJ}{h} [3069.5 - 2283.9 - 153.5 + 151.5] \frac{kJ}{kg} \\ &\quad - 0.141 (0.82 \times 3600) [2652.5 - 2283.9] = 2.16 \times 10^6 \frac{kJ}{h} \quad \leftarrow \dot{W}_{net} \end{aligned}$$

$$\begin{aligned} \text{(c) } \dot{Q}_{LOAD} &= \dot{m}_2 [h_2 - h_6] = 0.141 (0.82 \times 3600) [2652.5 - 251.1] \\ &= 10^6 \frac{kJ}{h} \quad \leftarrow \dot{Q}_{LOAD} \end{aligned}$$

(d) For a control volume enclosing the working fluid side of the condenser

$$\dot{Q}_{out} = (\dot{m}_1 - \dot{m}_2) h_3 + \dot{m}_2 h_7 - \dot{m}_1 h_4$$

$$= \dot{m}_1 [h_3 - h_4] + \dot{m}_2 [h_7 - h_3]$$

$$\begin{aligned} \text{①} \quad &= ((0.82) \times 3600) [2283.9 - 151.5] + [(0.141) \times (0.82 \times 3600)] [251.1 - 2283.9] \\ &= 5.45 \times 10^6 \frac{kJ}{h} \quad \leftarrow \dot{Q}_{out} \end{aligned}$$

1. An overall energy rate balance reads:

• Rate energy is added by heat transfer in the steam generator

$$8.61 \times 10^6 \text{ kJ/h}$$

• Disposition:

- net power developed

$$2.16 \times 10^6 \text{ kJ/h} \quad (25.1\%)$$

- energy to campus buildings

$$1.00 \times 10^6 \text{ kJ/h} \quad (11.6\%)$$

- energy to cooling water

$$5.45 \times 10^6 \text{ kJ/h} \quad (63.3\%)$$

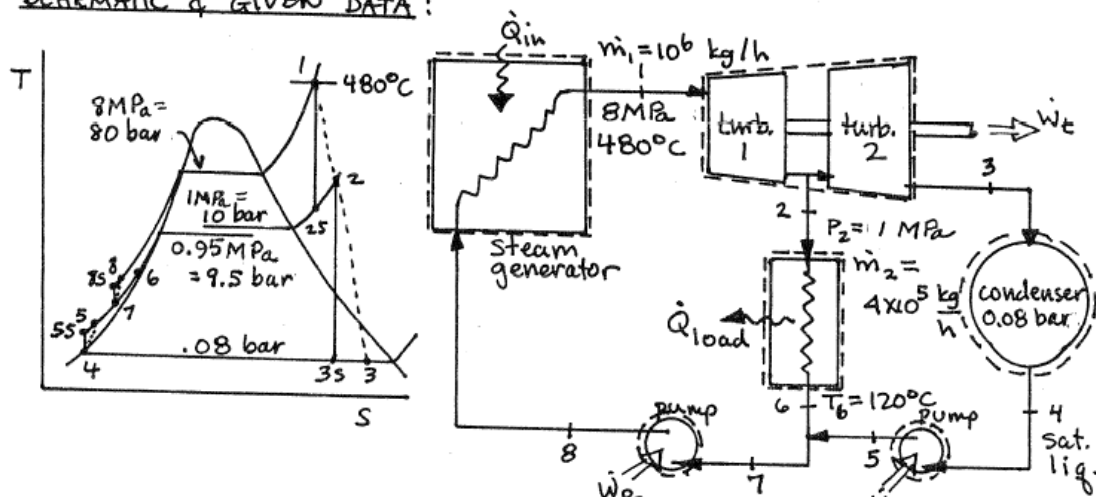
$$\underline{8.61 \times 10^6 \text{ kJ/h}}$$

PROBLEM 8.80

KNOWN: Water is the working fluid in a cogeneration cycle producing power and providing steam for a process heating load. Data are known at various locations.

FIND: Determine (a) the heating load, (b) the power developed by the turbine, and (c) the rate of heat transfer to the working fluid passing through the steam generator.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each control volume shown is at steady state. (2) The turbine stages and the pumps operate adiabatically, with $\eta_{t1} = \eta_{t2} = .86$ and $\eta_{p1} = \eta_{p2} = 0.8$. (3) Kinetic and potential energy effects are negligible. (4) The condenser operates at 0.08 bars, with the condensate leaving as saturated liquid.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 80 \text{ bar}$, $T_1 = 480^\circ\text{C} \Rightarrow h_1 = 3348.4 \text{ kJ/kg}$, $s_1 = 6.6586 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

State 2: First, $P_2 = 10 \text{ bar}$, $s_{2s} = s_1 \Rightarrow h_{2s} = 2811.5 \text{ kJ/kg}$. Using the turbine efficiency

$$\eta_{t1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 2886.7 \text{ kJ/kg}$$

Also, from Table A-4; $s_2 = 6.8133 \text{ kJ/kg}\cdot\text{K}$

State 3: Similarly, $P_3 = 0.08 \text{ bar}$, $s_{3s} = s_2 \Rightarrow h_{3s} = 2131.5 \text{ kJ/kg}$.

Thus $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 2237.2 \text{ kJ/kg}$

State 4: $P_4 = 0.08 \text{ bar}$, sat. liquid $\Rightarrow h_4 = 173.88 \text{ kJ/kg}$

$$\begin{aligned} \text{State 5: } h_{5s} &\approx h_4 + v_4(P_5 - P_4) \\ &= 173.88 \frac{\text{kJ}}{\text{kg}} + (1.0084 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (9.5 - 0.08) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \\ &= 173.88 + 0.95 = 174.83 \text{ kJ/kg} \end{aligned}$$

Using the pump efficiency, $h_5 = h_4 + (h_{5s} - h_4)/\eta_{p1} = 175.07 \text{ kJ/kg}$

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Problem 8-80 continued

State 6: $P_6 = 9.5 \text{ bar}$, $T_6 = 120^\circ\text{C} \Rightarrow$ estimate h_6 using

$$h_6 \approx h_f(T_6) + v_f(T_6) [P - P_{\text{sat}}(T_6)]$$

With data from Table A-2

$$h_6 \approx 503.71 + (1.0603 \times 10^{-3})(9.5 - 1.985) \left| \frac{10^5}{10^3} \right| = 504.5 \text{ kJ/kg}$$

State 7: To fix state 7, consider the mixing of streams 5 and 6 to form stream 7. From the given data, $\dot{m}_1 = 10^6 \text{ kg/h} = \dot{m}_7$ and $\dot{m}_2 = \dot{m}_6 = 4 \times 10^5 \text{ kg/h}$. Thus, from the mass balance

$$\dot{m}_5 = \dot{m}_7 - \dot{m}_6 = 6 \times 10^5 \text{ kg/h}$$

thus, applying an energy balance

$$0 = \dot{m}_5 h_5 + \dot{m}_6 h_6 - \dot{m}_7 h_7$$

or
$$h_7 = \frac{\dot{m}_5 h_5 + \dot{m}_6 h_6}{\dot{m}_7} = 306.84 \text{ kJ/kg}$$

State 8: $h_{8s} \approx h_7 + v_7 (P_8 - P_7)$. If $h_7 \approx 75^\circ\text{C}$, $v_7 \approx (1.0259 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}}$

Thus $h_{8s} \approx 306.84 + (1.0259 \times 10^{-3})(80 - 9.5) \left| \frac{10^5}{10^3} \right| = 314.1 \text{ kJ/kg}$

With the pump efficiency; $h_8 = h_7 + (h_{8s} - h_7)/\eta_{p2} = 315.88 \text{ kJ/kg}$

(a) The heating load is

$$\begin{aligned} \dot{Q}_{\text{load}} &= \dot{m}_2 (h_2 - h_6) = (4 \times 10^5 \frac{\text{kg}}{\text{h}})(2886.7 - 504.5) \frac{\text{kJ}}{\text{kg}} \\ &= 9.529 \times 10^8 \text{ kJ/h} \end{aligned} \quad \leftarrow \dot{Q}_{\text{load}}$$

(b) For the control volume enclosing the turbine

$$\begin{aligned} \dot{W}_t &= \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \\ &= (1 \times 10^6)(3348.4) - (4 \times 10^5)(2886.7) - (6 \times 10^5)(2237.2) \\ &= 8.514 \times 10^8 \text{ kJ/h} = 236,500 \text{ kW} \end{aligned} \quad \leftarrow \dot{W}_t$$

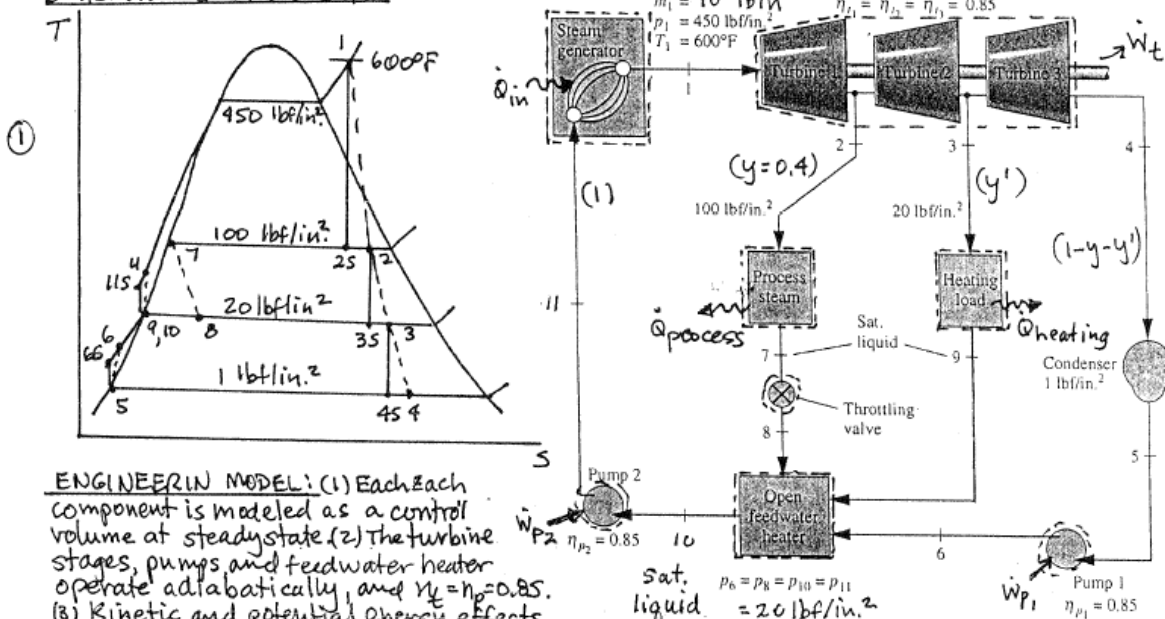
(c) For the working fluid passing through the steam generator

$$\begin{aligned} \dot{Q}_{\text{in}} &= \dot{m}_1 (h_1 - h_8) \\ &= (1 \times 10^6)(3348.4 - 315.88) = 3.032 \times 10^9 \text{ kJ/h} \end{aligned} \quad \leftarrow \dot{Q}_{\text{in}}$$

PROBLEM 8.81

KNOWN: A combined heat and power system provides turbine power, process steam, and steam for a space heating load. Data are given at key states.
FIND: Determine (a) the rates steam is extracted, (b) the rates of heat transfer for each load, and (c) the net power. Devise and evaluate an overall energy-based efficiency for the combined heat and power system.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is modeled as a control volume at steady state. (2) The turbine stages, pumps and feedwater heater operate adiabatically, and $\eta_t = \eta_p = 0.85$. (3) Kinetic and potential energy effects are negligible. (4) Saturated liquid exits the condenser.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 450 \text{ lbf/in}^2$, $T_1 = 600^\circ\text{F} \Rightarrow h_1 = 1302.5 \frac{\text{Btu}}{\text{lb}}$, $s_1 = 1.5732 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}$

State 2: $P_2 = 100 \text{ lbf/in}^2$, $s_{2s} = s_1 \Rightarrow x_{2s} = (s_{2s} - s_{f2})/s_{fg2} = 0.9733 \Rightarrow h_{2s} = 1164.1 \text{ Btu/lb}$

With the turbine-stage efficiency $h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 1184.9 \text{ Btu/lb}$

Using h_2 , $x_2 = (h_2 - h_{f2})/h_{fg2} = 0.9967 \Rightarrow s_2 = 1.5997 \text{ Btu/lb}\cdot^\circ\text{R}$

State 3: $P_3 = 20 \text{ lbf/in}^2$, $s_{3s} = s_2 \Rightarrow x_{3s} = 0.9052$, $h_{3s} = 1065.3 \text{ Btu/lb}$

Thus $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 1083.2 \text{ Btu/lb}$. Also, $x_3 = 0.9238$, $s_3 = 1.6256 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}$

State 4: $P_4 = 1 \text{ lbf/in}^2$, $s_{4s} = s_3 \Rightarrow x_{4s} = 0.8090$, $h_{4s} = 907.9 \text{ Btu/lb}$

Thus $h_4 = h_3 - \eta_{t3}(h_3 - h_{4s}) = 934.2 \text{ Btu/lb}$

State 5: $P_5 = 1 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_5 = 69.74 \text{ Btu/lb}$

State 6: $P_6 = 20 \text{ lbf/in}^2$; $h_{6s} \approx h_5 + v_5(P_6 - P_5)$

$$= 69.74 \frac{\text{Btu}}{\text{lb}} + (0.01614 \frac{\text{ft}^3}{\text{lb}})(20 - 1) \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1.8 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}} \right|$$

$$= 69.80 \text{ Btu/lb}$$

With the pump efficiency, $h_6 = h_5 + (h_{6s} - h_5)/\eta_p = 69.81 \text{ Btu/lb}$

State 7: $P_7 = 100 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_7 = 298.6 \text{ Btu/lb}$, **State 8:** $h_8 = h_7 = 298.6 \text{ Btu/lb}$

State 9, 10: $P_9 = P_{10} = 20 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_9 = h_{10} = 196.26 \text{ Btu/lb}$

State 11: $h_{11s} \approx h_{10} + v_{10}(P_{11} - P_{10}) = 196.26 + (0.01683)(450 - 20) \left| \frac{144}{778} \right| = 197.60 \text{ Btu/lb}$

$h_{11} = h_{10} + (h_{11s} - h_{10})/\eta_{p2} = 197.84 \text{ Btu/lb}$

Continued on next slide

Problem 8-81 continued

(a) For the steam extracted at state 2: $y = \dot{m}_2 / \dot{m}_1 \Rightarrow \dot{m}_2 = y \dot{m}_1 = (0.4)(10^4 \text{ lb/h})$
 $= 4 \times 10^3 \text{ lb/h} \leftarrow \dot{m}_2$

To find $\dot{m}_3$: $\dot{m}_3 = y' \dot{m}_1$

For the open feedwater heater: $0 = y h_8 + y' h_9 + (1 - y - y') h_6 - h_{10}$

Solving for y' :

$$y' = \frac{y(h_6 - h_8) + h_{10} - h_6}{(h_9 - h_6)} = \frac{(0.4)(69.81 - 298.6) + 196.26 - 69.81}{(196.26 - 69.81)} = 0.2763$$

Thus

$$\dot{m}_3 = (0.2763)(10^4 \text{ lb/h}) = 2.763 \times 10^3 \text{ lb/h} \leftarrow \dot{m}_3$$

(b) $\dot{Q}_{\text{process}} = \dot{m}_2 (h_2 - h_7) = (4 \times 10^3 \text{ lb/h})(1184.9 - 298.6) \frac{\text{Btu}}{\text{lb}} = 3.545 \times 10^6 \text{ Btu/h} \leftarrow \dot{Q}_{\text{process}}$

$$\dot{Q}_{\text{heating}} = \dot{m}_3 (h_3 - h_9) = (2.763 \times 10^3)(1083.2 - 196.26) = 2.451 \times 10^6 \text{ Btu/h} \leftarrow \dot{Q}_{\text{heating}}$$

(c) For the control volume enclosing the turbine stages

$$\begin{aligned} \dot{W}_t &= \dot{m}_1 [h_1 - y h_2 - y' h_3 - (1 - y - y') h_4] \\ &= (10^4 \frac{\text{lb}}{\text{h}}) [1302.5 - (0.4)(1184.9) - (0.2763)(1083.2) - (0.3237)(934.2)] \frac{\text{Btu}}{\text{lb}} \\ &= 2.268 \times 10^6 \text{ Btu/h} \end{aligned}$$

For the pumps

$$\begin{aligned} \dot{W}_p &= \dot{W}_{P_1} + \dot{W}_{P_2} = \dot{m}_1 [(1 - y - y')(h_6 - h_5) + (h_{11} - h_{10})] \\ &= (10^4 \frac{\text{lb}}{\text{h}}) [(0.3237)(69.80 - 69.74) + (197.84 - 196.26)] \frac{\text{Btu}}{\text{lb}} = 1.6 \times 10^4 \text{ Btu/h} \end{aligned}$$

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = 2.252 \times 10^6 \text{ Btu/h} \leftarrow \dot{W}_{\text{cycle}}$$

Energy-based Efficiency: The energy transfers $\dot{Q}_{\text{process}}$, $\dot{Q}_{\text{heating}}$, and $\dot{W}_{\text{cycle}}$ all have economic value. Thus, a rational energy-based efficiency is

$$\begin{aligned} \eta_{\text{overall}} &= \frac{(\dot{Q}_{\text{process}} + \dot{Q}_{\text{heating}} + \dot{W}_{\text{cycle}})}{\dot{Q}_{\text{in}}} = \frac{(\dot{Q}_{\text{process}} + \dot{Q}_{\text{heating}} + \dot{W}_{\text{cycle}})}{\dot{m}_1 (h_1 - h_{11})} \\ &= \frac{3.545 \times 10^6 + 2.451 \times 10^6 + 2.252 \times 10^6}{(10^4)(1302.5 - 197.84)} \\ &= \frac{8.248 \times 10^6}{1.1047 \times 10^7} = 0.747 \text{ (74.7\%)} \leftarrow \eta_{\text{overall}} \end{aligned}$$

1. In early printing of the sixth edition, Fig 8.81 was incorrect. The corrected version is used in this solution. We apologize for the error.

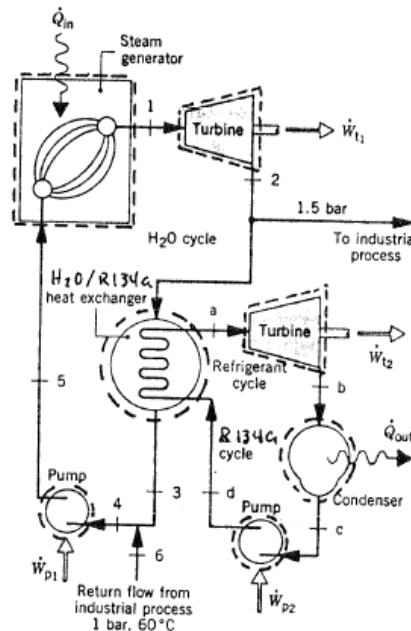
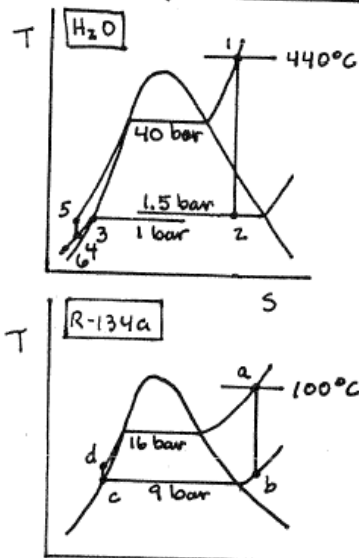
2. An exergetic efficiency would provide a more accurate picture of the economic value provided with the combined heating and power cycle.

PROBLEM 8.82

KNOWN: Water and Refrigerant 134a are the working fluids in a binary cycle used for cogeneration of power and process steam. Data are known at various locations.

FIND: Determine (a) the rate of heat transfer to the steam passing through the steam generator, (b) the net power output of the binary cycle, and (c) the rate of heat transfer to the industrial process.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All turbines and pumps operate isentropically. (3) The control volume around the inter-connecting heat exchanger is adiabatic. (4) Kinetic and potential energy effects are negligible. (5) Saturated liquid exits each condenser at the respective pressure.

ANALYSIS: First, fix each of the principal states. For the H_2O cycle:

State 1: $P_1 = 40 \text{ bar}$, $T_1 = 440^\circ\text{C} \Rightarrow h_1 = 3307.1 \text{ kJ/kg}$, $s_1 = 6.9041 \text{ kJ/kg}\cdot\text{K}$

State 2: $P_2 = 1.5 \text{ bar}$, $s_2 = s_1 \Rightarrow h_2 = 2570.8 \text{ kJ/kg}$

State 3: $P_3 = 1 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 417.46 \text{ kJ/kg}$

State 6: $P_6 = 1 \text{ bar}$, $T_6 = 60^\circ\text{C} \Rightarrow$ compressed liquid;

$$\begin{aligned} h_6 &\approx h_f(60) + v_f(60) (P - P_{\text{sat}} @ 60^\circ\text{C}) \\ &= 251.13 \frac{\text{kJ}}{\text{kg}} + (1.0172 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (1 - 1.994) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \\ &= 251.2 \text{ kJ/kg} \end{aligned}$$

State 4: For the mixing of streams 3 and 6 to form stream 4
 $0 = \dot{m}_3 h_3 + \dot{m}_6 h_6 - (\dot{m}_3 + \dot{m}_6) h_4$

or, with $\dot{m}_3 = 2.5 \text{ kg/s}$, $\dot{m}_6 = 2.5 \text{ kg/s}$

Continued on next slide

Problem 8-82 continued

$$h_4 = (0.5)(417.46) + (0.5)(251.2) = 334.33 \text{ kJ/kg}$$

$$\begin{aligned} \text{State 5: } T_4 &\approx 80^\circ\text{C}, h_5 \approx h_4 + v_4(P_5 - P_4) \\ &= 334.33 + (1.0291 \times 10^{-3})(40 - 1) \left| \frac{10^5}{10^3} \right| \\ &= 338.34 \text{ kJ/kg} \end{aligned}$$

For the R-134a cycle;

$$\text{State a: } P_a = 16 \text{ bar}, T_a = 100^\circ\text{C} \Rightarrow h_a = 327.46 \text{ kJ/kg}, s_a = 1.0467 \text{ kJ/kg}\cdot\text{K}$$

$$\text{State b: } P_b = 9 \text{ bar}, s_b = s_a \Rightarrow h_b = 312.74 \text{ kJ/kg}$$

$$\text{State c: } P_c = 9 \text{ bar}, \text{ sat. liquid} \Rightarrow h_c = 99.56 \text{ kJ/kg}$$

$$\text{State d: } h_d \approx h_c + v_c(P_d - P_c) = 99.56 + (0.8576 \times 10^{-3})(16 - 9) \left| \frac{10^5}{10^3} \right| = 100.16 \frac{\text{kJ}}{\text{kg}}$$

a) From mass and energy balances for the control volume enclosing steam passing through the steam generator

$$\begin{aligned} \dot{Q}_{in} &= \dot{m}_1(h_1 - h_5) \\ &= (5 \text{ kg/s})(3307.1 - 338.34) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 14,844 \text{ kW} \end{aligned}$$

b) For the H₂O cycle

$$\begin{aligned} \dot{W}_{H_2O} &= \dot{W}_{t1} - \dot{W}_{p1} = (5 \text{ kg/s}) [(3307.1 - 2570.8) - (338.34 - 334.33)] \frac{\text{kJ}}{\text{kg}} \\ &= 3661 \text{ kW} \end{aligned}$$

To determine the mass flow rate of R-134a, consider the inter-connecting heat exchanger

$$0 = \dot{m}_3(h_2 - h_3) + \dot{m}_R(h_d - h_a)$$

$$\begin{aligned} \text{or } \dot{m}_R &= \frac{\dot{m}_3(h_2 - h_3)}{(h_a - h_d)} = \frac{(2.5 \text{ kg/s})(2570.8 - 417.46)}{(327.46 - 100.16)} \\ &= 23.68 \text{ kg/s} \end{aligned}$$

$$\begin{aligned} \text{Thus } \dot{W}_R &= \dot{m}_R [(h_a - h_b) - (h_d - h_c)] \\ &= (23.68) [(327.46 - 312.74) - (100.16 - 99.56)] = 334.4 \text{ kW} \end{aligned}$$

The total power developed is

$$\dot{W}_{cycle} = 3661 + 334.4 = 3995 \text{ kW}$$

(c) For the industrial process

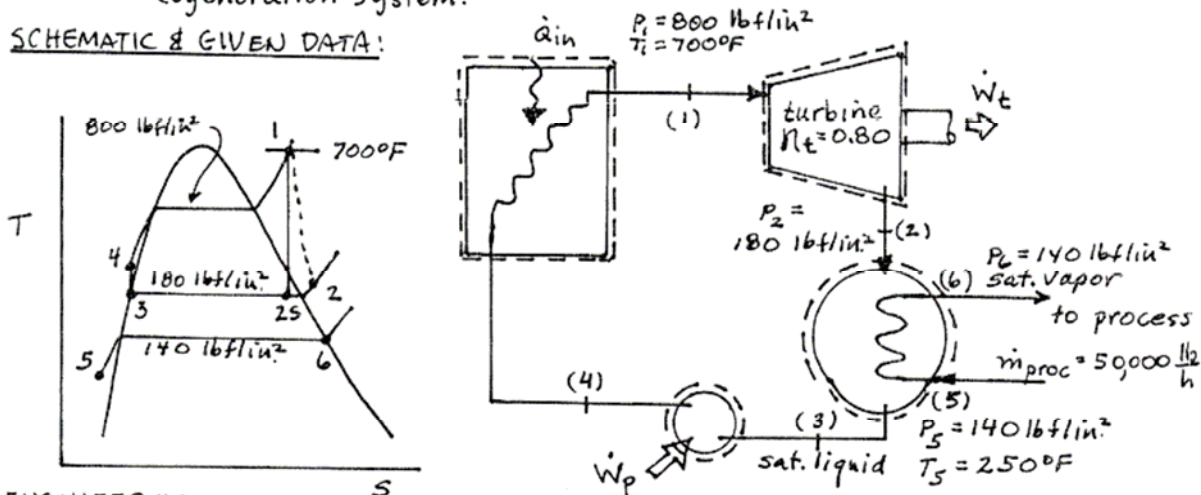
$$\begin{aligned} \dot{Q}_{process} &= \dot{m}_{process}(h_2 - h_6) \\ &= (2.5 \text{ kg/s})(2570.8 - 251.2) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 5799 \text{ kW} \end{aligned}$$

PROBLEM 8.83

KNOWN: Water is the working fluid in a Rankine cycle used for cogeneration. The cycle produces power, and energy rejected from the condensing steam is transferred to a separate process stream. Data are known at various locations.

FIND: Determine the mass flow rate for the Rankine cycle working fluid, and devise and evaluate an exergetic efficiency for the overall cogeneration system.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The turbine, pump, and condenser operate adiabatically. (3) For the turbine, $\eta_t = 0.80$, and the pump is assumed to operate isentropically. (4) Kinetic and potential energy effects are negligible. (5) Saturated liquid exits the condenser at the condenser pressure. (6) $T_6 = 530^\circ\text{R}$, $P_6 = 14.7 \text{ lbf/in}^2$.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 800 \text{ lbf/in}^2$, $T_1 = 700^\circ\text{F} \Rightarrow h_1 = 1338.0 \text{ Btu/lb}$, $s_1 = 1.5471 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2: First, $P_2 = 180 \text{ lbf/in}^2$, $s_{2s} = s_1 \Rightarrow h_{2s} = 1191.1 \text{ Btu/lb}$. With the turbine efficiency $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1220.5 \text{ Btu/lb}$

and, from Table A-4E; $s_2 \approx 1.5818 \text{ Btu/lb}\cdot^\circ\text{R}$

State 3: $P_3 = 180 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_3 = 346.3 \text{ Btu/lb}$, $s_3 = 0.5329 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}$

State 4: $h_4 \approx h_3 + v_4(P_4 - P_3)$

$$= 346.3 \frac{\text{Btu}}{\text{lb}} + (0.01827 \frac{\text{ft}^3}{\text{lb}})(800 - 180) \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}} \right|$$

$$= 346.3 + 2.10 = 348.4 \text{ Btu/lb}, s_4 = s_3 = 0.5329 \text{ Btu/lb}\cdot^\circ\text{R}$$

State 5: $P_5 = 140 \text{ lbf/in}^2$, $T_5 = 250^\circ\text{F} \Rightarrow$ compressed liquid. Thus

$$h_5 \approx h_f(250) + v_f(250)[P - P_{\text{sat}}@250^\circ\text{F}]$$

$$= 218.6 + (0.01700)(140 - 29.82) \left| \frac{144}{778} \right| = 218.9 \text{ Btu/lb}$$

and $s_5 \approx s_f(250) = 0.3677 \text{ Btu/lb}\cdot^\circ\text{R}$

Continued on next slide

Problem 8-83 continued

State 6: $P_6 = 140 \text{ lbf/in}^2$, sat. vapor $\Rightarrow h_6 = 1193.8 \text{ Btu/lb}$, $s_6 = 1.5761 \text{ Btu/lb} \cdot ^\circ\text{R}$

To find the cycle mass flow rate, use an energy balance for the condenser

$$0 = \dot{m}_{\text{cycle}}(h_2 - h_3) + \dot{m}_{\text{proc}}(h_5 - h_6)$$

or

$$\dot{m}_{\text{cycle}} = \dot{m}_{\text{proc}} \left(\frac{h_5 - h_6}{h_3 - h_2} \right) \\ = (50,000 \text{ lb/h}) \left(\frac{218.9 - 1193.8}{346.3 - 1220.5} \right) = 5.577 \times 10^4 \text{ lb/h} \leftarrow \dot{m}_{\text{cycle}}$$

For the cogeneration system, the cycle net work and the exergy transferred to the process stream are the outputs, and the exergy increase of the working fluid passing through the steam generator is the input. The difference between the input and output represents exergy destroyed due to irreversibilities. Thus, a reasonable exergetic efficiency is

$$\epsilon = \frac{\text{net rate of exergy output}}{\text{rate of exergy input}} \\ = \frac{\dot{W}_{\text{cycle}} + \dot{m}_{\text{proc}}(e_{f6} - e_{f5})}{\dot{m}_{\text{cycle}}(e_{f1} - e_{f4})} \leftarrow \epsilon (\text{definition})$$

Evaluating the various quantities in this expression

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = \dot{m}_{\text{cycle}}[(h_1 - h_2) - (h_4 - h_3)] \\ = (5.577 \times 10^4)[(1338.0 - 1220.5) - (2.10)] = 6.436 \times 10^6 \text{ Btu/h}$$

$$e_{f6} - e_{f5} = (h_6 - h_5) - T_0(s_6 - s_5) \\ = (1193.8 - 218.9) - (530)(1.5761 - 0.3677) = 334.4 \text{ Btu/lb}$$

$$e_{f1} - e_{f4} = (h_1 - h_4) - T_0(s_1 - s_4) \\ = (1338.0 - 348.4) - (530)(1.5471 - 0.5329) = 452.1 \text{ Btu/lb}$$

Thus

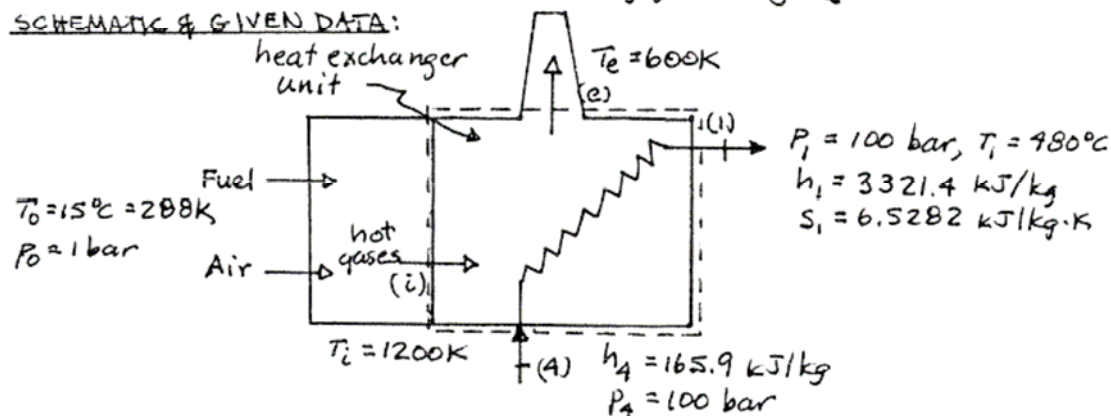
$$\epsilon = \frac{6.436 \times 10^6 \text{ Btu/h} + (5.577 \times 10^4 \text{ lb/h})(334.4 \text{ Btu/lb})}{(5.577 \times 10^4 \text{ lb/h})(452.1 \text{ Btu/lb})} \\ = \frac{6.436 \times 10^6 + 1.865 \times 10^6}{2.521 \times 10^7} = 0.329 (32.9\%) \leftarrow \epsilon (\text{value})$$

PROBLEM 8.84

KNOWN: Hot combustion gases enter the heat exchanger unit of the steam generator of Problem 8.17. The inlet and exit temperatures of the gas stream are provided. Other data are from the solution of Problem 8.17.

FIND: Determine (a) the net rate exergy is carried in by the gas stream, (b) the net rate exergy is carried out by the water stream, (c) the rate of exergy destruction, (d) the exergetic efficiency given by Eq. 7.27.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) For the steady-state control volume shown, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The combustion gases are modeled as air as an ideal gas. (4) Each stream experiences negligible pressure drop. (5) $T_0 = 288\text{ K}$, $P_0 = 1\text{ bar}$.

ANALYSIS: From steady-state mass balances on the water and air streams, respectively; $\dot{m}_4 = \dot{m}_1 \equiv \dot{m}$ and $\dot{m}_i = \dot{m}_e \equiv \dot{m}_{air}$. Thus, the energy rate balance is

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_4 - h_1) + \dot{m}_{air}(h_i - h_e)$$

$$\text{or} \quad \frac{\dot{m}_{air}}{\dot{m}} = \frac{h_1 - h_4}{h_i - h_e}$$

From Table A-22, $h_i = 1277.79\text{ kJ/kg}$ and $h_e = 607.02\text{ kJ/kg}$. Thus

$$\frac{\dot{m}_{air}}{\dot{m}} = \frac{3321.4 - 165.9}{1277.79 - 607.02} = 4.704$$

(a) The net rate exergy is carried in by the gas stream is

$$\frac{\dot{E}_{fi} - \dot{E}_{fe}}{\dot{m}} = \frac{\dot{m}_{air}}{\dot{m}} (e_{fi} - e_{fe}) = \frac{\dot{m}_{air}}{\dot{m}} [(h_i - h_e) - T_0(s_i - s_e)]$$

Since $P_i = P_e$; $s_i - s_e = s_i^\circ - s_e^\circ - R \ln(P_i/P_e) = s_i^\circ - s_e^\circ$. With data from Table A-22

$$\begin{aligned} \frac{\dot{E}_{fi} - \dot{E}_{fe}}{\dot{m}} &= (4.704) [(1277.79 - 607.02) - (288)(3.17888 - 2.40902)] \frac{\text{kJ}}{\text{kg}} \\ &= 2112 \text{ kJ/kg of steam flow} \end{aligned}$$

Net rate
exergy
carried
in

(b) The net rate exergy is carried out by the water stream is

$$\frac{\dot{E}_{f1} - \dot{E}_{f4}}{\dot{m}} = (e_{f1} - e_{f4}) = (h_1 - h_4) - T_0(s_1 - s_4)$$

Continued on next slide

Problem 8-84 continued

Assuming $s_4 \approx s_f(T_4)$ and interpolating in Table A-2 with h_4 ; $s_4 \approx 0.5672$

Thus $e_{f1} - e_{f4} = (3321.4 - 165.9) - (288)(6.5282 - 0.5672) = 1438.7 \frac{\text{kJ}}{\text{kg}}$ ← net rate of exergy carried out

(c) Applying the exergy rate balance to the heat exchanger unit

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W}_{cv} + \dot{m}(e_{f4} - e_{f1}) + \dot{m}_{air}(e_{f1} - e_{fe}) - \dot{E}_d$$

or
 ① $\frac{\dot{E}_d}{\dot{m}} = \frac{\dot{m}(e_{f4} - e_{f1})}{\dot{m}} + \frac{\dot{m}_{air}(e_{f1} - e_{fe})}{\dot{m}}$
 $= (-1438.7) + (2112) = 673.3 \text{ kJ/kg}$ ← $\dot{E}_d/\dot{m}$

(d) Applying Eq. 7.27

$$\epsilon = \frac{\dot{m}(e_{f1} - e_{f4})}{\dot{m}_{air}(e_{f1} - e_{fe})} = \frac{1438.7}{2112} = 0.681 \text{ (68.1\%)} \leftarrow \epsilon$$

1. Exergy destruction in this case is due to heat transfer from the higher-temperature combustion gas stream to the vaporizing water.

PROBLEM 8.85

KNOWN: The vapor power cycle of Problem 8.17 is to be analyzed from the perspective of exergy accounting.

FIND: Determine the rate of input to the working fluid passing through the steam generator and account for all outputs, losses, and destructions of this exergy.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.17.

ENGINEERING MODEL: Same as Problem 8.17. Also, $T_0 = 15^\circ\text{C} = 288\text{K}$, $p_0 = 1\text{ bar}$.

ANALYSIS: With data from the solution to Problem 8.17

State 1: $h_1 = 3321.4\text{ kJ/kg}$, $s_1 = 6.5282\text{ kJ/kg}\cdot\text{K}$

State 2: $h_2 = 2272.1\text{ kJ/kg}$, $s_2 = 7.8334\text{ kJ/kg}\cdot\text{K}$ ($x_2 = 0.8778$)

State 3: $h_3 = 151.53\text{ kJ/kg}$, $s_3 = 0.5210\text{ kJ/kg}\cdot\text{K}$

State 4: $h_4 = 165.9\text{ kJ/kg}$, $s_4 = 0.5672\text{ kJ/kg}\cdot\text{K}$ (Assume $s_4 \approx s_f(T_4)$)

Input $\frac{\dot{E}_{f1} - \dot{E}_{f4}}{\dot{m}} = (e_{f1} - e_{f4}) = (h_1 - h_4) - T_0(s_1 - s_4)$

$$= (3321.4 - 165.9) - (288)(6.5282 - 0.5672)$$

$$= 1438.7\text{ kJ/kg} \quad \leftarrow \text{Input}$$

Outputs

• Net Power: $\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = 1034.9\text{ kJ/kg}$

• Condenser: $\frac{\dot{E}_{f2} - \dot{E}_{f3}}{\dot{m}} = e_{f2} - e_{f3} = (h_2 - h_3) - T_0(s_2 - s_3)$

$$= (2272.1 - 151.53) - (288)(7.8334 - 0.5210)$$

$$= 14.6\text{ kJ/kg}$$

Destructions:

• Turbine: $\frac{\dot{E}_{d,\text{turb}}}{\dot{m}} = T_0(s_2 - s_1) = (288)(7.8334 - 6.5282)$

$$= 375.9\text{ kJ/kg}$$

• Pump: $\frac{\dot{E}_{d,\text{pump}}}{\dot{m}} = T_0(s_4 - s_3) = (288)(0.5672 - 0.5210)$

$$= 13.31\text{ kJ/kg}$$

Exergy Accounting Summary

Input 1438.7 kJ/kg

Outputs 1034.9

14.6

1049.5 kJ/kg

Destructions: 375.9

13.3

389.2 kJ/kg

Total: 1438.7 kJ/kg

PROBLEM 8.86

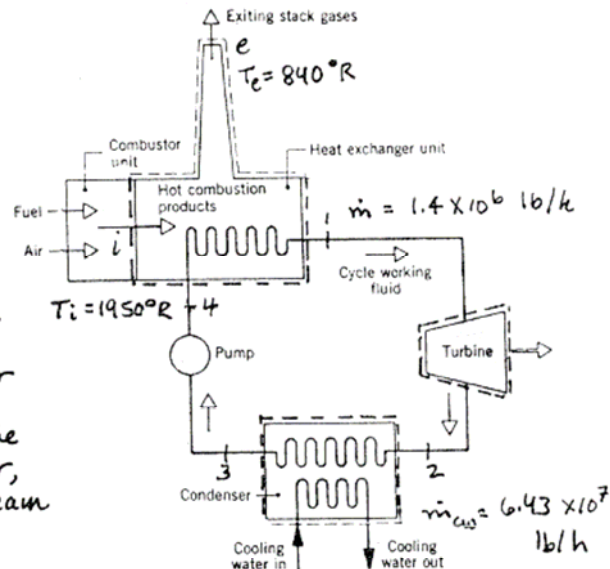
KNOWN: Hot combustion gases enter the heat exchanger unit of the steam generator of Problem 8.19. The inlet and exit temperatures of the gas stream are known. Other data are from Problem 8.19.

FIND: Determine the exergy destruction rates in (a) the heat exchanger unit, (b) the turbine and pump, and (c) the condenser. Also, calculate the net rate exergy is carried away by the cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:

$$T_0 = 520^\circ\text{R}$$

$$P_0 = 14.7 \text{ lbf/in}^2$$



ENGINEERING

MODEL: (1) Same as Problem 8.19. (2) $T_0 = 520^\circ\text{R}$, $P_0 = 14.7 \text{ lbf/in}^2$. (3) The combustion gases are modeled as air as an ideal gas. (4) The gas stream, the working fluid passing through the heat exchanger unit and the condenser, and the condenser cooling water stream each experience no pressure drop.

ANALYSIS: (a) For the control volume surrounding the heat exchanger unit, steady-state mass balances for the water and air stream reduce, respectively, to $\dot{m}_4 = \dot{m}_1 \equiv \dot{m}$ and $\dot{m}_i = \dot{m}_e \equiv \dot{m}_a$. Thus, the energy rate balance is

$$0 = \dot{Q}_{cw} - \dot{Q}_{cv} + \dot{m}(h_4 - h_1) + \dot{m}_a(h_i - h_e)$$

or

$$\dot{m}_a = \frac{\dot{m}(h_1 - h_4)}{(h_i - h_e)}$$

From Table A-22E, $h_i = 490.88 \text{ Btu/lb}$ and $h_e = 201.56 \text{ Btu/lb}$. Thus

$$\dot{m}_a = \frac{(1.4 \times 10^6 \text{ lb/h})(1547.7 \text{ Btu/lb} - 75.17 \text{ Btu/lb})}{(490.88 - 201.56) \text{ Btu/lb}} = 7.13 \times 10^6 \text{ lb/h}$$

The rate of exergy destruction can be found from $\dot{E}_d = T_0 \dot{\sigma}_{cv}$. The entropy rate balance for the control volume is

$$0 = \sum_j \left(\frac{\dot{Q}_j}{T_j} \right) + \dot{m}(s_4 - s_1) + \dot{m}_a(s_i - s_e) + \dot{\sigma}_{cv}$$

Thus

$$\dot{E}_d = T_0 [\dot{m}(s_1 - s_4) + \dot{m}_a(s_e - s_i)]$$

Since $P_i = P_e$, $s_e - s_i = s_e^\circ - s_i^\circ - R \ln(P_e/P_i) = s_e^\circ - s_i^\circ$. With values from Table A-22E, $s_e^\circ = 0.70747$ and $s_i^\circ = 0.92505$. Further, interpolating in

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Problem 8-86 continued

Table A-5E, $s_4 \approx 0.13364 \text{ Btu/lb} \cdot ^\circ\text{R}$, and with $s_1 = 1.6315 \text{ Btu/lb} \cdot ^\circ\text{R}$ from Problem 8.19,

$$\begin{aligned}\dot{E}_d &= (520^\circ\text{R}) \left[(1.4 \times 10^6 \frac{\text{lb}}{\text{h}}) (1.6315 - 0.13364) \text{ Btu/lb} \right. \\ &\quad \left. + (7.13 \times 10^6 \text{ lb/h}) (0.70747 - 0.92505) \right] \\ &= 2.84 \times 10^8 \text{ Btu/h} \quad \xrightarrow{\text{heat exchange unit}} \dot{E}_d\end{aligned}$$

(b) For the turbine, with $s_1 = 1.6315 \text{ Btu/lb} \cdot ^\circ\text{R}$ and $s_2 = 1.7676 \text{ Btu/lb} \cdot ^\circ\text{R}$ based on data from Problem 8.19

$$\begin{aligned}\dot{E}_d &= T_0 \dot{m} (s_2 - s_1) \\ &= (520)(1.4 \times 10^6)(1.7676 - 1.6315) = 9.91 \times 10^7 \text{ Btu/h} \quad \xrightarrow{\text{turbine}} \dot{E}_d\end{aligned}$$

Further, with $s_3 = 0.1327 \text{ Btu/lb} \cdot ^\circ\text{R}$ (sat. liq. at 1 lbf/in^2), the rate of exergy destruction for the pump is

$$\begin{aligned}\dot{E}_d &= T_0 \dot{m} (s_4 - s_3) \\ &= (520)(1.4 \times 10^6)(0.13364 - 0.1327) = 6.84 \times 10^5 \text{ Btu/h} \quad \xrightarrow{\text{pump}} \dot{E}_d\end{aligned}$$

(c) Similarly, for the condenser

$$\begin{aligned}\dot{E}_d &= T_0 [\dot{m} (s_3 - s_2) + \dot{m}_{cw} (s_{cw, \text{out}} - s_{cw, \text{in}})] \\ &= (520) [(1.4 \times 10^6)(0.1327 - 1.7676) + (6.43 \times 10^7)(.09332 - .05555)] \\ &= 7.27 \times 10^7 \text{ Btu/h} \quad \xrightarrow{\text{condenser}} \dot{E}_d\end{aligned}$$

where $s_{cw} \approx s_f(T)$.

Finally, the net rate exergy is carried out by the cooling water passing through the condenser is

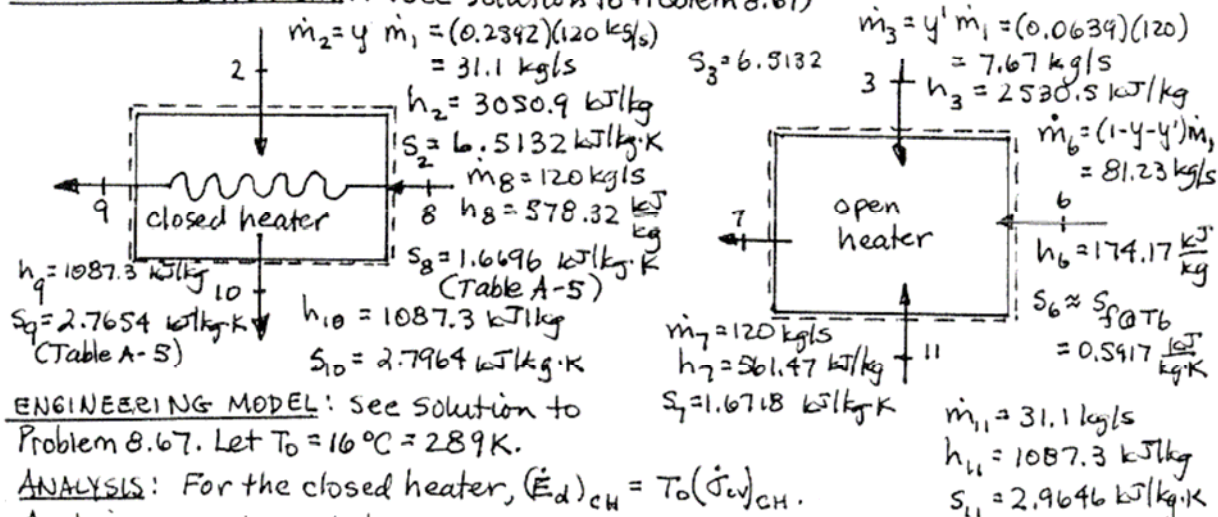
$$\begin{aligned}\dot{m}_{cw} (e_{f, cw, \text{out}} - e_{f, cw, \text{in}}) &= \dot{m}_{cw} [(h_{cw, \text{out}} - h_{cw, \text{in}}) - T_0 (s_{cw, \text{out}} - s_{cw, \text{in}})] \\ &= (6.43 \times 10^7) [(48.09 - 28.08) - (520)(.09332 - .05555)] \\ &= 2.38 \times 10^7 \text{ Btu/h} \quad \xrightarrow{\text{net rate exergy is carried out by cooling water}}\end{aligned}$$

PROBLEM 8.87

KNOWN: Feedwater heaters operate as in the regenerative vapor powercycle of Problem 8.67.

FIND: Determine the exergy destruction rates for each feedwater heater and express each as a percentage of the flow exergy increase of the working fluid passing through the steam generator.

SCHEMATIC & GIVEN DATA: (See solution to Problem 8.67)



ENGINEERING MODEL: See solution to Problem 8.67. Let $T_0 = 16^\circ\text{C} = 289 \text{ K}$.

ANALYSIS: For the closed heater, $(\dot{E}_d)_{CH} = T_0(\dot{S}_{cv})_{CH}$.
Applying an entropy balance

$$0 = \sum \left(\frac{\dot{Q}_j}{T_j} \right)_j + \dot{m}_2(s_2 - s_{10}) + \dot{m}_8(s_8 - s_9) + (\dot{S}_{cv})_{CH}$$

Thus

$$\begin{aligned}
 (\dot{E}_d)_{CH} &= T_0 [\dot{m}_2(s_{10} - s_2) + \dot{m}_8(s_9 - s_8)] \\
 &= (289 \text{ K}) [(31.1)(2.7964 - 6.5132) + (120)(2.7654 - 1.6696)] \frac{\text{kJ}}{\text{s}\cdot\text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\
 &= 4596 \text{ kW} \quad (\dot{E}_d)_{CH}
 \end{aligned}$$

Similarly for the open feedwater heater

$$0 = \sum \left(\frac{\dot{Q}_j}{T_j} \right)_j + \dot{m}_3 s_3 + \dot{m}_6 s_6 + \dot{m}_{11} s_{11} - \dot{m}_7 s_7 + (\dot{S}_{cv})_{OH}$$

And

$$\begin{aligned}
 (\dot{E}_d)_{OH} &= T_0 [\dot{m}_7 s_7 - \dot{m}_3 s_3 - \dot{m}_6 s_6 - \dot{m}_{11} s_{11}] \\
 &= (289) [(120)(1.6718) - (7.67)(6.5132) - (81.23)(0.5917) - (31.1)(2.9646)] \\
 &= 3005 \text{ kW} \quad (\dot{E}_d)_{OH}
 \end{aligned}$$

For the working fluid passing through the steam generator

$$\begin{aligned}
 \dot{E}_{f1} - \dot{E}_{f9} &= \dot{m}_1(e_{f1} - e_{f9}) = \dot{m}_1[(h_1 - h_9) - T_0(s_1 - s_9)] \\
 &= (120) [(3465.4 - 1087.3) - (289)(6.5132 - 2.7654)] = 1.554 \times 10^5 \text{ kW}
 \end{aligned}$$

Thus

$$\frac{(\dot{E}_d)_{CH}}{\dot{E}_{f1} - \dot{E}_{f9}} = \frac{4596}{1.554 \times 10^5} = 0.0296 (2.96\%) \quad \text{closed heater}$$

$$\frac{(\dot{E}_d)_{OH}}{\dot{E}_{f1} - \dot{E}_{f2}} = \frac{3005}{1.554 \times 10^5} = 0.0193 (1.93\%) \quad \text{open heater}$$

PROBLEM 8.88

KNOWN: The regenerative vapor power cycle of Problem 8.74 is analyzed on the basis of exergy accounting.

FIND: Determine the rate of exergy input to the working fluid passing through the steam generator and account for the disposition of this exergy.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.74.

ENGINEERING MODEL: Same as in Problem 8.74. Also, let $T_0 = 15^\circ\text{C} = 288\text{K}$ and $p_0 = 1\text{ bar}$.

ANALYSIS: Each principal state is fixed in the solution to problem 8.74. The specific enthalpies and entropies are:

State	$h(\text{kJ/kg})$	$s(\text{kJ/kg}\cdot\text{K})$
1	3424.6	6.1858
2	3082.6	6.2781
3	3545.3	6.9072
4	3025.7	7.0771
5	2645.6	7.2284
6	2296.9	7.4559
7	151.53	.5210
8	151.67	.5210
9	467.11	1.4336
10	500.64	1.4336
11	781.7	2.1396
12	762.81	2.1387
13	762.81	2.2025
14	1303.8	3.2064
15	1316.6	3.2068
16	1316.6	3.3609

($s_f @ 180^\circ\text{C}$)

($s_f @ 295^\circ\text{C}$)

From the solution to Problem 8.74 ; $\dot{m}_1 = 1.588 \times 10^6 \text{ kg/h} = 411.1 \text{ kg/s}$, $y' = 0.2956$, $y'' = 0.0519$, and $y''' = 0.0405$.

INPUTS

Boiler/superheater:

$$\begin{aligned} \dot{m}_1(e_{s1} - e_{s14}) &= \dot{m}_1 [(h_1 - h_{14}) - T_0(s_1 - s_{14})] \\ &= (411.1 \frac{\text{kg}}{\text{s}}) [(3424.6 - 1303.8) - 288(6.1858 - 3.2064)] \frac{\text{kJ}}{\text{kg}} \\ &= 5.191 \times 10^5 \frac{\text{kJ}}{\text{s}} = 519.1 \text{ MW} \end{aligned}$$

Reheater:

$$\begin{aligned} \dot{m}_1(1 - y') (e_{f3} - e_{f2}) &= \dot{m}_1(1 - y') [(h_3 - h_2) - T_0(s_3 - s_2)] \\ &= 81.5 \text{ MW} \end{aligned}$$

Total : 600.6 MW $\leftarrow$ Input

OUTPUT

$$\begin{aligned} \text{net output of cycle} &= \dot{m}_1 [\dot{w}_t / \dot{m}_1 - \dot{w}_p / \dot{m}_1] \\ &= (411.1) [1167.4 - 33.62] \frac{\text{kJ}}{\text{s}} \left(\frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right) \\ &= 466.0 \text{ MW} \rightarrow \text{Output} \end{aligned}$$

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Problem 8-88 continued

LOSS

$$\begin{aligned}\text{Condenser loss} &= \dot{m}_1 (1 - y' - y'' - y''') (e_{f6} - e_{f7}) \\ &= \dot{m}_1 (1 - y' - y'' - y''') [(h_6 - h_7) - T_0 (s_6 - s_7)] \\ &= 37.3 \text{ MW} \leftarrow \text{loss}\end{aligned}$$

DESTRUCTIONS

turbine:

$$\begin{aligned}\dot{E}_d &= \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) \\ &= \dot{m}_1 T_0 [(s_2 - s_1) + (1 - y')(s_4 - s_3) + (1 - y' - y'')(s_5 - s_4) + (1 - y' - y'' - y''')(s_6 - s_5)] \\ &= 53.27 \text{ MW}\end{aligned}$$

80-bar heater:

$$\begin{aligned}\dot{E}_d &= \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 [y'(s_{15} - s_2) + (s_{14} - s_{11})] \\ &= 18.82 \text{ MW}\end{aligned}$$

10-bar heater:

$$\begin{aligned}\dot{E}_d &= \dot{m}_1 T_0 [s_{11} - s_{10} + (y' + y'')s_{12} - y's_{16} - y''s_4] \\ &= 10.47 \text{ MW}\end{aligned}$$

Open heater:

$$\begin{aligned}\dot{E}_d &= \dot{m}_1 T_0 [s_9 - y'''s_5 - (y' + y'')s_{13} - (1 - y' - y'' - y''')s_8] \\ &= 6.7 \text{ MW}\end{aligned}$$

Traps:

$$\dot{E}_d = \dot{m}_1 T_0 [(s_{16} - s_{15})y'] = 5.39 \text{ MW}$$

$$\dot{E}_d = \dot{m}_1 T_0 [(y' + y'')(s_{13} - s_{12})] = 2.62 \text{ MW}$$

$$\text{Total: } 97.27 \text{ MW} \leftarrow \text{destroyed}$$

SUMMARY

Input: 600.6 MW

Output: 466.0

Loss: 37.3

Destroyed: $\frac{97.27}{600.57} \text{ MW}$

PROBLEM 8.89

KNOWN: The regenerative power cycle of Problem 8.75 is analyzed on the basis of exergy accounting.

FIND: Determine the rate of exergy input to the working fluid passing through the steam generator and account for its disposition.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.75

ENGINEERING MODEL: Same as Problem 8.75. Also, let $T_0 = 520^\circ\text{R}$, $p_0 = 1 \text{ lbf/in}^2$.

ANALYSIS: Each principal state is fixed in the solution to Problem 8.75. The specific enthalpies and entropies are:

State	h (Btu/lb)	s (Btu/lb $\cdot^\circ\text{R}$)
1	1542.5	1.6159
2	1404.2	1.6315
3	1272.0	1.6507
4	1232.1	1.6569
5	1532.1	1.9204
6	1341.5	1.9455
7	1118.0	1.9988
8	69.74	.1327
9	69.80	.1327
10	196.26	.3358
11	203.28	.3358
12	339.10	.5159
13	472.22	.6674
14	471.7	.6723
15	471.7	.6854
16	336.2	.5208
17	336.2	.5221

From the solution to Problem 8.75; $\dot{m}_1 = 5.286 \times 10^6 \text{ lb/h}$, $y' = .1428$, $y'' = .1245$, $y''' = .0434$.

INPUTS

Boiler/superheater:

$$\begin{aligned}\dot{m}_1 (e_{f1} - e_{f13}) &= \dot{m}_1 [(h_1 - h_{13}) - T_0 (s_1 - s_{13})] \\ &= (5.286 \times 10^6 \frac{\text{lb}}{\text{h}}) [(1542.5 - 472.22) - (520)(1.6159 - .6674)] \frac{\text{Btu}}{\text{lb}} \\ &= 3.05 \times 10^9 \text{ Btu/h}\end{aligned}$$

Reheater:

$$\begin{aligned}(1 - y' - y'') \dot{m}_1 (e_{f5} - e_{f4}) &= (1 - y' - y'') \dot{m}_1 [(h_5 - h_4) - T_0 (s_5 - s_4)] \\ &= 6.312 \times 10^8 \text{ Btu/h}\end{aligned}$$

$$\text{Total: } 3.681 \times 10^9 \text{ Btu/h} \xleftarrow{\text{Input}}$$

$$\text{OUTPUT: net output of cycle} = \dot{W}_{\text{cycle}} = 3 \times 10^9 \text{ Btu/h} \xleftarrow{\text{Output}}$$

Loss

$$\begin{aligned}\text{Condenser loss} &= \dot{m}_1 (1 - y' - y'' - y''') (e_{f7} - e_{f8}) \\ &= \dot{m}_1 (1 - y' - y'' - y''') [(h_7 - h_8) - T_0 (s_7 - s_8)] \\ &= 2.838 \times 10^8 \text{ Btu/h} \xleftarrow{\text{Loss}}\end{aligned}$$

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Problem 8-89 continued

DESTRUCTIONS

Turbine:

$$\begin{aligned}\dot{E}_d &= T_0 \dot{m}_1 (\dot{\sigma}_{\text{turb}}/\dot{m}_1) \\ &= T_0 \dot{m}_1 [(1-y'-y''-y''')s_7 + y'''s_6 - (1-y'-y'')(s_3-s_4) + y''s_3 + y's_2 - s_1] \\ &= 2.521 \times 10^8 \text{ Btu/h}\end{aligned}$$

600 lbf/in² heater:

$$\dot{E}_d = T_0 \dot{m}_1 [y'(s_{14}-s_2) + (s_{13}-s_{12})] = 3.99 \times 10^7 \text{ Btu/h}$$

160 lbf/in² heater:

$$\dot{E}_d = T_0 \dot{m}_1 [(y'+y'')s_{16} + s_{12} - s_{11} - y''s_3 - y's_{15}] = 4.377 \times 10^7 \text{ Btu/h}$$

Open heater:

$$\begin{aligned}\dot{E}_d &= T_0 \dot{m}_1 [s_{10} - y'''s_6 - (y'+y'')s_{17} - (1-y'-y''-y''')s_9] \\ &= 5.59 \times 10^7 \text{ Btu/h}\end{aligned}$$

Traps:

$$\dot{E}_d = T_0 \dot{m}_1 [y'(s_{15}-s_{14}) + (y'+y'')(s_{17}-s_{16})] = 6.097 \times 10^6 \text{ Btu/h}$$

$$\text{Total: } 3.978 \times 10^8 \text{ Btu/h} \quad \swarrow \text{destroyed}$$

Finally, to summarize:

$$\text{Input: } 3.681 \times 10^9 \text{ Btu/h}$$

$$\begin{array}{rcl}\text{Output:} & 3.00 & \times 10^9 \\ \text{Loss:} & 2.838 & \times 10^8 \\ \text{Destroyed:} & 3.978 & \times 10^8 \\ \hline & 3.682 & \times 10^8 \text{ Btu/h}\end{array}$$

PROBLEM 8.90

① KNOWN: The regenerative vapor power cycle of Problem 8.78 is analyzed from the perspective of exergy accounting.

FIND: Determine the rate of exergy input to the working fluid passing through the steam generator and perform calculations to account for this exergy.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.78.

ASSUMPTIONS: Same as in Problem 8.78. Also, let $T_0 = 520^\circ\text{R}$ and $p_0 = 14.7 \text{ lbf/in}^2$.

ANALYSIS: Each principal state is fixed in the solution to Problem 8.66. The specific enthalpies and entropies are:

State	$h \text{ (Btu/lb)}$	$s \text{ (Btu/lb} \cdot ^\circ\text{R)}$	
1	1388.5	1.5665	
2	1186.3	1.6015	$x_2 = 0.9983$
3	1187.8	1.6034	
4	1294.2	1.7234	
5	1002.5	1.7941	$x_5 = 0.9003$
6	69.74	0.1327	
7	70.04	0.1327	
8	542.4	0.6471	
9	542.4	0.7839	$x_9 = 0.2742$
10	298.6	0.4744	
11	123.1	0.22315	$s_f @ 155^\circ\text{F}$
12	125.83	0.22315	

From the solution to Problem 8.78; $\dot{m}_1 = 1.148 \times 10^7 \text{ lb/h}$, $y' = 0.1115$, $y'' = 0.8866$.

INPUT Boiler/superheater:

$$\begin{aligned} \dot{m}_1 (e_{f1} - e_{f12}) &= \dot{m}_1 [(h_1 - h_{12}) - T_0 (s_1 - s_{12})] \\ &= (1.148 \times 10^7 \frac{\text{lb}}{\text{h}}) [(1388.5 - 125.83) - (520)(1.5665 - 0.22315)] \frac{\text{Btu}}{\text{lb}} \\ &= 6.476 \times 10^9 \text{ Btu/h} \quad \text{Input} \end{aligned}$$

OUTPUT

$$\left(\begin{array}{c} \text{net power} \\ \text{output of} \\ \text{cycle} \end{array} \right) = 5 \times 10^9 \text{ Btu/h} \quad \text{Output}$$

LOSS

$$\begin{aligned} \left(\begin{array}{c} \text{condenser} \\ \text{loss} \end{array} \right) &= \dot{m}_1 y'' (e_{f5} - e_{f6}) \\ &= \dot{m}_1 y'' [(h_5 - h_6) - T_0 (s_5 - s_6)] \\ &= (1.148 \times 10^7) (0.8866) [(1002.5 - 69.74) - (520)(1.7941 - 0.1327)] \\ &= 7.006 \times 10^8 \text{ Btu/h} \quad \text{loss} \end{aligned}$$

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Problem 8-90 continued

DESTRUCTIONS

Turbine stages :

$$\begin{aligned}\dot{E}_d &= \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 [(1-y')(s_2-s_1) + y''(s_5-s_4)] \\ &= 5.598 \times 10^8 \text{ Btu/h}\end{aligned}$$

Heat Exchanger :

$$\begin{aligned}\dot{E}_d &= \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 [y''(s_4-s_3) + y'(s_8-s_1)] \\ &= 2.316 \times 10^7 \text{ Btu/h}\end{aligned}$$

Open heater :

$$\begin{aligned}\dot{E}_d &= \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 [s_{11} - y''s_7 - y's_9 - (1-y'-y'')s_{10}] \\ &= 1.026 \times 10^8 \text{ Btu/h}\end{aligned}$$

Trap :

$$\dot{E}_d = \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 [y'(s_9-s_8)] = 9.106 \times 10^7 \text{ Btu/h}$$

Flash Chamber: $\dot{E}_d = 0$

$$\text{Total: } 7.7662 \times 10^8 \frac{\text{Btu}}{\text{h}} \leftarrow \text{destroyed}$$

Finally, to summarize :

Input : $6.476 \times 10^9 \text{ Btu/h}$

Output: 5×10^9 (77.2%)
 Loss: 7.006×10^8 (10.8%)
 Destroyed: 7.7662×10^8 (12.0%)
 $\underline{6.477 \times 10^9 \text{ Btu/h}}$

PROBLEM 8.91

KNOWN: The regenerative vapor power cycle of Problem 8.46 is analyzed from the perspective of exergy accounting.

FIND: Determine the rate of transfer to the working fluid passing through the steam generator per kg of steam entering the first-stage turbine. Perform a complete exergy accounting.

SCHEMATIC & GIVEN DATA: See the solution to Problem 8.46.

ENGINEERING MODEL: Same as in Problem 8.46. Also, let $T_0 = 15^\circ\text{C} = 288\text{K}$ and $p_0 = 0.1\text{ MPa} = 1\text{ bar}$. Assume the cooling water enters at 288K and is an incompressible substance with $c_{cw} = 4.179\text{ kJ/kg}\cdot\text{K}$.

ANALYSIS: From Problem 8.46, the principal states are fixed. The specific enthalpies and entropies are

State	$h(\text{kJ/kg})$	$s(\text{kJ/kg}\cdot\text{K})$
1	3465.4	6.5132
2	2745.1	6.5132
3	2037.0	6.5132
4	173.88	0.5926
5	174.88	0.5926
6	762.81	2.1387
7	779.72	2.1387

$$s_1 = s_2 = s_3$$

$$s_5 = s_4$$

$$s_7 = s_6$$

$$\dot{W}_{\text{cycle}} = 1.498 \times 10^5\text{ kW}$$

$$\dot{m}_1 = 120\text{ kg/s}$$

$$y = 0.2287$$

$$\dot{m}_{cw} = 2292\text{ kg/s}$$

$$T_0 = 288\text{ K}$$

$$T_{cw,in} = 288\text{ K}$$

$$T_{cw,out} = 288 + 18 = 306\text{ K}$$

Input

• Steam generator: $\frac{\dot{E}_f - \dot{E}_7}{\dot{m}_1} = e_{f1} - e_{f7} = (h_1 - h_7) - T_0(s_1 - s_7)$

$$= (3465.4 - 779.72) - (288)(6.5132 - 2.1387) = 1425.8\frac{\text{kJ}}{\text{kg}}$$

Output

• Net work: $\dot{W}_{\text{cycle}}/\dot{m}_1 = \frac{1.498 \times 10^5}{120} = 1248.3\text{ kJ/kg}$

Loss to cooling water

$$\begin{aligned} \frac{(\dot{E}_f - \dot{E}_f)_{cw}}{\dot{m}_1} &= \frac{\dot{m}_{cw}}{\dot{m}_1} [(h_{out} - h_{in})_{cw} - T_0(s_{out} - s_{in})_{cw}] \\ &= \frac{\dot{m}_{cw}}{\dot{m}_1} [c_{cw} \Delta T_{cw} - T_0 c_{cw} \ln\left(\frac{T_{out}}{T_{in}}\right)_{cw}] \\ &= \frac{2292}{120} [(4.179)(18) - (288)(4.179) \ln\left(\frac{306}{288}\right)] = 43.1\text{ kJ/kg} \end{aligned}$$

DESTRUCTION

• open Heater: $\left(\frac{\dot{E}_d}{\dot{m}_1}\right)_{OH} = T_0 \left(\frac{\dot{\sigma}_{cr}}{\dot{m}_1}\right)_{OH} = T_0 [s_6 - y s_2 - (1-y) s_5]$

$$= (288) [(2.1387) - (0.2287)(6.5132) - (0.7713)(0.5926)]$$

$$= 55.3\text{ kJ/kg}$$

• Condenser: $\left(\frac{\dot{E}_d}{\dot{m}_1}\right)_{cond} = T_0 [(1-y)(s_4 - s_5) + \frac{\dot{m}_{cw}}{\dot{m}_1} c_{cw} \ln\left(\frac{T_{out}}{T_{in}}\right)_{cw}]$

$$= 288 [(0.7713)(0.5926 - 6.5132) + \left(\frac{2292}{120}\right)(4.179) \ln\left(\frac{306}{288}\right)]$$

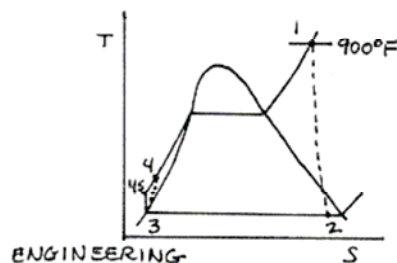
$$= 79.3\text{ kJ/kg}$$

PROBLEM 8.92

KNOWN: Water is the working fluid in a simple vapor power plant. Data are known at various locations. For the steam generator, specific exergy values are provided for the fuel, air, and stack gases.

FIND: Determine, as percentages of the exergy entering with the fuel, (a) the exergy exiting with the stack gases, (b) exergy destroyed in the steam generator, (c) net power developed, (d) exergy destroyed in the turbine and pump, (e) exergy exiting with the cooling water, (f) exergy destroyed in the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING-

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) There are no stray heat transfers for any component. (3) kinetic and potential energy effects are negligible. (4) Let $T_0 = 537^\circ\text{R}$, $P_0 = 14.7 \text{ lbf/in}^2$.

ANALYSIS: First, fix each of the principal states of the cycle.

State 1 $P_1 = 500 \text{ lbf/in}^2$, $T_1 = 900^\circ\text{F} \Rightarrow h_1 = 1466.5 \text{ Btu/lb}$, $s_1 = 1.6987 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2 $P_2 = 1 \text{ lbf/in}^2$, $x_2 = 0.97 \Rightarrow h_2 = 1074.66 \text{ Btu/lb}$, $s_2 = 1.9226 \text{ Btu/lb}\cdot^\circ\text{R}$

State 3 $P_3 = 1 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_3 = 69.74 \text{ Btu/lb}$, $s_3 = 0.1327 \text{ Btu/lb}\cdot^\circ\text{R}$

State 4 $P_4 = 500 \text{ lbf/in}^2$; $s_4 = 0.1327 \Rightarrow$ Table A-5E; $h_4 = 71.34 \text{ Btu/lb}$
With the pump efficiency; $h_4 = h_3 + \frac{(h_3 - h_4)}{\eta_p} = 71.74 \text{ Btu/lb}$
 $s_4 = 0.1334 \text{ Btu/lb}\cdot^\circ\text{R}$

ANALYSIS: (a) For the stack gases

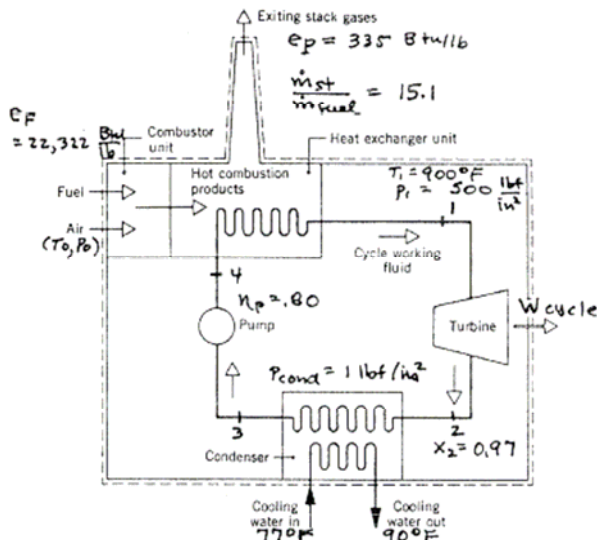
$$\frac{\dot{E}_P}{\dot{E}_F} = \frac{335 \text{ Btu/lb(fuel)}}{22,322 \text{ Btu/lb(fuel)}} = 0.015 \quad (1.5\%) \quad \leftarrow (a)$$

(b) An exergy rate balance for the steam generator gives

$$0 = \dot{E}_A^0 + \dot{E}_F - \dot{E}_P + \dot{m}_{st} [e_{st} - e_f] - \dot{E}_d$$

$$\begin{aligned} \Rightarrow \frac{\dot{E}_d}{\dot{m}_F} &= e_F - e_P + \frac{\dot{m}_{st}}{\dot{m}_F} [(h_4 - h_1) - T_0(s_4 - s_1)] \\ &= 22,322 - 335 + 15.1 [(71.74 - 1466.5) - 537(0.1334 - 1.6987)] \\ &= 13,619 \text{ Btu/lb(fuel)} \end{aligned}$$

$$\Rightarrow \% \text{ destroyed} = \frac{13,619}{22,322} = 0.61 \quad (61\%) \quad \leftarrow (b)$$



Continued on next slide

Problem 9-92 continued

(c) The net work developed per unit mass of fuel entering is

$$\frac{\dot{W}_{net}}{\dot{m}_F} = \frac{\dot{m}_{st}}{\dot{m}_F} [(h_1 - h_2) - (h_4 - h_3)] = 15.1 [(1466.5 - 1074.66) - (71.74 - 69.74)]$$

$$= 5887 \text{ Btu/lb(fuel)}$$

$$\% \text{ output as work} = \frac{5887}{22,322} = 0.264 \quad (26.4\%) \quad \leftarrow (c)$$

(d) The exergy destroyed in the turbine and pump can be found using $\dot{E}_d = T_0 \dot{\sigma}$. That is

$$\frac{\dot{E}_d}{\dot{m}_F} = \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) T_0 [(s_2 - s_1) + (s_4 - s_3)] = (15.1)(537) [(1.9226 - 1.6987) + (0.1334 - 0.1327)]$$

$$= 1821 \text{ Btu/lb(fuel)}$$

$$\% \text{ destroyed} = \frac{1821}{22,322} = 0.082 \quad (8.2\%) \quad \leftarrow (d)$$

(e) The exergy exiting with the cooling water per unit mass of fuel entering is given by

$$\left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left[\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) [e_{f,cw,out} - e_{f,cw,in}] \right] = \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) [(h_{cw,out} - h_{cw,in}) - T_0(s_{cw,out} - s_{cw,in})]$$

An energy balance on the condenser using $h_{cw} \approx h_f(T)$ gives

$$\frac{\dot{m}_{cw}}{\dot{m}_{st}} = \frac{h_2 - h_3}{h_f(40^\circ\text{F}) - h_f(77^\circ\text{F})} = 77.42$$

Then, with $s_{cw} \approx s_f(T)$

$$\left[\begin{array}{l} \text{Exergy exiting with} \\ \text{the cooling water per} \\ \text{unit mass of fuel} \\ \text{entering} \end{array} \right] = (15.1)(77.42) [(58.07 - 45.09) - (537)(0.1117 - 0.08775)]$$

$$= 139 \text{ Btu/lb(fuel)}$$

$$\% \text{ exiting with the cooling water} = \frac{139}{22,322} = 0.006 \quad (0.6\%) \quad \leftarrow (e)$$

(f) Using $\dot{E}_d = T_0 \dot{\sigma}$, the exergy destroyed in the condenser is

$$\frac{\dot{E}_d}{\dot{m}_F} = T_0 \left[\frac{\dot{m}_{st}}{\dot{m}_F} (s_3 - s_2) + \left(\frac{\dot{m}_{cw}}{\dot{m}_F} \right) \left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) [s_{cw,out} - s_{cw,in}] \right]$$

$$= (537)(15.1) [(0.1327 - 1.9226) + (77.42)(0.1117 - 0.08775)] = 521 \text{ Btu/lb(fuel)}$$

$$\% \text{ destroyed} = 521/22,322 = 0.023 \quad (2.3\%) \quad \leftarrow f$$

SUMMARY:

• Exergy in with fuel = 22,322 Btu/lb(fuel)

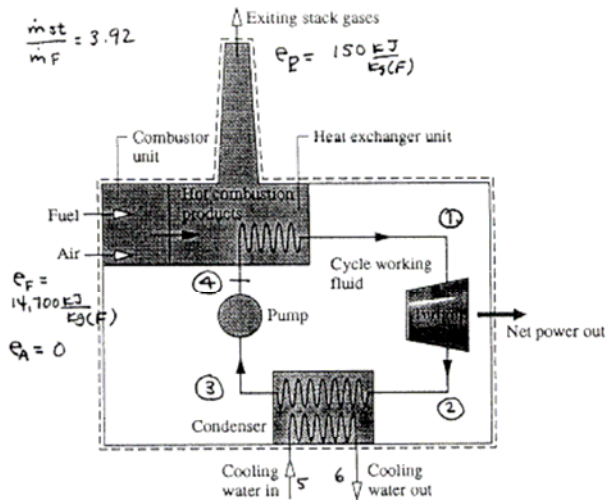
• Exergy Out:	
– Net work	5887 (26.4%)
– Cooling water	139 (0.6%)
– Stack gas	335 (1.5%)
• Exergy Destroyed:	
– Steam gen.	13,619 (61%)
– Turbine/pump	1821 (8.2%)
– Condenser	521 (2.3%)
	22,322 (100%)

PROBLEM 8.93

KNOWN: Water is the working fluid in a simple vapor powerplant. Data are known at various locations. For the steam generator, specific exergy values are provided for the fuel, air, and stack gases.

FIND: Develop an exergy accounting of the exergy entering the plant with the fuel.

SCHEMATIC & GIVEN DATA:



State	T(°C)	p(bar)	h(kJ/kg)	s(kJ/kg·K)
1	520	100	3425.1	6.6622
2	(x=.90)	0.08	2336.7	7.4651
3	(sat. liq)	0.08	173.9	0.5926
4	430°C	100	188.9	0.6061
5	20	—	84	0.2966
6	35	—	146.7	0.5053

ENGINEERING MODEL: 1. Control volumes enclosing each component are at steady state. 2. There are no stray heat transfers, and kinetic/potential energy effects can be ignored. 4. $T_0 = 20^\circ\text{C}$, $P_0 = 1\text{ atm}$

ANALYSIS: Begin by fixing each state and obtaining h, s data: Table A-4 gives $h_1 = 3425.1\text{ kJ/kg}$, $s_1 = 6.6622\text{ kJ/kg}\cdot\text{K}$. Then, with data from Table A-3

$h_2 = 173.9 + 0.9(2403.1) = 2336.7$, $s_2 = 0.5926 + 0.9(8.2287 - 0.5926) = 7.4651$.
 $h_3 = h_f(0.08\text{ bar}) = 173.9$, $s_3 = s_f = 0.5926$. Then, from Table A-5, $h_4 = 188.9$, $s_4 = 0.6061$. And with $h = h_f(T)$, $s = s_f(T)$ and data from Table A-2, $h_5 = 84$, $s_5 = 0.2966$, $h_6 = 146.7$, $s_6 = 0.5053$

For the stack gases:

$$\frac{\dot{E}_P}{\dot{E}_F} = \frac{150\text{ kJ/kg(F)}}{14,700\text{ kJ/kg(F)}} = 0.01 \quad (1\%)$$

Exergy Destruction - Steam Generator: An exergy rate balance reduces to read

$$0 = \dot{E}_{\text{Air}} + \dot{E}_F - \dot{E}_P + \dot{m}_{\text{st}}[e_{f4} - e_{f1}] - \dot{E}_d$$

$$\Rightarrow \frac{\dot{E}_d}{\dot{m}_F} = e_F - e_P + \frac{\dot{m}_{\text{st}}}{\dot{m}_F} [(h_4 - h_1) - T_0(s_4 - s_1)]$$

$$= 14,700 - 150 + 3.92 [(188.9 - 3425.1) - 293(0.6061 - 6.6622)] = 8820\text{ kJ/kg(F)}$$

Expressed as a percentage of the fuel exergy: $= \frac{8820}{14,700} = 0.6 \quad (60\%)$

Net Work Developed:

$$\dot{W}_{\text{net}}/\dot{m}_F = (\dot{m}_{\text{st}}/\dot{m}_F) [(h_1 - h_6) - (h_4 - h_3)] = 3.92 [(3425.1 - 2336.7) - (188.9 - 173.9)] = 4208\text{ kJ/kg(F)}$$

Expressed as a percentage of the fuel exergy: $= \frac{4208}{14,700} = 0.286 \quad (28.6\%)$

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Problem 9-93 continued

Exergy Destruction in Turbine/Pump: Using $\dot{E}_d = T_0 \dot{\sigma}$

$$\frac{\dot{E}_d}{\dot{m}_F} = \frac{\dot{m}_{st}}{\dot{m}_F} \left[T_0 [s_2 - s_1] - T_0 [s_4 - s_3] \right] = (3.92)(293) [(7.4651 - 6.6622) + (0.6061 - 0.5926)]$$

$$= 938 \frac{\text{kJ}}{\text{kg(F)}}, \quad \text{when expressed as a percentage of the fuel exergy: } = \frac{938}{14,700} = 0.064 \quad (6.4\%)$$

Exergy exiting with the cooling water:

$$= \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) [e_{f,6} - e_{f,5}] = \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) [(h_6 - h_5) - T_0 (s_6 - s_5)]$$

An energy rate balance on the condenser gives

$$\frac{\dot{m}_{cw}}{\dot{m}_{st}} = \frac{h_2 - h_3}{h_6 - h_5} = \frac{2336.7 - 173.9}{146.7 - 84} = 34.49$$

Thus,

$$(\text{Exergy exiting with c.w.}) = (3.92)(34.49) [(146.7 - 84) - 293(0.5053 - 0.2966)] = 210 \text{ kJ/kg(F)}$$

$$\text{when expressed as a percentage of the fuel exergy: } = \frac{210}{14,700} = 0.014 \quad (1.4\%)$$

Exergy Destruction in the Condenser: Using $\dot{E}_d = T_0 \dot{\sigma}$

$$\frac{\dot{E}_d}{\dot{m}_F} = T_0 \left[\frac{\dot{m}_{st}}{\dot{m}_F} (s_3 - s_2) + \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) (s_6 - s_5) \right]$$

$$= (293)(3.92) [(0.5926 - 7.4651) + 34.49(0.5053 - 0.2966)] = 374 \text{ kJ/kg(F)}$$

$$\text{when expressed as a percentage of the fuel exergy} = \frac{374}{14,700} = 0.025 \quad (2.5\%)$$

SUMMARY:

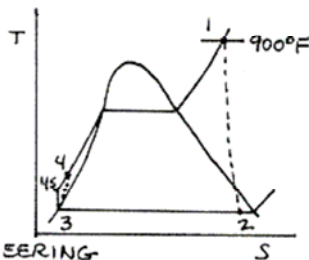
• Exergy entering with the fuel, per unit mass of fuel:	14,700 kJ/kg(F)	(100%)
• Exergy Out		
- net work	4208	(28.6%)
- cooling water	210	(1.4%)
- stack gases	150	(1.0%)
• Exergy Destroyed		
- turbine/pump	938	(6.4%)
- condenser	374	(2.5%)
- steam generator	8820	(60%)
	<u>14,700</u>	<u>(100%)</u>

PROBLEM 8.92

KNOWN: Water is the working fluid in a simple vapor power plant. Data are known at various locations. For the steam generator, specific exergy values are provided for the fuel, air, and stack gases.

FIND: Determine, as percentages of the exergy entering with the fuel, (a) the exergy exiting with the stack gases, (b) exergy destroyed in the Steam generator, (c) net power developed, (d) exergy destroyed in the turbine and pump, (e) exergy exiting with the cooling water, (f) exergy destroyed in the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL : (1) Each component is analyzed as a control volume at steady state. (2) There are no stray heat transfers for any component. (3) Kinetic and potential energy effects are negligible. (4) Let $T_o = 537^\circ\text{R}$, $P_o = 14.7 \text{ lbf/in}^2$

ANALYSIS: First, fix each of the principal states of the cycle.

State 1 $p_1 = 500 \text{ lbf/in}^2$, $T_1 = 900^\circ\text{F} \Rightarrow h_1 = 1466.5 \text{ Btu/lb}$, $s_1 = 1.6987 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2 $p_2 = 1 \text{ lbf/in}^2$, $x_2 = 97 \Rightarrow h_2 = 1074.66 \text{ Btu/lb}$, $s_2 = 1.9226 \text{ Btu/lb} \cdot ^\circ\text{R}$

State 3 $p_3 = 1 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_3 = 69.74 \text{ Btu/lb}$, $s_3 = 1.327 \text{ Btu/lb} \cdot ^\circ\text{R}$

State 4 $p_4 = 500 \text{ lbf/in}^2$; $s_{4s} = .1327 \Rightarrow$ Table A-5E; $h_{4s} = 71.34 \text{ Btu/lb}$
With the pump efficiency; $h_4 = h_3 + \left(\frac{h_{4s} - h_3}{\eta_P} \right) = 71.74 \text{ Btu/lb}$
 $s_4 \approx 0.1334 \text{ Btu/lb}$

ANALYSIS: (a) For the stack gases

$$\frac{\dot{m}_p}{\dot{m}_F} = \frac{335 \text{ Btu/lb(fuel)}}{22,322 \text{ Btu/lb(fuel)}} = 0.015 \quad (1.5\%). \quad \leftarrow (a)$$

(b) An energy rate balance for the steam generator gives

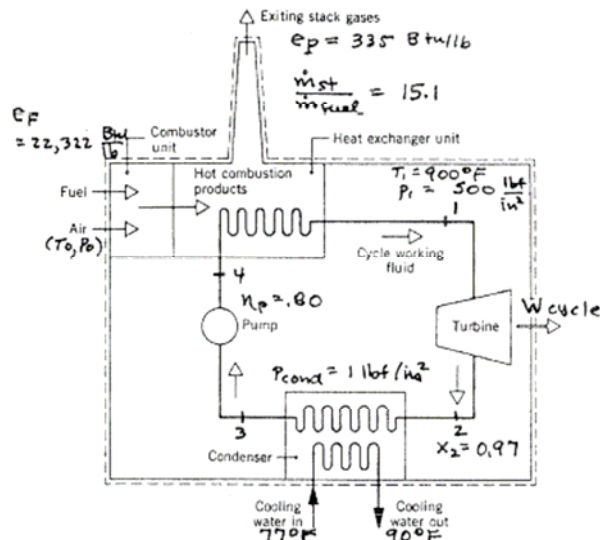
$$0 = \cancel{\dot{E}_A^0} + \dot{E}_F - \dot{E}_p + \dot{m}_s [e_{f4} - e_{f1}] - \dot{E}_d$$

$$\Rightarrow \frac{\dot{E}_d}{\dot{m}_F} = e_F - e_P + \frac{\dot{m}_{st}}{\dot{m}_F} [(h_4 - h_1) - T_0(s_4 - s_1)]$$

$$= 22,322 - 335 + 15.1 [(71.74 - 1466.5) - 537[0.1334 - 1.6987]]$$

$$= 13,619 \text{ Btu/lb (fuel)}$$

$$\Rightarrow \% \text{ destroyed} = \frac{13,614}{22,322} = 0.61 \quad (61\%) \quad \leftarrow (b)$$



Continued on next slide

Problem 9-92 continued

(c) The net work developed per unit mass of fuel entering is

$$\frac{\dot{W}_{net}}{\dot{m}_F} = \frac{\dot{m}_{st}}{\dot{m}_F} [(h_1 - h_2) - (h_4 - h_3)] = 15.1 [(1466.5 - 1074.66) - (71.74 - 69.74)]$$

$$= 5887 \text{ Btu/lb(fuel)}$$

$$\% \text{ output as work} = \frac{5887}{22,322} = 0.264 \quad (26.4\%) \quad \leftarrow (c)$$

(d) The exergy destroyed in the turbine and pump can be found using $\dot{E}_d = T_0 \dot{\sigma}$. That is

$$\frac{\dot{E}_d}{\dot{m}_F} = \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) T_0 [(s_2 - s_1) + (s_4 - s_3)] = (15.1)(537) [(1.9226 - 1.6987) + (0.1334 - 0.1327)]$$

$$= 1821 \text{ Btu/lb(fuel)}$$

$$\% \text{ destroyed} = \frac{1821}{22,322} = 0.082 \quad (8.2\%) \quad \leftarrow (d)$$

(e) The exergy exiting with the cooling water per unit mass of fuel entering is given by

$$\left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left[\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) [e_{f,cw,out} - e_{f,cw,in}] \right] = \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) [(h_{cw,out} - h_{cw,in}) - T_0(s_{cw,out} - s_{cw,in})]$$

An energy balance on the condenser using $h_{cw} \approx h_f(T)$ gives

$$\frac{\dot{m}_{cw}}{\dot{m}_{st}} = \frac{h_2 - h_3}{h_f(40^\circ\text{F}) - h_f(77^\circ\text{F})} = 77.42$$

Then, with $s_{cw} \approx s_f(T)$

$$\left[\begin{array}{l} \text{Exergy exiting with} \\ \text{the cooling water per} \\ \text{unit mass of fuel} \\ \text{entering} \end{array} \right] = (15.1)(77.42) [(58.07 - 45.09) - (537)(0.1117 - 0.08775)]$$

$$= 139 \text{ Btu/lb(fuel)}$$

$$\% \text{ exiting with the cooling water} = \frac{139}{22,322} = 0.006 \quad (0.6\%) \quad \leftarrow (e)$$

(f) Using $\dot{E}_d = T_0 \dot{\sigma}$, the exergy destroyed in the condenser is

$$\frac{\dot{E}_d}{\dot{m}_F} = T_0 \left[\frac{\dot{m}_{st}}{\dot{m}_F} (s_3 - s_2) + \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) [s_{cw,out} - s_{cw,in}] \right]$$

$$= (537)(15.1) [(0.1327 - 1.9226) + (77.42)(0.1117 - 0.08775)] = 521 \text{ Btu/lb(fuel)}$$

$$\% \text{ destroyed} = 521/22,322 = 0.023 \quad (2.3\%) \quad \leftarrow f$$

SUMMARY:

• Exergy in with fuel = 22,322 Btu/lb(fuel)

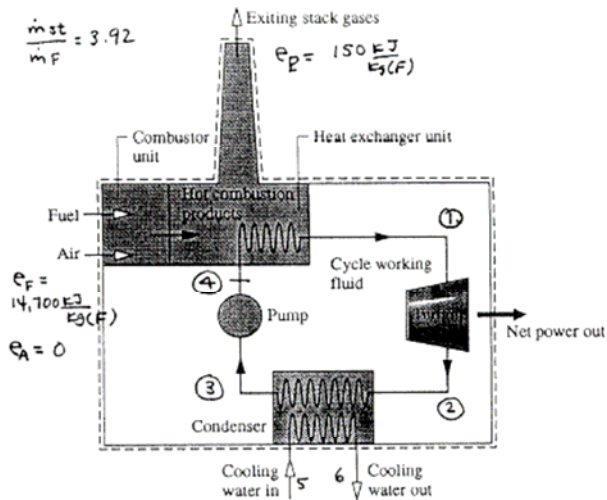
• Exergy Out:	
– Net work	5887 (26.4%)
– Cooling water	139 (0.6%)
– Stack gas	335 (1.5%)
• Exergy Destroyed:	
– Steam gen.	13,619 (61%)
– Turbine/pump	1821 (8.2%)
– Condenser	521 (2.3%)
	22,322 (100%)

PROBLEM 8.93

KNOWN: Water is the working fluid in a simple vapor powerplant. Data are known at various locations. For the steam generator, specific exergy values are provided for the fuel, air, and stack gases.

FIND: Develop an exergy accounting of the exergy entering the plant with the fuel.

SCHEMATIC & GIVEN DATA:



State	T(°C)	p(bar)	h($\frac{\text{kJ}}{\text{kg}}$)	s($\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$)
1	520	100	3425.1	6.6622
2	(x=.90)	0.08	2336.7	7.4651
3	(sat. liq)	0.08	173.9	0.5926
4	430°C	100	188.9	0.6061
5	20	—	84	0.2966
6	35	—	146.7	0.5053

ENGINEERING MODEL: 1. Control volumes enclosing each component are at steady state. 2. There are no stray heat transfers, and kinetic/potential energy effects can be ignored. 4. $T_0 = 20^\circ\text{C}$, $P_0 = 1\text{atm}$

ANALYSIS: Begin by fixing each state and obtaining h, s data: Table A-4 gives $h_1 = 3425.1\text{ kJ/kg}$, $s_1 = 6.6622\text{ kJ/kg}\cdot\text{K}$. Then, with data from Table A-3

$h_2 = 173.9\text{ kJ/kg} + 0.9(2403.1) = 2336.7$, $s_2 = 0.5926 + 0.9(8.2287 - 0.5926) = 7.4651$.
 $h_3 = h_f(0.08\text{ bar}) = 173.9$, $s_3 = s_f = 0.5926$. Then, from Table A-5, $h_4 = 188.9$, $s_4 = 0.6061$. And with $h = h_f(T)$, $s = s_f(T)$ and data from Table A-2, $h_5 = 84$, $s_5 = 0.2966$, $h_6 = 146.7$, $s_6 = 0.5053$

For the stack gases:

$$\frac{\dot{E}_P}{\dot{E}_F} = \frac{150\text{ kJ/kg(F)}}{14,700\text{ kJ/kg(F)}} = 0.01 \quad (1\%)$$

Exergy Destruction - Steam Generator: An exergy rate balance reduces to read

$$0 = \dot{E}_{\text{Air}} + \dot{E}_F - \dot{E}_P + \dot{m}_{\text{st}}[e_{f4} - e_{f1}] - \dot{E}_d$$

$$\Rightarrow \frac{\dot{E}_d}{\dot{m}_F} = e_F - e_P + \frac{\dot{m}_{\text{st}}}{\dot{m}_F}[(h_4 - h_1) - T_0(s_4 - s_1)]$$

$$= 14,700 - 150 + 3.92[(188.9 - 3425.1) - 293(0.6061 - 6.6622)] = 8820\text{ kJ/kg(F)}$$

Expressed as a percentage of the fuel exergy: $= \frac{8820}{14,700} = 0.6 \quad (60\%)$

Net Work Developed:

$$\dot{W}_{\text{net}}/\dot{m}_F = (\dot{m}_{\text{st}}/\dot{m}_F)[(h_1 - h_6) - (h_4 - h_3)] = 3.92[(3425.1 - 2336.7) - (188.9 - 173.9)] = 4208\text{ kJ/kg(F)}$$

Expressed as a percentage of the fuel exergy: $= \frac{4208}{14,700} = 0.286 \quad (28.6\%)$

Continued on next slide

Problem 9-93 continued

Exergy Destruction in Turbine/Pump: Using $\dot{E}_d = T_0 \dot{\sigma}$

$$\frac{\dot{E}_d}{\dot{m}_F} = \frac{\dot{m}_{st}}{\dot{m}_F} \left[T_0 [s_2 - s_1] - T_0 [s_4 - s_3] \right] = (3.92)(293) [(7.4651 - 6.6622) + (0.6061 - 0.5926)]$$

$$= 938 \frac{\text{kJ}}{\text{kg(F)}}, \quad \text{when expressed as a percentage of the fuel exergy: } = \frac{938}{14,700} = 0.064 \quad (6.4\%)$$

Exergy exiting with the cooling water:

$$= \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) [e_{f,6} - e_{f,5}] = \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) [(h_6 - h_5) - T_0 (s_6 - s_5)]$$

An energy rate balance on the condenser gives

$$\frac{\dot{m}_{cw}}{\dot{m}_{st}} = \frac{h_2 - h_3}{h_6 - h_5} = \frac{2336.7 - 173.9}{146.7 - 84} = 34.49$$

Thus,

$$(\text{Exergy exiting with c.w.}) = (3.92)(34.49) [(146.7 - 84) - 293(0.5053 - 0.2966)] = 210 \text{ kJ/kg(F)}$$

$$\text{when expressed as a percentage of the fuel exergy: } = \frac{210}{14,700} = 0.014 \quad (1.4\%)$$

Exergy Destruction in the Condenser: Using $\dot{E}_d = T_0 \dot{\sigma}$

$$\frac{\dot{E}_d}{\dot{m}_F} = T_0 \left[\frac{\dot{m}_{st}}{\dot{m}_F} (s_3 - s_2) + \left(\frac{\dot{m}_{st}}{\dot{m}_F} \right) \left(\frac{\dot{m}_{cw}}{\dot{m}_{st}} \right) (s_6 - s_5) \right]$$

$$= (293)(3.92) [(0.5926 - 7.4651) + 34.49(0.5053 - 0.2966)] = 374 \text{ kJ/kg(F)}$$

$$\text{when expressed as a percentage of the fuel exergy} = \frac{374}{14,700} = 0.025 \quad (2.5\%)$$

SUMMARY:

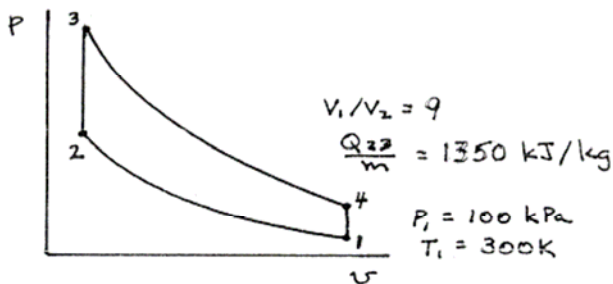
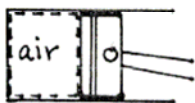
• Exergy entering with the fuel, per unit mass of fuel:	14,700 kJ/kg(F)	(100%)
• Exergy Out		
- net work	4208	(28.6%)
- cooling water	210	(1.4%)
- stack gases	150	(1.0%)
• Exergy Destroyed		
- turbine/pump	938	(6.4%)
- condenser	374	(2.5%)
- steam generator	8820	(60%)
	<u>14,700</u>	<u>(100%)</u>

PROBLEM 9.1

KNOWN: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the net work per unit mass of air, (b) the thermal efficiency, (c) the mean effective pressure, and (d) the maximum cycle temperature. (e) Plot each of these quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1.

ANALYSIS: Begin by fixing each principal state of the cycle (Table A-22).

State 1 $P_1 = 100 \text{ kPa}$, $T_1 = 300 \text{ K}$ $\Rightarrow u_1 = 214.07 \text{ kJ/kg}$, $v_{r1} = 621.2$

State 2 For isentropic compression

$$v_{r2} = v_{r1} \frac{V_2}{V_1} = \frac{621.2}{9} = 69.022$$

Thus, $T_2 = 702.7 \text{ K}$, $u_2 = 514.50 \text{ kJ/kg}$

State 3 The specific internal energy u_3 is found using the energy balance for process 2-3

$$m(u_3 - u_2) = Q_{23} - W_{23}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = 1350 \frac{\text{kJ}}{\text{kg}} + 514.50 \frac{\text{kJ}}{\text{kg}} = 1864.5 \frac{\text{kJ}}{\text{kg}}$$

Thus, $T_3 = 2191.9 \text{ K}$, $v_{r3} = 2.0385$

(d) T_3

State 4 For the isentropic expansion

$$v_{r4} = v_{r3} \frac{V_4}{V_3} = v_{r3} \frac{V_1}{V_2} = (2.0385)(9) = 18.3465$$

Finally, $T_4 = 1110.9 \text{ K}$, $u_4 = 854.81 \text{ kJ/kg}$

(a) To find the net work, note that $W_{\text{cycle}} = Q_{\text{cycle}}$, so

$$\begin{aligned} \frac{W_{\text{cycle}}}{m} &= \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (u_4 - u_1) \\ &= 1350 - (854.81 - 214.07) = 709.26 \frac{\text{kJ}}{\text{kg}} \quad \text{(a) } W_{\text{cycle}} \end{aligned}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{23}/m} = \frac{709.26}{1350} = 0.525 \text{ (52.5 \%)} \quad \text{(b) } \eta$$

(c) The displacement volume is $V_1 - V_2 = m(v_1 - v_2)$, so the mean effective pressure is given by

Continued on next slide

Problem 9-1 continued

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1 (1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right) (300 \text{ K})}{(100 \text{ kPa})} \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right|$$

$$= 0.861 \text{ m}^3/\text{kg}$$

Thus

$$mep = \frac{(709.26 \text{ kJ/kg})}{\left(0.861 \frac{\text{m}^3}{\text{kg}}\right) \left(1 - \frac{1}{9}\right)} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right|$$

$$= 926.7 \text{ kPa} \quad \leftarrow (c) \text{ mep}$$

The data for the required plots are obtained using IT, as follows:

IT Code

```
r = 9
p1 = 100 // kPa
T1 = 300 // K
Q23 / m = 1350 // kJ/kg
m = 1 // assume a unit mass of 1 kg.
```

```
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
s2 = s_TP("Air", T2, p2)
v2 = v_TP("Air", T2, p2)
u2 = u_T("Air", T2)
v3 = v2
m * (u3 - u2) = Q23 - W23
W23 = 0
u3 = u_T("Air", T3)
v3 = v_TP("Air", T3, p3)
s3 = s_TP("Air", T3, p3)
v4 = r * v3
s4 = s3
v4 = v_TP("Air", T4, p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)
```

```
Wcycle = Qcycle
Wcycle / m = Q23 / m - Q41 / m
Q41 / m = u4 - u1
eta = Wcycle / Q23
V1 = v1 * m
V2 = v2 * m
mep = Wcycle / (V1 - V2)
```

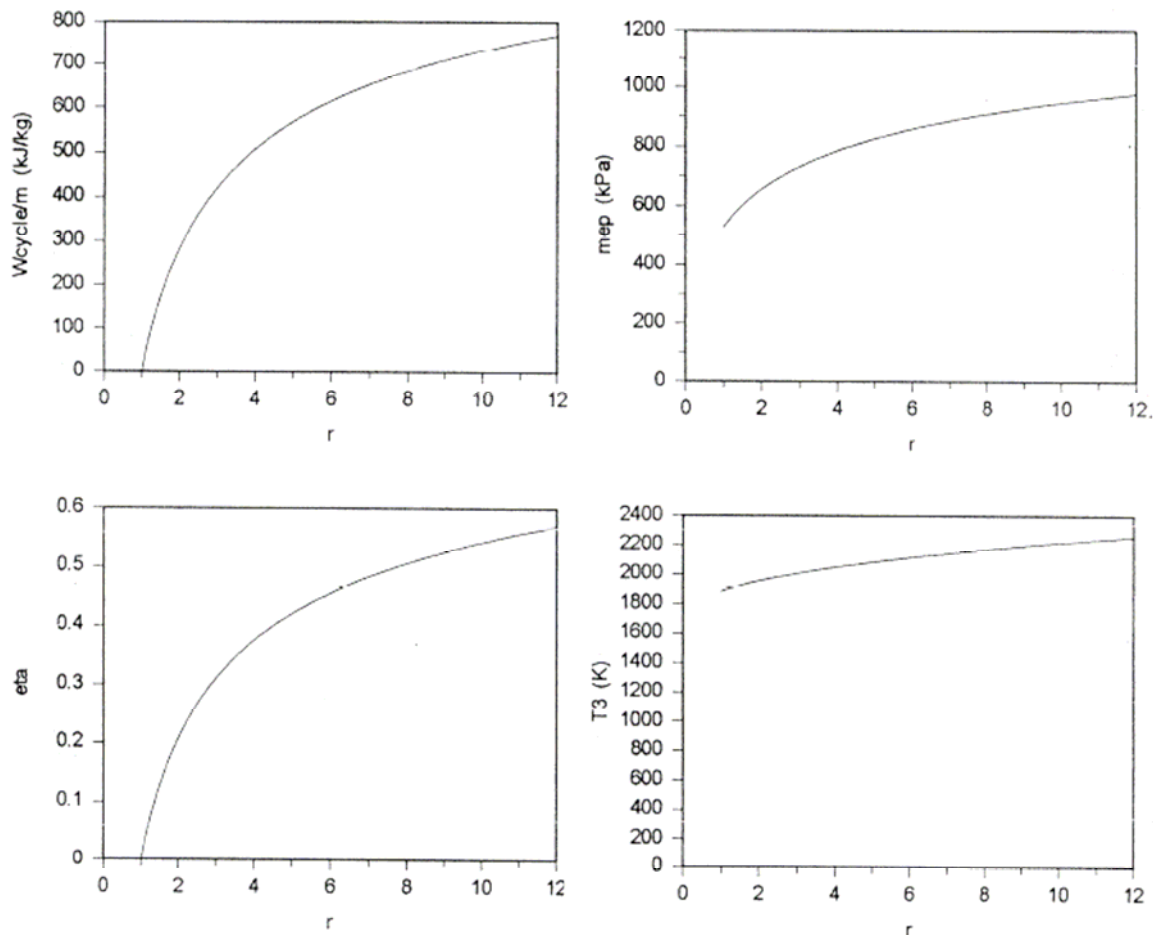
IT Results for $r = 9$

```
T2 = 702.8 K
T3 = 2194 K
T4 = 1111 K
u1 = 213.9 kJ/kg
u2 = 514.3 kJ/kg
u3 = 1864 kJ/kg
u4 = 854.4 kJ/kg
v1 = 0.861 m³/kg
Q41/m = 640.5 kJ/kg
Wcycle/m = 709.5 kJ/kg
eta = 0.5256
mep = 927.1 kPa
```

Continued on next slide

Problem 9-1 continued

PLOTS:



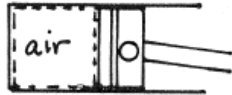
Thermal efficiency increases with increasing compression ratio. Since the heat addition is constant, the net work of the cycle increases, as expected. Also, the maximum cycle temperature increases, since the temperature at the end of compression, T_2 , goes up with increasing r . The mep increases as well for similar reasons.

PROBLEM 9.2

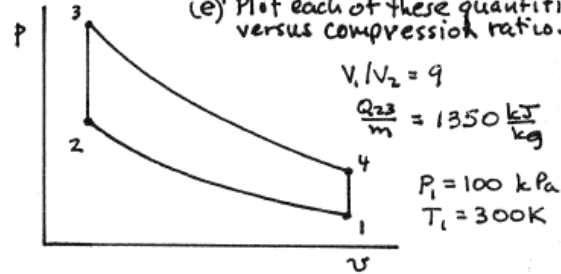
KNOWN: A cold air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the net work per unit mass of air, (b) the thermal efficiency, and (c) the mean effective pressure, and (d) the maximum cycle temperature.
(e) Plot each of these quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1. Also, assume constant specific heats evaluated at $T = 300\text{K}$.



ANALYSIS: First, determine the temperature at each principal state of the cycle. $T_1 = 300\text{K}$ is given.

State 2: For isentropic compression, $T_2 = (v_1/v_2)^{k-1} T_1 = 722.5\text{K}$
Where $k = 1.4$ from Table A-20.

State 3: The temperature T_3 is found using the energy balance for process 2-3:

$$m(u_3 - u_2) = Q_{23} - W_{23} \Rightarrow m c_v (T_3 - T_2) = Q_{23}$$

$$T_3 = \frac{(Q_{23}/m)}{c_v} + T_2 = \frac{1350 \text{ kJ/kg}}{0.718 \text{ kJ/kg}\cdot\text{K}} + 722.5 \text{ K} = 2602.7 \text{ K} \quad T_3$$

State 4: For the isentropic expansion,

$$T_4 = \left(\frac{v_2}{v_4}\right)^{k-1} T_3 = \left(\frac{v_2}{v_1}\right)^{k-1} T_3 = \left(\frac{1}{9}\right)^{k-1} (2602.7) = 1080.8 \text{ K}$$

(a) To find the net work, note that $W_{\text{cycle}} = Q_{\text{cycle}}$, so

$$\frac{W_{\text{cycle}}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - c_v (T_4 - T_1)$$

$$= 1350 - (0.718)(1080.8 - 300) = 789.4 \text{ kJ/kg} \quad W_{\text{cycle}}$$

(b) The thermal efficiency is $\eta = \frac{W_{\text{cycle}}/m}{Q_{23}/m} = 0.585 \text{ (58.5\%)} \quad \eta$

(c) The mean effective pressure is given by

$$p_{\text{mep}} = \frac{W_{\text{cycle}}}{v_1 - v_2} = \frac{W_{\text{cycle}}}{m(v_1 - v_2)} = \frac{W_{\text{cycle}}/m}{v_1(1 - v_2/v_1)}$$

Evaluating v_1 ,

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kJ/kg}\cdot\text{K})(300 \text{ K})}{(100 \text{ kPa})} \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}} \right|$$

$$= 0.861 \text{ m}^3/\text{kg}$$

Thus

$$p_{\text{mep}} = \frac{(789.4 \text{ kJ/kg})}{(0.861 \text{ m}^3/\text{kg})(1 - \frac{1}{9})} \left| \frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| = 1031.4 \text{ kPa} \quad p_{\text{mep}}$$

Continued on next slide

Problem 9-2 continued

The data for the required plots are obtained using IT, as follows:

IT Code

```
r = 9
p1 = 100 // kPa
T1 = 300 // K
Q23 / m = 1350 // kJ/kg
m = 1 // assume a unit mass of 1 kg.
k = 1.4
cv = 0.718 // kJ/kg·K
```

```
T2 = T1 * r^(k-1)
m * cv * (T3 - T2) = Q23 - W23
W23 = 0
T4 = T3 * (1 / r)^(k - 1)
```

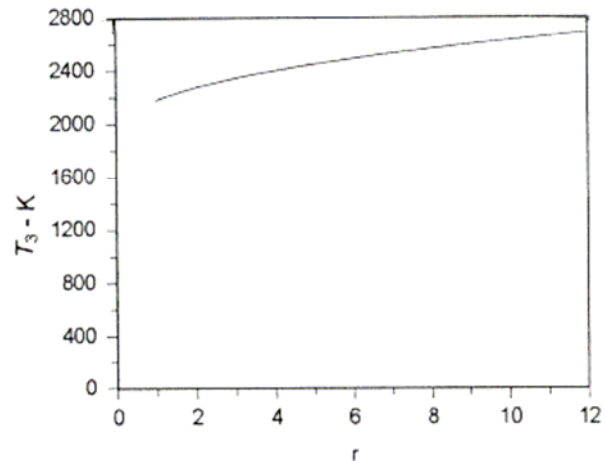
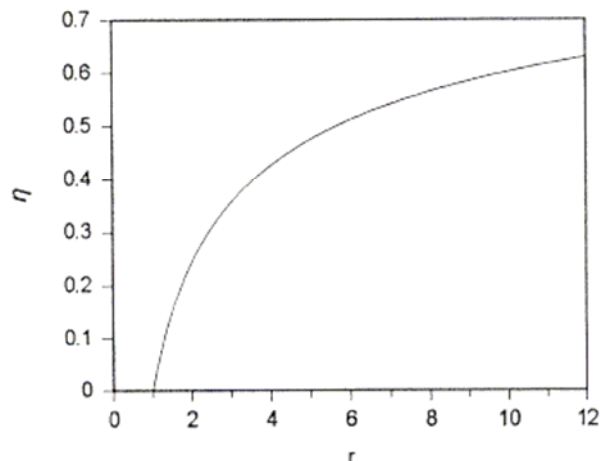
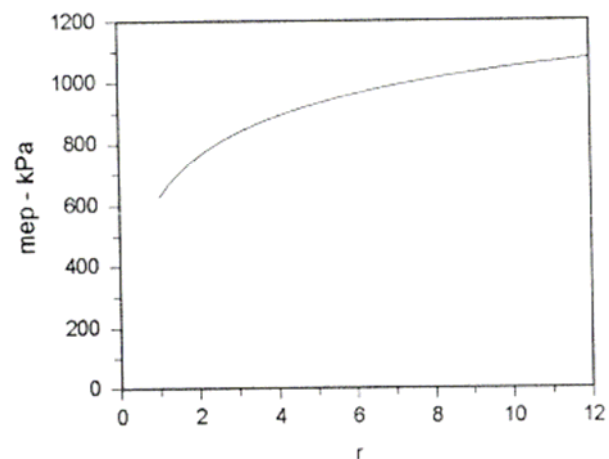
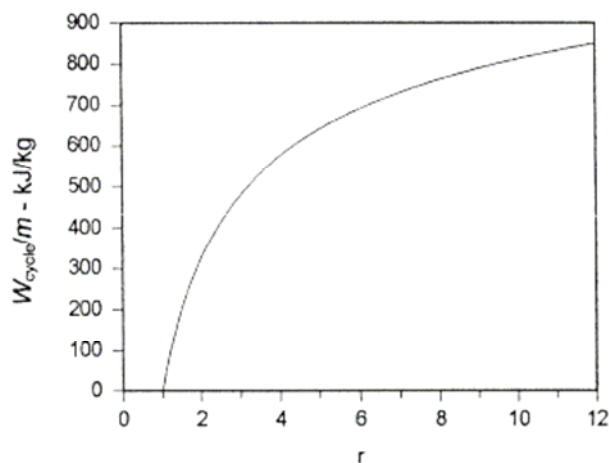
```
Wcycle = Qcycle
Wcycle / m = Q23 / m - Q41 / m
Q41 / m = cv * (T4 - T1)
eta = Wcycle / Q23
```

```
v1 = v_TP("Air", T1, p1)
V1 = v1 * m
mep = Wcycle / (V1 * (1 - 1 / r))
```

IT Results for $r = 9$

```
T2 = 722.5 K
T3 = 2603 K
T4 = 1081 K
Q41/m = 560.6 kJ/kg
Wcycle/m = 789.4 KJ/kg
η = 0.5848
mep = 1032 kPa
```

Plots:

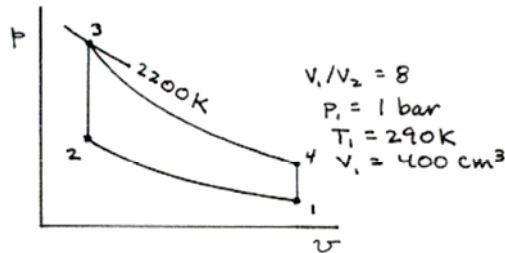
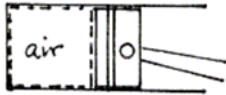


PROBLEM 9.3

KNOWN: An air-standard Otto cycle has a known compression ratio, a specified state at the beginning of compression, and a specified maximum temperature.

FIND: Determine (a) the heat added, (b) net work, (c) thermal efficiency, (d) mean effective pressure. Also, (e) develop a full accounting of the energy transferred during heat addition and (f) devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1.
Also, let $T_0 = 290\text{ K}$ and $p_0 = 1\text{ bar}$.

ANALYSIS: First, fix each of the principal states of the cycle (Table A-22).

State 1: $P_1 = 1\text{ bar}$, $T_1 = 290\text{ K} \Rightarrow u_1 = 206.91\text{ kJ/kg}$, $v_{r1} = 676.1$

State 2: For isentropic compression, $v_{r2} = v_{r1}(V_1/V_2) = \frac{676.1}{8} = 84.5125$
Thus, $T_2 = 652.4\text{ K}$, $u_2 = 475.11\text{ kJ/kg}$

State 3: $T_3 = 2200\text{ K} \Rightarrow u_3 = 1872.4\text{ kJ/kg}$, $v_{r3} = 2.012$

State 4: For isentropic expansion, $v_{r4} = v_{r3}(V_4/V_3) = v_{r3}(V_1/V_2) = 16.096$
Thus, $T_4 = 1159.3\text{ K}$, $u_4 = 897.3\text{ kJ/kg}$

(a) To find the heat addition, first determine the mass of air

$$m = \frac{P_1 V_1}{RT_1} = \frac{(1\text{ bar})(400\text{ cm}^3)}{\left(\frac{8.314\text{ kJ}}{28.97\text{ kg}\cdot\text{K}}\right)(290\text{ K})} \left| \frac{10^5\text{ N/m}^2}{1\text{ bar}} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right| \left| \frac{1\text{ m}^3}{10^6\text{ cm}^3} \right|$$

$$= 4.806 \times 10^{-4}\text{ kg}$$

Thus $m(u_3 - u_2) = Q_{23} - \cancel{W_{23}^0}$

$$Q_{23} = m(u_3 - u_2) = (4.806 \times 10^{-4})(1872.4 - 475.11) = 0.6715\text{ kJ} \leftarrow Q_{23}$$

(b) To find the net work, note that $W_{\text{cycle}} = Q_{\text{cycle}}$, so

$$W_{\text{cycle}} = Q_{23} - Q_{41} = Q_{23} - m(u_4 - u_1) = 0.6715 - (4.806 \times 10^{-4})(897.3 - 206.91) = 0.3397\text{ kJ} \leftarrow W_{\text{cycle}}$$

(c) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{23}} = \frac{0.3397}{0.6715} = 0.506 \text{ (50.6\%)} \leftarrow \eta$$

(d) The mean effective pressure is given by

$$mep = \frac{W_{\text{cycle}}}{V_1 - V_2} = \frac{W_{\text{cycle}}}{V_1(1 - V_2/V_1)}$$

Inserting values

$$mep = \frac{(0.3397\text{ kJ})}{(400\text{ cm}^3)(1 - \frac{1}{8})} \left| \frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}} \right| \left| \frac{1\text{ bar}}{10^5\text{ N/m}^2} \right| \left| \frac{10^6\text{ cm}^3}{1\text{ m}^3} \right|$$

$$= 9.71\text{ bar} \leftarrow mep$$

Continued on next slide

Problem 9-3 continued

PROBLEM 9-3 (contd.)

(e) Since the heat addition process 2-3 involves no work or volume change and occurs without internal irreversibilities, an exergy balance reduces to read

$$\textcircled{1} \quad E_3 - E_2 = \underbrace{\int_2^3 \left[1 - \frac{T_0}{T_b}\right] \delta Q}_{E_Q} - \underbrace{[W - p_0 \Delta V]}_{Q_{23}} - \cancel{E_d} \Rightarrow E_Q = E_3 - E_2.$$

Then, with Eq. 7.10, $E_3 - E_2 = (U_3 - U_2) + p_0(V_3 - V_2) - T_0(S_3 - S_2)$

$$\Rightarrow E_Q = Q_{23} - T_0(S_3 - S_2) = Q_{23} - mT_0 \left[s^\circ(T_3) - s^\circ(T_2) - R \ln \frac{P_3}{P_2} \right]$$

Since $V_3 = V_2$, $P_3/P_2 = T_3/T_2$

With data from Table A-22

$$E_Q = 0.6715 - (4.806 \times 10^{-4}) (290) \left[3.9191 - 2.49753 - \frac{8.314}{28.97} \ln \frac{2200}{652.4} \right] = 0.522 \text{ kJ}$$

The net exergy transfer accompanying work is obtained as follows:

Process 1-2: $E_W = W_{12} - p_0 \Delta V$, Process 2-3: $E_W = W_{23} - p_0 \Delta V$

$$\Rightarrow (E_W)_{\text{cycle}} = W_{\text{cycle}} = 0.3397 \text{ kJ}$$

The exergy transfer accompanying heat rejection is obtained as above:

$$E_Q = (U_1 - U_4) + p_0(V_1 - V_4) - T_0(S_1 - S_4) \quad \text{Since } V_1 = V_4, P_1/P_4 = T_1/T_4, \text{ and we get}$$

$$= m \left[(u_1 - u_4) - T_0(s_1^\circ - s_4^\circ - R \ln \frac{T_1}{T_4}) \right] = (4.806 \times 10^{-4}) \left[(206.94 - 897.3) - 290 \left(1.66802 - 3.13845 - \frac{8.314}{28.97} \ln \frac{290}{1157.3} \right) \right] = -0.1823 \text{ kJ}$$

In Summary:

Exergy In: $0.522 \text{ kJ (100\%)}$

Disposition of the Exergy:

• Exergy Out		
- net work	0.3397	(65.1%)
- heat transfer	0.1823	(34.9%)

$\textcircled{2}$ Exergy Destroyed $\frac{0}{0.522}$

f. An exergetic efficiency in this case can be written as the ratio of the desired exergy output (exergy transfer accompanying the net work developed) to the exergy added. By inspection of the above summary, we get $\epsilon = 65.1\%$

1. Alternatively, for the internally reversible process we can write

$$E_Q = \int_2^3 \left[1 - \frac{T_0}{T_b}\right] \delta Q = Q_{23} - T_0 \int_2^3 \left(\frac{\delta Q}{T}\right)_{\text{int}} = Q_{23} - T_0(S_3 - S_2)$$

2. Significant internal irreversibilities are present in actual internal combustion engines. Accordingly, significant exergy destruction occurs within such engines and the exergetic efficiency would be much less than determined here.

PROBLEM 9.4

The data for the required plots are obtained using IT, as follows. See Problem 9.3 for sample calculations for the case of $r = 8$.

IT Code

```
r = 8
p1 = 1 // bar
T1 = 290 // K
V1 = 400 // cm³
T3 = 2200 // K
```

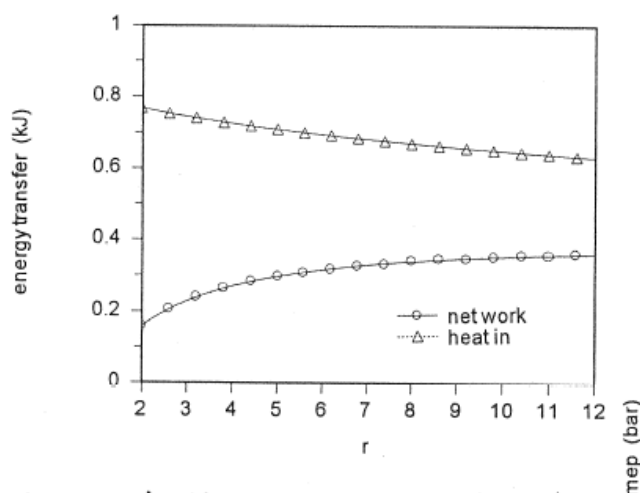
```
v1 = v_TP("Air", T1, p1)
v1 = (V1 / 10^6) / m
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
s2 = s_TP("Air", T2, p2)
v2 = v_TP("Air", T2, p2)
u2 = u_T("Air", T2)
v3 = v2
u3 = u_T("Air", T3)
v3 = v_TP("Air", T3, p3)
s3 = s_TP("Air", T3, p3)
v4 = r * v3
s4 = s3
v4 = v_TP("Air", T4, p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)
```

```
m * (u3 - u2) = Q23 - W23 // kJ
W23 = 0
Wcycle = Qcycle // kJ
Wcycle / m = Q23 / m - Q41 / m
Q41 / m = u4 - u1
eta = Wcycle / Q23
mep = Wcycle * (10^3 / 10^5) / ((V1 / 10^6) * (1 - 1 / r)) // bar
```

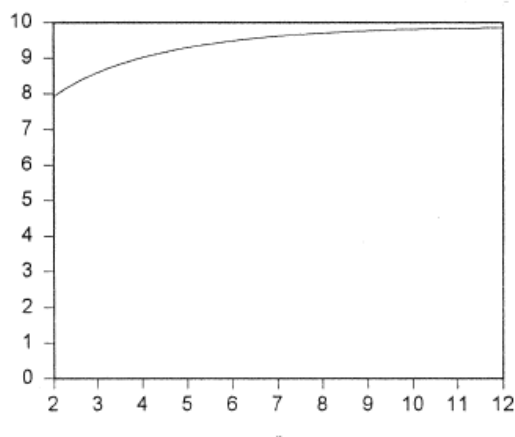
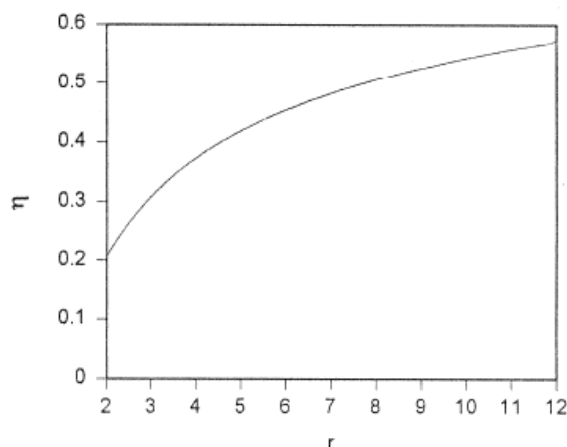
IT results for $r = 8$

```
Q4 = 0.3312 kJ
T2 = 652.4 K
T4 = 1158 K
u1 = 206.8 kJ
u2 = 474.9 kJ
u3 = 1871 kJ
u4 = 896 kJ
T3 = 2200 K
Q23 = 0.6707 kJ
Wcycle = 0.3395 kJ
eta = 0.5062
mep = 9.7 bar
```

PLOTS:



As seen in the accompanying plots, both η and mep increase with increasing compression ratio, r . Since T_3 is constant, the heat input decreases and the net work increases with r .

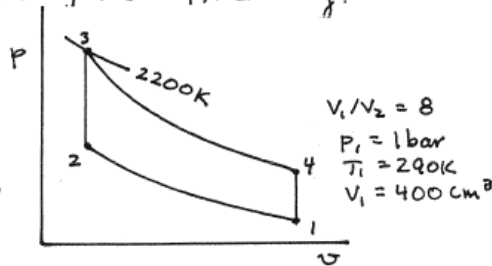
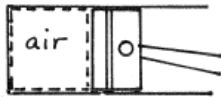


PROBLEM 9.5

KNOWN: A cold air-standard Otto cycle has a known compression ratio, a specified state at the beginning of compression, and a specified maximum temperature.

FIND: Determine (a) the heat added, (b) net work, (c) thermal efficiency, (d) mean effective pressure. Also, (e) develop a full exergy accounting of the exergy transferred during heat addition and (f) devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as Example 9.1
Also, assume constant specific heats evaluated at 300 K
And let $T_0 = 290 \text{ K}$, $P_0 = 1 \text{ bar}$.

ANALYSIS: First, determine the temperature at each principal state of the cycle. $T_1 = 290 \text{ K}$ is given.

State 2: For isentropic compression, $T_2 = (V_1/V_2)^{k-1} T_1 = 666.2 \text{ K}$
where $k = 1.4$ from Table A-20

State 3: $T_3 = 2200 \text{ K}$ is given.

State 4: For isentropic expansion,

$$T_4 = \left(\frac{V_3}{V_4}\right)^{k-1} T_3 = \left(\frac{V_2}{V_1}\right)^{k-1} T_3 = 957.6 \text{ K}$$

(a) To find the heat addition, first determine the mass of air

$$m = \frac{P_1 V_1}{R T_1} = \frac{(1 \text{ bar})(400 \text{ cm}^3)}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}}\right)(290 \text{ K})} \left\| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right\| \left\| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right\| \left\| \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} \right\| = 4.806 \times 10^{-4} \text{ kg}$$

Then

$$\begin{aligned} m(u_3 - u_2) &= Q_{23} - \cancel{W_{23}} \Rightarrow m c_v (T_3 - T_2) = Q_{23} \\ Q_{23} &= (4.806 \times 10^{-4} \text{ kg}) \left(0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right) (2200 - 666.2) \text{ K} \\ &= 0.5293 \text{ kJ} \end{aligned}$$

(b) To find the net work, note that $W_{\text{cycle}} = Q_{\text{cycle}}$, so

$$\begin{aligned} W_{\text{cycle}} &= Q_{23} - Q_{41} = Q_{23} - m c_v (T_4 - T_1) \\ &= 0.5293 - (4.806 \times 10^{-4} \text{ kg}) (0.718) (957.6 - 290) \\ &= 0.2989 \text{ kJ} \end{aligned}$$

(c) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{23}} = \frac{0.2989}{0.5293} = 0.565 \text{ (56.5\%)}$$

(d) The mean effective pressure is given by

$$mep = \frac{W_{\text{cycle}}}{V_1 - V_2} = \frac{W_{\text{cycle}}}{V_1 (1 - V_2/V_1)}$$

Inserting values

$$\begin{aligned} mep &= \frac{(0.2989 \text{ kJ})}{(400 \text{ cm}^3)(1 - 1/8)} \left\| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right\| \left\| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right\| \left\| \frac{10^6 \text{ cm}^3}{1 \text{ m}^3} \right\| \\ &= 8.54 \text{ bar} \end{aligned}$$

Continued on next slide

Problem 9-5 continued

(e) Since the heat addition process involves no internal irreversibilities, the exergy transfer accompanying heat transfer can be expressed as

$$E_Q = \int_2^3 \left[1 - \frac{T_0}{T_b}\right] \delta Q = Q_{23} - T_0 \int_2^3 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} = Q_{23} - T_0(S_3 - S_2)$$

The same result can be obtained by reducing an exergy balance:

$$E_3 - E_2 = \underbrace{\int_2^3 \left[1 - \frac{T_0}{T_b}\right] \delta Q}_{E_Q} - \cancel{[W - P_0 \Delta V]} - \cancel{E_d}^0 \Rightarrow E_Q = E_3 - E_2. \text{ Then, with Eq. 7.10}$$

$$E_Q = \underbrace{(U_3 - U_2)}_{Q_{23}} + P_0(V_3 - V_2) - T_0(S_3 - S_2) = Q_{23} - T_0(S_3 - S_2). \text{ With } c_p \text{ from Table A-20}$$

$$L = m[c_p \ln \frac{T_3}{T_2} - R \ln \frac{P_3}{P_2}]$$

$$E_Q = 0.5293 - \frac{(4.806)}{10^4} (290) \left[1.005 \ln \frac{2200}{666.2} - \frac{8.314}{28.97} \ln \frac{2200}{666.2} \right] = 0.4097 \text{ kJ} \quad \hookrightarrow = \frac{T_3}{T_2} \text{ since } V_3 = V_2$$

The net exergy transfer accompanying work is obtained as follows:

$$\text{Process 1-2: } E_W = W_{12} - P_0 \Delta V^0, \text{ Process 2-3: } E_W = W_{23} - P_0 \Delta V^0$$

$$\Rightarrow (E_W)_{\text{cycle}} = W_{\text{cycle}} = 0.2989 \text{ kJ}$$

The exergy transfer accompanying heat rejection is obtained as above:

$$E_Q = U_1 - U_4 + P_0(V_1 - V_4) - T_0(S_1 - S_4)$$

$$= m \left[c_v(T_1 - T_4) - T_0 \left(c_p \ln \frac{T_1}{T_4} - R \ln \frac{P_1}{P_4} \right) \right]$$

$$= (4.806 \times 10^{-4}) \left[0.718(290 - 957.6) - 290 \left(1.005 \ln \frac{290}{957.6} - \frac{8.314}{28.97} \ln \frac{290}{957.6} \right) \right]$$

$$= -0.1108 \text{ kJ}$$

In summary:

$$\text{Exergy In: } 0.4097 \text{ kJ} \quad (100\%)$$

Disposition of Exergy:

• Exergy out	0.2989	(73%)
- net work	0.1108	(27%)
- heat transfer		

① • Exergy Destroyed 0

(f) An exergetic efficiency in this case can be written as the ratio of the desired exergy output (exergy transfer accompanying the net work developed) to the exergy added. By inspection of the above summary, we get $\epsilon = 73\%$.

- Significant internal irreversibilities are present in actual internal combustion engines. Accordingly, significant exergy destruction occurs in such engines and the exergetic efficiency would be much less than determined here.

PROBLEM 9.6

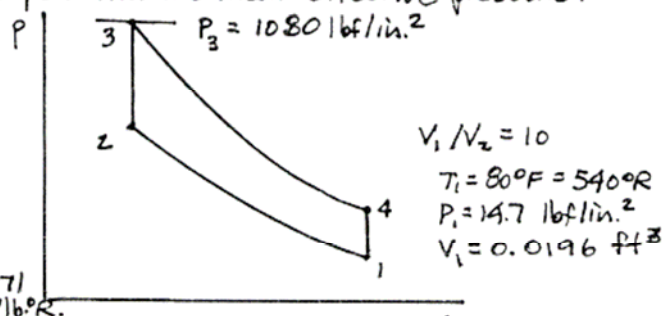
KNOWN: A four-cylinder, four-stroke engine operates at a known RPM. The processes in each cylinder are modeled as a cold air-standard Otto cycle with a specified state at the beginning of compression, a known compression ratio, and a specified maximum cycle pressure.

END: Determine the power developed and the mean effective pressure.

SCHEMATIC & GIVEN DATA:

①

Four-cylinder
four-stroke
2800 RPM



ENGINEERING MODEL: See

Example 9.1. Also, $k = 1.4$ and $c_v = 0.171 \text{ Btu/lb} \cdot ^\circ\text{R}$.

ANALYSIS: First, determine each temperature in the cycle.

State 2: Using Eq. 9.6, $T_2 = (V_1/V_2)^{k-1} T_1 = (10)^{0.4} (540^\circ\text{R}) = 1356.4^\circ\text{R}$

Also, $P_2 = (V_1/V_2)^k P_1 = (10)^{1.4} (14.7) = 369.2 \text{ lbf/in.}^2$

State 3: $V_3 = V_2 \Rightarrow T_3 = (P_3/P_2) T_2 = (1080/369.2)(1356.4) = 3967.8^\circ\text{R}$

State 4: $T_4 = (V_3/V_4)^{k-1} T_4 = (1/10)^{0.4} (3967.8) = 1579.6^\circ\text{R}$

To get the power, first determine the net work per cycle. The mass in the cylinder is

$$m = \frac{P_1 V_1}{R T_1} = \frac{(14.7 \text{ lbf/in.}^2)(0.0196 \text{ ft.}^3)}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)(540^\circ\text{R})} \left| \frac{144 \text{ in.}^2}{1 \text{ ft.}^2} \right| = 0.001441 \text{ lb}$$

$$W_{\text{cycle}} = W_{12} + W_{34}$$

$$= m c_v [(T_1 - T_2) + (T_3 - T_4)]$$

$$= (0.001441 \text{ lb})(0.171 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}) [(540 - 1356.4) + (3967.8 - 1579.6)]^\circ\text{R}$$

$$= 0.3873 \text{ Btu/cycle}$$

$$\dot{W}_{\text{net}} = (4 \text{ cyl}) \left(\frac{2800 \text{ cycle}}{2 \text{ min}} \right) (0.3872 \text{ Btu/cycle}) \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

$$= 51.1 \text{ hp} \leftarrow \dot{W}_{\text{net}}$$

The mean effective pressure is

$$mep = \frac{W_{\text{cycle}}}{V_1 - V_2} = \frac{W_{\text{cycle}}}{V_1 (1 - V_2/V_1)}$$

$$= \frac{0.3873 \text{ Btu}}{(0.0196 \text{ ft.}^3)(1 - 0.1)} \left| \frac{7.78 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}} \right| \left| \frac{1 \text{ ft.}^2}{144 \text{ in.}^2} \right| = 118.6 \text{ lbf/in.}^2 \leftarrow mep$$

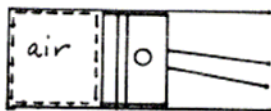
1. In early printing of the Sixth Edition, the maximum pressure and the volume at the beginning of compression were both given incorrectly. The correct values are $P_3 = 1080 \text{ lbf/in.}^2$ and $V_1 = 0.0196 \text{ ft.}^3$. We regret the error.

PROBLEM 9.7

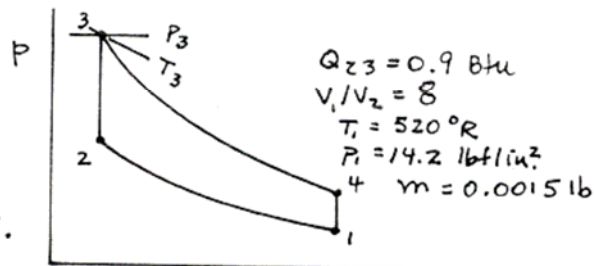
KNOWN: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition and the mass of air are given.

FIND: Determine (a) the maximum temperature, (b) the maximum pressure, and (c) the thermal efficiency. (d) Plot these quantities versus compression ratio ranging from 2 to 12.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1.



ANALYSIS: (a) To determine T_3 , begin by fixing state 2. For the isentropic compression

$$v_{r2} = v_{r1} \left(\frac{V_2}{V_1} \right) = \frac{158.58}{8} = 19.823$$

Where data for air is obtained from Table A-22E. Thus, for $v_{r2} = 19.823$, $T_2 \approx 1170.5^\circ\text{R}$, $u_2 \approx 203.58 \text{ Btu/lb}$. The energy balance for process 2-3 is

$$m(u_3 - u_2) = Q_{23} - W_{23}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = \frac{0.9 \text{ Btu}}{0.0015 \text{ lb}} + 203.58 \frac{\text{Btu}}{\text{lb}} = 803.58 \text{ Btu/lb}$$

Thus, from Table A-22E; $T_3 \approx 3954.9^\circ\text{R}$ T_3

(b) To find P_3 , first determine P_2 . For the isentropic compression

$$P_2 = P_1 \frac{P_{r2}}{P_{r1}} = (14.2) \left(\frac{21.922}{1.2147} \right) = 256.27 \text{ lbf/in}^2$$

Now, since $V_3 = V_2$; $P_3 = \frac{T_3}{T_2} \cdot P_2 = 865.89 \text{ lbf/in}^2$ P_3

(c) To determine the thermal efficiency, first find the net work. That is

$$\frac{W_{\text{cycle}}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (u_4 - u_1)$$

To fix state 4, consider the expansion process

$$v_{r4} = v_{r3} \left(\frac{V_4}{V_3} \right) = v_{r3} \left(\frac{V_1}{V_2} \right) = (169.55)(8) = 3.7564$$

Thus, $T_4 \approx 2084.1^\circ\text{R}$ and $u_4 \approx 385.26$, and the net work is

$$\frac{W_{\text{cycle}}}{m} = \left(\frac{0.9}{0.0015} \right) - (385.26 - 88.62) = 303.33 \text{ Btu/lb}$$

Finally, the thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{23}/m} = \frac{303.33}{(0.9/0.0015)} = 0.506 \text{ (50.6 \%)} \quad \eta$$

Continued on next slide

Problem 9-7 continued

(d) The data for the required plots are obtained using IT, as follows:

IT Code

```
r = 8
p1 = 14.2 // lbf/in2
T1 = 520 // °R
Q23 = 0.9 // Btu
m = 0.0015 // lb
```

```
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
s2 = s_TP("Air", T2, p2)
v2 = v_TP("Air", T2, p2)
u2 = u_T("Air", T2)
v3 = v2
m * (u3 - u2) = Q23 - W23
W23 = 0
u3 = u_T("Air", T3)
v3 = v_TP("Air", T3, p3)
s3 = s_TP("Air", T3, p3)
v4 = r * v3
s4 = s3
```

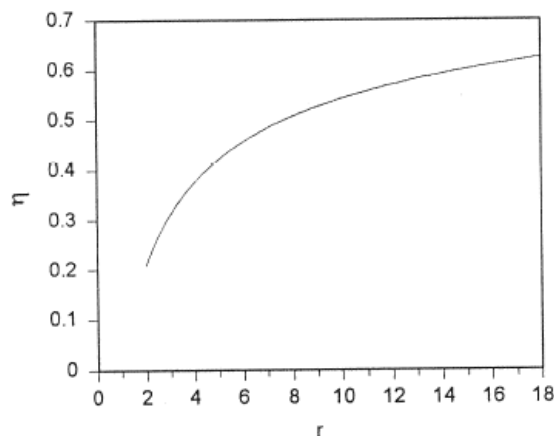
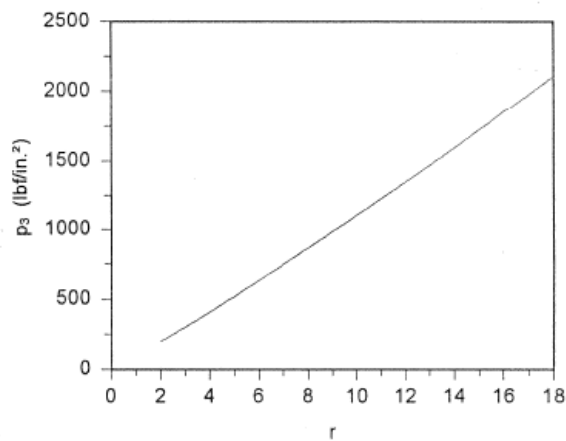
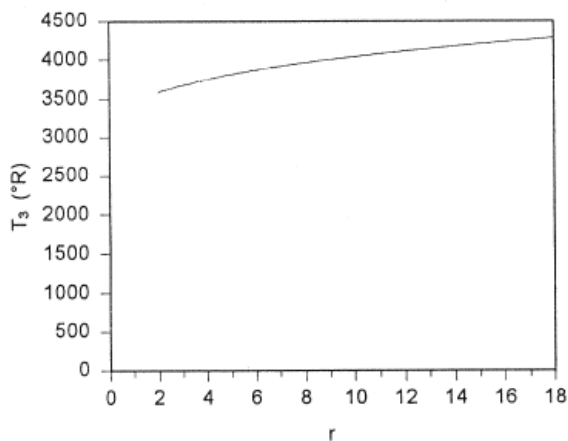
```
v4 = v_TP("Air", T4, p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)
```

```
Wcycle = Q23 - Q41
Q41 / m = u4 - u1
eta = Wcycle / Q23
```

IT Results for $r = 8$

```
T2 = 1170 °R
T3 = 3959 °R
T4 = 2084 °R
p2 = 255.6 lbf/in.2
p3 = 864.9 lbf/in.2
u1 = 88.73 Btu/lb
u2 = 203.5 Btu/lb
u3 = 803.5 Btu/lb
u4 = 385 Btu/lb
Wcycle = 0.4556 Btu
Q41 = 0.4444 Btu
η = 0.5063
```

PLOTS:

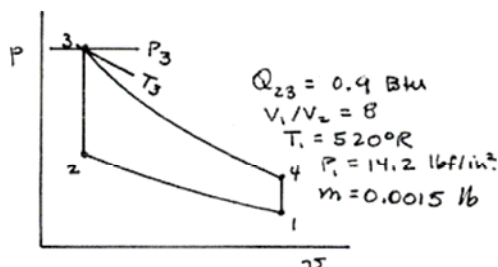


PROBLEM 9.8

KNOWN: A cold air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the maximum temperature, (b) the maximum pressure, and (c) the thermal efficiency. (d) Plot those quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1. Also, assume constant specific heats evaluated at 520°R.

ANALYSIS: To determine T_3 , begin by evaluating T_2 . For the isentropic compression

$$T_2 = \left(\frac{V_1}{V_2} \right)^{k-1} T_1 = (8)^{1.401} (520^\circ\text{R}) = 1197.1^\circ\text{R}$$

where $k = 1.401$ from Table A-20E. The energy balance for process 2-3 is

$$m(u_3 - u_2) = Q_{23} - W_{23} \Rightarrow Q_{23} = m c_v (T_3 - T_2)$$

$$\text{Thus } T_3 = \frac{Q_{23}}{m c_v} + T_2 = \frac{0.9 \text{ Btu}}{(0.0015 \text{ lb})(0.171 \text{ Btu/lb} \cdot ^\circ\text{R})} + 1197.1^\circ\text{R} = 4705.9^\circ\text{R}$$

(b) To find p_3 , first determine p_2 . For the isentropic compression

$$p_2 = \left(\frac{T_2}{T_1} \right)^{\frac{k}{k-1}} p_1 = 261.5 \text{ lbf/in}^2$$

$$\text{Now, since } V_3 = V_2; p_3 = \frac{T_3}{T_2} p_2 = 1028.0 \text{ lbf/in}^2$$

(c) The thermal efficiency can be determined using Eq. 9.8

$$\eta = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{(V_1/V_2)^{k-1}} = 1 - \frac{1}{8^{(1.401-1)}} = 0.566 \quad (56.6\%)$$

(d) The data for the required plots are obtained using IT, as follows:

IT Code

p1 = 14.2 // lbf/in.²

T1 = 520 // °R

r = 8

Q23 = 0.9 // Btu

m = 0.0015 // lb

cv = cv_T("Air", T1)

cp = cp_T("Air", T1)

k = cp / cv

IT results for r = 8

cv = 0.1711 Btu/lb·°R

k = 1.4

T2 = 1196 °R

p2 = 261.2 lbf/in.²

p3 = 1027 lbf/in.²

T3 = 4703 °R

η = 0.5651

(T2 / T1) = r^(k-1)

Q23 = m * cv * (T3 - T2)

(p2 / p1) = (T2 / T1)^(k / (k-1))

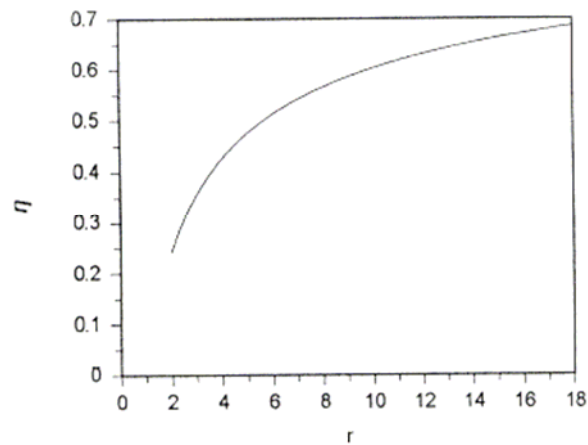
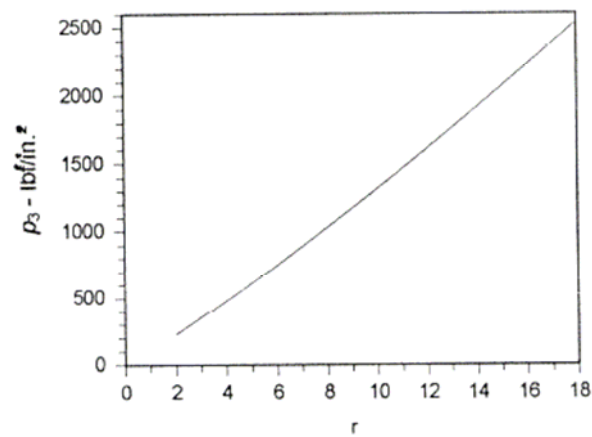
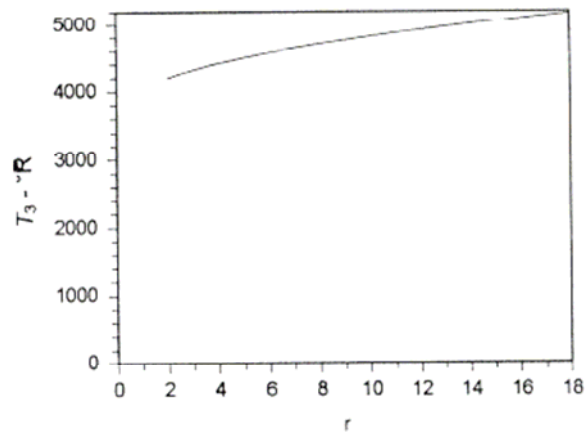
p3 / p2 = T3 / T2

eta = 1 - 1 / (r^(k-1))

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Problem 9-8 continued

PLOTS:

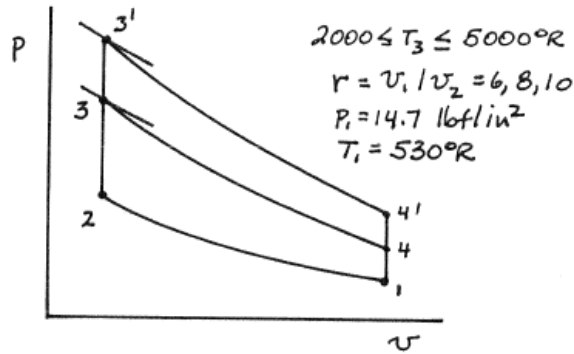
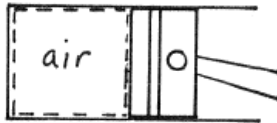


PROBLEM 9.9

KNOWN: An air-standard Otto cycle has known conditions at the beginning of compression. The maximum cycle temperature varies over a given range and the compression ratio takes on specified values.

FIND: Plot thermal efficiency and mean effective pressure versus maximum cycle temperature for each compression ratio.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:
Same as Example 9.1.

ANALYSIS: Sample calculation for $r = 8$, $T_3 = 2000^\circ\text{R}$ using Table A-22E. From Table A-22E, $u_1 = 90.33 \text{ Btu/lb}$, $v_{r1} = 151.38$. For the compression process

$$v_{r2} = \left(\frac{v_2}{v_1}\right) v_{r1} = \left(\frac{1}{8}\right) (151.38) = 18.9225$$

Thus, $T_2 = 1190.7^\circ\text{R}$ and $u_2 = 207.33 \text{ Btu/lb}$.

For various values of T_3 , u_3 and v_{r3} can be determined from Table A-22E. Then, state 4 is fixed using the relation for the isentropic expansion

$$v_{r4} = \left(\frac{v_4}{v_3}\right) v_{r3} = \left(\frac{v_1}{v_2}\right) v_{r3}$$

and u_4 can be evaluated from the tabular data. For $T_3 = 2000^\circ\text{R}$

$$u_3 = 367.61 \text{ Btu/lb}, v_{r3} = 4.258, v_{r4} = 34.064, u_4 = 164.23 \text{ Btu/lb}$$

Continuing for this case, the heat added to the cycle is

$$\frac{Q_{23}}{m} = u_3 - u_2 = 160.28 \text{ Btu/lb}$$

and the heat rejected is

$$\frac{Q_{41}}{m} = u_4 - u_1 = 73.9$$

Thus, the thermal efficiency is

$$\eta = 1 - \frac{Q_{41}/m}{Q_{23}/m} = 0.539 \text{ (53.9\%)} \leftarrow \eta$$

The mean effective pressure is

$$mep = \frac{W_{\text{cycle}}}{v_1 - v_2} = \frac{W_{\text{cycle}}/m}{v_1(1 - v_2/v_1)} = \frac{Q_{23}/m - Q_{41}/m}{v_1(1 - v_2/v_1)}$$

Continued on next slide

Problem 9-9 continued

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot ^\circ\text{R}}\right) (530^\circ\text{R})}{(14.7 \text{ lb}_f/\text{in}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 13.35 \text{ ft}^3/\text{lb}$$

Thus

$$m_{ep} = \frac{(160.28 - 73.9) \text{ Btu/lb}}{(13.35 \text{ ft}^3/\text{lb})(1 - \frac{1}{8})} \left| \frac{778 \text{ ft} \cdot \text{lb}_f}{1 \text{ Btu}} \right| \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 39.95 \text{ lb}_f/\text{in}^2 \leftarrow m_{ep}$$

The data for the required plots are obtained using IT, as follows:

IT Code

$r = 8$
 $p_1 = 14.7 \text{ // lb}_f/\text{in}^2$
 $T_1 = 530 \text{ // } ^\circ\text{R}$
 $T_3 = 2000 \text{ // } ^\circ\text{R}$
 $m = 1 \text{ // assume a unit mass of 1 lb.}$

$v_1 = v_TP(\text{"Air"}, T_1, p_1)$
 $s_1 = s_TP(\text{"Air"}, T_1, p_1)$
 $u_1 = u_T(\text{"Air"}, T_1)$
 $s_2 = s_1$
 $v_2 = v_1 / r$
 $s_2 = s_TP(\text{"Air"}, T_2, p_2)$
 $v_2 = v_TP(\text{"Air"}, T_2, p_2)$
 $u_2 = u_T(\text{"Air"}, T_2)$
 $v_3 = v_2$
 $u_3 = u_T(\text{"Air"}, T_3)$
 $v_3 = v_TP(\text{"Air"}, T_3, p_3)$
 $s_3 = s_TP(\text{"Air"}, T_3, p_3)$
 $v_4 = r * v_3$
 $s_4 = s_3$
 $v_4 = v_TP(\text{"Air"}, T_4, p_4)$
 $s_4 = s_TP(\text{"Air"}, T_4, p_4)$
 $u_4 = u_T(\text{"Air"}, T_4)$

$W_{\text{cycle}} = Q_{23} - Q_{41}$

$Q_{23} = m * (u_3 - u_2)$

$Q_{41} = m * (u_4 - u_1)$

$\eta = W_{\text{cycle}} / Q_{23}$

$V_1 = v_1 * m$

$V_2 = v_2 * m$

$m_{ep} = W_{\text{cycle}} * (778 / 144) / (V_1 - V_2)$

IT Results for $r = 8, T_3 = 2000^\circ\text{R}$

$T_2 = 1191^\circ\text{R}$

$T_4 = 953.6^\circ\text{R}$

$p_2 = 264.2 \text{ lb}_f/\text{in}^2$

$p_3 = 443.8 \text{ lb}_f/\text{in}^2$

$v_1 = 13.35 \text{ ft}^3/\text{lb}$

$Q_{23}/m = 160 \text{ Btu/lb}$

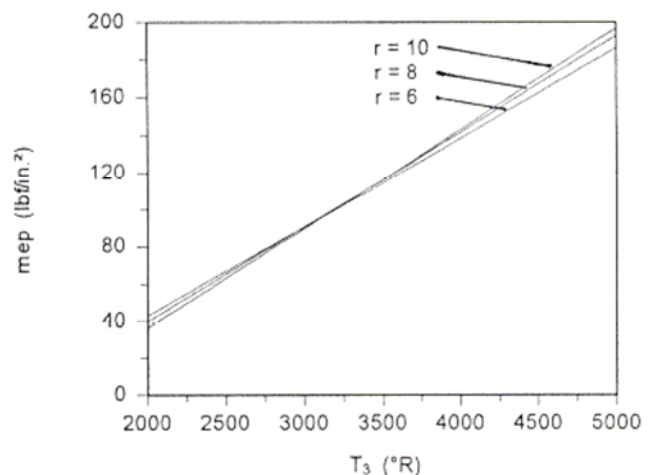
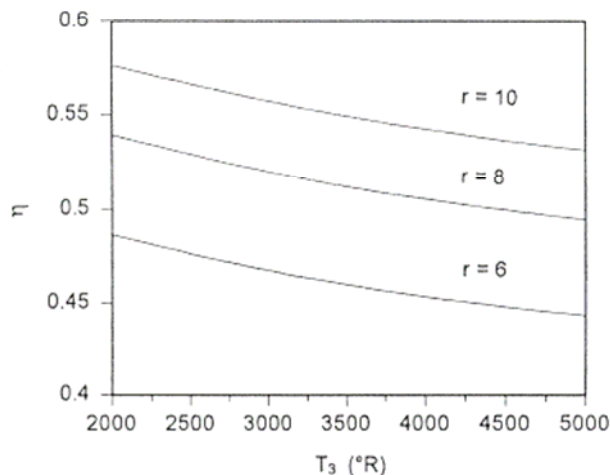
$Q_{41}/m = 73.73 \text{ Btu/lb}$

$W_{\text{cycle}}/m = 86.29 \text{ Btu/lb}$

$\eta = 0.5393$

$m_{ep} = 39.9 \text{ lb}_f/\text{in}^2$

LOTS:



1. Note that IT evaluates $s(T, p)$ directly, and thus does not utilize the special isentropic functions $v_r(T)$ and $p_r(T)$ as in Table A-22E.

PROBLEM 9.10

KNOWN: A cold air-standard Otto cycle has known conditions at the beginning of compression. The maximum cycle temperature varies over a given range and the compression ratio takes on specified values.

FIND: Plot thermal efficiency and mean effective pressure versus maximum cycle temperature for each compression ratio.

SCHEMATIC & GIVEN DATA: See solution to Problem 9.9.

ENGINEERING MODEL: Same as in Example 9.1. Also, assume constant specific heats.

ANALYSIS: First, determine the temperature at each principal state. T_1 is given, and $r = v_1/v_2$ is specified.

State 2: Isentropic compression; $T_2 = (v_1/v_2)^{k-1} T_1$

Also, for a particular value of T_3

State 4: $T_4 = (v_3/v_4)^{k-1} T_3 = (v_2/v_1)^{k-1} T_3$

Thus, with $k=1.4$, c_v can be determined from Table A-20 ($c_v = 0.171 \text{ Btu/lb} \cdot ^\circ\text{R}$).

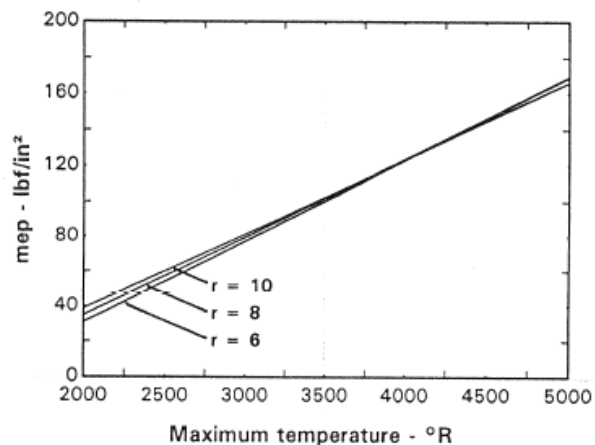
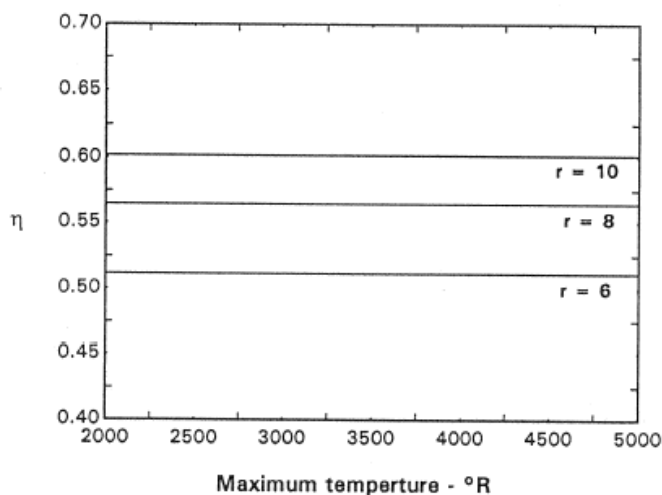
The thermal efficiency is

$$\eta = 1 - \frac{1}{r^{k-1}} \quad (1)$$

and $\frac{Q_{23}}{m} = c_v (T_3 - T_2)$; $\frac{Q_{41}}{m} = c_v (T_4 - T_1) \Rightarrow \frac{W_{\text{cycle}}}{m} = c_v [(T_3 - T_2) - (T_4 - T_1)]$

$$mep = \frac{W_{\text{cycle}}/m}{v_1 (1 - v_2/v_1)} = \frac{c_v [(T_3 - T_2) - (T_4 - T_1)]}{v_1 (1 - 1/r)}$$

Plotting for $r=6, 8, 10$ and maximum cycle temperatures ranging from 2000 to 5000°R:



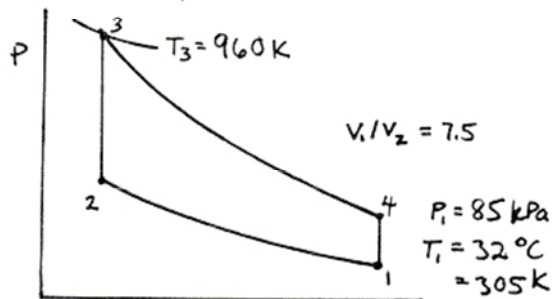
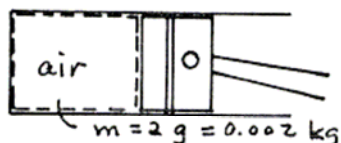
As evident from Eq (1) above, η depends only on r and not on the maximum cycle temperature. However, the mean effective pressure is affected by the maximum cycle temperature for the cold air-standard cycle.

PROBLEM 9.11

KNOWN: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition and the maximum cycle temperature are given.

FIND: Determine (a) the heat rejection, (b) the net work, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1.

ANALYSIS: From the given data, $T_1 = 305 \text{ K}$; $u_1 = 217.67 \text{ kJ/kg}$, $v_{r1} = 596.0$. Also, for $T_3 = 960 \text{ K}$; $u_3 = 725.02 \text{ kJ/kg}$, $v_{r3} = 28.40$. For the isentropic compression

$$v_{r2} = \frac{v_2}{v_1} v_{r1} = \left(\frac{1}{7.5}\right) 596.0 = 79.47$$

Thus, $T_2 = 367.4 \text{ K}$, $u_2 = 486.77 \text{ kJ/kg}$. Similarly, for the expansion

$$v_{r4} = \frac{v_4}{v_3} v_{r3} = \frac{v_1}{v_2} v_{r3} = (7.5) 28.40 = 213.0$$

Thus, $T_4 = 458.7 \text{ K}$, $u_4 = 329.01 \text{ kJ/kg}$.

(a) Consider an energy balance for process 2-3

$$Q_{23} = m(u_3 - u_2) = (0.002 \text{ kg})(725.02 - 486.77) \text{ kJ/kg} = 0.4765$$

And, for process 4-1

$$Q_{41} = m(u_4 - u_1) = (0.002 \text{ kg})(329.01 - 217.67) \text{ kJ/kg} = 0.2227 \text{ kJ} \leftarrow Q_{41}$$

(b) The net work is

$$W_{\text{cycle}} = Q_{23} - Q_{41} = 0.4765 - 0.2227 = 0.2538 \text{ kJ} \leftarrow W_{\text{cycle}}$$

(c) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{23}} = 0.533 \text{ (53.3\%)} \leftarrow \eta$$

(d) To determine the mean effective pressure, first find V_1

$$V_1 = \frac{mRT_1}{P_1} = \frac{(0.002 \text{ kg}) \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (305 \text{ K})}{(85 \text{ kPa})} \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| = 2.06 \times 10^{-3} \text{ m}^3$$

Thus

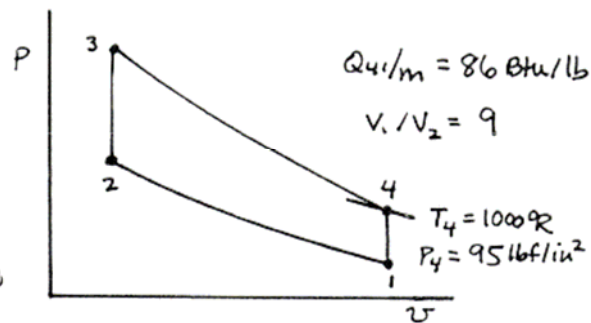
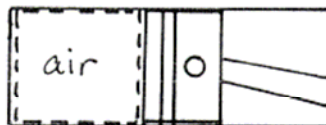
$$m_{\text{ep}} = \frac{W_{\text{cycle}}}{V_1 - V_2} = \frac{W_{\text{cycle}}}{V_1 (1 - V_2/V_1)} = \frac{(0.2538 \text{ kJ})}{(2.06 \times 10^{-3} \text{ m}^3) (1 - 1/7.5)} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| = 142.2 \text{ kPa} \leftarrow m_{\text{ep}}$$

PROBLEM 9.12

KNOWN: A cold air-standard Otto cycle has a known compression ratio and a specified state at the end of the expansion process. The heat rejection per unit mass of air is also given.

FIND: Determine (a) the net work per unit mass of air, (b) the thermal efficiency, and (c) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1. Also, $k = 1.4$.

ANALYSIS: Using the given heat rejection, $Q_{41}/m = u_4 - u_1$. Thus, with $\Delta u = c_v \Delta T$ and $c_v = 0.172 \text{ Btu/lb} \cdot ^\circ\text{R}$ from Table A-20E

$$T_1 = T_4 - \frac{Q_{41}}{m \cdot c_v} = 1000 - \frac{86}{0.172} = 500^\circ\text{R}$$

For the isentropic compression $T_2/T_1 = (v_1/v_2)^{k-1}$. Therefore

$$T_2 = (9)^{1.4-1} (500) = 1204.1^\circ\text{R}$$

Also, for the isentropic expansion

$$T_3 = \left(\frac{v_4}{v_3}\right)^{k-1} T_4 = \left(\frac{v_1}{v_2}\right)^{k-1} T_4 = 2408.2^\circ\text{R}$$

(a) The net work per unit mass of air is

$$\frac{W_{\text{cycle}}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = c_v (T_3 - T_2) - \frac{Q_{41}}{m} = 121.11 \text{ Btu/lb} \quad \frac{W_{\text{cycle}}}{m}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{23}/m} = \frac{W_{\text{cycle}}/m}{c_v (T_3 - T_2)} = \frac{121.11}{207.11} = 0.585 \text{ (58.5\%)} \quad \eta$$

(c) The mean effective pressure is given by

$$mep = \frac{W_{\text{cycle}}}{v_1 - v_2} = \frac{W_{\text{cycle}}/m}{v_1 (1 - v_2/v_1)}$$

Evaluating the specific volume ($v_1 = v_4$)

$$v_4 = \frac{RT_4}{P_4} = \frac{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}})(1000^\circ\text{R})}{(95 \text{ lbf/in}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 3.898 \text{ ft}^3/\text{lb}$$

Thus

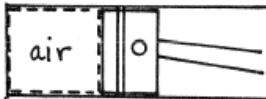
$$mep = \frac{(121.11 \text{ Btu/lb}) \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| \left| \frac{778 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}} \right|}{(3.898 \text{ ft}^3/\text{lb})(1 - 1/9)} = 188.8 \text{ lbf/in}^2 \quad mep$$

PROBLEM 9.13

KNOWN: An air-standard Otto cycle is modified by replacing the isentropic compression and expansion processes with polytropic processes having $n=1.3$. The compression ratio, state at the beginning of compression, and the maximum cycle temperature are all known.

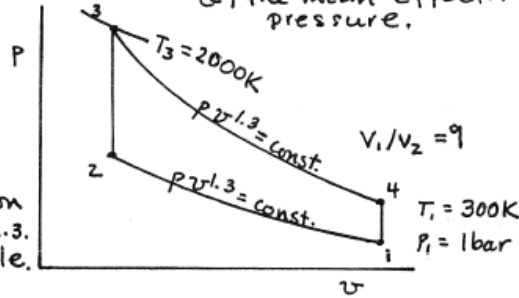
FIND: Determine (a) the heat transfer and work, per unit mass, for each process in the cycle, (b) the thermal efficiency, and (c) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The air is the closed system. (2) The compression and expansion processes are polytropic, each with $n=1.3$. (3) All processes are internally reversible. (4) The air behaves as an ideal gas. (5) Kinetic and potential energy effects are negligible.



ANALYSIS: Begin by fixing each of the four principal states. With $T_1 = 300\text{K}$, Table A-22 gives, $u_1 = 214.07\text{ kJ/kg}$. Next, using Eq. 3.56 for the polytropic compression

$$T_2 = T_1 \left[\frac{V_1}{V_2} \right]^{n-1} = (300\text{K}) [9]^{0.3} = 580\text{K} \quad \text{with Table A-22, } u_2 = 419.55\text{ kJ/kg.}$$

Also, since $T_3 = 2000\text{K}$, $u_3 = 1678.7\text{ kJ/kg}$. For the polytropic expansion

$$T_4 = T_3 \left[\frac{V_3}{V_4} \right]^{n-1} = 2000 \left[\frac{1}{9} \right]^{0.3} = 1035\text{K}, \quad \text{and } u_4 = 789.05\text{ kJ/kg.}$$

(a) Consider each process in series, beginning with process 1-2. With Eq. 3.57

$$\frac{W_{12}}{m} = \int_1^2 P dv = \frac{R(T_2 - T_1)}{1-n} = \frac{(8.314/28.97\text{ kJ/kg}\cdot\text{K})(580 - 300)\text{K}}{(1-1.3)} = -267.9\text{ kJ/kg}$$

An energy balance gives

$$\frac{Q_{12}}{m} = (u_2 - u_1) + \frac{W_{12}}{m} = (419.55 - 214.07) - 267.9 = -62.42\text{ kJ/kg}$$

Process 2-3: $W_{23} = 0$ and an energy balance gives

$$\frac{Q_{23}}{m} = (u_3 - u_2) = (1678.7 - 419.55) = 1259.15\text{ kJ/kg}$$

Process 3-4: Using Eq. 3.57

$$\frac{W_{34}}{m} = \int_3^4 P dv = \frac{R(T_4 - T_3)}{1-n} = \frac{(8.314/28.97)(1035 - 2000)}{-(0.3)} = 923.1\text{ kJ/kg}$$

An energy balance gives $Q_{34}/m = (u_4 - u_3) + W_{34}/m = (789.05 - 1678.7) + 923.1 = 33.45\text{ kJ/kg}$

Process 4-1: $W_{41} = 0$ and an energy balance gives

$$\textcircled{1} \quad \frac{Q_{41}}{m} = u_1 - u_4 = 214.07 - 789.05 = -574.98\text{ kJ/kg}$$

Continued on next slide

Problem 9-13 continued

(b) The thermal efficiency is $\eta = (w_{\text{cycle}}/m) / (Q_{\text{in}}/m)$, where

$$\frac{w_{\text{cycle}}}{m} = \frac{W_{12}}{m} + \frac{W_{34}}{m} = -267.9 + 923.1 = 655.2 \text{ kJ/kg}$$

$$\frac{Q_{\text{in}}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} = 1259.15 + 33.45 = 1292.6 \text{ kJ/kg}$$

Then

$$\eta = \frac{655.2}{1292.6} = 0.507 \quad (50.7\%)$$

← η

(c) The mean effective pressure is

$$mep = \frac{w_{\text{cycle}}/m}{v_1 - v_2} = \frac{w_{\text{cycle}}/m}{v_1 \left[1 - \frac{v_2}{v_1} \right]}$$

where

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8314 \text{ N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \right) (300 \text{ K})}{\frac{28.97}{10^5 \text{ N/m}^2}} = 0.861 \text{ m}^3/\text{kg}$$

Then

$$mep = \frac{655.2 \text{ kJ/kg}}{(0.861 \frac{\text{m}^3}{\text{kg}}) \left[1 - \frac{1}{9} \right]} \left| \frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| = 8.56 \text{ bar}$$

← mep

1. For any cycle $Q_{\text{cycle}} = W_{\text{cycle}}$. In this case, $w_{\text{cycle}}/m = 655.2 \text{ kJ/kg}$ and

$$\frac{Q_{\text{cycle}}}{m} = -62.42 + 1259.15 + 33.41 - 574.98 = 655.2 \text{ kJ/kg}$$

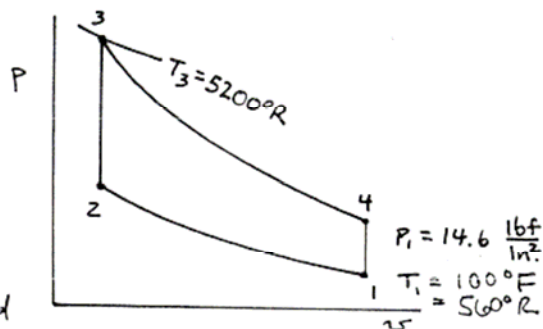
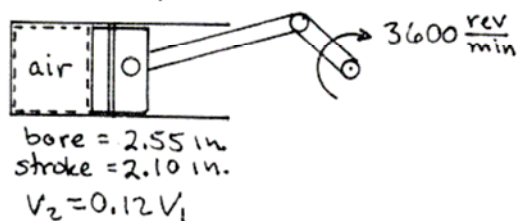
which serves to check the results obtained in part (a).

PROBLEM 9.14

KNOWN: The bore, stroke, clearance volume, and speed of crankshaft rotation of a four-stroke, four-cylinder internal combustion engine are known. The processes within the cylinders are modeled by an Otto cycle with a known state at the beginning of compression and maximum cycle temperature.

FIND: Determine the net work per cycle and the power developed by the engine.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: For the air-standard Otto cycle model, see Example 9.1.

ANALYSIS: Using the given data

$$V_1 - V_2 = \frac{\pi (\text{bore})^2}{4} \times (\text{stroke}) = \frac{\pi (2.55)^2 (2.10)}{4} \text{ in}^3 \left| \frac{1 \text{ ft}^3}{12^3 \text{ in}^3} \right| = 0.006207 \text{ ft}^3$$

With $V_2 = 0.12 V_1$, $V_1 = 0.007053 \text{ ft}^3$. Thus, the mass of air is

$$m = \frac{P_1 V_1}{R T_1} = \frac{(14.6 \frac{\text{lbf}}{\text{in}^2})(0.007053 \text{ ft}^3)}{\left(\frac{1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right) (560^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 0.000497 \text{ lb}$$

Next, fix each of the principal states of the cycle. From Table A-22E, at $T_1 = 560^\circ\text{R}$; $u_1 = 95.47 \text{ Btu/lb}$, $v_{r1} = 131.78$.

Thus, for the isentropic compression,

$$v_{r2} = \frac{V_2}{V_1} v_{r1} = (0.12) 131.78 = 15.81$$

Thus, we get $u_2 = 222.48 \text{ Btu/lb}$. At $T_3 = 5200^\circ\text{R}$; $u_3 = 1098.0 \text{ Btu/lb}$ and $v_{r3} = 0.1828$. For the isentropic expansion

$$v_{r4} = \frac{V_4}{V_3} v_{r3} = \frac{V_1}{V_2} v_{r3} = 1.523$$

Using this, we get $u_4 = 534.69 \text{ Btu/lb}$.

The net work per cycle is

$$\begin{aligned} W_{\text{cycle}} &= Q_{23} - Q_{41} = m [(u_3 - u_2) - (u_4 - u_1)] \\ &= (0.000497 \text{ lb}) [(1098.0 - 222.48) - (534.69 - 95.47)] \frac{\text{Btu}}{\text{lb}} \\ &= 0.217 \text{ Btu} \end{aligned}$$

For a four-stroke engine, the cycle is completed once for every two revolutions of the crankshaft. For four cylinders, the net power is

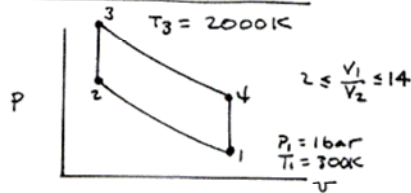
$$\dot{W}_{\text{net}} = (4) (1800 \text{ cycles/min}) (0.217 \text{ Btu/cycle}) \left| \frac{1 \text{ hp}}{42.42 \text{ Btu/min}} \right| = 36.8 \text{ hp}$$

PROBLEM 9.15

KNOWN: For an air-standard Otto cycle, the state at the beginning of compression and the maximum cycle temperature are known.

FIND: Plot the net work per unit of mass, the thermal efficiency, and the mean effective pressure versus compression ratio ranging from 2 to 14.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1.

ANALYSIS: The following reasoning is used to solve the problem:

1. Given values of P_1 , T_1 , r , and T_3 .
2. Fix state 2 using ideal gas relations and $S(T_1, P_1) = S(T_2, P_2)$. When using Table A-22, this is expressed in terms of $v_{r1}/v_{r2} = v_1/v_2$.
3. Use $v_2 = v_3$ and the ideal gas model to get P_3 .
4. Fix state 4 as in step 2; use $S(T_3, P_3) = S(T_4, P_4)$, or $v_{r4}/v_{r3} = v_4/v_3$.

Now, using energy balances, and the fact that $W_{23} = W_{41} = 0$

$$Q_{23} = m(u_3 - u_2) \quad (1)$$

$$Q_{41} = m(u_1 - u_4) \quad (2)$$

Further, for the cycle

$$W_{\text{cycle}}/m = Q_{23}/m + Q_{41}/m \quad (3)$$

and

$$\eta = W_{\text{cycle}}/Q_{23} \quad (4)$$

Finally

$$v_1 = \frac{RT_1}{P_1}; \text{ mep} = \frac{W_{\text{cycle}}}{m v_1 (1 - \frac{1}{r})} \quad (5)$$

Sample calculation: $r = 5$ From Table A-22, at $P_1 = 1 \text{ bar}$, $T_1 = 300 \text{ K}$;

$u_1 = 214.07 \text{ kJ/kg}$, $v_{r1} = 621.2$. Thus

$$v_{r2} = \frac{v_2}{v_1} v_{r1} = \left(\frac{1}{r}\right) v_{r1} = \left(\frac{1}{5}\right)(621.2) = 124.24 \Rightarrow \begin{cases} T_2 = 564.8 \text{ K} \\ u_2 = 408.0 \text{ kJ/kg} \end{cases}$$

Now, $T_3 = 2000 \text{ K} \Rightarrow u_3 = 1678.7 \text{ kJ/kg}$, $v_{r3} = 2.776$. Thus, for state 4

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} = \frac{v_1}{v_2} = 5 \Rightarrow v_{r4} = 13.88 \Rightarrow T_4 = 1216.32 \text{ K}, u_4 = 947.82 \text{ kJ/kg}.$$

Thus, with (1) - (5)

$$W_{\text{cycle}}/m = (u_3 - u_2) + (u_1 - u_4) = (1678.7 - 408.0) + (214.07 - 947.82) = 1270.7 - 733.75 = 536.95 \text{ kJ/kg} \leftarrow W_{\text{cycle}}/m$$

and

$$\eta = \frac{536.95}{1270.7} = 0.4226 \text{ (42.26\%)} \leftarrow \eta$$

$$\text{Finally } v_1 = \frac{(8.314 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})}{(1 \text{ bar})} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| = 0.861 \text{ m}^3/\text{kg}$$

$$\text{mep} = \frac{536.95 \text{ kJ/kg}}{(0.861 \text{ m}^3/\text{kg})(1 - \frac{1}{5})} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| = 7.795 \text{ bar} \leftarrow \text{mep}$$

Continued on next slide

Problem 9-15 continued

IT solution. The data for the required plots are obtained using IT, as follows:

IT Code

```
p1 = 1 // bar
T1 = 300 // K
r = 5
T3 = 2000 // K
m = 1 // Assume a unit mass of 1 kg.
```

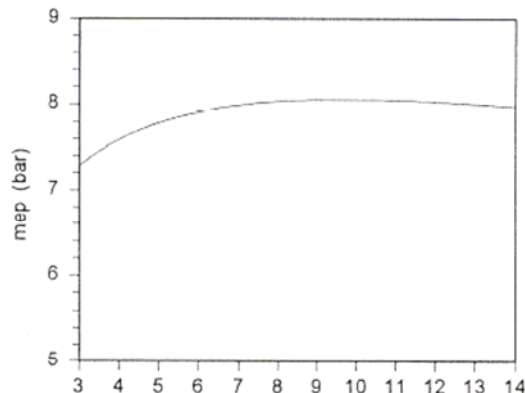
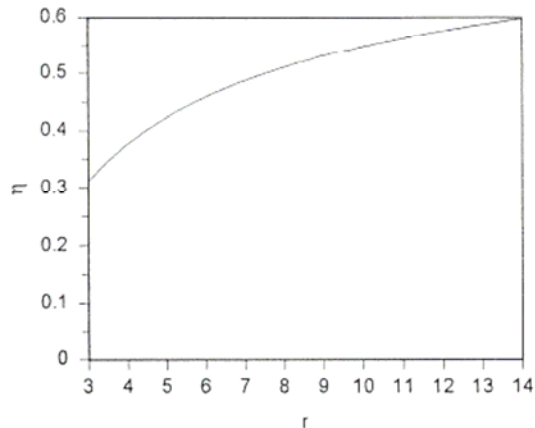
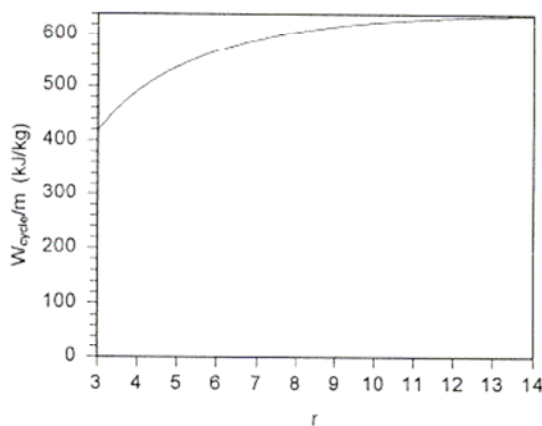
```
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
v2 = v1 / r
v2 = v_TP("Air", T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
v3 = v_TP("Air", T3, p3)
v3 = v2
u3 = u_T("Air", T3)
s3 = s_TP("Air", T3, p3)
v4 = v1
v4 = v_TP("Air", T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
u4 = u_T("Air", T4)
```

```
Q23 = m * (u3 - u2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / Q23
mep = (Wcycle / (m * v1 * (1 - 1/r))) / 100 // bar
```

IT Results for r = 5

```
T2 = 564.6 K
T4 = 1215 K
p3 = 33.33 bar
v1 = 0.861 m³/kg
Q23/m = 1269 kJ/kg
Q41/m = -732.3 kJ/kg
Wcycle/m = 536.3 kJ/kg
η = 0.4227
mep = 7.787 bar
```

Plots:

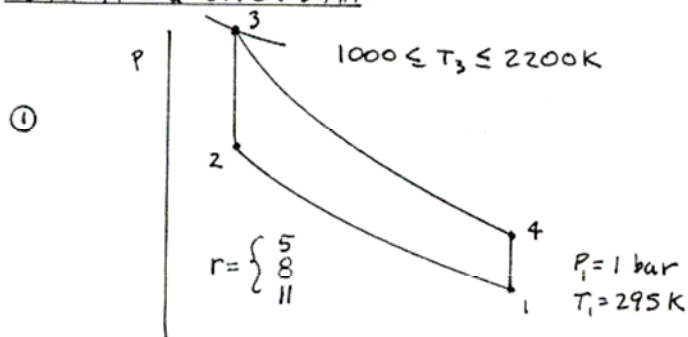


PROBLEM 9.16

KNOWN: An air-standard Otto cycle has known conditions at the beginning of compression. The maximum cycle temperature varies over a given range and the compression ratio takes on specified values.

FIND: Investigate the effect of maximum cycle temperature on the net work per unit mass of air.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1.

ANALYSIS: Sample calculation for $r=8$, $T_3=1000\text{ K}$ using Table A-22.

From Table A-22, $u_1 = 210.49\text{ kJ/kg}$, $v_{r1} = 647.9$. For the compression process

$$v_{r2} = \left(\frac{v_2}{v_1}\right) v_{r1} = \left(\frac{1}{8}\right)(647.9) = 80.99 \Rightarrow T_2 = 662.74\text{ K}, u_2 = 483.15\text{ kJ/kg}$$

Now, at $T_3 = 1000\text{ K}$; $u_3 = 758.94\text{ kJ/kg}$, $v_{r3} = 25.17$. For the expansion

$$v_{r4} = \left(\frac{v_4}{v_3}\right) v_{r3} = \left(\frac{v_1}{v_2}\right) v_{r3} = (8)(25.17) = 201.36 \Rightarrow u_4 = 336.50\text{ kJ/kg}$$

Since $W_{23} = W_{41} = 0$; $Q_{23}/m = (u_3 - u_2)$ and $Q_{41}/m = (u_1 - u_4)$. For the cycle, $W_{\text{cycle}} = Q_{\text{cycle}}$. Thus

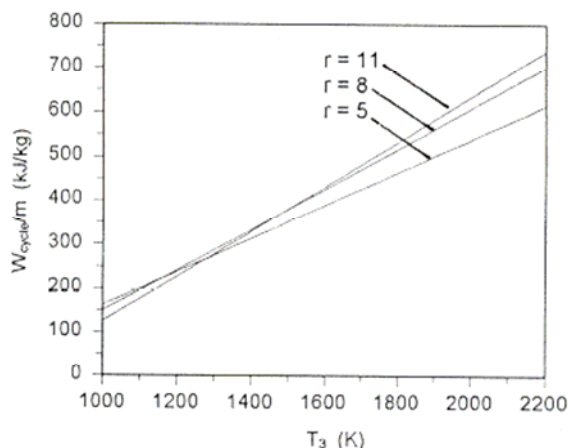
$$W_{\text{cycle}}/m = (u_3 - u_2) + (u_1 - u_4) = (758.94 - 483.15) + (210.49 - 336.50) = 149.78\text{ kJ/kg}$$

ITCode

```
p1 = 1 // bar
T1 = 295 // K
T3 = 1000 // K
r = 8
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
v2 = v1 / r
v2 = v_TP("Air", T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
v3 = v_TP("Air", T3, p3)
s3 = s_TP("Air", T3, p3)
u3 = u_T("Air", T3)
v3 = v2
v4 = v1
v4 = v_TP("Air", T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
u4 = u_T("Air", T4)
Wcycle = (u3 - u2) + (u1 - u4)
```

**IT Results for $r = 8$
and $T_3 = 1000\text{ K}$**

$u_1 = 210.4\text{ kJ/kg}$
$T_2 = 662.8\text{ K}$
$u_2 = 483\text{ kJ/kg}$
$u_3 = 758.6\text{ kJ/kg}$
$T_4 = 468.6\text{ K}$
$u_4 = 336.2\text{ kJ/kg}$
$W_{\text{cycle}}/m = 149.8\text{ kJ/kg}$



We see from the plot that the net work increases with T_3 at fixed compression ratio. Also, the plots cross each other, indicating the relation of W_{cycle} to r differs depending on T_3 .

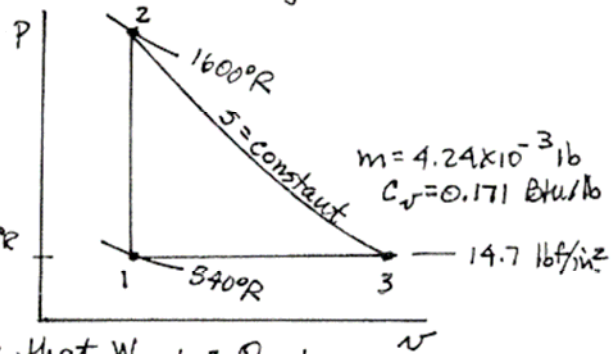
PROBLEM 9.17

KNOWN: An air-standard Otto cycle has a known state at the beginning of the constant volume heat addition and a known maximum temperature. The mass is given.

FIND: Determine (a) the net work, and (b) the thermal efficiency.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: (1) The cycle is modeled as processes in a closed system. (2) The expansion process is isentropic. (3) The air is modeled as an ideal gas with constant specific heats: $c_v = 0.171 \text{ Btu/lb}^\circ\text{R}$ and $c_p = 0.24 \text{ Btu/lb}^\circ\text{R}$. (4) Kinetic and potential energy effects are negligible.



ANALYSIS: (a) To find the net work, note that $W_{\text{cycle}} = Q_{\text{cycle}}$

and $W_{\text{cycle}} = Q_{12} + Q_{31}$

For process 1-2: $m(u_2 - u_1) = Q_{12} - W_{12}$

$$Q_{12} = m(u_2 - u_1) = m c_v (T_2 - T_1) = (4.24 \times 10^{-3} \text{ lb})(0.171 \frac{\text{Btu}}{\text{lb}^\circ\text{R}})(1600 - 540)^\circ\text{R} = 0.7685 \text{ Btu}$$

For Process 2-3: $T_3 = T_2 (P_3/P_2)^{\frac{k-1}{k}}$. $P_2 = (T_2/T_1) P_1 = 43.56 \text{ lbf/in}^2$
and $T_3 = (1600^\circ\text{R})(14.7/43.56)^{0.2857} = 1173.1^\circ\text{R}$

For Process 3-1: $m(u_1 - u_3) = Q_{31} - W_{31}$
 $W_{31} = m \int_3^1 p dv = m p (v_1 - v_3)$

$$Q_{31} = m(u_1 - u_3) + m p (v_1 - v_3) = m c_p (T_1 - T_3) = (4.24 \times 10^{-3})(0.24)(540 - 1173.1) = -0.6442 \text{ Btu}$$

Finally $W_{\text{cycle}} = Q_{12} + Q_{31} = 0.7685 + (-0.6442) = 0.1243 \text{ Btu} \leftarrow W_{\text{cycle}}$

(b) The thermal efficiency is

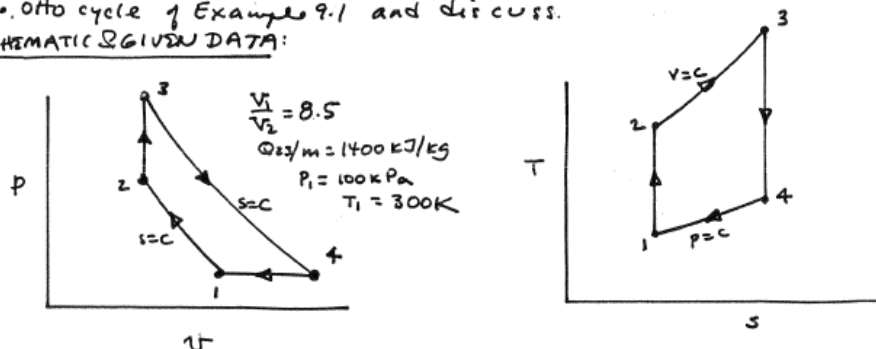
$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{W_{\text{cycle}}}{Q_{12}} = \frac{0.1243}{0.7685} = 0.162 (16.2\%) \leftarrow \eta$$

PROBLEM 9.18

KNOWN: An air-standard Atkinson cycle has a known compression ratio for isentropic compression and a specified state at the beginning of compression. The heat addition per unit of mass is also known.

FIND: (a) Sketch the cycle on T-s coordinates. Determine (b) the net work, (c) thermal efficiency, and (d) mean effective pressure. Compare with the Otto cycle of Example 9.1 and discuss.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1, except the compression from 4-1 is not adiabatic but rather it is at constant pressure.

ANALYSIS: For part (a), see above. For parts (b)-(d), begin by fixing each of the four principle states using data from Table A-22.

State 1: $P_1 = 100 \text{ kPa}$, $T_1 = 300 \text{ K} \Rightarrow u_1 = 214.07 \text{ kJ/kg}$, $v_{r1} = 621.2$, $P_{r1} = 1.3860$

State 2: For isentropic compression

$$v_{r2} = v_{r1} \frac{V_2}{V_1} = 621.2 \left(\frac{1}{8.5} \right) = 73.082$$

Thus, $T_2 = 688.2 \text{ K}$, $u_2 = 503.06 \frac{\text{kJ}}{\text{kg}}$, $P_{r2} = 27.034$

and $P_2 = P_1 \frac{P_{r2}}{P_{r1}} = (100 \text{ kPa}) \left(\frac{27.034}{1.3860} \right) = 1950.5 \text{ kPa}$

State 3: The specific internal energy u_3 is found using the energy balance for process 2-3

$$m(u_3 - u_2) = Q_{23} - W_{23}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = 1400 + 503.06 = 1903.06 \text{ kJ/kg}$$

Thus, $T_3 = 2231.3 \text{ K}$, $P_{r3} = 3342.4$

and $P_3 = P_2 \left(\frac{T_3}{T_2} \right) = 1950.5 \left(\frac{2231.3}{688.2} \right) = 6324 \text{ kPa}$

State 4: For the isentropic expansion

$$P_{r4} = P_{r3} \left(\frac{P_4}{P_3} \right) = P_{r3} \left(\frac{P_1}{P_3} \right) = 52.853$$

Thus, $T_4 = 821 \text{ K}$, $h_4 = 845.14 \text{ kJ/kg}$

(a) To find the net work per unit mass, note that $W_{\text{cycle}} = Q_{\text{cycle}}$, so

$$\frac{W_{\text{cycle}}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (h_4 - h_1)$$

$$= 1400 - (845.14 - 300.19)$$

$$= 855.1 \text{ kJ/kg} \leftarrow \frac{W_{\text{cycle}}}{m}$$

Continued on next slide

Problem 9-18 continued

(b) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{23}/m} = \frac{855.1}{1400} = 0.611 \text{ (61.1\%)} \quad \eta$$

(c) The overall displacement volume is $V_4 - V_2 = m(v_4 - v_2)$, so the mean effective pressure is given by

$$mep = \frac{W_{\text{cycle}}}{V_4 - V_2} = \frac{W_{\text{cycle}}/m}{v_4 - v_2} = \frac{W_{\text{cycle}}/m}{v_4(1 - v_2/v_4)}$$

Evaluating v_4

$$v_4 = \frac{RT_4}{P_4} = \frac{(8.314 \text{ kJ/kg} \cdot \text{K})(821 \text{ K})}{(100 \text{ kPa})} \left\| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right\| \left\| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right\|$$

$$= 2.356 \text{ m}^3/\text{kg}$$

Now, the specific volume ratio v_2/v_4 is

$$\frac{v_2}{v_4} = \frac{v_2}{v_1} \cdot \frac{v_1}{v_4} = \frac{v_2}{v_1} \cdot \frac{T_1}{T_4} = \left(\frac{1}{8.5}\right) \left(\frac{300}{821}\right) = 0.04299$$

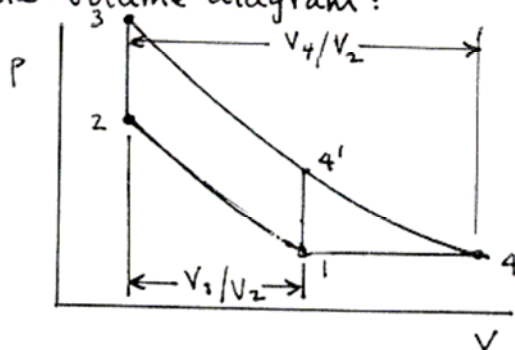
$P_1 = P_4$

Thus

$$mep = \frac{(855.1 \text{ kJ/kg})}{(2.356 \text{ m}^3/\text{kg})(1 - 0.04299)} \left\| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right\| \left\| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right\|$$

$$= 379.2 \text{ kPa} \quad mep$$

DISCUSSION: The particular Atkinson cycle in this problem has nearly the same compression ratio for process 1-2 as for the Otto cycle in Problem 9.1. However, the isentropic expansion (process 3-4) has a much higher volume ratio than for the Otto cycle. This is illustrated on the following pressure-volume diagram:



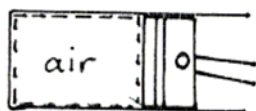
The results indicate that the work and thermal efficiency increase for the Atkinson cycle, as evidenced by the larger enclosed area. Further, the mean effective pressure decreases.

PROBLEM 9.19

KNOWN: Air undergoes a cold air-standard Atkinson cycle.

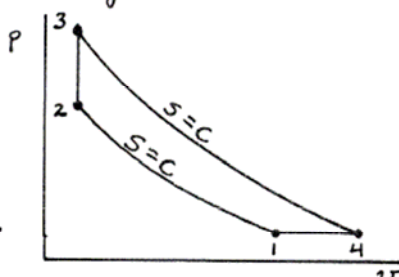
FIND: Derive an expression for thermal efficiency and compare with the cold air-standard Otto cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: See Example 9.1. Also, assume constant specific heats.



ANALYSIS: To find the net work, note that $W_{cycle} = Q_{cycle}$. Thus

$$\frac{W_{cycle}}{m} = \frac{Q_{in}}{m} - \frac{Q_{out}}{m} = c_v(T_3 - T_2) - c_p(T_4 - T_1)$$

Therefore, the thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = 1 - \frac{c_p(T_4 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{k T_1 (T_4/T_1 - 1)}{T_2 (T_3/T_2 - 1)} \quad (1)$$

$$\text{Process 1-2 is isentropic. Thus } T_2/T_1 = (v_1/v_2)^{k-1} \equiv r^{k-1} \quad (2)$$

$$\text{Further, for process 2-3: } \frac{T_3}{T_2} = \frac{P_3}{P_2} \equiv r_p \quad (3)$$

$$\text{And, for process 4-1: } \frac{T_4}{T_1} = \frac{v_4}{v_1} = \frac{v_4}{v_3} \cdot \frac{v_3}{v_1} = \frac{v_4}{v_3} \cdot \frac{1}{r} \quad (4)$$

$$\text{Now, for the isentropic compression: } \frac{1}{r} = \left(\frac{P_4}{P_2}\right)^{1/k} \quad (5)$$

$$\text{and, for the isentropic expansion: } \frac{v_4}{v_3} = \left(\frac{P_3}{P_4}\right)^{1/k} \quad (6)$$

Now, combining (4), (5), and (6)

$$\frac{T_4}{T_1} = \left(\frac{P_3}{P_4}\right)^{1/k} \left(\frac{P_4}{P_2}\right)^{1/k} = \left(\frac{P_3}{P_2}\right)^{1/k} = r_p^{1/k} \quad (7)$$

Finally, inserting the results of (2), (3), and (7) into (1)

$$\eta = 1 - \frac{k (r_p^{1/k} - 1)}{r^{k-1} (r_p - 1)} \quad \eta$$

To compare the thermal efficiency of the Atkinson and Otto cycles, recall that

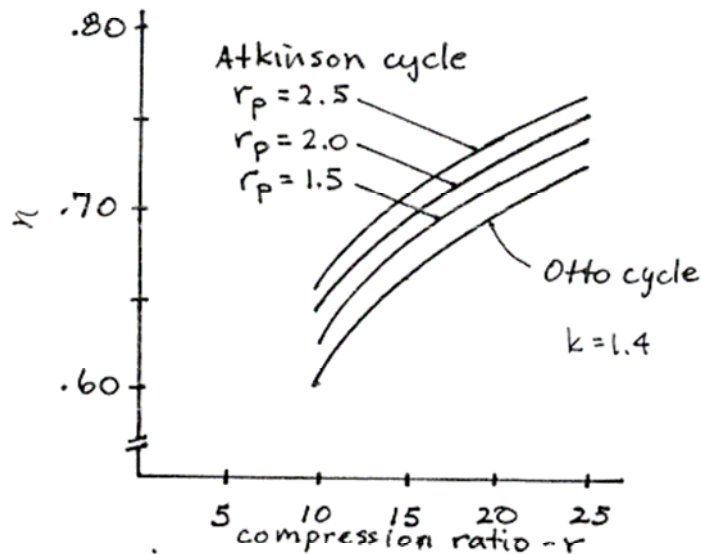
$$\eta_{Otto} = 1 - \frac{1}{r^{k-1}}$$

Note that the thermal efficiency of the Otto cycle depends on r and k , but not on the maximum cycle temperature (nor the

Continued on next slide

Problem 9-19 continued

pressure ratio, r_p .) The following graph can be used to compare the cycles further:



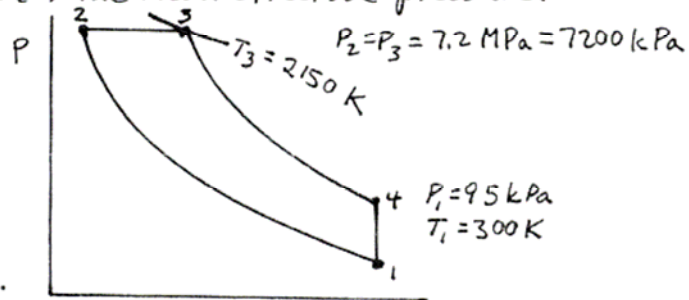
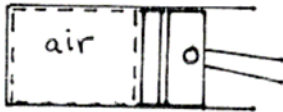
Thus, at any compression ratio, the Atkinson cycle has a higher thermal efficiency than the corresponding Otto cycle. Also, as the pressure ratio r_p increases at fixed compression ratio (r) the thermal efficiency increases as well.

PROBLEM 9.20

KNOWN: An air-standard Diesel cycle has a specified state at the beginning of compression and a known pressure and temperature at the end of heat addition.

FIND: Determine (a) the compression ratio, (b) the cut off ratio, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2.

ANALYSIS: Begin by fixing each principal state in the cycle (Table A-22).

State 1: $T_1 = 300 \text{ K}$, $P_1 = 95 \text{ kPa} \Rightarrow u_1 = 214.07 \text{ kJ/kg}$, $v_{r1} = 621.2$, $P_{r1} = 1.3860$

State 2: For the isentropic compression

$$P_{r2} = P_{r1} \left(\frac{P_2}{P_1} \right) = (1.3860) \left(\frac{7200}{95} \right) = 105.04$$

$$\text{Thus, } T_2 = 979.6 \text{ K}, v_{r2} = 26.793, h_2 = 1022.82 \text{ kJ/kg}$$

State 3: $T_3 = 2150 \text{ K}$, $P_3 = 7200 \text{ kPa} \Rightarrow h_3 = 2440.3 \text{ kJ/kg}$, $v_{r3} = 2.175$

State 4: For the isentropic expansion

$$\frac{v_{r4}}{v_{r3}} = \frac{v_{r1}}{v_{r2}} \cdot \frac{v_{r2}}{v_{r3}} = \frac{v_{r1}}{v_{r2}} \cdot \frac{T_2}{T_3} = \frac{v_{r1}}{v_{r2}} \cdot \frac{T_2}{T_3} = \frac{621.2}{26.793} \cdot \frac{979.6}{2150} = 10.56$$

$$v_{r4} = \frac{v_{r4}}{v_{r3}} \cdot v_{r3} = 22.98 \Rightarrow T_4 = 1031 \text{ K}, u_4 = 785.75 \text{ kJ/kg}$$

(a) The compression ratio is

$$r = \frac{v_1}{v_2} = \frac{v_{r1}}{v_{r2}} = \frac{621.2}{26.793} = 23.19$$

(b) The cutoff ratio is

$$r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{2150}{979.6} = 2.19$$

(c) The thermal efficiency is

$$\begin{aligned} \eta &= \frac{W_{\text{cycle}}/m}{Q_{23}/m} = \frac{(h_3 - h_2) - (u_4 - u_1)}{h_3 - h_2} \\ &= \frac{(2440.3 - 1022.82) - (785.75 - 214.07)}{(2440.3 - 1022.82)} \\ &= \frac{845.80}{1417.48} = 0.597 \text{ (59.7\%)} \end{aligned}$$

Continued on next slide

Problem 9-20 continued

(d) The mean effective pressure is given as

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}(m)}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97} \frac{kJ}{kg \cdot K}\right)(300 K)}{(95 \text{ kPa})} \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right|$$
$$= 0.9063 \text{ m}^3/\text{kg}$$

Thus

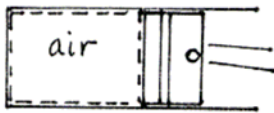
$$mep = \frac{(845.80 \text{ kJ/kg})}{(0.9063 \frac{\text{m}^3}{\text{kg}})(1 - 1/23.19)} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right|$$
$$= 975 \text{ kPa} \quad \leftarrow mep$$

PROBLEM 9.21

KNOWN: An air-standard Diesel cycle has a specified state at the beginning of compression and a known pressure and temperature at the end of heat addition.

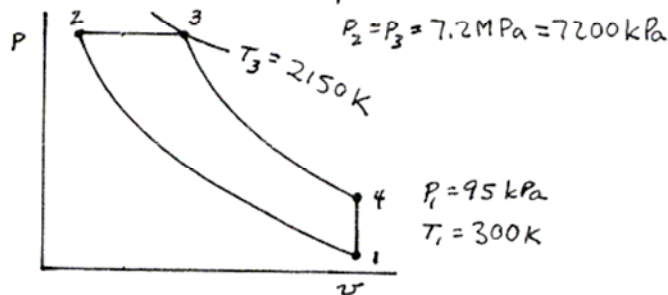
FIND: Determine (a) the compression ratio, (b) the cutoff ratio, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: See Example 9.2. Also, assume constant specific heats evaluated at 300 K.



ANALYSIS: First, determine the temperature at each principal state of the cycle. $T_1 = 300 \text{ K}$, $T_3 = 2150 \text{ K}$ are given.

State 2: For the isentropic compression, with $k = 1.4$ from Table A-20

$$T_2 = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} T_1 = 1033.1 \text{ K}$$

State 4: Similarly, for the expansion

$$\frac{v_4}{v_3} = \frac{v_1}{v_2} \cdot \frac{v_2}{v_3} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{k-1}} \cdot \frac{T_3}{T_2} = 10.57$$

and

$$T_4 = \left(\frac{v_3}{v_4} \right)^{k-1} T_3 = 837.0 \text{ K}$$

(a) The compression ratio is: $r = \frac{v_1}{v_2} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{k-1}} = 22.01$ ← r

(b) The cutoff ratio is: $r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = 2.081$ ← r

(c) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{23}/m} = \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

With $c_v = .718 \text{ kJ/kg} \cdot \text{K}$ and $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$ from Table A-20

$$\eta = \frac{(1.005)(2150 - 1033.1) - (.718)(837.0 - 300)}{(1.005)(2150 - 1033.1)} = \frac{736.9}{1122.5} = 0.657 \text{ (65.7\%)} \leftarrow \eta$$

(d) The mean effective pressure is given as

$$mep = \frac{W_{\text{cycle}}}{v_1 - v_2} = \frac{W_{\text{cycle}}}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{(8.314 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})}{(95 \text{ kPa})} \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| = 0.9063 \text{ m}^3/\text{kg}$$

Continued on next slide

Problem 9-21 continued

Thus

$$mep = \frac{(763.9 \text{ kJ/kg})}{(0.9063 \text{ m}^3/\text{kg}) (1 - 1/22.01)} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right|$$
$$= 883.0 \text{ kPa} \leftarrow mep$$

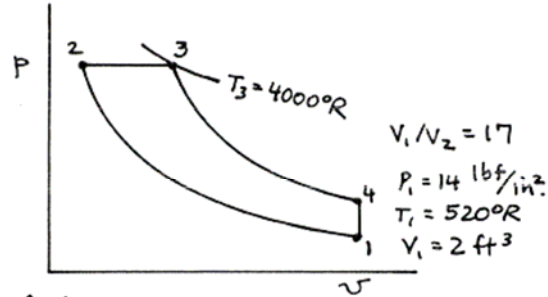
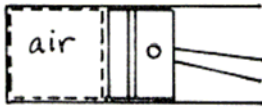
These results can be compared to those of Problem 9.20 to see some effects of the assumption of constant specific heats.

PROBLEM 9.22

KNOWN: An air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression. The maximum cycle temperature is given.

FIND: Determine (a) the net work, (b) the thermal efficiency, (c) the mean effective pressure, and (d) the cutoff ratio.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2.

ANALYSIS: Begin by fixing each principal state of the cycle (Table A-22E).

State 1: $T_1 = 520^\circ\text{R} \Rightarrow u_1 = 88.62 \text{ Btu/lb}, v_{r1} = 158.58$

State 2: For the isentropic compression

$$v_{r2} = \left(\frac{v_2}{v_1}\right) v_{r1} = \left(\frac{1}{17}\right) 158.58 = 9.3282$$

Thus, $T_2 = 1534.5^\circ\text{R}, h_2 = 378.32 \text{ Btu/lb}$

State 3: $T_3 = 4000^\circ\text{R} \Rightarrow h_3 = 1088.3 \text{ Btu/lb}, v_{r3} = .4518$

State 4: For the isentropic expansion

$$\frac{v_4}{v_3} = \frac{v_1}{v_2} \cdot \frac{v_2}{v_3} = \frac{v_1}{v_2} \cdot \frac{T_2}{T_3} = (17) \frac{1534.5}{4000} = 6.522$$

and

$$v_{r4} = \frac{v_4}{v_3} v_{r3} = 2.9466$$

Thus, $T_4 = 2253.7^\circ\text{R}, u_4 = 421.25 \text{ Btu/lb}$

(a) For the cycle, $W_{\text{cycle}} = Q_{\text{cycle}}$. Thus

$$W_{\text{cycle}} = Q_{23} - Q_{41} = m [(h_3 - h_2) - (u_4 - u_1)]$$

Evaluating m

$$m = \frac{P_1 V_1}{R T_1} = \frac{(14.0 \text{ lbf/in}^2)(2 \text{ ft}^3)}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}}\right)(520^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 0.1454 \text{ lb}$$

Thus

$$W_{\text{cycle}} = (0.1454 \text{ lb}) [(1088.3 - 378.32) - (421.25 - 88.62)] \frac{\text{Btu}}{\text{lb}} = 54.87 \text{ Btu} \quad \leftarrow W_{\text{cycle}}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{23}} = \frac{W_{\text{cycle}}}{m(h_3 - h_2)} = \frac{(54.87 \text{ Btu})}{(0.1454 \text{ lb})(1088.3 - 378.32) \text{ Btu/lb}} = 0.531 \text{ (53.1\%)} \quad \leftarrow \eta$$

Continued on next slide

Problem 9-22 continued

(c) The mean effective pressure is

$$\begin{aligned} mep &= \frac{W_{cycle}}{(V_1 - V_2)} = \frac{W_{cycle}}{V_1 (1 - V_2/V_1)} \\ &= \frac{(54.87 \text{ Btu})}{(2 \text{ ft}^3) (1 - 1/17)} \left| \frac{778 \text{ ft} \cdot \text{lb}}{1 \text{ Btu}} \right| \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| \\ &= 157.5 \text{ lbf/in}^2 \end{aligned}$$

mep

(d) The cutoff ratio is determined as follows:

$$r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{4000}{1534.5} = 2.61$$

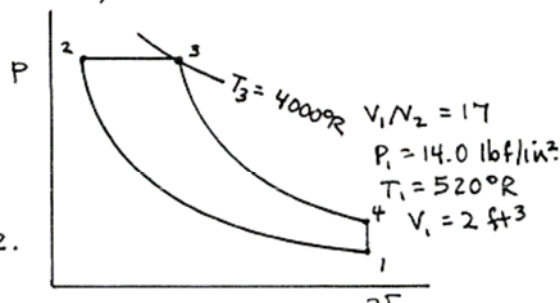
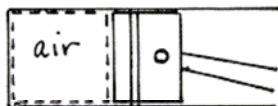
r_c

PROBLEM 9.23

KNOWN: A cold air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression. The maximum cycle temperature is given.

FIND: Determine (a) the net work, (b) the thermal efficiency, (c) the mean effective pressure, and (d) the cut off ratio.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2. Also, assume constant specific heats evaluated at 520°R

ANALYSIS: First, determine the temperature at each principal state of the cycle. $T_1 = 520^\circ\text{R}$ and $T_3 = 4000^\circ\text{R}$ are given.

State 2: For the isentropic compression, with $k = 1.401$ from Table A-20E

$$T_2 = (v_1/v_2)^{k-1} T_1 = 1619.6^\circ\text{R}$$

State 3: Similarly, for the expansion

$$\frac{v_4}{v_3} = \frac{v_1}{v_2} \cdot \frac{v_2}{v_3} = \frac{v_1}{v_2} \cdot \frac{T_2}{T_3} = (17) \frac{1619.6}{4000} = 6.8833$$

$$\text{and } T_4 = (v_3/v_4)^{k-1} T_3 = 1845.5^\circ\text{R}$$

(a) For the cycle, $W_{\text{cycle}} = Q_{\text{cycle}}$. Thus

$$W_{\text{cycle}} = Q_{23} - Q_{41} = m [c_p(T_3 - T_2) - c_v(T_4 - T_1)]$$

Evaluating m

$$m = \frac{P_1 v_1}{R T_1} = \frac{(14.0 \text{ lbf/in}^2)(2 \text{ ft}^3)}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}}\right)(520^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 0.1454 \text{ lb}$$

Thus, with $c_p = 0.240 \text{ Btu/lb} \cdot ^\circ\text{R}$ and $c_v = 0.171 \text{ Btu/lb} \cdot ^\circ\text{R}$

$$W_{\text{cycle}} = (0.1454) [(0.240)(4000 - 1619.6) - (0.171)(1845.5 - 520)] = 50.11 \text{ Btu} \quad W_{\text{cycle}}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{23}} = \frac{W_{\text{cycle}}}{m c_p(T_3 - T_2)} = \frac{(50.11)}{(0.1454)(0.240)(4000 - 1619.6)} = 0.603 (60.3\%) \quad \eta$$

(c) The mean effective pressure is

$$\begin{aligned} mep &= \frac{W_{\text{cycle}}}{v_1 - v_2} = \frac{W_{\text{cycle}}}{v_1(1 - v_2/v_1)} \\ &= \frac{(50.11 \text{ Btu})}{(2 \text{ ft}^3)(1 - 1/17)} \left| \frac{778 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}} \right| \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 143.8 \text{ lbf/in}^2 \quad mep \end{aligned}$$

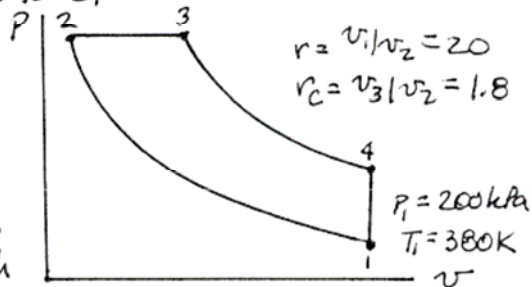
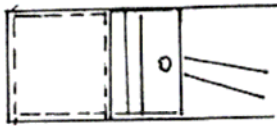
$$(d) \quad r_c = v_3/v_2 = T_3/T_2 = 4000/1619.6 = 2.47 \quad r_c$$

PROBLEM 9.24

KNOWN: A cold air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression. The cutoff ratio is also known.

FIND: Determine (a) the maximum temperature, (b) the heat addition per unit mass, (c) the net work per unit mass, (d) the thermal efficiency, (e) the mean effective pressure, and (f) the effects of varying compression ratio for compression ratios ranging from 5 to 25.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2.

Also, assume constant specific heats with

$k = 1.4$. From Table A-20: $c_v = 0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$, $c_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

ANALYSIS: First, determine the temperature at each principal state.

State 2 For the isentropic compression

$$T_2 = T_1 (v_1/v_2)^{k-1} = (380 \text{ K})(20)^{0.4} = 1259.5 \text{ K}$$

$$P_2 = P_1 (v_1/v_2)^k = (200 \text{ kPa})(20)^{1.4} = 13258 \text{ kPa}$$

State 3 $P_3 = P_2$; $\frac{T_3}{T_2} = \frac{v_3}{v_2} \Rightarrow T_3 = T_2 \left(\frac{v_3}{v_2} \right) = (1259.5)(1.8) = 2267.1 \text{ K}$

State 4 For the isentropic expansion

$$T_4 = T_3 (v_3/v_4)^{k-1} = T_3 (v_3/v_2 \cdot v_2/v_4)^{k-1} = T_3 (r_c \cdot \frac{1}{r})^{k-1} \quad \text{(a) } T_{\text{max}}$$

$$= (2267.1) (1.8 \cdot \frac{1}{20})^{0.4} = 865.3 \text{ K}$$

$$P_4 = P_3 (T_4/T_3)^{1/(k-1)} = (13258) (865.3/2267.1)^{1/0.4} = 455.4 \text{ kPa}$$

(b) The heat addition occurs during process 2-3. Thus

$$m(u_3 - u_2) = Q_{23} - W_{23}$$

and

$$Q_{23}/m = (u_3 - u_2) + p(v_3 - v_2) = h_3 - h_2 = c_p(T_3 - T_2)$$

$$= (1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(2267.1 - 1259.5) \text{ K} = 1012.6 \text{ kJ/kg} \quad \dot{Q}_{\text{in}}/m$$

(c) $W_{\text{cycle}}/m = \dot{Q}_{\text{in}}/m - \dot{Q}_{\text{out}}/m$. To get $\dot{Q}_{\text{out}}/m$

$$\dot{Q}_{\text{out}}/m = |\dot{Q}_4/m| = u_4 - u_1 = c_v(T_4 - T_1)$$

$$= (0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(865.3 - 380) \text{ K} = 348.45 \text{ kJ/kg}$$

$$\text{and } W_{\text{cycle}} = 1012.6 - 348.45 = 664.15 \text{ kJ/kg} \quad W_{\text{cycle}}/m$$

(d) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{\dot{Q}_{\text{in}}/m} = \frac{664.15}{1012.6} = 0.656 (65.6\%) \quad \eta$$

Continued on next slide

Problem 9-24 continued

(e) To find the mean effective pressure, first calculate v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.314 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(380 \text{ K})}{(200 \text{ kPa})} \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| = 0.5453 \frac{\text{m}^3}{\text{kg}}$$

Thus, the mep is

$$\begin{aligned} \text{mep} &= \frac{W_{\text{cycle}}/m}{(v_1 - v_2)} = \frac{W_{\text{cycle}}/m}{v_1 (1 - \frac{1}{r})} \\ &= \frac{(664.15 \text{ kJ/kg})}{(0.5453 \frac{\text{m}^3}{\text{kg}})(1 - \frac{1}{20})} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| = 128.2 \text{ kPa} \quad \text{mep} \end{aligned}$$

(f) The data for the required plots are obtained using IT as follows:

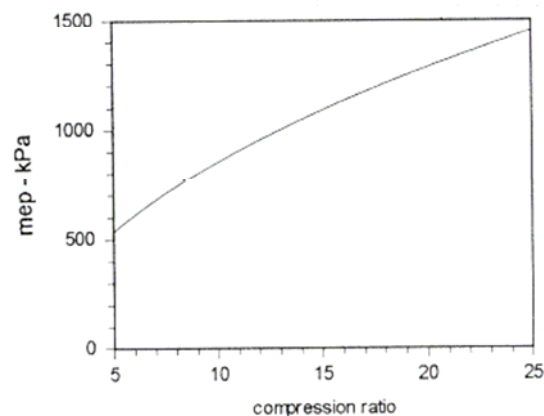
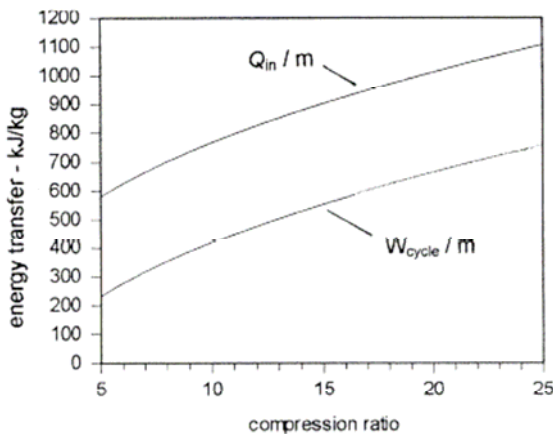
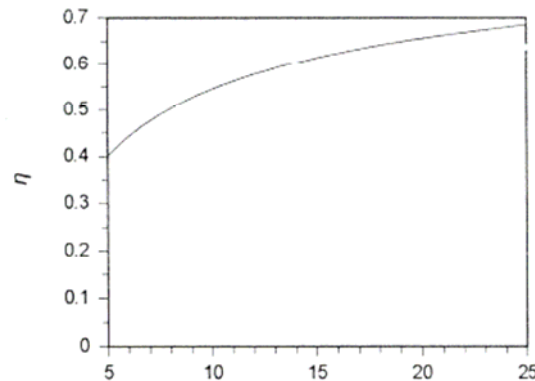
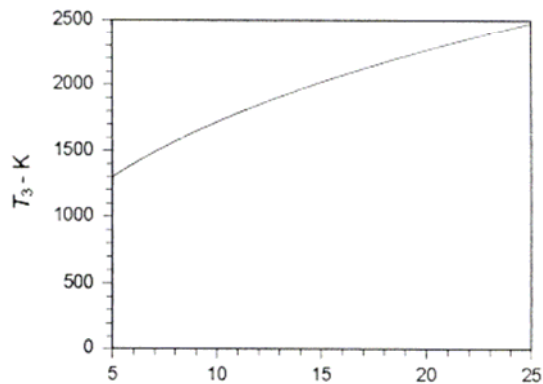
IT Code

```
T1 = 380 // K
p1 = 200 // kPa
r = v1 / v2
r = 20
v1 = v_TP("Air", T1, p1)
rc = v3 / v2
rc = 1.8
k = 1.4
cv = 0.718 // kJ/kg·K
cp = 1.005 // kJ/kg·K
m = 1 // assume a unit of mass of 1 kg
```

```
IT Results for r = 20, rc = 1.8
T2 = 1259 K
T3 = 2267 K
T4 = 865.3 K
p2 = 1.326E4 kPa
v1 = 0.5453 m³/kg
Qin / m = 1013 kJ/kg
Qout / m = 348.4 kJ/kg
Wcycle / m = 664.2 kJ/kg
η = 0.6559
mep = 1282 kPa
```

```
T2 / T1 = r^(k-1)
p2 / p1 = r^k
T3 / T2 = v3 / v2
T4 / T3 = (rc / r)^(k-1)
```

```
Qin / m = cp * (T3 - T2)
Qout / m = cv * (T4 - T1)
Wcycle / m = Qin / m - Qout / m
eta = Wcycle / Qin
mep = (Wcycle / m) / ((v1) * (1 - 1/r))
```

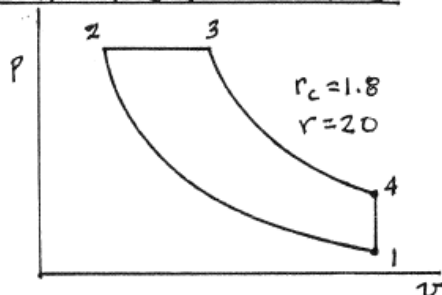


PROBLEM 9.25

KNOWN: Reconsider the cold air-standard Diesel cycle of Problem 9.24 from an exergy accounting viewpoint.

FIND: (a) Evaluate the exergy transfers accompanying heat and work for each process, and (2) devise and evaluate an exergetic efficiency for the cycle.

SCHEMATIC & GIVEN DATA:



Data from the solution to Problem 9.24:

State	P (kPa)	T (K)	v (m ³ /kg)
1	200	380	0.5453
2	13258	1259.5	0.02726
3	13258	2267.1	0.049075
4	455.4	865.3	0.5453

$$T_0 = 300 \text{ kPa}, p_0 = 100 \text{ kPa}$$

ENGINEERING MODEL: See Example 9.2. Also, assume constant specific heats with $k=1.4$, $c_v = 0.718 \text{ kJ/kg}\cdot\text{K}$, and $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$. Let $T_0 = 300 \text{ K}$, $p_0 = 100 \text{ kPa}$.

ANALYSIS:

(a) **Process 1-2:** $Q_{12} = 0 \Rightarrow (E_q)_{12} = 0$ $(E_q)_{12}/m$

The exergy transfer accompanying work is evaluated using Eq. 7.6.

$$\begin{aligned} \frac{(E_w)_{12}}{m} &= \frac{W_{12}}{m} - p_0(v_2 - v_1) = c_v(T_1 - T_2) - p_0(v_2 - v_1) \\ &= (0.718 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})(380 - 1259.5) \text{ K} - (100 \text{ kPa})(0.02726 - 0.5453) \frac{\text{m}^3}{\text{kg}} \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \\ &= -579.68 \text{ kJ/kg} \quad \leftarrow \frac{(E_w)_{12}}{m} \end{aligned}$$

Process 2-3: Using $W_{23}/m = \int_2^3 P dv = P_2(v_3 - v_2)$ and Eq. 7.6

$$\begin{aligned} \frac{(E_w)_{23}}{m} &= P_2(v_3 - v_2) - p_0(v_3 - v_2) = (P_2 - p_0)(v_3 - v_2) \\ &= (13258 - 100)(0.049075 - 0.02726) = 287.04 \text{ kJ/kg} \quad \leftarrow \frac{(E_w)_{23}}{m} \end{aligned}$$

With Eq. 7.5 and noting that all processes are internally reversible

$$(E_q)_{23} = \int_2^3 (1 - \frac{T_0}{T}) \delta Q = Q_{23} - T_0 \int_2^3 (\frac{\delta Q}{T})_{\text{int rev}} = Q_{23} - T_0 m(s_3 - s_2)$$

$$\begin{aligned} \text{Thus } \frac{(E_q)_{23}}{m} &= \frac{Q_{23}}{m} - T_0(s_3 - s_2) = c_p(T_3 - T_2) - T_0(c_p \ln \frac{T_3}{T_2} - R \ln \frac{P_3}{P_2}) \\ &= (1.005)(2267.1 - 1259.5) - (300)(1.005) \ln \frac{2267.1}{1259.5} \\ &= 835.42 \text{ kJ/kg} \quad \leftarrow \frac{(E_q)_{23}}{m} \end{aligned}$$

Process 3-4: $Q_{34} = 0 \Rightarrow (E_q)_{34}/m = 0$ $(E_q)_{34}/m$

$$\begin{aligned} \frac{(E_w)_{34}}{m} &= \frac{W_{34}}{m} - p_0(v_4 - v_3) = c_v(T_3 - T_4) - p_0(v_4 - v_3) \\ &= (0.718)(2267.1 - 865.3) - (100)(0.5453 - 0.049075) \\ &= 956.87 \text{ kJ/kg} \quad \leftarrow \frac{(E_w)_{34}}{m} \end{aligned}$$

Process 4-1: $W_{41} = 0 \Rightarrow (E_w)_{41}/m = 0$ $(E_w)_{41}/m$

Continued on next slide

Problem 9-25 continued

$$\begin{aligned} \frac{(E_g)_{41}}{m} &= Q_1 - Q_4 = (u_1 - u_4) + p_0(v_1 - v_4) - T_0(s_1 - s_4) \\ &= c_v(T_1 - T_4) - T_0 c_v \ln \frac{T_1}{T_4} = (0.718)(380 - 865.3) - (300)(0.718) \ln \frac{380}{865.3} \\ &= -171.19 \text{ kJ/kg} \end{aligned}$$

(b) An appropriate exergetic efficiency can be written as

$$\epsilon = \frac{(E_w)_{\text{cycle}}/m}{(E_w)_{\text{in}}/m}$$

$$\begin{aligned} \textcircled{1} \quad \frac{(E_w)_{\text{cycle}}}{m} &= \frac{(E_w)_{12}}{m} + \frac{(E_w)_{23}}{m} + \frac{(E_w)_{34}}{m} + \frac{(E_w)_{41}}{m} \\ &= -579.68 + 287.04 + 956.87 + 0 = 664.23 \text{ kJ/kg} \end{aligned}$$

$$\frac{(E_g)_{\text{in}}}{m} = \frac{(E_g)_{23}}{m} = 835.42 \text{ kJ/kg}$$

Thus

$$\textcircled{2} \quad \epsilon = \frac{664.23}{835.42} = 0.795 \text{ (79.5\%)} \quad \epsilon$$

1. Exergy Accounting Summary

	E_w/m	E_g/m
1-2:	-579.68	0
2-3:	287.04	835.42
3-4:	956.87	0
4-1:	0	-171.19
Cycle:	664.23	664.23

For the cycle:

$$(\Delta E)_{\text{cycle}} = (E_g)_{\text{cycle}} - (E_w)_{\text{cycle}} - (E_d)_{\text{cycle}}$$

$$(E_g)_{\text{cycle}} = (E_w)_{\text{cycle}}$$

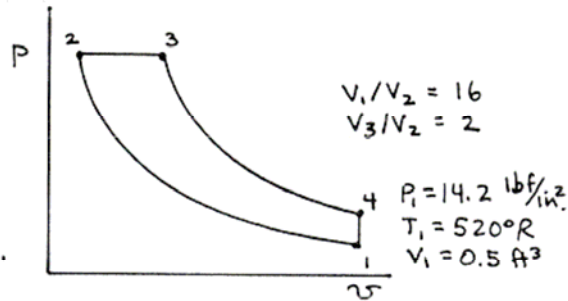
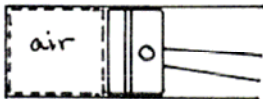
2. Significant irreversibilities are present in actual internal combustion engines. Accordingly, exergy destruction occurs and the exergetic efficiencies of real engines are much lower than calculated here for the ideal Diesel cycle.

PROBLEM 9.26

KNOWN: An air-standard Diesel cycle has known compression and cutoff ratios and a specified state at the beginning of compression.

FIND: Determine (a) the heat added, (b) the maximum temperature, (c) the thermal efficiency, (d) the mean effective pressure, and (e) plot these quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2.

ANALYSIS: Begin by fixing each principal state of the cycle (Table A-22E).

State 1: $T_1 = 520^\circ\text{R} \Rightarrow u_1 = 88.62 \text{ Btu/lb}$, $v_{r1} = 158.58$

State 2: For the isentropic compression

$$v_{r2} = \left(\frac{V_2}{V_1}\right) v_{r1} = \left(\frac{1}{16}\right) 158.58 = 9.911$$

Thus, $T_2 = 1502.5^\circ\text{R}$ and $h_2 = 369.85 \text{ Btu/lb}$

State 3: For the constant pressure process, $V_3/V_2 = T_3/T_2$. Thus $T_3 = 2(1502.5) = 3005^\circ\text{R}$, and $h_3 = 792.03 \text{ Btu/lb}$, $v_{r3} = 1.1743$ T_{max}

State 4: For the isentropic expansion

$$\frac{V_4}{V_3} = \frac{V_1}{V_2} \cdot \frac{V_2}{V_3} = (16)\left(\frac{1}{2}\right) = 8$$

$$\text{and } v_{r4} = \left(\frac{V_4}{V_3}\right) v_{r3} = 9.3944$$

Thus, $T_4 = 1530.7^\circ\text{R}$ and $u_4 = 272.36 \text{ Btu/lb}$

(a) The heat added is

$$Q_{23} = m(h_3 - h_2)$$

Evaluating m

$$m = \frac{P_1 V_1}{RT_1} = \frac{(14.2 \text{ lbf/in}^2)(0.5 \text{ ft}^3)}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}}\right)(520^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 0.0369 \text{ lb}$$

Thus

$$Q_{23} = (0.0369)(792.03 - 369.85) = 15.58 \text{ Btu} \quad \leftarrow Q_{23}$$

(b) $T_{\text{max}} = T_3 = 3004.6^\circ\text{R}$ (see above)

(c) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{23}} = 1 - \frac{Q_{41}}{Q_{23}} = 1 - \frac{6.78}{15.58} = 0.565 (56.5\%) \quad \leftarrow \eta$$

Continued on next slide

Problem 9-26 continued

(d) The mean effective pressure is given by

$$\begin{aligned}
 mep &= \frac{W_{cycle}}{V_1 - V_2} = \frac{Q_{23} - Q_{41}}{V_1 (1 - V_2/V_1)} \\
 &= \frac{(15.58 - 6.78) \text{ Btu}}{(0.5 \text{ ft}^3) (1 - 1/16)} \left\| \frac{778 \text{ ft} \cdot \text{lb}}{1 \text{ Btu}} \right\| \left\| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right\| \\
 &= 101.4 \text{ lbf/in}^2 \xrightarrow{\text{mep}}
 \end{aligned}$$

(e) The data for the required plots are obtained using IT, as follows

IT Code

```

p1 = 14.2 // lbf/in.^2
T1 = 520 // °R
V1 = 0.5 // ft^3
p3 = p2
v4 = v1
r = 16
rc = 2.5

```

```

v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
r = v1 / v2
v2 = v_TP("Air", T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)
rc = T3 / T2
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
rc = v3 / v2
v4 = v_TP("Air", T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
u4 = u_T("Air", T4)

```

```

v1 = V1 / m
Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / Q23
mep = ((Wcycle / m) / (v1 * (1 - 1/r))) * (778 / 144)

```

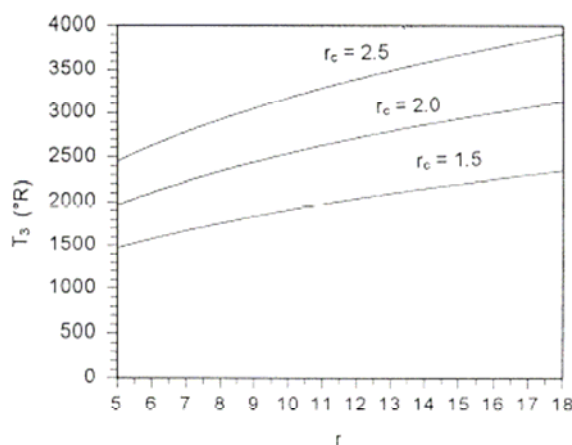
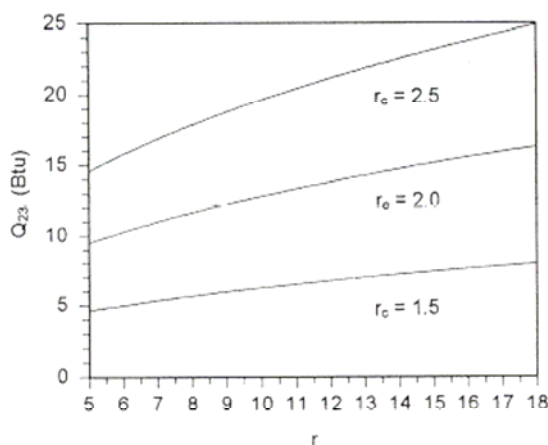
IT Results for r = 16 and rc = 2

```

T2 = 1502 °R
T4 = 1528 °R
h2 = 369.6 Btu/lb
h3 = 790.8 Btu/lb
p2 = 656.3 lbf/in.^2
p4 = 41.73 lbf/in.^2
u1 = 88.73 Btu/lb
u4 = 271.7 Btu/lb
m = 0.03687 lb
Q41 = -6.747 Btu
Wcycle = 8.783 Btu
Q23 = 15.53 Btu
T3 = 3004 °R
eta = 0.5655
mep = 101.2 lbf/in.^2

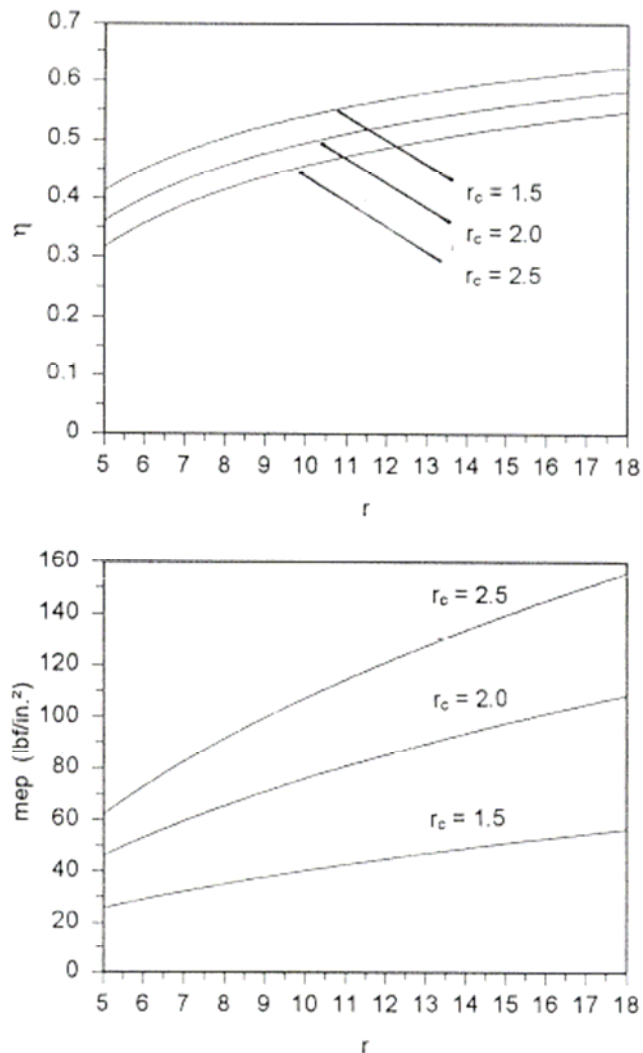
```

PLOTS:



Continued on next slide

Problem 9-26 continued



Discussion

- The thermal efficiency graph exhibit the same trends as in Fig 9.6 for fixed k .
- At constant compression ratio, the values of Q_{23} , T_3 , and mep all increase with increasing cut-off ratio r_c , as expected.

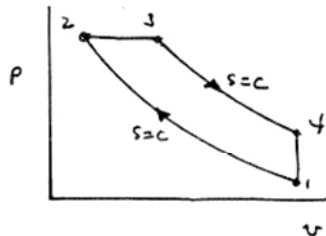
PROBLEM 9.27

KNOWN: Reconsider the air-standard Diesel cycle of Problem 9.26 from an exergy accounting viewpoint.

FIND: (a) Evaluate the exergy transfers accompanying heat and work for each process and (b) devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:

Data from Problem 9.26 further calculation and/or further interpolation in Table A-22E.



State	p (lbf/in ²)	T (°R)	v (ft ³ /lb)	u (Btu/lb)	s° (Btu/lb·°R)
1	14.2	520	13.562	88.62	0.59172
2	656.44	1502.5	0.84763	266.84	0.85459
3	656.44	3005	1.6953	586.16	1.05312
4	41.8	1530.7	13.562	272.36	0.85951

ENGINEERING MODEL: See Example 9.2. Also, $T_0 = 520^\circ\text{R}$, $P_0 = 14.2 \text{ lbf/in}^2$.

ANALYSIS: (a) Process 1-2: $Q_{12} = 0$, so there is no accompanying exergy transfer. With Eq. 7.6 the exergy transfer accompanying work is

$$E_W = W - P_0 \Delta V = m[(u_1 - u_2) - P_0(v_2 - v_1)] = (0.0369 \text{ lb})[(88.62 - 266.84) \text{ Btu/lb} - (14.2 \times 14 \frac{\text{lbf}}{\text{ft}^2})(-12.7143 \frac{\text{ft}^3}{\text{lb}}) \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}}] = -5.343 \text{ Btu}$$

Process 2-3: As above, using $W_{23} = \int_2^3 p dV = P_2(V_3 - V_2)$

$$E_W = P_2[V_3 - V_2] - P_0[V_3 - V_2] = m(P_2 - P_0)(v_3 - v_2) = (0.0369)(656.24 \times 14)(0.84763 - 13.562) \frac{1}{778} = 3.718 \text{ Btu}$$

With Eq. 7.5 and noting that process 2-3 is internally reversible

$$E_Q = \int_2^3 [1 - \frac{T_0}{T}] \delta Q = Q_{23} - T_0 \int_2^3 \frac{\delta Q}{T} = Q_{23} - T_0(s_3 - s_2) = Q_{23} - T_0 m[s_3^\circ - s_2^\circ - R \ln(P_3/P_2)] = 15.58 \text{ Btu} - (520^\circ\text{R})(0.0369 \text{ lb})(1.05312 - 0.85459) \frac{\text{Btu}}{16.0^\circ\text{R}} = 11.771 \text{ Btu}$$

Process 3-4: $Q_{34} = 0$, so there is no accompanying exergy transfer.

$$E_W = W - P_0 \Delta V = m[(u_3 - u_4) - P_0(v_4 - v_3)] = 0.0369 [586.16 - (14.2 \times 14)(11.8667) \frac{1}{778}] = 10.428 \text{ Btu}$$

Process 4-1: $W_{41} = 0$, so there is no accompanying exergy transfer.

$$E_Q = \int_4^1 [1 - \frac{T_0}{T}] \delta Q = Q_{41} - T_0 \int_4^1 \frac{\delta Q}{T} = Q_{41} - T_0(s_1 - s_4) = m[(u_1 - u_4) - T_0(s_1^\circ - s_4^\circ - R \ln(P_1/P_4))] = 0.0369 [88.62 - 272.36 - 520(0.59172 - 0.85951) \frac{1}{16.0^\circ\text{R}}] = -3.062 \text{ Btu}$$

Summary:

Exergy Input accompanying heat transfer in process 2-3

Disposition of the exergy input:

Transfer out - net work = $-5.343 + 3.718 + 10.428 = 8.80 \text{ Btu (74\%)}$
 Transfer out - heat transfer in process 4-1 = $3.06 \text{ " (26\%)}$
 Exergy destroyed = 0

11.86

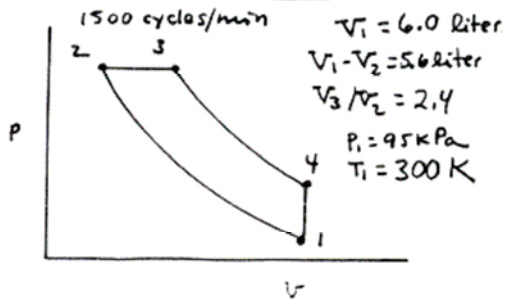
1. As expected, the exergy input and the disposition of the exergy agree to within roundoff. No exergy is destroyed because each process is internally reversible.

PROBLEM 9.28

KNOWN: Operating data are provided for an internal combustion engine modeled as an air-standard Diesel cycle.

FIND: Determine the net work per cycle, the power developed, and the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

See Example 9.2

ANALYSIS: Using given data, the compression ratio is obtained as follows:

$$v_2 = v_1 - 5.6 = 6.0 - 5.6 = 0.4 \text{ liter}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{6.0}{0.4} = 15$$

Next, data are obtained at each principal state:

State 1: From Table A-22 at $T_1 = 300$ K, $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$.

State 2: For the isentropic compression

$$v_{r2} = \frac{v_{r1}}{v_1/v_2} = \frac{621.2}{15} = 41.41 \Rightarrow h_2 = 869.63 \text{ kJ/kg}, T_2 = 843 \text{ K}$$

State 3: Since pressure is constant for process 2-3,

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} \Rightarrow T_3 = (2.4)(843) = 2023 \text{ K} \Rightarrow h_3 = 2280.85 \frac{\text{kJ}}{\text{kg}}, v_{r3} = 2.6743$$

State 4: For the isentropic expansion

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} = \frac{v_4}{v_2} \cdot \frac{v_2}{v_3} = (15) \left(\frac{1}{2.4} \right) = 6.25, \Rightarrow v_{r4} = (6.25)(2.6743) = 16.714$$

$$\Rightarrow u_4 = 884.96 \text{ kJ/kg}$$

The mass is obtained using the ideal gas model equation of state

$$m = \frac{P_1 v_1}{R T_1} = \frac{(95 \times 10^3 \text{ N/m}^2)(6.0 \text{ liters})}{\left(\frac{8314 \text{ N} \cdot \text{m}}{28.97 \text{ kg} \cdot \text{K}} \right)(300 \text{ K})} \left| \frac{10^{-3} \text{ m}^3}{1 \text{ liter}} \right| = 6.62 \times 10^{-3} \text{ kg}$$

For any cycle, $W_{\text{cycle}} = Q_{\text{cycle}}$. Processes 1-2 and 3-4 involve no heat transfer; so, $Q_{\text{cycle}} = Q_{23} - Q_{41}$, where $Q_{23} = m[h_3 - h_2]$, $Q_{41} = m(u_4 - u_1)$

$$\text{Thus, } W_{\text{cycle}} = (6.62 \times 10^{-3} \text{ kg}) [2280.85 - 869.63] - (884.96 - 214.07) \frac{\text{kJ}}{\text{kg}}$$

$$= 4.901 \text{ kJ per cycle}$$

The power developed is

$$W_{\text{cycle}} = (1500 \frac{\text{cycles}}{\text{min}}) (4.901 \frac{\text{kJ}}{\text{cycle}}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 122.53 \text{ kW}$$

The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{4.901 \text{ kJ/cycle}}{6.62 \times 10^{-3} (2280.85 - 869.63) \text{ kJ/cycle}} = \frac{4.901}{9.3423} = 0.525 (52.5\%) \leftarrow \eta$$

W_{cycle}

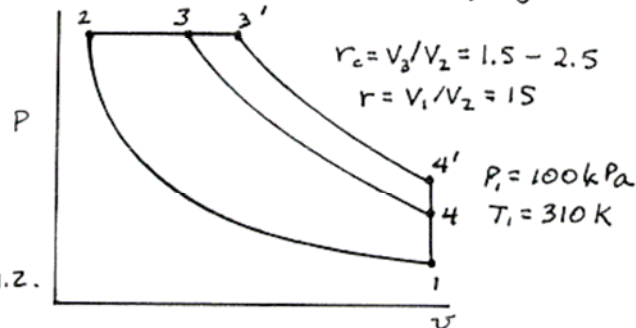
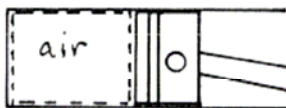
$\leftarrow$ power

PROBLEM 9.29

KNOWN: An air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression.

FIND: Plot (a) the maximum temperature, (b) the pressure at the end of expansion, (c) the net work per unit mass of air, and (d) the thermal efficiency versus cutoff ratio ranging from 1.5 to 2.5

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2.

ANALYSIS: A sample calculation will be done for $r_c = 1.5$. First, fix each of the principal states of the cycle:

(a) **State 1:** From Table A-22 at $T_1 = 310\text{ K}$; $u_1 = 221.25\text{ kJ/kg}$ and $v_{r1} = 572.3$.

State 2: For the isentropic compression

$$v_{r2} = \left(\frac{v_2}{v_1}\right) v_{r1} = \left(\frac{1}{15}\right) (572.3) = 38.153$$

Interpolating in Table A-22; $h_2 = 896.86\text{ kJ/kg}$, $T_2 = 867.7\text{ K}$

State 3: For each value of cutoff ratio, $r_c = v_3/v_2$, the value of T_3 can be determined from $T_3 = (v_3/v_2) T_2$. In this case

$$T_3 = 1301.5\text{ K}; h_3 = 1397.7\text{ kJ/kg}, v_{r3} = 11.235 \quad T_{\text{max}}$$

State 4: Note that for the isentropic expansion

$$v_{r4} = \left(\frac{v_4}{v_3}\right) v_{r3} = \left(\frac{v_1}{v_2} \cdot \frac{v_2}{v_3}\right) v_{r3} = \left(r \cdot \frac{1}{r_c}\right) v_{r3}$$

Thus $v_{r4} = 112.35$; $T_4 = 586.6\text{ K}$, $u_4 = 424.54\text{ kJ/kg}$

(b) The pressure at the end of the expansion is

$$P_4 = \left(\frac{T_4}{T_1}\right) P_1 = 189.2\text{ kPa} \quad P_4$$

(c) The net work per unit mass of air is

$$\frac{W_{\text{cycle}}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = (h_3 - h_2) - (u_4 - u_1) = 297.5\text{ kJ/kg} \quad W_{\text{cycle/m}}$$

(d) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle/m}}}{Q_{23/m}} = 0.594 \text{ (59.4\%)}$$

The data for the required plots are obtained using IT, as follows:

Continued on next slide

Problem 9-29 continued

IT Code

```
p1 = 100 // kPa
T1 = 310 // K
r = v1 / v2
r = 15
rc = v3 / v2
rc = 1.5
m = 1 // Assume a unit mass of 1 kg.
```

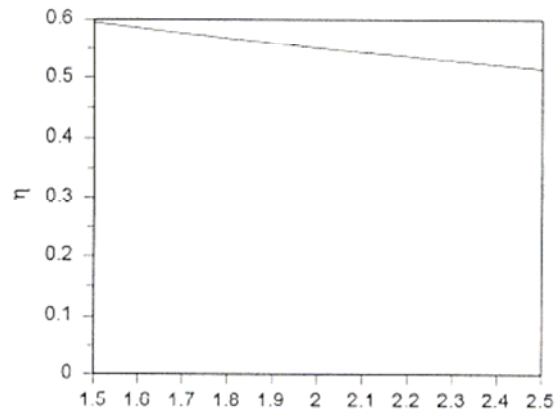
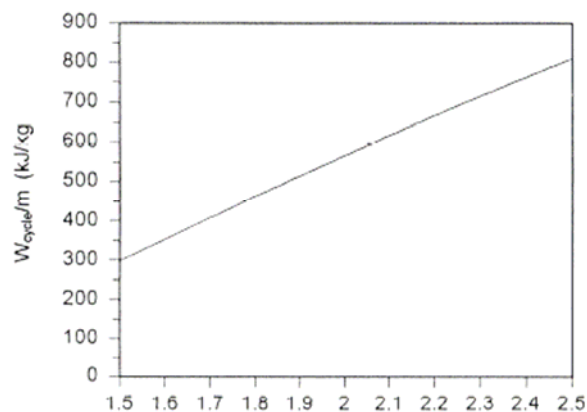
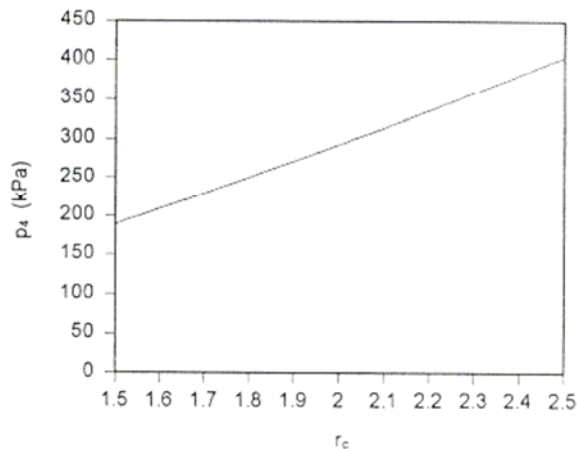
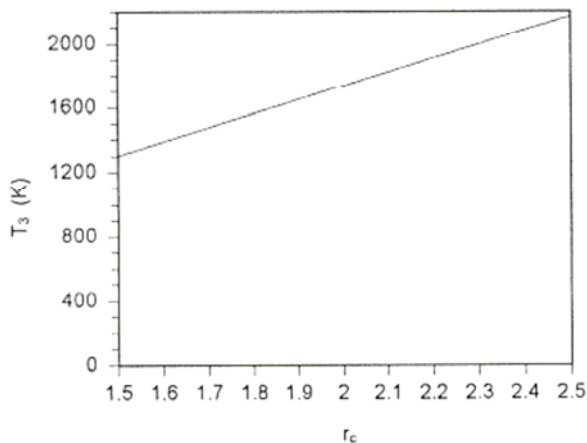
```
u1 = u_T("Air", T1)
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air", T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)
v3 = v_TP("Air", T3, p3)
p3 = p2
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
v4 = v_TP("Air", T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
v4 = v1
u4 = u_T("Air", T4)
```

```
Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / Q23
```

IT Results for $r_c = 1.5$

```
h2 = 896.6 kJ/kg
p2 = 4199 kPa
h3 = 1397 kJ/kg
T4 = 586.3 K
Q23/m = 500.4 kJ/kg
Q41/m = -203.1 kJ/kg
T3 = 1302 K
p4 = 189.1 kPa
Wcycle/m = 297.3 kJ/kg
η = 0.5941
```

PLOTS:

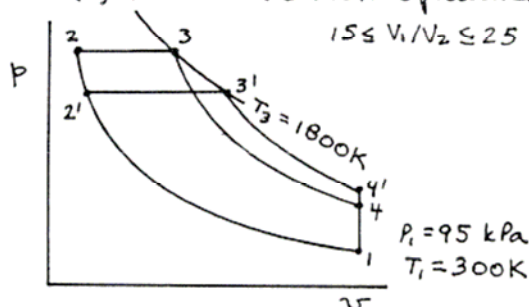
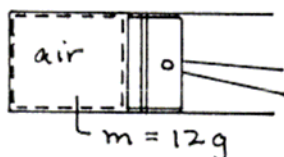


PROBLEM 9.30

KNOWN: An air-standard Diesel cycle has a known maximum temperature and a specified state at the beginning of compression. The mass of air is given.

FIND: Determine for various compression ratios (a) the net work, (b) the thermal efficiency, and (c) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2.

ANALYSIS: Consider first a sample calculation using data from Table A-22, for the case of $r = V_1/V_2 = 15$. From Table A-22 at $T_1 = 300\text{ K}$; $u_1 = 214.07\text{ kJ/kg}$, $v_{r1} = 621.2$. Also, at $T_3 = 1800\text{ K}$; $h_3 = 2003.3\text{ kJ/kg}$, $v_{r3} = 3.944$. For $r = 15$

$$v_{r2} = (V_2/V_1) v_{r1} = (1/15) 621.2 = 41.413$$

Interpolating in Table A-22; $h_2 = 869.63\text{ kJ/kg}$, $T_2 = 843.2\text{ K}$. Now, for process 3-4

$$v_{r4} = \left(\frac{V_4}{V_3}\right) v_{r3} = \left(\frac{V_1}{V_2}\right) \left(\frac{V_2}{V_3}\right) v_{r3} = \left(\frac{V_1}{V_2}\right) \left(\frac{T_2}{T_3}\right) v_{r3} = (15) \left(\frac{843.2}{1800}\right) (3.944) = 27.713$$

Interpolating in Table A-22; $u_4 = 731.99\text{ kJ/kg}$.

(a) The net work is calculated from $W_{\text{cycle}} = Q_{\text{cycle}}$, where $Q_{\text{cycle}} = Q_{23} + Q_{34}$. Thus

$$W_{\text{cycle}} = m [(h_3 - h_2) + (u_1 - u_4)]$$

$$= (12\text{ g}) \left| \frac{1\text{ kg}}{10^3\text{ g}} \right| [(2003.3 - 869.63) + (214.07 - 731.99)] \frac{\text{kJ}}{\text{kg}} = 7.389\text{ kJ} \leftarrow \frac{W_{\text{cycle}}}{(r=15)}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{23}} = \frac{W_{\text{cycle}}}{m(h_3 - h_2)} = \frac{7.389}{(12 \times 10^{-3})(2003.3 - 869.63)} = 0.543 \text{ (54.3\%)} \leftarrow \eta$$

(c) To get the mean effective pressure, first determine V_1 .

$$V_1 = \frac{mRT_1}{P_1} = \frac{(12\text{ g}) \left(\frac{8.314\text{ kJ}}{28.97\text{ kg} \cdot \text{K}} \right) (300\text{ K})}{(95\text{ kPa})} \left| \frac{1\text{ kg}}{10^3\text{ g}} \right| \left| \frac{1\text{ kPa}}{10^3\text{ N/m}^2} \right| \left| \frac{10^3\text{ N} \cdot \text{m}}{1\text{ kJ}} \right| = 1.088 \times 10^{-2}\text{ m}^3$$

Thus

$$m_{\text{ep}} = \frac{W_{\text{cycle}}}{V_1 - V_2} = \frac{W_{\text{cycle}}}{V_1 (1 - 1/r)}$$

$$= \frac{7.389\text{ kJ}}{(1.088 \times 10^{-2}\text{ m}^3) (1 - 1/15)} \left| \frac{10^3\text{ N} \cdot \text{m}}{1\text{ kJ}} \right| \left| \frac{1\text{ kPa}}{10^3\text{ N/m}^2} \right| = 727.6\text{ kPa} \leftarrow m_{\text{ep}} (r=15)$$

Continued on next slide

Problem 9-30 continued

The data for the required plots are obtained using IT, as follows:

IT Code

```
p1 = 95 // kPa
T1 = 300 // K
T3 = 1800 // K
r = 15
m = 12E-03 // kg

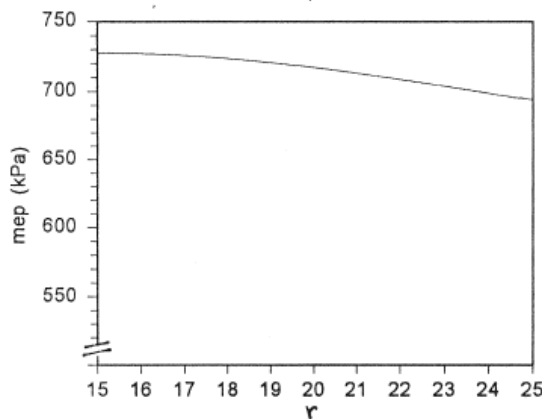
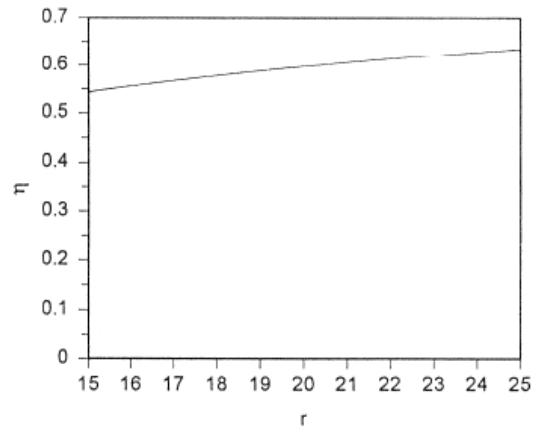
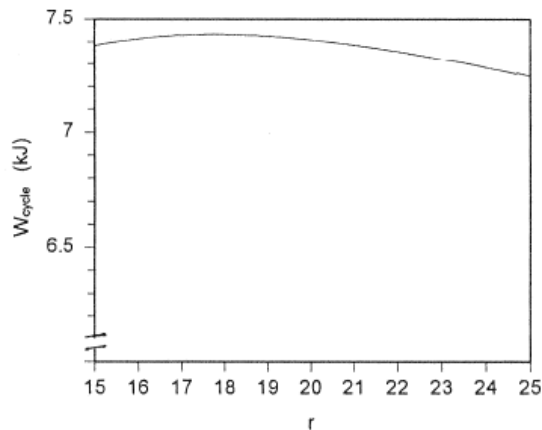
u1 = u_T("Air", T1)
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
r = v1 / v2
v2 = v_TP("Air", T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)
p3 = p2
h3 = h_T("Air", T3)
v3 = v_TP("Air", T3, p3)
s3 = s_TP("Air", T3, p3)
v4 = v_TP("Air", T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
v4 = v1
u4 = u_T("Air", T4)
```

```
Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / (m * (h3 - h2))
V1 = v1 * m
mep = Wcycle / (V1 * (1 - 1 / r)) // kPa
```

IT Results for $r = 15$

```
T2 = 843.4 K
p2 = 4006 kPa
h2 = 869.4 kJ/kg
T3 = 1800 K
h3 = 2002 kJ/kg
T4 = 967.2 K
p4 = 306.3 kPa
Q23 = 13.59 kJ
Q41 = -6.202 kJ
Wcycle = 7.384 kJ
eta = 0.5435
mep = 727.4 kPa
```

PLOTS:



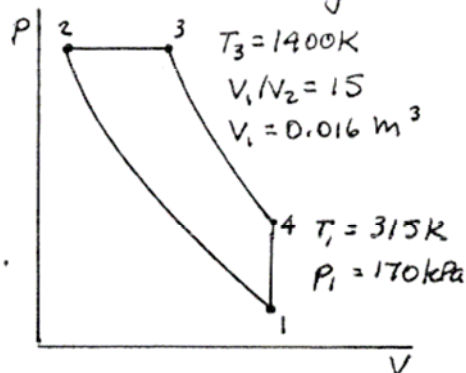
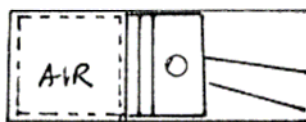
Note that the work doesn't vary greatly with r , but it does exhibit a maximum. Further, thermal efficiency increases with r because the average temperature of heat addition increases and the average temperature of heat rejection decreases, as can be verified by reference to the T - s diagram of the cycle. Also note that the mep decreases with increasing r .

PROBLEM 9.31

KNOWN: An air-standard Diesel cycle has a specified state at the beginning of compression. The compression ratio and maximum cycle temperature are known as is the volume at the beginning of compression.

FIND: Determine (a) the mass of air, (b) the heat addition and heat rejection per cycle, (c) the net work and thermal efficiency.

① SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = 315\text{K} \Rightarrow u_1 = 224.85\text{ kJ/kg}$

$$v_{r1} = 549.8$$

State 2 $v_{r2} = v_{r1} \left(\frac{V_2}{V_1} \right) = \frac{549.8}{15} = 36.653 \Rightarrow T_2 = 879.7\text{K}, h_2 = 910.18\text{ kJ/kg}$

State 3 $p_3 = p_2, T_3 = 1400\text{K} \Rightarrow h_3 = 1515.42\text{ kJ/kg}, v_{r3} = 8.919$

State 4 $v_{r4} = v_{r3} \left(\frac{V_4}{V_3} \right) = v_{r3} \left(\frac{V_4}{V_2} \cdot \frac{V_2}{V_3} \right)$

$$\begin{aligned} \because v_{r4} &= v_{r3} \left(\frac{V_1}{V_2} \cdot \frac{T_2}{T_3} \right) = (8.918) \left(15 \cdot \frac{879.7}{1400} \right) = 84.065 \\ &\Rightarrow T_4 = 653.7\text{K}, u_4 = 476.12\text{ kJ/kg} \end{aligned}$$

(a) The mass of air is

$$m = \frac{P_1 V_1}{R T_1} = \frac{(170\text{ kPa})(0.016\text{ m}^3)}{\left(\frac{8.314\text{ kJ}}{28.97\text{ kg} \cdot \text{K}} \right)(315\text{K})} \left| \frac{10^3\text{ N} \cdot \text{m}}{1\text{ kPa}} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N} \cdot \text{m}} \right| = 0.03\text{ kg} \leftarrow m$$

(b) The heat addition is

$$Q_{23} = m(h_3 - h_2) = (0.03\text{ kg})(1515.42 - 910.18) \frac{\text{kJ}}{\text{kg}} = 18.157\text{ kJ} \leftarrow Q_{23}$$

The heat rejected is

$$Q_{41} = m(u_4 - u_1) = (0.03)(476.12 - 224.85) = 7.538\text{ kJ} \leftarrow Q_{41}$$

(c) The net work of the cycle equals the net heat added, or

$$W_{\text{cycle}} = Q_{23} - Q_{41} = 18.157 - 7.538 = 10.619\text{ kJ} \leftarrow W_{\text{cycle}}$$

The thermal efficiency is $\eta = W_{\text{cycle}} / Q_{23} = 10.619 / 18.157 = 0.585 (58.5\%) \leftarrow \eta$

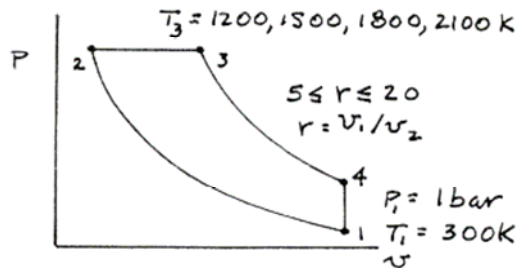
1. In early printings of the Sixth Edition, the value given for p_1 was incorrect. The correct value is $p_1 = 170\text{ kPa}$, as used in this solution. We regret the error.

PROBLEM 9.32

KNOWN: For an air-standard Diesel cycle, the state at the beginning of compression and several values for the maximum cycle temperature are known.

FIND: Plot the net work per unit mass, the mean effective pressure, and the thermal efficiency, each versus compression ratio ranging from 5 to 20.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2.

ANALYSIS: The following reasoning is used to solve the problem:

1. Given values: P_1 , T_1 , r , and T_3 .
2. Fix state 2 using ideal gas relations and $s(T_1, P_1) = s(T_2, P_2)$. When using Table A-22, this expressed in terms of $v_{r1}/v_{r2} = v_1/v_2$.
3. Using $P_2 = P_3$ and the ideal gas equation: $v_3 = (T_3/T_2)v_2$.
4. Fix the state at 4 as in step 2: $s(T_3, P_3) = s(T_4, P_4)$; or $v_{r4}/v_{r3} = v_4/v_3$. Also, note that $v_4 = v_1$.

Now, using energy balances and $W_{41} = 0$; $Q_{23} = m(h_3 - h_2)$, $Q_{41} = m(u_1 - u_4)$. (1)

Thus $W_{\text{cycle}} = Q_{23} + Q_{41} = m[(h_3 - h_2) + (u_1 - u_4)]$ (2)

Further $v_1 = \frac{RT_1}{P_1}$; $mep = \frac{W_{\text{cycle}}/m}{v_1(1 - 1/r)}$ (3)

and $\eta = \frac{W_{\text{cycle}}/m}{Q_{23}/m}$ (4)

Sample calculation. $r = 15$, $T_3 = 1200\text{K}$ From Table A-22 at $T_1 = 300\text{K}$
 $u_1 = 214.07\text{ kJ/kg}$, $v_{r1} = 621.2$. Thus $v_{r2} = (v_2/v_1)v_{r1} = (1/r)v_{r1} = 41.413$

Interpolating in Table A-22; $T_2 = 843.2\text{ K}$, $h_2 = 869.63\text{ kJ/kg}$. Now, at $T_3 = 1200\text{K}$,
 $h_3 = 1277.79\text{ kJ/kg}$, $v_{r3} = 14.470$. Further

$$v_{r4} = (v_4/v_3)v_{r3} = (v_1/v_2)(v_2/v_3)v_{r3} = (v_1/v_2)(T_2/T_3)v_{r3} = 152.514$$

Thus, $T_4 = 522.1\text{ K}$, $u_4 = 375.96\text{ kJ/kg}$.

Now, from (1)–(4)

$$Q_{23}/m = (1277.79 - 869.63) = 408.2\text{ kJ/kg}$$

$$W_{\text{cycle}} = (408.2) + (214.07 - 375.96) = 246.3\text{ kJ/kg}$$

$$\eta = 246.3/408.2 = 0.6034\text{ (60.34\%)}$$

Finally $v_1 = \frac{(8.314 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})(300\text{K})}{(1\text{ bar})} \left| \frac{1\text{ bar}}{10^5 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N}\cdot\text{m}}{1\text{ kJ}} \right| = 0.861\text{ m}^3/\text{kg}$

$$mep = \frac{(246.3\text{ kJ/kg})}{(0.861\text{ m}^3/\text{kg})(1 - 1/15)} \left| \frac{10^3 \text{ N}\cdot\text{m}}{1\text{ kJ}} \right| \left| \frac{1\text{ bar}}{10^5 \text{ N/m}^2} \right| = 3.065\text{ bar}$$

Continued on next slide

Problem 9-32 continued

The data for the required plots are obtained using IT, as follows:

IT Code

```
p1 = 1 // bar
T1 = 300 // K
T3 = 1200 // K
r = 15
m = 1 // Assume a unit mass of 1 kg.
```

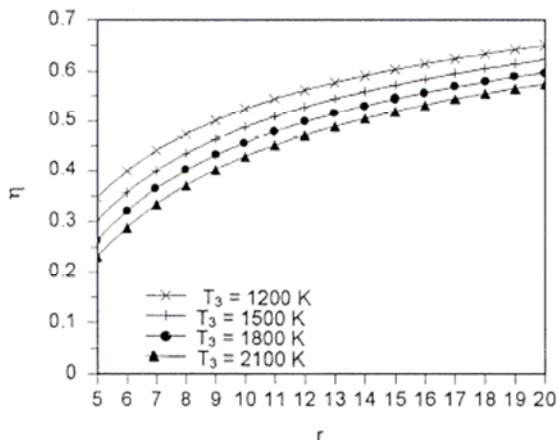
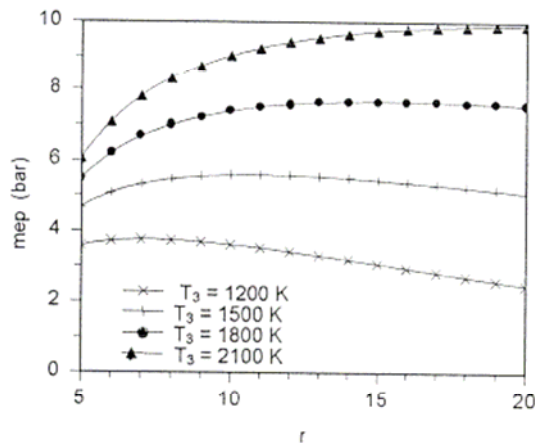
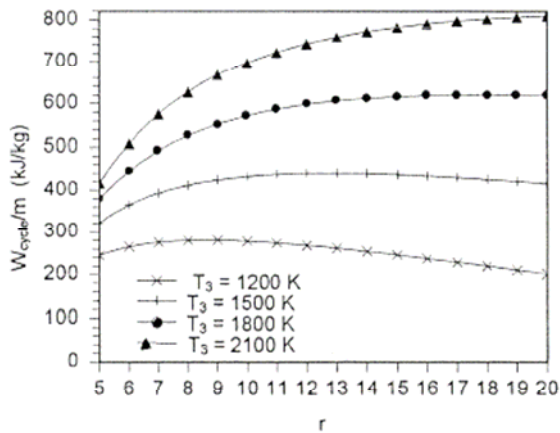
```
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
r = v1 / v2
v2 = v_TP("Air", T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
v3 / v2 = T3 / T2
v3 = v_TP("Air", T3, p3)
s3 = s_TP("Air", T3, p3)
v4 = v_TP("Air", T4, p4)
v4 = v1
s4 = s_TP("Air", T4, p4)
s4 = s3
u1 = u_T("Air", T1)
h2 = h_T("Air", T2)
h3 = h_T("Air", T3)
u4 = u_T("Air", T4)
```

```
Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
mep = (Wcycle / m) / (v1 * (1 - 1 / r)) / 100
eta = Wcycle / Q23
```

IT Results for $r = 15$ and $T_3 = 1200$ K

```
T2 = 843.4 K
p2 = 42.17 bar
h2 = 869.4 kJ/kg
h3 = 1277 kJ/kg
T4 = 521.9 K
p4 = 1.74 bar
Q23/m = 407.7 kJ/kg
Q41/m = -161.7 kJ/kg
Wcycle/m = 245.9 kJ/kg
mep = 3.06 bar
eta = 0.6033
```

PLOTS:



PROBLEM 9.33

KNOWN: Operating data are provided for an air-standard dual cycle.

FIND: Determine (a) the temperatures at the end of each heat addition process, (b) the net work per unit mass, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: See Example 9.3.

ANALYSIS: Using data from Table A-22, obtain data at each principal state.

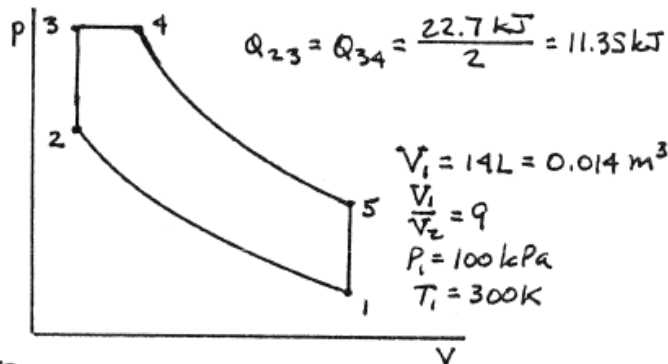
State 1: $T_1 = 300\text{ K}$, $p_1 = 100\text{ kPa}$

$$\Rightarrow u_1 = 214.07\text{ kJ/kg}, v_{r1} = 621.2$$

State 2: For the isentropic compression

$$v_{r2} = v_{r1} \left(\frac{v_2}{v_1} \right) = \frac{621.2}{9} = 69.022$$

$$\Rightarrow T_2 = 702.7\text{ K}, u_2 = 514.49\text{ kJ/kg}$$



State 3: For process 2-3; $Q_{23} = m(u_3 - u_2) \Rightarrow u_3 = \frac{Q_{23}}{m} + u_2$

The mass is

$$m = \frac{p_1 V_1}{R T_1} = \frac{(100\text{ kPa})(0.014\text{ m}^3)}{\left(\frac{8.314\text{ kJ}}{28.97\text{ kg} \cdot \text{K}} \right)(300\text{ K})} \left| \frac{10^3\text{ N/m}^2}{1\text{ kPa}} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N} \cdot \text{m}} \right| = 0.01626\text{ kg}$$

$$\text{Thus } u_3 = \frac{(11.35)}{(0.01626)} + 514.49 = 1212.52\text{ kJ/kg} \Rightarrow \begin{cases} T_3 = 1507.7\text{ K} \\ h_3 = 1645.31\text{ kJ/kg} \end{cases}$$

State 4: For the constant pressure heat addition, $Q_{34} = m(h_4 - h_3)$. Thus

$$h_4 = \frac{Q_{34}}{m} + h_3 = \frac{(11.35)}{(0.01626)} + 1645.31 = 2343.34\text{ kJ/kg}$$

$$\text{Thus } \Rightarrow T_4 = 2072.9\text{ K}, v_{r4} = 2.4639$$

$$\text{State 5: } v_{r5} = v_{r4} \left(\frac{v_5}{v_4} \right) = v_{r4} \left(\frac{v_1}{v_4} \right) = v_{r4} \left(\frac{v_1}{v_2} \cdot \frac{v_3}{v_4} \right) = (2.4639)(9) \left(\frac{1507.7}{2072.9} \right)$$

$$= 16.1298$$

$$\Rightarrow T_5 = 1158.5\text{ K}, u_5 = 896.6\text{ kJ/kg}$$

since $p_3 = p_4$; $T_3/T_4 = v_3/v_4$

$$(b) W_{\text{cycle}} = Q_{\text{cycle}} = Q_{23} + Q_{34} - Q_{51} = Q_{23} + Q_{34} - m(u_5 - u_1)$$

$$= (22.7) - (0.01626)(896.6 - 214.07) = 22.7 - 11.098 = 11.60\text{ kJ}$$

$$W_{\text{cycle}}/m = 713.4\text{ kJ/kg}$$

$$(c) \eta = W_{\text{cycle}}/Q_{\text{in}} = 11.60/22.7 = 0.511\text{ (51.1\%)} \quad \eta$$

(d) The mean effective pressure is

$$mep = \frac{W_{\text{cycle}}}{V_1(1 - v_2/v_1)}$$

$$= \frac{(11.60\text{ kJ})}{(0.014\text{ m}^3)(1 - \frac{1}{9})} \left| \frac{10^3\text{ N} \cdot \text{m}}{1\text{ kJ}} \right| \left| \frac{1\text{ kPa}}{10^3\text{ N/m}^2} \right| = 932.1\text{ kPa} \quad mep$$

PROBLEM 9.34

See Problem 9.33. The required data are obtained using IT , as follows:

IT Code

```
p1 = 100 // kPa
T1 = 300 // K
v1 / v2 = 9
V1 = 14 / 1000 // m³
Q23 + Q34 = 22.7 // kJ
Q23 / (Q23 + Q34) = ratio
ratio = .5
m = V1 / v1
```

```
u1 = u_T("Air", T1)
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air", T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
Q23 = m * (u3 - u2)
u2 = u_T("Air", T2)
u3 = u_T("Air", T3)
Q34 = m * (h4 - h3)
h3 = h_T("Air", T3)
h4 = h_T("Air", T4)
p3 / p2 = T3 / T2
p4 = p3
v4 = v_TP("Air", T4, p4)
s4 = s_TP("Air", T4, p4)
v5 = v1
```

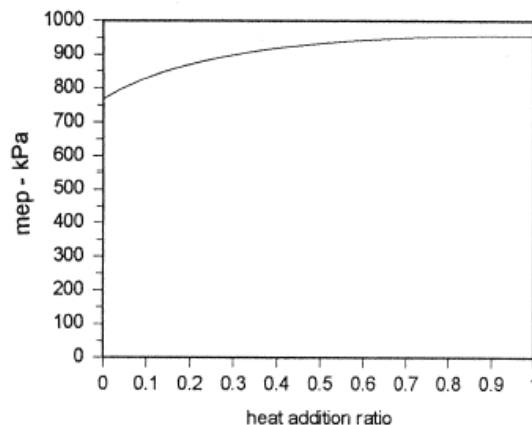
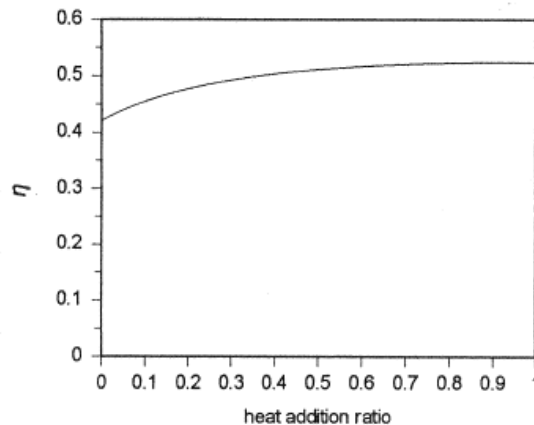
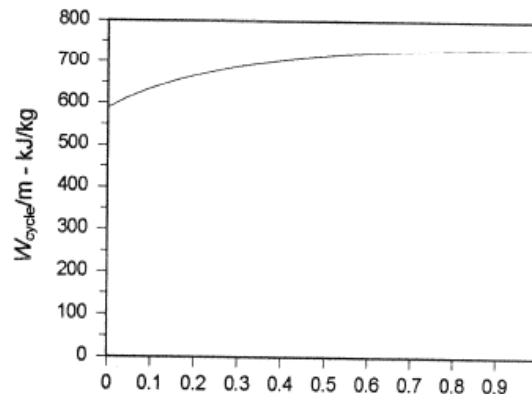
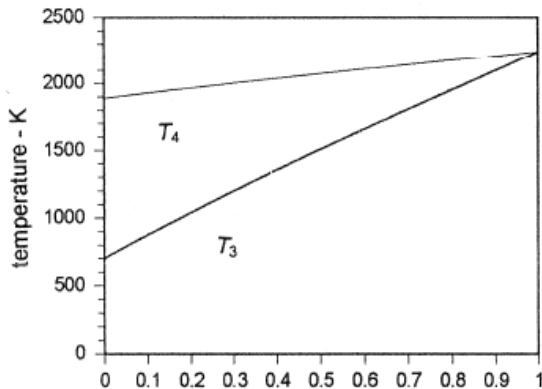
```
v5 = v_TP("Air", T5, p5)
s5 = s_TP("Air", T5, p5)
s5 = s4
u5 = u_T("Air", T5)

Wcycle = Q23 + Q34 - Q51
Q51 = m * (u5 - u1)
wcycle = Wcycle / m
eta = Wcycle / (Q23 + Q34)
mep = (Wcycle / m) / (v1 - v2)
```

IT Results for $Q_{23} = Q_{34} = 11.35$ kJ

```
T2 = 702.8 K
T3 = 1509 K
T4 = 2074 K
T5 = 1159 K
u1 = 213.9 kJ/kg
u2 = 514.3 kJ/kg
u3 = 1212 kJ/kg
h3 = 1645 kJ/kg
h4 = 2343 kJ/kg
u5 = 896.2 kJ/kg
Q51 = 11.09 kJ
Wcycle/m = 713.7 kJ/kg
eta = 0.5113
mep = 932.6 kPa
```

PLOTS:



PROBLEM 9.35

KNOWN: The dual cycle of Problem 9.33 is analyzed on a cold air-standard basis.

FIND: Determine (a) the temperatures at the end of each heat addition process, (b) the net work per unit mass, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA: For the cycle $T_1 = 300\text{ K}$, $p_1 = 100\text{ kPa}$, $V_1 = 0.014\text{ m}^3$, $V_1/V_2 = 9$, $Q_{23} = Q_{34} = 22.7\text{ kJ} = 11.35\text{ kJ}$. See the solution to Problem 9.33 for the p - V diagram. From the solution, $m = 0.01626\text{ kg}$.

ENGINEERING MODEL: See Example 9.3. Also, assume constant specific heats evaluated at 300 K. From Table A-19; $k = 1.4$, $c_v = 0.718\text{ kJ/kg}\cdot\text{K}$, $c_p = 1.005\text{ kJ/kg}\cdot\text{K}$.

ANALYSIS: For the isentropic compression $T_2 = T_1 (V_1/V_2)^{k-1} = (300)(9)^{1.4} = 722.47\text{ K}$

For the constant volume heat addition; $Q_{23} = m c_v (T_3 - T_2)$. Thus

$$T_3 = \frac{Q_{23}}{m c_v} + T_2 = \frac{(11.35)}{(0.01626)(0.718)} + 722.47 = 1694.66\text{ K} \leftarrow T_3$$

For the constant pressure process; $Q_{34} = m c_p (T_4 - T_3)$. Thus

$$T_4 = \frac{Q_{34}}{m c_p} + T_3 = \frac{(11.35)}{(0.01626)(1.005)} + 1694.66 = 2389.22\text{ K} \leftarrow T_4$$

For the isentropic expansion; $T_5 = T_4 (V_4/V_5)^{k-1} = T_4 \left(\frac{V_4}{V_3} \cdot \frac{V_2}{V_1} \right)^{k-1}$

$$= (2389.22) \left(\frac{2389.22}{1694.66} \cdot \frac{1}{9} \right)^{0.4} = 1138.23\text{ K}$$

Since $p_3 = p_4$; $V_4/V_3 = T_4/T_3$

$$(b) \quad Q_{51} = m c_v (T_5 - T_1) = (0.01626)(0.718)(1138.23 - 300) = 9.786\text{ kJ}$$

$$\text{Thus } W_{\text{cycle}} = Q_{23} + Q_{34} - Q_{51} = 22.7 - 9.786 = 12.914\text{ kJ}$$

$$W_{\text{cycle}}/m = 794.2\text{ kJ/kg} \leftarrow W_{\text{cycle}}/m$$

$$(c) \quad \eta = W_{\text{cycle}} / (Q_{23} + Q_{34}) = \frac{12.914}{22.7} = 0.569 \quad (56.9\%) \leftarrow \eta$$

$$(d) \quad m_{\text{ep}} = \frac{W_{\text{cycle}}}{V_1 (1 - V_2/V_1)} = \frac{(12.914\text{ kJ})}{(0.014\text{ m}^3)(1 - \frac{1}{9})} \left| \frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}} \right| \left| \frac{1\text{ kPa}}{10^3\text{ N/m}^2} \right|$$

$$= 1037.7\text{ kPa} \leftarrow m_{\text{ep}}$$

PROBLEM 9.36

KNOWN: An expression is given for the thermal efficiency of a cold air-standard dual cycle in terms of compression ratio, r , cutoff ratio, r_c , and pressure ratio for the constant volume heat addition, r_p .

FIND: Derive the expression.

ENGINEERING MODEL: See Example 9.3. Also, the specific heats are constant.

ANALYSIS: Begin with Eq. 9.14

$$\eta = 1 - \frac{(u_5 - u_1)}{(u_3 - u_2) + (h_4 - h_3)}$$

$$= 1 - \frac{c_v (T_5 - T_1)}{c_v (T_3 - T_2) + c_p (T_4 - T_3)} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + k (T_4 - T_3)} \quad (*)$$

Note that $r = v_1/v_2$, $r_c = v_4/v_3$, and $r_p = p_3/p_2$. Therefore

$$T_1 = \left(\frac{v_2}{v_1}\right)^{k-1} T_2 = \frac{T_2}{r^{k-1}}$$

$$T_3 = \left(\frac{p_3}{p_2}\right) T_2 = r_p T_2$$

$$T_4 = \left(\frac{v_4}{v_3}\right) T_3 = r_c T_3 = r_c r_p T_2$$

$$T_5 = \left(\frac{v_4}{v_5}\right)^{k-1} T_4 = \left(\frac{v_4}{v_3} \cdot \frac{v_2}{v_1}\right)^{k-1} T_4 = \left(r_c \cdot \frac{1}{r}\right)^{k-1} T_4 = r_p \left(\frac{r_c^k}{r^{k-1}}\right) T_2$$

Substituting these relations into (*) gives

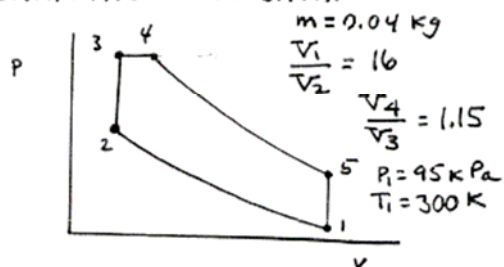
$$\eta = 1 - \frac{1}{r^{k-1}} \left[\frac{r_p r_c^k - 1}{(r_p - 1) + k r_p (r_c - 1)} \right] \leftarrow \text{result}$$

PROBLEM 9.37

KNOWN: Operating data are provided for an air-standard dual cycle.

FIND: Determine (a) the heat addition at constant volume and at constant pressure, (b) the net work, (c) the heat rejected, (d) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

See Example 9.3

ANALYSIS: Using data from Table A-22, fix each of the principal states.

State 1: At $T_1 = 300 \text{ K}$, $u_1 = 214.07 \text{ kJ/kg}$
 $v_{r1} = 621.2$

State 2: For the isentropic compression
 $v_{r2} = \frac{v_2}{v_1} v_{r1} = \frac{621.2}{16} = 38.825 \Rightarrow u_2 = 643.35 \text{ kJ/kg}$
 $T_2 = 862.4 \text{ K}$

State 3: For the constant volume heat addition, $T_3 = (P_3/P_2)T_2 = 1897.3 \text{ K}$.

$\Rightarrow u_3 = 1580.0 \text{ kJ/kg}$, $h_3 = 2124.0 \text{ kJ/kg}$.

State 4: For the constant pressure heat addition, $T_4 = \frac{v_4}{v_3} T_3 = (1.15)(1897.3) = 2182 \text{ K}$.

$\Rightarrow h_4 = 2480.6 \text{ kJ/kg}$, $v_{r4} = 2.0707$

State 5: For the isentropic expansion $\frac{v_5}{v_4} = \frac{v_1}{v_2} \frac{v_3}{v_4} = (16)\left(\frac{1}{1.15}\right) = 13.913$

$\Rightarrow v_{r5} = \frac{v_5}{v_4} v_{r4} = (13.913)(2.0707) = 28.810 \Rightarrow u_5 = 721.2 \text{ kJ/kg}$

(a) Applying energy balances

$$Q_{23} = m(u_3 - u_2) = 0.04 \text{ kg} (1580.0 - 643.35) \frac{\text{kJ}}{\text{kg}} = 37.47 \text{ kJ} \quad \leftarrow$$

$$Q_{34} = m(h_4 - h_3) = 0.04 \text{ kg} (2480.6 - 2124.0) \frac{\text{kJ}}{\text{kg}} = 14.26 \text{ kJ} \quad \leftarrow$$

(b) For any cycle, $W_{\text{cycle}} = Q_{\text{cycle}}$. Thus, since processes 1-2, 4-5 are adiabatic,

$$W_{\text{cycle}} = (Q_{23} + Q_{34}) - Q_{51} = (37.47 + 14.26) - \underbrace{(0.04)(721.2 - 214.07)}_{20.29 \text{ kJ}}$$

$$= 31.44 \text{ kJ} \quad \leftarrow$$

(c) The heat rejection is $Q_{51} = m(u_5 - u_1) = 20.29 \text{ kJ} \quad \leftarrow$

(d) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{31.44}{37.47 + 14.26} = 0.608 \quad (60.8\%) \quad \leftarrow$$

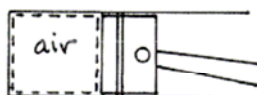
$\leftarrow Q_{23} + Q_{34}$

PROBLEM 9.38

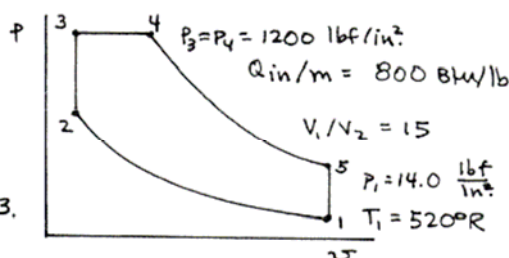
KNOWN: An air-standard dual cycle has a known compression ratio and a specified state at the beginning of compression. The total heat addition per unit mass and the pressure at the end of the constant volume heat addition are also known.

FIND: Determine the net work and the heat rejection, each per unit mass of air. Also, the thermal efficiency and cutoff ratio. Plot those quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.3.



ANALYSIS: Begin by fixing each of the principal states of the cycle (Table A-16E).

State 1: $T_1 = 520^\circ\text{R} \Rightarrow u_1 = 88.62 \text{ Btu/lb}, v_{r1} = 158.58$

State 2: For the isentropic compression, $v_{r2} = (V_2/V_1) v_{r1} = 10.572$.
Thus, $T_2 = 1468.8^\circ\text{R}$, $u_2 = 260.26 \text{ Btu/lb}$, and $P_{r2} = 51.561$

State 3: Since $V_3 = V_2$, $T_3 = (P_3/P_2) T_2$. To get P_2 , use
 $P_2 = (P_{r2}/P_{r1}) P_1 = 594.26 \text{ lbf/in}^2 \Rightarrow T_3 = 2965.9^\circ\text{R}$
Thus, $u_3 = 577.4 \text{ Btu/lb}$ and $h_3 = 780.7 \text{ Btu/lb}$

State 4: The total heat addition is known. Thus

$$\frac{Q_{in}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} = (u_3 - u_2) + (h_4 - h_3)$$

$$\text{or } h_4 = \frac{Q_{in}}{m} - (u_3 - u_2) + h_3 = 1263.56 \text{ Btu/lb}$$

Thus, $T_4 = 4577.6^\circ\text{R}$ and $v_{r4} = 0.2848$

State 5: For the isentropic expansion

$$\frac{v_{r5}}{v_{r4}} = \frac{V_1}{V_2} \cdot \frac{V_3}{V_4} = \frac{V_1}{V_2} \cdot \frac{T_3}{T_4} = 9.7187$$

$$\text{and } v_{r5} = \left(\frac{v_{r5}}{v_{r4}}\right) v_{r4} = 2.768 \Rightarrow T_5 = 2299.2^\circ\text{R}, u_5 = 431.0 \text{ Btu/lb}$$

Now, the heat rejection per unit mass is

$$(b) \quad \frac{Q_{51}}{m} = u_5 - u_1 = 431.0 - 88.62 = 342.4 \text{ Btu/lb} \quad \leftarrow \frac{Q_{51}}{m}$$

and for the cycle, $W_{cycle} = Q_{cycle}$. Thus

$$(a) \quad \frac{W_{cycle}}{m} = \frac{Q_{in}}{m} - \frac{Q_{51}}{m} = 457.6 \text{ Btu/lb} \quad \leftarrow \frac{W_{cycle}}{m}$$

(c) The thermal efficiency is

$$\eta = (W_{cycle}/m) / (Q_{in}/m) = (457.6) / (800) = 0.572 \text{ (57.2\%)} \quad \leftarrow \eta$$

(d) The cutoff ratio is

$$r_c = v_4/v_3 = T_4/T_3 = 4577.6/2965.9 = 1.543 \quad \leftarrow r_c$$

Continued on next slide

Problem 9-38 continued

The data for the required plots are obtained using IT, as follows:

IT Code

```
p1 = 14 // lbf/in.2
T1 = 520 // °R
r = v1 / v2
r = 15
p3 = 1200 // lbf/in.2
Qin / m = 800 // Btu/lb
m = 1 // Assume a unit mass of 1 lb.
```

```
u1 = u_T("Air", T1)
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air", T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
T3 / T2 = p3 / p2
u3 = u_T("Air", T3)
h3 = h_T("Air", T3)
v3 = v2
Qin / m = (u3 - u2) + (h4 - h3)
h4 = h_T("Air", T4)
v4 = v_TP("Air", T4, p4)
```

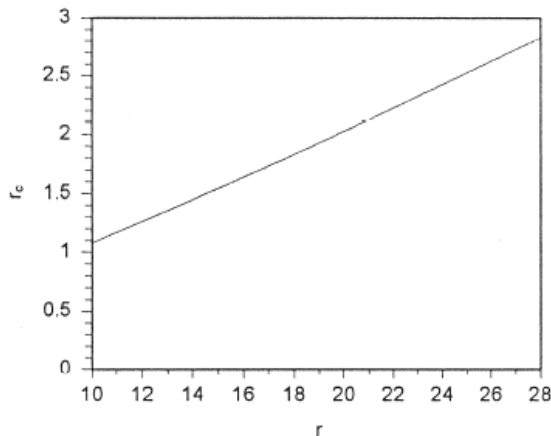
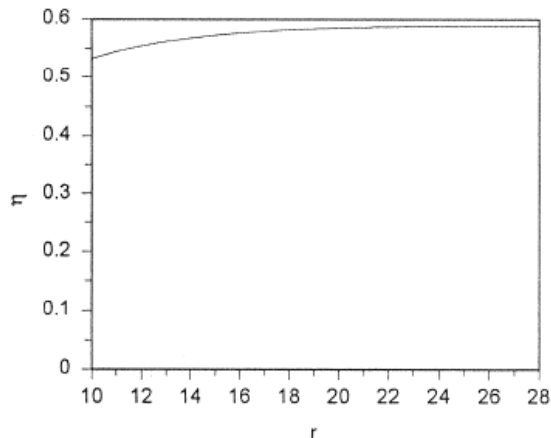
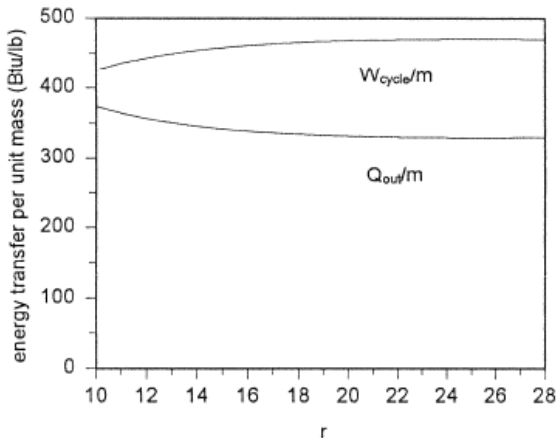
```
p4 = p3
s4 = s_TP("Air", T4, p4)
v5 = v_TP("Air", T5, p5)
s5 = s_TP("Air", T5, p5)
s5 = s4
v5 = v1
u5 = u_T("Air", T5)

Qout / m = u5 - u1
Wcycle / m = Qin / m - Qout / m
eta = Wcycle / Qin
rc = v4 / v3
```

IT Results for $r = 15$

```
T2 = 1469 °R
T3 = 2971 °R
T4 = 4582 °R
T5 = 2301 °R
Wcycle/m = 457.9 Btu/lb
Qout/m = 342.1 Btu/lb
η = 0.5724
rc = 1.542
```

PLOTS:



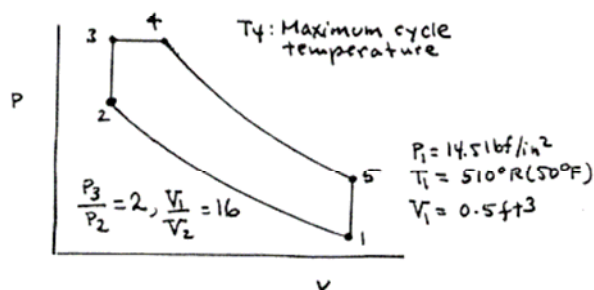
We see from the graphs that the thermal efficiency increases slightly with increasing r . Since Q_{in} is constant, it follows that W_{cycle} would increase and Q_{out} would decrease. Also, as r increases, the cutoff ratio increases, indicating that of the heat input occurs at constant pressure.

PROBLEM 9.39

KNOWN: Operating data are provided for an air-standard dual cycle.

FIND: For a maximum cycle temperature of 3000°R determine (a) the heat added, (b) the net work, (c) the thermal efficiency, (d) the mean effective pressure. (e) Plot the quantities of parts (a)-(d) versus maximum cycle temperature varying from 3000 to 4000°R .

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

See Example 9.3

ANALYSIS:

$T_4 = 3000^\circ\text{R}$... begin by fixing the state of each principal state using data from Table A-22E.

State 1: $T_1 = 510^\circ\text{R}$, $u_1 = 86.91 \text{ Btu/lb}$, $v_{r1} = 166.74$.

State 2: For the isentropic compression, $v_{r2} = (v_2/v_1)v_{r1} = (166.74/16) = 10.421$
 $\Rightarrow T_2 = 1476.1^\circ\text{R}$, $u_2 = 261.68 \text{ Btu/lb}$

State 3: For the constant volume heat addition, $T_3/T_2 = P_3/P_2$. Thus,
 $T_3 = 2T_2 = 2952.2^\circ\text{R} \Rightarrow u_3 = 574.33 \text{ Btu/lb}$, $h_3 = 776.69 \text{ Btu/lb}$.

State 4: With $T_4 = 3000^\circ\text{R}$, $h_4 = 790.68 \text{ Btu/lb}$, $v_{r4} = 1.18$

State 5: For the isentropic expansion $v_{r5} = (v_5/v_4)v_{r4}$, where

$$\frac{v_5}{v_4} = \frac{v_1}{v_2} \frac{v_3}{v_4} = \frac{v_1}{v_2} \frac{T_3}{T_4} = 16 \left(\frac{2952.2}{3000} \right) = 15.745 \Rightarrow v_{r5} = 18.579$$

$\leftarrow \text{since } P_3 = P_4,$
 $\frac{v_3}{v_4} = \frac{T_3}{T_4}$

$\Rightarrow u_5 = 208.76 \text{ Btu/lb}$

The mass of the system is obtained from

$$m = \frac{P_1 v_1}{R T_1} = \frac{(14.5 \times 144 \text{ lbf/ft}^2)(0.5 \text{ ft}^3)}{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right)(510^\circ\text{R})} = 0.03838 \text{ lb}$$

(a) The heat added is the sum $Q_{23} + Q_{34}$

$$\begin{aligned} Q_{23} &= m(u_3 - u_2) = 0.03838(574.33 - 261.68) = 12 \text{ Btu} \\ Q_{34} &= m(h_4 - h_3) = 0.03838(790.68 - 776.69) = 0.54 \text{ Btu} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} Q_{in} = 12.54 \text{ Btu} \leftarrow Q_{in}$$

(b) Since $w_{cycle} = Q_{cycle}$, and processes 1-2, 4-5 are adiabatic

$$w_{cycle} = Q_{in} - Q_{51} = Q_{in} - m(u_5 - u_1) = 12.54 - 0.03838(208.76 - 86.91) = 7.86 \text{ Btu}$$

$$(d) \quad mep = \frac{w_{cycle}}{(v_1 - v_2)} = \frac{w_{cycle}}{v_1 \left[1 - \frac{v_2}{v_1} \right]} = \frac{7.86 \times 778 \text{ ft} \cdot \text{lbf}}{0.5 \text{ ft}^3 \left[1 - \frac{1}{16} \right]} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 90.59 \frac{\text{lbf}}{\text{in}^2} \leftarrow mep$$

$$(c) \quad \eta = \frac{w_{cycle}}{Q_{in}} = \frac{7.86}{12.54} = 0.627 \quad (62.7\%) \quad \leftarrow \eta$$

Continued on next slide

Problem 9-39 continued

The data for the required plots are obtained using IT, as follows:

```

IT Code
p1 = 14.5 // lbf/in.²
T1 = 510 // °R
V1 = 0.5 // ft³
V1 / V2 = 16
p3 / p2 = 2
T4 = 3000 // °R

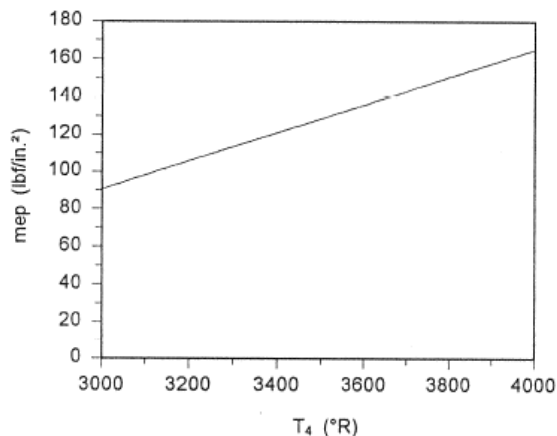
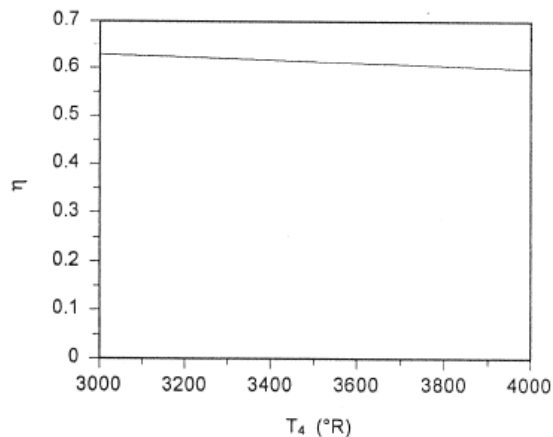
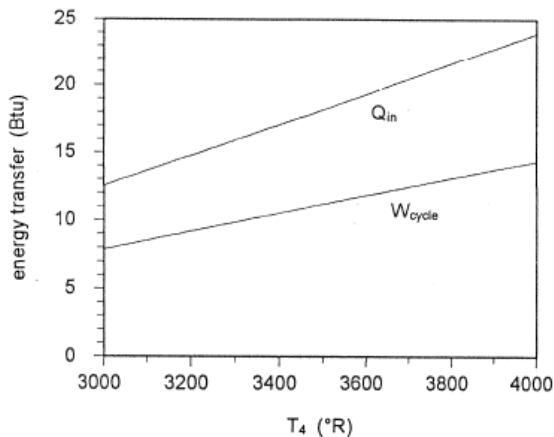
v1 = v_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air", T2, p2)
v1 / v2 = 16
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
T3 / T2 = p3 / p2
u3 = u_T("Air", T3)
h3 = h_T("Air", T3)
v3 = v2
h4 = h_T("Air", T4)
v4 = v_TP("Air", T4, p4)
p4 = p3
s4 = s_TP("Air", T4, p4)

v5 = v_TP("Air", T5, p5)
s5 = s_TP("Air", T5, p5)
s5 = s4
v5 = v1
u5 = u_T("Air", T5)

v1 = V1 / m
Qin = m * (u3 - u2) + m * (h4 - h3)
Q51 = m * (u1 - u5)
Wcycle = Qin + Q51
eta = Wcycle / Qin
mep = Wcycle / (V1 * (1 - V2 / V1)) * (778 / 144)

IT Results for T4 = 3000°R
T2 = 1477°R
T3 = 2954°R
T5 = 1196°R
m = 0.03838 lb
Qin = 12.49 Btu
Wcycle = 7.837 Btu
η = 0.6273
mep = 90.33 lbf/in.²
    
```

PLOTS:



As the maximum cycle temperature increases, we see that the net work and mep both increase as well. However, the heat addition also goes up, and as a result the thermal efficiency decreases slightly.

PROBLEM 9.40

KNOWN: An air-standard dual cycle has a known compression ratio and a specified state at the beginning of compression. The total heat addition is given.

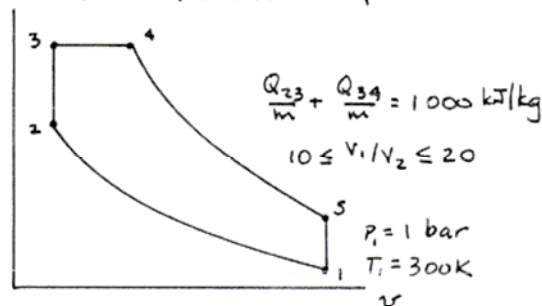
FIND: Plot the net work per unit mass, the mean effective pressure, and the thermal efficiency versus compression ratio ranging from 10 to 20 for various values of the ratio of constant volume to total heat input.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: See Example 9.3.

ANALYSIS: The following reasoning is used to solve the problem:

1. Given values: P_1 , T_1 , r , and $Q_{23}/(Q_{23}+Q_{34})$.
2. Fix state 2 using ideal gas relations and $S(T_1, P_1) = S(T_2, P_2)$. When using Table A-22, this is expressed in terms of $v_{r1}/v_{r2} = v_1/v_2$.
3. Use $Q_{23} = m(u_3 - u_2)$ to get T_3 . Then $v_3 = v_2$ gives $P_3 = (T_3/T_2)P_2$.
4. For state 4, use $P_4 = P_3$ and $Q_{34} = m(h_4 - h_3)$ to get T_4 , v_4 .
5. Fix state 5 as in step 2 using $S(T_4, P_4) = S(T_5, P_5)$ or $v_{r4}/v_{r5} = v_4/v_5$. Also, note that $v_5 = v_1$.



Now, using energy balances: $W_{cycle} = Q_{cycle} \Rightarrow \frac{W_{cycle}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} - (u_5 - u_1)$

Where $Q_{out}/m = u_5 - u_1$. Also

$$\eta = \frac{W_{cycle}/m}{(Q_{23} + Q_{34})/m} \quad \text{and} \quad mep = \frac{W_{cycle}/m}{v_1(1 - 1/r)}$$

Sample calculation. $r = 10$, $Q = Q_{23}/(Q_{23} + Q_{34}) = 0.5$ From Table A-22

at $T_1 = 300\text{ K}$; $u_1 = 214.07\text{ kJ/kg}$, $v_{r1} = 621.2$. Thus $v_{r2} = (v_2/v_1)v_{r1} = 62.12$.

Interpolating in Table A-22; $T_2 = 730\text{ K}$, $u_2 = 536.1\text{ kJ/kg}$. Now

$$u_3 = Q_{23}/m + u_2 = 1036.1\text{ kJ/kg} \Rightarrow T_3 = 1314.7\text{ K}, h_3 = 1413.46\text{ kJ/kg}$$

Further, $h_4 = Q_{34}/m + h_3 = 1913.46\text{ kJ/kg} \Rightarrow T_4 = 1727.1\text{ K}$, $v_{r4} = 4.526$

$$v_{r5} = (v_5/v_4)v_{r4} = (v_1/v_2)(v_3/v_4)v_{r4} = (v_1/v_2)(T_3/T_4)v_{r4} = 34.453$$

Thus, $T_5 = 898.8\text{ K}$, $u_5 = 673.55\text{ kJ/kg}$.

Now

$$W_{cycle}/m = 1000 - (673.55 - 214.07) = 540.5\text{ kJ/kg}$$

$$\eta = \frac{W_{cycle}/m}{(Q_{23} + Q_{34})/m} = 0.5405 \quad (54.05\%)$$

$$v_1 = \frac{RT_1}{P_1} = \frac{(8.314\text{ kJ/kg}\cdot\text{K}) (300\text{ K})}{(1\text{ bar})} \left| \frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}} \right| \left| \frac{1\text{ bar}}{10^5\text{ N/m}^2} \right| = 0.861\text{ m}^3/\text{kg}$$

$$mep = \frac{(540.5\text{ kJ/kg})}{(0.861\text{ m}^3/\text{kg})(1 - 1/10)} \left| \frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}} \right| \left| \frac{1\text{ bar}}{10^5\text{ N/m}^2} \right| = 6.975\text{ bar}$$

The data for the required plots are obtained using IT, as follows:

Continued on next slide

Problem 9-40 continued

IT Code

```
p1 = 1 // bar
T1 = 300 // K
(Q23 + Q34) / m = 1000 // kJ/kg
Q = Q23 / (Q23 + Q34)
Q = .5
r = 10
m = 1 // Assume a unit mass of 1
kg.
```

```
v1 = v_TP("Air", T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
v2 = v_TP("Air", T2, p2)
s2 = s_TP("Air", T2, p2)
u2 = u_T("Air", T2)
m * (u3 - u2) = Q23
u3 = u_T("Air", T3)
v3 = v2
v3 = v_TP("Air", T3, p3)
s3 = s_TP("Air", T3, p3)
h3 = h_T("Air", T3)
Q34 / m = h4 - h3
h4 = h_T("Air", T4)
```

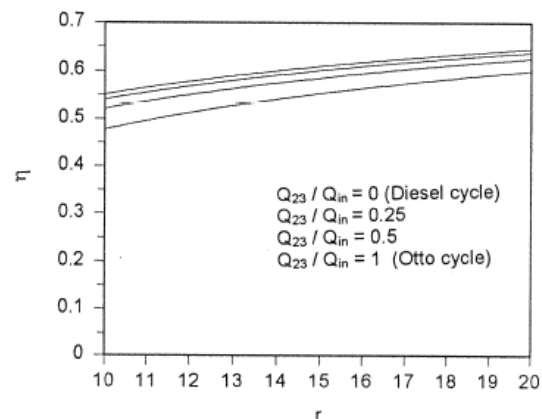
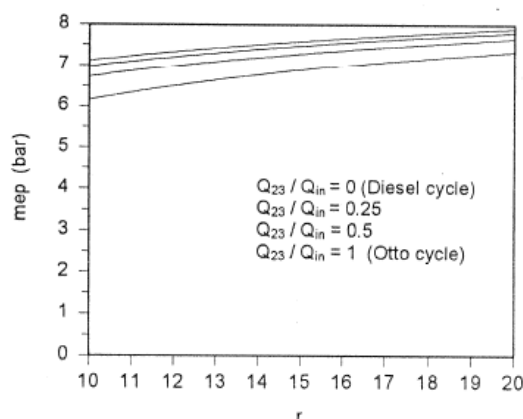
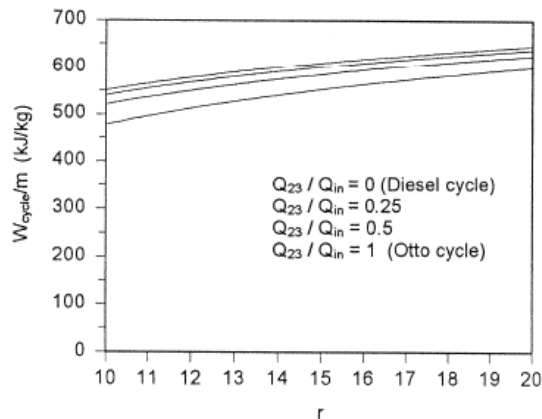
```
p4 = p3
v4 = v_TP("Air", T4, p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)
v5 = v1
s5 = s4
v5 = v_TP("Air", T5, p5)
s5 = s_TP("Air", T5, p5)
u5 = u_T("Air", T5)
```

```
Wcycle = Q23 / m + Q34 / m + Q51 / m
Q51 = m * (u1 - u5)
eta = Wcycle / (Q23 + Q34)
mep = (Wcycle / m) / (v1 * (1 - 1 / r)) / 100
```

IT Results for $Q_{23} / Q_{in} = 0.5$ and $r = 10$

```
T2 = 730.2 K
p2 = 24.34 bar
T3 = 1316 K
p3 = 43.86 bar
T4 = 1729 K
T5 = 899.3 K
Wcycle/m = 540.2 kJ/kg
eta = 0.5402
mep = 6.972 bar
```

PLOTS :

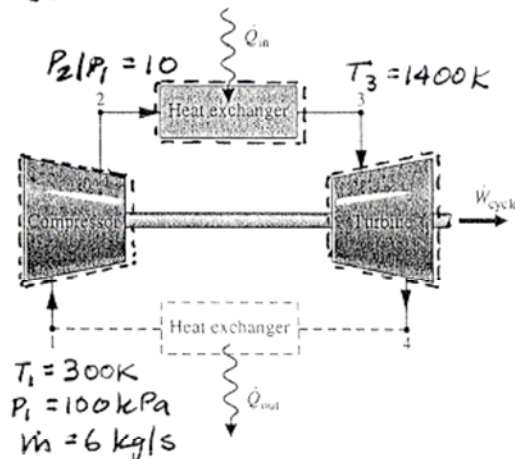
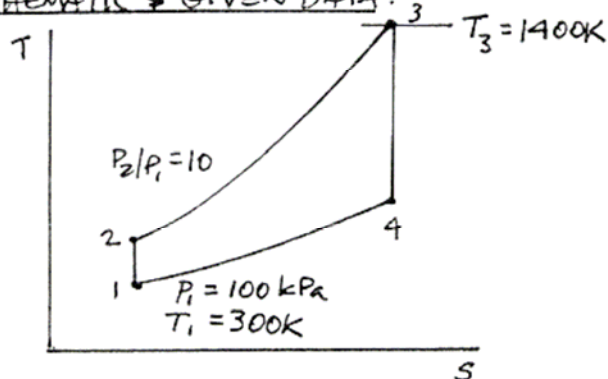


PROBLEM 9.41

KNOWN: Air enters a cold air-standard ideal Brayton cycle with a given flow rate and at a specified state. The compressor pressure ratio and maximum cycle temperature are known.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, and (c) the net power.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.4.

Also, assume $k = 1.4$ and $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$.

ANALYSIS: First, determine T_2 and T_4 , as follows:

$$(Eq. 9.23) \quad T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = (300) (10)^{\frac{1.4-1}{1.4}} = 579.2 \text{ K}$$

$$(Eq. 9.24) \quad T_4 = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = (1400) \left(\frac{1}{10} \right)^{\frac{1.4-1}{1.4}} = 725.13 \text{ K}$$

(a) To evaluate thermal efficiency, use

$$\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}}$$

$$\dot{Q}_{in}: \quad \dot{Q}_{in} = \dot{m} (h_3 - h_2) = \dot{m} c_p (T_3 - T_2) \\ = (6 \text{ kg/s}) (1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) (1400 - 579.2) \text{ K} = 4949.4 \text{ kJ/s}$$

$$\dot{Q}_{out}: \quad \dot{Q}_{out} = \dot{m} (h_4 - h_1) = \dot{m} c_p (T_4 - T_1) = (6) (1.005) (725.13 - 300) = 2563.5 \frac{\text{kJ}}{\text{s}}$$

$$\text{Thus} \quad \eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{2563.5}{4949.4} = 0.482 (48.2\%) \quad \leftarrow \eta$$

(b) The back work ratio is $bwr = \dot{W}_c / \dot{W}_t$

$$\dot{W}_c = \dot{m} (h_2 - h_1) = \dot{m} c_p (T_2 - T_1) = (6) (1.005) (579.2 - 300) \\ = 1683.6 \text{ kJ/s}$$

$$\dot{W}_t = \dot{m} (h_3 - h_4) = \dot{m} c_p (T_3 - T_4) = (6) (1.005) (1400 - 725.13) = 4069.5 \frac{\text{kJ}}{\text{s}}$$

$$\text{and} \quad bwr = \frac{\dot{W}_c}{\dot{W}_t} = \frac{1683.6}{4069.5} = 0.4137 \quad \leftarrow bwr$$

(c) The net power developed is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = (4069.5 - 1683.6) \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 2385.9 \text{ kW} \quad \leftarrow \dot{W}_{cycle}$$

PROBLEM 9.42

See Problem 9.41

IT Code

```
T1 = 300 // K
p1 = 100 // kPa
rp = 10
p2 = p1 * rp
p3 = p2
T3 = 1400 // K
p4 = p1
k = 1.4
cp = 1.005 // kJ/kg-K
mdot = 6 // kg/s
```

```
T2 = T1 * (p2 / p1) ^ ((k-1)/k)
T4 = T3 * (p4 / p3) ^ ((k-1)/k)
```

```
Qdotin = mdot * cp * (T3 - T2)
Qdotout = mdot * cp * (T4 - T1)
eta = 1 - Qdotout / Qdotin
Wdotc = mdot * cp * (T2 - T1)
Wdott = mdot * cp * (T3 - T4)
bwr = Wdotc / Wdott
Wdotcycle = Wdott - Wdotc
```

IT Results for $r_p = 10$, $T_3 = 1400$ K

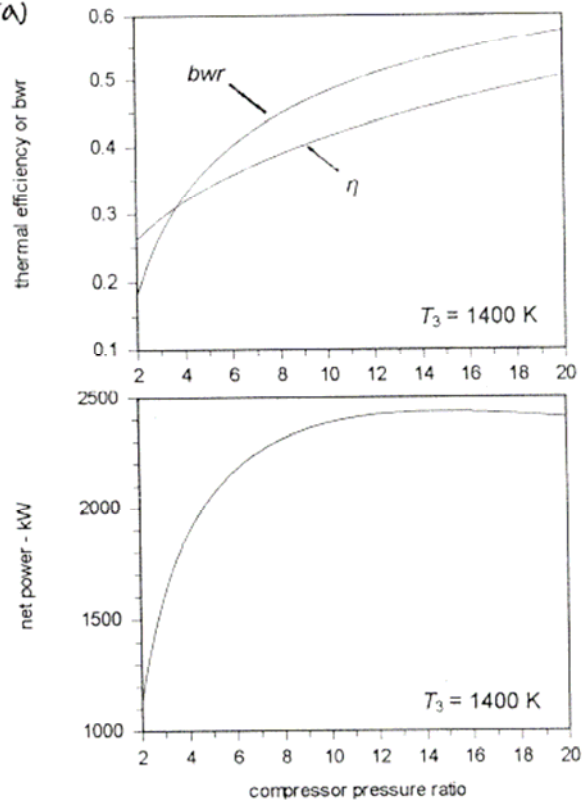
```
T2 = 579.2 K
T4 = 725.1 K
bwr = 0.4137
eta = 0.4821
Qdotin = 4949 kW
Qdotout = 2564 kW
Wdotc = 1684 kW
Wdott = 4069 kW
Wdotcycle = 2386 kW
```

DISCUSSION:

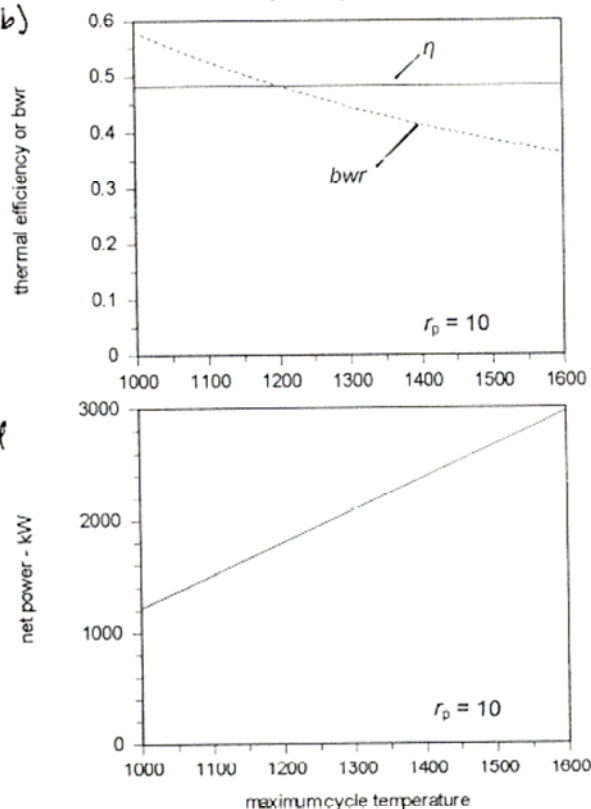
(a) Thermal efficiency and back work ratio both increase with increasing compressor pressure ratio at fixed T_3 . The net power exhibits a maximum in the range of r_p studied. See the discussion of Sec. 9.6.2 and Ex. 9.5.

(b) For fixed compression ratio, the thermal efficiency of the cold air-standard cycle is constant with T_3 , and the back work ratio increases. The net power increases as well as T_3 increases, since the compressor power is constant, but W_t increases.

(a)



(b)

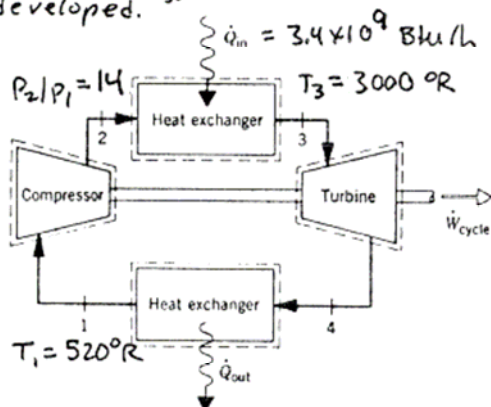
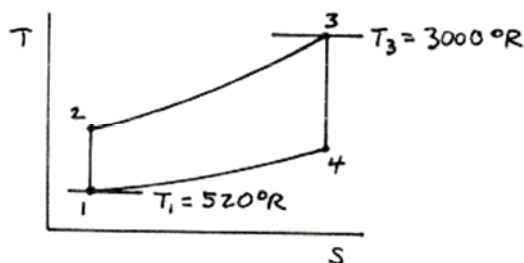


PROBLEM 9.43

KNOWN: The net power developed and the pressure ratio are known for an ideal air-standard Brayton cycle. The minimum and maximum temperatures are also known.

FIND: Determine (a) the thermal efficiency, (b) the mass flow rate of air, (c) the net power developed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.4.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1 $T_1 = 520^\circ\text{R} \Rightarrow h_1 = 124.27 \text{ Btu/lb}$, $P_{r1} = 1.2147$

State 2 For the isentropic compression, $P_{r2} = (P_2/P_1)P_{r1} = 17.0058$
Thus, $T_2 = 1092.5^\circ\text{R}$ and $h_2 = 264.12 \text{ Btu/lb}$

State 3 $T_3 = 3000^\circ\text{R} \Rightarrow h_3 = 790.68 \text{ Btu/lb}$, $P_{r3} = 941.4$

State 4 For the isentropic expansion, $P_{r4} = (P_4/P_3)P_{r3} = 67.24$
Thus, $T_4 = 1573^\circ\text{R}$ and $h_4 = 388.63 \text{ Btu/lb}$

(a) The thermal efficiency is

$$\eta = 1 - \frac{\dot{Q}_{\text{out}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)}$$

$$= 1 - \frac{(388.63 - 124.27)}{(790.68 - 264.12)} = 0.498 \text{ (49.8\%)} \quad \leftarrow \eta$$

(b) To determine the mass flow rate of air, start with the rate of heat addition

$$\dot{m} = \frac{\dot{Q}_{\text{in}}}{(h_3 - h_2)} = \frac{(3.4 \times 10^9 \text{ Btu/h})}{(790.68 - 264.12) \text{ Btu/lb}}$$

$$= 6.457 \times 10^6 \text{ lb/h} \quad \leftarrow \dot{m}$$

(c) The net power is

$$\dot{W}_{\text{cycle}} = \dot{m} [(h_3 - h_4) - (h_2 - h_1)]$$

$$= (6.457 \times 10^6) [(790.68 - 388.63) - (264.12 - 124.27)]$$

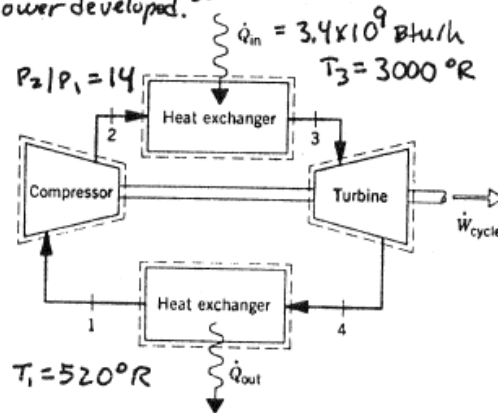
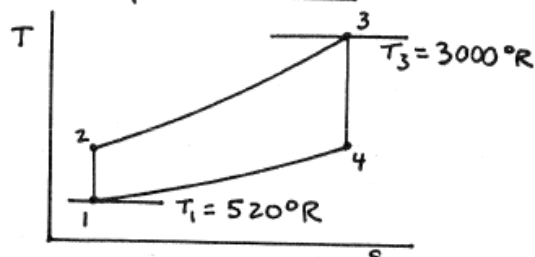
$$= 1.693 \times 10^9 \text{ Btu/h} \quad \leftarrow \dot{W}_{\text{cycle}}$$

Problem 9-44

KNOWN: The net power developed and the pressure ratio are known for a cold air-standard ideal Brayton cycle. The minimum and maximum temperatures are also known.

FIND: Determine (a) the thermal efficiency, (b) the mass flow rate of air, (c) the net power developed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.4. Also, assume constant specific heats evaluated at 520°R.

ANALYSIS: From Table A-20E, $k = 1.401$ and $c_p = 0.240 \text{ Btu/lb} \cdot ^\circ\text{R}$. $T_1 = 520^\circ\text{R}$ and $T_3 = 3000^\circ\text{R}$ are given. For the isentropic compression process

$$T_2 = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} T_1 = 1106.8^\circ\text{R}$$

Also, for the isentropic expansion

$$T_4 = \left(\frac{P_4}{P_3} \right)^{\frac{k-1}{k}} T_3 = 1409.5^\circ\text{R}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{1}{(P_2/P_1)^{(k-1)/k}} = 0.530 \text{ (53.0\%)} \quad \leftarrow \eta$$

(b) To determine the mass flow rate of air, start with the rate of heat addition

$$\dot{m} = \frac{\dot{q}_{in}}{c_p(T_3 - T_2)} = \frac{(3.4 \times 10^9 \text{ Btu/h})}{(0.240 \text{ Btu/lb} \cdot ^\circ\text{R})(3000 - 1106.8)^\circ\text{R}} = 7.483 \times 10^6 \text{ lb/h} \quad \leftarrow \dot{m}$$

(c) The net power is

$$\begin{aligned} \dot{W}_{cycle} &= \dot{m} [(h_3 - h_4) - (h_2 - h_1)] = \dot{m} c_p [(T_3 - T_4) - (T_2 - T_1)] \\ &= (7.483 \times 10^6)(0.240)[(3000 - 1409.5) - (1106.8 - 520)] \\ &= 1.803 \times 10^9 \text{ Btu/h} \quad \leftarrow \dot{W}_{cycle} \end{aligned}$$

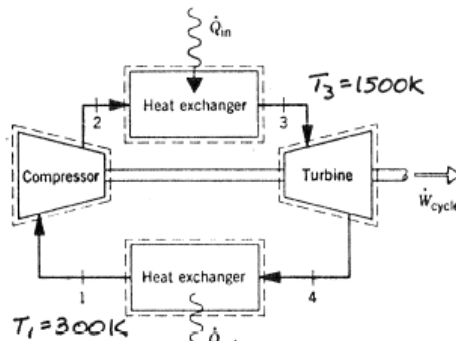
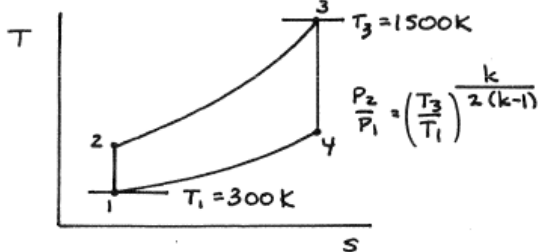
Discussion: The results of this problem can be compared with those of Problem 9.42 to see some effects of the assumption of constant specific heats.

PROBLEM 9.45

KNOWN: An ideal cold air-standard Brayton cycle has known minimum and maximum temperatures a pressure that corresponds to the maximum net work per unit mass of air flow.

FIND: Determine (a) the compressor and turbine work per unit of mass flow, (b) thermal efficiency, (c) Plot the thermal efficiency versus the maximum cycle temperature ranging from 1200 to 1800 K.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.5.

ANALYSIS: First, determine the unknown temperatures at principal states 2 and 4. From the solution to Example 9.5

$$\frac{P_2}{P_1} = \left(\frac{T_3}{T_1} \right)^{\frac{k}{2(k-1)}} = \left(\frac{1500}{300} \right)^{\frac{1.4}{2(1.4-1)}} = 16.719$$

where $k=1.4$ from Table A-20. For the isentropic compression

$$T_2 = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} T_1 = 670.8^\circ \text{R}$$

and for the isentropic expansion

$$T_4 = \left(\frac{P_4}{P_3} \right)^{\frac{k-1}{k}} T_3 = 670.8^\circ \text{R}$$

(a) The compressor work per unit mass of air flow is

$$\begin{aligned} \frac{\dot{W}_{\text{comp}}}{\dot{m}} &= h_2 - h_1 = c_p (T_2 - T_1) \\ &= (1.005 \text{ kJ/kg} \cdot \text{K})(670.8 - 300) \text{ K} = 372.6 \frac{\text{kJ}}{\text{kg}} \leftarrow \frac{\dot{W}_{\text{comp}}}{\dot{m}} \end{aligned}$$

similarly, for the turbine

$$\begin{aligned} \frac{\dot{W}_{\text{turb}}}{\dot{m}} &= c_p (T_3 - T_4) \\ &= (1.005)(1500 - 670.8) = 833.3 \text{ kJ/kg} \leftarrow \frac{\dot{W}_{\text{turb}}}{\dot{m}} \end{aligned}$$

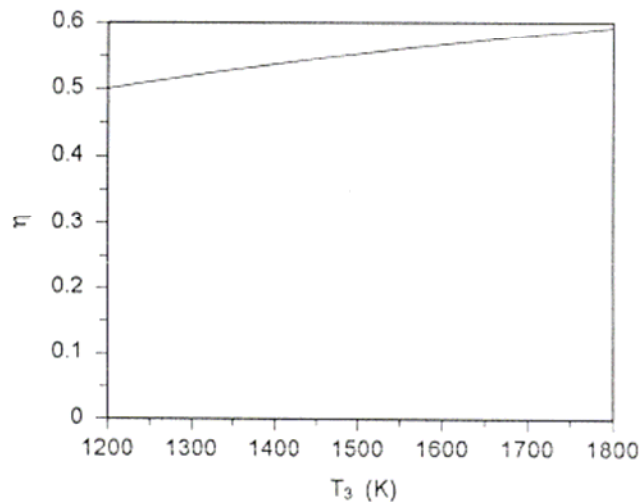
(b) Using Eq. 9.25 together with $P_2/P_1 = (T_3/T_1)^{\frac{k}{2(k-1)}}$, we get

$$\begin{aligned} \eta &= 1 - \frac{1}{(P_2/P_1)^{(k-1)/k}} = 1 - \frac{1}{\left[\left(\frac{T_3}{T_1} \right)^{\frac{k}{2(k-1)}} \right]^{\frac{k-1}{k}}} = 1 - \left(\frac{T_1}{T_3} \right)^{1/2} \\ &= 1 - \left(\frac{300}{1500} \right)^{1/2} = 0.553 \quad (55.3\%) \end{aligned}$$

Continued on next slide

Problem 9-45 continued

(c) The expression for η in part (b) is plotted using IT or any plotting package.



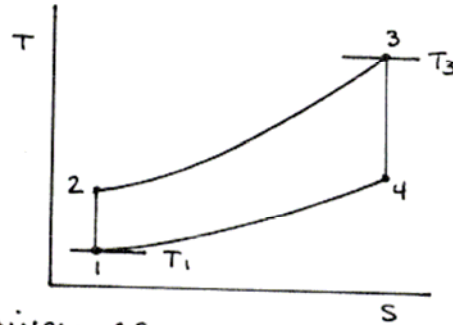
We see that thermal efficiency increases with increasing maximum cycle temperature. Also, note that for this idealized case η is independent of the value of k .

PROBLEM 9.46

KNOWN: An ideal cold air-standard Brayton cycle is analyzed.

FIND: Show that the back work ratio equals the ratio of absolute temperatures at the compressor inlet and turbine outlet.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.5.

ANALYSIS: The back work ratio is given as

$$bwr = \frac{h_2 - h_1}{h_3 - h_4} = \frac{c_p(T_2 - T_1)}{c_p(T_3 - T_4)}$$

or

$$bwr = \frac{T_1 (T_2/T_1 - 1)}{T_4 (T_3/T_4 - 1)}$$

For the isentropic processes

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = \frac{T_3}{T_4}$$

thus

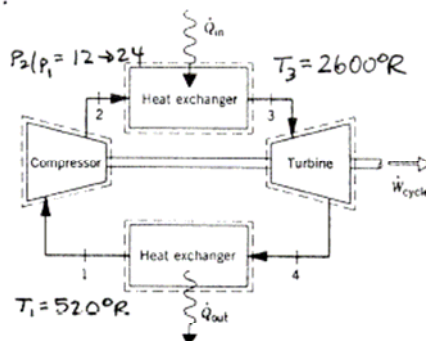
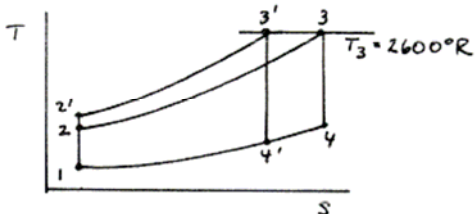
$$bwr = \frac{T_1}{T_4} \longleftarrow bwr$$

PROBLEM 9.47

KNOWN: An ideal air-standard Brayton cycle has a known compressor inlet temperature and a known maximum turbine inlet temperature.

FIND: Plot the net work per unit mass of air flow and the thermal efficiency versus compressor pressure ratio. Estimate the pressure ratio for maximum net work and the corresponding thermal efficiency and compare with a cold air-standard analysis.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.4.

ANALYSIS: Sample calculation. $P_2/P_1 = 12$ Using data from Table A-22E to fix each of the principal states:

State 1 $T_1 = 520^\circ\text{R} \Rightarrow h_1 = 124.27 \text{ Btu/lb}$, $Pr_1 = 1.2147$

State 2 For the isentropic compression, $Pr_2 = (P_2/P_1)Pr_1 = 14.5764$
Thus, $T_2 = 1047.5^\circ\text{R}$ and $h_2 = 252.84 \text{ Btu/lb}$

State 3 $T_3 = 2600^\circ\text{R} \Rightarrow h_3 = 674.49 \text{ Btu/lb}$, $Pr_3 = 513.5$

State 4 For the isentropic expansion, $Pr_4 = (P_4/P_3)Pr_3 = 42.792$
 $T_4 = 1399.2^\circ\text{R}$ and $h_4 = 342.69 \text{ Btu/lb}$

The net work per unit mass of air flow is

$$w_{\text{cycle}}/\dot{m} = (h_3 - h_4) - (h_2 - h_1) = 203.23 \text{ Btu/lb}$$

and the thermal efficiency is

$$\eta = \frac{w_{\text{cycle}}/\dot{m}}{q_{\text{in}}/\dot{m}} = \frac{w_{\text{cycle}}/\dot{m}}{(h_3 - h_2)} = 0.4820 \text{ (48.20\%)}$$

The data for the required plots are obtained using IT, as follows:

IT Code

```
T1 = 520 // °R
T3 = 2600 // °R
rp = 12
mdot = 1 // Assume a mass flow rate of unity.
p1 = 1 // Assume a value for pressure
// (any value would work).
```

```
h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2 = s1
p2 = p1 * rp
s2 = s_hP("Air", h2, p2)
p3 = p2
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
```

```
s4 = s3
p4 = p1
s4 = s_hP("Air", h4, p4)
Wdotnet = mdot * ((h3 - h4) - (h2 - h1))
eta = Wdotnet / Qdotin
Qdotin = mdot * (h3 - h2)
```

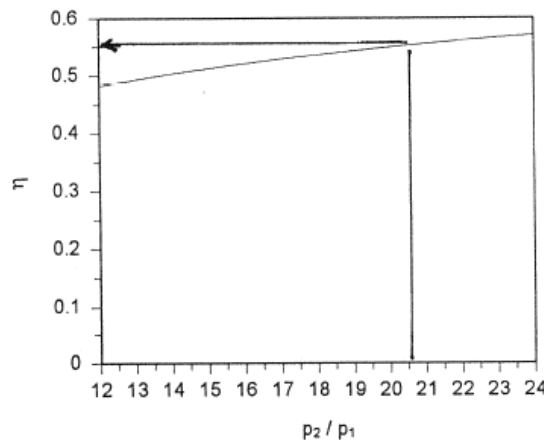
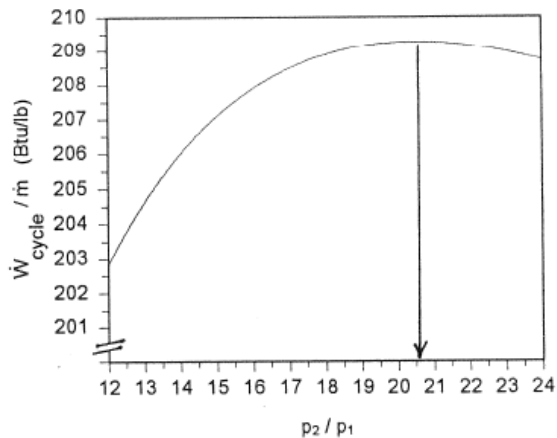
IT Results for $r_p = 12$

```
h1 = 124.3 Btu/lb
h2 = 252.9 Btu/lb
h3 = 673.6 Btu/lb
h4 = 342.2 Btu/lb
Wdotcycle / mdot = 202.8 Btu/lb
eta = 0.4821
```

Continued on next slide

Problem 9-47 continued

Plots:



From the plots: $(P_2/P_1)_{\text{max work}} \approx 20.6$

$(\eta)_{\text{max work}} \approx 55.2\%$

Using the result of Example 9.5, on a cold air-standard basis, the pressure ratio is

$$\left(\frac{P_2}{P_1}\right)_{\text{max work}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}} = 16.72$$

and the corresponding thermal efficiency is

$$\eta = 1 - \frac{1}{(P_2/P_1)^{(k-1)/k}} = 0.553$$

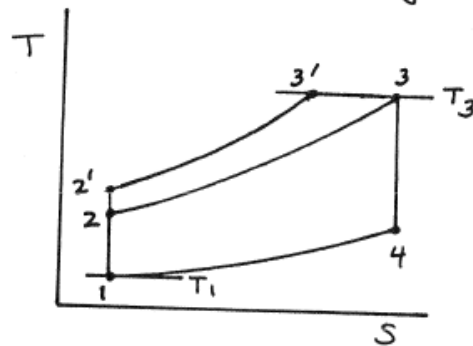
Discussion: The cold air-standard analysis predicts a pressure ratio for maximum net work that is quite different than the value obtained graphically. Interestingly, the corresponding thermal efficiencies are nearly equal.

PROBLEM 9.48

KNOWN: An ideal Brayton cycle is analyzed on a cold air-standard basis. The compressor inlet temperature is T_1 and the turbine inlet temperature is T_3 .

FIND: Show that the compressor exit temperature that maximizes net work per unit mass of air flow is given by $T_2 = (T_1 \cdot T_3)^{1/2}$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.5.

ANALYSIS: From the solution to Example 9.5, the compressor pressure ratio corresponding to maximum net work per unit mass of air flow is

$$\left(\frac{P_2}{P_1}\right)_{\max \text{ work}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$$

Further, for the isentropic compression

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$$

Equating these expressions yields the desired result

$$\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}} \Rightarrow \frac{T_2}{T_1} = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2}}$$

or

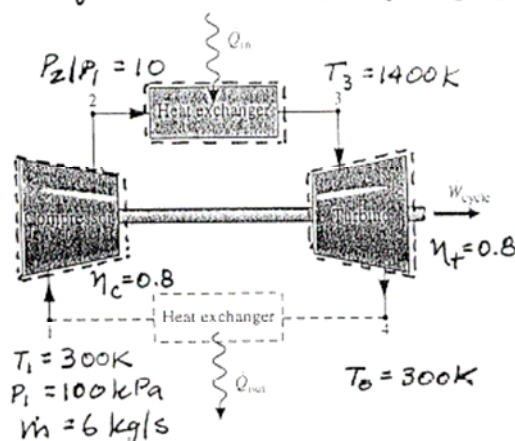
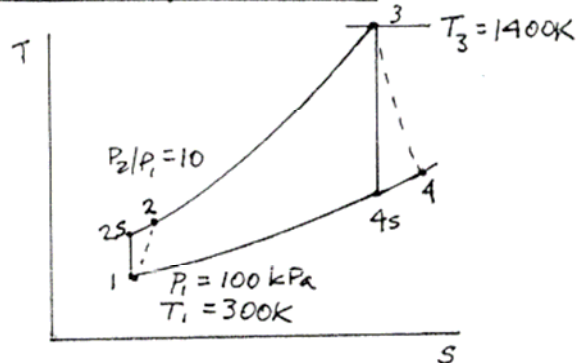
$$T_2 = (T_1 \cdot T_3)^{1/2}$$

PROBLEM 9.49

KNOWN: Air enters a cold air-standard Brayton cycle at a specified state and with a given mass flow rate. The compressor pressure ratio and maximum cycle temperature are known. The compressor and turbine have isentropic efficiencies of 0.8.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power, and (d) the rates of exergy destruction in the compressor and turbine for $T_0 = 300\text{ K}$. Plot these quantities vs. $\eta_c = \eta_t$ ranging from 70 to 100%.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.6. Also, assume $k = 1.4$ and $c_p = 1.005\text{ kJ/kg}\cdot\text{K}$.

ANALYSIS: First, determine T_2 and T_4 , as follows:

$$(Eq. 9.23) \quad T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = (300\text{ K})(10)^{\frac{1.4-1}{1.4}} = 579.2\text{ K}$$

Using the compressor efficiency: $\eta_c = (T_{2s} - T_1)/(T_2 - T_1)$. Thus

$$T_2 = T_1 + (T_{2s} - T_1)/\eta_c = 300 + (579.2 - 300)/0.8 = 649\text{ K}$$

Similarly

$$(9.24) \quad T_{4s} = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = (1400) \left(\frac{1}{10} \right)^{\frac{1.4-1}{1.4}} = 725.13\text{ K}$$

$$\text{and} \quad T_4 = T_3 - (T_3 - T_{4s})\eta_t = 1400 - (1400 - 725.13)(0.8) = 860.1\text{ K}$$

(a) To evaluate the thermal efficiency, use $\eta = 1 - \dot{Q}_{out}/\dot{Q}_{in}$.

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2) = \dot{m}c_p(T_3 - T_2) = (6 \frac{\text{kg}}{\text{s}})(1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})(1400 - 649)\text{ K} = 4528.5 \text{ kJ/s}$$

$$\dot{Q}_{out} = \dot{m}(h_4 - h_1) = \dot{m}c_p(T_4 - T_1) = (6)(1.005)(860.1 - 300) = 3377.4\text{ kJ/s}$$

$$\text{Thus} \quad \eta = 1 - \dot{Q}_{out}/\dot{Q}_{in} = 1 - 3377.4/4528.5 = 0.254 \text{ (25.4\%)} \quad \leftarrow \eta$$

(b) The back work ratio is: $bwr = \dot{W}_c/\dot{W}_t$

$$\dot{W}_t = \dot{m}(h_3 - h_4) = \dot{m}c_p(T_3 - T_4) = (6)(1.005)(1400 - 860.1) = 3255.6 \frac{\text{kJ}}{\text{s}}$$

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1) = (6)(1.005)(649 - 300) = 2104.5 \text{ kJ/s}$$

$$\text{and} \quad bwr = \dot{W}_c/\dot{W}_t = 2104.5/3255.6 = 0.6464$$

(c) The net power developed is

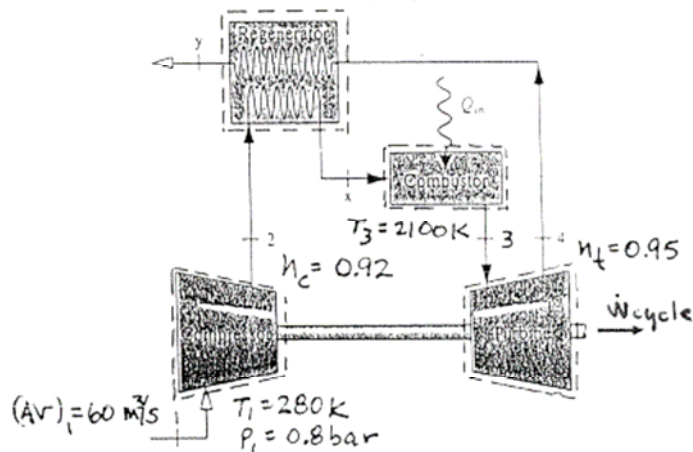
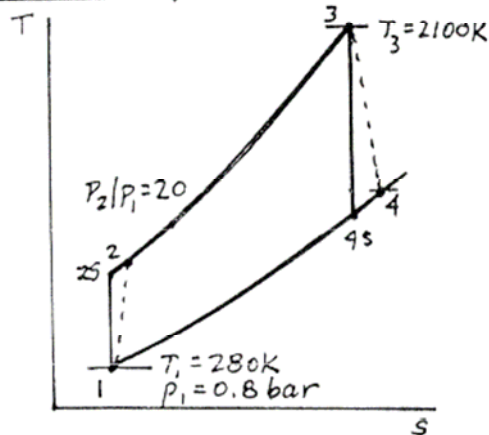
$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = (3255.6 - 2104.5) \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 1151.1 \text{ kW} \quad \leftarrow \dot{W}_{cycle}$$

PROBLEM 9.50

KNOWN: Air enters the compressor of an air-standard Brayton cycle at a specified state and a given volumetric flow rate. The compressor pressure ratio and maximum cycle temperature are known. The compressor and turbine isentropic efficiencies are known.

FIND: Determine (a) the net power, (b) the rate of heat addition, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.7. Also, $\eta_c = 0.92$, $\eta_t = 0.95$.

ANALYSIS: First, fix each of the principal states.

State 1: $T_1 = 280 \text{ K} \Rightarrow h_1 = 280.13 \text{ kJ/kg}$, $P_{r1} = 1.0889$

State 2: $P_{r2} = (P_2/P_1) P_{r1} = (20)(1.0889) = 21.778 \Rightarrow T_{2s} = 649.3 \text{ K}$, $h_{2s} = 659.13 \frac{\text{kJ}}{\text{kg}}$

Using the isentropic compressor efficiency; $\eta_c = (h_{2s} - h_1)/(h_2 - h_1)$
 $h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 280.13 + (659.13 - 280.13)/(0.92) = 692.09 \text{ kJ/kg}$

State 3: $T_3 = 2100 \text{ K}$; $h_3 = 2377.4 \text{ kJ/kg}$, $P_{r3} = 2559$

State 4: $P_{r4} = (P_4/P_3) P_{r3} = (\frac{1}{20})(2559) = 127.95 \Rightarrow T_{4s} = 1029.2 \text{ K}$, $h_{4s} = 1079.4 \frac{\text{kJ}}{\text{kg}}$

Using the isentropic turbine efficiency; $\eta_t = (h_3 - h_4)/(h_3 - h_{4s})$
 $h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 2377.4 - (0.95)(2377.4 - 1079.4) = 1144.3 \text{ kJ/kg}$

Now, determine the mass flow rate.

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{R T_1} = \frac{(60 \text{ m}^3/\text{s})(0.8 \text{ bar})}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}}\right)(280 \text{ K})} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 59.73 \text{ kg/s}$$

$$(a) \dot{W}_c = \dot{m}(h_2 - h_1) = (59.73 \frac{\text{kg}}{\text{s}})(692.09 - 280.13) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 2.461 \times 10^4 \text{ kW}$$

$$\dot{W}_t = \dot{m}(h_3 - h_4) = (59.73)(2377.4 - 1144.3) = 7.365 \times 10^4 \text{ kW}$$

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_c = 4.904 \times 10^4 \text{ kW} \leftarrow \dot{W}_{\text{cycle}}$$

$$(b) \dot{Q}_{\text{in}} = \dot{m}(h_3 - h_2) = (59.73)(2377.4 - 692.09) = 1.0066 \times 10^5 \text{ kW} \leftarrow \dot{Q}_{\text{in}}$$

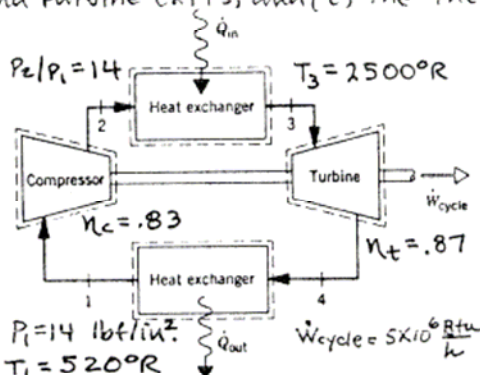
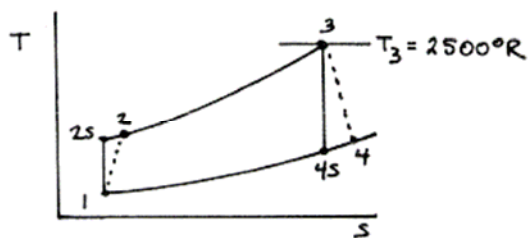
$$(c) \eta = \dot{W}_{\text{cycle}}/\dot{Q}_{\text{in}} = 4.904 \times 10^4 / 1.0066 \times 10^5 = 0.487 (48.7\%) \leftarrow \eta$$

PROBLEM 9.51

KNOWN: A simple gas turbine is analyzed on an air-standard basis. The conditions entering the compressor and the compressor and turbine efficiencies are known. The compressor pressure ratio and the temperature at the turbine inlet are specified as well.

FIND: determine (a) the volumetric flow rate entering the compressor, (b) the temperatures at the compressor and turbine exits, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.6.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1 $T_1 = 520^\circ\text{R} \Rightarrow h_1 = 124.27 \text{ Btu/lb}$, $P_{r1} = 1.2147$

State 2 For an isentropic compression; $P_{r2s} = (P_2/P_1)P_{r1} = 17.006$. Thus, $h_{2s} = 264.12 \text{ Btu/lb}$. Using the compressor efficiency,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 292.76 \text{ Btu/lb}, \quad T_2 = 1206^\circ\text{R}$$

State 3 $T_3 = 2500^\circ\text{R} \Rightarrow h_3 = 645.78 \text{ Btu/lb}$, $P_{r3} = 435.7$

State 4 For an isentropic expansion, $P_{r4s} = (P_4/P_3)P_{r3} = 31.121$. Thus, $h_{4s} = 313.36 \text{ Btu/lb}$. Using the turbine efficiency,

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 356.57 \text{ Btu/lb}, \quad T_4 = 1452^\circ\text{R}$$

The thermal efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{356.57 - 124.27}{645.78 - 292.76} = 0.342 \text{ (34.2\%)} \quad \leftarrow \eta$$

To find the volumetric flow rate at the inlet to the compressor, use

$$\dot{W}_{\text{cycle}} = \dot{m}[(h_3 - h_4) - (h_2 - h_1)] = \dot{m}[(645.78 - 356.57) - (292.76 - 124.27)] \frac{\text{Btu}}{\text{lb}} = \dot{m} [120.72 \text{ Btu/lb}] \frac{\text{Btu}}{\text{h}}$$

where

$$\dot{m} = \frac{(AV)_1 \rho_1}{RT_1} = \frac{(AV)_1 \left(\frac{1 \text{ lb}}{15.45 \text{ ft}^3} \right) (14 \times 144 \frac{\text{lbf}}{\text{ft}^2})}{(28.97 \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}) (520^\circ\text{R})} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 4.362 [(AV)_1 \frac{\text{ft}^3}{\text{min}}] \frac{\text{lb}}{\text{h}}$$

Thus

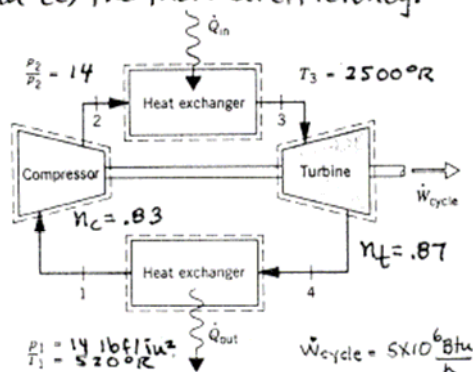
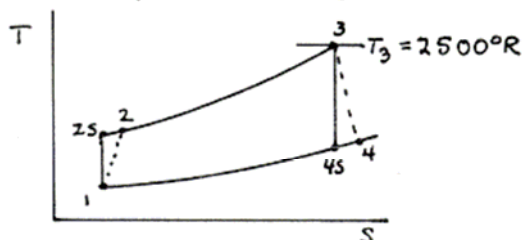
$$(AV)_1 = \frac{5 \times 10^6 \text{ Btu/h}}{(4.362 \frac{\text{lb/h}}{\text{ft}^3/\text{min}}) [120.72 \frac{\text{Btu}}{\text{lb}}]} = 9495 \frac{\text{ft}^3}{\text{min}} \quad \leftarrow (AV)_1$$

PROBLEM 9.52

KNOWN: A simple gas turbine is analyzed on a cold air-standard basis. The conditions entering the turbine and the turbine and compressor efficiencies are known. The compressor pressure ratio and the temperature at the turbine inlet are specified as well.

FIND: Determine (a) the volumetric flow rate entering the compressor, (b) the temperatures at the compressor and turbine exits, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.6. Also, assume constant specific heats evaluated at 520°R.

ANALYSIS: From Table A-20E, $k = 1.401$, $c_p = 0.240 \text{ Btu/lb} \cdot ^\circ\text{R}$. $T_1 = 520^\circ\text{R}$ is given and $T_3 = 2500^\circ\text{R}$ is given.

State 2 For an isentropic compression, $T_{2s} = (P_2/P_1)^{k-1/k} T_1 = 1106.8^\circ\text{R}$

Using the compressor efficiency,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)} \Rightarrow T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c} = 1227.0^\circ\text{R} \leftarrow T_2$$

State 4 For an isentropic expansion, $T_{4s} = (P_4/P_3)^{k-1/k} T_3 = 1174.6^\circ\text{R}$

Using the turbine efficiency,

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{c_p(T_3 - T_4)}{c_p(T_3 - T_{4s})} \Rightarrow T_4 = T_3 - \eta_t(T_3 - T_{4s}) = 1346.9^\circ\text{R} \leftarrow T_4$$

The thermal efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 0.3504 \quad (35.04\%) \leftarrow \eta$$

The net power developed is

$$\begin{aligned} \dot{W}_{\text{cycle}} &= \dot{m} [(h_3 - h_4) - (h_2 - h_1)] = \dot{m} c_p [(T_3 - T_4) - (T_2 - T_1)] \\ &= \dot{m} [(0.24 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}})(446.1^\circ\text{R})] \end{aligned}$$

where

$$\dot{m} = \frac{(AV)_1 P_1}{R T_1} = \frac{[(AV)_1] \frac{\text{ft}^3}{\text{min}} [14 \times 144 \frac{\text{lbf}}{\text{ft}^2}]}{(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}})(520^\circ\text{R})} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 4.362 [(AV)_1] \frac{\text{ft}^3}{\text{min}} \frac{\text{lb}}{\text{h}}$$

So,

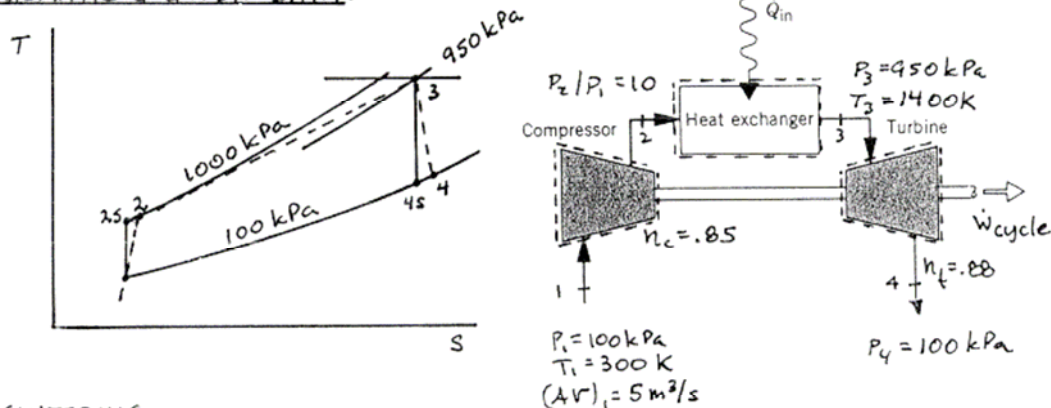
$$(AV)_1 = \frac{5 \times 10^6 \text{ Btu/h}}{4.362 \frac{\text{lb/h}}{\text{ft}^3/\text{min}} [(0.24)(446.1) \frac{\text{Btu}}{\text{lb}}]} = 10,706 \text{ ft}^3/\text{min} \leftarrow (AV)_1$$

PROBLEM 9.53

KNOWN: A simple gas turbine is analyzed on an air-standard basis from an exergy viewpoint. Data are known at various locations.

FIND: (a) Develop a full exergy accounting of the net exergy increase of the air passing through the gas turbine combustor, (b) devise and evaluate an exergetic efficiency for the gas turbine cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compressor and turbine are adiabatic. (3) Kinetic and potential energy effects are negligible. (4) The working fluid is air modeled as an ideal gas. (5) Let $T_0 = 300 \text{ K}$ and $p_0 = 100 \text{ kPa}$.

ANALYSIS: Data are obtained for each principal state from Table A-22:

State	T (K)	p (kPa)	h (kJ/kg)	s° (kJ/kg·K)
1	300	100	300.19	1.70203
2	621.1	1000	629.21	2.44539
3	1400	950	1515.42	3.36200
4	873.9	100	903.72	2.81558

The increase in flow exergy of the air passing through the heat exchanger is taken as the net input of exergy to the gas turbine.

$$\text{Input: } \dot{m}(e_{f3} - e_{f2}) = \dot{m}[(h_3 - h_2) - T_0(s_3 - s_2 - R \ln p_3/p_2)]$$

Evaluating $\dot{m}$:

$$\dot{m} = \frac{(\dot{A}V)_1 p_1}{RT_1} = \frac{(5 \text{ m}^3/\text{s})(100 \text{ kPa})}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}}\right)(300 \text{ K})} \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 5.807 \text{ kg/s}$$

Thus

$$\begin{aligned} \dot{m}(e_{f3} - e_{f2}) &= (5.807 \frac{\text{kg}}{\text{s}}) [(1515.42 - 629.21) \text{ kJ/kg} \\ &\quad - (300 \text{ K})(3.36200 - 2.44539 - \frac{8.314}{28.97} \ln \frac{950}{1000}) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 3524 \text{ kW (Input)} \end{aligned}$$

Continued on next slide

Problem 9-53 continued

Exergy is destroyed due to irreversibilities in the compressor and turbine. Thus, using $\dot{E}_d = T_0 \dot{\sigma}$, we get

Destructions:

$$\dot{E}_{d, \text{comp}} = T_0 \dot{m} (s_2 - s_1) = T_0 \dot{m} [(s_2^\circ - s_1^\circ) - R \ln P_2/P_1] = 143.8 \text{ kW}$$

$$\dot{E}_{d, \text{turb}} = T_0 \dot{m} [(s_4^\circ - s_3^\circ) - R \ln P_4/P_3] = 173.6 \text{ kW}$$

The net power developed by the cycle represents the output of exergy from the cycle, or

$$\begin{aligned} \dot{W}_{\text{cycle}} &= \dot{m} [(h_3 - h_4) - (h_2 - h_1)] \\ &= 1641.5 \text{ kW (Output)} \end{aligned}$$

Finally, the air enters the gas turbine at P_0 and T_0 . Thus, the change in flow exergy from inlet to exit represents the loss due to the hot air being discharged. Thus

$$\begin{aligned} \text{Loss: } \dot{m} (e_{f4} - e_{f1}) &= \dot{m} [(h_4 - h_1) - T_0 (s_4^\circ - s_1^\circ - R \ln P_4/P_1)] \\ &= 1564.8 \text{ kW} \end{aligned}$$

Summarizing

- Input: 3524 kW
- Disposition:
 - Net Power: 1641.5 kW (46.6%)
 - Destroyed: 317.4 (9%)
 - Loss: $\frac{1564.8}{3523.7} \text{ kW}$ (44.4%)

①

Since the objective of the gas turbine is to produce power, an exergetic efficiency expressed as the ratio of the desired output (net power) to the exergy input is, by inspection of the summary,

$$\epsilon = 46.6\%$$

-
1. Considerable exergy is carried out by the air discharged at 4. This might be exploited by the regenerative approach discussed in Sec. 9.7 or by means of a combined cycle as discussed in Sec. 9.10.

PROBLEM 9.54

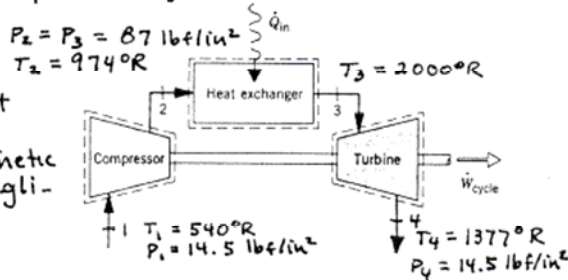
KNOWN: A simple gas turbine is analyzed on an air-standard basis from an exergy viewpoint. Data are known at various locations.

FIND: (a) Develop a full exergy accounting of the net exergy increase of the air passing through the gas turbine combustor, (b) devise and evaluate an exergetic efficiency for the gas turbine cycle.

SCHEMATIC & GIVEN DATA:

ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compressor and turbine are adiabatic. (3) Kinetic and potential energy effects are negligible. (4) The working fluid is air modeled as an ideal gas. (5) Let $T_0 = 540^\circ\text{R}$, $P_0 = 14.5 \text{ lbf/in}^2$.



ANALYSIS: Data are obtained at each principal state from Table A-22E:

State	T (°R)	p (lbf/in²)	h (Btu/lb)	s° (Btu/lb·°R)
1	540	14.5	129.06	0.60078
2	974	87	234.53	0.74387
3	2000	87	504.71	0.93205
4	1377	14.5	336.91	0.83169

The input of exergy to the gas turbine is equal to the increase in flow exergy of the working fluid passing through the heat exchanger

$$\text{Input: } (e_{f3} - e_{f2}) = (h_3 - h_2) - T_0 [s_3 - s_2] - R \ln P_3/P_2$$

$$= 168.6 \text{ Btu/lb}$$

Exergy is destroyed due to irreversibilities in the compressor and turbine. Thus, with $\dot{E}_d = T_0 \dot{\sigma}$

$$\text{Destructions: } (\dot{E}_d/\dot{m})_{\text{comp}} = T_0 [(s_2 - s_1) - R \ln P_2/P_1] = 10.94 \text{ Btu/lb}$$

$$(\dot{E}_d/\dot{m})_{\text{turb}} = T_0 [(s_4 - s_3) - R \ln P_4/P_3] = 12.13 \text{ Btu/lb}$$

Exergy is carried out via the net work:

$$W_{\text{cycle}}/\dot{m} = (h_3 - h_4) - (h_2 - h_1) = 62.33 \text{ Btu/lb}$$

Exergy also is carried out with the stream exiting at 4. The net exergy loss is then

$$e_{f4} - e_{f3} = (h_4 - h_1) - T_0 [(s_4 - s_1) - R \ln P_4/P_1]$$

$$= 83.16 \text{ Btu/lb}$$

Summary:

• Input :	168.6 Btu/lb
• Disposition:	
- network	62.33 Btu/lb (37%)
- destroyed	23.07 Btu/lb (13.7%)
- loss	83.16 Btu/lb (49.3%)
	168.6 Btu/lb

$$\epsilon = \frac{\text{net power output}}{\text{input}} = 37\%$$

①

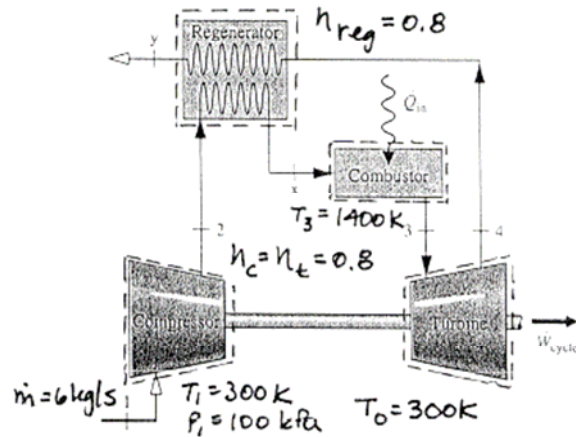
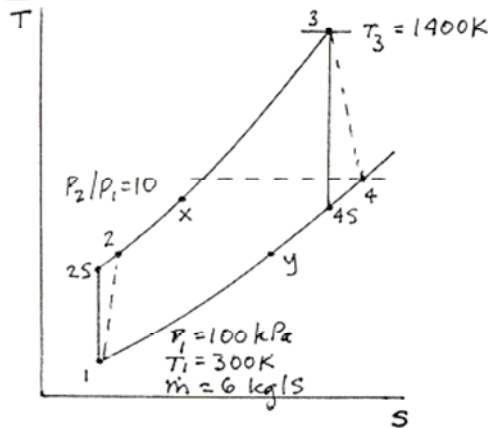
1. Considerable exergy is carried out by the air discharged at 4. This might be exploited by the regenerative approach discussed in Sec. 9.7 or by means of a combined cycle as discussed in Sec. 9.10.

PROBLEM 9.55

KNOWN: Air enters a cold air-standard Brayton cycle with regeneration at a specified state with a given mass flow rate. The compressor pressure ratio and maximum cycle temperature are known. The compressor and turbine have isentropic efficiencies of 0.8 and the regenerator effectiveness is 0.8.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power, and (d) rate of exergy destruction in the regenerator for $T_0 = 300\text{ K}$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.7. Also, $\eta_c = \eta_t = 0.8$ and $\eta_{reg} = 0.8$. The specific heats are constant, with $k = 1.4$ and $c_p = 1.005\text{ kJ/kg}\cdot\text{K}$.

ANALYSIS: The numbered states are the same as in Problem 9.49. The relevant data are: $T_2 = 649\text{ K}$ and $T_4 = 860.1\text{ K}$.

To find T_x , use the regenerator effectiveness;

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} = \frac{c_p(T_x - T_2)}{c_p(T_4 - T_2)} \Rightarrow T_x = T_2 + \eta_{reg}(T_4 - T_2) = 649 + (0.8)(860.1 - 649) = 817.9\text{ K}$$

For the control volume enclosing the regenerator

$$0 = \dot{m}[(h_2 - h_x) + (h_4 - h_y)]$$

$$\text{or } 0 = c_p(T_2 - T_x) + c_p(T_4 - T_y) \Rightarrow T_y = T_4 - (T_x - T_2) = 691.2\text{ K}$$

$$(a) \dot{Q}_{in} = \dot{m}c_p(T_3 - T_x) = (6\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(1400 - 817.9)\text{ K} = 3510.1\text{ kJ/s}$$

$$\dot{Q}_{out} = \dot{m}c_p(T_y - T_1) = (6)(1.005)(691.2 - 300) = 2358.9\text{ kJ/s}$$

$$\eta = 1 - \dot{Q}_{out}/\dot{Q}_{in} = 1 - 2358.9/3510.1 = 0.328 \text{ (32.8\%)} \quad \eta$$

$$(b) \dot{W}_c = \dot{m}c_p(T_2 - T_1) = 2104.5\text{ kJ/s}, \dot{W}_t = \dot{m}c_p(T_3 - T_4) = 3255.6\text{ kJ/s}$$

$$bwr = \dot{W}_c/\dot{W}_t = 0.6464 \quad bwr$$

$$(c) \dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = 1151.1\text{ kW} \quad \dot{W}_{cycle}$$

$$(d) \text{ For the regenerator: } 0 = \sum \left(\frac{\dot{Q}_j}{T_j} \right) + \dot{m}[(s_2 - s_x) + (s_4 - s_y)] + (\dot{S}_{cv})_{reg}$$

$$(\dot{E}_d)_{reg} = T_0(\dot{S}_{cv})_{reg} = T_0 \dot{m} \left[c_p \ln(T_x/T_2) - R \ln(P_2/P_1) + c_p \ln(T_y/T_4) - R \ln(P_4/P_1) \right]$$

$$= (300\text{ K})(6\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K}) \left[\ln\left(\frac{817.9}{649}\right) + \ln\left(\frac{691.2}{860.1}\right) \right] \left| \frac{\text{kJ}}{\text{K}\cdot\text{s}} \right|$$

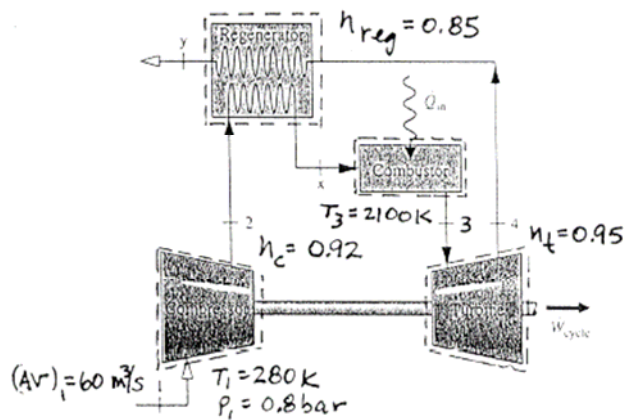
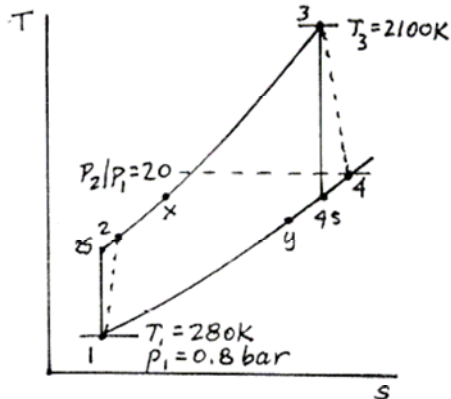
$$= 22.95\text{ kW} \quad (\dot{E}_d)_{reg}$$

PROBLEM 9.56

KNOWN: Air enters the compressor of a regenerative air-standard Brayton cycle at a specified state and a given volumetric flow rate. The compressor pressure ratio and maximum cycle temperature are known. The compressor and turbine isentropic efficiencies and the regenerator effectiveness are known.

FIND: Determine (a) the net power, (b) the rate of heat addition, and (c) the thermal efficiency. Plot these quantities vs. regenerator effectiveness ranging from 0 to 100%. Discuss.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.7. Also, $\eta_c = 0.92$, $\eta_t = 0.95$, and $\eta_{reg} = 0.85$.

ANALYSIS: First, fix each of the principal states.

State 1: $T_1 = 280 \text{ K} \Rightarrow h_1 = 280.13 \text{ kJ/kg}$, $P_{r1} = 1.0889$

State 2: $P_{r2} = (P_2/P_1) P_{r1} = (20)(1.0889) = 21.778 \Rightarrow T_{2s} = 649.3 \text{ K}$, $h_{2s} = 659.13 \frac{\text{kJ}}{\text{kg}}$
Using the isentropic compressor efficiency; $\eta_c = (h_{2s} - h_1)/(h_2 - h_1)$
 $h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 280.13 + (659.13 - 280.13)/(0.92) = 692.09 \text{ kJ/kg}$

State 3: $T_3 = 2100 \text{ K}$; $h_3 = 2377.4 \text{ kJ/kg}$, $P_{r3} = 2559$

State 4: $P_{r4} = (P_4/P_3) P_{r3} = (\frac{1}{20})(2559) = 127.95 \Rightarrow T_{4s} = 1029.2 \text{ K}$, $h_{4s} = 1079.4 \frac{\text{kJ}}{\text{kg}}$
Using the isentropic turbine efficiency; $\eta_t = (h_3 - h_4)/(h_3 - h_{4s})$
 $h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 2377.4 - (0.95)(2377.4 - 1079.4) = 1144.3 \text{ kJ/kg}$

State x: From the regenerator effectiveness; $\eta_{reg} = (h_x - h_2)/(h_4 - h_2)$
 $h_x = h_2 + \eta_{reg}(h_4 - h_2) = 692.09 + (0.85)(1144.3 - 692.09) = 1076.5 \frac{\text{kJ}}{\text{kg}}$

Now, determine the mass flow rate.

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{R T_1} = \frac{(60 \text{ m}^3/\text{s})(0.8 \text{ bar})}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}}\right)(280 \text{ K})} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 59.73 \text{ kg/s}$$

$$(a) \dot{W}_t = \dot{m}(h_3 - h_4) = (59.73 \frac{\text{kg}}{\text{s}})(2377.4 - 1144.3) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 7.365 \times 10^4 \text{ kW}$$

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (59.73)(692.09 - 280.13) = 2.461 \times 10^4 \text{ kW}$$

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = 4.904 \times 10^4 \text{ kW} \leftarrow \dot{W}_{cycle}$$

$$(b) \dot{Q}_{in} = \dot{m}(h_3 - h_x) = (59.73)(2377.4 - 1076.5) = 7.770 \times 10^4 \text{ kW} \leftarrow \dot{Q}_{in}$$

Continued on next slide

Problem 9-56 continued

(c) The thermal efficiency is $\eta = \dot{W}_{\text{cycle}} / \dot{Q}_{\text{in}} = 0.631$ (63.1%) η

IT Code

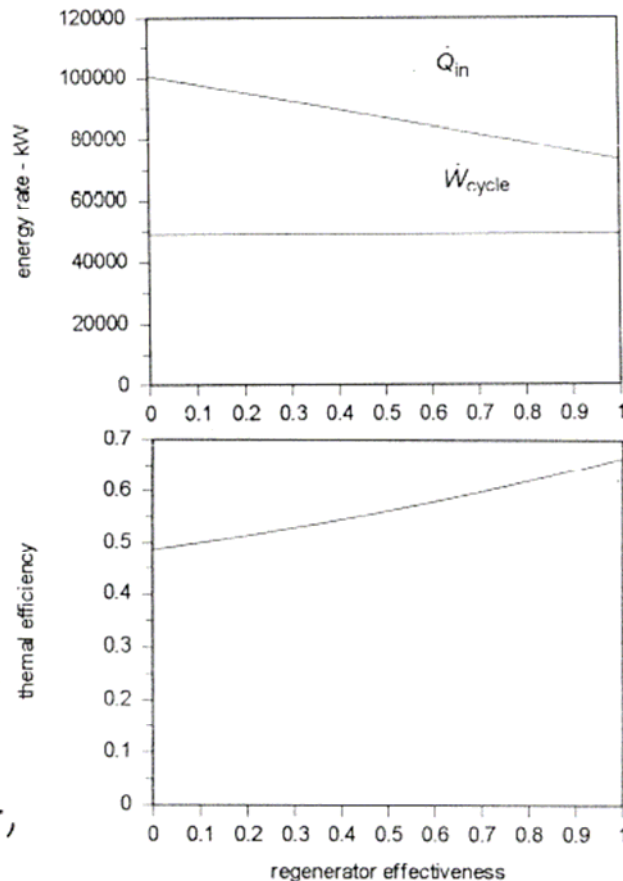
```
T1 = 280 // K
p1 = 0.8 // bar
r = 20
p2 = r * p1
p3 = p2
T3 = 2100 // K
p4 = p3 / r
eta_c = 0.92
eta_t = 0.95
eta_reg = 0.85
AV1 = 60 // m³/s
```

```
h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s1
s2s = s_hP("Air", h2s, p2)
h2 = h1 + (h2s - h1) / eta_c
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s3
s4s = s_hP("Air", h4s, p4)
h4 = h3 - eta_t * (h3 - h4s)
hx = h2 + eta_reg * (h4 - h2)

v1 = v_TP("Air", T1, p1)
mdot = AV1 / v1
Wdott = mdot * (h3 - h4)
Wdotc = mdot * (h2 - h1)
Wdotcycle = Wdott - Wdotc
Qdotin = mdot * (h3 - hx)
eta = Wdotcycle / Qdotin
```

IT Results for $\eta_{\text{reg}} = 0.85$, $\eta_c = 0.92$, and $\eta_t = 0.95$

```
m = 59.73 kg/s
Q_in = 7.768 x 10⁴ kW
W_cycle = 4.903 x 10⁴ kW
eta = 0.6312
h1 = 280 kJ/kg
h2 = 691.9 kJ/kg
h3 = 2376 kJ/kg
h4 = 1143 kJ/kg
hx = 1075 kJ/kg
```



Discussion

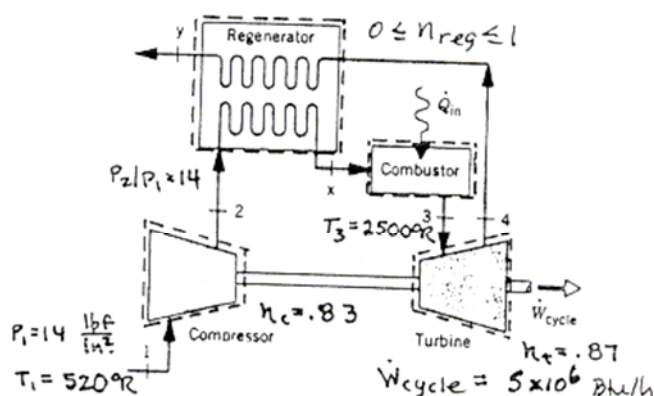
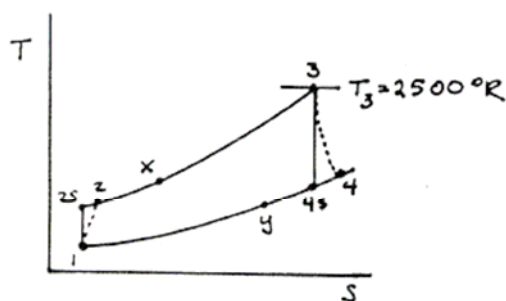
The states at the inlet and exit of the compressor and turbine, respectively, are not changed as the regenerator effectiveness changes. Hence, the power is constant. However, as the regenerator effectiveness increases, the specific enthalpy h_x increases and less heat addition is required in the combustor (external heat addition). The result is a substantial increase in the thermal efficiency.

PROBLEM 9.57

KNOWN: A regenerator is incorporated into the cycle of Problem 9.51.

FIND: Plot (a) the thermal efficiency and (b) the percent decrease in heat addition, each versus regenerator effectiveness ranging from 0 to 1.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.7.

ANALYSIS: Consider the sample case of $\eta_{reg} = 0.78$. The numbered states are the same as in Problem 9.51. Thus

$$h_1 = 124.27 \text{ Btu/lb}, h_2 = 292.76, h_3 = 645.78, h_4 = 356.57$$

The specific enthalpy h_x is found using the regenerator effectiveness;

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{reg}(h_4 - h_2) + h_2 = 342.53 \text{ Btu/lb}$$

From an energy balance on the regenerator

$$h_y = (h_2 - h_x) + h_4 = 306.8 \text{ Btu/lb}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_y - h_1}{h_3 - h_x} = 0.398 \text{ (39.8\%)}$$

(b) The percent decrease in heat addition is

$$\% \text{ decrease} = \frac{(h_3 - h_2) - (h_3 - h_x)}{(h_3 - h_2)} \times 100 = 14.1\%$$

The data for the required plots are obtained using IT, as follows:

IT Code

$$p1 = 14 \text{ // lbf/in.}^2$$

$$T1 = 520 \text{ // } ^\circ\text{R}$$

$$p2 / p1 = 14$$

$$p3 = p2$$

$$T3 = 2500 \text{ // } ^\circ\text{R}$$

$$p4 = p1$$

$$\eta_{c} = 0.83$$

$$\eta_{t} = 0.87$$

$$\eta_{reg} = 0.78$$

$$h1 = h_T(\text{"Air"}, T1)$$

$$s1 = s_{TP}(\text{"Air"}, T1, p1)$$

$$s2s = s_{hP}(\text{"Air"}, h2s, p2)$$

$$s2s = s1$$

$$h2 = h1 + (h2s - h1) / \eta_{c}$$

$$h3 = h_T(\text{"Air"}, T3)$$

$$s3 = s_{TP}(\text{"Air"}, T3, p3)$$

$$s4s = s_{hP}(\text{"Air"}, h4s, p4)$$

$$s4s = s3$$

$$h4 = h3 - (h3 - h4s) * \eta_{t}$$

$$h_x = \eta_{reg} * (h4 - h2) + h2$$

$$h_y = h2 - h_x + h4$$

$$\eta = 1 - (h_y - h1) / (h3 - h_x)$$

$$pct = (((h3 - h2) - (h3 - h_x)) / (h3 - h2)) * 100$$

PROBLEM 9.58

KNOWN: An ideal regenerative gas turbine is analyzed on a cold air-standard basis.

FIND: Show that the thermal efficiency can be expressed as

$$\eta = 1 - \left(\frac{T_1}{T_3} \right) (r)^{(k-1)/k}$$

where r is pressure ratio, and T_1 and T_3 denote the compressor and turbine inlet temperatures, respectively.

SCHEMATIC & GIVEN DATA:

ENGINEERING

MODEL: Same as in Example 9.7 except $\eta_{reg} = 100\%$. Also, assume constant specific heats.

ANALYSIS: The thermal efficiency of the ideal cold air-standard cycle can be expressed as

$$\eta = 1 - \frac{cp(T_y - T_1)}{cp(T_3 - T_x)}$$

With $\eta_{reg} = 100\%$, $T_x = T_4$ and $T_y = T_2$. Thus

$$\eta = 1 - \frac{T_2 - T_1}{T_3 - T_4} = 1 - \frac{T_1 (T_2/T_1 - 1)}{T_3 (1 - T_4/T_3)}$$

If $r = P_2/P_1 = P_3/P_4$, the isentropic relations apply;

$$\frac{T_2}{T_1} = r^{(k-1)/k} \quad \text{and} \quad \frac{T_4}{T_3} = r^{-(k-1)/k}$$

Thus

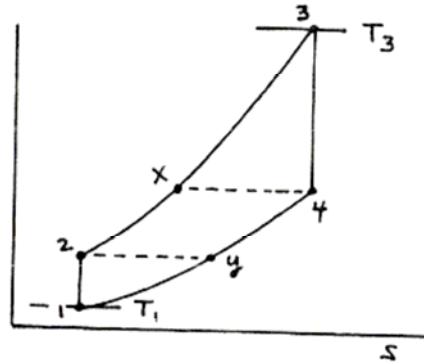
$$\eta = 1 - \frac{T_1 \left[\left(r^{(k-1)/k} - 1 \right) \right]}{T_3 \left[\left(1 - r^{-(k-1)/k} \right) \right]}$$

The bracketed term is of the form

$$\left[\frac{a-1}{1-\frac{1}{a}} \right] = \left[\frac{a-1}{\left(\frac{a-1}{a} \right)} \right] = a = r^{(k-1)/k}$$

Finally

$$\eta = 1 - \left(\frac{T_1}{T_3} \right) (r)^{(k-1)/k} \quad \leftarrow \eta$$



PROBLEM 9.59

KNOWN: Operating data are provided for an air-standard Brayton cycle.

FIND: Determine the thermal efficiency and net power developed, if
 (a) $\eta_t = \eta_c = 100\%$, (b) $\eta_t = 88\%$, $\eta_c = 84\%$, (c) $\eta_t = 88\%$, $\eta_c = 84\%$,
 and a regenerator with $\eta_{\text{regen}} = 80\%$ is incorporated.

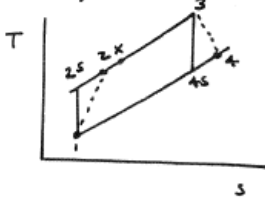
SCHEMATIC & GIVEN DATA:

For (a), refer to Fig. 9.10

$$P_2/P_1 = 10, P_1 = 14.716 \text{ lbf/in}^2, T_1 = 70^\circ\text{F}, \dot{m} = 9 \times 10^4 \text{ lb/h}, T_3 = 2200^\circ\text{R}$$

For (b), refer to Fig. 9.13b

For (c), refer to Figure below



ENGINEERING MODEL:

See Examples 9.4, 9.6, 9.7.

ANALYSIS: Using data from Table A-22E

State 1: $T_1 = 530^\circ\text{R}$

$$\Rightarrow h_1 = 126.66 \text{ Btu/lb}, P_{r1} = 1.2998$$

State 2s: $P_{r2s} = (P_2/P_1)P_{r1} = 12.998$

$$\Rightarrow h_{2s} = 244.68 \text{ Btu/lb}$$

State 3: $T_3 = 2200^\circ\text{R}, h_3 = 560.59 \text{ Btu/lb}, P_{r3} = 256.6$

State 4s: $P_{r4s} = (P_4/P_3)P_{r3} = 25.66 \Rightarrow h_{4s} = 296.7 \text{ Btu/lb}$

(a) **Ideal cycle.** Since $\dot{W}_{\text{cycle}} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}$ and $\eta = \dot{W}_{\text{cycle}} / \dot{Q}_{\text{in}}$, we get

$$\eta = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{(h_{4s} - h_1)}{(h_3 - h_{2s})} = 1 - \left(\frac{296.7 - 126.66}{560.59 - 244.68} \right) = 0.462 \text{ (46.2\%)} \quad \leftarrow (a)$$

The net power developed is

$$\begin{aligned} \dot{W}_{\text{cycle}} &= \dot{W}_t - \dot{W}_c = \dot{m} [(h_3 - h_{4s}) - (h_{2s} - h_1)] \\ &= (9 \times 10^4 \text{ lb/h}) [(560.59 - 296.7) - (244.68 - 126.66)] \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| \\ &= 5158 \text{ hp} \quad \leftarrow (a) \end{aligned}$$

(b) $\eta_t = 88\%$, $\eta_c = 84\%$. Using data given above

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 267.16 \frac{\text{Btu}}{\text{lb}}$$

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 328.4 \text{ Btu/lb}$$

$$\eta = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = 1 - \left(\frac{328.4 - 126.66}{560.59 - 267.16} \right) = 0.312 \text{ (31.2\%)} \quad \leftarrow (b)$$

$$\begin{aligned} \dot{W}_{\text{cycle}} &= \dot{W}_t - \dot{W}_c = \dot{m} [(h_3 - h_4) - (h_2 - h_1)] \\ &= (9 \times 10^4) [(560.59 - 328.4) - (267.16 - 126.66)] \left| \frac{1}{2545} \right| \\ &= 3242 \text{ hp} \quad \leftarrow (b) \end{aligned}$$

Continued on next slide

Problem 9-59 continued

(c) $\eta_t = 88\%$, $\eta_c = 84\%$, $\eta_{\text{regen}} = 80\%$

The specific enthalpy h_x is found using the regenerator effectiveness:

$$\eta_{\text{regen}} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{\text{regen}} (h_4 - h_2) + h_2 = 316.5 \text{ Btu/lb}$$

The addition of a regenerator does not affect the net power out, which remains the same as in (b):

$$\dot{W}_{\text{cycle}} = 3242 \text{ hp}$$

← (c)

The thermal efficiency is

$$\begin{aligned} \eta &= \frac{\dot{W}_{\text{cycle}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = \frac{(h_3 - h_2) - (h_2 - h_1)}{(h_3 - h_x)} \\ &= \frac{(560.59 - 328.4) - (267.16 - 126.66)}{(560.59 - 316.5)} \\ &= 0.376 \quad (37.6\%) \end{aligned}$$

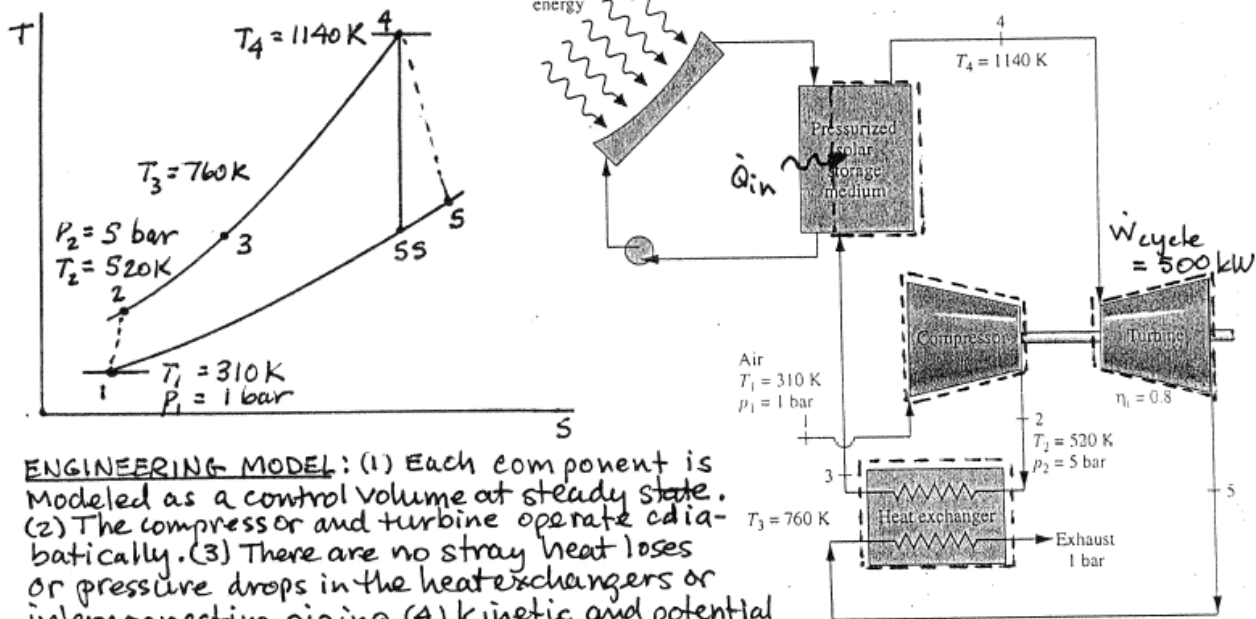
← (c)

PROBLEM 9.60

KNOWN: Data are known for a regenerative gas turbine power plant using solar energy as the source of heat addition.

FIND: Determine (a) the thermal efficiency, and (b) the mass flow rate for a net power output of 500 kW.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is modeled as a control volume at steady state. (2) The compressor and turbine operate adiabatically. (3) There are no stray heat losses or pressure drops in the heat exchangers or interconnecting piping. (4) Kinetic and potential energy effects are negligible. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: For states 1-4, the temperatures are known. Thus

$$\begin{aligned} T_1 = 310 \text{ K} &\Rightarrow h_1 = 310.24 \text{ kJ/kg} \\ T_2 = 520 \text{ K} &\Rightarrow h_2 = 523.63 \text{ kJ/kg} \\ T_3 = 760 \text{ K} &\Rightarrow h_3 = 778.18 \text{ kJ/kg} \\ T_4 = 1140 \text{ K} &\Rightarrow h_4 = 1207.57 \text{ kJ/kg} \end{aligned}$$

State 5: $Pr_5 = (P_5/P_4) Pr_4 = (1/5)(193.1) = 38.62 \Rightarrow h_{5s} = 774.49 \text{ kJ/kg}$

Using $\eta_t = (h_4 - h_5)/(h_4 - h_{5s}) \Rightarrow h_5 = h_4 - \eta_t(h_4 - h_{5s}) = 861.11 \text{ kJ/kg}$

(a) The thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_c/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_4 - h_5) - (h_2 - h_1)}{(h_4 - h_3)} = \frac{(1207.57 - 861.11) - (523.63 - 310.24)}{(1207.57 - 778.18)} = \frac{133.07}{429.39} = 0.31 \text{ (31\%)} \quad \eta$$

(b) For $\dot{W}_{cycle} = 500 \text{ kW}$

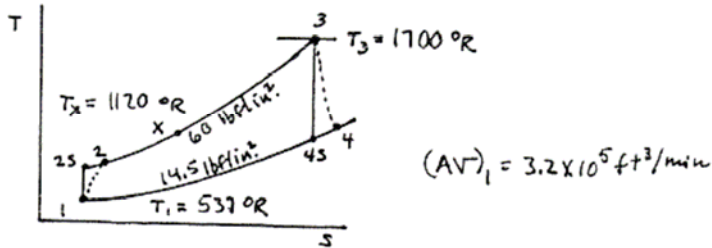
$$\dot{m} = \frac{\dot{W}_{cycle}}{(h_4 - h_5) - (h_2 - h_1)} = \frac{500 \text{ kW}}{133.07 \text{ kJ/kg}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 3.76 \text{ kg/s} \quad \dot{m}$$

PROBLEM 9.61

KNOWN: A regenerative gas turbine is analyzed on an air-standard basis. Data are known at various locations, and the net power output is given. The isentropic efficiencies of the compressor and turbine are also specified.

FIND: Determine (a) the thermal efficiency, (b) the regenerator effectiveness, and (c) the net power output.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.7, except $\eta_t = \eta_c = 0.84$.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1 $T_1 = 537^\circ\text{R} \Rightarrow h_1 = 128.34 \text{ Btu/lb}$, $P_{r1} = 1.3593$

State 2 For an isentropic compression, $P_{r2} = (P_2/P_1) P_{r1} = 5.625$. Thus, $h_{2s} = 192.76 \text{ Btu/lb}$. Using the compressor efficiency $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 205.03 \text{ Btu/lb}$

State 3 $T_3 = 1700^\circ\text{R} \Rightarrow h_3 = 422.59 \text{ Btu/lb}$, $P_{r3} = 90.95$

State 4 For an isentropic expansion, $P_{r4} = (P_4/P_3) P_{r3} = 21.980$. Thus, $h_{4s} = 284.01 \text{ Btu/lb}$. Using the turbine efficiency $\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 306.18 \text{ Btu/lb}$

State x $T_x = 1120^\circ\text{R} \Rightarrow h_x = 271.03 \text{ Btu/lb}$

(a) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_x} = 0.262 \text{ (26.2\%)} \quad \leftarrow \eta$$

(b) The regenerator effectiveness is

$$\eta_{\text{reg}} = \frac{h_x - h_2}{h_4 - h_2} = 0.652 \text{ (65.2\%)} \quad \leftarrow \eta_{\text{reg}}$$

(c) The net power output is

$$\dot{W}_{\text{cycle}} = \dot{m} [(h_3 - h_4) - (h_2 - h_1)]$$

where

$$\dot{m} = \frac{(AV)_1 P_1}{(RT_1)} = \frac{(3.2 \times 10^5 \text{ ft}^3/\text{min})(14.5 \times 144 \text{ lbf/ft}^2)}{(1545 \text{ ft} \cdot \text{lbf}/\text{lb} \cdot \text{mol} \cdot \text{R})(537^\circ\text{R})} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 1.400 \times 10^6 \frac{\text{lb}}{\text{h}}$$

Thus

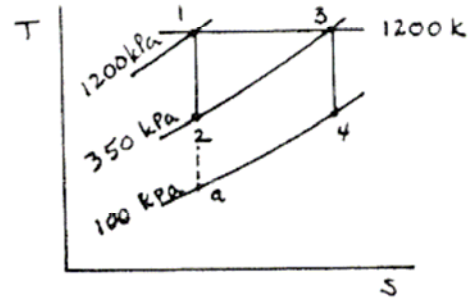
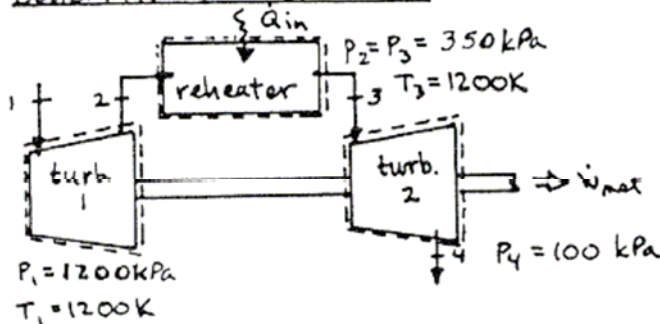
$$\begin{aligned} \dot{W}_{\text{cycle}} &= (1.400 \times 10^6 \frac{\text{lb}}{\text{h}}) [(422.59 - 306.18) - (205.03 - 128.34)] \frac{\text{Btu}}{\text{lb}} \\ &= 5.56 \times 10^7 \text{ Btu/h} \quad \leftarrow \dot{W}_{\text{cycle}} \end{aligned}$$

PROBLEM 9.62

KNOWN: Air expands in two stages through a turbine with reheat between the stages. The states are specified at the inlet and exit of each component.

FIND: Determine, per unit mass of air flowing, (a) the work developed by each stage, (b) the heat transfer for reheat, and (c) the increase in net work compared to a single stage of expansion with no reheat.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each control volume is at steady state. (2) The turbines operate isentropically. (3) Kinetic and potential energy effects are negligible. (4) The working fluid is air modeled as an ideal gas.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1 $T_1 = 1200 \text{ K} \Rightarrow h_1 = 1277.79 \text{ kJ/kg}$, $P_{r1} = 238.0$

State 2 $P_{r2} = (P_2/P_1) P_{r1} = 69.417 \Rightarrow h_2 = 912.11 \text{ kJ/kg}$

State 3 $T_3 = 1200 \text{ K} \Rightarrow h_3 = h_1 = 1277.79$, $P_{r3} = P_{r1} = 238.0$

State 4 $P_{r4} = (P_4/P_3) P_{r3} = 68 \Rightarrow h_4 = 906.85 \text{ kJ/kg}$

(a) The work developed by each stage is

$$\dot{W}_{t1}/\dot{m} = h_1 - h_2 = 365.68 \text{ kJ/kg}$$

$$\dot{W}_{t2}/\dot{m} = h_3 - h_4 = 370.94 \text{ kJ/kg}$$

(b) For the reheater

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = (1277.79 - 912.11) = 365.7 \text{ kJ/kg}$$

(c) To determine the work for a single stage of expansion, determine h_a , as follows.

$$P_{ra} = (P_a/P_1) P_{r1} = 19.833 \Rightarrow h_a = 638.58 \text{ kJ/kg}$$

Thus $\dot{W}/\dot{m} = (h_1 - h_a) = 639.21 \text{ kJ/kg}$

and $\% \text{ increase} = \frac{(365.68 + 370.94) - 639.21}{639.21} \times 100 = 15.2\%$

PROBLEM 9.63

KNOWN: The two-stage turbine with reheat in Problem 9.62 is reconsidered with turbine stage efficiencies less than 100%.

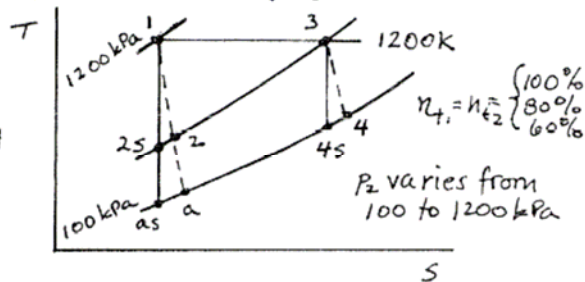
FIND: Plot (a) the work developed by each stage, (2) the heat transfer for the reheat process, and (c) the percent increase in work for the two-stage turbine with reheat compared to a single expansion, for each efficiency, and for interstage pressures ranging from 100 to 1200 kPa.

SCHEMATIC & GIVEN DATA:

See solution to Problem 9.62 for schematic.

ENGINEERING

MODEL: (1) Each control volume is at steady state. (2) The turbines operate adiabatically. (3) Kinetic and potential energy effects can be neglected. (4) The working fluid is air as an ideal gas.



ANALYSIS: Consider the sample case of $P_2 = 350 \text{ kPa}$ and $\eta_{t1} = \eta_{t2} = 0.8$. Fixing each of the principal states:

State 1: Problem 9.62: $T_1 = 1200 \text{ K}$, $h_1 = 1277.79 \text{ kJ/kg}$

State 2: With $h_{2s} = 912.11$ from Problem 9.62, and using the turbine efficiency
 $h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 985.25 \text{ kJ/kg}$

State 3: $h_3 = 1277.79 \text{ kJ/kg}$

State 4: With $h_{4s} = 906.85 \text{ kJ/kg}$; $h_4 = h_3 - \eta_{t2}(h_3 - h_{4s}) = 981.04 \text{ kJ/kg}$

State a: $h_{as} = 638.58 \text{ kJ/kg}$; $h_a = h_1 - \eta_{t1}(h_1 - h_{as}) = 766.42 \text{ kJ/kg}$

(a) The work developed by each stage is, respectively

$$\dot{W}_{t1}/\dot{m} = h_1 - h_2 = 292.54 \text{ kJ/kg}$$

$$\dot{W}_{t2}/\dot{m} = h_3 - h_4 = 296.75 \text{ kJ/kg}$$

(b) For the reheater

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = 292.54 \text{ kJ/kg}$$

(c) For comparison to a single stage, the percent increase in work is

$$\dot{W}/\dot{m} = h_1 - h_a = 511.37 \text{ kJ/kg}$$

$$\text{Thus } \% \text{ increase} = \frac{(292.54 + 296.75) - 511.37}{511.37} \times 100 = 15.2 \%$$

The plots on the next page show how each of these quantities vary with interstage pressure for each of three efficiency values. Note that only one curve is needed for the % increase in work, since the efficiency value cancels out if the same value is used for each stage.

The data for the required plots were obtained using IT, as follows:

Continued on next slide

Problem 9-63 continued

IT Code

```
p1 = 1200 // kPa
T1 = 1200 // K
p2 = 350 // kPa
T3 = 1200 // K
p3 = p2
p4 = 100 // kPa
eta_t = 0.6
```

```
h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 - (h1 - h2s) * eta_t
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eta_t
```

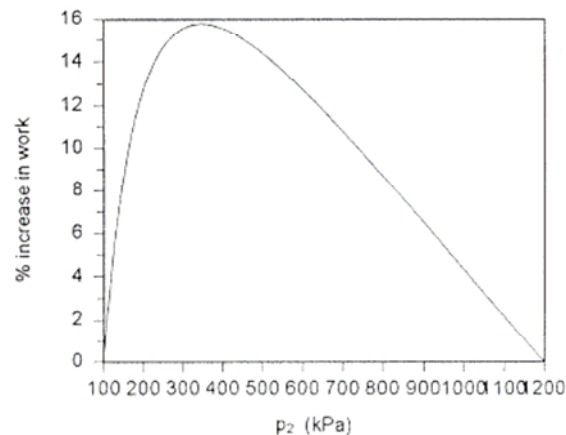
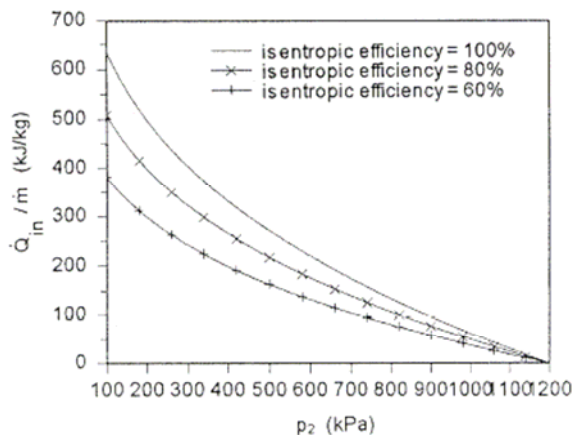
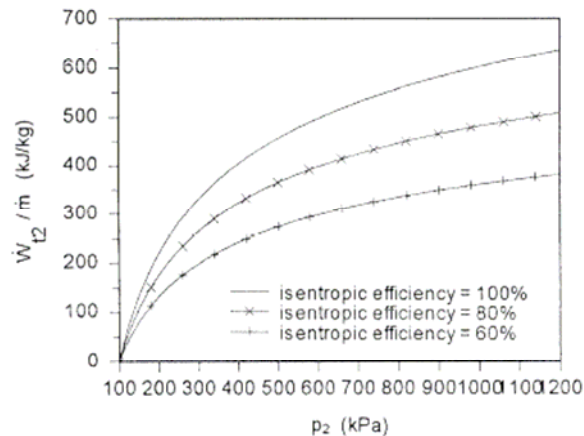
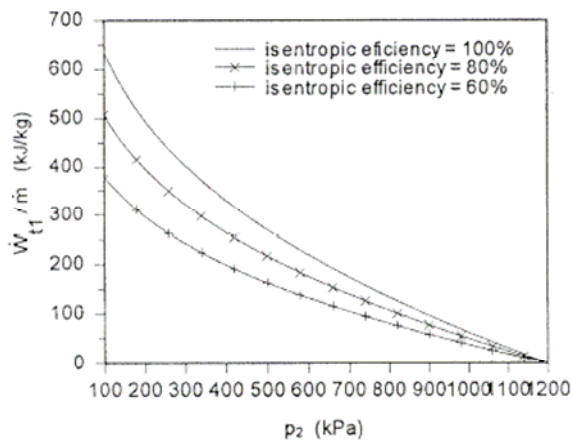
```
Wdot1 / mdot = (h1 - h2)
Wdot2 / mdot = (h3 - h4)
mdot = 1 // Assume a unit mass flow rate.
Qdot_in / mdot = h3 - h2
```

```
Wdots / mdot = h1 - ha
sas = s_hP("Air", has, p4)
sas = s1
ha = h1 - (h1 - has) * eta_t
pct = (((Wdot1 + Wdot2) - Wdots) / Wdots) * 100
```

IT Results for $\eta_t = 0.8$

```
h1 = 1277 kJ/kg
h2 = 984.9 kJ/kg
h2s = 911.9 kJ/kg
h3 = 1277 kJ/kg
h4 = 980.8 kJ/kg
h4s = 906.7 kJ/kg
ha = 768.7 kJ/kg
has = 641.7 kJ/kg
Wdot1 / mdot = 292.2 kJ/kg
Wdot2 / mdot = 296.3 kJ/kg
Qdot_in / mdot = 292.2 kJ/kg
% increase = 15.77
```

PLOTS:

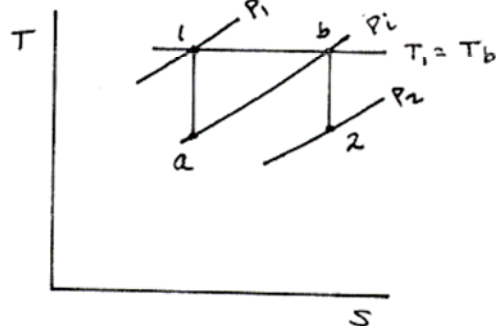
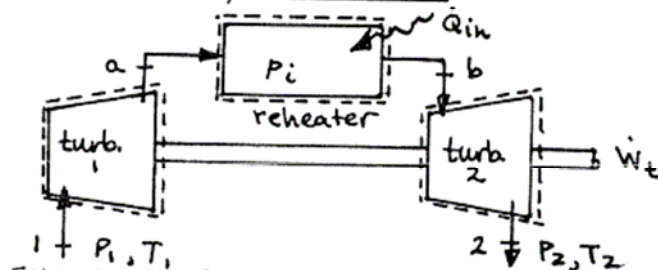


PROBLEM 9.64

KNOWN: A two-stage turbine with reheating operates at steady state under specified conditions.

FIND: Show that the maximum work is developed when the pressure ratio is the same across each stage.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The turbine expansions are isentropic. (3) There is no pressure drop through the reheater and the temperatures at the turbine inlets are equal. (4) Kinetic and potential energy effects are negligible. (5) The working fluid is air modeled as an ideal gas with constant c_p and k .

ANALYSIS: The turbine work developed is

$$\frac{\dot{W}_t}{\dot{m}} = (h_1 - h_a) + (h_b - h_2) = c_p [(T_1 - T_a) + (T_b - T_2)]$$

With $T_1 = T_b$

$$\frac{\dot{W}_t}{\dot{m}} = c_p [2T_1 - T_a - T_2] = c_p T_1 \left[2 - \frac{T_a}{T_1} - \frac{T_2}{T_1} \right]$$

For the isentropic expansions

$$\frac{T_a}{T_1} = \left(\frac{P_i}{P_1} \right)^{\frac{k-1}{k}} \quad \text{and} \quad \frac{T_2}{T_b} = \frac{T_2}{T_1} = \left(\frac{P_2}{P_i} \right)^{\frac{k-1}{k}}$$

Thus

$$\frac{\dot{W}_t}{\dot{m}} = c_p T_1 \left[2 - \left(\frac{P_i}{P_1} \right)^{\frac{k-1}{k}} - \left(\frac{P_2}{P_i} \right)^{\frac{k-1}{k}} \right]$$

To determine the pressure P_i that maximizes the total, form the derivative

$$\begin{aligned} \frac{\partial (\dot{W}_t/\dot{m})}{\partial P_i} &= -c_p T_1 \left(\frac{k-1}{k} \right) \left[\left(\frac{P_i}{P_1} \right)^{-\frac{k-1}{k}} \left(\frac{1}{P_i} \right) + \left(\frac{P_2}{P_i} \right)^{-\frac{k-1}{k}} \left(-\frac{P_2}{P_i^2} \right) \right] \\ &= -c_p T_1 \left(\frac{k-1}{k} \right) \left(\frac{1}{P_i} \right) \left[\left(\frac{P_i}{P_1} \right)^{\frac{k-1}{k}} - \left(\frac{P_2}{P_i} \right)^{\frac{k-1}{k}} \right] \end{aligned}$$

$$\text{For } \frac{\partial (\dot{W}_t/\dot{m})}{\partial P_i} = 0 \Rightarrow \frac{P_i}{P_1} = \frac{P_2}{P_i}$$

By checking the second derivative it can be verified that the total turbine work is a maximum.

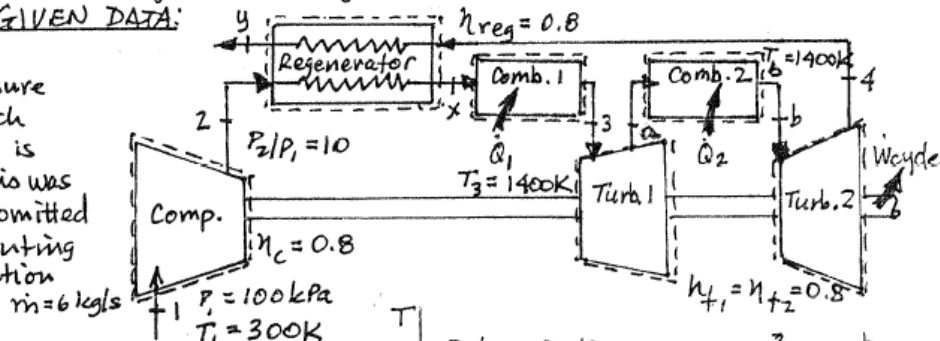
PROBLEM 9.65

KNOWN: Air enters a cold air-standard regenerative Brayton cycle with reheat at a specified state with a given mass flow rate. The compressor pressure ratio, maximum cycle temperature, and reheat temperature are known. The compressor and turbines each have isentropic efficiencies of 0.8 and the regenerator effectiveness is 0.8.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power, and (d) the rates of exergy destruction in the compressor, each turbine stage, and the regenerator, for $T_0 = 300 \text{ K}$.

SCHEMATIC & GIVEN DATA:

- ① **Note:** The pressure ratio across each turbine stage is the same. This was inadvertently omitted from early printing of the 6th Edition



ENGINEERING MODEL: See Example 9.11.

Also, $\eta_c = \eta_{t1} = \eta_{t2} = 0.8$ and $\eta_{reg} = 0.8$. The specific heats are constant, with $k = 1.4$ and $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$. Let $T_0 = 300 \text{ K}$.

ANALYSIS: From the solution to Problem 9.55: $T_2 = 649 \text{ K}$.

To find T_a , first find T_{as} as follows:

$$T_{as} = \left(\frac{P_a}{P_3} \right)^{\frac{k-1}{k}} (T_3) = \left(\frac{316.23}{1000} \right)^{\frac{1.4-1}{1.4}} (1400) = 1007.6 \text{ K}$$

Using the first-stage turbine efficiency: $\eta_{t1} = \frac{h_3 - h_a}{h_3 - h_{as}} = \frac{c_p(T_3 - T_a)}{c_p(T_3 - T_{as})}$

$$\text{Thus } T_a = T_3 - \eta_{t1}(T_3 - T_{as}) = 1400 - (0.8)(1400 - 1007.6) = 1086.1 \text{ K}$$

$$\text{Similarly, for the second-stage turbine } T_{4s} = \left(\frac{P_4}{P_5} \right)^{\frac{k-1}{k}} (T_5) = \left(\frac{100}{316.23} \right)^{\frac{1.4-1}{1.4}} (1400) = 1007.6 \text{ K}$$

$$T_4 = T_5 - \eta_{t2}(T_5 - T_{4s}) = 1086.1 \text{ K}$$

$$\text{The regenerator effectiveness is: } \eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} = \frac{c_p(T_x - T_2)}{c_p(T_4 - T_2)}$$

$$\text{or } T_x = T_2 + \eta_{reg}(T_4 - T_2) = 998.7 \text{ K}$$

For the control volume enclosing the regenerator

$$0 = (h_2 - h_x) + (h_4 - h_y) \Rightarrow 0 = c_p(T_2 - T_x) + c_p(T_4 - T_y)$$

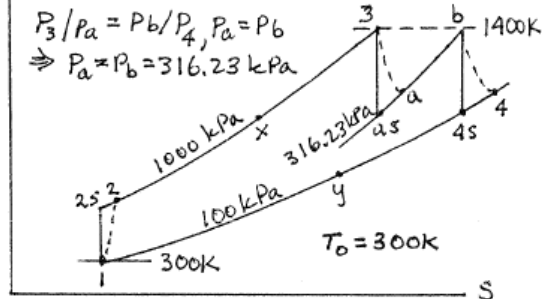
$$\text{or } T_y = T_4 - (T_x - T_2) = 736.4 \text{ K}$$

(a) The rate of heat addition is

$$\begin{aligned} \dot{Q}_{in} &= \dot{Q}_1 + \dot{Q}_2 = \dot{m} [c_p(T_3 - T_x) + c_p(T_6 - T_a)] \\ &= (6 \frac{\text{kg}}{\text{s}})(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) [(1400 - 998.7) + (1400 - 1086.1)] = 4312.7 \text{ kJ/s} \end{aligned}$$

And $\dot{Q}_{out} = \dot{m} c_p(T_y - T_1)$

$$= (6)(1.005)(736.4 - 300) = 2631.5 \text{ kJ/s}$$



Continued on next slide

Problem 9-65 continued

The thermal efficiency is

$$\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{2631.5}{4312.7} = 0.390 \text{ (39.0\%)} \leftarrow \eta$$

$$(b) \dot{W}_c = \dot{m} c_p (T_2 - T_1) = (6 \frac{\text{kg}}{\text{s}})(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(649 - 300) \text{K} = 2104.5 \text{ kJ/s}$$

$$\text{For turbine 1: } \dot{W}_{t1} = \dot{m} c_p (T_3 - T_a) = (6)(1.005)(1400 - 1086.1) = 1892.8 \frac{\text{kJ}}{\text{s}}$$

$$\text{Since } T_b = T_3 \text{ and } T_a = T_4; \dot{W}_{t2} = \dot{W}_{t1} = 1892.8 \text{ kJ/s}$$

$$\text{Thus } \dot{W}_t = (2)(1892.8) = 3785.6 \text{ kJ/s}$$

$$\text{and } bwr = \frac{\dot{W}_c}{\dot{W}_t} = \frac{2104.5}{3785.6} = 0.556 \leftarrow bwr$$

(c) The net power is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = (3785.6 - 2104.5) \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 1681.1 \text{ kW} \leftarrow \dot{W}_{cycle}$$

$$(d) \text{ For the compressor: } 0 = \sum_j (\dot{Q}_T)_j^0 + \dot{m}(s_1 - s_2) + (\dot{S}_{cv})_{comp}$$

$$\begin{aligned} (\dot{E}_d)_{comp} &= T_0 (\dot{S}_{cv})_{comp} = T_0 \dot{m} [c_p \ln(\frac{T_2}{T_1}) - R \ln(\frac{P_2}{P_1})] \\ &= (300 \text{ K})(6 \frac{\text{kg}}{\text{s}}) \left[(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \ln(\frac{649}{300}) - \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) \ln(\frac{1000}{100}) \right] \\ &= 206.5 \text{ kW} \leftarrow (\dot{E}_d)_{comp} \end{aligned}$$

$$\text{Turbine stage 1: } (\dot{E}_d)_{turb,1} = T_0 (\dot{S}_{cv})_{turb,1} = T_0 \dot{m} (s_a - s_3)$$

$$\begin{aligned} (\dot{E}_d)_{turb,1} &= T_0 \dot{m} [c_p \ln(\frac{T_a}{T_3}) - R \ln(\frac{P_a}{P_3})] \\ &= (300)(6) \left[(1.005) \ln(\frac{1086.1}{1400}) - \left(\frac{8.314}{28.97} \right) \ln(\frac{316.23}{1000}) \right] \\ &= 135.5 \text{ kW} \leftarrow (\dot{E}_d)_{turb,1} \end{aligned}$$

$$\begin{aligned} \text{Since } T_b = T_3, T_4 = T_a \text{ and } P_a/P_3 = P_4/P_b; (\dot{E}_d)_{turb,2} &= (\dot{E}_d)_{turb,1} \\ &= 135.5 \text{ kW} \leftarrow (\dot{E}_d)_{turb,2} \end{aligned}$$

Finally, for the regenerator;

$$0 = \sum_j (\dot{Q}_T)_j^0 + \dot{m} [s_2 - s_x + (s_4 - s_y)] + (\dot{S}_{cv})_{reg}$$

and

$$\begin{aligned} (\dot{E}_d)_{reg} &= T_0 (\dot{S}_{cv})_{reg} = T_0 \dot{m} [c_p \ln(\frac{T_x}{T_2}) - R \ln(\frac{P_x}{P_2}) + c_p \ln(\frac{T_y}{T_4}) - R \ln(\frac{P_y}{P_4})] \\ &= (300)(6)(1.005) \left[\ln(\frac{998.7}{649}) + \ln(\frac{736.4}{1086.1}) \right] \\ &= 76.8 \text{ kW} \leftarrow (\dot{E}_d)_{reg} \end{aligned}$$

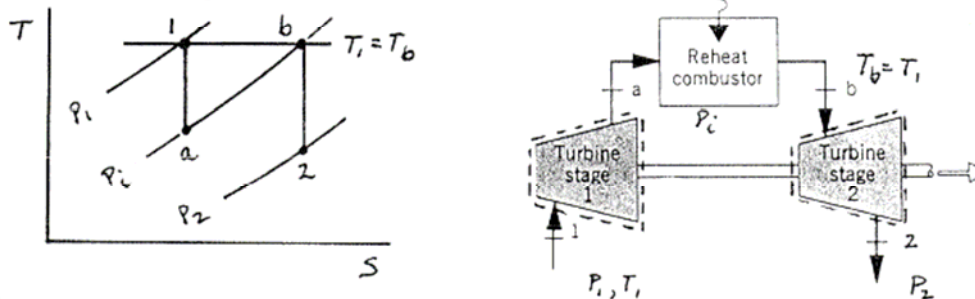
1. In early printings of the 6th Edition, the fact that the pressure ratios are equal across each turbine stage was inadvertently omitted. We regret the error.

PROBLEM 9.66

KNOWN: A two-stage turbine with reheat operates at steady state under specified conditions.

FIND: Show that the maximum total work output is obtained when the pressure ratio is the same across each stage.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The turbine stages are analyzed as control volumes at steady state. (2) The expansion processes are isentropic. (3) $P_a = P_b = P_i$. (4) $T_1 = T_b$. (5) Kinetic and potential energy effects are negligible. (6) The working fluid is air modeled as an ideal gas with constant specific heats.

ANALYSIS: The total turbine work output per unit mass of air flow is

$$\begin{aligned} \frac{\dot{W}_t}{\dot{m}} &= \frac{\dot{W}_{t1}}{\dot{m}} + \frac{\dot{W}_{t2}}{\dot{m}} = (h_1 - h_a) + (h_b - h_2) \\ \text{or} \quad &= c_p [(T_1 - T_a) + (T_b - T_2)] \\ &= c_p T_1 \left[\left(1 - \frac{T_a}{T_1}\right) + \left(\frac{T_b}{T_1} - \frac{T_2}{T_1}\right) \right] \end{aligned}$$

With $T_1 = T_b$,

$$\frac{\dot{W}_t}{\dot{m}} = c_p T_1 \left[2 - \frac{T_a}{T_1} - \frac{T_2}{T_1} \right]$$

For the isentropic processes: $T_a/T_1 = (P_a/P_1)^{\frac{k-1}{k}}$ and $T_2/T_b = (P_2/P_b)^{\frac{k-1}{k}}$.
Since $P_a = P_b = P_i$

$$\frac{\dot{W}_t}{\dot{m}} = c_p T_1 \left[2 - \left(\frac{P_i}{P_1}\right)^{\frac{k-1}{k}} - \left(\frac{P_2}{P_i}\right)^{\frac{k-1}{k}} \right]$$

Differentiating with respect to P_i and after re-arrangement

$$\frac{\partial (\dot{W}_t/\dot{m})}{\partial P_i} = c_p T_1 \left(\frac{k-1}{k} \right) \frac{1}{P_i} \left[\left(\frac{P_i}{P_1}\right)^{\frac{k-1}{k}} - \left(\frac{P_2}{P_i}\right)^{\frac{k-1}{k}} \right]$$

When the partial derivative is set to zero, the desired relationship is obtained

$$\frac{P_i}{P_1} = \frac{P_2}{P_i}$$

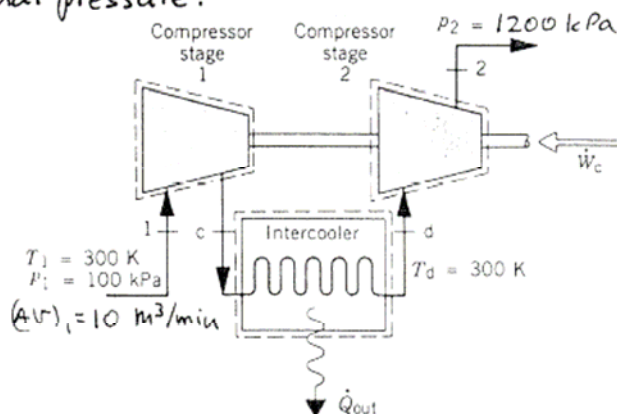
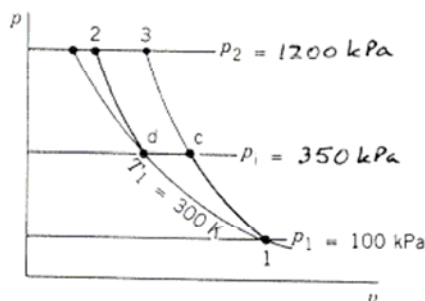
By checking the sign of the second derivative, it can be verified the the total turbine work output is indeed maximized at this pressure ratio.

PROBLEM 9.67

KNOWN: Air is compressed in a two-stage compressor with intercooling between the stages. Operating pressures and temperatures are given.

FIND: Determine the power to run the compressor and compare this to the power required for isentropic compression from the same inlet state to the same final pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as in Example 9.9.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1: $T_1 = 300 \text{ K} \Rightarrow h_1 = 300.19 \text{ kJ/kg}$, $P_{r1} = 1.3860$

State 2: $P_{r2} = (P_2/P_1) P_{r1} = 4.851 \Rightarrow h_2 = 429.77 \text{ kJ/kg}$

State 3: $T_d = T_1 = 300 \text{ K} \Rightarrow h_d = 300.19 \text{ kJ/kg}$, $P_{rd} = 1.3860$

State 4: $P_{r4} = (P_4/P_d) P_{rd} = 4.752 \Rightarrow h_4 = 427.21 \text{ kJ/kg}$

The mass flow rate is

$$\dot{m} = \frac{(AV)_1 P_1}{RT_1} = \frac{(10 \text{ m}^3/\text{min})(100 \text{ kPa})}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}}\right)(300 \text{ K})} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 0.1936 \text{ kg/s}$$

The compressor power is

$$\begin{aligned} \dot{W}_c &= \dot{W}_{c1} + \dot{W}_{c2} = \dot{m}(h_2 - h_1) + \dot{m}(h_4 - h_d) \\ &= (0.1936 \frac{\text{kg}}{\text{s}}) (429.77 - 300.19) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| + (0.1936)(427.21 - 300.19) \\ &= 25.087 \text{ kW} + 24.591 \text{ kW} = 49.68 \text{ kW} \end{aligned}$$

To find the power a single stage of compression, determine h_3 as follows:

$$P_{r3} = (P_2/P_1) P_{r1} = 16.632 \Rightarrow h_3 = 610.65 \text{ kJ/kg}$$

Thus $\dot{W}_c = \dot{m}(h_3 - h_1) = 60.11 \text{ kW}$

The decrease in power with two-stage, intercooled compression is

$$\% \text{ decrease} = \frac{60.11 - 49.68}{60.11} \times 100 = 17.35\%$$

% reduction in power

PROBLEM 9.68

KNOWN: The two-stage compressor with intercooling in Problem 9.67 is reconsidered with compressor stage efficiencies less than 100%.

FIND: Plot the (a) power input to each stage, (b) heat transfer rate for the intercooler, and (c) percent decrease in power for two-stage compression compared a single compression stage, for each efficiency value given, and for interstage pressures ranging from 100 to 1200 kPa.

SCHEMATIC & GIVEN DATA:

See solution to Problem 9.67 for schematic.

ENGINEERING MODEL: Same as in Example 9.9, except $\eta_{c1} = \eta_{c2} \leq 100\%$.

ANALYSIS: Consider the sample case of $P_c = P_d = 350$ kPa and $\eta_{c1} = \eta_{c2} = 0.8$.

Fixing each of the principal states:

State 1: $h_1 = 300.19$ kJ/kg (Problem 9.67)

State c: With $h_{c5} = 429.77$ kJ/kg from Problem 9.67, and using the compressor efficiency $h_c = h_1 + (h_{c5} - h_1)/\eta_{c1} = 462.16$ kJ/kg

State d: $h_d = h_1 = 300.19$ kJ/kg

State 2: $h_{25} = 427.21$ kJ/kg ; $h_2 = h_d + (h_{25} - h_d)/\eta_{c2} = 459.0$ kJ/kg

(a) The power input for each stage is, for $\dot{m} = 0.1936$ kg/s

$$\dot{W}_{c1} = \dot{m}(h_c - h_1) = 31.36 \text{ kW}$$

$$\dot{W}_{c2} = \dot{m}(h_2 - h_d) = 30.75 \text{ kW}$$

$$\text{also } \dot{W}_c = \dot{W}_{c1} + \dot{W}_{c2} = 62.11 \text{ kW}$$

(b) For the intercooler

$$\dot{Q}_{out} = \dot{m}(h_c - h_d) = 31.36 \text{ kW}$$

(c) Determine h_3 as follows: $h_{35} = 610.65$ kJ/kg (Problem 9.67). Thus

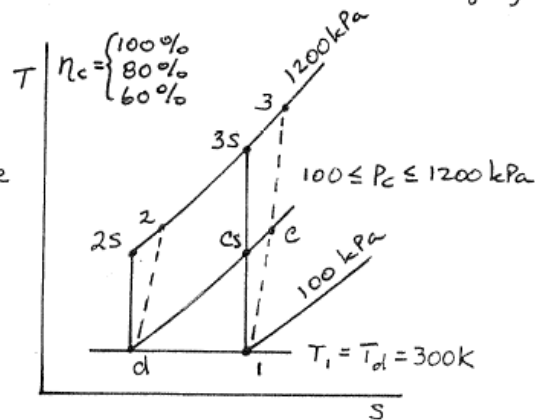
$$h_3 = h_1 + (h_{35} - h_1)/\eta_c = 688.3 \text{ kJ/kg}$$

$$\text{Thus } \dot{W} = \dot{m}(h_3 - h_1) = 75.14 \text{ kW}$$

Finally

$$\% \text{ decrease} = \frac{75.14 - 62.11}{75.14} \times 100 = 17.34\%$$

The plots on the next page show how each of these quantities varies with interstage pressure for each of the efficiency values. Note that only one curve is needed for the % decrease in power input, since the efficiency value cancels out if the same value is used for each stage.



Continued on next slide

Problem 9-68 continued

The data for the required plots are obtained using IT, as follows:

IT Code

```
p1 = 100 // kPa
T1 = 300 // K
pc = 350 // kPa
pd = pc
Td = T1
p2 = 1200 // kPa
p3 = p2
eta_c = 0.8
AV1 = 10 // m³/min

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
scs = s_hP("Air", hcs, pc)
scs = s1
hc = h1 + (hcs - h1) / eta_c
hd = h_T("Air", Td)
sd = s_TP("Air", Td, pd)
s2s = s_hP("Air", h2s, p2)
```

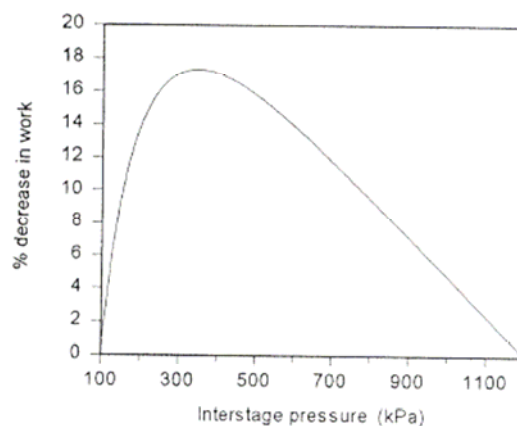
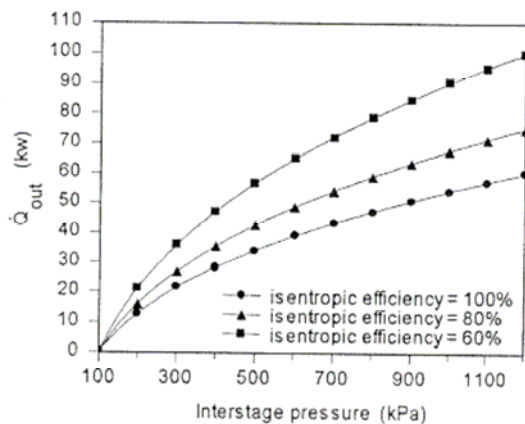
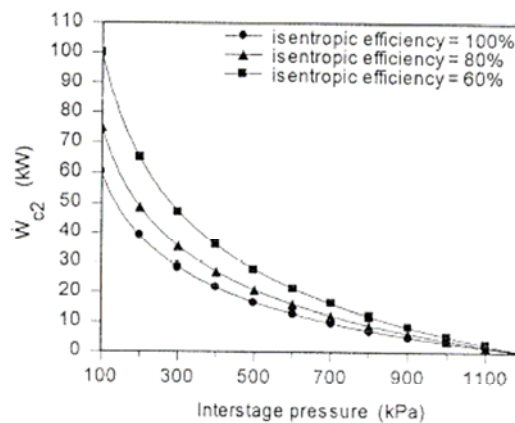
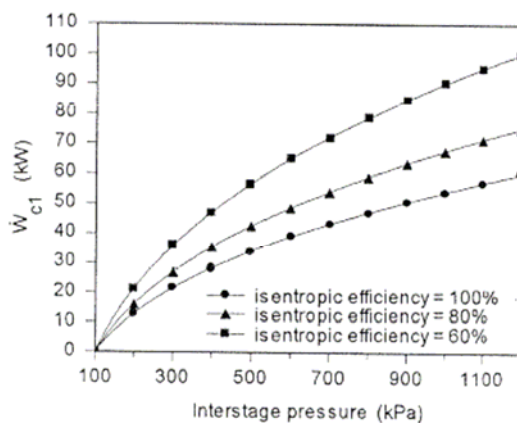
```
s2s = sd
h2 = hd + (h2s - hd) / eta_c
s3s = s_hP("Air", h3s, p3)
s3s = s1
h3 = h1 + (h3s - h1) / eta_c

mdot = (AV1 / v1) * (1 / 60) // kg/s
v1 = v_TP("Air", T1, p1)
Wdotc1 = mdot * (hc - h1)
Wdotc2 = mdot * (h2 - hd)
Wdotc = Wdotc1 + Wdotc2
Qdotout = mdot * (hc - hd)
Wdot = mdot * (h3 - h1)
pct = ((Wdot - Wdotc) / Wdot) * 100
```

IT Results for $p_2 = p_3 = 350$ kPa,

```
eta_c1 = eta_c2 = 80%
h1 = 300 kJ/kg
h2 = 458.9 kJ/kg
h2s = 427.1 kJ/kg
h3 = 688.2 kJ/kg
h3s = 610.5 kJ/kg
hc = 462.1 kJ/kg
hcs = 429.7 kJ/kg
hd = 300 kJ/kg
m = 0.1936 kg/s
Wc1 = 31.37 kW
Wc2 = 30.76 kW
Qout = 31.37 kW
% decrease = 17.32
```

PLOTS:

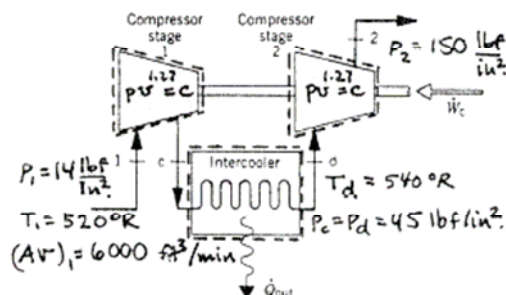
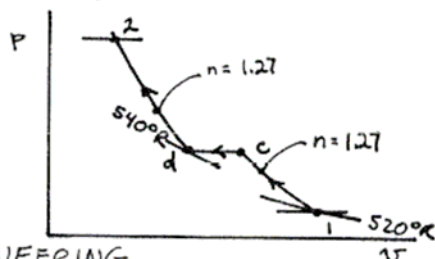


PROBLEM 9.69

KNOWN: Air is compressed at steady state in a two-stage compressor with intercooling between the stages. Operating pressures and temperatures are known.

FIND: Determine (a) the power and heat transfer rate for each compressor stage and (b) the intercooler heat transfer rate.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compression processes are polytropic with $n = 1.27$. (3) There is no pressure drop through the intercooler. (4) Kinetic and potential energy effects are negligible. (5) The air behaves as an ideal gas.

ANALYSIS: (a) For compressor stage 1, $T_c = (P_c/P_1)^{\frac{n-1}{n}} T_1 = 666.51^\circ\text{R}$. From Table A-22E, $h_1 = 124.27 \text{ Btu/lb}$ and $h_c = 159.49 \text{ Btu/lb}$.

Equation 6.66a, for a polytropic process of an ideal gas, applies

$$\frac{\dot{W}_{c1}}{\dot{m}} = \frac{nR}{n-1} (T_c - T_1) = \frac{(1.27) \left(\frac{1.986}{28.97} \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (666.51 - 520)^\circ\text{R}}{(1.27)} = 47.24 \text{ Btu/lb}$$

Evaluating $\dot{m}$

$$\dot{m} = \frac{(AV)_1 P_1}{RT_1} = \frac{(6000 \text{ ft}^3/\text{min}) (14 \text{ lbf/in}^2)}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right) (520^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 436.2 \text{ lb/min}$$

The power for stage 1 is

$$\dot{W}_{c1} = (436.2 \text{ lb/min}) (47.24 \text{ Btu/lb}) = 20,606 \text{ Btu/min} \quad \leftarrow \dot{W}_{c1}$$

Now, applying an energy balance to stage 1, the heat transfer rate is

$$\dot{Q}_{c1} = \dot{m} (h_c - h_1) - \dot{W}_{c1} = (436.2) (159.49 - 124.27) - 20,606 = -5243 \text{ Btu/min} \quad \leftarrow \dot{Q}_{c1}$$

For stage 2; $T_2 = (P_2/P_d)^{\frac{n-1}{n}} T_d = (150/45)^{\frac{1.27-1}{1.27}} (540) = 697.5^\circ\text{R}$. From

Table A-22E, $h_d = 129.06 \text{ Btu/lb}$, $h_2 = 166.96 \text{ Btu/lb}$. Then

$$\dot{W}_{c2} = \dot{m} \left[\frac{nR}{n-1} \right] (T_2 - T_d) = (436.2) \left[\frac{(1.27) \left(\frac{1.986}{28.97} \right)}{1.27} \right] (697.5 - 540) = 22,153 \text{ Btu/min} \quad \leftarrow \dot{W}_{c2}$$

$$\dot{Q}_{c2} = \dot{m} (h_2 - h_d) - \dot{W}_{c2} = (436.2) (166.96 - 129.06) - 22,153 = -5621 \text{ Btu/min} \quad \leftarrow \dot{Q}_{c2}$$

(b) $\dot{Q}_{\text{out}} = \dot{m} (h_c - h_d)$

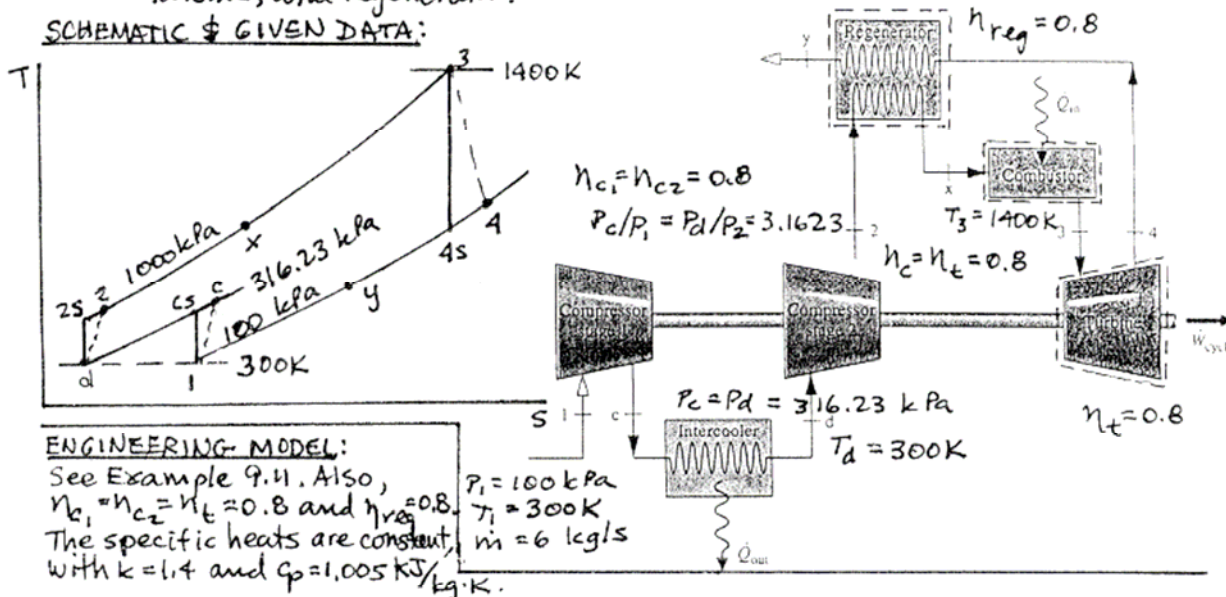
$$= 436.2 (159.49 - 129.06) = 13,274 \text{ Btu/min} \quad \leftarrow \dot{Q}_{\text{out}}$$

PROBLEM 9.70

KNOWN: Air enters a cold air-standard regenerative Brayton cycle with intercooling at a specified state with a given mass flow rate. The overall compressor pressure ratio is known, and the pressure ratios are the same across each compressor stage. The maximum cycle temperature is given. The compressor stage and turbine isentropic efficiencies are each 0.8 and the regenerator effectiveness is 0.8.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power, and (d) the rates of exergy destruction in the compressor stages, turbine, and regenerator.

SCHEMATIC & GIVEN DATA:



ANALYSIS: From the solution to Problem 9.55: $T_4 = 860.1 \text{ K}$.

For the first compressor stage: $T_{c1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} T_1 = (3.1623)^{\frac{1.4-1}{1.4}} (300) = 416.85 \text{ K}$

Using the first-stage compressor efficiency; $\eta_{c1} = (h_{c1} - h_1) / (h_2 - h_1)$ and $\Delta h = c_p \Delta T$, we get

$$T_c = T_1 + (T_{c1} - T_1) / \eta_{c1} = 300 + (416.85 - 300) / (0.8) = 446.1 \text{ K}$$

Since $P_d = P_c$ and $T_d = T_1 = 300 \text{ K}$; $T_2 = 446.1 \text{ K}$

From the regenerator effectiveness; $\eta_{reg} = (h_x - h_2) / (h_4 - h_2) = \frac{c_p(T_x - T_2)}{c_p(T_4 - T_2)}$

$$T_x = T_2 + (\eta_{reg})(T_4 - T_2) = 446.1 + (0.8)(860.1 - 446.1) = 777.3 \text{ K}$$

From an energy balance on the regenerator

$$T_y = T_4 - (T_x - T_2) = 860.1 - (777.3 - 446.1) = 528.9 \text{ K}$$

$$(a) \dot{Q}_{in} = \dot{m} c_p (T_3 - T_x) = (6 \frac{\text{kg}}{\text{s}}) (1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) (1400 - 777.3) \text{ K} = 3754.9 \text{ kJ/s}$$

$$\begin{aligned} \dot{Q}_{out} &= \dot{m} c_p (T_d - T_c) + \dot{m} c_p (T_y - T_1) \\ &= (6) (1.005) [(446.1 - 300) + (528.9 - 300)] = 2261.3 \text{ kJ/s} \end{aligned}$$

The thermal efficiency is

$$\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{2261.3}{3754.9} = 0.398 (39.8\%) \quad \eta$$

Continued on next slide

Problem 9-70 continued

$$(b) \dot{W}_c = \dot{W}_{c1} + \dot{W}_{c2} = 2 \cdot \dot{W}_{c1} = 2 \dot{m} c_p (T_c - T_1) \\ = (2)(6)(1.005)(446.1 - 300) = 1762 \text{ kJ/s}$$

$$\dot{W}_t = \dot{m} c_p (T_3 - T_4) = (6)(1.005)(1400 - 860.1) = 3255.6 \text{ kJ/s}$$

$$\text{Thus } bwr = \dot{W}_c / \dot{W}_t = 1762 / 3255.6 = 0.5412 \leftarrow bwr$$

(c) The net power is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = (3255.6 - 1762) \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 1493.6 \text{ kW} \leftarrow \dot{W}_{cycle}$$

(d) For the first compressor stage: $\phi = \sum \left(\frac{\dot{Q}}{T} \right)_j^o + \dot{m}(s_1 - s_c) + (\dot{\sigma}_{cv})_{c1}$

$$(\dot{E}_d)_{c1} = T_0 (\dot{\sigma}_{cv})_{c1} = T_0 \dot{m} (s_c - s_1) = T_0 \dot{m} \left[c_p \ln \left(\frac{T_c}{T_1} \right) - R \ln \left(\frac{P_c}{P_1} \right) \right] \\ = (300 \text{ K})(6 \frac{\text{kg}}{\text{s}}) \left[(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \ln \left(\frac{446.1}{300} \right) - \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) \ln (3.1623) \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ = 123.0 \text{ kW} \leftarrow (\dot{E}_d)_{c1}, (\dot{E}_d)_{c2}$$

$$(\dot{E}_d)_{c2} = (\dot{E}_d)_{c1} = 123.0 \text{ kW}$$

For the turbine

$$(\dot{E}_d)_t = T_0 \dot{m} (s_4 - s_3) = T_0 \dot{m} \left[c_p \ln \left(\frac{T_4}{T_3} \right) - R \ln \left(\frac{P_4}{P_3} \right) \right] \\ = (300)(6) \left[(1.005) \ln \left(\frac{860.1}{1400} \right) - \left(\frac{8.314}{28.97} \right) \ln \left(\frac{1}{10} \right) \right] = 308.2 \text{ kW} \leftarrow (\dot{E}_d)_t$$

Finally, for the regenerator: $\phi = \sum \left(\frac{\dot{Q}}{T} \right)_j^o + \dot{m} [(s_2 - s_x) + (s_4 - s_y)] + (\dot{\sigma}_{cv})_{reg}$

$$(\dot{E}_d)_{reg} = T_0 (\dot{\sigma}_{cv})_{reg} = T_0 \dot{m} \left[c_p \ln (T_x / T_2) - R \ln (P_x / P_2) + c_p \ln (T_y / T_4) - R \ln (P_y / P_4) \right] \\ = (300)(6)(1.005) \left[\ln (777.3 / 446.1) + \ln (528.9 / 860.1) \right] \\ = 124.9 \text{ kW} \leftarrow (\dot{E}_d)_{reg}$$

PROBLEM 9.71

KNOWN: A two-stage compressor with intercooling operates at steady state with known inlet conditions and a given exit pressure.

FIND: Referring to Example 9.10, derive an expression for the relation between pressure ratios across the two stages for the case where $T_d > T_i$.

SCHEMATIC & GIVEN DATA: See Example 9.10.

ENGINEERING MODEL: See Example 9.10, except $T_d > T_i$.

ANALYSIS: The total compressor work input per unit mass flow is

$$\begin{aligned}\frac{\dot{W}_c}{\dot{m}} &= (h_c - h_i) + (h_2 - h_d) = c_p (T_c - T_i) + c_p (T_2 - T_d) \\ &= c_p T_i \left(\frac{T_c}{T_i} - 1 \right) + c_p T_d \left(\frac{T_2}{T_d} - 1 \right)\end{aligned}$$

Introducing the isentropic relations

$$\frac{T_c}{T_i} = \left(\frac{P_i}{P_1} \right)^{\frac{k-1}{k}}, \quad \frac{T_2}{T_d} = \left(\frac{P_2}{P_i} \right)^{\frac{k-1}{k}}$$

we get

$$\frac{\dot{W}_c}{\dot{m}} = c_p T_i \left[\left(\frac{P_i}{P_1} \right)^{\frac{k-1}{k}} - 1 \right] + c_p T_d \left[\left(\frac{P_2}{P_i} \right)^{\frac{k-1}{k}} - 1 \right]$$

To determine the pressure P_i that minimizes the total work, form the derivative

$$\begin{aligned}\frac{\partial (\dot{W}_c / \dot{m})}{\partial P_i} &= c_p T_i \left(\frac{k-1}{k} \right) \left(\frac{P_i}{P_1} \right)^{-\frac{1}{k}} \left(\frac{1}{P_1} \right) - c_p T_d \left(\frac{k-1}{k} \right) \left(\frac{P_2}{P_i} \right)^{-\frac{1}{k}} \left(\frac{P_2}{P_i} \right) \\ &= c_p T_i \left(\frac{k-1}{k} \right) \frac{1}{P_i} \left\{ \left(\frac{P_i}{P_1} \right)^{\frac{k-1}{k}} - \left(\frac{T_d}{T_i} \right) \left(\frac{P_2}{P_i} \right)^{\frac{k-1}{k}} \right\}\end{aligned}$$

For the minimum, $\partial (\dot{W}_c / \dot{m}) / \partial P_i = 0$. Thus

$$0 = \left(\frac{P_i}{P_1} \right)^{\frac{k-1}{k}} - \left(\frac{T_d}{T_i} \right) \left(\frac{P_2}{P_i} \right)^{\frac{k-1}{k}}$$

and finally

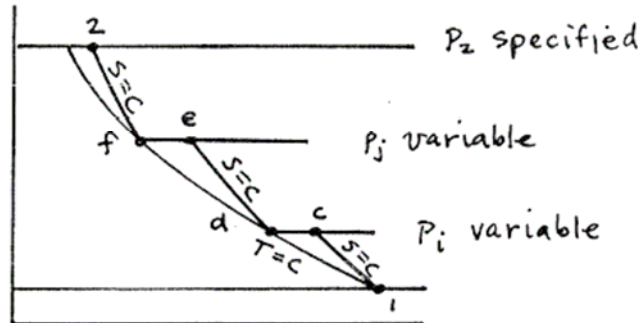
$$\frac{P_i}{P_1} = \left(\frac{T_d}{T_i} \right)^{\frac{k}{k-1}} \left(\frac{P_2}{P_i} \right)$$

PROBLEM 9.72

KNOWN: A multi-stage compressor with intercooling between the stages has known inlet conditions and a given exit pressure.

FIND: Show that the minimum total work input is required when the pressure ratio is the same across each stage.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.10.

ANALYSIS: From the result of Example 9.10;

$$\frac{P_i}{P_1} = \frac{P_j}{P_i}$$

similarly

$$\frac{P_j}{P_i} = \frac{P_2}{P_j}$$

Thus

$$\frac{P_i}{P_1} = \frac{P_j}{P_i} = \frac{P_2}{P_j}$$

Finally, solving for P_i and P_j

$$P_i = \sqrt[3]{P_1^2 P_2}$$

$$P_j = \sqrt[3]{P_2^2 P_1}$$

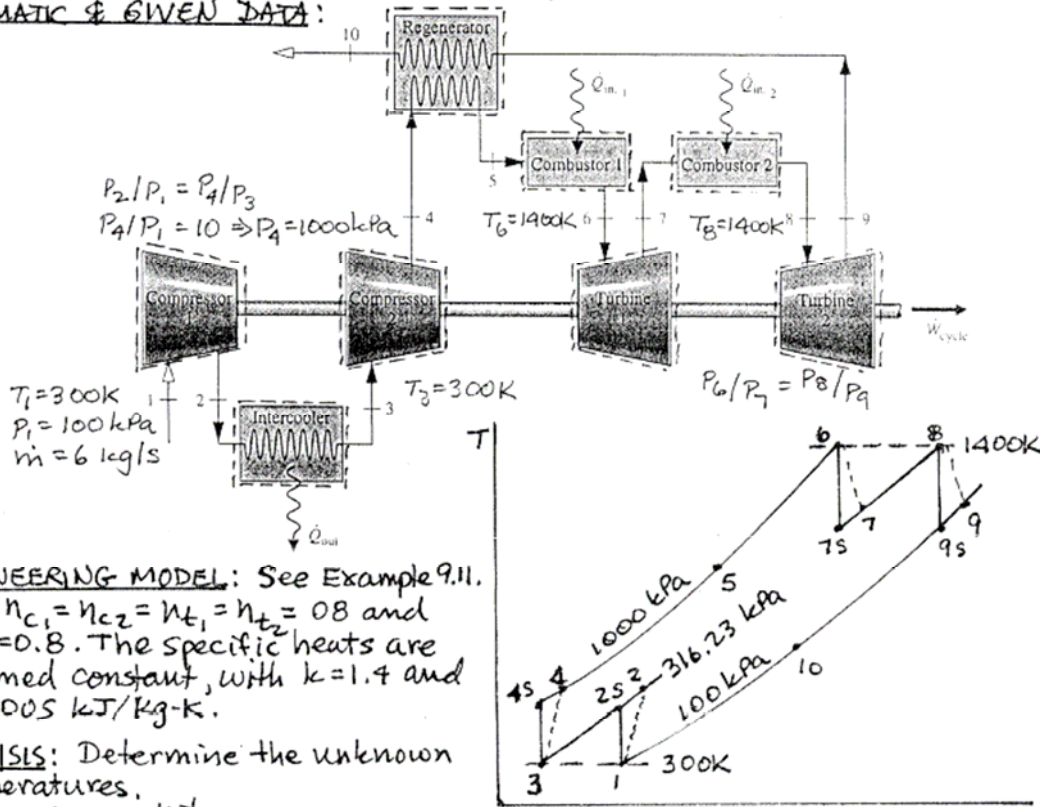
← optimum pressures

PROBLEM 9.13

KNOWN: Air enters a cold air-standard regenerative Brayton cycle with intercooling and reheat at a specified state and with a given mass flow rate. The overall compressor pressure ratio, maximum cycle temperature, and intercooler and reheat temperatures known. For the compressor and turbine stages, the isentropic efficiency is 0.8 and the regenerator effectiveness is 0.8.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power, and (d) exergy destruction rates in each compressor and turbine stage and in the regenerator.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.11.

Also, $\eta_{c1} = \eta_{c2} = \eta_{t1} = \eta_{t2} = 0.8$ and $\eta_{reg} = 0.8$. The specific heats are assumed constant, with $k = 1.4$ and $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$.

ANALYSIS: Determine the unknown temperatures.

$$T_{2s} = (P_2/P_1)^{\frac{k-1}{k}} (T_1) = 416.85 \text{ K. With the compressor stage efficiency}$$

$$T_2 = T_1 + (T_{2s} - T_1)/\eta_{c1} = 446.1 \text{ K}$$

$$\text{Since } T_3 = T_1 \text{ and } P_4/P_3 = P_2/P_1; T_4 = T_2 = 446.1 \text{ K}$$

$$T_{7s} = (P_7/P_6)^{\frac{k-1}{k}} (T_6) = 1007.6 \text{ K}$$

$$T_7 = T_6 - \eta_{t1} (T_6 - T_{7s}) = 1086.1 \text{ K}$$

$$\text{Since } T_8 = T_6 \text{ and } P_7/P_6 = P_9/P_8; T_9 = T_7 = 1086.1 \text{ K}$$

$$\text{For the regenerator; } T_5 = T_4 + \eta_{reg} (T_9 - T_4) = 958.1 \text{ K}$$

$$\text{and } T_{10} = T_9 - (T_5 - T_4) = 574.1 \text{ K}$$

$$(a) \dot{Q}_{in} = \dot{Q}_{in,1} + \dot{Q}_{in,2} = \dot{m} c_p [(T_6 - T_5) + (T_8 - T_7)]$$

$$= (6 \frac{\text{kg}}{\text{s}}) (1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}) [(1400 - 958.1) + (1400 - 1086.1)] \text{ K} = 4557.5 \frac{\text{kJ}}{\text{s}}$$

Continued on next slide

Problem 9-73 continued

$$\text{and } \dot{Q}_{\text{out, total}} = \dot{m} c_p (T_{10} - T_1) + \dot{m} c_p (T_2 - T_3) \\ = (6)(1.005) [(574.1 - 300) + (446.1 - 300)] = 2533.8 \text{ kJ/s}$$

The thermal efficiency is $\eta = 1 - \dot{Q}_{\text{out}} / \dot{Q}_{\text{in}} = 0.444$ (44.4%) $\leftarrow \eta$

(b) The power for each turbine stage is equal, Thus

$$\dot{W}_t = 2 \cdot \dot{W}_{t1} = 2 \dot{m} c_p (T_6 - T_7) = (2)(6)(1.005)(1400 - 1086.1) = 3785.6 \frac{\text{kJ}}{\text{s}}$$

Similarly, for the compressor stages

$$\dot{W}_c = 2 \cdot \dot{W}_{c1} = 2 \dot{m} c_p (T_2 - T_1) = (2)(6)(1.005)(446.1 - 300) = 1762.0 \frac{\text{kJ}}{\text{s}}$$

$$\text{and } \text{bwr} = \dot{W}_c / \dot{W}_t = 1762 / 3785.6 = 0.4654 \leftarrow \text{bwr}$$

(c) The net power is

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_c = (3785.6 - 1762) \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 2023.6 \text{ kW} \leftarrow \dot{W}_{\text{cycle}}$$

(d) For the first compressor stage

$$(\dot{E}_d)_{\text{comp},1} = T_0 \dot{m} (s_2 - s_1) = T_0 \dot{m} \left[c_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right) \right] \\ = (300 \text{ K}) \left(6 \frac{\text{kg}}{\text{s}} \right) \left[(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \ln \left(\frac{446.1}{300} \right) - \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) \ln \left(\frac{316.23}{100} \right) \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ = 123.0 \text{ kW} \leftarrow (\dot{E}_d)$$

$$(\dot{E}_d)_{\text{comp},2} = (\dot{E}_d)_{\text{comp},1} \quad \text{(compressor stages)}$$

For turbine stage 1

$$(\dot{E}_d)_{\text{turb},1} = T_0 \dot{m} (s_7 - s_6) = T_0 \dot{m} \left[c_p \ln \left(\frac{T_7}{T_6} \right) - R \ln \left(\frac{P_7}{P_6} \right) \right] \\ = (300 \text{ K}) (6) \left[(1.005) \ln \left(\frac{1086.1}{1400} \right) - \frac{8.314}{28.97} \ln \left(\frac{316.23}{1000} \right) \right] \\ = 135.5 \text{ kW} \quad (\dot{E}_d)$$

$$(\dot{E}_d)_{\text{turb},2} = (\dot{E}_d)_{\text{turb},1} \quad \text{(Turbine stages)}$$

For the regenerator

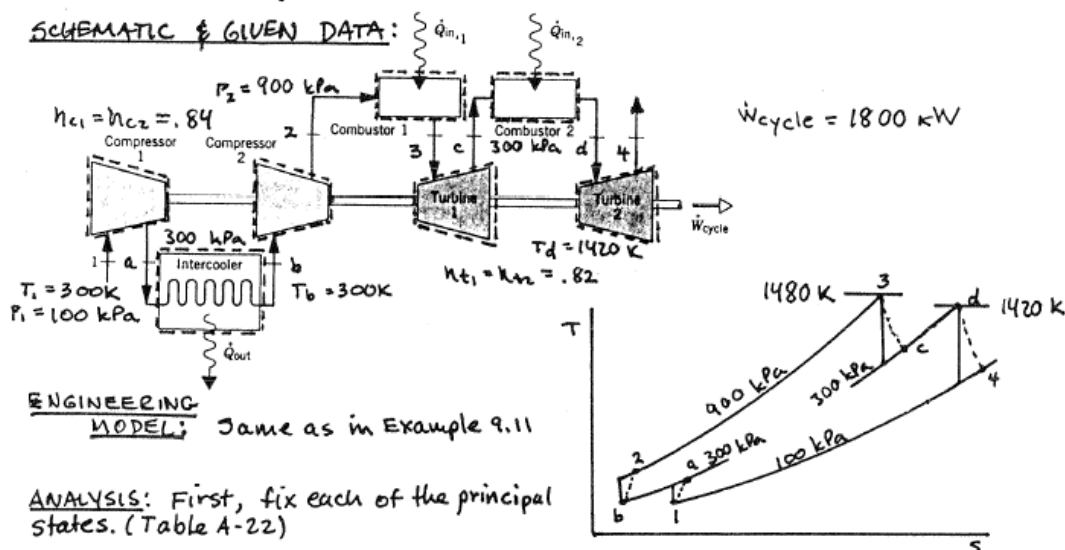
$$(\dot{E}_d)_{\text{reg}} = T_0 \dot{m} [(s_5 - s_4) + (s_{10} - s_9)] \\ = T_0 \dot{m} \left[c_p \ln \frac{T_5}{T_4} - R \ln \frac{P_5}{P_4} + c_p \ln \left(\frac{T_{10}}{T_9} \right) - R \ln \frac{P_{10}}{P_9} \right] \\ = (300 \text{ K}) (6) (1.005) \left[\ln \left(\frac{958.1}{446.1} \right) + \ln \left(\frac{574.1}{1086.1} \right) \right] = 229.5 \text{ kW} \quad (\dot{E}_d)_{\text{reg}}$$

PROBLEM 9.74

KNOWN: A gas turbine has two stages of compression with intercooling and a two-stage turbine with reheat. Data are known at various locations. Turbine and compressor stage efficiencies are known.

FIND: Determine (a) the volumetric flow rates at 1 and b, (2) the thermal efficiency, (3) the back work ratio.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as in Example 9.11

ANALYSIS: First, fix each of the principal states. (Table A-22)

State 1 $T_1 = 300 \text{ K} \Rightarrow h_1 = 300.19 \text{ kJ/kg}$, $P_{r1} = 1.3860$

State a For isentropic compression, $P_{ra} = (300/100) 1.3860 = 4.158$. Thus $h_{as} = 411.26 \text{ kJ/kg}$. Using the compressor efficiency

$$\eta_{c1} = \frac{h_{as} - h_1}{h_a - h_1} \Rightarrow h_a = h_1 + \frac{(h_{as} - h_1)}{\eta_{c1}} = 432.42 \text{ kJ/kg}$$

State b $h_b = 300.19 \text{ kJ/kg}$, $P_{rb} = 1.3860$

State 2 $P_{r2} = (900/300) 1.3860 = 4.158 \Rightarrow h_{2s} = 411.26 \text{ kJ/kg}$

Thus, with $\eta_{c2} = 0.84$, $h_2 = h_b + \frac{(h_{2s} - h_b)}{\eta_{c2}} = 432.42 \text{ kJ/kg}$

State 3 $T_3 = 1480 \text{ K} \Rightarrow h_3 = 1611.79 \text{ kJ/kg}$, $P_{r3} = 568.8$

State c For isentropic expansion, $P_{rc} = (P_2/P_3) P_{r3} = 189.60 \Rightarrow h_{cs} = 1201.5 \text{ kJ/kg}$

Using the turbine stage efficiency,

$$\eta_{t1} = \frac{h_3 - h_c}{h_3 - h_{cs}} \Rightarrow h_c = h_3 - \eta_{t1} (h_3 - h_{cs}) = 1275.4 \text{ kJ/kg}$$

State d $T_d = 1420 \text{ K} \Rightarrow h_d = 1539.44 \text{ kJ/kg}$, $P_{rd} = 478.0$

State 4 $P_{r4} = (100/300) 478.0 = 159.33 \Rightarrow h_{4s} = 1145.94 \text{ kJ/kg}$

Thus, $h_4 = h_d - \eta_{t2} (h_d - h_{4s}) = 1216.77 \text{ kJ/kg}$

(a) To determine the volumetric flow rate, first find $\dot{m}$ using

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (h_3 - h_c) + (h_d - h_4) - (h_a - h_1) - (h_2 - h_b) = 394.6 \text{ kJ/kg}$$

$$\Rightarrow \dot{m} = \frac{(1800 \text{ kW})}{(394.6 \text{ kJ/kg})} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 4.562 \frac{\text{kg}}{\text{s}}$$

$$\text{Then, } (AV)_1 = \dot{m} \left(\frac{RT_1}{P_1} \right) = (4.562 \frac{\text{kg}}{\text{s}}) \left(\frac{8314}{28.97} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) \left(\frac{300 \text{ K}}{10^5 \text{ N/m}^2} \right) = 3.93 \text{ m}^3/\text{s} \quad \leftarrow (AV)_1$$

Continued on next slide

Problem 9-74 continued

and

$$(AV)_b = \dot{m} \frac{RT_b}{P_b} = (4.562) \left(\frac{8314}{28.97} \right) \left(\frac{300}{3 \times 10^5} \right) = 1.31 \frac{\text{m}^3}{\text{s}} \quad \leftarrow (AV)_b$$

(b) The thermal efficiency is obtained as follows

The total heat added is

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_3 - h_2) + (h_d - h_c) = 1443.41 \text{ kJ/kg}$$

Thus

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{394.6}{1443.41} = 0.2734 \text{ (27.34\%)} \quad \leftarrow \eta$$

(c) The back work ratio is

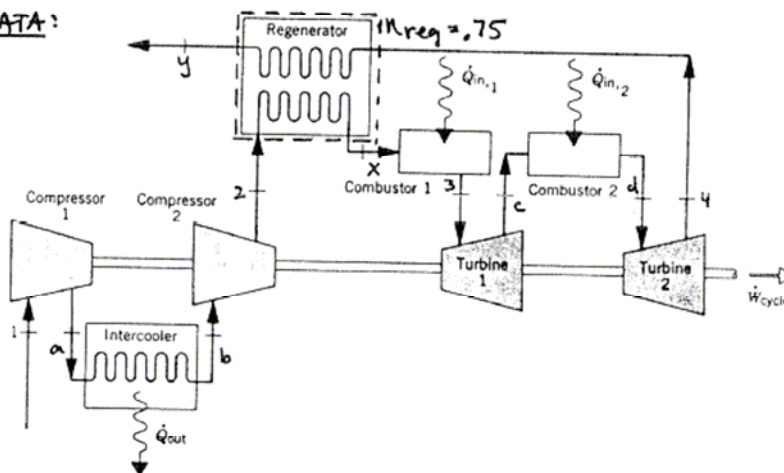
$$\begin{aligned} bwr &= \frac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{(h_a - h_1) + (h_2 - h_6)}{(h_3 - h_c) + (h_d - h_4)} \\ &= \frac{264.46}{659.06} = 0.401 \quad \leftarrow bwr \end{aligned}$$

PROBLEM 9.75

KNOWN: The gas turbine of Problem 9.74 is modified to include a regenerator with effectiveness 75%.

FIND: Answer the same questions for the modified gas turbine as in Problem 9.73.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as in Problem 9.74. Also, there is no pressure drop across the regenerator.

ANALYSIS: From the solution to Problem 9.74

$$\begin{aligned} h_1 = h_b &= 300.19 \text{ kJ/kg} & h_c &= 1275.4 \text{ kJ/kg} \\ h_a &= 432.42 \text{ kJ/kg} & h_d &= 1539.44 \text{ kJ/kg} \\ h_2 &= 432.42 \text{ kJ/kg} & h_4 &= 1216.77 \text{ kJ/kg} \\ h_3 &= 1611.79 \text{ kJ/kg} \end{aligned}$$

State x Using the regenerator effectiveness

$$\eta_{\text{reg}} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{\text{reg}}(h_4 - h_2) + h_2 = 1020.68 \text{ kJ/kg}$$

(a) The net work of the gas turbine is unchanged. Thus $\dot{W}_{\text{cycle}}/\dot{m} = 394.6 \text{ kJ/kg}$

Since the states at 1 and b remain the same, the volumetric flow rates also remain unchanged: $(AV)_1 = 3.93 \text{ m}^3/\text{s}$, $(AV)_b = 1.31 \text{ m}^3/\text{s}$. $\leftarrow (AV)$

(b) However, the heat addition becomes

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = (h_3 - h_x) + (h_d - h_c) = 855.15 \text{ kJ/kg}$$

① Thus $\eta = \frac{394.6}{855.15} = 0.461 \text{ (46.1\%)} \quad \leftarrow \eta$

(c) The back work ratio also is unchanged:

$$\text{bwr} = 0.411 \quad \leftarrow \text{bwr}$$

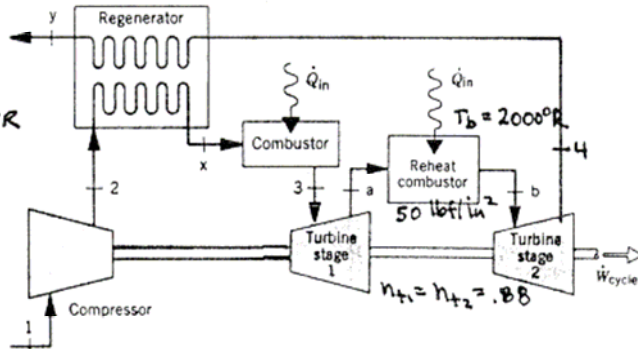
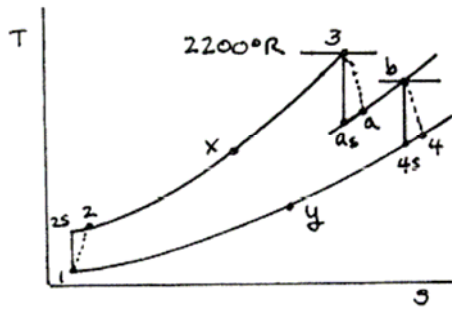
1. As a result of the introduction of the regenerator, there is a significant increase in thermal efficiency.

PROBLEM 9.76

KNOWN: Steady-state operating data are provided in Problem 9.59 (c) for a regenerative gas turbine cycle.

FIND: For each of three modifications of the cycle, determine the thermal efficiency and net power developed.

(a) SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as Example 9.7, except $n_{t1} = n_{t2} = .88$ and $n_c = .84$. Also, reheat occurs with no pressure drop.

ANALYSIS: From the solution of Problem 9.59, $h_1 = 126.66 \text{ Btu/lb}$, $h_2 = 267.16 \text{ Btu/lb}$, $h_3 = 560.59 \text{ Btu/lb}$, $p_{r3} = 256.6$. For turbine stage 1,

$$Pr_{as} = (P_b/P_3) Pr_3 = (50/147)(256.6) = 87.279 \Rightarrow h_{as} = 417.68 \text{ Btu/lb}$$

Thus $n_{t_1} = \frac{h_3 - h_a}{h_3 - h_{a5}} \Rightarrow h_a = h_3 - n_{t_1}(h_3 - h_{a5}) = 434.83 \text{ Btu/lb}$

Similarly, for turbine state 2; $p_{r6} = 174.0$ and $p_{r45} = 51.156 \Rightarrow h_{45} = 360.14 \frac{\text{Btu}}{\text{lb}}$
With $h_6 = 504.71 \text{ Btu/lb}$, $h_4 = h_6 - \eta_{t2}(h_6 - h_{45}) = 377.49 \text{ Btu/lb}$

The specific enthalpy h_x is calculated from

$$n_{\text{reg}} = \frac{h_x - h_z}{h_y - h_z} \Rightarrow h_x = n_{\text{reg}}(h_y - h_z) + h_z = 355.42 \text{ Btu/lb}$$

The thermal efficiency is found from

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_3 - h_x) + (h_b - h_a) = 275.05 \text{ Btu/lb}$$

and

and $\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (h_3 - h_a) + (h_b - h_4) - (h_2 - h_1) = 112.48 \text{ Btu/lb}$

Thus

Thus $\eta = \frac{\dot{W}_{\text{cycle}} / \dot{m}}{\dot{Q}_{\text{in}} / \dot{m}} = 0.409 \text{ (40.9 \%)} \leftarrow \eta$

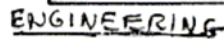
From Problem 9.59, $\dot{m} = 90,000 \text{ lb/h}$. Thus, the net power developed is

$$W_{\text{cycle}} = (90,000 \frac{\text{lb}}{\text{h}})(112.48) \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

$$= 3978 \text{ hp}$$

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(b) SCHEMATIC & GIVEN DATA:



ANALYSIS: States 1, 3, and 4 are the same as in Problem 9.59. Thus

For the first compressor stage, $P_{r_{a3}} = (P_a/P_i) P_{r_1} = (50/14.7)(1.2997) = 4.4207$
and $h_{a3} = 179.90 \text{ Btu/lb}$. Thus, with the compressor stage efficiency

Similarly, for stage 2; $Pr_{2s} = (P_2/P_b) Pr_b = 3.8211 \Rightarrow h_{2s} = 172.58 \text{ Btu/lb}$

For state x use the regenerator effectiveness

To find the thermal efficiency

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_x = 261.6 \text{ Btu/lb}$$

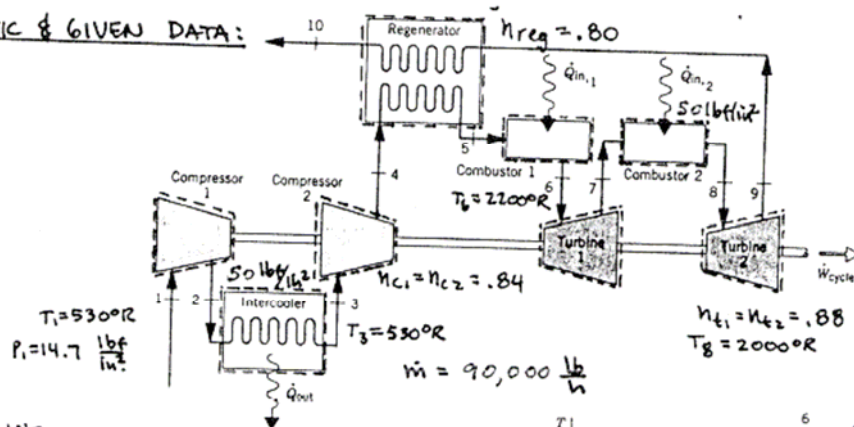
(b) The net power developed is

$$\dot{W}_{\text{cycle}} = \dot{m} \left(\frac{\dot{W}_{\text{cycle}}}{\dot{m}} \right) = (90,000 \frac{\text{lb}}{\text{h}}) (114.14 \frac{\text{Btu}}{\text{lb}}) \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

$$= 40.36 \text{ hp} \leftarrow \dot{W}_{\text{cycle}}$$



(c) SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: See Example 9.11

ANALYSIS: Fix each of the principal states.

 State 1 $T_1 = 530^\circ\text{R} \Rightarrow h_1 = 126.66 \text{ Btu/lb}$, $P_1 = 1.2997$

 State 2 For isentropic compression, $P_{r2} = 4.4207$ and $h_{2s} = 179.90 \text{ Btu/lb}$. With the compressor efficiency

$$\eta_{c1} = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_{c1}} = 190.04 \text{ Btu/lb}$$

 State 3 $T_3 = T_1 \Rightarrow h_3 = 126.66 \text{ Btu/lb}$, $P_{r3} = 1.2997$

 State 4 $P_{r4s} = (P_4/P_3) P_{r3} = 3.8211 \Rightarrow h_{4s} = 172.58 \text{ Btu/lb}$. With the compressor efficiency, $h_4 = h_3 + \frac{(h_{4s} - h_3)}{\eta_{c2}} = 181.33 \text{ Btu/lb}$

 State 6 $T_6 = 2200^\circ\text{R} \Rightarrow h_6 = 560.59 \text{ Btu/lb}$, $P_{r6} = 256.6$. For turbine stage 1 $P_{r7s} = (P_7/P_6) P_{r6} = 87.279 \Rightarrow h_{7s} = 417.68 \text{ Btu/lb}$. With the turbine efficiency

 State 7 $\eta_{t1} = \frac{h_6 - h_7}{h_6 - h_{7s}} \Rightarrow h_7 = h_6 - \eta_{t1} (h_6 - h_{7s}) = 434.83 \text{ Btu/lb}$

 State 8 $T_8 = 2000^\circ\text{R} \Rightarrow h_8 = 504.71 \text{ Btu/lb}$, $P_{r8} = 174.0$

 State 9 $P_{r9s} = 51.156 \Rightarrow h_{9s} = 360.14 \text{ Btu/lb}$
 $h_9 = h_8 - \eta_{t2} (h_8 - h_{9s}) = 377.49 \text{ Btu/lb}$

State 5 For the regenerator

$$\eta_{\text{reg}} = \frac{h_5 - h_4}{h_9 - h_4} \Rightarrow h_5 = \eta_{\text{reg}} (h_9 - h_4) + h_4 = 338.26 \text{ Btu/lb}$$

(a) The net work per unit mass of air flowing is

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (h_6 - h_7) + (h_8 - h_9) - (h_2 - h_1) - (h_4 - h_3) = 134.93 \text{ Btu/lb}$$

$$\text{and } \frac{\dot{Q}_{\text{in}}}{\dot{m}} = (h_6 - h_5) + (h_8 - h_7) = 292.27 \text{ Btu/lb}$$

$$\Rightarrow \eta = \frac{134.93}{292.27} = 0.462 \quad (46.2\%)$$

(b) The net power developed is

$$\dot{W}_{\text{cycle}} = \left(90,000 \frac{\text{lb}}{\text{h}} \right) \left(134.93 \frac{\text{Btu}}{\text{lb}} \right) \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

$$= 4772 \text{ hp}$$

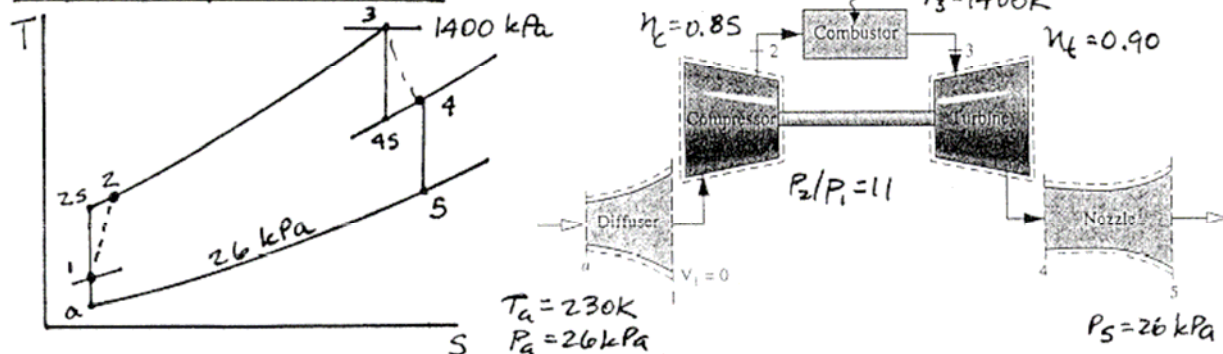
 $\dot{W}_{\text{cycle}}$

PROBLEM 9.77

KNOWN: A turbojet engine is analyzed on an air-standard basis. Data are known at various locations and the mass flow rate is specified.

FIND: Determine (a) the pressures and temperatures at each principal state, (b) the rate of heat addition, and (c) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as in Example 9.12, except $\eta_c = 0.85$ and $\eta_t = 0.90$.

ANALYSIS: (a) Fix each of the principal states.

State a: $T_a = 230 \text{ K}$, $P_a = 26 \text{ kPa} \Rightarrow h_a = 230.02 \text{ kJ/kg}$, $P_{r,a} = 0.5477$

State 1: From an energy balance for the diffuser: $0 = h_a + V_a^2/2 - h_1$. Thus

$$h_1 = h_a + \frac{V_a^2}{2} = 230.02 + \frac{220^2 \text{ m}^2/\text{s}^2}{2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 254.22 \frac{\text{kJ}}{\text{kg}}$$

For the isentropic process from a to 1: $P_1 = (P_{r,1}/P_{r,a}) P_a = \left(\frac{0.77759}{0.5477} \right) (26 \text{ kPa})$

State 2: $P_{r,2} = P_{r,1} (P_2/P_1) = 8.5535 \Rightarrow h_{2s} = 505.39 \frac{\text{kJ}}{\text{kg}} = 36.91 \text{ kPa}$

With the isentropic compressor efficiency

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 549.71 \text{ kJ/kg} \Rightarrow T_2 = 549.71 \text{ K}$$

State 3: $T_3 = 1400 \text{ K}$; $h_3 = 1515.42 \frac{\text{kJ}}{\text{kg}}$, $P_{r,3} = 450.5$ $P_2 = 11 (P_1) = 406.01 \text{ kPa}$

State 4: For a turbojet: $\dot{W}_c = \dot{W}_t \Rightarrow (h_2 - h_1) = (h_3 - h_4)$, Thus

$$h_4 = h_3 - (h_2 - h_1) = 1219.93 \text{ K}; T_4 = 1150.6 \text{ K}, P_{r,4} = 200.564$$

From the isentropic turbine efficiency; $\eta_t = (h_3 - h_4)/(h_3 - h_{4s})$

$$h_{4s} = h_3 - \frac{(h_3 - h_4)}{\eta_t} = 1187.10 \text{ kJ/kg}; P_{r,4s} = 181.32$$

$$\text{Thus } P_4 = P_3 \left(\frac{P_{r,4s}}{P_{r,3}} \right) = (406.01) \left(\frac{181.32}{450.5} \right) = 163.4 \text{ kPa}$$

State 5: $P_{r,5} = P_{r,4} \left(\frac{P_5}{P_4} \right) = (200.564) \left(\frac{26}{163.4} \right) = 31.913$

$$\text{Thus } h_5 = 734.06, T_5 = 719.3 \text{ K}$$

Continued on next slide

Problem 9-77 continued

(a) Summary

State	p (kPa)	T (K)	h (kJ/kg)
1	36.91	254.15	254.22
2	406.01	545.2	549.71
3	406.01	1400	1515.42
4	163.4	1129.8	1219.93
5	26	719.3	734.06

P, T

(b) $\dot{Q}_{in} = \dot{m} (h_3 - h_2)$

$= (25 \text{ kg/s}) (1515.42 - 549.71) \text{ kJ/kg} = 2.414 \times 10^4 \text{ kJ/s}$ $\leftarrow \dot{Q}_{in}$

(c) For the nozzle

$0 = (h_4 - h_5) + \left(\frac{V_4^2 - V_5^2}{2} \right)$

Thus

$V_5 = \sqrt{2(h_4 - h_5)}$

$= \sqrt{2(1219.93 - 734.06) \frac{\text{kJ}}{\text{kg}} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|}$

$= 985.8 \text{ m/s}$ $\leftarrow V_5$

PROBLEM 9.78

See Problem 9.77.

IT Code

pa = 26 // kPa

Ta = 230 // K

Va = 220 // m/s

rp = 11

T3 = 1400 // K

p5 = 26 // kPa

eta_t = 0.90

eta_c = 0.85

mdot = 25 // kg/s

ha = h_T("Air", Ta)

sa = s_TP("Air", Ta, pa)

h1 = ha + (Va^2 / 2) * (1 / 1000)

s1 = s_hP("Air", h1, p1)

s1 = sa

h1 = h_T("Air", T1)

p2 = p1 * rp

s2s = s_hP("Air", h2s, p2)

s2s = s1

h2 = h1 + (h2s - h1) / eta_c

h2 = h_T("Air", T2)

p3 = p2

h3 = h_T("Air", T3)

s3 = s_hP("Air", h3, p3)

s4s = s3

h2 - h1 = eta_t * (h3 - h4s)

s4s = s_hP("Air", h4s, p4)

h4 = h3 - (h3 - h4s) * eta_t

h4 = h_T("Air", T4)

s4 = s_hP("Air", h4, p4)

s5 = s4

s5 = s_hP("Air", h5, p5)

h5 = h_T("Air", T5)

h4 = h5 + (V5^2 / 2) * (1 / 1000)

Qdotin = mdot * (h3 - h2)

IT for T3 = 1400 K, rp = 11

Ta = 230 K

T1 = 254.1 K

T2 = 544.8 K

T4 = 1150 K

T5 = 719.3 K

ha = 229.8 kJ/kg

h1 = 254 kJ/kg

h2 = 549.2 kJ/kg

h2s = 504.9 kJ/kg

h3 = 1514 kJ/kg

h4 = 1219 kJ/kg

h4s = 1186 kJ/kg

h5 = 733.8 kJ/kg

pa = 26 kPa

p2 = 405.3 kPa

p3 = 405.3 kPa

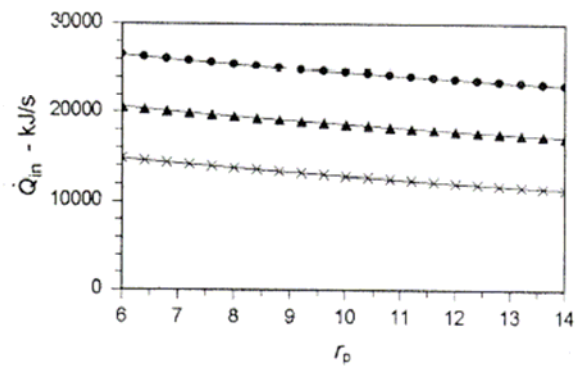
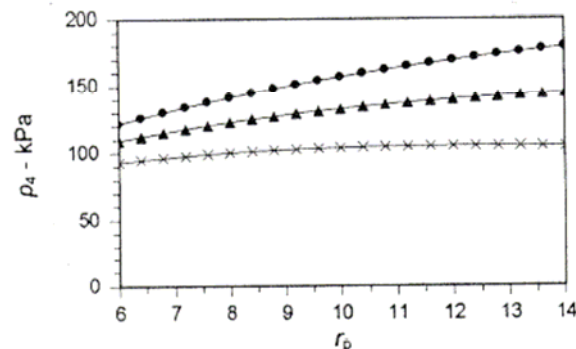
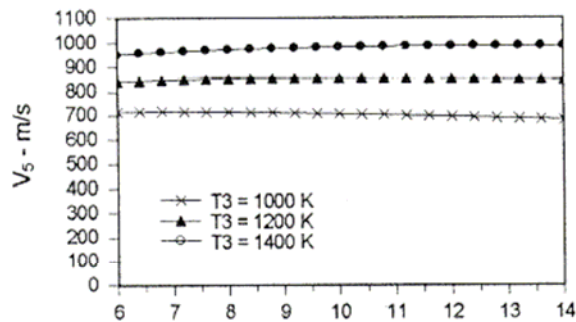
p4 = 163.2 kPa

p5 = 26 kPa

Qdotin = 2.412 x 10^4 KJ/s

V5 = 985 m/s

PLOTS:

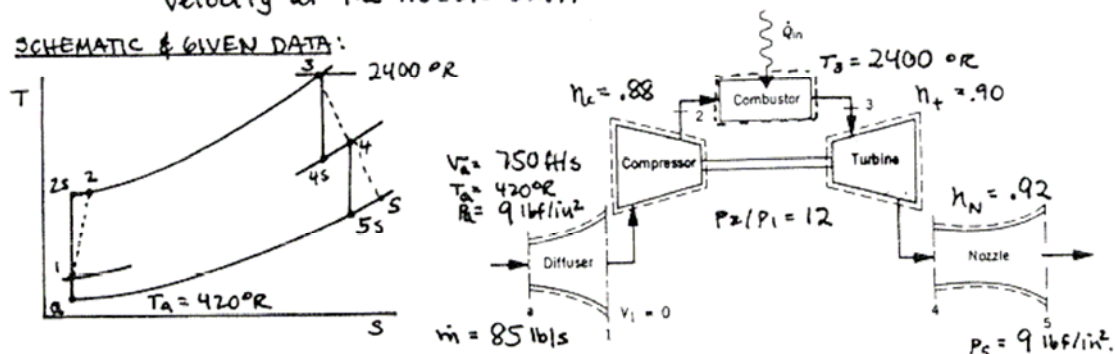


PROBLEM 9.79

KNOWN: A turbojet engine is analyzed on an air-standard basis. Data are known at various locations.

FIND: Determine (a) the rate of heat addition, (b) the pressure at the turbine exit, (c) the compressor power input, (d) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.12, except $\eta_c = 0.88$ and $\eta_t = 0.90$.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State a $T_a = 420^\circ\text{R} \Rightarrow h_a = 100.32 \text{ Btu/lb}$, $P_a = 0.5760$

State 1 For the diffuser, $0 = (h_a + V_a^2/2) - h_1$. Thus

$$h_1 = 100.32 \frac{\text{Btu}}{\text{lb}} + \frac{750^2 \text{ ft}^2/\text{s}^2}{2} \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft}/\text{s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 111.55 \frac{\text{Btu}}{\text{lb}}$$

$$P_1 = \left(\frac{P_1}{P_a} \right) P_a = \left(\frac{0.83492}{0.5760} \right) 9 = 13.05 \text{ lbf/in}^2$$

State 2 For isentropic compression, $P_{r2} = P_1 (P_2/P_1) = 10.019$ and $h_{2s} = 227.29 \text{ Btu/lb}$. With the compressor efficiency

$$h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c} \right) = 243.07 \text{ Btu/lb}, \quad P_2 = 12 P_1 = 156.6 \text{ lbf/in}^2$$

State 3 $T_3 = 2400^\circ\text{R} \Rightarrow h_3 = 617.22 \text{ Btu/lb}$, $P_{r3} = 367.6$

State 4 For a turbojet, $\dot{W}_c/\dot{m} = \dot{W}_t/\dot{m}$. Thus $h_2 - h_1 = \eta_t (h_3 - h_4)$.

$$h_{4s} = h_3 - (h_2 - h_1)/\eta_t = 471.09 \text{ Btu/lb} \Rightarrow P_{r4s} = 135.24$$

$$\text{Thus, } p_4 = (P_{r4s}/P_{r3}) P_3 = 57.61 \text{ lbf/in}^2 \quad (b)$$

Using the turbine efficiency, $h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 485.70 \text{ Btu/lb}$

$$\text{State 5s } P_{r5} = P_{r4} (P_5/P_4) = (151.24) (9/57.61) = 23.627 \Rightarrow h_{5s} = 289.93 \frac{\text{Btu}}{\text{lb}}$$

(a) The rate of heat addition is

$$\dot{Q}_{in} = \dot{m} (h_3 - h_2) = (85 \text{ lb/s}) (617.22 - 243.07) \text{ Btu/lb} \left| \frac{3600 \text{ s}}{1 \text{ hr}} \right| = 1.145 \times 10^8 \frac{\text{Btu}}{\text{hr}} \quad (a)$$

(c) The compressor power is $\dot{W}_c = \dot{m} (h_2 - h_1) = 85 (243.07 - 111.55) \text{ Btu/lb} \left| \frac{3600 \text{ s}}{1 \text{ hr}} \right| = 4.025 \times 10^7 \frac{\text{Btu}}{\text{hr}} \quad (c)$

(d) An energy rate balance for an isentropic expansion gives

$$(V_5^2)_s = 2(h_4 - h_{5s}) = 2(485.70 - 289.93) \frac{\text{Btu}}{\text{lb}} \left| \frac{778 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}} \right| \left| \frac{32.2 \text{ lb} \cdot \text{ft}/\text{s}^2}{1 \text{ lbf}} \right| = 9.809 \times 10^6 \frac{\text{ft}^2}{\text{s}^2}$$

Then, with the nozzle efficiency

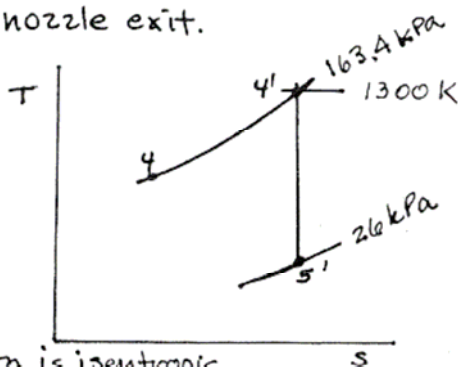
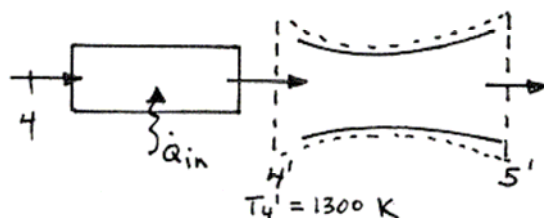
$$V_5 = (V_5)_s \sqrt{\eta_N} = 3004.0 \text{ ft/s} \quad (d)$$

PROBLEM 9.80

KNOWN: An afterburner is added to the turbojet in Problem 9.77.

FIND: Determine the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The nozzle expansion is isentropic.

ANALYSIS: For the isentropic process

$$Pr_{5'} = (P_5/P_4) Pr_{4'} = (26/163.4)(330.9) = 52.652$$

$$h_{5'} = 844.25 \text{ kJ/kg}$$

Thus, the nozzle exit velocity is

$$V_5 = \sqrt{2(h_{4'} - h_{5'})}$$

$$\textcircled{1} \quad = \sqrt{2(1395.97 - 844.25) \frac{\text{kJ}}{\text{kg}} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|} = 1050 \text{ m/s} \leftarrow$$

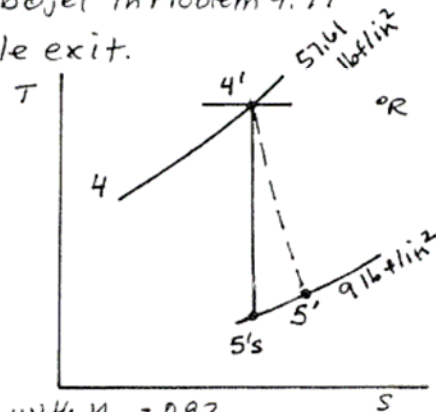
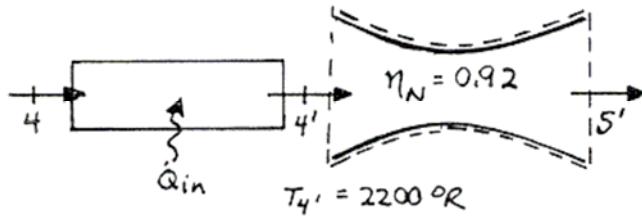
1. Comparing this result with that of Problem 9.77, we see that the use of an afterburner increases the exit velocity, and thus the engine thrust.

PROBLEM 9.81

KNOWN: An afterburner is added to the turbojet in Problem 9.79

FIND: Determine the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Nozzle operation is adiabatic, with $\eta_N = 0.92$.

ANALYSIS: For ideal expansion through the nozzle

$$Pr_{5s'} = (P_5/P_4) Pr_{4'}$$

From Table A-22E; $Pr_{4'} = 256.6$. Thus

$$Pr_{5s'} = 40.087 \Rightarrow h_{5s'} = 336.39 \text{ Btu/lb}$$

In the ideal case

$$0 = h_{4'} - \left(h_{5s'} + \left(\frac{V_s^2}{2} \right)_s \right)$$

$$\left(\frac{V_s^2}{2} \right)_s = h_{4'} - h_{5s'}$$

$$\begin{aligned} \text{or } (V_s)_s &= \sqrt{2(h_{4'} - h_{5s'})} \\ &= \sqrt{2(560.59 - 336.39) \frac{\text{Btu}}{\text{lb}} \left| \frac{778 \text{ ft} \cdot \text{lb}}{1 \text{ Btu}} \right| \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lb}} \right|} \\ &= 3352 \text{ ft/s} \end{aligned}$$

Using the nozzle efficiency

$$\frac{V_s^2}{2} = \eta_N \left(\frac{V_s^2}{2} \right)_s$$

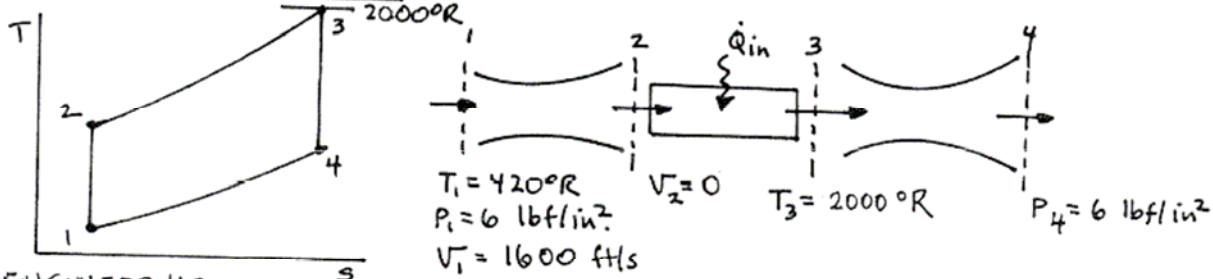
$$V_s = \sqrt{\eta_N} (V_s)_s = 3215 \text{ ft/s} \quad \leftarrow V_s$$

PROBLEM 9.82

KNOWN: Air enters the diffuser of a ramjet with known conditions. The temperature after combustion is specified.

FIND: Using an air-standard analysis, determine (a) the pressure at the diffuser exit and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The combustion process is modeled as a constant pressure heat addition. (3) The diffuser and nozzle operate isentropically. (4) Neglect kinetic energy at locations 2 and 3. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: (a) For the diffuser; $0 = (h_1 + \frac{V_1^2}{2}) - h_2$. Thus,

$$h_2 = h_1 + \frac{V_1^2}{2} = 100.32 + \frac{1600^2 \text{ ft}^2/\text{s}^2}{2} \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft}/\text{s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= 151.41 \text{ Btu/lb}$$

Thus, $Pr_2 = 2.4214$ and $P_2 = (Pr_2/Pr_1) P_1 = 25.22 \text{ lbf/in}^2$ ← P_2

(b) At $T_3 = 2000^\circ\text{R}$, Table A-22E gives $h_3 = 504.71 \text{ Btu/lb}$, $Pr_3 = 174$. For an isentropic expansion through the nozzle

$$Pr_4 = Pr_3 (P_4/P_3) = (174) (6/25.22) = 41.4$$

$$h_4 = 339.45 \text{ Btu/lb}$$

The velocity is found from

$$0 = h_3 - (h_4 + \frac{V_4^2}{2})$$

or $V_4 = \sqrt{2(h_3 - h_4)}$

$$= \sqrt{2(504.71 - 339.45) \frac{\text{Btu}}{\text{lb}} \left| \frac{32.2 \text{ lb} \cdot \text{ft}/\text{s}^2}{1 \text{ lbf}} \right| \left| \frac{778 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}} \right|}$$

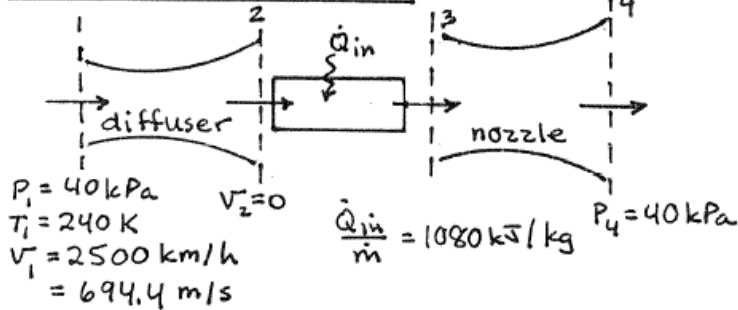
$$= 2878 \text{ ft/s} \leftarrow V_4$$

PROBLEM 9.83

KNOWN: Air enters the diffuser of a ramjet with known conditions. The heat addition per unit mass of air flowing is specified.

FIND: Using air-standard analysis, determine (a) the pressure at the diffuser exit and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The combustion process is modeled as a constant pressure heat addition. (3) The diffuser and nozzle operate isentropically. (4) Neglect kinetic energy at locations 2 & 3 and neglect potential energy throughout. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: (a) For the diffuser; $0 = (h_1 + \frac{V_1^2}{2}) - h_2$

$$\text{Thus } h_2 = h_1 + \frac{V_1^2}{2} = 240.02 \frac{\text{kJ}}{\text{kg}} + \frac{(694.4)^2 \text{ m}^2}{2 \text{ s}^2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 481.1 \text{ kJ/kg}$$

Interpolating in Table A-22; $T_2 \approx 479 \text{ K}$ and $P_{r2} = 7.197$

Therefore

$$P_2 = \left(\frac{P_{r2}}{P_{r1}} \right) P_1 = \left(\frac{7.197}{0.6355} \right) (40 \text{ kPa}) = 453.0 \text{ kPa} \leftarrow P_2$$

(b) State 3 is fixed by considering the heat addition process

$$0 = \dot{Q}_{in} + \dot{m} (h_2 - h_3)$$

$$\text{or } h_3 = \frac{\dot{Q}_{in}}{\dot{m}} + h_2 = 1080 + 481.1 = 1561.1 \text{ kJ/kg}$$

Again, from Table A-22; $T_3 \approx 1438 \text{ K}$, $P_{r3} = 504.0$

For isentropic expansion through the nozzle

$$P_{r4} = (P_4/P_3) P_{r3} = (40/453.0)(504.0) = 44.50 \Rightarrow h_4 = 805.78 \frac{\text{kJ}}{\text{kg}}$$

The velocity is obtained from; $0 = h_3 - (h_4 + \frac{V_4^2}{2})$

$$\text{or } V_4 = \sqrt{2(h_3 - h_4)}$$

$$= \sqrt{2(1561.1 - 805.78) \frac{\text{kJ}}{\text{kg}} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|}$$

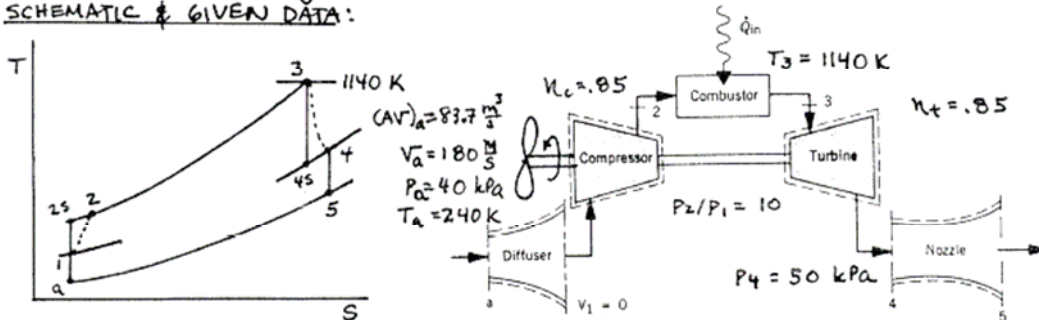
$$= 1229 \text{ m/s} \leftarrow V_4$$

PROBLEM 9.84

KNOWN: A turboprop engine is analyzed on an air-standard basis. Data are known at various locations.

FIND: Determine (a) the power delivered to the propeller, and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as in Example 9.12, except $\eta_c = \eta_t = 0.85$.

ANALYSIS: First, fix each of the principal states (Table A-22).

State a $T_a = 240 \text{ K} \Rightarrow h_a = 240.02 \text{ kJ/kg}$, $P_a = 0.6355$

State 1 An energy balance for the diffuser gives; $0 = (h_2 + \frac{V_2^2}{2}) - h_1$. Thus

$$h_1 = h_a + \frac{V_a^2}{2} = 240.02 \text{ kJ/kg} + \frac{180^2 \text{ m}^2/\text{s}^2}{2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 256.22 \text{ kJ/kg} \Rightarrow P_1 = 0.79902$$

$$P_1 = (P_1/P_a) P_a = 50.29 \text{ kPa}$$

State 2 For an isentropic compression, $P_{2s} = (P_2/P_1) P_1 = 7.9902 \Rightarrow h_{2s} = 495.65 \frac{\text{kJ}}{\text{kg}}$
 Using the compressor efficiency, $h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 537.9 \text{ kJ/kg}$

State 3 $T_3 = 1140 \Rightarrow h_3 = 1207.57 \text{ kJ/kg}$, $P_3 = 193.1$

State 4 For isentropic expansion, $P_{4s} = (P_4/P_3) P_3 = \left(\frac{50}{193.1} \right) 193.1 = 25.80$.
 Thus, $h_{4s} = 634.06 \text{ kJ/kg}$. Using the turbine efficiency,
 $h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 720.09 \text{ kJ/kg}$, $P_4 = 29.80$

State 5 $P_{5s} = (P_5/P_4) P_4 = 23.84 \Rightarrow h_5 = 676.17 \text{ kJ/kg}$

(a) The power delivered to the propeller is

$$\dot{W}_p = \dot{W}_t - \dot{W}_c = \dot{m} [(h_3 - h_4) - (h_2 - h_1)]$$

where

$$\dot{m} = \frac{(AV)_a P_a}{RT_a} = \frac{(83.7 \text{ m}^3/\text{s})(4 \times 10^4 \text{ N/m}^2)}{(8314 \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}})(240 \text{ K})} = 48.6 \frac{\text{kg}}{\text{s}}$$

$$\Rightarrow \dot{W}_p = (48.6 \frac{\text{kg}}{\text{s}}) [(1207.57 - 720.09) - (537.9 - 256.22)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| = 10 \text{ MW} \leftarrow \dot{W}_p$$

(b) An energy balance for the nozzle gives

$$V_5 = \sqrt{2(h_4 - h_5)}$$

$$= \sqrt{2(720.09 - 676.17) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right|}$$

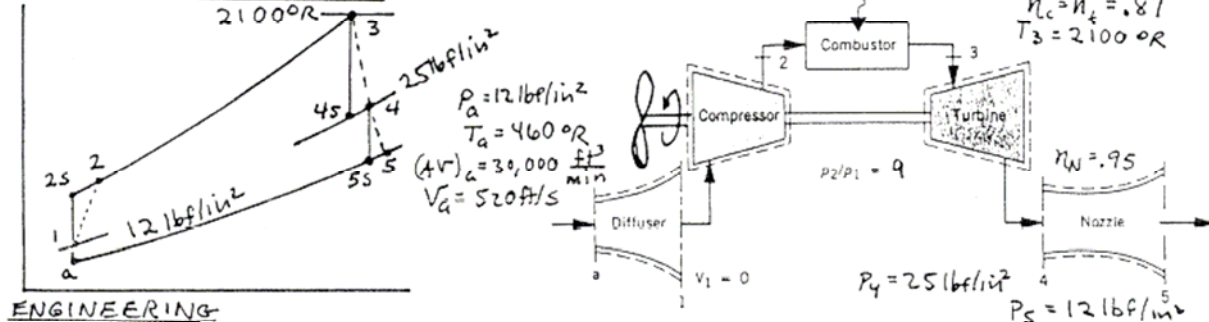
$$= 296.4 \text{ m/s} \leftarrow V_5$$

PROBLEM 9.85

KNOWN: A turboprop engine is analyzed on an air standard basis. Data are known at various locations.

FIND: Determine (a) the power delivered to the propeller, and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The diffuser process is isentropic and the turbine, compressor, and nozzle operate adiabatically. (3) Except at the inlet and exit of the engine, kinetic energy effects can be neglected. Potential energy effects are negligible throughout. (4) The working fluid is air modeled as an ideal gas.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State a: $T_a = 460^\circ\text{R} \Rightarrow h_a = 109.90 \text{ Btu/lb}$, $P_{r_a} = 0.7913$

State 1: An energy balance for the diffuser gives

$$h_1 = h_a + \frac{V_a^2}{2} = 109.90 \frac{\text{Btu}}{\text{lb}} + \left(\frac{520^2 \text{ ft}^2/\text{s}^2}{2} \right) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft}/\text{s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= 115.30 \text{ Btu/lb} \Rightarrow P_{r_1} = 0.93604$$

$$P_1 = (P_{r_1}/P_{r_a}) P_a = 14.195 \text{ lbf/in}^2$$

State 2: For ideal compression; $P_{r_2s} = (P_2/P_1) P_{r_1} = 8.42436 \Rightarrow h_{2s} = 216.36 \frac{\text{Btu}}{\text{lb}}$
Using the compressor efficiency

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 231.46 \text{ Btu/lb}$$

State 3: $T_3 = 2100^\circ\text{R} \Rightarrow h_3 = 532.55 \text{ Btu/lb}$, $P_{r_3} = 212.1$

State 4: For ideal expansion; $P_{r_4s} = (P_4/P_3) P_{r_3} = 41.505 \Rightarrow h_{4s} = 339.7 \text{ Btu/lb}$
Using the turbine efficiency

$$h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 364.77 \text{ Btu/lb} \Rightarrow P_{r_4} = 53.515$$

State 5s: $P_{r_5s} = (P_5/P_4) P_{r_4} = 25.687 \Rightarrow h_{5s} = 296.79 \text{ Btu/lb}$

(a) The power delivered to the propeller is

$$\dot{W}_p = \dot{W}_t - \dot{W}_c = \dot{m} [(h_3 - h_4) - (h_2 - h_1)]$$

Evaluating the mass flow rate

$$\dot{m} = \frac{(AV)_a P_a}{RT_a} = \frac{(30,000 \text{ ft}^3/\text{min})(12 \text{ lbf/in}^2)}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right)(460^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 2113.1 \text{ lb/min}$$

Continued on next slide

Problem 9-85 continued

Thus, the power is

$$\dot{W}_P = \left(2113.1 \frac{\text{lb}}{\text{min}}\right) \left(\frac{60 \text{ min}}{\text{h}}\right) [(532.55 - 364.77) - (231.46 - 115.30)] \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$
$$= 2571.6 \text{ hp} \longleftarrow \dot{W}_P$$

(b) For an ideal expansion through the nozzle

$$0 = h_4 - \left(h_{5s} + \frac{V_{5s}^2}{2}\right)$$

$$\text{or } V_{5s} = \sqrt{2(h_4 - h_{5s})}$$
$$= \sqrt{2(364.77 - 296.79) [32.2] [778]} = 1845.5 \text{ ft/s}$$

From the definition of nozzle efficiency

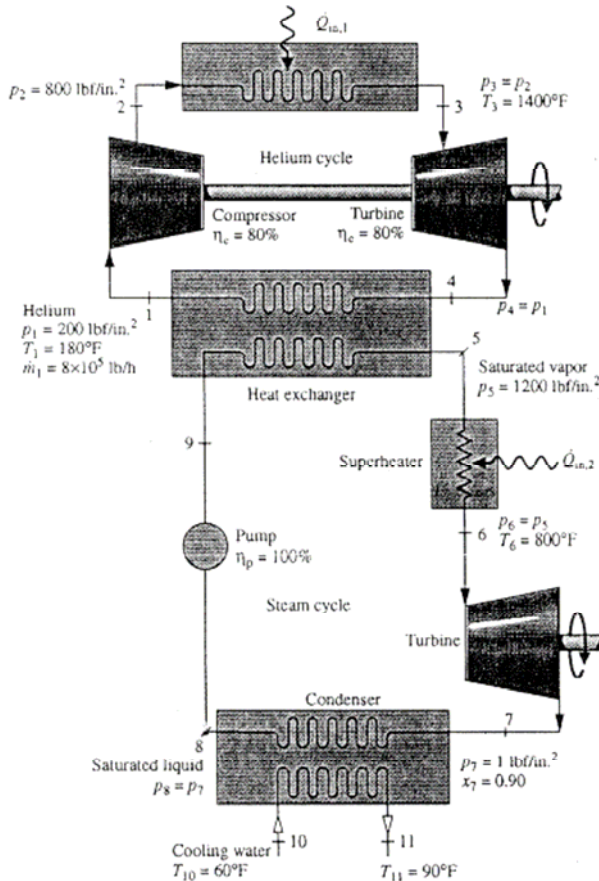
$$V_5 = \sqrt{\eta_N} V_{5s} = \sqrt{0.95} (1845.5)$$
$$= 1798.8 \text{ ft/s} \longleftarrow V_5$$

PROBLEM 9.86

KNOWN: Steady-state operating data are provided for a combined cycle power plant using helium and water as the working fluids.

FIND: Determine (a) the mass flow rates of the steam and cooling water, (b) the net power developed by the gas turbine and vapor cycles, (c) the thermal efficiency of the combined cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. Control volumes enclosing the principal components are at steady state.
2. There are no stray heat transfers, and kinetic and potential energy effects can be ignored.
3. Helium is modeled as an ideal gas with constant $c_p = 5/2 R$, $k = 1.67$ (Table A-2E).

ANALYSIS: Begin by evaluating data at principal states.

Helium: Using Eq. 6.45

$$T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 640 \text{ K} \left(\frac{800}{200} \right)^{\frac{1.67-1}{1.67}} = 1116.16^\circ \text{R}$$

$$T_{4s} = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = 1860 \left(\frac{200}{800} \right)^{\frac{1.67-1}{1.67}} = 1066.51^\circ \text{R}$$

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{c_p(T_3 - T_4)}{c_p(T_3 - T_{4s})}$$

$$\Rightarrow T_4 = T_3 - \eta_t(T_3 - T_{4s}) = 1860 - 0.8(1860 - 1066.51) = 1225.2^\circ \text{R}$$

$$\eta_c = \frac{h_2 - h_1}{h_2 - h_{1s}} = \frac{c_p(T_2 - T_1)}{c_p(T_2 - T_{1s})}$$

$$\Rightarrow T_2 = T_1 + \frac{(T_{2s} - T_1)}{\eta_c} = 640 + \frac{(1116.16 - 640)}{0.8} = 1235.2^\circ \text{R}$$

Steam: Table A-3E, $h_5 = 1183.9 \text{ Btu/lb}$. Table A-4E, $h_6 = 1378.4 \text{ Btu/lb}$. Table A-3E, $h_8 = 69.74 \text{ Btu/lb}$. $h_7 = h_f + x_7 h_{fg} = 69.74 + 0.9(1036) = 1002.14 \text{ Btu/lb}$.

Also

$$h_9 \approx h_8 + v_8 \Delta p = 69.74 \frac{\text{Btu}}{\text{lb}} + (0.01614 \frac{\text{ft}^3}{\text{lb}}) \left(\frac{1199 \times 144 \frac{\text{lbf}}{\text{ft}^2}}{\frac{\text{ft}^2}{\text{lb}} \right) \left(\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right) = 73.32 \text{ Btu/lb}$$

Cooling Water: $h_{10} \approx h_f(T_{10}) = 28.08 \text{ Btu/lb}$, $h_{11} \approx h_f(T_{11}) = 58.07 \text{ Btu/lb}$

- (a) Evaluate the mass flow rate of the steam by study of a control volume enclosing the heat exchanger. Mass and energy rate balances reduce to give

$$\dot{m}_S = \frac{\dot{m}_g [h_4 - h_1]}{[h_5 - h_9]} = \frac{\dot{m}_g c_p (T_4 - T_1)}{[h_5 - h_9]}$$

Continued on next slide

Problem 9-86 continued

Inserting values, $c_p = 5/2 R = \frac{5}{2} \left(\frac{1.986 \text{ Btu}}{4.003 \text{ lb} \cdot \text{°R}} \right) = 1.24 \text{ Btu/lb} \cdot \text{°R}$

$$\dot{m}_s = \frac{(8 \times 10^5 \text{ lb/h})(1.24 \text{ Btu/lb} \cdot \text{°R})[1225.2 - 640]}{[1183.9 - 73.32]} = 5.23 \times 10^5 \frac{\text{lb}}{\text{h}} \quad \leftarrow (a)$$

For a control volume enclosing the condenser, mass and energy rate balances reduce to give

$$\dot{m}_{cw} = \frac{\dot{m}_s (h_7 - h_8)}{h_{11} - h_{10}} = \frac{(5.23 \times 10^5)[1002.14 - 69.74]}{(58.07 - 28.08)} = 1.63 \times 10^7 \frac{\text{lb}}{\text{h}} \quad \leftarrow (a)$$

(b) The net power developed by the gas turbine is

$$\begin{aligned} \dot{W}_g &= \dot{W}_t - \dot{W}_c = \dot{m}_g [(h_3 - h_4) - (h_2 - h_1)] = \dot{m}_g c_p [(T_3 - T_4) - (T_2 - T_1)] \\ &= (8 \times 10^5 \text{ lb/h}) (1.24 \frac{\text{Btu}}{\text{lb} \cdot \text{°R}}) [(1860 - 1225.2) - (1235.2 - 640)] \text{°R} = 3.93 \times 10^7 \frac{\text{Btu}}{\text{h}} \quad \leftarrow (b) \end{aligned}$$

The net power developed by the vapor cycle is

$$\begin{aligned} \dot{W}_s &= \dot{W}_t - \dot{W}_p = \dot{m}_s [(h_6 - h_7) - (h_9 - h_8)] = (5.23 \times 10^5) [(1378.4 - 1002.14) - (73.32 - 69.74)] \\ &= 19.49 \times 10^7 \text{ Btu/h} \quad \leftarrow (b) \end{aligned}$$

(c) The thermal efficiency for the combined cycle is

$$\textcircled{1} \quad \eta = \frac{\dot{W}_g + \dot{W}_s}{\dot{Q}_{in}} = \frac{\dot{W}_g + \dot{W}_s}{\dot{Q}_{in,1} + \dot{Q}_{in,2}}$$

where

$$\begin{aligned} \dot{Q}_{in,1} &= \dot{m}_g (h_3 - h_2) = \dot{m}_g c_p (T_3 - T_2) = (8 \times 10^5 \frac{\text{lb}}{\text{h}}) (1.24 \frac{\text{Btu}}{\text{lb} \cdot \text{°R}}) (1860 - 1235.2) \text{°R} \\ &= 61.98 \times 10^7 \text{ Btu/h} \end{aligned}$$

$$\dot{Q}_{in,2} = \dot{m}_s (h_6 - h_5) = (5.23 \times 10^5) (1378.4 - 1183.9) = 10.17 \times 10^7 \text{ Btu/h}$$

Thus

$$\eta = \frac{(3.93 + 19.49) \times 10^7}{(61.98 + 10.17) \times 10^7} = 0.324 \quad (32.4\%) \quad \leftarrow (c)$$

1. An energy balance for the overall combined cycle gives

$$\dot{W}_g + \dot{W}_s = \dot{Q}_{in} - \dot{m}_{cw} [h_{11} - h_{10}]$$

Thus,

$$\eta = 1 - \frac{\dot{m}_{cw} [h_{11} - h_{10}]}{\dot{Q}_{in}} = 1 - \frac{(48.76 \times 10^7)}{(72.15 \times 10^7)} = 0.324$$

PROBLEM 9.87

KNOWN: Data are known at principal states of a combined gas turbine-vapor power plant. Use air standard analysis for the gas turbine.

FIND: Determine (a) the net power, and (b) the overall thermal efficiency. Develop a full accounting of the net rate of exergy transfer to the air passing through the gas turbine combustor.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: (1) Each component is a control volume at steady state. (2) The compressor, turbines, and pump operate adiabatically. (3) Kinetic and potential energy effects are negligible. (4) The air is modeled as an ideal gas. (5) There are no stray heat losses.

ANALYSIS: First, fix all of the principal states. For the air, use Table A-22.

$$T_1 = 300 \text{ K}; h_1 = 300.19 \frac{\text{kJ}}{\text{kg}}, s_1^0 = 1.70203$$

$$T_2 = 690 \text{ K}; h_2 = 702.52, s_2^0 = 2.55731$$

$$T_3 = 1580 \text{ K}; h_3 = 1733.17, s_3^0 = 3.50829$$

$$T_4 = 900 \text{ K}; h_4 = 932.93, s_4^0 = 2.84856$$

$$T_5 = 400 \text{ K}; h_5 = 400.98, s_5^0 = 1.99194$$

For the vapor cycle

$$\text{State 7: } p_7 = 100 \text{ bar}, T_7 = 520^\circ\text{C}$$

$$\Rightarrow h_7 = 3425.1 \text{ kJ/kg}, s_7 = 6.6622$$

$$\text{State 8: } p_8 = 0.08 \text{ bar}, s_{8s} = s_7 \Rightarrow x_{8s} = \frac{6.6622 - 0.5926}{8.2287 - 0.5926} = 0.7949; h_{8s} = 2084.1 \text{ kJ/kg}$$

$$h_8 = h_7 - \eta_t(h_7 - h_{8s}) = 2285.3 \text{ kJ/kg}; x_8 = 0.8786, s_8 = 7.3019 \text{ kJ/kg}\cdot\text{K}$$

$$\text{State 9: } p_9 = 0.08 \text{ bar, sat. liquid} \Rightarrow h_9 = 173.88 \text{ kJ/kg}, s_9 = 0.5926 \text{ kJ/kg}\cdot\text{K}$$

$$\text{State 6: } h_{6s} \approx h_9 + v_9(p_6 - p_9) = 173.88 \frac{\text{kJ}}{\text{kg}} + (1.0084 \times 10^{-3} \frac{\text{m}^3}{\text{kg}})(100 - 0.08) \text{ bar} \left(\frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right) = 183.40 \text{ kJ/kg}$$

$$\text{with } \eta_p; h_6 = h_9 + (h_{6s} - h_9)/\eta_p = 185.78 \text{ kJ/kg}$$

$$\text{Interpolating in Table A-5: } s_6 = 0.5968$$

(a) Begin by determining the mass flow rate of air, as follows.

$$\dot{W}_{\text{gas}} = \dot{m}_{\text{air}} [(h_3 - h_4) - (h_2 - h_1)]$$

$$\text{or } \dot{m}_{\text{air}} = \frac{\dot{W}_{\text{gas}}}{(h_3 - h_4) - (h_2 - h_1)} = \frac{(147000 \text{ kJ/s})}{[(1733.17 - 932.93) - (702.52 - 300.19)] \frac{\text{kJ}}{\text{kg}}} = 369.43 \text{ kg/s}$$

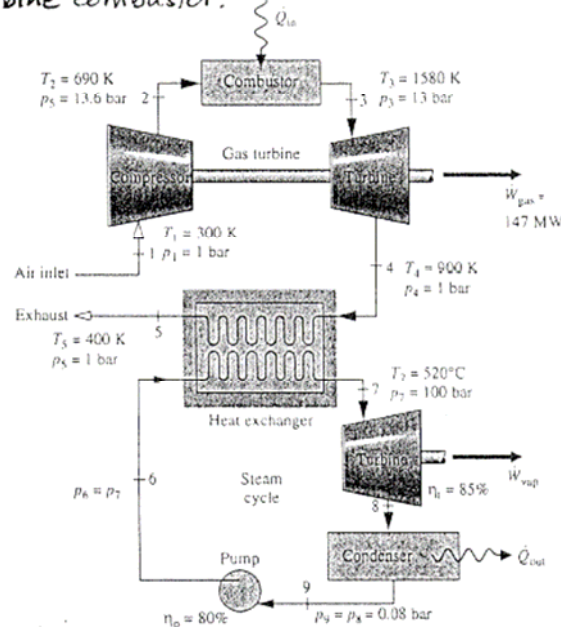
Energy and mass balances for the inter-connecting heat exchanger give

$$0 = \dot{m}_{\text{air}}(h_4 - h_5) + \dot{m}_{\text{st}}(h_6 - h_7)$$

$$\text{or } \dot{m}_{\text{st}} = \frac{\dot{m}_{\text{air}}(h_4 - h_5)}{(h_7 - h_6)} = \frac{(369.43)(932.93 - 400.98)}{(3425.1 - 185.78)} = 60.67 \text{ kg/s}$$

The power developed by the steam cycle is

$$\dot{W}_{\text{st}} = \dot{W}_{\text{vap}} - \dot{W}_{\text{p}} = \dot{m}_{\text{st}} [(h_7 - h_8) - (h_6 - h_9)] = (60.67 \frac{\text{kg}}{\text{s}}) [(3425.1 - 2285.3) - (185.78 - 173.88)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| = 68.43 \text{ MW}$$



Continued on next slide

Problem 9-87 continued

The net power for the plant is

$$\dot{W}_{\text{cycle}} = \dot{W}_{\text{gas}} + \dot{W}_{\text{vap}} = 147 + 68.43 = 215.43 \text{ MW} \leftarrow \dot{W}_{\text{cycle}}$$

(b) The heat addition is

$$\dot{Q}_{\text{in}} = \dot{m}_{\text{gas}} (h_3 - h_2) = (369.43) (1733.17 - 702.53) \left| \frac{1}{10^3} \right| = 380.75$$

$$\text{Thus } \eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{215.43}{380.75} = 0.566 \text{ (56.6\%)} \leftarrow \eta$$

Exergy Accounting

The net exergy increase of the air passing through the gas turbine combustor is

$$\begin{aligned} \dot{E}_{f3} - \dot{E}_{f2} &= \dot{m}_{\text{gas}} [(h_3 - h_2) - T_0 (s_3^\circ - s_2^\circ - R \ln P_3/P_2)] \\ &= (369.43 \frac{\text{kg}}{\text{s}}) [(1733.1 - 702.52) - (300)(3.50829 - 2.55731 - \frac{8.314}{28.97} \ln \frac{13}{13.6})] \frac{\text{kJ}}{\text{kg}} \\ &= 273.92 \text{ MW} \leftarrow \text{INPUT} \end{aligned}$$

Losses:

$$\begin{aligned} \text{• Air out: } (\dot{E}_{f5} - \dot{E}_{f1}) &= \dot{m}_{\text{gas}} [(h_5 - h_1) - T_0 (s_5^\circ - s_1^\circ - R \ln P_5/P_1)] \\ &= (369.43/1000) [(400.98 - 300.19) - (300)(1.99194 - 1.70203)] = 5.1 \text{ MW} \end{aligned}$$

$$\begin{aligned} \text{• Condenser: } (\dot{E}_{f8} - \dot{E}_{f9}) &= \dot{m}_{\text{st}} [(h_8 - h_9) - T_0 (s_8 - s_9)] \\ &= (60.67/1000) [(2285.3 - 175.88) - (300)(7.3019 - 0.5926)] \\ &= 6.05 \text{ MW} \end{aligned}$$

Destructions: Using $\dot{E}_d = T_0 \dot{\sigma}_{\text{cv}} = T_0 \dot{m} \Delta s$

$$\begin{aligned} \text{• Gas Turbine: } \dot{E}_d &= \dot{m}_{\text{gas}} T_0 (s_4 - s_3) = \dot{m}_{\text{gas}} T_0 [(s_4^\circ - s_3^\circ) - R \ln(P_4/P_3)] \\ &= (369.43/1000)(300) [(2.84856 - 3.50829) - \frac{8.314}{28.97} \ln(\frac{1}{13})] \\ &= 8.465 \text{ MW} \end{aligned}$$

$$\begin{aligned} \text{• Compressor: } \dot{E}_d &= \dot{m}_{\text{gas}} T_0 (s_2 - s_1) = \dot{m}_{\text{gas}} T_0 [(s_2^\circ - s_1^\circ) - R \ln(P_2/P_1)] \\ &= (\frac{369.43}{1000})(300) [(2.55731 - 1.70203) - \frac{8.314}{28.97} \ln(\frac{13.6}{1})] = 11.77 \text{ MW} \end{aligned}$$

$$\text{• Steam Turbine: } \dot{E}_d = \dot{m}_{\text{st}} T_0 (s_7 - s_8) = (\frac{60.67}{1000})(300) [(7.3019 - 6.6622)] = 11.64 \text{ MW}$$

$$\text{• Pump: } \dot{E}_d = \dot{m}_{\text{st}} T_0 (s_6 - s_9) = (\frac{60.67}{1000})(300) (0.5968 - 0.5926) = 0.08 \text{ MW}$$

$$\begin{aligned} \text{• Heat Exchanger: } \dot{E}_d &= T_0 [\dot{m}_{\text{gas}} (s_5^\circ - s_4^\circ - R \ln \frac{P_5}{P_4}) + \dot{m}_{\text{st}} (s_7 - s_6)] \\ &= (300) [\frac{369.43}{1000} (1.99194 - 2.84856) + (\frac{60.67}{1000}) (6.6622 - 0.5968)] \\ &= 15.46 \text{ MW} \end{aligned}$$

SUMMARY Input: 273.92 MW

Disposition: $\dot{W}_{\text{cycle}}: (215.43) + \text{Losses: } (11.15) + \text{Destructions: } (47.42)$
 $= 274 \text{ MW}$ (Which agrees within round-off)

Note, the exergetic efficiency is

$$\epsilon = \frac{\text{exergy output}}{\text{exergy input}} = \frac{215.43}{273.92} = 0.786 \text{ (78.6\%)}$$

PROBLEM 9.88

KNOWN: Operating data are provided for a combined gas turbine-vapor power plant.

FIND: Determine (a) the mass flow rates of the air, steam, and cooling water, (b) the net power developed by the gas turbine and the vapor cycle, respectively, (c) the thermal efficiency of the combined cycle, (d) the net rate exergy is carried out with the exhaust air, and (e) the net rate exergy is carried out with the cooling water.

Schematic & Given Data:

See Figure 9.23

<u>Gas Turbine</u>			<u>Vapor cycle</u>			<u>Cooling water</u>	
State	T(°C)	P(bar)	State	T(°C)	P(bar)	In @ state 10, T ₁₀ = 20°C	Out @ state 11, T ₁₁ = 35°C
1	25	1	6	-	125	<u>ENGINEERING MODEL:</u> 1. See items 1-5 of Example 9.13. 2. T ₀ = 20°C, P ₀ = 1 bar	
2	-	14	7	500	125		
3	1250	14	8	-	0.1		
4	-	1	9	sat. liq.	0.1		
5	200	1					
η _c = 85%, η _t = 87%			η _t = 90%, η _p = 100%				
Q̇ _{in} = 50 MW							

ANALYSIS: Begin by evaluating essential property data.

Gas Turbine: At 298 K, Table A-22 gives $h_1 = 298.2 \text{ kJ/kg}$, $P_{r1} = 1.3543$,
 $P_{r25} = P_{r1} \left(\frac{P_2}{P_1} \right) = 1.3543(14) = 18.96 \Rightarrow h_{25} = 632.4 \frac{\text{kJ}}{\text{kg}}$ $s_1^0 = 1.6953 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

Using the isentropic compressor efficiency,

$$h_2 = h_1 + \frac{(h_{25} - h_1)}{\eta_c} = 298.2 + \frac{(632.4 - 298.2)}{0.85} = 691.4 \frac{\text{kJ}}{\text{kg}}$$

$$\text{At } T_3 = 1523 \text{ K}, h_3 = 1663.9 \frac{\text{kJ}}{\text{kg}}, P_{r3} = 641.95.$$

$$P_{r45} = P_{r3} \left(\frac{P_4}{P_3} \right) = 641.95 \left(\frac{1}{14} \right) = 45.85 \Rightarrow h_{45} = 812.48 \text{ kJ/kg}$$

Using the isentropic turbine efficiency,

$$h_4 = h_3 - \eta_t (h_3 - h_{45}) = 1663.9 - 0.87(1663.9 - 812.48) = 923.2 \text{ kJ/kg}$$

$$\text{At } T_5 = 473 \text{ K}, h_5 = 475.3 \text{ kJ/kg}, s_5^0 = 2.1625 \text{ kJ/kg} \cdot \text{K}$$

① Vapor Cycle: Steam Tables give the following data:

$$P_7 = 125 \text{ bar}, T_7 = 500^\circ\text{C}; h_7 = 3341.8 \text{ kJ/kg}, s_7 = 6.4618 \text{ kJ/kg} \cdot \text{K}$$

$$s_{85} = s_7 \Rightarrow x_{85} = \frac{6.4618 - 0.6493}{7.5009} = 0.7749$$

$$h_{85} = h_f + x_{85}(h_{fg}) = 191.83 + (0.7749)(2392.8) = 2046.01 \text{ kJ/kg}$$

$$h_8 = h_7 - \eta_t (h_7 - h_{85}) = 3341.8 - 0.9(3341.8 - 2046.01) = 2175.6 \text{ kJ/kg}$$

$$h_9 = 191.83 \text{ kJ/kg}$$

$$h_6 \approx h_9 + v_9 \Delta p = 191.83 \frac{\text{kJ}}{\text{kg}} + \left(\frac{1.0102 \text{ m}^3}{\text{kg}} \right) \left(\frac{124.9 \times 10^5 \text{ N}}{\text{m}^2} \right) \left(\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right) = 204.5 \text{ kJ/kg}$$

$$\text{Cooling water: } h_{10} \approx h_f(20^\circ\text{C}) = 83.96 \text{ kJ/kg}, h_{11} \approx h_f(35^\circ\text{C}) = 146.68 \text{ kJ/kg}$$

$$s_{10} \approx s_f(20^\circ\text{C}) = 0.2966 \text{ kJ/kg} \cdot \text{K}, s_{11} \approx s_f(35^\circ\text{C}) = 0.5053 \text{ kJ/kg} \cdot \text{K}$$

Continued on next slide

Problem 9-88 continued

- (a) For a control volume enclosing the combustor, mass and energy rate balances reduce to give

$$\dot{Q}_{in} = \dot{m}_g (h_3 - h_2) \Rightarrow \dot{m}_g = \frac{\dot{Q}_{in}}{(h_3 - h_2)} = \frac{50 \times 10^3 \text{ kJ/s}}{(1663.9 - 691.4) \text{ kJ/kg}} = 51.41 \frac{\text{kg}}{\text{s}}$$

For a control volume enclosing the heat exchanger, we get

$$\dot{m}_s = \dot{m}_g \frac{(h_4 - h_5)}{h_7 - h_6} = \frac{(51.41 \text{ kg/s})(923.2 - 475.3)}{(3341.8 - 204.5)} = 7.34 \frac{\text{kg}}{\text{s}}$$

For a control volume enclosing the condenser

$$\dot{m}_{cw} = \frac{\dot{m}_s (h_8 - h_9)}{h_{11} - h_{10}} = \frac{7.34 (2175.6 - 191.83)}{(146.68 - 83.96)} = 232.16 \text{ kg/s}$$

- (b) The net power developed by the gas turbine is

$$\begin{aligned} \dot{W}_g &= \dot{W}_t - \dot{W}_c = \dot{m}_g [(h_3 - h_4) - (h_2 - h_1)] = (51.41 \frac{\text{kg}}{\text{s}}) [(1663.9 - 923.2) - (691.4 - 298.2)] \frac{\text{kJ}}{\text{kg}} \\ &= 17,865 \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| = 17.865 \text{ MW} \end{aligned}$$

For the vapor cycle, the net power developed is

$$\begin{aligned} \dot{W}_s &= \dot{W}_t - \dot{W}_p = \dot{m}_s [(h_7 - h_8) - (h_6 - h_9)] \\ &= (7.34) [(3341.8 - 2175.6) - (204.5 - 191.83)] \left| \frac{1}{10^3} \right| = 8.467 \text{ MW} \end{aligned}$$

- (c) The thermal efficiency is

$$\eta = \frac{\dot{W}_g + \dot{W}_s}{\dot{Q}_{in}} = \frac{17.865 + 8.467}{50} = 0.527 \quad (52.7\%)$$

- (d) The net rate exergy is carried out by the exhaust air is

$$\begin{aligned} \dot{E}_{f5} - \dot{E}_{f1} &= \dot{m}_g (e_{f5} - e_{f1}) = \dot{m}_g [(h_5 - h_1) - T_0 (s_5^o - s_1^o) - R \ln \frac{P_5}{P_1}] \\ &= (51.41 \frac{\text{kg}}{\text{s}}) [475.3 - 298.2 - 293 (2.1625 - 1.6953)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| = 2.07 \text{ MW} \end{aligned}$$

- (e) The net rate exergy is carried out by the cooling water is

$$\begin{aligned} \textcircled{2} \quad \dot{E}_{f11} - \dot{E}_{f10} &= \dot{m}_{cw} (e_{f11} - e_{f10}) = \dot{m}_{cw} [h_{11} - h_{10} - T_0 (s_{11} - s_{10})] \\ &= (232.16) [(146.68 - 83.96) - 293 (0.5053 - 0.2966)] \left| \frac{1}{10^3} \right| = 0.36 \text{ MW} \end{aligned}$$

- Since state 7 requires a double interpolation in Table A-4, steam table data are obtained from the source listed on p. B18. Alternatively, IT can be invoked to retrieve needed data.
- Considerable energy is carried out with the cooling water: $\dot{m}_{cw} (h_{11} - h_{10}) = 14.56 \text{ MW}$. But since the cooling water is at a relatively low temperature its utility, as gauged by exergy, is low. Also, note that considerable exergy is carried out at 5. This might be exploited by the regenerative approach discussed in Sec. 9.7 or by other means.

PROBLEM 9.89

KNOWN: Operating data are provided for a combined gas turbine - vapor power plant in which air and water vapor are the working fluids.

FIND: Determine (a) the mass flow rates of air and steam, (b) the thermal efficiency of the combined cycle, and (c) a full exergy accounting of the net exergy increase of the air passing through the combustor of the gas turbine.

SCHEMATIC & GIVEN DATA:

See Figure 9.23.

$$W_{\text{cycle}} = 100 \text{ MW}$$

Gas Cycle $\eta_t = 88\%$, $\eta_c = 84\%$

State	h (kJ/kg)	s° (kJ/kg·K)
1	300.19	1.70203
2	669.78	2.5088
3	1515.42	3.3620
4	858	2.76195
5	482.49	2.17760

Vapor Cycle $\eta_t = 90\%$, $\eta_p = 80\%$

State	h	s
6	183.96	0.5975
7	3138.3	6.3634
8	2104.74	6.7282
9	173.88	0.5926

ENGINEERING MODEL: See Example 9.13.

ANALYSIS: Begin by obtaining needed data at each of the principal states. These values are summarized in the tables above.

Gas Cycle

State 1: $T_1 = 300 \text{ K}$. Table A-22 gives $h_1 = 300.19 \text{ kJ/kg}$, $P_{r1} = 1.3860$, $s_1^\circ = 1.70203 \text{ kJ/kg·K}$.

State 2: $P_{r2s} = P_{r1} (P_2/P_1) = 1.3860 \left(\frac{1200 \text{ kPa}}{100 \text{ kPa}} \right) = 16.632 \Rightarrow h_{2s} = 610.65 \text{ kJ/kg}$

and $h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 669.78 \frac{\text{kJ}}{\text{kg}}$, $s_2^\circ = 2.5088 \text{ kJ/kg·K}$

State 3: $T_3 = 1400 \text{ K}$. $h_3 = 1515.42 \text{ kJ/kg}$, $P_{r3} = 450.5$, $s_3^\circ = 3.3620 \text{ kJ/kg·K}$

State 4: $P_{r4s} = (P_4/P_3) P_{r3} = \left(\frac{100}{1200} \right) (450.5) = 37.542 \Rightarrow h_{4s} = 768.38 \text{ kJ/kg}$

and $h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 1515.42 - 0.88 (1515.42 - 768.38) = 858 \text{ kJ/kg}$,
 $s_4^\circ = 2.76195 \text{ kJ/kg·K}$.

State 5: $T_5 = 480 \text{ K}$, $h_5 = 482.49 \text{ kJ/kg}$, $s_5^\circ = 2.17760 \text{ kJ/kg·K}$.

Vapor Cycle

State 7: $P_7 = 80 \text{ bar}$, $T_7 = 400^\circ \text{C}$. Table A-4 gives $h_7 = 3138.3 \text{ kJ/kg}$,
 $s_7 = 6.3634 \text{ kJ/kg·K}$

State 8: $s_{8s} = s_7 \Rightarrow x_{8s} = \frac{6.3634 - 0.5926}{8.2287 - 0.5926} = 0.7557$, $h_{8s} = 173.88 + 0.7557(2403.1) = 1989.9 \text{ kJ/kg}$

and $h_8 = h_7 - \eta_t (h_7 - h_{8s}) = 3138.3 - 0.9 (3138.3 - 1989.9) = 2104.74 \text{ kJ/kg}$

$x_8 = \frac{2104.74 - 173.88}{2403.1} = 0.8035$, $s_8 = 0.5926 + 0.8035(8.2287 - 0.5926) = 6.7282 \frac{\text{kJ}}{\text{kg·K}}$

State 9: Saturated liquid at 0.08 bar. $h_9 = 173.88 \frac{\text{kJ}}{\text{kg}}$, $s_9 = 0.5926 \frac{\text{kJ}}{\text{kg·K}}$

State 6: $h_{6s} \approx h_9 + v_9 \Delta P = 173.88 + \left(\frac{0.084}{10^3} \right) (79.92 \times 10^5) \left(\frac{1}{10^3} \right) = 181.94 \text{ kJ/kg}$

$h_6 = h_9 + \frac{(h_{6s} - h_9)}{\eta_p} = 173.88 + \frac{(181.94 - 173.88)}{0.8} = 183.96 \frac{\text{kJ}}{\text{kg}}$. Then, interpolating in

Table A-5 gives $s_6 = 0.5975 \text{ kJ/kg·K}$.

Continued on next slide

Problem 9-89 continued

(a) To find the mass flow rates of the air and steam, begin with mass and energy rate balances on the heat exchanger, which reduce to give

$$\frac{\dot{m}_s}{\dot{m}_g} = \frac{h_4 - h_5}{h_7 - h_6} = \frac{858 - 482.49}{3178.3 - 183.96} = 0.1271$$

For the gas turbine, the net power developed is

$$\dot{W}_g = \dot{m}_g [(h_3 - h_4) - (h_2 - h_1)] = \dot{m}_g (287.83 \frac{\text{kJ}}{\text{kg}}) \quad (1)$$

and for the vapor cycle

$$\dot{W}_s = \dot{m}_s [(h_7 - h_8) - (h_6 - h_9)] = \dot{m}_s (1023.5 \frac{\text{kJ}}{\text{kg}}) \quad (2)$$

The net power developed, $\dot{W}_{\text{net}} = \dot{W}_g + \dot{W}_s$, is given as 100 MW. Thus, collecting results we get

$$\dot{W}_{\text{net}} = \dot{m}_g [(287.83) + \frac{\dot{m}_s}{\dot{m}_g} (1023.5)] \Rightarrow$$

$$\dot{m}_g = \frac{100 \times 10^3 \text{ kJ/s}}{(287.83) + (0.1271)(1023.5)} = 239.3 \text{ kg/s}$$

$$\Rightarrow \dot{m}_s = 30.4 \text{ kg/s}$$

(b) The thermal efficiency is $\eta = (\dot{W}_g + \dot{W}_s) / \dot{Q}_{\text{in}}$, where $\dot{Q}_{\text{in}} = \dot{m}_g (h_3 - h_2)$. Thus

$$\eta = \frac{287.83 + 0.1271(1023.5)}{(1515.42 - 669.78)} = 0.494 \quad (49.4\%)$$

(c) The net energy increase of the air passing through the combustor of the gas turbine is

$$\dot{E}_{f3} - \dot{E}_{f2} = \dot{m}_g [(h_3 - h_2) - T_0 (s_3^0 - s_2^0 - R \ln(p_3/p_2))] = (239.3 \frac{\text{kg}}{\text{s}}) [(1515.42 - 669.78) - 300(3.3620 - 2.5088)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right|$$

$$= 141.1 \text{ MW}$$

Losses:

- air out: $(\dot{E}_{f5} - \dot{E}_{f1}) = \dot{m}_g [(h_5 - h_1) - T_0 (s_5^0 - s_1^0)] = (239.3) [482.49 - 300(2.1776 - 1.70203)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1}{10^3} \right| = 9.48 \text{ MW}$

- condenser: $(\dot{E}_{f8} - \dot{E}_{f9}) = \dot{m}_s [(h_8 - h_9) - T_0 (s_8 - s_9)] = 30.4 [(2104.74 - 173.88) - 300(6.7282 - 0.5926)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1}{10^3} \right| = 2.74 \text{ MW}$

Destructions: Using $\dot{E}_d = T_0 \dot{\sigma} = T_0 \dot{m} \Delta s$

- Gas turbine: $\dot{E}_d = \dot{m}_g T_0 (s_4 - s_3) = (239.3)(300) [2.76195 - 3.3620 - \frac{8.314 \ln(100/120)}{28.97}] \left| \frac{1}{10^3} \right| = 8.12 \text{ MW}$

- compressor: $\dot{E}_d = \dot{m}_g T_0 (s_2 - s_1) = (239.3)(300) [2.5088 - 1.70203 - \frac{8.314 \ln(120/100)}{28.97}] \left| \frac{1}{10^3} \right| = 6.72 \text{ MW}$

- Steam turbine: $\dot{E}_d = \dot{m}_s T_0 (s_8 - s_7) = (30.4)(300) [6.7282 - 6.3634] \left| \frac{1}{10^3} \right| = 3.33 \text{ MW}$

- pump: $\dot{E}_d = \dot{m}_s T_0 (s_6 - s_9) = (30.4)(300) [0.5926 - 0.5926] \left| \frac{1}{10^3} \right| = 0.04 \text{ MW}$

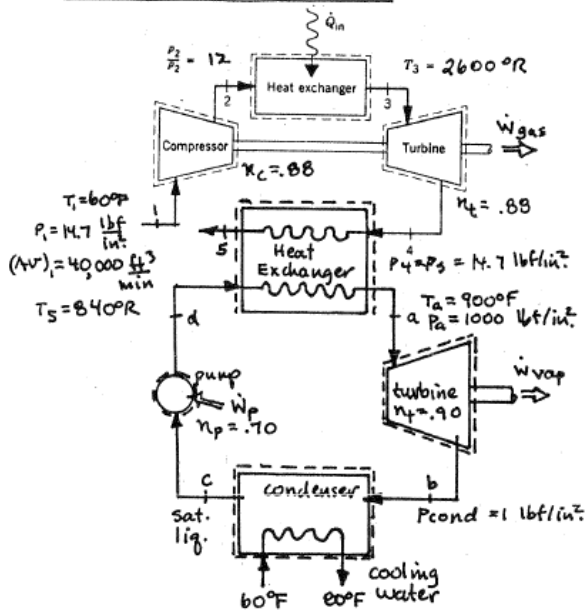
- heat exchanger: $\dot{E}_d = T_0 [\dot{m}_g (s_5 - s_4) + \dot{m}_s (s_7 - s_6)] = 300 [239.3(-0.58435) + 30.4(5.7659)] \left| \frac{1}{10^3} \right| = 10.63 \text{ MW}$

Summary: Input: 141.4 MW

Disposition: $\dot{W}_{\text{net}}: (100 \text{ MW}) + \text{Losses: } (12.22) + \text{Destructions: } (28.84) = 141.1 \text{ MW}$, which agrees to within round off

PROBLEM 9.90

KNOWN: Operating data are provided for a combined gas turbine-vapor power plant in which air and water vapor are the working fluids.
FIND: Determine (a) the mass flow rates of the air, steam, and cooling water, (b) the net power developed by the gas turbine and vapor cycle, respectively, (c) the thermal efficiency of the combined cycle, and (d) a full accounting of the net exergy increase of the air passing through the combustor of the gas turbine.
SCHEMATIC & GIVEN DATA:



Gas Cycle

State	h (Btu/lb)	s° (Btu/lb·°R)
1	124.27	0.59172
2	270.37	0.77820
3	674.49	1.00623
4	382.51	0.86289
5	201.56	0.70747

Vapor Cycle

State	h (Btu/lb)	s (Btu/lb·°R)
a	1448.1	1.6120
b	955.0	1.7095
c	69.74	.1327
d	74.16	.13497

ENGINEERING MODEL:

- See Example 9.13, items 1-5.
- $T_0 = 520^\circ\text{R}$, $P_0 = 14.7 \text{ lbf/in}^2$

ANALYSIS: First, fix each of the principal states. For the gas turbine cycle:

State 1 $T_1 = 520^\circ\text{R} \Rightarrow h_1 = 124.27 \text{ Btu/lb}$
 $P_{r1} = 1.2147$, $s_1^\circ = 0.59172 \text{ Btu/lb}^\circ\text{R}$

State 2 For isentropic compression,
 $P_{r2} = (P_2/P_1) P_{r1} = 14.5764 \Rightarrow h_{2s} = 252.84 \text{ Btu/lb}$

Using the compressor efficiency,
 $h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 270.37 \text{ Btu/lb}$, $s_2^\circ = 0.77820 \text{ Btu/lb}^\circ\text{R}$

State 3 $T_3 = 2600^\circ\text{R} \Rightarrow h_3 = 674.49 \text{ Btu/lb}$, $P_{r3} = 513.5$, $s_3^\circ = 1.00623 \text{ Btu/lb}^\circ\text{R}$

State 4 $P_{r4} = (P_4/P_3) P_{r3} = 42.792 \Rightarrow h_{4s} = 342.7 \text{ Btu/lb}$; With $\eta_t = 0.88$,
 $h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 382.51 \text{ Btu/lb}$, $s_4^\circ = 0.86289 \text{ Btu/lb}^\circ\text{R}$

State 5 $T_5 = 840^\circ\text{R} \Rightarrow h_5 = 201.56 \text{ Btu/lb}$, $s_5^\circ = 0.70747 \text{ Btu/lb}^\circ\text{R}$

For the vapor cycle:

State a $P_a = 1000 \text{ lbf/in}^2$, $T_a = 900^\circ\text{F} \Rightarrow h_a = 1448.1 \text{ Btu/lb}$, $s_a = 1.6120 \text{ Btu/lb}^\circ\text{R}$

State b $P_b = 1 \text{ lbf/in}^2$, $s_{bs} = s_a \Rightarrow h_{bs} = 900.26 \text{ Btu/lb}$. With $\eta_t = 0.90$,
 $h_b = h_a - \eta_t (h_a - h_{bs}) = 955.0 \text{ Btu/lb} \Rightarrow x_b = 0.8545$, $s_b = 1.7095 \text{ Btu/lb}^\circ\text{R}$

State c $P_c = 1 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_c = 69.74 \text{ Btu/lb}$, $s_c = .1327 \text{ Btu/lb}^\circ\text{R}$

State d $P_d = 1000 \text{ lbf/in}^2$, $s_{ds} = s_c \Rightarrow$ Table A-5E; $h_{ds} = 72.84 \text{ Btu/lb}$
 With $\eta_p = 0.70$, $h_d = h_c + \frac{(h_{ds} - h_c)}{\eta_p} = 74.16 \text{ Btu/lb}$, $s_d = 0.13497 \text{ Btu/lb}^\circ\text{R}$
 (Table A-5E)

Continued on next slide

Problem 9-90 continued

(a) The mass flow rate of the air is

$$\dot{m}_{air} = \frac{(AV)_1 P_1}{RT_1} = \frac{(40,000 \text{ ft}^3/\text{min}) (14.7 \frac{\text{lb}_f}{\text{in}^2})}{(\frac{1545 \cdot \text{ft} \cdot \text{lb}_f}{28.97 \text{ lb} \cdot ^\circ\text{R}}) (520^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{60 \text{ min}}{1 \text{ h}} \right|$$

$$= 1.832 \times 10^5 \text{ lb/h}$$

Reducing mass and energy rate balances for a control volume enclosing the heat exchanger

$$0 = \dot{m}_{air} (h_4 - h_5) + \dot{m}_{st} (h_d - h_a)$$

or

$$\dot{m}_{st} = \dot{m}_{air} \left(\frac{h_4 - h_5}{h_a - h_d} \right) = 2.413 \times 10^4 \text{ lb/h}$$

Combining energy and mass balances for the condenser

$$0 = \dot{m}_{st} (h_b - h_c) + \dot{m}_{cw} (h_{cw,in} - h_{cw,out})$$

Solving for $\dot{m}_{cw}$ and with $h_{cw} \approx h_f(T)$

$$\dot{m}_{cw} = \dot{m}_{st} \left(\frac{h_b - h_c}{h_f(80) - h_f(60)} \right) = 1.07 \times 10^6 \text{ lb/h} \leftarrow \dot{m}_{cw}$$

(b) the net power of the gas turbine cycle is

$$\dot{W}_{gas} = \dot{m}_{air} [(h_3 - h_4) - (h_2 - h_1)] = 2.67 \times 10^7 \text{ Btu/h}$$

For the vapor power cycle

$$\dot{W}_{vap} = \dot{m}_{st} [(h_a - h_b) - (h_d - h_c)] = 1.879 \times 10^7 \text{ Btu/h}$$

(c) The rate of heat transfer into the combined cycle is

$$\dot{Q}_{in} = \dot{m}_{air} (h_3 - h_2) = 7.403 \times 10^7 \text{ Btu/h}$$

thus

$$\eta = \frac{\dot{W}_{gas} + \dot{W}_{vap}}{\dot{Q}_{in}} = .5199 \text{ (51.99\%)} \leftarrow$$

(d) The net increase in exergy of the air passing through the combustor of the gas turbine is

$$\dot{E}_{f3} - \dot{E}_{f2} = \dot{m}_{air} [e_{f3} - e_{f2}] = \dot{m}_{air} [(h_3 - h_2) - T_0 (s_3^0 - s_2^0 - R \ln P_3/P_2)] = 5.231 \times 10^7 \frac{\text{Btu}}{\text{h}}$$

Losses:

- Air out: $\dot{E}_{f5} - \dot{E}_{f1} = \dot{m}_{air} [(h_5 - h_1) - T_0 (s_5^0 - s_1^0 - R \ln P_5/P_1)] = 3.133 \times 10^6 \text{ Btu/h}$

- Condenser:

$$\dot{E}_{f,cw,out} - \dot{E}_{f,cw,in} = \dot{m}_{cw} [h_f(80^\circ\text{F}) - h_f(60^\circ\text{F}) - T_0 (s_f(80^\circ) - s_f(60^\circ))] = 4.103 \times 10^5 \text{ Btu/h}$$

Continued on next slide

Problem 9-90 continued

Destructions: Using $\dot{E}_d = T_0 \dot{Q} = T_0 \dot{m}(\Delta s)$

$$\begin{aligned} \text{Gas turbine: } \dot{E}_d &= 2.573 \times 10^6 \text{ Btu/h} \\ \text{Compressor: } \dot{E}_d &= 1.537 \times 10^6 \text{ " } \\ \text{Steam turbine: } \dot{E}_d &= 1.223 \times 10^6 \text{ " } \\ \text{Pump: } \dot{E}_d &= 2.848 \times 10^4 \text{ " } \\ \hline \text{Subtotal: } \dot{E}_d &= 5.361 \times 10^6 \text{ Btu/h} \end{aligned}$$

$$\text{Condenser: } \dot{E}_d = T_0 [\dot{m}_{st}(s_c - s_b) + \dot{m}_{cw}(s_{cw,out} - s_{cw,in})] = 1.23 \times 10^6 \text{ Btu/h}$$

$$\text{Heat exchanger: } \dot{E}_d = T_0 [\dot{m}_{st}(s_a - s_d) + \dot{m}_{air}(s_5^o - s_4^o)] = 3.727 \times 10^6 \text{ Btu/h}$$

Output:

$$\dot{W}_{net} = \dot{W}_{gas} + \dot{W}_{vap} = 3.849 \times 10^7 \text{ Btu/h}$$

Summary

	Exergy Input	$5.231 \times 10^7 \text{ Btu/h}$	
	Disposition of the exergy input...		
①	- Net power developed	$3.849 \times 10^7 \text{ Btu/h}$	(73.5%)
	- Exergy Destruction	$10.318 \times 10^6 \text{ Btu/h}$	(19.7%)
	- Exergy Losses	$3.543 \times 10^6 \text{ Btu/h}$	(6.8%)
		<u>$5.235 \times 10^7 \text{ Btu/h}$</u>	

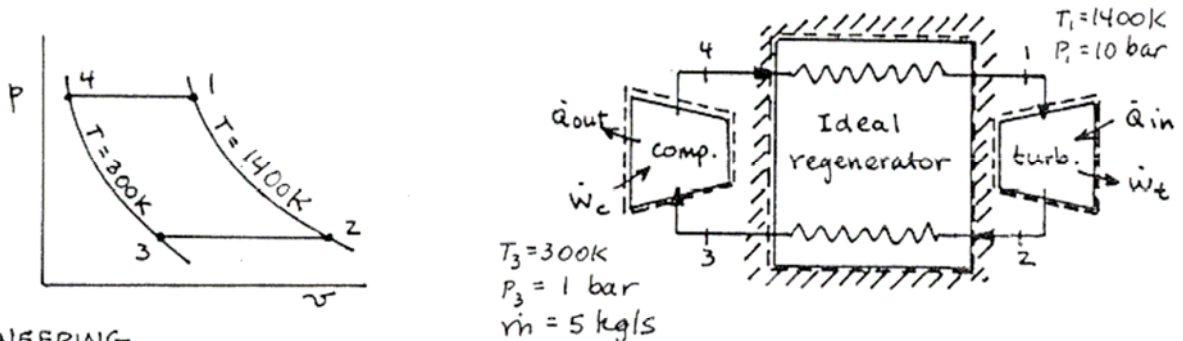
1. The exergy destruction associated with combustion is not considered in the present analysis, which is based on the air-standard model. See the discussion in note 2 of the solution to Example 9.13.

PROBLEM 9.91

KNOWN: Air is the working fluid in an Ericsson cycle with data known at various locations.

FIND: Determine (a) the net power developed, (b) the thermal efficiency, and (c) the backwork ratio.

SCHEMATIC & GIVEN DATA:



ENGINEERING-

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion processes are isothermal. (4) Kinetic and potential energy effects are negligible. (5) The air behaves as an ideal gas.

ANALYSIS: (a) The turbine power is evaluated using

$$\begin{aligned}\dot{W}_t &= -\dot{m} \int_1^2 v dp = -\dot{m} R T_1 \ln(P_2/P_1) \\ &= -(5 \text{ kg/s}) \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (1400 \text{ K}) \ln\left(\frac{1}{10}\right) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 4626 \text{ kW}\end{aligned}$$

and for the compressor

$$\dot{W}_c = \dot{m} R T_3 \ln P_4/P_3 = 991.2 \text{ kW}$$

Thus $\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_c = 3635 \text{ kW}$ $\dot{W}_{\text{cycle}}$

(b) The thermal efficiency is

$$\eta = 1 - \frac{T_3}{T_1} = 1 - \frac{300}{1400} = 0.786 \text{ (78.6\%)} \quad \eta$$

Alternatively, from an energy balance on the turbine; $\dot{Q}_{\text{in}} = \dot{W}_t$. Thus

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{3635}{4626} = 0.786$$

(c) The backwork ratio is

$$\text{bwr} = \frac{\dot{W}_c}{\dot{W}_t} = \frac{991.2}{4626} = 0.214 \quad \text{bwr}$$

PROBLEM 9.92

See Problem 9.91 for sample calculations. The data for the required plot are obtained using IT, as follows:

IT Code

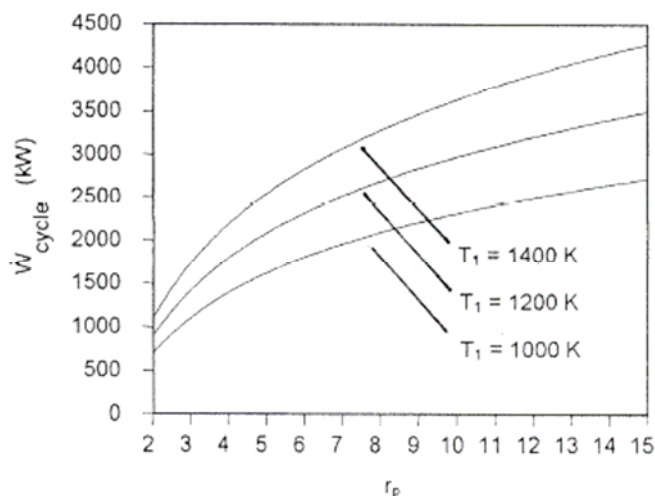
```
T1 = 1400 // K
T3 = 300 // K
p3 = 1 // bar
rp = 10
p4 = rp * p3
p1 = p4
p2 = p3
mdot = 5 // kg/s
R = 8.314 / 28.97 // kJ/kg·K

Wdot_t = - mdot * R * T1 * ln (p2 / p1)
Wdot_c = mdot * R * T3 * ln (p4 / p3)
Wdot_cycle = Wdot_t - Wdot_c
```

IT Results for $r_p = 10$, $T_1 = 1400$ K

```
W_t = 4626 kW
W_c = 991.2 kW
W_cycle = 3634 kW
```

PLOT :



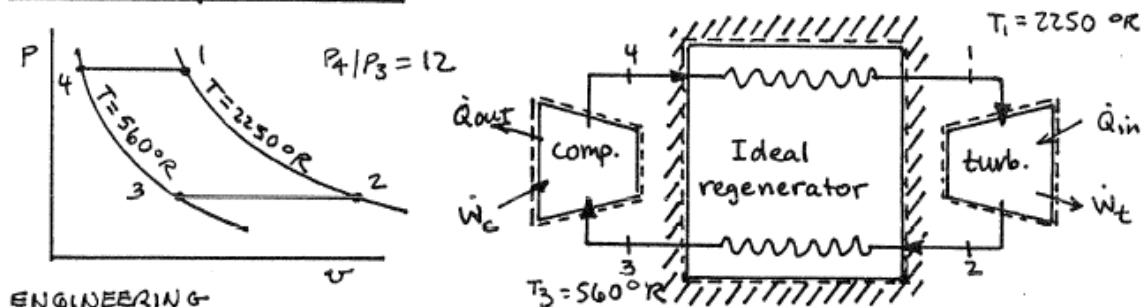
We see that the power increases significantly with compressor pressure ratio and with maximum cycle temperature for this idealized cycle.

PROBLEM 9.93

KNOWN: Air is the working fluid in an Ericsson cycle with data known at various locations.

FIND: Determine (a) the net work per unit mass of air flowing and (b) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion processes are isothermal. (4) Kinetic and potential energy effects are negligible. (5) The air behaves as an ideal gas.

ANALYSIS: (a) The turbine work is evaluated using Eq. 6.67

$$\begin{aligned}\frac{\dot{W}_t}{\dot{m}} &= - \int_1^2 v dp = -RT_1 \ln \frac{P_2}{P_1} \\ &= - \left(\frac{1.986}{28.97} \cdot \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (2250^\circ\text{R}) \ln \left(\frac{1}{12} \right) = 383.3 \text{ Btu/lb}\end{aligned}$$

For the compressor

$$\frac{\dot{W}_c}{\dot{m}} = RT_3 \ln \left(\frac{P_4}{P_3} \right) = \left(\frac{1.986}{28.97} \right) (560) \ln 12 = 95.4 \text{ Btu/lb}$$

Thus, the net work is

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_c}{\dot{m}} = 383.3 - 95.4 = 287.9 \frac{\text{Btu}}{\text{lb}} \quad \leftarrow \frac{\dot{W}_{\text{cycle}}}{\dot{m}}$$

(b) The thermal efficiency is

$$\eta = 1 - \frac{T_3}{T_1} = 0.751 \quad \leftarrow \eta$$

Alternatively, $\dot{Q}_{\text{in}}/\dot{m} = \frac{\dot{W}_t}{\dot{m}} + (h_2 - h_1) = 383.3 \text{ Btu/lb}.$

thus

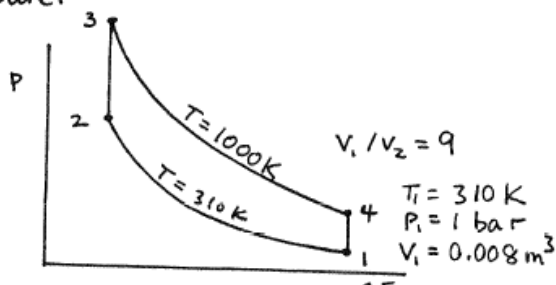
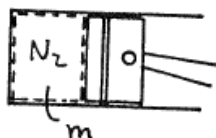
$$\eta = \frac{287.9}{383.3} = 0.751$$

PROBLEM 9.94

KNOWN: N_2 undergoes a Stirling cycle with a known compression ratio. Other data are also given for the cycle.

FIND: Determine (a) the net work, (b) the thermal efficiency, and (c) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The air is a closed system. (2) The compression and expansion are isothermal. (3) All processes are internally reversible. (4) An ideal regenerator is available to store the energy rejected at constant volume to be used as heat input during the constant volume heat transfer process. (5) The air behaves as an ideal gas.

ANALYSIS: The mass m is found as follows:

$$m = \frac{P_1 V_1}{R T_1} = \frac{(10^5 \text{ N/m}^2)(0.008 \text{ m}^3)}{\left(\frac{8.314}{28.01} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}\right)(290 \text{ K})} = 0.0093 \text{ kg}$$

(a) For the isothermal compression

$$\begin{aligned} W_{12} &= \int_1^2 p dV = m R T_1 \ln \frac{V_2}{V_1} \\ &= (0.0093 \text{ kg}) \left(\frac{8.314}{28.01} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (310 \text{ K}) \ln \left(\frac{1}{9} \right) = -1.88 \text{ kJ} \end{aligned}$$

Similarly, for the expansion

$$W_{34} = m R T_3 \ln \frac{V_4}{V_3} = (0.0093) \left(\frac{8.314}{28.01} \right) (1000) \ln(9) = 6.07 \text{ kJ}$$

$$\text{Thus, } W_{\text{cycle}} = W_{12} + W_{34} = 4.19 \text{ kJ} \quad \leftarrow W_{\text{cycle}}$$

(b) The thermal efficiency is

$$\eta = 1 - \frac{T_1}{T_3} = 1 - \frac{310}{1000} = 0.690 \quad \leftarrow \eta$$

$$\text{or } \eta = \frac{W_{\text{cycle}}}{Q_{34}} = \frac{4.19 \text{ kJ}}{6.07 \text{ kJ}} = 0.690$$

$Q_{34} = W_{34}$ (energy balance)

(c) The mean effective pressure is

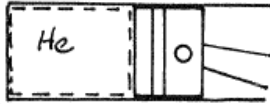
$$\begin{aligned} mep &= \frac{W_{\text{cycle}}}{V_1 - V_2} = \frac{W_{\text{cycle}}}{V_1 (1 - V_2/V_1)} \\ &= \frac{(4.19 \text{ kJ})}{(0.008 \text{ m}^3) (1 - \frac{1}{9})} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| = 5.89 \text{ bar} \quad \leftarrow mep \end{aligned}$$

PROBLEM 9.95

KNOWN: Helium is the working fluid in a Stirling cycle. Data are given for various states in the cycle.

FIND: Determine (a) the work and heat transfer for each process and (b) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The helium is a closed system. (2) The compression and expansion are isothermal. (3) All processes are internally reversible. (4) An ideal regenerator is available to store the energy transferred by heat during constant volume processes. (5) The helium behaves as an ideal gas.

ANALYSIS: (a) Considering each process

Process 1-2: The work is calculated using

$$\begin{aligned} \frac{W_{12}}{m} &= \int_1^2 p d\upsilon = RT_1 \ln \frac{\upsilon_2}{\upsilon_1} = RT_1 \ln \left(\frac{P_1}{P_2} \right) \\ &= \left(\frac{1545 \text{ ft} \cdot \text{lb}_f}{4.003 \text{ lb} \cdot ^\circ\text{R}} \right) \left(\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lb}_f} \right) (560^\circ\text{R}) \ln \left(\frac{15}{150} \right) = -639.7 \frac{\text{Btu}}{\text{lb}} \end{aligned}$$

$\leftarrow \frac{W_{12}}{m}$

From an energy balance

$$\frac{Q_{12}}{m} = (u_2 - u_1) + \frac{W_{12}}{m} = -639.7 \frac{\text{Btu}}{\text{lb}} \leftarrow \frac{Q_{12}}{m}$$

Process 3-4:

$$\begin{aligned} \frac{W_{34}}{m} &= RT_3 \ln \frac{\upsilon_4}{\upsilon_3} = RT_3 \ln \frac{\upsilon_1}{\upsilon_2} = RT_3 \ln \frac{P_2}{P_1} \\ &= \left(\frac{1545}{4.003} \right) \left(\frac{1}{778} \right) (1960) \ln \left(\frac{150}{15} \right) = 2238.9 \frac{\text{Btu}}{\text{lb}} \end{aligned}$$

$\leftarrow \frac{W_{34}}{m}$

and $\frac{Q_{34}}{m} = (u_4 - u_3) + \frac{W_{34}}{m} = 2238.9 \frac{\text{Btu}}{\text{lb}} \leftarrow \frac{Q_{34}}{m}$

Processes 2-3 and 4-1:

$$\left. \begin{aligned} \frac{Q_{23}}{m} &= u_3 - u_2 \\ \frac{Q_{41}}{m} &= u_1 - u_4 \end{aligned} \right\} \frac{Q_{23}}{m} = -\frac{Q_{41}}{m}$$

Thus, if the regenerator is ideal, there is no net exchange of energy with the surroundings.

(b) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{W_{12}/m + W_{34}/m}{Q_{34}/m} = 0.714 \text{ (71.4\%)}$$

Alternatively

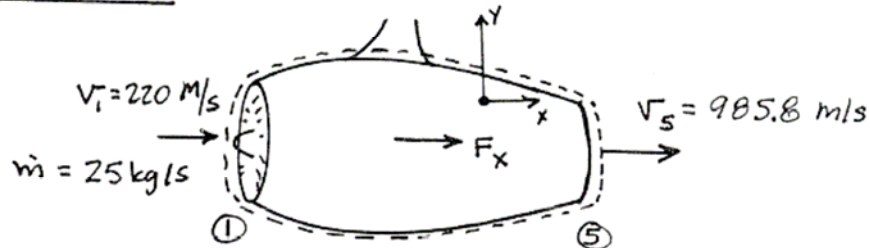
$$\eta = 1 - \frac{T_1}{T_3} = 1 - \frac{560}{1960} = 0.714$$

PROBLEM 9.96

KNOWN: Data are known for the turbojet engine of problem 9.77.

FIND: Calculate the thrust developed by the engine.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state. (2) The force F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$\begin{aligned} F_x &= \dot{m} (V_5 - V_1) \\ &= (25 \frac{\text{kg}}{\text{s}})(985.8 - 220) \frac{\text{m}}{\text{s}} \left| \frac{1 \text{ kN}}{10^3 \text{ kg} \cdot \text{m/s}^2} \right| \\ &= 19.15 \text{ kN} \end{aligned}$$

Thus, the thrust is the force developed by the control volume

$$F_{\text{thrust}} = -F_x = -19.15 \text{ kN} \quad \leftarrow \text{thrust}$$

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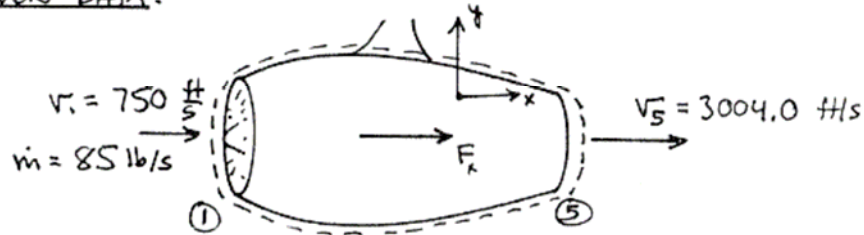
-
1. The effect of the surrounding pressure on the control volume boundary cancels.

PROBLEM 9.77

KNOWN: Conditions are known for the turbojet engine analyzed in Problem 9.79.

FIND: Calculate the thrust developed by the engine.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state. (2) The force F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$\begin{aligned} F_x &= \dot{m} (V_5 - V_1) \\ &= \left(85 \frac{\text{lb}}{\text{s}} \right) (3004.0 - 750) \frac{\text{ft}}{\text{s}} \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \\ &= 5950 \text{ lbf} \end{aligned}$$

The thrust is the force developed by the engine. Thus

$$F_{\text{thrust}} = -F_x = -5950 \text{ lbf} \quad \leftarrow \text{thrust}$$

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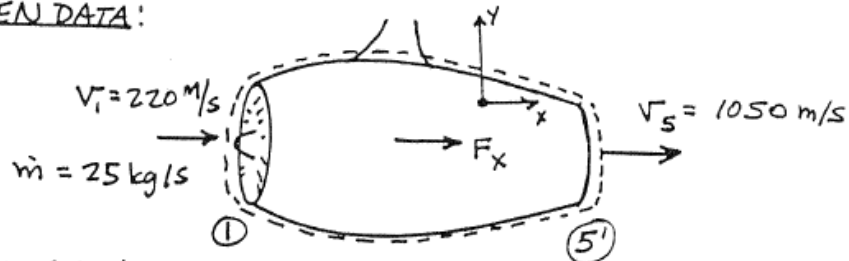
1. The effect of the surrounding pressure on the control volume cancels.

PROBLEM 9.98

KNOWN: Data are known for a turbojet with afterburner as in Problem 9.80.

FIND: Calculate the thrust developed by the engine.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) The force F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$\begin{aligned} \textcircled{1} \quad F_x &= \dot{m}(V_5 - V_1) \\ &= (25 \frac{\text{kg}}{\text{s}})(1050 - 220) \frac{\text{m}}{\text{s}} \left| \frac{1 \text{ kN}}{10^3 \text{ kg} \cdot \text{m/s}^2} \right| = 20.75 \text{ kN} \end{aligned}$$

Thus, the thrust is the force developed by the control volume

$$\textcircled{2} \quad F_{\text{thrust}} = -F_x = -20.75 \text{ kN} \quad \xleftarrow{\text{thrust}}$$

to the left

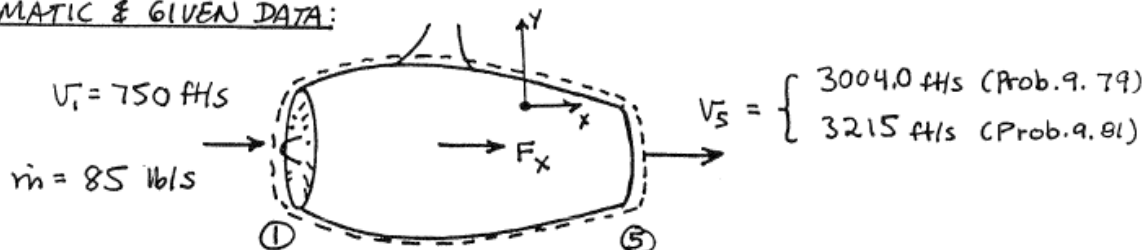
1. The effect of the surrounding pressure on the control volume boundary cancels.
2. Comparing this result with that of Problem 9.96, we see that the use of an afterburner increases the engine thrust.

PROBLEM 9.99

KNOWN: The turbojet in Problem 9.79 and the modified turbojet in Problem 9.81 are reconsidered.

FIND: Calculate the thrust developed by each engine. Discuss.

SCHEMATIC & GIVEN DATA:



- ASSUMPTIONS: (1) The control volume is at steady state. (2) The force F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot{m}(V_5 - V_1)$$

The thrust is the force developed by the control volume. Thus

$$F_{\text{thrust}} = -F_x = -\dot{m}(V_5 - V_1)$$

Inserting values, first for Problem 9.79 (turbojet)

$$F_{\text{thrust}} = -\left(85 \frac{\text{lb}}{\text{s}}\right)(3004.0 - 750) \frac{\text{ft}}{\text{s}} \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft}/\text{s}^2} \right|$$

thrust
(Prob. 9.79)

Similarly, for Problem 9.81 (turbojet + afterburner)

$$F_{\text{thrust}} = -\left(85\right)(3215 - 750) \left| \frac{1}{32.2} \right|$$

thrust
(Prob. 9.81)

The addition of an afterburner increases the velocity of gases leaving the engine. In this case, the result is a 9.4% increase in thrust developed by the engine.

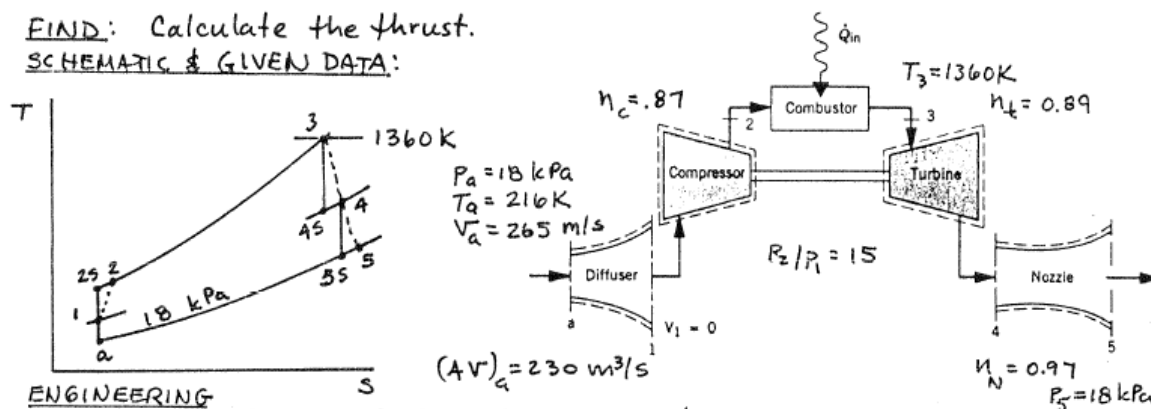
1. The effect of the surrounding pressure on the control volume boundary cancels.

PROBLEM 9.100

KNOWN: A turbojet engine is analyzed on an air-standard basis. Data are known at various locations.

FIND: Calculate the thrust.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: Same as in Example 9.12, except $\eta_c = 0.87$, $\eta_t = 0.89$, $\eta_N = 0.97$.

ANALYSIS: To determine the thrust, we begin by finding the velocity at the nozzle exit, V_5 . First, fix each of the principal states (Table A-22).

State a $T_a = 216 \text{ K} \Rightarrow h_a = 215.97 \text{ kJ/kg}$, $P_a = 0.44088$

State 1 Applying the energy balance to the diffuser; $0 = h_a + \frac{V_a^2}{2} - h_1$. Thus

$$h_1 = h_a + \frac{V_a^2}{2} = 215.97 \frac{\text{kJ}}{\text{kg}} + \frac{265^2 \text{ m}^2/\text{s}^2}{2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 251.08 \text{ kJ/kg}$$

Since the diffuser is assumed to operate isentropically

$$P_1 = (P_1/P_a) P_a = (0.74394/0.44088)(18 \text{ kPa}) = 30.373 \text{ kPa}$$

State 2 For isentropic compression, $P_{2s} = P_1$, $(P_2/P_1) = 11.159$ and $h_{2s} = 545.16 \frac{\text{kJ}}{\text{kg}}$ with the compressor efficiency

$$h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 589.10 \text{ kJ/kg}, \quad P_2 = 15 \cdot P_1 = 455.6 \text{ kPa}$$

State 3 $T_3 = 1360 \text{ K} \Rightarrow h_3 = 1467.49 \text{ kJ/kg}$, $P_{r3} = 399.1$

State 4 For a turbojet, $\dot{W}_c/\dot{m} = \dot{W}_t/\dot{m} \Rightarrow (h_2 - h_1) = \eta_t (h_3 - h_{4s})$. Thus

$$h_{4s} = 1087.69 \text{ kJ/kg} \Rightarrow P_{r4s} = 131.51 \text{ and } P_4 = (P_{r4s}/P_{r3}) P_3 = 150.13 \text{ kPa}$$

Now, with the turbine efficiency

$$h_4 = h_3 - (h_3 - h_{4s}) \eta_t = 1129.47 \text{ kJ/kg} \Rightarrow P_{r4} = 151.07$$

State 5 For isentropic expansion through the nozzle

$$P_{r5s} = (P_5/P_4) P_4 = 18.113 \Rightarrow h_{5s} = 625.61 \text{ kJ/kg}$$

Now, with the isentropic nozzle efficiency

$$V_5 = \sqrt{\eta_N} V_{5s} = \sqrt{\eta_N} 2(h_4 - h_{5s}) \\ = \sqrt{(0.97) 2(1129.47 - 625.61) \frac{\text{kJ}}{\text{kg}}} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| = 988.7 \text{ m/s}$$

Now, applying Eq. 9.31 to a control volume enclosing the entire engine

$$F_x = \dot{m}(V_5 - V_a)$$

where F_x is the net force acting on the control volume.

Continued on next slide

Problem 9-100 continued

The mass flow rate is found from $\dot{m} = (\dot{V})_a / v_a$, where

$$v_a = \frac{RT_a}{P_a} = \frac{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}}\right)(216 \text{ K})}{(18 \text{ kPa})} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| = 3.444 \text{ m}^3/\text{kg}$$

and $\dot{m} = \frac{230 \text{ m}^3/\text{s}}{3.444 \text{ m}^3/\text{kg}} = 66.78 \text{ kg/s}$

Thus $F_x = (66.78 \frac{\text{kg}}{\text{s}})(988.7 \text{ m/s} - 265 \text{ m/s}) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kN}}{10^3 \text{ N}} \right| = 48.33 \text{ kN}$

Finally, the thrust is the force developed by the engine, or

$$F_{\text{thrust}} = -F_x = -48.33 \text{ kN} \leftarrow F_{\text{thrust}}$$

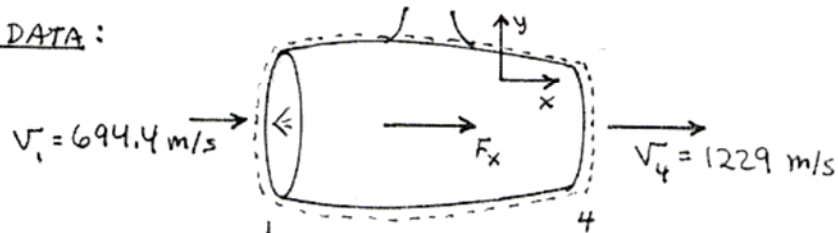
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PROBLEM 9.101

KNOWN: Conditions are known for the ramjet analyzed in Problem 9.83.

FIND: Determine the ratio of the thrust developed to the mass flow rate of air.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state. (2) The force F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot{m} (V_4 - V_1)$$

or

$$\begin{aligned} \frac{F_x}{\dot{m}} &= V_4 - V_1 \\ &= (1229 - 694.4) \frac{\text{m}}{\text{s}} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \\ &= 534.6 \frac{\text{N}}{\text{kg/s}} \end{aligned}$$

The thrust is the force developed by the engine. Thus

$$\frac{F_{\text{thrust}}}{\dot{m}} = - \frac{F_x}{\dot{m}} = - 534.6 \frac{\text{N}}{\text{kg/s}} \quad \xleftarrow{\quad F_{\text{thrust}}/\dot{m} \quad}$$

to the left

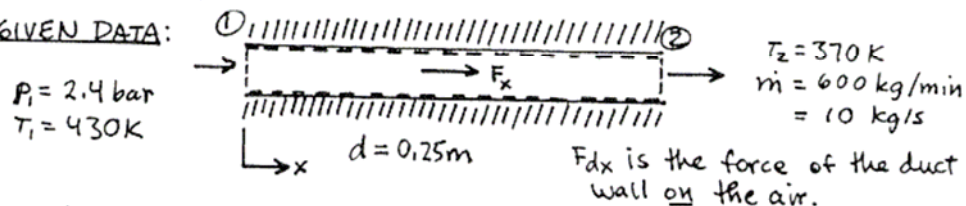
1. The effect of the surrounding pressure on the control volume boundary cancels.

PROBLEM 9.102

KNOWN: Air flows through a horizontal, well-insulated duct of constant cross-sectional area. Data are known at the inlet and exit.

FIND: Determine the magnitude and direction of the net horizontal force exerted by the duct wall on the air.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state, and $\dot{Q}_{cv} = \dot{W}_{cv} = 0$.
 (2) Potential energy effects are negligible. (3) The air behaves as an ideal gas.

ANALYSIS: To find the force F_x , we need to evaluate the velocities at ① and ② and P_2 . First

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}}\right)(430)}{(2.4 \text{ bars})} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| = 0.5142 \text{ m}^3/\text{kg}$$

$$A = \pi d^2/4 = 0.0491 \text{ m}^2$$

and
$$V_1 = \frac{\dot{m} v_1}{A} = \frac{(10 \text{ kg/s})(0.5142 \text{ m}^3/\text{kg})}{(0.0491 \text{ m}^2)} = 104.73 \text{ m/s}$$

To find V_2 , begin with the energy balance at steady state

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

or

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

From Table A-22; $h_1 = 431.43 \text{ kJ/kg}$ and $h_2 = 370.67 \text{ kJ/kg}$. Thus

$$V_2 = \sqrt{2(431.43 - 370.67) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| + 104.73^2 \text{ m}^2/\text{s}^2}$$

$$= 364.0 \text{ m/s}$$

Now, with $AV_1/v_1 = AV_2/v_2 = \dot{m}$

$$v_2 = \frac{AV_2}{\dot{m}} = \frac{(0.0491 \text{ m}^2)(364.0 \text{ m/s})}{(10 \text{ kg/s})} = 1.7872 \text{ m}^3/\text{kg}$$

And
$$P_2 = \frac{RT_2}{v_2} = \frac{\left(\frac{8.314}{28.97}\right)(370)}{(1.7872)} \left| \frac{10^3}{10^5} \right| = 0.594 \text{ bar} = 59.4 \text{ kPa}$$

Finally, using Eq. 9.31; $P_1 A - P_2 A + F_x = \dot{m}(V_2 - V_1)$. Thus

$$F_x = \dot{m}(V_2 - V_1) + (P_2 - P_1)A$$

$$= (10 \frac{\text{kg}}{\text{s}})(364.0 - 104.73) \frac{\text{m}}{\text{s}} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| + (0.594 - 2.4) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| (0.0491 \text{ m}^2)$$

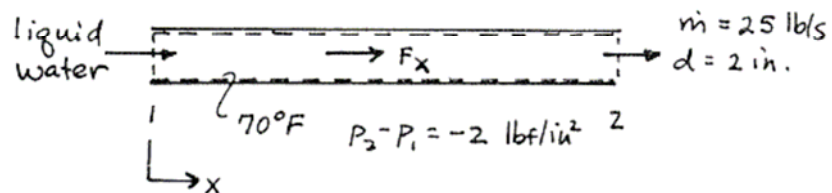
$$= -6275 \text{ N (to the left)} \quad \leftarrow F_x$$

PROBLEM 9.103

KNOWN: Water flows at steady state through a constant-diameter horizontal pipe. Data are known for flow through the pipe.

FIND: Determine the magnitude and direction of the horizontal force needed to hold the pipe in place.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state. (2) The water is modeled as incompressible, with $v = v_f @ 70^\circ\text{F}$. (3) The area of the pipe is constant.

ANALYSIS: Begin with the mass balance

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \Rightarrow V_1 = V_2$$

Now, applying Eq. 9.31

$$F_x + P_1 A - P_2 A = \dot{m} (V_x - V_1)$$

or

$$F_x = (P_2 - P_1) A = (P_2 - P_1) \left(\frac{\pi d^2}{4} \right)$$

$$= (-2 \text{ lbf/in}^2) \left(\frac{\pi (2 \text{ in})^2}{4} \right)$$

$$= -6.283 \text{ lbf (left)} \quad \xleftarrow{F_x}$$

This is the force of the pipe on the fluid, which acts to the left. Since the fluid force on the pipe is to the right, this is also the direction of the force required to hold the pipe in place.

PROBLEM 9.104

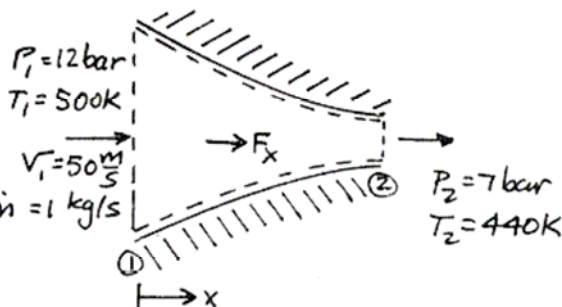
KNOWN: Air flows through a well-insulated, horizontal duct. Data are known at the inlet and exit.

FIND: Determine the force exerted by the air on the duct in the direction of flow.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL:

- (1) The control volume is at steady state with $\dot{W}_{cv} = \dot{Q}_{cv} = 0$.
- (2) Potential energy effects are negligible.
- (3) The air is modeled as an ideal gas.
- (4) The force F_x is the force of the duct on the air.



ANALYSIS: Applying Eq. 9.31

$$F_x + P_1 A_1 - P_2 A_2 = \dot{m} (V_2 - V_1)$$

$$\text{or } F_x = \dot{m} (V_2 - V_1) + P_2 A_2 - P_1 A_1 \quad (*)$$

Now, find V_2 using an energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} [(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)]$$

where $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Thus, with data from Table A-22

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2} = \sqrt{2(503.02 - 441.61) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| + 50^2 \text{ m}^2/\text{s}^2} = 354 \text{ m/s}$$

Next, calculate A_1 and A_2 using $\dot{m} = A V / v$ as follows:

$$A_1 = \frac{\dot{m} v_1}{V_1} = \frac{\dot{m} R T_1}{V_1 P_1} = \frac{(1 \text{ kg/s}) \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (500 \text{ K})}{(12 \text{ bar}) (50 \text{ m/s})} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| = 2.392 \times 10^{-3} \text{ m}^2$$

$$\text{Similarly } A_2 = \frac{(1) \left(\frac{8.314}{28.97} \right) (440)}{(7) (354)} \left| \frac{10^3}{10^5} \right| = 5.1 \times 10^{-4} \text{ m}^2$$

Inserting values in (*)

$$F_x = (1 \frac{\text{kg}}{\text{s}}) (354 - 50) \frac{\text{m}}{\text{s}} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| + [(7 \text{ bar}) (5.1 \times 10^{-4} \text{ m}^2) - (12) (2.39 \times 10^{-3})] \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| = -2009 \text{ N}$$

$$F_{\text{air/duct}} = -F_x = 2009 \text{ N (to the right)} \quad \leftarrow F_{\text{air/duct}}$$

PROBLEM 9.105

KNOWN: Ideal gas at a known temperature.

FIND: Determine the sonic velocity.

ANALYSIS: Eq. 9.37 gives the sonic velocity.

(a) Air at $60^\circ\text{F} = 520^\circ\text{R} \Rightarrow k = 1.4$ (Table A-20E)

$$c = \sqrt{kRT} = \sqrt{(1.4) \left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right) (520^\circ\text{R}) \left| \frac{32.2 \text{ ft} \cdot \text{lb}/\text{s}^2}{1 \text{ lbf}} \right|}$$
$$= 1118 \text{ ft/s} \longleftarrow c$$

(b) O_2 at $900^\circ\text{R} \Rightarrow k = 1.365$

$$c = \sqrt{(1.365) \left(\frac{1545}{32.00} \right) (900) | 32.2 |} = 1382 \text{ ft/s} \longleftarrow c$$

(c) Ar at $540^\circ\text{R} \Rightarrow$ For a monatomic gas, $k = 5/3$. Thus

$$c = \sqrt{(5/3) \left(\frac{1545}{39.94} \right) (540) | 32.2 |} = 1059 \text{ ft/s} \longleftarrow c$$

PROBLEM 9.106

KNOWN: A flash of lightning is sighted, and 3 seconds later thunder is heard.

FIND: Estimate how far away the strike was.

ENGINEERING MODEL: (1) The atmospheric air is at $20^\circ\text{C} = 293\text{K}$. (2) The ideal gas model is applicable.

ANALYSIS: From Table A-20, $k = 1.4$. With Eq. 9.37

$$c = \sqrt{(1.4) \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (293 \text{ K}) \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right|}$$
$$= 343 \text{ m/s}$$

Estimating the distance d

$$d = c \cdot t = (343 \text{ m/s})(3 \text{ s}) = 1029 \text{ m} = 0.64 \text{ mi} \quad \leftarrow \begin{array}{l} \text{estimated} \\ \text{distance} \end{array}$$

PROBLEM 9.107

KNOWN: Steam at a given state.

FIND: Estimate the sonic velocity using data from Table A-4 and using the ideal gas model. Compare.

ANALYSIS: Beginning with Eq. 9.36b

$$c = \sqrt{(-v^2) \left(\frac{\partial p}{\partial v} \right)_s} \approx \sqrt{(-v^2) \left(\frac{\Delta p}{\Delta v} \right)_s}$$

where the partial derivative is estimated by using differences in tabular data. At $T = 360^\circ\text{C}$, $p = 60 \text{ bar}$: $v = 0.04331 \text{ m}^3/\text{kg}$
 $s = 6.3782 \text{ kJ/kg} \cdot \text{K}$

Interpolating in adjacent table columns at the same entropy

T $^\circ\text{C}$	v m^3/kg	u kJ/kg	h kJ/kg	s $\text{kJ/kg} \cdot \text{K}$
-------------------------	-------------------------------	-----------------------	-----------------------	--------------------------------------

$p = 40 \text{ bar} = 4.0 \text{ MPa}$
 $(T_{\text{sat}} = 250.4^\circ\text{C})$

Sat.	0.04978	2602.3	2801.4	6.0701
280	0.05546	2680.0	2901.8	6.2568
320	0.06199	2767.4	3015.4	6.4553
360	0.06788	2845.7	3117.2	6.6215

$\rightarrow s = 6.3782$; $v = 0.05945 \text{ m}^3/\text{kg}$

$p = 80 \text{ bar} = 8.0 \text{ MPa}$
 $(T_{\text{sat}} = 295.06^\circ\text{C})$

Sat.	0.02352	2569.8	2758.0	5.7432
320	0.02682	2662.7	2877.2	5.9489
360	0.03089	2772.7	3019.8	6.1819
400	0.03432	2863.8	3138.3	6.3634
440	0.03742	2946.7	3246.1	6.5190

$\rightarrow s = 6.3782$; $v = 0.03461 \text{ m}^3/\text{kg}$

Using these values

$$c = \sqrt{-(0.04331^2) \frac{\text{m}^3}{\text{kg}} \frac{(80 - 40) \text{ bar}}{(0.03461 - 0.05945)} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{\text{kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|}$$

$$= 549.6 \text{ m/s} \leftarrow c$$

Using the function for H_2O in Table A-21: $c_p = 2.035 \text{ kJ/kg} \cdot \text{K}$ and with $c_p = kR/(k-1)$, $k = 1.293$. Thus, assuming the ideal gas model

$$c = \sqrt{kRT} = \sqrt{(1.293) \left(\frac{8.314}{18.02} \right) (633) \left| 10^3 \right|} = 614.5 \text{ m/s}$$

The ideal gas model appears to significantly over-estimate the sonic velocity at the given state.

PROBLEM 9.108

KNOWN: Carbon dioxide at 1 bar and a given velocity.

FIND: Plot the Mach number versus temperature ranging from 250 to 1000 K.

ENGINEERING MODEL: The carbon dioxide is modeled as an ideal gas, as can be verified readily.

ANALYSIS: Sample calculation for $T = 250$ K. From Eq. 9.37

$$c = \sqrt{kRT}$$

From Table A-20, at $T = 250$ K; $k = 1.314$. Thus

$$c = \sqrt{(1.314) \left(\frac{8.314 \text{ kJ}}{44.01 \text{ kg} \cdot \text{K}} \right) (250 \text{ K}) \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right|} = 249.1 \text{ m/s}$$

Now, with $V = 460$ m/s

$$M = V/c = 1.847$$

The data for the required plot are obtained using IT, as follows:

IT Code

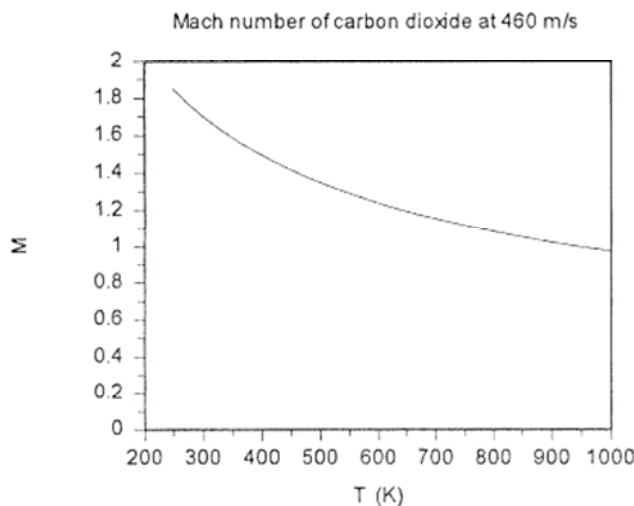
$T = 250 \text{ // K}$
 $V = 460 \text{ // m/s}$

$c_p = c_p(T(\text{"CO2"}, T))$
 $c_v = c_v(T(\text{"CO2"}, T))$
 $k = c_p / c_v$
 $R = 8.314 / 44.01$
 $c = \text{sqrt}(k * R * T * 1000)$
 $M = V / c$

IT Results for $T = 250$ K

$c_p = 0.7944 \text{ kJ/kg} \cdot \text{K}$
 $c_v = 0.6054 \text{ kJ/kg} \cdot \text{K}$
 $k = 1.312$
 $c = 248.9 \text{ m/s}$
 $M = 1.848$

PLOT:



Note that the sonic velocity increases with temperature. Thus, the Mach number decreases, as expected.

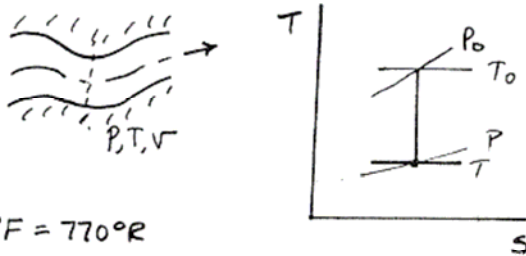
PROBLEM 9.109

KNOWN: An ideal gas flows through a duct. At a particular location, p , T , and V are known.

FIND: Determine M , T_0 , P_0 for three different gases at specified conditions.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: Each gas behaves as an ideal gas, as can be verified.



ANALYSIS:

(a) Air; $p = 100 \text{ lbf/in}^2$, $T = 310^\circ\text{F} = 770^\circ\text{R}$
 $V = 1400 \text{ ft/s}$

From Table A-20E at $T = 770^\circ\text{R}$; $k = 1.3935$. The sonic velocity is

$$c = \sqrt{kRT} = \sqrt{(1.3935) \left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right) (770^\circ\text{R}) \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right|} = 1357.4 \text{ ft/s}$$

and $M = V/c = 1400/1357.4 = 1.031$ (supersonic) (a) M

Using data from Table A-22E

$$h_0 = h + V^2/2 = 184.51 + \left(\frac{1400^2}{2} \frac{\text{ft}^2}{\text{s}^2} \right) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 223.63 \text{ Btu/lb} \Rightarrow T_0 = 929.9^\circ\text{R} \quad \text{(a) } T_0$$

and $P_0 = P \left(\frac{P_r(T_0)}{P_r(T)} \right) = (100) \left(\frac{9.4658}{4.829} \right) = 196.0 \text{ lbf/in}^2 \quad \text{(a) } P_0$

(b) Helium; $p = 20 \text{ lbf/in}^2$, $T = 520^\circ\text{R}$, $V = 900 \text{ ft/s}$

For the monatomic gas; $k = 5/3 = 1.667$, $c_p = \frac{5}{2} R = \frac{5}{2} \left(\frac{1545}{4.003} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right)$

$$c = \sqrt{kRT} = \sqrt{(1.667) \left(\frac{1545}{4.003} \right) (520) (32.2)} = 3281.9 \text{ ft/s} = 964.9 \text{ ft} \cdot \text{lbf/lb} \cdot ^\circ\text{R}$$

$M = 900/3281.9 = 0.2742$ (b) M

Thus $0 = c_p(T_0 - T) - V^2/2$

$$T_0 = T + \frac{V^2}{2c_p} = (520^\circ\text{R}) + \frac{900^2 \text{ ft}^2/\text{s}^2}{(2)(964.9 \text{ ft} \cdot \text{lbf/lb} \cdot ^\circ\text{R})} \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| = 533^\circ\text{R} \quad \text{(b) } T_0$$

$$P_0 = P \left(\frac{T_0}{T} \right)^{\frac{k}{k-1}} = (20) \left(\frac{533}{520} \right)^{2.5} = 21.27 \text{ lbf/in}^2 \quad \text{(b) } P_0$$

(c) Nitrogen; 50 lbf/in^2 , 600°R , $500 \text{ ft/s} \Rightarrow k = 1.399$

$$c = \sqrt{1.399 \left(\frac{1545}{28.01} \right) (600) \left| \frac{32.2}{1} \right|} = 1221.0 \text{ ft/s}; M = 500/1221 = 0.4095 \quad \text{(c) } M$$

Using data from Table A-23E; $\bar{h}_0 = \bar{h} + \frac{V^2}{2}$ (Mol. wt.)

$$\bar{h}_0 = 4167.9 \frac{\text{Btu}}{\text{lbmol}} + \frac{500^2 \text{ ft}^2/\text{s}^2}{2} \left(\frac{28.01 \text{ lb}}{\text{lbmol}} \right) \left| \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 4307.7 \text{ Btu/lbmol}$$

To get P_0 : $0 = \bar{s}^\circ(T_0) - \bar{s}^\circ(T) - \bar{R} \ln(P_0/p) \Rightarrow T_0 = 620^\circ\text{R} \quad \text{(c) } T_0$

$$\bar{R} \ln(P_0/p) = \frac{1}{2} [\bar{s}^\circ(T_0) - \bar{s}^\circ(T)] = \frac{(46.743 - 46.514)}{(1545/778)} = 0.1153 \Rightarrow P_0/p = 1.1222$$

$$\Rightarrow P_0 = 56.11 \text{ lbf/in}^2 \quad \text{(c) } P_0$$

PROBLEM 9.110

KNOWN: Air flows through a duct as in Problem 9.104.

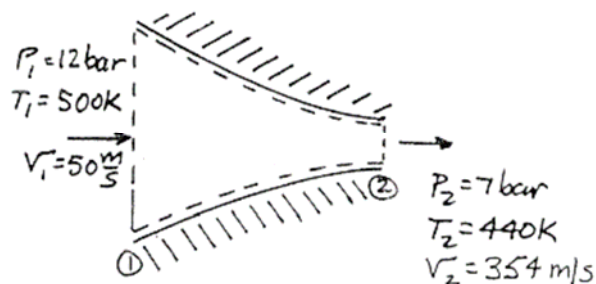
FIND: Determine the Mach number, stagnation temperature, and stagnation pressure at the inlet and exit of the duct.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: See

Solution to Problem 9.104.

ANALY



ANALYSIS: At state 1: $k_1 = 1.387$

From Table A-20. The sonic velocity is

$$c_1 = \sqrt{k_1 R T_1} = \sqrt{(1.387) \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (500 \text{ K})} \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| = 446.1 \text{ m/s}$$

Thus $M_1 = V_1 / c_1 = 50 / 446.1 = 0.112$ M_1

The stagnation enthalpy is

$$h_{01} = h_1 + V_1^2 / 2 = 503.02 \frac{\text{kJ}}{\text{kg}} + \left(\frac{50^2 \text{ m}^2/\text{s}^2}{2} \right) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 504.27 \text{ kJ/kg}$$

Interpolating in Table A-22; $T_{01} = 501.2 \text{ K}$

$$P_{01} = P_1 \left(\frac{Pr(T_{01})}{Pr(T_1)} \right) = (12 \text{ bar}) \left(\frac{8.4862}{8.411} \right) = 12.11 \text{ bar}$$
 T_{01}
 P_{01}

At state 2: $k_2 = 1.3918$

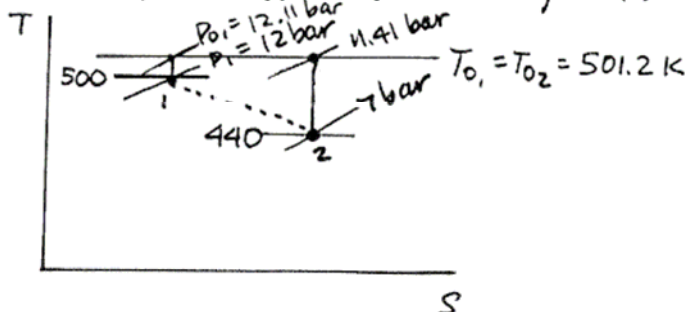
$$c_2 = \sqrt{(1.3918) \left(\frac{8.314}{28.97} \right) (440) | 10^3 |} = 419.2 \text{ m/s}$$

$$M_2 = 354 / 419.2 = 0.845$$
 M_2

$$h_{02} = h_{01} = 504.27 \text{ kJ/kg} \Rightarrow T_{02} = T_{01} = 501.2 \text{ K}$$
 T_{02}

$$P_{02} = P_2 \left(\frac{Pr(T_{02})}{Pr(T_2)} \right) = (7 \text{ bar}) \left(\frac{8.4862}{5.332} \right) = 11.14 \text{ bar}$$
 P_{02}

These states can be illustrated on a T-s diagram:



PROBLEM 9.111

KNOWN: Water vapor flows with known pressure, temperature, and velocity.

FIND: Using Fig. A-8E, determine (a) the stagnation enthalpy, (b) the stagnation temperature, and (c) the stagnation pressure.

SCHEMATIC & GIVEN DATA: $T = 600^\circ\text{F}$, $p = 500 \text{ lbf/in}^2$, $V = 1000 \text{ ft/s}$

ANALYSIS:

(a) $h \approx 1298 \text{ Btu/lb}$

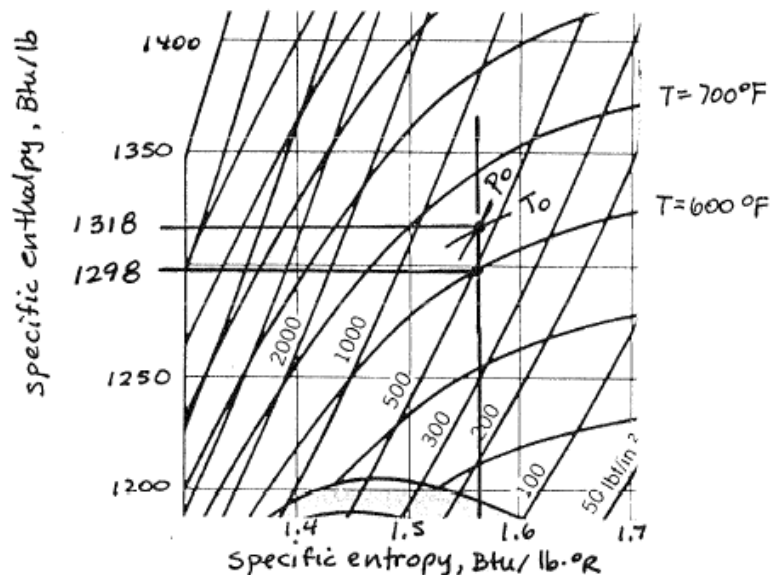
$$h_o = h + \frac{V^2}{2}$$

$$= 1298 \frac{\text{Btu}}{\text{lb}} + \left(\frac{1000^2 \text{ ft}^2/\text{s}^2}{2} \right) \left| \frac{1 \text{ lbf}}{32.2 \text{ ft} \cdot \text{lb/s}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right|$$

$$= 1318 \text{ Btu/lb}$$

(b) $T_o \approx 645^\circ\text{F}$

(c) $p_o \approx 600 \text{ lbf/in}^2$

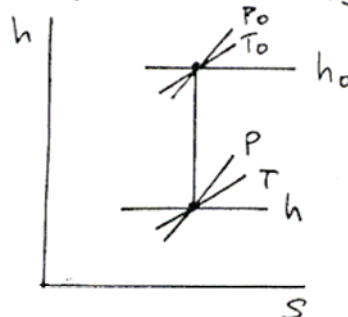
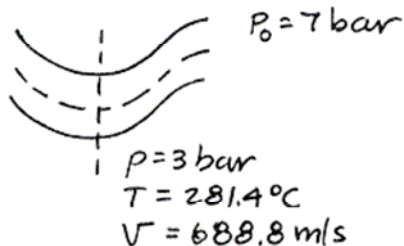


PROBLEM 9.112

KNOWN: Conditions for steam flowing at a particular location in a passageway are known. The stagnation pressure is known.

FIND: Determine the corresponding specific stagnation enthalpy and stagnation temperature.

SCHEMATIC & GIVEN DATA:



①

ANALYSIS: $h_0 = h + \frac{V^2}{2}$ From Table A-4; $h = 3031.5 \text{ kJ/kg}$

$$\text{Thus } h_0 = 3031.5 \frac{\text{kJ}}{\text{kg}} + \left(\frac{688.8^2 \text{ m}^2/\text{s}^2}{2} \right) \left| \frac{1 \text{ kg} \cdot \text{m}/\text{s}^2}{1 \text{ N}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 3268.7 \text{ kJ/kg} \longleftarrow h_0$$

$$\text{With } h_0 = 3268.7 \text{ and } p_0 = 7 \text{ bar} \Rightarrow T_0 = 400 \text{ K} \longleftarrow T_0$$

1. In early printings of the sixth edition, the values of T and V were given incorrectly. The values of $T = 281.4^\circ\text{C}$ and $V = 688.8 \text{ m/s}$ are correct. We regret the error.

PROBLEM 9.113

KNOWN: Isentropic flow of an ideal gas with constant specific heat ratio.

FIND: Develop the relation: $T^*/T_0 = 2/(k+1)$.

ANALYSIS: Begin with Eq. 9.50

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right) M^2$$

For $M=1$, $T=T^*$. Thus

$$\frac{T_0}{T^*} = 1 + \left(\frac{k-1}{2}\right) = \frac{k+1}{2}$$

① and

$$\frac{T^*}{T_0} = \frac{2}{k+1}$$

T^*/T_0

1. It follows directly that

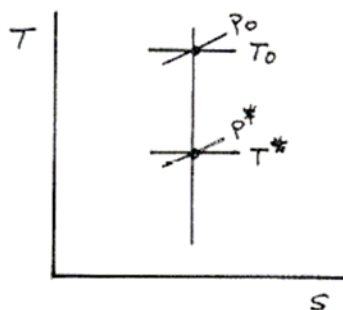
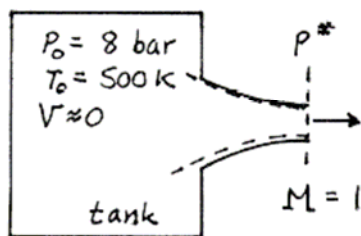
$$\frac{P^*}{P_0} = \left(\frac{T^*}{T_0}\right)^{\frac{k}{k-1}} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

PROBLEM 9.114

KNOWN: A gas expands isentropically through a converging nozzle from a large tank at known pressure and temperature.

FIND: Determine the critical pressure p^* and the corresponding temperature T^* if the gas is (a) air, (b) carbon dioxide, and (c) water vapor.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The gas expands isentropically. (3) The velocity in the tank is negligible. (4) The gas follows the ideal gas model. To avoid iteration, the specific heats are taken as constant, evaluated at 450 K.

ANALYSIS: With assumption (4), Eqs. 9.50 and 9.51 apply. Evaluating each one for $M = 1$, respectively, gives

$$\frac{T_0}{T^*} = \frac{k+1}{2}$$

$$\frac{P_0}{p^*} = \left(\frac{k+1}{2}\right)^{k/(k-1)}$$

(a) From Table A-20; at $T = 450$ K; $k = 1.391$ (air).

$$T^* = 418.2 \text{ K}$$

$$p^* = 4.238 \text{ bar}$$

(b) From Table A-20; at $T = 450$ K; $k = 1.239$ (CO_2)

$$T^* = 446.6 \text{ K}$$

$$p^* = 4.456 \text{ bar}$$

(c) Evaluating c_p using the relation in Table A-21 for H_2O ; $c_p = 1.926 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

Also

$$k = \frac{c_p}{c_p - R} = 1.315$$

Thus

$$T^* = 432.0 \text{ K}$$

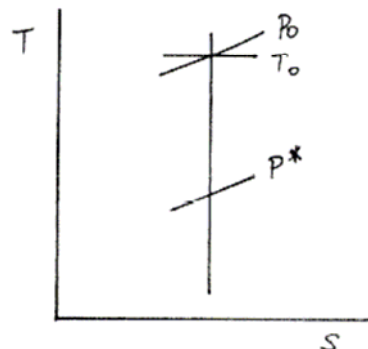
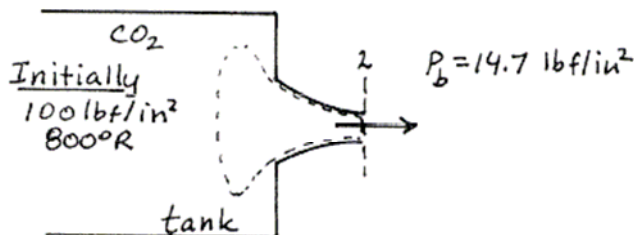
$$p^* = 4.344 \text{ bar}$$

PROBLEM 9.115

KNOWN: Carbon dioxide gas discharges from a large tank through a converging nozzle. The initial conditions in the tank and the pressure of the surroundings are known.

FIND: Estimate the pressure in the tank when the flow first ceases to be choked.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The tank pressure changes slowly, and the control volume can be considered to be at steady state for short periods of time. (2) The expansion can be considered isentropic at any time. (3) The carbon dioxide is modeled as an ideal gas with constant specific heats.

ANALYSIS: Under these assumptions, Eq. 9.51 can be applied, with $M=1$, to see if the flow is choked.

$$\frac{P_0}{P^*} = \left[1 + \frac{k-1}{2} (1) \right]^{k/(k-1)} = \left(\frac{k+1}{2} \right)^{k/(k-1)} = 1.80$$

where $k=1.246$ from Table A-20E at $T=760^\circ\text{R}$. For the initial conditions in the tank, $P^*=55.56 \text{ lbf/in}^2$. This is substantially greater than the pressure in the surroundings, $P_b=14.7 \text{ lbf/in}^2$.

As the pressure in the tank drops, the pressure at the nozzle exit also drops. Therefore, a plausible estimate of the tank pressure when the nozzle ceases to be choked is obtained when

$$P_b = P^* = 14.7 \text{ lbf/in}^2$$

Then

$$(P_0)_{\text{choking ceases}} = (14.7)(1.8) = 26.46 \text{ lbf/in}^2$$

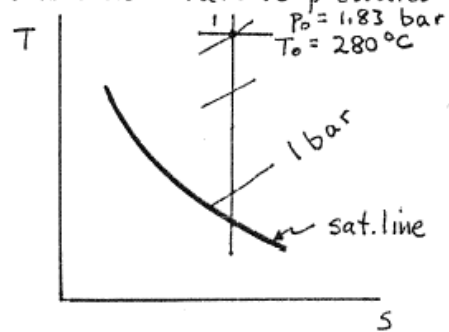
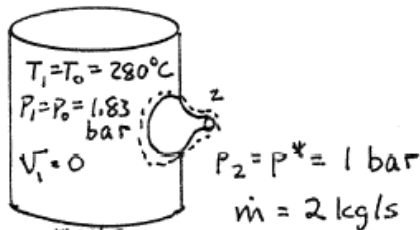
For tank pressures lower than this value, the exit plane pressure will be 14.7 lbf/in^2 , and the nozzle flow will not be choked.

PROBLEM 9.116

KNOWN: Steam expands isentropically through a converging nozzle from a large tank. The flow is choked, and the mass flow rate and exit plane pressure are known.

FIND: Determine the nozzle diameter at locations where various pressures are specified.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$.

(2) The steam expands isentropically. (3) The nozzle is choked. (4) The velocity in the tank is negligible, and potential energy effects can be ignored.

ANALYSIS: For each case to be considered, $P_1 = P_0 = 1.83$ bar and $T_1 = T_0 = 280^\circ\text{C}$.

Thus, from Table A-4, $h_1 = h_0 = 3031.9$ kJ/kg and $s_1 = s_0 = 7.8839$ kJ/kg·K.

Further, the exit velocity is found from

$$V_2 = \sqrt{2(h_0 - h_2)} \quad (1)$$

and

$$A_2 = \frac{\dot{m} V_2}{V_2} \Rightarrow d_2 = \sqrt{\frac{4A_2}{\pi}} \quad (2)$$

$P_2 = 1.5$ bar $h_2 = 2994.6$ kJ/kg, $v_2 = 1.6355$ m³/kg

$$V_2 = \sqrt{2(3031.9 - 2994.6) \frac{\text{kJ}}{\text{kg}} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|} = 273.13 \text{ m/s}$$

$$A_2 = \frac{(2 \text{ kg/s})(1.6355 \text{ m}^3/\text{kg})}{(273.13 \text{ m/s})} = 0.01198 \text{ m}^2$$

$$d_2 = \sqrt{\frac{(4)(0.01198) \text{ m}^2}{\pi} \left| \frac{10^4 \text{ cm}^2}{1 \text{ m}^2} \right|} = 12.348 \text{ cm} \leftarrow d_2 \text{ (1.5 bar)}$$

$P_2 = 1$ bar $h_2 = 2899.8$ kJ/kg, $v_2 = 2.2298$ m³/kg

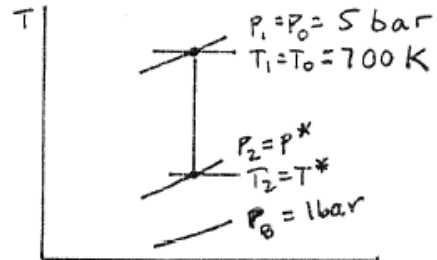
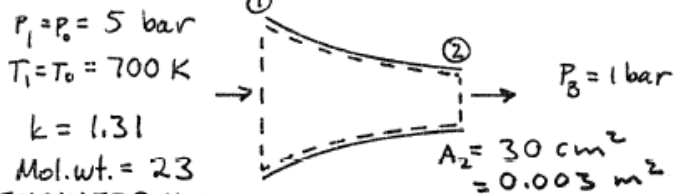
$$V_2 = 514.0 \text{ m/s}, A_2 = 0.008676 \text{ m}^2, d_2 = 10.510 \text{ cm} \leftarrow d_2 \text{ (1 bar)}$$

PROBLEM 9.117

KNOWN: An ideal gas mixture flows isentropically through a converging nozzle. The inlet conditions, back pressure, and exit area are specified.

FIND: Determine (a) the exit temperature, (b) the exit velocity, and (c) the mass flow rate.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The gas expands isentropically within the nozzle. (3) The velocity at the inlet is negligible, and potential energy effects can be ignored. (4) The mixture behaves as an ideal gas with constant specific heats.

ANALYSIS: First, it is necessary to determine if the nozzle is choked.

Using Eq. 9.51, with $M_1 = 1$

$$\frac{P_0}{P^*} = \left[1 + \left(\frac{k-1}{2} \right) (1)^2 \right]^{k/(k-1)} = \left(\frac{k+1}{2} \right)^{k/(k-1)} = 1.838$$

Thus, $P^* = P_0 / 1.838 = 2.720 \text{ bar}$. $P_0 < P^* \Rightarrow$ choked conditions $\Rightarrow P_2 = P^*$.

(a) $T_2 = T^*$. Using Eq. 9.50, with $M_2 = 1$

$$\frac{T^0}{T^*} = 1 + \left(\frac{k-1}{2} \right) (1)^2 = \frac{k+1}{2} = 1.155 \Rightarrow T^* = 606.1 \text{ K} \leftarrow T_2$$

(b) Since $M_2 = 1$, $V_2 = C_2 = \sqrt{kRT_2}$

$$V_2 = \sqrt{(1.31) \left(\frac{8.314 \text{ kJ}}{23 \text{ kg} \cdot \text{K}} \right) (606.1 \text{ K}) \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|}$$

$$= 535.7 \text{ m/s} \leftarrow V_2$$

(c) The mass flow rate is

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{P_2 A_2 V_2}{RT_2}$$

$$= \frac{(2.720 \text{ bar}) (0.003 \text{ m}^2) (535.7 \text{ m/s}) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|}{\left(\frac{8.314 \text{ kJ}}{23 \text{ kg} \cdot \text{K}} \right) (606.1 \text{ K})}$$

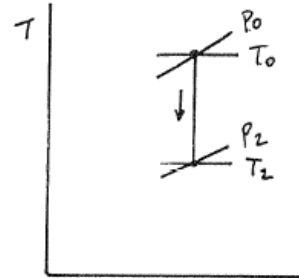
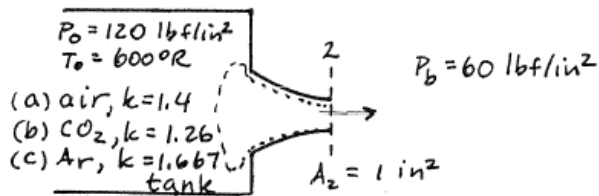
$$= 1.995 \text{ kg/s} \leftarrow \dot{m}$$

PROBLEM 9.118

KNOWN: An ideal gas expands isentropically through a converging nozzle from a large tank. The conditions in the tank, the pressure in the back, and the nozzle exit area are known.

FIND: Determine the mass flow rate if the gas is (a) air, (b) carbon dioxide, and (c) argon.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state. (2) The flow S is isentropic. (3) The gas behaves as an ideal gas with constant specific heats.

ANALYSIS: First, it is necessary to determine if the nozzle is choked. Using Eq. 9.51 with $M_2 = 1$

$$\frac{P_0}{P^*} = \left[1 + \left(\frac{k-1}{2} \right) (1)^2 \right]^{k/(k-1)} = \left(\frac{k+1}{2} \right)^{k/(k-1)} \quad (*)$$

(a) For air, with $k = 1.4$

$$\frac{P_0}{P^*} = 1.89293 \Rightarrow P^* = 63.39 \text{ lbf/in}^2$$

Since $P_b < P^*$, the nozzle is choked. Thus, $P_2 = P^* = 63.39 \text{ lbf/in}^2$.

Now, for the isentropic expansion

$$\frac{T_2}{T_0} = \left(\frac{P_2}{P_0} \right)^{\frac{k-1}{k}} \Rightarrow T_2 = 500^\circ\text{R}$$

Using the energy balance: $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} [(h_0 - h_2) + \frac{V_0^2 - V_2^2}{2} + g(z_0 - z_2)]$

$$\begin{aligned} V_2 &= \sqrt{2(h_0 - h_2)} = \sqrt{2c_p(T_0 - T_2)} \\ &= \sqrt{(2)(0.24 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}})(600 - 500)^\circ\text{R}} \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right| \left| \frac{778 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}} \right| \\ &= 1096.6 \text{ ft/s} \end{aligned}$$

All of the conditions are known at the exit. Thus

$$\begin{aligned} \dot{m} &= \frac{A_2 V_2}{v_2} = \frac{A_2 V_2 P_2}{R T_2} \\ &= \frac{(1 \text{ in}^2)(1096.6 \text{ ft/s})(63.39 \text{ lbf/in}^2)}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right)(500^\circ\text{R})} = 2.61 \text{ lb/s} \leftarrow \dot{m} \end{aligned}$$

Continued on next slide

Problem 9-118 continued

(b) For CO_2 , with $k=1.26$ ($c_p = 0.2187 \text{ Btu/lb} \cdot ^\circ\text{R}$)

$$\frac{P_0}{p^*} = 1.8081 \Rightarrow p^* = 66.37 \text{ lbf/in}^2$$

Again, $P_b < p^* \Rightarrow$ choked conditions, and $P_z = p^* = 66.37 \text{ lbf/in}^2$.

Thus

$$\frac{T_z}{T_0} = \left(\frac{P_z}{P_0}\right)^{\frac{k-1}{k}} \Rightarrow T_z = 531.0^\circ\text{R}$$

and

$$V_z = \sqrt{(2)(0.2187)(600 - 531.0) \left| \frac{32.2}{778} \right|} = 869.5 \text{ ft/s}$$

Finally

$$\dot{m} = \frac{(1)(869.5)(66.37)}{\left(\frac{1545}{44.01}\right)(531.0)} = 3.096 \text{ lb/s} \leftarrow \text{m}$$

(c) For Ar , $k=1.667$ ($c_p = 0.1243 \text{ Btu/lb} \cdot ^\circ\text{R}$)

$$\frac{P_0}{p^*} = 2.053 \Rightarrow p^* = 58.45 \text{ lbf/in}^2$$

In this case, $P_b > p^*$, which means the nozzle is not choked. Thus, the exit plane pressure is equal to the back region pressure. That is

$$P_z = P_b = 60 \text{ lbf/in}^2$$

and

$$\frac{T_z}{T_0} = \left(\frac{P_z}{P_0}\right)^{\frac{k-1}{k}} \Rightarrow T_z = 454.7^\circ\text{R}$$

The exit velocity is

$$V_z = \sqrt{(2)(0.1243)(600 - 454.7) \left| \frac{32.2}{778} \right|} = 951.3 \text{ ft/s}$$

Finally, the mass flow rate is

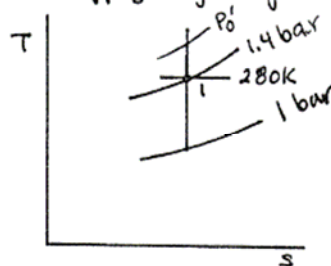
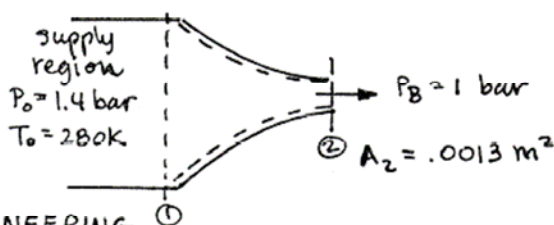
$$\dot{m} = \frac{(1)(951.3)(60)}{\left(\frac{1545}{39.94}\right)(454.7)} = 3.245 \text{ lb/s} \leftarrow \text{m}$$

PROBLEM 9.119

KNOWN: Air expands isentropically through a converging nozzle and discharges to the atmosphere. The inlet conditions are specified and the exit plane area is given.

FIND: Determine the mass flow rate. Is it possible to increase the mass flow rate by increasing the supply region pressure?

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air expands isentropically. (3) The air behaves as an ideal gas. (4) Because of the small pressure difference, the temperature range will be limited. Therefore, assume $k = 1.4$ is constant.

ANALYSIS: (a) First, determine if the nozzle is choked. Using Eq. 9.51 with $M = 1$ gives

$$\frac{P_0}{P^*} = \left(\frac{k+1}{2} \right)^{k/(k-1)} = 1.8929 \Rightarrow P^* = 0.7396 \text{ bar}$$

NOT CHOKED
∴ $P_2 = 1 \text{ bar}$

For the isentropic expansion

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 254.3 \text{ K}$$

The exit velocity is, with $c_p = 1.004 \text{ kJ/kg} \cdot \text{K}$

$$\begin{aligned} V_2 &= \sqrt{2 c_p (T_1 - T_2)} \\ &= \sqrt{2 (1.004 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) (280 - 254.3) \text{ K} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|} \\ &= 227.2 \text{ m/s} \end{aligned}$$

Thus, the mass flow rate is

$$\begin{aligned} \dot{m} &= \frac{A_2 V_2 P_2}{R T_2} = \frac{(0.0013 \text{ m}^2) (227.2 \text{ m/s}) (1 \text{ bar}) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|}{\left(\frac{8.314 \cdot \text{kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (254.3 \text{ K})} \\ &= 0.4047 \text{ kg/s} \end{aligned}$$

$\dot{m}$
(a)

(b) Next, consider the case for which $p_0 = 2 \text{ bar}$, $T_0 = 280 \text{ K}$ and $p_B = 1 \text{ bar}$. with Eq. 9.51

$$\frac{P_0}{P^*} = \left(\frac{k+1}{2} \right)^{k/(k-1)} = 1.8929 \Rightarrow P^* = 1.0566 \text{ bar}$$

$$P_B < P^* \Rightarrow \text{choked conditions} \Rightarrow P_2 = P^* = 1.0566 \text{ bar}$$

Continued on next slide

Problem 9-119 continued

Thus, $T_2 = T^*$. From Eq. 9.50 with $M=1$

$$\frac{T_0}{T^*} = \frac{k+1}{2} = 1.2 \Rightarrow T^* = 233.33 \text{ K}$$

The exit velocity is

$$V_2 = \sqrt{2(1.004)(280 - 233.33) \times 10^3} = 306.1 \text{ m/s}$$

Thus, the mass flow rate is

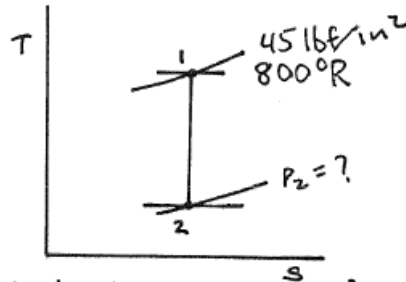
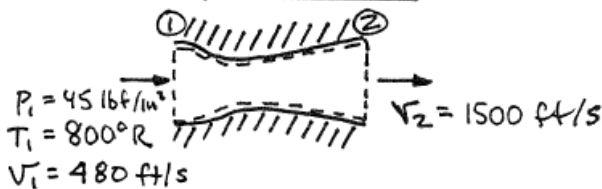
$$\dot{m} = \frac{(0.0013)(306.1)(1)}{\left(\frac{8.314}{28.97}\right)(233.33)} \left| \frac{10^5}{10^3} \right| = 0.5943 \text{ kg/s} \quad \leftarrow \dot{m}_{(b)}$$

PROBLEM 9.120

KNOWN: Air expands isentropically through a nozzle from known inlet conditions to a known exit velocity.

FIND: Determine (a) the exit pressure, (b) the ratio of exit area to inlet area, (c) whether the nozzle is converging-diverging or converging only.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air undergoes an isentropic process. (3) The air behaves as an ideal gas.

ANALYSIS: To fix the exit state, determine h_2 using an energy balance.

Thus

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$$

or

$$\begin{aligned} h_2 &= h_1 + \frac{V_1^2 - V_2^2}{2} \\ &= 191.81 \frac{\text{Btu}}{\text{lb}} + \left(\frac{480^2 - 1500^2}{2} \right) \frac{\text{ft}^2}{\text{s}^2} \left| \frac{1 \text{ lbf} \cdot \text{s}^2}{32.2 \text{ lb} \cdot \text{ft}} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\ &= 151.50 \frac{\text{Btu}}{\text{lb}} \Rightarrow P_{r2} = 2.4265, T_2 = 633.4^\circ\text{R} \end{aligned}$$

Finally

$$P_2 = P_1 \left(\frac{P_{r2}}{P_{r1}} \right) = (45 \frac{\text{lbf}}{\text{in}^2}) \left(\frac{2.4265}{5.526} \right) = 19.760 \frac{\text{lbf}}{\text{in}^2} \leftarrow P_2$$

(b) From the mass balance; $\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$. With $v = RT/P$,

$$\frac{A_1 V_1 P_1}{RT_1} = \frac{A_2 V_2 P_2}{RT_2} \Rightarrow \frac{A_2}{A_1} = \frac{V_1}{V_2} \cdot \frac{P_1}{P_2} \cdot \frac{T_2}{T_1} = 0.577 \leftarrow A_2/A_1$$

(c) To determine the nozzle type, calculate the inlet and exit Mach numbers.

$$\begin{aligned} M_1 &= \frac{V_1}{\sqrt{kRT_1}} = \frac{(480 \text{ ft/s})}{\sqrt{(1.39) \left(\frac{1545}{28.97} \right) \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} (800^\circ\text{R}) \left| \frac{32.2 \text{ lb} \cdot \text{ft}}{1 \text{ lbf} \cdot \text{s}^2} \right|}} \\ &= 0.347 \text{ (subsonic)} \end{aligned}$$

$$M_2 = \frac{(1500)}{\sqrt{(1.4) \left(\frac{1545}{28.97} \right) (633.4) \left| \frac{32.2}{\text{ft} \cdot \text{lbf}} \right|}} = 1.216 \text{ (supersonic)}$$

Thus, a converging-diverging nozzle is needed.

Nozzle
type

PROBLEM 9.121

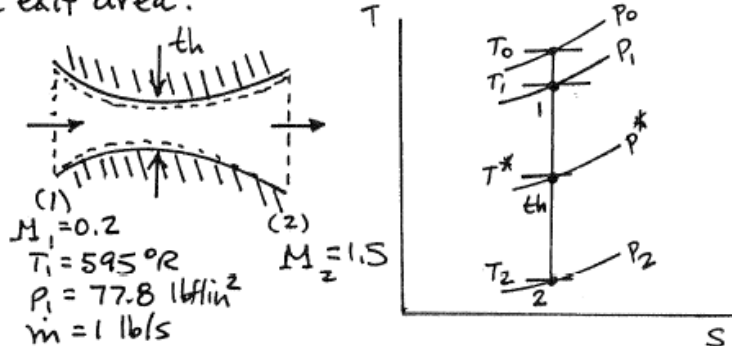
KNOWN: Air expands isentropically through a converging-diverging nozzle from known inlet conditions to a specified Mach number at the exit.

FIND: Determine (a) the stagnation pressure and temperature, (b) the throat area, and (c) the exit area.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$.

(2) The air undergoes an isentropic process. (3) The air behaves as an ideal gas with $k = 1.4$.



ANALYSIS: (a) Using data from Table 9.2 for the isentropic process, at $M_1 = 0.2$

$$P_1/P_0 = 0.97250 \Rightarrow P_0 = \frac{77.8}{0.97250} = 80 \text{ lbf/in}^2 \leftarrow P_0$$

$$T_1/T_0 = 0.99206 \Rightarrow T_0 = \frac{595}{0.99206} = 600^\circ\text{R} \leftarrow T_0$$

(b) Since $M_1 < 1$ and $M_2 > 1$, the flow is sonic at the throat. Thus

$$T_{th} = T^* = T_0(0.83333) = 500^\circ\text{R}$$

$$P_{th} = P^* = P_0(0.52828) = 42.26 \text{ lbf/in}^2$$

$$\text{Thus } v_{th} = \frac{RT^*}{P^*} = \frac{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}})(500^\circ\text{R})}{(42.26 \text{ lbf/in}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 4.382 \text{ ft}^3/\text{lb}$$

$$\text{and } V_{th} = \sqrt{kRT^*} = \sqrt{(1.4) \left(\frac{1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right) (500^\circ\text{R}) \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right|} = 1096.4 \text{ ft/s}$$

$$\text{Finally } A_{th} = \frac{\dot{m} v_{th}}{V_{th}} = \frac{(1 \text{ lb/s})(4.382 \text{ ft}^3/\text{lb})}{(1096.4 \text{ ft/s})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 0.576 \text{ in}^2 \leftarrow A_{th}$$

(c) From Table 9-2, at $M_2 = 1.5$; $A_2/A^* = 1.1762$. Thus

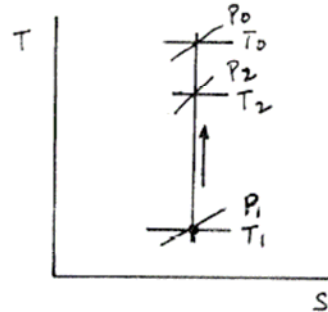
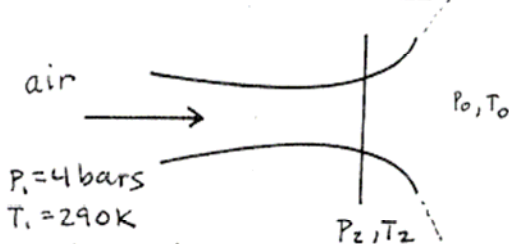
$$A_2 = (1.1762)(0.576) = 0.677 \text{ in}^2 \leftarrow A_2$$

PROBLEM 9.122

KNOWN: Air with $k=1.4$ flows isentropically through a diffuser.

FIND: Plot velocity, Mach number, and area ratio A/A^* for locations in the flow corresponding to a range of pressures.

SCHEMATIC & GIVEN DATA:



$P_1 = 4 \text{ bars}$
 $T_1 = 290 \text{ K}$
 $V_1 = 512 \text{ m/s}$

ENGINEERING

MODEL: (1) Isentropic flow. (2) Ideal gas model with $k=1.4$.

ANALYSIS: The following expressions are used to calculate the required quantities:

$$1. \quad c = \sqrt{kRT}$$

$$2. \quad M = V/c$$

$$3. \quad P_0/P = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)} \quad (\text{Eq. 9.51})$$

$$4. \quad T_0/T = \left[P_0/P\right]^{(k-1)/k} \quad (\text{Eq. 9.50})$$

$$5. \quad A/A^* = \frac{1}{M} \left[\left(\frac{2}{k+1}\right) \left(1 + \frac{k-1}{2} M^2\right)\right]^{(k+1)/(2(k-1))} \quad (\text{Eq. 9.52})$$

First, use given values of T_1, P_1, V_1 to get c_1, M_1, P_0, T_0 . Then, vary P to get $M, T, A/A^*$, and V .

The data for the required plots are obtained using IT, as follows:

IT Code

$p1 = 4 \text{ // bar}$

$T1 = 290 \text{ // K}$

$V1 = 512 \text{ // m/s}$

$k = 1.4$

$Rbar = 8.314 \text{ // kJ/kmol-K}$

$MW = 28.97 \text{ // kg/kmol}$

$c1 = \text{sqrt}(k * (Rbar / MW) * T1 * 1000)$

$M1 = V1 / c1$

$po / p1 = (1 + ((k - 1) / 2) * M1^2)^{(k / (k - 1))}$

$To / T1 = (po / p1)^{((k - 1) / k)}$

$p = 10$

$c = \text{sqrt}(k * (Rbar / MW) * T * 1000)$

$M = V / c$

$Aratio = (1 / M) * ((2 / (k + 1)) * (1 + ((k - 1) / 2) * M^2))^{((k + 1) / (2 * (k - 1)))}$

$po / p = (1 + ((k - 1) / 2) * M^2)^{(k / (k - 1))}$

$To / T = (po / p1)^{((k - 1) / k)}$

IT Results for $p = 10 \text{ bar}$

$T_0 = 420.5 \text{ K}$

$p_0 = 14.6 \text{ bar}$

$T = 290 \text{ K}$

$c = 341.3 \text{ m/s}$

$V = 260 \text{ m/s}$

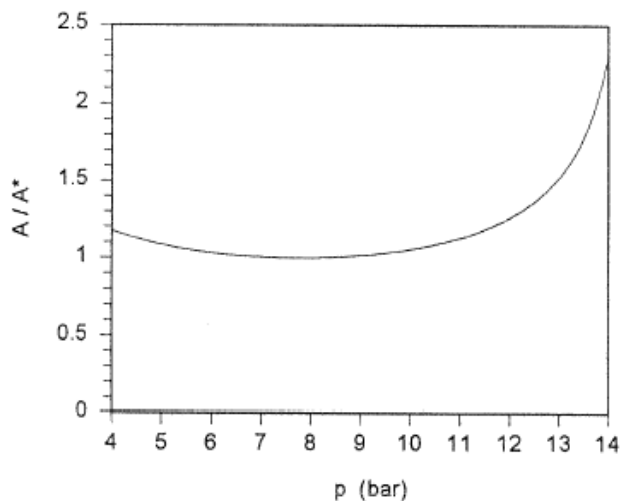
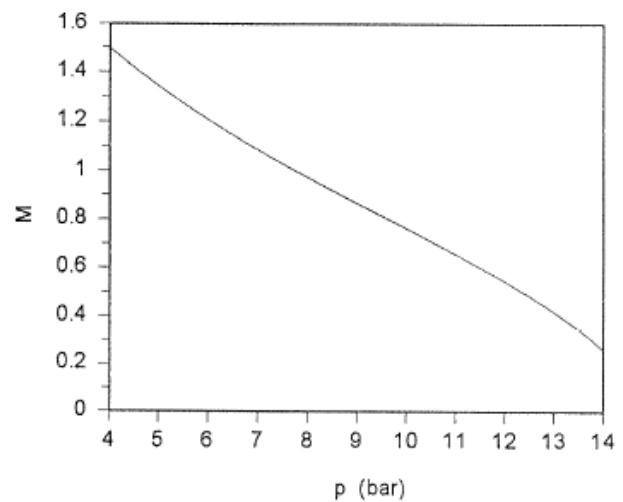
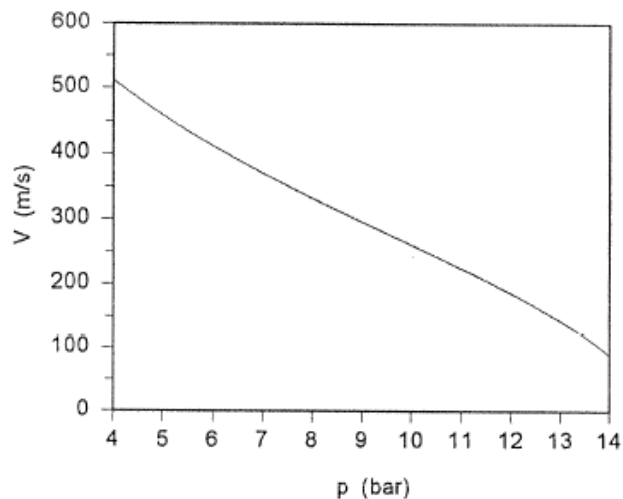
$M = 0.7616$

$A/A^* = 1.056$

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Problem 9-122 continued

Plots:



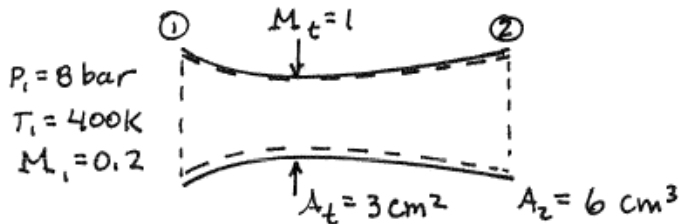
Note that the minimum value of A/A^* occurs at a Mach number of unity, as expected.

PROBLEM 9.123

KNOWN: A converging-diverging nozzle operates at steady state with isentropic flow of air.

FIND: Determine specified conditions at the nozzle exit and throat.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The air behaves as an ideal gas with $k = 1.4$.

ANALYSIS: Use Table 9.2.

First, find the stagnation state.

$$M_1 = 0.2 \Rightarrow T_1/T_0 = 0.99206 \Rightarrow T_0 = 403.2 \text{ K}$$

$$P_1/P_0 = 0.97250 \Rightarrow P_0 = 8.226 \text{ bar}$$

If the nozzle flow is choked, $M_t = 1$, and

$$T_t = T_0(0.83333) = 336.0 \text{ K}$$

$$P_t = P_0(0.52828) = 4.3456 \text{ bar}$$

Further,

$$v_{th} = \frac{RT_t}{P_t} = 0.2219 \text{ m}^3/\text{kg}$$

$$V_{th} = (1)\sqrt{kRT_{th}} = 367.4 \text{ m/s}$$

Thus, the mass flow rate is

$$\dot{m} = \frac{A_t V_{th}}{v_t} = 0.497 \text{ kg/s} \leftarrow \dot{m}$$

Now, with $A_2/A^* = 6 \text{ cm}^2 / 3 \text{ cm}^2 = 2$, there are two cases in Table 9.2.

Supersonic

$$M_2 \approx 2.20 \Rightarrow P_2 = (0.09352) P_0 = 0.769 \text{ bar} \leftarrow P_2$$

$$T_2 = (0.50813) T_0 = 204.9 \text{ K} \leftarrow T_2$$

Subsonic

$$M_2 \approx 0.3079 \Rightarrow P_2 = (0.93601) P_0 = 7.70 \text{ bar} \leftarrow P_2$$

$$T_2 = (0.98127) T_0 = 395.6 \text{ K} \leftarrow T_2$$

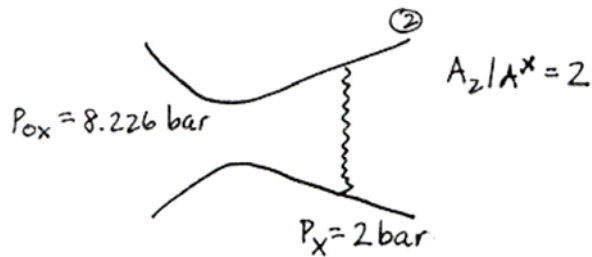
PROBLEM 9.124

$$M_x = 2.2 \Rightarrow M_y = 0.54706 \text{ and } p_y/p_x = 5.48$$

$$\text{with } p_x = 0.769; p_y = p_b = 4.214 \text{ bar} \leftarrow \frac{\text{back}}{\text{pressure}}$$

PROBLEM 9.125

(See Problem 9.123 for details.)



$$\frac{P_x}{P_{0x}} = \frac{2}{8.226} = 0.24313 \Rightarrow M_x = 1.579$$

Using this value in Table 9.3 gives $P_{0y}/P_{0x} = 0.90246$. Thus, with $P_{0x} = 8.226$; $P_{0y} = 7.424$ bar.

Now, for $A_2/A^* = 2$, and subsonic flow after the shock, from Table 9.1

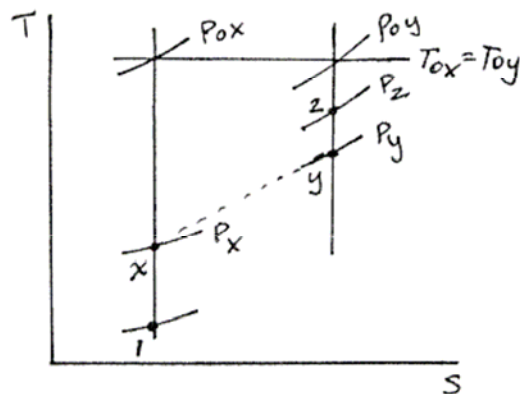
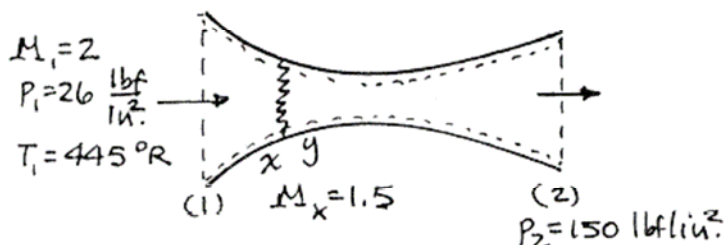
$$P_2/P_0 = 0.93601 \Rightarrow P_2 = 6.949 \text{ bar} \leftarrow \text{back pressure}$$

PROBLEM 9.126

KNOWN: Air flows through a converging-diverging duct with known conditions at the inlet and a known exit pressure. A normal shock stands at a location in the converging section where $M_x = 1.5$.

FIND: Determine the temperature and Mach number at the exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The flow is isentropic everywhere except in the immediate vicinity of the shock. (2) The air behaves as an ideal gas with $k = 1.4$.

ANALYSIS: First, determine the upstream stagnation conditions using data from Table 9-2. At $M_1 = 2$

$$P_1/P_{0x} = 0.12780 ; P_{0x} = \frac{26}{0.12780} = 203.44 \frac{\text{lbf}}{\text{in}^2}$$

$$T_1/T_{0x} = 0.55556 ; T_{0x} = \frac{445}{0.55556} = 801^\circ\text{R}$$

Now, from Table 9-3 at $M_x = 1.5$; $P_{0y}/P_{0x} = 0.92978 \Rightarrow P_{0y} = 189.15 \frac{\text{lbf}}{\text{in}^2}$

With $P_2 = 150$, we get

$$\frac{P_2}{P_{0y}} = \frac{150}{189.15} = 0.79302$$

Interpolating in Table 9-2 at $P/P_0 = 0.79302$

$$M_2 = 0.5847 \leftarrow M_2$$

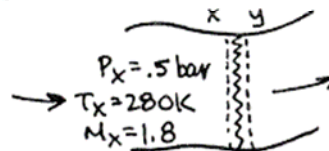
$$\text{and } T_2/T_0 = 0.93583 \Rightarrow T_2 = 749.6^\circ\text{R} \leftarrow T_2$$

PROBLEM 9.127

KNOWN: Air undergoes a normal shock. Conditions upstream are known.

FIND: Determine (a) P_y , (b) P_{0x} , (c) T_{0x} , and (d) the change in specific entropy across the shock, $s_y - s_x$.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: The air is an ideal gas with constant specific heat ratio, $k = 1.4$.

ANALYSIS: (a) Using Table 9.3, at $M_x = 1.8$;

$$P_y/P_x = 3.6133 \Rightarrow P_y = (3.6133)(0.5 \text{ bar}) = 1.8066 \text{ bar} \quad \leftarrow P_y$$

(b) Using Table 9.3, at $M_x = 1.8$;

$$P_x/P_{0x} = 0.17404 \Rightarrow P_{0x} = \frac{(0.5 \text{ bars})}{0.17404} = 2.873 \text{ bar} \quad \leftarrow P_{0x}$$

(c) Also from Table 9.2, $T_x/T_{0x} = 0.60680 \Rightarrow T_{0x} = 461.4 \text{ K} \quad \leftarrow T_{0x}$

(d) To calculate $s_y - s_x$,

$$s_y - s_x = c_p \ln(T_y/T_x) - R \ln(P_y/P_x)$$

These ratios can be read directly from Table 9.3;

$$\begin{aligned} s_y - s_x &= 1.004 \ln(1.5316) - \frac{8.314}{28.97} \ln(3.6133) \\ &= 0.05935 \text{ kJ/kg} \cdot \text{K} \quad \leftarrow \text{AS} \end{aligned}$$

The data for the required plots are obtained using IT, as follows:

IT Code

```
px = 0.5 // bar
Tx = 280 // K
Mx = 1.8
k = 1.4
```

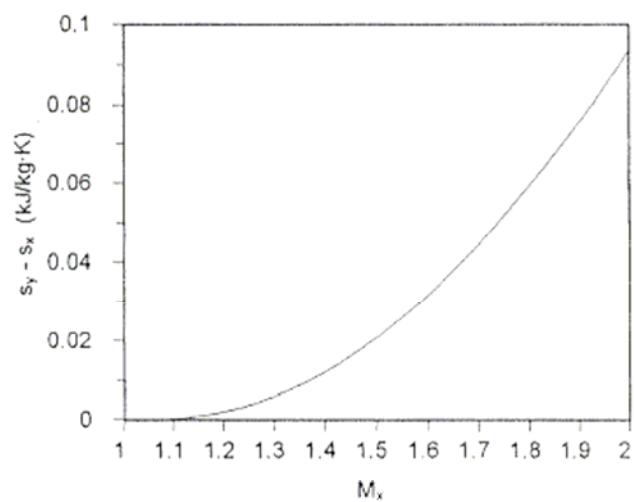
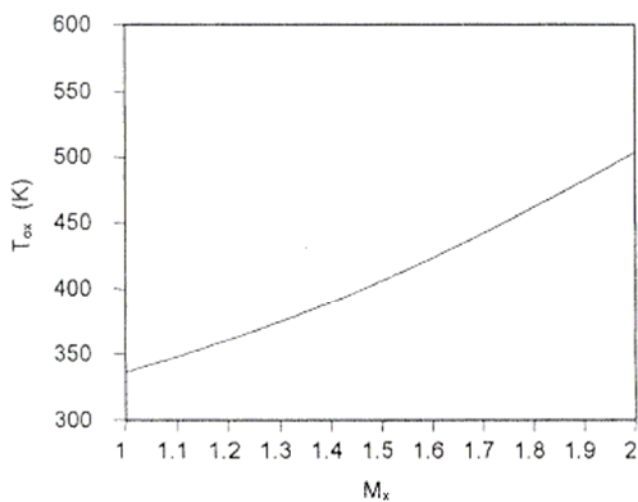
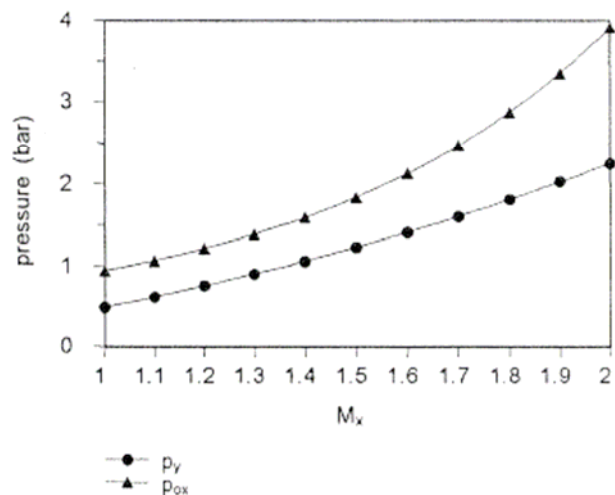
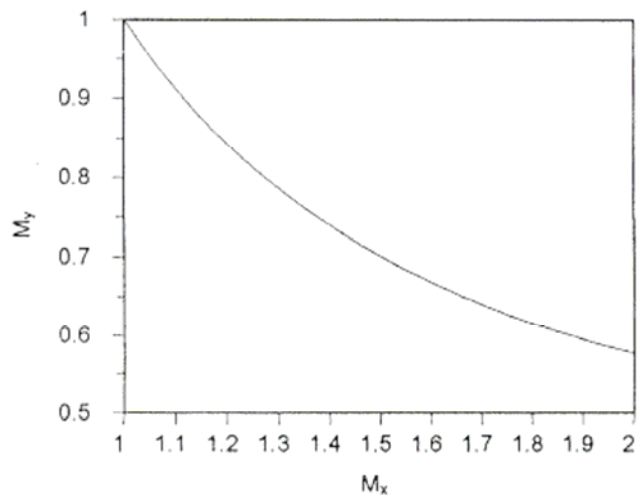
```
My^2 = (Mx^2 + (2/(k-1)))/(((2*k)/(k-1))*Mx^2 - 1)
py / px = (1 + k*Mx^2)/(1 + k*My^2)
Ty / Tx = (1 + ((k-1)/2)*Mx^2)/(1 + ((k-1)/2)*My^2)
pox / px = (1 + ((k-1)/2)*Mx^2)^(k/(k-1))
Tox / Tx = (pox / px)^(k/(k-1))
dels = 1.004 * ln(Ty/Tx) - (8.314 / 28.97) * ln(py/px)
```

IT Results for Mx = 1.8

```
My = 0.6165
Tox = 461.4 K
Ty = 428.8 K
pox = 2.873 bar
py = 1.807 bar
sy - sx = 0.05933 kJ/kg.K
```

Continued on next slide

Problem 9-127 continued

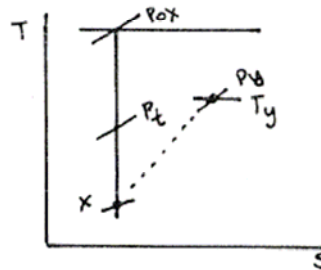
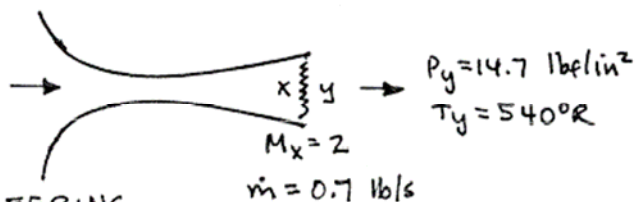


PROBLEM 9.128

KNOWN: Air flows through a converging-diverging nozzle and discharges to the atmosphere. The flow is isentropic up to the exit plane where a normal shock stands.

FIND: Determine (a) the stagnation pressure, p_{0x} , (b) the stagnation temperature, T_{0x} , and (c) the nozzle exit area.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL:

- (1) The air behaves as an ideal gas, with constant $k = 1.4$.
- (2) The flow up to the shock is isentropic.

ANALYSIS: (a) Using Table 9.3, at $M_x = 2.0$;

$$p_y/p_x = 4.5000 \Rightarrow p_x = 3.267 \text{ lbf/in}^2$$

and, from Table 9.2; $p_x/p_{0x} = 0.12780 \Rightarrow p_{0x} = 25.56 \frac{\text{lbf}}{\text{in}^2}$ ← p_{0x}

(b) Similarly, $T_y/T_x = 1.6875 \Rightarrow T_x = 320^\circ\text{R}$

and $T_x/T_{0x} = .55556 \Rightarrow T_{0x} = 576^\circ\text{R}$ ← T_{0x}

(c) The velocity at x is

$$V_x = M_x \sqrt{k R T_x} = 2 \sqrt{(1.4) \left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right) (320^\circ\text{R}) \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right|}$$

$$= 1754 \text{ ft/s}$$

Thus

$$A_x = \frac{\dot{m} R T_x}{V_x p_x} = \frac{(0.7 \text{ lb/s}) \left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot ^\circ\text{R}} \right) (320^\circ\text{R})}{(1754 \text{ ft/s}) (3.267 \text{ lbf/in}^2)} = 2.08 \text{ in}^2 \leftarrow A_x$$

PROBLEM 9.129

KNOWN: See Problem 9.128

FIND: Determine the throat area and the entropy produced per unit mass of air flowing.

ANALYSIS: From Table 9.2; $A_x/A^* = 1.6875 \Rightarrow A^* = 1.185 \text{ in}^2$ ← throat area

The entropy produced is

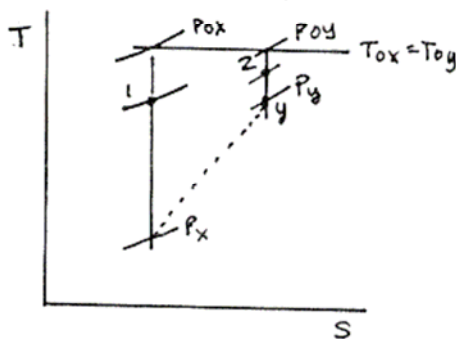
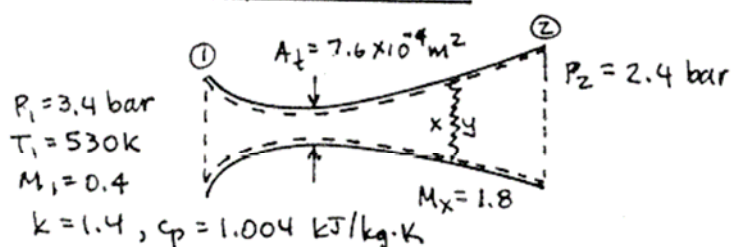
$$\begin{aligned}\dot{\sigma}/\dot{m} &= s_y - s_x = c_p \ln(T_y/T_x) - R \ln(p_y/p_x) \\ &= (0.240 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}) \ln(1.6875) - \left(\frac{1.986}{28.97} \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) \ln(4.5000) \\ &= 0.0225 \text{ Btu/lb} \cdot ^\circ\text{R} \leftarrow \dot{\sigma}/\dot{m}\end{aligned}$$

PROBLEM 9.130

KNOWN: Air flows through a converging-diverging nozzle. A normal shock stands in the diverging section, and the flow is isentropic elsewhere.

FIND: Determine (a) T_{0x} , (b) P_{0x} , (c) P_x , (d) P_y , (e) P_{0y} , and (f) T_{0y} . Also, determine the mass flow rate and the exit plane area.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The air behaves as an ideal gas with constant specific heats ($k = 1.4$).

ANALYSIS: Use Tables 9.2 and 9.3.

- (a) $M_1 = 0.4 \Rightarrow T_1/T_{0x} = 0.96899 \Rightarrow T_{0x} = 547.0 \text{ K}$
- (b) $P_1/P_{0x} = 0.89562 \Rightarrow P_{0x} = 3.796 \text{ bar}$
- (c) at $M_x = 1.8$; $P_x/P_{0x} = 0.17404 \Rightarrow P_x = 0.6607 \text{ bar}$
- (d) at $M_x = 1.8$; $P_y/P_x = 3.6133 \Rightarrow P_y = 2.387 \text{ bar}$
- (e) $M_y = 0.61650 \Rightarrow P_y/P_{0y} = 0.77359 \Rightarrow P_{0y} = 3.086 \text{ bar}$
- (f) $T_{0y} = T_{0x} = 547.0 \text{ K}$

To determine the mass flow rate, we first determine the conditions at the throat, where the area is known. That is

$$T^* = (0.83333)(547.0 \text{ K}) = 455.8 \text{ K}$$

$$V_{th} = C^* = \sqrt{kRT^*} = \sqrt{(1.4)\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg}\cdot\text{K}}\right)(455.8 \text{ K}) \left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right| \left|\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right|} = 427.9 \text{ m/s}$$

$$P^* = (0.52828)(3.796 \text{ bar}) = 2.005 \text{ bar}$$

Thus

$$\dot{m} = \frac{P^* A_{th} V_{th}}{RT^*} = \frac{(2.005 \text{ bar})(7.6 \times 10^{-4} \text{ m}^2)(427.9 \text{ m/s})}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg}\cdot\text{K}}\right)(455.8 \text{ K}) \left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|} = 0.498 \frac{\text{kg}}{\text{s}}$$

Now, downstream of the shock, $P_{0y} = 3.086 \text{ bar}$. Thus $P_2/P_{0y} = 0.7777 \Rightarrow M_2 = 0.61$

Also, $T_2/T_{0y} = 0.93064 \Rightarrow T_2 = 509.1 \text{ K}$. Now

$$V_2 = M_2 \sqrt{kRT_2} = (0.61) \sqrt{(1.4)\left(\frac{8.314}{28.97}\right)(509.1) \left|\frac{10^3}{1}\right|} = 275.9 \text{ m/s}$$

Finally

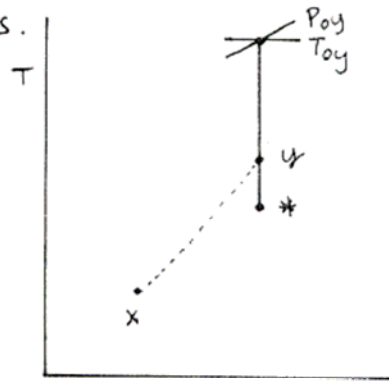
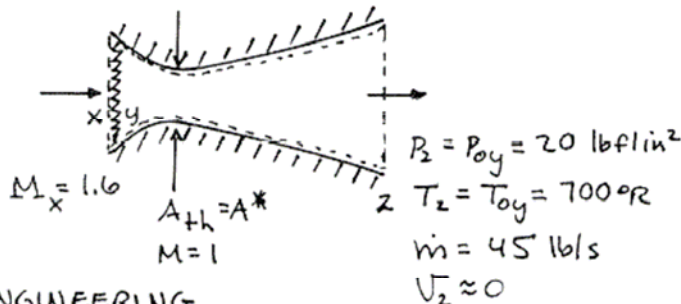
$$A_2 = \frac{\dot{m} RT_2}{P_2 V_2} = \frac{(0.498)\left(\frac{8.314}{28.97}\right)(509.1) \left|\frac{10^3}{1}\right|}{(2.4)(275.9)} = 9.84 \times 10^{-4} \text{ m}^2$$

PROBLEM 9.131

KNOWN: Air enters a converging-diverging channel at a supersonic Mach number. A normal shock stands at the inlet, and downstream of the shock, the flow is isentropic. The mass flow rate, and conditions at the exit are known.

FIND: Determine the inlet and throat areas.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state. (2) Downstream of the normal shock, the flow is isentropic. (3) Potential energy effects and the exit kinetic energy can be neglected. (4) The air behaves as an ideal gas with $k=1.4$.

ANALYSIS: Using data from Table 9.3; $M_y = 0.66844$. Thus, interpolating in Table 9.2; at $M = 0.66844$

$$T_y/T_{0y} = 0.91772$$

$$P_y/P_{0y} = 0.74083$$

$$A_y/A^* = 1.12398$$

With $T_{0y} = 700^\circ\text{R}$; $T_y = 642.4^\circ\text{R}$ and with $P_{0y} = 20 \text{ lbf/in}^2$; $P_y = 14.817 \text{ lbf/in}^2$
To find V_y

$$V_y = M_y \sqrt{kRT_y} = (0.66844) \sqrt{(1.4) \left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lb}}{\text{lb} \cdot ^\circ\text{R}} \right) (642.4^\circ\text{R}) \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{\text{lbf}} \right|}$$

$$= 830.7 \text{ ft/s}$$

$$\text{Also, } v_y = \frac{RT_y}{P_y} = \frac{\left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lb}}{\text{lb} \cdot ^\circ\text{R}} \right) (642.4^\circ\text{R})}{(14.817 \text{ lbf/in}^2)} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 16.057 \text{ ft}^3/\text{lb}$$

$$\text{Thus } A_y = \frac{\dot{m} v_y}{V_y} = \frac{(45 \text{ lb/s})(16.057 \text{ ft}^3/\text{lb})}{(830.7 \text{ ft/s})} = 0.8698 \text{ ft}^2 \quad A_y$$

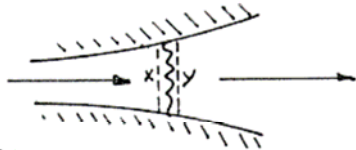
And, finally

$$A^* = A_y / 1.12398 = 0.7739 \text{ ft}^2 \quad A^*$$

PROBLEM 9.132 (Derivations)

FIND: Derive (a) Eq. 9.55, (b) Eq. 9.56, (c) Eq. 9.57

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure is at steady state. (2) The gas flowing is modeled as an ideal gas with constant specific heat ratio κ . (3) $A_x = A_y$.

ANALYSIS: (a) Conservation of mass is expressed by Eq. 9.46. Conservation of energy is expressed by Eq. 9.47 which can be rewritten as Eq. 9.53. Newton's Second Law of motion is expressed as Eq. 9.48 which can be rewritten as Eq. 9.48. Using ideal gas relations these can be combined as follows:

$$\begin{aligned} \rho_x V_x &= \rho_y V_y \\ \frac{P_x}{RT_x} M_x \sqrt{\kappa R T_x} &= \frac{P_y}{RT_y} M_y \sqrt{\kappa R T_y} \\ \frac{P_x M_x}{\sqrt{T_x}} &= \frac{P_y M_y}{\sqrt{T_y}} \\ \frac{M_x (1 + \frac{\kappa-1}{2} M_x^2)^{1/2}}{1 + \kappa M_x^2} &= \frac{M_y (1 + \frac{\kappa-1}{2} M_y^2)^{1/2}}{1 + \kappa M_y^2} \end{aligned} \quad (1)$$

One solution to this equation is $M_y = M_x$, corresponding to no change in properties in a flow through a control volume with $A_x = A_y$. This trivial solution is of no interest here. When the above equation is squared it becomes

$$\frac{M_x^2 (1 + \frac{\kappa-1}{2} M_x^2)}{(1 + \kappa M_x^2)^2} = \frac{M_y^2 (1 + \frac{\kappa-1}{2} M_y^2)}{(1 + \kappa M_y^2)^2}$$

or, when expressed as a quadratic equation in M_y^2

$$M_y^4 \left[\left(\frac{\kappa-1}{2} \right) - \kappa^2 L \right] + M_y^2 [1 - 2\kappa L] - L = 0$$

Solving the quadratic equation for M_y^2 gives Eq. 9.55.

(b). With Eq. 9.51

$$\begin{aligned} P_{0y} &= P_y \left[1 + \left(\frac{\kappa-1}{2} \right) M_y^2 \right]^{\kappa/(\kappa-1)} \\ P_{0x} &= P_x \left[1 + \left(\frac{\kappa-1}{2} \right) M_x^2 \right]^{\kappa/(\kappa-1)} \end{aligned}$$

Forming a ratio

$$\frac{P_{0y}}{P_x} = \frac{P_y}{P_x} \left[\frac{1 + \left(\frac{\kappa-1}{2} \right) M_y^2}{1 + \left(\frac{\kappa-1}{2} \right) M_x^2} \right]^{\kappa/(\kappa-1)}$$

Introducing Eq. (1) above together with Eq. 9.53

Continued on next slide

Problem 9-132 continued

$$\frac{P_{0y}}{P_{0x}} = \frac{M_x}{M_y} \left[\frac{1 + \left(\frac{\kappa-1}{2}\right) M_x^2}{1 + \left(\frac{\kappa-1}{2}\right) M_y^2} \right]^{1/2} \left[\frac{1 + \left(\frac{\kappa-1}{2}\right) M_y^2}{1 + \left(\frac{\kappa-1}{2}\right) M_x^2} \right]^{\kappa/(\kappa-1)}$$

which simplifies to yield Eq. 9.56.

(c) With Eq. 9.52

$$A_x^* = \frac{M_x A_x}{\left[\left(\frac{2}{\kappa+1}\right) \left(1 + \frac{\kappa-1}{2} M_x^2\right) \right]^{(\kappa+1)/2(\kappa-1)}}$$

$$A_y^* = \frac{M_y A_y}{\left[\left(\frac{2}{\kappa+1}\right) \left(1 + \frac{\kappa-1}{2} M_y^2\right) \right]^{(\kappa+1)/2(\kappa-1)}}$$

Then, since $A_y = A_x$

$$\frac{A_x^*}{A_y^*} = \frac{M_x}{M_y} \left[\frac{1 + \left(\frac{\kappa-1}{2}\right) M_y^2}{1 + \left(\frac{\kappa-1}{2}\right) M_x^2} \right]^{(\kappa+1)/2(\kappa-1)}$$

Finally, introducing Eq. 9.56 gives Eq. 9.57.

PROBLEMS 9.133 and 9.134

The computer programs to be written in these problems involve straightforward evaluations of equations in the text.

9.133 Eqs. 9.50, 9.51, and 9.52

9.134 Eqs. 9.53, 9.54, 9.55, and 9.56

As computers and software vary, the details of the programs are left to the reader.

PROBLEMS 9.133 and 9.134

The computer programs to be written in these problems involve straightforward evaluations of equations in the text.

9.133 Eqs. 9.50, 9.51, and 9.52

9.134 Eqs. 9.53, 9.54, 9.55, and 9.56

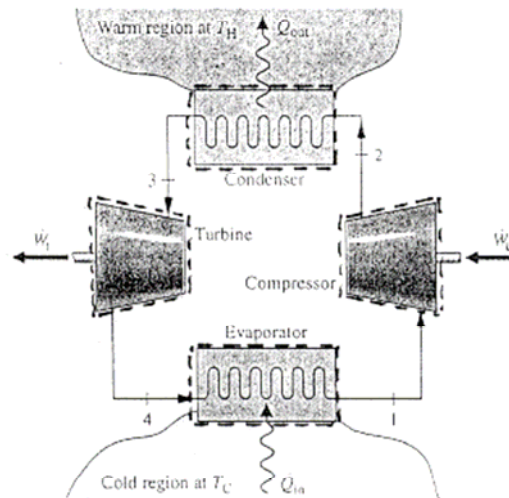
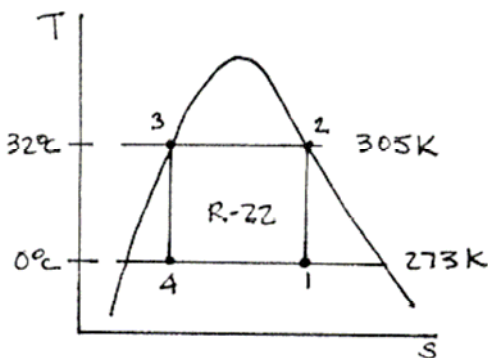
As computers and software vary, the details of the programs are left to the reader.

PROBLEM 10.1

KNOWN: R-22 is the working fluid in a Carnot refrigeration cycle. The states at the inlet and exit of the condenser and the evaporator temperature are specified.

FIND: Determine the coefficient of performance. Also, determine per unit mass of refrigerant flowing (a) the compressor work, (b) the turbine work, and (c) the heat transfer to the refrigerant passing through the evaporator.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible. (3) The compression and expansion are adiabatic. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 2 $T_2 = 32^\circ\text{C}$, sat. vapor $\Rightarrow h_2 = 259.32 \text{ kJ/kg}$, $s_2 = 0.8842 \text{ kJ/kg}\cdot\text{K}$

State 1 $T_1 = 0^\circ\text{C}$, $s_1 = s_2 \Rightarrow x_1 = 0.9428$, $h_1 = 238.21 \text{ kJ/kg}$

State 3 $T_3 = 32^\circ\text{C}$, sat. liquid $\Rightarrow h_3 = 84.14 \text{ kJ/kg}$, $s_3 = 0.3101 \text{ kJ/kg}\cdot\text{K}$

State 4 $T_4 = 0^\circ\text{C}$, $s_4 = s_3 \Rightarrow x_4 = 0.1771$, $h_4 = 81.39 \text{ kJ/kg}$

For the Carnot refrigeration cycle

$$\beta = \frac{T_C}{T_H - T_C} = \frac{273}{305 - 273} = 8.53 \quad \beta$$

(a) For the compressor

$$\frac{\dot{W}_c}{\dot{m}} = (h_2 - h_1) = 21.11 \text{ kJ/kg} \quad \dot{W}_c/\dot{m}$$

(b) For the turbine

$$\frac{\dot{W}_t}{\dot{m}} = (h_3 - h_4) = 2.75 \text{ kJ/kg} \quad \dot{W}_t/\dot{m}$$

(c) For the evaporator

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_1 - h_4) = 156.82 \text{ kJ/kg} \quad \dot{Q}_{in}/\dot{m}$$

1. Alternatively

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m} - \dot{W}_t/\dot{m}} = \frac{156.82}{21.11 - 2.75} = 8.54$$

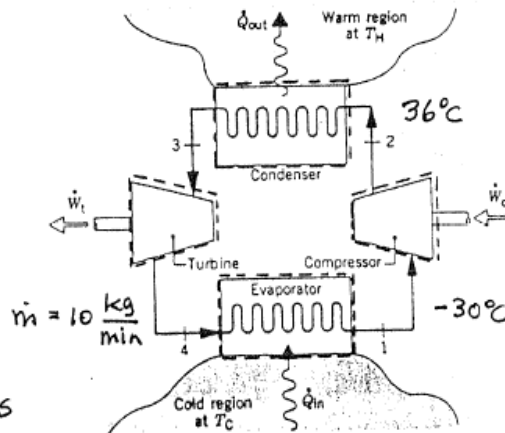
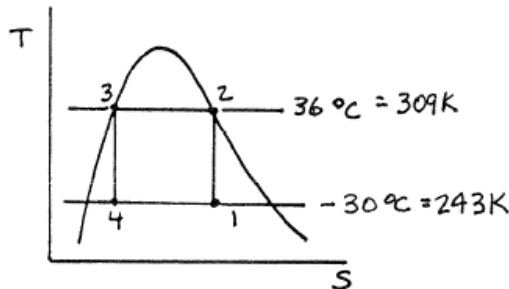
The difference is due to roundoff.

PROBLEM 10.2

KNOWN: Refrigerant 22 is the working fluid in a Carnot vapor refrigeration cycle. Data are known at various locations and the mass flow rate is specified.

FIND: Determine (a) the rate of heat transfer to the R-22 passing through the evaporator, (b) the net power input, (c) the coefficient of performance, and (d) the refrigerating capacity.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the R-22 are internally reversible. (3) The compression and expansion are adiabatic. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states (Table A-7).

State 2 $T_2 = 36^\circ\text{C}$, sat. vapor $\Rightarrow h_2 = 260.11 \text{ kJ/kg}$, $s_2 = 0.8790 \text{ kJ/kg}\cdot\text{K}$

State 1 $T_1 = -30^\circ\text{C}$, $s_2 = s_1 \Rightarrow x_1 = 0.8931$; $h_1 = 213.53 \text{ kJ/kg}$

State 3 $T_3 = 36^\circ\text{C}$, sat. liquid $\Rightarrow h_3 = 89.29 \text{ kJ/kg}$, $s_3 = 0.3265 \text{ kJ/kg}\cdot\text{K}$

State 4 $T_4 = -30^\circ\text{C}$, $s_4 = s_3 \Rightarrow x_4 = 0.3007$; $h_4 = 79.19 \text{ kJ/kg}$

(a) For the evaporator

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = (10 \frac{\text{kg}}{\text{min}})(213.53 - 79.19) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 22.39 \text{ kW} \quad \dot{Q}_{\text{in}}$$

(b) The net power input is

$$\dot{W}_{\text{cycle}} = \dot{W}_c - \dot{W}_t = \dot{m}[(h_2 - h_1) - (h_3 - h_4)] \\ = (10)[(260.11 - 213.53) - (89.29 - 79.19)] \left| \frac{1}{60} \right| = 6.08 \text{ kW} \quad \dot{W}_{\text{cycle}}$$

(c) The coefficient of performance is

$$\textcircled{1} \quad \beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{cycle}}} = \frac{22.39}{6.08} = 3.68 \quad \beta$$

(d) The refrigerating capacity is

$$\text{refrig. capacity} = \dot{Q}_{\text{in}} = 22.39 \text{ kW} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| \left| \frac{60 \text{ s}}{1 \text{ min}} \right| \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right| \\ = 6.37 \text{ tons} \quad \text{refrigerating capacity}$$

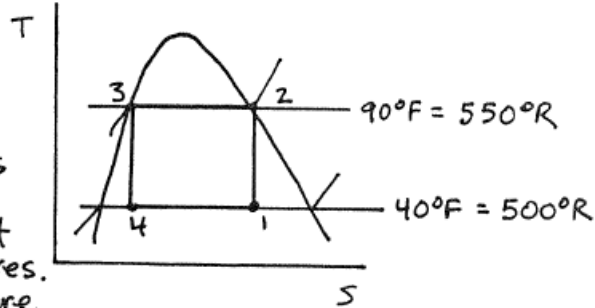
$$\text{i. Alternatively } \beta = T_C / (T_H - T_C) = 243 / (309 - 243) = 3.68$$

PROBLEM 10.3

KNOWN: A Carnot vapor refrigeration cycle operates between known reservoir temperatures.

FIND: Determine operating pressures in the condenser and evaporator for (a) Refrigerant 134a, (b) propane, (c) water, (d) refrigerant 22, and (e) ammonia as the working fluid. Calculate the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component operates at steady state. (2) All processes are internally reversible. (3) The condenser and evaporator operate at the respective reservoir temperatures. (4) The compression and expansion are adiabatic.

ANALYSIS: The operating pressures are the saturation pressures at the respective temperatures.

(a) Refrigerant 134a

① $P_{\text{cond}} = P_{\text{sat}} @ 90^\circ\text{F} = 118.99 \text{ lbf/in}^2 \leftarrow P_{\text{cond}}$
 $P_{\text{evap}} = P_{\text{sat}} @ 40^\circ\text{F} = 49.738 \text{ lbf/in}^2 \leftarrow P_{\text{evap}}$

(b) Propane

$P_{\text{cond}} = P_{\text{sat}} @ 90^\circ\text{F} = 165.2 \text{ lbf/in}^2 \leftarrow P_{\text{cond}}$
 $P_{\text{evap}} = P_{\text{sat}} @ 40^\circ\text{F} = 78.6 \text{ lbf/in}^2 \leftarrow P_{\text{evap}}$

(c) Water

② $P_{\text{cond}} = P_{\text{sat}} @ 90^\circ\text{F} = 0.6988 \text{ lbf/in}^2 \leftarrow P_{\text{cond}}$
 $P_{\text{evap}} = P_{\text{sat}} @ 40^\circ\text{F} = 0.1217 \text{ lbf/in}^2 \leftarrow P_{\text{evap}}$

(d) Refrigerant 22

$P_{\text{cond}} = P_{\text{sat}} @ 90^\circ\text{F} = 183.16 \text{ lbf/in}^2 \leftarrow P_{\text{cond}}$
 $P_{\text{evap}} = P_{\text{sat}} @ 40^\circ\text{F} = 83.278 \text{ lbf/in}^2 \leftarrow P_{\text{evap}}$

(e) Ammonia

$P_{\text{cond}} = P_{\text{sat}} @ 90^\circ\text{F} = 180.73 \text{ lbf/in}^2 \leftarrow P_{\text{cond}}$
 $P_{\text{evap}} = P_{\text{sat}} @ 40^\circ\text{F} = 73.359 \text{ lbf/in}^2 \leftarrow P_{\text{evap}}$

The coefficient of performance is

$$\beta_{\text{max}} = \frac{T_c}{T_H - T_c} = \frac{500}{550 - 500} = 10 \leftarrow \beta_{\text{max}}$$

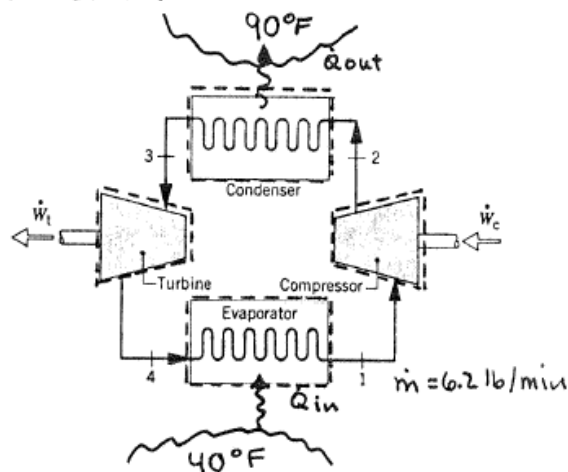
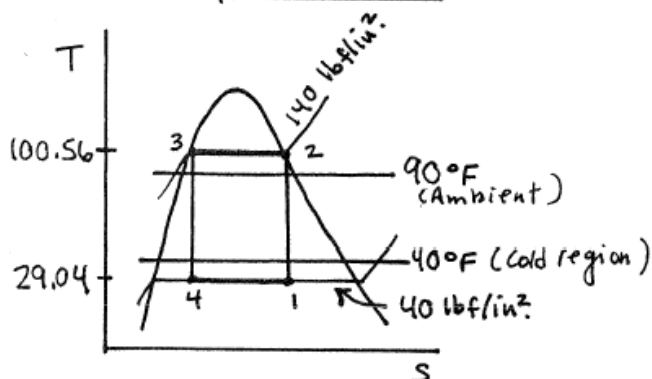
1. Refrigerant 134a, has lower operating pressures than propane, Refrigerant 22, and ammonia in this cycle.
2. Water operates at very low pressures and cannot be used to achieve temperatures lower than 32°F .

PROBLEM 10.4

KNOWN: Refrigerant 134a is the working fluid in a Carnot vapor refrigeration cycle. Operating data are specified.

FIND: Determine (a) the compressor and turbine power and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion are adiabatic. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states (Table A-11E).

State 2 $P_2 = 140 \text{ lbf/in}^2$, sat. vapor $\Rightarrow h_2 = 114.95 \frac{\text{Btu}}{\text{lb}}$, $s_2 = 0.2161 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$

State 1 $P_1 = 40 \text{ lbf/in}^2$, $s_1 = s_2 \Rightarrow x_1 = 0.9794$, $h_1 = 104.12 \text{ Btu/lb}$

State 3 $P_3 = 140 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_3 = 44.43 \frac{\text{Btu}}{\text{lb}}$, $s_3 = 0.0902 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$

State 4 $P_4 = 40 \text{ lbf/in}^2$, $s_4 = s_3 \Rightarrow x_4 = 0.2579$, $h_4 = 42.57 \text{ Btu/lb}$

(a) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = 6.2 \frac{\text{lb}}{\text{min}} (114.95 - 104.12) \frac{\text{Btu}}{\text{lb}} = 67.1 \text{ Btu/min} \quad \leftarrow \dot{W}_c$$

The turbine power is

$$\dot{W}_t = \dot{m}(h_3 - h_4) = (6.2)(44.43 - 42.57) = 11.53 \text{ Btu/min} \quad \leftarrow \dot{W}_t$$

(b) The evaporator heat transfer rate is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = 381.6 \text{ Btu/min}$$

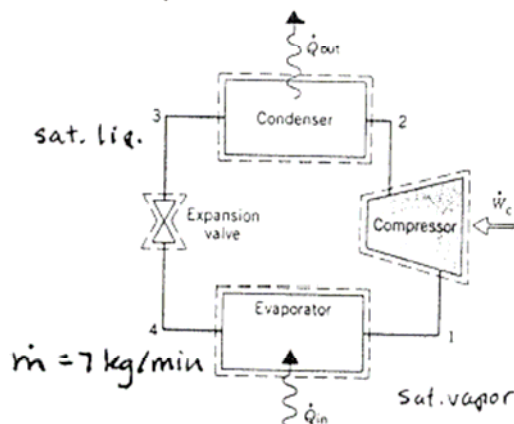
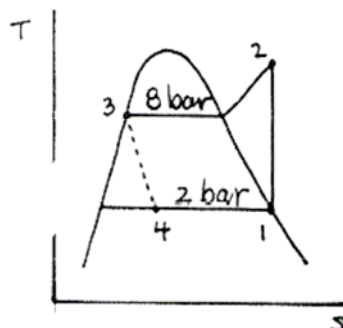
thus $\beta = \frac{\dot{Q}_{in}}{\dot{W}_c - \dot{W}_t} = 6.87 \quad \leftarrow \beta$

PROBLEM 10.6

KNOWN: Refrigerant 134a is the working fluid in an ideal vapor-compression refrigeration cycle. Operating data are known.

FIND: Determine (a) the compressor power, (b) the refrigerating capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The expansion through the valve is a throttling process. All other processes are internally reversible. (3) The compressor and valve operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $p_1 = 2 \text{ bar}$, sat. vapor $\Rightarrow h_1 = 241.30 \text{ kJ/kg}$, $s_1 = 0.9253 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

State 2 $p_2 = 8 \text{ bar}$, $s_2 = s_1 \Rightarrow h_2 = 269.92 \text{ kJ/kg}$

State 3 $p_3 = 8 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 93.42 \text{ kJ/kg}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 93.42 \text{ kJ/kg}$

(a) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(7 \frac{\text{kg}}{\text{min}}\right) (269.92 - 241.30) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 3.34 \text{ kW} \quad \leftarrow \dot{W}_c$$

(b) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(7 \frac{\text{kg}}{\text{min}}\right) (241.30 - 93.42) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right|$$

$$= 4.91 \text{ tons} \quad \leftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

$$\beta = \frac{(h_1 - h_4)}{(h_2 - h_1)} = \frac{(241.30 - 93.42)}{(269.92 - 241.30)} = 5.17 \quad \leftarrow \beta$$

PROBLEM 10.7

See solution to Problem 10.6. The data for the required plots are obtained using IT, as follows.

IT Code

```
p1 = 2 // bar
p2 = 8 // bar
p3 = p2
mdot = 7 // kg/min
```

```
h1 = hsat_Px("R134A", p1, 1)
s1 = ssat_Px("R134A", p1, 1)
s2 = s1
h2 = h_Ps("R134A", p2, s2)
h3 = hsat_Px("R134A", p3, 0)
h4 = h3
```

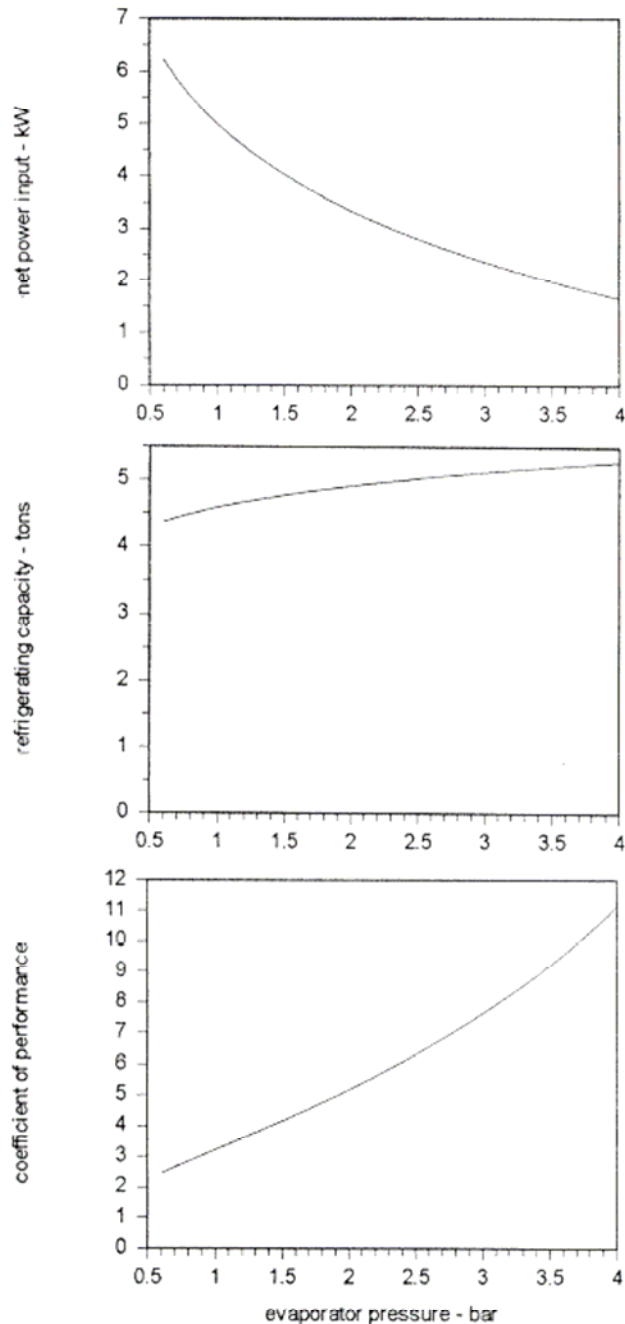
```
Wdotc = mdot * (h2 - h1) / (60) // kW
Qdotin = mdot * (h1 - h4) / (211) // tons
beta = (h1 - h4) / (h2 - h1)
```

IT Results

```
h1 = 241.3 kJ/kg
h2 = 269.9 kJ/kg
h3 = 93.42 kJ/kg
h4 = 93.42 kJ/kg
Wcycle = 3.337 kW
Qin = 4.906 tons
beta = 5.169
```

As the evaporator pressure increases at fixed condenser pressure, significantly less work is required for compression. Further, the refrigerating capacity increases slightly. As a result, the coefficient of performance increases significantly.

Plots:

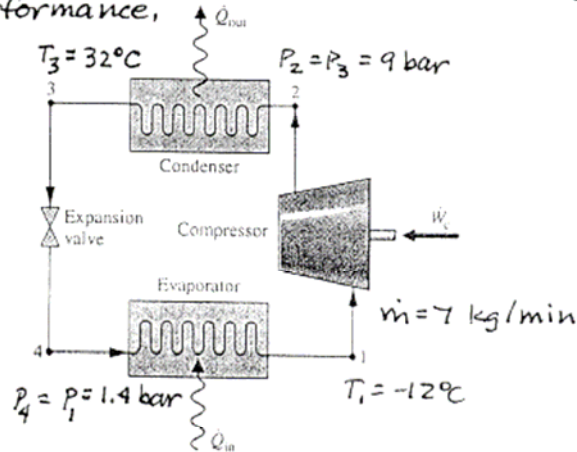
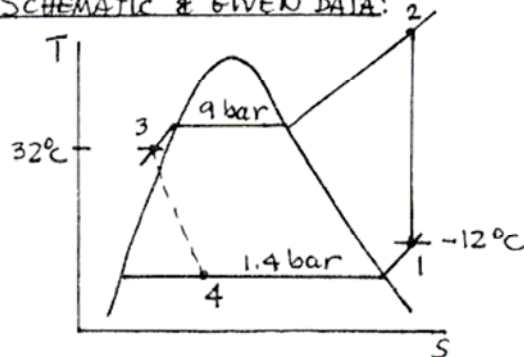


PROBLEM 10.8

KNOWN: R-134a is the working fluid in an ideal vapor compression refrigeration cycle. Operating data are known, and the refrigerant mass flow rate is given.

FIND: Determine (a) the compressor power, (b) the refrigeration capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.1, items 1-4.

ANALYSIS: First, fix each of the principal states.

State 1 $P_1 = 1.4 \text{ bar}$, $T_1 = -12^\circ\text{C} \Rightarrow h_1 = 241.73 \text{ kJ/kg}$, $s_1 = 0.95415 \text{ kJ/kg}\cdot\text{K}$

State 2 $P_2 = 9 \text{ bar}$, $s_2 = s_1 \Rightarrow h_2 = 281.56 \text{ kJ/kg}$

State 3 $P_3 = 9 \text{ bar}$, $T_3 = 32^\circ\text{C} \Rightarrow$ sub-cooled liquid. Thus, $h_3 \approx h_f @ T_3 = 94.39 \frac{\text{kJ}}{\text{kg}}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 94.39 \text{ kJ/kg}$

(a) For the compressor

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(7 \frac{\text{kg}}{\text{min}}\right)(281.56 - 241.73) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 4.65 \text{ kW} \leftarrow \dot{W}_c$$

(b) For the evaporator

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(7 \frac{\text{kg}}{\text{min}}\right)(241.73 - 94.39) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right|$$

$$= 4.89 \text{ tons} \leftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

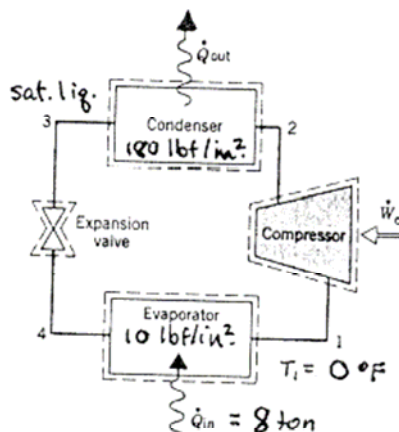
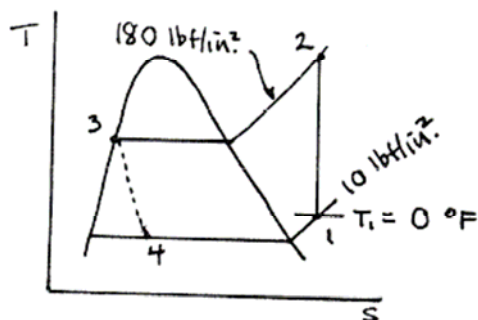
$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m}} = \frac{(h_1 - h_4)}{(h_2 - h_1)} = \frac{(241.73 - 94.39)}{(281.56 - 241.73)} = 3.70 \leftarrow \beta$$

PROBLEM 10.9

KNOWN: Refrigerant 134a is the working fluid in an ideal vapor-compression refrigeration cycle. Operating data are known, and the refrigerating capacity is given.

FIND: Determine (a) the compressor power, (b) the rate of heat transfer from the refrigerant passing through the condenser, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.1, items 1-4.

ANALYSIS: First, fix each of the principal states.

State 1 $P_1 = 10 \text{ lbf/in}^2$, $T_1 = 0^\circ\text{F} \Rightarrow h_1 = 102.94 \text{ Btu/lb}$, $s_1 = 0.2391 \text{ Btu/lb}\cdot\text{R}$

State 2 $P_2 = 180 \text{ lbf/in}^2$, $s_2 = s_1 \Rightarrow h_2 = 131.04 \text{ Btu/lb}$

State 3 $P_3 = 180 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_3 = 50.64 \text{ Btu/lb}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 50.64 \text{ Btu/lb}$

(a) The refrigerating capacity is used to find the mass flow rate.

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) \Rightarrow \dot{m} = \frac{\dot{Q}_{\text{in}}}{(h_1 - h_4)} = \frac{8 \text{ ton}}{(102.94 - 50.64) \frac{\text{Btu}}{\text{lb}}} \frac{200 \text{ Btu/min}}{1 \text{ ton}} = 30.59 \text{ lb/min}$$

The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (30.59 \frac{\text{lb}}{\text{min}}) \frac{60 \text{ min}}{1 \text{ h}} (131.04 - 102.94) \frac{\text{Btu}}{\text{lb}} \frac{1 \text{ hp}}{2545 \text{ Btu/h}} = 20.27 \text{ hp} \leftarrow \dot{W}_c$$

(b) For the condenser

$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3) = (30.59)(131.04 - 50.64) = 2459.4 \text{ Btu/min} \leftarrow \dot{Q}_{\text{out}}$$

(c) The coefficient of performance is

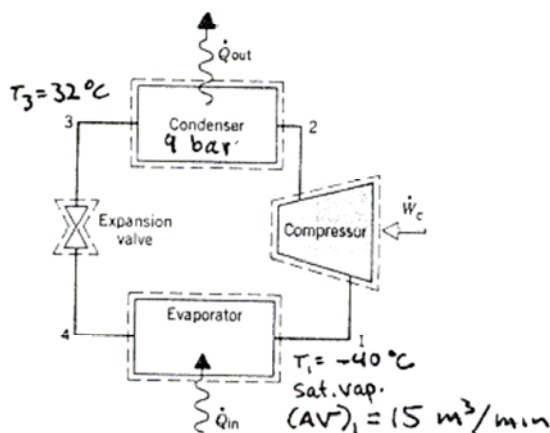
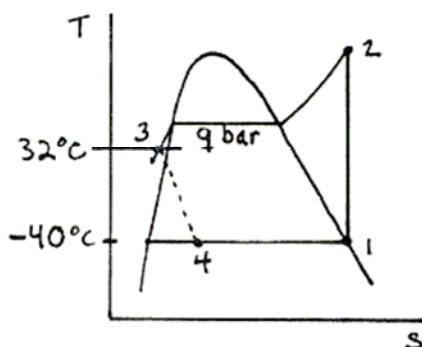
$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 1.86 \leftarrow \beta$$

PROBLEM 10.10

KNOWN: Refrigerant 22 is the working fluid in an ideal vapor-compression refrigeration cycle. Operating data are known.

FIND: Determine (a) the compressor power, (b) the refrigerating capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.1, items 1-4.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -40^\circ\text{C}$, sat. vapor $\Rightarrow h_1 = 233.27 \text{ kJ/kg}$, $s_1 = 1.0005 \text{ kJ/kg}\cdot\text{K}$

State 2 $p_2 = 9 \text{ bar}$, $s_2 = s_1 \Rightarrow h_2 = 287.54 \text{ kJ/kg}$

State 3 $h_3 \approx h_f(32^\circ\text{C}) = 84.14 \text{ kJ/kg}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 84.14 \text{ kJ/kg}$

(a) From Table A-7, $v_1 = 0.2052 \text{ m}^3/\text{kg}$. The mass flow rate is

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{15 \text{ m}^3/\text{min}}{0.2052 \text{ m}^3/\text{kg}} = 73.10 \text{ kg/min}$$

The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (73.10 \frac{\text{kg}}{\text{min}}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (287.54 - 233.27) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 66.12 \text{ kW} \leftarrow \dot{W}_c$$

(b) the refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = (73.10 \frac{\text{kg}}{\text{min}}) (233.27 - 84.14) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right|$$

$$= 51.67 \text{ tons} \leftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

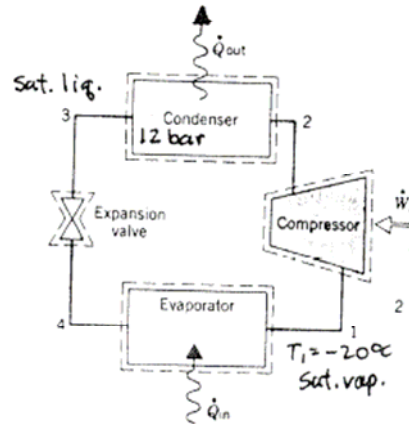
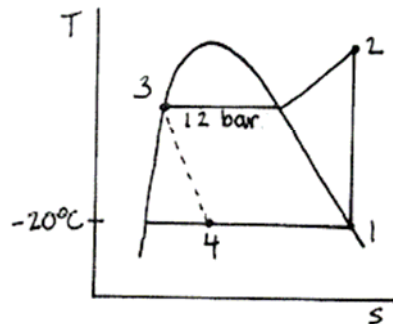
$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 2.75 \leftarrow \beta$$

PROBLEM 10.11

KNOWN: An ideal vapor-compression refrigeration cycle uses ammonia as the working fluid. Operating data are known.

FIND: Determine (a) the coefficient of performance and (b) the refrigerating capacity.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.1.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -20^\circ\text{C}$, sat. vapor $\Rightarrow h_1 = 1417.79 \text{ kJ/kg}$, $s_1 = 5.6144 \text{ kJ/kg}\cdot\text{K}$

State 2 $p_2 = 12 \text{ bar}$, $s_2 = s_1 \Rightarrow h_2 = 1689.5 \text{ kJ/kg}$

State 3 $p_3 = 12 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 327.01 \text{ kJ/kg}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 327.01 \text{ kJ/kg}$

(a) The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1417.79 - 327.01}{1689.5 - 1417.79} = 4.015 \quad \beta$$

(b) The refrigerating capacity is

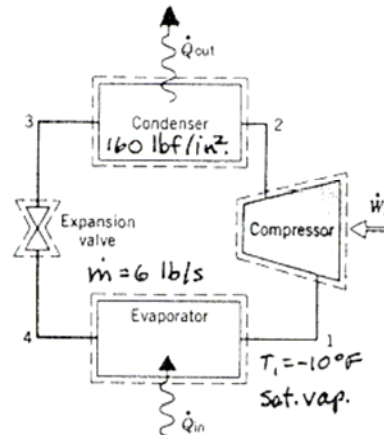
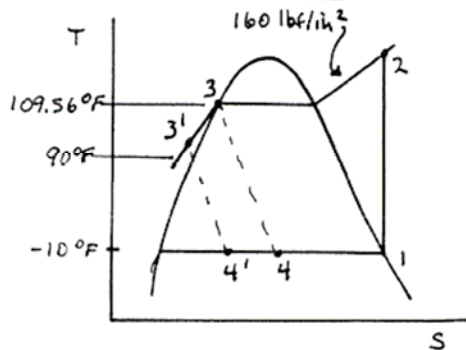
$$\begin{aligned} \dot{Q}_{in} &= \dot{m}(h_1 - h_4) \\ &= \left(3 \frac{\text{kg}}{\text{min}}\right)(1417.79 - 327.01) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right| \\ &= 15.51 \text{ tons} \quad \dot{Q}_{in} \end{aligned}$$

PROBLEM 10.12

KNOWN: An ideal vapor-compression refrigeration cycle uses refrigerant 134a as the working fluid. Operating data are known.

FIND: Plot the coefficient of performance and the refrigerating capacity for a range of condenser exit temperatures.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.1, items 1-4.

ANALYSIS: For a sample calculation, consider state 3 to be saturated liquid. Fixing each principal state:

State 1: $T_1 = -10^\circ\text{F}$, sat. vapor $\Rightarrow h_1 = 100.29 \text{ Btu/lb}$, $s_1 = 0.2236 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2: $p_2 = 160 \text{ lbf/in}^2$, $s_2 = s_1 \Rightarrow h_2 = 120.48 \text{ Btu/lb}$

State 3: $p_3 = 160 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_3 = 47.65 \text{ Btu/lb}$

State 4: Throttling process $\Rightarrow h_4 = h_3 = 47.65 \text{ Btu/lb}$

The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 2.61$$

The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m} (h_1 - h_4) = (6 \frac{\text{lb}}{\text{min}})(100.29 - 47.65) \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ ton}}{200 \text{ Btu/min}} \right| = 1.58 \text{ tons}$$

The data for the required plots are obtained using IT, as follows:

IT Code

T1 = -10 // °F
p3 = 160 // lbf/in²
mdot = 6 // lb/min
T3 = 109.5 // °F

x1 = 1
p1 = Psat_T("R134A", T1)
h1 = hsat_Px("R134A", p1, x1)
s1 = ssat_Px("R134A", p1, x1)
s2 = s1
p2 = p3
h2 = h_Ps("R134A", p2, s2)

h3 = h_PT("R134A", p3, T3)
h4 = h3

beta = (h1 - h4) / (h2 - h1)
Qdotin = mdot * (h1 - h4) * (1 / 200)

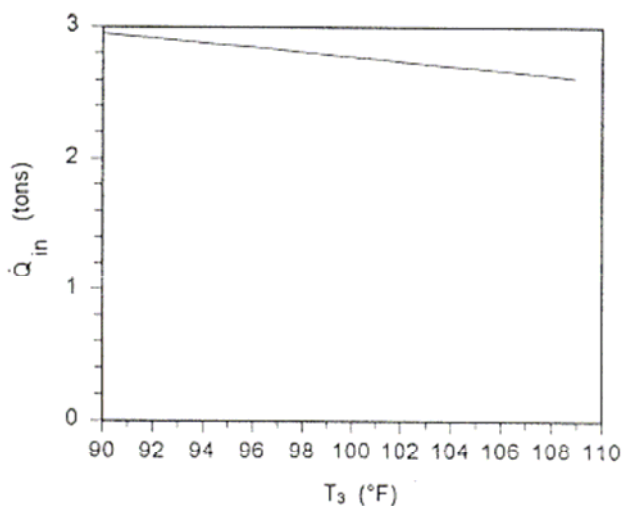
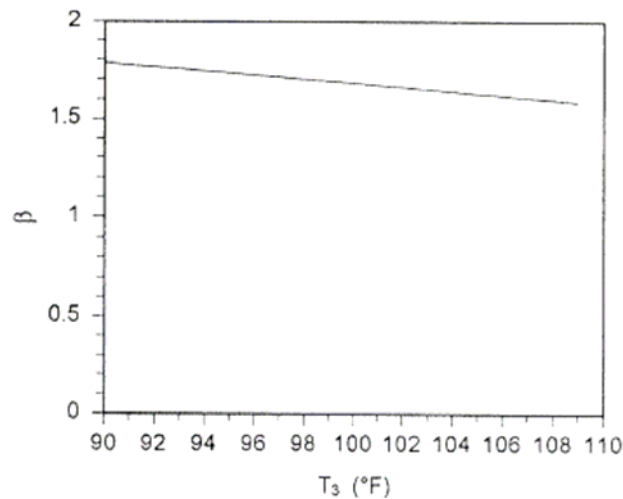
IT Results for $T_3 = 109.5^\circ\text{F}$

h1 = 100.3 Btu/lb
h2 = 120.5 Btu/lb
h3 = 47.63 Btu/lb
h4 = 47.63 Btu/lb
β = 2.608
 $\dot{Q}_{in} = 1.58 \text{ tons}$

Continued on next slide

Problem 10-12 continued

Plots:



As T_3 decreases (more subcooling) point 4 on the T-s diagram moves to the left, increasing the change in specific enthalpy across the evaporator. For constant mass flow rate, the refrigeration capacity increases accordingly. Also, since the compressor power is constant, the coefficient of performance is greater as subcooling increases.

PROBLEM 10.13

The data for the required plots are obtained using IT, as follows: (See the solution to Problem 10.11 for a sample calculation.)

IT Code

```
T1 = -20 // °C
p3 = 12 // bar
mdot = 3 // kg/min
```

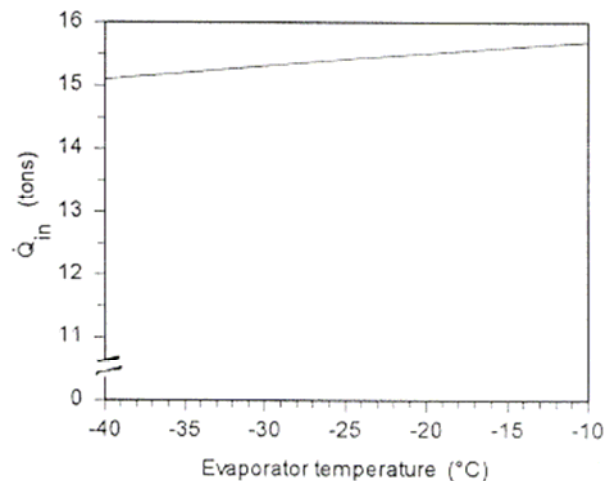
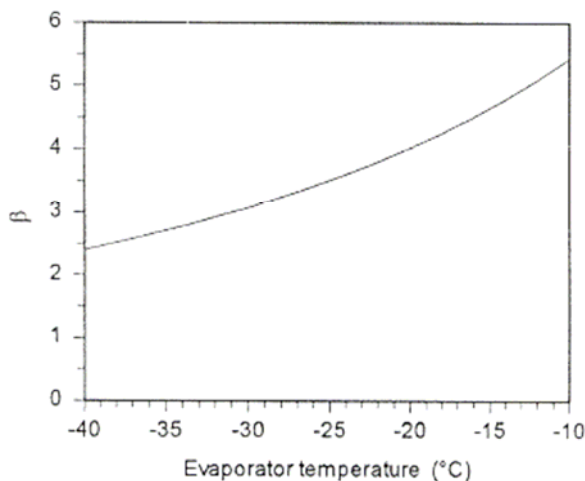
```
x1 = 1
p1 = Psat_T("Ammonia", T1)
h1 = hsat_Px("Ammonia", p1, x1)
s1 = ssat_Px("Ammonia", p1, x1)
s2 = s1
p2 = p3
s2 = s_PT("Ammonia", p2, T2)
h2 = h_PT("Ammonia", p2, T2)
x3 = 0
h3 = hsat_Px("Ammonia", p3, x3)
h4 = h3
```

```
beta = (h1 - h4) / (h2 - h1)
Qdotin = mdot * (h1 - h4) * (1 / 211) // tons
```

IT Results for $T_{\text{evap}} = -20^\circ\text{C}$

```
h1 = 1418 kJ/kg
h2 = 1689 kJ/kg
h3 = 327 kJ/kg
h4 = 327 kJ/kg
β = 4.019
Q̇in = 15.51 tons
```

PLOTS:

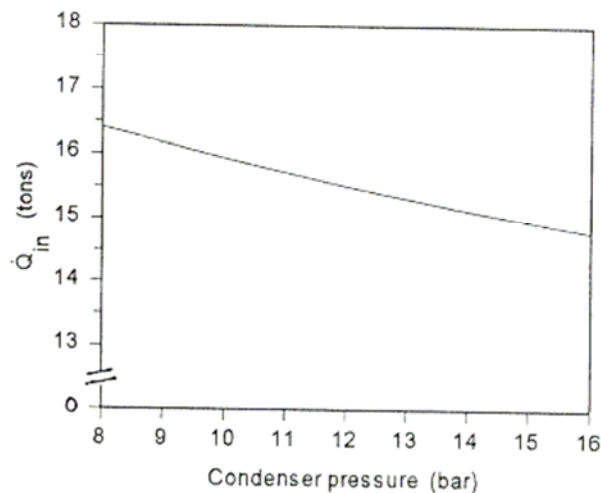
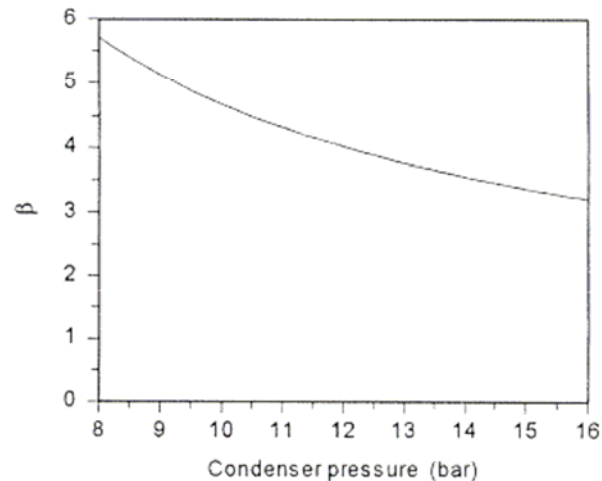


In this case, the condenser pressure is constant. Thus, $h_3 = h_4$ remains unchanged as evaporator temperature changes. Since h_1 decreases as evaporator temperature decreases, the change in specific enthalpy across the evaporator decreases as T_1 goes down. Accordingly, the capacity decreases. Furthermore, the compressor power is greater as the evaporator temperature goes down, which contributes to the dramatic decrease in coefficient of performance shown on the graph.

PROBLEM 10.14

The data for the required plots are obtained using the IT code given in the solution of Problem 10.13. In this case, the variable to sweep is P_3 , the condenser pressure.

PLOTS:



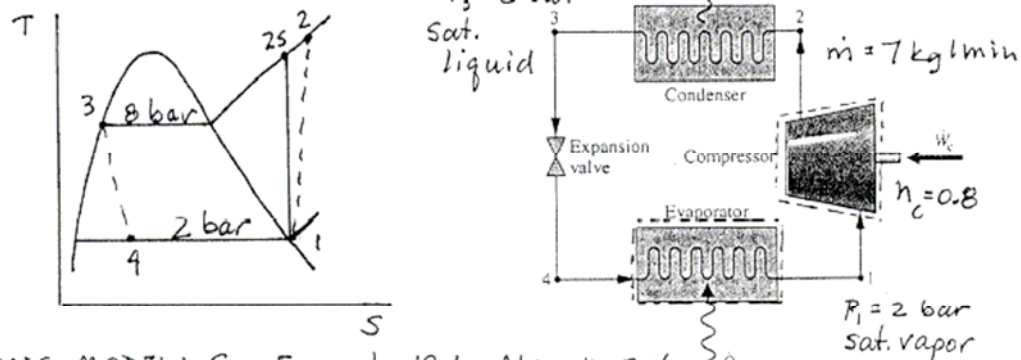
In this case, with the evaporator pressure constant and the compressor inlet state fixed, the specific enthalpy h_1 is constant. As condenser pressure is increased, state 4 on the T-s diagram moves to the right, thereby decreasing Δh across the evaporator. With mass flow rate fixed, the capacity decreases accordingly. Also, the compressor power increases, resulting in the dramatic decrease in coefficient of performance.

PROBLEM 10.15

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression refrigeration cycle. Operating data are known, and the refrigerant mass flowrate is given.

FIND: Determine (a) the compressor power, (b) the refrigeration capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.1. Also, $\eta_c = 80\%$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 2 \text{ bar}$, sat. vapor $\Rightarrow h_1 = 241.30 \text{ kJ/kg}$, $s_1 = 0.9253 \text{ kJ/kg} \cdot \text{K}$

State 2: $p_2 = 8 \text{ bar}$, $s_{2s} = s_1 \Rightarrow$ Interpolating in Table A-12; $h_{2s} = 269.92 \frac{\text{kJ}}{\text{kg}}$

Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 277.08 \text{ kJ/kg}$$

State 3: $p_3 = 8 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 36.84 \text{ kJ/kg}$

State 4: Throttling process $\Rightarrow h_4 = h_3 = 36.84 \text{ kJ/kg}$

(a) The compressor power is

$$\dot{W}_c = \dot{m} (h_2 - h_1) = \left(7 \frac{\text{kg}}{\text{min}} \right) (277.08 - 241.30) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 4.17 \text{ kW} \leftarrow \dot{W}_c$$

(b) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m} (h_1 - h_4) = \left(7 \frac{\text{kg}}{\text{min}} \right) (241.3 - 36.84) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right| = 6.783 \text{ tons} \leftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

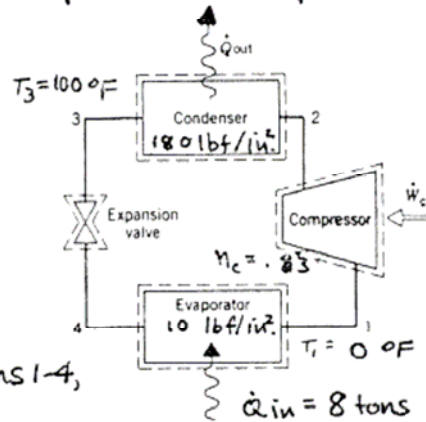
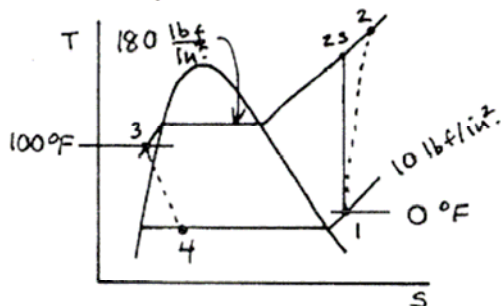
$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 5.71 \leftarrow \beta$$

PROBLEM 10.16

KNOWN: The ideal vapor-compression refrigeration cycle of Problem 10.9 is modified to have a compressor efficiency of 83 % and subcooled liquid exiting the condenser.

FIND: Answer the same questions as in Prob. 10.9 and determine (d) the exergy destruction rates in the compressor and expansion valve.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.3, items 1-4, except $\eta_c = 0.83$. Also, let $T_0 = 550^\circ\text{R}$.

ANALYSIS: First, fix each of the principal states.

From the solution of Prob. 10.9; $h_1 = 102.94 \text{ Btu/lb}$, $s_1 = 0.2391 \text{ Btu/lb}\cdot^\circ\text{R}$, and $h_{2s} = 131.04 \text{ Btu/lb}$. Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 136.80 \text{ Btu/lb}, s_2 = 0.2481 \text{ Btu/lb}\cdot^\circ\text{R}$$

State 3 $p_3 = 180 \text{ lbf/in}^2$, $T_3 = 100^\circ\text{F} \Rightarrow$ subcooled liq.; $h_3 \approx h_f(100^\circ\text{F}) = 44.23 \text{ Btu/lb}$
 $s_3 \approx s_f(100^\circ\text{F}) = 0.0898 \text{ Btu/lb}\cdot^\circ\text{R}$

State 4 $h_4 = h_3 = 44.23 \text{ Btu/lb}$ and $s_4 = 0.1029 \text{ Btu/lb}\cdot^\circ\text{R}$

(a) The mass flow rate is

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{(8 \text{ tons})}{(102.94 - 44.23) \frac{\text{Btu}}{\text{lb}}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 27.30 \text{ lb/min}$$

and the compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (27.30 \frac{\text{lb}}{\text{min}}) \left| \frac{60 \text{ min}}{1 \text{ h}} \right| (136.80 - 102.94) \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 21.79 \text{ hp}$$

(b) $\dot{Q}_{out} = \dot{m}(h_2 - h_3) = 2527.2 \text{ Btu/min}$

(c) $\beta = \frac{h_1 - h_4}{h_2 - h_1} = 1.73$

(d) For the compressor: $0 = \sum \left(\frac{\dot{Q}_j}{T_j} \right) + \dot{m}(s_2 - s_1) + \dot{\sigma}_{comp}$. Thus

$$(\dot{E}_d)_{comp} = T_0 \dot{\sigma}_{comp} = T_0 \dot{m}(s_2 - s_1) = (550^\circ\text{R})(27.30 \frac{\text{lb}}{\text{min}})(0.2481 - 0.2391) \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}} = 135.14 \text{ Btu/min}$$

similarly, for the valve

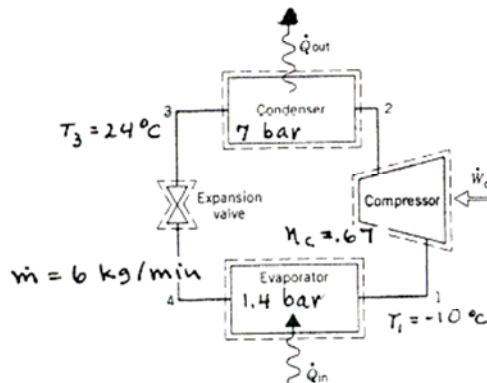
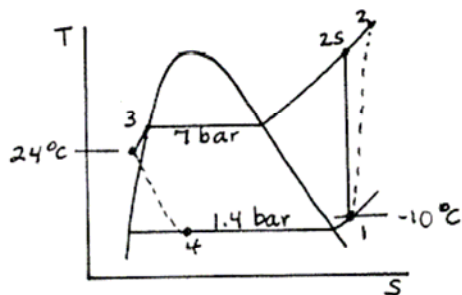
$$(\dot{E}_d)_{valve} = T_0 \dot{\sigma}_{valve} = T_0 \dot{m}(s_4 - s_3) = 196.70 \text{ Btu/min}$$

PROBLEM 10.17

KNOWN: A vapor-compression refrigeration system circulates R134 with a known mass flow rate. Data are given at various locations and the compressor efficiency is specified.

FIND: Determine (a) the coefficient of performance, (b) the refrigerating capacity, (c) the rates of exergy destruction in the compressor and valve, (d) the changes in specific flow exergy of the refrigerant passing through the evaporator and condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.3, except $\eta_c = 0.67$. Also, Let $T_0 = 294\text{K}$.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -10^\circ\text{C}$, $p_1 = 1.4\text{ bar} \Rightarrow h_1 = 243.40\text{ kJ/kg}$, $s_1 = .9606\text{ kJ/kg}\cdot\text{K}$

State 2 For isentropic compression, $p_2 = 7\text{ bar}$, $s_2 = s_1 \Rightarrow h_2 = 278.06\text{ kJ/kg}$
Using the compressor efficiency,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 295.13\text{ kJ/kg}$$

$$s_2 = 1.0135\text{ kJ/kg}\cdot\text{K}$$

State 3 $p_3 = 7\text{ bar}$, $T_3 = 24^\circ\text{C} \Rightarrow h_3 = h_{f@24} = 82.90\text{ kJ/kg}$, $s_3 = .3113\text{ kJ/kg}\cdot\text{K}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 82.90\text{ kJ/kg}$, $s_4 = .33011\text{ kJ/kg}\cdot\text{K}$

(a) The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 3.10 \leftarrow \beta$$

(b) The refrigerating capacity is

$$\begin{aligned} \dot{Q}_{in} &= \dot{m}(h_1 - h_4) \\ &= (6\text{ kg/min})(243.40 - 82.90)\text{ kJ/kg} \left| \frac{1\text{ ton}}{211\text{ kJ/min}} \right| = 4.564\text{ tons} \leftarrow \dot{Q}_{in} \end{aligned}$$

(c) For the compressor: $0 = \sum \left(\frac{\dot{Q}_j}{T_j} \right) + \dot{m}(s_2 - s_1) + \dot{\sigma}_{comp}$. Thus, the rate of exergy destruction is

$$\begin{aligned} (\dot{E}_d)_{comp} &= T_0 \dot{\sigma}_{comp} = T_0 \dot{m}(s_2 - s_1) \\ &= (294\text{ K})(6\text{ kg/min}) \left| \frac{1\text{ min}}{60\text{ s}} \right| (1.0135 - .9606)\text{ kJ/kg}\cdot\text{K} \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right| \\ &= 1.55\text{ kW} \leftarrow (\dot{E}_d)_{comp} \end{aligned}$$

Continued on next slide

Problem 10-17 continued

Similarly, for the valve

$$\begin{aligned} (\dot{E}_d)_{\text{valve}} &= T_o \dot{m} (s_4 - s_3) \\ &= (294 \text{ K})(6) \left| \frac{1}{60} \right| (.33011 - .3113) = .5530 \text{ kW} \leftarrow (\dot{E}_d)_{\text{valve}} \end{aligned}$$

(d) The change in specific flow exergy for refrigerant passing through the evaporator is

$$\begin{aligned} (e_{f1} - e_{f4}) &= (h_1 - h_4) - T_o (s_1 - s_4) \\ &= (243.40 - 82.90) - (294)(.9606 - .33011) \\ &= -24.86 \text{ kJ/kg} \leftarrow (e_{f1} - e_{f4}) \end{aligned}$$

①

For the condenser

$$(e_{f3} - e_{f2}) = (h_3 - h_2) - T_o (s_3 - s_2) = -5.78 \text{ kJ/kg} \leftarrow (e_{f3} - e_{f2})$$

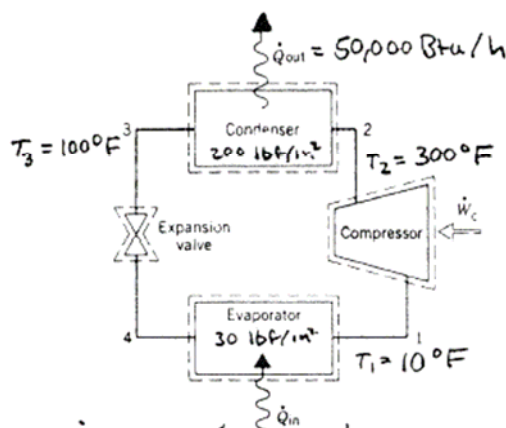
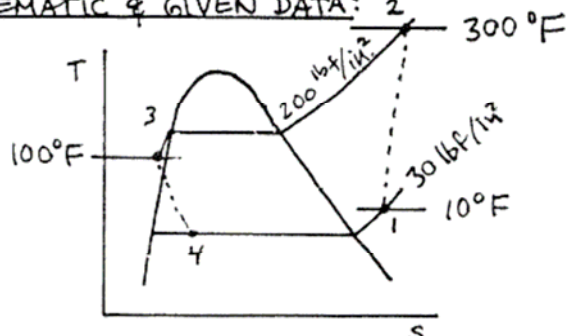
1. Although there is heat transfer to the refrigerant passing through the evaporator, the specific flow exergy decreases. This can be explained by noting that the state of the working fluid moves closer to the dead state as it is heated at a temperature below T_o .

PROBLEM 10.18

KNOWN: A vapor-compression refrigeration system uses ammonia as the working fluid. Data are known at various locations, and the heat transfer rate from the working fluid passing through the condenser is specified.

FIND: Determine (a) the compressor power and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as in Example 10.3, items 1-4, except compressor efficiency is not specified.

ANALYSIS: First, fix each of the principal states.

State 1 $P_1 = 30 \text{ lbf/in}^2$, $T_1 = 10^\circ\text{F} \Rightarrow h_1 = 617.07 \text{ Btu/lb}$, $s_1 = 1.3479 \text{ Btu/lb}^\circ\text{R}$

State 2 $P_2 = 200 \text{ lbf/in}^2$, $T_2 = 300^\circ\text{F} \Rightarrow h_2 = 763.74 \text{ Btu/lb}$, $s_2 = 1.3774 \text{ Btu/lb}^\circ\text{R}$

State 3 $P_3 = 200 \text{ lbf/in}^2$, $T_3 = 100^\circ\text{F} \Rightarrow h_3 = h_f @ 100^\circ\text{F} = 155.05 \text{ Btu/lb}$

State 4 $P_4 = 30 \text{ lbf/in}^2$; Throttling process $\Rightarrow h_4 = h_3 = 155.05 \text{ Btu/lb}$

(a) The compressor power is

$$\dot{W}_c = \dot{m} (h_2 - h_1)$$

Evaluating $\dot{m}$,

$$\dot{Q}_{\text{out}} = \dot{m} (h_2 - h_3) \Rightarrow \dot{m} = \frac{\dot{Q}_{\text{out}}}{(h_2 - h_3)} = \frac{(50,000 \text{ Btu/h})}{(763.74 - 155.05) \text{ Btu/lb}} = 82.14 \text{ lb/h}$$

thus

$$\dot{W}_c = \dot{m} (h_2 - h_1) = (82.14 \frac{\text{lb}}{\text{h}}) (763.74 - 617.07) \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

$$= 4.73 \text{ hp} \quad \leftarrow \quad \dot{W}_c$$

(b) The coefficient of performance is

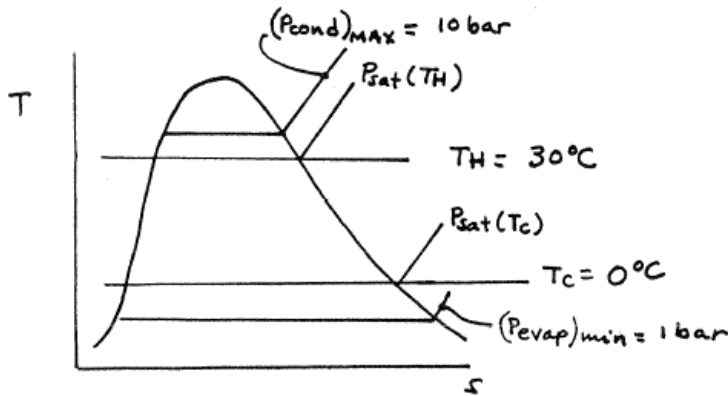
$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 3.15 \quad \leftarrow \quad \beta$$

PROBLEM 10.19

KNOWN: For a vapor-compression refrigeration cycle, the minimum and maximum allowed refrigeration pressures are 1 and 10 bar, respectively. The cold and warm region temperatures are also specified.

FIND: Among several candidate working fluids, determine which (if any) can be used for this duty.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The refrigeration cycle adheres to the model presented for the vapor-compression cycle in Sec. 10.2

ANALYSIS: For heat transfer to the working fluid passing through the evaporator from the cold region, we must have

$$1 \text{ bar} \leq P_{\text{evap}} \leq P_{\text{sat}}(T_c)$$

For heat transfer from the working fluid passing through the condenser to the warm region, we must have

$$P_{\text{sat}}(T_H) \leq P_{\text{cond}} \leq 10 \text{ bar}$$

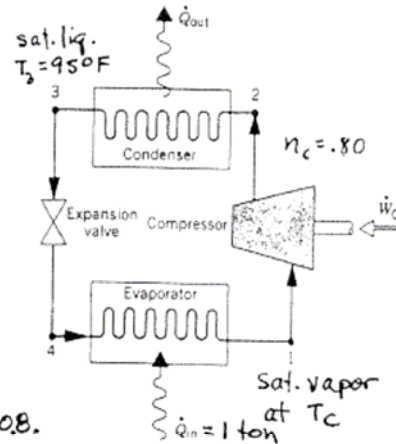
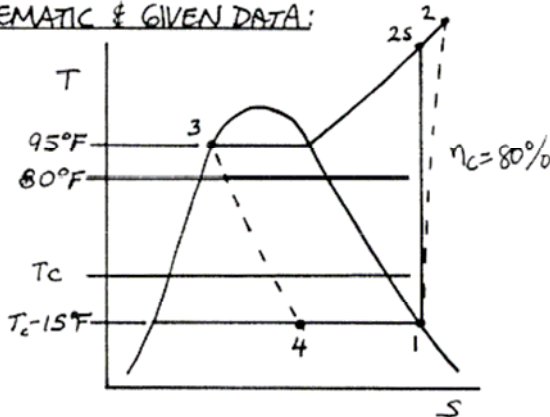
working fluid	table	$P_{\text{sat}}(0^\circ\text{C})$	$P_{\text{sat}}(30^\circ\text{C})$	feasible
R22	A-7	4.98 bar	11.93 bar	No - $P_{\text{sat}}(30^\circ\text{C})$ too high
R134a	A-10	2.93	7.7	yes
Ammonia	A-13	4.3	11.69	No - $P_{\text{sat}}(30^\circ\text{C})$ too high
Propane	A-16	4.74	10.8	No - $P_{\text{sat}}(30^\circ\text{C})$ too high

PROBLEM 10.20

KNOWN: A vapor-compression refrigeration cycle is used to maintain a cold region at temperature T_c . Refrigerant conditions are given at various points. The refrigerating capacity is specified.

FIND: Plot refrigerant mass flow rate, coefficient of performance, and refrigerating efficiency versus a range of T_c values and for the refrigerants: (a) R134a, (b) propane, (c) R22, (d) ammonia.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.2, except $\eta_c = 0.8$.

ANALYSIS: The procedure is the same for each refrigerant. The principal states are fixed as follows:

State 1: $T_1 = T_c - 15$, sat. vapor; h_g and s_g at T_1 .

State 2: Using T_3 , determine $P_3 = P_2 = P_{\text{sat}}@T_3$. Then, with P_2 and $s_1 = s_2$, determine h_2 . When using the superheated vapor table, double interpolation is likely. Next, use η_c to get h_2 ;

$$h_2 = h_1 + (h_{2s} - h_1) / \eta_c$$

State 3: $T_3 = 95^\circ\text{F}$. $h_3 = h_f@95^\circ\text{F}$

State 4: Throttling process: $h_4 = h_3$

To get the refrigerant mass flow rate

$$\dot{m} = \frac{\dot{Q}_{\text{in}}}{h_1 - h_4}$$

The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1}$$

$$\text{Finally; } \beta_{\text{carnot}} = \frac{T_c(^{\circ}\text{R})}{540 - T_c(^{\circ}\text{R})}$$

$$\text{and } \eta_{\text{ref}} = \beta / \beta_{\text{carnot}}$$

Continued on next slide

Problem 10-20 continued

The data for the required plots are obtained using IT, as follows. Shown here is the code for ammonia. The code for other refrigerants is similar.

IT Code

```
T1 = TC - 15 // °F
eta_c = 0.8
T3 = 95 // °F
Qdotin = 1 // ton
TC = -25 // °F
TH = 80 // °F
```

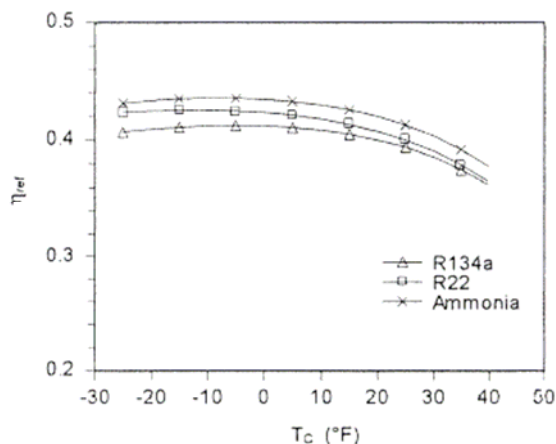
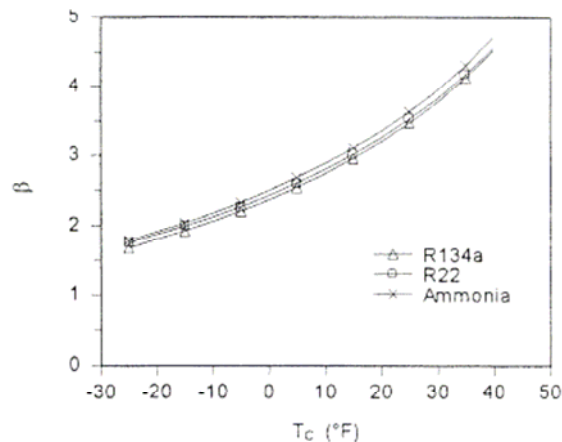
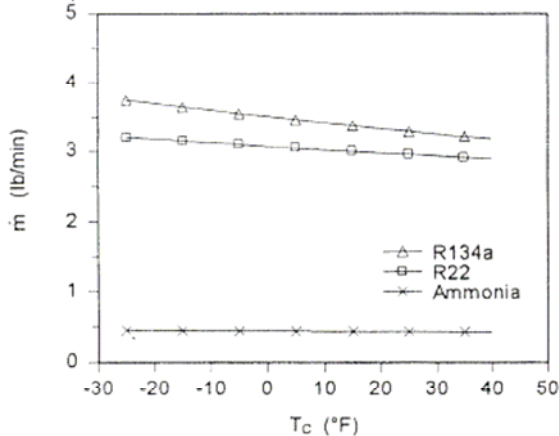
```
x1 = 1
p1 = Psat_T("Ammonia", T1)
h1 = hsat_Px("Ammonia", p1, x1)
s1 = ssat_Px("Ammonia", p1, x1)
s2s = s1
p2 = p3
s2s = s_PT("Ammonia", p2, T2s)
h2s = h_PT("Ammonia", p2, T2s)
h2 = h1 + (h2s - h1) / eta_c
p3 = Psat_T("Ammonia", T3)
x3 = 0
h3 = hsat_Px("Ammonia", p3, x3)
h4 = h3
```

```
mdot = (Qdotin * 200) / (h1 - h4)
beta = (h1 - h4) / (h2 - h1)
betacarnot = (TC + 460) / ((TH + 460) - (TC + 460))
eff = beta / betacarnot
```

IT Results for Ammonia, $T_C = -25^\circ\text{F}$

```
h1 = 597 Btu/lb
h2 = 847.8 Btu/lb
h2s = 797.7 Btu/lb
h3 = 149.1 Btu/lb
h4 = 149.1 Btu/lb
beta_carnot = 4.143
m = 0.4465 lb/min
beta = 1.785
eta_ref = 0.431
```

PLOTS:



Property data for propane are not available in IT. Using data from Tables A-16E, A-17E, and A-18E, we get the following results for a sample case:

$$T_c = -25^\circ\text{F} = 435^\circ\text{R}$$

$$\dot{m} = 1.985 \text{ lb/min}$$

$$\beta = 2.07$$

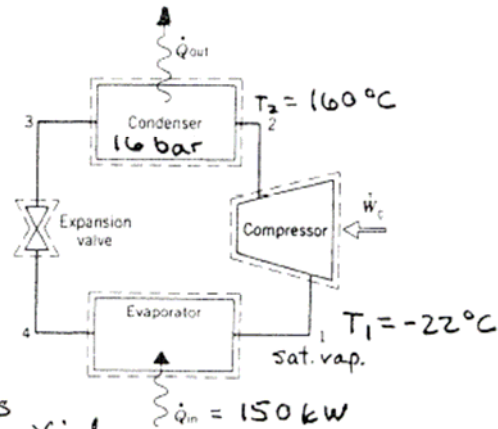
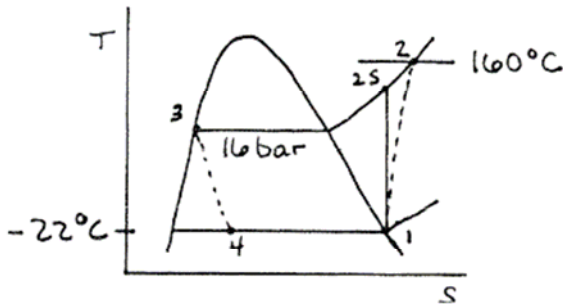
$$\eta_{ref} = 0.5$$

PROBLEM 10.21

KNOWN: Ammonia is the working fluid in a vapor-compression refrigeration system. Data are known at various locations and the refrigerating capacity is given.

FIND: Determine (a) the mass flow rate of refrigerant, (b) the compressor power, (c) the coefficient of performance, and (d) the isentropic compressor efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.3, items 1-4, except the compressor efficiency is unspecified.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -22^\circ\text{C}$, sat. vapor $\Rightarrow h_1 = 1415.08 \text{ kJ/kg}$, $s_1 = 5.6457 \text{ kJ/kg}\cdot\text{K}$

State 2 $P_2 = 16 \text{ bar}$, $T_2 = 160^\circ\text{C} \Rightarrow h_2 = 1798.45 \text{ kJ/kg}$

State 3 $P_3 = 16 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 376.46 \text{ kJ/kg}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 376.46 \text{ kJ/kg}$

(a) The mass flow rate is determined using the refrigerating capacity

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4)$$

Thus
$$\dot{m} = \frac{\dot{Q}_{\text{in}}}{h_1 - h_4} = \frac{(150 \text{ kW})}{(1415.08 - 376.46) \frac{\text{kJ}}{\text{kg}}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 0.1444 \frac{\text{kg}}{\text{s}} \quad \leftarrow \dot{m}$$

(b) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (0.1444 \frac{\text{kg}}{\text{s}})(1798.45 - 1415.08) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 55.36 \text{ kW} \quad \leftarrow \dot{W}_c$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_c} = \frac{150 \text{ kW}}{55.36 \text{ kW}} = 2.71 \quad \leftarrow \beta$$

(d) For isentropic compression, $P_2 = 16 \text{ bar}$, $s_{2s} = s_1 \Rightarrow h_{2s} = 1755.38 \text{ kJ/kg}$

Thus, the isentropic efficiency is

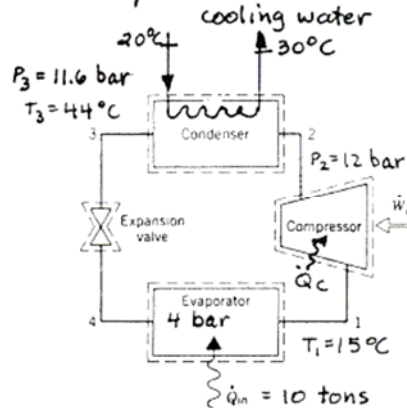
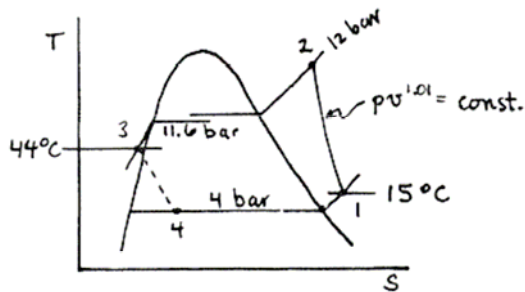
$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{1755.38 - 1415.08}{1798.45 - 1415.08} = 0.888 \quad (88.8\%) \quad \leftarrow \eta$$

PROBLEM 10.22

KNOWN: R134a is the working fluid in a vapor-compression refrigeration system having a water cooled condenser. The capacity is known and the compression process is described by $pv^{1.01} = \text{constant}$.

FIND: Determine (a) the mass flow rate of R134a, (b) power input and heat transfer rate for the compressor, (c) coefficient of performance, (d) cooling water mass flow rate, (e) rates of exergy destruction in the condenser and expansion valve, each expressed as a percentage of the power input.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compression is polytropic with $n=1.01$. (3) The expansion through the valve is a throttling process. (4) Heat transfer from the outside of the condenser can be neglected. (5) Kinetic and potential energy effects are negligible. (6) Let $T_0 = 293\text{K}$.

ANALYSIS: First, fix each of the principal states.

State 1 $P_1 = 4\text{ bar}, T_1 = 15^\circ\text{C} \Rightarrow h_1 = 258.15 \frac{\text{kJ}}{\text{kg}}, s_1 = .9348 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$
 $v_1 = 0.05258 \text{ m}^3/\text{kg}$

State 2 For the polytropic process, $P_2 = 12\text{ bar}$ and
 $v_2 = \left(\frac{P_1}{P_2}\right)^{1/1.01} v_1 = 0.01772 \text{ m}^3/\text{kg}$

From Table A-12; $h_2 = 281.33 \text{ kJ/kg}, s_2 = .9341 \text{ kJ/kg}\cdot\text{K}$

State 3 $T_3 = 44^\circ\text{C}$, compressed liquid $\Rightarrow h_3 \approx h_f(44^\circ\text{C}) = 112.22 \text{ kJ/kg}$
 $s_3 \approx s_f(44^\circ\text{C}) = 0.4054 \text{ kJ/kg}\cdot\text{K}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 112.22 \text{ kJ/kg}$

Equation 6.55 is used to determine the compressor work input

$$\frac{\dot{W}_c}{\dot{m}} = \int_1^2 v dp = \left(\frac{n}{n-1}\right) (P_2 v_2 - P_1 v_1)$$

$$= \left(\frac{1.01}{.01}\right) \left[(12\text{ bar}) (0.01772 \frac{\text{m}^3}{\text{kg}}) - (4) (0.05258) \right] \left| \frac{10^5 \text{ N/m}^2}{1\text{ bar}} \right| \left| \frac{1\text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right|$$

$$= 23.432 \text{ kJ/kg}$$

The mass flow rate is found using the refrigerating capacity

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{(10\text{ tons})}{(258.15 - 112.22) \frac{\text{kJ}}{\text{kg}}} \left| \frac{211 \text{ kJ/min}}{1\text{ ton}} \right| \left| \frac{1\text{ min}}{60\text{ s}} \right| = 0.241 \text{ kg/s} \leftarrow \dot{m}$$

Continued on next slide

Problem 10-22 continued

Thus, the compressor power input is

$$\dot{W}_c = \dot{m} \left(\frac{\dot{W}_c}{\dot{m}} \right) = (0.241 \frac{\text{kg}}{\text{s}}) (23.432 \frac{\text{kJ}}{\text{kg}}) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 5.647 \text{ kW} \leftarrow \dot{W}_c$$

The heat transfer rate is

$$\begin{aligned} \dot{Q}_{cv} &= -\dot{W}_{cv} + \dot{m}(h_2 - h_1) \\ &= -5.647 + (0.241)(281.33 - 258.15) = -0.0606 \text{ kW} \leftarrow \dot{Q}_{cv} \end{aligned}$$

The coefficient of performance is

$$\begin{aligned} \beta &= \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{(10 \text{ tons})}{(5.647 \text{ kW})} \left| \frac{211 \text{ kJ/min}}{1 \text{ ton}} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 6.227 \leftarrow \beta \end{aligned}$$

Evaluating $\dot{m}_{cw}$ from an energy balance

$$\begin{aligned} 0 &= \dot{m}(h_2 - h_3) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out}) \\ \dot{m}_{cw} &= \dot{m} \left(\frac{h_2 - h_3}{h_{cw,out} - h_{cw,in}} \right) = (0.241 \frac{\text{kg}}{\text{s}}) \left(\frac{281.33 - 112.22}{125.79 - 83.96} \right) \\ &= 0.9743 \text{ kg/s} \leftarrow \dot{m}_{cw} \end{aligned}$$

where $h_{cw} = h_f(T)$.

The rate of exergy destruction can be found using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$.

• For the condenser,

$$0 = \sum \left(\frac{\dot{Q}_j}{T_j} \right) + \dot{m}(s_2 - s_3) + \dot{m}_{cw}(s_{cw,in} - s_{cw,out}) + \dot{\sigma}_{cond}$$

$$\begin{aligned} \Rightarrow (\dot{E}_d)_{cond} &= T_0 \dot{\sigma}_{cond} = T_0 [\dot{m}(s_3 - s_2) + \dot{m}_{cw}(s_{cw,out} - s_{cw,in})] \\ &= (293 \text{ K}) \left[(0.241) \frac{\text{kg}}{\text{s}} (0.4054 - 0.9341) \frac{\text{kJ}}{\text{kg}} \right. \\ &\quad \left. + (0.9743)(.4369 - .2966) \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 2.718 \text{ kW} \end{aligned}$$

Expressed as a percentage of the compressor power, $\% = \left(\frac{2.718}{5.647} \right) (100) = 48 \%$ (e)

• For the valve,

$$\begin{aligned} (\dot{E}_d)_{valve} &= T_0 \dot{m}(s_4 - s_3). \text{ To find } s_4, \text{ use } h_4 = h_3 \text{ with data from} \\ \text{Table A-11 at 4 bar: } x_4 &= \frac{112.22 - 62}{190.32} = 0.2639 \Rightarrow s_4 = 0.2399 + 0.2639(0.9145 - 0.2399) = 0.4179 \text{ FJ/K} \cdot \text{K} \end{aligned}$$

Thus

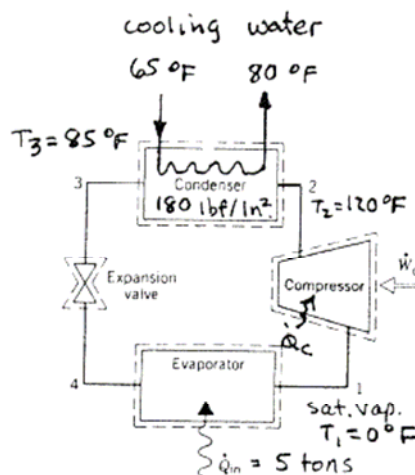
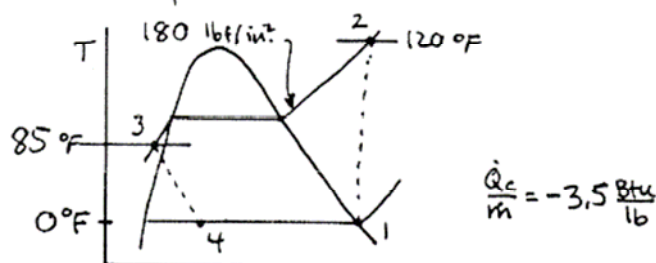
$$\begin{aligned} (\dot{E}_d)_{valve} &= (293)(0.241)(0.4179 - 0.4054) = 0.88 \text{ kW} \\ \% &= \left(\frac{0.88}{5.647} \right) (100) = 16 \end{aligned} \leftarrow (e)$$

PROBLEM 10.23

KNOWN: A vapor-compression refrigeration system uses propane as the working fluid. Data are known at various locations and the refrigerating capacity is given. The condenser is water-cooled.

FIND: Determine (a) the compressor power, (b) the mass flow rate of condenser cooling water, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) There are no pressure drops through the heat exchangers. (3) Expansion through the valve is a throttling process. (4) Kinetic and potential energy effects are negligible. (5) Heat transfer from the outside of the condenser can be neglected.

ANALYSIS: First, fix each of the principal states (Tables A-16E to A-18E).

State 1 $T_1 = 0^\circ\text{F}$, sat. vapor $\Rightarrow h_1 = 193.2 \text{ Btu/lb}$, $s_1 = 0.422 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2 $p_2 = 180 \text{ lbf/in}^2$, $T_2 = 120^\circ\text{F} \Rightarrow h_2 = 229.8 \text{ Btu/lb}$, $s_2 = 0.431 \text{ Btu/lb}\cdot^\circ\text{R}$

State 3 $p_3 = 180 \text{ lbf/in}^2$, $T_3 = 85^\circ\text{F} \Rightarrow$ compressed liquid; $h_3 \approx h_f(85^\circ\text{F}) = 74.41 \text{ Btu/lb}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 74.41 \text{ Btu/lb}$

(a) Evaluating the refrigerant mass flow rate

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_1 - h_4)} = \frac{5 \text{ tons}}{(193.2 - 74.41) \frac{\text{Btu}}{\text{lb}}} \frac{200 \text{ Btu/min}}{1 \text{ ton}} = 8.42 \frac{\text{lb}}{\text{min}}$$

$$\text{Thus } \dot{W}_c = \dot{m} [-(\dot{Q}_c/\dot{m}) + (h_2 - h_1)] = (8.42 \frac{\text{lb}}{\text{min}}) [-(-3.5) + (229.8 - 193.2)] \frac{\text{Btu}}{\text{lb}} = 337.6 \text{ Btu/min} \leftarrow \dot{W}_c$$

(b) An energy balance on the condenser gives, with $h_{cw} = h_f(T)$

$$\dot{m}_{cw} = \dot{m} \left(\frac{h_2 - h_3}{h_f(80) - h_f(65)} \right) = (8.42 \frac{\text{lb}}{\text{min}}) \left(\frac{229.8 - 74.41}{48.09 - 33.09} \right) = 87.23 \frac{\text{lb}}{\text{min}} \leftarrow \dot{m}_{cw}$$

(c) The coefficient of performance is

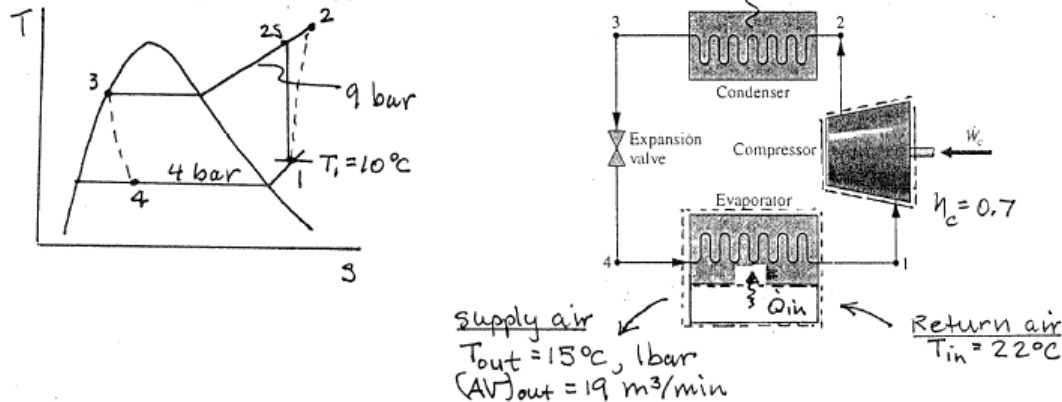
$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{5 \text{ tons}}{337.6 \text{ Btu/min}} \frac{200 \text{ Btu/min}}{1 \text{ ton}} = 2.96 \leftarrow \beta$$

PROBLEM 10.24

KNOWN: Data are known for a window-mounted air conditioner. The air conditioner operates on a vapor-compression refrigeration cycle with R-22 as the working fluid. Operating data for the cycle are known, and conditions of the air entering and leaving the unit are specified.

FIND: Determine (a) the compressor power, (b) the refrigeration capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) There are no pressure drops through the evaporator and condenser. (3) The compressor operates adiabatically, with $\eta_c = 70\%$, and the expansion through the valve is a throttling process. (4) Saturated liquid leaves the condenser. (5) The air behaves as an ideal gas with $c_{p,air} = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 4 \text{ bar}$, $T_1 = 10^\circ\text{C} \Rightarrow h_1 = 259.18 \text{ kJ/kg}$, $s_1 = 0.9795$

State 2: $p_2 = 9 \text{ bar}$, $s_{2s} = s_1 \Rightarrow h_{2s} = 280.68 \text{ kJ/kg}$

Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 289.89 \text{ kJ/kg}$$

State 3: $p_3 = 9 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 68.59 \text{ kJ/kg}$

State 4: Throttling process $\Rightarrow h_4 = h_3 = 68.59 \text{ kJ/kg}$

To get the mass flowrate of R-22, analyze the evaporator. First

$$\dot{m}_{air} = \frac{(AV)_{air} p}{R T_{out}} = \frac{(19 \frac{\text{m}^3}{\text{min}}) (1 \text{ bar})}{(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}}) (208 \text{ K})} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 31.83 \text{ kg/min}$$

For the control volume enclosing the evaporator

$$0 = \dot{m}_R (h_4 - h_1) + \dot{m}_{air} (h_{in} - h_{out})$$

$$\text{With } h_{in} - h_{out} = c_p (T_{in} - T_{out}) \quad (c_{p,air} = 1.005 \text{ kJ/kg} \cdot \text{K})$$

Continued on next slide

Problem 10-24 continued

$$\dot{m}_R = \frac{\dot{m}_{air} c_{p,air} (T_{in} - T_{out})}{(h_1 - h_4)}$$

$$= \frac{(31.83 \text{ kg/min})(1.005 \text{ kJ/kg} \cdot \text{K})(22 - 15) \text{ K}}{(259.18 - 68.59) \text{ kJ/kg}} = 1.175 \text{ kg/min}$$

(a) the compressor power is

$$\dot{W}_c = \dot{m}_R (h_2 - h_1) = (1.175 \frac{\text{kg}}{\text{min}}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (289.89 - 259.18) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 0.6 \text{ kW} \leftarrow \dot{W}_c$$

(b) The refrigeration capacity is

$$\dot{Q}_{in} = \dot{m}_R (h_1 - h_4)$$

$$= (1.175 \frac{\text{kg}}{\text{min}})(259.18 - 68.59) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right|$$

$$= 1.06 \text{ tons} \leftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

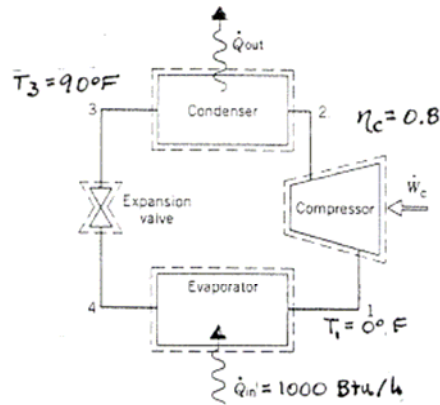
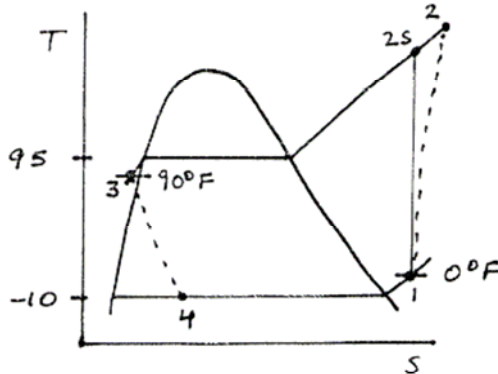
$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 6.21 \leftarrow \beta$$

PROBLEM 10.25

KNOWN: Data are known at various locations in a vapor-compression refrigeration cycle. The refrigerating capacity is 1000 Btu/h.

FIND: Determine the evaporator and condenser pressures, mass flow rate of refrigerant, compressor power input, and coefficient of performance for (a) Refrigerant 134a and (b) propane as the working fluid.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: See Example 10.3, assumptions 1-4.

ANALYSIS: First, fix each principal state.

(a) **Refrigerant 134a**

State 1: $P_1 = P_{\text{sat}} @ -10^\circ\text{F} = 16.674 \text{ lbf/in}^2$ $\xrightarrow{\text{Pevap(a)}}$

Interpolating in Table A-12E at $T_1 = 0^\circ\text{F} \Rightarrow h_1 = 102.2 \frac{\text{Btu}}{\text{lb}}, s_1 = 0.2281 \text{ Btu/lb}\cdot\text{R}$

State 2: $P_2 = P_{\text{sat}} @ 95^\circ\text{F} = 128.62 \text{ lbf/in}^2$ $\xrightarrow{\text{Pcond(a)}}$

$h_{2s} = 121.01 \text{ Btu/lb}$

$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 125.7 \text{ Btu/lb}$

State 3: $h_3 \approx h_f(T_3) = 40.72 \text{ Btu/lb}$

State 4: throttling process $\Rightarrow h_3 = h_4 = 40.72 \text{ Btu/lb}$

(b) **Propane**

State 1 $P_1 = P_{\text{sat}} @ -10^\circ\text{F} = 31.9 \text{ lbf/in}^2$ $\xrightarrow{\text{Pevap(b)}}$

Double interpolation in Table A-18E gives; $h_1 = 194.2 \text{ Btu/lb}, s_1 = 0.4331 \text{ Btu/lb}\cdot\text{R}$

State 2 $P_2 = P_{\text{sat}} @ 95^\circ\text{F} = 176.9 \text{ lbf/in}^2$ $\xrightarrow{\text{Pcond(b)}}$

For isentropic compression, $s_{2s} = s_1 = 0.4331 \Rightarrow h_{2s} = 230.61 \text{ Btu/lb}$.

Using the compressor efficiency

$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 239.7$

State 3 $h_3 \approx h_f(90^\circ\text{F}) = 77.74 \text{ Btu/lb}$

State 4 throttling process $\Rightarrow h_4 = h_3 = 77.74 \text{ Btu/lb}$

Continued on next slide

Problem 10-25 continued

Mass flow rate

The refrigerating capacity is used to find the mass flow rate.

$$\dot{m}_{R-134a} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{(1000 \text{ Btu/h})}{(102.2 - 40.72) \text{ Btu/lb}} \left| \frac{1 \text{ h}}{60 \text{ min}} \right| = 0.271 \text{ lb/min} \leftarrow \dot{m} (a)$$

Similarly, for propane; $\dot{m}_{\text{propane}} = 0.143 \text{ lb/min} \leftarrow \dot{m} (b)$

Compressor power input

For Refrigerant 134a

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (0.271 \frac{\text{lb}}{\text{min}})(125.7 - 102.2) \frac{\text{Btu}}{\text{lb}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 0.150 \text{ hp} \leftarrow \dot{W}_c (a)$$

Similarly, for propane; $\dot{W}_c = 0.153 \leftarrow \dot{W}_c (b)$

Coefficient of performance

$$\beta_{R-134a} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{102.2 - 40.72}{125.7 - 102.2} = 2.62 \leftarrow \beta (a)$$

Similarly, for propane; $\beta_{\text{propane}} = 2.56 \leftarrow \beta (b)$

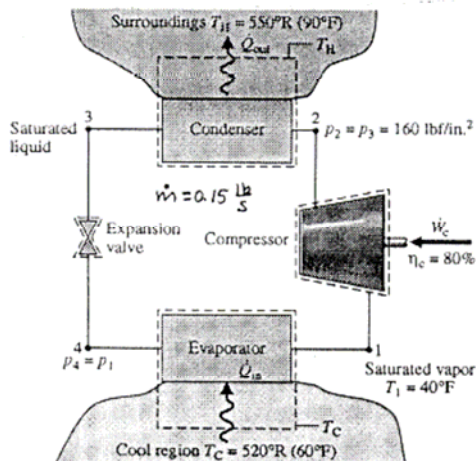
1. Note that the propane cycle operates at higher pressures than the R-134a cycle.
2. The propane cycle has a much lower mass flow rate than the R-134a cycle. However, the power and coefficients of performance are nearly the same for the two working fluids in this case.

PROBLEM 10.26

KNOWN: Steady-state operating data are provided for a vapor-compression air conditioning system.

FIND: Determine (a) the power required by the compressor, (b) the coefficient of performance, (c) the rates of exergy destruction in the compressor and valve, and the rates of exergy destruction and exergy transfer accompanying heat transfer for a control volume enclosing (d) the evaporator and a portion of the cool region, (e) the condenser and a portion of the surroundings.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volumes shown in the schematic are at steady state.
2. Kinetic and potential energy changes are negligible as are pressure changes for flow through the evaporator and condenser.
3. The expansion across the valve is a throttling process.
4. There is no stray heat transfer for any control volume.
5. $T_0 = 550^\circ\text{R}$.

ANALYSIS: For the principal states, essential property data can be obtained from the tables for R134a

State	T(°F)	p(lbf/in²)	h(Btu/lb)	s(Btu/lb·°R)	
1	40	49.7	107.39	0.2189	(Table A-10E)
2	-	160	120.33	0.2234	
3	-	160	47.65	0.0958	(Table A-10E)
4	-	49.7	47.65	0.0994	

Using $s_{2s} = s_1$, and interpolating in Table A-12E gives $h_{2s} = 117.74 \text{ Btu/lb}$. Then,

Using the isentropic efficiency $\eta_c = h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 107.39 + \frac{(117.74 - 107.39)}{0.8} = 120.33 \text{ Btu/lb}$

Interpolation in Table A-12E then yields $s_2 = 0.2234 \text{ Btu/lb·°R}$.

Since $h_4 = h_3$, $x_4 = \frac{47.65 - 24.05}{83.34} = 0.2832 \Rightarrow s_4 = 0.0522 + 0.2832(0.2189 - 0.0522) = 0.0994 \frac{\text{Btu}}{\text{lb·°R}}$

(a) An energy rate balance for the compressor gives the compressor power input:

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (0.15 \text{ lb/s})(120.33 - 107.39) \frac{\text{Btu}}{\text{lb}} = 1.94 \frac{\text{Btu}}{\text{s}} \quad \leftarrow (a)$$

(b) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{\dot{m}(h_1 - h_4)}{\dot{W}_c} = \frac{(0.15)(107.39 - 47.65) \text{ Btu/s}}{1.94 \text{ Btu/s}} = \frac{8.96 \text{ Btu/s}}{1.94 \text{ Btu/s}} = 4.62 \quad \leftarrow (b)$$

Continued on next slide

Problem 10-26 continued

To evaluate exergy destruction rates, we use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production obtained from an entropy rate balance. Thus

(c) For the compressor

$$(\dot{E}_d)_{comp} = T_0 \dot{m} (s_2 - s_1) = (550^\circ R)(0.15 \frac{\text{lb}}{\text{s}})(0.2234 - 0.2189) \frac{\text{Btu}}{\text{lb} \cdot ^\circ R} = 0.371 \frac{\text{Btu}}{\text{s}}$$

For the valve

$$(\dot{E}_d)_{valve} = T_0 \dot{m} (s_4 - s_3) = (550)(0.15)(0.0994 - 0.0958) = 0.297 \frac{\text{Btu}}{\text{s}}$$

(d) For a control volume enclosing the evaporator, as shown in the schematic, an entropy rate balance reads

$$0 = \frac{\dot{Q}_{in}}{T_c} + \dot{m} (s_4 - s_1) + \dot{\sigma}_{cv}$$

$$\Rightarrow (\dot{E}_d)_{evap} = T_0 \left[-\frac{\dot{Q}_{in}}{T_c} + \dot{m} (s_1 - s_4) \right] = 550^\circ R \left[-\frac{8.96 \text{ Btu/s}}{520^\circ R} + (0.15 \frac{\text{lb}}{\text{s}})(0.2189 - 0.0994) \frac{\text{Btu}}{\text{lb} \cdot ^\circ R} \right]$$

$$= 0.382 \frac{\text{Btu}}{\text{s}}$$

The transfer of exergy accompanying heat transfer is

$$\textcircled{1} \quad \dot{E}_q = \left[1 - \frac{T_0}{T_c} \right] \dot{Q}_{in} = \left[1 - \frac{550}{520} \right] (8.96) = -0.517 \frac{\text{Btu}}{\text{s}}. \text{ The exergy transfer is opposite to the direction of the heat transfer because heat transfer occurs at } T_c < T_0.$$

(e) For a control volume enclosing the condenser, as shown in the schematic, an entropy rate balance reads

$$0 = -\frac{\dot{Q}_{out}}{T_H} + \dot{m} (s_2 - s_3) + \dot{\sigma}_{cv}$$

$$\Rightarrow (\dot{E}_d)_{cond} = T_0 \left[\frac{\dot{Q}_{out}}{T_H} + \dot{m} (s_1 - s_2) \right] = 550 \left[\frac{10.9}{550} + 0.15(0.0958 - 0.2234) \right]$$

$$= 0.373 \text{ Btu/s}$$

The transfer of exergy accompanying heat transfer is

$$\dot{E}_q = \left[1 - \frac{T_0}{T_H} \right] \dot{Q}_{out} = 0, \text{ since } T_0 = T_H.$$

1. The exergy evaluations can be summarized as follows:

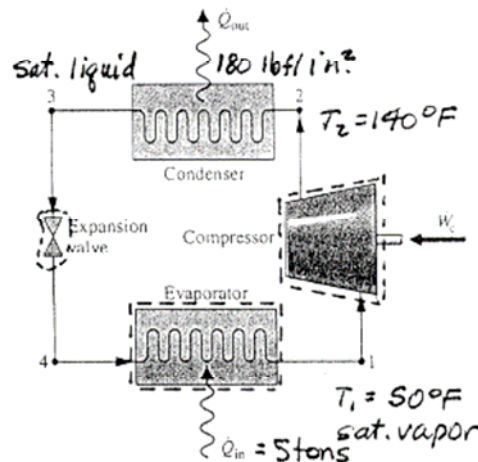
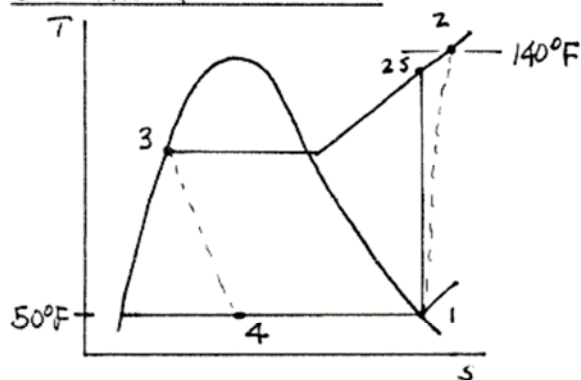
• Rate of exergy in:		• Disposition of the exergy:	
- compressor power input	1.94 $\frac{\text{Btu}}{\text{s}}$	- Transfer to cool region	0.517 $\frac{\text{Btu}}{\text{s}}$ (26.6%)
		- Transfer to warm region	0
		- Destructions	
		compressor	0.371 " (19.1%)
		valve	0.297 " (15.3%)
		evaporator	0.382 " (19.7%)
		condenser	0.373 " (19.2%)
			<u>1.94 $\frac{\text{Btu}}{\text{s}}$</u>

PROBLEM 10.27

KNOWN: Data are known at various locations in a vapor-compression refrigeration cycle. The refrigerating capacity is 5 tons.

FIND: Determine (a) the refrigerant mass flow rate, (b) the compressor isentropic efficiency, (c) the compressor power, and (d) the coefficient of performance. Plot each of these quantities for compressor exit temperatures ranging from 130°F to 140°F.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.3, items 1-4, except the compressor efficiency is not specified.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = 50^\circ\text{F}$, saturated vapor $\Rightarrow h_1 = 108.74 \text{ Btu/lb}$, $s_1 = 0.2183 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2 $P_2 = 180 \text{ lbf/in}^2$, $T_2 = 140^\circ\text{F} \Rightarrow h_2 = 123.21 \text{ Btu/lb}$

State 3 $P_3 = 180 \text{ lbf/in}^2$, Sat. liquid $\Rightarrow h_3 = 50.64 \text{ Btu/lb}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 50.64 \text{ Btu/lb}$

(a) With the refrigerating capacity

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_1 - h_4)} = \frac{5 \text{ tons}}{(108.74 - 50.64) \text{ Btu/lb}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 17.21 \text{ lb/min} \leftarrow \dot{m}$$

(b) $P_2 = 180 \text{ lbf/in}^2$, $s_{2s} = s_1 \Rightarrow h_{2s} = 118.42 \text{ Btu/lb}$

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{118.42 - 108.74}{123.21 - 108.74} = 0.669 \text{ (66.9\%)} \leftarrow \eta_c$$

$$\begin{aligned} \dot{W}_c &= \dot{m}(h_2 - h_1) = (17.21 \frac{\text{lb}}{\text{min}})(123.21 - 108.74) \frac{\text{Btu}}{\text{lb}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| \\ &= 5.87 \text{ hp} \leftarrow \dot{W}_c \end{aligned}$$

$$\begin{aligned} \beta &= \frac{\dot{Q}_{in}}{\dot{W}_c} = \left(\frac{5 \text{ tons}}{5.87 \text{ hp}} \right) \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| \\ &= 4.02 \leftarrow \beta \end{aligned}$$

Continued on next slide

Problem 10-27 continued

The required data for the plots are obtained using *IT*, as follows.

IT Code

```
T1 = 50 // °F
T2 = 140 // °F
p2 = 180 // lbf/in.2
p3 = p2
Qdotin = 5 // tons

p1 = Psat_T("R134A", T1)
h1 = hsat_Px("R134A", p1, 1)
s1 = ssat_Px("R134A", p1, 1)
h2 = h_PT("R134A", p2, T2)
s2s = s1
h2s = h_Ps("R134A", p2, s2s)
h3 = hsat_Px("R134A", p3, 0)
h4 = h3
```

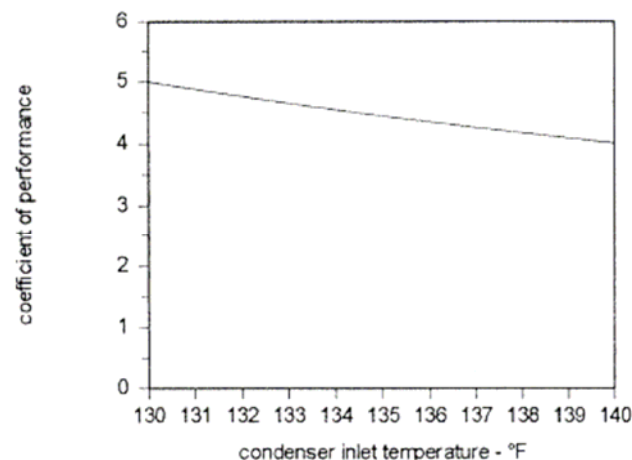
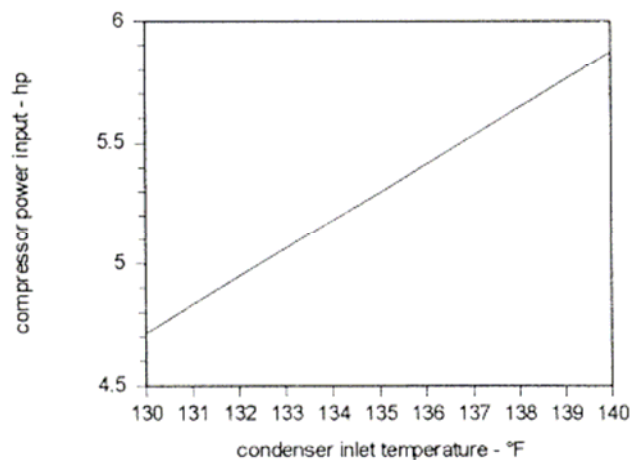
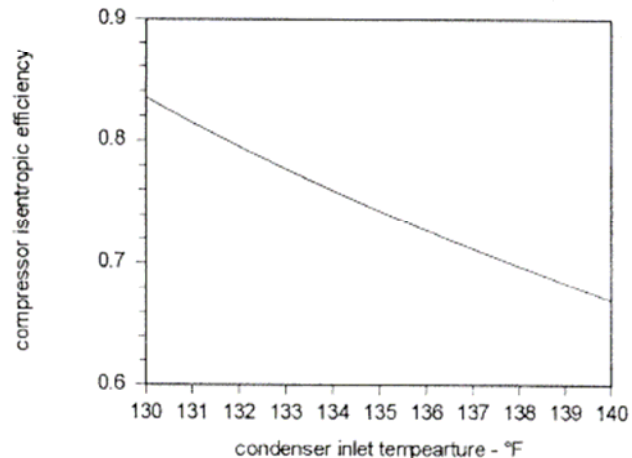
```
mdot = Qdotin / (h1 - h4) * 200 // conversion to tons
etac = (h2s - h1) / (h2 - h1)
Wdotc = mdot * (h2 - h1) * (60 / 2545) // conversion to hp
beta = (Qdotin / Wdotc) * (200 * 60 / 2545)
```

IT Results (for $T_2 = 140$ °F)

```
m = 17.21 lb/min
Wc = 5.873 hp
β = 4.014
ηc = 0.6699
h1 = 108.7 Btu/lb
h2 = 123.2 Btu/lb
h2s = 118.4 Btu/lb
h3 = 50.64 Btu/lb
h4 = 50.64 Btu/lb
```

Note: $\dot{m}$ doesn't vary with T_2 . Thus, no plot is included.

As T_2 increases, the compressor efficiency decreases and more input power is required. For fixed $\dot{Q}_{in}$, this results in a decrease in coefficient of performance.



Problem 10-28 continued

To find the refrigerant 22 mass flow rate, analyze the intermediate heat exchanger.

$$0 = \dot{m}_{R-134a}(h_2 - h_3) + \dot{m}_{R-22}(h_8 - h_5)$$

or

$$\dot{m}_{R-22} = \dot{m}_{R-134a} \left(\frac{h_2 - h_3}{h_5 - h_8} \right) = \left(54.66 \frac{\text{lb}}{\text{min}} \right) \left(\frac{111.34 - 24.14}{107.8 - 43.46} \right)$$

$$= 74.08 \text{ lb/min}$$

Thus, for the R-22 compressor

$$\dot{W}_{c2} = \dot{m}_{R-22}(h_6 - h_5) = \left(74.08 \frac{\text{lb}}{\text{min}} \right) (120.51 - 107.8) \frac{\text{Btu}}{\text{lb}} = 941.6 \text{ Btu/min} \leftarrow \dot{W}_{c2}$$

(b) The overall coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{c1} + \dot{W}_{c2}} = \frac{(20 \text{ tons})}{(766.3 + 941.6) \text{ Btu/min}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right|$$

$$= 2.34 \leftarrow \beta$$

(c) For the heat exchanger

$$0 = \sum_j \left(\frac{\dot{Q}_j}{T_j} \right) + \dot{m}_{R-134a}(s_2 - s_3) + \dot{m}_{R-22}(s_8 - s_5) + \dot{\sigma}_{HX}$$

Thus

$$\dot{E}_d = T_0 \dot{\sigma}_{HX} = T_0 [\dot{m}_{R-134a}(s_3 - s_2) + \dot{m}_{R-22}(s_5 - s_8)]$$

$$= (540^\circ\text{R}) [(54.66)(.0523 - .2266) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} + (74.08)(.22095 - .09088)]$$

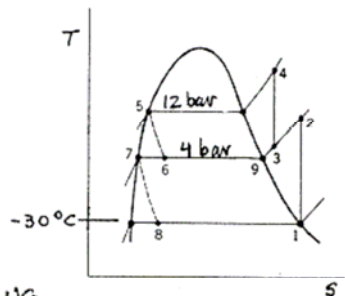
$$= 58.51 \text{ Btu/min}$$

PROBLEM 10.29

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression refrigeration system using two-stage compression with intercooling between the stages. The refrigerating capacity is given, and data at various locations are known.

FIND: Determine (a) the power input to each compressor, and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compressors operate isentropically. (3) There are no pressure drops for flow through any of the heat exchangers or the flash chamber. (4) The flash chamber and direct contact heat exchanger operate adiabatically. (5) The expansion in the valve is a throttling process. (6) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -30^\circ\text{C}$, sat. vapor $\Rightarrow h_1 = 229.14 \text{ kJ/kg}$, $s_1 = 0.9434 \text{ kJ/kg}\cdot\text{K}$.

State 2 $P_2 = 4 \text{ bar}$, $s_2 = s_1 \Rightarrow h_2 = 260.62 \text{ kJ/kg}$

State 5 $P_5 = 12 \text{ bar}$, sat. liquid $\Rightarrow h_5 = 115.76 \text{ kJ/kg}$

State 6 Throttling process $\Rightarrow h_6 = h_5 = 115.76 \text{ kJ/kg}$, $x_6 = 0.2825$

State 7 $P_7 = 4 \text{ bar}$, sat. liquid $\Rightarrow h_7 = 62.00 \text{ kJ/kg}$

State 8 Throttling process $\Rightarrow h_8 = h_7 = 62.00 \text{ kJ/kg}$

State 9 $P_9 = 4 \text{ bar}$, sat. vapor $\Rightarrow h_9 = 252.32 \text{ kJ/kg}$

State 3 The fraction of the flow into the flash chamber at 6 that exits as saturated vapor at 9 is equal to the quality at 6. The liquid leaving the flash chamber at 7 is the fraction $1 - x_6$. With these flow rate ratios

$$0 = (1 - x_6) h_2 + x_6 h_9 - 1 h_3$$

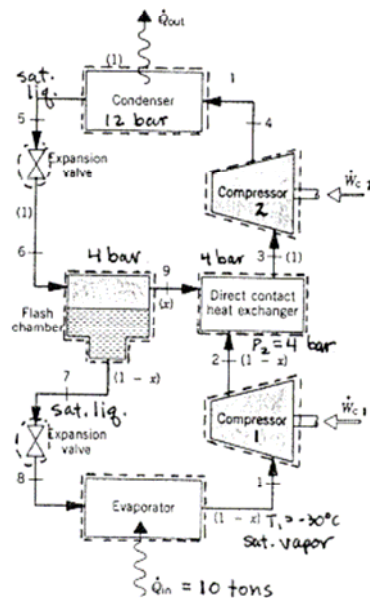
or

$$h_3 = (1 - x_6) h_2 + x_6 h_9$$

$$= (1 - 0.2825)(260.62) + 0.2825(252.32) = 258.27 \text{ kJ/kg}$$

and by interpolation in Table A-12 with h_3 we get $s_3 = 0.9352 \text{ kJ/kg}\cdot\text{K}$.

State 4 $P_4 = 12 \text{ bar}$, $s_4 = s_3 \Rightarrow h_4 = 281.69 \text{ kJ/kg}$



Continued on next slide

Problem 10-29 continued

(a) To determine the compressor power, first determine the mass flow rates. For the evaporator

$$\dot{Q}_{in} = \dot{m}_1 (h_1 - h_8)$$

$$\text{or } \dot{m}_1 = \frac{\dot{Q}_{in}}{(h_1 - h_8)} = \frac{(10 \text{ tons})}{(229.14 - 62.00) \frac{\text{kJ}}{\text{kg}}} \left| \frac{211 \text{ kJ/min}}{1 \text{ ton}} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right|$$

$$= 0.21 \text{ kg/s}$$

Also, since $1 - x_6$ is the fraction of the total flow passing through the evaporator

$$\frac{\dot{m}_1}{\dot{m}_3} = 1 - x_6 \Rightarrow \dot{m}_3 = \frac{\dot{m}_1}{1 - x_6} = 0.293 \text{ kg/s}$$

Thus,

$$\dot{W}_{c1} = \dot{m}_1 (h_2 - h_1) = (0.21 \frac{\text{kg}}{\text{s}}) (260.62 - 229.14) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 6.61 \text{ kW} \quad \leftarrow \dot{W}_{c1}$$

and

$$\dot{W}_{c2} = \dot{m}_3 (h_4 - h_3) = (0.293) (281.69 - 258.27) = 6.86 \text{ kW} \quad \leftarrow \dot{W}_{c2}$$

(b) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{c1} + \dot{W}_{c2}} = \frac{(10 \text{ tons})}{(6.61 + 6.86) \text{ kW}} \left| \frac{211 \text{ kJ/min}}{1 \text{ ton}} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

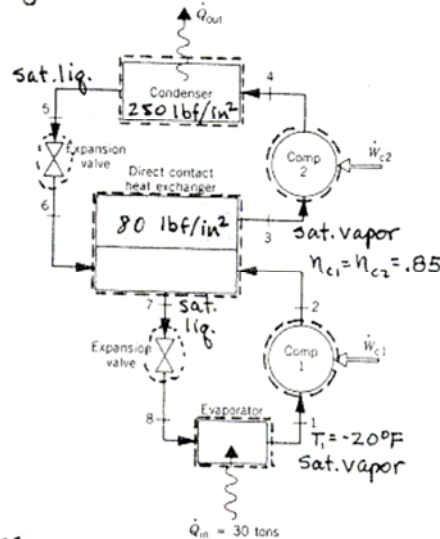
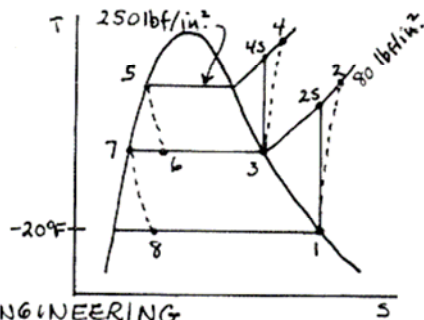
$$= 2.61 \quad \leftarrow \beta$$

PROBLEM 10.30

KNOWN: Ammonia is the working fluid in a two-stage vapor-compression refrigeration system using a direct contact heat exchanger to achieve intercooling. Data are known at various locations, and the refrigerating capacity is given.

FIND: Determine (a) the ratio of the mass flow rates $\dot{m}_3/\dot{m}_1$ (see figure), (b) the power input to each compressor stage, and (c) the coefficient of performance. (d) Plot each of the quantities in parts (a) - (c) versus the direct-contact heat exchanger pressure. Discuss.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compressors, valves, and direct contact heat exchanger each operate adiabatically. (3) The expansions through the valves are throttling processes. (4) There are no significant pressure drops for flow through the heat exchangers. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -20^\circ\text{F}$, sat. vapor $\Rightarrow h_1 = 604.61 \text{ Btu/lb}$, $s_1 = 1.3762 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2 For isentropic compression, $p_2 = 80 \text{ lbf/in}^2$, $s_2 = s_1 \Rightarrow h_{2s} = 692.14 \frac{\text{Btu}}{\text{lb}}$
Using the compressor efficiency,
 $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c} \right) = 707.59 \text{ Btu/lb}$

State 3 $p_3 = 80 \text{ lbf/in}^2$, sat. vapor $\Rightarrow h_3 = 623.32 \text{ Btu/lb}$, $s_3 = 1.2529 \text{ Btu/lb}\cdot^\circ\text{R}$

State 4 $p_4 = 250 \text{ lbf/in}^2$, $s_{4s} = s_3 \Rightarrow h_{4s} = 693.53 \text{ Btu/lb}$. Thus
 $h_4 = h_3 + \left(\frac{h_{4s} - h_3}{\eta_c} \right) = 705.92 \text{ Btu/lb}$

State 5 $p_5 = 250 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_5 = 167.77 \text{ Btu/lb}$

State 6 Throttling process $\Rightarrow h_6 = h_5 = 167.77 \text{ Btu/lb}$

State 7 $p_7 = 80 \text{ lbf/in}^2$, sat. liquid $\Rightarrow h_7 = 91.22 \text{ Btu/lb}$

State 8 Throttling process $\Rightarrow h_8 = h_7 = 91.22 \text{ Btu/lb}$

(a) The mass flow rate through the evaporator is

$$\dot{m}_1 = \frac{\dot{Q}_{in}}{h_1 - h_8} = \frac{(30 \text{ tons}) \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right|}{(604.61 - 91.22) \frac{\text{Btu}}{\text{lb}}} = 11.69 \frac{\text{lb}}{\text{min}}$$

Continued on next slide

Problem 10-30 continued

The mass flow rate into the second turbine stage is determined by using an energy balance on the direct contact heat exchanger

$$0 = \dot{m}_6 h_6 + \dot{m}_2 h_2 - \dot{m}_3 h_3 - \dot{m}_7 h_7$$

But $\dot{m}_6 = \dot{m}_3$ and $\dot{m}_7 = \dot{m}_2 = \dot{m}_1$

Thus $0 = \dot{m}_3 (h_6 - h_3) + \dot{m}_1 (h_2 - h_7)$

or $\dot{m}_3 = \dot{m}_1 \left(\frac{h_7 - h_2}{h_6 - h_3} \right) = (11.69) \left(\frac{91.22 - 707.59}{167.77 - 623.32} \right) = 15.82 \text{ lb/min}$

The mass flow rate ratio is

$$\dot{m}_3 / \dot{m}_1 = 1.353 \leftarrow \dot{m}_3 / \dot{m}_1$$

(b) The power input to the first compressor stage is

$$\begin{aligned} \dot{W}_{c1} &= \dot{m}_1 (h_2 - h_1) \\ &= (11.69 \frac{\text{lb}}{\text{min}}) \left| \frac{60 \text{ min}}{1 \text{ h}} \right| (707.59 - 604.61) \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| \\ &= 28.38 \text{ hp} \leftarrow \dot{W}_{c1} \end{aligned}$$

And, the power input to the second compressor stage is

$$\begin{aligned} \dot{W}_{c2} &= \dot{m}_3 (h_4 - h_3) \\ &= (15.82) (705.92 - 623.32) \left| \frac{60}{2545} \right| = 30.81 \text{ hp} \leftarrow \dot{W}_{c2} \end{aligned}$$

(c) The coefficient of performance is

$$\begin{aligned} \beta &= \frac{\dot{Q}_{in}}{\dot{W}_{c1} + \dot{W}_{c2}} = \frac{(30 \text{ tons})}{(28.38 + 30.81) \text{ hp}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \\ &= 2.390 \leftarrow \beta \end{aligned}$$

(d) The data for the required plots are obtained using IT, as follows:

```
IT Code
Qdotin = 30 // tons
T1 = -20 // °F
p2 = 80 // lbf/in²
p3 = p2
p4 = 250 // lbf/in²
eta_c1 = 0.85
eta_c2 = 0.85

s1 = ssat_Px("Ammonia", p1, x1)
s2s = s1
h2s = h_Ps("Ammonia", p2, s2s)
h2 = h1 + (h2s - h1) / eta_c1
x3 = 1
h3 = hsat_Px("Ammonia", p3, x3)
s3 = ssat_Px("Ammonia", p3, x3)
s4s = s3
h4s = h_Ps("Ammonia", p4, s4s)
h4 = h3 + (h4s - h3) / eta_c2
p5 = p4
h5 = hsat_Px("Ammonia", p5, 0)
h6 = h5

p7 = p3
h7 = hsat_Px("Ammonia", p7, 0)
h8 = h7

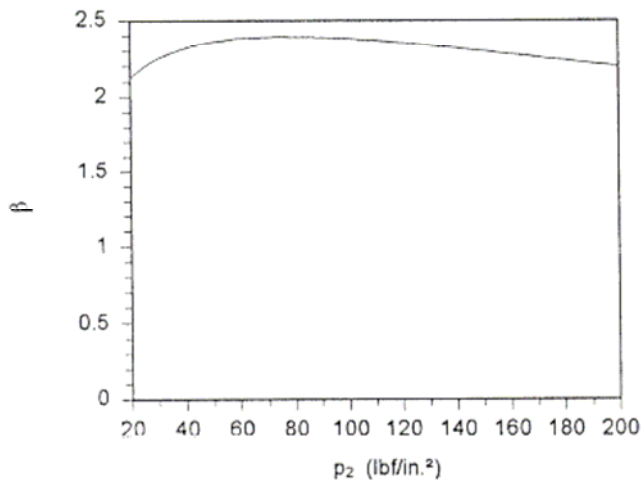
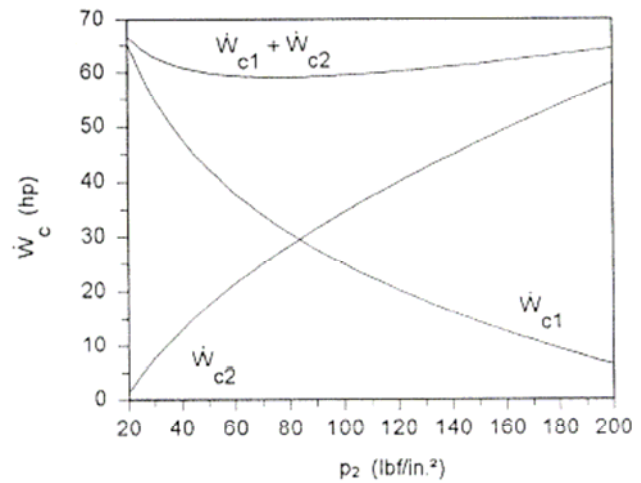
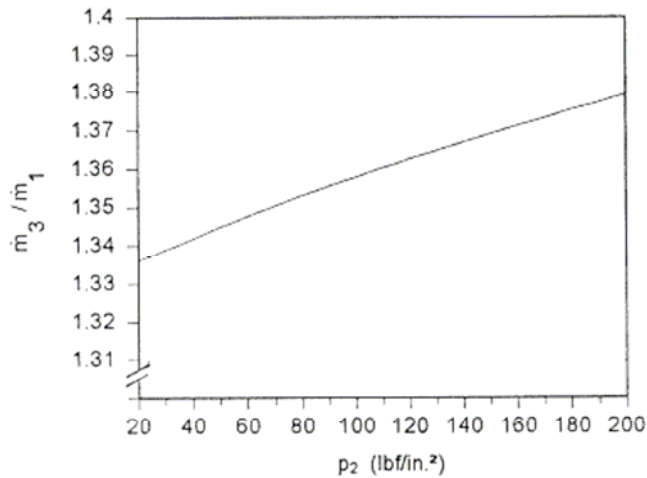
mdot1 = (Qdotin * 200) / (h1 - h8) // Btu/min
0 = mdot3 * (h6 - h3) + mdot1 * (h2 - h7)
mratio = mdot3 / mdot1
Wdotc1 = mdot1 * (h2 - h1) * (60 / 2545)
Wdotc2 = mdot3 * (h4 - h3) * (60 / 2545)
Wdotc = Wdotc1 + Wdotc2
beta = ((Qdotin * 200) / (Wdotc1 + Wdotc2)) * (60 / 2545)
```

IT Results for $p_2 = 80 \text{ lbf/in}^2$	
$h_1 = 604.2 \text{ Btu/lb}$	$h_7 = 91.16 \text{ Btu/lb}$
$h_2 = 707.1 \text{ Btu/lb}$	$h_8 = 91.16 \text{ Btu/lb}$
$h_{2s} = 691.7 \text{ Btu/lb}$	$\dot{m}_3 / \dot{m}_1 = 1.353$
$h_3 = 622.9 \text{ Btu/lb}$	$\dot{W}_{c1} = 28.37 \text{ hp}$
$h_4 = 705.2 \text{ Btu/lb}$	$\dot{W}_{c2} = 30.71 \text{ hp}$
$h_{4s} = 692.9 \text{ Btu/lb}$	$\dot{W}_c = 59.08 \text{ hp}$
$h_5 = 167.7 \text{ Btu/lb}$	$\beta = 2.394$
$h_6 = 167.7 \text{ Btu/lb}$	

Continued on next slide

Problem 10-30 continued

PLOTS:



From the plots we see that as the inter stage pressure increases, the fraction of the flow entering the first compressor stage that enters the second stage increases. Also, the power required by the first stage decreases and that required by the second stage increases. The curve that shows the total compressor power exhibits a minimum value. Accordingly, the coefficient of performance exhibits a maximum value.

PROBLEM 10.31

KNOWN: Ammonia is the working fluid in a two-stage, vapor-compression refrigeration system with two evaporators and a direct contact heat exchanger. Data are given at various locations, and the capacities of both evaporators are specified.

①

FIND: Determine (a) the evaporator temperatures, (b) the power input to each compressor stage, and (c) the overall coefficient of performance.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL:

(1) Each component operates as a control volume at steady state. (2) There are no pressure drops through the evaporators, condenser, or direct contact heat exchanger. (3) The compressors and heat exchanger operate adiabatically. The expansion through the valve is a throttling process. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $p_1 = 18 \text{ lbf/in}^2$, sat. vapor $\Rightarrow h_1 = 604.40 \frac{\text{Btu}}{\text{lb}}$
 $s_1 = 1.3775 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$

State 2 $p_2 = 70 \text{ lbf/in}^2$, $s_{2s} = s_1 \Rightarrow h_{2s} = 683.72 \text{ Btu/lb}$

With the compressor stage efficiency; $\eta_c = (h_{2s} - h_1) / (h_2 - h_1)$

$h_2 = h_1 + (h_{2s} - h_1) / \eta_{c1} = 604.40 + (683.72 - 604.40) / 0.8 = 703.55 \text{ Btu/lb}$

State 3 First, get $\dot{m}_1$ and $\dot{m}_8$. Note: $h_5 = h_6 = h_7 = 150.73 \text{ Btu/lb}$

$$\dot{m}_1 = \frac{\dot{Q}_{in,1}}{(h_1 - h_7)} = \frac{5 \text{ tons}}{(604.40 - 150.73) \frac{\text{Btu}}{\text{lb}}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 2.204 \text{ lb/min} = \dot{m}_2$$

$$\dot{m}_8 = \frac{\dot{Q}_{in,2}}{(h_8 - h_6)} = \frac{10 \text{ tons}}{(621.74 - 150.73)} = 4.246 \text{ lb/min}$$

$$h_8 = h_g @ 70 \text{ lbf/in}^2$$

For the direct contact heat exchanger: $0 = \dot{m}_2 h_2 + \dot{m}_8 h_8 - (\dot{m}_2 + \dot{m}_8) h_3$

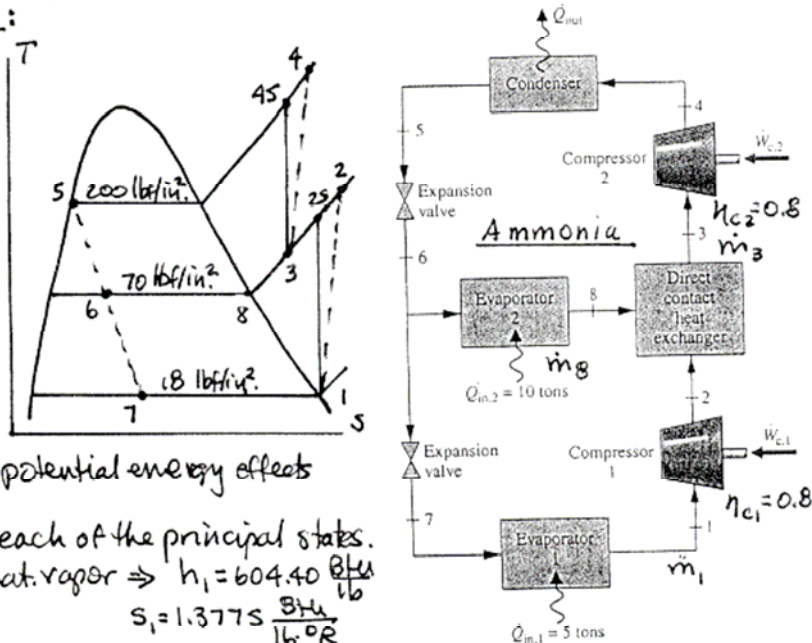
$$h_3 = \frac{\dot{m}_2 h_2 + \dot{m}_8 h_8}{(\dot{m}_2 + \dot{m}_8)} = \frac{(2.204)(703.55) + (4.246)(621.74)}{(2.204 + 4.246)} = 649.69 \frac{\text{Btu}}{\text{lb}}$$

State 4 $p_3 = 70 \text{ lbf/in}^2$, $h_3 = 649.69 \text{ Btu/lb} \Rightarrow s_3 = 1.3179 \text{ Btu/lb} \cdot ^\circ\text{R}$

$p_4 = 200 \text{ lbf/in}^2$, $s_{4s} = s_3 \Rightarrow h_{4s} = 720.82 \text{ Btu/lb}$. $\eta_{c2} = (h_{4s} - h_3) / (h_4 - h_3)$

$$h_4 = h_3 + (h_{4s} - h_3) / \eta_{c2} = 649.69 + (720.82 - 649.69) / 0.8 = 738.6 \text{ Btu/lb}$$

States 5, 6, 7, 8 $h_5 = h_6 = h_7 = 150.73 \text{ Btu/lb}$, $h_8 = 621.74 \text{ Btu/lb}$



Continued on next slide

Problem 10-31 continued

(a) From Table A-14E: $T_1 = -20.60^\circ\text{F}$, $T_8 = 37.67^\circ\text{F}$ T_1, T_8

$$\begin{aligned} \dot{W}_{c1} &= \dot{m}_1 (h_2 - h_1) \\ &= (2.204 \frac{\text{lb}}{\text{min}})(703.55 - 604.40) \frac{\text{Btu}}{\text{lb}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| \\ &= 5.15 \text{ hp} \end{aligned} \quad \dot{W}_{c1}$$

$$\dot{W}_{c2} = (4.246)(738.6 - 649.69) \left| \frac{60}{2545} \right| = 8.9 \text{ hp} \quad \dot{W}_{c2}$$

$$\begin{aligned} \beta &= \frac{(\dot{Q}_{in,1} + \dot{Q}_{in,2})}{(\dot{W}_{c1} + \dot{W}_{c2})} = \frac{(10 + 5) \text{ tons}}{(5.15 + 8.9) \text{ hp}} \left| \frac{12000 \text{ Btu/h}}{1 \text{ ton}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| \\ &= 5.03 \end{aligned} \quad \beta$$

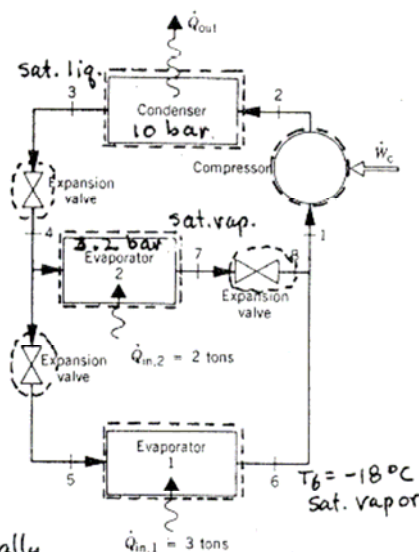
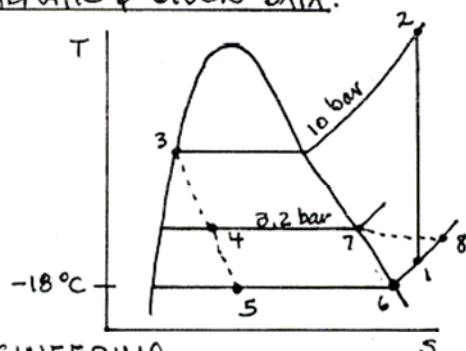
1. In early printings of the Sixth Edition, the identity of the working fluid was omitted inadvertently. We regret the error.

PROBLEM 10.32

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression refrigeration system with two evaporators. Data are known at various locations, and the refrigerating capacity of each evaporator is specified. The system uses only one compressor.

FIND: Determine (a) the mass flow rates through each evaporator, (b) the compressor power input, and (c) the heat transfer from the refrigerant passing through the condenser.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the expansion through each valve, which is a throttling process. (3) The compressor and valves operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 3 $p_3 = 10 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 105.29 \text{ kJ/kg}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 105.29 \text{ kJ/kg}$

State 5 Throttling process $\Rightarrow h_5 = h_4 = 105.29 \text{ kJ/kg}$

State 6 $T_6 = -18^\circ\text{C}$, sat. vapor $\Rightarrow h_6 = 236.53 \text{ kJ/kg}$

State 7 $p_7 = 3.2 \text{ bar}$, sat. vapor $\Rightarrow h_7 = 248.66 \text{ kJ/kg}$

State 8 Throttling process $\Rightarrow h_8 = h_7 = 248.66 \text{ kJ/kg}$

To fix states 1 and 2 requires the mass flow rates through the evaporators. Thus

$$(a) \quad \dot{Q}_{in,1} = \dot{m}_6 (h_6 - h_5)$$

$$\text{or} \quad \dot{m}_6 = \frac{\dot{Q}_{in,1}}{(h_6 - h_5)} = \frac{(3 \text{ tons})}{(236.53 - 105.29) \frac{\text{kJ}}{\text{kg}}} \left| \frac{211 \text{ kJ/min}}{1 \text{ ton}} \right|$$

$$= 4.823 \text{ kg/min} \quad \leftarrow \dot{m}_6$$

Continued on next slide

Problem 10-32 continued

and

$$\dot{m}_8 = \frac{\dot{Q}_{in,2}}{(h_7 - h_4)} = \frac{(2)(211)}{(248.46 - 105.29)} = 2.943 \frac{\text{kg}}{\text{min}} \leftarrow \dot{m}_8$$

State 1 Now, for the adiabatic mixing streams 6 and 8 to form Stream 1: $0 = \dot{m}_6 h_6 + \dot{m}_8 h_8 - (\dot{m}_6 + \dot{m}_8) h_1$. Thus

$$h_1 = \frac{\dot{m}_6 h_6 + \dot{m}_8 h_8}{\dot{m}_6 + \dot{m}_8} = 241.13 \text{ kJ/kg}$$

The specific entropy s_1 is found using double interpolation in Table A-12 to be; $s_1 \approx 0.9493 \text{ kJ/kg}\cdot\text{K}$

State 2 $p_2 = 10 \text{ bar}$, $s_2 = s_1 \Rightarrow h_2 = 282.3 \text{ kJ/kg}$

(b) The compressor power is

$$\begin{aligned} \dot{W}_c &= (\dot{m}_6 + \dot{m}_8)(h_2 - h_1) \\ &= (7.766 \frac{\text{kg}}{\text{min}}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (282.3 - 241.13) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 5.329 \text{ kW} \leftarrow \dot{W}_c \end{aligned}$$

For the condenser

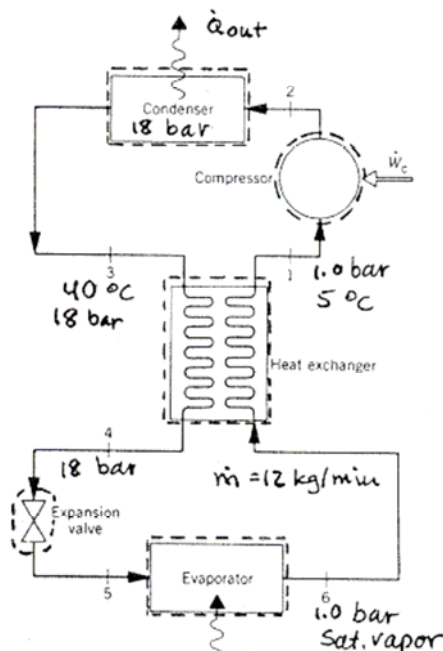
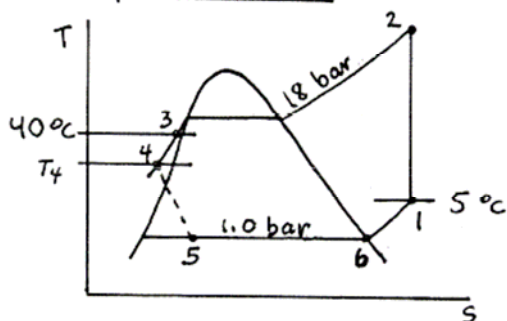
$$\begin{aligned} \dot{Q}_{out} &= (\dot{m}_6 + \dot{m}_8)(h_2 - h_3) \\ &= (7.766) \left| \frac{1}{60} \right| (282.3 - 105.29) = 22.91 \text{ kW} \leftarrow \dot{Q}_{out} \end{aligned}$$

PROBLEM 10.33

KNOWN: An ideal vapor-compression refrigeration cycle is modified to include a counterflow heat exchanger between the streams exiting the condenser and evaporator. Ammonia is the working fluid. Data are known at various locations, and the mass flow rate is given.

FIND: Determine (a) the refrigerating capacity, (b) the compressor power input, (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as Example 10.1, items 1-4. Also, assume no heat loss from the outside of the heat exchanger.

ANALYSIS: First, fix each of the principal states.

State 6 $p_6 = 1.0 \text{ bar}$, sat. vapor
 $\Rightarrow h_6 = 1398.41 \text{ kJ/kg}$

State 1 $p_1 = 1.0 \text{ bar}$, $T_1 = 5^\circ\text{C} \Rightarrow h_1 = 1483.25 \text{ kJ/kg}$, $s_1 = 6.1676 \text{ kJ/kg}\cdot\text{K}$ $\overset{\dot{Q}_{in}}{\curvearrowright}$

State 2 $p_2 = 18 \text{ bar}$, $s_2 = s_1 \Rightarrow h_2 = 2025.54 \text{ kJ/kg}$

State 3 $p_3 = 18 \text{ bar}$, $T_3 = 40^\circ\text{C} \Rightarrow \text{comp. liquid}$; $h_3 \approx h_f(40^\circ\text{C}) = 371.35 \text{ kJ/kg}$

State 4 Using an energy balance on the heat exchanger control volume

or $h_4 = h_6 - h_1 + h_3 = 286.51 \text{ kJ/kg}$

State 5 Throttling process $\Rightarrow h_5 = h_4 = 286.51 \text{ kJ/kg}$

(a) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_6 - h_5) = (12 \frac{\text{kg}}{\text{min}})(1398.41 - 286.5) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right| = 63.24 \text{ tons} \leftarrow \dot{Q}_{in}$$

(b) The compressor power is

$$\dot{W}_c = \dot{m} (h_2 - h_1) = (12 \frac{\text{kg}}{\text{min}}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (2025.54 - 1483.25) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 108,5 \text{ kW} \leftarrow \dot{W}_c$$

(c) The coefficient of performance is $\beta = (h_6 - h_5) / (h_2 - h_1) = 2.05$ ← β

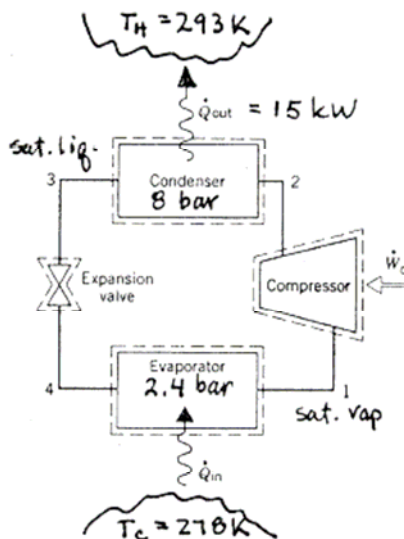
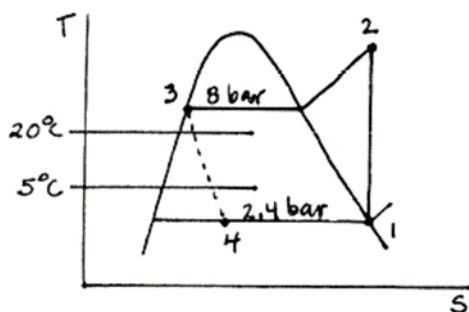
Discussion: The heat exchanger (1) tends to increase the capacity, (2) helps ensure superheated vapor entering the compressor, and (3) increases the average specific volume in the compressor, thereby increasing the power required.

PROBLEM 10.34

KNOWN: An ideal vapor-compression heat pump cycle uses Refrigerant 134a as the working fluid and provides a known energy output to heat a building. Data are known at various locations.

FIND: Determine (a) the compressor power, (b) the coefficient of performance, and (c) the maximum theoretical coefficient of performance for a heat pump operating between reservoirs at 20°C and 5°C.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.1.

ANALYSIS: First, fix each of the principal states.

State 1 $p_1 = 2.4 \text{ bar}$, sat. vapor $\Rightarrow h_1 = 244.09 \text{ kJ/kg}$, $s_1 = 0.9222 \text{ kJ/kg}\cdot\text{K}$

State 2 $p_2 = 8 \text{ bar}$, $s_2 = s_1 \Rightarrow h_2 = 268.97 \text{ kJ/kg}$

State 3 $p_3 = 8 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 93.42 \text{ kJ/kg}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 93.42 \text{ kJ/kg}$

(a) To determine the compressor, first find the mass flow rate from

$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3)$$

$$\text{or } \dot{m} = \frac{\dot{Q}_{\text{out}}}{h_2 - h_3} = \frac{(15 \text{ kW})}{(268.97 - 93.42) \frac{\text{kJ}}{\text{kg}}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 0.08544 \text{ kg/s}$$

$$\text{Thus, } \dot{W}_c = \dot{m}(h_2 - h_1) = (0.08544 \frac{\text{kg}}{\text{s}})(268.97 - 244.09) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 2.126 \text{ kW} \quad \dot{W}_c$$

(b) The coefficient of performance is

$$\gamma = \frac{\dot{Q}_{\text{out}}}{\dot{W}_c} = \frac{15}{2.126} = 7.055 \quad \gamma$$

(c) For a reversible heat pump operating between reservoirs at $T_H = 293 \text{ K}$ and $T_C = 278 \text{ K}$

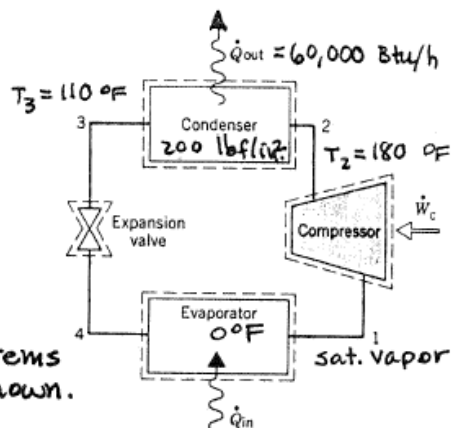
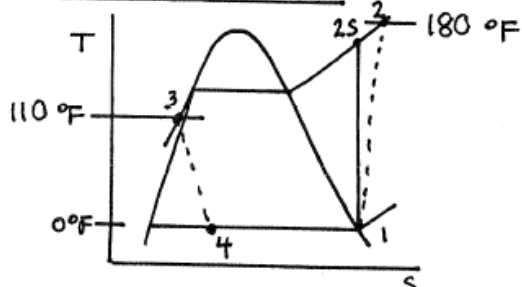
$$\gamma_{\text{max}} = \frac{T_H}{T_H - T_C} = 19.53 \quad \gamma_{\text{max}}$$

PROBLEM 10.35

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression heat pump system with known heating capacity. Data are known at various locations.

FIND: Determine (a) the mass flow rate, (b) the compressor power, (c) the isentropic compressor efficiency, and (d) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.3, items 1-4, except the compressor efficiency is unknown.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = 0^\circ\text{F}$, sat. vapor $\Rightarrow h_1 = 101.75 \text{ Btu/lb}$, $s_1 = 0.2224 \text{ Btu/lb}\cdot^\circ\text{R}$

State 2 $P_2 = 200 \text{ lbf/in}^2$, $T_2 = 180^\circ\text{F} \Rightarrow h_2 = 133.28 \text{ Btu/lb}$

State 3 $P_3 = 200 \text{ lbf/in}^2$, $T_3 = 110^\circ\text{F} \Rightarrow \text{comp. liq.}; h_3 \approx h_f(110) = 47.81 \text{ Btu/lb}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 47.81 \text{ Btu/lb}$

(a) The mass flow rate is

$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3)$$

$$\text{or } \dot{m} = \frac{\dot{Q}_{\text{out}}}{h_2 - h_3} = \frac{60,000 \text{ Btu/h}}{(133.28 - 47.81) \frac{\text{Btu}}{\text{lb}}} \left| \frac{1 \text{ h}}{60 \text{ min}} \right| = 11.70 \text{ lb/min} \leftarrow \dot{m}$$

(b) Thus, the compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1)$$

$$= (11.70 \frac{\text{lb}}{\text{min}}) \left| \frac{60 \text{ min}}{1 \text{ h}} \right| (133.28 - 101.75) \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2544 \text{ Btu/h}} \right| = 8.70 \text{ hp} \leftarrow \dot{W}_c$$

(c) The compressor efficiency is found as follows. First, for isentropic compression, $P_2 = 200 \text{ lbf/in}^2$, $s_2 = s_1 \Rightarrow h_{2s} = 121.80 \text{ Btu/lb}$. Then

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{121.80 - 101.75}{133.28 - 101.75} = 0.852 \text{ (85.2\%)} \leftarrow \eta_c$$

(d) The coefficient of performance is

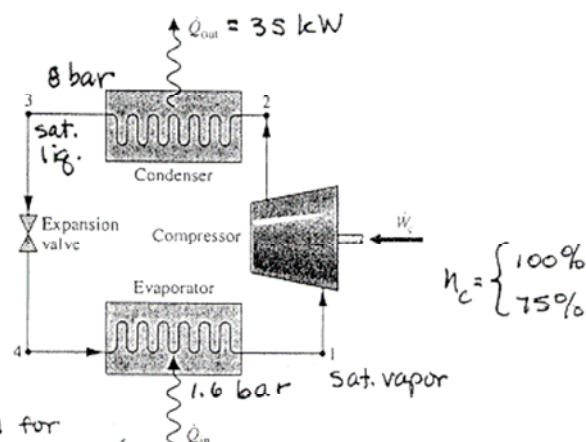
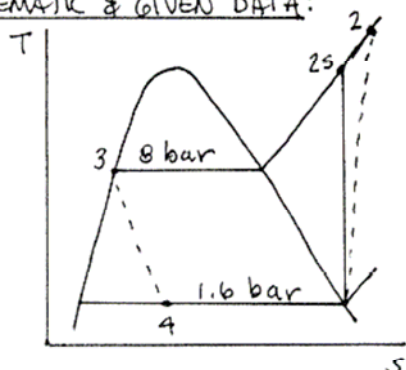
$$\gamma = \frac{h_2 - h_3}{h_2 - h_1} = 2.711 \leftarrow \gamma$$

PROBLEM 10.36

KNOWN: Data are given for a vapor-compression heat pump using R-134a as the working fluid. The rate of heat transfer to a dwelling is specified.

FIND: Determine, for isentropic compression, (a) the refrigerant mass flow rate, (b) the compressor power, and (c) the coefficient of performance. Repeat for $\eta_c = 0.75$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.1 for $\eta_c = 100\%$. See Example 10.3, items 1-4, for $\eta_c = 75\%$.

ANALYSIS: First, fix each of the principal states for $\eta_c = 100\%$.

State 1 $p_1 = 1.6 \text{ bar}$, sat. vapor $\Rightarrow h_1 = 237.97 \text{ kJ/kg}$, $s_1 = 0.9295 \text{ kJ/kg}\cdot\text{K}$

State 2s $p_2 = 8 \text{ bar}$, $s_{2s} = s_1 \Rightarrow h_{2s} = 271.22 \text{ kJ/kg}$

State 3 $p_3 = 8 \text{ bar}$, sat. liquid $\Rightarrow h_3 = 93.42 \text{ kJ/kg}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 93.42 \text{ kJ/kg}$

(a) Using the given value of $\dot{Q}_{out}$ to get $\dot{m}$

$$\dot{m} = \frac{\dot{Q}_{out}}{(h_{2s} - h_3)} = \frac{35 \text{ kW}}{(271.22 - 93.42) \text{ kJ/kg}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 0.197 \frac{\text{kg}}{\text{s}} \leftarrow \frac{\dot{m}}{(\eta_c = 100\%)}$$

(b) The compressor power is

$$\dot{W}_c = \dot{m}(h_{2s} - h_1) = (0.197)(271.22 - 237.97) = 6.55 \text{ kW} \leftarrow \frac{\dot{W}_c}{(\eta_c = 100\%)}$$

(c) The coefficient of performance for the heat pump is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{35}{6.55} = 5.34 \leftarrow \frac{\gamma}{(\eta_c = 100\%)}$$

$$\text{For } \eta_c = 75\%; \quad h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 237.97 + (271.22 - 237.97)/(0.75) = 282.30 \text{ kJ/kg}$$

$$\dot{m} = \frac{35}{(282.30 - 93.42)} = 0.1853 \text{ kg/s}$$

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (0.1853)(282.30 - 237.97) = 8.214 \text{ kW} \leftarrow \frac{\dot{W}_c}{(\eta_c = 75\%)}$$

$$\gamma = \frac{35}{8.214} = 4.26 \leftarrow \frac{\gamma}{(\eta_c = 75\%)}$$

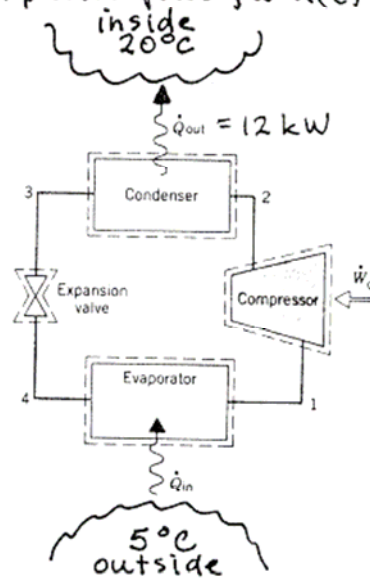
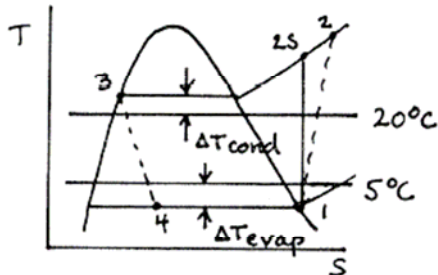
The presence of irreversibilities in the compressor increases the power input by over 25%.

PROBLEM 10.37

KNOWN: A vapor-compression heat pump with Refrigerant 22 as the working fluid is to be used to provide 12 kW to heat a house to an inside temperature of 20°C when the outside temperature is 5°C.

FIND: Specify evaporator and condenser pressures and calculate (a) the mass flow rate, (b) the compressor power, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.3, items 1-4.

ANALYSIS: The evaporator and condenser pressures must be chosen to allow for sufficient ΔT 's to avoid excessive heat exchanger sizes (surface area). For a preliminary design, assume

$$\Delta T_{\text{evap}} = 10^\circ\text{C} \Rightarrow T_1 = -5^\circ\text{C}$$

$$P_{\text{cond}} = 12 \text{ bar} \quad (T_{\text{sat}} = 30.25^\circ\text{C})$$

Also, let $\eta_c = 80\%$ for the purpose of this analysis. With these specifications

State 1 $h_1 = 248.82$, $s_1 = 0.9315 \text{ kJ/kg}\cdot\text{K}$

State 2 $h_{2s} = 273.01 \text{ kJ/kg} \Rightarrow h_2 = 279.06 \text{ kJ/kg}$

State 3 $h_3 = 81.90 \text{ kJ/kg}$ (sat. liq. at 12 bar)

State 4 $h_4 = h_3 = 81.90 \text{ kJ/kg}$

$$(a) \quad \dot{m} = \frac{\dot{q}_{\text{out}}}{h_2 - h_3} = \frac{(12 \text{ kW})}{(279.06 - 81.90) \text{ kJ/kg}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 3.652 \frac{\text{kg}}{\text{min}}$$

$$(b) \quad \dot{W}_c = \dot{m}(h_2 - h_1) = (3.652) \left| \frac{1}{60} \right| (279.06 - 248.82) = 1.84 \text{ kW}$$

$$(c) \quad \gamma = \frac{\dot{Q}_{\text{out}}}{\dot{W}_c} = \frac{12}{1.84} = 6.52$$

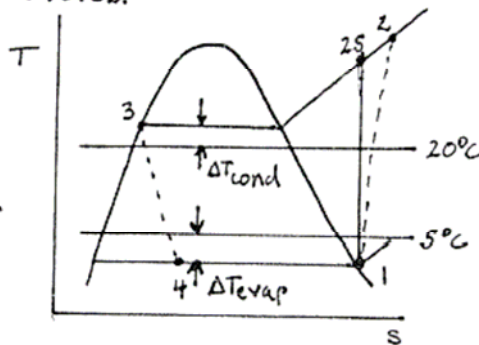
Discussion: These values are preliminary estimates. In practice, other factors, such as economics, would play an important role in the design process.

PROBLEM 10.38

KNOWN: Reconsider the cycle of Problem 10.37, but use Refrigerant 134a as the working fluid.

FIND: Answer the same questions as in Problem 10.36, and compare the results with those of Problem 10.36.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.3, items 1-4.

ANALYSIS: For a preliminary design, we use similar consideration to those in Problem 10.37. Thus, let

$$\Delta T_{\text{evap}} = 10^\circ\text{C} \Rightarrow T_1 = -5^\circ\text{C}$$

$$P_{\text{cond}} = 7 \text{ bar} \quad (T_{\text{sat}} = 26.72^\circ\text{C})$$

Also, let $\eta_c = 80\%$. With these specifications

State 1 $h_1 = 244.31$, $s_1 = .92195 \text{ kJ/kg}\cdot\text{K}$

State 2 $h_{2s} = 266.06 \text{ kJ/kg} \Rightarrow h_2 = 271.50 \text{ kJ/kg}$

State 3 $h_3 = 86.78 \text{ kJ/kg}$

State 4 $h_4 = h_3 = 86.78 \text{ kJ/kg}$

$$(a) \quad \dot{m} = \frac{\dot{Q}_{\text{out}}}{h_2 - h_3} = \frac{(12 \text{ kW})}{(271.50 - 86.78) \text{ kJ/kg}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 3.90 \text{ kg/min}$$

$$(b) \quad \dot{W}_c = \dot{m}(h_2 - h_1) = (3.90) \left| \frac{1}{60} \right| (271.50 - 244.31) = 1.77 \text{ kW}$$

$$(c) \quad \gamma = \frac{\dot{Q}_{\text{out}}}{\dot{W}_c} = \frac{12}{1.77} = 6.78$$

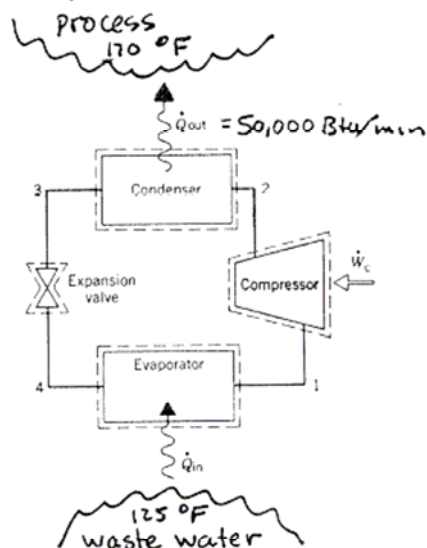
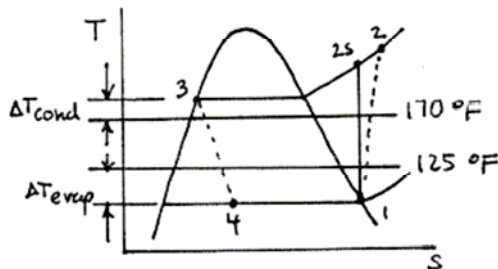
The performance of the two cycles is quite comparable in terms of $\dot{m}$, $\dot{W}_c$, and γ . The only significant difference is the somewhat higher operating pressures of the R-22 cycle.

PROBLEM 10.39

KNOWN: A Refrigerant 134a vapor-compression heat pump is proposed to develop process heat at a specified rate and temperature from a waste water stream at a known temperature.

FIND: Specify evaporator and condenser pressures and calculate (a) the mass flow rate, (b) the compressor power, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.3, items 1-4.

ANALYSIS: The evaporator and condenser pressures must be chosen to allow for sufficient ΔT 's to avoid excessive heat exchanger sizes (surface area). For a preliminary design, assume

$$P_{\text{evap}} = 180 \text{ lbf/in}^2 \rightarrow T_1 = 117.74^\circ\text{F}$$

$$P_{\text{cond}} = 400 \text{ lbf/in}^2 \rightarrow T_{\text{sat}} = 179.95^\circ\text{F}$$

Also, let $\eta_c = 80\%$ for the purpose of this analysis. With these specifications

State 1 $h_1 = 116.74 \text{ Btu/lb}$, $s_1 = 0.2154 \text{ Btu/lb} \cdot \text{R}$

State 2 $s_{2s} = s_1 \Rightarrow h_{2s} = 123.32 \text{ Btu/lb} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 124.97 \text{ Btu/lb}$

State 3 $h_3 = 50.64 \text{ Btu/lb}$

State 4 $h_4 = h_3 = 50.64 \text{ Btu/lb}$

(a) $\dot{m} = \frac{\dot{Q}_{\text{out}}}{h_2 - h_3} = \frac{50,000 \text{ Btu/min}}{(124.97 - 50.64) \text{ Btu/lb}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 4.036 \times 10^4 \frac{\text{lb}}{\text{h}} \leftarrow \dot{m}$

(b) $\dot{W}_c = \dot{m} (h_2 - h_1) = (4.036 \times 10^4 \frac{\text{lb}}{\text{h}}) \left(\frac{124.97 - 116.74}{8.23} \right) \frac{\text{Btu}}{\text{lb}} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 130.5 \text{ hp} \leftarrow \dot{W}_c$

(c) $\gamma = \frac{\dot{Q}_{\text{out}}}{\dot{W}_c} = \frac{(50,000 \text{ Btu/min}) \left| \frac{60 \text{ min}}{1 \text{ h}} \right|}{(4.036 \times 10^4 \frac{\text{lb}}{\text{h}}) (8.23) \frac{\text{Btu}}{\text{lb}}} = 9.03 \leftarrow \gamma$

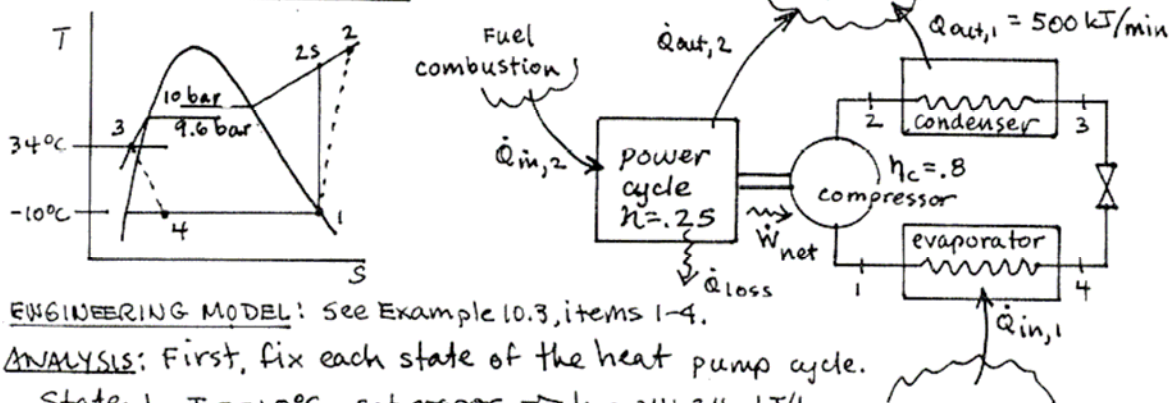
Discussion: The use of heat pumps to produce process heat from waste streams can enhance overall energy resource use in a plant. However, other factors such as economics weigh heavily in decisions to utilize such techniques.

PROBLEM 10.40

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression heat pump driven by a power cycle. Operating data are specified for the heat pump and the power cycle.

FIND: Determine (a) the heat pump compressor power and (b) the ratio of the total rate heat is delivered to the heated space to the rate of heat input to the power cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.3, items 1-4.

ANALYSIS: First, fix each state of the heat pump cycle.

State 1 $T_1 = -10^\circ\text{C}$, sat. vapor $\Rightarrow h_1 = 241.34 \text{ kJ/kg}$
 $s_1 = .9253 \text{ kJ/kg}\cdot\text{K}$

State 2 For isentropic compression, $p_2 = 10 \text{ bar}$, $s_2 = s_1 \Rightarrow h_{2s} = 274.63 \text{ kJ/kg}$
 Using the compressor efficiency
 $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c} \right) = 282.95 \text{ kJ/kg}$

State 3 $p_3 = 9.6 \text{ bar}$, $T_3 = 34^\circ\text{C} \Rightarrow \text{comp. liq.}; h_3 \approx h_f(34^\circ\text{C}) = 97.31 \text{ kJ/kg}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 97.31 \text{ kJ/kg}$

(a) The mass flow rate of refrigerant is

$$\dot{m} = \frac{\dot{Q}_{\text{out},1}}{h_2 - h_3} = \frac{(500 \text{ kJ/min})}{(282.95 - 97.31) \text{ kJ/kg}} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 0.04489 \text{ kg/s}$$

The compressor power becomes

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (0.04489 \text{ kg/s}) (282.95 - 241.34) \left| \frac{\text{kJ}}{\text{kg}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 1.868 \text{ kW} \quad \dot{W}_c$$

(b) For the power cycle, $\eta = 0.25$. With $\dot{W}_{\text{power cycle}} = \dot{W}_{\text{heat pump}} = 1.868 \text{ kW}$

$$\dot{Q}_{\text{in},2} = \frac{\dot{W}_{\text{cycle}}}{\eta} = 7.472 \text{ kW}$$

The total heat rejected is, $\dot{Q}_{\text{rej}} = 7.472 \text{ kW} - 1.868 \text{ kW} = 5.604 \text{ kW}$

Thus, $\dot{Q}_{\text{out},2} = (0.8) \dot{Q}_{\text{rej}} = 4.483 \text{ kW}$

$$\text{Finally, } \frac{\dot{Q}_{\text{out},1} + \dot{Q}_{\text{out},2}}{\dot{Q}_{\text{in},2}} = \frac{(500/60) + 4.483}{7.472} = 1.687 \quad (b)$$

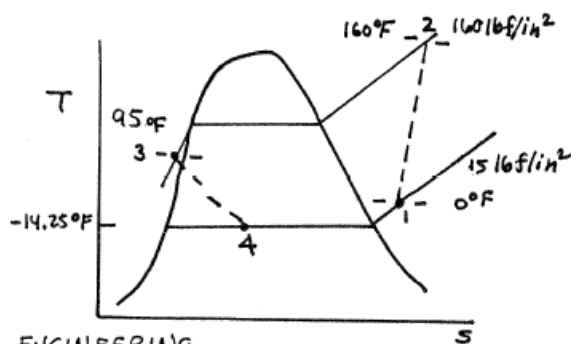
Discussion: The engine-driven heat pump delivers more energy to the heated space than could be obtained by burning the fuel directly.

PROBLEM 10.41

known: Operating data are provided for a vapor compression heat pump using R134a as the working fluid.

FIND: Determine (a) the isentropic compressor efficiency, and (b) the coefficient of performance. (c) Perform an exergy accounting of the compressor power input.

SCHEMATIC & GIVEN DATA:



State	$h(\text{Btu/lb})$	$s(\text{Btu/lb}\cdot^{\circ}\text{R})$
1	102.42	0.2303
2	129.78	0.2391
3	42.47	0.0867
4	42.47	0.0958

ENGINEERING MODEL: 1. Control volumes enclosing each of the principal components are at steady state. 2. Kinetic and potential energy effects can be ignored as can stray heat transfers and pressure drops for flow through the evaporator and condenser. 3. The expansion across the valve is a throttling process. 4. $T_0 = 480^\circ\text{R}$

ANALYSIS: First, fix each of the principal states and obtain the data shown in the table above.

State 1: $P_1 = 1516 \text{ lbf/in}^2$, $T_1 = 0^\circ\text{F}$. Table A-12E gives $h_1 = 102.42 \text{ Btu/lb}$, $s_1 = 0.2303 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}$
State 2: $P_2 = 16016 \text{ lbf/in}^2$, $T_2 = 160^\circ\text{F}$, Table A-12E gives $h_2 = 129.78 \text{ Btu/lb}$, $s_2 = 0.2391 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}$
 Also, $s_{2s} = s_1 \Rightarrow h_{2s} = 124.41 \text{ Btu/lb}$

State 3: $h_3 = h_f(95^\circ\text{F}) = 42.47 \text{ Btu/lb}$, $s_3 = s_f(95^\circ\text{F}) = 0.0867 \text{ Btu/lb}\cdot^\circ\text{R}$ from Table A-10E.

State 4: $h_4 = h_3 = 42.47 \text{ kJ/kg}$

$$\Rightarrow x_4 = \frac{42.47 - 7.40}{92.27} = 0.380 \Rightarrow S_u = 0.0171 + 0.380[0.2242 - 0.0171] = 0.0958 \frac{\text{Btu}}{\text{lb}^\circ\text{R}}$$

(a) The isentropic compressor efficiency is

$$n_c = \frac{h_{25} - h_1}{h_2 - h_1} = \frac{124.41 - 102.42}{129.78 - 102.42} = 0.804 \text{ (80.4\%)}$$

(b) The coefficient of performance is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{\dot{m}(h_2 - h_3)}{\dot{m}(h_2 - h_1)} = \frac{129.78 - 42.47}{129.78 - 102.42} = 3.19$$

(c) The compressor work input per unit of mass flowing is

$$\dot{W}_{c/m} = h_2 - h_1 = 27.36 \text{ Btu/lb}$$

Continued on next slide

Problem 10-41 continued

The rates of exergy destruction for the compressor and valve can be found using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$. That is,

- Compressor

$$(\dot{E}_d/m)_{\text{comp}} = T_0 (s_2 - s_1) = 480^\circ\text{R} (0.2391 - 0.2303) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 4.22 \text{ Btu/lb}$$

- Valve

$$(\dot{E}_d/m)_{\text{valve}} = T_0 (s_4 - s_3) = (480^\circ\text{R}) (0.0958 - 0.0867) \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} = 4.37 \text{ Btu/lb}$$

- ① The change in flow exergy as the refrigerant passes through the evaporator is

$$\begin{aligned} e_{f1} - e_{f4} &= h_1 - h_4 - T_0 (s_1 - s_4) \\ &= (102.42 - 42.47) - 480 (0.2303 - 0.0958) = -4.61 \text{ Btu/lb} \end{aligned}$$

The change in flow exergy as the refrigerant passes through the condenser is

$$\begin{aligned} e_{f3} - e_{f2} &= h_3 - h_2 - T_0 (s_3 - s_2) \\ &= (42.47 - 129.78) - 480 (0.0867 - 0.2391) = -14.16 \text{ Btu/lb} \end{aligned}$$

Summary:

• Exergy input - compressor work	27.36 $\frac{\text{Btu}}{\text{lb}}$	• Disposition of the exergy input:	
		- Exergy destruction	
		compressor	4.22 Btu/lb (15.4%)
		valve	4.37 Btu/lb (16.0%)
		- Exergy transfer out	
		evaporator	4.61 Btu/lb (16.8%)
		condenser	14.16 Btu/lb (51.8%)
			<u>27.36 Btu/lb</u>

The summary indicates that there are two significant sources of irreversibility: the compression process and the expansion process. The compressor irreversibility can be reduced by means of a compressor having a higher isentropic efficiency than found in part (a). Economic consequences should be carefully considered, however.

1. Since there is no work (except flow work) and no internal irreversibilities (pressure is constant), an exergy rate balance for the refrigerant side of the evaporator, and of the condenser, reads

$$0 = \dot{E}_q - \dot{W}_{cv} + m(\dot{e}_{f,in} - \dot{e}_{f,out}) - \dot{E}_d \Rightarrow \dot{E}_q = m(\dot{e}_{f,out} - \dot{e}_{f,in})$$

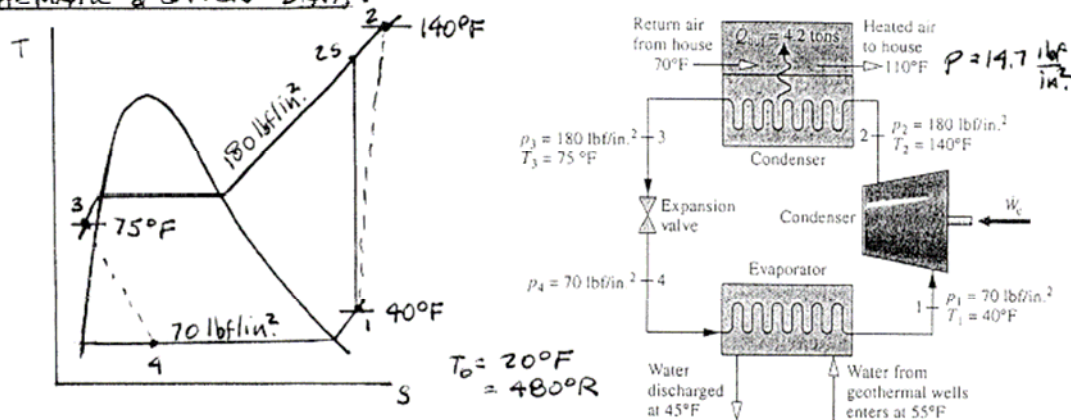
Thus, the change in flow exergy equals the exergy transfer accompanying heat transfer. Since heat transfer occurs at $T < T_0$ in the evaporator case, the exergy transfer accompanying heat is opposite to the direction of the heat transfer: from the refrigerant to the cold space. For the condenser case, heat transfer and exergy transfer are in the same direction: from the refrigerant to the heated space.

PROBLEM 10.42

KNOWN: Data are provided for a geothermal heat pump providing energy to heat a house. The heating capacity is known, and the working fluid is R-22.

FIND: Determine (a) the volumetric flow rate of air to heat the house, (b) the isentropic compressor efficiency, (c) the compressor power, (d) the coefficient of performance, and (e) the gal/min of water from the geothermal wells. Perform a full exergy accounting of the compressor power input and devise a second law efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.3, items 1-4, except the compressor efficiency is not specified. Also, assume the air behaves as an ideal gas with $c_p = 0.24 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$.

ANALYSIS: First, fix all of the principal states in the R-22 cycle.

State 1 $p_1 = 70 \text{ lbf/in}^2$, $T_1 = 40^\circ\text{F} \Rightarrow h_1 = 109.07 \text{ Btu/lb}$, $s_1 = 0.2254 \text{ Btu/lb} \cdot ^\circ\text{R}$

State 2 $p_2 = 180 \text{ lbf/in}^2$, $T_2 = 140^\circ\text{F} \Rightarrow h_2 = 122.11 \text{ Btu/lb}$, $s_2 = 0.2299 \text{ Btu/lb} \cdot ^\circ\text{R}$

State 3 $p_3 = 180 \text{ lbf/in}^2$, $T_3 = 75^\circ\text{F} \Rightarrow$ compressed liquid. Thus
 $h_3 \approx h_f @ 75^\circ\text{F} = 31.79 \text{ Btu/lb}$, $s_3 \approx s_f @ 75^\circ\text{F} = 0.0661 \text{ Btu/lb} \cdot ^\circ\text{R}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 31.79 \text{ Btu/lb}$. Also, $x_4 = 0.14551$ and
 $s_4 = 0.06766 \text{ Btu/lb} \cdot ^\circ\text{R}$

(a) Using the given value of $\dot{Q}_{\text{out}}$ to get $\dot{m}_{\text{air}}$

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_{\text{out}}}{(h_{\text{out}} - h_{\text{in}})_{\text{air}}} = \frac{\dot{Q}_{\text{out}}}{c_p (T_{\text{out}} - T_{\text{in}})_{\text{air}}} = \frac{4.2 \text{ tons}}{0.24 (110 - 70) \text{ Btu/lb}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right|$$

The volumetric flow rate of the air delivered is 87.5 lb/min

$$\begin{aligned} (AV)_{\text{air out}} &= \dot{m}_{\text{air}} v_{\text{air out}} = \frac{\dot{m}_{\text{air}} R T_{\text{out}}}{P} \\ &= \frac{(87.5 \text{ lb/min}) \left(\frac{1545 \text{ ft} \cdot \text{lb}}{2897 \text{ lb} \cdot ^\circ\text{R}} \right) (570^\circ\text{R})}{14.7 \text{ lbf/in}^2} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 1257 \frac{\text{ft}^3}{\text{min}} \leftarrow (AV)_{\text{air}} \end{aligned}$$

(b) $p_2 = 180 \text{ lbf/in}^2$, $s_{2s} = s_1 \Rightarrow h_{2s} = 119.46 \text{ Btu/lb}$. The compressor isentropic efficiency is

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{119.46 - 109.07}{122.11 - 109.07} = 0.797 (79.7\%) \leftarrow \eta_s$$

Continued on next slide

Problem 10-42 continued

(c) The mass flow rate of R-22 is

$$\dot{m}_R = \frac{\dot{Q}_{out}}{h_2 - h_3} = \frac{4.2 \text{ tons}}{(122.11 - 31.79) \text{ Btu/lb}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 9.3 \frac{\text{lb}}{\text{min}}$$

And the compressor power is

$$\dot{W}_c = \dot{m}_R (h_2 - h_1) = (9.3 \frac{\text{lb}}{\text{min}}) (122.11 - 109.07) \frac{\text{Btu}}{\text{lb}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 2.86 \text{ hp} \leftarrow \dot{W}_c$$

(d) The coefficient of performance is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{4.2 \text{ tons}}{2.86 \text{ hp}} \left| \frac{12000 \text{ Btu/h}}{1 \text{ ton}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 6.92 \leftarrow \gamma$$

(e) For the evaporator: $0 = \dot{m}_R (h_4 - h_1) + \dot{m}_w (h_{in} - h_{out})_w$

With $\Delta h_w = c_w \Delta T$, and $c_w = 1 \text{ Btu/lb} \cdot ^\circ\text{R}$, we get

$$\dot{m}_w = \frac{\dot{m}_R (h_1 - h_4)}{c_w (T_{in} - T_{out})_w} = \frac{(9.3 \text{ lb/min}) (109.07 - 31.79) \text{ Btu/lb}}{(1) (55 - 45) \text{ Btu/lb}} = 71.87 \frac{\text{lb}}{\text{min}}$$

Noting the $v_f @ 55^\circ\text{P} = 0.01602 \text{ ft}^3/\text{lb}$, the volumetric flow rate of water is

$$(\dot{A}V)_w = \dot{m}_w v_f = (71.87 \frac{\text{lb}}{\text{min}}) (0.01602 \frac{\text{ft}^3}{\text{lb}}) \left| \frac{1 \text{ gal}}{0.13368 \text{ ft}^3} \right| = 8.6 \text{ gal/min} \leftarrow (\dot{A}V)_w$$

Exergy Accounting

Power Input: $\dot{W}_c = (9.3)(122.11 - 109.07) = 121.27 \text{ Btu/min}$

Exergy Input from well water: $(\dot{E}_{f,in} - \dot{E}_{f,out})_w$

$$\begin{aligned} (\dot{E}_{f,in} - \dot{E}_{f,out})_w &= [(h_{in} - h_{out})_w - T_0 (s_{in} - s_{out})_w] \dot{m}_w \\ &= \left\{ c_{pw} (T_{in} - T_{out})_w - T_0 \left[c_{pw} \ln \left(\frac{T_{in}}{T_{out}} \right)_w - R \ln \left(\frac{P_{in}}{P_{out}} \right)_w \right] \right\} \dot{m}_w \\ &= \left\{ (1)(55 - 45) - (480)(1) \ln \left(\frac{515}{505} \right) \right\} (71.87) = 42.25 \text{ Btu/min} \end{aligned}$$

Destructions (Using Eq. 6.13 for water)

• Evaporator: $0 = \dot{m}_R (s_4 - s_1) + \dot{m}_w (c_w \ln \frac{T_{in}}{T_{out}})_w + (\dot{\sigma}_{cv})_{evap}$

$$\begin{aligned} (\dot{E}_d)_{evap} &= T_0 (\dot{\sigma}_{cv})_{evap} = T_0 [\dot{m}_R (s_1 - s_4) + \dot{m}_w c_w \ln \frac{T_{out}}{T_{in}}]_w \\ &= (480) [(9.3)(0.2254 - 0.06766) + (71.87)(1) \ln \left(\frac{505}{515} \right)] = 27.71 \frac{\text{Btu}}{\text{min}} \end{aligned}$$

• Compressor: $(\dot{E}_d)_c = T_0 (\dot{\sigma}_{cv})_{comp} = T_0 \dot{m} (s_2 - s_1) = (480)(9.3)(0.2299 - 0.2254) = 20.09 \text{ Btu/min}$

• Condenser: $0 = \dot{m}_R (s_2 - s_3) + \dot{m}_{air} [c_{pa} \ln \frac{T_{in}}{T_{out}}]_{air} - R \ln \left(\frac{P_{in}}{P_{out}} \right)_{air} + (\dot{\sigma}_{cv})_{cond}$

$$\begin{aligned} (\dot{E}_d)_{cond} &= T_0 (\dot{\sigma}_{cv})_{cond} = T_0 [\dot{m}_R (s_3 - s_2) + \dot{m}_{air} c_{pa} \ln \frac{T_{out}}{T_{in}}]_{air} \\ &= (480) [(9.3)(0.0661 - 0.2299) + (87.5)(0.24) \ln \left(\frac{570}{530} \right)] = 2.21 \text{ Btu/min} \end{aligned}$$

Continued on next slide

Problem 10-42 continued

$$\begin{aligned} \text{Valve: } 0(\dot{E}_d)_{\text{valve}} &= T_0(\dot{S}_{cv})_{\text{valve}} = T_0 \dot{m}_R (S_4 - S_3) \\ &= (480)(9.3)(0.06766 - 0.0661) = 6.96 \text{ Btu/min} \end{aligned}$$

Exergy delivered to air: $(\dot{E}_{f\text{out}} - \dot{E}_{f\text{in}})_{\text{air}}$

$$\begin{aligned} (\dot{E}_{f\text{out}} - \dot{E}_{f\text{in}})_{\text{air}} &= \dot{m}_{\text{air}} [(h_{\text{out}} - h_{\text{in}})_{\text{air}} - T_0 (s_{\text{out}} - s_{\text{in}})_{\text{air}}] \\ &= \dot{m}_{\text{air}} \left[c_{p,a} (T_{\text{out}} - T_{\text{in}}) - (480) \left\{ c_{p,a} \ln\left(\frac{T_{\text{out}}}{T_{\text{in}}}\right)_{\text{air}} - R \ln\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right)_{\text{air}} \right\} \right] \\ &= (87.5) \left[(0.24)(110 - 70) - (480)(0.24) \ln\left(\frac{570}{530}\right) \right] = 106.59 \text{ Btu/min} \end{aligned}$$

Summary

Inputs

Compressor	121.27 Btu/min
Evaporator	42.25
TOTAL	163.52 Btu/min

Destructions

Evaporator	27.71 Btu/min
Compressor	20.09
Condenser	2.21
Valve	6.96
SUB-TOTAL	56.97 Btu/min

← Exergy Accounting

Output

Heated air	106.59 Btu/min
TOTAL	163.56 Btu/min

The output of exergy is the exergy transferred to the heated air. The input that we pay for is the compressor power. Thus

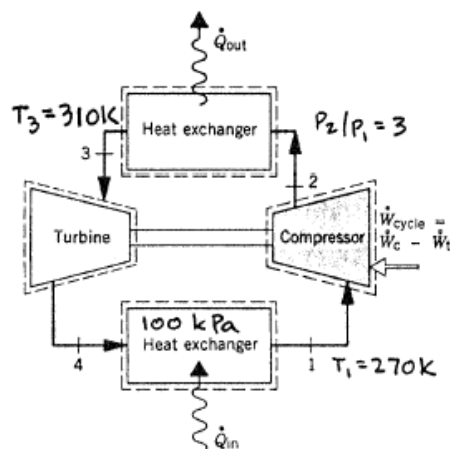
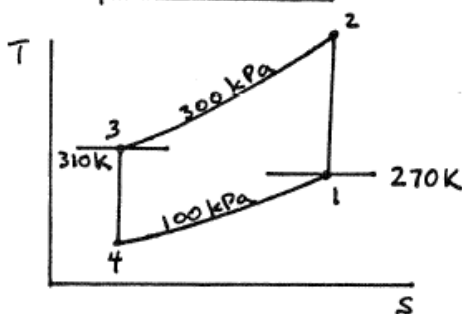
$$\varepsilon = \frac{\text{Output to heated air}}{\text{Compressor power}} = \frac{106.59}{121.27} = 0.879 \quad \leftarrow \text{Exergetic efficiency}$$

PROBLEM 10.43

KNOWN: Air is the working fluid in an ideal Brayton refrigeration cycle. Operating data are specified.

FIND: Determine (a) the net work input, per unit mass of air flow, (b) the refrigerating capacity, per unit mass of air flow, (c) the coefficient of performance, and (d) the coefficient of performance of a reversible cycle operating between reservoirs at $T_c = 270\text{ K}$ and $T_H = 310\text{ K}$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.4.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1 $T_1 = 270\text{ K} \Rightarrow h_1 = 270.11\text{ kJ/kg}$ $Pr_1 = 0.9590$

State 2 $Pr_2 = Pr_1 (P_2/P_1) = 2.877 \Rightarrow h_2 = 370.10\text{ kJ/kg}$

State 3 $T_3 = 310\text{ K} \Rightarrow h_3 = 310.24\text{ kJ/kg}$ $Pr_3 = 1.5546$

State 4 $Pr_4 = Pr_3 (P_4/P_3) = 0.5182 \Rightarrow h_4 = 226.25\text{ kJ/kg}$

(a) The net work is

$$\begin{aligned} \frac{\dot{W}_{\text{cycle}}}{\dot{m}} &= \frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}} = (h_2 - h_1) - (h_3 - h_4) \\ &= (370.10 - 270.11) - (310.24 - 226.25) \frac{\text{kJ}}{\text{kg}} \\ &= 16.0 \text{ kJ/kg} \end{aligned}$$

(b) The refrigerating capacity is

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_1 - h_4 = 270.11 - 226.25 = 43.86 \text{ kJ/kg}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{\text{in}}/\dot{m}}{\dot{W}_{\text{cycle}}/\dot{m}} = 2.74$$

(d) For a reversible cycle with $T_c = 270\text{ K}$, $T_H = 310\text{ K}$

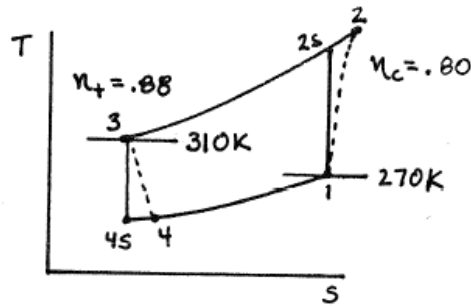
$$\beta_{\text{max}} = \frac{T_c}{T_H - T_c} = \frac{270}{310 - 270} = 6.75$$

PROBLEM 10.44

KNOWN: The ideal Brayton refrigeration cycle of Problem 10.43 is modified to include compressor and turbine efficiencies of 80 and 88%, respectively.

FIND: Determine (a) the coefficient of performance and (b) develop an exergy accounting of the net work input. Discuss.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.5. Also, let $T_0 = 310\text{K}$.

ANALYSIS: First, fix each of the principal states (Table A-22). From the solution to Problem 10.43; $h_1 = 270.11\text{ kJ/kg}$, $h_{2s} = 370.10\text{ kJ/kg}$, $h_3 = 310.24\text{ kJ/kg}$, $h_{4s} = 226.25\text{ kJ/kg}$.

State 2 Using the compressor efficiency,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 395.1\text{ kJ/kg}$$

State 4 Using the turbine efficiency,

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 236.3\text{ kJ/kg}$$

(a) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_{cycle}/\dot{m}} = \frac{(h_1 - h_4)}{(h_2 - h_1) - (h_3 - h_4)} = 0.662 \quad \leftarrow (3)$$

(b) The exergy destruction rate for the compressor is

$$(\dot{E}_d/\dot{m})_c = T_0 \dot{\sigma}_c/\dot{m} = T_0 [(s_2^\circ - s_1^\circ) - R \ln P_2/P_1]$$

From Table A-22; $s_1^\circ = 1.59634$ and $s_2^\circ = 1.95805$. Thus

$$\begin{aligned} (\dot{E}_d/\dot{m})_c &= (310) \left[(1.97703 - 1.59634) - \left(\frac{8.314}{28.97} \right) \ln(3) \right] \\ &= 20.27\text{ kJ/kg} \quad \leftarrow (\dot{E}_d/\dot{m})_c \end{aligned}$$

Similarly, for the turbine, with $s_3^\circ = 1.73498$ and $s_4^\circ = 1.46237$

$$\begin{aligned} (\dot{E}_d/\dot{m})_t &= T_0 [(s_4^\circ - s_3^\circ) - R \ln P_4/P_3] \\ &= (310) \left[(1.46237 - 1.73498) - \frac{8.314}{28.97} \ln\left(\frac{1}{3}\right) \right] \\ &= 13.23\text{ kJ/kg} \quad \leftarrow (\dot{E}_d/\dot{m})_t \end{aligned}$$

Continued on next slide

Problem 10-44 continued

The net change in exergy as air passes through the high-temperature heat exchanger is

$$\begin{aligned} \textcircled{1} \quad e_{f3} - e_{f2} &= (h_3 - h_2) - T_0(s_3 - s_2) = (h_3 - h_2) - T_0(s_3^0 - s_2^0 - R \ln P_3/P_2) \\ &= (310.24 - 395.1) - 310(1.73498 - 1.97703) = -9.83 \text{ kJ/kg} \quad \leftarrow \end{aligned}$$

The net change in exergy as air passes through the low-temperature heat exchanger is

$$\begin{aligned} e_{f1} - e_{f4} &= (h_1 - h_4) - T_0(s_1 - s_4) = (h_1 - h_4) - T_0(s_1^0 - s_4^0 - R \ln P_1/P_4) \\ &= (270.11 - 236.3) - 310(1.59634 - 1.46237) = -7.72 \text{ kJ/kg} \quad \leftarrow \end{aligned}$$

The net work input is

$$\begin{aligned} \dot{W}_{\text{net}}/\dot{m} &= (h_2 - h_1) - (h_3 - h_4) \\ &= (395.1 - 270.11) - (310.24 - 236.3) = 51.05 \text{ kJ/kg} \end{aligned}$$

Summary:

• Net exergy input by work	51.05 kJ/kg
• Disposition of the exergy input	
- Exergy transfers out	
✓ high-temp. HX	9.83 kJ/kg (19.3%)
✓ low-temp. HX	7.72 kJ/kg (15.1%)
- Exergy destruction	
✓ compressor	20.27 kJ/kg (39.7%)
✓ turbine	13.23 kJ/kg (25.9%)
	<u>51.05 kJ/kg</u>

In this case, nearly two-thirds of the exergy input is destroyed in achieving a fairly modest refrigerating effect, as gauged by the coefficient of performance.

1. For the heat exchangers there is no work (except for flow work). Also, there are no internal irreversibilities (pressure is constant). Thus, an exergy rate balance for the air side of each heat exchanger takes the form

$$0 = \dot{E}_q - \dot{W}_{\text{cv}} + \dot{m}(e_{f,\text{in}} - e_{f,\text{out}}) - \dot{E}_d$$

$\Rightarrow \dot{m}(e_{f,\text{out}} - e_{f,\text{in}}) = \dot{E}_q$. That is, the change in flow exergy is equal to the exergy transfer accompanying heat transfer. For the low-temperature heat exchanger, heat transfer occurs at $T < T_0$, and thus the exergy transfer is opposite to the heat transfer: from the air to the cold space. For the high-temperature heat exchanger, heat transfer and exergy transfer are in the same direction: from the air to the warm space.

PROBLEM 10.45

The data for the required plots are obtained using IT, as follows:

IT Code

```
T1 = 270 // K
p1 = 100 // kPa
rp = 3
T3 = 310 // K
eta_t = 1
eta_c = 1
mdot = 1 // Assume a unit mass
// flow rate of 1 kg/s.
```

```
h1 = h_T("Air", T1)
s1 = s_hP("Air", h1, p1)
s2s = s1
p2 = rp * p1
s2s = s_hP("Air", h2s, p2)
h2 = h1 + (h2s - h1) / eta_c
h3 = h_T("Air", T3)
p3 = p2
s3 = s_hP("Air", h3, p3)
s4s = s3
p4 = p1
s4s = s_hP("Air", h4s, p4)
h4 = h3 - (h3 - h4s) * eta_t
```

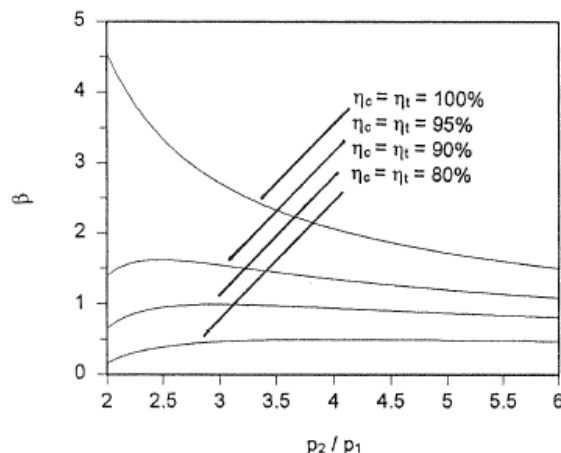
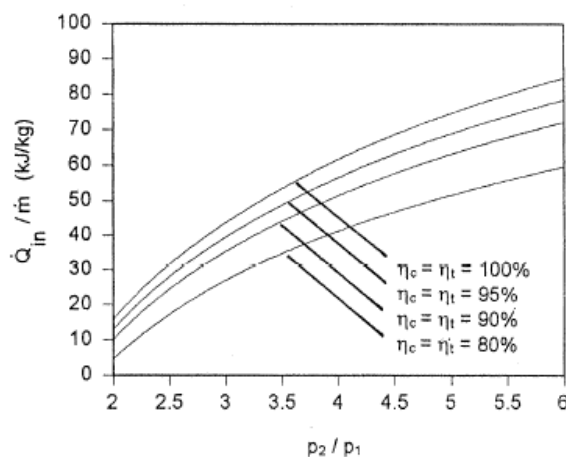
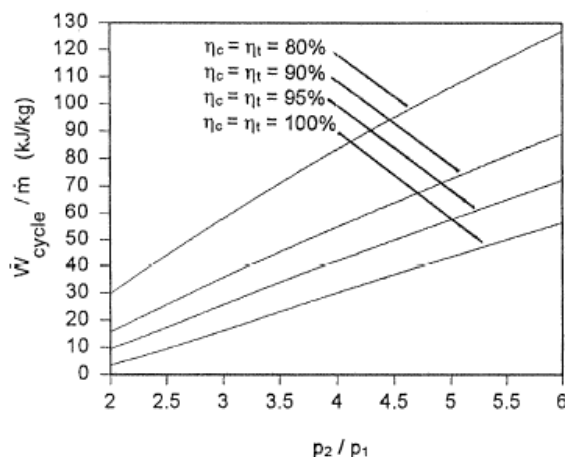
```
Wdotcycle / mdot = Wdotc / mdot - Wdott / mdot
Wdott / mdot = (h3 - h4)
Wdotc / mdot = (h2 - h1)
Qdotin / mdot = (h1 - h4)
beta = (h1 - h4) / ((h2 - h1) - (h3 - h4))
```

IT Results for $p_2/p_1 = 3$ and

$\eta_c = \eta_t = 100\%$

```
h1 269.9 kJ/kg
h2 369.9 kJ/kg
h3 310.1 kJ/kg
h4 226.2 kJ/kg
W_c / m = 100 kJ/kg
W_t / m = 83.88 kJ/kg
W_cycle / m = 16.13 kJ/kg
Q_in / m = 43.73 kJ/kg
beta = 2.711
```

PLOTS:



Discussion.

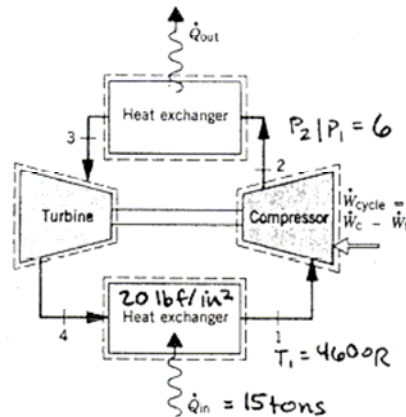
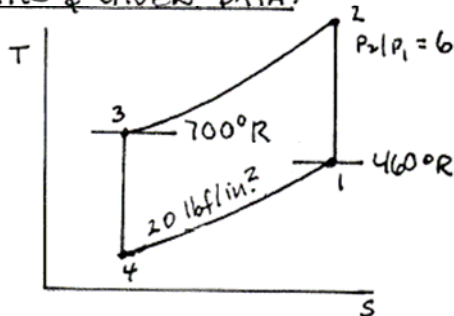
- Compressor and turbine irreversibility decrease the capacity and increase the work input to the cycle. As a result, the coefficient of performance decreases significantly for $\eta_c = \eta_t \leq 100\%$.
- Increasing compressor pressure ratio increases capacity but also the net work input. The coefficient of performance exhibits a maximum and then decreases with increasing pressure ratio.

PROBLEM 10.46

KNOWN: Air is the working fluid in an ideal Brayton refrigeration cycle. Operating data and the refrigerating capacity are specified.

FIND: Determine (a) the air mass flow rate, (b) the net power input, (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.4.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1 $T_1 = 460^\circ\text{R} \Rightarrow h_1 = 109.90 \text{ Btu/lb}$, $p_{r1} = 0.7913$

State 2 $p_{r2} = p_{r1} (P_2/P_1) = 4.7478 \Rightarrow h_2 = 183.62 \text{ Btu/lb}$

State 3 $T_3 = 700^\circ\text{R} \Rightarrow h_3 = 167.56 \text{ Btu/lb}$, $p_{r3} = 3.446$

State 4 $p_{r4} = p_{r3} (P_4/P_3) = (3.446)(\frac{1}{6}) = 0.5743 \Rightarrow h_4 = 100.23 \text{ Btu/lb}$

(a) The mass flow rate of air is

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{15 \text{ tons}}{(109.90 - 100.23) \frac{\text{Btu}}{\text{lb}}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 310.24 \frac{\text{lb}}{\text{min}} \quad \leftarrow \dot{m}$$

(b) Thus, the net power input is

$$\begin{aligned} \dot{W}_{cycle} &= \dot{m} [(h_2 - h_1) - (h_3 - h_4)] \\ &= (310.24) [(183.62 - 109.90) - (167.56 - 100.23)] \\ &= 1,942.1 \text{ Btu/min} \quad \leftarrow \dot{W}_{cycle} \end{aligned}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{15 \text{ tons}}{1,942.1 \text{ Btu/min}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 1.54 \quad \leftarrow \beta$$

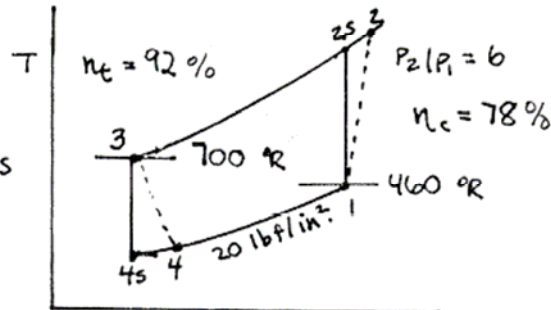
PROBLEM 10.47

KNOWN: The Brayton refrigeration cycle of Problem 10.46 is modified to include that the compressor and turbine have isentropic efficiencies of 78% and 92%, respectively.

FIND: Determine (a) the air mass flowrate, (b) the net power input, (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:

$$\dot{Q}_{in} = 15 \text{ tons}$$



ENGINEERING MODEL: See Example 10.5.

ANALYSIS: First, fix each of the principal states (Table A-22E).⁵ From the solution of Problem 10.46

$$h_1 = 109.80 \text{ Btu/lb}$$

$$h_{2s} = 183.62 \text{ Btu/lb}$$

$$h_3 = 167.56 \text{ Btu/lb}$$

$$h_{4s} = 100.23 \text{ Btu/lb}$$

State 2: Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c} \right) = 204.44 \text{ Btu/lb}$$

State 4: Using the turbine efficiency

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 105.62 \text{ Btu/lb}$$

(a) Using the refrigerating capacity

$$\begin{aligned} \dot{m} &= \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{(15 \text{ tons})}{(109.80 - 105.62) \frac{\text{Btu}}{\text{lb}}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| \\ &= 717.7 \frac{\text{lb}}{\text{min}} \end{aligned}$$

(b) The net power input is

$$\begin{aligned} \dot{W}_{net} &= \dot{m} [(h_2 - h_1) - (h_3 - h_4)] = (717.7) [(204.44 - 109.80) - (167.56 - 105.62)] \\ &= 23,469 \text{ Btu/min} \end{aligned}$$

(c) The coefficient of performance is

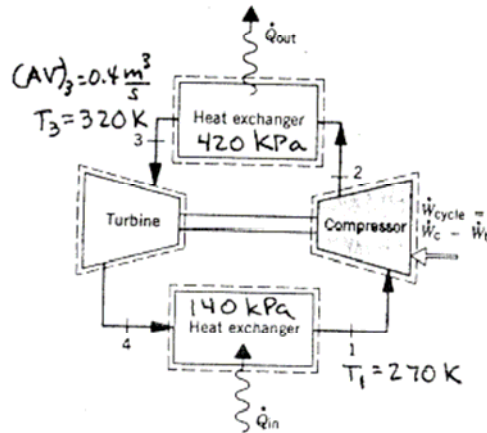
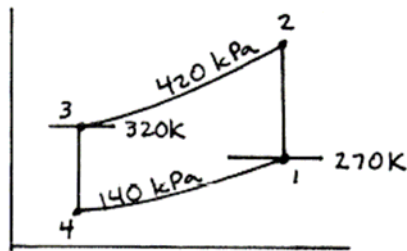
$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{net}} = \frac{(15 \times 200) \text{ Btu/min}}{(23,469) \text{ Btu/min}} = 0.128$$

PROBLEM 10.48

KNOWN: Operating data are provided for a Brayton refrigeration cycle using air as the working fluid.

FIND: Determine (a) the air mass flow rate, (b) the net power input, (c) the refrigerating capacity, and (d) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.4.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1 $T_1 = 270\text{ K} \Rightarrow h_1 = 270.11\text{ kJ/kg}$, $Pr_1 = 0.9590$

State 2 $Pr_2 = Pr_1 (P_2/P_1) = 2.877 \Rightarrow h_2 = 370.10\text{ kJ/kg}$

State 3 $T_3 = 320\text{ K} \Rightarrow h_3 = 320.29\text{ kJ/kg}$, $Pr_3 = 1.7375$

State 4 $Pr_4 = Pr_3 (P_4/P_3) = 0.5792 \Rightarrow h_4 = 233.61\text{ kJ/kg}$

(a) The mass flow rate is

$$\dot{m} = \frac{(AV)_3}{v_3} = \frac{(AV)_3 P_3}{R T_3} = \frac{(0.4\text{ m}^3/\text{s})(420\text{ kPa})}{\left(\frac{8.314\text{ kJ}}{28.97\text{ kg}\cdot\text{K}}\right)(320\text{ K})} \left| \frac{10^3\text{ N/m}^2}{1\text{ kPa}} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right| = 1.829\text{ kg/s}$$

Thus, the net power is

$$(b) \dot{W}_{\text{cycle}} = \dot{m} [(h_2 - h_1) - (h_3 - h_4)]$$

$$= (1.829\text{ kg/s}) [(370.1 - 270.11) - (320.29 - 233.61)] \frac{\text{kJ}}{\text{kg}} \left| \frac{1\text{ kW}}{1\text{ kJ/s}} \right| = 24.34\text{ kW}$$

(c) The refrigerating capacity is

$$\dot{Q}_{\text{in}} = \dot{m} (h_1 - h_4) = 66.76\text{ kW}$$

(d) The coefficient of performance is

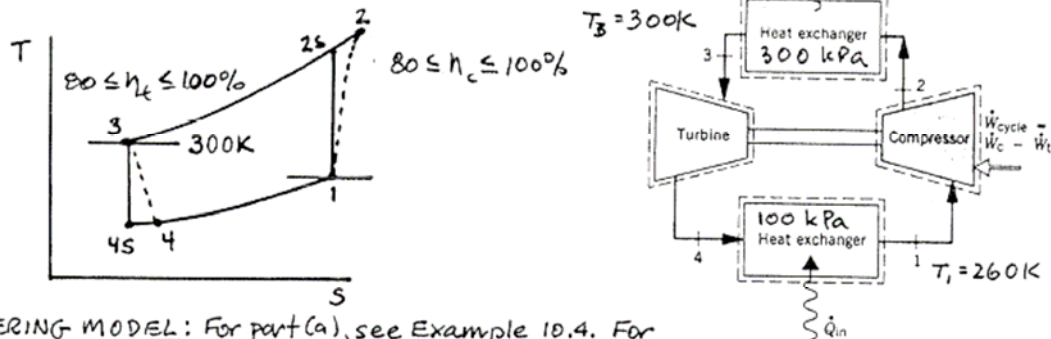
$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{cycle}}} = \frac{66.76\text{ kW}}{24.34\text{ kW}} = 2.743$$

PROBLEM 10.49

KNOWN: Air is the working fluid in a Brayton refrigeration cycle. Operating data are specified.

FIND: (a) Determine the net work per unit mass of air flow and the coefficient of performance for $\eta_c = \eta_t = 100\%$. (b) Plot the same quantities for $\eta_c = \eta_t$ ranging from 80% to 100%.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: For part (a), see Example 10.4. For part (b), see Example 10.5.

ANALYSIS: Sample calculations using data from Table A-22:

State 1: $T_1 = 260\text{ K} \Rightarrow h_1 = 260.09\text{ kJ/kg}$, $P_{r1} = 0.8405$

State 2: $P_{r2s} = P_{r1}(P_2/P_1) = 2.5215 \Rightarrow h_{2s} = 356.31\text{ kJ/kg}$

Using the isentropic compressor efficiency

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c; \quad \eta_c = 100\% \Rightarrow h_2 = 356.31\text{ kJ/kg} \quad (a)$$

$$\eta_c = 80\% \Rightarrow h_2 = 380.37\text{ kJ/kg} \quad (b)$$

State 3: $T_3 = 300\text{ K} \Rightarrow h_3 = 300.19\text{ kJ/kg}$, $P_{r3} = 1.3860$

State 4: $P_{r4s} = P_{r3}(P_4/P_3) = 0.462 \Rightarrow h_{4s} = 218.97\text{ kJ/kg}$

Using the isentropic turbine efficiency

$$h_4 = h_3 - (h_3 - h_{4s})\eta_t; \quad \eta_t = 100\% \Rightarrow h_4 = 218.97\text{ kJ/kg} \quad (a)$$

$$\eta_t = 80\% \Rightarrow h_4 = 235.21\text{ kJ/kg} \quad (b)$$

The net work is

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = \frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}} = (h_2 - h_1) - (h_3 - h_4)$$

$$\eta_c = \eta_t = 100\% \Rightarrow \dot{W}_{\text{cycle}}/\dot{m} = 15\text{ kJ/kg} \quad \leftarrow \frac{\dot{W}_{\text{cycle}}/\dot{m}}{\text{(part a)}}$$

$$\eta_c = \eta_t = 80\% \Rightarrow \dot{W}_{\text{cycle}}/\dot{m} = 55.3\text{ kJ/kg}$$

Now, the coefficient of performance is

$$\beta = \frac{\dot{Q}_{\text{in}}/\dot{m}}{\dot{W}_{\text{cycle}}/\dot{m}} = \frac{(h_1 - h_4)}{\dot{W}_{\text{cycle}}/\dot{m}}$$

$$\eta_c = \eta_t = 100\% \Rightarrow \beta = 2.74 \quad \leftarrow \beta \text{ (part a)}$$

$$\eta_c = \eta_t = 80\% \Rightarrow \beta = 0.45$$

Continued on next slide

Problem 10-49 continued

(b) The data for the required plots are obtained using iT , as follows:

iT Code

```
T1 = 260 // K
p1 = 100 // kPa
p2 = 300 // kPa
T3 = 300 // K
p3 = p2
p4 = p1
eta_c = 80 // %
eta_t = eta_c
mdot = 1 // Assume a unit mass
// flow rate of 1 kg/s.

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / (eta_c / 100)
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
```

$$h4 = h3 - (h3 - h4s) * (\eta_t / 100)$$

$$\dot{W}_{dot_c} / \dot{m} = (h2 - h1)$$

$$\dot{W}_{dot_t} / \dot{m} = (h3 - h4)$$

$$\dot{W}_{dotcycle} / \dot{m} = \dot{W}_{dot_c} / \dot{m} - \dot{W}_{dot_t} / \dot{m}$$

$$\beta = (h1 - h4) / (\dot{W}_{dotcycle} / \dot{m})$$

iT Results for $\eta_c = \eta_t = 80\%$

$$h_1 = 259.9 \text{ kJ/kg}$$

$$h_2 = 380.3 \text{ kJ/kg}$$

$$h_{2s} = 356.2 \text{ kJ/kg}$$

$$h_3 = 300 \text{ kJ/kg}$$

$$h_4 = 235.1 \text{ kJ/kg}$$

$$h_{4s} = 218.9 \text{ kJ/kg}$$

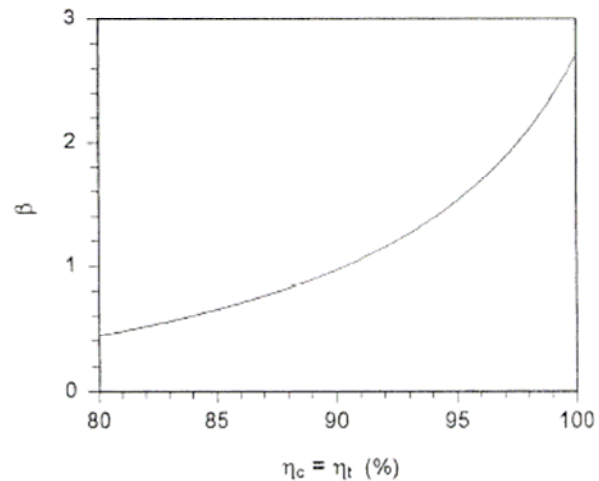
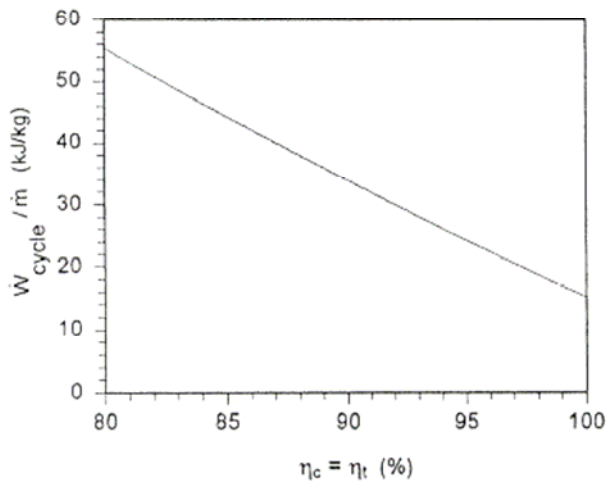
$$\dot{W}_c / \dot{m} = 120.4 \text{ kJ/kg}$$

$$\dot{W}_t / \dot{m} = 64.94 \text{ kJ/kg}$$

$$\dot{W}_{cycle} / \dot{m} = 55.45 \text{ kJ/kg}$$

$$\beta = 0.4471$$

Plots:



Discussion:

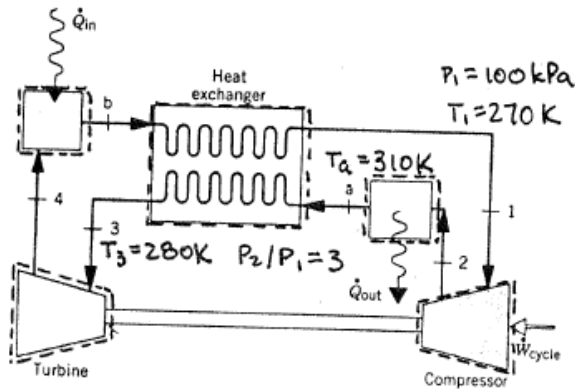
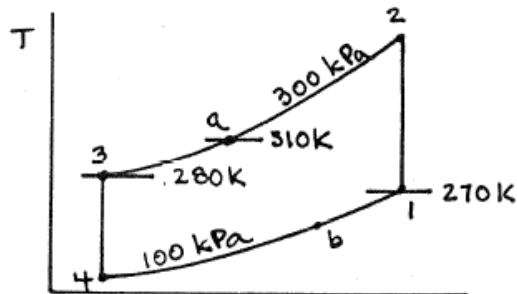
- As the isentropic compressor and turbine efficiencies decrease, the net work input to the cycle increases.
- As turbine efficiency decreases, point 4 on the accompanying T-s diagram moves to the right, thereby decreasing the capacity. The decrease in capacity, coupled with the increase in net work, result in a dramatic drop in coefficient of performance compared to the ideal case (isentropic compression and expansion).

PROBLEM 10.50

KNOWN: The ideal Brayton refrigeration cycle of Problem 10.43 is modified to include a regenerative heat exchanger.

FIND: Determine (a) the lowest temperature, (b) the net work input per unit mass of air flow, (c) the refrigerating capacity, per unit mass of air flow, (d) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.4.

Also, there is no heat transfer from the heat exchanger to its surroundings.

ANALYSIS: First, fix each of the principal states (Table A-22). From the solution to Problem 10.43; $h_1 = 270.11$ kJ/kg, $h_2 = 370.10$ kJ/kg, and $h_a = 310.24$ kJ/kg.

State 3 $T_3 = 280$ K $\Rightarrow h_3 = 280.13$ kJ/kg, $Pr_3 = 1.0889$

State 4 $Pr_4 = Pr_3(P_4/P_3) = 0.36297 \Rightarrow h_4 = 204.24$ kJ/kg, $T_4 = 204.3$ K

State b For the heat exchanger

$$0 = (h_b - h_1) + (h_a - h_3) \Rightarrow h_b = 240 \text{ kJ/kg} \quad T_{\min} \quad (a)$$

$$(b) \frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (h_2 - h_1) - (h_3 - h_4) = 24.1 \text{ kJ/kg} \quad \dot{W}_{\text{cycle}}/\dot{m}$$

(c) The refrigerating capacity becomes

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_b - h_4 = 35.76 \text{ kJ/kg} \quad \dot{Q}_{\text{in}}/\dot{m}$$

$$(d) \beta = \frac{\dot{Q}_{\text{in}}/\dot{m}}{\dot{W}_{\text{cycle}}/\dot{m}} = 1.484 \quad \beta$$

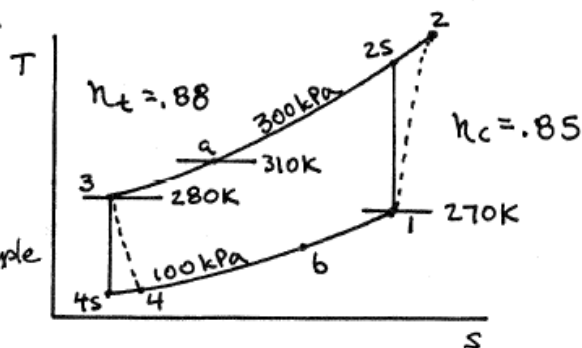
Discussion: Comparing these results with those of Problem 10.43, we see that introducing a regenerator has reduced T_4 significantly. However, comparing the other quantities indicates that the net work has increased, the capacity has decreased, and the coefficient of performance has gone down.

PROBLEM 10.51

KNOWN: Reconsider the cycle of Problem 10.50 and include that the compressor and turbine have efficiencies of 85 and 88%, respectively.

FIND: Answer the same questions as in Problem 10.50.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 10.5. Also, there is no heat transfer from the heat exchanger to its surroundings.

ANALYSIS: First, fix each of the principal states (Table A-22). From the solution to Problem 10.50

$$\begin{aligned} h_1 &= 270.11 \text{ kJ/kg} & h_3 &= 280.13 \text{ kJ/kg} \\ h_{2s} &= 370.10 \text{ kJ/kg} & h_{4s} &= 204.24 \text{ kJ/kg} \\ h_a &= 310.24 \text{ kJ/kg} & h_b &= 240 \text{ kJ/kg} \end{aligned}$$

State 2 using the compressor efficiency,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c} \right) = 387.7 \text{ kJ/kg}$$

State 4 Using the turbine efficiency

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 213.35 \text{ kJ/kg}$$

$$\therefore T_4 = 213.4 \text{ K} \quad \leftarrow T_{\min} \quad (a)$$

$$(b) \quad \frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (h_2 - h_1) - (h_3 - h_4) = 50.81 \text{ kJ/kg} \quad \leftarrow \frac{\dot{W}_{\text{cycle}}}{\dot{m}}$$

$$(c) \quad \frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_b - h_4 = 26.65 \text{ kJ/kg} \quad \leftarrow \frac{\dot{Q}_{\text{in}}}{\dot{m}}$$

$$(d) \quad \beta = \frac{26.65}{50.81} = 0.524 \quad \leftarrow \beta$$

Discussion: Comparing these results with those of Problem 10.50, we see the penalties associated with irreversibilities in the compressor and turbine: increased $T_{\min}$, increased net work, decreased capacity, and decreased coefficient of performance.

PROBLEM 10.52

The data for the required plots are obtained using IT, as follows. (See Problem 10.50 for sample calculations.)

IT Code

```
p1 = 100 // kPa
T1 = 270 // K
Ta = 310 // K
rp = 3
p2 = p1 * rp
p3 = p2
T3 = 280 // K
p4 = p1
eta_c = .8
eta_t = eta_c
mdot = 1 // Assume a unit mass flow rate.
```

```
h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / eta_c
ha = h_T("Air", Ta)
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eta_t
T4 = T_h("Air", h4)
O = (hb - h1) + (ha - h3)
```

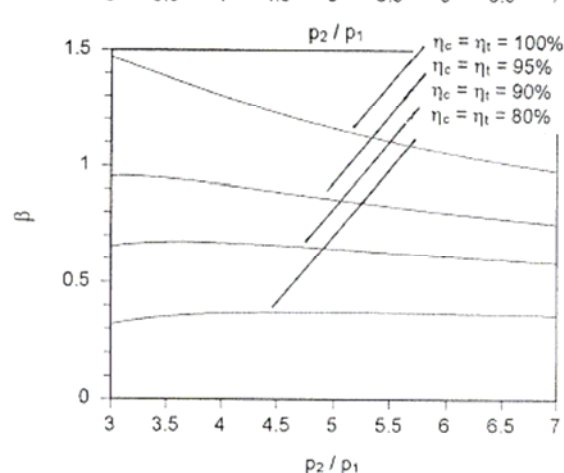
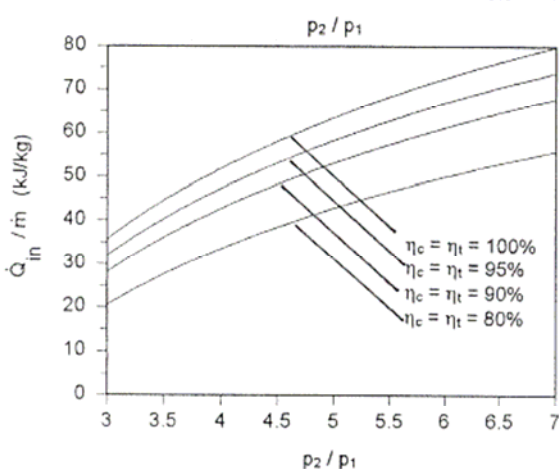
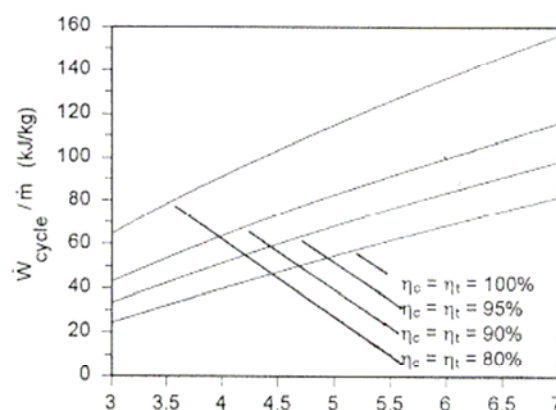
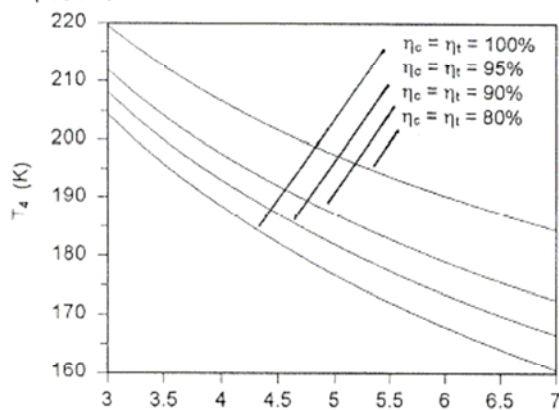
```
Wdotcycle / mdot = (h2 - h1) - (h3 - h4)
Qdotin / mdot = hb - h4
beta = Qdotin / Wdotcycle
```

IT Results for $p_2/p_1 = 3$

and $\eta_c = \eta_t = 100\%$

```
h1 = 269.9 kJ/kg
h2 = 369.9 kJ/kg
h2s = 369.9 kJ/kg
h3 = 280 kJ/kg
h4 = 204.2 kJ/kg
h4s = 204.2 kJ/kg
ha = 310.1 kJ/kg
hb = 239.8 kJ/kg
T4 = 204.5 K
Wdotcycle / m = 24.25 kJ/kg
Qdotin / m = 35.6 kJ/kg
beta = 1.468
```

PLOTS:

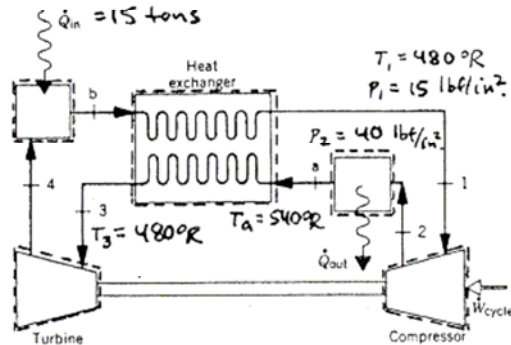
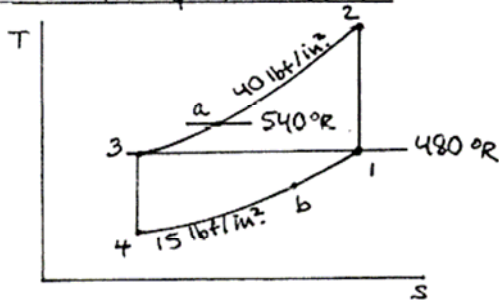


PROBLEM 10.53

KNOWN: Air is the working fluid in a Brayton refrigeration cycle with a regenerative heat exchanger. Data are known at various locations and the refrigeration capacity is given.

FIND: Determine (a) the volumetric flow rate at the turbine inlet, and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: See Example 10.4. Also, there is no heat transfer from the heat exchanger to its surroundings.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1 $T_1 = 480^\circ\text{R} \Rightarrow h_1 = 114.69 \text{ Btu/lb}$, $p_{r1} = 0.9182$

State 2 $p_{r2} = p_{r1} (p_2/p_1) = 2.449 \Rightarrow h_2 = 151.90 \text{ Btu/lb}$

State a $T_a = 540^\circ\text{R} \Rightarrow h_a = 129.06 \text{ Btu/lb}$

State 3 $T_3 = 480^\circ\text{R} \Rightarrow h_3 = 114.69 \text{ Btu/lb}$, $p_{r3} = 0.9182$

State 4 $p_{r4} = p_{r3} (p_4/p_3) = 0.3443 \Rightarrow h_4 = 86.52 \text{ Btu/lb}$

State b For the heat exchanger,

$$0 = (h_b - h_1) + (h_a - h_3) \Rightarrow h_b = 100.32 \text{ Btu/lb}$$

(a) With the given refrigeration capacity, the air mass flow rate is

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_b - h_4)} = \frac{(15 \text{ tons})}{(100.32 - 86.52) \text{ Btu/lb}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 217.4 \frac{\text{lb}}{\text{min}}$$

and the volumetric flow rate at the compressor inlet is

$$(AV)_1 = \frac{\dot{m} R T_1}{P_1} = \frac{(217.4) \frac{\text{lb}}{\text{min}} \left(\frac{1545}{28.97} \right) \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} (480)^\circ\text{R}}{(20) \text{ lbf/in}^2} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 1932 \frac{\text{ft}^3}{\text{min}} \leftarrow (AV)_1$$

(b) The coefficient of performance is

$$\beta = \frac{(h_b - h_4)}{(h_2 - h_1) - (h_3 - h_4)} = \frac{13.80}{37.21 - 28.17} = 1.527 \leftarrow \beta$$

PROBLEM 10.54

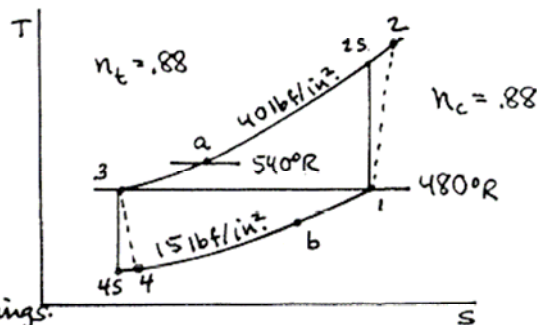
KNOWN: Reconsider the cycle of Problem 10.53, but include compressor and turbine efficiencies of 88 % each in the analysis.

FIND: Answer the same questions as in Problem 10.53.

SCHEMATIC & GIVEN DATA:

ENGINEERING

MODEL: See Example 10.5. Also, there is no heat transfer between the heat exchanger and its surroundings.



ANALYSIS: First, fix each of the principal states (Table A-2ZE). From the solution to Problem 10.53

$$\begin{aligned} h_1 &= 114.69 \text{ Btu/lb} & h_3 &= 114.69 \text{ Btu/lb} \\ h_{2s} &= 151.90 \text{ Btu/lb} & h_{4s} &= 86.52 \text{ Btu/lb} \\ h_a &= 129.06 \text{ Btu/lb} & h_b &= 100.32 \text{ Btu/lb} \end{aligned}$$

State 2 Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c} \right) = 156.97 \text{ Btu/lb}$$

State 4 Using the turbine efficiency

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 89.90 \text{ Btu/lb}$$

(a) With the given refrigeration capacity, the air mass flow rate is

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_b - h_a)} = \frac{(15 \text{ tons}) \left(\frac{200 \text{ Btu/min}}{1 \text{ ton}} \right)}{(100.32 - 89.90) \text{ Btu/lb}} = 287.9 \frac{\text{lb}}{\text{min}}$$

and the volumetric flow rate at the compressor inlet is

$$(AV)_1 = \frac{\dot{m} R T_1}{P_1} = \frac{(287.9) \frac{\text{lb}}{\text{min}} \left(\frac{1545}{28.97} \right) \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ \text{R}} (480) ^\circ \text{R}}{(20) \text{ lbf/in}^2} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 2559 \frac{\text{ft}^3}{\text{min}} \leftarrow (AV)_1$$

$$(b) \beta = \frac{(h_b - h_4)}{(h_2 - h_1) - (h_3 - h_4)} = \frac{10.42}{(156.97 - 114.69) - (114.69 - 89.90)} = 0.596 \leftarrow \beta$$

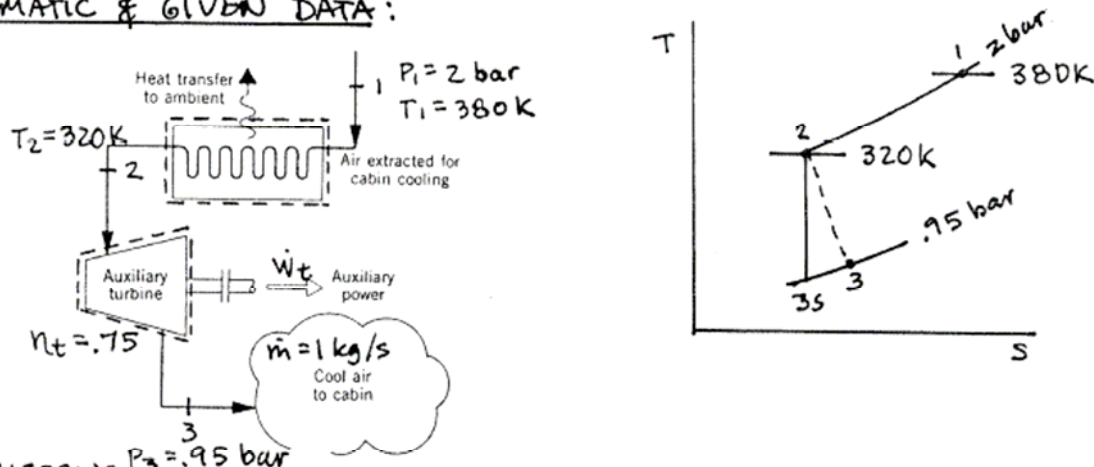
Note, the volumetric flow rate is higher and the coefficient of performance is lower than in Problem 10.53 due to irreversibilities in the compressor and turbine.

PROBLEM 10.55

KNOWN: Air is extracted from a main jet engine for cabin cooling. The air passes through a heat exchanger and turbine before being discharged into the cabin.

FIND: Determine (a) the power developed by the turbine and (b) the heat transfer rate from the air passing through the heat exchanger.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is at steady state. (2) The air experiences no pressure drop in passing through the heat exchanger. (3) Kinetic and potential energy effects can be neglected. (4) The turbine operates adiabatically. (5) The air is modeled as an ideal gas.

ANALYSIS: (a) From Table A-22; $h_2 = 320.29 \text{ kJ/kg}$ and $Pr_2 = 1.7375$. For an isentropic expansion, $Pr_{3s} = Pr_2 (P_3/P_2) = 0.82531$. Thus, $h_{3s} = 258.67 \text{ kJ/kg}$. Using the turbine efficiency

$$\begin{aligned} \dot{W}_t &= \dot{m} \eta_t (h_2 - h_{3s}) \\ &= (1 \frac{\text{kg}}{\text{s}})(0.75)(320.29 - 258.67) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 46.2 \text{ kW} \end{aligned}$$

(b) From Table A-22; $h_1 = 380.77 \text{ kJ/kg}$. Thus

$$\begin{aligned} 0 &= \dot{Q}_{cv} + \dot{m} (h_1 - h_2) \\ \text{or } \dot{Q}_{cv} &= \dot{m} (h_2 - h_1) = (1)(320.29 - 380.77)(1) \\ &= -60.48 \text{ kW} \end{aligned}$$

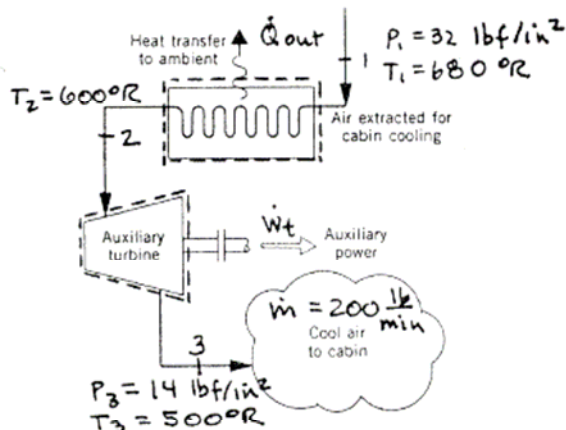
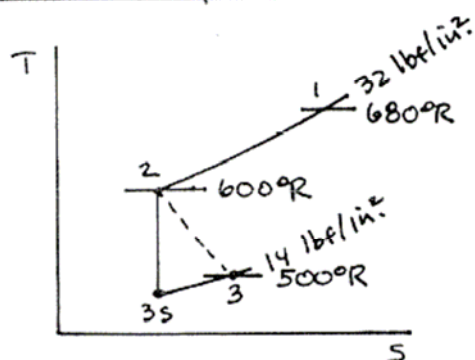
$$\dot{Q}_{out} = 60.48 \text{ kW}$$

PROBLEM 10.56

KNOWN: Air is extracted from a main jet engine for cabin cooling. The air passes through a heat exchanger and turbine before being discharged into the cabin.

FIND: Determine (a) the power developed by the turbine, (b) the isentropic turbine efficiency, and (c) the heat transfer rate from the air passing through the heat exchanger.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is at steady state. (2) The air passes through the heat exchanger with no pressure drop. (3) Kinetic and potential energy effects are negligible. (4) The turbine operates adiabatically. (5) The air is modeled as an ideal gas.

ANALYSIS: (a) From Table A-22E, $h_2 = 143.47$ Btu/lb and $h_3 = 119.48$ Btu/lb. Thus, the power developed by the turbine is

$$\dot{W}_t = \dot{m}(h_2 - h_3) = \left(200 \frac{\text{lb}}{\text{min}}\right) \left|\frac{60 \text{ min}}{1 \text{ h}}\right| (143.47 - 119.48) \frac{\text{Btu}}{\text{lb}} \left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 113.1 \text{ hp}$$

(b) For an isentropic expansion, $Pr_2 = 2.005$ and $Pr_{3s} = Pr_2(P_3/P_2) = 0.87719$. Thus, $h_{3s} = 113.14$ Btu/lb, and

$$\eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} = 0.791 \text{ (79.1 \%)}$$

(c) From Table A-22E, $h_1 = 162.73$ Btu/lb. Thus

$$\dot{Q}_{\text{out}} = \dot{m}(h_1 - h_2)$$

$$= \left(200 \frac{\text{lb}}{\text{min}}\right) (162.73 - 143.47) \frac{\text{Btu}}{\text{lb}}$$

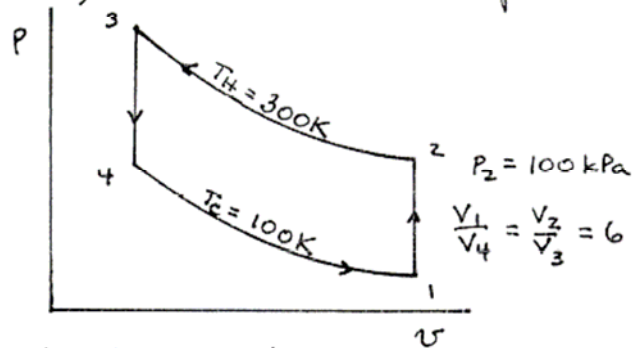
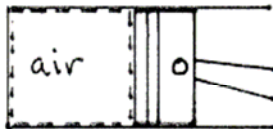
$$= 3852 \text{ Btu/min}$$

PROBLEM 10.57

KNOWN: Air undergoes a Stirling refrigeration cycle with a known compression ratio. Other data are also given for the cycle.

FIND: Determine (a) the heat transfer during the isothermal expansion, per unit mass of air, (b) the net work for the cycle, per unit mass of air, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The air is a closed system. (2) The compression and expansion are isothermal. (3) All processes are internally reversible. (4) An ideal regenerator is available to store the energy rejected during process 3-4 to be used as the energy input during process 1-2. (5) The air behaves as an ideal gas. (6) Kinetic and potential energy effects are negligible.

ANALYSIS: (a) The energy balance for process 4-1 reduces to

$$\frac{Q_{41}}{m} = (u_1 - u_4) + \frac{W_{41}}{m}$$

The work is calculated using Eq. 3.58

$$\begin{aligned} \frac{W_{41}}{m} &= \int_4^1 p dv = RT_c \ln \frac{v_1}{v_4} \\ &= \left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (100 \text{ K}) \ln(6) = 51.42 \text{ kJ/kg} \end{aligned}$$

Thus $\frac{Q_{41}}{m} = \frac{W_{41}}{m} = 51.42 \text{ kJ/kg}$ $\frac{Q_{41}}{m}$

(b) The work for process 2-3 is

$$\frac{W_{23}}{m} = \int_2^3 p dv = RT_h \ln \frac{v_3}{v_2} = \left(\frac{8.314}{28.97} \right) (300) \ln\left(\frac{1}{6}\right) = -154.26 \frac{\text{kJ}}{\text{kg}}$$

Thus $\frac{W_{\text{cycle}}}{m} = \left| \frac{W_{23}}{m} \right| - \frac{W_{41}}{m} = 154.26 - 51.42 = 102.84 \text{ kJ/kg}$ $\frac{W_{\text{cycle}}}{m}$

(c) The coefficient of performance is

① $\beta = \frac{Q_{41}/m}{W_{\text{cycle}}/m} = \frac{51.42}{102.84} = 0.5$ β

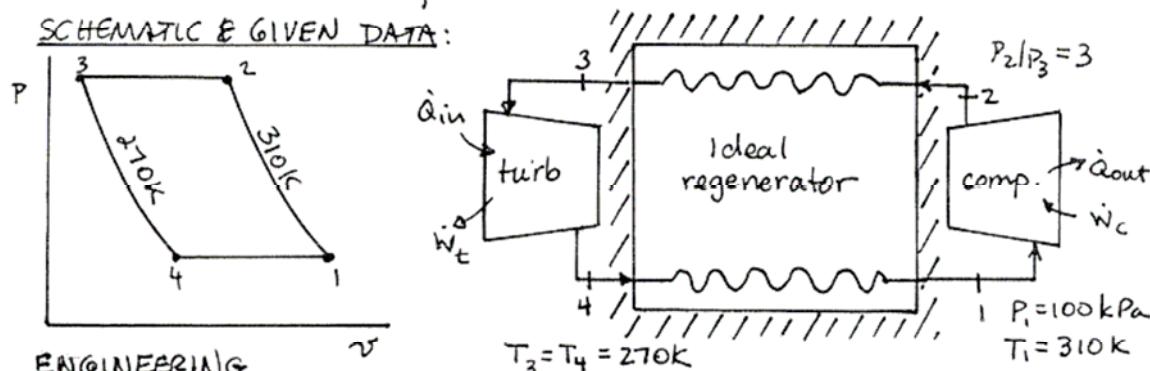
1. Alternatively, using Eq. 5.10, $\beta = \beta_{\text{max}} = \frac{T_c}{T_h - T_c} = 0.5$.

PROBLEM 10.58

KNOWN: Air is the working fluid in an Ericsson refrigeration cycle. Data are known at various locations.

FIND: Determine (a) the heat transfer for the isothermal expansion, (b) the net work per unit mass of air flow, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion processes are isothermal. (4) Kinetic and potential energy effects are negligible. (5) The air behaves as an ideal gas.

ANALYSIS: (a) Determine $\dot{Q}_{in}/\dot{m}$ from an energy balance on the turbine

$$0 = \dot{Q}_{in} - \dot{W}_t + \dot{m}(h_3 - h_4)$$

or

$$\frac{\dot{Q}_{in}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}}$$

The turbine work is evaluated using Eq. 6.56

$$\begin{aligned} \frac{\dot{W}_t}{\dot{m}} &= - \int_3^4 v dp = -RT_3 \ln \frac{P_4}{P_3} = RT_3 \ln \frac{P_2}{P_1} \\ &= \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (270 \text{ K}) \ln(3) = 85.13 \text{ kJ/kg} \end{aligned}$$

$$\frac{\dot{Q}_{in}}{\dot{m}} = 85.13 \text{ kJ/kg} \quad \underline{\dot{Q}_{in}/\dot{m}}$$

(b) The compressor work is evaluated similarly

$$\frac{\dot{W}_c}{\dot{m}} = RT_1 \ln \frac{P_2}{P_1} = \left(\frac{8.314}{28.97} \right) (310) \ln(3) = 97.69 \text{ kJ/kg}$$

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}} = 12.56 \text{ kJ/kg} \quad \underline{\dot{W}_{cycle}/\dot{m}}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_{cycle}/\dot{m}} = 6.78$$

PROBLEM 11.1

KNOWN: 100 lb of CO_2 is at 212°F in a 19.3-ft^3 cylinder.

FIND: Determine the pressure using (a) the van der Waals equation, (b) the compressibility chart, (c) the ideal gas model.

ANALYSIS: Using the given data $v = 0.193 \text{ ft}^3/\text{lb}$.

(a) The van der Waals equation is given by Eq. 11.2

$$p = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2}$$

From Table A-24E

$$a = 926 \text{ atm} \left(\frac{\text{ft}^3}{\text{lbmol}} \right)^2, \quad b = 0.686 \frac{\text{ft}^3}{\text{lbmol}}$$

$$p = \frac{(1545 \frac{\text{ft} \cdot \text{lb}_f}{\text{lbmol} \cdot ^\circ\text{R}})(672^\circ\text{R}) \left| \frac{\text{ft}^2}{144 \text{ in}^2} \right| \left| \frac{\text{atm}}{14.696 \text{ lb}_f/\text{in}^2} \right|}{(0.193 \frac{\text{ft}^3}{\text{lb}})(44.01 \frac{\text{lb}}{\text{lbmol}}) - 0.686 \frac{\text{ft}^3}{\text{lbmol}}} - \frac{926 \text{ atm} \left(\frac{\text{ft}^3}{\text{lbmol}} \right)^2}{\left[(0.193)(44.01) \frac{\text{ft}^3}{\text{lbmol}} \right]^2} = 50 \text{ atm} \quad \leftarrow \text{vdW}$$

(b) Compressibility Chart

From Table A-1E, $T_c = 548^\circ\text{R}$, $p_c = 72.9 \text{ atm}$. Thus, $T_R = 672/548 = 1.226$ and

$$v_R' = \frac{\bar{v} p_c}{\bar{R} T_c} = \frac{(0.193)(44.01) \frac{\text{ft}^3}{\text{lbmol}} [72.9 \text{ atm}] \left| \frac{14.696 \text{ lb}_f/\text{in}^2}{\text{atm}} \right| \left| \frac{144 \text{ in}^2}{\text{ft}^2} \right|}{(1545 \frac{\text{ft} \cdot \text{lb}_f}{\text{lbmol} \cdot ^\circ\text{R}})(548^\circ\text{R})} = 1.548$$

Then Fig. A-1 gives $p_R \approx 0.685 \Rightarrow p = 49.94 \text{ atm}$.

$\leftarrow$ z chart

(c) With the ideal gas equation of state

$$p = \frac{\bar{R}T}{\bar{v}} = \frac{(1545)(44)(14.696)}{(0.193)(44.01)} = 57.76 \text{ atm}$$

$\leftarrow$ ideal gas

Discussion: Methods (a), (b) suggest that the pressure level would be safe, but at the high end of the allowed range. The ideal gas model suggests that the pressure would not be satisfactory.

PROBLEM 11.2

- ① KNOWN: Ten lb of C_3H_8 has a volume of 2 ft^3 and a pressure of 600 lbf/in^2 .
FIND: Determine the temperature using (a) the Vander Waals equation, (b) the compressibility chart, (c) the ideal gas equation of state, and (d) the propane tables.

ANALYSIS: $p = \left(600 \frac{\text{lbf}}{\text{in}^2} \right) \left| \frac{1 \text{ atm}}{14.696 \text{ lbf/in}^2} \right| = 40.83 \text{ atm}$, $\bar{v} = \left(\frac{2 \text{ ft}^3}{10 \text{ lb}} \right) \left| \frac{44.09 \text{ lb}}{1 \text{ lbmol}} \right| = 8.818 \frac{\text{ft}^3}{\text{lbmol}}$

(a) With a and b from Table A-24E, the Vander Waals Equation gives

$$T = \frac{(p + a/\bar{v}^2)(\bar{v} - b)}{R}$$

$$= \frac{\left[40.83 \text{ atm} + \frac{2369 (\text{ft}^3/\text{lbmol})^2}{(8.818 \text{ ft}^3/\text{lbmol})^2} \right] (8.818 - 1.444) \frac{\text{ft}^3}{\text{lbmol}} \left| \frac{14.696 \text{ lbf/in}^2}{1 \text{ atm}} \right| \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|}{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lbmol} \cdot ^\circ\text{R}})}$$

$$= 720.1 ^\circ\text{R} \quad \leftarrow (a)$$

(b) From Table A-1E; $T_c = 66 ^\circ\text{R}$, $p_c = 42.1 \text{ atm}$. Thus $P_R = 40.83/42.1 = 0.97$, and

$$v_{R'} = \frac{\bar{v} p_c}{R T_c} = \frac{(8.818 \text{ ft}^3/\text{lbmol})(42.1 \text{ atm}) \left| \frac{14.696 \text{ lbf/in}^2}{1 \text{ atm}} \right| \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|}{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lbmol} \cdot ^\circ\text{R}})(66 ^\circ\text{R})} = 0.764$$

Then, Fig A-1 gives $T_R \approx 1.08 \Rightarrow T = 719.3 ^\circ\text{R} \quad \leftarrow (b)$

(c) The ideal gas equation gives

$$T = \frac{p \bar{v}}{R} = \frac{(600 \text{ lbf/in}^2)(8.818 \text{ ft}^3/\text{lbmol}) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|}{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lbmol} \cdot ^\circ\text{R}})} = 493 ^\circ\text{R} \quad \leftarrow (c)$$

(d) $p = 600 \text{ lbf/in}^2$, $\bar{v} = 0.2 \text{ ft}^3/\text{lb}$. Interpolating in Table A-18E, we get

$$T = 261.5 ^\circ\text{F} = 721.5 ^\circ\text{R} \quad \leftarrow (d)$$

Discussion. The answers for parts (a), (b) and (d) agree well. The ideal gas value deviates significantly from the others.

PROBLEM 11.3

KNOWN: 1000 kg of water vapor fills a 23.3-m^3 tank at 360°F .

FIND: Estimate the pressure using (a) the ideal gas equation, (b) the van der Waals equation, (c) the Redlich-Kwong equation, (d) the compressibility chart, (e) the steam tables.

ANALYSIS:

(a) ideal gas equation.

$$p = \frac{RT}{V} = \frac{(8314/18.02) \left(\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \right) (633\text{ K})}{(23.31/10^3) (\text{m}^3/\text{kg})} \left| \frac{1\text{ bar}}{10^5\text{ N/m}^2} \right| = 125.29\text{ bar} \quad \leftarrow (a)$$

(25% high)

(b) van der Waals equation. With a and b from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} = \frac{(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}})(633\text{ K})}{(0.42 - 0.0305) \text{ m}^3/\text{kmol}} \left| \frac{1\text{ bar}}{10^5\text{ N/m}^2} \right| - \frac{5.531 \text{ bar}(\text{m}^3/\text{kmol})^2}{(0.42 \text{ m}^3/\text{kmol})^2} = 103.76\text{ bar} \quad \leftarrow (b)$$

(~4% high)

where $\bar{v} = vM = \left(\frac{23.31}{10^3} \right) \left(\frac{\text{m}^3}{\text{kg}} \right) (18.02 \frac{\text{kg}}{\text{kmol}}) = 0.42 \frac{\text{m}^3}{\text{kmol}}$

(c) Redlich-Kwong equation. With a and b from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)T^{1/2}} = \frac{(8314)(633)}{(0.42 - 0.02111)} \left| \frac{1}{10^5} \right| - \frac{142.59}{(0.42)(0.42 + 0.02111)(633)^{1/2}} = 101.34\text{ bar} \quad \leftarrow (c)$$

(~1% high)

(d) Compressibility chart. From Table A-1, $T_c = 647.3\text{ K}$, $P_c = 220.9\text{ bar}$. Thus, $T_R = 633/647.3 = 0.978$. And

$$v_R = \frac{\bar{v} P_c}{\bar{R} T_c} = \frac{(0.42 \text{ m}^3/\text{kmol})(220.9 \times 10^5 \text{ N/m}^2)}{(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}})(647.3\text{ K})} = 1.724$$

Then, Fig A-1 gives $Z \approx 0.81$, so

$$p = Z \frac{RT}{\bar{v}} = 0.81(125.29) = 101.48\text{ bar} \quad \leftarrow (d)$$

(~1.5% high)

(e) Steam tables. Table A-4 gives 100 bar.

Discussion: All but the ideal gas model suggest that the pressure would be in the safe range. Using the steam table value, 100 bar, as the standard, the various methods depart by the following percentages:

ideal gas: 25% high
van der Waals: 4% high
Redlich-Kwong: 10% high
Compressibility chart: 1.5% high

PROBLEM 11.4

KNOWN: Water vapor is at $T = 500^\circ\text{C}$, $\rho = 24 \text{ kg/m}^3$.

FIND: Determine the pressure using (a) the steam tables, (b) the compressibility chart, (c) the Redlich-Kwong equation, (d) the van der Waals equation, (e) the ideal gas model.

ANALYSIS: (a) At 500°C and $v = 1/\rho = 0.0417 \text{ m}^3/\text{kg}$; Steam Table data gives $p = 80.1 \text{ bar}$. (a)

(b) Compressibility chart $T_R = \frac{773}{647} = 1.19$, $\bar{v}_R' = \frac{\bar{v} p_c}{R T_c} = \frac{[(0.0417)(18.02) \frac{\text{m}^3}{\text{kmol}}] [220.9 \times 10^5 \frac{\text{N}}{\text{m}^2}]}{[8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}] (647 \text{ K})}$

or $\bar{v}_R' = 3.08$. Then, Figure A-1 gives $Z \approx 0.93$. So

$$p = \frac{ZRT}{v} = \frac{0.93 \left[\frac{8314 \text{ N}\cdot\text{m}}{18.02 \text{ kg}\cdot\text{K}} \right] (773 \text{ K})}{(0.0417 \frac{\text{m}^3}{\text{kg}}) \left| 10^5 \text{ N/m}^2/\text{bar} \right|} = 79.5 \text{ bar} \quad (1\% \text{ low}) \quad (b)$$

(c) the Redlich-Kwong Equation with data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}(\bar{v} + b)\sqrt{T}} = \frac{(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}})(773 \text{ K}) \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right|}{\left[(0.0417)(18.02) - 0.02111 \right] \frac{\text{m}^3}{\text{kmol}}} - \frac{142.59 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2 (\text{K})^{1/2}}{(0.7514 \frac{\text{m}^3}{\text{kmol}}) (0.77251 \frac{\text{m}^3}{\text{kmol}}) (773 \text{ K})^{1/2}}$$

$$= 88.0 - 8.84 = 79.2 \text{ bar} \quad (\sim 1\% \text{ low}) \quad (c)$$

(d) the van der Waals Equation with data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2} = \frac{(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}})(773 \text{ K}) \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right|}{\left(0.7514 \frac{\text{m}^3}{\text{kmol}} - 0.0305 \frac{\text{m}^3}{\text{kmol}} \right)} - \frac{5.531 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2}{\left(0.7514 \frac{\text{m}^3}{\text{kmol}} \right)^2}$$

$$= 89.15 - 9.8 = 79.35 \text{ bar} \quad (\sim 1\% \text{ low}) \quad (d)$$

(e) ideal gas model

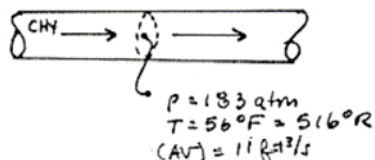
$$p = \frac{RT}{v} = \frac{(8314/18.02 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}})(773 \text{ K}) \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right|}{(0.0417 \text{ m}^3/\text{kg})} = 85.53 \text{ bar} \quad (\sim 7\% \text{ high}) \quad (e)$$

PROBLEM 11.5

KNOWN: CH₄ flows through a pipeline at a pressure of 183 atm, a temperature of 56°F, and a volumetric flow rate of 11 ft³/s.

FIND: Determine the mass flow rate using (a) the ideal gas equation, (b) the van der Waals equation, (c) the compressibility chart.

SCHEMATIC & GIVEN DATA:



ANALYSIS: The mass flow rate and volumetric flow rate are related by

$$\dot{m} = \frac{(AV)}{v} \quad (1)$$

(a) Ideal Gas Equation:

$$v = \frac{RT}{P} = \frac{(1545/16.04) \left(\frac{\text{ft} \cdot \text{lb}_f}{\text{lbmol} \cdot ^\circ\text{R}} \right) (516^\circ\text{R})}{(183 \text{ atm}) \left[14.696 \times 144 \frac{\text{lb}_f/\text{ft}^2}{\text{atm}} \right]} = 0.128 \frac{\text{ft}^3}{\text{lb}}$$

Thus, Eq. (1) gives

$$\dot{m} = \frac{11 \text{ ft}^3/\text{s}}{0.128 \text{ ft}^3/\text{lb}} = 85.9 \text{ lb/s} \quad (a)$$

(b) van der Waals Equation: With *a* and *b* from Table A-24E

$$P = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2} \Rightarrow 183 \text{ atm} = \left[\frac{(1545 \frac{\text{ft} \cdot \text{lb}_f}{\text{lbmol} \cdot ^\circ\text{R}})(516^\circ\text{R})}{(\bar{v} - 0.685) \text{ ft}^3/\text{lbmol}} \right] \left[\frac{\text{atm}}{(14.696)(144) \frac{\text{lb}_f/\text{ft}^2}{\text{atm}}} \right] - \frac{581}{\bar{v}^2}$$

or

$$183 = \frac{376.718}{\bar{v} - 0.685} - \frac{581}{\bar{v}^2}$$

Using an equation solver, $\bar{v} = 1.611 \text{ ft}^3/\text{lbmol}$. Thus, $v = \bar{v}/M = (1.611/16.04) = 0.1004 \text{ ft}^3/\text{lb}$, and Eq. (1) gives

$$\dot{m} = \frac{11 \text{ ft}^3/\text{s}}{0.1004 \text{ ft}^3/\text{lb}} = 109.6 \text{ lb/s} \quad (b)$$

(c) Generalized Chart. From Table A-1E $T_c = 344^\circ\text{R}$, $P_c = 45.8 \text{ atm}$. Thus

$$T_r = \frac{516}{344} = 1.5, \quad P_r = 4.0$$

Then, from Fig. A-2, $Z = 0.81$. Accordingly

$$v = Z \frac{RT}{P} = (0.81) \underbrace{(0.128)}_{\text{part (a)}} = 0.1037 \frac{\text{ft}^3}{\text{lb}}$$

Finally, with Eq. (1)

$$\dot{m} = \frac{11 \text{ ft}^3/\text{s}}{0.1037 \text{ ft}^3/\text{lb}} = 106.1 \text{ lb/s} \quad (c)$$

PROBLEM 11.6

KNOWN: Water vapor is at 20 MPa, 400°C.

FIND: v in m^3/kg using (a) steam tables, (b) compressibility chart, (c) Redlich-Kwong equation, (d) van der Waals equation, (e) ideal gas model.

ANALYSIS: (a) Steam Tables. Table A-4 gives, $v = 0.00994 \text{ m}^3/\text{kg}$. $\leftarrow$ (a)

(b) Compressibility Chart. With P_r, T_r from Table A-1

$$T_r = \frac{673}{647} = 1.04, P_r = \frac{20}{22.09} = 0.91. \text{ Figure A-1 gives } Z \sim 0.65. \text{ Then}$$

$$v = \frac{ZRT}{P} = 0.65 \left[\frac{(8314/18.02) \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \cdot 673 \text{ K}}{20 \times 10^6 \text{ N/m}^2} \right] = 0.0101 \text{ m}^3/\text{kg} \quad \leftarrow (b)$$

$$0.0155 \text{ m}^3/\text{kg} \quad \leftarrow (e)$$

(c) Redlich-Kwong Equation. With a, b from Table A-24

$$P = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)\sqrt{T}} \Rightarrow 200 \text{ bar} = \frac{(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}})(673 \text{ K})}{(\bar{v}-0.02111) \frac{\text{m}^3}{\text{kmol}}} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| - \frac{142.59 \text{ bar}}{\bar{v}(\bar{v}+0.02111)(673)^{1/2}}$$

or

$$200 = \frac{55.953}{(\bar{v}-0.02111)} - \frac{5.496}{\bar{v}(\bar{v}+0.02111)}. \text{ Using an equation solver, } \bar{v} = 0.18 \text{ m}^3/\text{kmol}.$$

$$\Rightarrow \bar{v} = \frac{0.18 \text{ m}^3/\text{kmol}}{18.02 \text{ kg/kmol}} = 0.00999 \text{ m}^3/\text{kg} \quad \leftarrow (c)$$

(d) van der Waals Equation. With a, b from Table A-24

$$P = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} \Rightarrow 200 \text{ bar} = \frac{(8314)(673)}{(\bar{v}-0.0305)} \left| \frac{1}{10^5} \right| - \frac{5.531}{\bar{v}^2}$$

or

$$200 = \frac{55.953}{\bar{v}-0.0305} - \frac{5.531}{\bar{v}^2}. \text{ Using an equation solver, } \bar{v} = 0.186 \text{ m}^3/\text{kmol}.$$

$$\Rightarrow \bar{v} = \frac{0.186}{18.02} = 0.01032 \frac{\text{m}^3}{\text{kg}} \quad \leftarrow$$

Discussion: Comparing the calculated values with the Steam Table result

compressibility chart $\sim 2\%$ high

Redlich-Kwong equation $\sim 1/2\%$ high

van der Waals equation $\sim 4\%$ high

ideal gas model $\sim 6\%$ high

PROBLEM 11.7

KNOWN: A vessel whose volume is 1 m^3 contains 4 kmol of CH_4 at 100°C .
FIND: Estimate the pressure using (a) the ideal gas model, (b) the Redlich-Kwong equation, (c) the Benedict-Webb-Rubin equation.

ANALYSIS: (a) Ideal Gas Model

$$P = \frac{\bar{R}T}{\bar{v}} = \frac{\left(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}\right)(373\text{K})}{(1\text{m}^3/4\text{kmol})} \left| \frac{1\text{bar}}{10^5 \frac{\text{N}}{\text{m}^2}} \right| = 124.04 \text{ bar} \quad \leftarrow (a)$$

(b) Redlich-Kwong, with data from Table A-24

$$P = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)(T)^{1/2}} = \frac{\left(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}\right)(373\text{K})}{(0.25 - 0.02965) \frac{\text{m}^3}{\text{kmol}}} \left| \frac{1\text{bar}}{10^5 \text{N/m}^2} \right| - \frac{32.11 \text{ bar}}{0.25(0.25 + 0.02965)(373)^{1/2}}$$

$$= 140.74 - 23.78 = 116.96 \text{ bar} \quad \leftarrow (b)$$

(c) Benedict-Webb-Rubin, with data from Table A-24

$$P = \frac{\bar{R}T}{\bar{v}} + \left[B\bar{R}T - A - \frac{C}{T^2} \right] \frac{1}{\bar{v}^2} + \left[\frac{b\bar{R}T - a}{\bar{v}^3} \right] + \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3 T^2} \left(1 + \frac{\gamma}{\bar{v}^2} \right) \exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$P = \frac{(0.08314 \frac{\text{bar}\cdot\text{m}^3}{\text{kmol}\cdot\text{K}})(373\text{K})}{0.25 \text{ m}^3/\text{kmol}} + \left[(0.04260)(0.08314)(373) - 1.8796 - \frac{2.287 \times 10^{-4}}{(373)^2} \right] \frac{1}{(0.25)^2}$$

$$+ \left[\frac{(0.00338)(0.08314)(373) - 0.0501}{(0.25)^3} \right] + \frac{(0.0501)(1.244 \times 10^{-4})}{(0.25)^6}$$

$$+ \left[\frac{(2.579 \times 10^3)}{(0.25)^3 (373)^2} \left(1 + \frac{0.0060}{(0.25)^2} \right) \exp\left(-\frac{0.0060}{(0.25)^2}\right) \right]$$

$$= 124.04 - 11.57 + 3.50 + 0.03 + 1.18 = 117.18 \text{ bar} \quad \leftarrow (c)$$

Discussion: The Redlich-Kwong and Benedict-Webb-Rubin equations suggest that the pressure is in the safe range. Using P_c and T_c from Table A-1,

$$P_R = \frac{120 \text{ bar}}{46.4 \text{ bar}} = 2.59, \quad T_R = \frac{373\text{K}}{191\text{K}} = 1.95 \quad \text{part (a)}$$

Figure A-2 gives $Z \approx 0.96$, and so $p = Z \bar{R}T/\bar{v} = (0.96)(124.04) \text{ bar} = 119.1 \text{ bar}$, which is also in the safe range

PROBLEM 11.8

KNOWN: A 10-m³ tank contains CH₄ at 100 atm and -18°C.

FIND: Determine the mass contained in the tank using (a) the ideal gas model, (b) the van der Waals equation, the Benedict-Webb-Rubin equation.

ANALYSIS: (a) Ideal Gas Model

$$\bar{v} = \frac{\bar{R}T}{P} = \frac{(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}})(255 \text{ K})}{(100 \text{ atm}) \left[1.01325 \times 10^5 \frac{\text{N}}{\text{m}^2} / \text{atm} \right]} = 0.209 \frac{\text{m}^3}{\text{kmol}}$$

Thus

$$m = \frac{V}{\bar{v}} = \frac{V}{\bar{v}/M} = \left(\frac{10 \text{ m}^3}{0.209 \frac{\text{m}^3}{\text{kmol}}} \right) \left(\frac{16.04 \text{ kg}}{\text{kmol}} \right) = 767.5 \text{ kg} \quad \leftarrow (a)$$

(b) van der Waals equation. With data from Table A-24

$$P = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} \Rightarrow 1.01325 \times 10^5 \frac{\text{N}}{\text{m}^2} = \frac{(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}})(255 \text{ K})}{(\bar{v}-0.0428) \left(\frac{\text{m}^3}{\text{kmol}} \right)} - \frac{2.293 \times 10^5 \text{ N/m}^2}{\bar{v}^2}$$

Using an equation solver, $\bar{v} = 0.14 \text{ m}^3/\text{kmol}$.

$$\Rightarrow m = \frac{VM}{\bar{v}} = \frac{(10)(16.04)}{0.14} = 1145.7 \text{ kg} \quad \leftarrow (b)$$

(c) Benedict-Webb-Rubin equation. With data from Table A-24

$$101.325 \text{ bar} = \frac{(0.08314)(255)}{\bar{v}} + \left[\frac{(0.0426)(0.08314)(255) - 1.8796 - \frac{(2.287)(10^4)}{(255)^2}}{\bar{v}^2} \right] + \left[\frac{(0.00338)(0.08314)(255) - 0.0501}{\bar{v}^3} + \frac{(0.0501)(1.244 \times 10^{-4})}{\bar{v}^6} + \left[\frac{2.579 \times 10^3}{\bar{v}^3 (255)^2} \left(1 + \frac{0.0060}{\bar{v}^2} \right) \exp \left(-\frac{0.0060}{\bar{v}^2} \right) \right] \right]$$

Using an equation solver, $\bar{v} = 0.149 \text{ m}^3/\text{kmol}$.

$$\Rightarrow m = \frac{VM}{\bar{v}} = \frac{(10)(16.04)}{0.149} = 1076.5 \text{ kg} \quad \leftarrow (c)$$

1. With P_c, T_c from Table A-1

$$P_R = \frac{100 \text{ atm}}{45.8 \text{ atm}} = 2.18, \quad T_R = \frac{255 \text{ K}}{191 \text{ K}} = 1.34$$

Figure A-2 gives $Z \sim 0.72$. Thus

$$\bar{v} = Z \frac{\bar{R}T}{P} = 0.72 \left(0.209 \right) = 0.15 \text{ m}^3/\text{kmol}$$

$$\Rightarrow m = \frac{VM}{\bar{v}} = \frac{(10)(16.04)}{0.15} = 1069.3 \text{ kg}$$

This agrees closely with the result of part (c).

PROBLEM 11.9

KNOWN: 165 kg of CH₄ is at 200 atm, 400 K.

FIND: Using the Benedict-Webb-Rubin equation, determine the volume in m³. Compare with the results obtained from the ideal gas model and the compressibility chart.

ANALYSIS: With the ideal gas equation of state

$$\bar{v} = \frac{\bar{R}T}{P} = \frac{(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}})(400 \text{ K})}{(200 \text{ atm})[1.01325 \times 10^5 \text{ N/m}^2/\text{atm}]} = 0.164 \frac{\text{m}^3}{\text{kmol}}$$

Thus,

$$V = m \frac{\bar{v}}{M} = (165 \text{ kg}) \frac{(0.164 \text{ m}^3/\text{kmol})}{(16.04 \text{ kg/kmol})} = 1.69 \text{ m}^3 \quad \leftarrow \text{ideal gas.}$$

With T_c, P_c from Table A-1, T_R = 400/191 = 2.09, P_R = 200/45.8 = 4.37. Then, Fig. A-2 gives Z ≈ 0.98. Thus

$$v = Z \frac{\bar{R}T}{P} = 0.98(0.164) = 0.161 \frac{\text{m}^3}{\text{kmol}}$$

So

$$V = m \frac{\bar{v}}{M} = (165) \frac{(0.161)}{(16.04)} = 1.66 \text{ m}^3 \quad \leftarrow \text{compressibility chart}$$

With constants from Table A-24 and $\bar{R} = 0.08314 \text{ bar}\cdot\text{m}^3/\text{kmol}\cdot\text{K}$, Eq. 11.12 gives with p = (200)(1.01325) = 202.65 bar

$$p = \frac{\bar{R}T}{\bar{v}} + \left[B\bar{R}T - A - \frac{C}{T^2} \right] \frac{1}{\bar{v}^2} + \frac{(b\bar{R}T - a)}{\bar{v}^3} + \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3 T^2} \left(1 + \frac{\gamma}{\bar{v}^2} \right) \exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$202.65 = \frac{(0.08314)(400)}{\bar{v}} + \left[(0.04260)(0.08314)(400) - 1.8796 - \frac{2.287 \times 10^4}{(400)^2} \right] \frac{1}{\bar{v}^2} + \frac{[(0.00338)(0.08314)(400) - 0.0501]}{\bar{v}^3} + \frac{(0.0501)(1.244 \times 10^{-4})}{\bar{v}^6} + \frac{2.579 \times 10^3}{\bar{v}^2 (400)^2} \left(1 + \frac{0.0060}{\bar{v}^2} \right) \exp\left(-\frac{0.0060}{\bar{v}^2}\right)$$

$$202.65 = \frac{33.256}{\bar{v}} - \frac{0.6058}{\bar{v}^2} + \frac{0.0623}{\bar{v}^3} + \frac{(0.0623 \times 10^4)}{\bar{v}^6} + \frac{0.0161(1 + \frac{0.0060}{\bar{v}^2}) \exp(-\frac{0.0060}{\bar{v}^2})}{\bar{v}^3}$$

Using an equation solver, $\bar{v} = 0.161 \text{ m}^3/\text{kmol}$, giving $V = 1.66 \text{ m}^3$. ← B-W-R equation

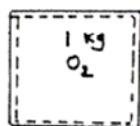
Discussion: The B-W-R equation and compressibility chart values agree. Since Z ≈ 0.98, the ideal gas value gives a reasonable approximation as well.

PROBLEM 11.10

KNOWN: A rigid tank contains 1 kg of O_2 at $p_1 = 40$ bar, $T_1 = 180$ K. The gas is cooled to a final state where $T_2 = 150$ K.

FIND: Determine the tank volume and p_2 using (a) the ideal gas equation, (b) the Redlich-Kwong equation, (c) the compressibility chart.

SCHEMATIC & GIVEN DATA:



$p_1 = 40$ bar, $T_1 = 180$ K
 $T_2 = 150$ K

ENGINEERING MODEL: As shown above, the O_2 is the system.

ANALYSIS: (a) Ideal Gas. The ideal gas equation of state gives

$$V = \frac{mRT}{P} = \frac{(1 \text{ kg})(8314/32) \left(\frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (180 \text{ K})}{(40 \times 10^5 \text{ N/m}^2)} = 5.846 \times 10^{-3} \text{ m}^3 \quad \leftarrow (a)$$

Since V is constant

$$\frac{p_1 V_1 = mRT_1}{p_2 V_2 = mRT_2} > \frac{T_1}{T_2} = \frac{p_2}{p_1} \Rightarrow p_2 = \frac{T_2}{T_1} p_1 = \frac{150}{180} (40 \text{ bar}) = 33.33 \text{ bar}$$

(b) Redlich-Kwong. With values for a and b from Table A-24 and $\bar{R} = 0.08314 \frac{\text{bar} \cdot \text{m}^3}{\text{kmol} \cdot \text{K}}$

$$p_1 = \frac{\bar{R} T_1}{\bar{v}_1 - b} - \frac{a}{\bar{v}_1(\bar{v}_1 + b)(T_1)^{1/2}} \Rightarrow 40 \text{ bar} = \frac{(0.08314 \frac{\text{bar} \cdot \text{m}^3}{\text{kmol} \cdot \text{K}})(180 \text{ K})}{(\bar{v}_1 - 0.02197) \text{ m}^3/\text{kmol}} - \frac{17.22}{\bar{v}_1(\bar{v}_1 + 0.02197)(180)^{1/2}}$$

$$\text{Simplifying } 40 = \frac{14.9652}{\bar{v}_1 - 0.02197} - \frac{1.2835}{\bar{v}_1(\bar{v}_1 + 0.02197)} \quad (1)$$

Since Eq. (1) is implicit in $\bar{v}_1$, an iterative solution is required if the result is pursued using hand calculations. Alternatively, an equation-solver such as IT can be used. The result is $\bar{v}_1 = 0.3015 \text{ m}^3/\text{kmol}$. Thus

$$V = m\bar{v}_1 = \frac{m\bar{v}_1}{M} = \frac{(1)(0.3015)}{32} = 9.422 \times 10^{-3} \text{ m}^3 \quad \leftarrow (b)$$

To find p_2 write, using $\bar{v}_2 = \bar{v}_1$

$$p_2 = \frac{\bar{R} T_2}{\bar{v}_2 - b} - \frac{a}{\bar{v}_2(\bar{v}_2 + b)(T_2)^{1/2}} \Rightarrow p_2 = \frac{(0.08314)(150)}{(0.3015 - 0.02197)} - \frac{17.22}{(0.3015)(0.3015 + 0.02197)(150)^{1/2}} \\ = 30.2 \text{ bar}$$

(c) Compressibility Chart. With T_c and p_c from Table A-1

$$(T_R)_1 = 180/154 = 1.169, (P_R)_1 = 40/50.5 = 0.792$$

Then Fig. A-1 gives $(V_R)_1 \approx 1.22$. Thus

$$v_1 = (V_R)_1 \left(\frac{RT_c}{P_c} \right) = 1.22 \left[\frac{(8314 \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}})(154 \text{ K})}{50.5 \times 10^5 \text{ N/m}^2} \right] = 9.666 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}$$

$$\text{Accordingly, } V = m v_1 = 9.666 \times 10^{-3} \text{ m}^3 \quad \leftarrow (c)$$

To find p_2 , use $V'_R = V'_1$ and $T_{R2} = 150/154 = 0.97$ to locate state 2 on Fig. A-1, giving $P_{R2} \approx 0.6$. Accordingly $p_2 = p_c P_{R2} = (50.5)(0.6) = 30.3 \text{ bar}$.

PROBLEM 11.11

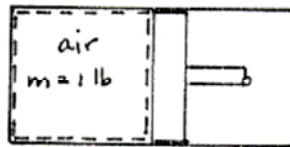
KNOWN: One lb of air initially at $V_1 = 0.4 \text{ ft}^3$ and $1000 \frac{\text{lb}_f}{\text{in}^2}$ expands isothermally until $V_2 = 2 \text{ ft}^3$.

FIND: Using the Van der Waals equation of state, determine (a) T , (b) p_2 , and (c) the work.

SCHEMATIC & GIVEN DATA:

ENGINEERING

MODEL: (1) The closed system consists of 1 lb of air. (2) The expansion takes place without irreversibilities. (3) The air is modeled by the Van der Waals equation of state.



$$\begin{aligned} V_1 &= 0.4 \text{ ft}^3 \\ P_1 &= 1000 \text{ lb}_f/\text{in}^2 \\ V_2 &= 2 \text{ ft}^3 \end{aligned}$$

ANALYSIS: $\bar{v}_1 = \frac{(0.4 \text{ ft}^3)}{(1 \text{ lb})} \left(\frac{28.97 \text{ ft} \cdot \text{lb}_f}{1 \text{ lb} \cdot \text{mol} \cdot ^\circ\text{R}} \right) = 11.588 \frac{\text{ft}^3}{\text{lbmol}}$, $\bar{v}_2 = \frac{(2)(28.97)}{(1)} = 57.94 \frac{\text{ft}^3}{\text{lbmol}}$

$$P_1 = (1000 \text{ lb}_f/\text{in}^2) \left| \frac{1 \text{ atm}}{14.696 \text{ lb}_f/\text{in}^2} \right| = 68.05 \text{ atm}$$

(a) With constants from Table A-24E and $\bar{R} = 0.73 \frac{\text{atm} \cdot \text{ft}^3}{\text{lbmol} \cdot ^\circ\text{R}}$

$$T_1 = \left(\frac{\bar{v}_1 - b}{\bar{R}} \right) \left(P_1 + \frac{a}{\bar{v}_1^2} \right) = \frac{(11.588 - 0.586) \text{ ft}^3/\text{lbmol}}{\left(0.73 \frac{\text{atm} \cdot \text{ft}^3}{\text{lbmol} \cdot ^\circ\text{R}} \right)} \left[68.05 \text{ atm} + \frac{345 \frac{\text{atm} \cdot \text{lbmol}^2}{\text{ft}^6}}{(11.588 \text{ ft}^3/\text{lbmol})^2} \right]$$

$$= 1064.3 ^\circ\text{R} \leftarrow T_1$$

(b) Since $T_2 = T_1$

$$P_2 = \frac{\bar{R} T_2}{\bar{v}_2 - b} - \frac{a}{\bar{v}_2^2} = \frac{(0.73)(1064.3)}{(11.588 - 0.586)} - \frac{345}{11.588^2} = 68.05 \text{ atm}$$

$$P_2 = (68.05 \text{ atm}) \left| \frac{14.696 \text{ lb}_f/\text{in}^2}{1 \text{ atm}} \right| = 1000 \text{ lb}_f/\text{in}^2 \leftarrow P_2$$

(c) The work can be evaluated from

$$W = \int_1^2 p dV = n \int_1^2 \left[\frac{\bar{R} T}{\bar{v} - b} - \frac{a}{\bar{v}^2} \right] d\bar{v} = n \left[\bar{R} T \ln \left(\frac{\bar{v}_2 - b}{\bar{v}_1 - b} \right) + a \left(\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1} \right) \right]$$

Thus

$$\begin{aligned} \frac{W}{n} &= \left(1.986 \frac{\text{Btu}}{\text{lbmol} \cdot ^\circ\text{R}} \right) (1064.3 ^\circ\text{R}) \left[\ln \left(\frac{57.94 - 0.586}{11.588 - 0.586} \right) \right] \\ &\quad + \left(345 \frac{\text{atm} \cdot \text{ft}^3}{\text{lbmol}} \right) \left[\frac{1}{57.94} - \frac{1}{11.588} \right] \left| \frac{14.696 \text{ lb}_f/\text{in}^2}{1 \text{ atm}} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lb}_f} \right| \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \\ &= 3425.2 \text{ Btu/lbmol} \end{aligned}$$

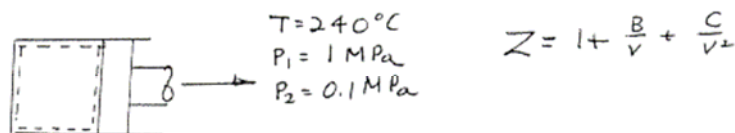
or $W = \frac{(1 \text{ lb})}{(28.97 \frac{\text{lb}}{\text{lbmol}})} (3425.2 \frac{\text{Btu}}{\text{lbmol}}) = 118.2 \text{ Btu} \leftarrow W$

PROBLEM 11.12

KNOWN: Water vapor expands isothermally and without irreversibilities from 240°C, 1 MPa to 0.1 MPa. A truncated virial equation of state is specified.

FIND: Determine the work.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The water vapor is the closed system. (2) The process is isothermal with no irreversibilities. (3) Volume change is the only work mode. (4) The p-v-T relation is given by the truncated virial series above.

ANALYSIS: The work is given by

$$\frac{W}{m} = \int_{v_1}^{v_2} p dv$$

Then, with

$$\frac{pv}{RT} = 1 + \frac{B}{v} + \frac{C}{v^2} \Rightarrow p = RT \left(\frac{1}{v} + \frac{B}{v^2} + \frac{C}{v^3} \right) \quad (1)$$

$$\frac{W}{m} = RT \left[\ln \frac{v_2}{v_1} - B \left(\frac{1}{v_2} - \frac{1}{v_1} \right) - \frac{C}{2} \left[\frac{1}{v_2^2} - \frac{1}{v_1^2} \right] \right] \quad (2)$$

where from Table A-4, $v_1 = 0.2275 \text{ m}^3/\text{kg}$, $v_2 = 2.359 \text{ m}^3/\text{kg}$.

The coefficients B, C can be evaluated using steam table data by noting that

$$Z = 1 + \frac{B}{v} + \frac{C}{v^2} \Rightarrow (Z-1)v = B + C(1/v)$$

① Thus, B and C are the intercept and slope, respectively, of a plot of $(Z-1)v$ vs $(1/v)$. The outcome of this procedure using steam table data is

$$B = -84.0 \times 10^{-4} \text{ m}^3/\text{kg}, \quad C = -1.25 \times 10^{-4} \text{ m}^6/\text{kg}^2.$$

as can be verified by using Eq. (1) to check steam table pressure values at 240°C.

Calculating

$$\begin{aligned} \frac{W}{m} &= \left(\frac{8314 \text{ J}}{18.02 \text{ kg}} \right) (513.15 \text{ K}) \left[\ln \left(\frac{2.359}{0.2275} \right) + \frac{84}{10^4} \left[\frac{1}{2.359} - \frac{1}{0.2275} \right] + \frac{1.25}{(2)(10^4)} \left[\frac{1}{(2.359)^2} - \frac{1}{(0.2275)^2} \right] \right] \\ &= 545.6 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

1. The use of appropriate computer software facilitates this step. Alternatively, table data can be plotted by hand, and the values of B and C determined graphically.

PROBLEM 11.13

KNOWN: The compressibility factor Z can be expressed in the forms

$$Z = 1 + \hat{B}(T)p + \hat{C}(T)p^2 + \hat{D}(T)p^3 + \dots \quad (3.29)$$

$$Z = 1 + \frac{B(T)}{\bar{v}} + \frac{C(T)}{\bar{v}^2} + \frac{D(T)}{\bar{v}^3} + \dots \quad (3.30)$$

FIND: Show that

$$\hat{B} = B/\bar{R}T, \quad \hat{C} = (C - B^2)/\bar{R}^2T^2$$

ANALYSIS: With $Z = p\bar{v}/\bar{R}T$, Eq. 3.30 gives

$$p = \frac{\bar{R}T}{\bar{v}} + \frac{\bar{R}TB}{\bar{v}^2} + \frac{\bar{R}TC}{\bar{v}^3} + \dots$$

Inserting this into Eq. 3.29

$$\begin{aligned} Z &= 1 + \hat{B} \left[\frac{\bar{R}T}{\bar{v}} + \frac{\bar{R}TB}{\bar{v}^2} + \frac{\bar{R}TC}{\bar{v}^3} + \dots \right] + \hat{C} \left[\frac{\bar{R}T}{\bar{v}} + \frac{\bar{R}TB}{\bar{v}^2} + \dots \right]^2 + \dots \\ &= 1 + \frac{\hat{B}\bar{R}T}{\bar{v}} + \frac{\hat{B}\bar{R}TB + \hat{C}(\bar{R}T)^2}{\bar{v}^2} + \underbrace{\dots}_{\text{terms in } \bar{v}^3 \text{ or higher}} \end{aligned}$$

Comparing this with Eq. 3.30

$$B = \hat{B}\bar{R}T \Rightarrow \hat{B} = B/\bar{R}T$$

and

$$\hat{B}\bar{R}TB + \hat{C}(\bar{R}T)^2 = C(T)$$

or

$$\begin{aligned} \left(\frac{B}{\bar{R}T}\right)\bar{R}TB + \hat{C}(\bar{R}T)^2 &= C(T) \\ \textcircled{1} \quad B^2 + \hat{C}(\bar{R}T)^2 &= C(T) \Rightarrow \hat{C} = \frac{C(T) - (B(T))^2}{\bar{R}^2T^2} \end{aligned}$$

1. This procedure can be continued to obtain expressions for $\hat{D}$ and higher terms.

PROBLEM 11.14

KNOWN: The van der Waals equation expressed in terms of Z is

$$Z = \frac{V_R'}{V_R' - 1/8} - \frac{27/64}{T_R V_R'}$$

FIND: Express this (a) as a virial series in V_R' , (b) as a virial series in P_R . (c) Dropping higher terms in the expression of part (b), obtain an approximate form, and (d) compare with data from the literature.

ANALYSIS: (a) The term $1/(V_R' - 1/8)$ can be written as

$$\frac{1}{(V_R' - 1/8)} = \frac{1}{V_R'} + \frac{1}{8(V_R')^2} + \frac{1}{(8)^2(V_R')^3} + \dots$$

Accordingly

$$\begin{aligned} Z &= V_R' \left[\frac{1}{V_R'} + \frac{1}{8(V_R')^2} + \frac{1}{(8)^2(V_R')^3} + \dots \right] - \frac{27/64}{T_R V_R'} \\ &= 1 + \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \frac{1}{V_R'} + \frac{1}{(8)^2(V_R')^2} + \dots \end{aligned}$$

terms in $(V_R')^3$ and higher

This is the desired form:

$$Z = 1 + \frac{B}{V_R'} + \frac{C}{(V_R')^2} + \dots \quad \leftarrow (a)$$

(b) A virial series in P_R has the form

$$Z = 1 + \hat{B}(T) P_R + \hat{C}(T) (P_R)^2 + \dots$$

With $Z = P_R V_R' / T_R$, the result of part (a) becomes

$$\frac{P_R V_R'}{T_R} = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \frac{1}{V_R'} + \frac{1}{(8)^2(V_R')^2} + \dots$$

or

$$P_R = \frac{T_R}{V_R'} + \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \frac{T_R}{(V_R')^2} + \frac{T_R}{(8)^2(V_R')^3} + \dots$$

Inserting this into the series in P_R

$$\begin{aligned} Z &= 1 + \hat{B} \left[\frac{T_R}{V_R'} + \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \frac{T_R}{(V_R')^2} + \frac{T_R}{(8)^2(V_R')^3} + \dots \right] + \hat{C} \left[\frac{T_R}{V_R'} + \dots \right]^2 \\ &= 1 + \frac{\hat{B} T_R}{V_R'} + \left[\frac{\hat{B} \left[\frac{1}{8} - \frac{27/64}{T_R} \right] T_R + \hat{C} T_R^2}{(V_R')^2} \right] + \dots \end{aligned}$$

terms in $(V_R')^3$ or higher

Comparing this with the result of part (a)

$$\hat{B} T_R = \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \Rightarrow \hat{B} = \frac{1}{T_R} \left[\frac{1}{8} - \frac{27/64}{T_R} \right]$$

$$\begin{aligned} \hat{B} \left[\frac{1}{8} - \frac{27/64}{T_R} \right] T_R + \hat{C} T_R^2 &= \frac{1}{(8)^2} \Rightarrow \left[\frac{1}{8} - \frac{27/64}{T_R} \right]^2 + \hat{C} T_R^2 = \frac{1}{(8)^2} \\ \Rightarrow \hat{C} &= \left[\frac{1}{(8)^2} - \left[\frac{1}{8} - \frac{27/64}{T_R} \right]^2 \right] \frac{1}{T_R^2} \end{aligned}$$

Continued on next slide

Problem 11-14 continued

(c) The series obtained in part (b) has the form

$$Z = 1 + \hat{B} P_R + \hat{C} (P_R)^2 + \dots$$

$\rightarrow$ Terms in $(P_R)^2, (P_R)^3, \dots$

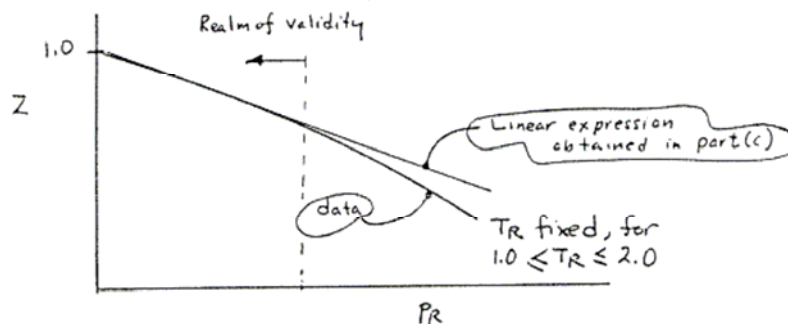
Dropping terms involving $(P_R)^2$ and higher

$$Z = 1 + \hat{B} P_R$$

or

$$Z = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \frac{P_R}{T_R}$$

- ① (d) Calculating compressibility factors from the result of part (c) and comparing them with values obtained from the literature, the realm of validity can be determined. This procedure is indicated by the following sketch:



1. See, for example, R.C. Reid and T.K. Sherwood, The Properties of Gases and Liquids, 2nd ed., McGraw-Hill, New York, 1966, pp. 587-595.

PROBLEM 11.15

KNOWN: The Berthelot equation of state has the form

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{T\bar{v}^3}$$

FIND: (a) Using Eqs. 11.3, evaluate a and b in terms of T_c and p_c , (b) Express the equation in terms of Z , T_R , and $\bar{v}_R$.

ANALYSIS: (a) Differentiating

$$\left(\frac{\partial p}{\partial \bar{v}}\right)_T = -\frac{\bar{R}T}{(\bar{v}-b)^2} + \frac{2a}{T\bar{v}^4}$$

At the critical point: $(\partial p / \partial \bar{v})_T = 0$. Thus

$$-\frac{\bar{R}T_c}{(\bar{v}_c-b)^2} + \frac{2a}{T_c\bar{v}_c^4} = 0 \Rightarrow -\bar{R}T_c^2\bar{v}_c^3 + 2a(\bar{v}_c-b)^2 = 0 \quad (1)$$

Differentiating

$$\left(\frac{\partial^2 p}{\partial \bar{v}^2}\right)_T = +\frac{2\bar{R}T}{(\bar{v}-b)^3} - \frac{6a}{T\bar{v}^5}$$

At the critical point: $(\partial^2 p / \partial \bar{v}^2)_T = 0$. Thus

$$\frac{2\bar{R}T_c}{(\bar{v}_c-b)^3} - \frac{6a}{T_c\bar{v}_c^5} = 0 \Rightarrow 2\bar{R}T_c^2\bar{v}_c^4 - 6a(\bar{v}_c-b)^3 = 0 \quad (2)$$

At the critical point:

$$p_c = \frac{\bar{R}T_c}{\bar{v}_c-b} - \frac{a}{T_c\bar{v}_c^3} \quad (3)$$

Combining (1) and (2), $b = \bar{v}_c/3$. Inserting this into Eq. (1)

$$-\bar{R}T_c^2\bar{v}_c^3 + 2a\left(\bar{v}_c - \frac{\bar{v}_c}{3}\right)^2 = 0 \Rightarrow a = 9\bar{R}T_c^2\bar{v}_c/8$$

Inserting the expressions for a and b into Eq. (3), $(p_c\bar{v}_c/\bar{R}T_c) = 3/8$, or $\bar{v}_c = \frac{3}{8} \frac{\bar{R}T_c}{p_c}$. Finally

$$a = \frac{9\bar{R}T_c^2}{8} \left[\frac{3}{8} \frac{\bar{R}T_c}{p_c} \right] = \frac{27}{64} \frac{\bar{R}^2 T_c^3}{p_c}, \quad b = \frac{1}{8} \frac{\bar{R}T_c}{p_c} \quad (a)$$

(b) With $Z = p\bar{v}/\bar{R}T$ and the above expressions for a and b

$$\frac{p\bar{v}}{\bar{R}T} = \frac{\bar{v}}{\bar{v}-b} - \frac{a}{\bar{R}T^2\bar{v}} = \frac{\bar{v}}{\bar{v}-\left[\frac{\bar{R}T_c}{8p_c}\right]} - \frac{(27/64)(\bar{R}^2 T_c^3/p_c)}{\bar{R}T^2\bar{v}}$$

or

$$Z = \frac{1}{1 - \frac{1}{8} \left[\frac{\bar{R}T_c}{\bar{v}p_c} \right]} - \frac{(27/64)(\bar{R}T_c/p_c\bar{v})}{T_R^2}$$

Thus

$$Z = \frac{\bar{v}_R'}{\bar{v}_R' - \frac{1}{8}} - \frac{27/64}{\bar{v}_R' T_R^2} \quad (b)$$

PROBLEM 11.16

KNOWN: The Beattie-Bridgeman equation of state can be expressed as

$$P = \frac{RT(1-\epsilon)(v+B)}{v^2} - \frac{A}{v^2}$$

where

$$A = A_0 \left(1 - \frac{a}{v}\right), \quad B = B_0 \left(1 - \frac{b}{v}\right), \quad \epsilon = \frac{c}{vT^3}$$

FIND: Express the equation in terms of P_R, T_R, v_R' , and dimensionless constants.

ANALYSIS: With $T = T_R T_c$, $P = P_R P_c$, and $v = v_R' (R T_c / P_c)$, the equation becomes

$$P_R P_c = \frac{R T_R T_c (1-\epsilon)(v_R' R T_c / P_c + B)}{(v_R')^2 (R T_c / P_c)^2} - \frac{A}{(v_R')^2 (R T_c / P_c)^2}$$

where

$$A = A_0 \left(1 - \frac{a}{v_R' (R T_c / P_c)}\right), \quad B = B_0 \left(1 - \frac{b}{v_R' (R T_c / P_c)}\right), \quad \epsilon = \frac{c}{v_R' T_R^3 (R T_c / P_c) (T_c)^3}$$

Define the following dimensionless quantities

$$a' \equiv \frac{a}{R T_c / P_c}, \quad b' \equiv \frac{b}{R T_c / P_c}, \quad c' \equiv \frac{c}{R T_c^4 / P_c}$$

$$A'_0 \equiv \frac{A_0}{R^2 T_c^2 / P_c}, \quad B'_0 \equiv \frac{B_0}{R T_c / P_c}, \quad A' \equiv \frac{A}{R^2 T_c^2 / P_c}, \quad B' \equiv \frac{B}{R T_c / P_c}$$

Then

$$A' = A'_0 \left(1 - \frac{a'}{v_R'}\right), \quad B' = B'_0 \left(1 - \frac{b'}{v_R'}\right), \quad \epsilon = \frac{c'}{v_R' T_R^3}$$

Upon simplification, the equation of state becomes

$$P_R = \frac{R T_R T_c \left(\frac{R T_c}{P_c}\right) (1-\epsilon) (v_R' + \frac{B'_0}{R T_c})}{P_c (R T_c / P_c)^2 (v_R')^2} - \frac{A}{P_c (v_R')^2 (R T_c / P_c)^2}$$

or

$$P_R = \frac{T_R (1-\epsilon) (v_R' + B')}{(v_R')^2} - \frac{A'}{(v_R')^2}$$

which is the required result.

PROBLEM 11.17

KNOWN: The Dieterici equation of state is

$$p = \left(\frac{RT}{v-b} \right) \exp \left(-\frac{a}{RTv} \right)$$

FIND: (a) Evaluate a and b in terms of T_c and P_c using Eqs. 11.3. (b) Express the equation in terms of Z , T_R , and V_R . (c) Convert the result of part (b) into a virial series in V_R .

ANALYSIS: (a) Differentiating twice with respect to v , we get

$$\begin{aligned} \left(\frac{\partial p}{\partial v} \right)_T &= -\frac{RT}{(v-b)^2} \exp \left(-\frac{a}{RTv} \right) + \left(\frac{RT}{v-b} \right) \exp \left(-\frac{a}{RTv} \right) \cdot \left(+\frac{a}{RTv^2} \right) \\ &= -\frac{RT}{(v-b)^2} \exp \left(-\frac{a}{RTv} \right) + \frac{a}{v^2(v-b)} \exp \left(-\frac{a}{RTv} \right) \end{aligned} \quad (1)$$

$$\begin{aligned} \left(\frac{\partial^2 p}{\partial v^2} \right)_T &= \frac{2RT}{(v-b)^3} \exp \left(-\frac{a}{RTv} \right) - \frac{a}{v^2(v-b)^2} \exp \left(-\frac{a}{RTv} \right) - \\ &\quad \frac{a(3v-2b)}{v^3(v-b)^2} \exp \left(-\frac{a}{RTv} \right) + \frac{a^2}{RTv^4(v-b)} \exp \left(-\frac{a}{RTv} \right) \end{aligned} \quad (2)$$

With $(\partial p / \partial v)_T = 0$ at the critical point, Eq. (1) gives

$$0 = -\frac{RT_c}{(v_c-b)^2} + \frac{a}{v_c^2(v_c-b)} \Rightarrow a = \frac{RT_c(v_c)^2}{(v_c-b)} \quad (3)$$

With $(\partial^2 p / \partial v^2)_T = 0$ at the critical point, Eq. (2) gives

$$0 = \frac{2RT_c}{(v_c-b)^3} - \frac{a}{v_c^2(v_c-b)^2} - \frac{a(3v_c-2b)}{v_c^3(v_c-b)^2} + \frac{a^2}{RT_c v_c^4(v_c-b)} \quad (4)$$

Inserting Eq. (3) into Eq. (4) and simplifying, $b = v_c/2$. Thus, $a = 2RT_c v_c$.

At the critical point, the equation of state reads

$$P_c = \frac{RT_c}{v_c-b} \exp \left(-\frac{a}{RT_c v_c} \right) = \frac{RT_c}{v_c - \frac{v_c}{2}} \exp \left(-\frac{2RT_c v_c}{RT_c v_c} \right)$$

Upon reduction, $v_c = 2RT_c / P_c e^2$, giving

$$a = \frac{4R^2 T_c^2}{P_c e^2}, \quad b = \frac{RT_c}{P_c e^2}$$

(b) With the given equation of state, the result of part (a), and $T = T_R T_c$, $v = v_R RT_c / P_c$, we get

$$\frac{pV}{RT} = \frac{v}{v-b} \exp \left(-\frac{a}{RTv} \right)$$

Continued on next slide

Problem 11-17 continued

Thus

$$Z = \frac{V_R' RT_C / P_C}{[V_R' RT_C / P_C - RT_C / P_C e^z]} \exp\left(\frac{\frac{4R^2 T_C^2}{P_C e^z}}{R(T_R T_C)(V_R' RT_C / P_C)}\right)$$

$$Z = \left[\frac{V_R'}{V_R' - 1/e^z} \right] \exp\left(-\frac{4}{T_R V_R' e^z}\right) \leftarrow (c)$$

(c) The term $1/(V_R' - 1/e^z)$ can be written as

$$\frac{1}{V_R' - 1/e^z} = \frac{1}{V_R'} + \frac{1}{e^z (V_R')^2} + \frac{1}{e^{2z} (V_R')^3} + \dots$$

The exponential term can also be written as a series. Collect results and reduce to read

$$Z = 1 + \frac{1}{e^z} \left[1 - \frac{4}{T_R}\right] \frac{1}{V_R'} + \frac{1}{e^{2z}} \left[1 - \frac{4}{T_R} - \frac{8}{T_R^2}\right] \frac{1}{(V_R')^2} + \dots \leftarrow (c)$$

PROBLEM 11.18

KNOWN: The Peng-Robinson equation of state is specified.

FIND: Evaluate the constants a, b, c in terms of P_c, T_c, Z_c using Eqs. 11.3

ANALYSIS: Evaluating the Peng-Robinson equation of state at the critical point and using Eqs. 11.3 gives the following three expressions

$$P_c = \frac{RT_c}{v_c - b} - \frac{a}{v_c^2 - c^2} \quad (1)$$

$$\frac{-RT_c}{(v_c - b)^2} + \frac{2av_c}{(v_c^2 - c^2)^2} = 0 \quad (2)$$

$$\frac{2RT_c}{(v_c - b)^3} + \frac{2a(v_c^2 - c^2) - 8av_c^2}{(v_c^2 - c^2)^3} = 0 \quad (3)$$

Solving these expressions simultaneously

$$\textcircled{1} \quad a = \frac{8v_c^3 P_c^2}{RT_c} = 8v_c^2 P_c Z_c \quad (4)$$

$$b = 3v_c - \frac{RT_c}{P_c} = v_c \left[3 - \frac{1}{Z_c} \right] \quad (5)$$

$$c^2 = v_c^2 \left[\frac{8P_c v_c}{RT_c} - 3 \right] = v_c^2 [8Z_c - 3] \quad (6)$$

as can be verified by substitution into Eqs. (1)-(3) and reduction of the resulting expressions.

1. Note that when $Z_c = 3/8$, Eqs. (4, 5) reduce to Eqs. (11.4a, 4b), respectively. The equation of state reduces to the van der Waals equation.

PROBLEM 11.19

KNOWN: The Carnahan-Starling-De Santis equation of state is provided, together with values of the constants in the equation for the cases of Refrigerants 12 and 13.

FIND: Specify which of the two refrigerants would allow the smaller amount of mass to be stored in a 10-m^3 vessel at 0.2 MPa , 80°C . Determine the mass.

ANALYSIS:

For the case of R12, using the values of the constants provided

$$b = (0.15376) + (-1.84195 \times 10^{-4})(353) + (-5.03644 \times 10^{-8})(353)^2 \\ = 8.2463 \times 10^{-2} \frac{\text{L}}{\text{mol}} = 8.2463 \times 10^{-2} \frac{\text{m}^3}{\text{kmol}}$$

$$a = (3.52412 \times 10^3) \exp\left((-2.7723 \times 10^{-3})(353) + (-0.67318 \times 10^{-6})(353)^2\right) \\ = 1217.896 \frac{\text{J} \cdot \text{L}}{(\text{mol})^2} = 1.2179 \times 10^6 \frac{\text{J} \cdot \text{m}^3}{(\text{kmol})^2}$$

Inserting values into the equation of state

$$\frac{(0.2 \times 10^6 \text{ N/m}^2) \bar{v}}{(8314 \frac{\text{J}}{\text{kmol} \cdot \text{K}})(353\text{K})} = \frac{1 + \beta + \beta^2 - \beta^3}{(1 + \beta)^3} - \frac{1.2179 \times 10^6 \frac{\text{J} \cdot \text{m}^3}{(\text{kmol})^2}}{(8314 \frac{\text{J}}{\text{kmol} \cdot \text{K}})(353\text{K})(\bar{v} + 8.2463 \times 10^{-2}) \frac{\text{m}^3}{\text{kmol}}}$$

where $\beta = 8.2463 \times 10^{-2} / \bar{v}$.

Using an equation solver, we get $\bar{v} = 14.21 \text{ m}^3/\text{kmol}$. Then, with $M = 120.92$ for R12

$$v = \frac{14.21}{120.92} = 0.1175 \text{ m}^3/\text{kg}$$

- ① This value is within 1% of the value obtained from the literature: $v = 0.1187 \text{ m}^3/\text{kg}$.

Similarly, for R13 at the same state, $v = 0.1391 \text{ m}^3/\text{kg}$. Thus, a smaller mass of R13 can be stored

$$m = \frac{V}{v} = \frac{10 \text{ m}^3}{0.1391 \text{ m}^3/\text{kg}} = 71.89 \text{ kg}$$

1. For example, R12 and R13 data can be obtained from C. Borgnakke and R.E. Sonntag, Thermodynamic and Transport Properties, Wiley, New York, 1997, Tables B.12, B.13.

PROBLEM 11.20

KNOWN: Two expressions for dp in terms of v and T are provided.

FIND: Determine which of the two expressions is the differential of an equation of state: $p = p(v, T)$, and obtain the equation.

ANALYSIS: For the first of the two equations, let

$$M = \frac{2(v-b)}{RT}, \quad N = \frac{(v-b)^2}{RT^2}$$

If this is the differential of an equation of state, the test for exactness must be satisfied: $\partial M / \partial T)_v = \partial N / \partial v)_T$. Checking

$$\left(\frac{\partial M}{\partial T} \right)_v = -\frac{2(v-b)}{RT^2}, \quad \left(\frac{\partial N}{\partial v} \right)_T = \frac{2(v-b)}{RT^2}$$

As these are unequal, the first expression given cannot be the differential of an equation of state.

For the second of the two equations, let

$$M = -\frac{RT}{(v-b)^2}, \quad N = \frac{R}{v-b}$$

Applying the test for exactness: $\partial M / \partial T)_v = -R / (v-b)^2$, $\partial N / \partial v)_T = -R / (v-b)^2$. As these are equal, a function $p = p(T, v)$ exists for which the second differential form is the exact differential. To obtain the function, write

$$dp = \left(\frac{\partial p}{\partial v} \right)_T dv + \left(\frac{\partial p}{\partial T} \right)_v dT$$

Comparing with the given differential form

$$\left(\frac{\partial p}{\partial v} \right)_T = -\frac{RT}{(v-b)^2}, \quad \left(\frac{\partial p}{\partial T} \right)_v = \frac{R}{v-b}$$

Integrating the second of these

$$p = \frac{RT}{v-b} + f(v)$$

Inserting this trial form into the first gives

$$-\frac{RT}{(v-b)^2} + \frac{df}{dv} = -\frac{RT}{(v-b)^2} \Rightarrow \frac{df}{dv} = 0 \Rightarrow f = \text{constant.}$$

Accordingly, the equation of state has the form: $p = \frac{RT}{(v-b)} + \text{constant.}$

PROBLEM 11.21

KNOWN: $\delta Q_{\text{int, rev}} = dU + p dV$

FIND: Use the given expression together with the test for exactness to demonstrate that $Q_{\text{int, rev}}$ is not a property.

ENGINEERING MODEL: (1) A simple compressible system of fixed mass is under consideration. (2) The system undergoes an internally reversible process.

ANALYSIS: For a simple compressible system of fixed mass, $U = U(V, T)$. The differential of this is

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV = M dT + N dV \quad (1)$$

As U , V , and T are properties, this is an exact differential. Applying the test for exactness:

$$\frac{\partial}{\partial V} \left[\underbrace{\left(\frac{\partial U}{\partial T}\right)_V}_M \right]_T = \frac{\partial}{\partial T} \left[\underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_N \right]_V \quad (2)$$

Inserting Eq. (1) into the given expression for $\delta Q_{\text{int, rev}}$

$$\begin{aligned} \delta Q_{\text{int, rev}} &= \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + p dV \\ &= \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p \right] dV \end{aligned} \quad (3)$$

If $Q_{\text{int, rev}}$ is a property, the last expression would be an exact differential. Applying the test for exactness to Eq. (3)

$$\begin{aligned} M &= \left(\frac{\partial U}{\partial T}\right)_V \Rightarrow \left(\frac{\partial M}{\partial V}\right)_T = \frac{\partial}{\partial V} \left[\left(\frac{\partial U}{\partial T}\right)_V \right]_T \\ N &= \left(\frac{\partial U}{\partial V}\right)_T + p \Rightarrow \left(\frac{\partial N}{\partial T}\right)_V = \frac{\partial}{\partial T} \left[\left(\frac{\partial U}{\partial V}\right)_T \right]_V + \left(\frac{\partial p}{\partial T}\right)_V \end{aligned}$$

Accordingly

$$\left(\frac{\partial M}{\partial V}\right)_T = \left(\frac{\partial N}{\partial T}\right)_V \Rightarrow \frac{\partial}{\partial V} \left[\left(\frac{\partial U}{\partial T}\right)_V \right]_T = \frac{\partial}{\partial T} \left[\left(\frac{\partial U}{\partial V}\right)_T \right]_V + \left(\frac{\partial p}{\partial T}\right)_V$$

↑
equal by Eq. (2)

Using Eq. (2) in this, as indicated, the test for exactness is satisfied only when

$$\left(\frac{\partial p}{\partial T}\right)_V \equiv 0$$

However, reference to Fig. 11.1 shows that this expression is not generally satisfied. It can be concluded therefore that the test for exactness is not satisfied and $Q_{\text{int, rev}}$ cannot be a property. The use of the symbol δ with $Q_{\text{int, rev}}$ is to signal this fact. ←

PROBLEM 11.22

KNOWN: An equation of state has the form $p = [RT/(v-b)] + a$

FIND: Show that Eq. 11.16 is satisfied.

ANALYSIS: Eq. 11.16 has the form

$$\left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -1$$

Identifying $p: x$, $v: y$, $T: z$, then

$$\left(\frac{\partial y}{\partial z}\right)_x = \left(\frac{\partial v}{\partial T}\right)_p$$

$$\left(\frac{\partial z}{\partial x}\right)_y = \left(\frac{\partial T}{\partial p}\right)_v$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial p}{\partial v}\right)_T$$

$$\text{Using the equation of state, } \left(\frac{\partial p}{\partial v}\right)_T = -RT(v-b)^{-2} \quad (1)$$

Solving,

$$v = b + \frac{RT}{p-a} \Rightarrow \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p-a} \Rightarrow \left(\frac{\partial v}{\partial T}\right)_p = \frac{v-b}{T} \quad (2)$$

Solving

$$T = \frac{(p-a)(v-b)}{R} \Rightarrow \left(\frac{\partial T}{\partial p}\right)_v = \frac{v-b}{R} \quad (3)$$

With (1)-(3), Eq. 11.16 takes the form

$$\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v \left(\frac{\partial p}{\partial v}\right)_T = \left(\frac{v-b}{T}\right) \left(\frac{v-b}{R}\right) \left(-\frac{RT}{(v-b)^2}\right) = -1$$

Thus, Eq. 11.16 is satisfied.

PROBLEM 11.23

KNOWN: $x = x(y, w)$, $y = y(z, w)$, $z = z(x, w)$.

FIND: Show that

$$\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w = 1$$

ANALYSIS: Forming differentials

$$dx = \left(\frac{\partial x}{\partial y}\right)_w dy + \left(\frac{\partial x}{\partial w}\right)_y dw \quad (1)$$

$$dy = \left(\frac{\partial y}{\partial z}\right)_w dz + \left(\frac{\partial y}{\partial w}\right)_z dw \quad (2)$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_w dx + \left(\frac{\partial z}{\partial w}\right)_x dw \quad (3)$$

Replacing dy in Eq. (1) with Eq. (2), and then replacing dz in the resulting expression by Eq. (3):

$$\begin{aligned} dx &= \left(\frac{\partial x}{\partial y}\right)_w \left[\left(\frac{\partial y}{\partial z}\right)_w dz + \left(\frac{\partial y}{\partial w}\right)_z dw \right] + \left(\frac{\partial x}{\partial w}\right)_y dw = \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w dz + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y \right] dw \\ &= \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left[\left(\frac{\partial z}{\partial x}\right)_w dx + \left(\frac{\partial z}{\partial w}\right)_x dw \right] + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y \right] dw \\ &= \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w dx + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial w}\right)_x + \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y \right] dw \end{aligned}$$

On rearrangement

$$\left[1 - \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w \right] dx = \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial w}\right)_x + \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y \right] dw$$

Since x and w are independent, hold w constant and vary x . That is, $dw=0$ and $dx \neq 0$.

Then

$$\left[1 - \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w \right] dx = 0 \Rightarrow \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w = 1. \quad \leftarrow$$

PROBLEM 11.24

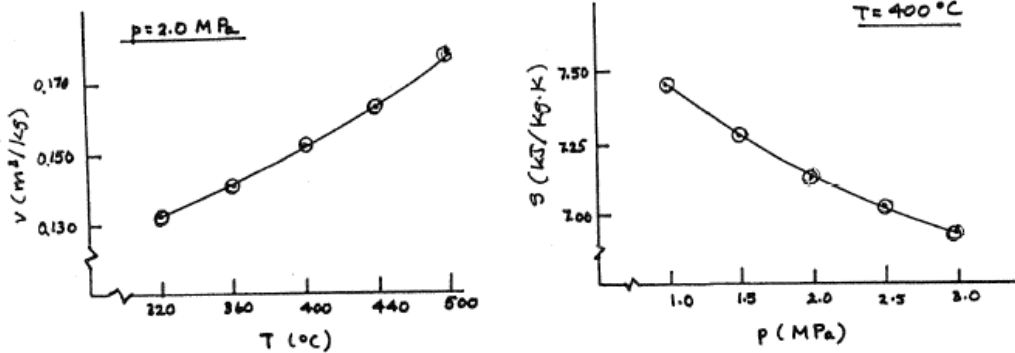
KNOWN:

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$$

(11.35)

FIND: Using Eq. (11.35), check the consistency of (a) the steam tables at 2 MPa, 400°C, (b) the R134a tables at 2 bar, 50°C.

ANALYSIS: We select a graphical approach here. (a) Using data from Table A-4



From the v vs. T plot, at 400°C

$$\left(\frac{\partial v}{\partial T}\right)_p \approx 0.25 \times 10^{-3} \frac{\text{m}^3}{\text{kg} \cdot \text{K}}$$

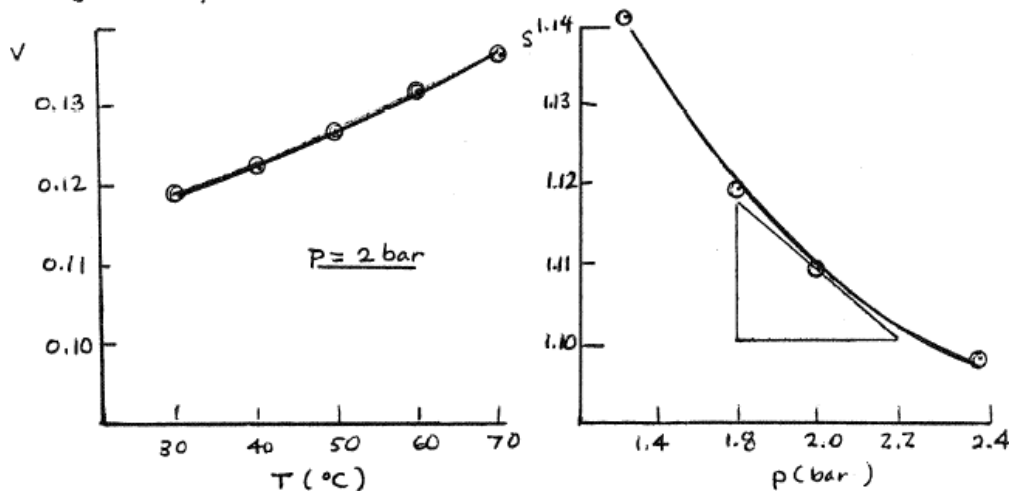
From the s vs p plot, at 2 MPa

$$\left(\frac{\partial s}{\partial p}\right)_T \approx -0.249 \frac{\text{kJ/kg} \cdot \text{K}}{\text{MPa}} = -0.249 \left(\frac{10^6 \text{ N} \cdot \text{m}}{\text{kg} \cdot \text{K}}\right) \left(\frac{1}{10^6 \text{ N/m}^2}\right) = -0.249 \times 10^{-3} \frac{\text{m}^3}{\text{kg} \cdot \text{K}}$$

These values are in good agreement, as required by 11.35.

(b)

Using data from Table A-12



From the v vs. T plot, at 50°C

$$\left(\frac{\partial v}{\partial T}\right)_p \approx 4.45 \times 10^{-4} \frac{\text{m}^3}{\text{kg} \cdot \text{K}}$$

From the s vs p plot, at 2 bar

$$\left(\frac{\partial s}{\partial p}\right)_T \approx 4.45 \times 10^{-4} \frac{\text{m}^3}{\text{kg} \cdot \text{K}}$$

In accordance with Eq. 11.35, the data are in agreement.

Continued on next slide

Problem 11-24 continued

An alternative solution using IT follows:

IT Code - part (a)

```
// (a) steam at 2 MPa, 400°C
p = 2000 // kPa
T = 400 // °C
dT = 0.001
dp = 0.001
T1 = T - dT
T2 = T + dT
p1 = p - dp
p2 = p + dp
v1 = v_PT("Water/Steam", p, T1)
v2 = v_PT("Water/Steam", p, T2)
dvdT_a = (v2 - v1) / (T2 - T1)
s1 = s_PT("Water/Steam", p1, T)
s2 = s_PT("Water/Steam", p2, T)
dsdp = (s2 - s1) / (p2 - p1)
```

IT Results - part (a)

```
(∂v/∂T)p = 0.2491 × 10-3 m³/kg·K
(∂s/∂p)T = - 0.2492 × 10-3 m³/kg·K
```

IT Code - part (b)

```
(b) R-134a at 2 bar, 50°C
p_b = 200 // kPa
T_b = 50 // °C
v = v_PT("R134A", p_b, T_b)
T1_b = T_b - dT
T2_b = T_b + dT
p1_b = p_b - dp
p2_b = p_b + dp
v1_b = v_PT("R134A", p_b, T1_b)
v2_b = v_PT("R134A", p_b, T2_b)
dvdT_b = (v2_b - v1_b) / (T2_b - T1_b)
s2_b = s_PT("R134A", p2_b, T_b)
s1_b = s_PT("R134A", p1_b, T_b)
dsdp_b = (s2_b - s1_b) / (p2_b - p1_b)
```

IT Results - part (b)

```
(∂v/∂T)p = 4.449 × 10-4 m³/kg·K
(∂s/∂p)T = - 4.449 × 10-4 m³/kg·K
```

The results of the IT solution compare favorably with the graphical results.

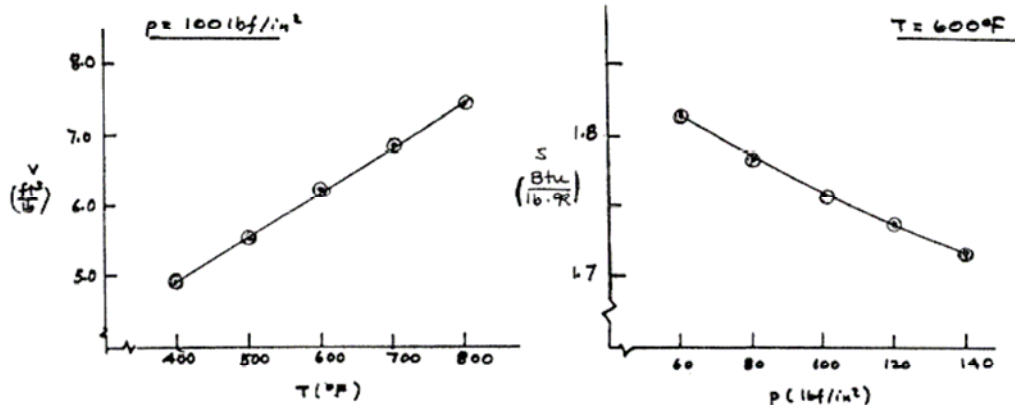
PROBLEM 11.25

KNOWN:

$$\left(\frac{\partial v}{\partial T}\right)_p = - \left(\frac{\partial s}{\partial p}\right)_T \quad (11.35)$$

FIND: Using Eq. 11.35, check the consistency of (a) the steam tables at 100 lbf/in², 600°F, (b) the R134a tables at 40 lbf/in², 100°F

ANALYSIS: We select a graphical approach here. (a) Using data from Table A-4E



From the v vs T plot at 600°F

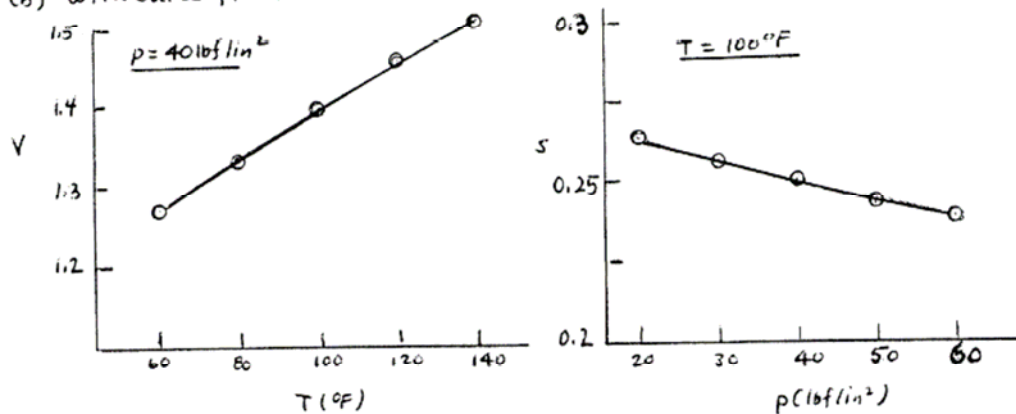
$$\left(\frac{\partial v}{\partial T}\right)_p = 6.24 \times 10^{-3} \frac{\text{ft}^3}{\text{lb} \cdot ^\circ\text{R}}$$

From the s vs p plot at 100 lbf/in²

$$\left(\frac{\partial s}{\partial p}\right)_T = -1.168 \times 10^{-3} \frac{\text{Btu/lb} \cdot ^\circ\text{R}}{\text{lbf/in}^2} \left(\frac{778.16 \text{ ft} \cdot \text{lbf}}{\text{Btu}} \right) \left(\frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) = -6.31 \times 10^{-3} \frac{\text{ft}^3}{\text{lb} \cdot ^\circ\text{R}}$$

These values are in good agreement, as required by Eq. 11.35.

(b) With data from Table A-12E



From the v vs. T plot, at 100°F:

$$\left(\frac{\partial v}{\partial T}\right)_p \approx 3.05 \times 10^{-3} \frac{\text{ft}^3}{\text{lb} \cdot ^\circ\text{R}}$$

From the s vs. p plot, at 40 lbf/in²:

$$\left(\frac{\partial s}{\partial p}\right)_T \approx -3.11 \times 10^{-3} \frac{\text{ft}^3}{\text{lb} \cdot ^\circ\text{R}}$$

again, we have good agreement, in accordance with Eq. 11.35.

Continued on next slide

Problem 11-25 continued

An alternative solution using IT follows:

IT Code - part (a)

```
// (a) steam at 100 lbf/in², 600°F
pa = 100 // lbf/in²
Ta = 600 // °F
dT_a = 0.1
T1a = Ta - dT_a
T2a = Ta + dT_a
dPa = 0.1
p1a = pa - dPa
p2a = pa + dPa
v1a = v_PT("Water/Steam", pa, T1a)
v2a = v_PT("Water/Steam", pa, T2a)
dvdTa = (v2a - v1a) / (T2a - T1a)
s1a = s_PT("Water/Steam", p1a, Ta)
s2a = s_PT("Water/Steam", p2a, Ta)
dsdPa = ((s2a - s1a) / (p2a - p1a)) * (778.17 / 144)
```

IT Results - part (a)

$(\partial v / \partial t)_p = 6.225 \times 10^{-3} \text{ ft}^3/\text{lb} \cdot ^\circ\text{R}$
 $(\partial s / \partial p)_T = -6.226 \times 10^{-3} \text{ ft}^3/\text{lb} \cdot ^\circ\text{R}$

IT Code - part (b)

```
// (b) R134a at 40 lbf/in², 100°F
pb = 40 // lbf/in²
Tb = 100 // °F
dT_b = 0.1
T1b = Tb - dT_b
T2b = Tb + dT_b
dPb = 0.1
p1b = pb - dPb
p2b = pb + dPb
v1b = v_PT("R134A", pb, T1b)
v2b = v_PT("R134A", pb, T2b)
dvdTb = (v2b - v1b) / (T2b - T1b)
s1b = s_PT("R134A", p1b, Tb)
s2b = s_PT("R134A", p2b, Tb)
dsdPb = ((s2b - s1b) / (p2b - p1b)) * (778.17 / 144)
```

IT Results - part (b)

$(\partial v / \partial t)_p = 3.042 \times 10^{-3} \text{ ft}^3/\text{lb} \cdot ^\circ\text{R}$
 $(\partial s / \partial p)_T = -3.041 \times 10^{-3} \text{ ft}^3/\text{lb} \cdot ^\circ\text{R}$

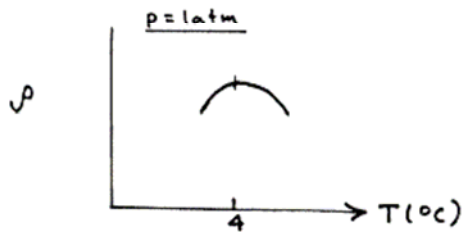
The results of the IT solution compare favorably with the graphical results.

PROBLEM 11.26

KNOWN: At 1 atm, liquid water has a state of maximum density at about 4°C.

FIND: Determine what can be concluded about $(\partial s / \partial p)_T$ at (a) 3°C, (b) 4°C, (c) 5°C?

SCHEMATIC & GIVEN DATA:



ANALYSIS: From Eq. 11.35

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

Since

$$\left(\frac{\partial p}{\partial T}\right)_p = \left(\frac{\partial v}{\partial T}\right)_p \frac{dp}{dv} = -\frac{1}{v^2} \left(\frac{\partial v}{\partial T}\right)_p \Rightarrow -\left(\frac{\partial v}{\partial T}\right)_p = \frac{1}{v^2} \left(\frac{\partial p}{\partial T}\right)_p$$

Thus

$$\left(\frac{\partial s}{\partial p}\right)_T = \frac{1}{v^2} \left(\frac{\partial p}{\partial T}\right)_p \Rightarrow \begin{array}{ll} \left(\frac{\partial s}{\partial p}\right)_T > 0 & \text{for } T < 4^\circ\text{C} \\ \left(\frac{\partial s}{\partial p}\right)_T = 0 & \text{for } T = 4^\circ\text{C} \\ \left(\frac{\partial s}{\partial p}\right)_T < 0 & \text{for } T > 4^\circ\text{C} \end{array}$$

PROBLEM 11.27

KNOWN: A gas enters a compressor and is compressed isentropically.

FIND: Determine if the specific enthalpy increases or decreases as the gas passes from inlet to exit.

ENGINEERING MODEL: The compression is isentropic.

ANALYSIS: From Eq. 11.27

$$\left. \frac{\partial h}{\partial p} \right|_s = v$$

Since v is a positive number and pressure p increases (compression) the specific enthalpy must also increase.

PROBLEM 11.28

KNOWN: A fundamental thermodynamic function $u = u(s, v)$ is provided.

FIND: Show that T, p, h, ψ , and g can each be determined using $u(s, v)$.

ANALYSIS: From Eqs. 11.24 and 11.25 T and p can be found by differentiation

$$T = \left(\frac{\partial u}{\partial s} \right)_v$$

$$p = - \left(\frac{\partial u}{\partial v} \right)_s$$

Then, by definition $h = u + p v$, giving

$$h = u(s, v) + \left(- \left(\frac{\partial u}{\partial v} \right)_s \right) v$$

Also, by definition $\psi = u - T s$, giving

$$\psi = u(s, v) - \left(\frac{\partial u}{\partial s} \right)_v s$$

Too, $g = h - T s$, giving $g = u(s, v) - v \left(\frac{\partial u}{\partial v} \right)_s - s \left(\frac{\partial u}{\partial s} \right)_v$

PROBLEM 11.29

KNOWN: The Helmholtz function is known for a certain substance.

FIND: Evaluate p , s , u , h , c_v , and c_p .

ANALYSIS: The Helmholtz function has the form

$$\Psi = -RT \ln \frac{v}{v'} - CT' \left[1 - \frac{T}{T'} + \frac{T}{T'} \ln \frac{T}{T'} \right]$$

where v', T' define the reference state and C is a constant. Note that at the reference state: when $v = v'$ and $T = T'$, $\Psi = 0$.

With Eq. 11.28

$$p = - \left(\frac{\partial \Psi}{\partial v} \right)_T = - \left(- \frac{RT}{v} \right) \Rightarrow p = RT/v \quad \leftarrow p$$

With Eq. 11.29

$$\begin{aligned} s &= - \left(\frac{\partial \Psi}{\partial T} \right)_v = - \left(-R \ln \frac{v}{v'} - CT' \left[-\frac{1}{T'} + \frac{1}{T'} \ln \frac{T}{T'} + \frac{T}{T'} \left(\frac{1}{T} \right) \right] \right) \\ &= R \ln \frac{v}{v'} + CT' \left[-\frac{1}{T'} + \frac{1}{T'} \ln \frac{T}{T'} + \frac{1}{T'} \right] \\ &= R \ln \frac{v}{v'} + C \ln \frac{T}{T'} \quad \leftarrow s \end{aligned}$$

Note that $s = 0$ when $v = v'$, $T = T'$.

Then, since $\Psi = u - Ts$, $u = \Psi + Ts$

$$\begin{aligned} u &= \left[-RT \ln \frac{v}{v'} - CT' \left[1 - \frac{T}{T'} + \frac{T}{T'} \ln \frac{T}{T'} \right] \right] + T \left[R \ln \frac{v}{v'} + C \ln \frac{T}{T'} \right] \\ &= -RT \ln \frac{v}{v'} - CT' + CT = C T \ln \frac{T}{T'} + RT \ln \frac{v}{v'} + C T \ln \frac{T}{T'} \end{aligned}$$

$$u = C [T - T'] \quad \leftarrow u$$

So, $u = 0$ when $T = T'$.

With $h = u + pv$,

$$h = C [T - T'] + RT \quad \leftarrow h$$

Further, $c_v = (\partial u / \partial T)_v$, so

$$c_v = C \quad \leftarrow c_v$$

and $c_p = (\partial h / \partial T)_p$, so

$$\textcircled{1} \quad c_p = C + R \quad \leftarrow c_p$$

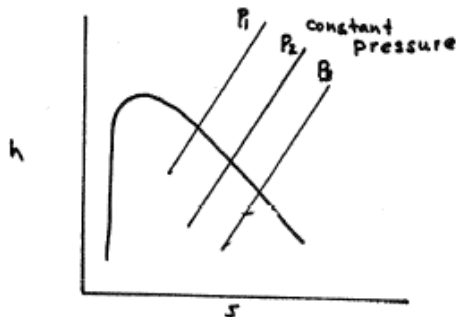
1. This substance adheres to the ideal gas model with constant c_v .

PROBLEM 11.30

KNOWN: The Mollier diagram gives a fundamental thermodynamic function $h = h(s, p)$.

FIND: Show that at any state fixed by s and p , the properties T, v, u, ψ , and g can be evaluated from the diagram.

SCHEMATIC & GIVEN DATA



ANALYSIS: From Eqs. 11.26 and 11.27 T and v can be found by differentiation (which can be performed graphically)

$$T = \left(\frac{\partial h}{\partial s} \right)_p, \quad v = \left(\frac{\partial h}{\partial p} \right)_s$$

Then, since $h = u + pv$

$$u = h - pv = h(s, p) - p \left(\frac{\partial h}{\partial p} \right)_s$$

And with $\psi = u - Ts$

$$\psi = h(s, p) - p \left(\frac{\partial h}{\partial p} \right)_s - s \left(\frac{\partial h}{\partial s} \right)_p$$

Finally, using $g = h - Ts$

$$g = h(s, p) - s \left(\frac{\partial h}{\partial s} \right)_p$$

PROBLEM 11.31

KNOWN:

$$c_p = -T \left(\frac{\partial^2 g}{\partial T^2} \right)_p$$

FIND: Derive the above expression.

ANALYSIS: Since T and p are regarded as independent, the following functions can be considered: $u(T, p)$, $h(T, p)$, $s(T, p)$, etc. As $c_p = (\partial h / \partial T)_p$, form the differential of $h(T, p)$

$$\begin{aligned} dh &= \left(\frac{\partial h}{\partial T} \right)_p dT + \left(\frac{\partial h}{\partial p} \right)_T dp \\ &= c_p dT + \left(\frac{\partial h}{\partial p} \right)_T dp \end{aligned} \quad (1)$$

Also, forming the differential of $s(T, p)$

$$ds = \left(\frac{\partial s}{\partial T} \right)_p dT + \left(\frac{\partial s}{\partial p} \right)_T dp \quad (2)$$

Substituting Eqs. (1) and (2) into Eq. 11.19: $dh = T ds + v dp$

$$[c_p dT + \left(\frac{\partial h}{\partial p} \right)_T dp] = T \left[\left(\frac{\partial s}{\partial T} \right)_p dT + \left(\frac{\partial s}{\partial p} \right)_T dp \right] + v dp$$

Collecting terms

$$[c_p - T \left(\frac{\partial s}{\partial T} \right)_p] dT = \left[T \left(\frac{\partial s}{\partial p} \right)_T + v - \left(\frac{\partial h}{\partial p} \right)_T \right] dp$$

As T and p are independent, fix p while varying T : $dp = 0$ and $dT \neq 0$; then

$$[c_p - T \left(\frac{\partial s}{\partial T} \right)_p] dT = 0 \Rightarrow [c_p - T \left(\frac{\partial s}{\partial T} \right)_p] = 0$$

or

$$c_p = T \left(\frac{\partial s}{\partial T} \right)_p \quad (3)$$

Next, introduce Eq. 11.31: $-s = (\partial g / \partial T)_p$ to obtain on differentiation

$$-\left(\frac{\partial s}{\partial T} \right)_p = \left(\frac{\partial^2 g}{\partial T^2} \right)_p \quad (4)$$

Finally, combining Eqs. (3) and (4)

$$c_p = -T \left(\frac{\partial^2 g}{\partial T^2} \right)_p$$

which is the result to be obtained.

PROBLEM 11.32

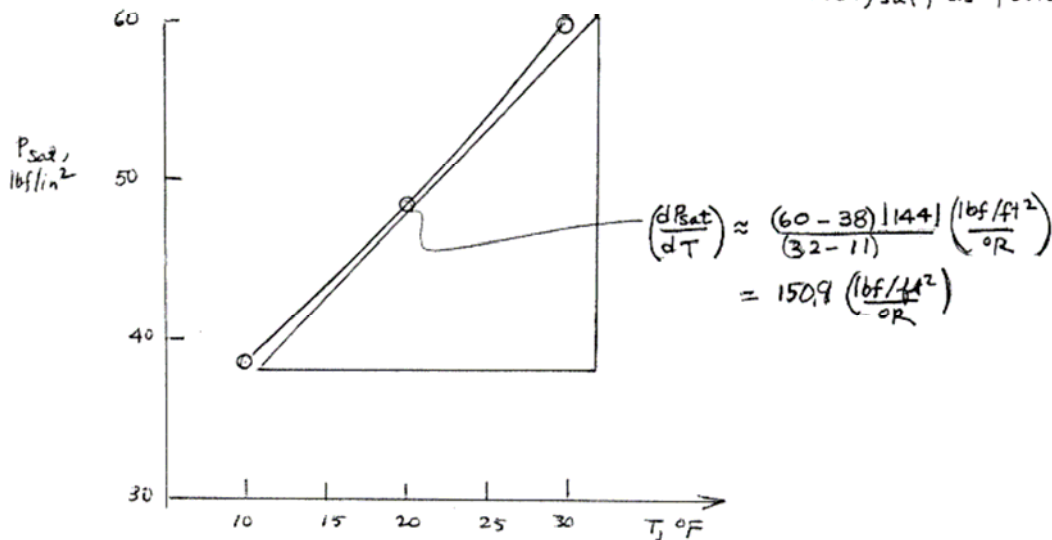
KNOWN: p-v-T data for saturated ammonia are available from Table A-13E.

FIND: Determine at 20°F h_{fg} , u_{fg} , s_{fg} , and compare with table values.

ANALYSIS: From Table A-13, $h_{fg} = 552.95 \text{ Btu/lb}$, $u_{fg} = 500.46 \text{ Btu/lb}$, $s_{fg} = 1.1528 \text{ Btu/lb} \cdot ^\circ\text{R}$.

The value of h_{fg} can be determined from saturated p-v-T data using the Clapeyron equation, Eq. 11.40: $h_{fg} = T v_{fg} \left(\frac{dp}{dT} \right)_{\text{sat}}$. (1)

A graphical method can be used to obtain $(dp/dT)_{\text{sat}}$, as follows:



Inserting values into Eq. (1)

$$h_{fg} = \frac{(479.67^\circ\text{R})(5.878 \frac{\text{ft}^3}{\text{lb}})(150.9 \frac{\text{lbf}}{\text{ft}^2})}{1778.17 \text{ ft} \cdot \text{lbf} / \text{Btu}} = 546.7 \text{ Btu/lb} \leftarrow h_{fg}$$

Then, with Eq. 11.38

$$s_{fg} = \frac{h_{fg}}{T} = \frac{546.7 \text{ Btu/lb}}{479.67^\circ\text{R}} = 1.1397 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \leftarrow s_{fg}$$

Finally, with $h = u + pv$

$$u_{fg} = h_{fg} - P_{\text{sat}} v_{fg} = 546.7 - \frac{(48.224)(144)(5.878)}{1778.17 \text{ ft} \cdot \text{lbf} / \text{Btu}} = 494.2 \frac{\text{Btu}}{\text{lb}} \leftarrow u_{fg}$$

Each of these is about 1% less than the corresponding table value.

The values obtained using the graphical approach are sensitive to the accuracy of the slope $(dp/dT)_{\text{sat}}$ determined graphically, as above.

Alternatively, an IT solution like that of Example 11.4 can be employed:

Continued on next slide

Problem 11-32 continued

IT Code

```
T = 20 // °F
dT = 0.01
T1 = T - dT
T2 = T + dT

p2 = Psat_T("Ammonia", T2)
p1 = Psat_T("Ammonia", T1)
dpdT_sat = ((p2 - p1) / (T2 - T1)) * 144
p = Psat_T("Ammonia", T)
vg = vsat_Px("Ammonia", p, 1)
vf = vsat_Px("Ammonia", p, 0)

hfg = ((T + 459.67) * (vg - vf) * dpdT_sat) / 778.17
sg = ssat_Px("Ammonia", p, 1)
sf = ssat_Px("Ammonia", p, 0)
sfg = hfg / (T + 459.67)
ufg = hfg - p * (vg - vf) * 144 / 778.67
```

IT Results

```
(∂p/∂T)sat = 152.5 lbf/ft2·°R
hfg = 552.6 Btu/lb
sfg = 1.152 Btu/lb·°R
ufg = 500.2 Btu/lb
```

PROBLEM 11.33

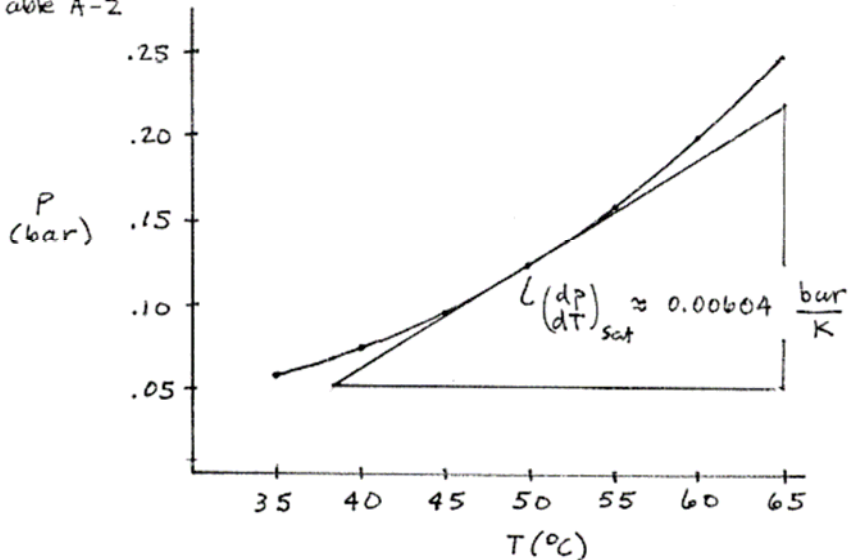
KNOWN: p-v-T data for water are available from the steam tables.

FIND: Determine at 50°C, (a) $h_g - h_f$, (b) $u_g - u_f$, and (c) $s_g - s_f$, and compare with table values.

ANALYSIS: The value of $(h_g - h_f)$ can be calculated using saturation data and the Clapeyron equation, Eq. 11.40

$$h_g - h_f = T(v_g - v_f) \left(\frac{dp}{dT} \right)_{\text{sat}} \quad (1)$$

To obtain the value for $(dp/dT)_{\text{sat}}$, prepare the plot shown below using data from Table A-2



Inserting values into Eq. (1)

$$h_g - h_f = (50 + 273.15) \text{ K} (12.032 - 1.0121 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (0.00604 \frac{\text{bar}}{\text{K}}) \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 2348 \text{ kJ/kg}$$

With Eq. 11.38

$$s_g - s_f = \frac{h_g - h_f}{T} = \frac{2348}{323.15} = 7.266 \text{ kJ/kg} \cdot \text{K}$$

Further, using $u = h - pv$

$$u_g - u_f = (h_g - h_f) - p(v_g - v_f)$$

$$= (2348 \frac{\text{kJ}}{\text{kg}}) - (0.1235 \text{ bar})(12.032 - 1.021 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 2199.4 \text{ kJ/kg}$$

The respective table values are $h_g - h_f = 2382.7 \text{ kJ/kg}$, $s_g - s_f = 7.3725 \text{ kJ/kg} \cdot \text{K}$, and $u_g - u_f = 2234.2 \text{ kJ/kg}$. The percent difference between calculated and table values is about 1.5% for all quantities.

An alternative solution using IT follows:

Continued on next slide

Problem 11-33 continued

IT Code

T = 50 // °C

dT = 0.001

T1 = T - dT

T2 = T + dT

p1 = Psat_T("Water/Steam", T1)

p2 = Psat_T("Water/Steam", T2)

dPdT_sat = ((p2 - p1) / (T2 - T1)) // bar/K

p = Psat_T("Water/Steam", T)

vg = vsat_Px("Water/Steam", p, 1)

vf = vsat_Px("Water/Steam", p, 0)

hfg = (T + 273.15) * (vg - vf) * dPdT_sat * 100

sg = ssat_Px("Water/Steam", p, 1)

sf = ssat_Px("Water/Steam", p, 0)

sfg = sg - sf

ufg = hfg - p * (vg - vf) * 100

IT Results

$(\partial p / \partial T)_{\text{sat}} = 0.006127 \text{ bar}$

$h_{fg} = 2382 \text{ kJ/kg}$

$s_{fg} = 7.372 \text{ kJ/kg}\cdot\text{K}$

$u_{fg} = 2234 \text{ kJ/kg}$

The IT results compare very favorably with the table data.

PROBLEM 11.34

KNOWN: h_{fg} , v_{fg} , and P_{sat} at 10°F are known from the R134a tables.

FIND: Estimate P_{sat} at 20°F . Comment on the accuracy of this estimate.

ENGINEERING MODEL: (1) The ratio h_{fg}/v_{fg} does not change significantly with temperature over the interval from 10° to 20°F . (2) v_f can be ignored relative to v_g , and v_g can be evaluated using the ideal gas model.

ANALYSIS: Table A-10E at 20°F gives $P_{\text{sat}} = 33.137 \text{ lbf/in}^2$.

Method 1: Assuming h_{fg}/v_{fg} is constant, Eq. 11.40 gives upon integration

$$(\Delta P)_{\text{sat}} = \frac{h_{fg}}{v_{fg}} \ln \frac{T_2}{T_1}$$

With data from Table A-10E at 10°F

$$(\Delta P)_{\text{sat}} = \left[\frac{88.53 \text{ Btu/lb}}{(1.7251 - 0.012) \text{ ft}^3/\text{lb}} \right] \left[\frac{778 \text{ ft lbf}}{18 \text{ Btu}} \right] \left[\frac{1 \text{ ft}^2}{144 \text{ in}^2} \right] \ln \left(\frac{480}{470} \right)$$

$$= 5.878 \text{ lbf/in}^2$$

At 10°F , $P_{\text{sat}} = 26.651 \text{ lbf/in}^2$. So

$$P_{\text{sat}}(20^\circ\text{F}) \approx 26.651 + 5.878 = 32.529 \text{ lbf/in}^2$$

This value is about 1.8% less than the table value.

Method 2: With assumption 2, Eq. 11.40 can be converted to the Clausius-Clapeyron equation: Eq. 11.42, which upon integration gives

$$\ln \frac{P_2}{P_1} = -\frac{h_{fg}}{R} \left[\frac{1}{T_2} - \frac{1}{T_1} \right]$$

Thus

$$\ln \frac{P_2}{P_1} = -\frac{(88.53 \text{ Btu/lb})}{\left(\frac{1.986 \text{ Btu}}{120.92 \text{ lb} \cdot ^\circ\text{R}} \right)} \left[\frac{1}{480} - \frac{1}{470} \right] \left(\frac{1}{^\circ\text{R}} \right)$$

$$\Rightarrow \frac{P_2}{P_1} = 1.27 \Rightarrow P_2 = 33.85 \text{ lbf/in}^2$$

This is about 2% above the table value.

Discussion. Even though the assumptions are only satisfied approximately, satisfactory results are obtained with both methods. Excellent agreement with the table value is achieved with the following IT approach based on integrating Eq. 11.40 without making the above assumptions:

IT Code

```
hg = hsat_Px("R134A", p, 1)
hf = hsat_Px("R134A", p, 0)
vf = vsat_Px("R134A", p, 0)
vg = vsat_Px("R134A", p, 1)
p = Psat_T("R134A", T)
```

// Clapeyron Eq. 11.40

```
dPdT_sat = ((hg - hf) / ((vg - vf) * (T + 459.67))) * (778.17 / 144)
psat = Integral(dPdT_sat, T) + 26.651
```

IT Result (Sweep T from 10 to 20°F in steps of 0.1.)

```
Psat(20°F) = 33.14 lbf/in.²
```

PROBLEM 11.35

KNOWN: h_{fg} , v_{fg} , and P_{sat} at 26°C are known from the ammonia tables.

FIND: Estimate P_{sat} at 30°C . Comment on the accuracy of this estimate.

ENGINEERING MODEL: (1) The ratio h_{fg}/v_{fg} does not change significantly with temperature over the interval from 26°C to 30°C . (2) v_f can be ignored relative to v_g , and v_g can be evaluated using the ideal gas model.

ANALYSIS: Table A-13 gives at 26°C ,

$$P_{sat} = 10.3602 \text{ bar}, \quad v_{fg} = 0.1229 \frac{\text{m}^3}{\text{kg}}, \quad h_{fg} = 1160.7 \text{ kJ/kg}$$

Method #1. With assumption (1), Eq. 11.40 gives on integration

$$(\Delta p)_{sat} = \frac{h_{fg}}{v_{fg}} \ln \frac{T_2}{T_1} = \left(\frac{1160.7 \text{ kJ/kg}}{0.1229 \text{ m}^3/\text{kg}} \right) \left| \frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| \ln \left(\frac{303}{299} \right) \\ = 1.255 \text{ bar}$$

$$\Rightarrow P_{sat}(30^\circ\text{C}) \approx 10.3602 + 1.255 = 11.615 \text{ bar}.$$

Comparing with the table value: 11.6865 bar , this is about 0.6% low.

Method #2. With assumption (2), Eq. 11.40 can be converted to the Clausius - Clapeyron equation: Eq. 11.42, which on integration gives

$$\ln \frac{P_2}{P_1} = - \frac{h_{fg}}{R} \left[\frac{1}{T_2} - \frac{1}{T_1} \right] = - \left(\frac{1160.7 \text{ kJ/kg}}{\frac{8314 \text{ N}\cdot\text{m}}{17.04 \text{ kg}\cdot\text{K}}} \right) \left(\frac{1}{303 \text{ K}} - \frac{1}{299 \text{ K}} \right) \left| \frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}} \right| \\ \Rightarrow \frac{P_2}{P_1} = 1.11 \Rightarrow P_2 = 11.5 \text{ bar}$$

This is about 1.6% below the table value.

Discussion. Even though the assumptions are only satisfied approximately, satisfactory results are obtained with both methods. Excellent agreement with the table value is achieved with the following IT approach based on integrating Eq. 11.40 without making the above assumptions.

IT Code

```
hg = hsat_Px("Ammonia", p, 1)
hf = hsat_Px("Ammonia", p, 0)
vg = vsat_Px("Ammonia", p, 1)
vf = vsat_Px("Ammonia", p, 0)
p = Psat_T("Ammonia", T)
```

// Clapeyron Eq. 11.40

```
dPdT_sat = ((hg - hf) / ((vg - vf) * (T + 273.15))) / 100
psat = Integral(dPdT_sat, T) + 10.3602
```

IT Result (Sweep T from 26 to 30 in steps of 0.1.)

```
psat(30°C) = 11.69 bar
```

PROBLEM 11.36

KNOWN: Triple point data for water are available from Table A-6E.

FIND: Estimate P_{sat} at -40°F and compare with the table value, which is 0.0019 lbf/in^2 from Table A-6E.

ENGINEERING MODEL: (1) The value of h_{ig} is constant independent of temperature over the interval from the triple point temperature to -40°F , (2) v_{f} can be ignored relative to v_{g} and v_{g} can be evaluated using the ideal gas model.

ANALYSIS: With the assumptions listed, Eq. 11.41 can be converted to give

$$\left(\frac{d \ln P}{dT}\right)_{\text{sat}} = \frac{h_{\text{ig}}}{RT^2}$$

which gives upon integration

$$\ln \frac{P_2}{P_1} = \frac{h_{\text{ig}}}{R} \left[\frac{1}{T_1} - \frac{1}{T_2} \right]$$

With data from Table A-6E: $h_{\text{ig}} = 1218.7 \text{ Btu/lb}$, $P_1 = 0.0887 \text{ lbf/in}^2$

$$\ln \frac{P_2}{P_1} = \frac{(1218.7 \text{ Btu/lb})}{\left(\frac{1.986 \text{ Btu}}{18.02 \text{ lb} \cdot ^\circ\text{R}}\right)} \left[\frac{1}{492^\circ\text{R}} - \frac{1}{420^\circ\text{R}} \right]$$

$$\Rightarrow \frac{P_2}{P_1} = 0.02122$$

$$\Rightarrow P_2 = 0.00188 \text{ lbf/in}^2$$

Discussion: The calculated value agrees closely with the table value. This is an expected outcome, for by reference to Table A-6E data the two assumptions are closely adhered to in this case.

PROBLEM 11.37

KNOWN: For water at 0°C

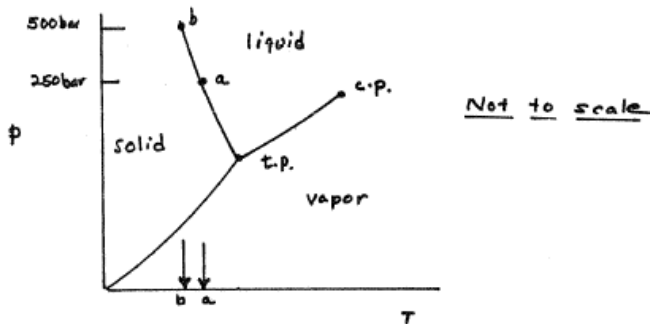
$$v_i = 1.0911 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$v_f = 1.0002 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$h_{if} = 333.4 \text{ kJ/kg}$$

FIND: Estimate the melting temperature of ice at (a) 250 bar, (b) 500 bar.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: h_{if} and v_{if} do not vary significantly with temperature.

ANALYSIS: With the assumptions listed, Eq 11.41 gives upon integration

$$\Delta p = \left(\frac{h_f - h_i}{v_f - v_i} \right) \ln \frac{T_2}{T_1}$$

$$\text{or} \quad = \left[\frac{333.4 \text{ kJ/kg}}{\left(\frac{1.0002 - 1.0911}{10^3} \right) \frac{\text{m}^3}{\text{kg}}} \right] \left[\frac{10^3 \text{ N}\cdot\text{m}}{\text{kJ}} \right] \left[\frac{\text{bar}}{10^5 \text{ N/m}^2} \right] \ln \frac{T_2}{T_1}$$

$$\Delta p = -(36,677.7 \text{ bar}) \ln \frac{T_2}{T_1}$$

At 0°C , the pressure is negligible relative to P_2 : $\Delta p \approx P_2$.

(a) $P_2 = 250 \text{ bar}$

$$\ln \frac{T_2}{273.15} = -\frac{250}{36,677.7} \Rightarrow T_2 = 271.29 \text{ K } (-1.86^\circ\text{C}) \quad \leftarrow \text{ (a)}$$

(b) $P_2 = 500 \text{ bar}$

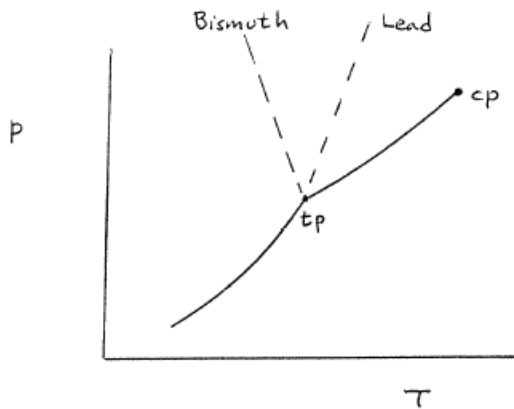
$$\ln \frac{T_2}{273.15} = -\frac{500}{36,677.7} \Rightarrow T_2 = 269.45 \text{ K } (-3.7^\circ\text{C}) \quad \leftarrow \text{ (b)}$$

PROBLEM 11.38

KNOWN: Bismuth and lead are the substances under consideration.

FIND: Confirm that the line representing the two-phase solid-liquid region on the phase diagram slopes to the left for bismuth and to the right for lead.

SCHEMATIC & GIVEN DATA:



ANALYSIS: The key expression is Eq 11.41

$$\left(\frac{dP}{dT}\right)_{\text{sat}} = \frac{h'' - h'}{T(v'' - v')}$$

where ' denotes solid and '' denotes liquid. As $(h'' - h')$ would be positive, the slope is determined by the difference $(v'' - v')$.

The CRC Handbook of Chemistry and Physics, 37th Ed, 1955-1956, gives a table of the change in volume due to fusion for selected substances, p. 2138. For the substances under consideration

Bismuth: change in volume = $-0.0034 \text{ cm}^3/\text{g}$

Lead: " " " = $+0.0034 \text{ cm}^3/\text{g}$

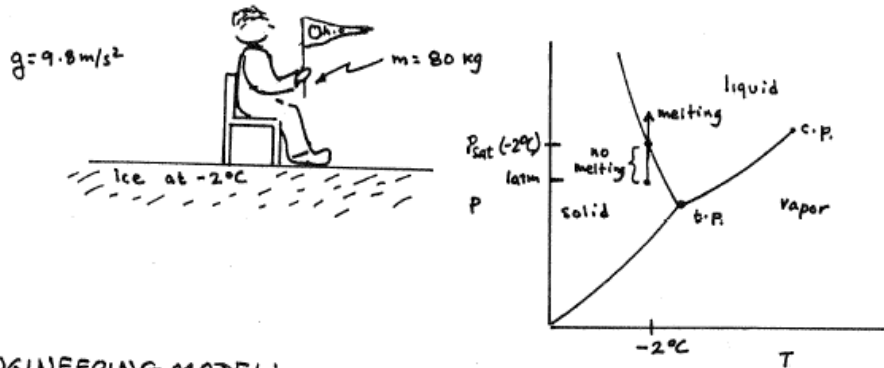
Accordingly, $(dP/dT)_{\text{sat}} < 0$ for bismuth and $(dP/dT)_{\text{sat}} > 0$ for lead.

PROBLEM 11.39

KNOWN: An occupied chair of total mass equal to 80 kg is at rest on an ice rink. The ice temperature is -2°C .

FIND: Determine the minimum total area the tips of the chair legs can have before the ice in contact with the legs would melt.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

- (1) The ice temperature remains at -2°C . (2) The local acceleration of gravity is 9.8 m/s^2 . (3) At 0°C , $h_{if} = 333.4 \text{ kJ/kg}$, $v_f = 1.0911 \times 10^{-3} \text{ m}^3/\text{kg}$, $v_i = 1.0002 \times 10^{-3} \text{ m}^3/\text{kg}$, h_{if} and v_{if} remain constant with temperature.

ANALYSIS: As shown by the phase diagram, melting would take place if the pressure exerted on the ice (at -2°C) would exceed $P_{\text{sat}}(-2^{\circ}\text{C})$.

The pressure $P_{\text{sat}}(-2^{\circ}\text{C})$ can be estimated using Eq. 11.41, together with the assumption that h_{if} and v_{if} remain constant with temperature, for then the equation becomes on integration

$$\Delta p = \left(\frac{h_{if} - h_i}{v_f - v_i} \right) \ln \frac{T_2}{T_1}$$

Inserting values for h_{if} , v_i , v_f and taking $T_1 = 273.15 \text{ K}$ (0°C), $T_2 = 271.15 \text{ K}$ (-2°C)

$$\Delta p = \left[\frac{(333.4 \frac{\text{kJ}}{\text{kg}}) \left(\frac{1000 \text{ N} \cdot \text{m}}{\text{kJ}} \right)}{\left(\frac{1.0002 - 1.0911}{1000} \right) \left(\frac{\text{m}^3}{\text{kg}} \right)} \right] \left| \frac{\text{bar}}{10^5 \text{ N/m}^2} \right| \ln \frac{271.15}{273.15} = 269.54 \text{ bar}$$

Since $P_{\text{sat}}(0^{\circ}\text{C}) = 0.61 \text{ kPa}$, $P_{\text{sat}}(-2^{\circ}\text{C}) = 269.55 \text{ bar}$

The pressure exerted on the ice by the chair/occupant and the atmosphere is

$$p = P_{\text{atm}} + \frac{mg}{A}$$

where A is the total area of the tips of the 4 chair legs. The pressure p can be no greater than 269.54 bar . Thus the minimum total area A is

$$269.55 \text{ bar} = 1.01325 \text{ bar} + \frac{(80 \text{ kg})(9.8 \text{ m/s}^2)}{A (\text{m}^2)} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{\text{bar}}{10^5 \text{ N/m}^2} \right|$$

or

$$A_{\text{min}} = (2.92 \times 10^{-5} \text{ m}^2) \left| \frac{100 \text{ cm}^2}{\text{m}^2} \right|$$

$$= 0.292 \text{ cm}^2$$

A_{min}

PROBLEM 11.40

KNOWN: Over a certain temperature interval the saturation pressure-temperature curve of a substance is represented by

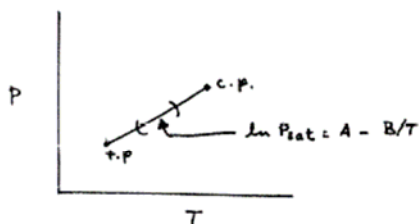
$$\ln P_{\text{sat}} = A - B/T$$

where A and B are constants.

FIND: (a) Obtain expressions for h_{fg} and s_{fg} in terms of p - v - T data and B .

(b) Using the result of (a), determine h_{fg} and s_{fg} for water vapor at 25°C and compare with steam table data.

SCHEMATIC & GIVEN DATA:



ANALYSIS: (a) The Clapeyron equation takes the form

$$\left(\frac{dp}{dT}\right)_{\text{sat}} = \frac{h_{fg}}{T v_{fg}} \quad (1)$$

To obtain $(dp/dT)_{\text{sat}}$, differentiate the given expression for $\ln P_{\text{sat}}$

$$\frac{d}{dT} (\ln P_{\text{sat}}) = + \frac{B}{T^2}$$

$$\frac{1}{P_{\text{sat}}} \frac{dP_{\text{sat}}}{dT} = \frac{B}{T^2}$$

or

$$\left(\frac{dp}{dT}\right)_{\text{sat}} = \frac{P_{\text{sat}}(T) B}{T^2} \quad (2)$$

Combining Eqs. (1) and (2)

$$h_{fg} = \frac{P_{\text{sat}}(T) v_{fg}(T) B}{T} \quad (3) \quad \leftarrow h_{fg}$$

Then, with Eq. 11.38: $s_{fg} = h_{fg}/T$

$$s_{fg} = \frac{P_{\text{sat}}(T) v_{fg}(T) B}{T^2} \quad (4) \quad \leftarrow s_{fg}$$

(b) Using standard computer routines, the values of A and B are found such that

$$\ln P_{\text{sat}} = \underbrace{28.83593}_A - \underbrace{\frac{5299.62}{T}}_B \quad (P_{\text{sat}}: \text{Pa}, T: \text{K}) \quad (5)$$

Then, with Eq. (3), B from Eq. (5), and P_{sat}, v_{fg} from Table A-2 at 25°C

$$\textcircled{1} \quad h_{fg} = \frac{(0.03169 \times 10^{-5} \frac{\text{m}^3}{\text{kg}})(43.359 \frac{\text{m}^3}{\text{kg}})(5299.62 \text{ K})}{298.15 \text{ K}} \left| \frac{\text{KJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 2442.4 \frac{\text{KJ}}{\text{kg}} \quad (\text{table value: } 2442.8 \text{ KJ/kg})$$

Similarly, Eq. (4) gives

$$s_{fg} = \frac{(0.03169 \times 10^{-5})(43.359)(5299.62)}{(298.15)^2} \left| \frac{1}{10^3} \right| = 8.192 \frac{\text{KJ}}{\text{kg}\cdot\text{K}} \quad (\text{table value: } 8.1906 \text{ KJ/kg}\cdot\text{K})$$

1. In this case, Excellent agreement is realized with steam table data.

PROBLEM 11.41

KNOWN: Data for water from Table A-2 is available for use.

FIND: Determine the constants A and B to give the best fit in a least-squares sense in the interval from 20° to 30°C by the equation $\ln P_{\text{sat}} = A - B/T$. Using this equation determine dP_{sat}/dT at 25°C. Calculate h_{fg} at 25°C and compare with the steam table value.

ANALYSIS: Using standard computer routines, the values of A and B are found so that

$$\ln P_{\text{sat}} = -1.999728 - \frac{35.767963}{T} \quad (P_{\text{sat}}: \text{bar}, T: ^\circ\text{C}) \quad (1)$$

or

$$\ln P_{\text{sat}} = 25.83593 - \frac{5299.62}{T} \quad (P_{\text{sat}}: \text{Pa}, T: \text{K}) \quad (2)$$

Using Eq. (2)

$$P_{\text{sat}} = \exp \left(25.83593 - \frac{5299.62}{T} \right)$$

Thus

$$\frac{dP_{\text{sat}}}{dT} = \frac{5299.62}{T^2} \exp \left(25.83593 - \frac{5299.62}{T} \right)$$

At 298.15 K, $dP_{\text{sat}}/dT = 188.88 \text{ Pa/K}$.

Using Eq. 11.40 and v_{fg} from Table A-2 at 25°C

$$\begin{aligned} h_{fg} &= T v_{fg} \left(\frac{dP}{dT} \right)_{\text{sat}} \\ &= (298.15 \text{ K}) \left(\frac{43360 - 1.0029}{10^3} \right) \left(\frac{\text{m}^3}{\text{kg}} \right) \left(\frac{188.88 \text{ N/m}^2}{\text{K}} \right) \left| \frac{\text{kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \\ &= 2441.7 \text{ kJ/kg} \end{aligned}$$

Table A-2 gives at 25°C, 2442.3 kJ/kg. Accordingly the calculated and table values of h_{fg} agree extremely well.

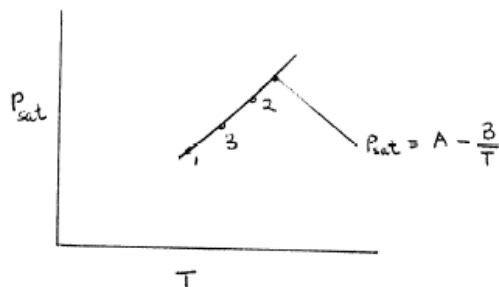
PROBLEM 11.42

KNOWN: The saturation pressure - temperature curve for two-phase liquid-vapor states is represented by $\ln P_{\text{sat}} = A - B/T$ where A, B are constants.

FIND: Show that any three points on the curve are related by

$$\frac{P_{\text{sat},3}}{P_{\text{sat},1}} = \left[\frac{P_{\text{sat},2}}{P_{\text{sat},1}} \right]^{\tau} \quad \text{where} \quad \tau = \frac{T_2 [T_3 - T_1]}{T_3 [T_2 - T_1]}$$

SCHEMATIC & GIVEN DATA:



ANALYSIS: Using the given expression

$$\begin{aligned} \ln(P_{\text{sat},1}) &= A - \frac{B}{T_1} \Rightarrow A = \ln(P_{\text{sat},1}) + \frac{B}{T_1} \\ \ln(P_{\text{sat},2}) &= A - \frac{B}{T_2} \Rightarrow \ln(P_{\text{sat},2}) = \ln(P_{\text{sat},1}) + \frac{B}{T_1} - \frac{B}{T_2} \\ &\Rightarrow \ln\left(\frac{P_{\text{sat},2}}{P_{\text{sat},1}}\right) = B\left[\frac{1}{T_1} - \frac{1}{T_2}\right] \\ \ln(P_{\text{sat},3}) &= A - \frac{B}{T_3} \Rightarrow \ln\left(\frac{P_{\text{sat},3}}{P_{\text{sat},1}}\right) = B\left[\frac{1}{T_1} - \frac{1}{T_3}\right] \end{aligned}$$

Combining,

$$\frac{\ln\left(\frac{P_{\text{sat},2}}{P_{\text{sat},1}}\right)}{\ln\left(\frac{P_{\text{sat},3}}{P_{\text{sat},1}}\right)} = \frac{\frac{1}{T_1} - \frac{1}{T_2}}{\frac{1}{T_1} - \frac{1}{T_3}} = \frac{\frac{T_2 - T_1}{T_1 T_2}}{\frac{T_3 - T_1}{T_1 T_3}} = \frac{T_3}{T_2} \cdot \frac{T_2 - T_1}{T_3 - T_1} = \frac{1}{\tau}$$

where $\tau \equiv \frac{T_2}{T_3} \cdot \frac{T_2 - T_1}{T_3 - T_1}$. Then

$$\ln\left(\frac{P_{\text{sat},3}}{P_{\text{sat},1}}\right) = \tau \ln\left(\frac{P_{\text{sat},2}}{P_{\text{sat},1}}\right)$$

$$\Rightarrow \frac{P_{\text{sat},3}}{P_{\text{sat},1}} = \left[\frac{P_{\text{sat},2}}{P_{\text{sat},1}} \right]^{\tau}$$

PROBLEM 11.43

KNOWN: The result of Problem 11.42 is applicable.

FIND: Using saturation pressure temperature data from Table A-2

(a) evaluate P_{sat} at 30°C and compare with the tabulated value.

(b) evaluate T_{sat} at 0.006 MPa and compare with the tabulated value.

ANALYSIS: The result of Problem 11.42 is

$$\frac{P_{\text{sat},3}}{P_{\text{sat},1}} = \left[\frac{P_{\text{sat},2}}{P_{\text{sat},1}} \right]^{\tau} \quad \text{where} \quad \tau = \frac{T_2}{T_3} \left[\frac{T_3 - T_1}{T_2 - T_1} \right]$$

(a) $T_1 = 20^\circ\text{C}$, $T_2 = 30^\circ\text{C}$, $T_3 = 40^\circ\text{C}$. From Table A-2

$$P_{\text{sat},1} = 0.02339 \text{ bar}$$

$$P_{\text{sat},3} = 0.07384 \text{ bar}$$

$$\tau = \frac{T_2}{T_3} \left(\frac{T_3 - T_1}{T_2 - T_1} \right) = \frac{303.15}{313.15} \left(\frac{20}{10} \right) = 1.9361$$

Accordingly

$$\ln \frac{P_{\text{sat},2}}{P_{\text{sat},1}} = \frac{1}{\tau} \ln \frac{P_{\text{sat},3}}{P_{\text{sat},1}} = \frac{1}{1.9361} \ln \left(\frac{0.07384}{0.02339} \right)$$

$$\Rightarrow \frac{P_{\text{sat},2}}{P_{\text{sat},1}} = 1.8108 \Rightarrow P_{\text{sat},2} = 0.04235 \text{ bar} \quad \leftarrow$$

Table A-2 gives $P_{\text{sat},2} = 0.04246 \text{ bar}$. Accordingly the percent difference is

$$\% = \left(\frac{0.04235 - 0.04246}{0.04246} \right) (100) = -0.26$$

(b) $T_1 = 20^\circ\text{C}$, $P_2 = 0.006 \text{ MPa}$, $T_3 = 40^\circ\text{C}$. $P_{\text{sat},1}$ and $P_{\text{sat},3}$ are given in part (a).

Accordingly

$$\tau = \frac{\ln \left(\frac{P_{\text{sat},3}}{P_{\text{sat},1}} \right)}{\ln \left(\frac{P_2}{P_{\text{sat},1}} \right)} = \frac{\ln \left(\frac{0.07384}{0.02339} \right)}{\ln \left(\frac{0.06}{0.02339} \right)} = 1.22033$$

And

$$\tau = \frac{T_2 [20]}{313.15 [T_2 - 293.15]}$$

so

$$20 T_2 = 1.22033 [313.15 [T_2 - 293.15]]$$

$$\Rightarrow T_2 = 309.34 \text{ K} \quad (36.19^\circ\text{C}) \quad \leftarrow$$

Table A-3 gives $T = 36.16^\circ\text{C}$. The percent difference is

$$\% = \left(\frac{36.19 - 36.16}{36.16} \right) (100) = 0.08$$

PROBLEM 11.44

KNOWN: Four exercises dealing with slopes are described.

FIND: (a) Using steam table data, evaluate the ratio of the slope of the vaporization line to the slope of the sublimation line. (b) On a T - s diagram, show that the slope of a constant specific volume line is greater than the slope of a constant pressure line through the same superheated vapor state. (c) For an h - s diagram, obtain an expression for the slope of a constant pressure line in terms of p - v - T data only. (d) For a p - h diagram, obtain an expression for the slope of an isentropic line in terms of p - v - T data only.

ANALYSIS: (a) Using the Clapeyron equation, Eq. 11.40

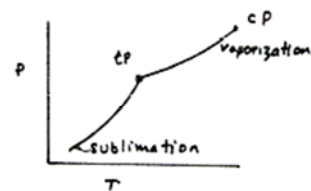
$$\left(\frac{dp}{dT}\right)_{\text{sat}} = \frac{1}{T} \frac{\Delta h}{\Delta v}$$

Accordingly at the triple point

$$\frac{\left(\frac{dp}{dT}\right)_{\text{sat}}^{\text{subl.}}}{\left(\frac{dp}{dT}\right)_{\text{sat}}^{\text{vap}}} = \frac{[h_g - h_i] / [v_g - v_i]}{[h_g - h_f] / [v_g - v_f]}$$

With data from Tables A-2 and A-6 at 0.01°C

$$\frac{\left(\frac{dp}{dT}\right)_{\text{sat}}^{\text{subl.}}}{\left(\frac{dp}{dT}\right)_{\text{sat}}^{\text{vap}}} = \frac{[2834.8] / (206100 - 1.0908)}{[2501.3] / (206136 - 1.0002)} = 1.134 \quad \leftarrow (a)$$



(b) With Eqs. 11.46 and 11.55

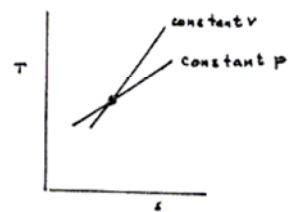
$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{c_v}{T}, \quad \left(\frac{\partial s}{\partial T}\right)_p = \frac{c_p}{T}$$

Or, using Eq. 11.15

$$\left(\frac{\partial T}{\partial s}\right)_v = \frac{T}{c_v}, \quad \left(\frac{\partial T}{\partial s}\right)_p = \frac{T}{c_p}$$

Forming a ratio

$$\frac{\left(\partial T / \partial s\right)_v}{\left(\partial T / \partial s\right)_p} = \frac{T / c_v}{T / c_p} = \frac{c_p}{c_v} = k > 1$$



$\leftarrow (b)$

(c) and (d) Eq. 11.19 reads

$$dh = Tds + vdp$$

The function $h(s, p)$ gives

$$dh = \left(\frac{\partial h}{\partial s}\right)_p ds + \left(\frac{\partial h}{\partial p}\right)_s dp$$

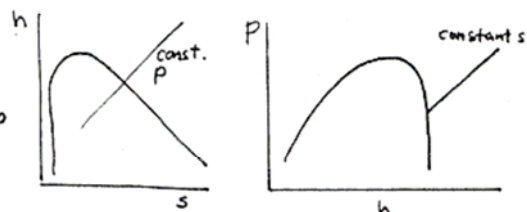
By comparison

$$(11.26): \quad \left(\frac{\partial h}{\partial s}\right)_p = T$$

$$(11.27): \quad \left(\frac{\partial h}{\partial p}\right)_s = v$$

With Eq. 11.15, the second of these gives

$$\left(\frac{\partial p}{\partial h}\right)_s = \frac{1}{v}$$



$\leftarrow (c)$

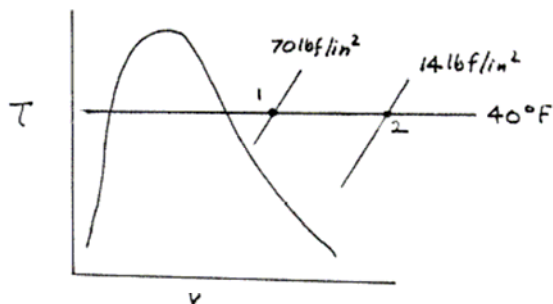
$\leftarrow (d)$

PROBLEM 11.45

KNOWN: NH_3 undergoes a process from 70 lbf/in^2 , 40°F to 14 lbf/in^2 , 40°F .

FIND: Using p-v-T data, evaluate $(h_2 - h_1)$ and $(s_2 - s_1)$, and compare with table values.

SCHEMATIC & GIVEN DATA:



ANALYSIS: Since the states under consideration are in the superheated vapor region, temperature and pressure can be selected as the independent properties that fix the state. Then, Eqs. 11.59 and 11.60 give, respectively

$$\textcircled{1} \quad s(T, P_2) - s(T, P_1) = \int_{P_1}^{P_2} \left(\frac{\partial v}{\partial T} \right)_P (T, P) dP \quad (1)$$

$$h(T, P_2) - h(T, P_1) = \int_{P_1}^{P_2} [v(T, P) - T \left(\frac{\partial v}{\partial T} \right)_P (T, P)] dP \quad (2)$$

Consequently, the analysis reduces to the evaluation of $(\partial v / \partial T)_P$ as a function of pressure at 40°F , and the subsequent use of these data to obtain the required integrals. A numerical procedure using data obtained from the source listed for Table A-13 is suggested. The values of Δh and Δs obtained directly from table data are, respectively

$$h_2 - h_1 = 14.46 \text{ Btu/lb}$$

$$s_2 - s_1 = 0.2103 \text{ Btu/lb} \cdot ^\circ\text{R}$$

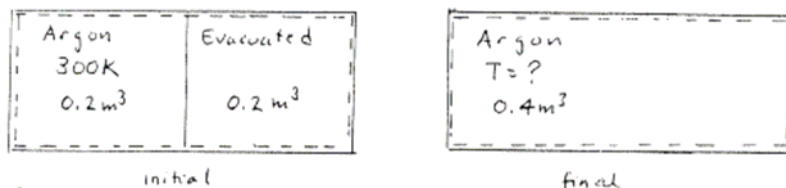
1. Since the integrations are at fixed temperature, the terms of Eqs. 11.59 and 11.60 involving the specific heat c_p drop out of consideration, thereby allowing the required property changes to be found using only p-v-T data. These integrations require appropriate software.

PROBLEM 11.46

KNOWN: One kmol of Ar initially occupies one side of a container divided by a partition. The other side is evacuated. The partition is removed and the Ar expands to fill the entire volume.

FIND: Find the final temperature of the argon using (1) the van der Waals equation of state, (2) the ideal gas model.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system is shown in the figure. (2) For the system, $Q=W=0$, kinetic and potential energy effects are absent, and the partition can be ignored. (3) The argon is modeled by the van der Waals equation of state and then by the ideal gas model.

ANALYSIS: Using indicated assumptions, the energy balance reduces to give $\Delta U=0$, or since mass is fixed $\Delta u=0$. Invoking Eq. 11.51

$$\Delta u = \int_1^2 c_v dT + \int_1^2 \left[T \left(\frac{\partial p}{\partial T} \right)_v - p \right] dv = 0 \quad (1)$$

- **van der Waals Equation.** Since $p = \frac{RT}{v-b} - \frac{a}{v^2}$

$$\left(\frac{\partial p}{\partial T} \right)_v = \frac{R}{v-b} \quad \Rightarrow \quad \left[T \left(\frac{\partial p}{\partial T} \right)_v - p \right] = \frac{RT}{v-b} - \left[\frac{RT}{v-b} - \frac{a}{v^2} \right] = \frac{a}{v^2}$$

To investigate c_v , the test for exactness applied to Eq. 11.48 gives

$$\frac{\partial (c_v/T)}{\partial v} = \frac{\partial}{\partial T} \left(\frac{\partial p}{\partial T} \right)_v \Rightarrow \frac{\partial (c_v/T)}{\partial v} = \frac{\partial}{\partial T} \left(\frac{R}{v-b} \right) = 0 \Rightarrow \frac{\partial c_v}{\partial v} = 0. \text{ This implies}$$

c_v is independent of v and can depend only on T . Accordingly, it can be concluded that $c_v = 3/2 R$, which is the limiting value for a monatomic gas such as Ar. Inserting results into Eq. (1), and integrating

$$\frac{3}{2} R [T_2 - T_1] - a \left[\frac{1}{v_2} - \frac{1}{v_1} \right] = 0$$

$$\Rightarrow T_2 - T_1 = \frac{2a}{3R} \left[\frac{1}{v_2} - \frac{1}{v_1} \right]$$

$$= \frac{2 \left(\frac{27}{64} \frac{R^2 T_c^2}{P_c} \right)}{3R} \left[\frac{1}{v_2} - \frac{1}{v_1} \right] = \frac{9}{32} \left[\frac{R T_c^2}{P_c} \right] \left[\frac{1}{v_2} - \frac{1}{v_1} \right]$$

With $\bar{v}_1 = 0.2 \text{ m}^3/\text{kmol}$, $\bar{v}_2 = 0.4 \text{ m}^3/\text{kmol}$, and data from Table A-1

$$T_2 - T_1 = \frac{9}{32} \left[\frac{8314 \frac{\text{J}}{\text{kmol} \cdot \text{K}}}{\text{kmol} \cdot \text{K}} \right] \left[15/\text{K} \right]^2 \left[\frac{1}{48.6 \times 10^{-3} \text{ m}^3/\text{mol}} \right] \left[\frac{1}{0.4} - \frac{1}{0.2} \right] \frac{\text{kmol}}{\text{m}^3}$$

$$= -27.43 \text{ K} \Rightarrow T_2 = 272.6 \text{ K}$$

• **ideal gas model.** Since $p = RT/v$, $(\partial p / \partial T)_v = R/v$ and $[T(\partial p / \partial T)_v - p] = 0$. Thus, with $c_v = 5/2 R$, Eq. (1) becomes

$$\frac{5}{2} R [T_2 - T_1] = 0 \Rightarrow T_2 = T_1 = 300 \text{ K}$$

PROBLEM 11.47

KNOWN: A gas obeys the equation of state $p(v-b) = RT$

FIND: Obtain the relationship between c_p and c_v for this gas.

ANALYSIS:

Method 1. Using Eq. 11.47

$$\left(\frac{\partial u}{\partial v}\right)_T = T \left(\frac{\partial p}{\partial T}\right)_v - p \quad (1)$$

The given equation of state can be rearranged to read

$$p = \frac{RT}{v-b}$$

Then

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b} \quad (2)$$

Combining Eqs. (1) and (2)

$$\left(\frac{\partial u}{\partial v}\right)_T = \frac{RT}{v-b} - p \equiv 0 \Rightarrow u = u(T) \quad (u \text{ depends on } T \text{ only})$$

Since $h = u + pv$, the above equation indicates that

$$h(T, p) = u(T) + pv$$

Then, since $c_p = (\partial h / \partial T)_p$ and $c_v = (\partial u / \partial T)_v = du/dT$

$$c_p = \frac{du}{dT} + \frac{\partial(pv)}{\partial T}_p = c_v + p \left(\frac{\partial v}{\partial T}\right)_p \quad (3)$$

The equation of state can be expressed as

$$v = \frac{RT}{p} + b \Rightarrow \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p}$$

Inserting this into Eq. (3)

$$\textcircled{1} \quad c_p = c_v + p \left(\frac{R}{p}\right) = c_v + R$$

Method 2. Using Eq. 11.68

$$c_p - c_v = -T \left(\frac{\partial v}{\partial T}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T$$

From above, $(\partial v / \partial T)_p = R/p$, and using $p = RT/(v-b)$

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2}$$

Collecting results

$$\begin{aligned} c_p - c_v &= -T \left(\frac{R}{p}\right)^2 \left(-\frac{RT}{(v-b)^2}\right) \\ &= -T \left(\frac{R}{RT/(v-b)}\right)^2 \left(-\frac{RT}{(v-b)^2}\right) = R \end{aligned}$$

or

$$c_p = c_v + R$$

1. Method 1 also brings out the conclusion that the specific internal energy, u , can only vary with temperature. Thus, c_v (and c_p) can only vary with temperature as well.

PROBLEM 11.48

KNOWN: The p-v-T relation for a certain gas is

$$v = \frac{RT}{P} + B - \frac{A}{RT}$$

where A and B are constants.

FIND: Obtain expressions for $[h(P_2, T) - h(P_1, T)]$, $[u(P_2, T) - u(P_1, T)]$, and $[s(P_2, T) - s(P_1, T)]$.

ANALYSIS: As T and p are the independent properties that fix the state and an equation of state explicit in v is known, Eqs. 11.59 and 11.60 are convenient to use here. Each of these requires $(\partial v / \partial T)_p$. Thus, with the given equation of state

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{P} + \frac{A}{RT^2}$$

Then, Eq. 11.59 reads

$$\begin{aligned} \textcircled{1} \quad s(P_2, T) - s(P_1, T) &= - \int_{P_1}^{P_2} \left(\frac{\partial v}{\partial T}\right)_p dp = - \int_{P_1}^{P_2} \left[\frac{R}{P} + \frac{A}{RT^2} \right] dp \\ &= -R \ln \frac{P_2}{P_1} - \frac{A}{RT^2} [P_2 - P_1] \quad \leftarrow \Delta s \end{aligned}$$

Similarly, Eq. 11.60 gives

$$\begin{aligned} h(P_2, T) - h(P_1, T) &= \int_{P_1}^{P_2} \left[v - T \left(\frac{\partial v}{\partial T}\right)_p \right] dp \\ &= \int_{P_1}^{P_2} \left[\left(\frac{RT}{P} + B - \frac{A}{RT} \right) - T \left(\frac{R}{P} + \frac{A}{RT^2} \right) \right] dp \\ &= \int_{P_1}^{P_2} \left(B - \frac{2A}{RT} \right) dp = \left[B - \frac{2A}{RT} \right] (P_2 - P_1) \quad \leftarrow \Delta h \end{aligned}$$

Then, with $h = u + Pv$

$$\begin{aligned} h(P_2, T) - h(P_1, T) &= [u(P_2, T) - u(P_1, T)] + [P_2 v(P_2, T) - P_1 v(P_1, T)] \\ \left[B - \frac{2A}{RT} \right] (P_2 - P_1) &= [u(P_2, T) - u(P_1, T)] + P_2 \left[\frac{RT}{P_2} + B - \frac{A}{RT} \right] - P_1 \left[\frac{RT}{P_1} + B - \frac{A}{RT} \right] \\ \left(B - \frac{2A}{RT} \right) (P_2 - P_1) &= [u(P_2, T) - u(P_1, T)] + (P_2 - P_1) \left[B - \frac{A}{RT} \right] \\ \therefore u(P_2, T) - u(P_1, T) &= (P_2 - P_1) \left[\left(B - \frac{2A}{RT} \right) - \left(B - \frac{A}{RT} \right) \right] \\ &= - \frac{A}{RT} (P_2 - P_1) \quad \leftarrow \Delta u \end{aligned}$$

1. Since the integrations are at fixed temperature, the terms of Eqs. 11.59 and 11.60 involving c_p drop out of consideration, thereby allowing the required property changes to be evaluated in terms of T_1 , T_2 , P_1 , and P_2 .

PROBLEM 11.49

KNOWN: The equations of state are the van der Waals equation and the Redlich-Kwong equation.

FIND: For each equation of state, determine $[h(v_2, T) - h(v_1, T)]$, $[u(v_2, T) - u(v_1, T)]$, $[s(v_2, T) - s(v_1, T)]$.

ANALYSIS: Since the equations of state are explicit in pressure, and temperature is fixed, it is convenient to use Eqs. 11.50 and 11.51, which reduce to read

$$u(v_2, T) - u(v_1, T) = \int_{v_1}^{v_2} \left[T \left(\frac{\partial p}{\partial T} \right)_v - p \right] dv \quad (1)$$

$$s(v_2, T) - s(v_1, T) = \int_{v_1}^{v_2} \left(\frac{\partial p}{\partial T} \right)_v dv \quad (2)$$

Also, Δh can be found using Eq. 11.61:

$$h_2 - h_1 = (u_2 - u_1) + (p_2 v_2 - p_1 v_1) \quad (3)$$

(a) van der Waals. This equation is explicit in pressure. So $(\partial p / \partial T)_v = R / (v - b)$. Eq. (1) then becomes

$$u(v_2, T) - u(v_1, T) = \int_{v_1}^{v_2} \left(\frac{RT}{v-b} - p \right) dv = \int_{v_1}^{v_2} \left(\frac{RT}{v-b} - \left(\frac{RT}{v-b} - \frac{a}{v^2} \right) \right) dv = \int_{v_1}^{v_2} \frac{a}{v^2} dv$$

$$= -a \left(\frac{1}{v_2} - \frac{1}{v_1} \right) \quad \leftarrow \Delta u$$

Thus

$$h(v_2, T) - h(v_1, T) = p(v_2, T) v_2 - p(v_1, T) v_1 - a \left(\frac{1}{v_2} - \frac{1}{v_1} \right) \quad \leftarrow \Delta h$$

Also, Eq. (2) becomes

$$s(v_2, T) - s(v_1, T) = \int_{v_1}^{v_2} \left(\frac{R}{v-b} \right) dv = R \ln \left(\frac{v_2 - b}{v_1 - b} \right) \quad \leftarrow \Delta s$$

(b) Redlich-Kwong. This equation is explicit in pressure. So, $(\partial p / \partial T)_v = \frac{R}{v-b} + \frac{a}{2v(v+b)T^{3/2}}$. Eq. (1) then becomes

$$u(v_2, T) - u(v_1, T) = \int_{v_1}^{v_2} \left(\left(\frac{RT}{v-b} + \frac{a}{2v(v+b)T^{3/2}} \right) - \left(\frac{RT}{v-b} - \frac{a}{v(v+b)T^{1/2}} \right) \right) dv$$

$$= \frac{3}{2} \int_{v_1}^{v_2} \frac{a}{v(v+b)T^{1/2}} dv = -\frac{3a}{2bT^{1/2}} \ln \left[\frac{v_2+b}{v_1+b} \cdot \frac{v_1}{v_2} \right] \quad \leftarrow \Delta u$$

Thus

$$h(v_2, T) - h(v_1, T) = p(v_2, T) v_2 - p(v_1, T) v_1 - \frac{3a}{2bT^{1/2}} \ln \left[\frac{v_2+b}{v_1+b} \cdot \frac{v_1}{v_2} \right] \quad \leftarrow \Delta h$$

Also, Eq. (2) becomes

$$s(v_2, T) - s(v_1, T) = \int_{v_1}^{v_2} \left(\frac{R}{v-b} + \frac{a}{2v(v+b)T^{3/2}} \right) dv$$

$$= R \ln \left(\frac{v_2 - b}{v_1 - b} \right) - \frac{a}{2bT^{3/2}} \ln \left[\frac{v_2+b}{v_1+b} \cdot \frac{v_1}{v_2} \right] \quad \leftarrow \Delta s$$

PROBLEM 11.50

KNOWN: At certain states the p-v-T data of a gas can be described as

$$Z = 1 - \frac{AP}{T^4}$$

where A is a constant.

FIND: Obtain an expression for (a) $(\partial p / \partial T)_v$ in terms of p, T, A and R, (b) the quantity $[s(p_2, T) - s(p_1, T)]$, and (c) $[h(p_2, T) - h(p_1, T)]$.

ANALYSIS: Since $Z = pV/RT$, the equation can be expressed as

$$\frac{pV}{RT} = 1 - \frac{AP}{T^4} \Rightarrow v = \frac{RT}{p} - \frac{AR}{T^3} \quad (1)$$

(a) To obtain $(\partial p / \partial T)_v$, Eq. (1) can be solved for p and then differentiated:

$$\begin{aligned} p &= \frac{RT}{v + AR/T^3} \\ \left(\frac{\partial p}{\partial T}\right)_v &= \frac{R[v + \frac{AR}{T^3}] - RT[-\frac{3AR}{T^4}]}{(v + \frac{AR}{T^3})^2} = \frac{RV + \frac{4AR^2}{T^3}}{(v + \frac{AR}{T^3})^2} = \frac{R[\frac{RT}{p} - \frac{AR}{T^3}] + \frac{4AR^2}{T^3}}{(RT/p)^2} \\ &= \frac{p^2}{R^2} \left[\frac{RT}{p} + \frac{3AR^2}{T^3} \right] = \frac{p^2}{RT} \left[\frac{R}{p} + \frac{3AR}{T^4} \right] \quad (a) \end{aligned}$$

Alternatively, Eq. 11.16 can be used:

$$\begin{aligned} \left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v &= -1 \\ \left(\frac{\partial p}{\partial v}\right)_T &= \frac{-RT}{[v + \frac{AR}{T^3}]^2} = -\frac{p^2}{RT} \\ \text{Then using Eq. 11.15} \quad \left(\frac{\partial p}{\partial T}\right)_v &= -\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \\ \left(\frac{\partial v}{\partial T}\right)_p &= \frac{R}{p} + \frac{3AR}{T^4} \\ &= -\left[-\frac{p^2}{RT}\right] \left[\frac{R}{p} + \frac{3AR}{T^4}\right] = \frac{p^2}{RT} \left[\frac{R}{p} + \frac{3AR}{T^4}\right] \quad (a) \end{aligned}$$

(b) Using Eq. 11.59

$$\begin{aligned} s(p_2, T) - s(p_1, T) &= \int_{p_1}^{p_2} -\left(\frac{\partial v}{\partial T}\right)_p dp = -\int_{p_1}^{p_2} \left[\frac{R}{p} + \frac{3AR}{T^4}\right] dp \\ &= -R \ln \frac{p_2}{p_1} + \frac{3AR}{T^4} [p_2 - p_1] \quad (b) \end{aligned}$$

(c) Using Eq. 11.60

$$\begin{aligned} h(p_2, T) - h(p_1, T) &= \int_{p_1}^{p_2} [v - T\left(\frac{\partial v}{\partial T}\right)_p] dp \\ \text{From (a)} \quad \left(\frac{\partial v}{\partial T}\right)_p &= \frac{R}{p} + \frac{3AR}{T^4} \Rightarrow T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p} + \frac{3AR}{T^3} \Rightarrow [v - T\left(\frac{\partial v}{\partial T}\right)_p] = \left[\frac{RT}{p} - \frac{AR}{T^3}\right] - \left[\frac{RT}{p} + \frac{3AR}{T^3}\right] = -\frac{4AR}{T^3} \\ \text{So} \quad h(p_2, T) - h(p_1, T) &= \int_{p_1}^{p_2} \left(-\frac{4AR}{T^3}\right) dp = -\frac{4AR}{T^3} (p_2 - p_1) \quad (c) \end{aligned}$$

PROBLEM 11.51

KNOWN: The p-v-T behavior of a gas is described by

$$Z = 1 + \frac{B(T)P}{RT}$$

FIND: Obtain expressions for (a) $[h(P_2, T) - h(P_1, T)]$, (b) $[u(P_2, T) - u(P_1, T)]$
(c) $[s(P_2, T) - s(P_1, T)]$.

ANALYSIS: Since $Z = pV/RT$, the equation becomes

$$\frac{pV}{RT} = 1 + \frac{B(T)P}{RT}$$

$$V = \frac{RT}{P} + B(T) \quad (1)$$

As T and p are the independent properties that fix the state and an equation of state explicit in v is known, Eqs. 11.59 and 11.60 are convenient to use here. Each of these requires $(\partial v / \partial T)_p$. Thus, from Eq. (1)

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{P} + \frac{dB}{dT}$$

Eq. 11.59 then gives

$$\begin{aligned} s(P_2, T) - s(P_1, T) &= - \int_{P_1}^{P_2} \left(\frac{\partial v}{\partial T}\right)_p dp = - \int_{P_1}^{P_2} \left[\frac{R}{P} + \frac{dB}{dT} \right] dp \\ &= -R \ln \frac{P_2}{P_1} - \frac{dB}{dT} [P_2 - P_1] \longleftarrow \Delta s \end{aligned}$$

Similarly, using Eq. 11.60

$$\begin{aligned} h(P_2, T) - h(P_1, T) &= \int_{P_1}^{P_2} \left[v - T \left(\frac{\partial v}{\partial T}\right)_p \right] dp \\ &= \int_{P_1}^{P_2} \left[\left(\frac{RT}{P} + B(T) \right) - T \left(\frac{R}{P} + \frac{dB}{dT} \right) \right] dp \\ &= \int_{P_1}^{P_2} \left[B - T \frac{dB}{dT} \right] dp \\ &= \left[B - T \frac{dB}{dT} \right] [P_2 - P_1] \longleftarrow \Delta h \end{aligned}$$

Then

$$\begin{aligned} \Delta h &= \Delta u + \Delta(Pv) = \Delta u + [P_2 v(P_2, T) - P_1 v(P_1, T)] \\ &= \Delta u + P_2 \left[\frac{RT}{P_2} + B \right] - P_1 \left[\frac{RT}{P_1} + B \right] \\ &= \Delta u + B(P_2 - P_1) \\ \Rightarrow \Delta u &= \underbrace{\left[B - T \frac{dB}{dT} \right] (P_2 - P_1)}_{\Delta h} - B(P_2 - P_1) \Rightarrow \Delta u = -T \frac{dB}{dT} (P_2 - P_1) \longleftarrow \Delta u \end{aligned}$$

PROBLEM 11.52

KNOWN: The p - v - T behavior of a gas is described by

$$Z = 1 + \frac{B(T)}{v} + \frac{C(T)}{v^2}$$

FIND: Obtain an expression for $[s(v_2, T) - s(v_1, T)]$.

ANALYSIS: Since $Z = pv/RT$, the equation becomes

$$p = \frac{RT}{v} + \frac{RT B(T)}{v^2} + \frac{RT C(T)}{v^3} \quad (1)$$

As T and v are the independent properties that fix the state and an equation of state explicit in p is known, Eq. 11.50 is convenient to use here. This expression requires $(\partial p / \partial T)_v$. Thus, from Eq. (1)

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v} + \frac{R B(T) + RT B'(T)}{v^2} + \frac{R C(T) + RT C'(T)}{v^3}$$

Then, with Eq. 11.50

$$\begin{aligned} s(v_2, T) - s(v_1, T) &= \int_{v_1}^{v_2} \left(\frac{\partial p}{\partial T}\right)_v dv \\ &= \int_{v_1}^{v_2} \left[\frac{R}{v} + \frac{[RB + RTB']}{v^2} + \frac{[RC + RTC']}{v^3} \right] dv \\ &= R \ln \frac{v_2}{v_1} - [RB + RTB'] \left[\frac{1}{v_2} - \frac{1}{v_1} \right] - \frac{[RC + RTC']}{2} \left[\frac{1}{v_2^2} - \frac{1}{v_1^2} \right] \\ &= R \ln \frac{v_2}{v_1} - R[B + TB'] \left[\frac{1}{v_2} - \frac{1}{v_1} \right] - \frac{R}{2} [C + TC'] \left[\frac{1}{v_2^2} - \frac{1}{v_1^2} \right] \leftarrow \Delta s \end{aligned}$$

or

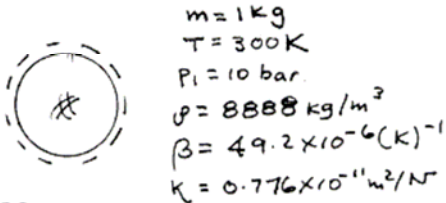
$$s(v_2, T) - s(v_1, T) = R \ln \frac{v_2}{v_1} + R \left[\frac{1}{v_1} - \frac{1}{v_2} \right] \left\{ \frac{d(BT)}{dT} + \frac{1}{2} \frac{d(CT)}{dT} \left[\frac{1}{v_1} + \frac{1}{v_2} \right] \right\}$$

PROBLEM 11.53

KNOWN: A 1-kg copper sphere is at 300K and an initial pressure of 10 bar.

FIND: If the volume of the sphere is not allowed to vary by more than 0.1%, determine the maximum allowed pressure the sphere can attain, in bar.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The copper sphere is the system. (2) Temperature remains constant at 300K. (3) ρ , β , K do not vary as pressure changes.

ANALYSIS: As the volume would decrease as pressure increases, the restriction on volume change reads

$$\frac{(\Delta V)_{\text{MAX}}}{V} = -0.001 \quad (1)$$

With Eq. 11.63

$$\textcircled{1} \quad \left(\frac{\partial V}{\partial P} \right)_T = -K V = -K/\rho$$

and at fixed T

$$\Delta V = - \int_{P_1}^{P_2} \frac{K}{\rho} dP = -\frac{K}{\rho} (P_2 - P_1)$$

and

$$\Delta V = \left[-\frac{K}{\rho} (P_2 - P_1) \right] m = -K \Delta P V$$

$$\Rightarrow \frac{\Delta V}{V} = -K \Delta P \quad (2)$$

Combining (1), (2)

$$-K(\Delta P)_{\text{MAX}} = -0.001$$

or

$$(\Delta P)_{\text{MAX}} = \left(\frac{0.001}{0.776 \times 10^{11} \text{ m}^2/\text{N}} \right) \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| = 1289 \text{ bar}$$

$$\textcircled{2} \quad \Rightarrow (P_2)_{\text{MAX}} = 1299 \text{ bar} \quad \leftarrow$$

1. Alternatively, this equation can be expressed as

$$\left(\frac{\partial \ln V}{\partial P} \right)_T = -K$$

Since K is constant with P (assumption 3), integration gives

$$\frac{V_2}{V_1} = \exp(-K(P_2 - P_1))$$

so

$$\Delta V = V_2 - V_1 = V_1 [\exp(-K(P_2 - P_1)) - 1]$$

2. β is not required for the solution.

PROBLEM 11.54

KNOWN: A 1-lb copper sphere is at 80°F and an initial pressure of 1 atm.

FIND: If the volume of the sphere is not allowed to vary by more than 0.1%, determine the maximum allowed pressure the sphere can attain, in atm.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} m &= 1 \text{ lb} \\ T &= 80^\circ\text{F} \\ P_1 &= 1 \text{ atm} \\ \rho &= 555 \text{ lb/ft}^3 \\ \beta &= 2.75 \times 10^{-5} (\text{°F})^{-1} \\ K &= 3.72 \times 10^{-10} \text{ ft}^2/\text{lb} \end{aligned}$$

ENGINEERING MODEL: (1) The copper sphere is the system. (2) Temperature remains constant at 80°F. (3) ρ , β , K do not vary as pressure changes.

ANALYSIS: As the volume would decrease as pressure increases, the restriction on volume change reads

$$\frac{(\Delta V)_{\text{MAX}}}{V} = -0.001 \quad (1)$$

With Eq. 11.62

$$\textcircled{1} \quad \left(\frac{\partial V}{\partial P} \right)_T = -K V = -K/\rho$$

and at fixed T

$$\Delta V = - \int_{P_1}^{P_2} \frac{K}{\rho} dP = - \frac{K}{\rho} \Delta P$$

and

$$\Delta V = \left[- \frac{K}{\rho} \Delta P \right] m = -K \Delta P V$$

$$\Rightarrow \frac{\Delta V}{V} = -K \Delta P \quad (2)$$

Combining (1), (2)

$$-K (\Delta P)_{\text{MAX}} = -0.001$$

$$\begin{aligned} (\Delta P)_{\text{MAX}} &= \left[\frac{0.001}{3.72 \times 10^{-10} \text{ ft}^2/\text{lb}} \right] \left| \frac{1 \text{ atm}}{2116 \text{ lb/ft}^2} \right| \\ &= 1270 \text{ atm} \end{aligned}$$

$$\textcircled{2} \quad \Rightarrow (P_2)_{\text{MAX}} = 1271 \text{ atm} \quad \leftarrow$$

1. Alternatively, this equation can be expressed as

$$\left(\frac{\partial \ln V}{\partial P} \right)_T = -K$$

Since K is constant with pressure (assumption 3), integration gives

$$\frac{V_2}{V_1} = \exp(-K(P_2 - P_1)) \Rightarrow \Delta V = V_2 - V_1 = V_1 [\exp(-K(P_2 - P_1)) - 1]$$

2. β is not required for the solution

PROBLEM 11.55

KNOWN: Three cases are under consideration: (a) an ideal gas, (b) a gas whose equation of state is $p(V-b) = RT$, (c) a gas obeying the van der Waals equation.

FIND: Derive expressions for β and κ for each case.

ANALYSIS: From Eqs. 11.62 and 11.63

$$\beta = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p \quad \text{and} \quad \kappa = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T$$

(a) Ideal Gas: $V = RT/p$

$$\left(\frac{\partial V}{\partial T} \right)_p = \frac{R}{p} \quad \text{and} \quad \left(\frac{\partial V}{\partial p} \right)_T = -\frac{RT}{p^2}$$

Thus

$$\beta = \frac{1}{V} \left[\frac{R}{p} \right] = \frac{1}{T} \quad \leftarrow (a)$$

$$\kappa = -\frac{1}{V} \left[-\frac{RT}{p^2} \right] = \frac{1}{p} \left[\frac{RT}{pV} \right] = \frac{1}{p}$$

(b) $V = (RT/p) + b$

$$\left(\frac{\partial V}{\partial T} \right)_p = \frac{R}{p} \quad , \quad \left(\frac{\partial V}{\partial p} \right)_T = -\frac{RT}{p^2}$$

Thus

$$\beta = \frac{1}{V} \left[\frac{R}{p} \right] = \frac{R}{V} \left[\frac{V-b}{RT} \right] = \frac{1}{T} \left[\frac{V-b}{V} \right] \quad \leftarrow (b)$$

$$\kappa = -\frac{1}{V} \left[-\frac{RT}{p^2} \right] = \frac{1}{p} \left[\frac{RT}{pV} \right] = \frac{1}{p} \left[\frac{RT/V}{RT/(V-b)} \right] = \frac{1}{p} \left[\frac{V-b}{V} \right]$$

Note: When $b=0$ these expressions reduce to those of part (a).

(c) van der Waals: $p = \frac{RT}{(V-b)} - \frac{a}{V^2}$.

As the van der Waals equation is not explicit in V , the required partial derivatives are not so easily found as in parts (a) and (b). Thus, following the procedure explained in Example 11.2, an expression for $(\partial V/\partial T)_p$ is obtained as

$$\left(\frac{\partial V}{\partial T} \right)_p = \frac{-R/(V-b)}{[2a/V^3 - RT/(V-b)^2]}$$

So

$$\beta = \frac{-R(V-b)V^2}{2a(V-b)^2 - RTV^3}$$

Using Eqs. 11.15 and 11.16

$$\left(\frac{\partial V}{\partial p} \right)_T = \frac{-\left(\partial V/\partial T \right)_p}{\left(\partial p/\partial T \right)_V} = \frac{\frac{R/(V-b)}{[2a/V^3 - RT/(V-b)^2]}}{R/(V-b)} = \frac{1}{[2a/V^3 - RT/(V-b)^2]} \quad \leftarrow (c)$$

Thus

$$\kappa = -\frac{1}{V[2a/V^3 - RT/(V-b)^2]} = \frac{-V^2(V-b)^2}{[2a(V-b)^2 - RTV^3]}$$

PROBLEM 11.56

FIND: Derive expressions for β and K in terms of $T, p, Z, (\partial Z/\partial T)_p, (\partial Z/\partial p)_T$.
For gas states with $P_R < 3, T_R < 2$, determine the sign of K .

ANALYSIS: From Eqs. 11.62 and 11.63

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p, \quad K = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T$$

The compressibility factor is

$$Z(T, p) = \frac{pv}{RT}$$

$$\Rightarrow v = \frac{RT}{p} \cdot Z(T, p)$$

Then

$$\left(\frac{\partial v}{\partial T} \right)_p = \frac{R}{p} Z + \frac{RT}{p} \left(\frac{\partial Z}{\partial T} \right)_p$$

and

$$\left(\frac{\partial v}{\partial p} \right)_T = -\frac{RT}{p^2} Z + \frac{RT}{p} \left(\frac{\partial Z}{\partial p} \right)_T$$

Accordingly

$$\begin{aligned} \beta &= \frac{1}{v} \left[\frac{R}{p} Z + \frac{RT}{p} \left(\frac{\partial Z}{\partial T} \right)_p \right] \\ &= \frac{R}{pv} \left(\frac{pv}{RT} \right) + \frac{1}{Z} \left(\frac{\partial Z}{\partial T} \right)_p \\ &= \frac{1}{T} + \frac{1}{Z} \left(\frac{\partial Z}{\partial T} \right)_p \end{aligned} \quad \longleftarrow \beta$$

And

$$\begin{aligned} K &= -\frac{1}{v} \left[-\frac{RT}{p^2} Z + \frac{RT}{p} \left(\frac{\partial Z}{\partial p} \right)_T \right] \\ &= \left[\frac{1}{p} \cdot \frac{RT}{pv} \cdot Z - \frac{RT}{pv} \left(\frac{\partial Z}{\partial p} \right)_T \right] \\ &= \frac{1}{p} - \frac{1}{Z} \left(\frac{\partial Z}{\partial p} \right)_T \end{aligned} \quad \longleftarrow K$$

Referring to Figure A-2, for $P_R < 3, T_R < 2$ all isotherms have a negative slope: $(\partial Z/\partial p)_T < 0$. Thus, it is evident that K must be positive at all such states.

PROBLEM 11.51

FIND: Show that $k > \alpha$.

ANALYSIS: From the development of Sec. 11.5.2, the following expressions are obtained

$$c_p - c_v = v \frac{T \beta^2}{\kappa} \quad (11.69)$$

$$k = \frac{\kappa}{\alpha} \quad (11.73)$$

where $k = c_p/c_v$. Following the discussion of Eq. 11.69 $c_p > c_v$ except when $\beta = 0$ and as $T \rightarrow 0$. Accordingly, except for such special circumstances, $k > 1$ and so $k/\alpha > 1$, as was to be established.

PROBLEM 11.58

FIND: Prove that $(\partial \beta / \partial p)_T = -(\partial K / \partial T)_p$.

ANALYSIS: Consider $v = v(T, p)$ and form the differential

$$dv = \left(\frac{\partial v}{\partial T} \right)_p dT + \left(\frac{\partial v}{\partial p} \right)_T dp$$

With Eqs. 11.62 and 11.63 this becomes

$$dv = \underbrace{(v\beta)}_M dT + \underbrace{(-vK)}_N dp \quad (1)$$

Since this is an exact differential, the test for exactness: Eq. 11.14 must be satisfied. That is

$$\left(\frac{\partial M}{\partial p} \right)_T = \left(\frac{\partial N}{\partial T} \right)_p$$

Thus

$$\frac{\partial (v\beta)}{\partial p} \Big|_T = \frac{\partial (-vK)}{\partial T} \Big|_p$$

$$\beta \left(\frac{\partial v}{\partial p} \right)_T + v \left(\frac{\partial \beta}{\partial p} \right)_T = -K \left(\frac{\partial v}{\partial T} \right)_p - v \left(\frac{\partial K}{\partial T} \right)_p$$

$$\underbrace{\frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial v}{\partial p} \right)_T}_{\text{cancel}} + v \left(\frac{\partial \beta}{\partial p} \right)_T = \underbrace{- \frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_p \left(\frac{\partial v}{\partial T} \right)_p}_{\text{cancel}} - v \left(\frac{\partial K}{\partial T} \right)_p$$

The underlined terms cancel, leaving

$$\textcircled{1} \quad \left(\frac{\partial \beta}{\partial p} \right)_T = - \left(\frac{\partial K}{\partial T} \right)_p$$

1. The desired result is obtained more directly by writing Eq. (1) as

$$d \ln v = \underbrace{\beta}_{M} dT + \underbrace{(-K)}_N dp$$

and then applying the test for exactness.

PROBLEM 11.59

KNOWN: Data are provided for aluminum at 0°C.

FIND: Determine the percent error in c_v that would result if it were assumed that $c_p = c_v$.

SCHEMATIC & GIVEN DATA:

$$\begin{array}{lcl} \text{Al} & T = 0^\circ\text{C} & \beta = 7.14 \times 10^{-8} \text{ K}^{-1} \\ & \rho = 2700 \text{ kg/m}^3 & \kappa = 1.34 \times 10^{-13} \text{ m}^2/\text{N} \\ & c_p = 0.9211 \text{ kJ/kg}\cdot\text{K} & \end{array}$$

ANALYSIS: Eq 11.69 can be used to find $c_p - c_v$:

$$\begin{aligned} c_p - c_v &= v \frac{T\beta^2}{\kappa} = \frac{T\beta^2}{\rho\kappa} \\ &= \frac{(273 \text{ K})(7.14 \times 10^{-8} \text{ K}^{-1})^2}{(2700 \text{ kg/m}^3)(1.34 \times 10^{-13} \text{ m}^2/\text{N})} \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 3.846 \times 10^{-6} \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

Thus, $c_p \approx c_v$. That is, the difference is negligible. Specifically

$$\% \text{ error} = \left(\frac{3.846 \times 10^{-6}}{0.9211} \right) (100) = 4.2 \times 10^{-4} \% \quad \leftarrow \text{error}$$

Note that specific heat data reported for solids and liquids are normally c_p values, since these are more easily determined for solids and liquids than c_v .

PROBLEM 11.60

KNOWN: Data are provided for mercury at 0°C, 1 bar.

FIND: Estimate the temperature rise for an isentropic process to 1000 bar.

SCHEMATIC & GIVEN DATA:



$$\begin{array}{ll}
 T_1 = 0^\circ\text{C} & \bar{c}_p = 28.0 \text{ kJ/kmol}\cdot\text{K} \\
 P_1 = 1 \text{ bar} & \bar{v} = 0.0147 \text{ m}^3/\text{kmol} \\
 P_2 = 1000 \text{ bar} & \beta = 17.8 \times 10^{-5} \text{ K}^{-1} \\
 s_1 = s_2 &
 \end{array}$$

ENGINEERING MODEL: The temperature change is small, and $\bar{c}_p$, $\bar{v}$, and β can be taken as constant.

ANALYSIS: Begin with Eq. 11.33: $\left. \frac{\partial T}{\partial p} \right|_s = \frac{\partial \bar{v}}{\partial s} \bigg|_p$. The derivative on the right hand side of this expression can be expanded as follows:

$$\begin{aligned}
 \left. \frac{\partial T}{\partial p} \right|_s &= \left. \frac{\partial \bar{v}}{\partial T} \right|_p \left. \frac{\partial T}{\partial s} \right|_p \\
 \left. \frac{\partial \bar{v}}{\partial T} \right|_p &= \bar{v} \beta \quad \left(\text{from Eq. 11.62} \right) \\
 \left. \frac{\partial T}{\partial s} \right|_p &= \frac{T}{\bar{c}_p} \quad \left(\text{from Eq. 11.55} \right)
 \end{aligned}$$

Thus

$$\left. \frac{\partial T}{\partial p} \right|_s = \frac{\bar{v} \beta T}{\bar{c}_p}$$

Using the assumption, we can integrate

$$\int_{T_1}^{T_2} dT = \int_{P_1}^{P_2} \frac{\bar{v} \beta T}{\bar{c}_p} dp \Rightarrow \Delta T = \left(\frac{\bar{v} \beta T}{\bar{c}_p} \right) \Delta p$$

Inserting values

$$\begin{aligned}
 \Delta T &= \frac{(0.0147 \text{ m}^3/\text{kmol})(17.8 \times 10^{-5} \text{ K}^{-1})(273.15 \text{ K})(999 \text{ bar})}{(28.0 \text{ kJ/kmol}\cdot\text{K})} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| \\
 &= 2.55 \text{ K} \quad \leftarrow \Delta T
 \end{aligned}$$

PROBLEM 11.61

KNOWN: The p - v - T data for a certain gas can be represented as

$$Z = 1 - \frac{AP}{T^4}$$

where A is a constant.

FIND: Obtain an expression for C_p and verify that the expression reduces to Eq. 3.47a when $Z = 1$.

ANALYSIS: Using Eq. 11.68

$$C_p - C_v = -T \left(\frac{\partial v}{\partial T} \right)_p^2 \left(\frac{\partial p}{\partial v} \right)_T$$

With Eq. 11.15

$$\left(\frac{\partial p}{\partial v} \right)_T = \frac{1}{(\partial v / \partial p)_T}$$

Accordingly

$$C_p - C_v = -T \frac{(\partial v / \partial T)_p^2}{(\partial v / \partial p)_T} \quad (1)$$

This requires an equation of state explicit in v . With $Z = pv/RT$, the given equation becomes

$$v = \frac{RT}{p} - \frac{AR}{T^3}$$

Thus

$$\left(\frac{\partial v}{\partial T} \right)_p = \frac{R}{p} + \frac{3AR}{T^4} \quad (2)$$

$$\left(\frac{\partial v}{\partial p} \right)_T = -\frac{RT}{p^2} \quad (3)$$

Combining Eqs. (1)-(3)

$$\begin{aligned} C_p - C_v &= \frac{T \left[\frac{R}{p} + \frac{3AR}{T^4} \right]^2}{RT/p^2} \\ &= R \left[1 + \frac{3AP}{T^4} \right]^2 \end{aligned}$$

With $(AP/T^4) = 1 - Z$

$$C_p - C_v = R [1 + 3(1 - Z)]^2 = R(4 - 3Z)^2$$

With $k = C_p/C_v$, $C_v = C_p/k$ and this expression can be written as

$$C_p = \frac{kR}{k-1} [4 - 3Z]^2$$

If $Z = 1$, this becomes Eq. 3.47a:

$$C_p = \frac{kR}{k-1}$$

PROBLEM 11.62

KNOWN: A gas obeys the van der Waals equation of state.

FIND: (a) Show that $(\partial c_v / \partial v)_T = 0$. (b) Develop an expression for $(c_p - c_v)$.

(c) Develop expressions for $[u(T_2, v_2) - u(T_1, v_1)]$, $[s(T_2, v_2) - s(T_1, v_1)]$.

(d) Complete the results of (c) if $c_v = a + bT$ (Note: a, b are constants, but do not correspond to the constants of the Equation of State.)

ANALYSIS: (a) $c_v = (\partial u / \partial T)_v$. Then

$$\left(\frac{\partial c_v}{\partial v}\right)_T = \frac{\partial}{\partial v} \left[\left(\frac{\partial u}{\partial T}\right)_v \right] = \frac{\partial}{\partial T} \left[\left(\frac{\partial u}{\partial v}\right)_T \right]$$

Inserting Eq. 11.47

$$\left(\frac{\partial c_v}{\partial v}\right)_T = \frac{\partial}{\partial T} \left[T \left(\frac{\partial p}{\partial T}\right)_v - p \right] = \left(\frac{\partial p}{\partial T}\right)_v + T \left(\frac{\partial^2 p}{\partial T^2}\right)_v - \left(\frac{\partial p}{\partial T}\right)_v = T \left(\frac{\partial^2 p}{\partial T^2}\right)_v$$

Using the van der Waals equation

$$\frac{\partial p}{\partial T} = \frac{R}{v-b}, \quad \frac{\partial^2 p}{\partial T^2} = 0 \Rightarrow \left(\frac{\partial c_v}{\partial v}\right)_T = 0. \quad \leftarrow (a)$$

That is, c_v is independent of v and depends only on T : $c_v = c_v(T)$.

(b) Begin with Eq. 11.68, and note that $(\partial v / \partial T)_p$ is evaluated for the van der Waals equation in Ex 11.2. Further

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

Thus

$$\begin{aligned} c_p - c_v &= -T \left[\frac{R/(v-b)}{2a/v^3 - RT/(v-b)^2} \right]^2 \left[\frac{2a}{v^3} - \frac{RT}{(v-b)^2} \right] \\ &= \frac{-R^2 T}{(v-b)^2 \left[\frac{2a}{v^3} - \frac{RT}{(v-b)^2} \right]} \\ &= \frac{R}{1 - 2a(v-b)^2 / RTv^3} \quad \leftarrow (c_p - c_v) \end{aligned}$$

(c) Using the result of part (a), Eqs. 11.50 and 11.51 give

$$\begin{aligned} s(T_2, v_2) - s(T_1, v_1) &= \int_{T_1}^{T_2} \frac{c_v(T)}{T} dT + \int_{v_1}^{v_2} \left(\frac{R}{v-b} \right) dv = \int_{T_1}^{T_2} \frac{c_v(T)}{T} dT + R \ln \left(\frac{v_2 - b}{v_1 - b} \right) \quad \leftarrow \Delta s \\ u(T_2, v_2) - u(T_1, v_1) &= \int_{T_1}^{T_2} c_v(T) dT + \int_{v_1}^{v_2} \left[T \left(\frac{R}{v-b} \right) - \left(\frac{RT}{v-b} - \frac{a}{v^2} \right) \right] dv \\ &= \int_{T_1}^{T_2} c_v(T) dT - a \left[\frac{1}{v_2} - \frac{1}{v_1} \right] \quad \leftarrow \Delta u \end{aligned}$$

To evaluate these expressions requires only the function $c_v(T)$.

(d) If $c_v = a + bT$, then

$$\begin{aligned} \Delta s &= \int_{T_1}^{T_2} \left(\frac{a + bT}{T} \right) dT + R \ln \left(\frac{v_2 - b}{v_1 - b} \right) = a \ln \frac{T_2}{T_1} + b(T_2 - T_1) + R \ln \left(\frac{v_2 - b}{v_1 - b} \right) \\ \Delta u &= \int_{T_1}^{T_2} (a + bT) dT - a \left[\frac{1}{v_2} - \frac{1}{v_1} \right] = a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2) - a \left[\frac{1}{v_2} - \frac{1}{v_1} \right] \end{aligned}$$

PROBLEM 11.63

KNOWN: For air $c_v = 0.1965 \text{ Btu/lb}\cdot\text{OR}$ at $T_1 = 1000^\circ\text{F}$, $v_1 = 36.8 \text{ ft}^3/\text{lb}$.

FIND: Using the Berthelot equation of state, determine c_v at $T_2 = 1000^\circ\text{F}$, $v_2 = 0.0555 \text{ ft}^3/\text{lb}$.

ENGINEERING MODEL: The Berthelot Eqn. describes the behavior of air at the specified states:

$$p = \frac{RT}{v-b} - \frac{a}{Tv^2} \quad \text{where} \quad a = \frac{27}{64} \frac{R^2 T_c^3}{P_c}, \quad b = \frac{1}{8} \frac{RT_c}{P_c}$$

ANALYSIS: $c_v = (\partial u / \partial T)_v$. Also, Eq. 11.47 gives $(\partial u / \partial v)_T = T(\partial p / \partial T)_v - p$. Thus

$$\frac{\partial}{\partial T} \left[\left(\frac{\partial u}{\partial v} \right)_T \right]_v = \frac{\partial}{\partial T} \left[T \left(\frac{\partial p}{\partial T} \right)_v - p \right]$$

$$\frac{\partial}{\partial v} \left[\left(\frac{\partial u}{\partial T} \right)_v \right]_T = T \left(\frac{\partial^2 p}{\partial T^2} \right)_v + \left(\frac{\partial p}{\partial T} \right)_v - \left(\frac{\partial p}{\partial T} \right)_v$$

or

$$\left(\frac{\partial c_v}{\partial v} \right)_T = T \left(\frac{\partial^2 p}{\partial T^2} \right)_v \quad (1)$$

Differentiation of the equation of state gives

$$\left(\frac{\partial p}{\partial T} \right)_v = \frac{R}{v-b} + \frac{a}{T^2 v^2}, \quad \left(\frac{\partial^2 p}{\partial T^2} \right)_v = -\frac{2a}{T^3 v^2}$$

Inserting this into Eq. (1) and integrating

$$\begin{aligned} c_v(T, v_2) - c_v(T, v_1) &= \left[\frac{2a}{T^2 v} \right]_{v_1}^{v_2} \\ &= \frac{2a}{T^2} \left[\frac{1}{v_2} - \frac{1}{v_1} \right] \end{aligned}$$

The value of a is determined using T_c and P_c from Table A-1E

$$\begin{aligned} a &= \frac{27}{64} \frac{R^2 T_c^3}{P_c} = \frac{27}{64} \frac{(1545/28.97 \frac{\text{ft}\cdot\text{lb}}{\text{lb}\cdot\text{OR}})^2 (239^\circ\text{R})^3}{(37.2) | 14.696 \times 144 | \text{ lb/ft}^2} \\ &= 208,081 \frac{\text{lb}\cdot\text{ft}^4 \cdot (\text{OR})^2}{(\text{lb})^2} \left| \frac{\text{Btu}}{778 \text{ ft}\cdot\text{lb}} \right| \\ &= 267.46 \frac{\text{ft}^3 \cdot (\text{OR})^2 \cdot \text{Btu}}{(\text{lb})^2} \end{aligned}$$

Accordingly, for $T = 1460^\circ\text{R}$

$$\begin{aligned} c_v(T, 0.0555 \text{ ft}^3/\text{lb}) &= 0.1965 + \frac{2(267.46)}{(1460)^2} \left[\frac{1}{0.0555} - \frac{1}{36.8} \right] \\ &= 0.1965 + 0.0045 \\ &= 0.2010 \text{ Btu/lb}\cdot\text{OR} \end{aligned}$$

PROBLEM 11.64

KNOWN: The specific heat ratio can be expressed as

$$\kappa = \frac{c_p \kappa}{c_p \kappa - T v \beta^2}$$

FIND: Derive this expression and use it together with steam table data to evaluate κ at 200 lbf/in², 500°F.

ANALYSIS: Begin with Eq. 11.69, divide each side by c_p , giving

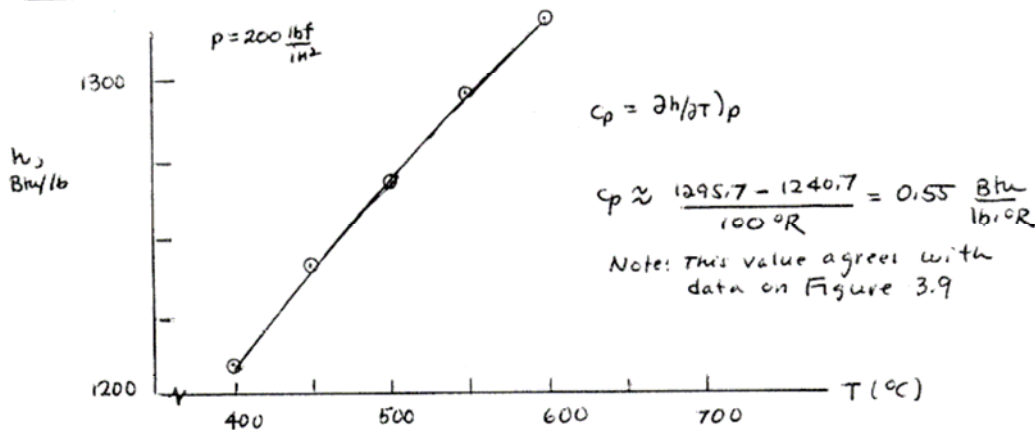
$$1 - \frac{c_v}{c_p} = \frac{T v \beta^2}{c_p \kappa} \Rightarrow 1 - \frac{1}{\kappa} = \frac{T v \beta^2}{c_p \kappa}$$

$$\text{Then } 1 - \frac{T v \beta^2}{c_p \kappa} = \frac{1}{\kappa} \Rightarrow \kappa = \frac{c_p \kappa}{c_p \kappa - T v \beta^2} = \frac{1}{1 - (T v \beta^2 / c_p \kappa)} \quad (1)$$

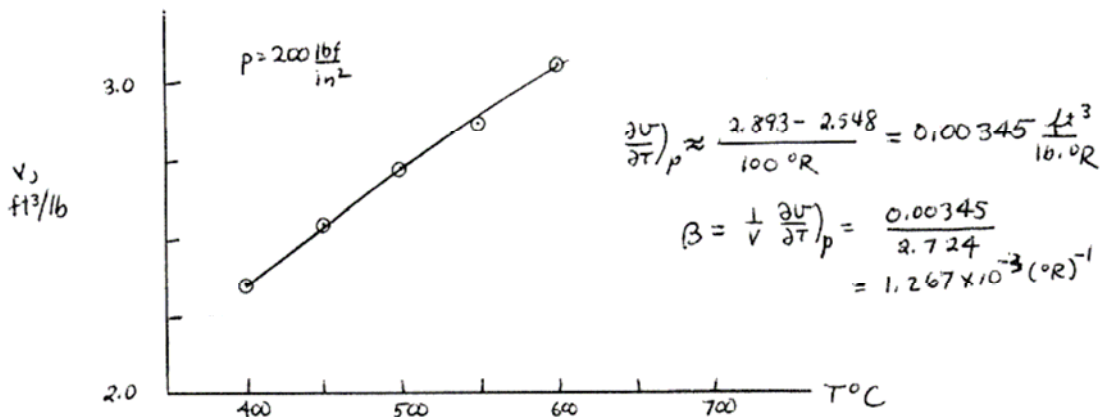
To evaluate κ requires c_p , κ , β , or alternatively c_p , $\left(\frac{\partial v}{\partial T}\right)_p$, $\left(\frac{\partial v}{\partial p}\right)_T$

GRAPHICAL SOLUTION:

To find c_p



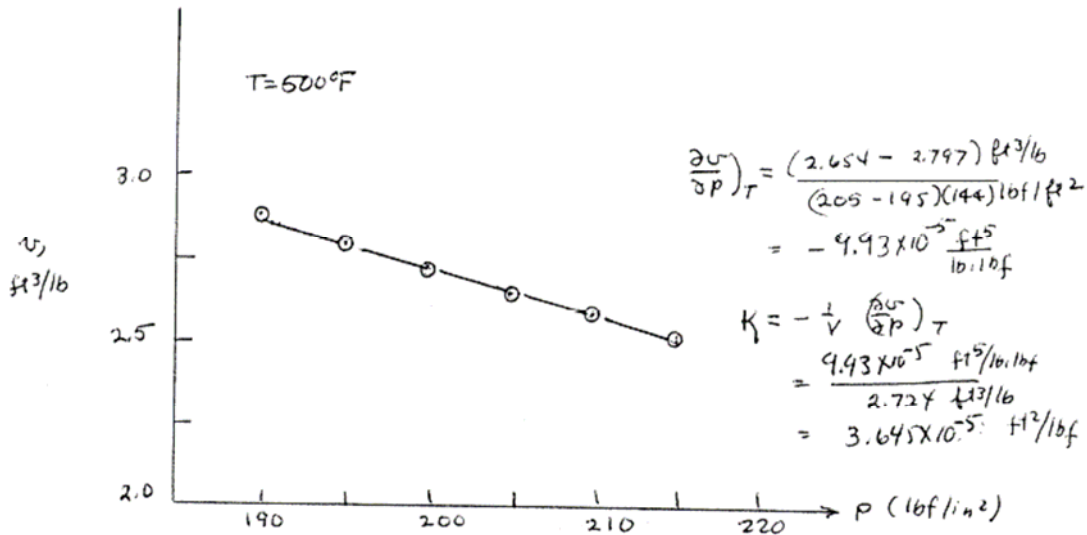
To find $\left(\frac{\partial v}{\partial T}\right)_p$



Continued on next slide

Problem 11-64 continued

To find $(\partial v / \partial p)_T$



With these values

$$\frac{v T \beta^2}{c_p \kappa} = \frac{(2.724 \text{ ft}^3/\text{lb})(959.67^\circ\text{R})(1.267 \times 10^{-3} (\text{°R})^{-1})^2}{(0.55 \frac{\text{Btu}}{\text{lb} \cdot \text{°R}})(778.17 \frac{\text{ft} \cdot \text{lbf}}{\text{Btu}})(3.645 \times 10^{-5} \frac{\text{ft}^2}{\text{lb} \cdot \text{lbf}})} = 0.269$$

and Eq. (1) gives $\kappa = 1.37$, which agrees with the value determined from Fig. 6 of Steam Tables (English Units) by Keenan et al., Wiley, New York, 1969.

IT Code

p = 200 // lbf/in²

T = 500 // °F

dT = 0.001

dp = 0.001

T1 = T - dT

T2 = T + dT

p1 = p - dp

p2 = p + dp

h1 = h_PT("Water/Steam", p, T1)

h2 = h_PT("Water/Steam", p, T2)

c_p = (h2 - h1) / (T2 - T1)

v2 = v_PT("Water/Steam", p, T2)

v1 = v_PT("Water/Steam", p, T1)

v = v_PT("Water/Steam", p, T)

beta = (1 / v) * ((v2 - v1) / (T2 - T1))

v22 = v_PT("Water/Steam", p2, T)

v11 = v_PT("Water/Steam", p1, T)

kappa = -(1 / v) * ((v22 - v11) / (p2 - p1)) / 144

k = 1 / (1 - (v * (T + 459.67) * beta^2) / (c_p * kappa * 778.17))

IT Results

β = 0.001265 (1/°R)

c_p = 0.548 Btu / lb · °R

κ = 3.656 × 10⁻⁵ ft² / lb · lbf

k = 1.366

PROBLEM 11.65

KNOWN: Liquid water at 40°C , 1 atm is under consideration. Data are available from Table 11.2.

FIND: Determine (a) c_v , (b) velocity of sound.

ANALYSIS: (a) Using Eq. 11.69

$$c_v = c_p - v \frac{T\beta^2}{\kappa} = c_p - \frac{T\beta^2}{\rho\kappa} \quad (1)$$

From Table 11.2 at 40°C

$$\rho = 992.22 \frac{\text{kg}}{\text{m}^3}, \quad \beta = \frac{385.4}{10^6} (\text{K})^{-1}, \quad \kappa = \frac{44.24}{10^6} (\text{bar})^{-1}$$

① From Table A.19, $c_p = 4.18 \text{ kJ/kg} \cdot \text{K}$.

Inserting values in Eq. (1)

$$c_v = \left(4.18 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) - \frac{(313.15 \text{ K}) \left(\frac{385.4 \times 10^{-6}}{\text{K}} \right)^2 (\text{bar})}{\left(992.22 \frac{\text{kg}}{\text{m}^3} \right) (\text{K})^2 \left(\frac{44.24 \times 10^{-6}}{\text{bar}} \right)} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$\textcircled{2} \quad = (4.18 - 0.11) = 4.07 \text{ kJ/kg} \cdot \text{K} \quad \leftarrow$$

(b) With Eqs. 11.65, 11.73

$$c = \sqrt{\frac{v}{\alpha}} = \sqrt{\frac{\kappa}{\rho\beta}}$$

With the result of (a), $\kappa = \frac{c_p}{c_v} = \frac{4.18}{4.07} = 1.027$

$$c = \sqrt{\frac{(1.027) (\text{bar})}{\left(992.22 \frac{\text{kg}}{\text{m}^3} \right) \left(\frac{44.24 \times 10^{-6}}{\text{bar}} \right)}} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|$$

$$= 1530 \text{ m/s} \quad \leftarrow$$

1. Following the discussion of Sec. 3.3.6, we take c_p at 1 atm, 40°C as the saturated liquid value at 40°C .

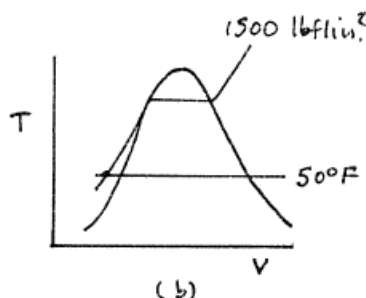
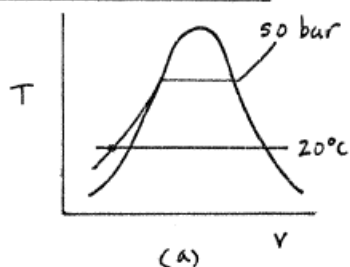
2. The result of part (a) shows that for liquid water at the given state, c_p and c_v are closely equal.

PROBLEM 11.66

KNOWN: Liquid water is at specified states.

FIND: Using steam table data, estimate the velocity of sound at (a) 20°C, 50 bar, (b) 50°F, 1500 lbf/in.²

SCHEMATIC & GIVEN DATA:



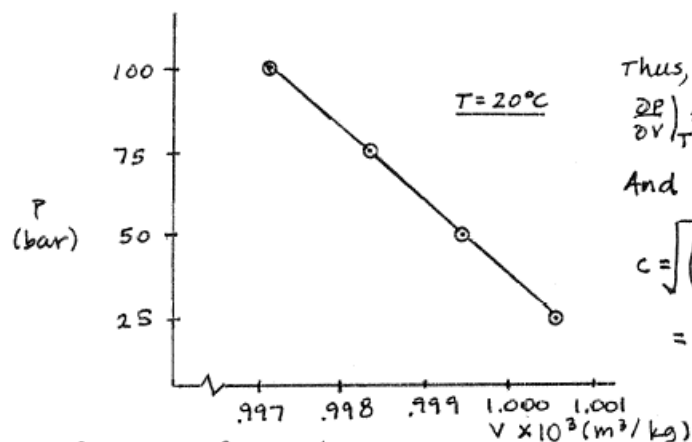
ENGINEERING

MODEL: For liquid water in the range of temperatures under consideration, the ratio of specific heats is $k \approx 1.0$. (See discussion in Sec. 11.52)

ANALYSIS: With Eq. 11.74 and the above assumption

$$c = \sqrt{-k v^2 \left(\frac{\partial P}{\partial v} \right)_T} \approx \sqrt{-v^2 \left(\frac{\partial P}{\partial v} \right)_T}$$

(a) With data from Table A-5



Thus,

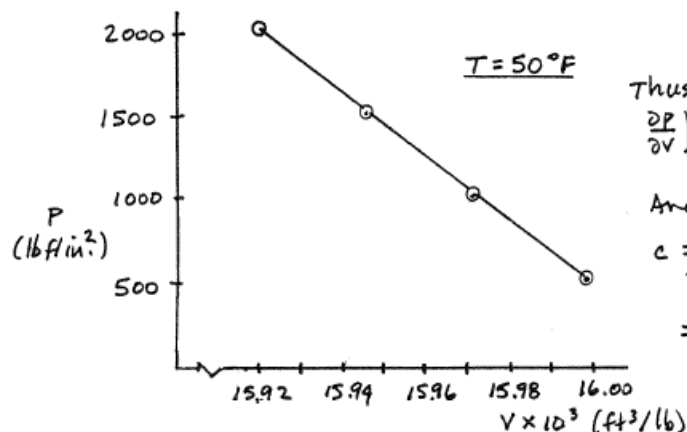
$$\left(\frac{\partial P}{\partial v} \right)_T \approx \frac{(75 - 25) 10^5 \text{ N/m}^2}{\frac{(0.9984 - 1.0006) \text{ m}^3}{10^3 \text{ kg}}} = -22727 \times 10^8 \frac{\text{N} \cdot \text{kg}}{\text{m}^5}$$

And

$$c = \sqrt{\left(\frac{0.9995 \text{ m}^3}{10^3 \text{ kg}} \right)^2 (22727 \times 10^8) \frac{\text{N} \cdot \text{kg}}{\text{m}^5} \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|}$$

$$= 1507 \text{ m/s} \quad \leftarrow (a)$$

(b) With data from Table A-5E



Thus,

$$\left(\frac{\partial P}{\partial v} \right)_T \approx \frac{(2000 - 1000) 144 \text{ lbf/ft}^2}{(0.015920 - 0.015972) \text{ ft}^3/\text{lb}} = -2.7692 \times 10^9 \frac{\text{lbf} \cdot \text{lb}}{\text{ft}^5}$$

And

$$c = \sqrt{\left(\frac{15.946 \text{ ft}^3}{10^3 \text{ lb}} \right)^2 (2.7692 \times 10^9) \frac{\text{lbf} \cdot \text{lb}}{\text{ft}^5} \left| \frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right|}$$

$$= 4762 \text{ ft/s} \quad \leftarrow (b)$$

PROBLEM 11.67

KNOWN: At a point within a stream of air $T = 500^\circ\text{F}$, $p = 1\text{ atm}$, $V = 2115\text{ ft/s}$.

FIND: Determine the Mach number.

ENGINEERING MODEL: Air is modeled as an ideal gas.

ANALYSIS: The Mach number is $M = V/c$, where c is the velocity of sound. For an ideal gas, c is given by Eq. 9.37, obtained by reduction of Eq. 11.74. Then

$$M = \frac{V}{c} = \frac{V}{\sqrt{kRT}} = \frac{2115\text{ ft/s}}{\sqrt{(1.383) \left(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot ^\circ\text{R}} \right) (960^\circ\text{R}) \left| \frac{32.2 \text{ lb} \cdot \text{ft}}{\text{lb}_f \cdot \text{s}^2} \right|}} = \frac{2115}{1510} = 1.4$$

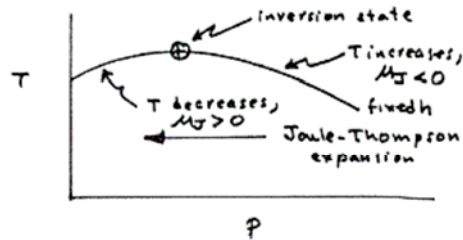
where k is obtained from Table A-20E.

PROBLEM 11.68

KNOWN: A gas obeys the equation of state, $p(v-b) = RT$, where b is a positive constant.

FIND: Determine if the temperature can be reduced in a Joule-Thompson expansion.

SCHEMATIC & GIVEN DATA



$$\mu_J = \left(\frac{\partial T}{\partial p} \right)_h$$

ANALYSIS: Using Eq. 11.77

$$\mu_J = \frac{1}{c_p} \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right]$$

With the given equation of state $(\partial v / \partial T)_p = R/p$. Thus

$$\begin{aligned} \mu_J &= \frac{1}{c_p} \left[\frac{RT}{p} - v \right] \\ &= \frac{1}{c_p} \left[\frac{RT}{p} - \left(\frac{RT}{p} + b \right) \right] = -\frac{b}{c_p} < 0 \end{aligned}$$

Accordingly, the Joule-Thompson coefficient is negative, and so the temperature can only increase in a Joule-Thompson expansion. $\leftarrow$

PROBLEM 11.69

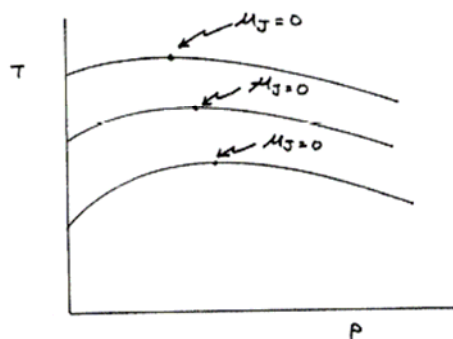
KNOWN: A gas is described by the equation of state

$$v = \frac{RT}{P} - \frac{A}{T} + B$$

where A and B are constants.

FIND: (a) Obtain an expression for the temperatures at the Joule-Thompson inversion states. (b) Obtain an expression for $(c_p - c_v)$.

SCHEMATIC & GIVEN DATA:



At Joule-Thompson inversion states,
 $\mu_J = 0$

ANALYSIS: (a) At the Joule-Thompson inversion states, the Joule-Thompson coefficient vanishes. Thus, Eq. 11.77 becomes

$$0 = \frac{1}{c_p} \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] \Rightarrow T \left(\frac{\partial v}{\partial T} \right)_P - v \equiv 0$$

With the given equation of state

$$\left(\frac{\partial v}{\partial T} \right)_P = \frac{R}{P} + \frac{A}{T^2} \quad (1)$$

$$T \left(\frac{\partial v}{\partial T} \right)_P = \frac{RT}{P} + \frac{A}{T}$$

Accordingly

$$\left(\frac{RT}{P} + \frac{A}{T} \right) - \left(\frac{RT}{P} - \frac{A}{T} + B \right) = 0 \Rightarrow \frac{2A}{T} - B \Rightarrow T = \frac{2A}{B}$$

- ① (b) To find $c_p - c_v$, employ Eq. 11.66. This requires $(\partial v / \partial T)_P$ and $(\partial P / \partial T)_V$. $(\partial v / \partial T)_P$ is given by Eq. (1). Rewriting the equation of state

$$P = RT \left[v + \frac{A}{T} - B \right]^{-1}, \quad \left(\frac{\partial P}{\partial T} \right)_V = R \left[v + \frac{A}{T} - B \right]^{-1} - RT \left[v + \frac{A}{T} - B \right]^{-2} \left[-\frac{A}{T^2} \right]$$

$$= \frac{R(v-B) + 2RA/T}{\left[v + \frac{A}{T} - B \right]^2}$$

$$\textcircled{2} \Rightarrow c_p - c_v = T \left[\frac{R}{P} + \frac{A}{T^2} \right] \left[\frac{R(v-B) + 2RA/T}{\left(v + \frac{A}{T} - B \right)^2} \right]$$

- Alternatively, $(c_p - c_v)$ can be evaluated from Eq. 11.68.
- For an ideal gas $A = B = 0$. Then, this expression for $(c_p - c_v)$ reduces to $(c_p - c_v) = R$, which is Eq. 3.44.

PROBLEM 11.70

KNOWN: Three equations of state are under consideration: (a) van der Waals equation, (b) Redlich-Kwong equation, (c) Dieterici equation.

FIND: Determine the maximum Joule-Thompson inversion temperature in terms of the critical temperature, T_c , predicted by each of these equations.

ANALYSIS: Eq. 11.77 can be rearranged to give

$$\mu_J = \frac{[T(\partial v/\partial T)_p - v]}{c_p}$$

The maximum inversion temperature is determined in accordance with $\lim_{p \rightarrow 0} \mu_J = 0$. Since $\lim_{p \rightarrow 0} c_p(T, p) \rightarrow c_p(T)$ (ideal gas), this reduces to a consideration of

$$\lim_{p \rightarrow 0} [T(\partial v/\partial T)_p - v] = 0$$

Also, note that the limit as pressure tends to zero corresponds to the limit as $v \rightarrow \infty$. Accordingly, the condition to be considered is

$$\lim_{v \rightarrow \infty} [T(\partial v/\partial T)_p - v] = 0$$

(a) van der Waals. The partial derivative $(\partial v/\partial T)_p$ is evaluated in Example 11.2 as

$$(\partial v/\partial T)_p = \frac{-R/(v-b)}{[\frac{2a}{v^3} - \frac{RT}{(v-b)^2}]} = \frac{RV^3(v-b)}{RTV^3 - 2a(v-b)^2}$$

Thus

$$\begin{aligned} [T(\partial v/\partial T)_p - v] &= \frac{RTV^3(v-b)}{RTV^3 - 2a(v-b)^2} - v \\ &= \frac{+RTv^4 - bRTv^3 - vRTV^3 + 2av(v-b)^2}{RTV^3 - 2a(v-b)^2} \\ &= \frac{2av(v-b)^2 - bRTv^3}{RTV^3 - 2a(v-b)^2} = \frac{2a - \frac{bRT}{(1-(b/v))^2}}{\frac{RT}{(1-(b/v))^2} - \frac{2a}{v}} \end{aligned}$$

Considering the limit as $v \rightarrow \infty$

$$\lim_{v \rightarrow \infty} [T(\partial v/\partial T)_p - v] = \frac{2a - bRT}{RT} = 0 \Rightarrow T = \frac{2a}{Rb}$$

Introducing Eqs. 11.4a and 11.4b

$$T = \frac{2(\frac{27}{8} \frac{RT_c^3}{P_c})}{(\frac{1}{8} \frac{RT_c}{P_c})} = \frac{27}{4} T_c = 6.75 T_c \quad \leftarrow (a)$$

(b) Redlich-Kwong. To evaluate $(\partial v/\partial T)_p$, use the relation

$$(\partial v/\partial T)_p (\partial p/\partial v)_T (\partial T/\partial p)_v = -1 \Rightarrow (\partial v/\partial T)_p = - \frac{(\partial p/\partial T)_v}{(\partial p/\partial v)_T}$$

For the Redlich-Kwong equation, the partial derivatives are

$$(\partial p/\partial T)_v = \frac{R}{(v-b)} + \frac{a}{2v(v+b)T^{3/2}}, \quad (\partial p/\partial v)_T = -\frac{RT}{(v-b)^2} + \frac{a(2v+b)}{v^2(v+b)^2 T^{1/2}}$$

So

$$[T(\partial v/\partial T)_p - v] = -T \left[\frac{\frac{R}{(v-b)} + \frac{a}{2v(v+b)T^{3/2}}}{-\frac{RT}{(v-b)^2} + \frac{a(2v+b)}{v^2(v+b)^2 T^{1/2}}} \right] - v$$

Continued on next slide

Problem 11-70 continued

Upon rearrangement

$$\left[T \left(\frac{\partial V}{\partial T} \right)_P - V \right] = \frac{-\frac{RT}{(v-b)^2} - \frac{a}{2v(v+b)T^{1/2}} - \frac{a(2v+b)}{v(v+b)^2 T^{1/2}} + \frac{vRT}{(v-b)^2}}{\left(\frac{a(2v+b)}{v^2(v+b)^2 T^{1/2}} - \frac{RT}{(v-b)^2} \right)}$$

$$= \frac{\frac{bRT}{(v-b)^2} - \frac{a}{2(v+b)T^{1/2}} \left[5 + 3 \frac{b}{v} \right]}{\left(\frac{a(2v+b)}{v^2(v+b)^2 T^{1/2}} - \frac{RT}{(v-b)^2} \right)} = \left[\frac{bRT}{(1-\frac{b}{v})^2} - \frac{a \left[5 + 3 \frac{b}{v} \right]}{2(1+\frac{b}{v})^2 T^{1/2}} \right] \left[\frac{a(2+\frac{b}{v})}{v(v+b)^2 T^{1/2}} - \frac{RT}{(1-\frac{b}{v})^2} \right]^{-1}$$

Considering the limit as $v \rightarrow \infty$

$$\lim_{v \rightarrow \infty} \left[T \left(\frac{\partial V}{\partial T} \right)_P - V \right] = \frac{bRT - \frac{5a}{2T^{1/2}}}{-RT} = 0 \Rightarrow T = \left(\frac{5a}{2bR} \right)^{2/3}$$

With Eqs. 11.8

$$\textcircled{1} \quad T = \left(\frac{5(0.42748) R^2 T_c^{5/2} / P_c}{(2)(0.08664) \left(\frac{RT_c}{P_c} \right) R} \right)^{2/3} = 5.34 T_c \quad \leftarrow (b)$$

(c) Dieterici. As above

$$\left(\frac{\partial V}{\partial T} \right)_P = - \frac{(\partial P / \partial T)_V}{(\partial P / \partial V)_T}$$

For the Dieterici equation, the partial derivatives are

$$\left(\frac{\partial P}{\partial T} \right)_V = \frac{R}{(v-b)} \exp\left(-\frac{a}{RTv}\right) \left[1 + \frac{a}{RTv} \right], \quad \left(\frac{\partial P}{\partial V} \right)_T = \frac{RT}{(v-b)^2} \exp\left(-\frac{a}{RTv}\right) \left[\frac{a}{RTv^2} - \frac{1}{(v-b)} \right]$$

Thus

$$\left[T \left(\frac{\partial V}{\partial T} \right)_P - V \right] = \frac{\left[1 + \frac{a}{RTv} \right]}{\left[\frac{a}{RTv^2} - \frac{1}{(v-b)} \right]} - v = \frac{\frac{v}{v-b} - \frac{2a}{RTv} - 1}{\frac{a}{RTv^2} - \frac{1}{(v-b)}}$$

As $\lim_{v \rightarrow \infty} \left[T \left(\frac{\partial V}{\partial T} \right)_P - V \right] = \frac{0}{0}$, apply L'Hospital's rule to obtain

$$\lim_{v \rightarrow \infty} \left[\frac{\frac{-b}{(v-b)^2} + \frac{2a}{RTv^2}}{\frac{-2a}{RTv^3} + \frac{1}{(v-b)^2}} \right] = \lim_{v \rightarrow \infty} \left[\frac{\frac{-b}{(1-\frac{b}{v})^2} + \frac{2a}{RT}}{\frac{-2a}{RTv} + \frac{1}{(1-\frac{b}{v})^2}} \right] = -b + \frac{2a}{RT}$$

$$\Rightarrow T = \frac{2a}{bR}$$

Introducing the expressions for a and b given in Problem 11.67

$$T = \frac{2 \left(\frac{4R^2 T_c^2}{R e^2} \right)}{R \left(\frac{RT_c}{R e^2} \right)} = 8 T_c \quad \leftarrow (c)$$

- The maximum inversion temperature for a wide range of gases is about $5T_c$. Of the three equations of state considered here, only the Redlich-Kwong equation predicts a maximum inversion temperature in the neighborhood of $5T_c$. 11-70a

PROBLEM 11.71

KNOWN: A certain gas obeys the van der Waals equation of state and has a specific heat c_v

①

$$c_v = A + BT + CT^2$$

where A , B , and C are constants.

FIND: Derive an equation for u_J as a function of v and T . Evaluate the Joule-Thompson inversion state temperatures in terms of v , R , and the van der Waals constants a and b .

ANALYSIS: Using Eq. 11.77: $u_J = [T(\partial v/\partial T)_p - v]/c_p$. This requires $(\partial v/\partial T)_p$ which is evaluated for the van der Waals equation in Example 11.2 as

$$\left(\frac{\partial v}{\partial T}\right)_p = -\frac{R/(v-b)}{[2a/v^3 - RT/(v-b)^2]} = -\frac{Rv^3(v-b)}{2a(v-b)^2 - RTv^3}$$

Thus

$$\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{-RTv^3(v-b)}{2a(v-b)^2 - RTv^3} - v = \frac{2av(v-b)^2 - bRTv^3}{RTv^3 - 2a(v-b)^2}$$

An expression for c_p is also required. This can be found using Eq. 11.66

$$c_p - c_v = T\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial T}\right)_v$$

From the van der Waals equation, $(\partial p/\partial T)_v = R/(v-b)$. Thus

$$\begin{aligned} c_p &= c_v + T\left[\frac{-Rv^3(v-b)}{2a(v-b)^2 - RTv^3}\right]\frac{R}{(v-b)} \\ &= (A + BT + CT^2) - \frac{R^2 T v^3}{2a(v-b)^2 - RTv^3} \\ &= \frac{(A + BT + CT^2)[RTv^3 - 2a(v-b)^2] + R^2 Tv^3}{RTv^3 - 2a(v-b)^2} \end{aligned}$$

Collecting results

$$\begin{aligned} u_J &= \left\{ \frac{\left(\frac{2av(v-b)^2 - bRTv^3}{RTv^3 - 2a(v-b)^2}\right)}{\frac{(A + BT + CT^2)[RTv^3 - 2a(v-b)^2] + R^2 Tv^3}{RTv^3 - 2a(v-b)^2}} \right\} \\ &= \frac{2av(v-b)^2 - bRTv^3}{(A + BT + CT^2)(RTv^3 - 2a(v-b)^2) + R^2 Tv^3} \quad \leftarrow u_J \end{aligned}$$

At Joule-Thompson inversion states, $u_J = 0$. Accordingly, the inversion temperatures T_i are determined by

$$\textcircled{2} \quad 0 = 2av(v-b)^2 - bRT_i v^3 \Rightarrow T_i = \frac{2a(v-b)^2}{bRv^2} = \frac{2a}{bR} \left[1 - \left(\frac{b}{v}\right)\right]^2 \quad \leftarrow T_i$$

1. For a van der Waals gas, $c_v = f(T)$. See Problem 11.62a.

2. In the limit as $p \rightarrow 0$, $v \rightarrow \infty$. Thus, the maximum inversion temperature is $(T_i)_{\max} = \frac{2a}{bR}$, which is in agreement with the result of Problem 11.70(a).

PROBLEM 11.72

KNOWN: Equation 11.77 is written in an alternative form.

FIND: Verify the alternative form. (a) Using this result, obtain an expression for μ_J for a gas obeying

$$v = \frac{RT}{p} - \frac{Ap}{T^2} \quad \text{where } A \text{ is a constant.}$$

(b) Using the expression obtained in (b), determine c_p for CO_2 at a state where $T = 400\text{ K}$, $p = 1\text{ atm}$, $\mu_J = 0.57\text{ K/atm}$.

ANALYSIS: The alternative expression for μ_J is

$$\mu_J = \frac{T^2}{c_p} \left(\frac{\partial(v/T)}{\partial T} \right)_p \Rightarrow \mu_J = \frac{T^2}{c_p} \left[\frac{1}{T} \left(\frac{\partial v}{\partial T} \right)_p - \frac{v}{T^2} \right] = \frac{1}{c_p} \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right]$$

which agrees with Eq. 11.77.

(b) Using the given relationship, we get $(v/T) = (R/p) - (Ap/T^3)$. Thus

$$\left(\frac{\partial(v/T)}{\partial T} \right)_p = \frac{3Ap}{T^4} \Rightarrow \mu_J = \frac{T^2}{c_p} \left[\frac{3Ap}{T^4} \right] = \frac{3Ap}{c_p T^2} \quad \leftarrow$$

(c) Solving for c_p , and using $A = 2.78 \times 10^{-3} \text{ m}^5 \cdot \text{K}^2 / \text{kg} \cdot \text{N}$ and other known data

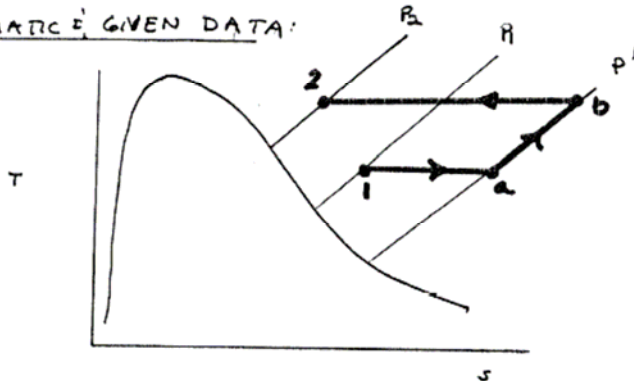
$$\begin{aligned} c_p &= \frac{3Ap}{\mu_J T^2} = \frac{3(2.78 \times 10^{-3} \text{ m}^5 \cdot \text{K}^2 / \text{kg} \cdot \text{N})(1.01325 \times 10^5 \text{ N/m}^2)}{(0.57 \frac{\text{K}}{\text{atm}}) \left| \frac{1 \text{ atm}}{1.01325 \times 10^5 \text{ N/m}^2} \right| (400 \text{ K})^2} \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= 0.939 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad \leftarrow \end{aligned}$$

PROBLEM 11.73

KNOWN: For a gas obeying the van der Waals equation, at pressure p' c_v is given by $c_v = A + BT$, where A and B are constants.

FIND: Develop an expression for $[s(T_2, p_2) - s(T_1, p_1)]$

SCHEMATIC & GIVEN DATA:



$$A + 1, \quad v_1 = v(T_1, p_1)$$

$$A + a, \quad v_a = v(T_1, p')$$

$$A + b, \quad v_b = v(T_2, p')$$

$$A + 2, \quad v_2 = v(T_2, p_2)$$

$$c_v = A + BT \quad \text{when } p = p'$$

ASSUMPTION: The gas is described by the van der Waals equation and the given c_v function.

ANALYSIS: For any choice of states 1, 2 for which the assumption is satisfied.

$$s(T_2, p_2) - s(T_1, p_1) = [s(T_2, p_2) - s(T_2, p')] + [s(T_2, p') - s(T_1, p')] + [s(T_1, p') - s(T_1, p_1)]$$

$(b \rightarrow 2) \qquad \qquad \qquad (a \rightarrow b) \qquad \qquad \qquad (1 \rightarrow a)$

The specific volumes v_1, v_a, v_b, v_2 can be found by solving the van der Waals equation using the respective temperature and pressure.

Using the van der Waals equation, $(\partial p / \partial T)_v = R / (v - b)$. Thus, with Eq. 11.50

$$s_a - s_1 = \int_1^a \left(\frac{\partial p}{\partial T} \right)_v dv = \int_1^a \frac{R}{v - b} dv = R \ln \left(\frac{v_a - b}{v_1 - b} \right) \quad (1)$$

Similarly

$$s_2 - s_b = R \ln \left(\frac{v_2 - b}{v_b - b} \right) \quad (2)$$

Then for process $a \rightarrow b$, Eq. 11.50 gives

$$s_b - s_a = \int_{T_1}^{T_2} \frac{c_v}{T} dT = \int_{T_1}^{T_2} \frac{A + BT}{T} dT = A \ln \frac{T_2}{T_1} + B(T_2 - T_1) \quad (3)$$

Then, summing Eqs. (1) - (3)

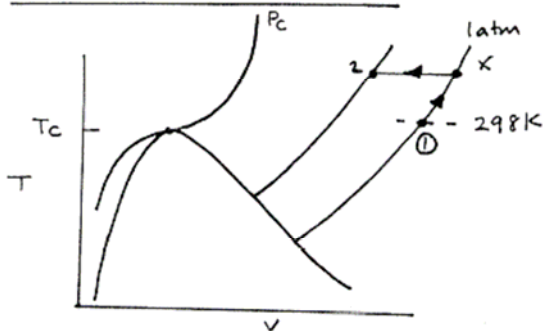
$$s_2 - s_1 = \underbrace{R \ln \left(\frac{v_a - b}{v_1 - b} \right)}_{(1 \rightarrow a)} + \underbrace{A \ln \frac{T_2}{T_1} + B(T_2 - T_1)}_{(a \rightarrow b)} + \underbrace{R \ln \left(\frac{v_2 - b}{v_b - b} \right)}_{(b \rightarrow 2)}$$

PROBLEM 11.74

KNOWN: Air is modeled by the van der Waals equation and $c_p(T)$ from Table A-21.

FIND: Write a computer program evaluating Δh between a state where $(T_1 = 298\text{K}, P_1 = 1\text{atm})$ and (T_2, P_2) .

SCHEMATIC & GIVEN DATA:



From Table A-1, $P_c = 37.2\text{atm}$, $T_c = 133\text{K}$

ENGINEERING MODEL: The ideal gas model is valid for the path 1-X (check Fig. A-1) and the specific heat function of Table A-21 is valid for $298 \leq T \leq 1000\text{K}$.

ANALYSIS: The evaluation of Δh is conveniently accomplished by considering two paths: 1-X and X-2. As noted, the ideal gas model applies for path 1-X, and so

$$h_X - h_1 = \int_{T_1}^{T_2} c_p(T) dT = R \left[\alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5) \right] \quad (1)$$

where $\alpha, \beta, \gamma, \delta,$ and ϵ are obtained from Table A-21

Since the van der Waals equation is explicit in pressure, it is convenient to evaluate $h_2 - h_X$ using

$$h_2 - h_X = (u_2 - u_X) + P_2 v_2 - P_X v_X \quad (2)$$

where $(u_2 - u_X)$ is evaluated from Eq. 11.51, which reduces to give

$$u_2 - u_X = \int_{v_X}^{v_2} \left[T \left(\frac{\partial P}{\partial T} \right)_v - P \right] dv = \int_{v_X}^{v_2} \left(T \left(\frac{R}{v - b} \right) - P \right) dv = \int_{v_X}^{v_2} \frac{a}{v^2} dv = -a \left[\frac{1}{v_2} - \frac{1}{v_X} \right] \quad (3)$$

With Eqs. (2) and (3)

$$h_2 - h_X = P_2 v_2 - P_X v_X - a \left[\frac{1}{v_2} - \frac{1}{v_X} \right] \quad (4)$$

The value of v_X can be obtained using the ideal gas equation of state: $v_X = RT_1/P_1 = RT_2/P_1$. The value of v_2 can be obtained iteratively from the van der Waals equation using the specified values for P_2, T_2 .

The overall change $(h_2 - h_1)$ is then obtained from Eqs. (1) and (4):

$$(h_2 - h_1) = \underbrace{(h_2 - h_X)}_{\text{Eq. (4)}} + \underbrace{(h_X - h_1)}_{\text{Eq. (1)}}$$

This provides the required relations for finding Δh . Computer program details are left to the reader.

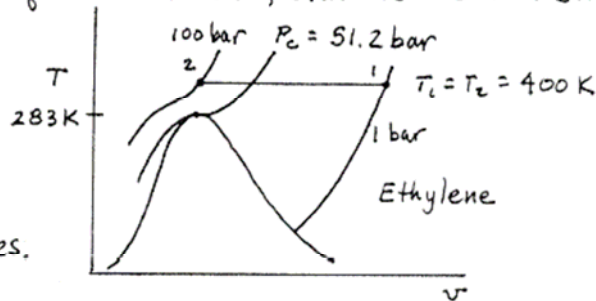
- For temperatures out of this range, other representations of $c_p(T)$ may be required.

PROBLEM 11.75

KNOWN: Two states of ethylene (C_2H_4) are under consideration: $T_1 = T_2 = 400\text{ K}$, $P_1 = 1\text{ bar}$, $P_2 = 100\text{ bar}$.

FIND: Using the Redlich-Kwong equation of state, evaluate $\Delta \bar{S}$ and $\Delta \bar{h}$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The Redlich-Kwong equation of state is applicable at these states.

ANALYSIS: The results of Problem 11.49(b) are quoted for present use:

$$\bar{h}_2 - \bar{h}_1 = P(\bar{v}_2, T) \bar{v}_2 - P(\bar{v}_1, T) \bar{v}_1 - \frac{3a}{2bT^{1/2}} \ln \left[\frac{\bar{v}_2 + b}{\bar{v}_1 + b} \cdot \frac{\bar{v}_1}{\bar{v}_2} \right] \quad (1)$$

$$\bar{s}_2 - \bar{s}_1 = \bar{R} \ln \left(\frac{\bar{v}_2 - b}{\bar{v}_1 - b} \right) - \frac{a}{2bT^{3/2}} \ln \left[\frac{\bar{v}_2 + b}{\bar{v}_1 + b} \cdot \frac{\bar{v}_1}{\bar{v}_2} \right] \quad (2)$$

where the values of a and b are obtained using Eq. 11.8 as

$$a = 77.76 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2 \text{ K}^{1/2}, \quad b = 0.03981 \frac{\text{m}^3}{\text{kmol}}$$

The values of $\bar{v}_1$ and $\bar{v}_2$ can be found using IT to solve the Redlich-Kwong equation knowing the pressure and temperature at each state. The results are

$$\bar{v}_1 = 33.18 \text{ m}^3/\text{kmol}, \quad \bar{v}_2 = 0.2636 \text{ m}^3/\text{kmol}$$

Inserting numerical values into Eq. (2)

$$\begin{aligned} \bar{s}_2 - \bar{s}_1 &= \left(8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \right) \ln \left(\frac{0.2636}{33.18} \right) - \frac{77.76 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2 \text{ K}^{1/2}}{2 \left(0.03981 \frac{\text{m}^3}{\text{kmol}} \right) (400 \text{ K})^{3/2}} \ln \left[\frac{0.30341}{33.21981} \cdot \frac{33.18}{0.2636} \right] \\ &= -43.20 \text{ kJ/kmol} \cdot \text{K} \quad \Delta \bar{S} \end{aligned}$$

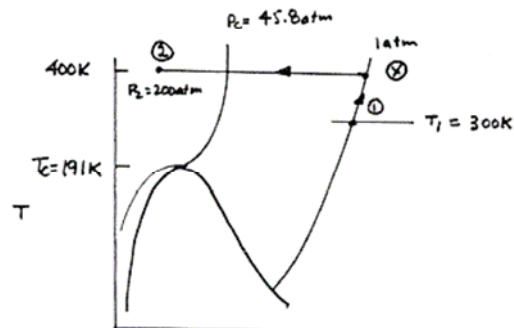
$$\begin{aligned} \bar{h}_2 - \bar{h}_1 &= \left[(100 \text{ bar})(0.2636 \text{ m}^3/\text{kmol}) - (1)(33.18) \right] \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &\quad - \frac{(3)(77.76 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2 \text{ K}^{1/2})}{(2)(0.03981 \text{ m}^3/\text{kmol})(400 \text{ K})^{1/2}} \ln \left[\frac{0.30341}{33.21981} \cdot \frac{33.18}{0.2636} \right] \\ &= -2270.6 \text{ kJ/kmol} \quad \Delta \bar{h} \end{aligned}$$

PROBLEM 11.76

KNOWN: Two states of CH_4 are under consideration: $T_1 = 300\text{K}$, $P_1 = 1\text{atm}$, $T_2 = 400\text{K}$, $P_2 = 200\text{atm}$.

FIND: Using the Benedict-Webb-Rubin equation of state together with an appropriate specific heat relation, determine $\Delta \bar{h}$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The ideal gas model applies along path 1-x.

ANALYSIS: The evaluation of $\Delta \bar{h}$ can be accomplished conveniently by considering two paths: 1-x followed by x-2. For $P = 1\text{atm}$, $P_R = 1/45.8 = 0.022$. Reference to Fig A-1 suggests that at this pressure the ideal gas model is applicable. Thus, since T_1 and $T_x = T_2$ are in the range of Table A-21

$$\bar{h}_x - \bar{h}_1 = \int_{T_1}^{T_x=T_2} \bar{c}_p(T) dT = \bar{R} \left[\alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5) \right] \quad (1)$$

where $\alpha, \beta, \gamma, \delta$, and ϵ are given in the table.

Since the Benedict-Webb-Rubin equation is explicit in pressure, it is convenient to evaluate $\bar{h}_2 - \bar{h}_x$ using

$$\bar{h}_2 - \bar{h}_x = (\bar{u}_2 - \bar{u}_x) + P_2 \bar{v}_2 - P_x \bar{v}_x \quad (2)$$

where $(\bar{u}_2 - \bar{u}_x)$ is evaluated from Eq. 11.51, which reduces to give

$$\bar{u}_2 - \bar{u}_x = \int_{\bar{v}_x}^{\bar{v}_2} \left[T \left(\frac{\partial p}{\partial T} \right)_v - p \right] d\bar{v}$$

Differentiating Eq. 11.12

$$\left(\frac{\partial p}{\partial T} \right)_v = \frac{\bar{R}}{\bar{v}} + \left[B\bar{R} + \frac{2C}{T^3} \right] \frac{1}{\bar{v}^2} + \frac{b\bar{R}}{\bar{v}^3} - \frac{2C}{\bar{v}^3 T^3} \left(1 + \frac{\gamma}{\bar{v}^2} \right) \exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$T \left(\frac{\partial p}{\partial T} \right)_v = \frac{\bar{R}T}{\bar{v}} + \left[B\bar{R}T + \frac{2C}{T^2} \right] \frac{1}{\bar{v}^2} + \frac{b\bar{R}T}{\bar{v}^3} - \frac{2C}{\bar{v}^3 T^2} \left(1 + \frac{\gamma}{\bar{v}^2} \right) \exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$\left[T \left(\frac{\partial p}{\partial T} \right)_v - p \right] = \left[A + \frac{3C}{T^2} \right] \frac{1}{\bar{v}^2} + \frac{a}{\bar{v}^3} - \frac{a\alpha}{\bar{v}^6} - \frac{3C}{\bar{v}^3 T^2} \left(1 + \frac{\gamma}{\bar{v}^2} \right) \exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

Thus

$$\begin{aligned} \bar{u}_2 - \bar{u}_x &= \left(A + \frac{3C}{T^2} \right) \left[\frac{1}{\bar{v}_x} - \frac{1}{\bar{v}_2} \right] + \frac{a}{2} \left[\frac{1}{\bar{v}_x^2} - \frac{1}{\bar{v}_2^2} \right] + \frac{a\alpha}{5} \left[\frac{1}{\bar{v}_x^5} - \frac{1}{\bar{v}_2^5} \right] \\ &\quad - \frac{3C}{T^2} \int_{\bar{v}_x}^{\bar{v}_2} \frac{1}{\bar{v}^3} \exp\left(-\frac{\gamma}{\bar{v}^2}\right) d\bar{v} - \frac{3C\gamma}{T^2} \int_{\bar{v}_x}^{\bar{v}_2} \frac{1}{\bar{v}^5} \exp\left(-\frac{\gamma}{\bar{v}^2}\right) d\bar{v} \end{aligned} \quad (3)$$

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Problem 11-76 continued

In Eq. (3)

$$\int_{\bar{v}_x}^{\bar{v}_2} \frac{1}{\bar{v}^3} \exp\left(-\frac{\gamma}{\bar{v}^2}\right) d\bar{v} = \frac{1}{2\gamma} \left[\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \exp\left(-\frac{\gamma}{\bar{v}_x^2}\right) \right]$$

$$\int_{\bar{v}_x}^{\bar{v}_2} \frac{1}{\bar{v}^5} \exp\left(-\frac{\gamma}{\bar{v}^2}\right) d\bar{v} = \frac{1}{2\gamma^2} \left(\frac{\gamma}{\bar{v}_2^2} + 1 \right) \exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \frac{1}{2\gamma^2} \left(\frac{\gamma}{\bar{v}_x^2} + 1 \right) \exp\left(-\frac{\gamma}{\bar{v}_x^2}\right)$$

Collecting results

$$\bar{u}_2 - \bar{u}_x = \left[A + \frac{3C}{T_2} \right] \left[\frac{1}{\bar{v}_x} - \frac{1}{\bar{v}_2} \right] + \frac{a}{2} \left[\frac{1}{\bar{v}_x^2} - \frac{1}{\bar{v}_2^2} \right] + \frac{a^2}{5} \left[\frac{1}{\bar{v}_x^5} - \frac{1}{\bar{v}_2^5} \right] -$$

$$\frac{3C}{2\gamma T_2^2} \left[\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \exp\left(-\frac{\gamma}{\bar{v}_x^2}\right) \right] - \frac{3C}{2\gamma T_2^2} \left[\left(\frac{\gamma}{\bar{v}_2^2} + 1 \right) \exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \left(\frac{\gamma}{\bar{v}_x^2} + 1 \right) \exp\left(-\frac{\gamma}{\bar{v}_x^2}\right) \right]$$

Finally

$$\bar{u}_2 - \bar{u}_x = \left[A + \frac{3C}{T_2} \right] \left[\frac{1}{\bar{v}_x} - \frac{1}{\bar{v}_2} \right] + \frac{a}{2} \left[\frac{1}{\bar{v}_x^2} - \frac{1}{\bar{v}_2^2} \right] + \frac{a^2}{5} \left[\frac{1}{\bar{v}_x^5} - \frac{1}{\bar{v}_2^5} \right] - \frac{3C}{2\gamma T_2^2} \left(\frac{\gamma}{\bar{v}_2^2} + 2 \right) \exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) + \frac{3C}{2\gamma T_2^2} \left(\frac{\gamma}{\bar{v}_x^2} + 2 \right) \exp\left(-\frac{\gamma}{\bar{v}_x^2}\right) \quad (4)$$

The value of $\bar{v}_x$ can be found using the ideal gas equation of state: $\bar{v}_x = RT_x/P_x = RT_2/P_1$. The value of $\bar{v}_2$ can be obtained by solving the Benedict-Webb-Rubin equation. The results are

$$\bar{v}_x = 32.807 \frac{\text{m}^3}{\text{kmol}}, \quad \bar{v}_2 = 0.1607 \frac{\text{m}^3}{\text{kmol}}$$

The values of the Benedict-Webb-Rubin constants are obtained from Table A-24. Accordingly, all values that are required to evaluate Eqs. (1), (2), and (4) are known. The final result is

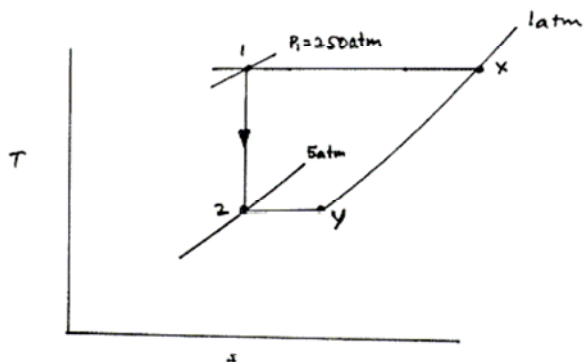
$$\bar{h}_2 - \bar{h}_1 \approx 2305 \frac{\text{kJ}}{\text{kmol}} \quad \left(\approx 144 \frac{\text{kJ}}{\text{kg}} \right) \quad \longleftarrow \Delta \bar{h}$$

PROBLEM 11.77

KNOWN: Nitrogen expands isentropically from $T_1 = 400\text{ K}$, $P_1 = 2.50\text{ atm}$ to $P_2 = 5\text{ atm}$.

FIND: Using the Redlich-Kwong Equation of state together with an appropriate specific heat relation, determine T_2 .

SCHEMATIC & GIVEN DATA:



ANALYSIS: For an isentropic expansion, $\bar{s}_2 - \bar{s}_1 = 0$. In the present case it is convenient to consider three paths in series: constant temperature to 1 atm ($1-x$), constant pressure to T_2 ($x-y$), constant temperature to state 2: $y-2$. Then

$$0 = \bar{s}_2 - \bar{s}_1 = (\bar{s}_2 - \bar{s}_y) + (\bar{s}_y - \bar{s}_x) + (\bar{s}_x - \bar{s}_1) \quad (1)$$

Since $P_c = 33.5\text{ atm}$, $P_R = 1/33.5 = 0.03$, indicating that the ideal gas model is appropriate for path $x-y$. As $1-x$ and $y-2$ are at fixed temperature, and the Redlich-Kwong equation is explicit in pressure, it is convenient to select Eq. 11.50 which reduces to give

$$\bar{s}_x - \bar{s}_1 = \int_1^x \left(\frac{\partial p}{\partial T} \right)_v d\bar{v}, \quad \bar{s}_2 - \bar{s}_y = \int_y^2 \left(\frac{\partial p}{\partial T} \right)_v d\bar{v}$$

In Problem 11.49b, an expression for these integrals is obtained. Thus

$$\bar{s}_x - \bar{s}_1 = \bar{R} \ln \left[\frac{\bar{v}_x - b}{\bar{v}_1 - b} \right] - \frac{a}{2bT_1^{3/2}} \ln \left[\frac{\bar{v}_x + b}{\bar{v}_1 + b} \cdot \frac{\bar{v}_1}{\bar{v}_x} \right] \quad (2)$$

$$\bar{s}_2 - \bar{s}_y = \bar{R} \ln \left[\frac{\bar{v}_2 - b}{\bar{v}_y - b} \right] - \frac{a}{2bT_2^{3/2}} \ln \left[\frac{\bar{v}_2 + b}{\bar{v}_y + b} \cdot \frac{\bar{v}_y}{\bar{v}_2} \right] \quad (3)$$

The values of the constants a and b can be obtained from Table A-24. The value of $\bar{v}_x$ can be found using the ideal gas equation of state: $\bar{v}_x = \bar{R}T_1/P_x$, where $P_x = 1\text{ atm}$. Similarly, $\bar{v}_y = \bar{R}T_2/P_y$, where $P_y = 1\text{ atm}$. The value of $\bar{v}_1$ can be obtained by solving the Redlich-Kwong equation of state, knowing $P_1 = 2.50\text{ atm}$, $T_1 = 400\text{ K}$. Similarly, $\bar{v}_2$ can be found by solving the Redlich-Kwong equation of state, knowing T_2 and $P_2 = 5\text{ atm}$.

To obtain $(\bar{s}_y - \bar{s}_x)$, Eq. 6.19 can be used:

$$\bar{s}_y - \bar{s}_x = \int_{T_x=T_1}^{T_y=T_2} \frac{\bar{c}_p(T)}{T} dT \quad (4)$$

where $\bar{c}_p(T)$ is the ideal gas specific heat c_p . The $c_p(T)$ relation given in Table A-21 is valid for $300 < T < 1000\text{ K}$, so a relation that is more suited for the temperatures under present consideration is needed. For example, Table 1-23 of the Handbook of Tables for Applied Engineering Science, 2nd Ed.,

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Problem 11-77 continued

CRC Press, Cleveland, OH, 1977 (R.E. Bolz and G.L. Tive, Eds.) gives c_p at 1 atm for $250 < T < 1500$ K. Another source of data is p. 2-91 of the Handbook of Heat Transfer, McGraw-Hill, New York, 1973 (W.M. Rohsenow and J.P. Hartnett, Eds.). Also, note that the data of Table A-23 suggests that c_p of N_2 varies only slightly for temperatures below about 300 K. Using tabular data from the literature, and appropriate computer software, a polynomial expression for c_p can be developed. To illustrate, the equation form of Table A-21 might be employed. Then, Eq. (4) would give

$$s_y - s_x = R \left[\alpha \ln \frac{T_2}{T_1} + \beta (T_2 - T_1) + \frac{\gamma}{2} (T_2^2 - T_1^2) + \frac{\delta}{3} (T_2^3 - T_1^3) + \frac{\epsilon}{4} (T_2^4 - T_1^4) \right] \quad (5)$$

where α , β , γ , δ , and ϵ are determined as indicated above.

Collecting Eqs. (1), (2), (3), and (5)

$$0 = \left\{ \bar{R} \ln \left[\frac{\bar{v}_x - b}{\bar{v}_y - b} \right] - \frac{a}{2bT_1^{3/2}} \left[\frac{\bar{v}_x + b}{\bar{v}_1 + b} \cdot \frac{\bar{v}_1}{\bar{v}_x} \right] \right\} + \bar{R} \left[\alpha \ln \frac{T_2}{T_1} + \beta (T_2 - T_1) + \frac{\gamma}{2} (T_2^2 - T_1^2) + \frac{\delta}{3} (T_2^3 - T_1^3) + \frac{\epsilon}{4} (T_2^4 - T_1^4) \right] + \left\{ \bar{R} \ln \left[\frac{\bar{v}_2 - b}{\bar{v}_y - b} \right] - \frac{a}{2bT_2^{3/2}} \ln \left[\frac{\bar{v}_2 + b}{\bar{v}_y + b} \cdot \frac{\bar{v}_y}{\bar{v}_2} \right] \right\} \quad (6)$$

The following values are known: T_1 , $\bar{v}_1$, $\bar{v}_x$, a , b , α , β , γ , δ , ϵ , $\bar{R}$. As noted above, $\bar{v}_y$ and $\bar{v}_2$ can be determined from the appropriate equation of state using T_2 and $p_y = 1$ atm and $p_2 = 5$ atm, respectively. Accordingly, the computational procedure is to specify T_2 , find $\bar{v}_y$ and $\bar{v}_2$, and check for closure using Eq. (6). The use of a computer is clearly indicated. The final result is $T_2 \approx 125$ K.

PROBLEM 11.78

KNOWN: A substance has the property relations

$$(\text{vapor phase}) \quad v = \frac{RT}{P} - \frac{B}{T^2}$$

$$(\text{sat. p vs. } T) \quad \ln P_{\text{sat}} = 12 - \frac{2400}{T}$$

$$(\text{ideal gas}) \quad c_p = \text{constant} \quad 0 < T < 300^\circ\text{F}$$

$$(\text{Joule-Thomson}) \quad \mu_J (10 \text{ lbf/in}^2, 200^\circ\text{F}) = 0.004^\circ\text{R} \cdot \text{ft}^2/\text{lbf}$$

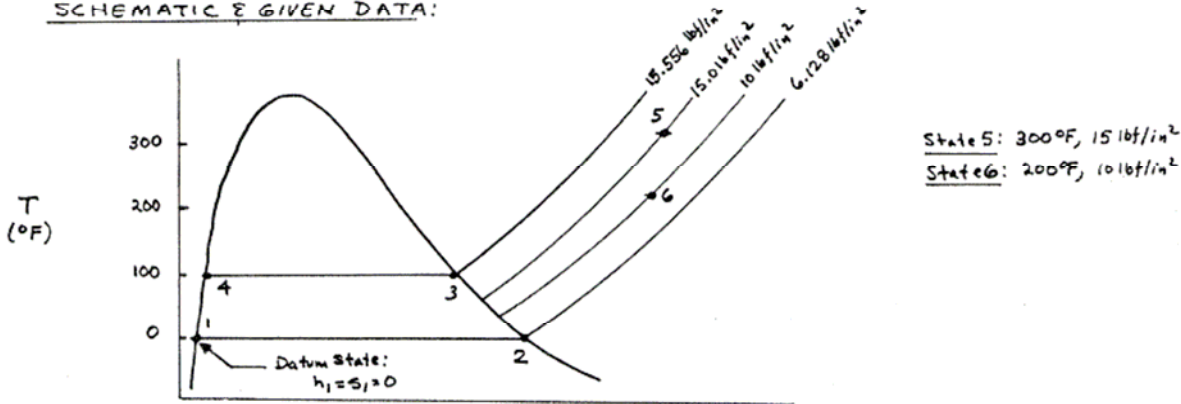
$$\text{where } v: \text{ft}^3/\text{lb}, T: ^\circ\text{R}, p: \text{lbf/ft}^2, R = 50 \text{ ft} \cdot \text{lbf}/\text{lb} \cdot ^\circ\text{R}, B = 100 \text{ ft}^5(^{\circ}\text{R})^2/\text{lb} \cdot \text{lbf}$$

FIND: (a) Complete the table

T (°F)	p (lbf/in ²)	v (ft ³ /lb)		h (Btu/lb)		s (Btu/lb·°R)	
-	-	v _f	v _g	h _f	h _g	s _f	s _g
0	-	0.03	-	0	-	0	-
100	-	0.03	-	-	-	-	-

(b) Evaluate v, h, s at 300°F, 15 lbf/in²

SCHEMATIC & GIVEN DATA:



ANALYSIS: (a) Using the given expression for $\ln P_{\text{sat}}$

$$\text{at } 0^\circ\text{F}: \quad \ln P_{\text{sat}} = 12 - \frac{2400}{460} \Rightarrow P_2 = P_1 = 882.37 \text{ lbf/ft}^2 \quad (6.128 \text{ lbf/in}^2)$$

$$\text{at } 100^\circ\text{F}: \quad \ln P_{\text{sat}} = 12 - \frac{2400}{560} \Rightarrow P_3 = P_4 = 2240.12 \text{ lbf/ft}^2 \quad (15.556 \text{ lbf/in}^2)$$

State 2: With the above pressure value, and the vapor phase equation of state,

$$v_2 = \frac{RT_2}{P_2} - \frac{B}{T_2^2} = \frac{(50 \text{ ft} \cdot \text{lbf}/\text{lb} \cdot ^\circ\text{R})(460^\circ\text{R})}{882.37 \text{ lbf/ft}^2} - \frac{(100 \text{ ft}^5(^{\circ}\text{R})^2/\text{lb} \cdot \text{lbf})(882.37 \text{ lbf/ft}^2)}{(460^\circ\text{R})^2}$$

$$= 25.65 \text{ ft}^3/\text{lb}$$

Similarly

$$v_3 = \frac{RT_3}{P_3} - \frac{B}{T_3^2} = \frac{(50)(560)}{2240.12} - \frac{(100)(2240.12)}{(560)^2} = 11.79 \text{ ft}^3/\text{lb}$$

$$v_5 = \frac{RT_5}{P_5} - \frac{B}{T_5^2} = \frac{(50)(760)}{2160} - \frac{(100)(2240.12)}{(760)^2} = 17.20 \text{ ft}^3/\text{lb}$$

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Problem 11-78 continued

Using the Clapeyron equation, Eq. 11.40,

$$h_2 = h_1 + T_2(v_g - v_f) \left(\frac{dP}{dT} \right)_{\text{sat}}$$

Since

$$P_{\text{sat}} = \exp \left(12 - \frac{2400}{T} \right)$$

$$\frac{dP_{\text{sat}}}{dT} = \frac{2400}{T^2} \exp \left(12 - \frac{2400}{T} \right) = \frac{2400 P_{\text{sat}}}{T^2}$$

So

$$\begin{aligned} h_2 &= T_2(v_g - v_f) \left[\frac{2400 P_2}{T_2^2} \right] = \frac{(2400)(v_g - v_f) P_2}{T_2} \\ &= \frac{(2400)(25.65 - 0.08) \left(\frac{11^3}{16} \right) (882.37 \text{ lbf/ft}^2) \left| \frac{8 \text{ ft}}{778 \text{ ft}^2/\text{lbf}} \right|}{460^\circ \text{R}} = 151.60 \frac{\text{Btu}}{\text{lb}} \end{aligned}$$

Then, with Eq. 11.38

$$s_2 - s_1 = \frac{h_2 - h_1}{T} \Rightarrow s_2 = \frac{151.60}{460} = 0.330 \frac{\text{Btu}}{\text{lb} \cdot ^\circ \text{R}}$$

State 3. With Eq. 11.82

$$h(T_3, P_3) = h^*(T_3) + \int_0^{P_3} \left[v - T \left(\frac{\partial v}{\partial T} \right)_P \right] dP \quad (1)$$

$$h(T_2, P_2) = h^*(T_2) + \int_0^{P_2} \left[v - T \left(\frac{\partial v}{\partial T} \right)_P \right] dP \quad (2)$$

where with the given equation of state

$$\left[v - T \left(\frac{\partial v}{\partial T} \right)_P \right] = \left(\frac{RT}{P} - \frac{BP}{T^2} \right) - T \left(\frac{R}{P} + \frac{2BP}{T^3} \right) = -\frac{3BP}{T^2} \quad (3)$$

Accordingly, with Eqs. (1)-(3)

$$\begin{aligned} h_3 - h_2 &= h^*(T_3) - h^*(T_2) + \int_0^{P_3} \left(-\frac{3BP}{T^2} \right) dP - \int_0^{P_2} \left(-\frac{3BP}{T^2} \right) dP \\ &= \int_{T_2}^{T_3} c_{p0} dT - \frac{3BP^2}{2T^2} + \frac{3BP^2}{2T^2} = c_{p0}[T_3 - T_2] - \frac{3B}{2} \left[\left(\frac{P_3}{T_3} \right)^2 - \left(\frac{P_2}{T_2} \right)^2 \right] \quad (4) \end{aligned}$$

where c_{p0} is given in the problem statement as constant.

To calculate $h_3 - h_2$ requires the value of c_{p0} . This can be determined as follows: Applying the test for exactness to Eq. 11.57 gives

$$\left(\frac{\partial c_p}{\partial P} \right)_T = -T \frac{\partial^2 v}{\partial T^2} \bigg|_P$$

Then, paralleling the development of Eq. 11.82

$$c_p(T, P) - c_{p0}(T) = -T \int_0^P \left(\frac{\partial^2 v}{\partial T^2} \right)_P dP$$

From the given equation of state $(\partial^2 v / \partial T^2)_P = -6BP / T^4$. Thus,

$$c_p(T, P) - c_{p0}(T) = -T \int_0^P \left(-\frac{6BP}{T^4} \right) dP = \frac{3BP^2}{T^3}$$

Rearranging this

$$c_{p0} = c_p(T, P) - \frac{3BP^2}{T^3} \quad (5)$$

Thus c_{p0} can be found from the value of c_p at any state. The value

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Problem 11-78 continued

of C_p at State 5 can be found using the Joule-Thomson coefficient. Thus, with Eq. 11.77

$$C_p = \frac{1}{M_J} \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right] = \frac{3BP}{M_J T^2}$$

At $T = 660^\circ R$, $p = 1440 \text{ lbf/ft}^2$ this gives

$$C_p = \frac{3(100 \text{ ft}^3 \cdot (^\circ R)^2 / 16.1 \text{ lbf}) (1440 \text{ lbf/ft}^2)}{(0.00408 \cdot \text{ft}^3 / \text{lbf}) (660^\circ R)^2} \left| \frac{\text{Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| = 0.3187 \frac{\text{Btu}}{16^\circ R}$$

Then, with Eq. (5)

$$C_{p0} = 0.3187 - \frac{(3)(100)(1440)^2}{(660)^3} \left| \frac{1}{778} \right| = 0.3159 \frac{\text{Btu}}{16^\circ R}$$

Returning to Eq. (4)

$$\begin{aligned} h_3 - h_2 &= 0.3159(100) - \frac{3(100 \text{ ft}^3 \cdot (^\circ R)^2 / 16.1 \text{ lbf})}{2} \left[\left(\frac{2240.12}{560} \right)^2 - \left(\frac{882.37}{460} \right)^2 \right] \left(\frac{1 \text{ lbf/ft}^2}{^\circ R} \right) \left| \frac{\text{Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \\ &= 31.59 - 2.38 = 29.21 \text{ Btu/lb} \end{aligned}$$

Then, with the previously determined value for h_2 , $h_3 = 180.81 \text{ Btu/lb}$.

With Eq. 11.90

$$s(T, p) - s^*(T, p) = \int_0^p \left[\frac{R}{p} - \left(\frac{\partial v}{\partial T} \right)_p \right] dp$$

Using the given equation of state

$$s(T, p) - s^*(T, p) = \int_0^p \left[\frac{R}{p} - \left(\frac{R}{p} + \frac{2BP}{T^3} \right) \right] dp = - \frac{BP^2}{T^3} \quad (6)$$

Using Eq. (6),

$$s_3 - s_3^* = - \frac{BP_3^2}{T_3^3}, \quad s_2 - s_2^* = - \frac{BP_2^2}{T_2^3}$$

Accordingly

$$\begin{aligned} s_3 - s_2 &= s_3^* - s_2^* - B \left[\frac{P_3^2}{T_3^3} - \frac{P_2^2}{T_2^3} \right] \\ &= C_{p0} \ln \frac{T_3}{T_2} - R \ln \frac{P_3}{P_2} - B \left[\frac{P_3^2}{T_3^3} - \frac{P_2^2}{T_2^3} \right] \end{aligned}$$

Substituting values

$$\begin{aligned} s_3 - s_2 &= 0.3159 \ln \left[\frac{560}{460} \right] - \left(\frac{50}{778} \right) \ln \frac{2240.12}{882.37} - \\ &\quad \left(\frac{100}{778} \right) \left[\frac{(2240.12)^2}{(560)^3} - \frac{(882.37)^2}{(460)^3} \right] \\ &= -0.00038 \frac{\text{Btu}}{16^\circ R} \end{aligned}$$

Thus, with the previously determined value for s_2 , $s_3 = 0.3296 \text{ Btu/}16^\circ R$.

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Problem 11-78 continued

State 4. Using the Clapeyron equation

$$h_3 - h_4 = T_3 (v_3 - v_4) \left(\frac{dp}{dT} \right)_{\text{sat}}$$

Thus

$$h_4 = h_3 - \frac{2400(v_3 - v_4) P_3}{T_3} = 180.81 - \frac{2400(11.79 - 0.03)(2240.12)}{(560)(778)} \\ = 35.69 \text{ Btu/lb}$$

And with Eq. 11.38

$$s_3 - s_4 = \frac{h_3 - h_4}{T_3} = 0.259 \Rightarrow s_4 = 0.0705 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}$$

SUMMARY:

T (°F)	P (lbf/in²)	v _f	v _g	h _f	h _g	s _f	s _g
0	6.128	0.03	25.65	0	151.6	0	0.330
100	15.556	0.03	11.79	35.69	180.81	0.0705	0.3296

(b) To evaluate v₅, h₅, s₅.

With the equation of state, v₅ = 17.20 ft³/lb (see page 1).

Following the same procedures as used to evaluate h₃ and s₃

$$h_5 = h_2 + c_{p0}(T_5 - T_2) - \frac{3B}{2} \left[\left(\frac{P_5}{T_5} \right)^2 - \left(\frac{P_2}{T_2} \right)^2 \right] \\ = 151.6 + 0.3159(300) - \frac{(3)(100)}{(2)(778)} \left[\left(\frac{2160}{760} \right)^2 - \left(\frac{882.37}{460} \right)^2 \right] \\ = 245.52 \frac{\text{Btu}}{\text{lb}} \quad \leftarrow h_5$$

$$s_5 = s_2 + c_{p0} \ln \frac{T_5}{T_2} - R \ln \frac{P_5}{P_2} - B \left[\frac{P_5^2}{T_5^3} - \frac{P_2^2}{T_2^3} \right] \\ = 0.3296 + 0.3159 \ln \left(\frac{760}{460} \right) - \left(\frac{50}{778} \right) \ln \frac{2160}{882.37} - \\ \left(\frac{100}{778} \right) \left[\frac{(2160)^2}{(760)^3} - \frac{(882.37)^2}{(460)^3} \right] \\ = 0.4303 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \quad \leftarrow s_5$$

PROBLEM 11.79

FIND: Beginning with Eq. 11.90 derive Eq. 11.91.

ANALYSIS:

$$s(T, p) - s^*(T, p) = \int_0^p \left[\frac{R}{p} - \left(\frac{\partial v}{\partial T} \right)_p \right] dp$$

With $v = ZRT/p$

$$\left(\frac{\partial v}{\partial T} \right)_p = \frac{RZ}{p} + \frac{RT}{p} \left(\frac{\partial Z}{\partial T} \right)_p$$

Thus

$$\left[\frac{R}{p} - \left(\frac{\partial v}{\partial T} \right)_p \right] = \frac{R}{p} (1 - Z) - \frac{RT}{p} \left(\frac{\partial Z}{\partial T} \right)_p$$

With $T = T_c T_R$, $p = p_c p_R$ $(\partial Z / \partial T)_p = Y_{Tc} (\partial Z / \partial T_R)_{p_R}$

$$\left[\frac{R}{p} - \left(\frac{\partial v}{\partial T} \right)_p \right] = \frac{R}{p_R p_c} (1 - Z) - \frac{RT_R T_c}{p_R p_c} \cdot \frac{1}{T_c} \left(\frac{\partial Z}{\partial T_R} \right)_{p_R}$$

Collecting results

$$\begin{aligned} s(T, p) - s^*(T, p) &= \int_0^p \frac{R}{p_R p_c} (1 - Z) dp - \int_0^p \frac{RT_R T_c}{p_R p_c} \left(\frac{\partial Z}{\partial T_R} \right)_{p_R} dp \\ &= R \left[\int_0^{p_R} (1 - Z) \frac{dp_R}{p_R} - \int_0^{p_R} T_R \left(\frac{\partial Z}{\partial T_R} \right)_{p_R} \frac{dp_R}{p_R} \right] \end{aligned}$$

So

$$\frac{h^*(T) - h(T, p)}{R T_c T_R} \quad (\text{Eq. 11.84})$$

$$\frac{s^*(T, p) - s(T, p)}{R} = \frac{h^*(T) - h(T, p)}{R T_c T_R} + \int_0^{p_R} (Z - 1) \frac{dp_R}{p_R}$$

Or on a per mole basis

$$\frac{\bar{s}^*(T, p) - \bar{s}(T, p)}{R} = \frac{\bar{h}^*(T) - \bar{h}(T, p)}{R T_c T_R} + \int_0^{p_R} (Z - 1) \frac{dp_R}{p_R} \quad \leftarrow$$

PROBLEM 11.80

FIND: (a) Derive an expression giving $[u(T, v) - u^*(T)]$. (b) Derive an expression giving $[s(T, v) - s^*(T, v)]$.

ANALYSIS: (a) Eq. 11.47 gives

$$\left(\frac{\partial u}{\partial v}\right)_T = T \left(\frac{\partial p}{\partial T}\right)_v - p$$

Integrating from specific volume v' to v at fixed T

$$u(T, v) - u(T, v') = \int_{v'}^v \left[T \left(\frac{\partial p}{\partial T}\right)_v - p \right] dv$$

Adding and subtracting $u^*(T)$ on the left side

$$[u(T, v) - u^*(T)] - [u(T, v') - u^*(T)] = \int_{v'}^v \left[T \left(\frac{\partial p}{\partial T}\right)_v - p \right] dv \quad (1)$$

As pressure tends to zero at fixed temperature, the internal energy of a substance approaches that of its ideal gas model. However, $\lim_{p \rightarrow 0} (at \text{ fixed } T)$ is equivalent to $\lim_{v' \rightarrow \infty} (at \text{ fixed } T)$. Accordingly, Eq. (1) becomes in the limit as $v' \rightarrow \infty$ (at fixed T)

$$\lim_{v' \rightarrow \infty} \left\{ [u(T, v) - u^*(T)] - [u(T, v') - u^*(T)] \right\} = \int_{\infty}^v \left[T \left(\frac{\partial p}{\partial T}\right)_v - p \right] dv$$

or

$$u(T, v) - u^*(T) = \int_{\infty}^v \left[T \left(\frac{\partial p}{\partial T}\right)_v - p \right] dv \quad \leftarrow (a)$$

(b) Eq. 11.34 gives

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

Integrating from specific volume v' to v at fixed T

$$s(T, v) - s(T, v') = \int_{v'}^v \left(\frac{\partial p}{\partial T}\right)_v dv \quad (2)$$

For an ideal gas $p = RT/v$, so $(\partial p / \partial T)_v = R/v$. Using this, Eq. (2) gives for an ideal gas

$$s^*(T, v) - s^*(T, v') = \int_{v'}^v \frac{R}{v} dv \quad (3)$$

Subtracting Eq. (3) from Eq. (2)

$$[s(T, v) - s^*(T, v)] - [s(T, v') - s^*(T, v')] = \int_{v'}^v \left[\left(\frac{\partial p}{\partial T}\right)_v - \frac{R}{v} \right] dv \quad (4)$$

As noted in part (a), as specific volume tends to infinity at fixed temperature, the specific entropy of a substance approaches that of its ideal gas model. Thus, in the limit as $v' \rightarrow \infty$ (at fixed T), Eq. (4) gives

$$s(T, v) - s^*(T, v) = \int_{\infty}^v \left[\left(\frac{\partial p}{\partial T}\right)_v - \frac{R}{v} \right] dv \quad \leftarrow (b)$$

PROBLEM 11.81

KNOWN: The equation of state is

$$Z = 1 + B(T_R) P_R$$

FIND: Derive expressions for the enthalpy and entropy departures.

ANALYSIS: Beginning with Eq. 11.84

$$\frac{\bar{h}^*(T) - \bar{h}(T, P)}{RT_c} = T_R^2 \int_0^{P_R} \left(\frac{\partial Z}{\partial T_R} \right)_{P_R} \frac{dP_R}{P_R}$$

where

$$\left(\frac{\partial Z}{\partial T_R} \right)_{P_R} = \frac{dB}{dT_R} P_R$$

Thus

$$\begin{aligned} \frac{\bar{h}^* - \bar{h}}{RT_c} &= T_R^2 \int_0^{P_R} \left(\frac{dB}{dT_R} P_R \right) \frac{dP_R}{P_R} \\ &= T_R^2 \left(\frac{dB}{dT_R} \right) P_R \end{aligned}$$

Beginning with Eq. 11.91

$$\begin{aligned} \frac{\bar{s}^*(T, P) - \bar{s}(T, P)}{R} &= \frac{\bar{h}^* - \bar{h}}{RT_c T_R} + \underbrace{\int_0^{P_R} (Z-1) \frac{dP_R}{P_R}}_{B P_R} \\ &= \frac{T_R^2 \left(\frac{dB}{dT_R} \right) P_R}{T_R} + B P_R \\ &= P_R T_R \frac{dB}{dT_R} + B P_R \\ &= P_R \frac{d[TRB]}{dT_R} \end{aligned}$$

PROBLEM 11.82

KNOWN: An expression is provided for the enthalpy departure function when an equation of state explicit in pressure is in hand.

FIND: (a) Derive the expressions, (b) Using (a), obtain an expression for the enthalpy departure for a gas obeying the Redlich-Kwong equation, (c) Using (b), evaluate $\Delta \bar{h}$ for CO_2 in a process at 300K from 50 to 20 bar.

ANALYSIS: (a) Beginning with $h = h(T, v)$, write $dh = \left(\frac{\partial h}{\partial T}\right)_v dT + \left(\frac{\partial h}{\partial v}\right)_T dv$. Then, at fixed T

$$dh|_T = \left(\frac{\partial h}{\partial v}\right)_T dv \Rightarrow h(T, v) - h(T, v') = \int_{v'}^v \left(\frac{\partial h}{\partial v}\right)_T dv \quad (1)$$

With $h = u + pv$, $(\partial h / \partial v)_T = (\partial u / \partial v)_T + (\partial(pv) / \partial v)_T$. Introducing Eq. 11.47, this becomes

$$\left(\frac{\partial h}{\partial v}\right)_T = \left[T \left(\frac{\partial p}{\partial T}\right)_v - p\right] + \left(\frac{\partial(pv)}{\partial v}\right)_T \quad (2)$$

Inserting Eq. (2) into Eq. (1)

$$h(T, v) - h(T, v') = \int_{v'}^v \left[T \left(\frac{\partial p}{\partial T}\right)_v - p\right] dv + [pv](T, v) - [pv](T, v') \quad (3)$$

In the limit as $p \rightarrow 0$, $v' \rightarrow \infty$ and $[pv](T, v') \rightarrow RT$, $h(T, v') \rightarrow \bar{h}^*(T)$, so

$$h(T, v) - \bar{h}^*(T) = \int_{\infty}^v \left[T \left(\frac{\partial p}{\partial T}\right)_v - p\right] dv + pv - RT$$

or

$$\bar{h}^*(T) - h(T, v) = RT \left[1 - \frac{pv}{RT} - \frac{1}{RT} \int_{\infty}^v \left[T \left(\frac{\partial p}{\partial T}\right)_v - p\right] dv \right]$$

With $T = T_R T_C$, and expressing this on a molar basis

$$\frac{\bar{h}^*(T) - \bar{h}(T, \bar{v})}{R T_C} = T_R \left[1 - Z - \frac{1}{R T} \int_{\infty}^{\bar{v}} \left[T \left(\frac{\partial p}{\partial T}\right)_v - p\right] d\bar{v} \right] \quad (a)$$

(b) Using the Redlich-Kwong equation of state,

$$T \left(\frac{\partial p}{\partial T}\right)_v = \frac{RT}{\bar{v} - b} + \frac{1}{2} \frac{a}{\bar{v}(\bar{v} + b)\sqrt{T}} = p + \frac{3}{2} \frac{a}{\bar{v}(\bar{v} + b)\sqrt{T}} \Rightarrow T \left(\frac{\partial p}{\partial T}\right)_v - p = \frac{3}{2} \frac{a}{\bar{v}(\bar{v} + b)\sqrt{T}}$$

$$\therefore \frac{\bar{h}^* - \bar{h}(T, \bar{v})}{R T_C} = T_R \left[1 - Z - \frac{3/2 a}{R T^{3/2}} \int_{\infty}^{\bar{v}} \frac{d\bar{v}}{\bar{v}(\bar{v} + b)} \right]$$

$$\left[-\frac{1}{b} \ln \left(1 + \frac{b}{\bar{v}} \right) \right]_{\infty}^{\bar{v}} = -\frac{1}{b} \ln \left(1 + \frac{b}{\bar{v}} \right)$$

So

$$\frac{\bar{h}^* - \bar{h}}{R T_C} = T_R \left[1 - Z + \frac{3a}{2b R T^{3/2}} \ln \left(1 + \frac{b}{\bar{v}} \right) \right] \quad (4) \quad (b)$$

(c) At 300K, $p_1 = 50 \text{ bar}$, $p_2 = 20 \text{ bar}$. Obtaining a, b from Table A-24, the Redlich-Kwong equation can be solved to give $\bar{v}_1 = 0.3475 \text{ m}^3/\text{kmol}$, $\bar{v}_2 = 1.119 \text{ m}^3/\text{kmol}$. The corresponding values of Z are $Z_1 = 0.697$, $Z_2 = 0.897$. With Eq. (4)

$$\bar{h}_2 - \bar{h}_1 = RT \left[Z_2 - Z_1 + \frac{3a}{2b R T^{3/2}} \ln \left[\frac{1 + b/\bar{v}_1}{1 + b/\bar{v}_2} \right] \right] \quad (5)$$

Inserting values

$$\bar{h}_2 - \bar{h}_1 = \left(8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \right) (300\text{K}) \left[0.897 - 0.697 + \frac{3(64.43 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2 (\text{K})^{1/2}) | 10^5 \text{ N/m}^2/\text{bar} |}{2(0.02963 \frac{\text{m}^3}{\text{kmol}}) (8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}}) (300)^{3/2}} \ln \left[\frac{1 + \frac{0.02963}{0.3475}}{1 + \frac{0.02963}{1.119}} \right] \right]$$

$$= 1548 \frac{\text{kJ}}{\text{kmol}} \quad (c)$$

PROBLEM 11.83

KNOWN: The equation of state has the form

$$Z = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \frac{P_R}{T_R}$$

FIND: For water vapor at 550°C, 20 MPa, evaluate v and c_p . Compare with data from Table A-4 and Fig. 3.9, respectively.

ANALYSIS: At 550°C, 20 MPa, Table A-4 gives $v = 0.01655 \text{ m}^3/\text{kg}$. Fig. 3.9 gives $c_p \approx 2.95 \text{ kJ/kg} \cdot \text{K}$.

The equation of state can be expressed alternatively as

$$\frac{Pv}{RT} = 1 + \left[\frac{1}{8} - \frac{27/64}{T/T_c} \right] \left(\frac{P/P_c}{T/T_c} \right) \Rightarrow v = \frac{RT}{P} + \left[\frac{1}{8} - \frac{(27/64)T_c}{T} \right] \left(\frac{RT_c}{P_c} \right)$$

Substituting values from Table A-1

$$v = \left(\frac{8314 \text{ N} \cdot \text{m}}{18.02 \text{ kg} \cdot \text{K}} \right) \left(\frac{823.15 \text{ K}}{200 \times 10^5 \text{ Pa}} \right) + \left(\frac{1}{8} - \frac{(27/64)(647.3)}{823.15} \right) \left(\frac{8314 \text{ N} \cdot \text{m}}{18.02 \text{ kg} \cdot \text{K}} \right) \left(\frac{647.3 \text{ K}}{220.9 \times 10^5 \text{ Pa}} \right)$$

$$= 0.0162 \text{ m}^3/\text{kg}$$

Comparing with the steam table value

$$\% = \frac{(0.0162 - 0.01655)}{0.01655} (100) = -2.1$$

Differentiating the given equation of state

$$\left(\frac{\partial Z}{\partial T_R} \right)_{P_R} = -\frac{1}{8} P_R T_R^{-2} + \frac{27}{32} P_R T_R^{-3} \Rightarrow \frac{1}{P_R} \left(\frac{\partial Z}{\partial T_R} \right)_{P_R} = -\frac{1}{8} T_R^{-2} + \frac{27}{32} T_R^{-3}$$

Substituting this into Eq. 11.84

$$\frac{h^*(T) - h(T, P)}{RT_c} = T_R^2 \int_0^{P_R} \left[-\frac{1}{8} T_R^{-2} + \frac{27}{32} T_R^{-3} \right] dP_R = \left[-\frac{1}{8} + \frac{27}{32} T_R^{-1} \right] P_R$$

or

$$h(T, P) - h^*(T) = RT_c \left[\frac{1}{8} - \frac{27}{32} \frac{1}{T_R} \right] P_R$$

But with $c_p = (\partial h / \partial T)_P$, $c_{p0} = dh^*/dT$, this yields upon differentiation

$$c_p - c_{p0} = \frac{\partial}{\partial T_R} \left[RT_c \left[\frac{1}{8} - \frac{27}{32} \frac{1}{T_R} \right] P_R \right] \frac{dT_R}{dT}$$

$$= \left[\frac{27}{32} R \right] \left(\frac{P_R}{T_R^2} \right)$$

Using P_c, T_c from Table A-1, $P_R = (200/220.9) = 0.905$, $T_R = (823.15/647.3) = 1.272$. So

$$c_p - c_{p0} = \left(\frac{27}{32} \right) \left(\frac{8314 \text{ kJ}}{18.02 \text{ kg} \cdot \text{K}} \right) \left(\frac{0.905}{(1.272)^2} \right) = 0.218 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Evaluating c_{p0} from the appropriate expression given in Table A-21,

$c_{p0} = 2.163 \text{ kJ/kg} \cdot \text{K}$. Finally

$$c_p = 2.163 + 0.218 = 2.381 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Comparing with the value from Fig. 3.9

$$\textcircled{1} \quad \% = \frac{(2.381 - 2.95)}{2.95} (100) = -19.3$$

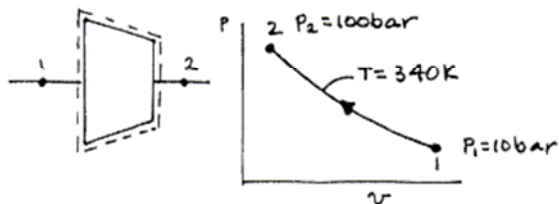
1. The simple equation of state used does a fair job in representing the p - v - T variation but does not suffice to yield other properties, such as c_p , accurately.

PROBLEM 11.84

KNOWN: Steady-state operating data are provided for a compressor for which ethylene is the working fluid.

FIND: Evaluate per kg of ethylene flowing the (a) work required, (b) heat transfer.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is at steady state.
2. Compression occurs without internal irreversibilities at fixed temperature.
3. Kinetic and potential energy effects can be ignored.

ANALYSIS: For this case, Eq. 6.51 is applicable. Since temperature is constant, $(\dot{Q}_{cv}/\dot{m})_{int, rev} = T(s_2 - s_1)$, where $(s_2 - s_1)$ is obtained from Eq. 11.92.

From Table A-1, $P_c = 51.2$ bar, $T_c = 283$ K. Thus, $T_R = (340/283) = 1.2$, $P_{R1} = (10/51.2) = 0.2$, $P_{R2} = (100/51.2) = 1.95$. Then, from Fig. A-5, we get $(\frac{\bar{s}^* - \bar{s}}{R})_1 = 0.08$, $(\frac{\bar{s}^* - \bar{s}}{R})_2 = 1.25$.

Thus

$$\begin{aligned} \bar{s}_2 - \bar{s}_1 &= \underbrace{\bar{s}^0(T_2) - \bar{s}^0(T_1)}_{=0 \text{ } T_1=T_2} - \bar{R} \ln \frac{P_2}{P_1} - \bar{R} \left[\left(\frac{\bar{s}^* - \bar{s}}{R} \right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{R} \right)_1 \right] = -\bar{R} \left[\ln \frac{P_2}{P_1} + \left(\frac{\bar{s}^* - \bar{s}}{R} \right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{R} \right)_1 \right] \\ &= -(8.314 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \left[\ln 10 + 1.25 - 0.08 \right] = -28.87 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \end{aligned}$$

$$\Rightarrow \left(\frac{\dot{Q}_{cv}}{\dot{m}} \right)_{int, rev} = 340 \text{ K} \left[-28.87 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \right] \left[\frac{1}{28.05 \text{ kg/kmol}} \right] = -350 \text{ kJ/kg} \quad \leftarrow (b)$$

Reducing mass and energy rate balances gives $(\dot{W}_{cv}/\dot{m}) = (\dot{Q}_{cv}/\dot{m}) + (h_1 - h_2)$. From Fig. A-4, we get $(\frac{\bar{h}^* - \bar{h}}{RT_c})_1 = 0.15$, $(\frac{\bar{h}^* - \bar{h}}{RT_c})_2 = 2$. Then, with Eq. 11.85

$$\bar{h}_2 - \bar{h}_1 = \underbrace{\bar{h}_2^* - \bar{h}_1^*}_{=0 \text{ } T_1=T_2} - \bar{R} T_c \left[\left(\frac{\bar{h}^* - \bar{h}}{RT_c} \right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{RT_c} \right)_1 \right] = -(8.314)(283) [2 - 0.15] = -4352.8 \frac{\text{kJ}}{\text{kmol}}$$

$$\Rightarrow h_1 - h_2 = \frac{4352.8}{28.05} = 155 \frac{\text{kJ}}{\text{kg}}$$

Finally,

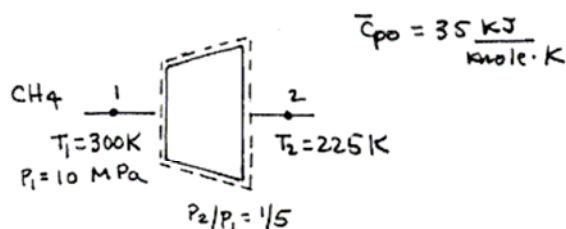
$$\left(\frac{\dot{W}_{cv}}{\dot{m}} \right)_{int, rev} = -350 + 155 = -195 \text{ kJ/kg} \quad \leftarrow (a)$$

PROBLEM 11.85

KNOWN: Steady-state operating data are provided for the adiabatic expansion of CH₄ through a turbine.

FIND: Determine the work developed per unit mass flowing and compare with the value obtained using the ideal gas model only.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is at steady state.
2. $\dot{Q}_{cv} = 0$, kinetic & potential energy effects can be ignored.
3. The ideal gas specific heat $\bar{c}_{p0}$ is constant.

ANALYSIS: Reducing mass and energy rate balances, we get

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = \cancel{\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)} + (h_1 - h_2) \Rightarrow \left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = h_1 - h_2, \text{ where } (h_1 - h_2) \text{ can be found using Eq. 11.85:}$$

$$h_1 - h_2 = \frac{1}{M} \left[\underbrace{(\bar{h}_1^* - \bar{h}_2^*)}_{\bar{c}_{p0}(T_1 - T_2)} - \bar{R}T_c \left(\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 \right) \right]$$

With data from Table A-1, $M = 16.04\text{ kg/kmol}$, $T_c = 191\text{ K}$, $P_c = 46.4\text{ bar}$. Thus

$$T_{R1} = \frac{300}{191} = 1.57, \quad T_{R2} = \frac{225}{191} = 1.18, \quad P_{R1} = \frac{100}{46.4} = 2.16, \quad P_{R2} = \frac{20}{46.4} = 0.43.$$

Then, from Fig. A-4

$$\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 = 1, \quad \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 = 0.34$$

So

$$\begin{aligned} \left(\frac{\dot{W}_{cv}}{\dot{m}}\right) &= \left[\frac{1}{16.04\text{ kg/kmol}} \right] \left[(35\text{ kJ/kmol}\cdot\text{K})(300 - 225)\text{ K} - (8.314\text{ kJ/kmol}\cdot\text{K})(191\text{ K})[1 - 0.34] \right] \\ &= \left(\frac{1}{16.04}\right)[2625 - 1048] = 98.3\text{ kJ/kg} \end{aligned}$$

The ideal gas value is

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = \frac{1}{M} [\bar{h}_1^* - \bar{h}_2^*] = \frac{1}{M} [\underbrace{\bar{c}_{p0}(T_1 - T_2)}_{2625}] = 163.7\text{ kJ/kg}$$

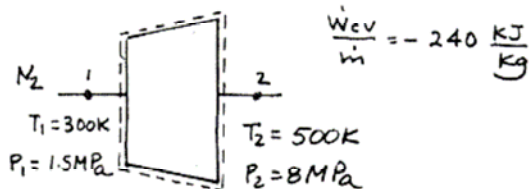
which is 62% greater than the value found using Eq. 11.85.

PROBLEM 11.86

KNOWN: Steady-state operating data is provided for a compressor for which N_2 is the working fluid.

FIND: Determine the heat transfer per unit mass of N_2 flowing.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is at steady state.
2. Kinetic & potential energy changes can be ignored.

ANALYSIS: Reducing mass and energy rate balances, we get

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1), \text{ where } (h_2 - h_1) \text{ can be found using Eq. 11.85.}$$

With data from Table A-1, $M = 28.01$, $T_c = 126K$, $P_c = 33.9 \text{ bar}$,

$$TR_1 = \frac{300}{126} = 2.38, \quad TR_2 = \frac{500}{126} = 3.97, \quad PR_1 = \frac{15}{33.9} = 0.44, \quad PR_2 = \frac{80}{33.9} = 2.36.$$

From Figure A-4

$$\left(\frac{\bar{h}^* - \bar{h}}{RT_c} \right)_1 = 0.12, \quad \left(\frac{\bar{h}^* - \bar{h}}{RT_c} \right)_2 = 0.09$$

From Table A-23, $\bar{h}_1^* = 8723 \text{ kJ/kmol}$, $\bar{h}_2^* = 14,581 \text{ kJ/kmol}$.

Thus

$$h_2 - h_1 = \frac{1}{M} \left[\bar{h}_2^* - \bar{h}_1^* - RT_c \left[\left(\frac{\bar{h}^* - \bar{h}}{RT_c} \right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{RT_c} \right)_1 \right] \right]$$

$$\begin{aligned} \textcircled{1} \quad &= \frac{1}{28.01} \left[\underbrace{(14,581 - 8723)}_{5858} - \underbrace{(8.314)(126) [0.09 - 0.12]}_{-31.4} \right] \\ &= 210.3 \text{ kJ/kg} \end{aligned}$$

Finally,

$$\frac{\dot{Q}_{cv}}{\dot{m}} = -240 \frac{kJ}{kg} + 210.3 \frac{kJ}{kg} = -29.7 \frac{kJ}{kg}$$

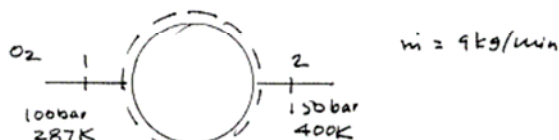
! In this case, the ideal gas model contribution dominates. As can be seen from Fig A-2, the large TR values give $Z \sim 1$ without regard for PR .

PROBLEM 11.87

KNOWN: O_2 enters a control volume at steady state with a mass flow rate of 9 kg/min at 100 bar , 287 K and is compressed adiabatically to 150 bar , 400 K .

FIND: Determine the power required and the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible.

ANALYSIS: Reducing mass, energy, and entropy balances

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2, \quad \frac{\dot{Q}_{cv}}{\dot{m}} = s_1 - s_2$$

Using Eq. 11.85 and Eq. 11.92 to evaluate these differences via the generalized plots,

$$T_{R1} = \frac{287 \text{ K}}{154 \text{ K}} = 1.86, \quad P_{R1} = \frac{100 \text{ bar}}{50.5 \text{ bar}} = 1.98 \Rightarrow \left[\frac{h^* - \bar{h}}{R T_c} \right]_1 \approx 0.63, \quad \left[\frac{s^* - \bar{s}}{R} \right]_1 \approx 0.27$$

$$T_{R2} = \frac{400}{154} = 2.60, \quad P_{R2} = \frac{150}{50.5} = 2.97 \Rightarrow \left[\frac{h^* - \bar{h}}{R T_c} \right]_2 \approx 0.43, \quad \left[\frac{s^* - \bar{s}}{R} \right]_2 \approx 0.17$$

For O_2 , Table A-23 gives $\bar{h}_1^* - \bar{h}_2^* = 8355 - 11,711 = -3356 \text{ kJ/kmol}$. Then

$$\begin{aligned} \frac{\dot{W}_{cv}}{\dot{m}} &= \frac{\bar{h}_1^* - \bar{h}_2^* - R T_c \left[\left(\frac{h^* - \bar{h}}{R T_c} \right)_1 - \left(\frac{h^* - \bar{h}}{R T_c} \right)_2 \right]}{M} \\ &= \frac{-3356 - (8.314)(154)(0.63 - 0.43)}{32} \\ &= -104.9 - 8 = -112.9 \text{ kJ/kg} \end{aligned}$$

Thus

$$\dot{W}_{cv} = \left(9 \frac{\text{kg}}{\text{min}} \right) \left(-112.9 \frac{\text{kJ}}{\text{kg}} \right) \left| \frac{\text{min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -16.94 \text{ kW} \quad \leftarrow \dot{W}_{cv}$$

With $\Delta \bar{s}^* = \bar{s}^*(T_2) - \bar{s}^*(T_1) - R \ln P_2/P_1$ and data from Table A-23

$$\Delta \bar{s}^* = 213.765 - 203.91 - 8.314 \ln \frac{150}{100} = 6.484 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

Accordingly

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\Delta \bar{s}^* - R \left[\left(\frac{s^* - \bar{s}}{R} \right)_2 - \left(\frac{s^* - \bar{s}}{R} \right)_1 \right]}{M} = \frac{6.484 - 8.314(0.17 - 0.27)}{32} = 0.2286 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Finally

$$\textcircled{1} \quad \dot{Q}_{cv} = \left(9 \frac{\text{kg}}{\text{min}} \right) \left(0.2286 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) \left| \frac{\text{min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 0.034 \frac{\text{kW}}{\text{K}} \quad \leftarrow \dot{Q}_{cv}$$

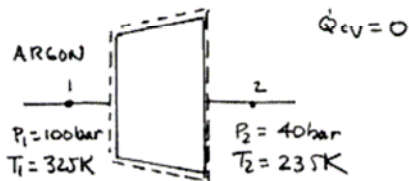
1. Owing to the approximate nature of retrieving data from the generalized charts, extreme accuracy in the calculated results should not be expected. This applies with greater effect for entropy evaluations, which are generally sensitive to roundoff.

PROBLEM 11.88

KNOWN: Steady-state operating data are provided for a turbine in which argon is the working fluid.

FIND: Determine per unit mass of argon flowing (a) the work, (b) the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is at steady state.
2. $\dot{Q}_{cv} = 0$, and kinetic & potential energy changes can be ignored.

ANALYSIS: Reducing mass, energy and entropy balances, we get

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad (1) \quad , \quad \frac{\dot{Q}_{cv}}{\dot{m}} = s_2 - s_1 \quad (2)$$

where Eqs. 11.85, 11.92 are used to find $(h_1 - h_2)$, $(s_2 - s_1)$. With data from Table A-1, $M = 39.94 \text{ kg/kmol}$, $T_c = 151 \text{ K}$, $P_c = 48.6 \text{ bar}$. Then,

$$T_{R1} = \frac{325}{151} = 2.15, \quad T_{R2} = \frac{235}{151} = 1.56, \quad P_{R1} = \frac{100}{48.6} = 2.06, \quad P_{R2} = \frac{40}{48.6} = 0.82$$

Also, from Table A-21, $\bar{c}_p^* = \frac{5}{2} \bar{R}$, and from Figs. A-4, A-5

$$\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right)_1 = 0.4, \quad \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right)_2 = 0.38, \quad \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right)_1 = 0.2, \quad \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right)_2 = 0.18$$

Then,

$$\begin{aligned} \frac{\dot{W}_{cv}}{\dot{m}} &= \frac{1}{M} \left[\underbrace{(\bar{h}_1^* - \bar{h}_2^*)}_{2.5\bar{R}(T_1 - T_2)} - \bar{R}T_c \left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right)_1 \right] \right] = \frac{8.314}{39.94} [2.5(90) - 151[0.4 - 0.38]] \\ &= 46.2 \text{ kJ/kg} \end{aligned} \quad \leftarrow (a)$$

And with Eq. 6.23

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}} &= \frac{1}{M} \left[\left(\bar{c}_p^* \ln \frac{T_2}{T_1} - \bar{R} \ln \frac{P_2}{P_1} \right) - \bar{R} \left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right)_1 \right] \right] \\ \textcircled{1} \quad &= \frac{8.314}{39.94} \left[\left(2.5 \ln \frac{235}{325} - \ln \frac{40}{100} \right) + [0.2 - 0.18] \right] = 0.026 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad \leftarrow (b) \end{aligned}$$

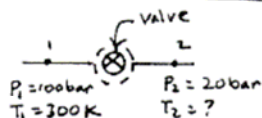
1. Owing to the approximate nature of retrieving data from the generalized charts, extreme accuracy in the calculated results should not be expected. This applies with greater effect for entropy evaluations, which are generally sensitive to roundoff.

PROBLEM 11.89

KNOWN: O_2 undergoes a throttling process from 100 bar, 300 K to 20 bar.

FIND: Determine the temperature after throttling, and compare with the value obtained using the ideal gas model.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

The expansion across the valve adheres to the throttling process model of Section 4.10, for which $h_2 = h_1$.

ANALYSIS: Using Eq. 11.85 with $h_2 = h_1$, we get

$$\bar{h}^*(T_2) - \bar{R}T_c \left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right]_2 = \bar{h}^*(T_1) - \bar{R}T_c \left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right]_1 \quad (1)$$

With data from Table A-1, $T_c = 154 \text{ K}$, $P_c = 50.5 \text{ bar}$,

$$P_{R1} = \frac{100}{50.5} = 1.98, \quad P_{R2} = \frac{20}{50.5} = 0.4, \quad T_{R1} = \frac{300}{154} = 1.95$$

Fig. A-4 gives $\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right]_1 = 0.5$. Table A-23 gives $\bar{h}_1^* = 8736 \text{ kJ/kmol}$.

Thus, Eq. (1) becomes

$$\begin{aligned} \bar{h}^*(T_2) - (8.314)(154) \left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right]_2 &= 8736 \frac{\text{kJ}}{\text{kmol}} - \underbrace{(8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}})(154 \text{ K})(0.5)}_{640.2} \\ &= 8096 \frac{\text{kJ}}{\text{kmol}} \end{aligned} \quad (2)$$

Equation (2) can then be used in a trial procedure with data from

① Table A-23: $\bar{h}_2^*$ and from Fig. A-4: $\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right]_2$. Iterating, $T_2 = 280 \text{ K}$. $\leftarrow T_2$

1. At $T_2 = 280 \text{ K}$, Table A-23 gives $\bar{h}_2^* = 8150 \text{ kJ/kmol}$. Using $T_{R2} = (280/154) = 1.82$ and $P_{R2} = 0.4$, Fig. A-4 gives $\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right]_2 = 0.1$.

With these values the left side of Eq. 2 is

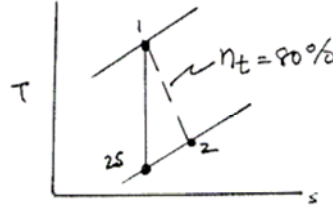
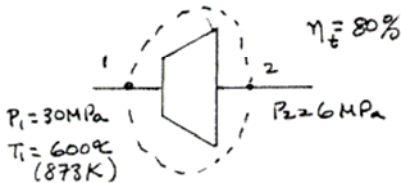
$$\bar{h}^*(T_2) - (8.314)(154) \left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right]_2 = 8150 - (8.314)(154)(0.1) = 8022 \frac{\text{kJ}}{\text{kmol}}$$

which is within 1% of the value 8096 kJ/kmol on the right side of Eq. (2). The approximate nature of obtaining data from the generalized charts does not justify additional iteration.

PROBLEM 11.90

KNOWN: Steady state operating data are provided for a steam turbine.
FIND: Using the generalized charts, determine the work developed per kg of steam flowing, and compare with the steam table result.

SCHEMATIC GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible.

ANALYSIS: Reducing mass and energy rate balances, $\dot{W}_{cv}/\dot{m} = h_1 - h_2$. Then, with the isentropic turbine efficiency

$$\frac{\dot{W}_{cv}}{\dot{m}} = \eta_t (h_1 - h_{2s})$$

STEAM TABLE SOLUTION. With steam table data, $h_1 = 3443.9 \text{ kJ/kg}$, $s_1 = 6.233 \text{ kJ/kg} \cdot \text{K}$. Then, with $s_{2s} = s_1$, $h_{2s} = 2981.6 \text{ kJ/kg}$, $T_{2s} = 329^\circ\text{C}$. Finally

$$\frac{\dot{W}_{cv}}{\dot{m}} = 0.8 (3443.9 - 2981.6) = 369.8 \text{ kJ/kg}$$

← steam tables

GENERALIZED CHART SOLUTION.

$$T_{R1} = \frac{873 \text{ K}}{647.3 \text{ K}} = 1.35, \quad P_{R1} = \frac{300 \text{ bar}}{220.9 \text{ bar}} = 1.36 \Rightarrow \left(\frac{\bar{h}^* - \bar{h}}{RT_c} \right)_1 \approx 0.91, \quad \left(\frac{\bar{s}^* - \bar{s}}{R} \right)_1 \approx 0.5$$

(Fig. A-4) (Fig. A-5)

$$P_{R2} = \frac{60}{220.9} = 0.27$$

To fix state 2s, use $\bar{s}_2 = \bar{s}_{2s} = 0$. That is, with Eq. 11.92

$$0 = [\bar{s}^0(T_{2s}) - \bar{s}^0(T_1) - \bar{R} \ln \frac{P_2}{P_1}] - \bar{R} \left[\left(\frac{\bar{s}^* - \bar{s}}{R} \right)_{2s} - \left(\frac{\bar{s}^* - \bar{s}}{R} \right)_1 \right]$$

With $\bar{s}^0(T_1)$ from Table A-23, this takes the form

$$\bar{s}^0(T_{2s}) - \bar{R} \left[\left(\frac{\bar{s}^* - \bar{s}}{R} \right)_{2s} \right] = 227.11 + 8.314 \ln \frac{6}{30} - 8.314 (0.5) = 209.57 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad (1)$$

① Eq. (1) can be used iteratively to determine state 2s. We get $T_{2s} = 600 \text{ K}$. Thus, $(T_{R2s}) = 0.93$. Table A-23 gives $\bar{h}_1^* = 30754 \text{ kJ/kmol}$, $\bar{h}_{2s}^* = 20402$, and Fig. A-4 gives

$$\left(\frac{\bar{h}^* - \bar{h}}{RT_c} \right)_{2s} = 0.45. \quad \text{Thus, Eq. 11.85 yields}$$

$$\bar{h}_1 - \bar{h}_{2s} = \bar{h}_1^* - \bar{h}_{2s}^* - \bar{R} T_c \left[\left(\frac{\bar{h}^* - \bar{h}}{RT_c} \right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{RT_c} \right)_{2s} \right] = 30754 - 20402 - \frac{8.314 (647.3)}{2476} [0.91 - 0.45]$$

$$= 7876 \text{ kJ/kmol}$$

$$\text{And } \frac{\dot{W}_{cv}}{\dot{m}} = \frac{0.8}{18.02} (7876) = 349.7 \frac{\text{kJ}}{\text{kg}}$$

← generalized charts

which is about 5% less than the steam table result.

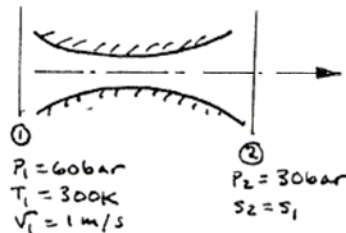
1. Using $T_{2s} = 600 \text{ K}$, Table A-23 gives $\bar{s}_{2s}^0 = 212.92$. Figure A-5 gives $\left(\frac{\bar{s}^* - \bar{s}}{R} \right)_{2s} = 0.3$. Thus, the left side of Eq. (1) is $[(212.92) - (8.314)(0.3)] = 204.4 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$, which is about 0.4% greater than the value on the right side of Eq. (1).

PROBLEM 11.91

KNOWN: O_2 expands isentropically through a nozzle. Steady-state operating data are provided.

FIND: Determine the velocity at the exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume enclosing the nozzle is at steady state.
2. The expansion is isentropic.
3. Potential energy change can be ignored.
 $W_{cv} = Q_{cv} = 0$.

ANALYSIS: Reducing mass and energy rate balances $V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$, (1) where $(h_1 - h_2)$ can be evaluated using Eq. 11.85. First, state 2 is fixed using $s_2 = s_1$ and Eq. 11.92.

From Table A-1, $M = 32$, $T_c = 154 \text{ K}$, $P_c = 50.5 \text{ bar}$. Thus,

$$P_{r1} = \frac{60}{50.5} = 1.19, \quad P_{r2} = \frac{30}{50.5} = 0.59, \quad T_{r1} = \frac{300}{154} = 1.95$$

When $s_2 = s_1$, Eq. 11.92 reduces to read

$$\bar{s}^0(T_2) - \bar{R} \left[\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right]_2 = \underbrace{\bar{s}^0(T_1)}_{\substack{\text{Table A-23} \\ 205.213}} + \bar{R} \ln \frac{P_2}{P_1} - \underbrace{\bar{R} \left[\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right]_1}_{\substack{\text{Fig A-5} \\ = 0.14}} = 205.213 + 8.314(\ln 0.5 - 0.14) = 198.29 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \quad (2)$$

- ① This result can be used to determine T_2 using trial with data from Table A-23, Fig A-5. We get $T_2 = 245 \text{ K}$.

Then, to find $(h_1 - h_2)$ use the following expression with data from Table A-23, Fig A-4,

$$\begin{aligned} \bar{h}_1 - \bar{h}_2 &= \bar{h}_1^* - \bar{h}_2^* - \bar{R} T_c \left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c} \right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c} \right)_2 \right] = 8736 - 7130 - (8.314)(154) [0.35 - 0.26] \\ &= 1491 \frac{\text{kJ}}{\text{kmol}} \end{aligned}$$

Then, returning to Eq. (1)

$$V_2 = \sqrt{\left(1 \frac{\text{m}}{\text{s}} \right)^2 + 2 \left[\frac{1491 \text{ kJ/kmol}}{32 \text{ kg/kmol}} \right] \left[\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right] \left[\frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right]} = 305.3 \text{ m/s}$$

1. At 245 K , $T_{r2} = 1.59$. Then, with $P_{r2} = 0.59$, Fig. A-5 gives

$$\left[\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right]_2 = 0.12. \text{ Table A-23 gives } \bar{s}^0(T_2) = 199.29. \text{ Thus, the left-side of}$$

Eq. (2) reads

$$\bar{s}^0(T_2) - \bar{R} \left[\frac{\bar{s}^* - \bar{s}}{\bar{R}} \right]_2 = 199.29 - 8.314(0.12) = 198.29 \text{ kJ/kmol} \cdot \text{K},$$

which agrees with the term on the right side of Eq. (2).

PROBLEM 11.92

KNOWN: A quantity of N_2 expands at a constant pressure of 80 bar from $T_1 = 220\text{ K}$ to $T_2 = 300\text{ K}$.

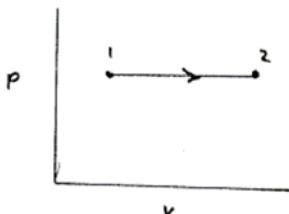
FIND: Determine w/n and q/n .

SCHEMATIC & GIVEN DATA:



$$T_1 = 220\text{ K}, T_2 = 300\text{ K}$$

$$P = 80\text{ bar}$$



ENGINEERING MODEL: The system shown in the accompanying figure experiences no change in kinetic or potential energy.

ANALYSIS: As pressure is fixed the work is given by

$$\frac{W}{n} = \int_1^2 P d\bar{v} = P[\bar{v}_2 - \bar{v}_1]$$

Then, since $Z = P\bar{v}/RT$, this equation becomes

$$\frac{W}{n} = \bar{R} [T_2 Z_2 - T_1 Z_1] \quad (1)$$

where Z is the compressibility factor. From Table A-1, $T_c = 126\text{ K}$, $P_c = 33.9\text{ bar}$, so

$$P_R = \frac{80}{33.9} = 2.36, T_{R1} = \frac{220}{126} = 1.75, T_{R2} = 2.38. \text{ From Fig. A-2, } Z_1 = 0.91 \text{ and}$$

$Z_2 = 0.99$. Then, Eq. (1) gives

$$\frac{W}{n} = (8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}) [(300)(0.99) - (220)(0.91)] \text{ K} = 805 \frac{\text{kJ}}{\text{kmol}} \quad \leftarrow \frac{w}{n}$$

An energy balance reduces to $\Delta U = Q - W$ or $Q = \Delta U + W$. Thus

$$\frac{Q}{n} = (\bar{u}_2 - \bar{u}_1) + \frac{W}{n} = (\bar{u}_2 - \bar{u}_1) + (P\bar{v}_2 - P\bar{v}_1) = \bar{h}_2 - \bar{h}_1$$

Then, with Eq. 11.85

$$\frac{Q}{n} = \bar{h}^*(T_2) - \bar{h}^*(T_1) - \bar{R}T_c \left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} \right)_1 \right]$$

with data from Table A-23 and Fig. A-4

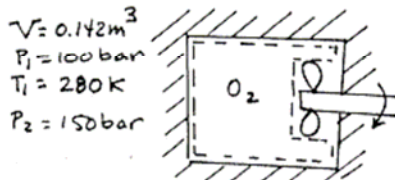
$$\frac{Q}{n} = 8123 - 6391 - (8.314)(126) [0.43 - 0.86] = 2782 \frac{\text{kJ}}{\text{kmol}} \quad \leftarrow \frac{q}{n}$$

PROBLEM 11.93

KNOWN: O_2 contained in a closed, rigid, insulated vessel is stirred by paddle wheel. State data are provided.

FIND: Determine (a) the final temperature, (b) the work, and (c) the amount of exergy destroyed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system shown in the accompanying figure experiences no changes in kinetic and potential energy between the initial and final states. (2) The generalized property charts provide data with sufficient accuracy. (3) $T_0 = 7^\circ\text{C}$ (280 K)

ANALYSIS: From Table A-1, $T_c = 154 \text{ K}$, $P_c = 50.5 \text{ bar}$. Thus, $P_{R1} = \frac{100}{50.5} = 1.98$, $P_{R2} = \frac{150}{50.5} = 2.97$, $T_{R1} = 1.82$. The gen. compressibility chart gives $Z_1 = 0.932$, $Z_2 = 1.01$

(a) Since volume is constant, $V_2 = V_1 \Rightarrow \left(\frac{ZRT}{P}\right)_2 = \left(\frac{ZRT}{P}\right)_1 \Rightarrow$

$$T_2 = \left(\frac{Z_1}{Z_2}\right) \left(\frac{P_2}{P_1}\right) T_1 = \left(\frac{0.932}{1.01}\right) \left(\frac{150}{100}\right) (280) = 387.6 \text{ K}$$

(b) An energy balance reduces to give $\Delta U = -W \Rightarrow W = -\Delta U$, or

$$W = -m[u_2 - u_1] = -m[(h - Pv)_2 - (h - Pv)_1] = -m[(h_2 - h_1) - v(P_2 - P_1)] \text{ or}$$

since $Pv = ZRT$, $W = -m[(h_2 - h_1) - R(Z_2 T_2 - Z_1 T_1)]$. To find m , use

$$Z_1 = \frac{P_1 V}{m R T_1} \Rightarrow m = \frac{P_1 V}{Z_1 R T_1} = \frac{(100 \times 10^5 \text{ N/m}^2)(0.142 \text{ m}^3)}{(0.932) \left(\frac{8.314 \text{ J/mole} \cdot \text{K}}{32}\right) (280 \text{ K})} = 20.94 \text{ kg}$$

$(h_2 - h_1)$ is evaluated using Eq. 11.85 with data from Table A-23, Fig. A-4:

$$h_2 - h_1 = \frac{1}{M} \left[h_2^* - h_1^* - \bar{R} T_c \left[\left(\frac{h_2^* - \bar{h}}{\bar{R} T_c} \right)_2 - \left(\frac{h_2^* - \bar{h}}{\bar{R} T_c} \right)_1 \right] \right] = \frac{1}{32} \left[11,337 - 8150 - (8.314)(154) [1.45 - 0.66] \right] = 108 \text{ kJ/kg}$$

Then

$$W = -20.94 \left[108 - \frac{8.314}{32} (1.01)(387.6) - (0.932)(280) \right] = -1551 \text{ kJ}$$

(c) Using $E_d = T_0 \sigma$, where from an entropy balance $\sigma = m(s_2 - s_1)$, together with Eq. 11.92 and data from Table A-23, Fig. A-5, we get

$$E_d = m T_0 (s_2 - s_1) = \frac{m T_0}{M} \left[(s^\circ(T_2) - s^\circ(T_1)) - \bar{R} \ln \frac{P_2}{P_1} \right] - \bar{R} \left[\left(\frac{s^\circ - \bar{s}}{\bar{R}} \right)_2 - \left(\frac{s^\circ - \bar{s}}{\bar{R}} \right)_1 \right]$$

$$= \frac{(20.94)(280)}{32} \left[212.82 - 203.19 - 8.314 \ln 1.5 - 8.314 [0.18 - 0.29] \right]$$

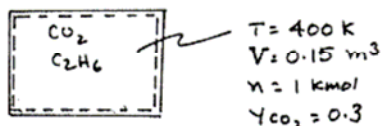
$$= 1314.4 \text{ kJ}$$

PROBLEM 11.94

KNOWN: A 1 kmol mixture of CO_2 and C_2H_6 occupies a volume of 0.15 m^3 at $T = 400 \text{ K}$. $y_{\text{CO}_2} = 0.3$. Pressure should not exceed 180 bar.

FIND: Determine p using (a) the ideal gas equation of state, (b) Kay's rule and the compressibility chart, (c) the additive pressure rule and the compressibility chart.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The mixture is a closed system.

ANALYSIS: (a) Ideal Gas.

$$p = \frac{n \bar{R} T}{V} = \frac{(1 \text{ kmol})(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}})(400 \text{ K})}{0.15 \text{ m}^3} \left| \frac{\text{bar}}{10^5 \text{ N/m}^2} \right| = 221.7 \text{ bar} \quad \leftarrow (a)$$

(b) Kay's Rule plus Z chart. From Table A-1

		$T_c \text{ (K)}$	$P_c \text{ (bar)}$
1.	CO_2	304	73.9
2.	C_2H_6	305	48.8

Then, with Eq. 11.97

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.3)(304) + (0.7)(305) = 304.7 \text{ K}$$

$$P_c = y_1 P_{c1} + y_2 P_{c2} = (0.3)(73.9) + (0.7)(48.8) = 56.33 \text{ bar}$$

Accordingly, $T_R = 400/304.7 = 1.313$ and

$$V_R' = \frac{V P_c}{\bar{R} T_c} = \frac{(0.15 \text{ m}^3/\text{kmol})(56.33 \times 10^5 \text{ N/m}^2)}{(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}})(304.7 \text{ K})} = 0.334$$

With these values, Fig. A-2 gives $Z \approx 0.67$, $P_R \approx 2.65$. With the Z value

$$p = Z \frac{\bar{R} T}{V} = Z P_{\text{ideal}} = 0.67(221.7) = 148.5 \text{ bar} \quad \leftarrow (b)$$

With the P_R value

$$p = P_c P_R = 56.33(2.65) = 149.3 \text{ bar}$$

(c) Additive Pressure Rule plus Z chart. Selecting Eq. 11.99b

$$Z = y_1 Z_1(T, V) + y_2 Z_2(T, V)$$

$$T_{R1} = 400/304 = 1.32, \quad V_{R1}' = \frac{(0.15 \text{ m}^3/0.3 \text{ kmol})(73.9 \times 10^5 \text{ N/m}^2)}{(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}})(304 \text{ K})} = 1.46 \Rightarrow Z_1 \approx 0.9$$

$$T_{R2} = 400/305 = 1.31, \quad V_{R2}' = \frac{(0.15 \text{ m}^3/0.7 \text{ kmol})(48.8 \times 10^5 \text{ N/m}^2)}{(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}})(305 \text{ K})} = 0.412 \Rightarrow Z_2 \approx 0.69$$

$$\text{Then, } Z = (0.3)(0.9) + (0.7)(0.69) = 0.753$$

and

$$p = Z \frac{n \bar{R} T}{V} = Z P_{\text{ideal}} = 0.753(221.7 \text{ bar}) = 166.9 \text{ bar} \quad \leftarrow$$

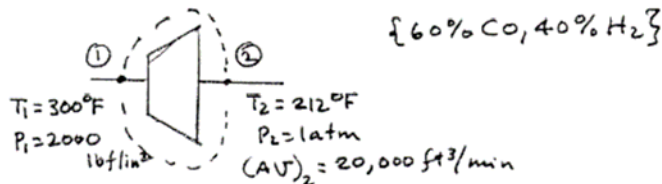
Discussion: At this state, where $Z = 0.67$, the ideal gas model is not applicable. Although methods (b), (c) yield different predictions, the methods do suggest that operation in the safe range might be realized.

PROBLEM 11.95

KNOWN: A gaseous mixture with the molar composition {60% CO, 40% H₂} enters a turbine operating at steady state at 300°F, 2000 lbf/in² and exits at 212°F, 1 atm with a volumetric flow rate of 20,000 ft³/min.

FIND: Using Kay's rule estimate the volumetric flow rate at the turbine inlet, and compare with the value yielded by the ideal gas model.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The control volume shown in the figure is at steady state.

ANALYSIS: At steady state a mass rate balance reduces to give

$$\frac{(AV)_1}{v_1} = \frac{(AV)_2}{v_2} \Rightarrow (AV)_1 = \frac{v_1}{v_2} (AV)_2 \quad (1)$$

Ideal Gas Model. With $v = RT/p$, Eq. (1) gives

$$(AV)_1 = \left(\frac{T_1}{T_2} \right) \left(\frac{P_2}{P_1} \right) (AV)_2 = \left(\frac{760^\circ R}{672^\circ R} \right) \left(\frac{14.7 \text{ lbf/in}^2}{2000 \text{ lbf/in}^2} \right) (20,000 \frac{\text{ft}^3}{\text{min}}) = 166.3 \frac{\text{ft}^3}{\text{min}}$$

Kay's Rule. With $Z = Pv/RT$, $v = ZRT/p$, and Eq. (1) becomes

$$(AV)_1 = \frac{Z_1}{Z_2} \left(\frac{T_1}{T_2} \right) \left(\frac{P_2}{P_1} \right) (AV)_2 = \frac{Z_1}{Z_2} (166.3 \frac{\text{ft}^3}{\text{min}}) \quad (2)$$

$\underbrace{\frac{Z_1}{Z_2}}_{\text{ideal gas value}} \underbrace{\left(\frac{T_1}{T_2} \right) \left(\frac{P_2}{P_1} \right)}_{\text{ideal gas value}}$

With T_c, P_c data from Tables A-1 using the corrections for HL footnoted in Sec. 3.4

$$\begin{array}{ll} \text{CO: } T_c = 133 \text{ K} & \text{H}_2: T_c = 33.2 + 8 = 41.2 \text{ K} \\ P_c = 34.5 \text{ atm} & P_c = 12.8 + 8 = 20.8 \text{ atm} \end{array}$$

Then with Kay's rule

$$\begin{aligned} T_c &= (0.6)(133) + (0.4)(41.2) = 96.3 \text{ K} \\ P_c &= (0.6)(34.5) + (0.4)(20.8) = 29 \text{ atm} \end{aligned}$$

Thus,

$$TR1 = \frac{760/1.8}{96.3} = 4.38, \quad PR1 = \frac{2000/14.7}{29} = 4.69 \Rightarrow Z_1 \approx 1.08 \quad (\text{Fig. A-2})$$

$$TR2 = \frac{672/1.8}{96.3} = 3.88, \quad PR2 = \frac{1/29}{0.03} = 0.03 \Rightarrow Z_2 \approx 1.0 \quad (\text{Fig. A-1})$$

Finally, using Eq. (2)

$$(AV)_1 = \left(\frac{1.08}{1.0} \right) (166.3 \frac{\text{ft}^3}{\text{min}}) = 179.6 \frac{\text{ft}^3}{\text{min}} \quad \leftarrow$$

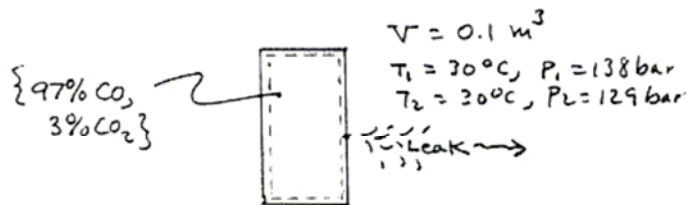
The ideal gas value is about 8% less than the volumetric flow rate predicted by Kay's rule.

PROBLEM 11.96

KNOWN: A 0.1-m^3 cylinder contains a gaseous mixture with the molar composition $\{97\% \text{CO}, 3\% \text{CO}_2\}$ initially at 30°C and 138 bar . Mixture leaks from the cylinder, eventually the cylinder contents are at 129 bar , 30°C .

FIND: Using Kay's rule, estimate the amount of mixture that leaks from the cylinder.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The control volume shown on the accompanying sketch is the system.

ANALYSIS: With $Z = PV/nRT$; $n = PV/ZRT$. For the control volume, the amount of mixture that leaks out equals Δn . Thus

$$n_1 - n_2 = \frac{P_1 V}{Z_1 R T} - \frac{P_2 V}{Z_2 R T} = \frac{V}{R T} \left[\frac{P_1}{Z_1} - \frac{P_2}{Z_2} \right] \quad (1)$$

With data from Table A-1

$$\begin{array}{ll} \text{CO: } T_c = 133 \text{ K} & \text{CO}_2: T_c = 304 \text{ K} \\ P_c = 35 \text{ bar} & P_c = 73.9 \text{ bar} \end{array}$$

Kay's Rule

$$\begin{aligned} T_c &= 0.97(133) + 0.03(304) = 138.1 \text{ K} \\ P_c &= 0.97(35) + 0.03(73.9) = 36.2 \text{ bar} \end{aligned}$$

Thus

$$P_{R1} = \frac{138}{36.2} = 3.81, \quad T_{R1} = \frac{303}{138.1} = 2.19, \quad Z_1 \approx 0.98$$

$$P_{R2} = \frac{129}{36.2} = 3.56, \quad T_{R2} = T_{R1} = 2.19, \quad Z_2 \approx 0.97$$

Accordingly, Eq. (1) gives

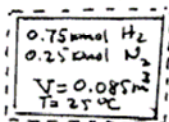
$$\begin{aligned} n_1 - n_2 &= \frac{0.1 \text{ m}^3}{\left(\frac{8314 \text{ J} \cdot \text{m}}{\text{kmol} \cdot \text{K}} \right) (303 \text{ K})} \left[\frac{138}{0.98} - \frac{129}{0.97} \right] \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \\ &= 0.031 \text{ kmol} \quad \leftarrow \text{leak} \end{aligned}$$

PROBLEM 11.97

KNOWN: A gaseous mixture consisting of 0.75 kmol H_2 and 0.25 kmol N_2 occupies 0.085 m^3 at 25°C .

FIND: Estimate the pressure using (a) the ideal gas equation of state, (b) Kay's rule with the gen. compressibility chart, (c) van der Waals equation, (d) rule of additive pressure using the gen. compressibility chart.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

(a) Ideal gas equation of state

$$p = \frac{n \bar{R} T}{V} = \frac{(1 \text{ kmol}) \left(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}} \right) (298 \text{ K})}{0.085 \text{ m}^3} \left| \frac{1 \text{ bar}}{10^5 \frac{\text{N}}{\text{m}^2}} \right| = 291.5 \text{ bar} \quad (a)$$

(b) Kay's rule plus gen. compressibility chart. With data from Table A-1 and using the corrections for H_2 indicated in the footnote of Sec. 3.4.

$$H_2: T_{c1} = 41.2 \text{ K} \quad N_2: T_{c2} = 126 \text{ K}$$

$$P_{c1} = 21.08 \text{ bar} \quad P_{c2} = 33.9 \text{ bar}$$

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.75)(41.2) + (0.25)(126) = 62.4 \text{ K}$$

$$P_c = y_1 P_{c1} + y_2 P_{c2} = (0.75)(21.08) + (0.25)(33.9) = 24.3 \text{ bar}$$

$$\therefore T_R = \frac{298}{62.4} = 4.78, \quad v_R' = \frac{(V/n) P_c}{\bar{R} T_c} = \frac{(0.085 \text{ m}^3/\text{kmol}) (24.3 \times 10^5 \text{ N/m}^2)}{(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}}) (62.4 \text{ K})} = 0.398$$

$$\text{Figure A-3 gives } Z \approx 1.3. \text{ Then, } p = Z \left[\frac{n \bar{R} T}{V} \right] = Z p_{id} = 1.3 (291.5) = 379 \text{ bar} \quad (b)$$

(c) van der Waals equation. From Table A-24, for N_2

$$a_2 = 1.366 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2, \quad b_2 = 0.0386 \text{ m}^3/\text{kmol}. \text{ Using Eqs. 11.4 for } H_2$$

$$a_1 = 0.247, \quad b_1 = 0.0265. \text{ Then, with Eqs. 11.96}$$

$$a = [y_1 a_1 + y_2 a_2] = [0.75(0.247) + 0.25(1.366)] = 0.442$$

$$b = [y_1 b_1 + y_2 b_2] = [0.75(0.0265) + 0.25(0.0386)] = 0.0296$$

Then, Eq. 11.2 gives

$$p = \frac{(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}}) (298 \text{ K})}{(0.085 - 0.0296) \frac{\text{m}^3}{\text{kmol}}} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| - \frac{0.442 \text{ bar} (\text{m}^3/\text{kmol})^2}{(0.085 \text{ m}^3/\text{kmol})^2} = 386 \text{ bar} \quad (c)$$

(d) Additive pressure rule plus gen. compressibility chart. Using T_{c1}, P_{c1} for H_2 from part (b), $T_{R1} = 298/41.2 = 7.23$,

$$v_{R1} = \frac{V P_{c1}}{\bar{R} T_{c1}} = \frac{(0.085 \frac{\text{m}^3}{\text{kmol}}) (21.08 \times 10^5 \text{ N/m}^2)}{(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}}) (41.2 \text{ K})} = 0.697 \quad \left. \vphantom{\frac{V P_{c1}}{\bar{R} T_{c1}}} \right\} \text{Fig. A-2, } Z_1 \approx 1.05$$

For N_2 , $T_{R2} = 298/126 = 2.37$

$$v_{R2} = \frac{(0.085) (33.9 \times 10^5)}{(8314) (126)} = 1.1 \quad \left. \vphantom{\frac{(0.085) (33.9 \times 10^5)}{(8314) (126)}} \right\} \text{Fig. A-2, } Z_2 \approx 0.99$$

Then, with Eq. 11.99b,

$$Z = 0.75(1.05) + (0.25)(0.99) = 1.04$$

$$\Rightarrow p = Z \frac{n \bar{R} T}{V} = Z p_{id} = 1.04 (291.5) = 303 \text{ bar} \quad (d)$$

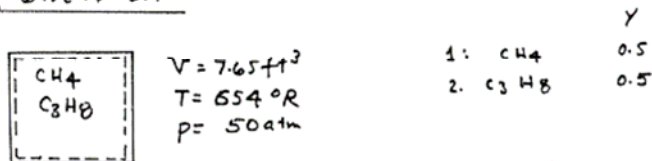
1. Parts (a) and (d) are roughly in agreement. Also, parts (c) and (d) agree fairly well. A conservative choice dictated by the application at hand would govern the value that would be used for the estimated pressure.

PROBLEM 11.98

KNOWN: A gaseous mixture of 0.5 lbmol CH₄ and 0.5 lbmol of C₃H₈ occupies a volume of 7.65 ft³ at 194 °F. The measured pressure is 50 atm.

FIND: Estimate the pressure using each of six specified procedures and compare with the measured pressure.

SCHEMATIC & GIVEN DATA



ENGINEERING MODEL: The mixture is a closed system.

ANALYSIS: (a) Ideal Gas Equation of State.

$$p = \frac{n \bar{R} T}{V} = \frac{(1.0 \text{ lbmol})(1545 \text{ ft}^2/\text{lbmol} \cdot \text{R})(654 \text{ R})}{7.65 \text{ ft}^3} \left| \frac{\text{atm}}{14.696 \times 10^4 \text{ lbf/ft}^2} \right| = 62.4 \text{ atm} \quad (a)$$

(b) Kay's Rule plus Z chart. With data from Table A-1E and Eqs. 11.97

$$T_c = y_1 T_{c1} + y_2 T_{c2} = 0.5(344 + 666) = 505 \text{ R}$$

$$p_c = y_1 p_{c1} + y_2 p_{c2} = 0.5(45.8 + 42.1) = 43.95 \text{ atm}$$

Then

$$T_R = 654/505 = 1.3$$

$$V_R' = \frac{\bar{V} p_c}{\bar{R} T_c} = \frac{(7.65/1 \text{ lbmol}) \left(\frac{1545}{14.696 \times 10^4} \right) (43.95 \times 14.696 \times 10^4 \text{ lbf/ft}^2)}{(1545 \text{ ft}^2/\text{lbmol} \cdot \text{R})(505 \text{ R})} = 0.911$$

Fig. A-2, $Z \approx 0.825$. Thus

$$p = Z \frac{n \bar{R} T}{V} = Z p_{\text{ideal}} = (0.825)(62.4) = 51.48 \text{ atm} \quad (b)$$

(c) Vander Waals Equation of State. With data from Table A-24E and Eqs. 11.96

$$a = [y_1 (a_1)^{1/2} + y_2 (a_2)^{1/2}]^2 = 0.25 [(581)^{1/2} + (2369)^{1/2}]^2 = 1323.9 \text{ atm} \left(\frac{\text{ft}^3}{\text{lbmol}} \right)^2$$

$$b = y_1 b_1 + y_2 b_2 = 0.5 [0.685 + 1.444] = 1.065 \frac{\text{ft}^3}{\text{lbmol}}$$

Then, Eq. 11.2 gives

$$p = \frac{(1545 \text{ ft}^2/\text{lbmol} \cdot \text{R})(654 \text{ R})}{(7.65 - 1.065) \left(\frac{\text{ft}^3}{\text{lbmol}} \right)} \left| \frac{\text{atm}}{14.696 \times 10^4 \text{ lbf/ft}^2} \right| - \frac{1323.9 \text{ atm}}{(7.65)^2} = 49.9 \text{ atm} \quad (c)$$

(d) Additive Pressure Rule plus Vander Waals Equation. Selecting Eq. 11.99a

$$p = p_1(T, \frac{V}{n_1}) + p_2(T, \frac{V}{n_2})$$

With data from Table A-24E and Eq. 11.2

$$p_1 = \frac{(1545)(654)}{\left[\left(\frac{7.65}{0.5} \right) - 0.685 \right] (14.696 \times 10^4)} - \frac{581}{(7.65/0.5)^2} = 30.19 \text{ atm}$$

$$p_2 = \frac{(1545)(654)}{\left[\left(\frac{7.65}{0.5} \right) - 1.444 \right] (14.696 \times 10^4)} - \frac{2369}{(7.65/0.5)^2} = 24.34 \text{ atm}$$

Then

$$p = 30.19 + 24.34 = 54.5 \text{ atm} \quad (d)$$

Continued on next slide

Problem 11-98 continued

(e) Additive Pressure Rule plus Z chart. Selecting Eq. 11.99a

$$Z = y_1 Z_1(T, V) + y_2 Z_2(T, V) = 0.5 [Z_1 + Z_2]$$

$$T_{R1} = \frac{654}{344} = 1.90, \quad V_{R1} = \frac{\left(\frac{7.65}{0.5}\right) (45.8 \times 14.696 \times 144)}{(1545)(344)} = 2.79 \Rightarrow Z_1 \approx 0.98$$

$$T_{R2} = \frac{654}{666} = 0.982, \quad V_{R2} = \frac{\left(\frac{7.65}{0.5}\right) (42.1) (14.696) (144)}{(1545)(666)} = 1.325 \Rightarrow Z_2 \approx 0.77$$

$$\text{Then } Z = 0.5(0.98 + 0.77) = 0.875 \Rightarrow p = Z \frac{nRT}{V} = Z p_{ideal} = (0.875)(62.4) = 54.6 \text{ atm} \leftarrow (e)$$

(f) Additive Volume Rule plus Vander Waals Equation. Selecting Eq. 11.100a

$$V = V_1(P, T) + V_2(P, T) \quad (1)$$

With data from Table A-24E Eq 11-2 gives

$$p = \frac{(1545)(654)}{\left(\frac{V_1}{0.5} - 0.685\right)(14.696 \text{ RM})} - \frac{581}{\left(\frac{V_1}{0.5}\right)^2} \Rightarrow p = \frac{238.73}{V_1 - 0.3425} - \frac{145.25}{V_1^2} \quad (2)$$

$$p = \frac{(1545)(654)}{\left(\frac{V_2}{0.5} - 1.444\right)(14.696 \text{ RM})} - \frac{2369}{\left(\frac{V_2}{0.5}\right)^2} \Rightarrow p = \frac{238.73}{V_2 - 0.722} - \frac{592.25}{V_2^2} \quad (3)$$

Setting Eqs. (2) and (3) equal

$$\frac{238.73}{V_1 - 0.3425} - \frac{145.25}{V_1^2} = \frac{238.73}{V_2 - 0.722} - \frac{592.25}{V_2^2}$$

Introducing Eq. (1): $V_1 = V - V_2 = 7.65 - V_2$

$$\frac{238.73}{7.3075 - V_2} - \frac{145.25}{(7.65 - V_2)^2} = \frac{238.73}{V_2 - 0.722} - \frac{592.25}{V_2^2} \quad (4)$$

Using an equation solver, $V_2 = 1.76 \text{ ft}^3$. Eq. (3) then gives, $p = 38.8 \text{ atm} \leftarrow$

Discussion: Comparing the calculated values with the measured value, 50 atm, the result is summarized as follows.

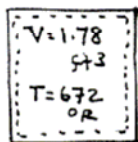
(a)	Ideal Gas Equation	62.4 atm	+ 24.8 %
(b)	Kay's Rule / Z chart	51.48 atm	+ 3.0 %
(c)	vander Waals Equation	49.9 atm	—
(d)	Additive pressure / vander Waals	54.5 atm	+ 9.0 %
(e)	Additive pressure / Z chart	54.6 atm	+ 9.0 %
(f)	Additive volume / vander Waals	38.8 atm	- 22.4 %

PROBLEM 11.99

KNOWN: One lbmol of { 69.5% CO₂, 30.5 C₂H₄ } occupies 1.78 ft³ at 212°F.

FIND: Estimate the pressure using (a) the ideal gas equation of state, (b) Kay's rule with gen. compressibility chart, (c) additive pressure rule with gen. compressibility chart, (d) van der Waals equation.

SCHEMATIC & GIVEN DATA:



$$\text{CO}_2: y_1 = 0.695$$

$$\text{C}_2\text{H}_4: y_2 = 0.305$$

ENGINEERING MODEL: The mixture is the closed system, as shown.

ANALYSIS: (a) Ideal Gas Model

$$p = \frac{n \bar{R} T}{V} = \frac{(1 \text{ lbmol}) (1545 \frac{\text{ft} \cdot \text{lb}_f}{\text{lbmol} \cdot ^\circ \text{R}}) (672 ^\circ \text{R})}{(1.78 \text{ ft}^3) (144) (\frac{\text{ft}^2}{\text{ft}^2}) (\frac{\text{lb}_f}{\text{ft}^2} / \text{atm})} = 275.5 \text{ atm} \quad \leftarrow (a)$$

(b) Kay's rule with gen. compressibility chart. With data from Table A-1E, Eqs. 11.97 give

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.695)(548 ^\circ \text{R}) + (0.305)(510 ^\circ \text{R}) = 536.4 ^\circ \text{R}$$

$$P_c = y_1 P_{c1} + y_2 P_{c2} = (0.695)(72.9 \text{ atm}) + (0.305)(50.5 \text{ atm}) = 66.07 \text{ atm}$$

$$\text{Then, } T_R = (672 / 536.4) = 1.25$$

$$V_R = \frac{V P_c}{\bar{R} T_c} = \frac{(1.78 \text{ ft}^3 / \text{lbmol}) (66.07 \times 14.7 \times 144 \text{ lb}_f / \text{ft}^2)}{(1545 \frac{\text{ft} \cdot \text{lb}_f}{\text{lbmol} \cdot ^\circ \text{R}}) (536.4 ^\circ \text{R})} = 0.3 \quad \left. \begin{array}{l} \text{Fig. A-2} \\ Z \sim 0.61 \end{array} \right\} \quad (b)$$

$$\Rightarrow p = Z \frac{n \bar{R} T}{V} = Z P_{id} = 0.61 (275.5 \text{ atm}) = 168.1 \text{ atm.}$$

(c) Additive pressure rule with gen. compressibility chart.

$$T_{R1} = 672 / 548 = 1.23$$

$$V_{R1} = \frac{(1.78)}{(0.695)} \frac{(72.9 \times 14.7 \times 144)}{(1545)(548)} = 0.47 \quad \left. \begin{array}{l} \text{Fig. A-2} \\ Z_1 \approx 0.68 \end{array} \right\}$$

$$T_{R2} = 672 / 510 = 1.32$$

$$V_{R2} = \frac{(1.78)}{(0.305)} \frac{(50.5 \times 14.7 \times 144)}{(1545)(510)} = 0.79 \quad \left. \begin{array}{l} \text{Fig. A-2} \\ Z_2 \approx 0.81 \end{array} \right\}$$

With Eq. 11.99b,

$$Z = (0.695)(0.68) + (0.305)(0.81) = 0.72$$

$$\therefore p = Z P_{id} = (0.72)(275.5 \text{ atm}) = 198.4 \text{ atm} \quad \leftarrow (c)$$

(d) van der Waals equation. From Table A-24E, for CO₂

$$a_1 = 926 \text{ atm}(\text{ft}^3 / \text{lbmol})^2, \quad b_1 = 0.686 \text{ ft}^3 / \text{lbmol. Then, using Eqs. 11.4 for C}_2\text{H}_4,$$

$$a_2 = 1158, \quad b_2 = 0.921. \text{ With these values, Eqs. 11.96 give}$$

$$a = [(0.695)(926)^{1/2} + (0.305)(1158)^{1/2}]^2 = 994 \text{ atm}(\text{ft}^3 / \text{lbmol})^2$$

$$b = [(0.695)(0.686) + (0.305)(0.921)] = 0.758 \text{ ft}^3 / \text{lbmol}$$

Eq. 11.2 then gives

$$\textcircled{1} \quad p = \frac{(1545 \text{ ft} \cdot \text{lb}_f / \text{lbmol} \cdot ^\circ \text{R})(672 ^\circ \text{R})}{(1.78 - 0.758)(\text{ft}^3 / \text{lbmol})} \left| \frac{1 \text{ atm}}{(14.7 \times 14.7) \text{ lb}_f / \text{ft}^2} \right| - \frac{994 \text{ atm}(\text{ft}^3 / \text{lbmol})^2}{(1.78 \text{ ft}^3 / \text{lbmol})^2} = 166.2 \text{ atm} \quad \leftarrow (d)$$

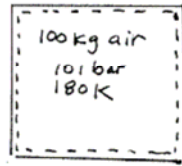
1. Although the results of (b) and (d) are in fair agreement, the results obtained with the four methods are scattered. A conservative choice dictated by the application at hand would govern the value that would be used for the estimated pressure.

PROBLEM 11.100

KNOWN: 100 kg of air with the molar composition {79% N_2 , 21% O_2 } fills a 0.36-m^3 vessel at a measured pressure and temperature of 101 bar, 180 K, respectively.

FIND: Compare the measured pressure with the pressure predicted by (a) the ideal gas model, (b) Kay's rule, (c) additive pressure rule with the Redlich-Kwong equation, (d) additive volume rule with the Redlich-Kwong equation.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The closed system is the 100 kg of air, as shown.

- ① ANALYSIS: (a) Ideal gas model, $p = m \bar{R} T / M V$, where $M = y_1 M_1 + y_2 M_2$ or $M = (0.79)(28.01) + (0.21)(32) = 28.85$. Thus

$$p = \frac{(100 \text{ kg}) \left(\frac{8314 \text{ N}\cdot\text{m}}{28.85 \text{ kg}\cdot\text{K}} \right) (180 \text{ K})}{0.36 \text{ m}^3} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| = 144.1 \text{ bar} \quad (\sim 43\% \text{ high})$$

(b) With data from Table A-1

$$\begin{array}{ll} N_2: T_c = 126 \text{ K} & O_2: T_c = 154 \text{ K} \\ p_c = 33.9 \text{ bar} & p_c = 50.5 \text{ bar} \end{array} \Rightarrow \begin{array}{l} T_c = (0.79)(126) + (0.21)(154) = 131.9 \text{ K} \\ p_c = (0.79)(33.9) + (0.21)(50.5) = 37.39 \text{ bar} \end{array}$$

$$\begin{array}{l} \text{Then, } T_R = 180/131.9 = 1.36 \\ \bar{V}_R = \frac{\bar{V} p_c}{\bar{R} T_c} = \left[\frac{0.36 \text{ m}^3}{(100/28.85) \text{ kmol}} \right] \left[\frac{37.39 \times 10^5 \text{ N/m}^2}{(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}})(131.9 \text{ K})} \right] = 0.354 \end{array} \left. \vphantom{\begin{array}{l} \bar{V}_R = \frac{\bar{V} p_c}{\bar{R} T_c} = \left[\frac{0.36 \text{ m}^3}{(100/28.85) \text{ kmol}} \right] \left[\frac{37.39 \times 10^5 \text{ N/m}^2}{(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}})(131.9 \text{ K})} \right] = 0.354} \right\} \begin{array}{l} \text{Figure A-2} \\ Z \sim 0.7 \end{array}$$

With $Z = pV / mRT \Rightarrow$

$$p = Z \left(\frac{m \bar{R} T}{V} \right)_{\text{ideal gas value}} = 0.7(144.1 \text{ bar}) = 100.9 \text{ bar} \quad (\text{within } 0.1\% \text{ of the measured value})$$

- (c) Additive pressure model: $p = p_1 + p_2$, where p_1 denotes the pressure of N_2 if it occupied the full volume at 180 K and p_2 denotes the pressure of O_2 if it occupied the full volume at 180 K.

$$\text{With } n = \frac{m}{M} = \frac{100 \text{ kg}}{28.85 \text{ kg/kmol}} = 3.466 \text{ kmol. Then}$$

$$n_{N_2} = 0.79(3.466) = 2.738 \text{ kmol}, \quad n_{O_2} = 0.728 \text{ kmol}$$

Then, with data from Table A-24 for the Redlich-Kwong equation

$$\text{Nitrogen: } \bar{V} = 0.36 \text{ m}^3 / 2.738 \text{ kmol} = 0.131 \text{ m}^3/\text{kmol}$$

$$\begin{aligned} p_1 &= \frac{(8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}})(180 \text{ K})}{(0.131 - 0.02677) \text{ m}^3/\text{kmol}} \left| \frac{\text{bar}}{10^5 \text{ N/m}^2} \right| - \frac{(15.53) \text{ bar}}{(0.131)(0.131 + 0.02677)(180)^{1/2}} \\ &= 143.58 \text{ bar} - 56.01 \text{ bar} = 87.57 \text{ bar} \end{aligned}$$

Continued on next slide

Problem 11-100 continued

Oxygen: $\bar{v} = 0.36/0.728 = 0.495 \text{ m}^3/\text{kmol}$

$$P_2 = \frac{(8314)(180)}{(0.495 - 0.02197)} \left| \frac{1}{10^5} \right| - \frac{17.22}{(0.495)(0.495 + 0.02197)(180)^{1/2}}$$

$$= 31.64 - 5.02 = 26.62 \text{ bar}$$

Then, $p = p_1 + p_2 = 87.57 + 26.62 = 114.2 \text{ bar}$ (13% higher than the measured value)

(d) Additive volume model: $V = V_1 + V_2$, where V_1 is the volume occupied by N_2 if it were at the mixture temperature and pressure and V_2 is the volume occupied by O_2 if it were at the mixture temperature and pressure. Thus, $V = n_1 \bar{v}_1(T, p) + n_2 \bar{v}_2(T, p)$, or

$$\frac{V}{n} = y_1 \bar{v}_1(T, p) + y_2 \bar{v}_2(T, p) \quad (1)$$

Also, with the Redlich-Kwong equation, when each gas is at the same T, p

$$p = \frac{\bar{R}T}{\bar{v}_1 - b_1} - \frac{a_1}{\bar{v}_1(\bar{v}_1 + b_1)\sqrt{T}} = \frac{\bar{R}T}{\bar{v}_2 - b_2} - \frac{a_2}{\bar{v}_2(\bar{v}_2 + b_2)\sqrt{T}} \quad (2)$$

Eqs. (1), (2) are simultaneous equations for $\bar{v}_1, \bar{v}_2$. Inserting values

$$\left\{ \begin{array}{l} 0.1039 = 0.79 \bar{v}_1 + 0.21 \bar{v}_2 \quad (1)' \\ \frac{(8314)(180)}{(\bar{v}_1 - 0.02677)(10^5)} - \frac{15.53}{\bar{v}_1(\bar{v}_1 + 0.02677)(180)^{1/2}} = \frac{(8314)(180)}{(\bar{v}_2 - 0.02197)(10^5)} - \frac{17.22}{\bar{v}_2(\bar{v}_2 + 0.02197)(180)^{1/2}} \quad (2)' \end{array} \right.$$

With an equation solver, $\bar{v}_1 = 0.11, \bar{v}_2 = 0.078$, using $\bar{v}_1$, Eq. (2) gives $p = 102.86 \text{ bar}$. Using $\bar{v}_2$, Eq. (2) gives $p = 102.49 \text{ bar}$. Thus, $p \approx 102.7 \text{ bar}$, which is about 1.7% higher than the measured value.

$$1. \quad M_{\text{mix}} = \frac{m_{\text{mix}}}{n_{\text{mix}}} = \frac{m_1 + m_2}{n_{\text{mix}}} = \frac{M_1 n_1 + M_2 n_2}{n_{\text{mix}}} = y_1 M_1 + y_2 M_2$$

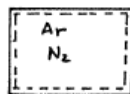
PROBLEM 11.101

KNOWN: A gaseous mixture with the molar analysis below is contained in a tank at 20 atm, 320°R.

1. Argon : $y_1 = 0.50$
2. N_2 : $y_2 = 0.50$

FIND: Estimate the specific volume using each of four specified procedures.

SCHEMATIC & GIVEN DATA:



$$T = 320^\circ R \quad y_1 = y_2 = 0.5$$

$$P = 20 \text{ atm}$$

ENGINEERING MODEL: The mixture is the closed system, as shown.

ANALYSIS: (a) Ideal Gas Model

$$v = \frac{(\bar{R}/M)T}{P} = \frac{\left(\frac{1545}{33.975} \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot ^\circ R}\right)(320^\circ R)}{(20)(14.696)(144) \text{ lb}_f/\text{ft}^2} = 0.344 \text{ ft}^3/\text{lb}$$

Where the mixture molecular weight is found using

$$M_{\text{mix}} = \frac{m_{\text{mix}}}{n_{\text{mix}}} = \frac{m_1 + m_2}{n_{\text{mix}}} = \frac{M_1 n_1 + M_2 n_2}{n_{\text{mix}}} = y_1 M_1 + y_2 M_2$$

$$= 0.5(39.94 + 28.01) = 33.975$$

(b) Kay's Rule plus Z chart. With data from Table A-1E and Eqs. 11.97

$$T_c = y_1 T_{c1} + y_2 T_{c2} = 0.5[272 + 227] = 249.5^\circ R$$

$$P_c = y_1 P_{c1} + y_2 P_{c2} = 0.5[47.97 + 33.5] = 40.74 \text{ atm}$$

Then

$$\left. \begin{aligned} T_R &= \frac{320}{249.5} = 1.283 \\ P_R &= \frac{20}{40.74} = 0.49 \end{aligned} \right\} : Z \approx 0.92, \quad v_R' \approx 2.40$$

With $Z = 0.92$

$$v = Z \frac{\bar{R} T}{P} = Z v_{\text{ideal}} = 0.92(1.03) = 0.95 \text{ ft}^3$$

With $v_R' = 2.40$

$$v = v_R' \frac{RT_c}{P_c} = \frac{2.4 \left(\frac{1545}{33.975} \right) (249.5)}{(40.74)(14.696)(144)} = 0.316 \frac{\text{ft}^3}{\text{lb}}$$

(b)

(c). Redlich-Kwong Equation. Using data from Table A-24E for N_2

$$a_2 = 5280 \text{ atm} \left(\frac{\text{ft}^3}{\text{lbmol}} \right)^2 (^\circ R)^{1/2}, \quad b_2 = 0.4286 \text{ ft}^3/\text{lbmol}$$

For argon use Eqs. 11.8 together with critical data from Table A-1E

$$a_1 = 0.42748 \left[\frac{0.73 \text{ atm} \cdot \text{ft}^3}{\text{lbmol} \cdot ^\circ R} \right]^2 [272^\circ R]^{3/2} = 5794.5 \text{ atm} \left(\frac{\text{ft}^3}{\text{lbmol}} \right)^2 (^\circ R)^{1/2}$$

$$b_1 = \frac{0.08664 (0.73)(272)}{47.97} = 0.3586 \text{ ft}^3/\text{lbmol}$$

Then, using Eqs. 11.96

$$a = \left[y_1 a_1^{1/2} + y_2 a_2^{1/2} \right]^2 = 0.25 \left[(5280)^{1/2} + (5794.5)^{1/2} \right]^2 = 5534.3 \text{ atm} \left(\frac{\text{ft}^3}{\text{lbmol}} \right)^2 (^\circ R)^{1/2}$$

$$b = y_1 b_1 + y_2 b_2 = 0.5[0.4286 + 0.3586] = 0.3936 \text{ ft}^3/\text{lbmol}$$

Continued on next slide

Problem 11-101 continued

Then, the Redlich-Kwong equation of state becomes

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)T^{1/2}}$$

$$20 \text{ atm} = \frac{(0.73 \text{ atm} \cdot \text{ft}^3 / \text{lbmol} \cdot ^\circ\text{R}) (320^\circ\text{R})}{(\bar{v} - 0.3936 \text{ ft}^3 / \text{lbmol})} - \frac{5534.3 \text{ atm} (\frac{\text{ft}^3}{\text{lbmol}})^2 (^\circ\text{R})^{1/2}}{\bar{v} (\bar{v} + 0.3936 \text{ ft}^3 / \text{lbmol})^2 (320^\circ\text{R})^{1/2}}$$

or

$$20 = \frac{233.6}{\bar{v} - 0.3936} - \frac{309.37}{\bar{v}(\bar{v} + 0.3936)}$$

With an equation solver, $\bar{v} = 10.73 \text{ ft}^3 / \text{lbmol}$. Finally

$$v = \frac{10.73}{33.975} = 0.316 \text{ ft}^3 / \text{lb} \quad \leftarrow (c)$$

(d) Additive volume rule with gen. compressibility chart. With data from Table A-1E

$$\begin{array}{l} \text{ARGON: } T_{R1} = \frac{320}{272} = 1.18 \\ P_{R1} = \frac{20}{47.97} = 0.42 \end{array} \left. \vphantom{\begin{array}{l} T_{R1} \\ P_{R1} \end{array}} \right\} Z_1 = 0.92 \quad \begin{array}{l} \text{N}_2: T_{R2} = \frac{320}{227} = 1.41 \\ P_{R2} = \frac{20}{33.5} = 0.6 \end{array} \left. \vphantom{\begin{array}{l} T_{R2} \\ P_{R2} \end{array}} \right\} Z_2 = 0.94$$

Then, with Eq. 11.100b, $Z = (0.5)(0.92) + (0.5)(0.94) = 0.93$. Since $Z = p\bar{v}/mRT$

$$\textcircled{1} \quad v = \frac{Z \bar{R}T}{p} = Z v_{id} = 0.93 (0.344 \frac{\text{ft}^3}{\text{lb}}) = 0.32 \text{ ft}^3 / \text{lb} \quad \leftarrow (d)$$

-
1. Methods (b)-(d) give estimates for specific volume of about $0.32 \text{ ft}^3 / \text{lb}$.

PROBLEM 11.102

KNOWN: A mixture rule is provided for the Carnahan-Starling-DeSantis equation of state.

FIND: Evaluate the pressure, in kPa, for a mixture of R12 and R13, in which R12 is 40% by mass, at $v = 0.005 \text{ m}^3/\text{kg}$, $T = 180^\circ\text{C}$.

ANALYSIS: Rearranging the equation of state

$$P = \frac{\bar{R}T}{\bar{v}} \left\{ \frac{1 + \beta + \beta^2 - \beta^3}{(1 + \beta)^3} - \frac{a}{\bar{R}T(\bar{v} + b)} \right\}$$

The term in brackets $\{\dots\}$ is seen to "correct" the pressure that an ideal gas would exhibit at the same state: $P_{id} = \bar{R}T/\bar{v}$. Begin by calculating this pressure.

The molar masses are $M_{12} = 120.92$, $M_{13} = 104.5$. Considering a typical 1 kg of mixture, then, $m_{12} = 0.4 \text{ kg}$, $m_{13} = 0.6 \text{ kg}$, and so

$$n_{12} = \frac{0.4}{120.92}, \quad n_{13} = \frac{0.6}{104.5} \Rightarrow y_{12} = \frac{\left(\frac{0.4}{120.92}\right)}{\left(\frac{0.4}{120.92}\right) + \left(\frac{0.6}{104.5}\right)} = 0.3655$$

$$y_{13} = 1 - y_{12} = 0.6345$$

Accordingly, for the mixture

$$\textcircled{1} \quad M = y_{12}M_{12} + y_{13}M_{13} = 110.5 \frac{\text{kg}}{\text{kmol}}$$

Then $\bar{v} = Mv = \left(110.5 \frac{\text{kg}}{\text{kmol}}\right)\left(0.005 \frac{\text{m}^3}{\text{kg}}\right) = 0.5525 \text{ m}^3/\text{kmol}$. Using this

$$P_{id} = \frac{\bar{R}T}{\bar{v}} = \left(\frac{8314 \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}}{0.5525 \frac{\text{m}^3}{\text{kmol}}}\right)(453 \text{ K}) \left|\frac{1 \text{ kPa}}{10^3 \text{ N/m}^2}\right| = 6817 \text{ kPa}$$

Turning next to the term in brackets $\{\dots\}$, the individual values of a , b must be determined for each component. That is, for R12

$$\begin{aligned} a_{12} &= a_0 \exp(a_1 T + a_2 T^2) = 3524.12 \exp\left(-2.77230\left(\frac{453}{10^3}\right) - 0.67318\left(\frac{453}{10^3}\right)^2\right) \\ &= 3524.12 \exp(-1.25585 - 0.13814) = 874.276 \frac{\text{J}\cdot\text{L}}{(\text{mol})^2} \\ &= 874.276 \frac{\text{J}\cdot\text{m}^3}{(\text{kmol})^2} \end{aligned}$$

$$\begin{aligned} b_{12} &= b_0 + b_1 T + b_2 T^2 = 0.15376 - \frac{1.84195}{10}\left(\frac{453}{10^3}\right) - \frac{5.03644}{10^2}\left(\frac{453}{10^3}\right)^2 \\ &= 0.15376 - 0.08344 - 0.01034 = 0.05998 \frac{\text{m}^3}{\text{kmol}} \end{aligned}$$

And for R13

$$\begin{aligned} a_{13} &= 2298.13 \exp(-3.41820(.453) - 1.52430(.453)^2) \\ &= 2298.13 \exp(-1.54848 - 0.3128) = 357.298 \frac{\text{J}\cdot\text{L}}{(\text{mol})^2} = 357.298 \frac{\text{J}\cdot\text{m}^3}{(\text{kmol})^2} \end{aligned}$$

$$\begin{aligned} b_{13} &= 0.12814 - \frac{1.84474}{10}(.453) - \frac{10.7951}{10^2}(.453)^2 \\ &= 0.12814 - 0.08357 - 0.02215 = 0.0224 \frac{\text{m}^3}{\text{kmol}} \end{aligned}$$

Continued on next slide

Problem 11-102 continued

Next, using the given mixture rule, mixture values for a & b are calculated as follows:

$$b = y_{12} b_{12} + y_{13} b_{13} = (0.3655)(0.05998) + (0.6345)(0.0224) \\ = 0.03614 \frac{\text{m}^3}{\text{kmol}}$$

$$a = y_{12}^2 a_{12} + 2 y_{12} y_{13} (1-f) (a_{12} a_{13})^{1/2} + y_{13}^2 a_{13} \\ = [(0.3655)^2 (.874276) + 2(.3655)(.6345)(1-.035) [(0.874276)(.357298)]^{1/2} + \\ (0.6345)^2 (.357298)] \times 10^6 \\ = [0.116795 + 0.250159 + 0.143845] \times 10^6 = 510,799 \frac{\text{J} \cdot \text{m}^3}{(\text{kmol})^2}$$

Also,

$$\beta = \frac{b}{4\bar{v}} = \frac{0.03614}{4(0.5525)} = 0.01635$$

Accordingly

$$\left\{ \frac{1+\beta+\beta^2-\beta^3}{(1+\beta)^3} - \frac{a}{RT(\bar{v}+b)} \right\} = 0.96831 - \frac{510,799 \frac{\text{J} \cdot \text{m}^3}{(\text{kmol})^2}}{\left(8314 \frac{\text{J}}{\text{kmol} \cdot \text{K}}\right) (453\text{K}) (0.58864 \frac{\text{m}^3}{\text{kmol}})} \\ = 0.7379$$

Collecting results

$$p = (6817 \text{ kPa})(0.7379)$$

②

$$= 5030 \text{ kPa}$$

← P

1. See Sec. 12.1, Eq. 12.9.

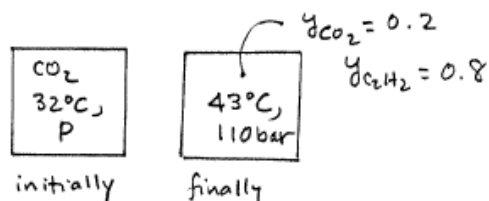
2. The calculated value is within 9% of the value read from a thermodynamic diagram for the mixture: $\approx 5500 \text{ kPa}$. See "Thermodynamic Diagrams for Refrigerant Mixtures" by J.S. Gallagher et al. ASHRAE TRANS., 1988 (Part 2), 2119-2135.

PROBLEM 11.103

KNOWN: A rigid vessel initially contains CO_2 at 32°C and pressure p . C_2H_4 enters the tank until a mixture of 20% CO_2 , 80% C_2H_4 (molar basis) is contained at 43°C , 110 bar.

FIND: Determine the pressure p .

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. Kay's rule together with the gen. compressibility chart suffices.

ANALYSIS: The pressure p can be found, in principle, from the gen. Compressibility chart using $p = P_R P_C$, where $P_C = 73.9$ bar from Table A-1. P_R is fixed by $T_R = T/T_C = 305/304 = 1.0$ and

$$V_R' = \frac{\bar{V} P_C}{R T_C} = \frac{\bar{V} (73.9 \times 10^5 \text{ N/m}^2)}{(8314 \text{ N}\cdot\text{m}/\text{kmol}\cdot\text{K})(304 \text{ K})} = 2.924 \bar{V} \quad (1)$$

when $\bar{V}$ is the molar specific volume of CO_2 initially. The specific volume $\bar{V}$ is determined next.

$$\text{For the final mixture, } \frac{\bar{V}}{n_{\text{mix}}} = Z_{\text{mix}} \frac{\bar{R} T_{\text{mix}}}{P_{\text{mix}}} = Z_{\text{mix}} \left(\frac{(8314)(316)}{110 \times 10^5} \right) = 0.239 Z_{\text{mix}}$$

Since the amount of CO_2 present initially and finally is the same, we can write $n_{\text{CO}_2} = 0.2 n_{\text{mix}}$. Thus

$$\bar{V} = \frac{\bar{V}}{n_{\text{CO}_2}} = \frac{\bar{V}}{0.2 n_{\text{mix}}} = 5 \left(\frac{\bar{V}}{n_{\text{mix}}} \right).$$

$$\text{Combining the last two expressions, } \bar{V} = 5(0.239) Z_{\text{mix}}. \quad (2)$$

To find Z_{mix} , use Kay's rule with Table A-1 data:

$$T_C = (y T_C)_{\text{CO}_2} + (y T_C)_{\text{C}_2\text{H}_4} = (0.2)(304) + (0.8)(283) = 287.2 \text{ K}$$

$$P_C = (y P_C)_{\text{CO}_2} + (y P_C)_{\text{C}_2\text{H}_4} = (0.2)(73.9) + (0.8)(51.2) = 55.74 \text{ bar}$$

Then, for the final mixture

$$(T_R)_{\text{mix}} = \frac{316}{287.2} = 1.1, \quad (P_R)_{\text{mix}} = \frac{110}{55.74} = 1.97 \Rightarrow Z_{\text{mix}} = 0.4 \quad (\text{Fig. A-2})$$

Accordingly, Eq. (2) gives $\bar{V} = 5(0.239)(0.4) = 0.478 \text{ m}^3/\text{kmol}(\text{CO}_2)$. And Eq. (1) gives

$$V_R' = (2.924)(0.478) = 1.4$$

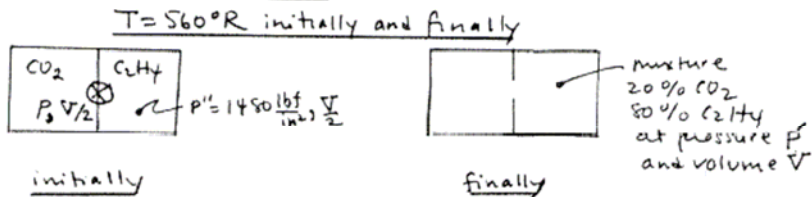
With $T_R = 1$, Fig. A-1 gives $P_R = 0.57$. Then $p = (0.57)(73.9 \text{ bar}) = 42.1 \text{ bar}$

PROBLEM 11.104

KNOWN: Two equal-volume tanks are connected by a valve. Initially, one tank contains CO_2 at 560°R , pressure p . The other tank contains C_2H_4 at 560°R , 1480 lbf/in^2 . The gases are allowed to mix until a final mixture of 20% CO_2 , 80% C_2H_4 is achieved at 560°R and pressure p' .

FIND: Determine the pressures p and p' .

SCHEMATIC & GIVEN DATA:



ANALYSIS: Let's begin by evaluating the specific volume of the ethylene in the initial condition. With data from Table A-1E for C_2H_4 , $T_R = (560^\circ\text{R}/510^\circ\text{R}) = 1.1$, $P_R = (1480/14.7) = 100.68$. Then, Fig. A-2 gives $Z = 0.4$. Using this

$$\bar{v}_{\text{C}_2\text{H}_4} = \frac{Z \bar{R} T}{p} = \frac{(0.4)(1545 \text{ ft} \cdot \text{lbf}/\text{lbmol} \cdot ^\circ\text{R})(560^\circ\text{R})}{(1480 \times 14.7 \text{ lbf/ft}^2)} = 1.62 \frac{\text{ft}^3}{\text{lbmol}}$$

Since the amount of C_2H_4 present initially and finally is the same, we can write $n_{\text{C}_2\text{H}_4} = 0.8 n$, where n is the total amount of mixture at the final condition. Accordingly, the specific volume $\bar{v}_{\text{C}_2\text{H}_4}$ is

$$\bar{v}_{\text{C}_2\text{H}_4} = \frac{V/2}{n_{\text{C}_2\text{H}_4}} = \frac{V/2}{0.8 n} = \frac{1}{1.6} \left(\frac{V}{n} \right) \Rightarrow \frac{V}{n} = 1.6 \bar{v}_{\text{C}_2\text{H}_4} = 1.6 (1.62 \frac{\text{ft}^3}{\text{lbmol}}) = 2.59 \frac{\text{ft}^3}{\text{lbmol}}$$

where V/n is the specific volume of the mixture at the final condition. Next, using Kay's rule

$$T_c = (y_{T_c})_{\text{CO}_2} + (y_{T_c})_{\text{C}_2\text{H}_4} = (0.2)(548) + (0.8)(510) = 518^\circ\text{R}$$

$$P_c = (y_{P_c})_{\text{CO}_2} + (y_{P_c})_{\text{C}_2\text{H}_4} = (0.2)(72.9) + (0.8)(50.5) = 55 \text{ atm}$$

Thus, for the final condition, $T_R = (560/518) = 1.08$ and

$$P_R = \frac{\bar{v} P_c}{\bar{R} T_c} = \frac{(2.59 \text{ ft}^3/\text{lbmol})(55 \times 14.7 \times 144 \text{ lbf/ft}^2)}{(1545 \text{ ft} \cdot \text{lbf}/\text{lbmol} \cdot ^\circ\text{R})(518^\circ\text{R})} = 0.377$$

Referring again to Fig. A-2, we get $P_R = 1.4$, and thus $p' = P_R P_c = 1.4 (55 \text{ atm}) = 77 \text{ atm}$
or $p' = (77 \times 14.7) = 1132 \text{ lbf/in}^2$.

Considering next CO_2 in the initial condition, its specific volume is

$$\bar{v}_{\text{CO}_2} = \frac{V/2}{n_{\text{CO}_2}} = \frac{V/2}{0.2 n} = \left(\frac{1}{0.4} \right) \left(\frac{V}{n} \right) = \left(\frac{1}{0.4} \right) (2.59) = 6.48 \text{ ft}^3/\text{lbmol}$$

Accordingly, for the CO_2 , $T_R = (560/344) = 1.63$ and

$$P_R = \frac{(6.48)(72.9 \times 14.7 \times 144)}{(1545 \times 560)} = 1.16$$

Referring to Fig. A-1, $P_R = 0.67$. Then, $p = P_R P_c = 0.67 (72.9 \times 14.7) = 718 \text{ lbf/in}^2$.

PROBLEM 11.105

KNOWN: A binary solution at 25°C consists of 59 kg of C₂H₅OH and 41 kg of water. The partial molar volumes are 0.0573 and 0.0172 m³/kmol, respectively.

FIND: Determine the total volume using (a) the partial molar volumes and (b) the molar specific volumes of the pure components, each a liquid at 25°C.

ENGINEERING MODEL: The solution is the closed system.

ANALYSIS: (a) With Eq. 11.104

$$\begin{aligned} V &= n_1 \bar{V}_1 + n_2 \bar{V}_2 \\ &= \left(\frac{59 \text{ kg}}{46.07 \text{ kg/kmol}} \right) \left(0.0573 \frac{\text{m}^3}{\text{kmol}} \right) + \left(\frac{41 \text{ kg}}{18.02 \text{ kg/kmol}} \right) \left(0.0172 \frac{\text{m}^3}{\text{kmol}} \right) \\ &= 0.0734 + 0.0391 = 0.1125 \text{ m}^3 \end{aligned}$$

(b) Table A-2 gives for liquid water at 25°C, $v = \frac{1.0029 \text{ m}^3}{\text{kg}}$. A handbook value for the specific volume of liquid C₂H₅OH at 25°C is $\frac{1.2738 \text{ m}^3}{10^3 \text{ kg}}$. Thus

$$\begin{aligned} V &= m_1 v_1 + m_2 v_2 \\ &= (59 \text{ kg}) \left(\frac{1.2738 \text{ m}^3}{10^3 \text{ kg}} \right) + (41 \text{ kg}) \left(\frac{1.0029 \text{ m}^3}{10^3 \text{ kg}} \right) \\ &= 0.0752 + 0.0411 = 0.1163 \text{ m}^3 \quad \left(\begin{array}{l} \text{about 3\% above the value} \\ \text{obtained previously} \end{array} \right) \end{aligned}$$

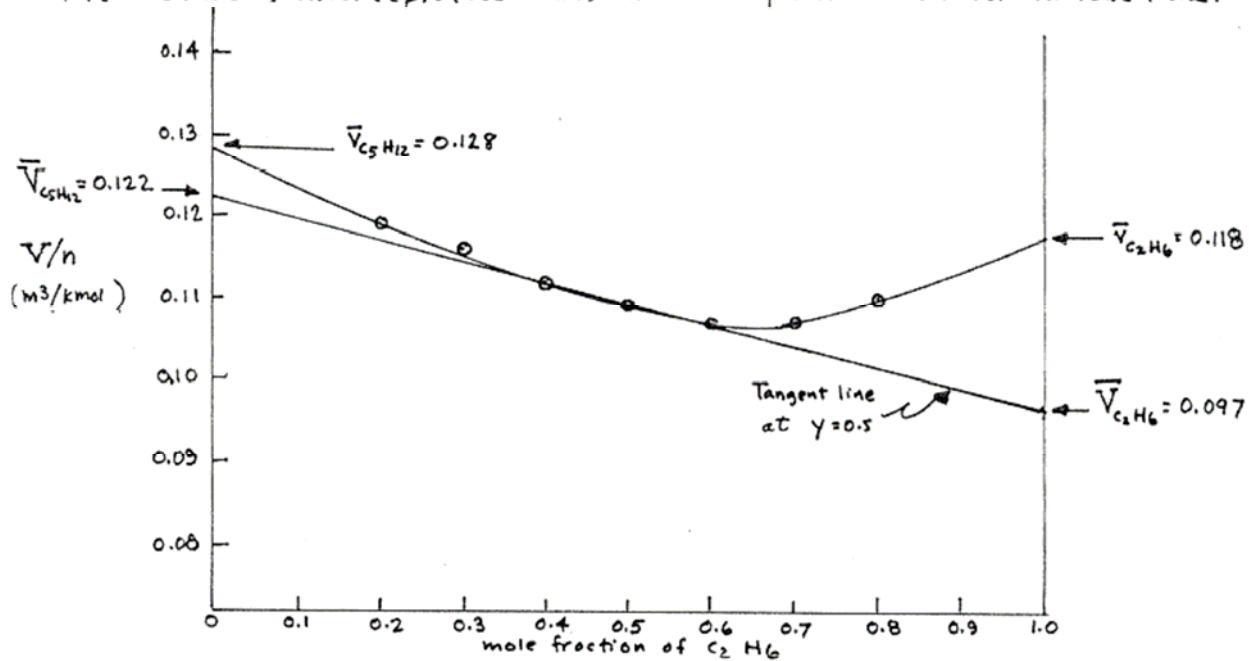
PROBLEM 11.106

KNOWN: Data are provided for a solution of C_2H_6 and C_5H_{12} .

FIND: Estimate (a) the specific volumes of C_2H_6 and C_5H_{12} , (b) the partial molar volumes of C_2H_6 and C_5H_{12} for an equimolar solution.

ENGINEERING MODEL: The solution is the closed system.

ANALYSIS: Using the data provided, the following plot can be developed. The method of intercepts (Sec. 11.9.1) allows the partial molar volumes to be found.



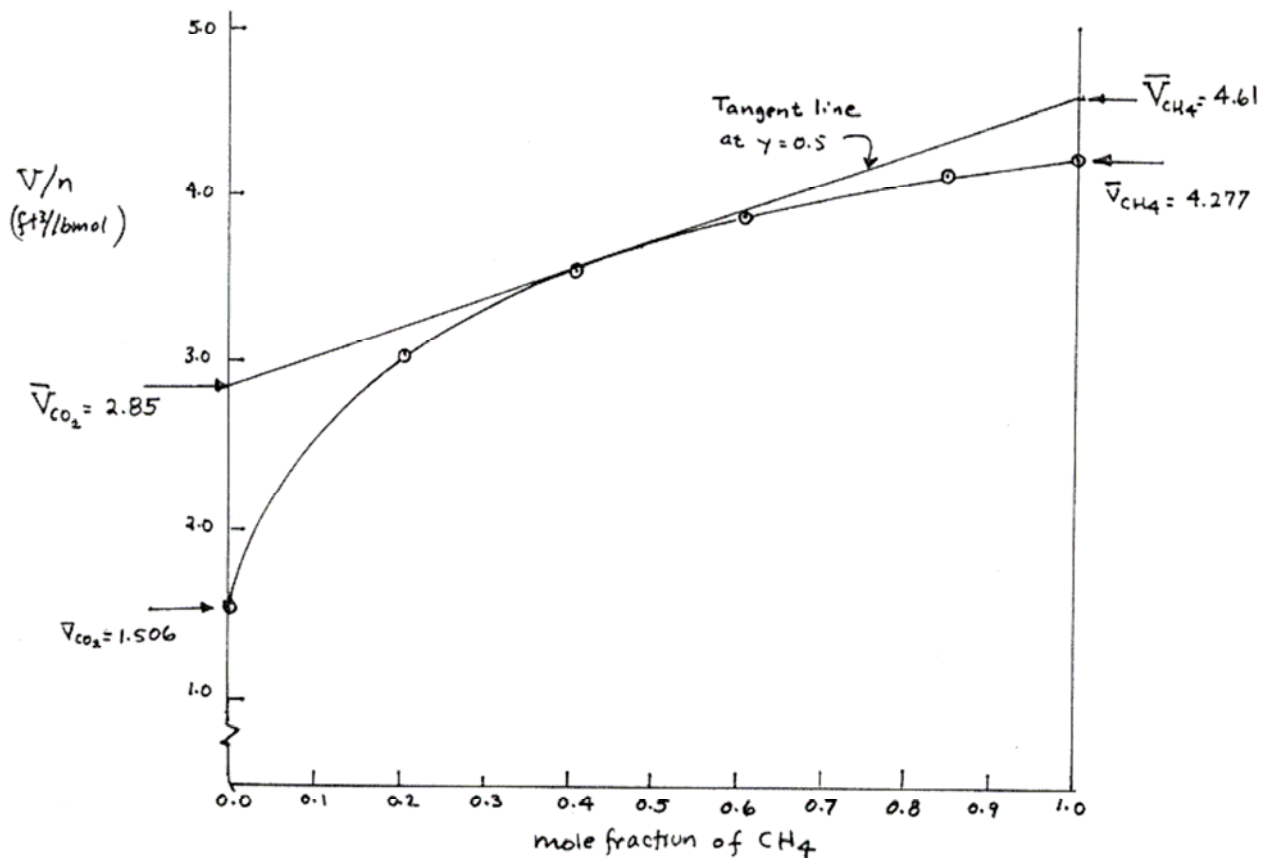
PROBLEM 11.107

KNOWN: Data are provided for a mixture of CO_2 and CH_4 .

FIND: Estimate (a) the specific volumes of CO_2 and CH_4 , (b) the partial molar volumes of CO_2 and CH_4 for an equimolar mixture.

ENGINEERING MODEL: The mixture is the closed system.

ANALYSIS: Using the data provided, the following plot can be developed. The method of intercepts (Sec. 11.9.1) allows the partial molar volumes to be found.



PROBLEM 11.108

KNOWN: Water as a saturated vapor at (a) 280°C, (b) 500°F is under consideration.

FIND: Using p-v-T data from the steam tables determine the fugacity at each state and compare with the value yielded by the generalized fugacity chart.

ANALYSIS: (a) Saturated vapor at 280°C.

From Table A-1 $T_c = 647.3 \text{ K}$, $P_c = 220.9 \text{ bar}$. Then $T_R = 553/647.3 = 0.854$
 $(P_R = 64.12/220.9 = 0.29)$ Then, Fig. A-6 gives $f/p \approx 0.825$. Thus

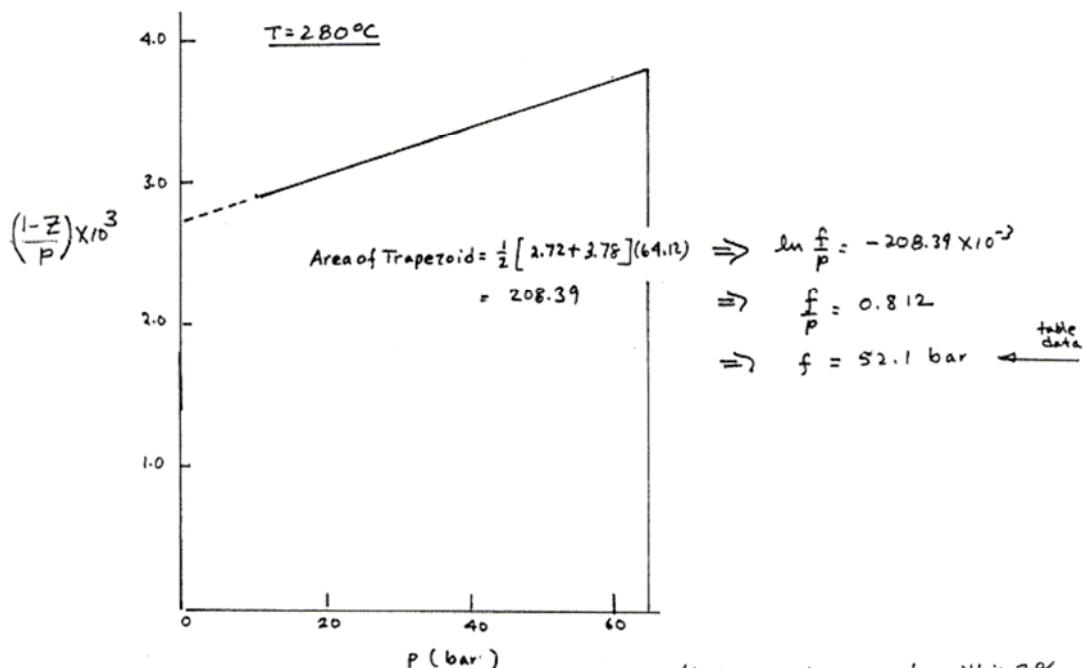
$$f = 0.825(64.12) = 52.9 \text{ bar} \quad \leftarrow \text{chart}$$

From the discussion of Sec. 11.9.4

$$\ln \frac{f}{p} = \int_0^P \left(\frac{Z-1}{p} \right) dp$$

Graphical Solution:

Using p-v-T data from the steam tables the following plot can be drawn at $T = 280^\circ\text{C}$



The answers obtained using these two graphical methods agree to within 2%.

(b) Saturated vapor at 500°F

From Table A-1E $T_c = 1165^\circ\text{R}$, $P_c = 218 \text{ atm}$. Then, $T_R = 960/1165 = 0.82$ ($P_R = \frac{680/14.696}{218} = 0.212$). Fig. A-6 gives $f/p \approx 0.87$. Thus

$$f = 0.87(680) = 592 \text{ lbf/in}^2 \quad \leftarrow \text{chart}$$

From the discussion of Sec. 11.9.4

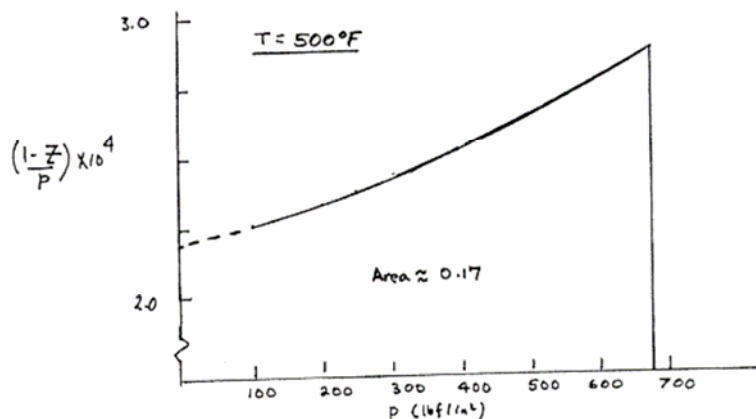
$$\ln \frac{f}{p} = \int_0^P \left(\frac{Z-1}{p} \right) dp$$

Continued on next slide

Problem 11-108 continued

Graphical Solution:

Using p-v-T data from the steam tables the following plot can be drawn at $T = 500^\circ\text{F}$



Thus $\ln \frac{f}{p} = -0.17 \Rightarrow \frac{f}{p} = 0.844 \Rightarrow f = 574 \frac{\text{lbf}}{\text{in}^2}$

The answers obtained using these two graphical methods agree to within 3%.

The following IT code provides an alternative to using Steam Table data for part (a). A similar approach can be used for part (b).

IT Code

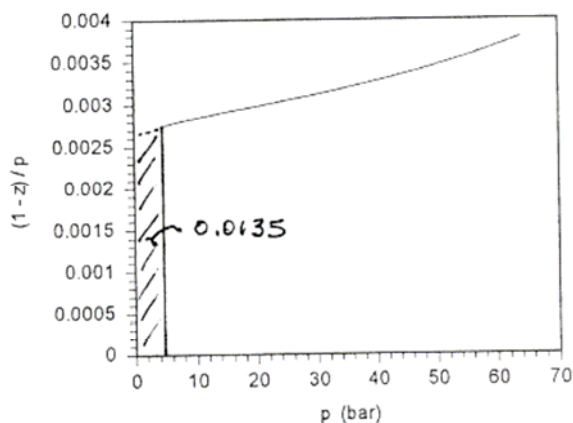
```
T = 280 // °C
psat = 64.12 // bar

R = 8.314/18.02 // kJ/kg·K
Z = (p * v * 100) / (R * (T + 273.15))
v = v_PT("Water/Steam", p, T)
Int = -(Z - 1) / p
Inf_p = Integral(Int, p)
f_p = exp(Inf_p + 0.0135)
f = f_p * p
```

/* The constant 0.0135 is added to account for the area from 0 to 5 bar. The computer cannot evaluate Int for that range because the pressure is too low. The value was determined by extrapolating the data from the BROWSE table to low pressure. */

IT Results

```
ln f/p = -0.1898
f/p = 0.816
f = 52.31 bar
```



PROBLEM 11.109

KNOWN: Systems at specified states are given.

FIND: For each case determine the fugacity, in atm.

ANALYSIS:

(a) Butane 555 K, 150 bar

For Butane $T_c = 425 \text{ K}$, $p_c = 38 \text{ bar}$. Then

$$T_R = \frac{555}{425} = 1.31, \quad P_R = \frac{150}{38} = 3.95$$

$$\text{Figure A-6, } \frac{f}{p} = 0.62 \Rightarrow f = 0.62(150 \text{ bar}) \left| \frac{1 \text{ atm}}{1.01325 \text{ bar}} \right| = 91.8 \text{ atm} \quad \leftarrow (a)$$

(b) Methane 120°F, 800 lbf/in²

For Methane $T_c = 344^\circ \text{R}$, $p_c = 45.8 \text{ atm}$. Then

$$T_R = \frac{580}{344} = 1.69, \quad P_R = \frac{(800 \text{ lbf/in}^2 / 14.7 \frac{\text{lbf/in}^2}{\text{atm}})}{45.8 \text{ atm}} = 1.19$$

$$\text{Figure A-6, } \frac{f}{p} = 0.95 \Rightarrow f = 51.7 \text{ atm} \quad \leftarrow (b)$$

(c) Benzene 890°R, 135 atm

For Benzene $T_c = 1013^\circ \text{R}$, $p_c = 48.7 \text{ atm}$. Then

$$T_R = \frac{890}{1013} = 0.88, \quad P_R = \frac{135 \text{ atm}}{48.7 \text{ atm}} = 2.77$$

$$\text{Figure A-6, } \frac{f}{p} = 0.18 \Rightarrow f = 24.3 \text{ atm} \quad \leftarrow (c)$$

PROBLEM 11.110

KNOWN: The equation of state has the form

$$Z = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \frac{P_R}{T_R}$$

FIND: Using the equation of state evaluate the fugacity of ammonia at 750 K, 100 atm and compare with the value obtained from Fig. A-6.

ANALYSIS: Eq. 11.124 is the applicable expression

$$\ln \frac{f}{P} = \int_0^{P_R} (Z-1) d \ln P_R = \int_0^{P_R} \left(\frac{Z-1}{P_R} \right) dP_R$$

The equation of state gives

$$\frac{Z-1}{P_R} = \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \frac{1}{T_R}$$

Combining these two results

$$\ln \frac{f}{P} = \int_0^{P_R} \frac{1}{T_R} \left[\frac{1}{8} - \frac{27/64}{T_R} \right] dP_R = \frac{P_R}{T_R} \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \quad (1)$$

For ammonia, Table A-1 gives $T_c = 406 \text{ K}$, $P_c = 112.8 \text{ bar}$. Thus

$$P_R = \frac{(100 \text{ atm})(1.01325 \text{ bar/atm})}{112.8 \text{ bar}} = 0.898$$

$$T_R = \frac{750 \text{ K}}{406 \text{ K}} = 1.847$$

Inserting values into Eq. (1)

$$\ln \frac{f}{P} = \frac{0.898}{1.847} \left[\frac{1}{8} - \frac{27/64}{1.847} \right] \Rightarrow \frac{f}{P} = 0.951$$

Thus, $f = 95.1 \text{ atm}$. ←

By inspection of Fig. A-6, $f/P = 0.97$, giving $f = 97 \text{ atm}$.

$$\% \text{ difference} = \left(\frac{95.1 - 97}{97} \right) (100) = -1.96 \quad \leftarrow$$

PROBLEM 11.111

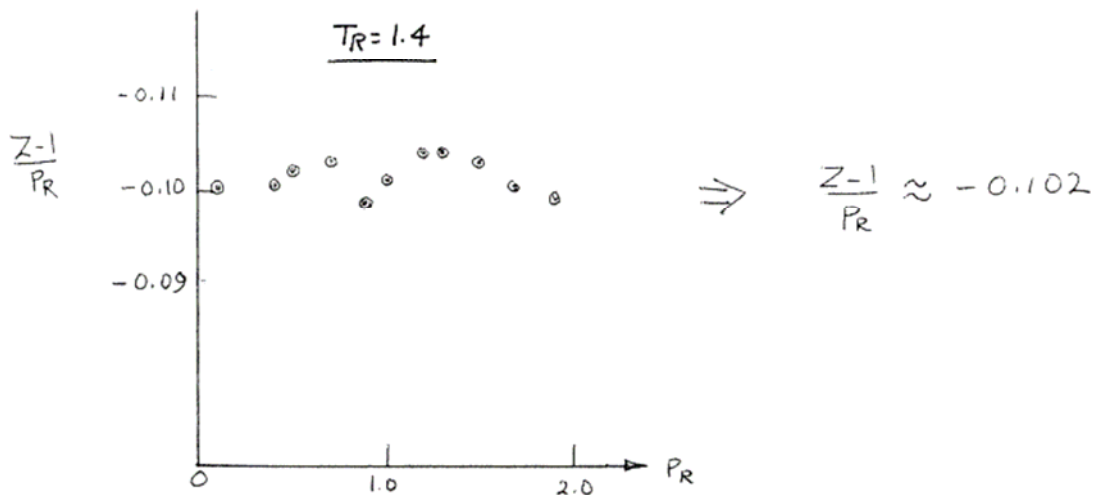
KNOWN: Tabulated compressibility data from the literature is to be used.

FIND: Evaluate f/p at $T_R = 1.4$, $P_R = 2.0$, and compare with the value from Fig. A-6.

ANALYSIS: From Fig. A-6, $f/p = 0.824$. For calculation, Eq. 11.124 is the applicable expression:

$$\ln \frac{f}{p} = \int_0^{P_R} (Z-1) d \ln P_R = \int_0^{P_R} \left(\frac{Z-1}{P_R} \right) dP_R$$

The plan is to evaluate $(Z-1)/P_R$ at $T_R = 1.4$ in the range $0 < P_R \leq 2.0$, and then plot vs P_R . (Data from "The Properties of Gases and Liquids" 2nd ed., by R.C. Reid and T.K. Sherwood, McGraw-Hill, 1966, pp. 587-595.)



Accordingly

$$\ln \frac{f}{p} \approx (-0.102)(2.0) = -0.204$$

$$\Rightarrow \frac{f}{p} \approx 0.815$$

Comparison

$$\% = \left(\frac{0.815 - 0.824}{0.824} \right) (100) = -1.1$$

PROBLEM 11.112

KNOWN: A truncated virial expansion is specified:

$$Z = 1 + \hat{B}(T_R) P_R + \hat{C}(T_R) P_R^2 + \hat{D}(T_R) P_R^3$$

FIND: (a) Using tabulated compressibility data, evaluate $\hat{B}, \hat{C}, \hat{D}$ for $0 < P_R < 1.0$ and $T_R = 1.0, 1.2, 1.4, 1.6, 1.8, 2.0$. (b) Obtain an expression for $\ln f/p$ in terms of T_R, P_R . Then, using the coefficients of part (a), evaluate f/p at selected states and compare with tabulated values.

ANALYSIS: (a) Eq. 11.124 is the applicable expression:

$$\ln \frac{f}{p} = \int_0^{P_R} (Z-1) d \ln P_R = \int_0^{P_R} \left(\frac{Z-1}{P_R} \right) dP_R$$

Using the given equation of state

$$\begin{aligned} \ln \frac{f}{p} &= \int_0^{P_R} (\hat{B}(T_R) + \hat{C}(T_R) P_R + \hat{D}(T_R) P_R^2) dP_R \\ &= \hat{B}(T_R) P_R + \hat{C}(T_R) \frac{P_R^2}{2} + \hat{D}(T_R) \frac{P_R^3}{3} \end{aligned} \quad (1)$$

where the coefficients $\hat{B}, \hat{C}, \hat{D}$ are obtained for selected T_R values and P_R ranging from 0 to 1.0 by linear regression using tabulated compressibility data. The coefficients evaluated this way are

T_R	$\hat{B}$	$\hat{C}$	$\hat{D}$
1	-0.66404	1.274314	-1.30854
1.0	-0.32199	0.081732	-0.14687
1.1	-0.24781	-0.02425	-0.02729
1.2	-0.17771	0.012029	-0.04160
1.3	-0.12188	-0.05410	0.034815
1.4	-0.09219	-0.02849	0.021567
1.5	-0.07307	-0.00706	0.010698
1.6	-0.06488	0.028939	-0.01516
1.8	-0.05442	0.068857	-0.04020
2	-0.03951	0.070563	-0.04350

← (a)

(b) As a sample calculation, at $P_R = 0.5, T_R = 1.2, f/p = 0.92$. Inserting values into Eq. (1)

$$\ln \frac{f}{p} = -0.17771(0.5) + 0.012029 \frac{(0.5)^2}{2} - 0.04160 \frac{(0.5)^3}{3}$$

$$\Rightarrow \frac{f}{p} = 0.915$$

← (b)

The % difference is

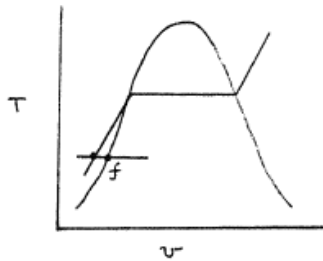
$$\% = \left(\frac{0.915 - 0.92}{0.92} \right) (100) = -0.54$$

PROBLEM 11.113

KNOWN : Approximate expressions for the fugacity of a liquid are specified.

FIND : Derive the expressions and comment.

SCHEMATIC & GIVEN DATA :



ANALYSIS : Beginning with Eq. 11.122

$$RT \left(\frac{\partial \ln f}{\partial p} \right)_T = v$$

Then, approximating v as $v \approx v_f(T)$

$$\left(\frac{\partial \ln f}{\partial p} \right)_T \approx \frac{v_f(T)}{RT} \Rightarrow \ln \frac{f}{f_{sat}^L} \approx \frac{v_f(T)}{RT} [p - p_{sat}]$$

or

$$f(T, p) \approx f_{sat}^L(T) \exp \left\{ \frac{v_f(T)}{RT} (p - p_{sat}) \right\}$$

Letting $x = \frac{v_f(T)}{RT} [p - p_{sat}]$ and expanding $\exp(x)$

$$\frac{f(T, p)}{f_{sat}^L(T)} \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Accordingly, $f(T, p) \approx f_{sat}^L$ when the terms in powers of x are small enough to ignore. For example, considering the case of water at 27°C , Table A-2 gives $p_{sat} = 0.03567 \text{ bar}$, $v_f = 1.0035 \times 10^{-3} \text{ m}^3/\text{kg}$

$$x = \frac{v_f(T)}{RT} [p - p_{sat}] = \frac{1.0035 \times 10^{-3} \text{ m}^3/\text{kg}}{\left(\frac{8314 \text{ N}\cdot\text{m}}{18.02 \text{ kg}\cdot\text{K}} \right) (300.15 \text{ K})} \left[\frac{p - 0.03567 \text{ bar}}{\left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right|} \right]$$

$$= (7.25 \times 10^{-4}) [p - 0.03567]$$

The pressure range for which $x < 10^{-n}$, where n is an integer, is

$$7.25 \times 10^{-4} [p - 0.03567] < 10^{-n}$$

$$\Rightarrow [p - 0.03567] < 0.13793 \times 10^{4-n}$$

n	$p \text{ (bar)}$
2	< 13.83
3	< 1.41

PROBLEM 11.114

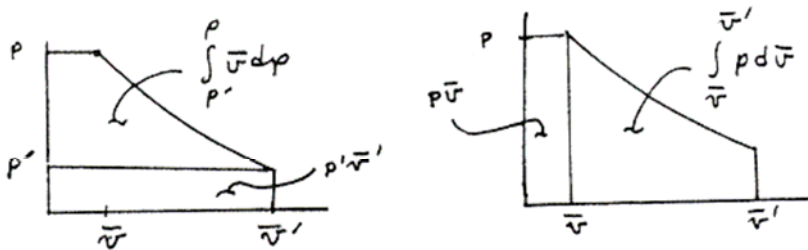
KNOWN: The Redlich-Kwong equation of state describes the p - $\bar{v}$ - T behavior of a gas.

FIND: (a) Evaluate $\ln f$ for the gas. (b) Using the result of part (a), evaluate f , in bar, for Refrigerant 134a at 90°C , 10 bar, and compare with the value obtained from Fig. A-6, the generalized fugacity chart.

ANALYSIS: (a) Beginning with Eq. 11.122, integrate at fixed T from $(p', \bar{v}')$ to $(p, \bar{v})$

$$\bar{R}T \ln f \Big|_{p'}^p = \int_{p'}^p \bar{v} dp \quad (1)$$

Considering areas,



$$\begin{aligned} \int_{p'}^p \bar{v} dp + p' \bar{v}' &= \int_{\bar{v}}^{\bar{v}'} p d\bar{v} + p \bar{v} \\ \Rightarrow \int_{p'}^p \bar{v} dp &= p \bar{v} - p' \bar{v}' + \int_{\bar{v}}^{\bar{v}'} p d\bar{v} \\ &= p \bar{v} - p' \bar{v}' - \int_{\bar{v}'}^{\bar{v}} p d\bar{v} \end{aligned} \quad (2)$$

Combining Eqs. (1) and (2)

$$\bar{R}T \ln f \Big|_{p'}^p = p \bar{v} - p' \bar{v}' - \int_{\bar{v}'}^{\bar{v}} p d\bar{v} \quad (3)$$

Introducing the Redlich-Kwong equation in the integral

$$\begin{aligned} \bar{R}T \ln f \Big|_{p'}^p &= p \bar{v} - p' \bar{v}' - \int_{\bar{v}'}^{\bar{v}} \left[\frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\sqrt{T} \bar{v}(\bar{v}+b)} \right] d\bar{v} \\ \Rightarrow \ln \frac{f}{f'} &= \frac{p \bar{v}}{\bar{R}T} - \frac{p' \bar{v}'}{\bar{R}T} - \ln \left[\frac{\bar{v}-b}{\bar{v}'-b} \right] - \frac{a}{b \bar{R}T^{3/2}} \ln \left[\frac{\bar{v}+b}{\bar{v}} \left(\frac{1}{1+b/\bar{v}'} \right) \right] \\ \text{or} \quad \ln f &= \frac{p \bar{v}}{\bar{R}T} - \frac{p' \bar{v}'}{\bar{R}T} + \ln \left[\frac{f'(\bar{v}'-b)}{\bar{v}-b} \right] - \frac{a}{b \bar{R}T^{3/2}} \ln \left[\frac{\bar{v}+b}{\bar{v}} \left(\frac{1}{1+b/\bar{v}'} \right) \right] \end{aligned}$$

Continued on next slide

Problem 11-114 continued

Next, consider the limit as $p' \rightarrow 0$ ($\bar{v}' \rightarrow \infty$) at fixed T :

$$f' \rightarrow p' \rightarrow 0, p'\bar{v}' \rightarrow \bar{R}T, p'(\bar{v}'-b) \rightarrow \bar{R}T, 1 + \frac{b}{\bar{v}'} \rightarrow 1.$$

Thus

$$\ln f = Z - 1 + \ln \left[\frac{\bar{R}T}{\bar{v}-b} \right] - \frac{a}{b\bar{R}T^{3/2}} \ln \left(\frac{\bar{v}+b}{\bar{v}} \right) \quad (4)$$

Eq (4) can be rewritten using the Redlich-Kwong equation:

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)T^{1/2}} \Rightarrow \underbrace{\frac{p\bar{v}}{\bar{R}T}}_Z = \frac{\bar{v}}{\bar{v}-b} - \frac{a}{(\bar{v}+b)\bar{R}T^{3/2}}$$

$$\Rightarrow Z = \frac{\bar{v}}{\bar{v}-b} - \frac{a}{(\bar{v}+b)\bar{R}T^{3/2}} \Rightarrow Z-1 = \frac{b}{\bar{v}-b} - \frac{a}{(\bar{v}+b)\bar{R}T^{3/2}}$$

Thus

$$\ln f = \frac{b}{\bar{v}-b} + \ln \left[\frac{\bar{R}T}{\bar{v}-b} \right] - \frac{a}{\bar{R}T^{3/2}} \left[\frac{1}{(\bar{v}+b)} + \frac{1}{b} \ln \left(\frac{\bar{v}+b}{\bar{v}} \right) \right] \quad (5) \leftarrow$$

(b) From Table A-1, for R134a, $T_c = 374\text{K}$, $P_c = 40.7\text{bar}$. Thus

$$T_R = \frac{363\text{K}}{374\text{K}} = 0.97, P_R = \frac{10\text{bar}}{40.7\text{bar}} = 0.25$$

Figure A-6 gives, $\frac{f}{p} \sim 0.92 \Rightarrow f = 9.2\text{bar} \leftarrow$

Evaluating a and b for R134a via Eqs. 11.8

$$a = 196.39 \text{ bar} \left(\frac{\text{m}^3}{\text{kmol}} \right)^2 (\text{K})^{1/2}, b = 0.0662 \text{ m}^3/\text{kmol}$$

Solving the Redlich-Kwong equation for $\bar{v}$ when $T = 363\text{K}$ and $p = 10\text{bar}$ gives $\bar{v} = 2.724 \text{ m}^3/\text{kmol}$.

Finally, inserting values into Eq. (5), using $\bar{R} = 0.08314 \frac{\text{bar} \cdot \text{m}^3}{\text{kmol} \cdot \text{K}}$

$$\ln f = \frac{0.0662}{2.6578} + \ln \left[\frac{(0.08314)(363)}{2.6578} \right] - \frac{196.39}{(0.08314)(363)^{3/2}} \left[\frac{1}{2.7902} + \frac{1}{0.0662} \ln \left(\frac{2.7902}{2.724} \right) \right]$$

$$= 2.208 \Rightarrow f = 9.1\text{bar} \leftarrow$$

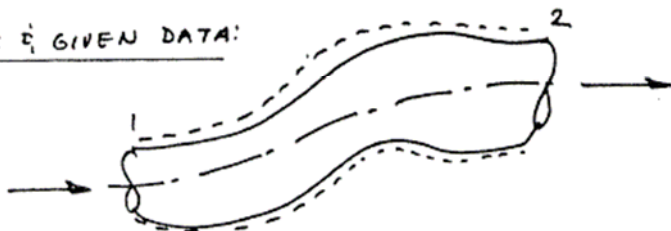
The calculated and chart values for f agree to within $\sim 1\%$.

PROBLEM 11.115

KNOWN: Under consideration is a one-inlet one-exit control volume at steady state through which the flow is internally reversible and isothermal.

FIND: Derive an expression for the work per unit of mass flowing in terms of the fugacity and other relevant quantities.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying sketch is at steady state. (2) The flow is internally reversible and isothermal. (3) A single component (or a mixture with known properties such as air) is flowing.

ANALYSIS:

mass rate balance: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$

energy rate balance: $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right]$

entropy rate balance (with assumption 2):

$$0 = \frac{\dot{Q}_{cv}}{T} + \dot{m}(s_1 - s_2) + \cancel{\dot{Q}_{cv}}$$

Eliminating $\dot{Q}_{cv}$ between the energy and entropy rate balances

$$\frac{\dot{W}_{cv}}{\dot{m}} = T(s_2 - s_1) + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2)$$

The Gibbs function is $g = h - Ts$, thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = (g_1 - g_2) + \left(\frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2)$$

For a single component, the chemical potential equals the Gibbs function per mole. Then, with Eq. 11.121

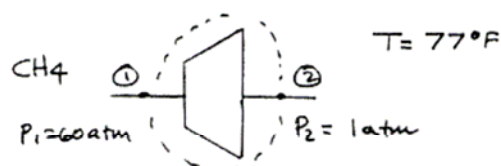
$$\frac{\dot{W}_{cv}}{\dot{m}} = -RT \ln \frac{f_2}{f_1} + \left(\frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2) \quad \leftarrow$$

PROBLEM 11.116

KNOWN: Methane expands isothermally and without irreversibilities through a turbine operating at steady state.

FIND: Using the generalized fugacity chart, determine the work developed per lb of CH₄ flowing.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The control volume shown in the figure is at steady state. (2) The expansion is isothermal and internally reversible. (3) Kinetic and potential energy effects can be ignored.

ANALYSIS: Using the result of Problem 11.115 and data from Table A-1E and

Figure A-6, $P_{R1} = \frac{60 \text{ atm}}{45.8 \text{ atm}} = 1.31$, $T_{R1} = T_{R2} = \frac{537^\circ\text{R}}{344^\circ\text{R}} = 1.56$

$$P_{R2} = \frac{1}{45.8} = 0.02$$

$$\left(\frac{f}{P}\right)_1 \approx 0.92, \left(\frac{f}{P}\right)_2 \approx 1.0 \Rightarrow \left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{int, rev} = -RT \ln \frac{f_2}{f_1} = -\left(\frac{1.986 \text{ Btu}}{16.04 \text{ lb} \cdot 0.92}\right) (537^\circ\text{R}) \ln \left(\frac{1.0 \times 1 \text{ atm}}{0.92 \times 60 \text{ atm}}\right)$$

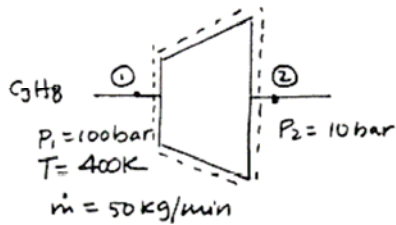
$$= 266.7 \text{ Btu/lb} \leftarrow$$

PROBLEM 11.117

KNOWN: Propane expands isothermally and without irreversibilities through a turbine operating at steady state.

FIND: Using the generalized fugacity chart, determine the power developed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume shown operates at steady state.
2. The expansion is isothermal and internally reversible.
3. Kinetic & potential energy effects can be ignored.

ANALYSIS: Using the result of Problem 11.115 and data from Table A-1 and Fig. A-6, $P_{R1} = \frac{100 \text{ bar}}{42.7 \text{ bar}} = 2.34$, $P_{R2} = \frac{10}{42.7} = 0.23$, $T_R = \frac{400}{370} = 1.08$

$\left(\frac{f}{P}\right)_1 = 0.48$, $\left(\frac{f}{P}\right)_2 = 0.96$. Then

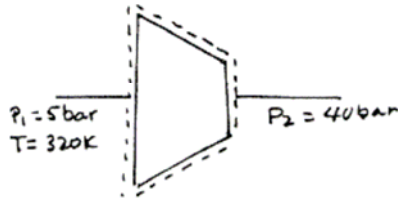
$$\begin{aligned}
 \dot{W}_{cv} &= -\dot{m} R T \ln \left(\frac{f_2}{f_1} \right) = -\dot{m} \left(\frac{\bar{R}}{M} \right) T \ln \left(\left(\frac{f_2}{P_2} \right) \left(\frac{P_2}{P_1} \right) \left(\frac{P_1}{f_1} \right) \right) \\
 &= - \left(\frac{50}{60} \frac{\text{kg}}{\text{s}} \right) \left(\frac{8.314}{44.09} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (400 \text{ K}) \ln \left((0.96)(0.1) \left(\frac{1}{0.48} \right) \right) \left| \frac{\text{kW}}{\text{kJ/s}} \right| = 101.1 \text{ kW} \leftarrow \dot{W}_{cv}
 \end{aligned}$$

PROBLEM 11.118

KNOWN: C_2H_6 is compressed isothermally without internal irreversibilities.

FIND: Determine the work and heat transfer, each per unit mass of C_2H_6 flowing.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is at steady state.
2. Kinetic & potential energy effects can be ignored.
3. The process occurs isothermally and without internal irreversibilities.

ANALYSIS: Using the result of Prob. 11.115

$$\frac{\dot{W}_{cv}}{\dot{m}} = -RT \ln \left(\left(\frac{f}{P} \right)_2 \left(\frac{P_2}{P_1} \right) \left(\frac{P_1}{f_1} \right) \right)$$

With data from Table A-1 and Fig. A-6,

$$T_R = \frac{320 \text{ K}}{305 \text{ K}} = 1.05 \quad P_{R1} = \frac{5 \text{ bar}}{48.8 \text{ bar}} = 0.1, \quad P_{R2} = \frac{40}{48.8} = 0.82, \quad \left(\frac{f}{P} \right)_1 = 0.98, \quad \left(\frac{f}{P} \right)_2 = 0.79$$

Thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = - \left(\frac{8.314 \text{ kJ}}{30.07 \text{ kg} \cdot \text{K}} \right) (320 \text{ K}) \ln \left((0.79) \left(\frac{40}{5} \right) (0.98) \right) = -164.9 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow \frac{\dot{W}_{cv}}{\dot{m}}$$

Reducing mass and energy rate balances gives $\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$.
Invoking Eq. 11.85 and Fig. A-4 data

$$h_2 - h_1 = \underbrace{\bar{h}_2^* - \bar{h}_1^*}_{=0 \text{ since } T \text{ is const}} - \bar{R} T_c \left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c} \right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c} \right)_1 \right]$$

$$\Rightarrow h_2 - h_1 = - \left(\frac{8.314}{30.07} \right) (305) [(1.0) - (0.09)] = -76.7 \frac{\text{kJ}}{\text{kg}}$$

Then

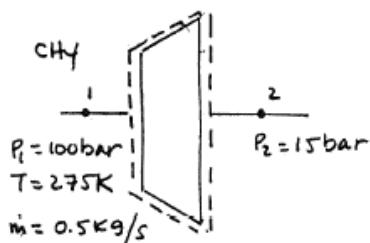
$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1) = -164.9 \frac{\text{kJ}}{\text{kg}} + (-76.7) \frac{\text{kJ}}{\text{kg}} = -241.6 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow \frac{\dot{Q}_{cv}}{\dot{m}}$$

PROBLEM 11.119

KNOWN: Steady-state operating data are provided for methane expanding isothermally and without internal irreversibilities through a turbine.

FIND: Determine the power developed and the rate of heat transfer.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is at steady state.
2. Kinetic & potential energy effects can be ignored.

ANALYSIS: Using the result of Prob. 11.115 together with data from Table A-1 and Fig. A-6: $M = 16.04$, $T_r = \frac{275 \text{ K}}{191 \text{ K}} = 1.44$, $P_{r1} = \frac{100 \text{ bar}}{46.4 \text{ bar}} = 2.16$, $P_{r2} = \frac{15}{46.4} = 0.32$, $\left(\frac{f}{p}\right)_1 = 0.82$, $\left(\frac{f}{p}\right)_2 = 0.97$.

Then

$$\dot{W}_{cv} = -\dot{m} R T \ln\left(\frac{f_2}{f_1}\right) = -\dot{m} \left(\frac{\bar{R}}{M}\right) T \ln\left(\left(\frac{f}{p}\right)_2 \left(\frac{p_2}{p_1}\right) \left(\frac{p}{f}\right)_1\right)$$

$$= -(0.5 \frac{\text{kg}}{\text{s}}) \left(\frac{8.314}{16.04}\right) (275) \ln\left(0.97 \left(\frac{15}{100}\right) \left(\frac{1}{0.82}\right)\right) = 123.3 \text{ kW} \quad \leftarrow \dot{W}_{cv}$$

Mass and energy rate balances reduce to give $\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$.

$(h_2 - h_1)$ can be found using Eq. 11.85 and data from Fig. A-4

$$\bar{h}_2 - \bar{h}_1 = \underbrace{\bar{h}_2^* - \bar{h}_1^*}_{=0 \text{ since } T \text{ is const}} - \bar{R} T_c \left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c}\right)_1 \right]$$

$$\Rightarrow h_2 - h_1 = -\left(\frac{8.314}{16.04}\right) (191) [0.17 - (1.18)] = 100 \frac{\text{kJ}}{\text{kg}}$$

Thus

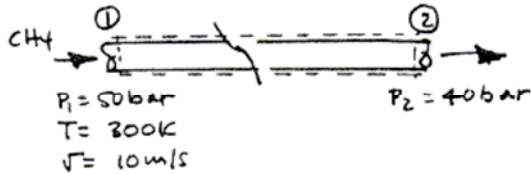
$$\dot{Q}_{cv} = (123.3 \text{ kW}) + (0.5 \frac{\text{kg}}{\text{s}}) \left(100 \frac{\text{kJ}}{\text{kg}}\right) \left|\frac{\text{kW}}{\text{kJ/s}}\right| = 173.3 \text{ kW} \quad \leftarrow \dot{Q}_{cv}$$

PROBLEM 11.120

KNOWN: CH₄ flows isothermally and without irreversibilities through a horizontal pipe. Steady-state data are provided.

FIND: Determine the exit velocity.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The control volume enclosing the pipe is at steady state.
 2. For the control volume, potential energy effects can be ignored, and $\dot{W}_{cv} = 0$.

ANALYSIS: Using the result of Problem 11.115

$$\left(\frac{\dot{W}_{cv}}{\dot{m}} \right)_{int, rev}^0 = -RT \ln \left(\frac{f_2}{f_1} \right) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)$$

$$\Rightarrow V_2^2 = V_1^2 - 2RT \ln \left(\frac{f_2}{f_1} \right) \quad (1)$$

With data from Table A-1 and Fig A-6

$$M = 16.04, \quad T_R = \frac{300 \text{ K}}{191 \text{ K}} = 1.57, \quad P_{R1} = \frac{50}{46.4} = 1.08, \quad P_{R2} = \frac{40}{46.4} = 0.86$$

$$\left(\frac{f}{P} \right)_1 = 0.93, \quad \left(\frac{f}{P} \right)_2 = 0.95$$

Thus, Eq. (1) gives

$$\begin{aligned} V_2^2 &= (10 \text{ m/s})^2 - 2 \left(\frac{8314 \text{ N} \cdot \text{m}}{16.04 \text{ kg} \cdot \text{K}} \right) (300 \text{ K}) \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \ln \left((0.93) \left(\frac{40}{50} \right) \left(\frac{1}{0.93} \right) \right) \\ &= (10 \text{ m/s})^2 - (-62.820 \text{ m}^2/\text{s}^2) = 162.8 \text{ m}^2/\text{s}^2 \end{aligned}$$

$$\therefore V_2 = 12.76 \text{ m/s} \quad \leftarrow \quad V_2$$

PROBLEM 11.121

KNOWN: Ethane (C_2H_6) is (a) pure at 310 K, 20.4 atm, (b) a component of an ideal solution at 310 K, 20.4 atm with a mole fraction of 0.35.

FIND: In each case, evaluate the fugacity.

ANALYSIS: (a) From Table A-1, $T_c = 305$ K, $p_c = 48.8$ bar. Then, $T_R = 310/305 = 1.02$,

$$P_R = \frac{20.4 \text{ atm}}{48.8 \text{ bar}} \left| \frac{1.01325 \text{ bar}}{1 \text{ atm}} \right| = 0.42$$
 Fig. A-6 gives $\frac{f}{p} = 0.89 \Rightarrow f = 18.2 \text{ atm} \leftarrow (a)$

(b) For ethane as a member of an ideal solution, Eq. 11.134 is applicable:

$\bar{f}_i = y_i f_i$, where f_i is the fugacity of pure i at the same T, p . Thus,

$$\bar{f}_i = 0.35(18.2 \text{ atm}) = 6.37 \text{ atm}.$$

$\leftarrow (b)$

PROBLEM 11.122

KNOWN: The fugacity of the solute (denoted by 2) in a dilute binary liquid solution at T, p is proportional to its mole fraction in the solution: $\bar{f}_2 = K y_2$, where K is a constant.

FIND: Show that the fugacity of the solvent (denoted by 1) is $\bar{f}_1 = y_1 f_1$, where y_1 is the mole fraction of the solvent and f_1 is the fugacity of pure 1 at T, p .

ANALYSIS: The intensive state of the solution is fixed by T, p, y_1 , and y_2 . But with T, p assumed known, either y_1 or y_2 can be viewed as the independent variable since $y_1 + y_2 = 1$. Accordingly, the Gibbs-Duhem equation (Eq. 11.113) reduces at fixed T, p and y_2 as the independent variable to read

$$n_1 \frac{d\mu_1}{dy_2} + n_2 \frac{d\mu_2}{dy_2} = 0 \Rightarrow y_1 \frac{d\mu_1}{dy_2} + y_2 \frac{d\mu_2}{dy_2} = 0$$

Introducing $\mu_i = \bar{R}T \ln \bar{f}_i + G_i(T)$ (Eq. 11.125)

$$y_1 \frac{d \ln \bar{f}_1}{dy_2} + y_2 \frac{d \ln \bar{f}_2}{dy_2} = 0 \quad (1)$$

$$\text{Since } \bar{f}_2 = K y_2 \Rightarrow \ln \bar{f}_2 = \ln y_2 + \ln K, \text{ and so } \frac{d \ln \bar{f}_2}{dy_2} = \frac{1}{y_2} \quad (2)$$

Combining Eqs. (1), (2)

$$y_1 \frac{d \ln \bar{f}_1}{dy_2} + 1 = 0 \Rightarrow \frac{d \ln \bar{f}_1}{dy_2} = -\frac{1}{y_1} \leftarrow (1 = y_1 + y_2) \\ = -\frac{1}{(1 - y_2)}$$

$$\text{Accordingly} \quad \ln \bar{f}_1 = \ln(1 - y_2) + \ln K \Rightarrow \bar{f}_1 = K(1 - y_2) \\ \bar{f}_1 = K y_1 \quad (3)$$

Since $\bar{f}_1 \rightarrow f_1$ as $y_1 \rightarrow 1$, $K = f_1$, and thus Eq. (3) becomes

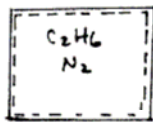
$$\bar{f}_1 = y_1 f_1 \quad \leftarrow$$

PROBLEM 11.123

KNOWN: A tank contains 310 kg of a gaseous mixture with the molar analysis {70% C₂H₆, 30% N₂} at 311 K, 170 atm.

FIND: Determine the tank volume using the compressibility chart and (a) Kay's rule, (b) the ideal solution model. Compare with the measured value: 1 m³.

SCHEMATIC & GIVEN DATA:



$T = 311 \text{ K}$
 $p = 170 \text{ atm}$
 $m = 310 \text{ kg}$
 1. C₂H₆ $y_1 = 0.7$
 2. N₂ $y_2 = 0.3$

ENGINEERING MODEL: The mixture is the closed system.

ANALYSIS: (a) Kay's Rule plus Z chart. With data from Table A-1, Eqs. 11.97 give

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.7)(305) + (0.3)(126) = 251.3 \text{ K}$$

$$p_c = y_1 p_{c1} + y_2 p_{c2} = (0.7)(48.2) + (0.3)(33.5) = 43.79 \text{ atm}$$

Thus, $T_R = 311/251.3 = 1.24$, $p_R = 170/43.79 = 3.88$. Then, Fig. A-2 gives, $Z \approx 0.65$, so

$$V = Z \left[\frac{mRT}{p} \right] = 0.65 \left[\frac{(310 \text{ kg}) (8314/29.45) (\text{N} \cdot \text{m}/\text{kg} \cdot \text{K}) (311 \text{ K})}{170 \times 1.01325 \times 10^5 \text{ N/m}^2} \right] = 1.03 \text{ m}^3 \quad \leftarrow (a)$$

① where $M = y_1 M_1 + y_2 M_2 = (0.7)(30.07) + (0.3)(28.0) = 29.45 \text{ kg/kmol}$ of mixture.

(b) Ideal Solution Model plus Z chart. Using Eqs. 11.136 for an ideal solution

$$V = n_1 \bar{v}_1 + n_2 \bar{v}_2 = n [y_1 \bar{v}_1 + y_2 \bar{v}_2]$$

where $\bar{v}_i$ is the molar specific volume of pure i at the temperature and pressure of the solution. Dividing by n , the number of moles of solution gives

$$\frac{V}{n} = y_1 \bar{v}_1 + y_2 \bar{v}_2 \quad (1)$$

To find $\bar{v}_1$, calculate $T_{R1} = 311/305 = 1.02$, $p_{R1} = 170/48.2 = 3.53$. Then, from Fig. A-2

$$Z_1 \approx 0.51. \text{ Accordingly } \bar{v}_1 = \frac{Z_1 \bar{R} T}{p} = \frac{(0.51) (8314 \text{ N} \cdot \text{m}/\text{kmol} \cdot \text{K}) (311 \text{ K})}{(170 \times 1.01325 \times 10^5 \text{ N/m}^2)} = 0.0766 \frac{\text{m}^3}{\text{kmol}}$$

To find $\bar{v}_2$, calculate $T_{R2} = 311/126 = 2.47$, $p_{R2} = 170/33.5 = 5.07$. From Fig. A-2, $Z_2 \approx 1.05$. Accordingly

$$\bar{v}_2 = \frac{Z_2 \bar{R} T}{p} = \frac{(1.05) (8314) (311)}{(170 \times 1.01325 \times 10^5)} = 0.1576 \frac{\text{m}^3}{\text{kmol}}$$

Then, with Eq. (1), $V = n [y_1 \bar{v}_1 + y_2 \bar{v}_2]$, or

$$V = \left(\frac{310 \text{ kg}}{29.45 \text{ kg/kmol}} \right) [(0.7)(0.0766) + (0.3)(0.1576)] \frac{\text{m}^3}{\text{kmol}} = 1.06 \text{ m}^3 \quad \leftarrow (b)$$

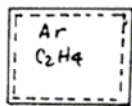
1. Using Eq. 12.9 of Sec. 12.1

PROBLEM 11.124

KNOWN: A tank contains 157 lb of a mixture of 75% Ar and 25% C_2H_4 (molar basis) at 77°F, 81.42 atm.

FIND: Estimate the tank volume using (a) the ideal gas equation of state, (b) Kay's rule with the gen. compressibility chart, (c) ideal solution model with the compressibility chart.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} T &= 537^\circ R \\ P &= 81.42 \text{ atm} \\ m &= 157 \text{ lb} \end{aligned}$$

$$\begin{aligned} 1. \text{ Argon } Y_1 &= 0.75 \\ 2. \text{ } C_2H_4 \text{ } Y_2 &= 0.25 \end{aligned}$$

ENGINEERING MODEL: The mixture is the closed system.

(a) Ideal gas model.

$$V_{id} = \frac{mRT}{P} = \frac{(157 \text{ lb})(1545/36.97)(ft \cdot lb / lb \cdot ^\circ R)(537^\circ R)}{(81.42 \times 14.7 \times 144) \text{ lb} / \text{ft}^2} = 20.44 \text{ ft}^3 \quad \leftarrow (a)$$

① where $M = Y_1 M_1 + Y_2 M_2 = (0.75)(39.94) + (0.25)(28.05) = 36.97 \text{ lb} / \text{lbmol}$

(b) Kay's rule with gen. compressibility chart. With data from Table A-1E, Eqs. 11.97 give

$$\begin{aligned} T_c &= Y_1 T_{c1} + Y_2 T_{c2} = (0.75)(272) + (0.25)(510) = 331.5^\circ R \\ P_c &= Y_1 P_{c1} + Y_2 P_{c2} = (0.75)(47.97) + (0.25)(50.5) = 48.61 \text{ atm} \end{aligned}$$

Thus, $T_R = 537/331.5 = 1.62$, $P_R = 1.67$. Fig. A-2 gives $Z \approx 0.91$, so

$$V = Z \frac{mRT}{P} = Z V_{id} = 0.91(20.44 \text{ ft}^3) = 18.6 \text{ ft}^3 \quad \leftarrow (b)$$

(c) Ideal solution with gen. compressibility chart. Using Eq. 11.136

$$V = n_1 \bar{V}_1 + n_2 \bar{V}_2 = n_1 \bar{v}_1 + n_2 \bar{v}_2$$

where $\bar{v}_i$ is the molar specific volume of pure i at the temperature and pressure of the solution. Dividing by n , the number of moles of solution gives

$$\frac{V}{n} = Y_1 \bar{v}_1 + Y_2 \bar{v}_2 \quad (1)$$

To find $\bar{v}_1$, calculate $T_{R1} = 537/272 = 1.97$, $P_{R1} = 81.42/47.97 = 1.7$. From Fig. A-2, $Z_1 \approx 0.97$. Accordingly

$$\bar{v}_1 = \frac{Z_1 \bar{R}T}{P} = \frac{(0.97)(1545)(537)}{[(81.42)(14.696)(144)]} = 4.671 \frac{\text{ft}^3}{\text{lbmol}}$$

To find $\bar{v}_2$, calculate $T_{R2} = 537/510 = 1.053$, $P_{R2} = 81.42/50.5 = 1.612$. From Fig. A-2, $Z_2 \approx 0.32$. Accordingly

$$\bar{v}_2 = \frac{Z_2 \bar{R}T}{P} = \frac{(0.32)(1545)(537)}{[(81.42)(14.696)(144)]} = 1.541 \frac{\text{ft}^3}{\text{lbmol}}$$

with Eq. (1)

$$V = n[Y_1 \bar{v}_1 + Y_2 \bar{v}_2] = \left(\frac{157}{36.97}\right) [0.75(4.671) + 0.25(1.541)] = 16.51 \text{ ft}^3 \quad \leftarrow (c)$$

1. Using Eq. 12.9 of Sec. 12.1

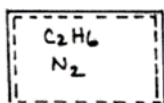
11-124

PROBLEM 11.125

KNOWN: A tank contains 2130 kg of a mixture of 70% C₂H₆, 30% N₂ at 400K, 200 atm.

FIND: Estimate the tank volume using (a) the ideal gas equation of state, (b) Kay's rule with the gen. compressibility chart, (c) the ideal solution model with the gen. compressibility chart.

SCHEMATIC & GIVEN DATA:



T = 400K
P = 200 atm
m = 2130 kg

1. C₂H₆, y₁ = 0.7
2. N₂, y₂ = 0.3

ENGINEERING MODEL: The mixture is the closed system.

ANALYSIS: (a) Ideal Gas Model.

$$V_{id} = \frac{mRT}{P} = \frac{(2130 \text{ kg}) \left(\frac{8314}{29.45} \right) \left(\frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (400 \text{ K})}{(200 \times 1.01325 \times 10^5 \text{ N/m}^2)} = 11.87 \text{ m}^3 \quad \leftarrow (a)$$

① where $M = y_1 M_1 + y_2 M_2 = (0.7)(30.07) + (0.3)(28.01) = 29.45 \text{ kg/kmol}$.

(b) Kay's rule with gen. compressibility chart. With Eqs. 11.97

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.7)(305) + (0.3)(126) = 251.3 \text{ K}$$

$$P_c = y_1 P_{c1} + y_2 P_{c2} = (0.7)(48.2) + (0.3)(33.5) = 43.79 \text{ atm}$$

Then, $T_R = 400/251.3 = 1.59$, $P_R = 200/43.79 = 4.57$. Fig. A-2 gives $Z \approx 0.87$. So

$$V = Z \left[\frac{mRT}{P} \right] = Z V_{id} = (0.87)(11.87) = 10.33 \text{ m}^3 \quad \leftarrow (b)$$

(c) Ideal Solution plus Z chart. Using Eqs. 11.136 for an ideal solution

$$V = n_1 \bar{V}_1 + n_2 \bar{V}_2 = n_1 \bar{v}_1 + n_2 \bar{v}_2$$

where $\bar{v}_i$ is the molar specific volume of pure i at the temperature and pressure of the solution. Dividing by n , the number of moles of solution gives

$$\frac{V}{n} = y_1 \bar{v}_1 + y_2 \bar{v}_2 \quad (1)$$

To find $\bar{v}_1$, calculate $T_{R1} = 400/305 = 1.31$, $P_{R1} = 200/48.2 = 4.15$. Then, Fig. A-2 gives $Z_1 \approx 0.7$. Accordingly

$$\bar{v}_1 = \frac{Z_1 \bar{R} T}{P} = \frac{(0.7) \left(\frac{8314}{29.45} \right) (400)}{[(200)(1.01325 \times 10^5)]} = 0.1149 \frac{\text{m}^3}{\text{kmol}}$$

To find $\bar{v}_2$, calculate $T_{R2} = 400/126 = 3.175$, $P_{R2} = 200/33.5 = 5.97$. Fig. A-2 gives

$Z_2 \approx 1.08$. Accordingly

$$\bar{v}_2 = \frac{Z_2 \bar{R} T}{P} = \frac{(1.08) \left(\frac{8314}{29.45} \right) (400)}{[(200)(1.01325 \times 10^5)]} = 0.1772 \frac{\text{m}^3}{\text{kmol}}$$

With Eq. (1)

$$V = n [y_1 \bar{v}_1 + y_2 \bar{v}_2] = \left(\frac{2130}{29.45} \right) [(0.7)(0.1149) + (0.3)(0.1772)] = 9.66 \text{ m}^3 \quad \leftarrow (c)$$

1. Using 12.9 of Sec. 12.1

11-125

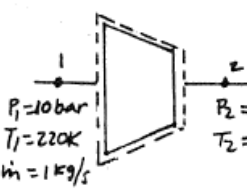
PROBLEM 11.126

KNOWN: Steady-state operating data are provided for a compressor handling an equimolar mixture of O_2 and N_2 .

FIND: Determine (a) the power required, (b) the rate of entropy production.

SCHEMATIC & GIVEN DATA:

Mixture: $y_{O_2} = y_{N_2} = 0.5$



	10 bar, 220 K		60 bar, 400 K	
	h (kJ/kg)	s (kJ/kg·K)	h (kJ/kg)	s (kJ/kg·K)
O_2	195.6	5.521	358.2	5.601
N_2	224.1	5.826	409.8	5.911

ENGINEERING MODEL: The control volume shown is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, kinetic & potential energy effects can be ignored. (3) The mixture adheres to the ideal solution model.

ANALYSIS: Reducing mass, energy, and entropy balances, we get

$$\dot{W}_{cv} = \dot{m} (h_1 - h_2) = \frac{\dot{m}}{M_{mix}} [\bar{h}_1 - \bar{h}_2], \quad \dot{S}_{cv} = \dot{m} (s_2 - s_1) = \frac{\dot{m}}{M_{mix}} (\bar{s}_2 - \bar{s}_1)$$

① where $M_{mix} = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = 30.01$.

(a) With Eq. 11.138

$$\begin{aligned} \dot{W}_{cv} &= \frac{\dot{m}}{M_{mix}} [y_{O_2} (h_1 - h_2)_{O_2} + y_{N_2} (h_1 - h_2)_{N_2} M_{N_2}] \\ &= \left(\frac{1 \text{ kg/s}}{30.01 \text{ kg/kmol}} \right) \left[(0.5) \left[\underbrace{(195.6 - 358.2)}_{-162.6} (32) + \underbrace{(224.1 - 409.8)}_{-185.7} (28.01) \right] \right] \frac{\text{kJ}}{\text{kmol}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 173.4 \text{ kW} \end{aligned}$$

$\leftarrow \dot{W}_{cv}$

(b) Similarly

$$\begin{aligned} \dot{S}_{cv} &= \left(\frac{1}{30.01} \right) \left[(0.5) \left[\underbrace{(5.601 - 5.521)}_{0.08} (32) + \underbrace{(5.911 - 5.826)}_{0.085} (28.01) \right] \right] \\ &= 0.082 \frac{\text{KW}}{\text{K}} \end{aligned}$$

$\leftarrow \dot{S}_{cv}$

1. Using Eq. 12.9 of Sec. 12.1

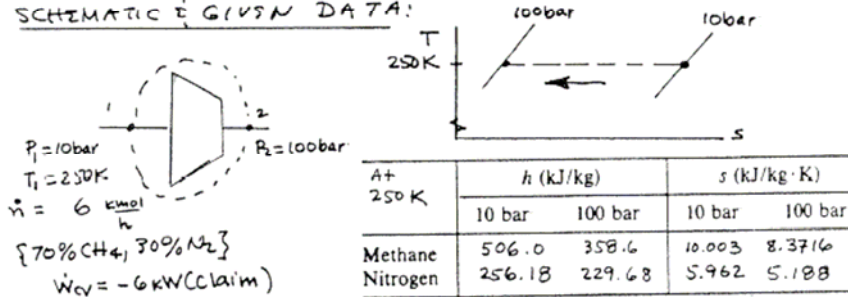
2. Since composition remains constant, this approach suffices. For the special case of ideal gas mixture processes at constant composition, see the discussion of Sec. 12.4.1 leading to Eq. 12.36.

PROBLEM 11.127

KNOWN: A gaseous mixture with the molar analysis {70% CH₄, 30% N₂} enters a compressor operating at steady state and exits at a higher pressure. During compression the temperature of the mixture departs by no more than 0.1 K from a temperature of 250 K. The power required is claimed to be 6 kW.

FIND: Determine if the claimed power requirement can be correct.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

- (1) The control volume shown in the figure is at steady state.
- (2) Kinetic and potential energy changes can be ignored.
- (3) The mixture is modeled as an ideal solution.
- (4) The temperature at which heat transfer takes place is 250 K.

ANALYSIS: Reducing mass, energy, and entropy balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{n}(\bar{h}_1 - \bar{h}_2); \quad 0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{n}(\bar{s}_1 - \bar{s}_2) + \dot{\sigma}_{cv}$$

Combining these expressions, we get

$$(-\dot{W}_{cv}) = \dot{n}[(\bar{h}_2 - \bar{h}_1) - T_b(\bar{s}_2 - \bar{s}_1)] + T_b\dot{\sigma}_{cv}$$

Since states 1 and 2 are fixed, the minimum theoretical power input corresponds to $\dot{\sigma}_{cv} = 0$: $(-\dot{W}_{cv})_{min} = \dot{n}[(\bar{h}_2 - \bar{h}_1) - T_b(\bar{s}_2 - \bar{s}_1)]$, where T_b is the temperature at which heat transfer occurs. Since the mixture is modeled as an ideal solution, use of Eq. 11.138 gives

$$\begin{aligned} (\bar{h}_2 - \bar{h}_1) &= y_{CH_4}[\bar{h}_2 - \bar{h}_1]_{CH_4} + y_{N_2}[\bar{h}_2 - \bar{h}_1]_{N_2} \\ &= (0.7)[358.6 - 506.0](16.04) + (0.3)[229.68 - 256.18](28.01) = -1877.7 \frac{\text{kJ}}{\text{kmol}} \end{aligned}$$

Similarly,

$$\begin{aligned} \textcircled{1} (\bar{s}_2 - \bar{s}_1) &= y_{CH_4}[\bar{s}_2 - \bar{s}_1]_{CH_4} + y_{N_2}[\bar{s}_2 - \bar{s}_1]_{N_2} \\ &= (0.7)[8.3716 - 10.003](16.04) + (0.3)[5.188 - 5.962](28.01) = -24.82 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \end{aligned}$$

Collecting results

$$\begin{aligned} (-\dot{W}_{cv})_{min} &= \left(6 \frac{\text{kmol}}{\text{h}}\right) \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left[-1877.7 - 250(-24.82) \right] \left(\frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \right) \left| \frac{\text{K}}{\text{K}} \right| \left(\frac{\text{h}}{\text{h}} \right) \\ &= 7.2 \text{ kW} \end{aligned}$$

Since the claimed value for $(-\dot{W}_{cv})$ is less than the minimum theoretical value, the claim cannot be valid. $\leftarrow$

1. Since composition remains constant, this approach suffices. For the special case of ideal gas mixture processes at constant composition, see the discussion of Sec. 12.4.1 leading to Eq. 12.36

Continued on next slide

Problem 11-128 continued

Also, for $dp = dT = 0$, the Gibbs-Duhem equation reduces to

$$\sum_{i=1}^2 n_i d\mu_i = 0 \quad (2)$$

for a binary mixture.

Also for a binary mixture,

$$\begin{aligned} y_1 + y_2 &= 1 \\ y_1 &= 1 - y_2 \\ \Rightarrow dy_1 &= -dy_2 \end{aligned} \quad (3)$$

The difference in the chemical potential of i between a specified state of the mixture and the reference state is given by Eq. 11.139

$$\mu_i - \mu_i^\circ = RT \ln \left(\frac{\bar{f}_i}{f_i^\circ} \right)$$

Inserting the activity coefficient and rearranging

$$\mu_i = \mu_i^\circ + RT \ln (\gamma_i y_i)$$

Performing the differentiation indicated in (1)

$$\frac{d\mu_i}{dy_i} = RT \left. \frac{d \ln \gamma_i}{dy_i} \right|_{T,P} + \frac{RT}{y_i}$$

$$\Rightarrow d\mu_i = RT \left[dy_i \left. \frac{d \ln \gamma_i}{dy_i} \right|_{T,P} + \frac{dy_i}{y_i} \right]$$

Substituting this result into (2)

$$n_1 RT \left[dy_1 \left. \frac{d \ln \gamma_1}{dy_1} \right|_{T,P} + \frac{dy_1}{y_1} \right] + n_2 RT \left[dy_2 \left. \frac{d \ln \gamma_2}{dy_2} \right|_{T,P} + \frac{dy_2}{y_2} \right] = 0$$

Continued on next slide

Problem 11-128 continued

Recalling that $dy_2 = -dy_1$

$$n_1 RT \left[dy_1 \left. \frac{d \ln \gamma_1}{d y_1} \right|_{T,p} + \frac{dy_1}{y_1} \right] + n_2 RT \left[-dy_1 \left. \frac{d \ln \gamma_2}{d y_2} \right|_{T,p} - \frac{dy_1}{y_2} \right] = 0$$

$$n_1 \left[dy_1 \left. \frac{d \ln \gamma_1}{d y_1} \right|_{T,p} + \frac{dy_1}{y_1} \right] = n_2 \left[dy_1 \left. \frac{d \ln \gamma_2}{d y_2} \right|_{T,p} + \frac{dy_1}{y_2} \right]$$

$$n_1 \left[\left. \frac{d \ln \gamma_1}{d y_1} \right|_{T,p} + \frac{1}{y_1} \right] = n_2 \left[\left. \frac{d \ln \gamma_2}{d y_2} \right|_{T,p} + \frac{1}{y_2} \right]$$

$$y_i = \frac{n_i}{n} \Rightarrow \frac{n_i}{y_i} = n$$

$$\Rightarrow n_1 \left[\left. \frac{d \ln \gamma_1}{d y_1} \right|_{T,p} \right] + n = n_2 \left[\left. \frac{d \ln \gamma_2}{d y_2} \right|_{T,p} \right] + n$$

$$n_1 \left[\left. \frac{d \ln \gamma_1}{d y_1} \right|_{T,p} \right] = n_2 \left[\left. \frac{d \ln \gamma_2}{d y_2} \right|_{T,p} \right]$$

Dividing through by n gives the desired relation

$$y_1 \left[\left. \frac{d \ln \gamma_1}{d y_1} \right|_{T,p} \right] = y_2 \left[\left. \frac{d \ln \gamma_2}{d y_2} \right|_{T,p} \right]$$

This expression is of particular value in minimizing the number of experimental data necessary to evaluate the properties of a system and for detecting inconsistent or erroneous measurements.

PROBLEM 12.1

KNOWN: The problem concerns a mixture of two gases.

FIND: Determine (a) when the analysis in terms of mass fractions would be identical to the analysis in terms of mole fractions, (b) when the apparent molecular weight of the mixture would equal the average of the molecular weights of the gases.

ANALYSIS: (a) Using Eq. 12.3

$$mf_1 = \frac{m_1}{m}, \quad mf_2 = \frac{m_2}{m}$$

and Eqs. 12.1, 6

$$y_1 = \frac{n_1}{n} = \frac{m_1/M_1}{\left[\frac{m_1}{M_1} + \frac{m_2}{M_2} \right]} = \frac{m_1}{m} \left[\frac{m/M_1}{\frac{m_1}{M_1} + \frac{m_2}{M_2}} \right]$$

$$\Rightarrow y_1 = mf_1 \left[\frac{\frac{m_1+m_2}{M_1}}{\frac{m_1}{M_1} + \frac{m_2}{M_2}} \right]$$

For $y_1 = mf_1$, the term in brackets must equal unity. That is

$$\frac{m_1+m_2}{M_1} = \frac{m_1}{M_1} + \frac{m_2}{M_2} \Rightarrow \frac{m_2}{M_1} = \frac{m_2}{M_2} \Rightarrow M_1 = M_2 \leftarrow$$

Accordingly, for $y_1 = mf_1$, $y_2 = mf_2$, it is necessary for $M_1 = M_2$.

(b) Using Eq. 12.9, $M = y_1 M_1 + y_2 M_2$. Then, since $1 = y_1 + y_2$

$$M = y_1 M_1 + (1-y_1) M_2 \Rightarrow M = y_1 (M_1 - M_2) + M_2$$

If $M = (M_1 + M_2)/2$, the last expression becomes

$$\frac{M_1 + M_2}{2} = y_1 (M_1 - M_2) + M_2$$

or

$$y_1 (M_1 - M_2) = \frac{M_1 + M_2}{2} - M_2$$

$$= \frac{M_1 - M_2}{2}$$

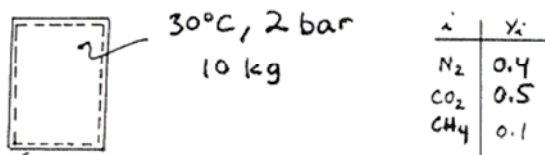
Accordingly, $y_1 = 1/2$. In sum, for $M = (M_1 + M_2)/2$, it is necessary for the mole fractions to be equal: $y_1 = y_2 = 1/2$. $\leftarrow$

PROBLEM 12.2

KNOWN: The molar analysis of a gas mixture is specified.

FIND: Determine the analysis in terms of mass fractions, the partial pressure of each component, and the volume occupied by 10 kg of mixture.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) Calculations are based on 1 kmol of mixture in part (a).

ANALYSIS (a) Considering a typical 1 kmol of mixture

i	n_i	M_i	$m_i = n_i M_i$	$m_{f,i}$
N ₂	0.4	28.01	11.204	0.3218
CO ₂	0.5	44.01	22.005	0.6321
CH ₄	0.1	16.04	1.604	0.0461
			34.813 $\frac{\text{kg}}{\text{kmol}}$	1.0000

← (a)

(b) With Eq. 12.12

$$P_{N_2} = y_{N_2} P = (0.4)(2 \text{ bar}) = 0.8 \text{ bar}$$

$$P_{CO_2} = y_{CO_2} P = (0.5)(2 \text{ bar}) = 1.0 \text{ bar}$$

$$P_{CH_4} = y_{CH_4} P = (0.1)(2 \text{ bar}) = 0.2 \text{ bar}$$

← (b)

(c) With the ideal gas equation of state applied to the overall mixture.

$$\begin{aligned}
 V &= \frac{m (\bar{R}/M) T}{P} \\
 &= \frac{(10 \text{ kg}) \left(\frac{8314}{34.813} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (303 \text{ K})}{2 \times 10^5 \text{ N/m}^2} = 3.618 \text{ m}^3
 \end{aligned}$$

← (c)

1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Equivalently, Eq. 12.9 can be used: $M = \sum y_i M_i$

$$M = y_{N_2} M_{N_2} + y_{CO_2} M_{CO_2} + y_{CH_4} M_{CH_4}$$

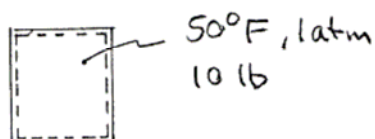
$$= (0.4)(28.01) + (0.5)(44.01) + (0.1)(16.04) = 34.813 \frac{\text{kg}}{\text{kmol}}$$

PROBLEM 12.3

KNOWN: The molar analysis of a gas mixture is specified.

FIND: Determine the analysis in terms of mass fractions, the partial pressure of each component, the volume occupied by 10 lb of mixture.

SCHEMATIC & GIVEN DATA:



i	y_i
Ar	0.2
CO ₂	0.35
O ₂	0.45

ENGINEERING

MODEL: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) Calculations are based on 1 lbmol of mixture in part (a).

ANALYSIS: (a) Considering a typical 1 lbmol of mixture

i	n_i	M_i	$m_i = n_i M_i$	mf_i
Ar	0.2	39.94	7.988	0.2114
CO ₂	0.35	44.01	15.404	0.4076
O ₂	0.45	32.00	14.400	0.3810
37.792 $\frac{\text{lb}}{\text{lbmol}}$				

(a)

(b) $P_{Ar} = y_{Ar} P = (0.2)(14.7) = 2.940 \text{ lbf/in}^2$

$P_{CO_2} = y_{CO_2} P = (0.35)(14.7) = 5.145 \text{ lbf/in}^2$

$P_{O_2} = y_{O_2} P = (0.45)(14.7) = 6.615 \text{ lbf/in}^2$

(b)

(c) Using the ideal gas equation of state

$$V = \frac{m \bar{R} / M T}{P} = \frac{(10 \text{ lb}) \left(\frac{1545}{37.792} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ \text{R}} \right) (510^\circ \text{R})}{(14.7 \times 144 \text{ lbf/ft}^2)}$$

$$= 98.50 \text{ ft}^3$$

(c)

1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Equivalently, Eq. 12.9 can be used: $M = \sum y_i M_i$

$$M = y_{Ar} M_{Ar} + y_{CO_2} M_{CO_2} + y_{O_2} M_{O_2}$$

$$= (0.2)(39.94) + (0.35)(44.01) + (0.45)(32.00) = 37.792 \frac{\text{lb}}{\text{lbmol}}$$

PROBLEM 12.4

KNOWN: The molar analysis of a gas mixture is specified.

FIND: Determine the analysis in terms of mass fractions, the partial pressure of each component, the volume occupied by 50 kg of mixture.

SCHEMATIC & GIVEN DATA:



25°C, 0.1 MPa
m = 50 kg

i	y _i
N ₂	0.6
CO ₂	0.3
O ₂	0.1

ENGINEERING

MODEL: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) Calculations are based on 1 kmol of mixture in part (a).

ANALYSIS: (a) Considering a typical 1 kmol of mixture

i	n _i	M _i	m _i = n _i · M _i	mf _i
N ₂	0.6	28.01	16.806	0.5061
CO ₂	0.3	44.01	13.203	0.3976
O ₂	0.1	32.00	3.200	0.0964
33.209 $\frac{\text{kg}}{\text{kmol}}$				1.000

①

(b) $P_{N_2} = y_{N_2} p = 0.6 (0.1 \text{ MPa}) = 0.06 \text{ MPa}$

$P_{CO_2} = y_{CO_2} p = 0.3 (0.1 \text{ MPa}) = 0.03 \text{ MPa}$

$P_{O_2} = y_{O_2} p = 0.1 (0.1 \text{ MPa}) = 0.01 \text{ MPa}$

(c) Using the ideal gas equation of state

$$V = \frac{m R T}{P} = \frac{(50 \text{ kg}) \left(\frac{8314 \text{ N} \cdot \text{m}}{33.209 \text{ kg} \cdot \text{K}} \right) (298 \text{ K})}{(0.1 \times 10^6 \text{ N/m}^2)}$$

$$= 37.3 \text{ m}^3$$

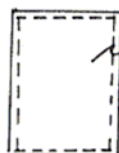
1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Equivalently, Eq. 12.9 can be used: $M = \sum y_i M_i$.

PROBLEM 12.5

KNOWN: An analysis on a mass basis is specified for a gas mixture.

FIND: Determine the analysis in terms of mole fractions, the partial pressure of each component, the volume occupied by 20 lb of mixture.

SCHEMATIC & GIVEN DATA:



400F, 20 lbf/in²
m = 20 lb

i	mf _i
CO ₂	0.6
CO	0.25
O ₂	0.15

ENGINEERING MODEL: (1) The mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) Calculations are based on 1 lb of mixture in part (a).

ANALYSIS: (a) Considering a typical 1 lb of mixture

①

i	m _i	M _i	n _i = m _i /M _i	y _i
CO ₂	0.6	44.01	0.01363	0.5002
CO	0.25	28.01	0.00893	0.3277
O ₂	0.15	32.00	0.00469	0.1721
			0.02725	

← (a)

$$M = \frac{m}{n} = \frac{1}{0.02725} = 36.697$$

(b) $P_{CO_2} = y_{CO_2} p = (0.5002)(20) = 10.004 \text{ lbf/in}^2$

$P_{CO} = y_{CO} p = (0.3277)(20) = 6.554 \text{ lbf/in}^2$

$P_{O_2} = y_{O_2} p = (0.1721)(20) = 3.442 \text{ lbf/in}^2$

← (b)

(c) Using the ideal gas equation of state

$$V = \frac{m R T}{P} = \frac{(20 \text{ lb}) \left(\frac{1545}{36.697} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ \text{R}} \right) (500^\circ \text{R})}{(20 \times 14.7 \text{ lbf/ft}^2)} = 146.2 \text{ ft}^3 \quad \leftarrow (c)$$

1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Equivalently, Eq. 12.9 can be used:

$$\begin{aligned} M &= y_{CO_2} M_{CO_2} + y_{CO} M_{CO} + y_{O_2} M_{O_2} \\ &= (0.5002)(44.01) + (0.3277)(28.01) + (0.1721)(32) \\ &= 36.7 \end{aligned}$$

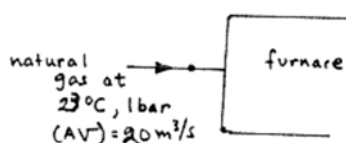
The difference between this value and the one obtained above is due to round off.

PROBLEM 12.6

KNOWN: An analysis on a molar basis is specified for a natural gas.

FIND: Determine the analysis in terms of mass fractions, the partial pressure of each component, the mass flow rate for a volumetric flow rate of $20 \text{ m}^3/\text{s}$.

SCHEMATIC & GIVEN DATA:



i	Y_i
C_3H_8	0.4
C_2H_6	0.4
CH_4	0.2

ENGINEERING MODEL: (1) The mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the full volume of the mixture at the mixture temperature. (3) In part (a), calculations are based on 1 kmol of mixture.

ANALYSIS: (a) Considering a typical 1 kmol of mixture

i	n_i	M_i	$m_i = n_i M_i$	mf_i
C_3H_8	0.4	44.09	17.636	0.5365
C_2H_6	0.4	30.07	12.028	0.3659
CH_4	0.2	16.04	3.208	0.0976
			32.872	1.0000

← (a)

(b) $P_{\text{C}_3\text{H}_8} = Y_{\text{C}_3\text{H}_8} P = 0.4 (1 \text{ bar}) = 0.4 \text{ bar}$

$P_{\text{C}_2\text{H}_6} = Y_{\text{C}_2\text{H}_6} P = 0.4 (1 \text{ bar}) = 0.4 \text{ bar}$

$P_{\text{CH}_4} = Y_{\text{CH}_4} P = 0.2 (1 \text{ bar}) = 0.2 \text{ bar}$

← (b)

(c) Using the ideal gas equation of state, $v = RT/P$

$$\dot{m} = \frac{(AV)}{v}$$

$$= \frac{(AV)P}{(\bar{R}/M)T} = \frac{(20 \text{ m}^3/\text{s})(10^5 \text{ N/m}^2)}{\left(\frac{8314}{32.872} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(296 \text{ K})}$$

$$= 26.71 \frac{\text{kg}}{\text{s}}$$

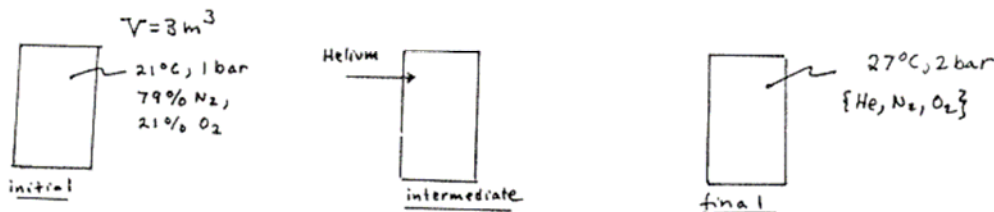
← $\dot{m}$

PROBLEM 12.7

KNOWN: A rigid vessel whose volume is 3 m^3 initially contains a mixture of N_2 and O_2 at a specified pressure, temperature, and molar analysis. Helium is allowed to flow in until the pressure attains a specified value.

FIND: If the final temperature is 21°C , determine the mass of each component present.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

ANALYSIS: Using the ideal gas equation of state, $V = n\bar{R}T/p$. Since the total volume is constant: $V_1 = V_2$

$$n_1 \frac{\bar{R} T_1}{P_1} = n_2 \frac{\bar{R} T_2}{P_2} \quad (1)$$

where 1 denotes the initial total number of moles present, temperature, and pressure, and 2 denotes the final total number of moles present, temperature, and pressure. From Eq. 1

$$n_2 = \frac{T_1}{T_2} \frac{P_2}{P_1} n_1 \quad (2)$$

Initially

$$n_1 = \frac{P_1 V}{\bar{R} T_1} = \frac{(1.05 \text{ N/m}^2)(3 \text{ m}^3)}{\left(\frac{8314 \text{ N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}\right)(294 \text{ K})} = 0.1227 \text{ kmol (mix)}$$

The amounts of N_2 and O_2 present initially and finally are

$$n_{\text{O}_2} = y_{\text{O}_2} n_1 = (0.21)(0.1227) = 0.0258 \text{ kmol } (\text{O}_2)$$

$$n_{\text{N}_2} = y_{\text{N}_2} n_1 = (0.79)(0.1227) = 0.0969 \text{ kmol } (\text{N}_2)$$

And with Eq. (2) the total amount of mixture finally is

$$n_2 = \left(\frac{294 \text{ K}}{300 \text{ K}}\right) \left(\frac{2 \text{ bar}}{1 \text{ bar}}\right) (0.1227 \text{ kmol}) = 0.2405 \text{ kmol (mix)}$$

Then, since $n_2 = \underbrace{n_{\text{O}_2} + n_{\text{N}_2}}_{n_1} + n_{\text{He}}$;

$$n_{\text{He}} = 0.2405 - \underbrace{0.0258 + 0.0969}_{n_1} = 0.1178 \text{ kmol (He)}$$

Finally, with molecular weights from Table A-1 we get

$$m_{\text{O}_2} = (32)(0.0258) = 0.8256 \text{ kg } (\text{O}_2)$$

$$m_{\text{N}_2} = (28.01)(0.0969) = 2.7142 \text{ kg } (\text{N}_2)$$

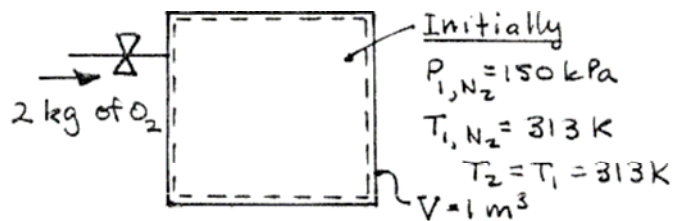
$$m_{\text{He}} = (4.003)(0.1178) = 0.4716 \text{ kg (He)}$$

PROBLEM 12.8

KNOWN: A known amount of oxygen (O_2) is added to Nitrogen (N_2) in a rigid tank. The initial state of the nitrogen is given, and the final temperature is given as equal to the initial temperature.

FIND: Determine the final molar analysis and the final pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The final contents of the tank constitute the system.
2. $T_2 = T_1$
3. The gases behave as ideal gases.

ANALYSIS: First, determine the moles of N_2 present

$$n_{N_2} = \frac{P_{1,N_2} V}{\bar{R} T} = \frac{(150 \text{ kPa})(1 \text{ m}^3)}{(8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}})(313 \text{ K})} \left| \frac{10^3 \text{ N/m}^2}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 0.0576 \text{ kmol}$$

For the O_2 ; $n_{O_2} = (2 \text{ kg}) \left(\frac{1 \text{ kmol}}{32.00 \text{ kg}} \right) = 0.0625 \text{ kmol}$

Thus

$$n_{\text{tot}} = 0.0576 + 0.0625 = 0.1201 \text{ kmol}$$

$$y_{N_2} = \frac{0.0576}{0.1201} = 0.48, \quad y_{O_2} = \frac{0.0625}{0.1201} = 0.52 \quad \leftarrow y_{N_2}, y_{O_2}$$

The final pressure is

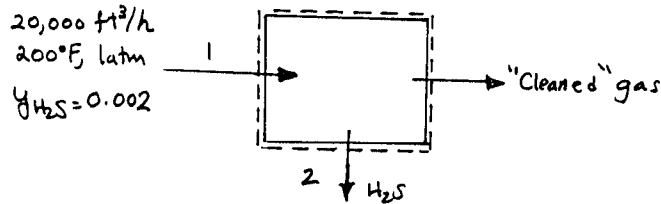
$$P_2 = \frac{n_{\text{tot}} \bar{R} T}{V} = \frac{(0.1201)(8.314)(313)}{(1)} \left| \frac{10^3}{10^3} \right| = 312.5 \text{ kPa} \quad \leftarrow P_2$$

PROBLEM 12.9

KNOWN: A flue gas containing H_2S enters a scrubber at a specified state and volumetric flow rate. The scrubber removes 92% (molar basis) of the H_2S .

FIND: Determine the rate H_2S is removed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The control volume shown in the figure is at steady state.

ANALYSIS: The molar flow rate of the entering mixture, $\dot{n}_1$, can be found as follows:

$$\begin{aligned}\dot{n}_1 &= \frac{(AV)_1}{\bar{v}_1} = \frac{(AV)_1 P_1}{RT_1} = \frac{(20,000 \text{ ft}^3/\text{h})(14.7 \times 144 \text{ lbf/ft}^2)}{(1545 \text{ ft}^3 \text{ lbf/lbmol} \cdot ^\circ\text{R})(660^\circ\text{R})} \\ &= 41.518 \frac{\text{lbmol}(\text{mixture})}{\text{h}}\end{aligned}$$

The rate H_2S enters is then

$$\dot{n}_{\text{H}_2\text{S},1} = (0.002)(41.518) = 0.083 \frac{\text{lbmol}(\text{H}_2\text{S})}{\text{h}}$$

If 92% is removed by the scrubber

$$\dot{n}_{\text{H}_2\text{S},2} = (0.92)(0.083) = 0.0764 \frac{\text{lbmol}(\text{H}_2\text{S})}{\text{h}}$$

Then with $M = 34.08$ for H_2S

$$\dot{m}_{\text{H}_2\text{S},2} = (0.0764)(34.08) = 2.6 \frac{\text{lb}(\text{H}_2\text{S})}{\text{h}} \longleftarrow$$

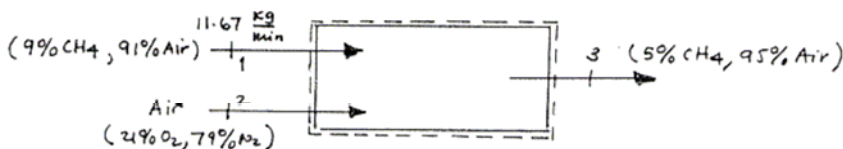
H_2S should be removed because it can combine with water to form a corrosive product. Acid rain can be traced to the presence of sulfur in combustion gases.

PROBLEM 12.10

KNOWN: A control volume at steady state has two entering streams and a single exiting stream. The molar analysis of each entering stream is provided. The mole fraction of CH_4 in the exiting stream is also provided.

FIND: Determine (a) the molar flow rate of the entering air stream,
(b) the mass flow rate of O_2 in the exiting stream.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) Air has the molar analysis: 21% O_2 , 79% N_2 .

ANALYSIS: (a) For the control volume, a mass rate balance reads $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$, or $\dot{m}_2 = \dot{m}_3 - \dot{m}_1$. Also, $\dot{m} = M\dot{n}$. So, $\dot{n}_2 = \frac{\dot{m}_3 - \dot{m}_1}{M_2}$. (1)

where $M_2 = (0.21)(32) + (0.79)(28.01) = 28.848$.

Since the rate at which CH_4 enters at 1 equals the rate CH_4 exits at 3, the relationship between the molar flow rates at 1 and 3 is, $0.09\dot{n}_1 = 0.05\dot{n}_3$.

Or, with $\dot{n} = \dot{m}/M$

$$0.09 \frac{\dot{m}_1}{M_1} = 0.05 \frac{\dot{m}_3}{M_3} \Rightarrow \dot{m}_3 = \frac{0.09}{0.05} \frac{M_3}{M_1} \dot{m}_1 \quad (2)$$

where

$$M_1 = 0.09(16.04) + 0.91[28.848] = 27.696$$

$$M_3 = 0.05(16.04) + 0.95[28.848] = 28.208$$

Then, Eq. (2) gives

$$\dot{m}_3 = \left(\frac{0.09}{0.05} \right) \left(\frac{28.208}{27.696} \right) (11.67) = 21.39 \frac{\text{kg}}{\text{min}}$$

Then, Eq. (1) gives

$$\dot{n}_2 = \frac{21.39 - 11.67}{28.848} = 0.337 \frac{\text{kmol(air)}}{\text{min}}$$

(b) The molar flow rate of O_2 in the exiting stream is

$$\dot{n}_{\text{O}_2} = y_{\text{O}_2} \dot{n}_3$$

① where y_{O_2} is the mole fraction of O_2 in the exiting mixture and $\dot{n}_3$ is the molar flow rate of the mixture. With $\dot{m}_{\text{O}_2} = \dot{n}_{\text{O}_2} M_{\text{O}_2}$, this becomes

$$\begin{aligned} \dot{m}_{\text{O}_2} &= M_{\text{O}_2} (y_{\text{O}_2} \dot{n}_3) = M_{\text{O}_2} y_{\text{O}_2} \left(\frac{\dot{m}_3}{M_3} \right) \\ &= (32)(.1995) \left(\frac{21.39}{28.208} \right) = 4.84 \frac{\text{kg(O}_2\text{)}}{\text{min}} \end{aligned}$$

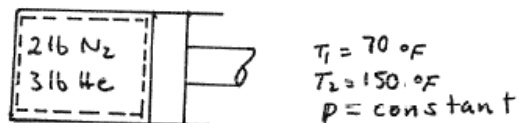
1. Note that the exiting mixture is (5% CH_4 , 95% Air) = (5% CH_4 , $(0.95)(21\% \text{O}_2)$, $(95)(79\% \text{N}_2)$)
19.95%

PROBLEM 12.11

KNOWN: A gas mixture of 2 lb of N_2 and 3 lb of He is maintained at a constant pressure.

FIND: Determine the mixture analysis in terms of mass fractions and mole fractions, the heat transfer required to increase the mixture temperature from 70 to 150 °F, and the change in entropy of the mixture for this process.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) For the system shown in the accompanying figure, changes in kinetic and potential energy are negligible. (2) The overall mixture acts as an ideal gas. Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) The specific heats of the gases are constants.

ANALYSIS: (a) The analysis of the mixture in terms of mass fractions is determined with $(mf)_i = m_i/m$

$$(mf)_{N_2} = \frac{2}{2+3} = 0.40 \quad , \quad (mf)_{He} = \frac{3}{2+3} = 0.60 \quad \leftarrow mf$$

(b) The molar analysis of the mixture is determined with $n_i = m_i/M_i$

$$\left. \begin{aligned} n_{N_2} &= \frac{2}{28.01} = 0.0714 \text{ lbmol} \\ n_{He} &= \frac{3}{4.003} = 0.7494 \text{ lbmol} \end{aligned} \right\} \Rightarrow \left. \begin{aligned} y_{N_2} &= \frac{0.0714}{0.0714 + 0.7494} = 0.0870 \\ y_{He} &= \frac{0.7494}{0.0714 + 0.7494} = 0.9130 \end{aligned} \right\} \leftarrow y$$

(c) With assumption 1 an energy balance reduces to $\Delta U = Q - W$ where $W = \int p dV = p \Delta V$. Thus,

$$Q = \Delta U + p \Delta V = (U_2 - U_1) + p(V_2 - V_1) = (U_2 + pV_2) - (U_1 + pV_1) = \Delta H$$

The change in enthalpy of the mixture is the sum of the enthalpy changes of the components:

$$\Delta H = m_{N_2} \Delta h_{N_2} + m_{He} \Delta h_{He}$$

With assumption 3, $\Delta h_{N_2} = c_{p,N_2}(T_2 - T_1)$ and $\Delta h_{He} = c_{p,He}(T_2 - T_1)$. Thus, with c_p data for N_2 from Table A-20E and for He, according to Table A-21E, $c_p = (2.5) \left(\frac{1.986}{4.003} \right) = 1.240 \text{ Btu/lb} \cdot ^\circ\text{R}$

$$\begin{aligned} Q &= [m_{N_2} c_{p,N_2} + m_{He} c_{p,He}] (T_2 - T_1) = [(2 \text{ lb})(0.248 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}}) + (3 \text{ lb})(1.240 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}})] (80^\circ\text{R}) \\ &= 337.3 \text{ Btu} \quad \leftarrow Q \end{aligned}$$

(d) Since pressure remains constant, the entropy change for the mixture is

$$\Delta S = m \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right] = m c_p \ln \frac{T_2}{T_1}$$

where $c_p = (m_{N_2} c_{p,N_2} + m_{He} c_{p,He}) / m = [(2)(0.248) + (3)(1.240)] / 5 = 0.8432 \text{ Btu/lb} \cdot ^\circ\text{R}$

Thus

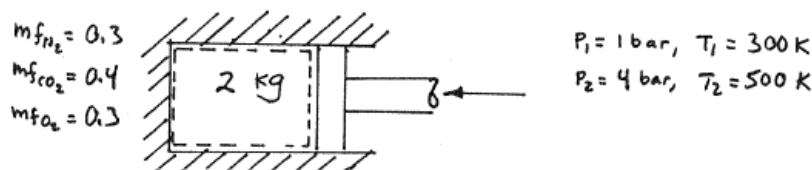
$$\Delta S = (5 \text{ lb}) \left(0.8432 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) \ln \frac{610}{530} = 0.593 \frac{\text{Btu}}{^\circ\text{R}} \quad \leftarrow \Delta S$$

PROBLEM 12.12

KNOWN: A mixture having a specified analysis on a mass basis is compressed adiabatically between specified states.

FIND: Determine the work and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) For the system shown in the accompanying figure, $Q = 0$ and changes in kinetic and potential energy are negligible. (2) The overall mixture acts as an ideal gas. Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

ANALYSIS: An energy balance reduces with assumption 1 to give $\Delta U = -W$. The internal energy change of the mixture, ΔU , equals the sum of the internal energy changes of the components. Thus, on a unit mass of mixture basis

$$-\frac{W}{m} = (mf)_{N_2} \Delta u_{N_2} + (mf)_{CO_2} \Delta u_{CO_2} + (mf)_{O_2} \Delta u_{O_2}$$

With data from the ideal gas tables and molecular weights from Table A-1

$$-\frac{W}{m} = 0.3 \left[\frac{10423 - 6229}{28.01} \right] + 0.4 \left[\frac{13521 - 6939}{44.01} \right] + 0.3 \left[\frac{10614 - 6242}{32} \right]$$

Thus

$$\frac{W}{m} = -(44.92 + 59.82 + 40.99) = -145.73 \text{ kJ/kg}$$

Then, with $m = 2 \text{ kg}$

$$W = -291.46 \text{ kJ} \quad \leftarrow \text{_____} \quad W$$

In this calculation, the relationship $\Delta u = \Delta \bar{u}/M$ is used for each of the three components.

An entropy rate balance reduces as follows

$$\Delta S = \int_1^2 \frac{\delta Q}{T_b} + \sigma \Rightarrow \sigma = \Delta S$$

The entropy change of the mixture, ΔS , equals the sum of the entropy changes of the components:

$$\Delta S = (\Delta S)_{N_2} + (\Delta S)_{CO_2} + (\Delta S)_{O_2} \Rightarrow \frac{\Delta S}{m} = (mf)_{N_2} \left(\frac{\Delta \bar{s}}{M} \right)_{N_2} + (mf)_{CO_2} \left(\frac{\Delta \bar{s}}{M} \right)_{CO_2} + (mf)_{O_2} \left(\frac{\Delta \bar{s}}{M} \right)_{O_2}$$

where each of the $\Delta \bar{s}$ terms is determined using Eq. 12.36. Thus, with 5° data

$$\begin{aligned} \frac{\sigma}{m} &= 0.3 \left[\frac{206.630 - 191.682 - 8.314 \ln 4}{28.01} \right] + 0.4 \left[\frac{234.814 - 213.915 - 8.314 \ln 4}{44.01} \right] + 0.3 \left[\frac{220.589 - 205.213 - 8.314 \ln 4}{32} \right] \\ &= (0.03665 + 0.08519 + 0.03610) \frac{\text{kJ/K}}{\text{kg}} \\ &= 0.15794 \frac{\text{kJ/K}}{\text{kg}} \end{aligned}$$

or

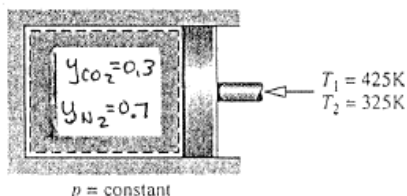
$$\sigma = 0.3159 \frac{\text{kJ}}{\text{K}} \quad \leftarrow \text{_____} \quad \sigma$$

PROBLEM 12.13

KNOWN: An ideal gas mixture with known molar analysis is compressed at constant pressure in a piston-cylinder assembly.

FIND: Determine the heat transfer and work, each per kg of mixture.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

- (1) The mixture is the closed system.
- (2) Pressure is constant.
- (3) The gases follow the ideal gas model with specific heats evaluated at 375 K.
- (4) $\Delta KE = \Delta PE = 0$.

ANALYSIS: First, determine the properties of the mixture. Interpolating in Table A-20: $c_{p,CO_2} = 1.045 \text{ kJ/kg}\cdot\text{K}$ and $c_{p,N_2} = 1.0425 \text{ kJ/kg}\cdot\text{K}$

Using the method of Example 12.1 to get the mass fractions, for $n = 1 \text{ kmol}$

Component	$m_i \times M_i = m_i$	$m_{f,i}$
CO_2	$0.3 \times 44.01 = 13.203$	0.4024
N_2	$0.7 \times 28.01 = 19.607$	0.5976
	1.0	
	32.810	
	$\hookrightarrow M_{mix} = 32.810 \text{ kg/kmol}$	

Finally $c_{p,mix} = m_{f,CO_2} c_{p,CO_2} + m_{f,N_2} c_{p,N_2} = 0.9920 \text{ kJ/kg}\cdot\text{K}$

From the energy balance

$$m(u_2 - u_1) = Q - W \quad \text{and} \quad p(V_2 - V_1) = m p(v_2 - v_1)$$

so

$$Q = m(u_2 - u_1) + m p(v_2 - v_1) = m(h_2 - h_1)$$

With $h_2 - h_1 = c_{p,mix}(T_2 - T_1)$

$$Q/m = c_{p,mix}(T_2 - T_1) = (0.9920 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})(325 - 425) \text{ K} = -99.2 \text{ kJ/kg} \quad \text{Q/m}$$

For the work

$$\begin{aligned} \textcircled{1} \quad \frac{W}{m} &= p(v_2 - v_1) = R_{mix}(T_2 - T_1) \\ &= \left(\frac{8.314}{32.81} \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \right) (325 - 425) \text{ K} = -25.34 \text{ kJ/kg} \quad \text{W/m} \end{aligned}$$

1. Alternatively, from the energy balance

$$W/m = Q/m - (u_2 - u_1) = Q/m - c_v(T_2 - T_1)$$

With $c_{v,mix} = 0.7385 \text{ kJ/kg}\cdot\text{K}$ (as can be verified)

$$W/m = -99.2 - (0.7385)(-100) = -25.35 \text{ kJ/kg}$$

which agrees within round off with the value calculated above.

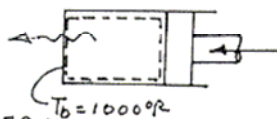
PROBLEM 12.14

KNOWN: A specified mixture is compressed isothermally.

FIND: Determine (a) the work, (b) the heat transfer, and (c) the amount of entropy produced, for each of two choices for the boundary.

SCHEMATIC & GIVEN DATA:

System #1



$$\begin{aligned} n_{N_2} &= 0.6 \text{ lbmol} \\ n_{O_2} &= 0.4 \text{ lbmol} \\ T &= 1000^\circ\text{R} \\ P_1 &= 1 \text{ atm} \\ P_2 &= 3 \text{ atm} \end{aligned}$$

System #2



ENGINEERING

MODEL: (1) For the system shown in the accompanying figure, changes in kinetic and potential energy are negligible. (2) The overall mixture acts as an ideal gas. Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) The immediate surroundings experience no net change in state. (4) In system #1, heat transfer occurs at $T_b = 1000^\circ\text{R}$. In system #2, heat transfer occurs at $T_b = 500^\circ\text{R}$.

ANALYSIS: (a), (b). With assumption (1), an energy balance reduces to give

$\Delta U = Q - W$, where ΔU is the change in internal energy of the mixture.

Since the overall mixture acts as an ideal gas $\Delta U = 0$ because the mixture temperature does not change. Thus, $Q = W$. The work is

$$W = \int_1^2 p dV = \int_1^2 \frac{n \bar{R} T}{V} dV = n \bar{R} T \ln \frac{V_2}{V_1}$$

With $V = nRT/p$ and $T_1 = T_2$

$$W = n \bar{R} T \ln \frac{P_2}{P_1} = [(0.6 + 0.4) \text{ lbmol}] \left[1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lbmol} \cdot ^\circ\text{R}} \right] (1000^\circ\text{R}) \left(\ln \frac{3}{1} \right) \left(\frac{\text{Btu}}{778 \text{ ft} \cdot \text{lbf}} \right) = -2181.7 \text{ Btu} \quad \leftarrow W$$

(c) An entropy balance for the system #1 takes the form

$$\Delta S = \frac{Q}{T_b} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T} = \Delta S + n \bar{R} \ln \frac{P_2}{P_1} \quad (1)$$

where ΔS is the sum of the entropy changes of the mixture components:

$$\Delta S = n_{N_2} [\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln \frac{P_2}{P_1}] + n_{O_2} [\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln \frac{P_2}{P_1}] = -n \bar{R} \ln \frac{P_2}{P_1} \quad (2)$$

$\approx 0 \text{ since } T_2 = T_1$ $\approx 0 \text{ since } T_2 = T_1$

Inserting this into Eq. (1) gives $\sigma = 0$. That is the process is internally reversible. $\leftarrow \sigma$

Taking an enlarged system that includes the mixture and enough of its immediate surroundings so that heat transfer occurs at $T_b = 500^\circ\text{K}$, an entropy balance takes the form

$$\Delta S + \Delta S_{\text{sur}}^{\text{immed.}} = \frac{Q}{T_b} + \sigma$$

where σ now accounts for the entropy produced in the enlarged system and $\Delta S_{\text{sur}}^{\text{immed.}}$ vanishes by assumption 3. Thus, with Eq. (2) and $Q = W = n \bar{R} T \ln P_2/P_1$

$$\begin{aligned} \sigma &= \Delta S - \frac{Q}{T_b} = -n \bar{R} \ln \frac{P_2}{P_1} - \frac{Q}{T_b} \\ &= 0 \left[\frac{1}{T} - \frac{1}{T_b} \right] = -2181.7 \text{ Btu} \left[\frac{1}{1000^\circ\text{R}} - \frac{1}{500^\circ\text{R}} \right] = 2.182 \frac{\text{Btu}}{^\circ\text{R}} \quad \leftarrow \sigma \end{aligned}$$

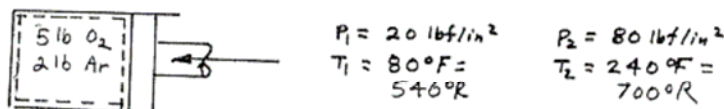
1. In this case entropy is produced in the immediate surroundings owing to heat transfer from the mixture at 1000°R to the surroundings at 500°R . This is an external irreversibility for system #1.

PROBLEM 12.15

KNOWN: A specified mixture of O_2 and Ar is compressed between two specified states.

FIND: Determine if the process can be adiabatic.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

ANALYSIS: Since W is not specified and cannot be determined from the given data, an energy balance cannot be invoked to determine if heat transfer occurs. However, if the process were adiabatic an entropy balance would give

$$\Delta S = \int \frac{\delta Q}{T} + \sigma \Rightarrow \Delta S \geq 0 \Rightarrow \text{(the entropy change cannot be negative)}$$

where ΔS is the entropy change of the mixture; the sum of the entropy changes of the components. That is

$$\Delta S = n_{O_2}(\Delta \bar{s})_{O_2} + n_{Ar}(\Delta \bar{s})_{Ar}$$

Since argon is a monatomic gas, Table A-21E indicates $\bar{C}_{p,Ar} = 5/2 \bar{R}$, so

$$\Delta S = n_{O_2} \left[\bar{s}_{O_2}^0(T_2) - \bar{s}_{O_2}^0(T_1) - \bar{R} \ln \frac{P_2}{P_1} \right] + n_{Ar} \left[\bar{C}_{p,Ar} \ln \frac{T_2}{T_1} - \bar{R} \ln \frac{P_2}{P_1} \right]$$

$\uparrow = \frac{n_{O_2}}{M_{O_2}}$
 $\uparrow = \frac{n_{Ar}}{M_{Ar}}$

where $\bar{s}_{O_2}^0$ is obtained from Table A-24E. In writing this, note that the mole fractions cancel in each of the log terms: the ratio of partial pressures equals the ratio of mixture pressures.

Inserting values

$$\begin{aligned} \Delta S &= \left(\frac{5}{32} \text{ lbmol} \right) \left(50.858 - 49.021 - 1.986 \ln \frac{80}{20} \right) \frac{8 \text{ Btu}}{\text{lbmol} \cdot ^\circ \text{R}} + \\ &\quad \left(\frac{2}{39.94} \right) (1.986) \left[\frac{5}{2} \ln \frac{700}{540} - \ln \frac{80}{20} \right] \\ &= (-0.1432 - 0.0733) \frac{8 \text{ Btu}}{^\circ \text{R}} \\ &= -0.2165 \text{ Btu}/^\circ \text{R} \end{aligned}$$

As ΔS is negative, the process cannot be adiabatic. There must be a heat transfer from the mixture.

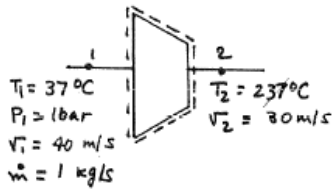
PROBLEM 12.16

KNOWN: A mixture with a specified molar analysis enters a compressor operating at steady state at a specified mass flow rate and state and exits at a specified temperature and velocity. The rate of heat transfer from the compressor to its surroundings is also specified.

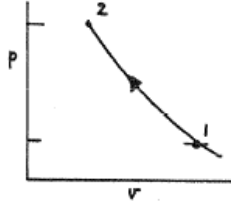
FIND: (a) Determine the power input to the compressor. (b) If the compression were polytropic, evaluate the polytropic exponent n and the exit pressure.

SCHEMATIC & GIVEN DATA:

① $\dot{Q}_{cv} = 0.5 \dot{W}_{cv}$



i	Yi
CO ₂	0.5
CO	0.333
O ₂	0.167



ENGINEERING MODEL: (1) The control volume is steady state. (2) Potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model. (4) The mixture composition remains constant.

ANALYSIS: (a) At steady state, mass and energy rate balances reduce to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}$$

Since $\dot{Q}_{cv} = 0.5 \dot{W}_{cv}$, this becomes

$$0.95 \frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} \quad (1)$$

In this equation $(h_1 - h_2)$ represents a change in the specific enthalpy of the mixture, per unit of mass of mixture. This can be evaluated as follows

$$h_1 - h_2 = \frac{\bar{h}_1 - \bar{h}_2}{M} = \frac{1}{M} \left[y_{CO_2} (\bar{h}_1 - \bar{h}_2)_{CO_2} + y_{CO} (\bar{h}_1 - \bar{h}_2)_{CO} + y_{O_2} (\bar{h}_1 - \bar{h}_2)_{O_2} \right]$$

where M is the mixture molecular weight:

$$M = y_{CO_2} M_{CO_2} + y_{CO} M_{CO} + y_{O_2} M_{O_2} = 0.5(44.01) + 0.333(28.01) + 0.167(32) = 36.68$$

With data from the ideal gas tables

$$h_1 - h_2 = \frac{1}{36.68 \frac{\text{kg}(\text{mole})}{\text{kg}(\text{mole})}} \left[0.5 \frac{\text{kmol}(\text{CO}_2)}{\text{kmol}(\text{mixture})} (9807 - 18126) \frac{\text{kJ}}{\text{kmol}(\text{CO}_2)} + 0.333 (9014 - 14898) + 0.167 (9030 - 15062) \right]$$

$$= -194.37 \text{ kJ/kg(mix)}$$

Substituting values into Eq. (1)

$$0.95 \frac{\dot{W}_{cv}}{\dot{m}} = (-194.37) \frac{\text{kJ}}{\text{kg(mix)}} + \frac{[(40 \text{ m/s})^2 - (30 \text{ m/s})^2]}{2} \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = -194.02 \frac{\text{kJ}}{\text{kg} \cdot \text{mix}}$$

0.35 kJ/kg

or

$$\frac{\dot{W}_{cv}}{\dot{m}} = -204.23 \text{ kJ/kg(mix)} \quad \leftarrow \frac{\dot{W}_{cv}}{\dot{m}}$$

(b) If the process adheres to $p v^n = \text{constant}$, the work required would be given by Eq. 6.55a:

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\frac{n R}{n-1} (T_2 - T_1) \Rightarrow \frac{n-1}{n} = \frac{R(T_2 - T_1)}{(-\dot{W}_{cv}/\dot{m})}$$

Continued on next slide

Problem 12-16 continued

Inserting known values

$$\frac{n-1}{n} = \frac{(8.314/36.68)(200)}{204.23} \Rightarrow n = 1.285 \quad \leftarrow n$$

Then, with Eq. 3.56

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{n/(n-1)} \Rightarrow P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{n/(n-1)}$$

or, with $n = 1.285$

$$P_2 = (1 \text{ bar}) \left(\frac{510\text{K}}{310\text{K}} \right)^{n/(n-1)} = 9.4 \text{ bar} \quad \leftarrow P_2$$

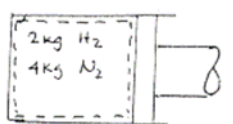
-
1. Since a power input is required W_{cv} takes on a negative value. Thus, in this expression Q_{cv} is negative, which is in accord with a heat loss during the compression.

Problem 12-17

KNOWN: A mixture of H_2 and N_2 is compressed in a polytropic process.

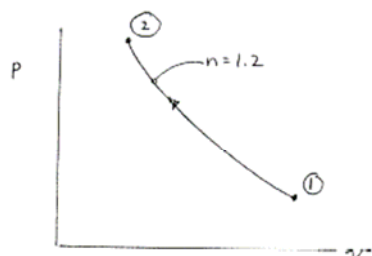
FIND: Determine the heat transfer and entropy change.

SCHEMATIC & GIVEN DATA:



$$p v^n = \text{const.}, n = 1.2$$

$$T_1 = 22^\circ\text{C}, T_2 = 150^\circ\text{C}$$



ENGINEERING

MODEL: (1) The system is shown in the figure above. (2) For the system, the composition remains constant and kinetic/potential energy effects are negligible. (3) Each component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. The overall mixture acts as an ideal gas. (4) The compression obeys $p v^{1.2} = \text{constant}$. (5) Specific heats are constant at the mean temperature.

ANALYSIS: An energy balance reduces to give $Q = \Delta U + W$, where

$$W = \int_1^2 p dv = \frac{m(\bar{R}/M)(T_2 - T_1)}{1 - n}$$

In this expression, $m = 2 + 4 = 6 \text{ kg}$ and M is given by $y_1 M_1 + y_2 M_2$, where

$$H_2: n_1 = \frac{2}{2.016} = 0.991 \text{ kmol} \Rightarrow y_1 = \frac{0.991}{1.134} = 0.874 \Rightarrow M = (0.874)(2.016) + (0.126)(28.01) = 5.29$$

$$N_2: n_2 = \frac{4}{28.01} = 0.143 \text{ kmol} \Rightarrow y_2 = 1 - 0.874 = 0.126$$

Then,

$$W = (6 \text{ kg}) \left(\frac{8.314 \text{ kJ}}{5.29 \text{ kg K}} \right) (128 \text{ K}) / (1 - 1.2) = -6035.1 \text{ kJ}$$

To find ΔU , write $\Delta U = m_1 c_{v1} (T_2 - T_1) + m_2 c_{v2} (T_2 - T_1) = (m_1 c_{v1} + m_2 c_{v2}) (T_2 - T_1)$ where the c_v terms are evaluated from Table A-20 at 359 K: $c_{v1} = 0.745 \text{ kJ/kg K}$, $c_{v2} = 10.311 \text{ kJ/kg K}$. Then

$$\Delta U = (4)(0.745) + (2)(10.311) \left(\frac{\text{kJ}}{\text{kg K}} \right) (128 \text{ K}) = (23.602) (128) = 3021.1 \text{ kJ}$$

Collecting results, $Q = \Delta U + W = 3021.1 + (-6035.1) = -3014 \text{ kJ} \leftarrow Q$

To find ΔS combine Eqs. (3.56) and (6.21) to obtain

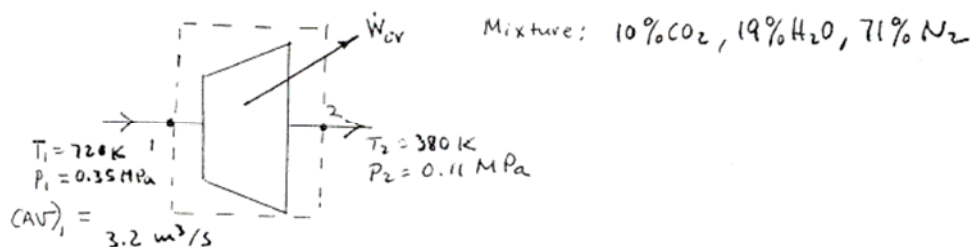
$$\begin{aligned} \Delta S &= m \left[c_v \ln \frac{T_2}{T_1} + \frac{\bar{R}}{M} \ln \frac{v_2}{v_1} \right] = m \left[c_v \ln \frac{T_2}{T_1} + \frac{\bar{R}}{M} \ln \left(\frac{T_2}{T_1} \right)^{\frac{1}{1-n}} \right] \\ &= m \left[c_v \ln \frac{T_2}{T_1} - \frac{\bar{R}/M}{n-1} \ln \frac{T_2}{T_1} \right] \\ &= m \left[c_v - \frac{\bar{R}/M}{n-1} \right] \ln \frac{T_2}{T_1} \\ &= 6 \text{ kg} \left[\left(\frac{23.602 \text{ kJ}}{6 \text{ kg K}} \right) - \frac{(8.314/5.29) \text{ kJ}}{(1.2-1) \text{ kg K}} \right] \ln \frac{423}{295} \\ &= 6 [3.934 - 7.858] \ln \frac{423}{295} \\ &= -8.485 \text{ kJ/K} \leftarrow \Delta S \end{aligned}$$

PROBLEM 12.18

KNOWN: A specified mixture expands through a turbine for which steady state operating data are provided.

FIND: Determine the power developed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

ANALYSIS: Reducing mass and energy balances at steady state

$$\dot{W}_{cv} = \dot{m} [h_1 - h_2] = \dot{m} \left[\frac{\bar{h}_1 - \bar{h}_2}{M_{mix}} \right] = \dot{m} \left[\frac{y_{CO_2} (\bar{h}_1 - \bar{h}_2)_{CO_2} + y_{H_2O} (\bar{h}_1 - \bar{h}_2)_{H_2O} + y_{N_2} (\bar{h}_1 - \bar{h}_2)_{N_2}}{M_{mix}} \right]$$

And

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{(\bar{R}/M_{mix}) T_1} \Rightarrow \frac{\dot{m}}{M_{mix}} = \frac{(AV)_1 P_1}{\bar{R} T_1} = \frac{(3.2 \text{ kg/s}) (0.35 \times 10^6 \frac{\text{N}}{\text{m}^2})}{(8314 \frac{\text{J}}{\text{kmol} \cdot \text{K}}) (720 \text{ K})} = 0.1871 \frac{\text{kmol (mixture)}}{\text{s}}$$

Then, with data from the ideal gas tables

$$\begin{aligned} \dot{W}_{cv} &= 0.1871 \left[0.10 (28,121 - 12,552) + 0.19 (24,840 - 12,672) + 0.71 (21,220 - 11,055) \right] \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 2074.2 \text{ kW} \end{aligned}$$

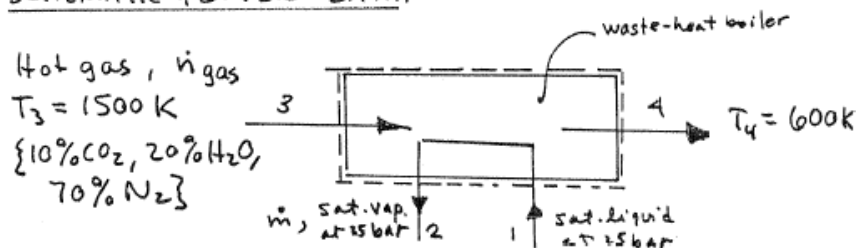
$\dot{W}_{cv}$

PROBLEM 12.19

KNOWN: Operating data are provided for a waste-heat boiler operating at steady state.

FIND: Determine the mass flow rate of the steam generated, in kg per kmol of entering hot gas.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{\text{cv}} = 0$, $\dot{W}_{\text{cv}} = 0$, and kinetic/potential energy effects are negligible. (3) The entering and exiting gas mixtures are modeled as ideal gas mixtures.

ANALYSIS: With the indicated assumptions, the mass and energy rate balances reduce to read

$$0 = \dot{m}[h_1 - h_2] + \dot{n}_{\text{gas}} [y_{\text{CO}_2}(\bar{h}_{\text{CO}_2}(T_3) - \bar{h}_{\text{CO}_2}(T_4)) + y_{\text{H}_2\text{O}}(\bar{h}_{\text{H}_2\text{O}}(T_3) - \bar{h}_{\text{H}_2\text{O}}(T_4)) + y_{\text{N}_2}(\bar{h}_{\text{N}_2}(T_3) - \bar{h}_{\text{N}_2}(T_4))]$$

or

$$\frac{\dot{m}}{\dot{n}_{\text{gas}}} = \frac{y_{\text{CO}_2}[\bar{h}_{\text{CO}_2}(T_3) - \bar{h}_{\text{CO}_2}(T_4)] + y_{\text{H}_2\text{O}}[\bar{h}_{\text{H}_2\text{O}}(T_3) - \bar{h}_{\text{H}_2\text{O}}(T_4)] + y_{\text{N}_2}[\bar{h}_{\text{N}_2}(T_3) - \bar{h}_{\text{N}_2}(T_4)]}{h_2 - h_1}$$

With data from Table A-23 for the numerator and from Table A-3 for the denominator

$$\frac{\dot{m}}{\dot{n}_{\text{gas}}} = \frac{0.1[71,078 - 22,280] + 0.2[57,999 - 20,402] + 0.7[47,073 - 17,563] \frac{\text{kJ}}{\text{kmol}(\text{mix})}}{1841.0 \frac{\text{kJ}}{\text{kg}(\text{steam})}}$$

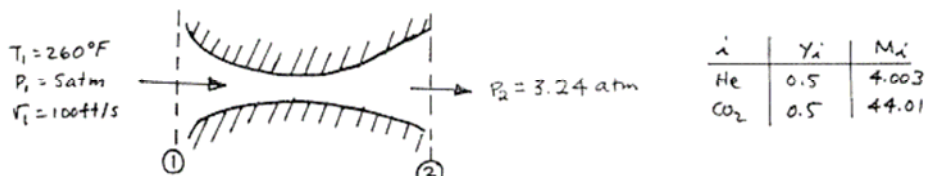
$$= 17.96 \frac{\text{kg}(\text{steam})}{\text{kmol}(\text{mix})} \quad \leftarrow \frac{\dot{m}}{\dot{n}_{\text{gas}}}$$

PROBLEM 12.20

KNOWN: An equimolar mixture of He and CO₂ expands isentropically from a specified inlet state to a specified exit pressure.

FIND: Determine the exit temperature and velocity.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The nozzle operates at steady state and the expansion is isentropic. (2) Potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

ANALYSIS: Since He is monatomic, Table A-21 gives $\bar{c}_{p,He} = 5/2 \bar{R}$, independent of temperature.

The expansion is isentropic. Thus $\bar{s}_2 - \bar{s}_1 = 0$. This can be expressed as

$$y_{He} (\bar{s}_2 - \bar{s}_1)_{He} + y_{CO_2} (\bar{s}_2 - \bar{s}_1)_{CO_2} = 0$$

or

$$y_{He} \left[\bar{c}_{p,He} \ln \frac{T_2}{T_1} - \bar{R} \ln \frac{P_2}{P_1} \right] + y_{CO_2} \left[\bar{s}_{CO_2}^0(T_2) - \bar{s}_{CO_2}^0(T_1) - \bar{R} \ln \frac{P_2}{P_1} \right] = 0$$

Solving

$$\ln \frac{P_2}{P_1} = \frac{y_{He} \bar{c}_{p,He} \ln \frac{T_2}{T_1} + y_{CO_2} (\bar{s}_{CO_2}^0(T_2) - \bar{s}_{CO_2}^0(T_1))}{\bar{R}}$$

With data from Table A-23E at $T_1 = 720^\circ R$

$$\ln \left(\frac{3.24}{5.0} \right) = 0.5 \left[\frac{5}{2} \ln \frac{T_2}{720} + \frac{\bar{s}_{CO_2}^0(T_2) - 53.78}{1.986} \right]$$

- ① This is a single equation in one unknown, T_2 . Solving iteratively using table data gives $T_2 = 640^\circ R$ T_2

Reduction of mass and energy rate balances at steady state result in

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right)$$

Thus

$$h_2 - h_1 + \frac{(V_2^2 - V_1^2)}{2} = 0 \Rightarrow \frac{\bar{h}_2 - \bar{h}_1}{M} + \frac{(V_2^2 - V_1^2)}{2} = 0$$

where M is the mixture molecular weight: $M = y_{He} M_{He} + y_{CO_2} M_{CO_2} = 24.01$, and

$$\bar{h}_2 - \bar{h}_1 = y_{He} (\bar{h}_2 - \bar{h}_1)_{He} + y_{CO_2} (\bar{h}_2 - \bar{h}_1)_{CO_2} = y_{He} \bar{c}_{p,He} (T_2 - T_1) + y_{CO_2} (\bar{h}_2 - \bar{h}_1)_{CO_2}$$

With $\bar{c}_{p,He} = 5/2 \bar{R}$ and $\bar{h}$ for CO₂ from Table A-23E

$$0 = \frac{(0.5) \left[\frac{(5/2)(1.986)}{24.01} (640 - 720) \right] + \frac{[4974.9 - 5748.4]}{16} + \left[\frac{V_2^2 - (100 \text{ ft/s})^2}{2} \right] \left(\frac{16 \text{ ft}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \right) \left(\frac{8 \text{ in}}{778 \text{ ft} \cdot \text{lb}} \right)$$

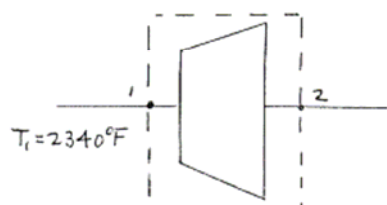
Solving, $V_2 = 1110 \text{ ft/s}$

1. An iterative solution using table data can be avoided using IT. The results for T_2 and V_2 obtained using IT agree with the values given here.

PROBLEM 12.21

KNOWN: An equimolar mixture of He and Ar expands adiabatically through a turbine for which steady state operating data are known.
FIND: Determine the power developed, in Btu per lb of mixture flowing.

SCHEMATIC & GIVEN DATA:



Mixture: (He: H, Ar: A) $y_H = y_A = 0.5$

$$P_2/P_1 = 1/7.5$$

$$\eta_t = 80\%$$

ENGINEERING

MODEL: (1) The control volume shown operates at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

ANALYSIS: Using the isentropic turbine efficiency η_t , $\dot{W}_{cv}/\dot{m} = \eta_t (\dot{W}_{cv}/\dot{m})_{s,c}$. Accordingly, the value of $(\dot{W}_{cv}/\dot{m})_{s,c}$ is required. Reducing mass and energy balances at steady state

$$\left(\frac{\dot{W}_{cv}}{\dot{m}} \right)_{s,c} = h_1 - h_2 = \frac{1}{M} [\bar{h}_1 - \bar{h}_2] = \frac{1}{M} [y_H [\bar{h}_1 - \bar{h}_{2s}]_H + y_A [\bar{h}_1 - \bar{h}_{2s}]_A]$$

By reference to Table A-21E, for each of the monatomic gases He and Ar, $\bar{c}_p = \frac{5}{2} \bar{R}$. Moreover, $y_A = y_H = 0.5$. So, in summary

$$\left(\frac{\dot{W}_{cv}}{\dot{m}} \right)_{s,c} = \frac{1}{M} \left[\frac{5}{2} \bar{R} \right] [T_1 - T_{2s}] \quad (1)$$

where $M = (M_{He} + M_{Ar})/2 = (4.003 + 39.94)/2 = 21.97$ and T_{2s} is the exit temperature for an isentropic expansion.

For an isentropic expansion, $\bar{s}_2 - \bar{s}_1 = 0$. Thus,

$$y_A [\bar{s}_1 - \bar{s}_{2s}]_A + y_H [\bar{s}_1 - \bar{s}_{2s}]_H = 0 \Rightarrow y_A [\bar{c}_{pA} \ln \frac{T_{2s}}{T_1} - \bar{R} \ln \frac{P_2}{P_1}] + y_H [\bar{c}_{pH} \ln \frac{T_{2s}}{T_1} - \bar{R} \ln \frac{P_2}{P_1}] = 0$$

With $y_A = y_H = 0.5$, $\bar{c}_{pH} = \bar{c}_{pA} = \frac{5}{2} \bar{R}$,

$$0.5 \left[\frac{5}{2} \bar{R} \ln \frac{T_{2s}}{T_1} - \bar{R} \ln \frac{P_2}{P_1} \right] + 0.5 \left[\frac{5}{2} \bar{R} \ln \frac{T_{2s}}{T_1} - \bar{R} \ln \frac{P_2}{P_1} \right] = 0$$

Reducing this

$$\frac{5}{2} \ln \frac{T_{2s}}{T_1} = \ln \frac{P_2}{P_1}$$

$$\Rightarrow \ln \frac{T_{2s}}{T_1} = 0.4 \ln (1/7.5) \Rightarrow T_{2s} = 1251^\circ R$$

Finally, substituting values into Eq. (1)

$$\left(\frac{\dot{W}_{cv}}{\dot{m}} \right) = (0.80) \left[\frac{(5/2)(1.986)}{21.97} \frac{\text{Btu}}{\text{lb} \cdot ^\circ R} \right] [2800 - 1251]^\circ R$$

$$= 280.05 \frac{\text{Btu}}{\text{lb}}$$

$\dot{W}_{cv}/\dot{m}$

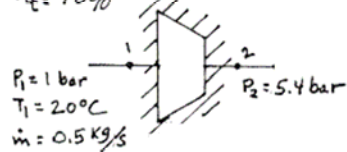
PROBLEM 12.22

KNOWN: A gas mixture with a specified molar analysis enters a compressor at a specified mass flow rate and state and exits at a specified pressure. The isentropic compressor efficiency is also known.

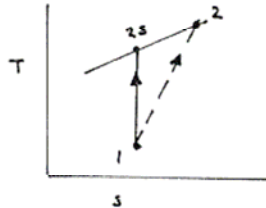
FIND: Determine the temperature at the exit, the power required, and the rate of entropy production.

SCHEMATIC & GIVEN DATA:

$$\eta_c = 78\%$$



i	Y _i	M _i
O ₂	0.6	32
N ₂	0.4	28.01



ENGINEERING MODEL: (1) The compressor is well insulated and operates at steady state. (2) Kinetic and potential energy effects can be ignored. (3) The mixture adheres to the idealizations of the Dalton model and composition remains constant.

ANALYSIS: The temperature T_2 can be determined using the isentropic compressor efficiency, but first the temperature at state 2s, T_{2s} , must be found. Since $\bar{s}_{2s} - \bar{s}_1 = 0$, Eq. 12.36 can be invoked to write

$$y_{O_2}(\bar{s}_{2s} - \bar{s}_1)_{O_2} + y_{N_2}(\bar{s}_{2s} - \bar{s}_1)_{N_2} = y_{O_2}(\bar{s}_{2s}^0 - \bar{s}_1^0 - \bar{R} \ln \frac{P_2}{P_1})_{O_2} + y_{N_2}(\bar{s}_{2s}^0 - \bar{s}_1^0 - \bar{R} \ln \frac{P_2}{P_1})_{N_2} = 0$$

Accordingly, with $\bar{s}^0$ data from the ideal gas tables

$$y_{O_2}(\bar{s}_{2s}^0)_{O_2} + y_{N_2}(\bar{s}_{2s}^0)_{N_2} = y_{O_2}(\bar{s}_1^0)_{O_2} + y_{N_2}(\bar{s}_1^0)_{N_2} + \bar{R} \ln \frac{P_2}{P_1}$$

$$= (0.6)(204.524) + (0.4)(190.998) + 8.314 \ln 5.4 = 213.13 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

① Solving iteratively using table data, $T_{2s} = 470 \text{ K}$.

In accord with listed assumptions, the isentropic compressor efficiency is

$$\eta_c = \frac{\bar{h}_{2s} - \bar{h}_1}{\bar{h}_2 - \bar{h}_1}$$

Thus

$$\bar{h}_2 - \bar{h}_1 = \frac{\bar{h}_{2s} - \bar{h}_1}{\eta_c} = \frac{y_{O_2}(\bar{h}_{2s} - \bar{h}_1)_{O_2} + y_{N_2}(\bar{h}_{2s} - \bar{h}_1)_{N_2}}{\eta_c} = \frac{0.6(13842 - 8533) + 0.4(13693 - 8521)}{0.78}$$

$$= 6736 \text{ kJ/kmol}$$

Accordingly

$$y_{O_2}(\bar{h}_2 - \bar{h}_1)_{O_2} + y_{N_2}(\bar{h}_2 - \bar{h}_1)_{N_2} = 6736 \text{ kJ/kmol}$$

or

$$y_{O_2}(\bar{h}_2)_{O_2} + y_{N_2}(\bar{h}_2)_{N_2} = y_{O_2}(\bar{h}_1)_{O_2} + y_{N_2}(\bar{h}_1)_{N_2} + 6736$$

$$= (0.6)(8533) + (0.4)(8521) + 6736 = 15,264 \text{ kJ/kmol}$$

$$\text{Solving, } T_2 = 520 \text{ K} (247^\circ \text{C})$$

_____ T_2

The power required is

$$\dot{W}_c = -\dot{m} (h_2 - h_1) = -\dot{m} \left(\frac{\bar{h}_2 - \bar{h}_1}{M} \right)$$

where M is the mixture molecular weight: $M = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = (0.6)(32) + (0.4)(28.01)$
 $= 30.4$. Therefore

Continued on next slide

Problem 12-22 continued

$$\dot{W}_C = - (0.5 \frac{\text{kg}}{\text{s}}) \left[\frac{6736 \text{ kJ/kmol}}{30.4 \text{ kg/kmol}} \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -110.8 \text{ kW} \quad \leftarrow \dot{W}_C$$

Mass and entropy rate balances reduce at steady state to give

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m} (s_1 - s_2) + \dot{\sigma}_{cv}$$

Thus

$$\begin{aligned} \dot{\sigma}_{cv} &= \dot{m} (s_2 - s_1) \\ &= \dot{m} \left(\frac{\bar{s}_2 - \bar{s}_1}{M} \right) = \dot{m} \left[\frac{y_{O_2} (\bar{s}_2 - \bar{s}_1)_{O_2} + y_{N_2} (\bar{s}_2 - \bar{s}_1)_{N_2}}{M} \right] \end{aligned}$$

With $\bar{s}^\circ$ data from Table A-23

$$\begin{aligned} \dot{\sigma}_{cv} &= (0.5 \frac{\text{kg}}{\text{s}}) \left[\frac{0.6 (221.812 - 204.524) + 0.4 (207.792 - 190.998) - 8.314 \ln 5.4}{30.4} \right] \left(\frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 0.0505 \frac{\text{KW}}{\text{K}} \quad \leftarrow \dot{\sigma}_{cv} \end{aligned}$$

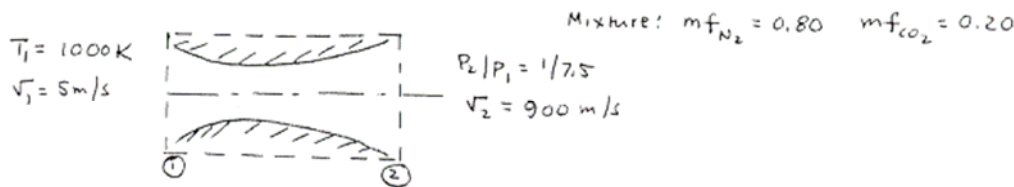
1. An iterative solution with table data can be avoided by using IT. The results for T_{2s} , T_2 , $\dot{W}_{cv}$, and $\dot{\sigma}_{cv}$ obtained using IT agree with the values given here.

PROBLEM 12.23

KNOWN: A specified mixture expands through a nozzle for which steady state operating data are provided.

FIND: Determine the isentropic nozzle efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

ANALYSIS: By definition of isentropic nozzle efficiency, $\eta_{nozzle} = (V_2^2/2)/(V_{2s}^2/2) = (V_2/V_{2s})^2$, where V_{2s} is the exit velocity for an isentropic expansion.

Reducing mass and energy rate balances at steady state

$$\frac{V_{2s}^2}{2} = \frac{V_1^2}{2} + h_1 - h_{2s} = \frac{V_1^2}{2} + \left[mf_{N_2} \frac{[\bar{h}_1 - \bar{h}_{2s}]_{N_2}}{M_{N_2}} + mf_{CO_2} \frac{[\bar{h}_1 - \bar{h}_{2s}]_{CO_2}}{M_{CO_2}} \right] \quad (1)$$

where h_{2s} denotes the enthalpy at the exit for an isentropic expansion.

Consider next, $S_2 - S_1 = 0 \Rightarrow mf_{N_2} [S_1 - S_{2s}]_{N_2} + mf_{CO_2} [S_1 - S_{2s}]_{CO_2} = 0$

or

$$0.8 \left[\frac{\bar{S}^0(T_{2s}) - \bar{S}^0(T_1) - \bar{R} \ln P_2/P_1}{M_{N_2}} \right]_{N_2} + 0.2 \left[\frac{\bar{S}^0(T_{2s}) - \bar{S}^0(T_1) - \bar{R} \ln P_2/P_1}{M_{CO_2}} \right]_{CO_2} = 0$$

With data from Table A-23

$$0.8 \left[\frac{\bar{S}_{N_2}^0(T_{2s}) - 228.057 + 8.314 \ln 7.5}{28.01} \right] + 0.2 \left[\frac{\bar{S}_{CO_2}^0(T_{2s}) - 269.215 + 8.314 \ln 7.5}{44.01} \right] = 0$$

$$0.8 \left[\frac{\bar{S}_{N_2}^0(T_{2s}) - 211.305}{28.01} \right] + 0.2 \left[\frac{\bar{S}_{CO_2}^0(T_{2s}) - 252.463}{44.01} \right] = 0$$

① Solving iteratively using $\bar{S}^0$ data from Table A-23, $T_{2s} = 616.5 \text{ K}$.

Returning to Eq. (1) and using $\bar{h}$ data from Table A-23

$$\begin{aligned} \frac{V_{2s}^2}{2} &= \frac{(5 \text{ m/s})^2}{2} + \left[0.8 \frac{(30,129 - 17,909)}{28.01} + 0.2 \frac{(42,769 - 22,826)}{44.01} \right] \left(\frac{\text{kJ}}{\text{kg}} \right) \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right| \\ &= 12.5 \left(\frac{\text{m}}{\text{s}} \right)^2 + 439.650 \left(\frac{\text{m}}{\text{s}} \right)^2 \Rightarrow V_{2s} = 937.7 \text{ m/s} \end{aligned}$$

Finally

$$\eta_{nozzle} = \left(\frac{900}{937.7} \right)^2 = 0.92 \quad (92\%)$$

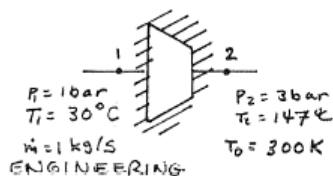
1. An iterative solution with table data can be avoided by using IT. The result for η_{nozzle} obtained using IT agrees with the value given here.

PROBLEM 12.24

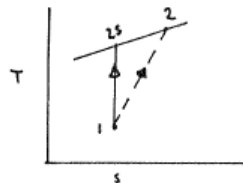
KNOWN: A mixture with a specified molar analysis enters a compressor operating at steady state at a specified mass flow rate and state and exits at a specified temperature and pressure.

FIND: Determine the power required, the isentropic compressor efficiency, and the rate of exergy destruction.

SCHEMATIC & GIVEN DATA



i	Y _i	M _i
N ₂	0.6	28.01
CO ₂	0.4	44.01



MODEL: (1) The compressor is well insulated and operates at steady state. (2) Kinetic and potential energy effects can be ignored. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant. (4) For the environment, $T_0 = 300\text{ K}$.

ANALYSIS: Reduction of the mass and energy rate balances at steady state gives

$$\dot{W}_c = \dot{m} [h_1 - h_2] = \dot{m} \left[\frac{Y_{N_2}(\bar{h}_1 - \bar{h}_2)_{N_2} + Y_{CO_2}(\bar{h}_1 - \bar{h}_2)_{CO_2}}{M} \right]$$

where M is the mixture molecular weight: $M = Y_{N_2} M_{N_2} + Y_{CO_2} M_{CO_2} = 0.6(28.01) + 0.4(44.01) = 34.41$. Then, with data from Table A-23

$$\dot{W}_c = (1 \text{ kg/s}) \left[\frac{0.6(8810.3 - 12225) + 0.4(9543.8 - 14206)}{34.41} \right] \left(\frac{\text{kJ}}{\text{kg}} \right) \left(\frac{\text{kg}}{\text{s}} \right) = -113.7 \text{ kW} \leftarrow \dot{W}_c$$

To determine the isentropic compressor efficiency requires the power for an isentropic compression from state 1 to the pressure P_2 — that is, from 1 to state 2s. For this process, $\bar{s}_{2s} - \bar{s}_1 = 0$. Or

$$Y_{N_2}[\bar{s}_{2s} - \bar{s}_1]_{N_2} + Y_{CO_2}[\bar{s}_{2s} - \bar{s}_1]_{CO_2} = 0 \Rightarrow Y_{N_2}[\bar{s}_{2s}^0 - \bar{s}_1^0 - \bar{R} \ln \frac{P_2}{P_1}]_{N_2} + Y_{CO_2}[\bar{s}_{2s}^0 - \bar{s}_1^0 - \bar{R} \ln \frac{P_2}{P_1}]_{CO_2} = 0$$

Then, with data from Table A-23

$$Y_{N_2}(\bar{s}_{2s}^0)_{N_2} + Y_{CO_2}(\bar{s}_{2s}^0)_{CO_2} = (0.6)(191.969) + (0.4)(214.281) + 8.314 \ln 3 = 210.029 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

① Solving this equation by iteration with table data; $T_{2s} = 399\text{ K}$. Accordingly

$$\begin{aligned} (\dot{W}_c)_s &= \dot{m} (h_1 - h_{2s}) = \dot{m} \left[\frac{Y_{N_2}(\bar{h}_1 - \bar{h}_{2s})_{N_2} + Y_{CO_2}(\bar{h}_1 - \bar{h}_{2s})_{CO_2}}{M} \right] \\ &= (1) \left[\frac{0.6(8810.3 - 11610.7) + 0.4(9543.8 - 13330.8)}{34.41} \right] = -92.85 \text{ kW} \end{aligned}$$

The isentropic compressor efficiency is then

$$\eta_c = \frac{(\dot{W}_c)_s}{(\dot{W}_c)} = \frac{(-92.85)}{(-113.7)} = 0.817 \text{ (81.7\%)} \leftarrow \eta_c$$

The rate of exergy destruction can be found using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production obtained from an entropy rate balance, which reduces at steady state as follows:

$$\begin{aligned} 0 &= \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) \\ \Rightarrow \dot{\sigma}_{cv} &= \dot{m} \left[\frac{Y_{N_2}(\bar{s}_2 - \bar{s}_1)_{N_2} + Y_{CO_2}(\bar{s}_2 - \bar{s}_1)_{CO_2}}{M} \right] \end{aligned}$$

Continued on next slide

Problem 12-24 continued

The terms $(\bar{s}_2 - \bar{s}_1)_{N_2}$ and $(\bar{s}_2 - \bar{s}_1)_{CO_2}$ are evaluated using Eq. 12.36, so

$$\begin{aligned}\dot{\sigma}_{cv} &= \dot{m} \left[\frac{y_{N_2} (\bar{s}_{N_2}^0(T_2) - \bar{s}_{N_2}^0(T_1) - \bar{R} \ln \frac{P_2}{P_1}) + y_{CO_2} (\bar{s}_{CO_2}^0(T_2) - \bar{s}_{CO_2}^0(T_1) - \bar{R} \ln \frac{P_2}{P_1})}{M} \right] \\ &= \dot{m} \left[\frac{y_{N_2} (\bar{s}_{N_2}^0(T_2) - \bar{s}_{N_2}^0(T_1)) + y_{CO_2} (\bar{s}_{CO_2}^0(T_2) - \bar{s}_{CO_2}^0(T_1)) - \bar{R} \ln \frac{P_2}{P_1}}{M} \right]\end{aligned}$$

Then, with $\bar{s}^0$ data from Table A-23

$$\begin{aligned}\dot{\sigma}_{cv} &= (1 \frac{kg}{s}) \left[\frac{0.6(201.499 - 191.969) + 0.4(227.258 - 214.284) - 8.314 \ln 3}{34.41} \right] \left| \frac{KJ}{kg \cdot K} \right| \left| \frac{1 kW}{1 kJ/s} \right| \\ &= 0.0516 kW/K\end{aligned}$$

Multiplying this by T_0

$$\textcircled{2} \quad \dot{E}_d = T_0 \dot{\sigma}_{cv} = (300 K) \left(0.0516 \frac{KW}{K} \right) = 15.48 KW \quad \leftarrow \dot{E}_d$$

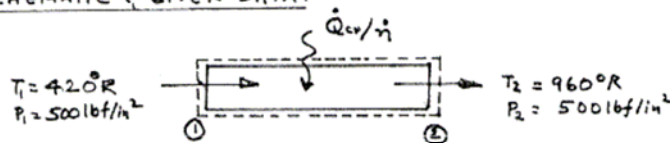
-
1. An iterative solution using table data can be avoided by using IT. The results for $\dot{W}_c$, T_{2s} , h_c , and $\dot{E}_d$ obtained using IT agree with the values given here.
 2. When expressed as a percentage of the power input to the compressor
 $\% = [\dot{E}_d / (-\dot{W}_{cv})](100) = (15.48 / 113.7)(100) = 13.6\%$

PROBLEM 12.25

KNOWN: An equimolar mixture of N_2 and CO_2 enters a heat exchanger at a known temperature and pressure and exits with no change in composition at a known temperature and pressure.

FIND: Determine the rate of heat transfer to the mixture per lbmol of mixture flowing using (a) ideal gas mixture principles; (b) the generalized enthalpy chart together with Kay's rule.

SCHEMATIC & GIVEN DATA:



i	Y _i
N_2	0.5
CO_2	0.5

ENGINEERING MODEL:

(1) For the control volume shown in the accompanying figure $W_{cv} = 0$ and kinetic and potential energy effects are negligible. (2) The control volume is at steady state. (3) The composition of the mixture remains constant. (4) In part (a), the mixture adheres to the Dalton model. (5) In Part (b), Kay's rule applies.

ANALYSIS: At steady state, mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_2 - h_1 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] \Rightarrow \dot{Q}_{cv} = \dot{m} (h_2 - h_1)$$

or, when expressed in terms of the molar flow rate of the mixture

$$\dot{Q}_{cv} = \dot{n} (\bar{h}_2 - \bar{h}_1) \Rightarrow \frac{\dot{Q}_{cv}}{\dot{n}} = \bar{h}_2 - \bar{h}_1 \quad (1)$$

(a) When the ideal gas mixture principles of Chap. 12 are used, the change in specific enthalpy of the mixture can be expressed in terms of the enthalpy changes of the components evaluated at the inlet and exit temperatures:

$$\frac{\dot{Q}_{cv}}{\dot{n}} = y_{N_2} [\bar{h}_{N_2}(T_2) - \bar{h}_{N_2}(T_1)] + y_{CO_2} [\bar{h}_{CO_2}(T_2) - \bar{h}_{CO_2}(T_1)]$$

With data from Table A-23

$$\frac{\dot{Q}_{cv}}{\dot{n}} = 0.5 [6693.1 - 2916.1] + 0.5 [8243.8 - 3035.7] = 4492.6 \text{ Btu/lbmol} \quad (a)$$

(b) Invoking Eq. 11.85 for the mixture:

$$\frac{\dot{Q}_{cv}}{\dot{n}} = \bar{h}_2^* - \bar{h}_1^* - \bar{R} T_c \left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c} \right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c} \right)_1 \right] \quad (2)$$

The underlined term is the enthalpy change for the mixture evaluated in part (a) using ideal gas mixture principles. The remaining terms can be evaluated from Figure A-4 using T_c and P_c for the mixture calculated via Kay's rule:

$$T_c = y_{N_2}(T_c)_{N_2} + y_{CO_2}(T_c)_{CO_2} = 0.5(227^\circ R + 548^\circ R) = 388^\circ R$$

$$P_c = y_{N_2}(P_c)_{N_2} + y_{CO_2}(P_c)_{CO_2} = 0.5(33.5 \text{ atm} + 72.9 \text{ atm}) = 53.2 \text{ atm}$$

Thus

$$(T_R)_1 = \frac{420}{388} = 1.08, \quad (T_R)_2 = \frac{960}{388} = 2.47, \quad P_{R1} = P_{R2} = \left(\frac{500/14.7}{53.2} \right) = 0.64$$

Fig. A-4 gives $\left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c} \right)_1 = 0.7$, $\left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c} \right)_2 = 0.1$. And from Eq. (2) we get

$$\textcircled{1} \quad \left(\frac{\dot{Q}_{cv}}{\dot{n}} \right) = 4492.6 \text{ Btu/lbmol} - (1.986 \text{ Btu/lbmol} \cdot ^\circ R)(388^\circ R)(0.1 - 0.7) = 4954.9 \text{ Btu/lbmol}$$

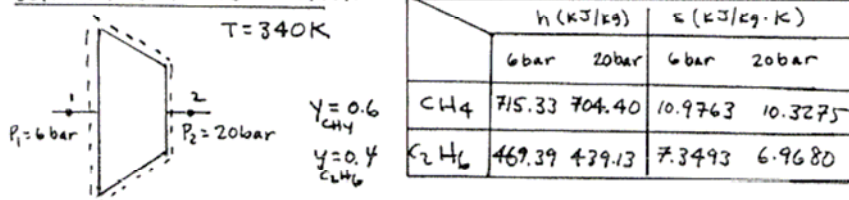
1. The result of part (b) is about 10% greater than that of part (a).

PROBLEM 12.26

KNOWN: Natural gas with a specified composition is compressed isothermally without internal irreversibilities from 6 to 20 bar. The compressor operates at steady state.

FIND: Determine the work and heat transfer per unit mole of mixture flowing (a) using ideal gas mixture principles, (b) using ideal solution principles with given data for the pure components.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume operates at steady state with negligible kinetic and potential energy changes. (2) The mixture composition remains constant. (3) In part (a) the mixture adheres to the Dalton model. The ideal solution model applies in part (b).

ANALYSIS: At steady state, mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} (h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}) \Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m} (h_1 - h_2)$$

Mass and entropy rate balances reduce to give

$$0 = \frac{\dot{Q}_{cv}}{T} + \dot{m} (s_1 - s_2) + \cancel{\dot{S}_{cv}} \Rightarrow \dot{Q}_{cv} = \dot{m} T (s_2 - s_1) = \dot{m} T (\bar{s}_2 - \bar{s}_1) \quad (1)$$

Combining these results

$$\dot{W}_{cv} = \dot{m} [h_1 - h_2 - T(s_1 - s_2)]$$

or, in terms of the molar flow rate of the mixture $\dot{n}$

$$\dot{W}_{cv} = \dot{n} [\bar{h}_1 - \bar{h}_2 - T(\bar{s}_1 - \bar{s}_2)] \Rightarrow \frac{\dot{W}_{cv}}{\dot{n}} = \bar{h}_1 - \bar{h}_2 - T(\bar{s}_1 - \bar{s}_2) \quad (2)$$

(a) When the ideal gas mixture principles of Chap. 12 are used, the enthalpy change of the mixture vanishes since temperature remains constant. The specific entropy change of the mixture can be expressed in terms of the specific entropy changes of the components using Eq. 12.26:

$$\textcircled{1} \quad \bar{s}_1 - \bar{s}_2 = y_{CH_4} \left[\underbrace{\bar{s}_1^0 - \bar{s}_2^0}_{=0 \text{ since } T_1 = T_2} - \bar{R} \ln \frac{P_1}{P_2} \right]_{CH_4} + y_{C_2H_6} \left[\underbrace{\bar{s}_1^0 - \bar{s}_2^0}_{=0 \text{ since } T_1 = T_2} - \bar{R} \ln \frac{P_1}{P_2} \right]_{C_2H_6} = -\bar{R} \ln \frac{P_1}{P_2}$$

Collecting results, Eq. (1) becomes

$$\frac{\dot{W}_{cv}}{\dot{n}} = \bar{R} T \ln \frac{P_1}{P_2} = \left(8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \right) (340 \text{ K}) \ln \frac{6}{20} = -3403 \frac{\text{kJ}}{\text{kmol}} \quad (a)$$

$$\text{Since } (h_1 - h_2) = 0, \quad (\dot{Q}_{cv}/\dot{n}) = (\dot{W}_{cv}/\dot{n}) = -3403 \frac{\text{kJ}}{\text{kmol}}$$

(b) Using the ideal solution model together with the data above for the pure components, we get

$$\bar{h}_1 - \bar{h}_2 = 0.6 [(16.04) [715.3 - 704.4]] + 0.4 [(30.07) [462.39 - 439.13]] = 385 \frac{\text{kJ}}{\text{kmol}}$$

Continued on next slide

Problem 12-26 continued

$$\textcircled{1} \quad \bar{s}_1 - \bar{s}_2 = 0.6 [(6.04)(10.9763 - 10.3275)] + 0.4 [(30.07)(7.3493 - 6.9680)] = 10.83 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

Then, with Eq. (2)

$$\textcircled{2} \quad \left(\frac{\dot{W}_{cv}}{\dot{n}} \right) = \bar{h}_1 - \bar{h}_2 - T(\bar{s}_1 - \bar{s}_2) = 385 - 340(10.83) = -3297 \frac{\text{kJ}}{\text{kmol}}$$

and with Eq. (1)

$$\left(\frac{\dot{Q}_{cv}}{\dot{n}} \right) = T(\bar{s}_2 - \bar{s}_1) = 340(-10.83) = -3682 \frac{\text{kJ}}{\text{kmol}}$$

1. Since composition remains constant, this approach suffices. See discussion leading up to Eq. 12.36.

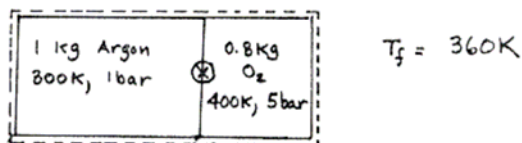
2. Comparing the results of part (a) in terms of magnitudes,
 $\left(\frac{\dot{W}_{cv}}{\dot{n}} \right)$ in part (a) is about 3% greater than the part (b) result.
 $\left(\frac{\dot{Q}_{cv}}{\dot{n}} \right)$ in part (a) is about 8% less than the part (b) result.

PROBLEM 12.27

KNOWN: Argon at a specified state is allowed to mix with O_2 at a specified state to achieve a mixture at a known temperature.

FIND: Determine (a) the volume of each gas initially, (b) the final pressure, (c) the heat transfer, and (d) the entropy change of each gas.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) For the system shown in the accompanying figure there is no change in kinetic or potential energy between initial and final states. (2) Each gas and the overall mixture behaves as an ideal gas. The Dalton mixture model applies.

ANALYSIS: (a) The volume of each tank can be determined using the ideal gas equation of state together with known initial data for each gas:

$$V_{Ar} = \frac{n_{Ar}(\bar{R}/M_{Ar})T_{Ar}}{P_{Ar}} = \frac{(1 \text{ kg}) \left(\frac{8314}{39.94} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (300 \text{ K})}{(10^5 \text{ N/m}^2)} = 0.624 \text{ m}^3$$

$$V_{O_2} = \frac{n_{O_2}(\bar{R}/M_{O_2})T_{O_2}}{P_{O_2}} = \frac{(0.8) \left(\frac{8314}{32.0} \right) (400)}{(5 \times 10^5)} = 0.166 \text{ m}^3$$

$\leftarrow V_{Ar}, V_{O_2}$

where P_{Ar} and P_{O_2} represent the initial pressures of the argon and oxygen, respectively, and not partial pressures.

(b) The final pressure, P_f , can also be determined using the ideal gas equation of state:

$$P_f = \frac{n \bar{R} T_f}{V}$$

where $V = 0.624 \text{ m}^3 + 0.166 \text{ m}^3 = 0.79 \text{ m}^3$ and

$$n = n_{Ar} + n_{O_2} = \frac{m_{Ar}}{M_{Ar}} + \frac{m_{O_2}}{M_{O_2}} = \frac{1}{39.94} + \frac{0.8}{32.0} = 0.025 + 0.025 = 0.05 \text{ kmol}$$

Finally

$$P_f = \frac{(0.05 \text{ kmol}) \left(\frac{8314}{\text{kmol} \cdot \text{K}} \right) (360 \text{ K})}{0.79 \text{ m}^3} \left| \frac{\text{bar}}{10^5 \text{ N/m}^2} \right| = 1.89 \text{ bar} \quad \leftarrow P_f$$

(c) An energy balance reduces to give $\Delta U = Q - W$, or

$$Q = \Delta U = \Delta U_{Ar} + \Delta U_{O_2} = n_{Ar} \bar{c}_{v,Ar} [T_f - T_{Ar}] + n_{O_2} [\bar{u}_{O_2}(T_f) - \bar{u}_{O_2}(T_{O_2})]$$

From Table A-21, for argon $\bar{c}_p = 5/2 \bar{R}$. Then, with Eq. 3.45, $\bar{c}_v = \bar{c}_p - \bar{R} = 3/2 \bar{R}$. With $\bar{u}$ data from Table A-23

$$Q = (0.025) \left[\frac{3}{2} \times 8.314 \right] [360 - 300] + (0.025) [7518 - 8384]$$

$$= -29.4 \text{ kJ} \quad \leftarrow Q$$

Continued on next slide

Problem 12-27 continued

(d) Argon initially is at 300 K, 1 bar and finally at 360 K and the partial pressure, $y_{Ar} P_f = (0.5)(1.896 \text{ bar}) = 0.945 \text{ bar}$. Thus, with $\bar{c}_p = 5/2 \bar{R}$

$$(\Delta S)_{Ar} = (0.025) \left[(2.5)(8.314) \ln \frac{360}{300} - (8.314) \ln \left(\frac{0.945}{1.00} \right) \right] = 0.083 \frac{\text{kJ}}{\text{K}} \quad \leftarrow (\Delta S)_{Ar}$$

Oxygen initially is at 400 K, 5 bar and finally at 360 K and the partial pressure 0.945 bar. With $\bar{s}^\circ$ data from Table A-23

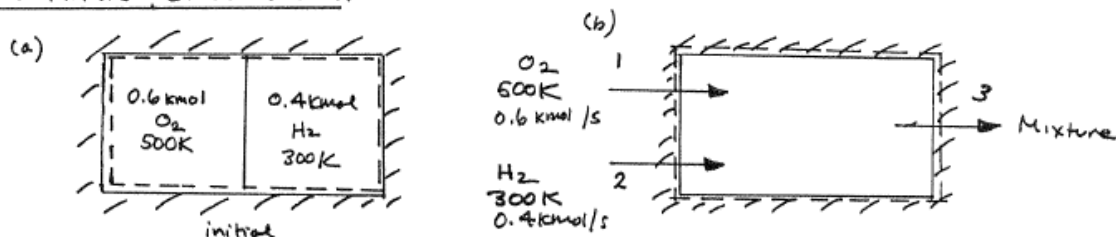
$$(\Delta S)_{O_2} = 0.025 \left[210.604 - 213.765 - 8.314 \ln \frac{0.945}{5} \right] = 0.2673 \frac{\text{kJ}}{\text{K}} \quad \leftarrow (\Delta S)_{O_2}$$

PROBLEM 12.28

KNOWN: Two cases involving the mixing of O_2 and H_2 are specified.

FIND: For each case determine the mixture temperature.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) For case (a), a closed system consisting of the O_2 and H_2 is the system. For case (b), a control volume at steady state is the system. (2) For each system, $Q = 0$, $\dot{W} = 0$ (except for flow work in (b)), and there are no significant kinetic/potential energy effects. (3) Each gas and the overall mixture behaves as an ideal gas. The Dalton mixture model applies. (4) Appropriate constant specific heats are employed.

ANALYSIS: (a) An energy balance reduces to read $\Delta U = \cancel{Q} - \cancel{W}$, or $\Delta U = 0$. That is

$$[n \Delta \bar{u}]_{O_2} + [n \Delta \bar{u}]_{H_2} = 0 \quad (1)$$

Since the initial temperature difference is just 200 K, and c_v does not vary significantly over such an interval, an appropriate constant c_v can be used for each gas. Thus, with data from Table A-20 at 400 K, Eq. (1) reads

$$[n(c_v M)(T_f - T_i)]_{O_2} + [n(c_v M)(T_f - T_i)]_{H_2} = 0$$

$$\Rightarrow (0.6)(0.681 \times 32)[T_f - 500] + (0.4)(10.352 \times 2.016)[T_f - 300] = 0 \Rightarrow T_f = 422 K$$

(b) An energy rate balance reduces to read

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{n}_{O_2} \bar{h}_{O_2}(T_1) + \dot{n}_{H_2} \bar{h}_{H_2}(T_2) - [\dot{n}_{O_2} \bar{h}_{O_2}(T_3) + \dot{n}_{H_2} \bar{h}_{H_2}(T_3)]$$

$$\text{or} \quad 0 = \dot{n}_{O_2} [\bar{h}_{O_2}(T_1) - \bar{h}_{O_2}(T_3)] + \dot{n}_{H_2} [\bar{h}_{H_2}(T_2) - \bar{h}_{H_2}(T_3)] \quad (2)$$

Then, with c_p for each gas from Table A-20 at 400 K we get

$$0 = \dot{n}_{O_2} [(c_p M)_{O_2} (T_1 - T_3)] + \dot{n}_{H_2} [(c_p M)_{H_2} (T_2 - T_3)]$$

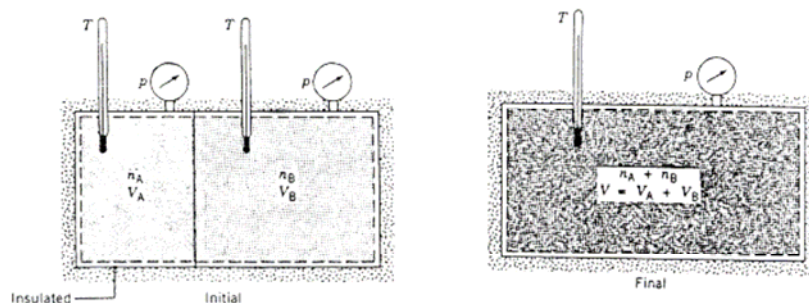
$$0 = (0.6) [(0.441 \times 32)(500 - T_3)] + (0.4) [(4.476)(2.016)(300 - T_3)] \Rightarrow T_f = 422 K$$

PROBLEM 12.29

KNOWN: A system consists initially of n_A moles of gas A at pressure p and temperature T and n_B moles of gas B separate from gas A but at the same pressure and temperature. The gases are allowed to mix at fixed total volume. The final temperature is T and the final pressure is p .

FIND: (a) Obtain an expression for the entropy produced in terms of $\bar{R}$, n_A and n_B . (b) Using the result of part (a) demonstrate that $\sigma > 0$. (c) Would entropy be produced if the gases were identical?

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system consists of gases A and B together.
2. Each gas behaves as an ideal gas. The final mixture also acts as an ideal gas with each mixture component occupying the total volume and exhibiting the mixture temperature.
3. No heat or work interactions with the surroundings occur.

Analysis: An entropy balance for the closed system reduces to

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \sigma$$

where the entropy transfer term drops out in the adiabatic process. S_1 and S_2 denote the initial and final entropies of the system, respectively.

The initial entropy of the system is the sum of the entropies of the gases when separate. That is

$$S_1 = n_A \bar{s}_A(T, p) + n_B \bar{s}_B(T, p)$$

Since each gas initially exists separately at T and p , each of the specific entropies appearing in this expression is evaluated at T, p .

The entropy of the system at the final state equals the sum of the entropies of the gases A and B evaluated at the conditions at which they exist in the mixture. That is

$$S_2 = n_A \bar{s}_A(T, y_A p) + n_B \bar{s}_B(T, y_B p)$$

In this expression, the specific entropy of each gas is evaluated at its partial pressure in the mixture and at the mixture temperature.

Combining the last three equations and solving for the entropy production

$$\sigma = n_A [\bar{s}_A(T, y_A p) - \bar{s}_A(T, p)] + n_B [\bar{s}_B(T, y_B p) - \bar{s}_B(T, p)]$$

The specific entropy changes can be evaluated using Eq. 6.25b. Since there is no change in temperature

$$\bar{s}_A(T, y_A p) - \bar{s}_A(T, p) = -\bar{R} \ln \frac{y_A p}{p} = -\bar{R} \ln y_A$$

$$\bar{s}_B(T, y_B p) - \bar{s}_B(T, p) = -\bar{R} \ln \frac{y_B p}{p} = -\bar{R} \ln y_B$$

Introducing the foregoing into the entropy production expression gives

$$\sigma = -\bar{R}(n_A \ln y_A + n_B \ln y_B)$$

① ②

Since y_A and y_B are each less than unity, σ would be positive, in accordance with the second law.

← (a)

← (b)

Continued on next slide

Problem 12-29 continued

If the gases were identical, the initial entropy of the system would be

$$S_1 = n_A s(T, p) + n_B s(T, p)$$

where n_A and n_B denote the amounts of the gas present initially in the two tanks. The final entropy of the system would be

$$S_2 = (n_A + n_B) s(T, p)$$

Clearly, $\sigma = 0$ for this case. No entropy would be produced. (c) ←

1. The irreversibility in this case is due to the *free expansion* each individual gas would undergo from its initial volume, V_A or V_B , and initial pressure p to the total volume V and its partial pressure in the mixture, p_A or p_B . The mixture would be formed spontaneously; however, once formed, energy would be required from the surroundings to separate the gases and return them to their respective initial conditions.
2. For the case of mixing several components, each initially at the same temperature and pressure, the expression for the entropy production takes the form

$$\sigma = -\bar{R} \sum_{i=1}^j n_i \ln y_i$$

where n_i and y_i are the number of moles and the mole fraction, respectively, of component i . The result obtained in the solution is a special case of this equation.

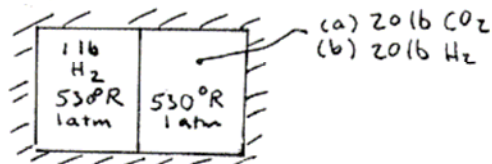
PROBLEM 12.30

KNOWN: Hydrogen (H_2) mixes adiabatically with (a) CO_2 , (b) H_2 , each at the same T, p , initially.

FIND: Determine the amount of entropy produced.

SCHEMATIC & GIVEN DATA:

initially:



ENGINEERING MODEL:

1. The system is the total quantity of gas.
2. Each gas behaves as an ideal gas and the final mixture is described by the Dalton model.
3. $Q = 0$

ANALYSIS: An entropy balance reduces with assumption 3 to give $\sigma = \Delta S$.
(a) In this case each gas undergoes a process from $530^\circ R, 1 \text{ atm}$ to $530^\circ R$ and its partial pressure in the mixture. To obtain the respective partial pressures ...

$$\left. \begin{aligned} n_{CO_2} &= \frac{20 \text{ lb}}{44.01 \text{ lb/lbmol}} = 0.4544 \text{ lbmol} \\ n_{H_2} &= \frac{1}{2.016} = 0.4960 \text{ lbmol} \\ n &= 0.9504 \text{ lbmol} \end{aligned} \right\} \begin{aligned} y_{CO_2} &= \frac{0.4544}{0.9504} = 0.4781, \quad P_{CO_2} = 0.4781 \text{ atm} \\ y_{H_2} &= \frac{0.4960}{0.9504} = 0.5219, \quad P_{H_2} = 0.5219 \text{ atm} \end{aligned}$$

Thus,

$$\sigma = n_{CO_2} \left[\underbrace{\Delta \bar{s}_{CO_2}^0}_{=0 \text{ since } T \text{ is constant}} - \bar{R} \ln \frac{P_{CO_2}}{P} \right] + n_{H_2} \left[\underbrace{\Delta \bar{s}_{H_2}^0}_{=0 \text{ since } T \text{ is constant}} - \bar{R} \ln \frac{P_{H_2}}{P} \right]$$

$$= (0.4544) \left[-1.986 \ln \frac{0.4781}{1} \right] + (0.4960) \left[-1.986 \ln \frac{0.5219}{1} \right]$$

$$= 1.307 \frac{\text{Btu}}{^\circ R}$$

(b) If H_2 is present initially in each subcompartment, the initial entropy is

$$S_i = n' \bar{s}(T, p) + n \bar{s}(T, p)$$

where n' and n denote the amounts present. At the final condition

$$S_f = (n' + n) \bar{s}(T, p)$$

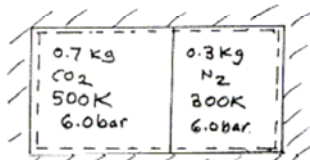
Clearly, then, $\sigma = 0$. That is, no entropy is produced when samples of the same gas at the same temperature and pressure are allowed to mix adiabatically.

PROBLEM 12.31

KNOWN: CO_2 and N_2 , initially separate, are allowed to mix without heat or work interactions with the surroundings until a final equilibrium state is attained.

FIND: Determine the final temperature and pressure, and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The consists of both the CO_2 and the N_2 . (2) For the system, $Q = 0$, $W = 0$, and kinetic/potential energy effects are absent. (3) Each gas behaves as an ideal gas. The mixture adheres to the Dalton model. (4) The specific heats are taken as constant at 400 K.

ANALYSIS: An energy balance reduces to $\Delta U = 0$ or $\Delta U = 0$. That is

$$n_{\text{CO}_2} \Delta \bar{u}_{\text{CO}_2} + n_{\text{N}_2} \Delta \bar{u}_{\text{N}_2} = 0 \quad \text{or} \quad m_{\text{CO}_2} \Delta u_{\text{CO}_2} + m_{\text{N}_2} \Delta u_{\text{N}_2} = 0$$

As the final temperature will fall between 300 and 500 K, an interval over which the variation in c_v is nearly linear, c_v for each gas is taken as constant at 400 K. This eliminates an iterative solution with the ideal gas tables. From Table A-20, $c_{v,\text{CO}_2} = 0.75 \text{ kJ/kg} \cdot \text{K}$, $c_{v,\text{N}_2} = 0.747 \text{ kJ/kg} \cdot \text{K}$. The energy balance becomes

$$m_{\text{CO}_2} c_{v,\text{CO}_2} [T_f - 500] + m_{\text{N}_2} c_{v,\text{N}_2} [T_f - 300] = 0$$

$$(0.7)(0.75)(T_f - 500) + (0.3)(0.747)(T_f - 300) = 0 \Rightarrow T_f = 440.17 \text{ K} \leftarrow T_f$$

The final pressure is found using the ideal gas equation of state

$$P_f = \frac{n \bar{R} T_f}{V} = \frac{(n_{\text{CO}_2} + n_{\text{N}_2}) \bar{R} T_f}{V}, \quad V = V_{\text{CO}_2} + V_{\text{N}_2} = \left(\frac{n \bar{R} T}{P} \right)_{\text{CO}_2} + \left(\frac{n \bar{R} T}{P} \right)_{\text{N}_2}$$

Combining

$$P_f = P \left[\frac{(n_{\text{CO}_2} + n_{\text{N}_2}) T_f}{n_{\text{CO}_2} T_{\text{CO}_2} + n_{\text{N}_2} T_{\text{N}_2}} \right], \quad \left. \begin{aligned} n_{\text{CO}_2} &= \frac{0.7}{44.01} = 0.01591 \\ n_{\text{N}_2} &= \frac{0.3}{28.01} = 0.01071 \end{aligned} \right\} n = 0.02662$$

$$= 6 \text{ bar} \left[\frac{(0.02662)(440.17)}{(0.01591)(500) + (0.01071)(300)} \right] = 6.295 \text{ bar} \leftarrow P_f$$

Reducing an entropy balance, $\Delta S = 0$, or

$$0 = m_{\text{CO}_2} \left[c_{p,\text{CO}_2} \ln \frac{T_f}{T_{\text{CO}_2}} - \frac{\bar{R}}{M_{\text{CO}_2}} \ln \frac{y_{\text{CO}_2} P_f}{P} \right] +$$

$$m_{\text{N}_2} \left[c_{p,\text{N}_2} \ln \frac{T_f}{T_{\text{N}_2}} - \frac{\bar{R}}{M_{\text{N}_2}} \ln \frac{y_{\text{N}_2} P_f}{P} \right]$$

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Problem 12-31 continued

Substituting values

$$\begin{aligned} \sigma &= (0.7 \text{ kg}) \left[\left(0.939 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) \ln \frac{440.17}{500} - \frac{8.314}{44.01} \ln \left(\frac{(0.5977)(6.295)}{6} \right) \right] \\ &\quad (0.3) \left[\left(1.044 \right) \ln \frac{440.17}{300} - \frac{8.314}{28.01} \ln \left(\frac{(0.4023)(6.295)}{6} \right) \right] \end{aligned}$$

where $y_{\text{CO}_2} = 0.01591/0.02662 = 0.5977$, $y_{\text{N}_2} = 0.01071/0.02662 = 0.4023$,
and c_{p,CO_2} , c_{p,N_2} are evaluated at 400 K. Calculating

$$\begin{aligned} \sigma &= -0.0221 \frac{\text{kJ}}{\text{K}} + 0.1969 \frac{\text{kJ}}{\text{K}} \\ &= 0.1748 \frac{\text{kJ}}{\text{K}} \end{aligned}$$

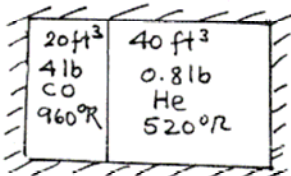
PROBLEM 12.32

KNOWN: Carbon monoxide and helium, initially separate at different states, are allowed to mix adiabatically to equilibrium.

FIND: Determine (a) the final temperature, (b) the final pressure, (c) the exergy destruction.

SCHEMATIC & GIVEN DATA:

Initially,



ENGINEERING MODEL:

1. The system is the total quantity of gas present.
2. For the system, $Q=0$, and kinetic and potential energy effects can be ignored.
3. Each gas behaves as an ideal gas. The mixture adheres to the Dalton model.
4. $T_0 = 520^\circ\text{R}$.

ANALYSIS: To begin, the number of moles of each gas and initial pressures are found...

$$\begin{aligned} n_{\text{CO}} &= \frac{4}{28.01} = 0.1428 \\ n_{\text{He}} &= \frac{0.8}{4.003} = 0.1999 \end{aligned} \quad \left\{ \begin{aligned} n &= 0.3427 \Rightarrow y_{\text{CO}} = \frac{0.1428}{0.3427} = 0.4167 \\ y_{\text{He}} &= \frac{0.1999}{0.3427} = 0.5833 \end{aligned} \right.$$

Using the ideal gas model equation of state the initial pressures are

$$\begin{aligned} P_{\text{CO}} &= \frac{n_{\text{CO}} \bar{R} T_{\text{CO}}}{V_{\text{CO}}} = \frac{(0.1428)(1545)(960)}{(20)(144)} = 73.542 \frac{\text{lb}}{\text{in}^2} \\ P_{\text{He}} &= \frac{n_{\text{He}} \bar{R} T_{\text{He}}}{V_{\text{He}}} = \frac{(0.1999)(1545)(520)}{(40)(144)} = 27.882 \frac{\text{lb}}{\text{in}^2} \end{aligned} \quad \left\{ \begin{aligned} \text{note:} \\ \text{these are the initial} \\ \text{pressures, not partial} \\ \text{pressures.} \end{aligned} \right.$$

(a) An energy balance reduces to $\Delta U = 0$, where ΔU is

$$\Delta U = \Delta U_{\text{CO}} + \Delta U_{\text{He}} = n_{\text{CO}} [\bar{u}_{\text{CO}}(T_f) - \bar{u}_{\text{CO}}(T_{\text{CO}})] + n_{\text{He}} \bar{c}_{v,\text{He}} [T_f - T_{\text{He}}] = 0$$

Since helium is monatomic, Table A-21 gives $\bar{c}_p = 5/2 \bar{R}$. So, with $\bar{c}_p = \bar{c}_v + \bar{R}$, we get $\bar{c}_{v,\text{He}} = 1.5 \bar{R}$. For CO, $\bar{u}$ is obtained from Table A-23. Rearranging

$$\begin{aligned} n_{\text{CO}} \bar{u}_{\text{CO}}(T_f) + n_{\text{He}} \bar{c}_{v,\text{He}} T_f &= n_{\text{CO}} \bar{u}_{\text{CO}}(T_{\text{CO}}) + n_{\text{He}} \bar{c}_{v,\text{He}} T_{\text{He}} \\ &= (0.1428)(4799.5) + (0.1999)(1.5 \times 1.986)(520) = 994.89 \end{aligned}$$

① Solving iteratively using table data, $T_f = 762^\circ\text{R}$ (302 °F) $\leftarrow T_f$

(b) Using the ideal gas equation of state applied to the final mixture

$$P_f = \frac{n \bar{R} T_f}{V} = \frac{(0.3427)(1545)(762)}{(60)(144)} = 46.696 \frac{\text{lb}}{\text{in}^2} \quad \leftarrow P_f$$

(c) The exergy destruction can be found using $E_d = T_0 \sigma$, where σ is the amount of entropy produced, obtained from an entropy balance: $\sigma = \Delta S$. Thus

$$E_d = T_0 \Delta S = T_0 [\Delta S_{\text{CO}} + \Delta S_{\text{He}}], \text{ where}$$

$$\Delta S_{\text{CO}} = n_{\text{CO}} \left[\bar{s}_{\text{CO}}^*(T_f) - \bar{s}_{\text{CO}}^*(T_{\text{CO}}) - \bar{R} \ln \frac{y_{\text{CO}} P_f}{P_{\text{CO}}} \right] = 0.1428 \left[49.715 - 51.353 - 1.986 \ln \frac{19.458}{73.542} \right] = 0.1432 \text{ Btu/}^\circ\text{R}$$

$$\Delta S_{\text{He}} = n_{\text{He}} \left[\bar{c}_{p,\text{He}} \ln \frac{T_f}{T_{\text{He}}} - \bar{R} \ln \frac{y_{\text{He}} P_f}{P_{\text{He}}} \right] = (0.1999) [1.986] \left(2.5 \ln \frac{762}{520} - \ln \frac{27.238}{27.882} \right) = 0.3885$$

$$\text{Thus, } E_d = (520^\circ\text{R}) [0.1432 + 0.3885] \frac{\text{Btu}}{^\circ\text{R}} = 276.5 \text{ Btu} \quad \leftarrow E_d$$

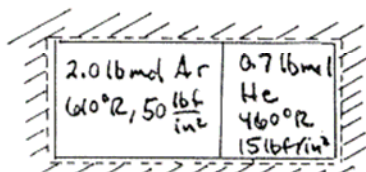
1. An iterative solution using table data can be avoided using IT. The results agree with those given here.

PROBLEM 12.33

KNOWN: Argon and helium, each initially at different temperatures and pressures are allowed to mix adiabatically to a final equilibrium state.

FIND: Determine the final temperature and pressure, and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system shown in the accompanying figure experiences no change in kinetic or potential energy, and $Q = 0$. (2) Each gas behaves as an ideal gas. The final mixture adheres to the Dalton model.

ANALYSIS: (a) An energy balance reduces as follows, $\Delta U = \cancel{Q} - \cancel{W}$. Thus

$$\Delta U = \Delta U_{Ar} + \Delta U_{He} = n_{Ar} \bar{c}_{v,Ar} [T_f - T_{Ar}] + n_{He} \bar{c}_{v,He} [T_f - T_{He}] = 0 \quad (1)$$

where T_{Ar} , T_{He} denote the initial temperatures of the argon and helium, respectively. The final temperature is denoted by T_f . From Table A-2E, for monatomic gases $\bar{c}_p = \frac{5}{2} \bar{R}$. Then, since $\bar{c}_p = \bar{c}_v + \bar{R}$ (Eq. 3.47), $\bar{c}_v = \frac{3}{2} \bar{R}$, and Eq. (1) reduces to give

$$T_f = \frac{n_{Ar} T_{Ar} + n_{He} T_{He}}{n_{Ar} + n_{He}} = \frac{(2.0)(610) + (0.7)(460)}{2.7} = 571.11 \text{ °R } (111 \text{ °F}) \leftarrow T_f$$

(b) The ideal gas equation of state gives $P_f = n \bar{R} T_f / V$, where V is the total volume:

$$V = \frac{n_{Ar} \bar{R} T_{Ar}}{P_{Ar}} + \frac{n_{He} \bar{R} T_{He}}{P_{He}}$$

where P_{Ar} , P_{He} denote the initial pressures of the argon and helium, and not partial pressures. Thus

$$P_f = \frac{(n_{Ar} + n_{He}) \bar{R} T_f}{\left[\frac{n_{Ar} \bar{R} T_{Ar}}{P_{Ar}} + \frac{n_{He} \bar{R} T_{He}}{P_{He}} \right]} = \frac{(2.7)(571.11)}{\left[\frac{(2.0)(610)}{(50)} + \frac{(0.7)(460)}{(15)} \right]} = 33.62 \frac{\text{lbf}}{\text{in}^2} = 2.29 \text{ atm} \leftarrow P_f$$

(c) An entropy balance reduces as follows

$$\Delta S = \int \frac{\delta Q}{T} + \sigma \Rightarrow \sigma = \Delta S = \Delta S_{Ar} + \Delta S_{He} \quad (2)$$

The argon undergoes a process from 610°R, 50 lbf/in² to 571.11°R and the partial pressure of argon in the final mixture: $y_{Ar} P_f = \left(\frac{2}{2.7} \right) (33.62) = 24.90 \text{ lbf/in}^2$. Thus, with $\bar{c}_p = \frac{5}{2} \bar{R}$

$$\Delta S_{Ar} = (2 \text{ lbmol}) \left(1.986 \frac{\text{Btu}}{\text{lbmol} \cdot \text{°R}} \right) \left[\frac{5}{2} \ln \frac{571.11}{610} - \ln \frac{24.90}{50} \right] = 2.115 \frac{\text{Btu}}{\text{°R}}$$

The helium undergoes a process from 460°R, 15 lbf/in² to 571.11°R and the partial pressure of helium in the final mixture: $y_{He} P_f = \left(\frac{0.7}{2.7} \right) (33.62) = 8.72 \text{ lbf/in}^2$. Thus

$$\Delta S_{He} = (0.7) \left(1.986 \right) \left[\frac{5}{2} \ln \frac{571.11}{460} - \ln \frac{8.72}{15} \right] = 1.506 \frac{\text{Btu}}{\text{°R}}$$

Inserting values into Eq. (2)

$$\sigma = 2.115 + 1.506 = 3.621 \frac{\text{Btu}}{\text{°R}} \leftarrow \sigma$$

PROBLEM 12.34

KNOWN: CO_2 and O_2 , each at initially different temperatures and pressures are allowed to mix adiabatically while a specified amount of energy is added by electrical work.

FIND: Determine the final temperature and pressure, the change in exergy and the exergy destruction.

SCHEMATIC & GIVEN DATA:

0.5 kmol CO_2 300 K, 2 bar	1.0 kmol O_2 425 K, 5 bar
---	--

$$W = -500 \text{ kJ}$$

$$T_0 = 293 \text{ K}$$

ENGINEERING

MODEL: (1) For the system shown in the accompanying figure $Q = 0$, and kinetic and potential energy effects are negligible. (2) Each gas behaves as an ideal gas and the final mixture adheres to the Dalton model.

ANALYSIS: (a) An energy balance reduces to the form $\Delta U = Q - W$, or

$$-W = \Delta U = \Delta U_{\text{CO}_2} + \Delta U_{\text{O}_2} = n_{\text{CO}_2} [\bar{u}_{\text{CO}_2}(T_f) - \bar{u}_{\text{CO}_2}(T_{\text{CO}_2})] + n_{\text{O}_2} [\bar{u}_{\text{O}_2}(T_f) - \bar{u}_{\text{O}_2}(T_{\text{O}_2})]$$

where T_f is the final temperature and $T_{\text{CO}_2}, T_{\text{O}_2}$ denote the initial temperatures of the CO_2 and O_2 , respectively. Solving for T_f

$$n_{\text{CO}_2} \bar{u}_{\text{CO}_2}(T_f) + n_{\text{O}_2} \bar{u}_{\text{O}_2}(T_f) = n_{\text{CO}_2} \bar{u}_{\text{CO}_2}(T_{\text{CO}_2}) + n_{\text{O}_2} \bar{u}_{\text{O}_2}(T_{\text{O}_2}) - W$$

Inserting known values and data from Tables A-24, A-27

$$0.5 \bar{u}_{\text{CO}_2}(T_f) + 1.0 \bar{u}_{\text{O}_2}(T_f) = 12902 \text{ kJ}$$

① Solving iteratively using table data $T_f = 387 \text{ K}$. ← T_f

(b) The ideal gas equation of state gives $P_f = n \bar{R} T_f / V$, where V is the total volume:

$$V = \frac{n_{\text{CO}_2} \bar{R} T_{\text{CO}_2}}{P_{\text{CO}_2}} + \frac{n_{\text{O}_2} \bar{R} T_{\text{O}_2}}{P_{\text{O}_2}}$$

Combining results, the final pressure is

$$P_f = \frac{(n_{\text{CO}_2} + n_{\text{O}_2}) T_f}{\left(\frac{n_{\text{CO}_2} T_{\text{CO}_2}}{P_{\text{CO}_2}} + \frac{n_{\text{O}_2} T_{\text{O}_2}}{P_{\text{O}_2}} \right)} = \frac{(1.5)(387)}{\left(\frac{(0.5)(300)}{2} + \frac{(1)(425)}{5} \right)} = 3.628 \text{ bar} \quad \leftarrow P_f$$

where $P_{\text{CO}_2}, P_{\text{O}_2}$ denote the initial pressures of the CO_2 and O_2 , respectively, and not partial pressures.

(c) The change in exergy is found from Eq. 7.10, which reduces to $\Delta E = \Delta U + P_0 \Delta V - T_0 \Delta S$. Since $W = -\Delta U$, $\Delta E = (-W) - T_0 \Delta S$, where

Continued on next slide

Problem 12-34 continued

$$\Delta S = \Delta S)_{CO_2} + \Delta S)_{O_2} = n_{CO_2} \left[\bar{s}_{CO_2}^0(T_f) - \bar{s}_{CO_2}^0(T_{O_2}) - \bar{R} \ln \frac{y_{CO_2} P_f}{P_{O_2}} \right] +$$

$$n_{O_2} \left[\bar{s}_{O_2}^0(T_f) - \bar{s}_{O_2}^0(T_{O_2}) - \bar{R} \ln \frac{y_{O_2} P_f}{P_{O_2}} \right]$$

with data from Table A-23

$$\Delta S = (0.5) \left[223.864 - 213.915 - 8.314 \ln \left(\frac{(1/3)(3.628)}{2} \right) \right] +$$

$$(1.0) \left[212.768 - 215.598 - 8.314 \ln \left(\frac{(2/3)(3.628)}{5} \right) \right] = 10.274 \frac{kJ}{K}$$

Then

$$\Delta E = 500 - 293(10.274) = -2510.3 \text{ kJ}$$

← ΔE

(d) The exergy destruction can be found using $E_d = T_0 \sigma$, where σ is obtained from an entropy balance. Thus

$$E_d = T_0 \Delta S = (293 \text{ K})(10.274 \frac{kJ}{K}) = 3010.3 \text{ kJ}$$

← E_d

Alternatively, an exergy balance can be used:

$$\Delta E = \int_1^2 \left[1 - \frac{T_0}{T_b} \right] \delta Q - [W - P_0 V] - E_d \Rightarrow E_d = (-W) - \Delta E$$

or

$E_d = 500 \text{ kJ} - (-2510.3) = 3010.3 \text{ kJ}$. The exergy destruction occurs because different gases at different initial states are allowed to mix.

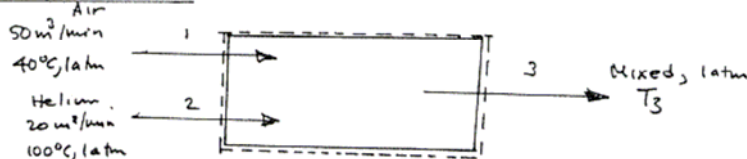
1. An iterative solution using table data can be avoided by using IT. The results agree with those given here.

PROBLEM 12.35

KNOWN: $50 \text{ m}^3/\text{min}$ of air at 400°C , 1 atm is mixed adiabatically and at steady state with $20 \text{ m}^3/\text{min}$ of helium at 1000°C , 1 atm to form a mixed stream at 1 atm .

FIND: Determine (a) the temperature of the exiting mixture, (b) the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the figure is at steady state.

(2) For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and effects of kinetic/potential energy can be ignored. (3) The entering gases each can be modeled as an ideal gas. The exiting mixture adheres to the Dalton model. (4) The specific heat $c_{p, \text{air}}$ is taken as a constant at 350 K .

ANALYSIS: (a) Mass rate balances at steady state indicate that

$$\dot{m}_a = \dot{m}_a \quad (\equiv \dot{m}_a)$$

$$\dot{m}_h = \dot{m}_h \quad (\equiv \dot{m}_h)$$

An energy rate balance reduces at steady state to read

$$0 = [\dot{m}_a h_a(T_1) + \dot{m}_h h_h(T_2)] - [\dot{m}_a h_a(T_3) + \dot{m}_h h_h(T_3)]$$

or

$$0 = \dot{m}_a [h_a(T_3) - h_a(T_1)] + \dot{m}_h [h_h(T_3) - h_h(T_2)] \quad (1)$$

For Helium Table A-21 indicates that $c_{p,h} = 5/2 R$. Since the final temperature will fall between T_1 and T_2 , and the difference is relatively narrow, c_p for air is also taken as constant. The value at 350 K is used: $c_{p, \text{air}} = 1.008 \text{ kJ/kg} \cdot \text{K}$ (Table A-2a). Thus, Eq. (1) gives

$$0 = \dot{m}_a c_{p, \text{air}} (T_3 - T_1) + \dot{m}_h c_{p,h} (T_3 - T_2) = 0 \quad (2)$$

The mass flow rates are found from the volumetric flow rates

$$\dot{m}_a = \frac{(AV)_1 P_1}{RT_1} = \frac{(50 \text{ m}^3/\text{min})(1.013 \times 10^5 \text{ N/m}^2)}{(8314 \text{ N} \cdot \text{m}/\text{kg} \cdot \text{K})(313 \text{ K})} \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 0.9398 \frac{\text{kg}}{\text{s}}$$

$$\dot{m}_h = \frac{(20)(1.013 \times 10^5)}{(8314)(373)(60)} = 0.0436 \frac{\text{kg}}{\text{s}}$$

Solving Eq. (2)

$$0 = (0.9398 \frac{\text{kg}}{\text{s}})(1.008 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(T_3 - 313) + (0.0436 \times \frac{5}{2} \times \frac{8.314}{4})(T_3 - 573) \Rightarrow T_3 = 324.6 \text{ K} \quad (52^\circ\text{C})$$

(b) Reducing an entropy rate balance

$$0 = \dot{m}_a s_a(T_1, P) + \dot{m}_h s_h(T_2, P) - [\dot{m}_a s_a(T_3, y_a P) + \dot{m}_h s_h(T_3, y_h P)] + \dot{\sigma}$$

Rearranging

$$\dot{\sigma} = \dot{m}_a \left[c_{p, \text{air}} \ln \frac{T_3}{T_1} - \frac{\bar{R}}{M_{\text{air}}} \ln \frac{y_a P}{P} \right] + \dot{m}_h \left[c_{p,h} \ln \frac{T_3}{T_2} - \frac{\bar{R}}{M_h} \ln \frac{y_h P}{P} \right]$$

Continued on next slide

Problem 12-35 continued

The mole fractions are evaluated as follows:

$$\dot{n}_a = \frac{\dot{m}_a}{M_{air}} = \frac{0.9398}{28.97} = 0.0324 \frac{\text{kmol}}{\text{s}} > \dot{n} = 0.0433$$

$$\dot{n}_h = \frac{\dot{m}_h}{M_h} = \frac{0.0436}{4} = 0.0109 \frac{\text{kmol}}{\text{s}}$$

So $y_a = \frac{\dot{n}_a}{\dot{n}} = \frac{0.0324}{0.0433} = 0.748$, $y_h = \frac{\dot{n}_h}{\dot{n}} = \frac{0.0109}{0.0433} = 0.252$

Solving for $\dot{Q}$

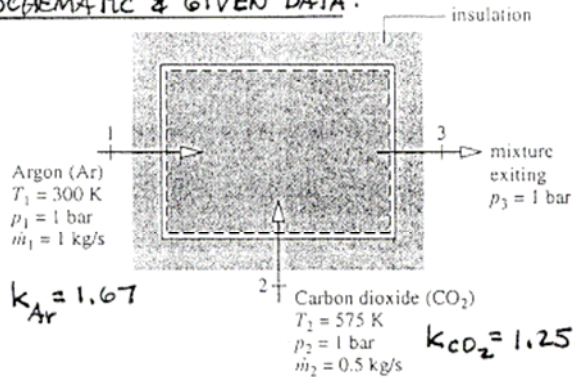
$$\begin{aligned} \dot{Q} &= (0.9398 \frac{\text{kg}}{\text{s}}) \left[(1.008 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \ln \frac{324.6}{313} - \frac{8.314}{2897} \ln 0.748 \right] \left| \frac{\text{kJ}}{\text{kg} \cdot \text{s}} \right| + \\ &\quad (0.0436) \left[(5.196) \ln \frac{324.6}{273} - \frac{8.314}{4} \ln 0.252 \right] \frac{\text{kJ}}{\text{K}} \\ &= (0.9398)(0.12) + (0.0436)(2.1428) = 0.2062 \frac{\text{kJ}}{\text{s}} \end{aligned}$$

PROBLEM 12.36

KNOWN: Argon and carbon dioxide enter an insulated mixing chamber operating at steady state. Data are known at the inlets and exit.

FIND: Determine (a) the molar analysis of the exiting mixture, (2) the exit temperature, and (c) the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The ideal gas model applies, and the specific heats are constant.

Using Eq. 3.47a

$$c_{p,Ar} = \frac{kR}{k-1} = \frac{(1.67)(8.314)}{(0.67)} = 0.5189 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$c_{p,CO_2} = \frac{(1.25)(8.314)}{(0.25)} = 0.9446 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

ANALYSIS: Based on the entering mass flow rates, the gravimetric analysis of the exiting mixture is

$$m_{f,Ar} = \frac{\dot{m}_1}{\dot{m}_3} = \frac{1}{1.5} = 0.6667, \quad m_{f,CO_2} = \frac{\dot{m}_2}{\dot{m}_3} = \frac{0.5}{1.5} = 0.3333$$

Following the method of Example 12.2 for 1 kg/s of mixture

Component	$m_{f,i} \div M_i = n_i$	y_i	
Ar	$0.6667 \div 39.94 = 0.01669$	0.6880	} molar analysis
CO ₂	$0.3333 \div 44.01 = 0.00757$	0.3120	
	0.02426	1.0000	

(b) The mass and energy rate balances reduce to

$$0 = \dot{m}_1 h_{Ar,1} + \dot{m}_2 h_{CO_2,2} - \dot{m}_3 h_{mix}$$

With $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ and $h_{mix} = m_{f,Ar} h_{Ar,3} + m_{f,CO_2} h_{CO_2,3}$ we get

$$0 = \dot{m}_1 h_{Ar,1} + \dot{m}_2 h_{Ar,2} - \underbrace{\dot{m}_3 m_{f,Ar}}_{=\dot{m}_1} h_{Ar,3} - \underbrace{\dot{m}_3 m_{f,CO_2}}_{=\dot{m}_2} h_{CO_2,3}$$

$$0 = \dot{m}_1 (h_{Ar,1} - h_{Ar,3}) + \dot{m}_2 (h_{CO_2,2} - h_{CO_2,3})$$

$$\quad \quad \quad c_{p,Ar}(T_1 - T_3) \quad \quad \quad c_{p,CO_2}(T_2 - T_3)$$

Solving T_3 and inserting values

$$T_3 = \frac{\dot{m}_1 c_{p,Ar} T_1 + \dot{m}_2 c_{p,CO_2} T_2}{\dot{m}_1 c_{p,Ar} + \dot{m}_2 c_{p,CO_2}}$$

$$= \frac{(1)(0.5189)(300) + (0.5)(0.9446)(575)}{(1)(0.5189) + (0.5)(0.9446)} = 431 \text{ K} \leftarrow T_3$$

Continued on next slide

Problem 12-36 continued

(c) Reducing the entropy balance

$$\begin{aligned}
 0 &= \cancel{\sum \left(\frac{\dot{Q}}{T} \right)} + \dot{m}_1 s_{Ar,1} + \dot{m}_2 s_{CO_2,2} - \dot{m}_{fAr} \dot{m}_3 s_{Ar,3} - \dot{m}_{fCO_2} \dot{m}_3 s_{CO_2,3} + \dot{\sigma}_{cv} \\
 &= \dot{m}_1 (s_{Ar,1} - s_{Ar,3}) + \dot{m}_2 (s_{CO_2,2} - s_{CO_2,3}) + \dot{\sigma}_{cv} \\
 &= \dot{m}_1 \left(c_{pAr} \ln \frac{T_1}{T_3} - R_{Ar} \ln \frac{P_1}{P_{Ar,3}} \right) + \dot{m}_2 \left(c_{pCO_2} \ln \frac{T_2}{T_3} - R_{CO_2} \ln \frac{P_2}{P_{CO_2,3}} \right)
 \end{aligned}$$

Solving for $\dot{\sigma}_{cv}$, and introducing $P_{Ar,3} = y_{Ar} P_3$, $P_{CO_2,3} = y_{CO_2} P_3$

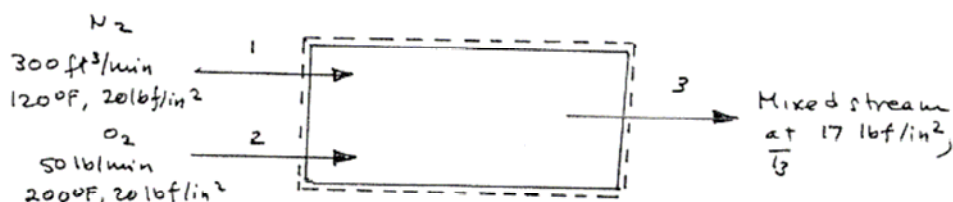
$$\begin{aligned}
 \dot{\sigma}_{cv} &= \dot{m}_1 \left[c_{pAr} \ln \frac{T_3}{T_1} - R_{Ar} \ln y_{Ar} \right] + \dot{m}_2 \left[c_{pCO_2} \ln \frac{T_3}{T_2} - R_{CO_2} \ln y_{CO_2} \right] \\
 &= (1 \text{ kg/s}) \left[(0.3189 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \ln \left(\frac{431}{300} \right) - \frac{8.314}{39.94} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln(0.6888) \right] \\
 &\quad + (0.5) \left[(0.9446) \ln \left(\frac{431}{575} \right) - \frac{8.314}{44.01} \ln(0.3120) \right] \\
 &= 0.2397 \text{ kJ/s} \cdot \text{K} \quad \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 0.2397 \frac{\text{kW}}{\text{K}} \longleftarrow \dot{\sigma}_{cv}
 \end{aligned}$$

PROBLEM 12.37

KNOWN: $300 \text{ ft}^3/\text{min}$ of N_2 at 120°F , 20 lbf/in^2 is mixed adiabatically at steady state with 50 lb/min of O_2 at 200°F , 20 lbf/in^2 to form a mixed stream at 17 lbf/in^2 .

FIND: Determine (a) the temperature of the exiting mixture and (b) the rate of exergy destruction.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

- (1) The control volume shown in the figure is at steady state.
- (2) For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$ and all kinetic/potential energy effects can be ignored.
- (3) The entering gases can each be modeled as an ideal gas. The exiting mixture adheres to the Dalton model.
- (4) The specific heats c_p can be taken as constants.
- (5) For the exergy reference environment, $T_0 = 5000 \text{ R}$.

ANALYSIS: (a) At steady state $\dot{m}_{\text{N}_2,1} = \dot{m}_{\text{N}_2,3}$, $\dot{m}_{\text{O}_2,2} = \dot{m}_{\text{O}_2,3}$. The energy rate balance reduces to read

$$0 = \dot{m}_{\text{N}_2} h_{\text{N}_2}(T_1) + \dot{m}_{\text{O}_2} h_{\text{O}_2}(T_2) - [\dot{m}_{\text{N}_2} h_{\text{N}_2}(T_3) + \dot{m}_{\text{O}_2} h_{\text{O}_2}(T_3)]$$

or on rearrangement and with assumption (4)

$$0 = \dot{m}_{\text{N}_2} c_{p,\text{N}_2} [T_3 - T_1] + \dot{m}_{\text{O}_2} c_{p,\text{O}_2} [T_3 - T_2] \quad (1)$$

As T_3 will fall between T_1 and T_2 , and the temperature difference narrow, c_p for each gas is taken as constant at the average temperature of 160°F . With data from Table A-20E, $c_{p,\text{N}_2} = 0.249 \text{ Btu/lb} \cdot ^\circ\text{R}$, $c_{p,\text{O}_2} = 0.222 \text{ Btu/lb} \cdot ^\circ\text{R}$. The mass flow rate of N_2 is found as follows:

$$\dot{m}_{\text{N}_2} = \frac{(AV)_1 P_1}{RT_1} = \frac{(300 \text{ ft}^3/\text{min})(20 \times 144 \text{ lbf/ft}^2)}{\left(\frac{1545}{28.01} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}}\right)(580^\circ\text{R})} = 27 \text{ lb/min}$$

Substituting values in Eq. (1) and solving

$$0 = (27 \frac{\text{lb}}{\text{min}})(0.249 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}})[T_3 - 580]^\circ\text{R} + (50)(0.222)(T_3 - 660) \Rightarrow T_3 = 630 \text{ K} \quad (170^\circ\text{F})$$

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Problem 12-37 continued

(b) An entropy rate balance reads

$$0 = \dot{m}_{N_2} s_{N_2}(T_1, P_1) + \dot{m}_{O_2} s_{O_2}(T_2, P_2) - [\dot{m}_{N_2} s_{N_2}(T_3, y_{N_2} P_3) + \dot{m}_{O_2} s_{O_2}(T_3, y_{O_2} P_3)] + \dot{\sigma}$$

or

$$\begin{aligned} \dot{\sigma} &= \dot{m}_{N_2} [s_{N_2}(T_3, y_{N_2} P_3) - s_{N_2}(T_1, P_1)] + \dot{m}_{O_2} [s_{O_2}(T_3, y_{O_2} P_3) - s_{O_2}(T_2, P_2)] \\ &= \dot{m}_{N_2} \left[c_{p, N_2} \ln \frac{T_3}{T_1} - \frac{R}{M_{N_2}} \ln \frac{y_{N_2} P_3}{P_1} \right] + \dot{m}_{O_2} \left[c_{p, O_2} \ln \frac{T_3}{T_2} - \frac{R}{M_{O_2}} \ln \frac{y_{O_2} P_3}{P_2} \right] \end{aligned}$$

To obtain the mole fractions

$$\begin{aligned} \dot{n}_{N_2} &= \frac{\dot{m}_{N_2}}{M_{N_2}} = \frac{27}{28.01} = 0.9639 \\ \dot{n}_{O_2} &= \frac{\dot{m}_{O_2}}{M_{O_2}} = \frac{50}{32} = 1.5625 \end{aligned} \quad \left. \vphantom{\begin{aligned} \dot{n}_{N_2} \\ \dot{n}_{O_2} \end{aligned}} \right\} \dot{n} = 2.5264$$

$$y_{N_2} = \frac{0.9639}{2.5264} = 0.3815, \quad y_{O_2} = \frac{1.5625}{2.5264} = 0.6185$$

Thus

$$\begin{aligned} \dot{\sigma} &= (27 \frac{\text{lb}}{\text{min}}) \left[0.249 \frac{\text{Btu}}{\text{lb} \cdot \text{R}} \ln \frac{630}{580} - \frac{1.986}{28.01} \ln \left(\frac{(0.3815)(17)}{20} \right) \right] + \\ &\quad (50) \left[0.222 \ln \frac{630}{660} - \frac{1.986}{32} \ln \left(\frac{(0.6185)(17)}{20} \right) \right] \\ &= (2.712 + 1.480) \frac{\text{Btu}}{\text{min} \cdot \text{R}} = 4.192 \frac{\text{Btu}}{\text{min} \cdot \text{R}} \end{aligned}$$

The rate of exergy destruction can be found as follows

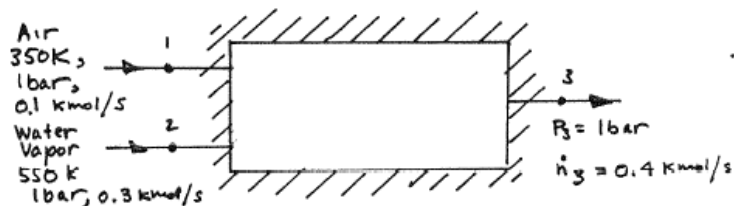
$$\dot{E}_d = T_0 \dot{\sigma} = (500^\circ \text{R}) (4.192 \frac{\text{Btu}}{\text{min} \cdot \text{R}}) = 2096 \frac{\text{Btu}}{\text{min}} \quad \leftarrow \dot{E}_d$$

PROBLEM 12.38

KNOWN: Air at a specified temperature and pressure enters an insulated chamber at steady state and mixes with water vapor entering at a specified temperature and pressure. The mixture exits at a known pressure and molar analysis.

FIND: Determine the temperature of the exiting mixture and the rate entropy is produced.

SCHEMATIC & GIVEN DATA:



At 3, based on the given molar flow rates

i	y _i
air	0.25
H ₂ O	0.75

ENGINEERING MODEL: (1) The chamber is well insulated and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) Each gas can be modeled as an ideal gas and the exiting mixture adheres to the Dalton model.

ANALYSIS: An energy rate balance at steady state, expressed on a molar basis, reduces with assumptions 1 and 2 to give

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{n}_1 \bar{h}_1 + \dot{n}_2 \bar{h}_2 - \dot{n}_3 \bar{h}_3 \Rightarrow \bar{h}_3 = \left(\frac{\dot{n}_1}{\dot{n}_3} \right) \bar{h}_1 + \left(\frac{\dot{n}_2}{\dot{n}_3} \right) \bar{h}_2$$

where $\dot{n}_1$, $\dot{n}_2$, and $\dot{n}_3$ are the molar flow rates of the air, water vapor, and mixture, respectively. Since $\dot{n}_1/\dot{n}_3 = 0.25$ and $\dot{n}_2/\dot{n}_3 = 0.75$, this becomes

$$\bar{h}_3 = 0.25 \bar{h}_1 + 0.75 \bar{h}_2 = 0.25 \bar{h}_{\text{air}}(T_1) + 0.75 \bar{h}_{\text{H}_2\text{O}}(T_2)$$

The enthalpy of the mixture, per kmol of mixture, is $\bar{h}_3 = 0.25 \bar{h}_{\text{air}}(T_3) + 0.75 \bar{h}_{\text{H}_2\text{O}}(T_3)$.

① Accordingly, with data from Table A-23,

$$0.25 [28.97 \bar{h}_{\text{air}}(T_3)] + 0.75 \bar{h}_{\text{H}_2\text{O}} = (0.25)(28.97)(350.49) + (0.75)(18601) = 16489 \text{ kJ}$$

where $\bar{h}_{\text{air}} = M_{\text{air}} h_{\text{air}}$.

An iterative solution using data from Table A-23 gives

② $T_3 = 507 \text{ K} (234^\circ\text{C})$.

T_3

(b) An entropy rate balance at steady state, expressed on a molar basis, reads

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{n}_1 \bar{s}_1 + \dot{n}_2 \bar{s}_2 - \dot{n}_3 \bar{s}_3 + \dot{\sigma}_{cv}$$

Thus

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_3} = \bar{s}_3 - \left(\frac{\dot{n}_1}{\dot{n}_3} \bar{s}_1 + \frac{\dot{n}_2}{\dot{n}_3} \bar{s}_2 \right)$$

The specific entropy of the mixture is

$$\bar{s}_3 = 0.25 \bar{s}_{\text{air}}(T_3, y_{\text{air}} P_3) + 0.75 \bar{s}_{\text{H}_2\text{O}}(T_3, y_{\text{H}_2\text{O}} P_3)$$

Continued on next slide

Problem 12-38 continued

Combining the last two equations and using s° data from Table A-23

$$\begin{aligned}\frac{\dot{Q}_{cv}}{\dot{n}_3} &= 0.25 \left[\bar{s}_{Air}(T_3, y_{Air} P_3) - \bar{s}_{Air}(T_1, P_1) \right] + 0.75 \left[\bar{s}_{H_2O}(T_3, y_{H_2O} P_3) - \bar{s}_{H_2O}(T_1, P_1) \right] \\ &= (0.25)(28.97) \left[\bar{s}_{Air}^\circ(T_3) - \bar{s}_{Air}^\circ(T_1) - \frac{\bar{R}}{M_{Air}} \ln \frac{y_{Air} P_3}{P_1} \right] + \\ &\quad 0.75 \left[\bar{s}_{H_2O}^\circ(T_3) - \bar{s}_{H_2O}^\circ(T_1) - \bar{R} \ln \frac{y_{H_2O} P_3}{P_2} \right]\end{aligned}$$

Continuing the calculation

$$\begin{aligned}\frac{\dot{Q}_{cv}}{\dot{n}_3} &= (0.25)(28.97) \left[2.2338 - 1.85708 - \frac{8.314}{28.97} \ln \frac{(0.25)(1)}{(1)} \right] + \\ &\quad 0.75 \left[206.902 - 209.795 - 8.314 \ln \frac{(0.75)(1)}{(1)} \right] \\ &= 5.6098 + (-0.3759) = 5.2339 \frac{KJ}{Kmol \cdot K}\end{aligned}$$

or

$$\dot{Q}_{cv} = (0.4 \frac{Kmol}{s}) \left(5.2339 \frac{KJ}{Kmol \cdot K} \right) \left| \frac{1 KW}{1 KJ/s} \right| = 2.094 \frac{KW}{K} \quad \leftarrow \dot{Q}_{cv}$$

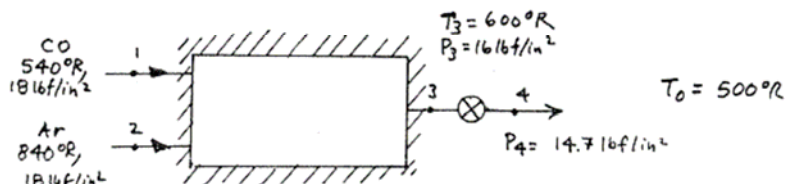
1. Table A-23 can be used here for water vapor because water vapor is modeled as an ideal gas at each of the three states.
2. An iterative solution using table data can be avoided in this case by using constant c_p for dry air and water vapor evaluated at the average of the inlet temperatures: 450 K. Alternatively, IT could be used.

PROBLEM 12.39

KNOWN: Streams of CO and Ar, each at known states, form a mixture that expands across a valve to a specified pressure.

FIND: Determine (a) the mass and molar analysis of the mixture, (b) the temperature at the valve exit, (c) the rates of exergy destruction for the mixing chamber and the valve, each per unit mass of mixture.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

- (1) The mixing chamber is well insulated and at steady state.
- (2) Kinetic and potential energy effects can be ignored.
- (3) The expansion across the valve is a throttling process.
- (4) Each gas can be modeled as an ideal gas and the mixture adheres to the Dalton model.
- (5) $T_0 = 500^\circ\text{R}$.

ANALYSIS: (a) Reducing mass and energy rate balances for a control volume enclosing the mixing chamber, we get $0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$, or expressed alternatively

$$0 = \dot{m}_{\text{CO}} h_{\text{CO}}(540^\circ\text{R}) + \dot{m}_{\text{Ar}} h_{\text{Ar}}(840^\circ\text{R}) - [\dot{m}_{\text{CO}} h_{\text{CO}}(600^\circ\text{R}) + \dot{m}_{\text{Ar}} h_{\text{Ar}}(600^\circ\text{R})]$$

$$\Rightarrow \dot{m}_{\text{CO}} \left[\frac{\bar{h}_{\text{CO}}(600^\circ\text{R}) - \bar{h}_{\text{CO}}(540^\circ\text{R})}{M_{\text{CO}}} \right] = \dot{m}_{\text{Ar}} \left[\frac{\bar{h}_{\text{Ar}}(840) - \bar{h}_{\text{Ar}}(600)}{M_{\text{Ar}}} \right]$$

For Ar from Table A-21, $\bar{C}_p = 2.5 \bar{R}$. Data for CO is obtained from Table A-23. Thus

$$\dot{m}_{\text{CO}} \left[\frac{4168 - 3750.3}{28.01} \right] = \dot{m}_{\text{Ar}} \left[\frac{(2.5)(1.986)[240]}{39.94} \right] \Rightarrow \frac{\dot{m}_{\text{CO}}}{\dot{m}_{\text{Ar}}} = 2$$

Accordingly, $(m_f)_{\text{CO}} = 2/3$, $(m_f)_{\text{Ar}} = 1/3$

Considering a typical unit mass of mixture

$$n_{\text{CO}} = \frac{2/3 \text{ lb}}{28.01 \text{ lb/lbmol}} = 0.0238, \quad n_{\text{Ar}} = \frac{1/3}{39.94} = 0.0083 \Rightarrow \begin{aligned} y_{\text{CO}} &= \frac{0.0238}{0.0238 + 0.0083} = 0.741 \\ y_{\text{Ar}} &= 0.259 \end{aligned}$$

(b) Since the expansion across the valve is a throttling process, $h_4 = h_3$. And since the enthalpy of an ideal gas depends only on temperature, $T_4 = T_3 = 600^\circ\text{R}$.

(c) The rate of exergy destruction can be found using $\dot{E}_d = T_0 \dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production. For a control volume enclosing the mixing chamber

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m}_1 s_{\text{CO}}(T_1, P_1) + \dot{m}_2 s_{\text{Ar}}(T_2, P_2) - \dot{m}_3 s_3 + \dot{\sigma}_{\text{cv}}$$

Thus, with $\dot{Q}_j = 0$ from Table A-23

$$\begin{aligned} \frac{\dot{\sigma}_{\text{cv}}}{\dot{m}_3} &= s_3 - \left(\frac{\dot{m}_1}{\dot{m}_3} \right) s_{\text{CO}}(T_1, P_1) - \left(\frac{\dot{m}_2}{\dot{m}_3} \right) s_{\text{Ar}}(T_2, P_2) \\ &= [(m_f)_{\text{CO}} s_{\text{CO}}(T_3, y_{\text{CO}} P_3) + (m_f)_{\text{Ar}} s_{\text{Ar}}(T_3, y_{\text{Ar}} P_3)] - \left(\frac{\dot{m}_1}{\dot{m}_3} \right) s_{\text{CO}}(T_1, P_1) - \left(\frac{\dot{m}_2}{\dot{m}_3} \right) s_{\text{Ar}}(T_2, P_2) \\ &= \frac{2}{3} [s_{\text{CO}}(T_3, y_{\text{CO}} P_3) - s_{\text{CO}}(T_1, P_1)] + \frac{1}{3} [s_{\text{Ar}}(T_3, y_{\text{Ar}} P_3) - s_{\text{Ar}}(T_2, P_2)] \end{aligned}$$

Continued on next slide

Problem 12-39 continued

That is

$$\begin{aligned}\frac{\dot{Q}_{cv}}{m_3} &= \frac{2}{3} \left[\frac{\bar{s}_{co}^o(T_3) - \bar{s}_{co}^o(T_1) - \bar{R} \ln \frac{y_{co} P_3}{P_1}}{M_{co}} \right] + \frac{1}{3} \left[\frac{\bar{c}_{p,Ar} \ln \frac{T_3}{T_2} - \bar{R} \ln \frac{y_{Ar} P_3}{P_2}}{M_{Ar}} \right] \\ &= \frac{2}{3} \left[\frac{48.044 - 47.310 - 1.986 \ln \left(\frac{0.781 \times 16}{18} \right)}{28.01} \right] + \frac{1}{3} \left[\frac{(1.986) \left[2.5 \ln \frac{600}{840} - \ln \left(\frac{0.259 \times 16}{18} \right) \right]}{39.94} \right] \\ &= 0.0372 + 0.0104 = 0.0476 \frac{\text{Btu/or}}{\text{lb(mix)}}$$

Thus, for the mixing chamber

$$(\dot{E}_d)_{mix} = T_0 \frac{\dot{Q}_{cv}}{m_3} = (500^\circ\text{R})(0.0476 \frac{\text{Btu/or}}{\text{lb(mix)}}) = 23.8 \frac{\text{Btu}}{\text{lb mix}} \leftarrow$$

For a control volume enclosing the valve, an entropy rate balance at steady state reduces to

$$\begin{aligned}\frac{\dot{Q}_{cv}}{m_3} &= S_4 - S_3 \\ &= \left[(mf)_{co} \bar{s}_{co}(T_4, y_{co} P_4) + (mf)_{Ar} \bar{s}_{Ar}(T_4, y_{Ar} P_4) \right] \\ &\quad - \left[(mf)_{co} \bar{s}_{co}(T_3, y_{co} P_3) + (mf)_{Ar} \bar{s}_{Ar}(T_3, y_{Ar} P_3) \right] \\ &= (mf)_{co} \left[\frac{\bar{s}_{co}^o(T_4) - \bar{s}_{co}^o(T_3) - \bar{R} \ln \frac{y_{co} P_4}{y_{co} P_3}}{M_{co}} \right] + (mf)_{Ar} \left[\frac{\bar{c}_{p,Ar} \ln \frac{T_4}{T_3} - \bar{R} \ln \frac{y_{Ar} P_4}{y_{Ar} P_3}}{M_{Ar}} \right]\end{aligned}$$

Since $T_3 = T_4$, the terms involving temperature vanish, leaving

$$\begin{aligned}\frac{\dot{Q}_{cv}}{m_3} &= -\bar{R} \left[\frac{(mf)_{co}}{M_{co}} + \frac{(mf)_{Ar}}{M_{Ar}} \right] \ln \frac{P_4}{P_3} \\ &= -1.986 \left[0.0238 + 0.0083 \right] \ln \frac{14.7}{16} = 0.0054 \frac{\text{Btu/or}}{\text{lb(mix)}}$$

Then

$$\textcircled{1} \quad (\dot{E}_d)_{valve} = (500)(0.0054) = 2.7 \frac{\text{Btu}}{\text{lb mix}} \leftarrow$$

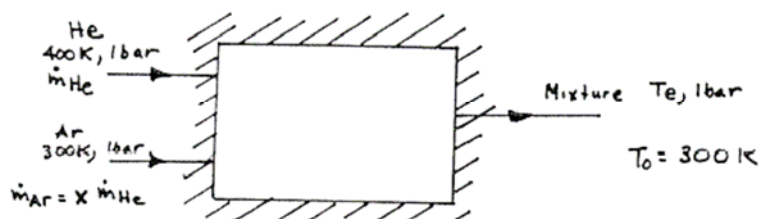
-
1. Significant exergy destruction accompanies the mixing of different substances initially at different states and the expansion of substances across valves.

PROBLEM 12.40

KNOWN: Helium at 400 K, 1 bar enters an insulated mixing chamber where it mixes with argon entering at 300 K, 1 bar. The mixture exits at a pressure of 1 bar. The argon mass flow rate is x times that of helium.

FIND: Plot versus x the exit temperature and the rate of energy destruction per unit mass of He entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The mixing chamber is well insulated and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) Each gas can be modeled as an ideal gas and the mixture adheres to the Dalton model.

ANALYSIS: At steady state the mass flow rate of the mixture exiting equals the sum of the incoming flow rates:

$$\dot{m}_{\text{mix}} = \dot{m}_{\text{He}} + \dot{m}_{\text{Ar}} = (1+x)\dot{m}_{\text{He}} \quad (1)$$

The energy rate balance reduces at steady state to give

$$0 = \cancel{\dot{Q}_{\text{cv}}} - \cancel{\dot{W}_{\text{cv}}} + \dot{m}_{\text{He}} h_{\text{He}}(400\text{K}) + \dot{m}_{\text{Ar}} h_{\text{Ar}}(300\text{K}) - [\dot{m}_{\text{He}} h_{\text{He}}(T_e) + \dot{m}_{\text{Ar}} h_{\text{Ar}}(T_e)]$$

Thus

$$\dot{m}_{\text{He}} [h_{\text{He}}(T_e) - h_{\text{He}}(400\text{K})] + \dot{m}_{\text{Ar}} [h_{\text{Ar}}(T_e) - h_{\text{Ar}}(300\text{K})] = 0$$

For the monatomic gases He and Ar, $c_p = 5/2 \bar{R}/M$ (Table A-21). Accordingly,

$$\dot{m}_{\text{He}} \left[\frac{5}{2} \frac{\bar{R}}{M_{\text{He}}} (T_e - 400) \right] + x \dot{m}_{\text{He}} \left[\frac{5}{2} \frac{\bar{R}}{M_{\text{Ar}}} (T_e - 300) \right] = 0$$

or

$$\frac{T_e - 400}{M_{\text{He}}} + \frac{x}{M_{\text{Ar}}} [T_e - 300] = 0$$

giving

$$T_e = \frac{\frac{400}{M_{\text{He}}} + \frac{300x}{M_{\text{Ar}}}}{\frac{1}{M_{\text{He}}} + \frac{x}{M_{\text{Ar}}}} = \frac{400\left(\frac{M_{\text{Ar}}}{M_{\text{He}}}\right) + 300x}{\left(\frac{M_{\text{Ar}}}{M_{\text{He}}}\right) + x} \quad (2)$$

where $M_{\text{Ar}} = 39.94$, $M_{\text{He}} = 4.003$:

The entropy rate balance reduces at steady state to give

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}_{\text{He}} s_{\text{He}}(400\text{K}, 1\text{bar}) + \dot{m}_{\text{Ar}} s_{\text{Ar}}(300\text{K}, 1\text{bar}) - [\dot{m}_{\text{He}} s_{\text{He}}(T_e, y_{\text{He}} \cdot 1\text{bar}) + \dot{m}_{\text{Ar}} s_{\text{Ar}}(T_e, y_{\text{Ar}} \cdot 1\text{bar})] + \dot{Q}_{\text{cv}}$$

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Problem 12-40 continued

Since $y_{Ar} + y_{He} = 1$, $y_{He} = 1 - y_{Ar}$, and $\dot{E}_d / \dot{m}_{He} = T_o \dot{\sigma}_w / \dot{m}_{He}$. Thus

$$\frac{\dot{E}_d}{\dot{m}_{He}} = T_o \left\{ \frac{\bar{R}}{M_{He}} \left[\frac{5}{2} \ln \frac{T_e}{400} - \ln(1 - y_{Ar}) \right] + x \frac{\bar{R}}{M_{Ar}} \left[\frac{5}{2} \ln \frac{T_e}{300} - \ln y_{Ar} \right] \right\} \quad (3)$$

To find y_{Ar} , notice that the mass fractions of He and Ar in the exiting mixture are, respectively

$$(mf)_{He} = \frac{\dot{m}_{He}}{\dot{m}_{mix}} = \frac{\dot{m}_{He}}{(1+x)\dot{m}_{He}} = \frac{1}{1+x}$$

$$(mf)_{Ar} = \frac{\dot{m}_{Ar}}{\dot{m}_{mix}} = \frac{x \dot{m}_{He}}{(1+x)\dot{m}_{He}} = \frac{x}{1+x}$$

The number of moles of He and Ar in a typical unit mass of mixture is

$$n_{He} = \frac{1/(1+x)}{M_{He}}, \quad n_{Ar} = \frac{x/(1+x)}{M_{Ar}}$$

Thus

$$y_{Ar} = \frac{n_{Ar}}{n_{He} + n_{Ar}} = \frac{\left(\frac{x}{(1+x) M_{Ar}} \right)}{\left(\frac{1}{(1+x) M_{He}} + \frac{x}{(1+x) M_{Ar}} \right)} = \frac{x}{\left(\frac{M_{Ar}}{M_{He}} + x \right)} \quad (4)$$

The data for the required plots are obtained using IT, as follows:

IT Code

$\dot{m}_{He} = 1$
 $M_{He} = 4.003 \text{ // kg/kmol}$
 $M_{Ar} = 39.94 \text{ // kg/kmol}$
 $T_o = 300 \text{ // K}$
 $Rbar = 8.314 \text{ // kJ/kmol}\cdot\text{K}$
 $x = 1$

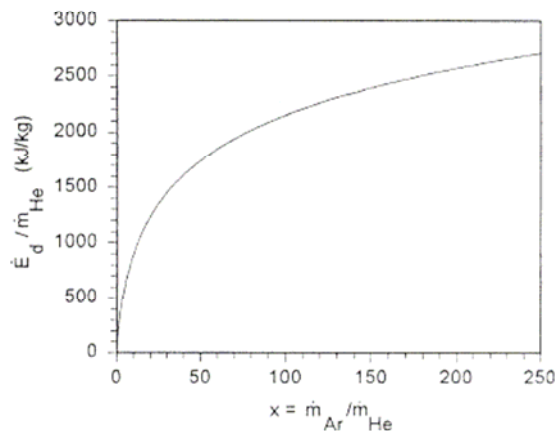
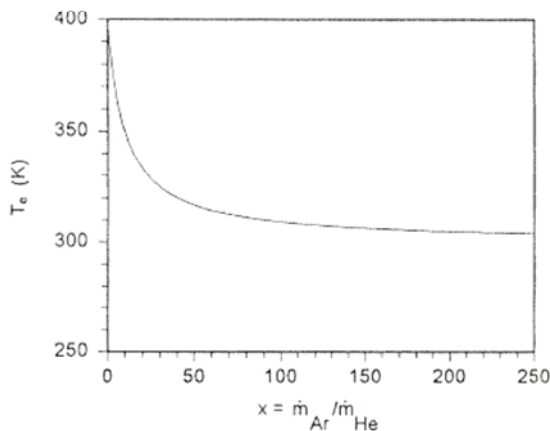
$\dot{m}_{He} + \dot{m}_{Ar} = (1+x) \cdot \dot{m}_{He}$
 $T_e = (400 \cdot (M_{Ar} / M_{He}) + 300 \cdot x) / ((M_{Ar} / M_{He}) + x)$

$\dot{E}_{dtd} / \dot{m}_{He} = T_o \cdot (A + B)$
 $A = (Rbar / M_{He}) \cdot (2.5 \cdot \ln(T_e / 400) - \ln(1 - y_{Ar}))$
 $B = x \cdot (Rbar / M_{Ar}) \cdot (2.5 \cdot \ln(T_e / 300) - \ln(y_{Ar}))$
 $y_{Ar} = x / ((M_{Ar} / M_{He}) + x)$

IT Results for $x = 1$

$y_{Ar} = 0.0911$
 $T_e = 390.9 \text{ K}$
 $\dot{E}_d / \dot{m}_{He} = 214.6 \text{ kJ/kg}$

PLOTS:



As the fraction of argon increases, the exit temperature approaches 300K, which is the temperature of the entering argon stream, as expected. Also, as the argon fraction increases, the exergy destruction increases due to the temperature difference between the two streams.

PROBLEM 12.41

KNOWN: Oxygen is allowed to flow into a 1000 ft^3 tank, initially filled with N_2 , until the pressure in the tank achieves a specified value, while the temperature in the tank remains constant.

FIND: Determine (a) the mass of O_2 that enters, (b) the heat transfer

SCHEMATIC & GIVEN DATA:



Final pressure in 1000 ft^3 tank = 15 lbf/in^2

ENGINEERING MODEL: (1) For the control volume shown, $\dot{W}_{cv} = 0$. (2) Heat transfer with the surroundings maintains the tank contents at 70°F . (3) Kinetic and potential energy effects are negligible. (4) Each pure component behaves as an ideal gas. The mixture adheres to the ideal gas model.

ANALYSIS: (a) Initially, the tank is filled with N_2 at $T = 530^\circ\text{R}$, $p = 5 \text{ lbf/in}^2$:

$P_1 V = n_{\text{N}_2} \bar{R} T$. Finally, the tank is filled N_2 and O_2 at $T = 530^\circ\text{R}$, $P_2 = 15 \text{ lbf/in}^2$:

$P_2 V = (n_{\text{N}_2} + n_{\text{O}_2}) \bar{R} T$. Combining these expressions:

$$\frac{P_2}{P_1} = \frac{n_{\text{N}_2} + n_{\text{O}_2}}{n_{\text{N}_2}} \Rightarrow \frac{15}{5} = 1 + \frac{n_{\text{O}_2}}{n_{\text{N}_2}} \Rightarrow n_{\text{O}_2} = 2 n_{\text{N}_2}$$

The amount of N_2 initially present is

$$n_{\text{N}_2} = \frac{P_1 V}{\bar{R} T} = \frac{(5 \times 14.7 \text{ lbf/in}^2)(1000 \text{ ft}^3)}{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lbmol} \cdot ^\circ\text{R}})(530^\circ\text{R})} = 0.879 \text{ lbmol}(\text{N}_2) \Rightarrow n_{\text{O}_2} = 2(0.879) = 1.758 \text{ lbmol}(\text{O}_2)$$

$$\Rightarrow m_{\text{O}_2} = \left(\frac{32 \text{ lb}}{\text{lbmol}} \right) (1.758 \text{ lbmol}) = 56.26 \text{ lb}(\text{O}_2)$$

(b) An energy rate balance reduces to

$$\frac{d\dot{U}_{cv}}{dt} = \dot{Q}_{cv} + \dot{n}_{\text{O}_2} h_{\text{O}_2}$$

Then, since O_2 enters at a constant temperature (530°R), h_{O_2} is constant. Thus, integration gives

$$\Delta \dot{U}_{cv} = \dot{Q}_{cv} + n_{\text{O}_2} h_{\text{O}_2}(T) \Rightarrow \dot{Q}_{cv} = \Delta \dot{U}_{cv} - n_{\text{O}_2} h_{\text{O}_2}(T)$$

where

$$\Delta \dot{U}_{cv} = [n_{\text{O}_2} \bar{u}_{\text{O}_2}(T_2) + n_{\text{N}_2} \bar{u}_{\text{N}_2}(T_2)] - n_{\text{N}_2} \bar{u}_{\text{N}_2}(T_1) = n_{\text{O}_2} \bar{u}_{\text{O}_2}(T)$$

since $T_1 = T_2 = T$. Collecting results

$$\begin{aligned} \textcircled{1} \quad \dot{Q}_{cv} &= n_{\text{O}_2} [\bar{u}_{\text{O}_2}(T) - \bar{h}_{\text{O}_2}(T)] = n_{\text{O}_2} [- (p\bar{v})_{\text{O}_2}] \quad \leftarrow = RT \\ &\quad \left[\bar{u}_{\text{O}_2}(T) + (p\bar{v})_{\text{O}_2} \right] \\ &= -n_{\text{O}_2} \bar{R} T = -1.758 \text{ lbmol}(\text{O}_2) \left[\frac{1.986 \text{ Btu}}{\text{lbmol} \cdot ^\circ\text{R}} \right] (530^\circ\text{R}) \\ &= -1850.4 \text{ Btu} \end{aligned}$$

$\leftarrow \dot{Q}_{cv}$

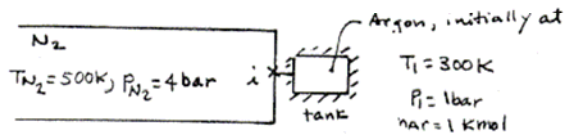
1. To maintain the temperature at 530°R , the energy exiting by heat transfer exactly matches the energy entering by flow work.

PROBLEM 12.42

KNOWN: An insulated, rigid tank initially containing Ar at a known temperature and pressure is connected to a large vessel containing N_2 at a known temperature and pressure. Nitrogen is allowed to flow into the tank, forming an Ar- N_2 mixture at T, p .

FIND: Plot T and p versus the amount of N_2 in the tank.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. For a control volume enclosing the tank, $Q_{cv} = W_{cv} = 0$. Kinetic, potential energy can be ignored.
2. At the control volume inlet, N_2 enters at 500 K, 4 bar.
3. Each pure component behaves as an ideal gas. The mixture adheres to the Dalton model.

ANALYSIS: Since the tank volume remains constant
 $\downarrow$ amount of N_2 in tank finally

$$V = \frac{n_{Ar} \bar{R} T_1}{p_1} = \frac{[n_{Ar} + n_{N_2}] \bar{R} T}{p} \Rightarrow p = p_1 \left[1 + \frac{n_{N_2}}{n_{Ar}} \right] \left[\frac{T}{T_1} \right] \quad (1)$$

where $p \leq 4$ bar.

Mass and energy rate balances reduce to give (see Sec. 4.4)

$$\Delta U_{cv} = \bar{h}_{N_2}(T_{N_2}) n_{N_2}$$

$$\text{or } \{ [n_{N_2} \bar{u}_{N_2}(T) + n_{Ar} \bar{u}_{Ar}(T)] - [n_{Ar} \bar{u}_{Ar}(T_1)] \} = \bar{h}_{N_2}(T_{N_2}) n_{N_2}$$

$$\Rightarrow n_{N_2} \bar{u}_{N_2}(T) + n_{Ar} [\bar{u}_{Ar}(T) - \bar{u}_{Ar}(T_1)] = \bar{h}_{N_2}(T_{N_2}) n_{N_2}$$

From Table A-23
 $\bar{c}_v = 1.5 \bar{R}$

$$\Rightarrow n_{N_2} \bar{u}_{N_2}(T) + n_{Ar} [1.5 \bar{R} (T - T_1)] = \bar{h}_{N_2}(T_{N_2}) n_{N_2} \quad (2)$$

Sample Calculation: When $n_{N_2} = 0.2$ kmol, Eq. (2) reads with data from Table A-23

$$(0.2 \text{ kmol}) \bar{u}_{N_2}(T) + (1 \text{ kmol}) (1.5 \times 8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}) (T - 300) \text{ K} = (0.2 \text{ kmol}) (14581 \frac{\text{kJ}}{\text{kmol}})$$

$$(0.2) \bar{u}_{N_2}(T) + 12.471 T = 6657.5 \text{ kJ}$$

Solving iteratively with Table A-23 data, $T = 400$ K. Then, Eq. (1) gives

$$p = (1 \text{ bar}) \left[1 + \frac{0.2}{1.0} \right] \left[\frac{400}{300} \right] = 1.6 \text{ bar.}$$

The data for the required plots are obtained using the following simple IT code:

IT Code

```
p1 = 1 // bar
T1 = 300 // K
TN2 = 500 // K
n_N2 = 0.2 // kmol
Rbar = 8.314 // kJ/kmol.K
```

```
p = p1 * (1 + n_N2) * (T / T1)
n_Ar = 1 // kmol
n_N2 * u_T("N2", T) + n_Ar * 1.5 * Rbar * (T - T1) = n_N2 * h_T("N2", TN2)
```

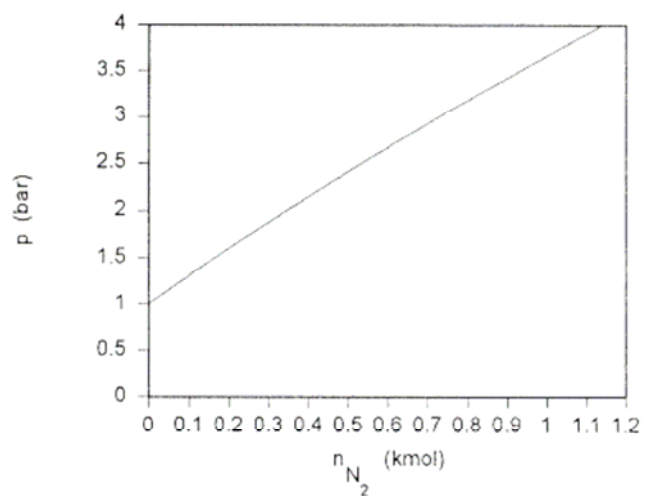
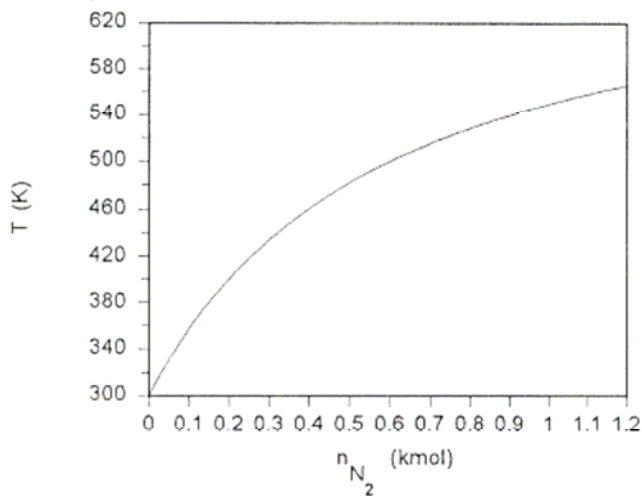
IT Results for $n_{N_2} = 0.2$ kmol

```
T = 400.4 K
p = 1.602 bar
```

Continued on next slide

Problem 12-42 continued

PLOTS:



Note

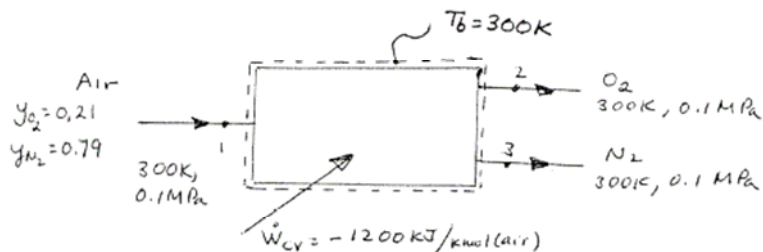
- As more N_2 is introduced, the pressure and temperature both increase.
- The tank temperature increases to values greater than the temperature of the nitrogen entering, due to flow work.
- When n_{N_2} reaches 1.13 kmol, the pressure in the tank reaches 4 bar and the process ceases.

PROBLEM 12.43

KNOWN: A stream of dry air at 300K, 0.1 MPa enters a device operating at steady state and is separated into pure O_2 , N_2 streams, each at 300K, 0.1 MPa. Operating data are provided.

FIND: Determine if the claimed work input value can be correct.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume shown is at steady state with negligible kinetic/potential energy effects. (2) Ideal gas principles apply to the air and the pure O_2 , N_2 streams exiting. (3) Heat transfer with the surroundings occurs at $T_b = 300K$. (4) $\dot{W}_{cv} = -1200 kJ/kmol(air)$

ANALYSIS: The plan is to evaluate the entropy production rate. This requires the heat transfer rate. Accordingly, reduction of an energy rate balance gives in terms of molar flow rates

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{n}_{air} \bar{h}_{air} - \dot{n}_{O_2} \bar{h}_{O_2} - \dot{n}_{N_2} \bar{h}_{N_2}$$

Then, with $\bar{h}_{air} = y_{O_2} \bar{h}_{O_2} + y_{N_2} \bar{h}_{N_2}$,

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{air}} - \frac{\dot{W}_{cv}}{\dot{n}_{air}} + [y_{O_2} \bar{h}_{O_2} + y_{N_2} \bar{h}_{N_2}] - \left(\frac{\dot{n}_{O_2}}{\dot{n}_{air}} \right) \bar{h}_{O_2} - \left(\frac{\dot{n}_{N_2}}{\dot{n}_{air}} \right) \bar{h}_{N_2}$$

And, since all streams are at 300K, this reduces to give $\frac{\dot{Q}_{cv}}{\dot{n}_{air}} = \frac{\dot{W}_{cv}}{\dot{n}_{air}} = -1200 \frac{kJ}{kmol(air)}$

Next, an entropy rate balance at steady state reads

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{n}_{air} \bar{s}_{air} - \dot{n}_{O_2} \bar{s}_{O_2} - \dot{n}_{N_2} \bar{s}_{N_2} + \dot{\sigma}_{cv}$$

$$\downarrow \bar{s}_{O_2}(T, p) \quad \downarrow \bar{s}_{N_2}(T, p)$$

$$\downarrow [y_{O_2} \bar{s}_{O_2}(T, y_{O_2}, p) + y_{N_2} \bar{s}_{N_2}(T, y_{N_2}, p)]$$

Thus,

$$0 = \frac{(\dot{Q}_{cv}/\dot{n}_{air})}{T_b} + [y_{O_2} \bar{s}_{O_2}(T, y_{O_2}, p) + y_{N_2} \bar{s}_{N_2}(T, y_{N_2}, p)] - y_{O_2} \bar{s}_{O_2}(T, p) - y_{N_2} \bar{s}_{N_2}(T, p) + \frac{\dot{\sigma}_{cv}}{\dot{n}_{air}}$$

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{air}}{T_b} + y_{O_2} \left[\underbrace{\bar{s}_{O_2}(T, y_{O_2}, p) - \bar{s}_{O_2}(T, p)}_{-R \ln y_{O_2}} \right] + y_{N_2} \left[\underbrace{\bar{s}_{N_2}(T, y_{N_2}, p) - \bar{s}_{N_2}(T, p)}_{-R \ln y_{N_2}} \right] + \frac{\dot{\sigma}_{cv}}{\dot{n}_{air}}$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{air}} = -\frac{\dot{Q}_{cv}/\dot{n}_{air}}{T_b} + R [y_{O_2} \ln y_{O_2} + y_{N_2} \ln y_{N_2}]$$

$$= \frac{1200 kJ/kmol}{300 K} + 8.314 [0.21 \ln 0.21 + 0.79 \ln 0.79]$$

$$= (4.00 - 4.273) kJ/kmol \cdot K$$

$$= -0.273 kJ/kmol \cdot K$$

Accordingly, since $\dot{\sigma}_{cv} < 0$, the assumed value for the work input cannot be valid.

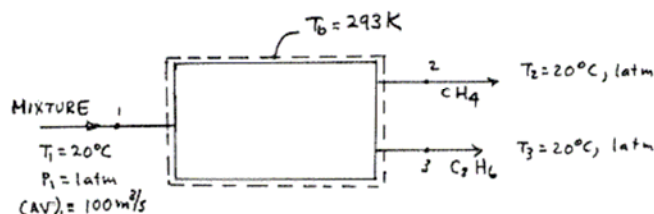
PROBLEM 12.44

KNOWN: Data are provided for a device operating at steady state that separates a natural gas into components.

FIND: Plot the minimum theoretical power input versus y , the mole fraction of C_2H_6 .

SCHEMATIC & GIVEN DATA:

Mixture	
component	mole fraction
C_2H_6	y
CH_4	$1-y$
$0.05 \leq y \leq 0.15$	



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates isothermally and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) Ideal gas principles apply for the pure components. The mixture adheres to the Dalton model.

ANALYSIS: The mass flow rate at 1 is obtained using the given volumetric flow rate and the ideal gas equation of state:

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{(R/M)T_1}$$

where $M = y_{CH_4} M_{CH_4} + y_{C_2H_6} M_{C_2H_6} = (0.94)(16.04) + (0.06)(30.07) = 16.88$. Thus,

$$\dot{m}_1 = \frac{(1.01325 \times 10^5 \text{ N/m}^2)(100 \text{ m}^3/\text{s})}{\left(\frac{8314}{16.88} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}\right)(293 \text{ K})} = 70.2 \text{ kg/s}$$

The molar flow rate is

$$\dot{n}_1 = \frac{70.2}{16.88} = 4.1588 \frac{\text{kmol}}{\text{s}}$$

Then

$$\begin{cases} \dot{n}_{C_2H_6} = y(4.1588) \\ \dot{n}_{CH_4} = (1-y)(4.1588) \end{cases} \quad (1)$$

With assumptions 1 and 2, an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{n}_{CH_4} \bar{h}_{CH_4}(T_1) + \dot{n}_{C_2H_6} \bar{h}_{C_2H_6}(T_1)] - \dot{n}_{CH_4} \bar{h}_{CH_4}(T_2) - \dot{n}_{C_2H_6} \bar{h}_{C_2H_6}(T_3)$$

Since $T_1 = T_2 = T_3$, the underlined term vanishes, leaving $\dot{W}_{cv} = \dot{Q}_{cv}$.

An entropy rate balance reduces at steady state to

$$0 = \frac{\dot{Q}_{cv}}{T_b} + [\dot{n}_{CH_4} \bar{s}_{CH_4}(T_1, y_{CH_4} P_1) + \dot{n}_{C_2H_6} \bar{s}_{C_2H_6}(T_1, y_{C_2H_6} P_1)] - \dot{n}_{CH_4} \bar{s}_{CH_4}(T_2, P_2) - \dot{n}_{C_2H_6} \bar{s}_{C_2H_6}(T_3, P_3) + \dot{\sigma}_{cv}$$

or, upon rearrangement, and inserting $\dot{W}_{cv} = \dot{Q}_{cv}$

$$-\dot{W}_{cv} = T_b \left\{ \dot{n}_{CH_4} (\bar{s}_{CH_4}(T_1, y_{CH_4} P_1) - \bar{s}_{CH_4}(T_2, P_2)) + \dot{n}_{C_2H_6} (\bar{s}_{C_2H_6}(T_1, y_{C_2H_6} P_1) - \bar{s}_{C_2H_6}(T_3, P_3)) + \dot{\sigma}_{cv} \right\}$$

In this equation $(-\dot{W}_{cv})$ is the work input. Since $\dot{\sigma}_{cv} \geq 0$, it follows that $(-\dot{W}_{cv})$ is a minimum when $\dot{\sigma}_{cv} = 0$. Accordingly

Continued on next slide

Problem 12-44 continued

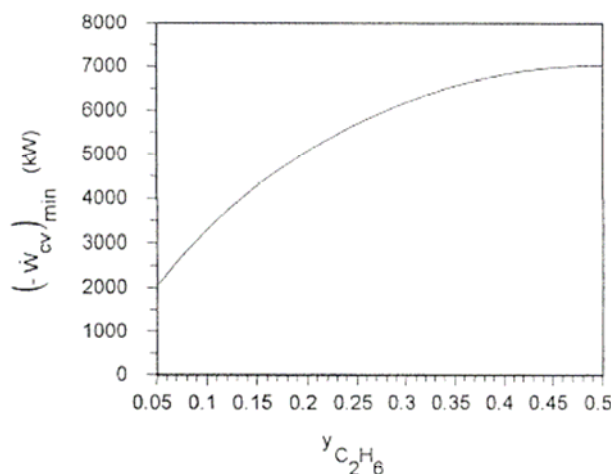
$$\begin{aligned}
 (-\dot{W}_{cv})_{\min} &= T_b \left\{ \dot{n}_{CH_4} \underbrace{(\bar{s}_{CH_4}^o(T_1) - \bar{s}_{CH_4}^o(T_2))}_{=0 \text{ since } T_1 = T_2} - \bar{R} \ln \frac{y_{CH_4} P_1}{P_2} \right\} + \\
 &\quad \dot{n}_{C_2H_6} \left(\underbrace{\bar{s}_{C_2H_6}^o(T_1) - \bar{s}_{C_2H_6}^o(T_2)}_{=0 \text{ since } T_1 = T_2} - \bar{R} \ln \frac{y_{C_2H_6} P_1}{P_2} \right) \} \\
 &= -\bar{R} T_b \left\{ \dot{n}_{CH_4} \ln y_{CH_4} + \dot{n}_{C_2H_6} \ln y_{C_2H_6} \right\} \quad (2)
 \end{aligned}$$

The governing equations are Equations (1) and (2). That is

$$\begin{aligned}
 (-\dot{W}_{cv})_{\min} &= \left(-8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \right) (293 \text{ K}) \left(4.1588 \frac{\text{kmol}}{\text{s}} \right) \left\{ (1-y) \ln(1-y) + y \ln y \right\} \left| \frac{\text{kJ}}{\text{kJ/s}} \right| \\
 \textcircled{1} \quad &= -(10,131 \text{ kW}) \left\{ (1-y) \ln(1-y) + y \ln y \right\} \quad (3)
 \end{aligned}$$

Eq. (3) can be plotted using available plotting software. The result using π is presented below.

PLOT:



We see from the plot that more power is required as the mole fraction of C_2H_6 increases.

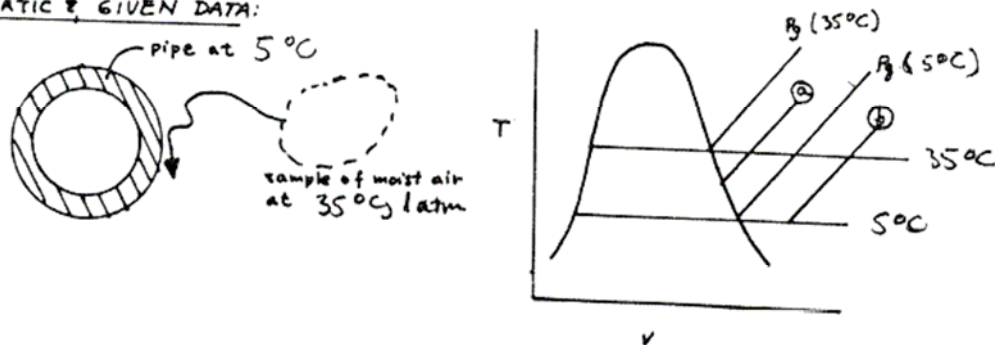
-
- Owing to inevitable irreversibilities, the actual power required would exceed the minimum theoretical value obtained from Eq. (3).

PROBLEM 12.45

KNOWN: A water pipe at 5°C runs between buildings through air at 35°C

FIND: Determine the maximum relative humidity the air can have before condensation occurs on the wall.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system consists of a sample of moist air initially at 35°C . (2) As the system comes close to the pipe at 5°C , the system undergoes a cooling process at fixed total pressure from 35°C to 5°C .

ANALYSIS: As the sample of moist air is cooled at fixed total pressure, the partial pressure of the water vapor remains constant as long as no condensation occurs, for $P_v = y_v p$ and y_v remains constant.

Accordingly, if the initial pressure is less than $P_g(5^\circ\text{C})$, such as (b) shown on the $T-v$ diagram, the sample would be cooled to 5°C without condensation. However, if the initial partial pressure is greater than $P_g(5^\circ\text{C})$, such as (a) shown on the $T-v$ diagram, the system would be cooled until a saturated mixture is attained. Subsequent cooling to 5°C would involve condensation. It can be concluded, therefore, that the partial pressure must be less than, or equal to, $P_g(5^\circ\text{C})$. Thus

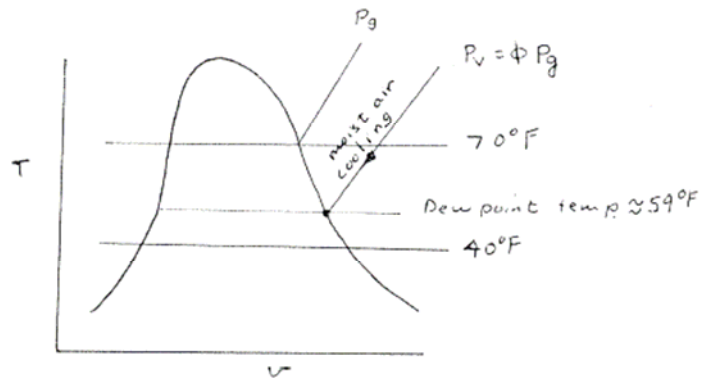
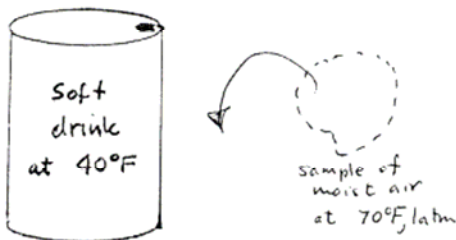
$$\phi = \frac{P_v}{P_g(35^\circ\text{C})} \leq \frac{P_g(5^\circ\text{C})}{P_g(35^\circ\text{C})} = \frac{0.00872 \text{ bar}}{0.05628 \text{ bar}} = 0.155 \text{ (15.5\%)} \quad \phi_{\max}$$

PROBLEM 12.46

KNOWN: A can of soft drink at 40°F is placed in a room where $T=70^\circ\text{F}$; $\phi=70\%$.

FIND: Explain why beads of moisture form on the can's surface.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system consists of a sample of moist air initially at 70°F .
 (2) As the system comes close to the can at 40°F , the system cools at a fixed total pressure.

ANALYSIS: As the sample of moist air is cooled at fixed total pressure, the partial pressure of the water vapor remains constant as long as no condensation occurs. Once the dew point temperature is attained, however, further cooling results in condensation. Accordingly, in this case beads of moisture form because the dew point temperature is greater than 40°F . That is

$$P_v = \phi P_g(70^\circ\text{F}) = (0.7)(0.3632 \frac{\text{lb}_f}{\text{in}^2}) = 0.2542 \frac{\text{lb}_f}{\text{in}^2}$$

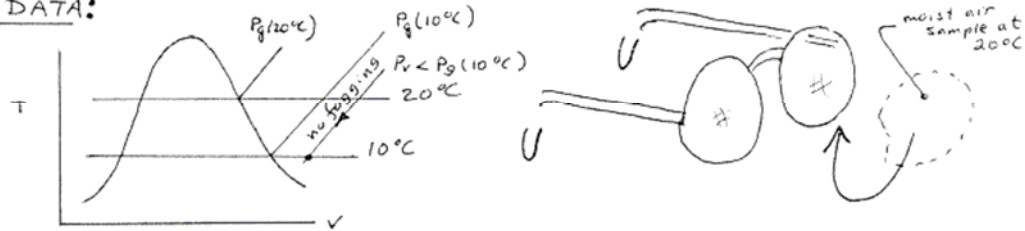
Interpolating in Table A-2E gives $T_{\text{dew point}} \approx 59^\circ\text{F}$

PROBLEM 12.47

KNOWN: Eyeglasses at 10°C brought into a dwelling at 20°C do not fog up.

FIND: Determine if the relative humidity in the dwelling can be 55%.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system consists of a sample of moist air initially at 20°C . (2) As the system comes close to the eyeglasses at 10°C , the system cools at a fixed mixture pressure.

ANALYSIS: As the sample of moist air is cooled at fixed total pressure, the partial pressure of the water vapor remains constant as long as no condensation occurs. That is, condensation would not occur if the partial pressure P_v satisfies $P_v < P_g(10^\circ\text{C})$. If $\phi = 55\%$

$$P_v = \phi P_g(20^\circ\text{C}) = 0.55(0.02339 \text{ bar}) = 0.0129 \text{ bar}$$

However, at 10°C , $P_g = 0.01228 \text{ bar}$. Since no fogging occurs, the relative humidity value cannot be correct.

COMMENT: The maximum relative humidity for fogging not to occur is

$$\phi_{\text{MAX}} = \frac{P_g(10^\circ\text{C})}{P_g(20^\circ\text{C})} = \frac{0.01228}{0.02339} = 0.525 \text{ (52.5\%)}$$

PROBLEM 12.48

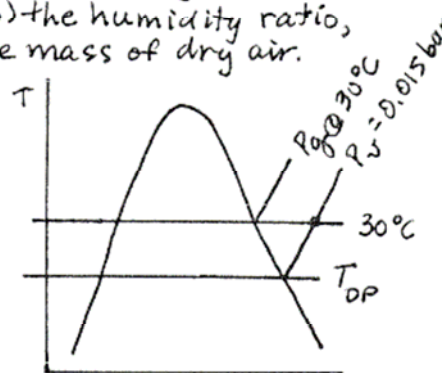
KNOWN: Conditions are known for a large room containing moist air

FIND: Determine (a) the relative humidity, (b) the humidity ratio, (c) the dew point temperature, and (d) the mass of dry air.

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} T &= 30^\circ\text{C} \\ P &= 102 \text{ kPa} \\ &= 1.02 \text{ bar} \\ P_v &= 1.5 \text{ kPa} \\ &= 0.015 \text{ bar} \\ m_v &= 10 \text{ kg} \end{aligned}$$



ENGINEERING MODEL: (1) The air in the room is the closed system.

(2) The moist air is treated as an ideal gas mixture.

ANALYSIS:

(a) The relative humidity is $\phi = P_v / P_g @ 30^\circ\text{C}$. With data from Table A-2

$$\phi = \frac{0.015 \text{ bar}}{0.04246 \text{ bar}} = 0.353 \text{ (35.3\%)} \leftarrow \phi$$

(b) The specific humidity is

$$\omega = 0.622 \left(\frac{P_v}{P - P_v} \right) = 0.622 \left(\frac{1.5}{102 - 1.5} \right) = 0.009284 \frac{\text{kg(v)}}{\text{kg(a)}} \leftarrow \omega$$

(c) The dew point temperature is the saturation temperature at $P_v = 0.015 \text{ bar}$. Thus

$$T_{DP} \approx 13^\circ\text{C} \leftarrow T_{DP}$$

(d) To get the mass of dry air, use $\omega = m_v / m_a$.

$$\textcircled{1} \quad m_a = \frac{m_v}{\omega} = \frac{10 \text{ kg}}{0.009284} = 1077 \text{ kg(a)} \leftarrow m_a$$

1. The volume of the room can be found as follows:

$$\begin{aligned} V &= \frac{m_a R_a T}{P_a} \\ &= \frac{(1077 \text{ kg}) \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (303 \text{ K})}{(102 - 1.5) \text{ kPa}} \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| = 932 \text{ m}^3 \end{aligned}$$

Approximate dimensions: Assume 3 m ceiling height

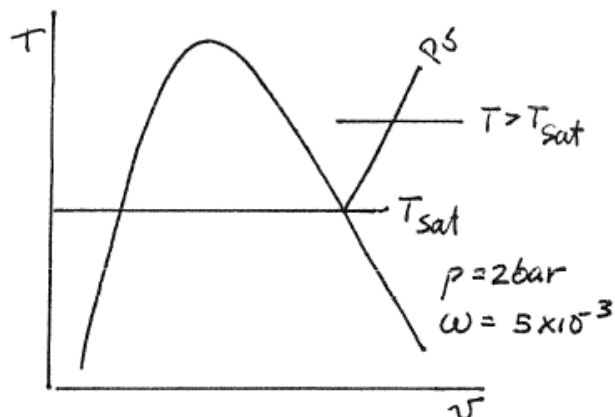
$$3 \text{ m} \times 17.5 \text{ m} \times 17.5 \text{ m}$$

PROBLEM 12.49

KNOWN: moist air with known ω is cooled at constant pressure.

FIND: At what temperature is the moist air saturated?

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL

The moist air behaves as an ideal gas mixture.

ANALYSIS: Using $\omega = 0.622(P_v/p - p_v)$ and solving for p_v

$$p_v = \frac{\omega p}{0.622 + \omega}$$

$$= \frac{(5 \times 10^{-3})(2 \text{ bar})}{(0.622 + 5 \times 10^{-3})} = 0.01595 \text{ bar}$$

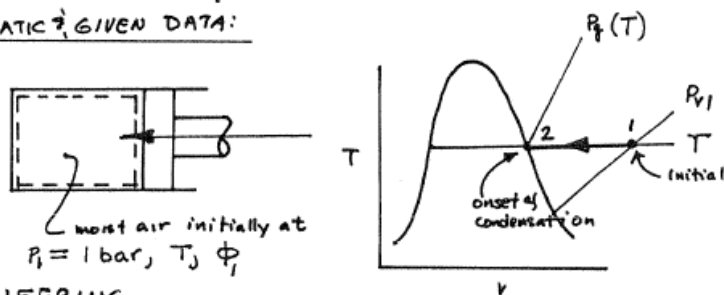
From Table A-2 at $p_v = 0.01598 \text{ bar}$; $T_{\text{sat}} \approx 14^\circ\text{C}$ T_{sat}

PROBLEM 12.50

KNOWN: A fixed amount of air initially at 1 bar, relative humidity ϕ_1 , and temperature T is compressed isothermally until the onset of condensation.

FIND: Determine the mixture pressure when condensation begins, if (a) $\phi_1 = 60\%$, (b) $\phi_1 = 90\%$

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system consists of a fixed amount of moist air, as illustrated in the accompanying figure. (2) The moist air acts as an ideal gas mixture, with each component adhering to the Dalton model.

ANALYSIS: As long as there is no condensation, the mole fraction of the water vapor, y_v , remains constant. Thus, the partial pressure of the water vapor at each state visited during the isothermal compression is $p_v = y_v p$, where p is the corresponding mixture pressure. For example, $p_{v1} = y_v p_1$, where $p_1 = 1 \text{ bar}$. The onset of condensation occurs when the mixture becomes saturated: state 2. At state 2, $p_g = y_v p_2$, where p_2 is the mixture pressure. Forming the ratio of partial pressures

$$\frac{p_{v1}}{p_g} = \frac{y_v p_1}{y_v p_2} = \frac{p_1}{p_2} = \phi_1$$

When $\phi_1 = 60\%$

$$p_2 = \frac{p_1}{\phi_1} = \frac{1 \text{ bar}}{0.6} = 1.67 \text{ bar} \quad \leftarrow (a)$$

When $\phi_1 = 90\%$

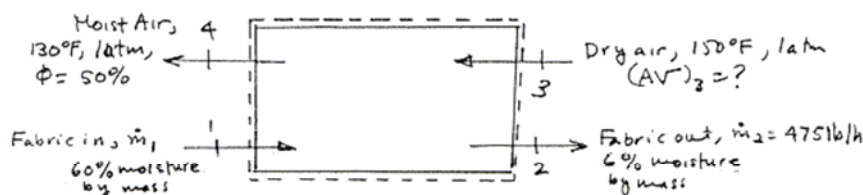
$$p_2 = \frac{p_1}{\phi_1} = \frac{1 \text{ bar}}{0.9} = 1.11 \text{ bar} \quad \leftarrow (b)$$

PROBLEM 12.51

KNOWN: Damp fabric containing 60% moisture by mass enters a dryer operating at steady state. Fabric exits the dryer with a moisture content of 6% at the rate of 475 lb/h. Dry air at 150°F, 1 atm enters the dryer and moist air at 130°F, 1 atm, $\phi = 50\%$ exits.

FIND: Determine the volumetric flow rate of the dry air.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) Ideal gas principles apply to the dry and moist air streams.

ANALYSIS: The volumetric flow rate of the dry air can be found from the molar flow rate of dry air:

$$\dot{n}_3 = \frac{(AV)_3}{\bar{V}_3} \Rightarrow (AV)_3 = \dot{n}_3 \left[\frac{RT_3}{P_3} \right] \quad (1)$$

$T_3 = 610^\circ R$
 $P_3 = 1 \text{ atm}$

The molar flow rate of dry air at 3 and 4 must be equal: $\dot{n}_4 = \dot{n}_3$.

At 4, $\dot{n}_4 = y_{a4} \dot{n}_4$ and $\dot{n}_{v4} = y_{v4} \dot{n}_4$. Combining these

$$\dot{n}_3 = \dot{n}_4 = y_{a4} \left[\frac{\dot{n}_{v4}}{y_{v4}} \right] = (1 - y_{v4}) \left[\frac{\dot{n}_{v4}}{y_{v4}} \right] \quad (2)$$

The mole fraction of moisture at 4 can be found using ϕ_4 as follows:

$$\phi_4 = \frac{P_{v4}}{P_{g4}} = \frac{y_{v4} P_4}{P_{g4}} \Rightarrow y_{v4} = \frac{\phi_4 P_{g4}}{P_4} = \frac{(0.5)(2.225 \text{ lbf/in}^2)}{14.7 \text{ lbf/in}^2} = 0.0757. \quad (3)$$

(Table A-2E at 130°F)

The molar flow rate of vapor at 4 can be found from an overall mass balance for water. If $\dot{m}_1$ is the total mass flow rate of fabric plus moisture at 1,

$$0.60 \dot{m}_1 + 0 = \dot{m}_{v4} + (0.06)(475) \Rightarrow \dot{m}_{v4} = 0.6 \dot{m}_1 - 28.5 \quad (4)$$

moisture in at ① moisture in at ③ moisture out at ④ moisture out at ②

The mass flow rate $\dot{m}_1$ can be found from a mass balance on the fabric:

$$0.4 \dot{m}_1 = (0.94)(475) \Rightarrow \dot{m}_1 = 1116.25 \text{ lb/h}$$

fabric in at ① fabric out at ②

Using this in Eq. (4), gives $\dot{m}_{v4} = 641.25 \text{ lb/h}$. Thus $\dot{n}_{v4} = 641.25 / 18.02 = 35.59 \text{ lbmol/h}$

Then, Eq. (2) gives $\dot{n}_3 = (1 - 0.0757)(35.59 / 0.0757) = 434.56 \text{ lbmol/h}$

Finally, with Eq. (1)

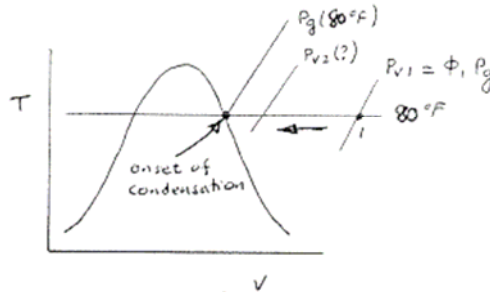
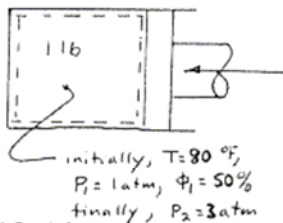
$$(AV)_3 = \frac{(434.56 \text{ lbmol/h}) \left(\frac{1545 \text{ lbf} \cdot \text{ft}}{\text{lbmol} \cdot ^\circ R} \right) (610^\circ R)}{(14.7 \times 144 \text{ lbf/ft}^2)} = 193,477 \frac{\text{ft}^3}{\text{h}}$$

PROBLEM 12.52

KNOWN: One lb of moist air initially at 80°F , 1 atm , $\phi = 50\%$ is compressed isothermally to 3 atm .

FIND: Determine if condensation occurs. If so, find the amount condensed. If not, find ϕ_2 .

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system consists of a 1 lb sample of dry air and water; (2) For the vapor phase, ideal gas mixture principles apply.

ANALYSIS: If no condensation occurs, $P_{v2} \leq P_g(80^\circ\text{F})$. To check this, write

$$w_1 = w_2 \Rightarrow 0.622 \frac{P_{v1}}{P_1 - P_{v1}} = 0.622 \frac{P_{v2}}{P_2 - P_{v2}} \Rightarrow \frac{P_{v1}}{P_1} = \frac{P_{v2}}{P_2} \Rightarrow P_{v2} = \frac{P_2}{P_1} P_{v1}$$

Then, with $P_{v1} = \phi_1 P_g(80^\circ\text{F})$, this expression gives

$$P_{v2} = \left(\frac{3\text{ atm}}{1\text{ atm}} \right) (0.50 P_g(80^\circ\text{F})) = 1.5 P_g(80^\circ\text{F})$$

However, for no condensation $P_{v2} \leq P_g$, and so condensation occurs in this case. $\leftarrow$

At the final state, the system consists of a gas phase (dry air plus saturated water vapor) and a liquid phase (saturated liquid). The amount condensed is

$$\text{Amount Condensed} = m_{v1} - m_{v2} = m_a (w_1 - w_2)$$

where

$$w_1 = 0.622 \frac{P_{v1}}{P_1 - P_{v1}} = \frac{0.622 (0.50) P_g}{P_1 - (0.50) P_g} = \frac{(0.622)(0.50)(0.5073 \text{ lb}/\text{lb(a)})}{[14.7 - (0.50)(0.5073)] \text{ lb}/\text{lb(a)}} = 0.01756 \frac{\text{lb(v)}}{\text{lb(a)}}$$

$$w_2 = 0.622 \frac{P_g}{P_2 - P_g} = \frac{0.622 (0.5073)}{(3)(14.7) - (0.5073)} = 0.00724 \frac{\text{lb(v)}}{\text{lb(a)}}$$

The mass of dry air is found from $m_{\text{mix}} = m_a + m_{v1} = m_a (1 + w_1)$, giving

$$m_a = \frac{m_{\text{mix}}}{1 + w_1} = \frac{1 \text{ lb}}{1.01756 (\text{lb}/\text{lb(a)})} = 0.983 \text{ lb(a)}$$

Collecting results

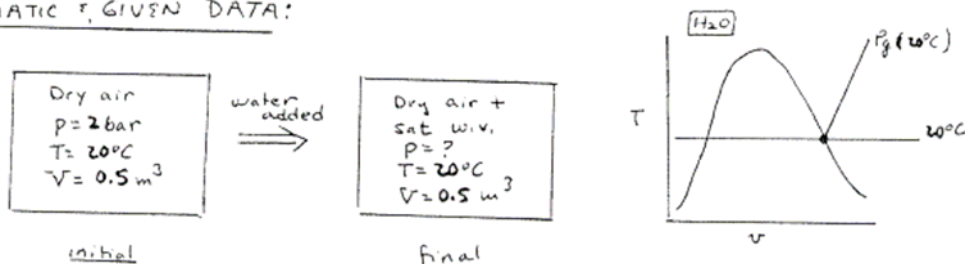
$$\begin{aligned} \text{Amount Condensed} &= 0.983 [0.01756 - 0.00724] \\ &= 0.01014 \text{ lb(v)} \end{aligned}$$

PROBLEM 12.53

KNOWN: Water is added to a vessel initially containing dry air at 20°C until the air is saturated at 20°C .

FIND: Determine (a) the mass of water added and (b) the final pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system is the final tank contents (2) Ideal gas principles apply.

ANALYSIS: (a) Since the water vapor is saturated at 20°C at the final state, its partial pressure is $p_g(20^\circ\text{C})$. Thus

$$m_v = \frac{p_g V}{(\bar{R}/M)T} \Rightarrow m_v = \frac{(0.02339 \times 10^5 \text{ N/m}^2)(0.5 \text{ m}^3)}{\left(\frac{8314 \text{ N}\cdot\text{m}}{18.02 \text{ kg}\cdot\text{K}}\right)(293 \text{ K})} = 0.00865 \text{ kg} \quad \leftarrow (a)$$

(b) For the overall mixture at the final state

$$p = \frac{(n_a + n_v)\bar{R}T}{V} = \frac{n_a\bar{R}T}{V} + \frac{n_v\bar{R}T}{V}$$

$\begin{matrix} \swarrow & \searrow \\ \text{Since } T \text{ \& } V \text{ do} & \text{This is the pressure} \\ \text{not change in the} & \text{of the w.v. at the} \\ \text{process, this is the} & \text{final state} \Rightarrow p_g = 0.02339 \text{ bar} \\ \text{initial pressure} = 2 \text{ bar} & \end{matrix}$

Thus

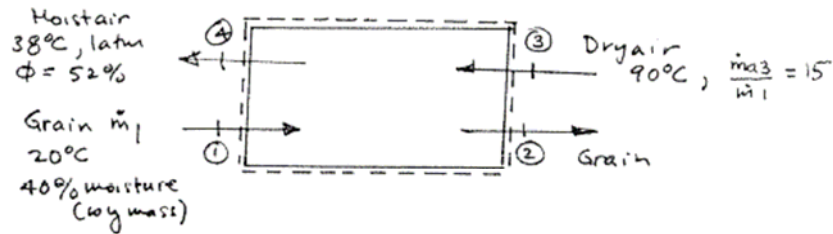
$$p = 2.0234 \text{ bar} \quad \leftarrow (b)$$

PROBLEM 12.54

KNOWN: Grain containing 40% moisture by mass enters a dryer operating at steady state. Dry air also enters and a moist air stream exits.

FIND: For the grain that exits the dryer, determine the percent moisture by mass.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) Ideal gas principles apply to the dry air and moist air streams.

ANALYSIS: The percent moisture by mass of the exiting grain is evaluated as

$$(1) \quad \% = (100) \left(\frac{\dot{m}_{\text{moisture},2}}{\dot{m}_2} \right) \quad \leftarrow \begin{array}{l} \text{rate moisture exits at 2} \\ \text{total rate grain plus moisture exits at 2} \end{array}$$

Mass balance on water:

$$\dot{m}_{\text{moisture},1} + \underbrace{0}_{\substack{\text{moisture in} \\ \text{dry air enters at 3}}} = \dot{m}_{\text{moisture},2} + \dot{m}_{v4} \Rightarrow \dot{m}_{\text{moisture},2} = 0.4\dot{m}_1 - \dot{m}_{v4} \quad (2)$$

Mass balance on dry air: $\dot{m}_{a4} = \dot{m}_{a3} = 15\dot{m}_1$. Additionally $w_4 = \dot{m}_v / \dot{m}_{a4}$. So

$\dot{m}_{v4} = w_4 \dot{m}_{a4} = w_4 (15\dot{m}_1)$. To find w_4 , calculate $P_{v4} = \phi_4 P_{g4}$ or

$$P_{v4} = 0.52(0.06632 \text{ bar}) = 0.0345 \text{ bar and}$$

$$w_4 = \frac{0.622 P_{v4}}{P_4 - P_{v4}} = \frac{0.622(0.0345)}{1.01325 - 0.0345} = 0.022 \frac{\text{kg}(v)}{\text{kg}(a)}$$

Thus

$$\dot{m}_{v4} = (0.022)(15)\dot{m}_1 = 0.33\dot{m}_1 \quad (3)$$

Also, the total mass flow rates at 1, 2 differ by the amount of moisture removed. That is

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_{v4} = \dot{m}_1 - 0.33\dot{m}_1 = 0.67\dot{m}_1 \quad (4)$$

Collecting Eqs. (1), (2), (3), (4)

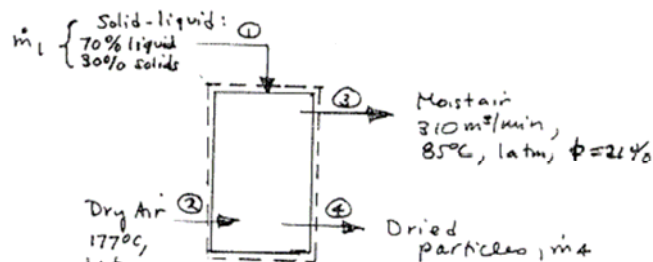
$$\% = \left(\frac{0.4\dot{m}_1 - 0.33\dot{m}_1}{0.67\dot{m}_1} \right) (100) = 10.4 \quad \leftarrow$$

PROBLEM 12.55

KNOWN: Operating data are provided for a spray dryer at steady state. Dry air enters the dryer at one location. Dried particles exit the dryer.

FIND: Determine (a) the volumetric flow rate of the entering dry air, (b) the rate that dried particles exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

- (1) The control volume shown in the figure is at steady state.
- (2) Ideal gas principles apply to the dry air and moist air streams.

ANALYSIS: (a) The volumetric flow rate of the dry air can be found from the molar flow rate of dry air:

$$\dot{n}_{a2} = \frac{(AV)_2}{V_2} \Rightarrow (AV)_2 = \dot{n}_{a2} \left(\frac{RT_2}{P_2} \right) \quad (1)$$

From a mass balance on dry air: $\dot{n}_{a2} = \dot{n}_{a3}$. To find $\dot{n}_{a3}$

$$\dot{n}_{a3} = \frac{(AV)_3}{V_{a3}} = \frac{(AV)_3 P_{a3}}{RT_3} \quad (2)$$

where $P_{a3} = P_3 - P_{v3}$. Using ϕ_3 , $P_{v3} = \phi_3 P_{g3} = 0.21(0.5783 \text{ bar}) = 0.12144 \text{ bar}$. So $P_{a3} = 1.01325 - 0.12144 = 0.8918 \text{ bar}$. Combining (1), (2), and P_{a3}

$$(AV)_2 = (AV)_3 \left(\frac{P_{a3}}{P_2} \right) \left(\frac{T_2}{T_3} \right) = \left(310 \frac{\text{m}^3}{\text{min}} \right) \left(\frac{0.8918}{1.01325} \right) \left(\frac{452}{358} \right) = 342.96 \frac{\text{m}^3}{\text{min}} \quad (a)$$

- (b) The rate that dried particles exit at 4 must equal the rate at which particles enter at 1. That is

$$\dot{m}_4 = 0.3 \dot{m}_1$$

where $\dot{m}_1$ is the total mass flow rate of particles plus moisture entering at 1.

$$\text{Since } \dot{m}_{\text{moisture},1} = 0.7 \dot{m}_1, \quad \dot{m}_1 = \frac{\dot{m}_{\text{moisture},1}}{0.7}$$

Since 1 is the only location that moisture enters, a mass balance on water gives $\dot{m}_{\text{moisture},1} = \dot{m}_{v3}$. Collecting results

$$\dot{m}_4 = 0.3 \dot{m}_1 = 0.3 \left[\frac{\dot{m}_{\text{moisture},1}}{0.7} \right] = \frac{0.3 \dot{m}_{v3}}{0.7} \quad (3)$$

To find $\dot{m}_{v3}$, use

$$\dot{m}_{v3} = \frac{(AV)_3}{V_{v3}} = \frac{(AV)_3 P_{v3}}{RT_3} = \frac{(310 \text{ m}^3/\text{min})(0.12144 \times 10^5 \text{ N/m}^2)}{\left(\frac{8314 \text{ N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \right) (358 \text{ K})} = 22.79 \frac{\text{kg}}{\text{min}}$$

Then Eq. (3) gives

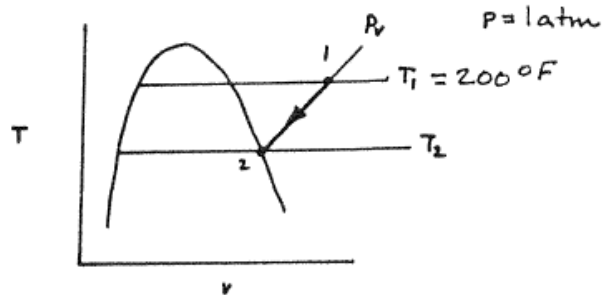
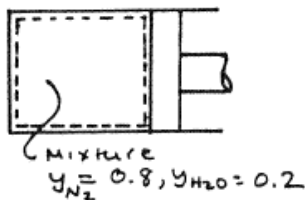
$$\dot{m}_4 = \frac{(0.3)(22.79)}{0.7} = 9.77 \frac{\text{kg}}{\text{min}} \quad (b)$$

PROBLEM 12.56

KNOWN: A mixture of nitrogen and water vapor having a molar analysis 80% N_2 , 20% water vapor is at 200°F, 1 atm.

FIND: If the mixture is cooled at constant pressure, determine the temperature at which water vapor begins to condense.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system consists of a fixed amount of a nitrogen/water vapor mixture, as illustrated in the accompanying figure. (2) The mixture acts like an ideal gas, with each component adhering to the Dalton Model.

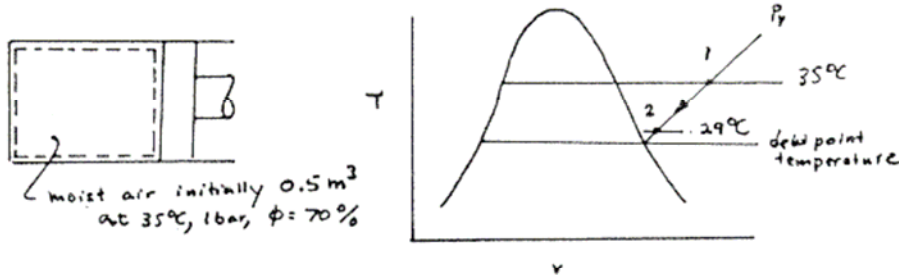
ANALYSIS: As long as there is no condensation, the mole fraction of the water vapor, y_v , remains constant. Moreover, as cooling occurs at fixed mixture pressure, the partial pressure of the water vapor remains constant: $P_v = y_v P$. Thus, cooling takes place at constant P_v , and the onset of condensation occurs at state 2 shown on the accompanying figure. The corresponding temperature is the saturation temperature, T_2 . Therefore, $P_v = y_{H_2O} P = (0.2)(14.7 \text{ lbf/in}^2) = 2.94 \text{ lbf/in}^2$. Interpolating in Table A-3E gives $T_2 = 140.5^\circ \text{F}$ ← T_2

PROBLEM 12.57

KNOWN: A system consisting initially of 0.5 m^3 of air at 35°C , 1 bar, $\phi = 70\%$ is cooled at constant pressure to 29°C .

FIND: Determine the work and heat transfer for the process.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) As shown in the figure, the system consists of the specified quantity of moist air. (2) The mixture acts as an ideal gas, with each component adhering to the Dalton model.

ANALYSIS: The first step is to determine whether condensation occurs. As long as there is no condensation the mole fraction of the water vapor, y_v , remains constant. Thus, as the cooling occurs at fixed mixture pressure, the partial pressure of the water vapor remains constant until a saturated mixture would be attained: $P_v = y_v P$. The onset of condensation in this case corresponds, therefore, to the dew point temperature. Using given data

$$P_v = \phi, p_g(35^\circ\text{C}) = (0.7)(0.05628 \text{ bar}) = 0.0394 \text{ bar} \Rightarrow T_{dp} = 28.7^\circ\text{C}$$

Accordingly, condensation does not take place.

Since the mixture undergoes a constant pressure process

$$W = \int p dV = p(V_2 - V_1) = P V_1 \left[\frac{V_2}{V_1} - 1 \right]$$

The total amount of mixture remains constant, so the ideal gas equation of state gives

$$\left. \begin{array}{l} P V_1 = n \bar{R} T_1 \\ P V_2 = n \bar{R} T_2 \end{array} \right\} \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

Thus

$$W = P_1 V_1 \left[\frac{T_2}{T_1} - 1 \right] = (10^5 \frac{\text{N}}{\text{m}^2})(1 \text{ m}^3) \left[\frac{302}{308} - 1 \right] \left| \frac{\text{kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = -1.948 \text{ kJ} \leftarrow W$$

With assumption 3, an energy balance gives $\Delta U = Q - W$, or $Q = \Delta U + W$. The change in internal energy of the system is the sum of the internal energy changes of the dry air and water vapor: $\Delta U = (\Delta U)_a + (\Delta U)_v$. Thus

$$Q = \{ m_a [u_a(T_2) - u_a(T_1)] + m_v [u_v(T_2) - u_v(T_1)] \} + W$$

Continued on next slide

Problem 12-57 continued

In accordance with the discussion of Sec. 12.5.2, $u_v \approx u_g(T)$ from the steam tables. For air u_a is obtained from Table A-22. The mass amounts m_a and m_v are obtained from the ideal gas equation of state:

$$m_v = \frac{P_{v1} V_1}{\bar{R}/M_v T_1} = \frac{(0.0394 \times 10^5 \text{ N/m}^2)(0.5 \text{ m}^3)}{(8314/18.02 \text{ N}\cdot\text{m/kg}\cdot\text{K})(308 \text{ K})} = 0.01387 \text{ kg}(v)$$

$$m_a = \frac{P_{a1} V_1}{\bar{R}/M_a T_1} = \frac{(P_1 - P_{v1}) V_1}{\bar{R}/M_a T_1} = \frac{(0.9606 \times 10^5)(0.5)}{(8314/28.97)(308)} = 0.54338 \text{ kg}(a)$$

Substituting values into the expression for Q

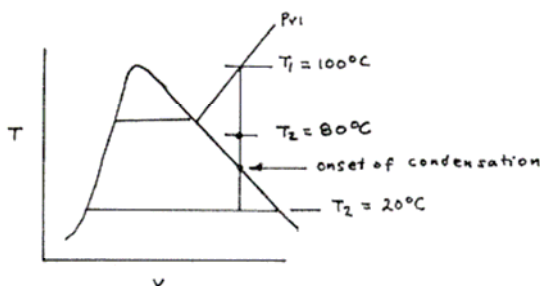
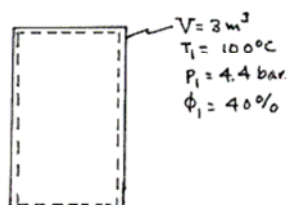
$$\begin{aligned} Q &= \left\{ 0.54338 \text{ kg}(a) [215.51 - 219.8] \frac{\text{kJ}}{\text{kg}(a)} + (0.01387) [2415.2 - 2423.4] \frac{\text{kJ}}{\text{kg}(v)} \right\} + \\ &\quad (-1.948 \text{ kJ}) \\ &= \{-2.331 - 0.114\} + (-1.948) = -4.393 \text{ kJ} \end{aligned} \quad \longleftarrow Q$$

PROBLEM 12.58

KNOWN: A tank with a volume of 3 m^3 initially contains air at 100°C , 4.4 bars, and $\phi = 60\%$. The tank contents are cooled to (a) 80°C , (b) 20°C .

FIND: Determine in each case the heat transfer.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) As shown by the accompanying figure, the system consists of the tank contents. For the system, the changes in kinetic and potential energy are zero in value. (2) The gas mixture acts as an ideal gas, with each component adhering to the Dalton model.

ANALYSIS: Begin by determining if condensation occurs during cooling.

As the total volume of the tank and the total mass of the water present are constant, the water undergoes a constant specific volume process as the tank contents are cooled. This is illustrated by the above $T-v$ diagram. Using the ideal gas equation of state

$$v_{v1} = \frac{(R/M_v) T_1}{P_{v1}} \quad \text{where } P_{v1} = \phi_1 P_g(T_1)$$

Thus, with data from Table A-2, $P_{v1} = (0.60)(1.014 \text{ bars}) = 0.6084 \text{ bars}$. And

$$v_{v1} = \frac{\left(\frac{8314}{18.02} \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}}\right)(373 \text{ K})}{(0.6084 \times 10^5 \text{ N/m}^2)} = \left(4.2429 \frac{\text{m}^3}{\text{kg}}\right) \left|\frac{10^6 \text{ cm}^3/\text{m}^3}{10^3 \text{ g/kg}}\right| = 4242.9 \frac{\text{cm}^3}{\text{g}}$$

Interpolating in Table A-2 with $v_g = v_{v1}$ gives $T = 74.4^\circ\text{C}$ as the temperature at which condensation would be begun.

(a) $T = 80^\circ\text{C}$. In this case there is no condensation, an energy balance $\Delta U = Q - W^0$ reduces to $Q = \Delta U$, where $U_1 = m_a u_a(T_1) + m_v u_g(T_1)$; $U_2 = m_a u_a(T_2) + m_v u_g(T_2)$. Then, with data from Tables A-2, A-22

$$\begin{aligned}
 Q &= m_a (u_a(T_2) - u_a(T_1)) + m_v (u_g(T_2) - u_g(T_1)) \\
 &= 11.194(252.19 - 266.63) + 0.7071(2482.2 - 2506.5) \\
 &= (-161.64) + (-17.18) = -178.82 \text{ kJ}
 \end{aligned}$$

(b) $T = 20^\circ\text{C}$. In this case condensation does occur. An energy balance reduces to give $\Delta U = Q - W^0$, or $Q = U_2 - U_1$, where

$$U_1 = m_v u_g(T_1) + m_a u_a(T_1)$$

$$U_2 = m_{v2} u_g(T_2) + m_{w2} u_f(T_2) + m_a u_a(T_2)$$

where m_{w2} is the mass of water that condenses. Collecting results

$$Q = m_a [u_a(T_2) - u_a(T_1)] + [m_{v2} u_g(T_2) + m_{w2} u_f(T_2) - m_v u_g(T_1)]$$

Continued on next slide

Problem 12-58 continued

Using the ideal gas equation of state

$$m_a = \frac{P_{a1} V}{\frac{R}{M_a} T_1} = \frac{(P_1 - P_{v1}) V}{\frac{R}{M_a} T_1} = \frac{[(4.4 - 0.4056) \times 10^5](3)}{\left(\frac{8314}{28.97}\right)(373)} = 11.194 \text{ kg(a)}$$

The mass of water vapor initially is

$$m_{v1} = \frac{V}{v_{v1}} = \left(\frac{3 \text{ m}^3}{4.2429 \text{ m}^3/\text{kg}} \right) = 0.7071 \text{ kg(v)}$$

Since the water undergoes a constant specific volume process, the quality of the two-phase liquid-vapor mixture at T_2 is

$$x_2 = \frac{v_{v2} - v_f}{v_g - v_f} = \frac{4242.9 - 1.0018}{5779.1 - 1.0018} = 0.0734$$

Thus, the amount of water vapor present in the final mixture is

$$m_{v2} = x_2 m_{v1} = (0.0734)(0.7071) = 0.0519 \text{ kg(v)}$$

The mass of condensate is

$$m_{w2} = m_{v1} - m_{v2} = 0.7071 - 0.0519 = 0.6552 \text{ kg(l)}$$

Finally, with data from Tables A-2 and A-22

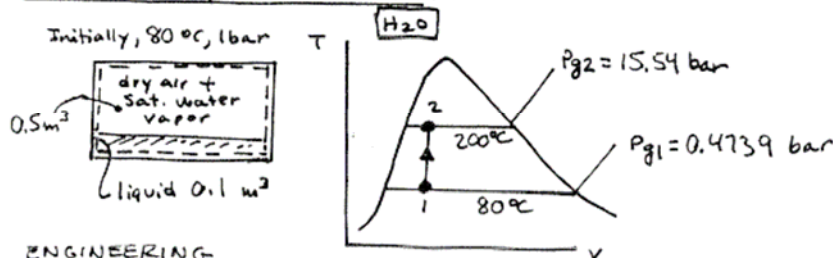
$$\begin{aligned} Q &= 11.194(209.06 - 266.63) + 0.0519(2402.9) + 0.6552(83.95) - 0.7071(2506.5) \\ &= (-644.44 + 124.71 + 55. - 1772.35) = -2237.08 \text{ kJ} \quad \leftarrow Q \end{aligned}$$

PROBLEM 12.59

KNOWN: A closed, rigid tank, initially containing moist air in equilibrium with liquid water, is heated.

FIND: Determine (a) the final pressure, (b) the heat transfer.

SCHEMATIC & GIVEN DATA



ENGINEERING

MODEL: (1) The system consists of the tank contents. (2) The gas phase adheres to ideal gas principles. (3) $W=0$ and kinetic & potential energy effects are absent.

ANALYSIS: (a) Since total volume is constant, the water present as liquid and water vapor, undergoes a constant specific volume process — as shown in the schematic. In stating this, the Dalton model is used: the water vapor occupies the full volume filled with water vapor and dry air. The initial amounts of liquid and vapor are

$$m_{\text{vap}} = \frac{V_{\text{vap}}}{v_g(80^\circ\text{C})} = \frac{0.5 \text{ m}^3}{3.407 \text{ m}^3/\text{kg}} = 0.1468 \text{ kg}, \quad m_{\text{liq}} = \frac{0.1 \text{ m}^3}{(1.0291/10^3) \text{ m}^3/\text{kg}} = 97.1723 \text{ kg}$$

The total mass of water is then $m_{\text{water}} = 97.3191 \text{ kg}$. The final specific volume is

$$v = \frac{0.60 \text{ m}^3}{97.3191 \text{ kg}} = 6.165 \times 10^{-3} \frac{\text{m}^3}{\text{kg}} \Rightarrow \text{at state 2 the water is a two-phase liquid-vapor mixture with quality}$$

$$x = \left[\frac{(6.165 \times 10^{-3}) - (1.1565 \times 10^{-3})}{0.1274 - (1.1565 \times 10^{-3})} \right] = 0.03967 \Rightarrow m_{v2} = x \cdot m_{\text{water}} = (0.03967)(97.1723) = 3.855 \text{ kg}$$

The mass of dry air is

$$m_a = \frac{P_{a1} V_{\text{gas}}}{R T_1} = \frac{[(1 - 0.4739) \times 10^5 \text{ N/m}^2] [0.5 \text{ m}^3]}{(0.287) (353 \text{ K})} = 0.2597 \text{ kg} \Rightarrow \omega_2 = \frac{m_{v2}}{m_a} = \frac{3.855}{0.259} = 14.846$$

Rearranging Eq. 12.43

$$P_2 = P_{r2} \left[1 + \frac{0.622}{\omega_2} \right] = (15.54 \text{ bar}) \left[1 + \frac{0.622}{14.846} \right] = 16.19 \text{ bar} = 1.62 \text{ MPa}$$

(b) An energy balance reduces to $\Delta U = Q - W$, where

$$U_1 = m_{\text{liq}} u_f(80^\circ\text{C}) + m_{\text{vap}} u_g(80^\circ\text{C}) + m_a u_a(80^\circ\text{C})$$

$$U_2 = m_{\text{water}} [u_f(200^\circ\text{C}) + x u_g(200^\circ\text{C})] + m_a u_a(200^\circ\text{C})$$

$$\text{or } Q = m_{\text{water}} [u_f(200^\circ\text{C}) + x u_g(200^\circ\text{C})] - [m_{\text{liq}} u_f(80^\circ\text{C}) + m_{\text{vap}} u_g(80^\circ\text{C})] + m_a [u_a(200^\circ\text{C}) - u_a(80^\circ\text{C})]$$

$$Q = (97.3191) [850.65 + (0.03967)(2595.3)] - [(97.1723)(334.86) + (0.1468)(2482.2)] + (0.2597) [339.5 - 252.2]$$

$$= 92,804 - 32,904 + 23$$

$$= 59,923 \text{ kJ}$$

← Q

PROBLEM 12.60

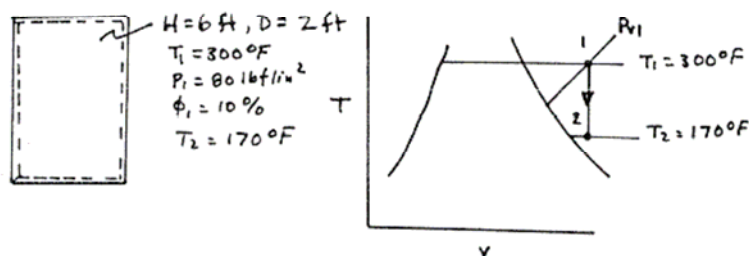
KNOWN: A cylindrical tank having specified dimensions initially contains moist air at known conditions. The tank contents are cooled.

FIND: Determine (a) whether condensation occurs, (b) the final pressure, (c) the heat transfer, and (d) the change in entropy.

KNOWN: A tank with a volume of 100 ft^3 initially contains air at 300°F , 80 lbf/in^2 , $\phi_1 = 10\%$. The tank contents are cooled to 170°F .

FIND: Determine the final pressure, the heat transfer, and the entropy change.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) As shown by the accompanying figure, the system consists of the tank contents for which there are no changes in kinetic or potential energy. (2) The mixture acts as an ideal gas and each component adheres to the Dalton model.

ANALYSIS: (a) The first step is to determine if condensation occurs. As the total volume of the tank and the total mass of water present are constant, the water undergoes a constant specific volume process as the tank contents are cooled (see T-v diagram). Using the ideal gas equation of state

$$v_{v1} = \frac{(\bar{R}/M_v)T_1}{P_{v1}} \quad \text{where} \quad P_{v1} = \phi_1 P_1 = (0.10)(80 \text{ lbf/in}^2) = 8 \text{ lbf/in}^2$$

Thus

$$v_{v1} = \frac{(1545 \text{ ft}^3 \cdot \text{lbf} / (\text{lb} \cdot ^\circ\text{R})) (760^\circ\text{R})}{(8 \text{ lbf/in}^2) (144 \text{ in}^2/\text{ft}^2)} = 67.56 \text{ ft}^3/\text{lb}$$

Interpolating in Table A-2E with $v_{v1} = v_g$ gives $T = 166.3^\circ\text{F}$ as the temperature at which condensation would begin. Accordingly, there is no condensation in the present case. (a)

(b) Treating the overall mixture as an ideal gas at states 1 and 2

$$P_1 = \frac{mRT_1}{V}, \quad P_2 = \frac{mRT_2}{V} \Rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow P_2 = (80 \text{ lbf/in}^2) \left(\frac{630^\circ\text{R}}{760^\circ\text{R}} \right) = 66.32 \text{ lbf/in}^2$$

(c) The volume of moist air is $V = \left(\frac{\pi D^2}{4} \right) (H) = \frac{\pi (2 \text{ ft})^2 (6 \text{ ft})}{4} = 18.85 \text{ ft}^3$. The mass of dry air is

$$m_a = \frac{P_a V}{(\bar{R}/M_a) T_1} = \frac{[(80 - 8) \text{ lbf/in}^2] (18.85 \text{ ft}^3)}{(1545/28.97) (760)} = 4.91 \text{ lb(a)}$$

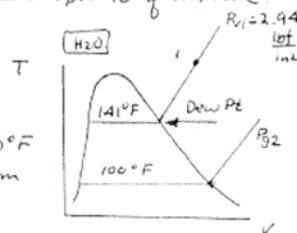
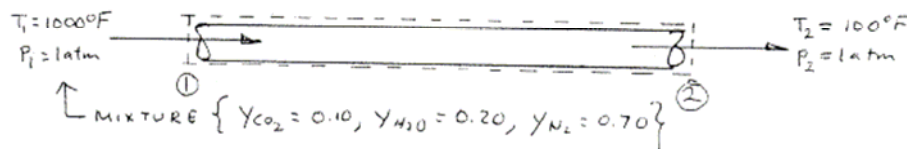
An energy balance reduces to $\Delta U = Q - W$, or $Q = \Delta U$. Thus

PROBLEM 12.61

KNOWN: A gaseous mixture including water vapor enters an exhaust pipe at 1000°F, 1 atm and exits at 100°F, 1 atm.

FIND: Determine the heat transfer rate at steady state, in Btu per lb of mixture.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown is at steady state. (2) For the control volume, $\dot{W}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) For the gas phase, ideal gas principles are applicable.

ANALYSIS: To begin, check if condensation would occur by determining the dew point temperature. For the water vapor entering, $P_{v1} = Y_{v1} P_1 = 0.2 \text{ atm} = 2.94 \text{ lbf/in}^2$. Thus, the dew point temperature is $\approx 141^\circ\text{F}$, and so condensation would occur. At the exit, there is a saturated gas phase and saturated liquid.

Next, consider the gas phase exiting. The partial pressure of the water vapor is P_{g2} and the mole fraction of the water vapor is $Y_v = n_v / (n_{\text{dry}} + n_v)$, where n_v is the amount of water vapor exiting per lbmol of entering mixture and n_{dry} is the amount of CO_2 and N_2 per lbmol of entering mixture. Accordingly

$$P_v = Y_v P \Rightarrow P_{g2} = \left(\frac{n_v}{0.8 + n_v} \right) P_2 \Rightarrow 0.9533 \left(\frac{n_v}{0.8 + n_v} \right) 14.7 \Rightarrow n_v = 0.0553 \frac{\text{lbmol (vapor)}}{\text{lbmol (mix, 1)}}$$

Accordingly, at ② there would be $(0.2 - 0.0553) = 0.1447 \text{ lbmol (condensate) / lbmol (mix, 1)}$

Reducing an energy balance at steady state

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}_{\text{mix},1}} + [0.1 \bar{h}_{\text{CO}_2} + 0.2 \bar{h}_{\text{H}_2\text{O}(v)} + 0.7 \bar{h}_{\text{N}_2}]_1 - [0.1 \bar{h}_{\text{CO}_2} + 0.0553 \bar{h}_{\text{H}_2\text{O}(v)} + 0.7 \bar{h}_{\text{N}_2}]_2 - [0.1447 \bar{h}_{\text{H}_2\text{O}(l)}]_2$$

① Then, with $h_{\text{H}_2\text{O}(v)} = h_g$ and $h_{\text{H}_2\text{O}(l)} = h_f$ at ② and $\bar{h}_{\text{H}_2\text{O}(v)}$ at ① from the steam tables, and data for CO_2 and N_2 from the ideal gas tables:

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}_{\text{mix},1}} &= 0.1 [\bar{h}_{\text{CO}_2}(T_2) - \bar{h}_{\text{CO}_2}(T_1)] + 0.7 [\bar{h}_{\text{N}_2}(T_2) - \bar{h}_{\text{N}_2}(T_1)] + \\ &\quad M_{\text{H}_2\text{O}} [0.0553 h_g(T_2) + 0.1447 h_f(T_2) - 0.2 h_g(T_1)] \\ &= 0.1 [4236 - 14081] + 0.7 [3870 - 10347] + 18.02 [0.0553(1105) + 0.1447(68) - 0.2(1535)] \\ &= -984.5 - 4519.9 - 4253.6 \\ &= -9758 \frac{\text{Btu}}{\text{lbmol (mix in)}} \end{aligned}$$

For the entering mixture $M_{\text{mix}} = (0.1)(44.01) + (0.2)(18.02) + (0.7)(28.01) = 27.6 \frac{\text{lb (mix)}}{\text{lbmol (mix)}}$. Accordingly,

$$\frac{\dot{Q}_{cv}}{\dot{m}_{\text{mix},1}} = \frac{-9758}{27.6} = -353.6 \frac{\text{Btu}}{\text{lb (mix in)}}$$

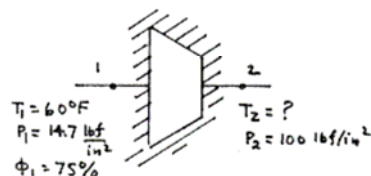
1. For the entering mixture, $P_{v1} = 2.94 \text{ lbf/in}^2$. At this pressure, the enthalpy of water vapor varies little with pressure and is determined mainly by temperature.

PROBLEM 12.62

KNOWN: Steady-state operating data are provided for a compressor.

FIND: (a) For $\eta_c = 0.8$, determine the exit temperature, the work and exergy destruction per unit mass of dry air flowing. (b) Plot the quantities of part (a) versus η_c ranging from 0.7 to 1.0.

SCHEMATIC & GIVEN DATA:



Note: Since T_2 would be greater than T_1 , as a consequence of compression, there is no possibility of condensation.

ENGINEERING

MODEL: (1) The compressor operates at steady state and is well insulated. (2) Changes in kinetic and potential energy effects from inlet to exit can be ignored. (3) The moist air acts as an ideal gas and each component adheres to the Dalton model. (4) $T_0 = 520^\circ\text{R}$.

ANALYSIS: At steady state mass rate balances give $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_a$ and $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_v$. Using assumptions 1-3 an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_a(T_1) + \dot{m}_v h_v(T_1)] - [\dot{m}_a h_a(T_2) + \dot{m}_v h_v(T_2)]$$

or

$$\dot{W}_{cv} = \dot{m}_a [h_a(T_1) - h_a(T_2)] + \dot{m}_v [h_v(T_1) - h_v(T_2)]$$

Noting that $w = \dot{m}_v / \dot{m}_a$ this becomes

$$\frac{\dot{W}_{cv}}{\dot{m}_a} = [h_a(T_1) - h_a(T_2)] + w [h_v(T_1) - h_v(T_2)] \quad (1)$$

This applies both for the actual compression and an isentropic compression from the same initial state. For the isentropic compression

$$\left(\frac{\dot{W}_{cv}}{\dot{m}_a} \right)_s = [h_a(T_1) - h_a(T_{2s})] + w [h_v(T_1) - h_v(T_{2s})] \quad (2)$$

where T_{2s} is the temperature at the exit for an isentropic compression.

The actual work requirement can be found using the given isentropic compression efficiency together with Eq. (2):

$$\frac{\dot{W}_{cv}}{\dot{m}_a} = \frac{(\dot{W}_{cv}/\dot{m}_a)_s}{\eta_c} \quad (3)$$

To determine the numerator of Eq. (3) requires T_{2s} which can be found in an iterative procedure using the requirement that the specific entropy of the mixture does not change from inlet to exit in an isentropic compression. When expressed per unit mass of dry air this condition takes the form

$$[s_a(T_1, P_{a1}) + w s_v(T_1, P_{v1})] = [s_a(T_{2s}, P_{a2}) + w s_v(T_{2s}, P_{v2})]$$

or

$$[s_a(T_{2s}, P_{a2}) - s_a(T_1, P_{a1})] + w [s_v(T_{2s}, P_{v2}) - s_v(T_1, P_{v1})] = 0$$

$$\left[s_a^\circ(T_{2s}) - s_a^\circ(T_1) - \frac{\bar{R}}{M_a} \ln \frac{P_{a2}}{P_{a1}} \right] + w \left[\frac{\bar{s}_v^\circ(T_{2s}) - \bar{s}_v^\circ(T_1) - \bar{R} \ln \frac{P_{v2}}{P_{v1}}}{M_v} \right] = 0$$

With values from Tables A-22E, 23E at 520°R

$$0 = \left[s_a^\circ(T_{2s}) - 0.59172 - \frac{1.986}{28.97} \ln \frac{100}{14.7} \right] + w \left[\frac{\bar{s}_v^\circ(T_{2s}) - (4.8421 + 1.986 \ln \frac{100}{14.7})}{18.02} \right] \quad (4)$$

Continued on next slide

Problem 12-62 continued

ω is found from Eq. 12.43 using $P_v = \phi P_g(T_1) = 0.75(0.2563) = 0.1922 \text{ lbf/in}^2$

Thus

$$\omega = 0.622 \left(\frac{0.1922}{14.7 - 0.1922} \right) = 0.622 \left(\frac{0.1922}{14.5078} \right) = 0.0082 \frac{\text{lb(v)}}{\text{lb(a)}}$$

With this Eq. (4) gives upon rearrangement

$$s_2^0(T_{2s}) + \left(\frac{0.0082}{18.02} \right) \bar{s}_v^0(T_{2s}) = 0.7452 \quad (5)$$

Solving Eq. (5) iteratively, $T_{2s} \approx 895^\circ \text{R}$.

With T_{2s} known, Eq. (2) yields

$$\begin{aligned} \left(\frac{\dot{W}_{cv}}{\dot{m}a} \right)_s &= (124.27 - 215.03) + 0.0082 \left(\frac{4122 - 7188.9}{18.02} \right) \\ &= -90.76 - 1.40 = -92.16 \text{ Btu/lb(a)} \end{aligned}$$

Equation (3) then gives

$$\left(\frac{\dot{W}_{cv}}{\dot{m}a} \right) = \frac{-92.16}{0.8} = -115.2 \frac{\text{Btu}}{\text{lb(a)}} \longleftarrow \frac{\dot{W}_{cv}}{\dot{m}a}$$

Finally, with $(\dot{W}_{cv}/\dot{m}a)$ known, Eq. (1) can be used iteratively to find T_2 .

Thus, upon rearrangement

$$\begin{aligned} h_a(T_2) + \frac{\omega}{M_v} \bar{h}_v(T_2) &= h_a(T_1) + \frac{\omega}{M_v} \bar{h}_v(T_1) - \left(\frac{\dot{W}_{cv}}{\dot{m}a} \right) \\ &= 124.27 + \frac{0.0082}{18.02} (4122) + 115.2 \\ &= 241.35 \text{ Btu/lb(a)} \end{aligned}$$

Solving this expression iteratively gives $T_2 \approx 987^\circ \text{R} \longleftarrow T_2$

The energy destruction can be obtained using $(\dot{E}_d/\dot{m}a) = T_0 (\dot{\sigma}_{cv}/\dot{m}a)$, where $\dot{\sigma}_{cv}$ is obtained from an entropy balance:

$$\begin{aligned} (\dot{\sigma}_{cv}/\dot{m}a) &= \left[s_2^0(T_2) - s_2^0(T_1) - \frac{\bar{R}}{M_a} \ln \frac{P_2}{P_1} \right] + \omega \left[\frac{s_v^0(T_2) - s_v^0(T_1) - \bar{R} \ln \frac{P_2}{P_1}}{M_v} \right] \\ &= \left[0.7472 - 0.59172 - \frac{1.986}{28.97} \ln \frac{100}{14.7} \right] + (0.0082) \left[\frac{50.079 - 44.821 - 1.986 \ln(100/14.7)}{18.02} \right] \\ &= 0.0240 + 0.0007 = 0.0247 \text{ Btu/lb(a) \cdot } ^\circ \text{R} \end{aligned}$$

$$\text{Then, } (\dot{E}_d/\dot{m}a) = 520^\circ \text{R} (0.0247 \frac{\text{Btu}}{\text{lb(a) \cdot } ^\circ \text{R}}) = 12.84 \frac{\text{Btu}}{\text{lb(a)}} \longleftarrow \frac{\dot{E}_d}{\dot{m}a}$$

b) The data for the required plots are obtained using IT, as follows:

Continued on next slide

Problem 12-62 continued

IT Code

```
T1 = 520
phi1 = 0.75
p1 = 14.7
p2 = 100
w = w_Tphi(T1, phi1, p1)
mdota = 1 // Assume a unit mass flow rate.
```

```
Wdot / mdota = (ha1 - ha2) + w * (hv1 - hv2)
Wdots / mdota = (ha1 - ha2s) + w * (hv1 - hv2s)
Wdot / mdota = (Wdots / mdota) / eta_c
eta_c = 0.8
```

```
ha1 = h_T("Air", T1)
ha2 = h_T("Air", T2)
hv1 = h_T("H2O", T1)
hv2 = h_T("H2O", T2)
ha2s = h_T("Air", T2s)
hv2s = h_T("H2O", T2s)
```

```
pg1 = Psat_T("Water/Steam", T1)
pv1 = phi1 * pg1
yv = pv1 / p1
ya = 1 - yv
pa1 = p1 - pv1
pv2 = yv * p2
pa2 = p2 - pv2
```

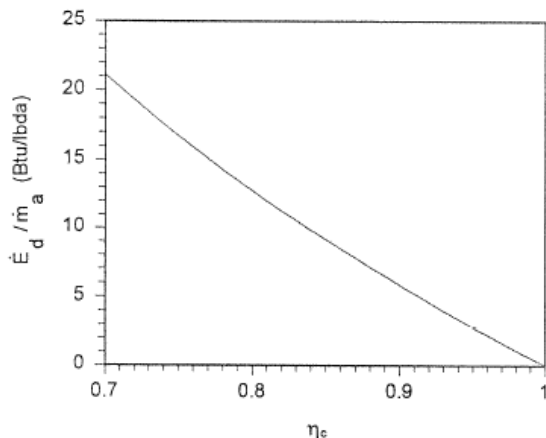
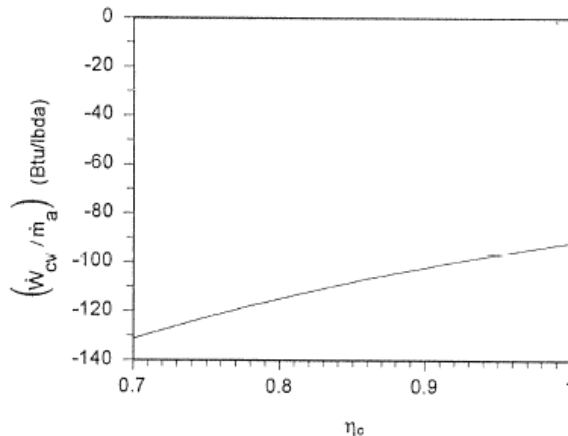
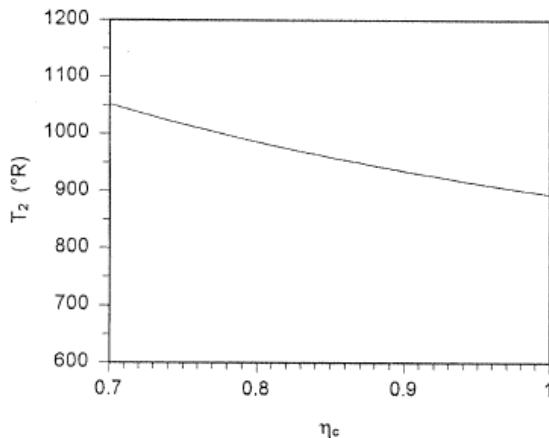
```
sa1 = s_TP("Air", T1, pa1)
sa2s = s_TP("Air", T2s, pa2)
sv1 = s_TP("H2O", T1, pv1)
sv2s = s_TP("H2O", T2s, pv2)
(sa1 - sa2s) + w * (sv1 - sv2s) = 0
sa2 = s_TP("Air", T2, pa2)
sv2 = s_TP("H2O", T2, pv2)
```

```
Edot_d / mdota = To * ((sa2 - sa1) + w * (sv2 - sv1))
To = 520
```

IT Results for $\eta_c = 0.8$

```
pa1 = 14.51 lbf/in.2
pa2 = 98.68 lbf/in.2
pv1 = 0.1945 lbf/in.2
pv2 = 1.323 lbf/in.2
T2s = 894.4 °R
w = 0.008341
 $\left(\dot{W}_{cv} / \dot{m}_a\right)_s = -91.98 \text{ Btu} / \text{lb}(\text{da})$ 
T2 = 986.1 °R
 $\left(\dot{W}_{cv} / \dot{m}_a\right) = -115 \text{ Btu} / \text{lb}(\text{da})$ 
 $\dot{E}_d / \dot{m}_a = 12.73 \text{ Btu} / \text{lb}(\text{da})$ 
```

PLOTS:



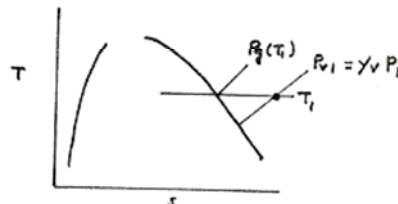
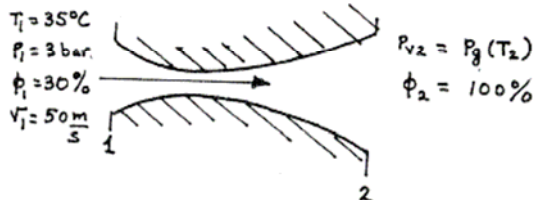
From the plots, we see that lower isentropic compressor efficiency corresponds to increased exit temperature, power input, and exergy destruction, as expected.

PROBLEM 12.63

KNOWN: Moist air expands isentropically through a nozzle. Conditions at the inlet are specified.

FIND: Determine the lowest exit pressure that can be obtained without condensation and the corresponding exit velocity.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The nozzle is well insulated and at steady state. (2) There are no significant potential energy effects. (3) The moist air acts as an ideal gas, with each component adhering to the Dalton model. (4) At the exit the moist air is saturated.

ANALYSIS: As the expansion is isentropic, $s_2 - s_1 = 0$, where

$$s_1 = m_{fa} s_a(T_1, y_{a1} P_1) + m_{fv} s_v(T_1, P_{v1})$$

$$s_2 = m_{fa} s_a(T_2, y_{a2} P_2) + m_{fv} s_v(T_2, P_{v2})$$

Since $P_{v2} = P_g(T_2)$, it follows that $y_{v2} P_2 = P_g(T_2)$ or $P_2 = P_g(T_2) / y_{v2}$. Also, at 2 the specific entropy of the water vapor is $s_g(T_2)$. Collecting results

$$m_{fa} [s_a(T_2, y_{a2} P_2) - s_a(T_1, y_{a1} P_1)] + m_{fv} [s_g(T_2) - s_v(T_1, P_{v1})] = 0$$

$$\underbrace{P_{v1}}_{= P_g(T_2) / y_{v1}} = s_g(T_1) - \frac{\bar{R}}{M_v} \ln \frac{P_{v1}}{P_g(T_1)} = s_g(T_1) - \frac{\bar{R}}{M_v} \ln \phi_1$$

or

$$m_{fa} \left[s_a^\circ(T_2) - s_a^\circ(T_1) - \frac{\bar{R}}{M_a} \ln \frac{P_g(T_2)}{P_{v1}} \right] + m_{fv} \left[s_g(T_2) - s_g(T_1) + \frac{\bar{R}}{M_v} \ln \phi_1 \right] = 0$$

Further, with $m_{fa} = \frac{m_a}{m_a + m_v} = \frac{1}{1 + \omega}$, $m_{fv} = \frac{m_v}{m_a + m_v} = \frac{\omega}{1 + \omega}$, the last equation is

$$\left[s_a^\circ(T_2) - s_a^\circ(T_1) - \frac{\bar{R}}{M_a} \ln \frac{P_g(T_2)}{P_{v1}} \right] + \omega \left[s_g(T_2) - s_g(T_1) + \frac{\bar{R}}{M_v} \ln \phi_1 \right] = 0 \quad (1)$$

With $\phi_1 = 0.30$ and $P_g(T_1)$ from Table A-2, $P_{v1} = 0.3(0.05628 \text{ bars}) = 0.01688 \text{ bar}$.
Too

$$\omega = 0.622 \frac{P_{v1}}{P_1 - P_{v1}} = 0.622 \frac{(0.01688)}{(3 - 0.01688)} = 0.00352 \frac{\text{kg(v)}}{\text{kg(a)}}$$

Substituting known values into Eq. (1)

$$\left[s_a^\circ(T_2) - 1.7284 - \frac{8.314}{28.97} \ln \frac{P_g(T_2)}{0.01688} \right] + 0.00352 \left[s_g(T_2) - 8.3531 + \frac{8.314}{18.02} \ln 0.3 \right] = 0$$

$$\left[s^\circ(T_2) - 1.7284 - 0.287 \ln \frac{P_g(T_2)}{0.01688} \right] + 0.00352 \left[s_g(T_2) - 8.9086 \right] = 0$$

Solving this equation iteratively using data from Tables A-2 and A-22,
① $T_2 \approx 10.4^\circ\text{C}$.

Continued on next slide

Problem 12-63 continued

With $P_{v2} = y_v P_2$ and $P_{v1} = y_v P_1$, $P_{v2}/P_{v1} = P_2/P_1$. Also, $P_{v2} = P_g(T_2)$, so

$$P_2 = \frac{P_g(T_2)}{P_{v1}} P_1 = \left(\frac{0.0126}{0.01688} \right) 3 \text{ bar} = 2.24 \text{ bar} \quad \longleftarrow P_2$$

To evaluate V_2 , write an energy rate balance for a control volume enclosing the nozzle. With $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and no kinetic and potential energy effects, this reduces to

$$0 = \left[\dot{m}_a h_a(T_1) + \dot{m}_v h_v(T_1) + (\dot{m}_a + \dot{m}_v) \frac{V_1^2}{2} \right] - \left[\dot{m}_a h_a(T_2) + \dot{m}_v h_v(T_2) + (\dot{m}_a + \dot{m}_v) \frac{V_2^2}{2} \right]$$

Dividing by $\dot{m}_a$, and using $\omega = \dot{m}_v/\dot{m}_a$, together with $h_v(T_1) \approx h_g(T_1)$ and $h_v(T_2) = h_g(T_2)$,

$$0 = \left[h_a(T_1) + \omega h_g(T_1) + (1 + \omega) \frac{V_1^2}{2} \right] - \left[h_a(T_2) + \omega h_g(T_2) + (1 + \omega) \frac{V_2^2}{2} \right]$$

Solving for $V_2^2/2$

$$\frac{V_2^2}{2} = \frac{V_1^2}{2} + \frac{h_a(T_1) - h_a(T_2) + \omega(h_g(T_1) - h_g(T_2))}{1 + \omega}$$

or

$$V_2 = \sqrt{V_1^2 + \frac{2}{1 + \omega} [h_a(T_1) - h_a(T_2) + \omega(h_g(T_1) - h_g(T_2))]}$$

$$= \sqrt{\left(50 \frac{\text{m}}{\text{s}}\right)^2 + \frac{2}{1.00352} \left[308.23 - 283.54 + 0.00352 (2565.3 - 2520.5) \right] \frac{\text{kJ}}{\text{kg}} \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \left| \frac{\text{kg} \cdot \text{m/s}^2}{1 \text{ N}} \right|}$$

$$= 228.1 \text{ m/s} \quad \longleftarrow V_2$$

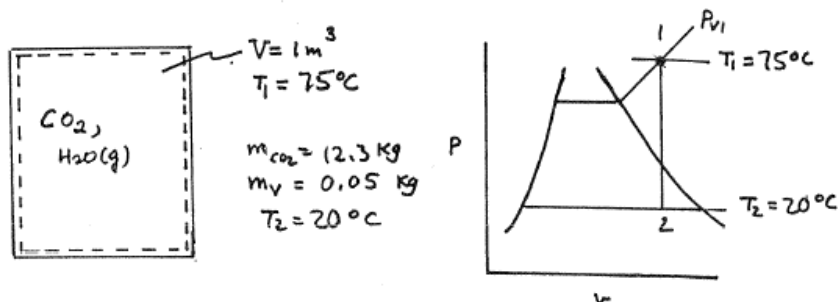
1. An iterative solution using table data can be avoided using IT. The results obtained using IT agree with the values given here.

PROBLEM 12.64

KNOWN: A tank having a known volume initially contains 12.3 kg of CO_2 and 0.05 kg of water vapor at 75°C , 15 bar. The tank contents are cooled to 20°C .

FIND: Determine the heat transfer.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) As shown on the figure, the system is the tank contents. (2) There are no changes in kinetic or potential energy. (3) The gas mixture behaves as an ideal gas, with each component adhering to the Dalton model.

ANALYSIS: The first step is to determine if condensation occurs. As the total volume of the tank and the total mass of water present are constant, the water undergoes a constant specific volume process as the tank contents are cooled (see T - v diagram). The specific volume of the water is $v = 1\text{m}^3/0.05\text{kg} = 20.00\text{m}^3/\text{kg}$. By inspection of v_g data in Table A-2, condensation would occur at a temperature between 38 and 40°C . Accordingly, there is condensation in this case.

An energy balance reduces to give $\Delta U = Q - W$, or $Q = \Delta U$:

$$Q = [m_{\text{CO}_2} u_{\text{CO}_2}(T_2) + m_{v2} u_g(T_2) + m_{w2} u_f(T_2)] - [m_{\text{CO}_2} u_{\text{CO}_2}(T_1) + m_{v1} u_g(T_1)]$$

$$= \frac{m_{\text{CO}_2} [u_{\text{CO}_2}(T_2) - u_{\text{CO}_2}(T_1)]}{M_{\text{CO}_2}} + [m_{v2} u_g(T_2) - m_{v1} u_g(T_1) + m_{w2} u_f(T_2)] \quad (1)$$

To find m_{v2} and m_{w2} , note that the quality of the two-phase liquid-vapor mixture of water at the final state is

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{20.00 - 0.0010018}{57.791 - 0.0010018} = 0.3461$$

Accordingly, $m_{v2} = x_2 m_{v1} = (0.3461)(0.05) = 0.01731\text{ kg}$. So, $m_{w2} = 0.03269\text{ kg}$.

Returning to Eq.(1) and using data from Tables A-2 and A-23

$$Q = \left(\frac{12.3}{44.01}\right)(6739 - 8377) + [(0.01731)(2402.9) - (0.05)(2475.9) + (0.03269)(83.95)]$$

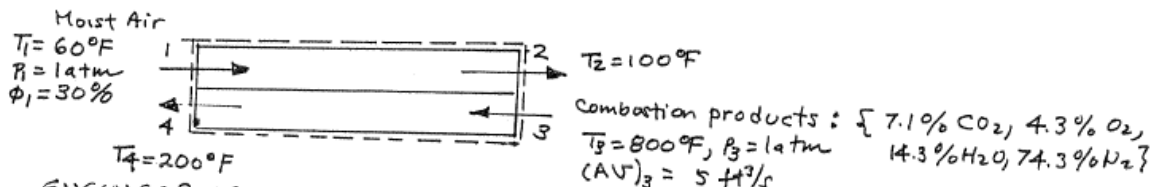
$$= -457.79 + [41.59 - 123.80 + 2.74] = -537.3\text{ kJ} \quad Q$$

PROBLEM 12.65

KNOWN: Steady state operating data are provided for a counterflow heat exchanger having combustion products flowing on one side and moist air flowing on the other.

FIND: Determine the mass flow rate of the moist air stream.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, kinetic/potential energy effects can be ignored, and there is no significant pressure drop on either side. (3) The combustion products and the moist air adhere to the Dalton model.

ANALYSIS: At steady state mass rate balances read $\dot{m}_1 = \dot{m}_2$, $\dot{m}_3 = \dot{m}_4$ (or $\dot{n}_3 = \dot{n}_4$). It follows that $\omega_2 = \omega_1$. The molar flow rate of the combustion products is

$$\dot{n}_3 = \frac{(\dot{AV})_3}{v_3} = \frac{(\dot{AV})_3 P_3}{RT_3} = \frac{(5 \text{ ft}^3/\text{s})(14.7 \times 144 \text{ lbf/ft}^2)}{(1545 \frac{\text{ft} \cdot \text{lbf}}{\text{lbmol} \cdot \text{R}})(1260 \text{ R})} = 0.00544 \frac{\text{lbmol}}{\text{s}}$$

Reducing an energy rate balance

$$0 = \dot{m}_a [(h_a + \omega h_g)_1 - (h_a + \omega h_g)_2] - \dot{n}_3 [y_{\text{CO}_2} \Delta \bar{h}_{\text{CO}_2} + y_{\text{O}_2} \Delta \bar{h}_{\text{O}_2} + y_{\text{H}_2\text{O}} \Delta \bar{h}_{\text{H}_2\text{O}} + y_{\text{N}_2} \Delta \bar{h}_{\text{N}_2}]$$

Or, since $\omega_1 = \omega_2$

$$\dot{m}_a = \frac{-\dot{n}_3 [y_{\text{CO}_2} \Delta \bar{h}_{\text{CO}_2} + y_{\text{O}_2} \Delta \bar{h}_{\text{O}_2} + y_{\text{H}_2\text{O}} \Delta \bar{h}_{\text{H}_2\text{O}} + y_{\text{N}_2} \Delta \bar{h}_{\text{N}_2}]}{[(h_{a2} - h_{a1}) + \omega (h_{g1} - h_{g2})]} \quad (1)$$

For the combustion products, ideal gas table data gives

	CO ₂	O ₂	H ₂ O	N ₂
y	0.071	0.043	0.143	0.743
$\bar{h}(1260 \text{ R})$	11,661	9,097	19,355	8,859
$\bar{h}(660 \text{ R})$	5,165	4,594	5,250	4,586
$y \Delta \bar{h}$	-461.2	-193.6	-730.0	-3174.8

From Table A-22E

$$h_a(520 \text{ R}) = 124.27 \text{ Btu/lb}$$

$$h_a(560 \text{ R}) = 133.86$$

From Table A-2E

$$h_g(60 \text{ °F}) = 1087.7 \text{ Btu/lb}$$

$$h_g(100 \text{ °F}) = 1105.0 \text{ Btu/lb}$$

$$\text{Also, } P_{v1} = \phi_1 P_{g1} = 0.30(2.563 \text{ lbf/in}^2) = 0.07689 \frac{\text{lbf}}{\text{in}^2}$$

$$\text{So } \omega = \frac{0.622(0.07689)}{(14.7 - 0.07689)} = 0.0033$$

Inserting values into Eq. (1)

$$\dot{m}_a = \frac{0.00544 [461.2 + 193.6 + 730 + 3174.8]}{(133.86 - 124.27) + 0.0033(1105 - 1087.7)} = 2.57 \text{ lb/s}$$

The total mass flow rate of the moist air stream is

$$\dot{m} = (1 + \omega) \dot{m}_a = (1.0033)(2.57 \frac{\text{lb}}{\text{s}}) = 2.58 \frac{\text{lb}}{\text{s}}$$

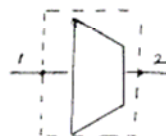
PROBLEM 12.66

KNOWN: Steady state operating data are provided for a compressor handling moist air.

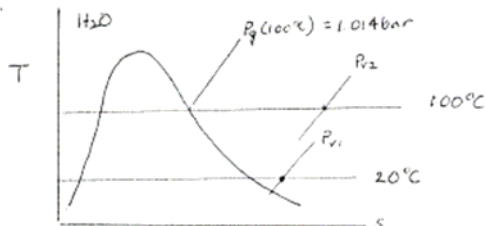
FIND: Determine (a) the relative humidity at the exit, (b) the power input, and (c) the rate of entropy production.

SCHEMATIC & GIVEN DATA:

$$\begin{aligned} T_1 &= 20^\circ\text{C} \\ P_1 &= 1.05 \text{ bar} \\ \phi_1 &= 85\% \\ (\dot{A}V)_1 &= 0.3 \text{ m}^3/\text{s} \end{aligned}$$



$$\begin{aligned} T_2 &= 100^\circ\text{C} \\ P_2 &= 2.0 \text{ bar} \end{aligned}$$



ENGINEERING MODEL: (1) The control volume is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The moist air adheres to ideal gas mixture principles.

ANALYSIS: As the partial pressures are only small fractions of a bar in magnitude, the final state in an adiabatic compression would be far enough from the vapor dome to eliminate the chance of condensation.

(a) Mass rate balances reduce to give $\dot{m}_{a1} = \dot{m}_{a2}$, $\dot{m}_{v1} = \dot{m}_{v2}$, or $\omega_2 = \omega_1$. Thus

$$(.622) \frac{P_{v1}}{P_1 - P_{v1}} = (.622) \frac{P_{v2}}{P_2 - P_{v2}} \Rightarrow \frac{P_{v2}}{P_2} = \frac{P_{v1}}{P_1} \Rightarrow P_{v2} = \phi_1 P_g(T_1) \frac{P_2}{P_1} = (0.85)(0.02339) \left(\frac{2}{1.05} \right) = 0.0379 \text{ bar}$$

$$\text{Accordingly, } \phi_2 = P_{v2}/P_g(T_2) = 0.0379/1.014 = 0.037 \text{ (3.7\%)} \quad \leftarrow (a)$$

(b) Reducing an energy rate balance gives after rearrangement

$$\textcircled{1} \quad \dot{W}_{cv} = \dot{m}_a [(h_a + \omega h_v)_1 - (h_a + \omega h_v)_2] = \dot{m}_a [h_{a1} - h_{a2} + \omega (h_{v1} - h_{v2})]$$

where

$$\dot{m}_a = \frac{(\dot{A}V)_1}{V_{a1}} = \left(\frac{\dot{A}V)_1}{\left(\frac{R}{M_a} \frac{T}{P_{a1}} \right)} = \frac{(0.3 \text{ m}^3/\text{s})(1.05 - 0.0199) \times 10^5 \text{ N/m}^2}{\left(\frac{8314 \text{ N} \cdot \text{m}}{\text{kmol} \cdot \text{K}} \right) (293 \text{ K})} = 0.368 \text{ kg(a)/s}$$

$$\omega = .622 \frac{P_{v1}}{P_1 - P_{v1}} = .622 \frac{(0.0199)}{1.0301} = 0.012 \text{ kg(v)/kg(a)}$$

Since there is no condensed phase present, it suffices to use the ideal gas tables, Table A-25 for the water and Table A-22 for the air.

$$\dot{W}_{cv} = -0.368 \left[373.7 - 293.2 + 0.0120 \left(\frac{12433 - 9733}{18} \right) \right] = -30.29 \text{ kW} \quad \text{Power input} = 30.29 \text{ kW} \quad \leftarrow (b)$$

(c) An entropy rate balance reduces to give

$$\begin{aligned} \dot{S}_{cv} &= \dot{m}_a [(s_{a2} - s_{a1}) + \omega (s_{v2} - s_{v1})] \\ &= \dot{m}_a \left[[s_a(T_2) - s_a(T_1) - \frac{R}{M_a} \ln \frac{P_{a2}}{P_{a1}}] + \omega [s_v(T_2) - s_v(T_1) - \frac{R}{M_v} \ln \frac{P_{v2}}{P_{v1}}] \right] \end{aligned}$$

Or with s° data from Table A-22 and S° data from Table A-23

$$\dot{S}_{cv} = 0.368 \left[[1.921 - 1.678 - \frac{8.314}{29} \ln \left(\frac{2.0}{1.05} \right)] + 0.012 \left[\frac{196.284 - 158.139 - 8.314 \ln(2.9/0.01)}{18} \right] \right]$$

$$= 0.368 [0.058 + 0.012 [0.155]]$$

$$= 0.022 \frac{\text{kJ/s}}{\text{K}} = 0.022 \frac{\text{KW}}{\text{K}} \quad \leftarrow (c)$$

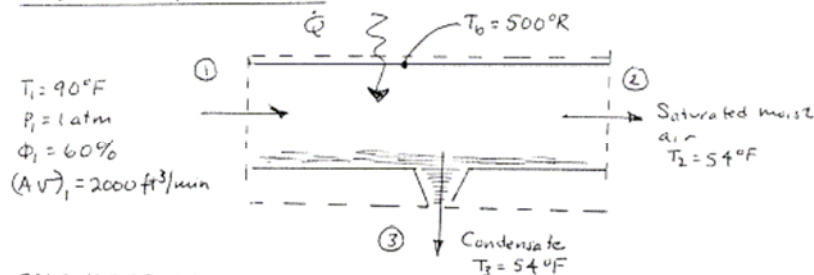
1. Parts (b) and (c) also could be solved on the basis of constant specific heats c_{pa} , c_{pv} evaluated at the mean temperature, for example.

PROBLEM 12.67

KNOWN: Operating data are provided for a control volume at steady state.

FIND: Determine (a) the heat transfer rate, (b) entropy production rate.

SCHEMATIC & GIVEN DATA



ENGINEERING

MODEL: (1) The control volume is at steady state. (2) For the control volume $\dot{W}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The moist air adheres to ideal gas principles. (4) The pressure is uniform at 1 atm.

ANALYSIS: Mass rate balances reduce to give $\dot{m}_a = \dot{m}_2 \equiv \dot{m}_a$, $\dot{m}_v = \dot{m}_2 + \dot{m}_3$.

Thus, $\dot{m}_3 = \dot{m}_a (w_1 - w_2)$. Calculations

$$w_1 = 0.622 \left[\frac{\phi_1 P_{g1}}{P - \phi_1 P_{g1}} \right] = 0.622 \left[\frac{(0.6)(0.6988)}{14.7 - 0.4193} \right] = 0.01826 \frac{\text{lb(v)}}{\text{lb(a)}}$$

$$w_2 = 0.622 \left[\frac{P_{g2}}{P - P_{g2}} \right] = 0.622 \left[\frac{0.2064}{14.7 - 0.2064} \right] = 0.00886 \frac{\text{lb(v)}}{\text{lb(a)}}$$

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{(AV)_1}{\frac{R}{M_a} \frac{T_1}{P_{a1}}} = \frac{(2000 \text{ ft}^3/\text{min})(14.7 \times 144 \text{ lbf/ft}^2)}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{29} \right) (550^\circ\text{R})} = 140.36 \frac{\text{lb(a)}}{\text{min}}$$

An energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_a [(h_a + w h_v)_1 - (h_a + w h_v)_2] - \dot{m}_3 h_f(T_3)$$

or

$$\dot{Q}_{cv} = \dot{m}_a [(h_{a2} - h_{a1}) + w_2 h_{g2} - w_1 h_{g1} + (w_1 - w_2) h_{f3}] - \dot{Q}_{cv}$$

$$= 140.36 [0.24(54 - 90) + 0.00886(1085.1) - 0.01826(1100.7) + (0.0094)(22.1)] = -2655.5 \text{ Btu/min}$$

An entropy rate balance reduces to and

$$0 = \frac{\dot{Q}_{cv}}{T_0} + [\dot{m}_a s_{a1} + \dot{m}_v s_{v1}] - [\dot{m}_a s_{a2} + \dot{m}_v s_{v2}] - \dot{m}_3 s_3 + \dot{Q}_{cv}$$

or

$$\dot{Q}_{cv} = -\frac{\dot{Q}_{cv}}{T_0} + \dot{m}_a [s_{a2} - s_{a1}] + \dot{m}_v s_{v2} - \dot{m}_v s_{v1} + \dot{m}_3 s_3$$

$\uparrow (w_2 \dot{m}_a)$ $\uparrow (w_1 \dot{m}_a)$ $\uparrow (w_1 - w_2) \dot{m}_a$

At (3) the liquid is saturated, $s_3 = s_f(T_3)$. At (2) the vapor is saturated, $s_{v2} = s_{g2}$. At (1), $s_{v1} = s_{g1} - \frac{R}{M} \ln \phi_1$ (see Sec. 12.5.2 for discussion). Thus

$$\dot{Q}_{cv} = -\frac{\dot{Q}_{cv}}{T_0} + \dot{m}_a \left[\left(c_{pa} \ln \frac{T_2}{T_1} - \frac{R}{M} \ln \frac{P_{a2}}{P_{a1}} \right) + w_2 s_{g2} - w_1 \left(s_{g1} - \frac{R}{M} \ln \phi_1 \right) + (w_1 - w_2) s_{f3} \right]$$

$$= \frac{2655.5}{500} + 140.36 \left[\left(0.24 \ln \frac{514}{550} - \frac{1.986}{29} \ln \frac{14.444}{14.281} \right) + (0.00886)(2.1131) - 0.01826 \left(2.0083 - \frac{1.986}{18} \ln(0.6) \right) + (0.0094)(0.04391) \right]$$

$$= 5.311 + 140.36 [-0.0172 + 0.0187 - 0.0377 + 0.0004]$$

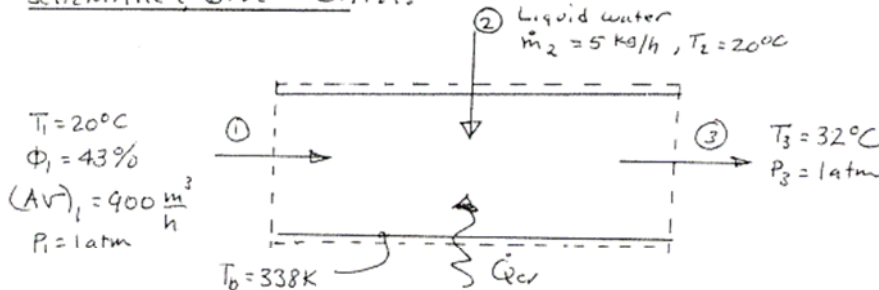
$$= 0.286 \frac{\text{Btu/min}}{^\circ\text{R}}$$

PROBLEM 12.68

KNOWN: Operating data are provided for a control volume at steady state.

FIND: Determine (a) the heat transfer rate, (b) the entropy production rate.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) For the control volume $\dot{W}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The moist air adheres to ideal gas mixture principles.

ANALYSIS: Mass rate balances at steady state read $\dot{m}_{a1} = \dot{m}_{a3} \equiv \dot{m}_a$, $\dot{m}_{v1} + \dot{m}_2 = \dot{m}_{v3}$. Calculating, $P_{a1} = P_1 - P_{v1} = P_1 - \Phi_1 P_{g1} = 1.01325 - (0.43)(0.02339) = 1.0032$ bar.

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{(AV)_1}{\frac{R}{M_a} T_1} = \frac{(400 \text{ m}^3/\text{h}) (1.0032 \times 10^5 \text{ N/m}^2)}{\left(\frac{8314 \text{ N} \cdot \text{m}}{29 \text{ kg} \cdot \text{K}}\right) (293 \text{ K})} = 1074.9 \frac{\text{kg}}{\text{h}}, \quad \frac{\dot{m}_2}{\dot{m}_a} = \frac{5}{1074.9} = 0.00465 \frac{\text{kg}(\text{H}_2\text{O})}{\text{kg}(\text{a})}$$

$$\omega_1 = 0.622 \frac{P_{v1}}{P_1 - P_{v1}} = \frac{(0.622)(0.43)(0.02339)}{1.0032} = 0.0062 \frac{\text{kg}(\text{H}_2\text{O})}{\text{kg}(\text{a})}, \quad \omega_3 = \frac{\dot{m}_2}{\dot{m}_a} + \omega_1 = 0.00965 + 0.0062 = 0.01085$$

An energy rate balance reduces to read

$$0 = \dot{Q}_{cv} + \dot{m}_a [(h_a + \omega h_v)_1 + \frac{\dot{m}_2}{\dot{m}_a} h_2 - (h_a + \omega h_v)_3]$$

or

$$\begin{aligned} \dot{Q}_{cv} &= \dot{m}_a \left[(h_{a3} - h_{a1}) + \omega_3 h_{g3} - \omega_1 h_{g1} - \frac{\dot{m}_2}{\dot{m}_a} h_{f2} \right] \\ &= 1074.9 \left[1.005(32 - 20) + (0.01085)(2559.9) - 0.0062(2538.1) - 0.00465(84) \right] \\ &= (25474.5 \frac{\text{kJ}}{\text{h}}) \left| \frac{\text{h}}{3600 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 7.08 \text{ kW} \end{aligned}$$

An entropy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}}{T_b} + [\dot{m}_a s_{a1} + \dot{m}_{v1} s_{v1}] + \dot{m}_2 s_2 - [\dot{m}_a s_{a3} + \dot{m}_{v3} s_{v3}] + \dot{J}_{cv}$$

or

$$\dot{J}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_a [s_{a3} - s_{a1}] + \dot{m}_{v3} s_{v3} - \dot{m}_{v1} s_{v1} - \dot{m}_2 s_2$$

$\left(\omega_3 \dot{m}_a \right) \left(s_{g3} - \frac{\bar{R}}{M_v} \ln \Phi_3 \right) - \left(\omega_1 \dot{m}_a \right) \left(s_{g1} - \frac{\bar{R}}{M_v} \ln \Phi_1 \right) - \left(\frac{\dot{m}_2}{\dot{m}_a} \dot{m}_a \right) s_{f2}$

So

$$\begin{aligned} \dot{J}_{cv} &= -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_a \left[\left(c_{pa} \ln \frac{T_3}{T_1} - \frac{\bar{R}}{M} \ln \frac{P_{a3}}{P_{a1}} \right) + \omega_3 \left(s_{g3} - \frac{\bar{R}}{M_v} \ln \Phi_3 \right) - \omega_1 \left(s_{g1} - \frac{\bar{R}}{M_v} \ln \Phi_1 \right) - \frac{\dot{m}_2}{\dot{m}_a} s_{f2} \right] \\ &= -\frac{(7.08 \text{ kW})}{338 \text{ K}} + \left(\frac{1074.9 \text{ kg}}{3600 \text{ s}} \right) \left[\left(1.005 \ln \frac{305}{293} - \frac{8.314}{29} \ln \frac{0.99508}{1.0032} \right) + 0.01085 \left(8.4127 - \frac{8.314}{18} \ln 0.365 \right) \right. \\ &\quad \left. - 0.0062 \left(8.6672 - \frac{8.314}{18} \ln 0.43 \right) - 0.00465 (0.2966) \right] \frac{\text{kJ}}{\text{kg}(\text{a})} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= -0.0209 + (0.294) [0.0424 + 0.0963 - 0.0562 - 0.0014] = 0.0033 \frac{\text{kW}}{\text{K}} \end{aligned}$$

PROBLEM 12.69

KNOWN: The dry-bulb and wet-bulb temperatures are provided for each of two cases.

FIND: Using Eq. 12.48, determine w and ϕ for each case.

ENGINEERING MODEL: (1) The wet bulb temperature can be used in place of the adiabatic saturation temperature. (2) $p = 1 \text{ atm}$.

ANALYSIS: (a) $T_{db} = 24^\circ\text{C}$, $T_{wb} = 16^\circ\text{C}$, $p = 1 \text{ atm}$

w' can be found from Eq. 12.48. The value of w' in this expression is found using p_g at the wet bulb temperature from Table A-2

$$w' = 0.622 \left[\frac{0.01818}{1.01325 - 0.01818} \right] = 0.0114 \frac{\text{kg}(v)}{\text{kg}(a)}$$

Equation 12.48 then gives with $c_{pa} = 1.005 \text{ kJ/kg}\cdot\text{K}$ and data from Table A-2

$$w = \frac{c_{pa}(T_{wb} - T) + w'[h_g(T_{wb}) - h_f(T_{wb})]}{h_g(T) - h_f(T_{wb})}$$

$$= \frac{(1.005)(16 - 24) + 0.0114[2463.6]}{(2545.4 - 67.2)} = \frac{-8.04 + 28.09}{2478.2} = 0.0081 \frac{\text{kg}(v)}{\text{kg}(a)}$$

Solving Eq. 12.43 for P_v

$$P_v = \frac{w P}{0.622 + w} = \frac{(0.0081)(1.01325 \text{ bar})}{0.622 + 0.0081} = 0.013 \text{ bar}$$

Then

$$\phi = \frac{P_v}{P_g(T)} = \frac{0.013}{0.02985} = 0.436 \quad (43.6\%)$$

(b) $T_{db} = 75^\circ\text{F}$, $T_{wb} = 60^\circ\text{F}$, $p = 1 \text{ atm}$

w can be found from Eq. 12.48. The value of w' in this expression is found using p_g at the wet bulb temperature from Table A-2E

$$w' = 0.622 \left[\frac{0.2563}{14.7 - 0.2563} \right] = 0.01104 \frac{\text{lb}(v)}{\text{lb}(a)}$$

Equation 12.48 then gives with $c_{pa} = 0.24 \text{ Btu/lb}\cdot^\circ\text{R}$ and data from Table A-2E

$$w = \frac{c_{pa}(T_{wb} - T) + w'[h_g(T_{wb}) - h_f(T_{wb})]}{h_g(T) - h_f(T_{wb})}$$

$$= \frac{0.24(60 - 75) + 0.01104[1059.6]}{1094.25 - 28.08} = \frac{-3.6 + 11.698}{1066.17} = 0.0076 \frac{\text{lb}(v)}{\text{lb}(a)}$$

Solving Eq. 12.43 for P_v

$$P_v = \frac{w P}{0.622 + w} = \frac{(0.0076)(14.696)}{0.622 + 0.0076} = 0.1774 \text{ lbf/in}^2$$

Then

$$\phi = \frac{P_v}{P_g(T)} = \frac{0.1774}{0.4302} = 0.412 \quad (41.2\%)$$

Continued on next slide

Problem 12-69 continued

(c) Referring to Figure A-9 with the data of part (a), $\phi = 44\%$,
 $\omega = 0.0081 \text{ kg(v)/kg(a)}$.

Referring to Figure A-9E with the data of part (b), $\phi = 42\%$,
 $\omega = 0.0078 \text{ lb(v)/lb(a)}$.

(d) IT Solution

$T_a = 24 \text{ // } ^\circ\text{C}$
 $T_{wb_a} = 16 \text{ // } ^\circ\text{C}$
 $p = 1.01325 \text{ // bar}$
 $w_a = w_{TTwb}(T_a, T_{wb_a}, p)$
 $\phi_{a_a} = \phi_{Tw}(T_a, w_a, p)$

IT Results - Part (a)

$\phi = 0.435$
 $\omega = 0.00807$

$T_b = 75 \text{ // } ^\circ\text{F}$
 $T_{wb_b} = 60 \text{ // } ^\circ\text{F}$
 $p = 1 \text{ // atm}$
 $w_b = w_{TTwb}(T_b, T_{wb_b}, p)$
 $\phi_{b_b} = \phi_{Tw}(T_b, w_b, p)$

IT Results - Part (b)

$\phi = 0.4124$
 $\omega = 0.00759$

PROBLEM 12.70

FIND: Using the psychrometric chart, determine

- ϕ , ω , $h_a + \omega h_g$ when $T_{db} = 30^\circ\text{C}$, $T_{wb} = 25^\circ\text{C}$.
- ω , $h_a + \omega h_g$, T_{wb} when $T_{db} = 30^\circ\text{C}$, $\phi = 60\%$
- $T_{\text{dew point}}$ for $T_{db} = 30^\circ\text{C}$, $T_{wb} = 20^\circ\text{C}$
- Repeat (a)-(c) using IT.

ANALYSIS: Using the psychrometric chart ($p = 1 \text{ atm}$)

(a) $T_{db} = 30^\circ\text{C}$, $T_{wb} = 25^\circ\text{C}$

$$\phi = 68\%, \omega = 0.0182, h_a + \omega h_g = 76.3 \text{ kJ/kg(a)}$$

(b) $T_{db} = 30^\circ\text{C}$, $\phi = 60\%$

$$\omega = 0.0163, h_a + \omega h_g = 71.5 \text{ kJ/kg(a)}, T_{wb} \approx 24^\circ\text{C}$$

(c) $T_{db} = 30^\circ\text{C}$, $T_{wb} = 20^\circ\text{C}$, $T_{\text{dew pt}} \approx 15^\circ\text{C}$

(d) Using IT for parts (a), (b), (c)

$$p = 1.01325 \text{ // bar}$$

$$T_a = 30 \text{ // } ^\circ\text{C}$$

$$T_{wb_a} = 25 \text{ // } ^\circ\text{C}$$

$$T_{wb_a} = \text{Twb_Tphi}(T_a, \phi_a, p)$$

$$w_a = w_TTwb(T_a, T_{wb_a}, p)$$

$$h_a = h_a_Tw(T_a, w_a)$$

IT Results for part (a)

$$h_a + \omega h_v = 76.13 \text{ kJ / kg(a)}$$

$$\phi = 0.6706$$

$$\omega = 0.018$$

$$T_b = 30 \text{ // } ^\circ\text{C}$$

$$\phi_b = 0.6$$

$$w_b = w_Tphi(T_b, \phi_b, p)$$

$$h_b = h_a_Tphi(T_b, \phi_b, p)$$

$$T_{wb_b} = \text{Twb_Tphi}(T_b, \phi_b, p)$$

IT Results for part (b)

$$T_{wb} = 23.83 \text{ } ^\circ\text{C}$$

$$h_a + \omega h_v = 71.15 \text{ kJ / kg(a)}$$

$$\omega = 0.01605$$

$$T_c = 30 \text{ // } ^\circ\text{C}$$

$$T_{wb_c} = 20 \text{ // } ^\circ\text{C}$$

$$T_{wb_c} = \text{Twb_Tphi}(T_c, \phi_c, p)$$

$$p_g = \text{Psat_T}(\text{"Water/Steam"}, T_c)$$

$$p_v = \phi_c \cdot p_g$$

$$T_{\text{dewpt}} = \text{Tsat_P}(\text{"Water/Steam"}, p_v)$$

IT Results for part (c)

$$p_g = 0.04246 \text{ bar}$$

$$p_v = 0.0169 \text{ bar}$$

$$\phi = 0.398$$

$$T_{\text{dew point}} = 14.86 \text{ } ^\circ\text{C}$$

PROBLEM 12.71

FIND: Using the psychrometric chart, determine

- T_{dew} for $T_{\text{db}} = 80^\circ\text{F}$, $T_{\text{wb}} = 70^\circ\text{F}$
- ω , $h_a + \omega h_g$, T_{wb} for $T_{\text{db}} = 80^\circ\text{F}$, $\phi = 70\%$
- ϕ , ω , $h_a + \omega h_g$ for $T_{\text{db}} = 80^\circ\text{F}$, $T_{\text{wb}} = 65^\circ\text{F}$
- Repeat (a)-(c) using IT

ANALYSIS: Using the psychrometric chart ($p = 1 \text{ atm}$)

(a) $T_{\text{db}} = 80^\circ\text{F}$, $T_{\text{wb}} = 70^\circ\text{F}$

$$T_{\text{dew}} = 65.5^\circ\text{F}$$

(b) $T_{\text{db}} = 80^\circ\text{F}$, $\phi = 70\%$

$$\omega = 0.0155$$

$$h_a + \omega h_g = 36.1 \text{ Btu/lb(a)}$$

$$T_{\text{wb}} = 72.5^\circ\text{F}$$

(c) $T_{\text{db}} = 80^\circ\text{F}$, $T_{\text{wb}} = 65^\circ\text{F}$

$$\phi = 45\%$$

$$\omega = 0.0099$$

$$h_a + \omega h_g = 30.8 \text{ Btu/lb(a)}$$

(d) Using IT for parts (a), (b), (c)

$$p = 1 \text{ // atm}$$

$$T_a = 80 \text{ // } ^\circ\text{F}$$

$$T_{\text{wb}_a} = 70 \text{ // } ^\circ\text{F}$$

$$T_{\text{wb}_a} = \text{Twb_Tphi}(T_a, \phi_a, p)$$

$$p_g = \text{Psat_T}(\text{"Water/Steam"}, T_a)$$

$$p_v = \phi_a * p_g$$

$$T_{\text{dewpt}} = \text{Tsat_P}(\text{"Water/Steam"}, p_v)$$

$$T_c = 80 \text{ // } ^\circ\text{F}$$

$$T_{\text{wb}_c} = 65 \text{ // } ^\circ\text{F}$$

$$T_{\text{wb}_c} = \text{Twb_Tphi}(T_c, \phi_c, p)$$

$$w_c = w_TTwb(T_c, T_{\text{wb}_c}, p)$$

$$h_c = h_a_Tphi(T_c, \phi_c, p)$$

IT Results for part (c)

$$\phi = 0.447$$

$$\omega = 0.009739$$

$$h_a + \omega h_g = 29.84 \text{ Btu/lb(a)}$$

IT Results for part (a)

$$p_g = 0.03452 \text{ lbf/in.}^2$$

$$p_v = 0.02114 \text{ lbf/in.}^2$$

$$\phi = 0.6124$$

$$T_{\text{dew point}} = 65.46^\circ\text{F}$$

$$T_b = 80 \text{ // } ^\circ\text{F}$$

$$\phi_b = 0.7$$

$$w_b = w_Tphi(T_b, \phi_b, p)$$

$$h_b = h_a_Tphi(T_b, \phi_b, p)$$

$$T_{\text{wb}_b} = \text{Twb_Tphi}(T_b, \phi_b, p)$$

IT Results for part (b)

$$\omega = 0.01539$$

$$h_a + \omega h_g = 36.03 \text{ Btu/lb(a)}$$

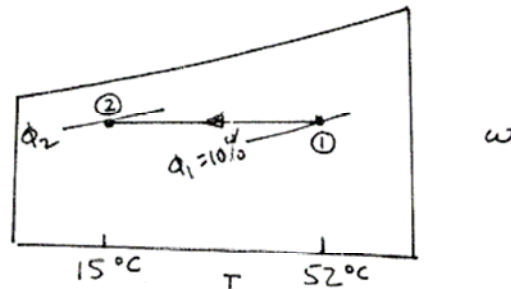
$$T_{\text{wb}} = 72.39^\circ\text{F}$$

PROBLEM 12.72

KNOWN: Moist air at 52°C , 1 atm , $\phi = 10\%$ is cooled at constant pressure to 15°C .

FIND: Using the psychrometric chart, determine whether condensation occurs. If so, find the amount condensed per kg of dry air. If not, determine ϕ at the final state.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: For moist air, ideal gas principles apply.

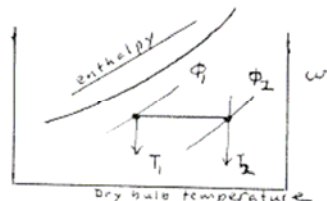
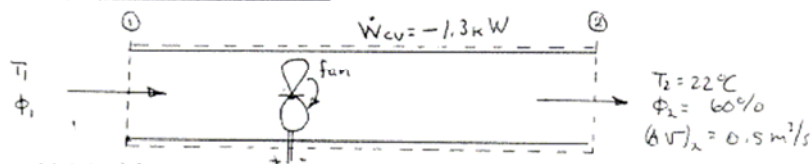
ANALYSIS: As long as there is no condensation the mole fraction of water vapor, and thus P_v remains constant during constant-pressure cooling: $P_v = Y_v p$. Since both P_v and p are constant, Eq. 12.43 shows w remains constant, as shown on the schematic. With the given data, Fig. A-9 indicates that no condensation occurs and $\phi_2 = 80\%$.

PROBLEM 12.73

KNOWN: Steady state operating data are given for a fan within an insulated duct.

FIND: Determine the temperature and relative humidity at the duct inlet.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The moist air adheres to ideal gas mixture principles. (4) As the psychrometric chart is to be used in the solution, pressure is taken at inlet and exit.

ANALYSIS: From mass rate balances on the dry air and water vapor, $\dot{m}_1 = \dot{m}_2$. Reducing an energy rate balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_a [(h_a + w h_v)_1 - (h_a + w h_v)_2]$$

Thus

$$(h_a + w h_v)_1 = (h_a + w h_v)_2 + \dot{W}_{cv} / \dot{m}_a \quad (1)$$

Using ϕ_2, T_2 , the psychrometric chart gives

$$w_2 = 0.0099 \text{ kg(w)/kg(a)}$$

$$(h_a + w h_v)_2 = 47.3 \text{ kJ/kg(a)}$$

$$v_{a2} = 0.849 \text{ m}^3/\text{kg(a)}$$

Then

$$\dot{m}_a = \frac{(\dot{A}V)_2}{v_{a2}} = \frac{0.5 \text{ m}^3/\text{s}}{0.849 \text{ m}^3/\text{kg(a)}} = 0.589 \frac{\text{kg(a)}}{\text{s}}$$

and inserting values in Eq. (1)

$$(h_a + w h_v)_1 = 47.3 \frac{\text{kJ}}{\text{kg(a)}} + \frac{(-1.3 \text{ kJ/s})}{0.589 \text{ kg(a)/s}} = 45.09 \frac{\text{kJ}}{\text{kg(a)}}$$

Then, using the known values for $(h_a + w h_v)_1$ and $w_1 (= w_2)$, Fig A-9 gives

$$T_1 \approx 20^\circ\text{C}$$

$$\phi_1 \approx 68\%$$

$$\rightarrow T_2, \phi_2$$

PROBLEM 12.74

KNOWN: Moist air adheres to ideal gas principles.

FIND: Derive expressions in SI and English units for the mixture enthalpy per unit of mass as used on Figs. A-9, A-9E.

ANALYSIS:

SI CASE.

Turning to the discussion of Fig. A-9 in Sec. 12.9, note that the enthalpy of the dry air is determined relative to a datum of 0°C . That is

$$h_a(T) = \cancel{h_a(T_{\text{ref}})} + c_{pa} [T - T_{\text{ref}}]$$

$\xrightarrow{1.005 \frac{\text{kJ}}{\text{kg}(\text{a})\cdot\text{K}}}$
 $\xleftarrow{T(^{\circ}\text{C})}$

The enthalpy of the water vapor is evaluated as $h_v \approx h_g$ at the dry-bulb temperature. A plot of h_g vs $T(^{\circ}\text{C})$ using data from Tables A-2, 6 shows a nearly linear variation; with h_g being given closely by

$$h_g \approx 2507.1 + \underbrace{1.82 T(^{\circ}\text{C})}_{\approx c_{pv}}$$

ENGLISH CASE.

The enthalpy of the dry air is determined relative to a datum of 0°F . That is

$$h_a(T) = \cancel{h_a(T_{\text{ref}})} + c_{pa} [T - T_{\text{ref}}]$$

$\xrightarrow{0.24 \frac{\text{Btu}}{\text{lb}(\text{a})\cdot^{\circ}\text{R}}}$
 $\xleftarrow{T(^{\circ}\text{F})}$

Plotting h_g vs $T(^{\circ}\text{F})$ using data from Tables A-2E, 6E shows a nearly linear variation, with h_g being given closely by

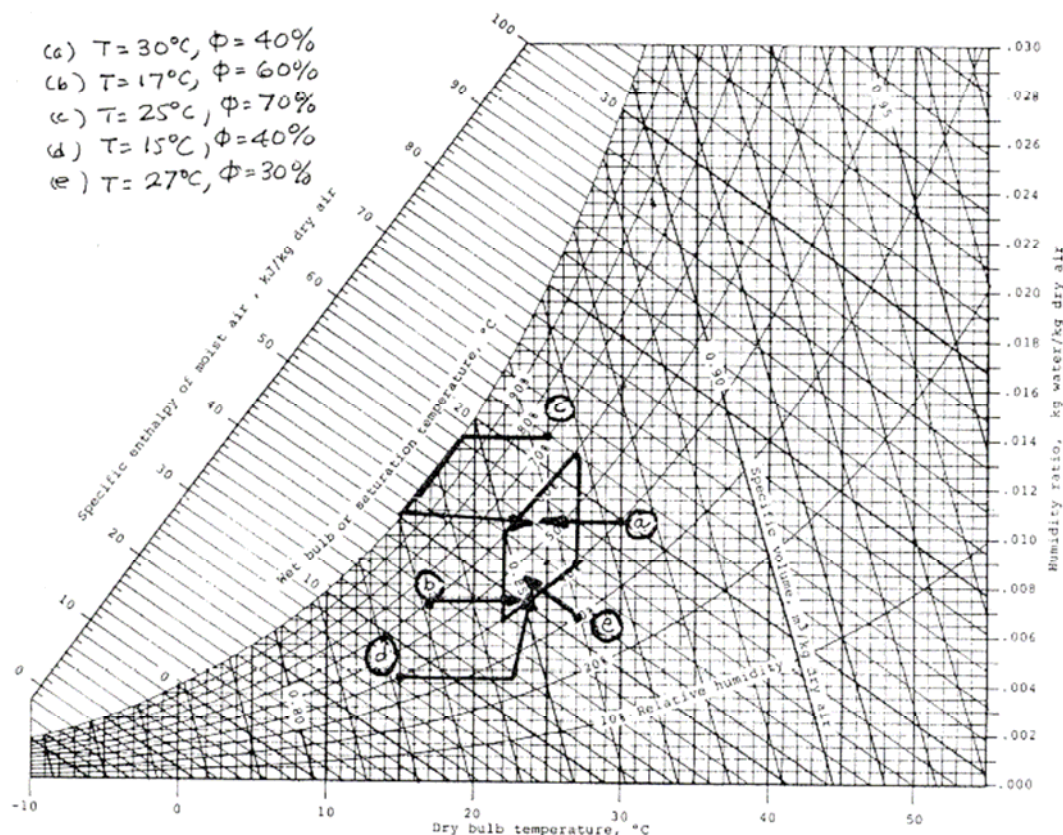
$$h_g \approx 1061 + \underbrace{0.444 T(^{\circ}\text{F})}_{\approx c_{pv}}$$

PROBLEM 12.75

KNOWN: Five cases are provided for the condition of the moist-air stream entering an air-conditioning system, together with the constraints that the exiting moist-air stream must satisfy.

FIND: In each case develop a schematic of equipment and processes from Sec. 12.10 that would achieve the desired result.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The principles of Sec. 12.9 apply.

ANALYSIS:

- (a) Cooling at fixed moisture.
- (b) Heating at fixed moisture.
- ① (c) Dehumidifying followed by reheating. Figure 12.11.
- (d) Heating and humidifying. Figure 12.12.
- (e) Evaporative cooling. Figure 12.13.

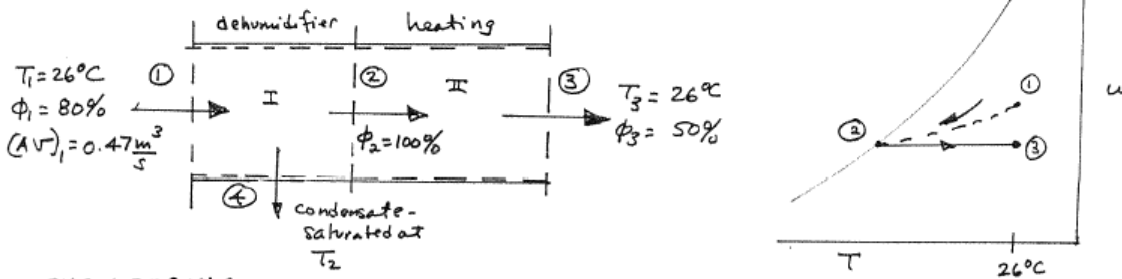
1. The alternatives presented are not the only ones that might be considered. Depending on the condition at the exit, other processes can be proposed.

PROBLEM 12.76

KNOWN: Steady state operating data are provided for an air-conditioning system as shown in Fig 12.11.

FIND: Determine (a) the energy removed by heat transfer in the dehumidifier section, in tons, (b) the energy added by heat transfer in the heating section, in kW.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and kinetic/potential energy effects are negligible. (3) $p = p_{atm}$ throughout. (a) At (2), $\phi = 100\%$. At (4) the condensate is saturated at T_2 . (5) The moist air is modeled as an ideal gas mixture.

ANALYSIS: Since no moisture is added or removed in the heating section, $w_2 = w_3$. And the values of w_1 and w_3 can be found from the psychrometric chart, or by calculation:

$$w_1 = 0.622 \frac{p_{v1}}{p - p_{v1}} \quad w_1 = 0.622 \frac{\phi_1 p_{g1}}{p_1 - \phi_1 p_{g1}} = \frac{(0.622)(0.8)(0.03763)}{1.01325 - (0.8)(0.03763)} = 0.017$$

$$\phi_1 = p_{v1}/p_{g1}$$

$$w_3 = \frac{(0.622)(0.5)(0.03763)}{1.01325 - (0.5)(0.03763)} = 0.0105$$

Since $w_2 = w_3$, and the mixture is saturated at (2)

$$p_{g2} = \frac{w_2 p}{w_2 + 0.622} = \frac{(0.0105)(1.01325)}{0.0105 + 0.622} = 0.0168 \text{ bar} \Rightarrow T_2 \approx 15^\circ\text{C}$$

Also,

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{(AV)_1 p_{a1}}{\left(\frac{R}{M_a}\right) T_1} = \frac{(0.47 \text{ m}^3/\text{s})(0.9863 \times 10^5 \text{ N/m}^2)}{\left(\frac{8314 \text{ N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}\right)(299 \text{ K})} = 0.54 \frac{\text{kg(a)}}{\text{s}}$$

(a) An energy rate balance for control volume I reads

$$0 = \dot{Q}_{cv} + \dot{m}_a [h_{a1} + w_1 h_{g1}] - \dot{m}_a [h_{a2} + w_2 h_{g2}] - \dot{m}_4 h_{f4}$$

A mass balance for water gives $\dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_4 \Rightarrow \dot{m}_4 = \dot{m}_{v1} - \dot{m}_{v2} = \dot{m}_a (w_1 - w_2)$

So

$$\dot{Q}_{cv} = \dot{m}_a [(h_{a2} - h_{a1}) + w_2 h_{g2} - w_1 h_{g1} + (w_1 - w_2) h_{f4}]$$

With data from Tables A-2, A-22

$$\dot{Q}_{cv} = (0.54 \text{ kg(a)/s}) \left[(288.2 - 299.2) + 0.0105(2528.9) - 0.017(2549) \right. \\ \left. + (0.017 - 0.0105)(62.99) \right] \frac{\text{kJ}}{\text{kg(a)}}$$

Continued on next slide

Problem 12-76 continued

That is

$$\dot{Q}_{cv} = 0.54 [-11 + 26.55 - 43.33 + 0.41] = -14.78 \frac{\text{kJ}}{\text{s}}$$

Then

$$\dot{Q}_{cv} = (-14.78 \frac{\text{kJ}}{\text{s}}) \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right| \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = -4.2 \text{ tons} \leftarrow$$

(b) An energy rate balance for control volume II reads

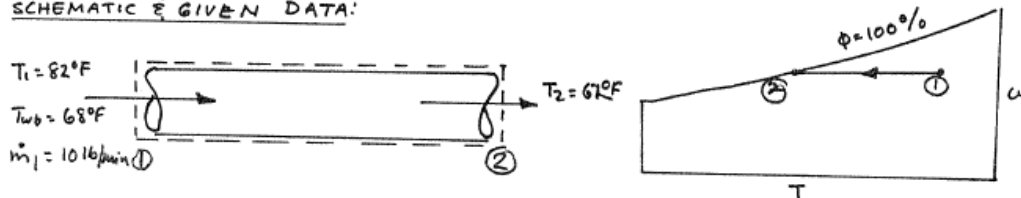
$$\begin{aligned} \dot{Q}_{cv} &= \dot{m}_a [(h_{a3} - h_{a2}) + w (h_{g3} - h_{g2})] \\ &= (0.54 \frac{\text{kg(a)}}{\text{s}}) \left[(299.2 - 288.2) + 0.0105 [2549 - 2528.9] \right] \frac{\text{kJ}}{\text{kg(a)}} \\ &= (0.54) [11 + 0.21] \left(\frac{\text{kJ}}{\text{s}} \right) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \\ &= 6.05 \text{ kW} \leftarrow \end{aligned}$$

PROBLEM 12.77

KNOWN: Moist air enters a duct with a known mass flow rate and is cooled at constant pressure to a specified temperature.

FIND: Using table data, determine (a) the relative humidity at the duct inlet and (b) the net rate of heat transfer. (c) Check the results of parts (a), (b) using the psychrometric chart. (d) Check the results of parts (a), (b) using I.T.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The moist air adheres to ideal gas mixture principles. (3) The wet-bulb temperature can be used in place of T_{as} to evaluate w .

ANALYSIS: (a) To find ϕ_1 , first evaluate w_1 using Eqs. 12.52 and 12.53. Thus

$$w' = 0.622 \frac{P_g(T_{wb})}{P - P_g(T_{wb})} = 0.622 \left(\frac{0.3391}{14.696 - 0.3391} \right) = 0.01469 \frac{\text{lb(v)}}{\text{lb(a)}}$$

and

$$w_1 = \frac{0.24[68 - 82] + 0.01469[1055.1]}{1097.3 - 36.09} = 0.0114 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Solving Eq. 12.43

$$P_{v1} = \frac{w_1 P}{0.622 + w_1} = \frac{(0.0114)(14.696)}{0.622 + 0.0114} = 0.2645 \frac{\text{lb(f)}}{\text{in}^2}$$

Then

$$\phi_1 = \frac{P_{v1}}{P_g(82^\circ\text{F})} = \frac{0.2645}{0.5415} = 0.489 \quad (48.9\%) \quad \leftarrow \phi_1$$

(b) Before applying an energy balance it is necessary to determine if condensation occurs upon cooling. Using P_{v1} , the dew point temperature is $\approx 61^\circ\text{F}$. Accordingly, no condensation takes place. At steady state an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_v h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}]$$

or

$$\dot{Q}_{cv} = \dot{m}_a [(h_{a2} - h_{a1}) + w(h_{g2} - h_{g1})]$$

The total mass flow rate is $\dot{m} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1 + w)$. Thus, $\dot{m}_a = \dot{m}/(1 + w)$, so

$$\begin{aligned} \dot{Q}_{cv} &= \frac{\dot{m}}{1 + w} [(h_{a2} - h_{a1}) + w(h_{g2} - h_{g1})] \\ &= \left(\frac{10}{1.0114} \frac{\text{lb(a)}}{\text{min}} \right) [0.24(62 - 82) + 0.0114(1088.6 - 1097.3)] \frac{\text{Btu}}{\text{lb(a)}} \\ &= \left(\frac{10}{1.0114} \right) (-4.9) = -48.4 \frac{\text{Btu}}{\text{min}} \quad \leftarrow \dot{Q}_{cv} \end{aligned}$$

Continued on next slide

Problem 12-77 continued

(c) In the psychrometric chart solution, inspection of Fig. A-9E using T_1 and ϕ_1 T_{wb} , $\phi_1 \approx 50\%$. Also, the energy rate balance takes the form $\leftarrow \phi_1$

$$\dot{Q}_{cv} = \dot{m}_{1+2} [(h_a + whg)_2 - (h_a + whg)_1]$$

Fig. A-9E gives $(h_a + whg)_2 \approx 27.3 \text{ Btu/lb(a)}$ and $(h_a + whg)_1 \approx 32.2 \text{ Btu/lb(a)}$. Accordingly

$$\therefore \dot{Q}_{cv} = \left(\frac{10}{1.0114} \right) [27.3 - 32.2] = -48.4 \frac{\text{Btu}}{\text{lb}} \leftarrow \dot{Q}_{cv}$$

(d)

IT Code

T1 = 82 // °F

Twb1 = 68 // °F

mdot1 = 10 // lb/min

T2 = 62 // °F

p = 1 // atm

// Check for condensation.

Twb1 = Twb_Tphi(T1, phi1, p)

pg1 = Psat_T("Water/Steam", T1)

pv1 = phi1 * pg1

Tdp1 = Tsat_P("Water/Steam", pv1)

// Result: Tdp = 61.03°F

// No condensation.

w = w_TTwb(T1, Twb1, p)

h1 = ha_Tw(T1, w)

h2 = ha_Tw(T2, w)

mdota1 = mdot1 / (1 + w)

Qdot = mdota1 * (h2 - h1)

IT Results

$h_1 = 32.24 \text{ Btu/lb(a)}$

$h_2 = 27.34 \text{ Btu/lb(a)}$

$\omega = 0.01148$

$T_{dp1} = 61.03^\circ\text{F}$

$\dot{m}_a = 9.886 \text{ lb/min}$

$\phi_1 = 0.491$

$\dot{Q}_{cv} = -48.39 \text{ Btu/min}$

$\leftarrow \phi_1$
 $\leftarrow \dot{Q}_{cv}$

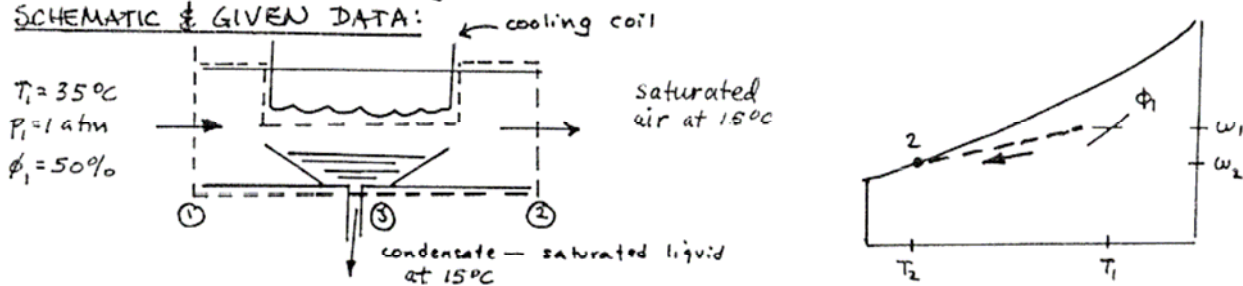
PROBLEM 12.78

KNOWN: Operating data are provided for a dehumidifier at steady state.

FIND: Using table data, determine (a) the heat transfer from the moist air, and (b) the amount of water condensed, each per kg of dry air flowing.

Check the results of parts (a) and (b) using (c) the psychrometric chart, and (d) using IT.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air at 2 is saturated. The condensate at 3 is a saturated liquid. (3) Pressure remains constant throughout at 1 atm.

ANALYSIS: (a) At steady state mass rate balances give $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_a$ and $\dot{m}_1 = \dot{m}_2 + \dot{m}_w$. Thus, the rate water is condensed per unit mass of dry air is

$$\frac{\dot{m}_w}{\dot{m}_a} = w_1 - w_2 \quad (1)$$

To find w_1 , use $P_{v1} = \phi_1 P_g(T_1) = (0.5)(0.05628) = 0.02814$ bar. Then

$$w_1 = 0.622 \left[\frac{0.02814}{1.01325 - 0.02814} \right] = 0.01777 \frac{\text{kg(v)}}{\text{kg(a)}}$$

Since the moist air is saturated at 2, $P_{v2} = P_g(T_2) = 0.01705$ bar, and

$$w_2 = 0.622 \left[\frac{0.01705}{1.01325 - 0.01705} \right] = 0.01065 \frac{\text{kg(v)}}{\text{kg(a)}}$$

These values are closely the same as obtained from Fig A-9. Substituting into Eq. (1)

$$\frac{\dot{m}_w}{\dot{m}_a} = 0.01777 - 0.01065 = 0.00717 \frac{\text{kg(v)}}{\text{kg(a)}} \quad \leftarrow \frac{\dot{m}_w}{\dot{m}_a}$$

At steady state an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_v h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}] - \dot{m}_w h_f$$

Or, with Eq. (1)

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = (h_{a2} - h_{a1}) + w_2 h_{g2} - w_1 h_{g1} + (w_1 - w_2) h_f \quad (2)$$

With $c_{pa} = 1.005$ kJ/kg·K and data from Table A-2 for water vapor

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}_a} &= 1.005 [15 - 35] + 0.01065 (2528.9) - 0.01777 (2565.3) + 0.00717 (62.99) \\ &= -20.1 + 26.93 - 45.59 + 0.452 = -38.31 \frac{\text{kJ}}{\text{kg(a)}} \quad \leftarrow \frac{\dot{Q}_{cv}}{\dot{m}_a} \end{aligned}$$

Continued on next slide

Problem 12-78 continued

(c) In the psychrometric chart solution, the energy rate equation, $E_f(z)$, takes the form

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = (h_a + w h_g)_2 - (h_a + w h_g)_1 + (w_1 - w_2) h_{f3}$$

From Fig A-9; $(h_a + w h_g)_1 \approx 80.6 \text{ kJ/kg(a)}$ and $(h_a + w h_g)_2 \approx 42 \text{ kJ/kg(a)}$. Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = 42 - 80.6 + 0.452 = -38.15 \quad \leftarrow \frac{\dot{Q}_{cv}}{\dot{m}_a}$$

(d)

IT Code

```
T1 = 35 // °C
phi1 = 0.5
T2 = 15 // °C
phi2 = 1
T3 = 15 // °C
p = 1.01325 // bar
mdota = 1 // Assume a unit mass flow rate.
```

```
w1 = w_Tphi(T1, phi1, p)
w2 = w_Tphi(T2, phi2, p)
h1 = ha_Tphi(T1, phi1, p)
h2 = ha_Tphi(T2, phi2, p)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 0)
```

```
mdotw / mdota = w1 - w2
Qdotcv / mdota = h2 - h1 + (w1 - w2) * h3
```

IT Results

```
w1 = 0.01775 kg(v)/kg(a)
w2 = 0.01066 kg(v)/kg(a)
h1 = 80.66 kJ/kg(a)
h2 = 42.02 kJ/kg(a)
h3 = 61.93 kJ/kg(a)
```

```
 $\dot{m}_w / \dot{m}_a = 0.007085 \text{ kg/kg(a)}$  ←
```

```
 $\dot{Q}_{cv} / \dot{m}_a = -38.21 \text{ kJ/kg(a)}$  ←
```

PROBLEM 12.79

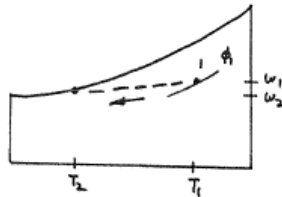
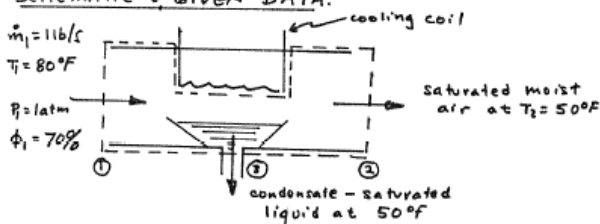
KNOWN: Steady-state operating data are provided for a dehumidifier.

FIND: Using table data, determine (a) the rate of heat transfer from the moist air, (b) the rate water is condensed, (c) Check results using the psychrometric chart. (d) Check results using I.T.

KNOWN: Operating data are provided for a dehumidifier at steady state.

FIND: Determine the heat transfer from the moist air and the rate water is condensed, in tons and lb/s, respectively.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air at 2 is saturated. The condensate at 3 is saturated liquid. (3) Pressure remains constant throughout at 1 atm.

ANALYSIS: At steady state mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$ and $\dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_w$. Thus, the rate water is condensed per unit mass of dry air is

$$\frac{\dot{m}_w}{\dot{m}_a} = w_1 - w_2 \quad (1)$$

To find w_1 , use $P_{v1} = \phi_1 P_g(T_1) = 0.7(0.5073) = 0.3551 \text{ lbf/in}^2$. Then

$$w_1 = 0.622 \left[\frac{0.3551}{14.696 - 0.3551} \right] = 0.0154 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Since the moist air is saturated at 2, $P_{v2} = P_g(T_2) = 0.1780 \text{ lbf/in}^2$, and

$$w_2 = 0.622 \left[\frac{0.1780}{14.696 - 0.1780} \right] = 0.0076 \frac{\text{lb(v)}}{\text{lb(a)}}$$

These values for w_1 and w_2 are closely the same as obtained from Fig. A-9E. Substituting into Eq. (1)

$$\frac{\dot{m}_w}{\dot{m}_a} = 0.0154 - 0.0076 = 0.0078 \frac{\text{lb(v)}}{\text{lb(a)}}$$

At the inlet $\dot{m}_1 = \dot{m}_{v1} + \dot{m}_{a1} = \dot{m}_{a1}(1 + w_1)$. Thus

$$\dot{m}_{a1} = \frac{\dot{m}_1}{1 + w_1} = \frac{1 \text{ lb/s}}{1.0154} = 0.985 \frac{\text{lb(a)}}{\text{s}}$$

Thus

$$\dot{m}_w = \left[0.0078 \frac{\text{lb(v)}}{\text{lb(a)}} \right] \left(0.985 \frac{\text{lb(a)}}{\text{s}} \right) = 0.0077 \frac{\text{lb}}{\text{s}} \quad (b)$$

At steady state an energy rate balance reduces with Eq. (1) to give

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = (h_{a2} - h_{a1}) + w_2 h_{g2} - w_1 h_{g1} + (w_1 - w_2) h_{f3} \quad (2)$$

With $c_p = 0.24 \text{ Btu/lb} \cdot ^\circ\text{R}$

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}_a} &= 0.24(50 - 80) + 0.0076(1083.3) - 0.0154(1096.4) + 0.0078(18.06) \\ &= -7.2 + 8.23 - 16.88 + 0.14 = -15.71 \text{ Btu/lb(a)} \end{aligned}$$

Thus

$$\dot{Q}_{cv} = \left[-15.71 \frac{\text{Btu}}{\text{lb(a)}} \right] \left[0.985 \frac{\text{lb(a)}}{\text{s}} \right] \left| \frac{1 \text{ ton}}{200 \text{ Btu/min}} \right| \left| \frac{60 \text{ s}}{\text{min}} \right| = -4.64 \text{ tons} \quad (a)$$

Continued on next slide

Problem 12-79 continued

(c) By inspection of Fig. A-9E, $\omega_1 = 0.0155$, $\omega_2 = 0.0076$, $(h_a + \omega h_g)_1 = 36.1 \text{ Btu/lb(a)}$, $(h_a + \omega h_g)_2 = 20.2 \text{ Btu/lb(a)}$. Thus, Eq. (1) gives

$$\frac{\dot{m}_w}{\dot{m}_a} = \omega_1 - \omega_2 = 0.0078 \frac{\text{lb(v)}}{\text{lb(a)}}$$
, which agrees well with the table result. Rearranging Eq. (2) to read

$$\begin{aligned} (\dot{Q}_{cv}/\dot{m}_a) &= (h_a + \omega h_g)_2 - (h_a + \omega h_g)_1 + (\omega_1 - \omega_2) h_{f3} \\ &= 20.2 - 36.1 + 0.14 = -15.76 \text{ Btu/lb(a)} \end{aligned}$$

This also agrees with the table result, as expected.

(d) IT Code

```
T1 = 80 // °F
phi1 = 0.7
T2 = 50 // °F
phi2 = 1
T3 = 50 // °F
p = 1 // atm
mdot1 = 1 // lb/s

w1 = w_Tphi(T1, phi1, p)
w2 = w_Tphi(T2, phi2, p)
h1 = ha_Tphi(T1, phi1, p)
h2 = ha_Tphi(T2, phi2, p)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 0)

mdota = mdot1 / (1 + w1)
mdotw / mdota = w1 - w2
Qdotcv / mdota = (h2 - h1 + (w1 - w2) * h3) * (60 / 200) // tons
```

IT Results

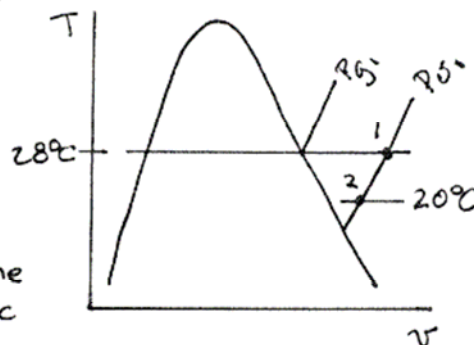
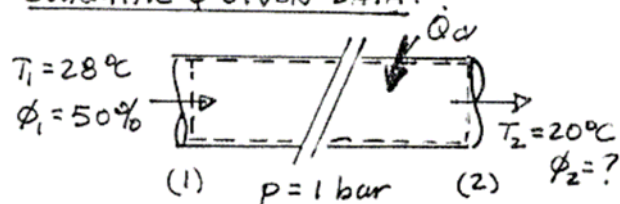
```
omega1 = 0.01539 lb(v)/lb(a)
omega2 = 0.007624 lb(v)/lb(a)
h1 = 36.03 Btu/lb(a)
h2 = 20.23 Btu/lb(a)
h3 = 17.67 Btu/lb(a)
m_a = 0.9848 lb/s
m_w = 0.007645 lb/s
Q_cv = -4.628 tons
```

PROBLEM 12.80

KNOWN: Moist air is cooled as it flows through a duct.

FIND: Determine the rate of heat transfer per unit mass of dry air flowing and the relative humidity at the exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state, and $\dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The moist air behaves as an ideal gas mixture. (4) Let $c_{pa} = 1.005 \text{ kJ/kg}\cdot\text{K}$.

ANALYSIS: First, check for condensation: $\phi_1 = 0.5 = \frac{P_{v,1}}{P_{g@T_1}}$

$$P_{v,1} = (0.5)(0.03782 \text{ bar}) = 0.01891 \text{ bar}$$

$\therefore T_{DP,1} \approx 17^\circ\text{C} \Rightarrow \text{no condensation} \Rightarrow \omega_1 = \omega_2 \text{ and } P_{v,1} = P_{v,2}$

The mass and energy rate balances reduce to

$$0 = \dot{Q}_{cv} + \dot{m}_a [(h_{a,1} - h_{a,2}) + \omega(h_{v,1} - h_{v,2})]$$

or

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = c_{pa}(T_2 - T_1) + \omega(h_{v,2} - h_{v,1})$$

Using P_v calculated above; $\omega = .622 \frac{0.01891}{(1 - 0.01891)} = 0.01199$

Thus

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{m}_a} &= (1.005)(20 - 28) \frac{\text{kJ}}{\text{kg}} + (0.01199)(2538.1 - 2552.6) \frac{\text{kJ}}{\text{kg}} \\ &= -8.214 \text{ kJ/kg(a)} \end{aligned} \quad \leftarrow \quad \dot{Q}_{cv}/\dot{m}_a$$

The relative humidity at the exit is

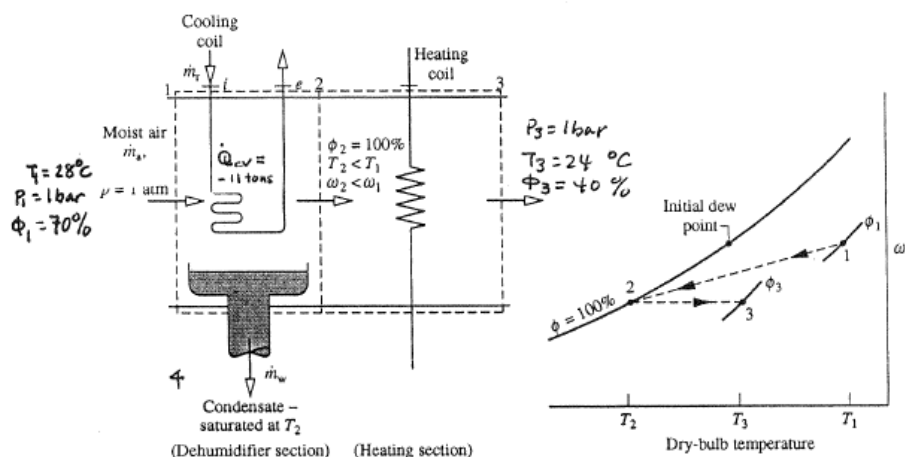
$$\phi_2 = \frac{P_v}{P_{g@T_2}} = \frac{0.01891}{0.02339} = 0.808 \text{ (80.8\%)} \quad \leftarrow \quad \phi_2$$

PROBLEM 12.81

KNOWN: Steady-state operating data are provided for an air conditioner wherein moisture is removed from the moist air, which is then heated.

FIND: Determine (a) the temperature of the air exiting the dehumidifier, (b) the volumetric flow rate at the air conditioner inlet, (c) the rate water is condensed, and (d) the rate of heat transfer to the air passing through the heating unit.

SCHEMATIC & GIVEN DATA



ENGINEERING MODEL: (1) The control volumes shown are at steady state. The coils are not included in the control volumes enclosing the two individual sections. (2) For each control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be ignored. (3) At 2, the moist air is saturated. At 4, the liquid is saturated at T_2 .

ANALYSIS: At steady state, mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_{a3} \equiv \dot{m}_a$, $\dot{m}_{v1} = \dot{m}_w + \dot{m}_{v2}$, $\dot{m}_{v2} = \dot{m}_{v3}$. Accordingly, $\omega_2 = \omega_3$, and

$$\dot{m}_w = \dot{m}_a (\omega_1 - \omega_2) = \dot{m}_a (\omega_1 - \omega_3) \quad (1)$$

Using $\omega_2 = \omega_3$,

$$0.622 \left[\frac{P_g(T_2)}{P - P_g(T_2)} \right] = 0.622 \left[\frac{P_{v3}}{P - P_{v3}} \right] \Rightarrow P_g(T_2) = P_{v3}$$

where $P_{v3} = \phi_3 P_{g3} = 0.4 (0.02985 \text{ bar}) = 0.01194 \text{ bar}$. Interpolating in Table A-2 with $P_g(T_2) = 0.01194 \text{ bar}$ gives $T_2 \approx 9.6^\circ\text{C}$ (a)

Then, with $P_{v1} = \phi_1 P_{g1} = (0.7)(0.03782) = 0.02647 \text{ bar}$

$$\omega_1 = 0.622 \left[\frac{0.02647}{1 - 0.02647} \right] = 0.01691 \frac{\text{kg(v)}}{\text{kg(a)}}, \quad \omega_2 = \omega_3 = 0.622 \left[\frac{0.01194}{1 - 0.01194} \right] = 0.007516$$

An energy rate balance on the dehumidifier reads

$$\begin{aligned} 0 &= \dot{Q}_{cv} - \dot{Q}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}] - \dot{m}_w h_w \\ 0 &= \dot{Q}_{cv} + \dot{m}_a [h_{a1} + \omega_1 h_{v1}] - \dot{m}_a [h_{a2} + \omega_2 h_{v2}] - \dot{m}_a (\omega_1 - \omega_2) h_w \\ 0 &= \dot{Q}_{cv} + \dot{m}_a \left[\underbrace{(h_{a1} - h_{a2})}_{c_p a (T_1 - T_2)} + \omega_1 \underbrace{h_{g1}}_{h_{g1}} - \omega_2 \underbrace{h_{g2}}_{h_{g2}} - (\omega_1 - \omega_2) \underbrace{h_{f2}}_{h_{f2}} \right] \end{aligned}$$

Continued on next slide

Problem 12-81 continued

Solving

$$\begin{aligned}\dot{m}a &= \frac{\dot{Q}_{cv}}{c_{pa}(T_2 - T_1) + \omega_2 h_{g2} - \omega_1 h_{g1} + (\omega_1 - \omega_2) h_{f2}} \\ &= \frac{(-11 \text{ tons}) \left| \frac{211 \text{ kJ/min}}{\text{ton}} \right|}{\left[1.005[9.6 - 20] + 0.007516(2519.06) - 0.01691(2552.6) + (0.0094)(40.33) \right] \frac{\text{kJ}}{\text{kg(a)}}} \\ &= 54.81 \text{ kg(a)/min}\end{aligned}$$

The volumetric flow rate at 1 is then

$$(AV)_1 = \dot{m}a v_{a1} = \dot{m}a \frac{RT_1}{(P_1 - P_{v1})} = \left(54.81 \frac{\text{kg(a)}}{\text{min}} \right) \frac{\left(\frac{8314 \text{ N}\cdot\text{m}}{\text{kmol}\cdot\text{K}} \right) (301 \text{ K})}{(0.97353 \times 10^5 \text{ N/m}^2)} = 48.63 \frac{\text{m}^3}{\text{min}} \leftarrow$$

Using Eq. (1), the rate water is condensed is

$$\dot{m}_w = (54.81)(0.01691 - 0.007516) = 0.515 \frac{\text{kg(l)}}{\text{min}}$$

Finally, an energy rate balance on the heating section reduces to give

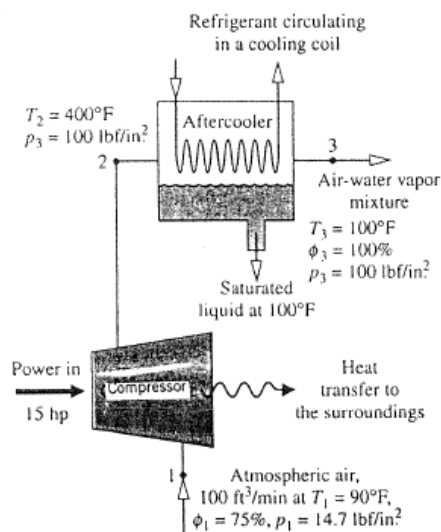
$$\begin{aligned}\dot{Q}_{cv} &= \dot{m}a [(h_{a3} - h_{a2}) + \omega_3 (h_{g3} - h_{g2})] \\ &= 54.81 \left[(1.005)(24 - 9.6) + 0.007516(2545.4 - 2519.06) \right] \\ &= 804.1 \frac{\text{kJ}}{\text{min}} \leftarrow\end{aligned}$$

PROBLEM 12.82

KNOWN: Steady-state operating data are provided for a compressor followed by an aftercooler.

FIND: Determine (a) the rate of heat transfer from the compressor to its surroundings, (b) the mass flow rate of the condensate, (c) the rate of heat transfer from the moist air to the refrigerant.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

(1) The compressor and aftercooler operate at steady state with negligible changes in kinetic and potential energy. (2) There is no stray heat transfer between the aftercooler and its surroundings.

ANALYSIS: At steady state mass rate balances on control volumes enclosing the compressor and aftercooler give $\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_a$, $\dot{m}_v = \dot{m}_v$, $\dot{m}_v = \dot{m}_v + \dot{m}_w$.

Accordingly, $\omega_1 = \omega_2$ and $\dot{m}_w/\dot{m}_a = (\omega_2 - \omega_3) = (\omega_1 - \omega_3)$.

To find ω_1 and ω_3 , $P_{v1} = \phi_1 P_{g1} = (0.75)(0.6988) = 0.5241 \text{ lbf/in}^2$ and $P_{v3} = \phi_3 P_{g3} = 0.9503 \text{ lbf/in}^2$. Thus

$$\omega_1 = 0.622 \left[\frac{0.5241}{14.7 - 0.5241} \right] = 0.023 \frac{\text{lb(v)}}{\text{lb(a)}}, \quad \omega_3 = 0.622 \left[\frac{0.9503}{100 - 0.9503} \right] = 0.00597 \frac{\text{lb(v)}}{\text{lb(a)}}$$

The mass flow rate of the dry air can be evaluated using the volumetric flow rate at 1 and the ideal gas equation of state:

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{P_{a1}(AV)_1}{RT_1} = \frac{[(14.7 - 0.5241) \text{ lbf/in}^2](100 \text{ ft}^3/\text{min})}{(1545/28.97 \text{ ft} \cdot \text{lbf/lb} \cdot \text{mol})(550^\circ \text{R})} = 6.96 \frac{\text{lb(a)}}{\text{min}}$$

The mass flow rate of condensate is then

$$\dot{m}_w = \dot{m}_a(\omega_1 - \omega_3) = 6.96(0.023 - 0.00597) = 0.1185 \frac{\text{lb}}{\text{min}} \quad \leftarrow (b)$$

An energy rate balance at steady state for a control volume enclosing the compressor gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_v h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}]$$

Continued on next slide

Problem 12-82 continued

or $\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}_a [(h_{a2} - h_{a1}) + w_1 (h_{v2} - h_{v1})]$
 with data from Tables A-2E, 22E $\uparrow h_{a2} \uparrow h_{v1}$

$$\dot{Q}_{cv} = (-15 \text{ hp}) \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right| \left| \frac{1 \text{ h}}{60 \text{ min}} \right| + (6.96 \frac{\text{lb(a)}}{\text{min}}) [(206.46 - 131.46) + 0.023(1202 - 1100.7)] \frac{\text{Btu}}{\text{lb(a)}}$$

$$= -636.25 + 538.22 = -98.03 \text{ Btu/min} \quad \leftarrow (a)$$

An energy rate balance at steady state for a control volume enclosing the aftercooler, but excluding the refrigerant line

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}] - [\dot{m}_a h_{a3} + \dot{m}_{v3} h_{v3}] - \dot{m}_w h_f$$

Thus, with $h_v \approx h_g$

$$-\dot{Q}_{cv} = \dot{m}_a \{ (h_{a2} - h_{a3}) + w_2 h_{g2} - w_3 h_{g3} \} - \dot{m}_w h_{f4}$$

$$= 6.96 \{ (206.46 - 133.86) + (0.023)(1202) - (0.00597)(1105) \} - (0.1185)(68.05)$$

$$= 6.96 \{ 72.6 + 27.65 - 6.60 \} - 8.06$$

$$= 643.74 \frac{\text{Btu}}{\text{min}}$$

1 ton of refrigeration = 200 Btu/min (Sec. 10.2.1). Thus

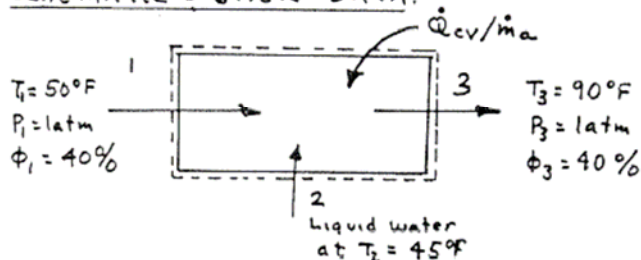
$$-\dot{Q}_{cv} = (643.74 \frac{\text{Btu}}{\text{min}}) \left| \frac{1 \text{ ton}}{200 \text{ Btu/min}} \right| = 3.22 \text{ tons} \quad \leftarrow (c)$$

PROBLEM 12.83

KNOWN: Operating data are provided for an air conditioner at steady state.

FIND: Determine the amount of liquid water injected and the heat transfer rate, each per lb of dry air.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The liquid water enters as saturated liquid and the moist air streams adhere to ideal gas mixture principles.

ANALYSIS: (a) At steady state, mass rate balances give $\dot{m}_a = \dot{m}_2 = \dot{m}_a$, $\dot{m}_v1 + \dot{m}_w = \dot{m}_v2$. Accordingly, the rate liquid is injected is

$$\dot{m}_w = \dot{m}_v3 - \dot{m}_v1 = \dot{m}_a (\omega_3 - \omega_1) \quad (1)$$

To find ω_1 and ω_3 , $P_{v1} = \phi_1 P_g(T_1) = 0.4(0.17816\text{ lbf/in}^2) = 0.0712\text{ lbf/in}^2$ and

$P_{v3} = \phi_3 P_g(T_3) = 0.4(0.6988) = 0.2795\text{ lbf/in}^2$. Then

$$\omega_1 = 0.622 \left[\frac{0.0712}{14.696 - 0.0712} \right] = 0.00303 \frac{\text{lb(v)}}{\text{lb(a)}}, \quad \omega_3 = 0.622 \left[\frac{0.2795}{14.696 - 0.2795} \right] = 0.01206$$

To get $\dot{m}_a$, use $\dot{m}_a = (AV)_3/\nu_{a3}$ with the ideal gas equation

$$\dot{m}_a = \frac{(P - P_{v3})(AV)_3}{R_a T_3} = \frac{(14.696 - 0.2795) \frac{\text{lbf}}{\text{in}^2} (1000 \text{ ft}^3/\text{min}) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|}{\left(\frac{1545}{28.97} \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} \right) (90 + 460) ^\circ\text{R}} = 70.77 \text{ lb/min}$$

Inserting values into (1)

$$\dot{m}_w = 70.77 (0.01206 - 0.00303) = 0.639 \text{ lb/min} \quad \leftarrow \dot{m}_w$$

(b) An energy balance at steady state reduces to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_v1 h_{v1}] + \dot{m}_w h_2 - [\dot{m}_a h_{a3} + \dot{m}_v3 h_{v3}]$$

or

$$\dot{Q}_{cv} = \dot{m}_a [(h_{a3} - h_{a1}) + \omega_3 h_{g3} - \omega_1 h_{g1}] - \dot{m}_w h_2$$

With data from Tables A-2E and A-2ZE

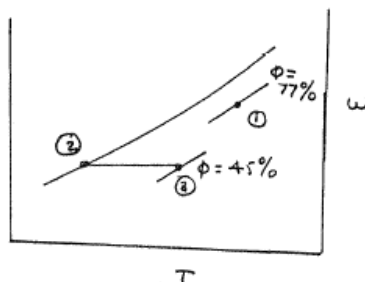
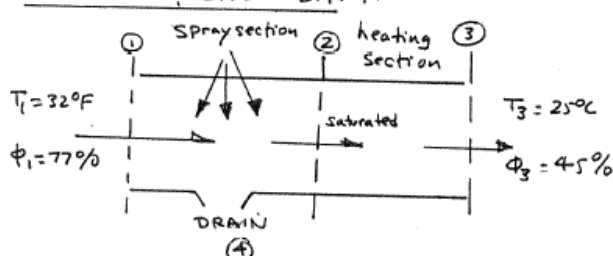
$$\begin{aligned} \dot{Q}_{cv} &= (70.77 \frac{\text{lb}}{\text{min}}) [(131.46 - 121.88) + (0.01206)(1100.7) - (0.00303)(1083.3)] \frac{\text{Btu}}{\text{lb}} \\ &\quad - (0.639)(13.04) \frac{\text{Btu}}{\text{min}} \\ &= (1376.8 \frac{\text{Btu}}{\text{min}}) \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 8.26 \times 10^4 \text{ Btu/h} \quad \leftarrow \dot{Q}_{cv} \end{aligned}$$

PROBLEM 12.84

KNOWN: An air-conditioning system operating at steady state consists of a spray section followed by a reheater. Moist air at 32°C , $\phi = 77\%$ enters the system and passes through the spray section, exiting saturated. The moist air is then heated at fixed moisture content to 25°C , $\phi = 45\%$.

FIND: Determine (a) the temperature of the moist air leaving the spray section, (b) the change in the amount of water vapor contained in the moist air passing through the system. Locate the principal states on a psychrometric chart.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Two control volumes at steady state are under consideration: one enclosing the overall system and one enclosing the heating section. (2) $p = 1 \text{ atm}$ throughout. (3) At 2 the moist air is saturated. (4) Ideal gas principles apply to the moist air.

ANALYSIS: (a) As no moisture is added or removed from the heating section, $w_2 = w_3$. To find w_3

$$w_3 = 0.622 \frac{p_{v3}}{p - p_{v3}} = 0.622 \left(\frac{\phi_3 p_{g3}}{p - \phi_3 p_{g3}} \right) = 0.622 \left(\frac{(0.45)(0.03169)}{1.01325 - (0.45)(0.03169)} \right) = 0.0089$$

Then, since the moist air is saturated at 2

$$w_2 = 0.622 \frac{p_{g2}}{p - p_{g2}} \Rightarrow p_{g2} = \frac{w_2 p}{w_2 + 0.622} = \frac{(0.0089)(1.01325)}{0.0089 + 0.622} = 0.01429 \text{ bar}$$

Interpolating in Table A-2, $T_2 \approx 12.3^\circ\text{C}$ $\leftarrow T_2$

(b) Considering next the overall system

$$\textcircled{1} \quad \left[\begin{array}{l} \text{change in the} \\ \text{amount of water} \\ \text{vapor contained} \\ \text{in the moist air} \end{array} \right] = \dot{m}_{v3} - \dot{m}_{v1} = \dot{m}_a (w_3 - w_1)$$

Thus

$$\left[\begin{array}{l} \text{change in the} \\ \text{amount of water} \\ \text{vapor contained} \\ \text{in the moist air} \\ \text{per unit mass of} \\ \text{dry air} \end{array} \right] = w_3 - w_1, \text{ where } w_1 = 0.622 \left[\frac{\phi_1 p_{g1}}{p - \phi_1 p_{g1}} \right] \\ = 0.622 \left[\frac{(0.77)(0.04759)}{(1.01325) - (0.77)(0.04759)} \right] \\ = 0.0233 \\ = 0.0089 - 0.0233 = -0.0144 \text{ kg/kg(d.a.)} \leftarrow (\text{Removed})$$

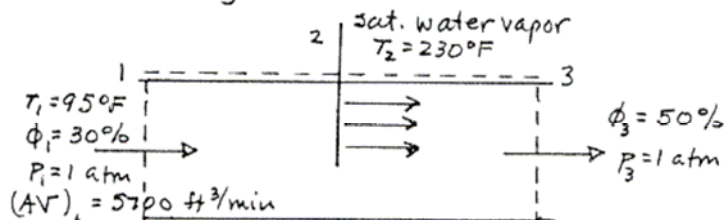
1. Note that neither the temperature nor the flow rate of the spray is provided. The analysis in this case uses the mass balance but not the energy balance.

PROBLEM 12.85

KNOWN: Operating data are provided for a steam-spray humidification device at steady state.

FIND: Determine the temperature of the exiting moist air stream and the rate of steam injection.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air streams adhere to the ideal gas model.

ANALYSIS: Governing Equations. The state at 3 will be determined using mass and energy rate balances. For mass, $\dot{m}_a = \dot{m}_{a3} \equiv \dot{m}_a$ and $\dot{m}_{v1} + \dot{m}_2 = \dot{m}_{v3}$. Thus

$$\dot{m}_2 = \dot{m}_a (\omega_3 - \omega_1) \quad (1)$$

To get $\dot{m}_a$, use

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{(p - p_{v1})(AV)_1}{R_a T_1} \quad (2)$$

The energy balance at steady state reduces as follows:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_a [(h_{a1} + \omega_1 h_{v1}) + (\omega_3 - \omega_1) h_2 - (h_{a3} + \omega_3 h_{v3})]$$

$$\text{or } (h_{a3} + \omega_3 h_{g3}) = (h_{a1} + \omega_1 h_{g1}) + (\omega_3 - \omega_1) h_{g2} \quad (3)$$

Evaluating Properties. To find ω_1 , use $p_{v1} = \phi_1 p_g(T_1)$ and $\omega_1 = 0.622 (p_{v1} / (p - p_{v1}))$. With T_1 and p known, ω_1 , h_{a1} , and h_{g1} can be determined. Also, h_{g2} can be determined since T_2 is known.

The value of ω_3 is related to T_3 using $p_{v3} = \phi_3 p_g(T_3)$ and $\omega_3 = 0.622 (p_{v3} / (p - p_{v3}))$. Also, h_{a3} and h_{g3} are related directly to T_3 .

Solution Procedure. We see from (3) and the discussion of property evaluation that T_3 can be found using an iterative procedure with data from Tables A-22E and A-2E. The following IT code provides an alternative to the iterative procedure:

IT Code

```
T1 = 95 // °F
p = 14.696
phi1 = 0.3
AV1 = 5700 // ft³/min
T2 = 230 // °F
x2 = 1
phi3 = 0.5
```

```
w1 = w_Tphi(T1, phi1, p)
h1 = ha_Tw(T1, w1)
va1 = va_Tw(T1, w1, p)
psat = Psat_T("Water/Steam", T2)
h2 = hsat_Px("Water/Steam", psat, x2)
w3 = w_Tphi(T3, phi3, p)
h3 = ha_Tphi(T3, phi3, p)

mdota = AV1 / va1
mdotv = mdota * (w3 - w1)
0 = mdota * (h1 - h3) + mdotv * h2
```

IT Results

```
ω1 = 0.01053 lb(v)/lb(a)
ω3 = 0.01877 lb(v)/lb(a)
h1 = 34.37 Btu/lb(a)
h2 = 1157 Btu/lb(a)
h3 = 43.9 Btu/lb(a)
ṁa = 400.9 lb/min
ṁ2 = 3.304 lb/min ← ṁ2
T3 = 96.79°F ← T3
```

To check the validity of the IT results, Fig A-9E gives $h_1 = 34.4$, $\omega_1 = 0.0106$, which agree closely with the respective IT answers. Also, referring to Fig A-9E with ω_3 and h_3 from IT, we get $T_3 \approx 96.5^\circ\text{F}$ which also agrees well with the IT value.

PROBLEM 12.86

KNOWN: Consider the steam-spray humidifier of Problem 12.85.

FIND: Determine the exergy destruction rate.

SCHEMATIC & GIVEN DATA: See solution to Problem 12.85 for the schematic.

State 1	State 2	State 3
$T_1 = 95^\circ\text{F}$	$T_2 = 230^\circ\text{F}$	$T_3 = 96.79^\circ\text{F}$
$P_1 = 14.696 \text{ lbf/in.}^2$	Sat. vapor	$P_3 = 14.696 \text{ lbf/in.}^2$
$\omega_1 = 0.01053 \text{ lb(v)/lb(a)}$	$\dot{m}_2 = 3.304 \text{ lb/min}$	$\omega_3 = 0.01877 \text{ lb(v)/lb(a)}$
$\dot{m}_a = 400.9 \text{ lb/min}$		

ENGINEERING MODEL: See Problem 12.85. Also let $T_0 = 95^\circ\text{F} = 550^\circ\text{R}$.

ANALYSIS: The exergy destruction rate can be evaluated using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is found from an entropy rate balance

$$0 = \sum_j \dot{Q}_j \frac{T_j}{T_0} + \dot{m}_a [s_a(T_1, P_{a1}) + \omega_1 s_v(T_1, P_{v1})] + \dot{m}_2 s_g(T_2) - \dot{m}_a [s_a(T_3, P_{a3}) + \omega_3 s_v(T_3, P_{v3})] + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \underbrace{\dot{m}_a [s_a(T_3, P_{a3}) + \omega_3 s_v(T_3, P_{v3})]}_{s(T_3, \omega_3, P_3)} - \underbrace{\dot{m}_a [s_a(T_1, P_{a1}) + \omega_1 s_v(T_1, P_{v1})]}_{s(T_1, \omega_1, P_1)} - \dot{m}_2 s_g(T_2) \quad (1)$$

As noted, the terms in brackets are each uniquely determined from the given data.

① These functions are provided by IT. Thus, $\dot{E}_d$ is determined using IT as follows:

IT Code

T1 = 95 // °F
 p1 = 14.696 // lbf/in.²
 w1 = 0.01053 // lb(v)/lb(a)
 mdota = 400.9 // lb/min
 T2 = 230 // °F
 x2 = 1
 T3 = 96.79 // °F
 p3 = 14.696 // lbf/in.²
 w3 = 0.01877 // lb(v)/lb(a)
 To = 555 // °R

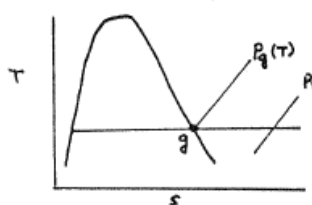
s1 = sa_Tw(T1, w1, p1)
 s3 = sa_Tw(T3, w3, p3)
 psat = Psat_T("Water/Steam", T2)
 s2 = ssat_Px("Water/Steam", psat, x2)
 sigmadot = mdota * (s3 - s1 - (w3 - w1) * s2)
 Edotd = To * sigmadot

IT Results

$\dot{\sigma}_{cv} = 1.526 \text{ Btu/min} \cdot ^\circ\text{R}$

$\dot{E}_d = 846.8 \text{ Btu/min} \leftarrow \dot{E}_d$

1. For states of water vapor where the ideal gas model applies, Eq. 6.18 can be invoked to write



$$s(T, P_v) - s(T, P_g) = \int_{P_g}^{P_v} \frac{v_g dP}{T} - R \ln \frac{P_v}{P_g}$$

or

$$s(T, P_v) = s_g(T) - \frac{\bar{R}}{M_v} \ln \frac{P_v}{P_g} \quad \underbrace{\ln \frac{P_v}{P_g}}_{\phi}$$

Further, applying Eq. 6.20b to the dry air

$$s(T, P_a) - s(T_{ref}, P_{ref}) = s^o(T) - s^o(T_{ref}) - R_a \ln \frac{P_a}{P_{ref}}$$

$$s(T, P_a) = s^o(T) - s^o(T_{ref}) - \frac{\bar{R}}{M_a} \ln \frac{(P - P_v)}{P_{ref}}$$

Thus

$$s(T, \omega, p) = [s^o(T) - s^o(T_{ref}) - \frac{\bar{R}}{M_a} \ln \frac{(P - P_v)}{P_{ref}}] + \omega [s_g(T) - \frac{\bar{R}}{M_v} \ln \phi]$$

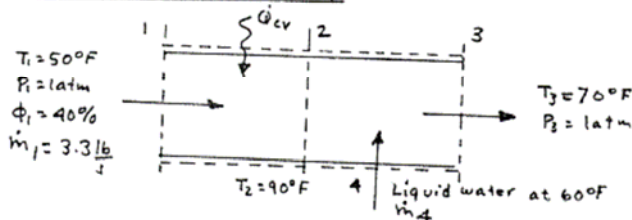
The functions in IT are based on this relationship. The development of a hand calculation to verify the IT result is left to the reader.

PROBLEM 12.87

KNOWN: Operating data are provided for an air conditioner at steady state.

FIND: Determine (a) the rate of heat transfer to the air passing through the heating section, (b) the rate water is injected, (c) the relative humidity at the exit of the humidification section.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volumes in the accompanying figure are at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. For the humidification unit, $\dot{Q}_{cv} = 0$. (2) The liquid enters saturated and the moist air streams adhere to the ideal gas model.

ANALYSIS: (a) At steady state, mass rate balances give $\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_a$, $\dot{m}_1 = \dot{m}_2$, $\dot{m}_3 = \dot{m}_2 + \dot{m}_w$. Thus $\dot{m}_w = \dot{m}_3 - \dot{m}_2 = \dot{m}_a (w_3 - w_1)$.

An energy rate balance for the first control volume reduces to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_v h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}]$$

or

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = [h_{a2} - h_{a1}] + w [h_{g2} - h_{g1}] \quad (1)$$

The humidity ratio is found from Eq 12.43: $P_{v1} = \phi_1 P_g(T_1) = 0.40(0.178) = 0.0712 \text{ lbf/in}^2$. So

$$w_1 = 0.622 \frac{P_{v1}}{P - P_{v1}} = 0.622 \left[\frac{0.0712}{14.696 - 0.0712} \right] = 0.00303 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Then, Eq. (1) gives with $c_{pa} = 0.24 \text{ Btu/lb} \cdot ^\circ\text{R}$

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = 0.24[90 - 50] + 0.00303[1100.7 - 1083.3] = 9.65 \text{ Btu/lb(a)}$$

To find $\dot{m}_a$, write $\dot{m}_1 = \dot{m}_v + \dot{m}_a = \dot{m}_a(1 + w_1) \Rightarrow$

$$\dot{m}_a = \frac{\dot{m}_1}{1 + w_1} = \frac{3.3 \text{ lb/s}}{1.00303} = 3.29 \frac{\text{lb(a)}}{\text{s}}$$

So

$$\dot{Q}_{cv} = \left(3.29 \frac{\text{lb(a)}}{\text{s}} \right) (9.65 \frac{\text{Btu}}{\text{lb(a)}}) = 31.75 \text{ Btu/s} \quad (a)$$

(c) The relative humidity ϕ_3 can be found from w_3 which is obtained from an energy rate balance on the the second control volume:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}] + \dot{m}_4 h_4 - [\dot{m}_a h_{a3} + \dot{m}_v h_{v3}]$$

or with $\dot{m}_4 = \dot{m}_a(w_3 - w_1)$ and $w_2 = w_1$

$$0 = (\dot{h}_{a2} + w_1 \dot{h}_{g2}) + (w_3 - w_1) \dot{h}_{f4} - [\dot{h}_{a3} + w_3 \dot{h}_{g3}]$$

Solving for w_3

$$w_3 = \frac{(\dot{h}_{a2} - \dot{h}_{a3}) + w_1(\dot{h}_{g2} - \dot{h}_{g3})}{\dot{h}_{g3} - \dot{h}_{f4}} = \frac{0.24[90 - 70] + 0.00303[1100.7 - 28.08]}{1092 - 28.08} = 0.00757 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Then, solving Eq 12.43

$$P_{v3} = \frac{w_3 P_3}{0.622 + w_3} = \frac{(0.00757)(14.696)}{0.62957} = 0.1767 \frac{\text{lbf}}{\text{in}^2} \Rightarrow \phi_3 = \frac{P_{v3}}{P_{g3}} = \frac{0.1767}{0.7632} = 0.487 \text{ (48.7\%)} \quad (c)$$

(b) A mass rate balance on water reads $\dot{m}_4 + \dot{m}_v = \dot{m}_v$, or

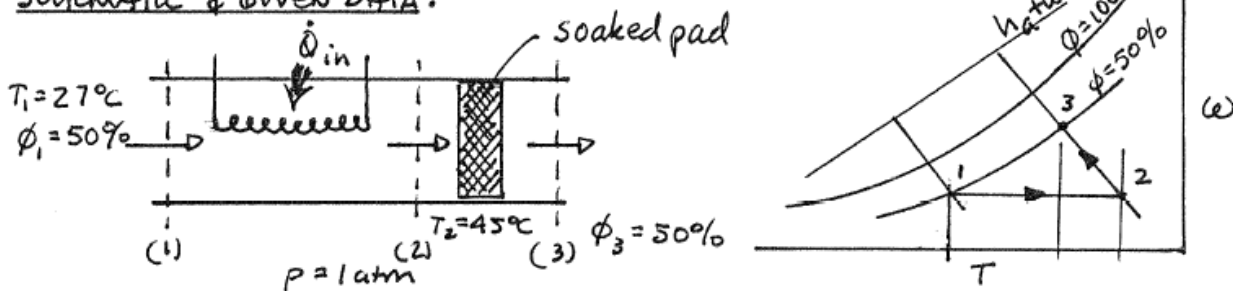
$$\dot{m}_4 = \dot{m}_v - \dot{m}_v = \dot{m}_a(w_3 - w_1) = 3.29 \frac{\text{lb(a)}}{\text{s}} [0.00757 - 0.00303] \frac{\text{lb(v)}}{\text{lb(a)}} = 0.0149 \frac{\text{lb}}{\text{s}} \quad (b)$$

PROBLEM 12-88

KNOWN: Moist air is heated and then passes through a soaked pad evaporative cooling unit. Data are known at various locations.

FIND: Determine (a) the entering humidity ratio, (b) the rate of heat transfer in the heating section per unit mass of moist air flowing, and (c) the humidity ratio and temperature at the exit of the evaporative cooling section.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each section is considered a control volume at steady state, with $\dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The moist air mixture is modeled as an ideal gas mixture. (4) For the cooling section, $\dot{Q}_{cv} = 0$ and the process is modeled as occurring at constant wet bulb temperature, hence, $h_2 \approx h_3$ on the psychrometric chart.

ANALYSIS: (a) From Fig. A-9; $\omega_1 = 0.0112 \text{ kg(v)/kg(a)}$ ← ω_1

(b) For the heating section, $\omega_1 = \omega_2 \equiv \omega$. Thus

$$0 = \dot{Q}_{in} + \dot{m}_a [(h_a + w h_v)_1 - (h_a + w h_v)_2]$$

Further, at location (1): $\dot{m}_1 = \dot{m}_a + \dot{m}_v = \dot{m}_a (1 + \omega)$

Thus $\dot{m}_a = \dot{m}_1 / (1 + \omega)$

$$\dot{Q}_{in} = [\dot{m}_1 / (1 + \omega)] [(h_a + w h_v)_2 - (h_a + w h_v)_1]$$

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = \left(\frac{1}{1 + \omega} \right) [(h_a + w h_v)_2 - (h_a + w h_v)_1]$$

$$= \left(\frac{1}{1 + 0.0112} \right) [(74.3) - (55.5)] = 18.59 \frac{\text{kJ}}{\text{kg (mixture)}} \leftarrow \frac{\dot{Q}_{in}}{\dot{m}_1}$$

(c) From Fig. A-9; $\omega_3 = 0.016$ ← ω_3

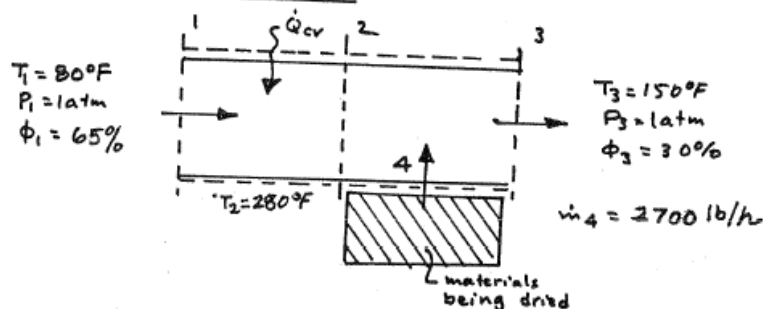
$T_3 = 33^\circ\text{C}$ ← T_3

PROBLEM 12.89

KNOWN: Operating data are provided for an industrial dryer at steady state.

FIND: Determine the mass flow rate of dry air required and the rate of heat transfer to the moist air flowing through the dryer.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volumes in the accompanying figure are at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air streams adhere to the ideal gas model.

ANALYSIS: At steady state mass rate balances give $\dot{m}_a = \dot{m}_2 = \dot{m}_3 = \dot{m}_a$ and $\dot{m}_v = \dot{m}_2$, $\dot{m}_v = \dot{m}_2 + \dot{m}_4 = \dot{m}_3 + \dot{m}_4$. The last of these can be expressed as

$$w_3 \dot{m}_a = w_1 \dot{m}_a + \dot{m}_4 \Rightarrow \dot{m}_a = \frac{\dot{m}_4}{w_3 - w_1}$$

To find w_1 and w_3 : $P_{v1} = \phi_1 P_g(T_1) = (0.65)(0.5073) = 0.3297 \text{ lbf/in}^2$, $P_{v3} = \phi_3 P_g(T_3) = 0.3(3.722) = 1.1166 \text{ lbf/in}^2$. Then with Eq. 12.43

$$w_1 = 0.622 \left[\frac{0.3297}{14.696 - 0.3297} \right] = 0.01427 \frac{\text{lb(v)}}{\text{lb(a)}}, \quad w_3 = 0.622 \left[\frac{1.1166}{14.696 - 1.1166} \right] = 0.05115 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Thus

$$\dot{m}_a = \frac{2700 \text{ lb(v)/h}}{(0.05115 - 0.01427) \frac{\text{lb(v)}}{\text{lb(a)}}} = 73,210 \frac{\text{lb(a)}}{\text{h}} \quad \leftarrow \dot{m}_a$$

An energy rate balance on the heating unit reduces to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_v h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}]$$

or

$$\dot{Q}_{cv} = \dot{m}_a [(h_{a2} - h_{a1}) + w (h_{g2} - h_{g1})]$$

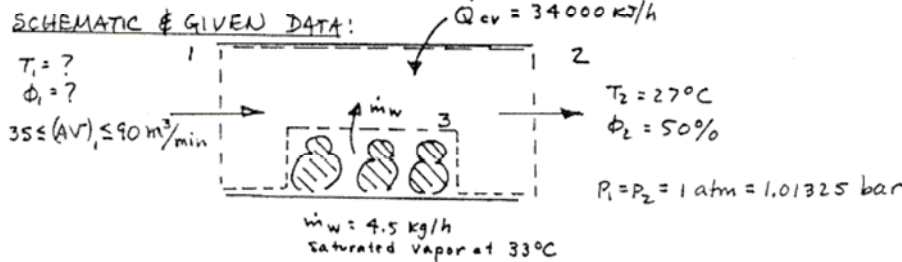
$$= 73,210 \left[177.23 - 129.06 + 0.01427 (1174.1 - 1096.4) \right]$$

$$= 3.609 \times 10^6 \text{ Btu/h} \quad \leftarrow \dot{Q}_{cv}$$

PROBLEM 12.90

KNOWN: Data are provided for moist air being supplied and removed from an occupied class room.

FIND: (a) For a specified supply air volumetric flow rate, determine the supply air temperature and relative humidity. (b) Plot supply air temperature and relative humidity versus supply air volumetric flow rate ranging from 35 to 90 m³/min.



ENGINEERING MODEL: (1) The control volume in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moisture entering from the occupants is saturated vapor at 33°C . The moist air streams adhere to the ideal gas model. Pressure is uniform throughout at 1 atm.

ANALYSIS: At steady state, mass rate balances give: $\dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$; $\dot{m}_{v1} + \dot{m}_w = \dot{m}_{v2}$. Thus $\dot{m}_w = \dot{m}_a(\omega_2 - \omega_1)$, or

$$\dot{m}_a = \frac{\dot{m}_w}{(\omega_2 - \omega_1)} \quad (1)$$

The humidity ratio ω_2 is found using $P_{v2} = \phi_2 P_g(T_2)$ and $\omega_2 = 0.622 [P_{v2}/(p - P_{v2})]$. To determine ω_1 and T_1 , we use an energy rate balance, which reduces at steady state to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] + \dot{m}_w h_3 - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}]$$

or

$$0 = \dot{Q}_{cv} + \dot{m}_a [(h_{a1} - h_{a2}) + \omega_1 h_{g1} - \omega_2 h_{g2}] + \dot{m}_w h_{g3} \quad (2)$$

With T_2 and T_3 known, the specific enthalpies h_{g2} and h_{g3} can be determined. Thus Eq. (1) has two unknowns: ω_1 and $\dot{m}_a$. Equation (2) has the same two unknowns and also has T_1 as an unknown.

Another expression relating these quantities is obtained from $(AV)_1 = \dot{m}_a v_{a1}$, as follows:

$$(AV)_1 = \frac{\dot{m}_w}{(\omega_2 - \omega_1)} \left[\frac{(\bar{R}/M_a) T_1}{P_{a1}} \right] = \frac{\dot{m}_w (\bar{R}/M_a) T_1}{(\omega_2 - \omega_1) (p - P_{v1})} \quad (3)$$

$$\text{where } \omega_1 = 0.622 \left[\frac{P_{v1}}{(p - P_{v1})} \right]. \quad (4)$$

(a) **Sample calculation** using table data, $(AV)_1 = 40 \text{ m}^3/\text{min}$. To evaluate ω_2

$$P_{v2} = \phi_2 P_g(T_2) = 0.5 (0.03567) = 0.01784 \text{ bar. Thus}$$

$$\omega_2 = 0.622 \left[\frac{(0.01784)}{(1.01325 - 0.01784)} \right] = 0.0111 \frac{\text{kg}(v)}{\text{kg}(a)}$$

Thus

$$\dot{m}_a = \frac{4.5 \text{ kg}(v)/\text{h}}{(0.0111 - \omega_1) \frac{\text{kg}(v)}{\text{kg}(a)}} \quad (a)$$

Solving Eq. (2) for ω_1

$$\omega_1 = \frac{\omega_2 [\dot{Q}_{cv}/\dot{m}_w + h_{g3} - h_{g2}] + (h_{a1} - h_{a2})}{(\dot{Q}_{cv}/\dot{m}_w + h_{g3} - h_{g1})}$$

Continued on next slide

Problem 12-90 continued

From Table A-2E, $h_{g2} = 2550.8$ and $h_{g3} = 2561.7$ kJ/kg. With $h_{a1} - h_{a2} = c_{pa}(T_1 - T_2)$ and $c_{pa} = 1.005$ kJ/kg·K, we get

$$\omega_1 = \frac{(0.0111) \left[(34000/4.5) + 2561.7 - 2550.8 \right] + 1.005(T_1 - 27)}{[(34000/4.5) + 2561.7 - h_{g1}]} \quad (b)$$

From Eq. (3)

$$40 \frac{\text{m}^3}{\text{min}} = \frac{(4.5 \text{ kg/h}) \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg} \cdot \text{K}} \right) (T_1 + 273) \text{ K}}{(0.0111 - \omega_1)(1.01325 - p_{v1}) \text{ bar}} \left| \frac{1 \text{ h}}{60 \text{ min}} \right| \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| \left| \frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}} \right| \quad (c)$$

From Eq. (4)

$$p_{v1} = \frac{\omega_1 p}{0.622 + \omega_1} = \frac{\omega_1 (1.01325)}{0.622 + \omega_1} \quad (d)$$

Solving (a) - (d) simultaneously using an iterative procedure with data for $h_g(T_1)$ from Table A-2: $T_1 = 15.1^\circ\text{C}$ T_1

$$\omega_1 = 0.009546$$

$$p_{v1} = 0.01532 \text{ bar}$$

To get ϕ_1

$$\phi_1 = p_{v1} / p_{g(T_1)} = 0.01532 / 0.01762 = 0.869 \text{ (86.9\%)} \quad \leftarrow \phi_1$$

(b) The data for the required plots are obtained using IT, as follows:

IT Code

```
p = 1.01325 // bar
T2 = 27
phi2 = 0.5
T3 = 33 // °C
Qdot = 34000 // kJ/h
mdotw = 4.5 // kg/h
AV1 = 40 // m³/min
```

```
phi1 = phi_Tw(T1, w1, p)
h1 = ha_Tw(T1, w1)
h2 = ha_Tphi(T2, phi2, p)
w2 = w_Tphi(T2, phi2, p)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 1)
```

```
va1 = va_Tw(T1, w1, p)
mdota = (AV1 * 60) / va1
mdota * w1 + mdotw = mdota * w2
0 = Qdot + mdota * (h1 - h2) + mdotw * h3
```

IT Results for $(AV)_1 = 40 \text{ m}^3/\text{min}$

$$\omega_2 = 0.01115 \text{ kg(v)/kg(a)}$$

$$h_2 = 55.54 \text{ kJ/kg}$$

$$h_3 = 2561 \text{ kJ/kg}$$

$$\dot{m}_a = 2891 \text{ kg(a)/min}$$

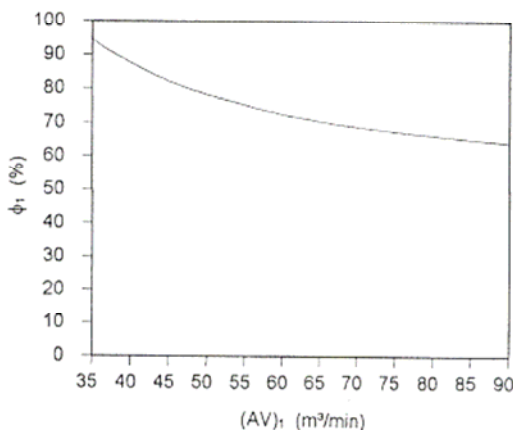
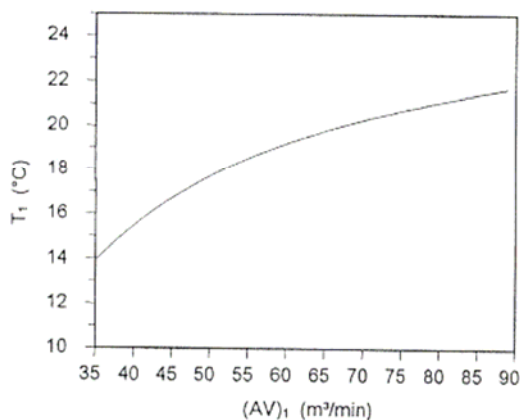
$$h_1 = 39.79 \text{ kJ/kg}$$

$$\omega_1 = 0.009592 \text{ kg(v)/kg(a)}$$

$$T_1 = 15.47^\circ\text{C}$$

$$\phi_1 = 0.8758 \text{ (87.58\%)}$$

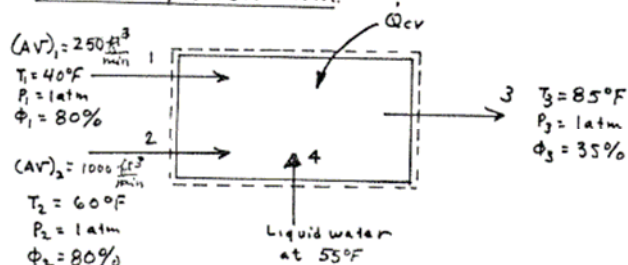
PLOTS:



PROBLEM 12.91

KNOWN: Data are provided for a device for heating and humidifying air.
FIND: Determine the rate of heat transfer to the device and the rate liquid water is injected.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume in the accompanying figure is at steady-state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The liquid enters as saturated liquid. The moist air streams adhere to the ideal gas model.

ANALYSIS: At steady state, mass rate balances give

$$\text{air: } \dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$$

$$\text{water: } \dot{m}_{w1} + \dot{m}_{w2} + \dot{m}_w = \dot{m}_{w3} \Rightarrow \dot{m}_w = \dot{m}_{w3} - \dot{m}_{w1} - \dot{m}_{w2} = \omega_3 \dot{m}_{a3} - \omega_1 \dot{m}_{a1} - \omega_2 \dot{m}_{a2}$$

Combining these results

$$\dot{m}_w = \omega_3 (\dot{m}_{a1} + \dot{m}_{a2}) - \omega_1 \dot{m}_{a1} - \omega_2 \dot{m}_{a2} = \dot{m}_{a1} (\omega_3 - \omega_1) + \dot{m}_{a2} (\omega_3 - \omega_2) \quad (1)$$

To find $\omega_1, \omega_2, \omega_3$: $P_{a1} = \phi_1 P_g(T_1) = 0.8 (0.1217) = 0.09736 \text{ lbf/in}^2$, $P_{a2} = \phi_2 P_g(T_2) = 0.8 (0.2563) = 0.20504 \text{ lbf/in}^2$, $P_{a3} = \phi_3 P_g(T_3) = 0.35 (0.5967) = 0.20885 \text{ lbf/in}^2$. Then

$$\omega_1 = 0.622 \left[\frac{0.09736}{14.696 - 0.09736} \right], \quad \omega_2 = 0.622 \left[\frac{0.20504}{14.696 - 0.20504} \right], \quad \omega_3 = 0.622 \left[\frac{0.20885}{14.696 - 0.20885} \right]$$

$$= 0.004148 \frac{\text{lb(v)}}{\text{lb(a)}} \quad = 0.0088 \frac{\text{lb(v)}}{\text{lb(a)}} \quad = 0.00897 \frac{\text{lb(v)}}{\text{lb(a)}}$$

The volumetric flow rates at 1 and 2, together with the ideal gas equation of state give

$$\dot{m}_{a1} = \frac{(AV)_1}{v_{a1}} = \frac{P_{a1}(AV)_1}{\frac{R}{M_a} T_1} = \frac{(14.696 - 0.09736) \text{ lbf/in}^2 (250 \text{ ft}^3/\text{min})}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot \text{R}} \right) (500 \text{ R})} = 19.71 \frac{\text{lb(a)}}{\text{min}}$$

$$\dot{m}_{a2} = \frac{(AV)_2}{v_{a2}} = \frac{P_{a2}(AV)_2}{\frac{R}{M_a} T_2} = \frac{(14.696 - 0.20504) (1000) (1000)}{\left(\frac{1545}{28.97} \right) (520)} = 75.24 \frac{\text{lb(a)}}{\text{min}}$$

Substituting values into Eq. (1), the rate liquid is injected is

$$\dot{m}_w = (19.71) [0.00897 - 0.004148] + (75.24) [0.00897 - 0.0088] = 0.1078 \frac{\text{lb}}{\text{min}} \leftarrow \dot{m}_w$$

An energy rate equation at steady state gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_{a1} h_{a1} + \dot{m}_{w1} h_{w1}] + [\dot{m}_{a2} h_{a2} + \dot{m}_{w2} h_{w2}] + \dot{m}_w h_w - [\dot{m}_{a3} h_{a3} + \dot{m}_{w3} h_{w3}]$$

Thus

$$\dot{Q}_{cv} = \dot{m}_{a3} [h_a(T_3) + \omega_3 h_g(T_3)] - \dot{m}_{a1} [h_a(T_1) + \omega_1 h_g(T_1)] - \dot{m}_{a2} [h_a(T_2) + \omega_2 h_g(T_2)] - \dot{m}_w h_f(T_4)$$

$$= 44.95 [130.26 + 0.00897 (1098.53)] - 19.71 [119.48 + 0.004148 (1078.9)]$$

$$- 75.24 [124.27 + 0.0088 (1087.7)] - 0.1078 (23.10)$$

$$= 44.95 [140.11] - 19.71 [128.96] - 75.24 [133.84] - 2.5$$

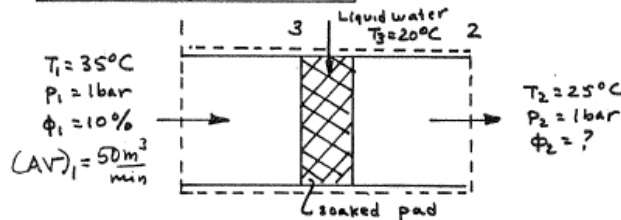
$$= 787.5 \frac{\text{Btu}}{\text{min}} \leftarrow \dot{Q}_{cv}$$

PROBLEM 12.92

KNOWN: Steady-state operating data are provided for an evaporative cooler.

FIND: Determine (a) the rate at which liquid enters, (b) the relative humidity at the exit, (c) the rate of exergy destruction.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) All of the entering liquid evaporates into the moist air stream. (3) The liquid enters as saturated liquid. (4) $T_0 = 293$.

ANALYSIS: At steady state mass rate balances give $\dot{m}_a = \dot{m}_2 = \dot{m}_a$, $\dot{m}_v + \dot{m}_w = \dot{m}_2$. Thus $\dot{m}_w = \dot{m}_a (\omega_2 - \omega_1)$ or $\dot{m}_w / \dot{m}_a = \omega_2 - \omega_1$. To find ω_1 , use ϕ_1 to obtain $P_{v1} = \phi_1 P_g(T_1) = (0.10)(0.05628) = 0.005628$ bar. Thus

$$\omega_1 = 0.622 \left[\frac{0.005628}{1 - 0.005628} \right] = 0.00352 \frac{\text{kg}(v)}{\text{kg}(a)}$$

The value of ω_2 can be obtained using an energy rate balance:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_v h_{v1}] + \dot{m}_w h_3 - [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}]$$

or

$$0 = \dot{m}_a [(h_{a1} - h_{a2}) + \omega_1 h_{g1} + (\omega_2 - \omega_1) h_{f3} - \omega_2 h_{g2}]$$

Solving for ω_2

$$\omega_2 = \frac{(h_{a1} - h_{a2}) + \omega_1 (h_{g1} - h_{f3})}{h_{g2} - h_{f3}} = \frac{1.005(35 - 25) + 0.00352(2565.3 - 83.96)}{2547.2 - 83.96} = 0.00763 \frac{\text{kg}(v)}{\text{kg}(a)}$$

The mass flow rate of dry air is

$$\dot{m}_a = \frac{(AV)_1 (P_{a1})}{RT_1} = \frac{(50 \text{ m}^3/\text{min}) [0.994372 \times 10^5 \text{ N/m}^2]}{(8.314 \text{ N} \cdot \text{m} / \text{kg} \cdot \text{K}) (308 \text{ K})} = 56.25 \frac{\text{kg}(a)}{\text{min}}$$

$$\text{Thus, } \dot{m}_w = \dot{m}_a (\omega_2 - \omega_1) = (56.25)(0.00763 - 0.00352) = 0.231 \frac{\text{kg}(l)}{\text{min}} \quad \leftarrow (a)$$

Solving Eq. 12.43

$$P_{v2} = \frac{\omega_2 P}{0.622 + \omega_2} = \frac{(0.00763)(1)}{0.622 + 0.00763} = 0.01212 \text{ bars}, \Rightarrow \phi_2 = \frac{P_{v2}}{P_g(T_2)} = \frac{0.01212}{0.03169} = 0.382 \quad \leftarrow (b)$$

(39.2%)

The rate of exergy destruction is found from $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production obtained from an entropy balance:

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_a} = [s_a(T_2, P_{a2}) - s_a(T_1, P_{a1})] + \omega_2 s_v(T_2, P_{v2}) - \omega_1 s_v(T_1, P_{v1}) - (\omega_2 - \omega_1) s_{f3}$$

Using Eq. 6.19, $s_v(T, P_v) = s_g(T) - \frac{R}{M_v} \ln \phi$ and applying Eq. 6.27 to the dry air

$$\begin{aligned} \frac{\dot{\sigma}_{cv}}{\dot{m}_a} &= \left[c_{pa} \ln \frac{T_2}{T_1} - \frac{R}{M_a} \ln \frac{P_{a2}}{P_{a1}} \right] + \omega_2 \left[s_g(T_2) - \frac{R}{M_v} \ln \phi_2 \right] - \omega_1 \left[s_g(T_1) - \frac{R}{M_v} \ln \phi_1 \right] - (\omega_2 - \omega_1) s_{f3} \\ &= c_{pa} \ln \frac{T_2}{T_1} - \frac{R}{M_a} \ln \frac{P_{a2}}{P_{a1}} + \omega_2 [s_g(T_2) - s_f(T_2)] - \omega_1 [s_g(T_1) - s_f(T_1)] + \frac{R}{M_v} [\omega_1 \ln \phi_1 - \omega_2 \ln \phi_2] \\ &= 1.005 \ln \frac{298}{308} - \frac{8.314}{28.97} \ln \frac{0.994372}{0.994372} + 0.00763 [8.558 - 0.2966] - 0.00352 [8.3531 - 0.2966] + \\ &\quad \frac{8.314}{18.02} [0.00352 \ln 0.1 - 0.00763 \ln 0.382] \end{aligned}$$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_a} = 0.003 \frac{\text{KJ/Kg}(a)}{\text{K}}$$

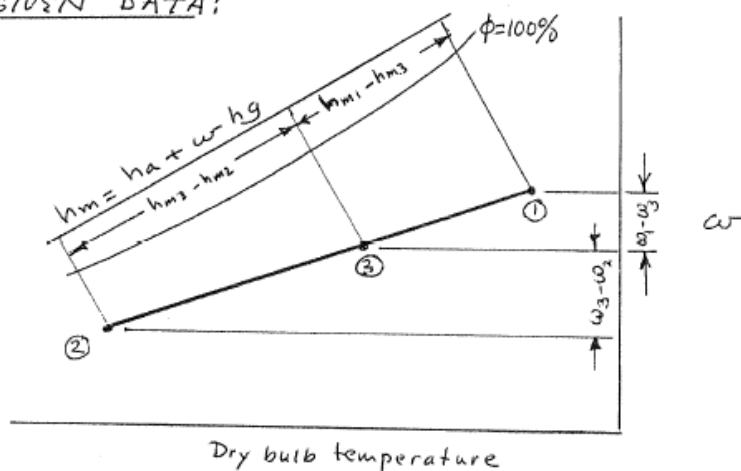
$$\Rightarrow \dot{E}_d = (56.25)(293)(0.003) = 49.44 \text{ KJ/min} \quad \leftarrow (c)$$

PROBLEM 12.93

KNOWN: The case of adiabatic mixing of two moist air streams is under consideration.

FIND: Show on a psychrometric chart that exit state 3 is on a straight line connecting inlet states 1 and 2.

SCHEMATIC & GIVEN DATA:



ANALYSIS: From Sec. 12.10.4, and writing for the mixture $h_m = h_a + w h_g$

$$(12.56b) \quad \omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$$

$$(12.56c) \quad \dot{m}_{a1} h_{m1} + \dot{m}_{a2} h_{m2} = \dot{m}_{a3} h_{m3} \text{ where } h_m = h_a + w h_g$$

Since $\dot{m}_{a3} = \dot{m}_{a1} + \dot{m}_{a2}$, Eq. 12.56b can be rearranged as

$$\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 (\dot{m}_{a1} + \dot{m}_{a2})$$

$$\Rightarrow (\omega_1 - \omega_3) \dot{m}_{a1} + (\omega_2 - \omega_3) \dot{m}_{a2} = 0$$

$$\Rightarrow \frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3} \quad (1)$$

Similarly, Eq. 12.56c gives

$$\frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{h_{m3} - h_{m2}}{h_{m1} - h_{m3}} \quad (2)$$

Combining Eqs. (1), (2)

$$\frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3} = \frac{h_{m3} - h_{m2}}{h_{m1} - h_{m3}} \quad (3)$$

Since the w and h_m scales are linear, the following proportionality applies for the sketch above

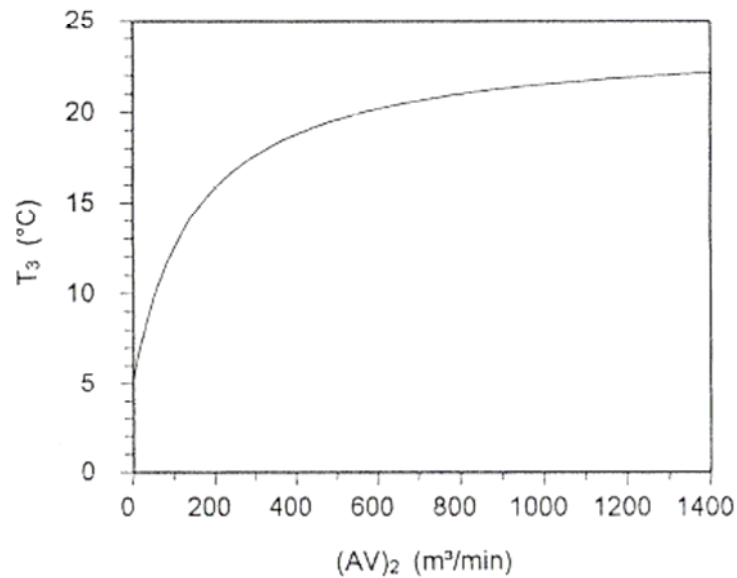
$$\frac{\text{Line } 2-3}{\text{Line } 3-1} = \frac{h_{m3} - h_{m2}}{h_{m1} - h_{m3}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3}$$

which corresponds to Eq. (3).

PROBLEM 12.94

See Example 12.16 for the IT code. To obtain data for the plot, use the **Explore** button to sweep AV_2 from 0 to 1400 m^3/min in steps of 10.

PLOT:



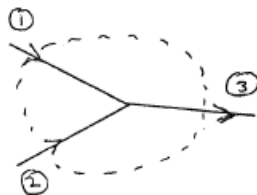
Discussion: When $(AV)_2$ approaches 0, there is no flow in stream 2, and the exit temperature approaches the value of T_1 . When $(AV)_2$ becomes large, the effect of stream 1 is reduced and the exit temperature approaches the value of T_2 .

PROBLEM 12.95

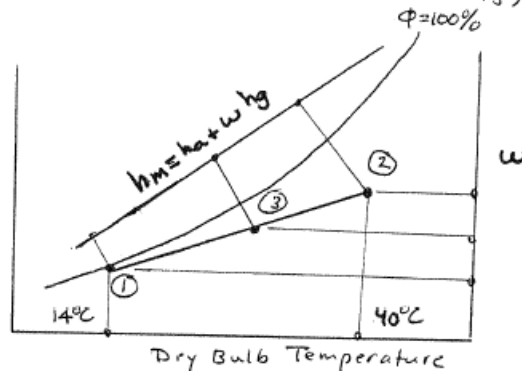
KNOWN: Steady state operating data are provided for the adiabatic mixing of two moist air streams.

FIND: Using the graphical procedure of Problem 12.93, find ϕ_3, T_3 .

SCHEMATIC & GIVEN DATA:



$$\begin{aligned} (AV)_1 &= 35 \text{ m}^3/\text{min}, \quad T_1 = 14^\circ\text{C}, \quad \phi_1 = 80\% \\ (AV)_2 &= 80 \text{ m}^3/\text{min}, \quad T_2 = 40^\circ\text{C}, \quad \phi_2 = 40\% \end{aligned}$$



ENGINEERING MODEL: (1) The control volume shown is at steady state. (2) For the control volume $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. Changes in kinetic and potential energy can be neglected. (3) The pressure remains constant at 1 atm.

ANALYSIS: Inspection of Fig. A-9 gives

$$\begin{aligned} h_{m1} &= 34.0 \text{ kJ/kg(a)} & h_{m2} &= 88.2 \\ \omega_1 &= 0.008 \text{ kg(v)/kg(a)} & \omega_2 &= 0.0188 \\ v_{a1} &= 0.824 \text{ m}^3/\text{kg(a)} & v_{a2} &= 0.913 \end{aligned}$$

Then

$$m_{a1} = \frac{(AV)_1}{v_{a1}} = \frac{35 \text{ m}^3/\text{min}}{0.824 \text{ m}^3/\text{kg(a)}} = 42.48 \frac{\text{m}^3}{\text{kg(a)}}$$

$$m_{a2} = \frac{80}{0.913} = 87.62 \frac{\text{m}^3}{\text{kg(a)}}$$

Therefore, the relation of Problem 12.87 becomes

$$\frac{42.48}{87.62} = \frac{\omega_3 - 0.0188}{0.008 - \omega_3} = \frac{h_{m3} - 88.2}{34.0 - h_{m3}}$$

Solving, $\omega_3 = 0.0153 \text{ kg(v)/kg(a)}$, $h_{m3} = 70.50 \text{ kJ/kg(a)}$. Using these values, Fig. A-9 gives

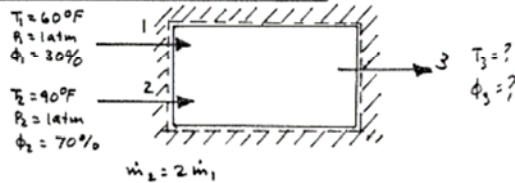
$$\begin{aligned} \phi_3 &\approx 52\% \leftarrow \phi_3 \\ T_3 &\approx 32^\circ\text{C} \leftarrow T_3 \end{aligned}$$

PROBLEM 12.96

KNOWN: A moist air stream at 60°F, 1 atm, $\phi = 30\%$ is mixed adiabatically with a stream of moist air at 90°F, 1 atm, $\phi = 70\%$. The mass flow rate of the 90°F stream is twice that of the 60°F stream.

FIND: Using the result of Problem 12.74, determine the temperature and relative humidity of the exiting stream.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The pressure at 3 is 1 atm.

ANALYSIS: At steady state, mass rate balances give

$$\text{mixture: } \dot{m}_1 + \dot{m}_2 = \dot{m}_3 \Rightarrow \dot{m}_3 = 3\dot{m}_1 \quad (1)$$

$$\text{air: } \dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$$

$$\text{water vapor: } \dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \Rightarrow \dot{m}_{a1}\omega_1 + \dot{m}_{a2}\omega_2 = \dot{m}_{a3}\omega_3 \quad (2)$$

$$\text{Also, at each location } \dot{m} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1 + \omega) \Rightarrow \dot{m}_a = \dot{m}/(1 + \omega) \quad (3)$$

Combining Eqs. (1), (2), (3) together with $\dot{m}_2 = 2\dot{m}_1$

$$\frac{\dot{m}_1}{(1 + \omega_1)}\omega_1 + \frac{\dot{m}_2}{(1 + \omega_2)}\omega_2 = \left(\frac{\dot{m}_3}{1 + \omega_3}\right)\omega_3 \Rightarrow \left(\frac{\omega_1}{1 + \omega_1}\right) + 2\left(\frac{\omega_2}{1 + \omega_2}\right) = 3\left(\frac{\omega_3}{1 + \omega_3}\right) \quad (4)$$

To find ω_1 and ω_2 , $P_{v1} = \phi_1 P_g(T_1) = (0.3)(0.2563) = 0.07689 \text{ lbf/in}^2$, $P_{v2} = \phi_2 P_g(T_2) = (0.7)(0.6988) = 0.4892 \text{ lbf/in}^2$, so

$$\omega_1 = 0.622 \left[\frac{0.07689}{14.696 - 0.07689} \right] = 0.0033 \frac{\text{lb(v)}}{\text{lb(a)}}, \quad \omega_2 = 0.622 \left[\frac{0.4892}{14.696 - 0.4892} \right] = 0.0214 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Substituting values into Eq. (4)

$$\left(\frac{0.0033}{1.0033}\right) + 2\left(\frac{0.0214}{1.0214}\right) = 3\left(\frac{\omega_3}{1 + \omega_3}\right) \Rightarrow \omega_3 = 0.0153 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Solving Eq. 12.43

$$P_{v3} = \frac{\omega_3 P}{0.622 + \omega_3} = \frac{(0.0153)(14.696)}{0.6373} = 0.3528 \frac{\text{lbf}}{\text{in}^2} \Rightarrow \phi_3 = \frac{P_{v3}}{P_g(T_3)} = \frac{0.3528}{P_g(T_3)} \quad (5)$$

To find T_3 , write an energy rate balance for the control volume. With assumption

(1) this becomes

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1}] + [\dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}] - [\dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3}]$$

or

$$\dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}] = \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}] \quad (6)$$

$$\frac{\dot{m}_3}{1 + \omega_3}[h_{a3} + \omega_3 h_{g3}] = \frac{\dot{m}_1}{1 + \omega_1}[h_{a1} + \omega_1 h_{g1}] + \frac{\dot{m}_2}{1 + \omega_2}[h_{a2} + \omega_2 h_{g2}]$$

With the result of Problem 12.74

$$(h_a + \omega h_g)_1 = 0.24(60) + 0.0033(1061 + 0.444(60)) = 17.99 \text{ Btu/lb(a)}$$

$$(h_a + \omega h_g)_2 = 0.24(90) + 0.0214(1061 + 0.444(90)) = 45.16 "$$

$$(h_a + \omega h_g)_3 = 0.24 T_3 + 0.0153(1061 + 0.444 T_3)$$

Thus, Eq. (6) becomes with $\dot{m}_3 = 3\dot{m}_1$, $\dot{m}_2 = 2\dot{m}_1$

$$0.24 T_3 + 0.0153[1061 + 0.444 T_3] = \frac{(1.0153)(17.99) + 2\left(\frac{1.0153}{1.0214}\right)(45.16)}{3} = 36.0$$

or

$$0.2468 T_3 = 19.77 \Rightarrow T_3 = 80 \text{ } ^\circ\text{F}$$

Then, with Eq. (5)

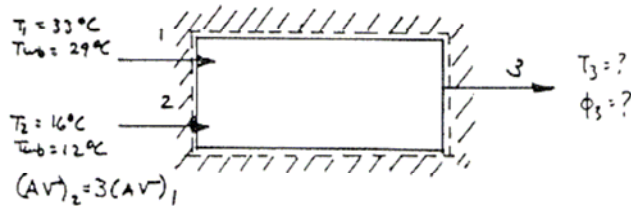
$$\phi_3 = \frac{0.3528}{0.5073} = 0.695 \text{ (69.5\%)} \quad \phi_3$$

PROBLEM 12.97

KNOWN: Atmospheric air having dry-bulb and wet-bulb temperatures of 33 and 29°C mixes adiabatically with moist air having dry-bulb and wet-bulb temperatures of 16 and 12°C. The volumetric flow rate of the 16°C stream is twice that of the other.

FIND: For the exiting stream, determine the relative humidity and the temperature.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The pressure is uniform throughout at 1 atm. (3) The wet-bulb temperature can be used in place of the adiabatic saturation temperature.

ANALYSIS: At steady state, mass rate balances give

air: $\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$

water: $\dot{m}_{w1} + \dot{m}_{w2} = \dot{m}_{w3} \Rightarrow \omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$

Combining these

$$\omega_3 = \frac{\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{\omega_1 + \left(\frac{\dot{m}_{a2}}{\dot{m}_{a1}}\right) \omega_2}{1 + \left(\frac{\dot{m}_{a2}}{\dot{m}_{a1}}\right)} \quad (1)$$

The mass flow rate ratio is obtained using the given volumetric flow rate relation:

$$\dot{m}_{a1} = \frac{(AV)_1}{v_{a1}} = \frac{P_{a1} (AV)_1}{\frac{R}{M_a} T_1}, \quad \dot{m}_{a2} = \frac{P_{a2} (AV)_2}{\frac{R}{M_a} T_2} \Rightarrow \frac{\dot{m}_{a2}}{\dot{m}_{a1}} = \frac{P_{a2}}{P_{a1}} \frac{T_1}{T_2} \frac{(AV)_2}{(AV)_1} = 3 \frac{P_{a2}}{P_{a1}} \frac{T_1}{T_2} \quad (2)$$

The terms ω_1 and ω_2 can be calculated using Eqs. 12.52 and 12.53: First, to find ω_1

$$\omega_1 = 0.622 \left[\frac{0.04008}{0.97817} \right] = 0.0256, \quad \omega_1 = \frac{1.005[29.33] + 0.0256[2422.8]}{2561.7 - 121.61} = 0.0239 \frac{\text{kg}(v)}{\text{kg}(a)}$$

To find ω_2

$$\omega_2 = 0.622 \left[\frac{0.01402}{0.98598} \right] = 0.0087, \quad \omega_2 = \frac{1.005[12.167] + 0.0087[2523.4]}{2530.8 - 50.41} = 0.0072 \frac{\text{kg}(v)}{\text{kg}(a)}$$

Solving Eq. 12.43

$$P_{v1} = \frac{\omega_1 P}{0.622 + \omega_1} = \frac{(0.0239)(1.01325)}{0.6459} = 0.0375 \text{ bar} \Rightarrow P_{a1} = 0.97576 \text{ bar}$$

$$P_{v2} = \frac{\omega_2 P}{0.622 + \omega_2} = \frac{(0.0072)(1.01325)}{0.6292} = 0.01159 \text{ bar} \Rightarrow P_{a2} = 1.00166 \text{ bar}$$

Inserting values into Eq. (2) gives

$$\frac{\dot{m}_{a2}}{\dot{m}_{a1}} = 3 \left(\frac{1.00166}{0.97576} \right) \left(\frac{306}{289} \right) = 3.261$$

Then, with known values Eq. (1) yields

$$\omega_3 = \frac{0.0239 + (3.261)(0.0072)}{4.261} = 0.0111 \frac{\text{kg}(v)}{\text{kg}(a)}$$

Accordingly

$$P_{v3} = \frac{\omega_3 P}{0.622 + \omega_3} = \frac{(0.0111)(1.01325)}{0.6331} = 0.0178 \text{ bar.}$$

Then

$$\phi_3 = \frac{P_{v3}}{P_g(T_3)} = \frac{0.0178}{P_g(T_3)} \quad (3)$$

12-97

Continued on next slide

Problem 12-97 continued

To find T_3 write an energy rate balance for the control volume.
With assumption (1)

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_1 h_1 + \dot{m}_v h_{v1}] + [\dot{m}_2 h_2 + \dot{m}_v h_{v2}] - [\dot{m}_3 h_3 + \dot{m}_v h_{v3}]$$

Thus

$$0 = \dot{m}_1 [h_1 + w_1 h_{g1}] + \dot{m}_2 [h_2 + w_2 h_{g2}] - \dot{m}_3 [h_3 + w_3 h_{g3}]$$

or

$$\begin{aligned} h_3 + w_3 h_{g3} &= \frac{\dot{m}_1 [h_1 + w_1 h_{g1}] + \dot{m}_2 [h_2 + w_2 h_{g2}]}{\dot{m}_1 + \dot{m}_2} \\ &= \frac{(h_1 + w_1 h_{g1}) + \left(\frac{\dot{m}_2}{\dot{m}_1}\right) (h_2 + w_2 h_{g2})}{\left(1 + \frac{\dot{m}_2}{\dot{m}_1}\right)} \end{aligned}$$

With data from Tables A-2 and A-22

$$\begin{aligned} h_3 + w_3 h_{g3} &= \frac{[306.2 + 0.0239(2561.7)] + (3.261)[289.2 + 0.0072(2530.8)]}{4.261} \\ &= 321.5 \text{ kJ/kg(a)} \end{aligned}$$

- ① This is a single equation in one unknown, T_3 . Solving iteratively using table data gives $T_3 \approx 20^\circ\text{C}$. $\leftarrow T_3$

Returning to Eq. (7)

$$\phi_3 = \frac{0.0178}{0.02339} = 0.761 \quad (76.1\%) \quad \leftarrow \phi_3$$

1. An alternative solution that avoids iteration with table data is obtained using IT, as follows:

IT Code

```
T1 = 33 // °C
Twb1 = 29 // °C
T2 = 16 // °C
Twb2 = 12 // °C
AV2 = R * AV1
R = 3
AV1 = 1
p = 1.01325 // bar

w1 = w_TTwb(T1, Twb1, p)
phi1 = phi_Tw(T1, w1, p)
h1 = ha_Tw(T1, w1)
w2 = w_TTwb(T2, Twb2, p)
phi2 = phi_Tw(T2, w2, p)
h2 = ha_Tw(T2, w2)
phi3 = phi_Tw(T3, w3, p)
h3 = ha_Tw(T3, w3)
```

$\dot{m}dota1 = AV1 / va1$

$\dot{m}dota2 = AV2 / va2$

$va1 = va_Tw(T1, w1, p)$

$va2 = va_Tw(T2, w2, p)$

$\dot{m}dota1 + \dot{m}dota2 = \dot{m}dota3$

$\dot{m}dota1 * w1 + \dot{m}dota2 * w2 = \dot{m}dota3 * w3$

$0 = \dot{m}dota1 * h1 + \dot{m}dota2 * h2 - \dot{m}dota3 * h3$

IT Results

$w_1 = 0.02391 \text{ kg(v)/kg(a)}$

$h_1 = 94.37 \text{ kJ/kg(a)}$

$w_2 = 0.007106 \text{ kg(v)/kg(a)}$

$h_2 = 34.04 \text{ kJ/kg(a)}$

$w_3 = 0.01105 \text{ kg(v)/kg(a)}$

$h_3 = 48.2 \text{ kJ/kg(a)}$

$T_3 = 20.08^\circ\text{C}$

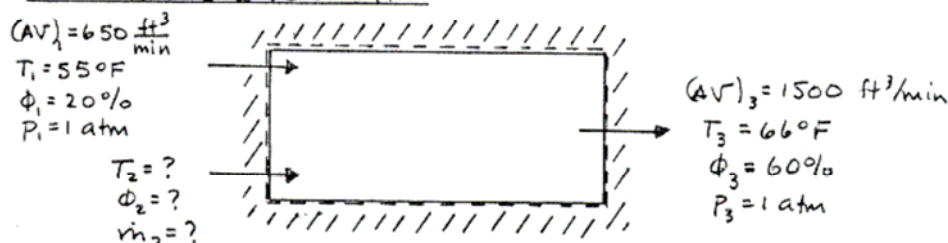
$\phi_3 = 0.7508 \text{ (75.08\%)}$

PROBLEM 12.98

KNOWN: A stream consisting of $650 \text{ ft}^3/\text{min}$ of air at 55°F , 1 atm , $\phi = 20\%$ mixes adiabatically with a second stream. A single stream consisting of $1500 \text{ ft}^3/\text{min}$ of air exits at 66°F , 1 atm , $\phi = 60\%$.

FIND: Determine for the second entering stream the relative humidity, temperature, and mass flow rate.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Each stream can be modeled as an ideal gas mixture.

ANALYSIS: At steady state, mass rate balances give

$$\text{air: } \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\text{water: } \dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \Rightarrow \dot{m}_1 \omega_1 + \dot{m}_2 \omega_2 = \dot{m}_3 \omega_3 \Rightarrow \omega_2 = \frac{\dot{m}_3 \omega_3 - \dot{m}_1 \omega_1}{\dot{m}_3 - \dot{m}_1} \quad (1)$$

The mass flow rates are determined using the given volumetric flow rates and $P_{v1} = \phi_1 P_g(T_1) = (0.2)(0.21415) = 0.0428 \text{ lbf/in}^2$, $P_{v3} = \phi_3 P_g(T_3) = (0.6)(0.3165) = 0.1899 \text{ lbf/in}^2$

$$\dot{m}_1 = \frac{(AV)_1}{v_{a1}} = \frac{P_{a1}(AV)_1}{R_a T_1} = \frac{(14.696 - 0.0428) \frac{\text{lbf}}{\text{in}^2} (650 \frac{\text{ft}^3}{\text{min}})}{(\frac{1545}{28.97} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}})(515^\circ\text{R})} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| = 49.94 \frac{\text{lb(a)}}{\text{min}}$$

$$\dot{m}_3 = \frac{P_{a3}(AV)_3}{R_a T_3} = \frac{(14.696 - 0.1899)(1500)}{(\frac{1545}{28.97})(526)} \left| \frac{144}{1} \right| = 111.7 \frac{\text{lb(a)}}{\text{min}}$$

The values of ω_1 and ω_3 are

$$\omega_1 = 0.622 \left[\frac{0.0428}{14.696 - 0.0428} \right] = 0.0018 \frac{\text{lb(v)}}{\text{lb(a)}}, \quad \omega_3 = 0.622 \left[\frac{0.1899}{14.696 - 0.1899} \right] = 0.008143 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Accordingly, Eq. (1) gives

$$\omega_2 = \frac{(111.7)(0.008143) - (49.94)(0.0018)}{111.7 - 49.94} = 0.01327 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Solving Eq. 12.43

$$P_{v2} = \frac{\omega_2 P_2}{0.622 + \omega_2} = \frac{(0.01327)(14.696)}{0.6353} = 0.307 \frac{\text{lbf}}{\text{in}^2} \Rightarrow \phi_2 = \frac{P_{v2}}{P_g(T_2)} = \frac{0.307}{P_g(T_2)} \quad (2)$$

The temperature T_2 can be obtained from an energy rate balance which reduces with assumptions listed above to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_1 h_{a1} + \dot{m}_{v1} h_{v1}] + [\dot{m}_2 h_{a2} + \dot{m}_{v2} h_{v2}] - [\dot{m}_3 h_{a3} + \dot{m}_{v3} h_{v3}]$$

or

$$0 = \dot{m}_1 [h_{a1} + \omega_1 h_{g1}] + \dot{m}_2 [h_{a2} + \omega_2 h_{g2}] - \dot{m}_3 [h_{a3} + \omega_3 h_{g3}]$$

Solving for $h_{a2} + \omega_2 h_{g2}$

$$h_{a2} + \omega_2 h_{g2} = \frac{\dot{m}_3 [h_{a3} + \omega_3 h_{g3}] - \dot{m}_1 [h_{a1} + \omega_1 h_{g1}]}{\dot{m}_3 - \dot{m}_1}$$

Continued on next slide

With data from Tables A-2E and A-22E

$$h_{a2} + w_2 h_{g2} = \frac{(111.7)[125.71 + (0.008143)(1090.3)] - (49.94)[123.07 + (0.0018)(1085.5)]}{111.7 - 49.94}$$

$$= 142.32 \frac{\text{Btu}}{\text{lb(a)}}$$

① This equation has a single unknown, T_2 . Solving iteratively using table data gives $T_2 \approx 74.8^\circ\text{F}$

Returning to Eq. (2)

$$\phi_2 = \frac{0.307}{0.4273} = 0.718 \text{ (71.8 \%)} \leftarrow \phi_2$$

To get the mass flow rate at 2

$$\dot{m}_2 = \dot{m}_{a2} + \dot{m}_{v2} = \dot{m}_{a2}(1 + w_2) = (\dot{m}_{a3} - \dot{m}_{a1})(1 + w_2)$$

$$= (111.7 - 49.94)(1 + 0.01327) = 62.58 \frac{\text{lb}}{\text{min}} \leftarrow \dot{m}_2$$

1. An alternative solution that avoids iteration with table data is obtained using IT, as follows:

IT Code

```
T1 = 55 // °F
phi1 = 0.2
p = 14.696
AV1 = 650 // ft³/min
T3 = 66 // °F
phi3 = .6
AV3 = 1500 // ft³/min
```

```
w1 = w_Tphi(T1, phi1, p)
w2 = w_Tphi(T2, phi2, p)
w3 = w_Tphi(T3, phi3, p)
h1 = ha_Tw(T1, w1)
h2 = ha_Tw(T2, w2)
h3 = ha_Tw(T3, w3)

mdota1 = AV1 / va1
va1 = va_Tw(T1, w1, p)
mdota2 = AV2 / va2
va2 = va_Tw(T2, w2, p)
mdota3 = AV3 / va3
va3 = va_Tw(T3, w3, p)
mdota3 = mdota1 + mdota2
mdota1 * w1 + mdota2 * w2 = mdota3 * w3
mdota1 * h1 + mdota2 * h2 = mdota3 * h3
mdot2 = (mdota3 - mdota1) * (1 + w2)
```

IT Results

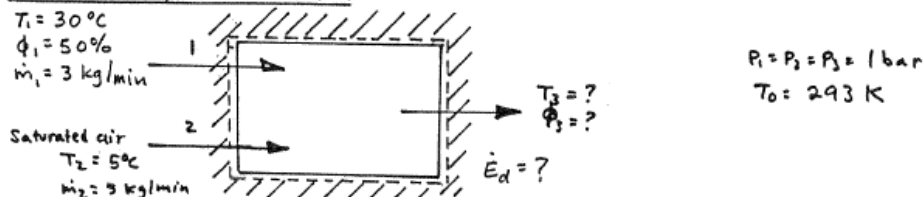
```
w1 = 0.001816 lb(v)/lb(a)
w2 = 0.01328 lb(v)/lb(a)
w3 = 0.008154 lb(v)/lb(a)
m_a1 = 49.96 lb(a)/min
m_a2 = 61.79 lb(a)/min
m_a3 = 111.7 lb(a)/min
(AV)2 = 850 ft³/min
T2 = 74.71°F
phi2 = 0.7215 (72.15%)
m2 = 62.61 lb/min
```


PROBLEM 12.99

KNOWN: Moist air at $T_1 = 30^\circ\text{C}$, $P_1 = 1 \text{ bar}$, $\phi_1 = 50\%$, $\dot{m}_1 = 3 \text{ kg/min}$ mixes adiabatically with saturated air at $T_2 = 5^\circ\text{C}$, $P_2 = 1 \text{ bar}$, $\dot{m}_2 = 5 \text{ kg/min}$ to produce a single mixed stream at $P_3 = 1 \text{ bar}$.

FIND: Determine ϕ_3 , T_3 , and the exergy destruction rate.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Each stream can be modeled as an ideal gas mixture.

ANALYSIS: At steady state, mass rate balances reduce to give $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ and $\dot{m}_{a3} = \dot{m}_{a1} + \dot{m}_{a2}$, $\dot{m}_{v3} = \dot{m}_{v1} + \dot{m}_{v2}$. Also, for each stream $\dot{m} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1 + \omega)$. Accordingly, $\dot{m}_{a3}\omega_3 = \dot{m}_{a1}\omega_1 + \dot{m}_{a2}\omega_2$, or

$$\omega_3 = \frac{\dot{m}_{a1}\omega_1 + \dot{m}_{a2}\omega_2}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{\left(\frac{\omega_1}{1+\omega_1}\right)\dot{m}_1 + \left(\frac{\omega_2}{1+\omega_2}\right)\dot{m}_2}{\left(\frac{\dot{m}_1}{1+\omega_1}\right) + \left(\frac{\dot{m}_2}{1+\omega_2}\right)}$$

To find ω_1 and ω_2 , $P_{v1} = \phi_1 P_g(T_1) = 0.5(0.04246) = 0.02123 \text{ bar}$, $P_{v2} = \phi_2 P_g(T_2) = 0.00872 \text{ bar}$. Then

$$\omega_1 = 0.622 \left(\frac{0.02123}{1 - 0.02123} \right) = 0.01349 \frac{\text{kg(v)}}{\text{kg(a)}}, \quad \omega_2 = 0.622 \left(\frac{0.00872}{1 - 0.00872} \right) = 0.00547 \frac{\text{kg(v)}}{\text{kg(a)}}$$

Accordingly

$$\omega_3 = \frac{\left(\frac{0.01349}{1.01349}\right)(3) + \left(\frac{0.00547}{1.00547}\right)(5)}{\left(\frac{3}{1.01349}\right) + \left(\frac{5}{1.00547}\right)} = 0.008463 \frac{\text{kg(v)}}{\text{kg(a)}}$$

Solving Eq. 12.43

$$P_{v3} = \frac{P_3 \omega_3}{0.622 + \omega_3} = \frac{(1)(0.008463)}{0.63046} = 0.01342 \Rightarrow \phi_3 = \frac{P_{v3}}{P_g(T_3)} = \frac{0.01342}{P_g(T_3)} \quad (1)$$

To find T_3 , an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1}] + [\dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}] - [\dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3}]$$

$$\text{or } 0 = \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}] - \dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}]$$

Then

$$h_{a3} + \omega_3 h_{g3} = \frac{\dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}]}{\dot{m}_{a1} + \dot{m}_{a2}}$$

$\dot{m}_{a1} = \dot{m}_1 / (1 + \omega_1) = 2.96 \text{ kg(a)/min}$, $\dot{m}_{a2} = \dot{m}_2 / (1 + \omega_2) = 4.9728 \text{ kg(a)/min}$. With data from Tables A-2, A-22

$$h_{a3} + \omega_3 h_{g3} = \frac{(2.96)[303.21 + (0.01349)(2556.3)] + 4.9728[278.1 + (0.00547)(2510.6)]}{(2.96 + 4.9728)} = 308.95 \text{ kJ/kg(a)}$$

① This equation has a single unknown, T_3 . Solving iteratively using table data gives $T_3 = 14.4^\circ\text{C}$

PROBLEM 12.99

Returning to Eq. (1), $\phi_3 = \frac{0.01342}{0.01641} = 0.818 \text{ (81.8\%)} \leftarrow \phi_3$

The exergy destruction rate is obtained from $\dot{E}_d = T_o \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance which at steady state reduces to

$$0 = \sum \frac{\dot{Q}_j^o}{T_j} + [\dot{m}_1 s_a(T_1, p_{a1}) + \dot{m}_1 s_v(T_1, p_{v1})] + [\dot{m}_2 s_a(T_2, p_{a2}) + \dot{m}_2 s_g(T_2)] - [\dot{m}_3 s_a(T_3, p_{a3}) + \dot{m}_3 s_v(T_3, p_{v3})] + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}_3 [s_a(T_3, p_{a3}) + \omega_3 s_v(T_3, p_{v3})] - \dot{m}_1 [s_a(T_1, p_{a1}) + \omega_1 s_v(T_1, p_{v1})] - \dot{m}_2 [s_a(T_2, p_{a2}) + \omega_2 s_g(T_2)]$$

$$= \dot{m}_1 [s_a(T_3, p_{a3}) - s_a(T_1, p_{a1})] + \dot{m}_2 [s_a(T_3, p_{a3}) - s_a(T_2, p_{a2})] + \dot{m}_3 \omega_3 s_v(T_3, p_{v3})$$

From Eq. 6.19, $s_v(T, p_v) = s_g(T) - \bar{R}/M_v \ln \phi$, and using Eq. 6.23 for the dry air

$$\dot{\sigma}_{cv} = \dot{m}_1 \left[c_{pa} \ln \frac{T_3}{T_1} - \frac{\bar{R}}{M_a} \ln \frac{p_{a3}}{p_{a1}} \right] + \dot{m}_2 \left[c_{pa} \ln \frac{T_3}{T_2} - \frac{\bar{R}}{M_a} \ln \frac{p_{a3}}{p_{a2}} \right]$$

$$+ \dot{m}_3 \omega_3 \left[s_g(T_3) - \frac{\bar{R}}{M_v} \ln \phi_3 \right] - \dot{m}_1 \omega_1 \left[s_g(T_1) - \frac{\bar{R}}{M_v} \ln \phi_1 \right] - \dot{m}_2 \omega_2 s_g(T_2)$$

From previous calculations: $p_{a1} = 0.97877 \text{ bar}$, $p_{a2} = 0.99128 \text{ bar}$, and $p_{a3} = 1 - 0.01342 = 0.98658 \text{ bar}$. Evaluating $\dot{\sigma}_{cv}$ with $c_{pa} = 1.005 \text{ kJ/kg} \cdot \text{K}$

$$\dot{\sigma}_{cv} = 2.96 \left[1.005 \ln \frac{287.4}{303} - \frac{8.314}{28.97} \ln \frac{0.98658}{0.97877} \right] + 4.9728 \left[1.005 \ln \frac{287.4}{278} - \frac{8.314}{28.97} \ln \frac{0.98658}{0.99128} \right] +$$

$$(7.9323)(0.008463) \left[8.79544 - \frac{8.314}{18.02} \ln 0.818 \right] - (2.96)(0.01342) \left[8.4533 - \frac{8.314}{18.02} \ln 0.5 \right] -$$

$$(4.9728)(0.00547)(9.0257)$$

$$= 0.009828 \frac{\text{kJ/min}}{\text{K}}$$

The exergy destruction rate is then

$$\dot{E}_d = (293 \text{ K})(0.009828 \frac{\text{kJ/min}}{\text{K}}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 0.048 \text{ kW} \leftarrow \dot{E}_d$$

1. An alternative solution that avoids iteration with table data is obtained using IT, as follows:

IT Code

T1 = 30 // °C
p = 1 // bar
phi1 = 0.5
mdot1 = 3 // kg/min
T2 = 5 // °C
phi2 = 1
mdot2 = 5 // kg/min

w1 = w_Tphi(T1, phi1, p)
w2 = w_Tphi(T2, phi2, p)
mdota1 = mdot1 / (1 + w1)
mdota2 = mdot2 / (1 + w2)
mdota3 = mdota1 + mdota2
mdota3 * w3 = mdota1 * w1 + mdota2 * w2
mdota3 * h3 = mdota1 * h1 + mdota2 * h2
h1 = ha_Tw(T1, w1)
h2 = ha_Tw(T2, w2)
h3 = ha_Tw(T3, w3)
w3 = w_Tphi(T3, phi3, p)

0 = mdota1 * s1 + mdota2 * s2 - mdota3 * s3 + sigmadot
Edotd = To * sigmadot / 60
To = 293
s1 = sa_Tw(T1, w1, p)
s2 = sa_Tw(T2, w2, p)
s3 = sa_Tw(T3, w3, p)

IT Results

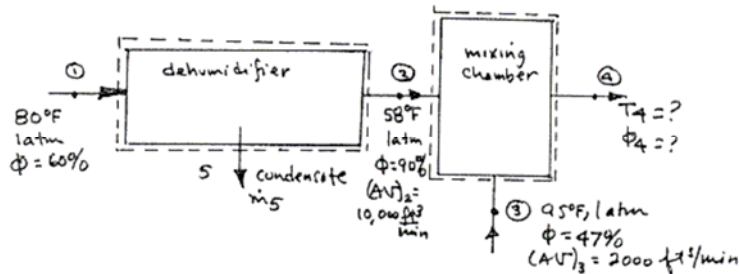
$\omega_1 = 0.0135 \text{ kg(v)/kg(a)}$
 $\omega_2 = 0.005486 \text{ kg(v)/kg(a)}$
 $\omega_3 = 0.008477 \text{ kg(v)/kg(a)}$
 $\dot{m}_{a1} = 2.96 \text{ kg(a)/min}$
 $\dot{m}_{a2} = 4.973 \text{ kg(a)/min}$
 $\dot{m}_{a3} = 7.933 \text{ kg(a)/min}$
 $\dot{\sigma}_{cv} = 0.01009 \text{ kJ/min} \cdot \text{K}$
 $T_3 = 14.41^\circ \text{C}$
 $\phi_3 = 0.8167 \text{ (81.67\%)}$
 $\dot{E}_d = 0.04926 \text{ kW}$

PROBLEM 12.100

KNOWN: Steady state operating data are provided for a system consisting of a dehumidifier followed by adiabatic mixing with another air stream.

FIND: Determine (a) the rate water is removed from the moist air passing through the dehumidifier, (b) the temperature and relative humidity of the stream exiting the mixing chamber.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volumes shown in the figure are at steady state. (2) For the control volumes $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and all kinetic/potential energy effects can be ignored. (3) The moist air streams are modeled as ideal gas mixtures. (4) $p = 1 \text{ atm}$ throughout

ANALYSIS: (a) Mass rate balances for dry air and water applied to the dehumidifier reduce to give $\dot{m}_5 = \dot{m}_1(\omega_1 - \omega_2)$.

Using ϕ_1 , ϕ_2 and p_g at T_1 , T_2

$$\omega_1 = \frac{0.622(0.6)(0.5073)}{14.7 - (0.6)(0.5073)} = 0.01315, \quad \omega_2 = \frac{0.622(0.9)(0.2386)}{14.7 - (0.9)(0.2386)} = 0.00922$$

Also

$$\dot{m}_{a2} = \frac{(\dot{A}V)_2}{v_{a2}} = \frac{(\dot{A}V)_2}{\frac{R}{M_a} T_2} = \frac{[(14.7 - (0.9)(0.2386)) \text{ lbf/in}^2]}{(15.45 \text{ ft} \cdot \text{lbf}) / (16.02 \text{ lb} \cdot \text{R}) (518 \text{ R})} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 45,302.5 \frac{\text{lb}}{\text{h}}$$

$$\text{Thus, } \dot{m}_5 = 45,302.5 [0.01315 - 0.00922] = 178 \text{ lb} \quad (a)$$

(b) Following the development of Sec. 12.9.4, mass rate and energy rate balances give

$$\omega_4 = \frac{\omega_2 \dot{m}_{a2} + \omega_3 \dot{m}_{a3}}{\dot{m}_{a2} + \dot{m}_{a3}}, \quad (h_a + \omega h_g)_4 = \frac{\dot{m}_{a2} (h_{a2} + \omega_2 h_{g2}) + \dot{m}_{a3} (h_{a3} + \omega_3 h_{g3})}{\dot{m}_{a2} + \dot{m}_{a3}} \quad (1), (2)$$

For stream 3

$$\omega_3 = \frac{0.622(0.47)(0.8165)}{14.7 - (0.47)(0.8165)} = 0.0167, \quad \dot{m}_{a3} = \frac{(\dot{A}V)_3}{\frac{R}{M_a} T_3} = \frac{(2000)(14.7162)(144)(60)}{(15.45 \text{ ft} \cdot \text{lbf}) / (28.97 \text{ lb} \cdot \text{R}) (535)} = 8358 \frac{\text{lb}}{\text{h}}$$

Thus, Eq. (1) gives

$$\omega_4 = \frac{(0.00922)(45302.5) + (0.0167)(8358)}{(45302.5 + 8358)} = 0.01039$$

① Using the result of Problem 12.70, Eq. (2) gives

$$0.24 T_4 (^\circ\text{F}) + 0.01039 [1061 + 0.444 T_4 (^\circ\text{F})] = \frac{(45,302.5)}{(53,660.5)} [(0.24)(58) + 0.00922 (1061 + (0.444)(58))] + \frac{(8358)}{(53,660.5)} [(0.24)(95) + 0.0167 (1061 + (0.444)(95))]$$

$$= 26.63 \text{ Btu/lb (da)}$$

Continued on next slide

Problem 12-100 continued

or

$$0.245 T_4(^{\circ}\text{F}) = 15.606 \Rightarrow T_4 = 63.7^{\circ}\text{F} \leftarrow T_4$$

To find ϕ_4 , solve Eq. 12.43 to give

$$P_{v4} = \frac{\omega_4 P_4}{\omega_4 + 0.622} = \frac{(0.01039)(14.7)}{(0.01039 + 0.622)} = 0.2415 \frac{\text{lb}_f}{\text{in}^2}$$

Then

$$\textcircled{2} \quad \phi_4 = \frac{P_{v4}}{P_{g4}} = \frac{0.2415}{0.2922} = 0.826 \quad (82.6\%)$$

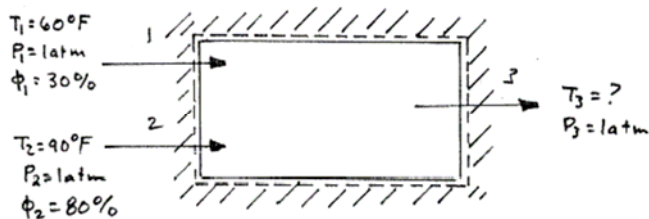
-
1. Using this expression avoids an iterative solution of Eq. (2) using table data.
 2. The problem is readily solved using the psychrometric chart, thereby reducing considerably the computation required throughout.

PROBLEM 12.101

KNOWN: A stream of air at 60°F , 1 atm , $\phi = 30\%$ is mixed adiabatically with a stream at 90°F , 1 atm , $\phi = 80\%$. A single mixed stream exits.

FIND: Plot the temperature of the exiting stream versus the ratio of the mass flow rates of the dry air present in the two incoming streams.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.

ANALYSIS: (a) At steady state mass rate balances reduce to give

$$\text{air: } \dot{m}_a1 + \dot{m}_a2 = \dot{m}_a3$$

$$\text{water: } \dot{m}_w1 + \dot{m}_w2 = \dot{m}_w3 \Rightarrow w_1 \dot{m}_a1 + w_2 \dot{m}_a2 = w_3 \dot{m}_a3$$

Thus

$$w_3 = \frac{w_1 \dot{m}_a1 + w_2 \dot{m}_a2}{\dot{m}_a1 + \dot{m}_a2} = \frac{\left(\frac{\dot{m}_a1}{\dot{m}_a2}\right) w_1 + w_2}{\left(\frac{\dot{m}_a1}{\dot{m}_a2}\right) + 1} = \frac{r w_1 + w_2}{r + 1} \quad (1)$$

where $r = \dot{m}_a1 / \dot{m}_a2$. To find w_1 and w_2 , $P_{v1} = \phi_1 P_g(T_1) = 0.3(0.2563) = 0.07689 \text{ lbf/in}^2$, $P_{v2} = \phi_2 P_g(T_2) = 0.8(0.6988) = 0.55904 \text{ lbf/in}^2$. Then

$$w_1 = 0.622 \frac{P_{v1}}{P - P_{v1}} = \frac{0.622(0.07689)}{14.696 - 0.07689} = 0.00327 \frac{\text{lb(a)}}{\text{lb(v)}}$$

$$w_2 = 0.622 \frac{P_{v2}}{P - P_{v2}} = \frac{0.622(0.55904)}{14.13696} = 0.024597 \frac{\text{lb(a)}}{\text{lb(v)}}$$

At steady state an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a1 h_{a1} + \dot{m}_w1 h_{w1}] + [\dot{m}_a2 h_{a2} + \dot{m}_w2 h_{w2}] - [\dot{m}_a3 h_{a3} + \dot{m}_w3 h_{w3}]$$

Thus

$$0 = \dot{m}_a1 [h_{a1} + w_1 h_{g1}] + \dot{m}_a2 [h_{a2} + w_2 h_{g2}] - \dot{m}_a3 [h_{a3} + w_3 h_{g3}]$$

or

$$\begin{aligned} h_{a3} + w_3 h_{g3} &= \frac{\dot{m}_a1 [h_{a1} + w_1 h_{g1}] + \dot{m}_a2 [h_{a2} + w_2 h_{g2}]}{\dot{m}_a1 + \dot{m}_a2} \\ &= \frac{\left(\frac{\dot{m}_a1}{\dot{m}_a2}\right) [h_{a1} + w_1 h_{g1}] + [h_{a2} + w_2 h_{g2}]}{\left(\frac{\dot{m}_a1}{\dot{m}_a2}\right) + 1} \\ &= \frac{r [h_{a1} + w_1 h_{g1}] + [h_{a2} + w_2 h_{g2}]}{r + 1} \quad (2) \end{aligned}$$

Combining Eqs. (1) and (2)

$$h_{a3} + \left[\frac{r w_1 + w_2}{r + 1}\right] h_{g3} = \frac{r [h_{a1} + w_1 h_{g1}] + [h_{a2} + w_2 h_{g2}]}{r + 1} \quad (3)$$

Using the result of Problem 12.74, $(h_a + w h_g) = 0.24 T(^{\circ}\text{F}) + w(1061 + 0.444 T(^{\circ}\text{F}))$

Continued on next slide

Problem 12-101 continued

Accordingly

$$h_{a1} = 0.24(60) = 14.4 \text{ Btu/lb(a)}, \quad h_{a2} = 0.24(90) = 21.6 \text{ Btu/lb(a)}$$

$$h_{g1} = 1061 + 0.444(60) = 1087.64 \text{ Btu/lb(v)}$$

$$h_{g2} = 1061 + 0.444(90) = 1100.96 \text{ Btu/lb(v)}$$

Equation (3) becomes

$$0.24 T_3 + \left[\frac{r\omega_1 + \omega_2}{r+1} \right] [1061 + 0.444 T_3] = \frac{17.957 r + 48.6803}{r+1}$$

or

$$\left\{ 0.24 + 0.444 \left[\frac{r\omega_1 + \omega_2}{r+1} \right] \right\} T_3 = \frac{14.488 r + 22.583}{r+1}$$

where $\omega_1 = 0.00327$ and $\omega_2 = 0.024597$. For $r=2$, $T_3 = 70.26^\circ\text{F}$ ← T_3

(b) The data for the required plot are obtained using IT, as follows:

IT Code

T1 = 60 // °F

p = 1 // atm

phi1 = 0.3

T2 = 90 // °F

phi2 = 0.8

r = 2

w1 = w_Tphi(T1, phi1, p)

w2 = w_Tphi(T2, phi2, p)

w3 = w_Tphi(T3, phi3, p)

h1 = ha_Tw(T1, w1)

h2 = ha_Tw(T2, w2)

h3 = ha_Tw(T3, w3)

r * w1 + w2 = (1 + r) * w3

r * h1 + h2 = (1 + r) * h3

IT Results

$\omega_1 = 0.003269 \text{ lb(v)/lb(a)}$

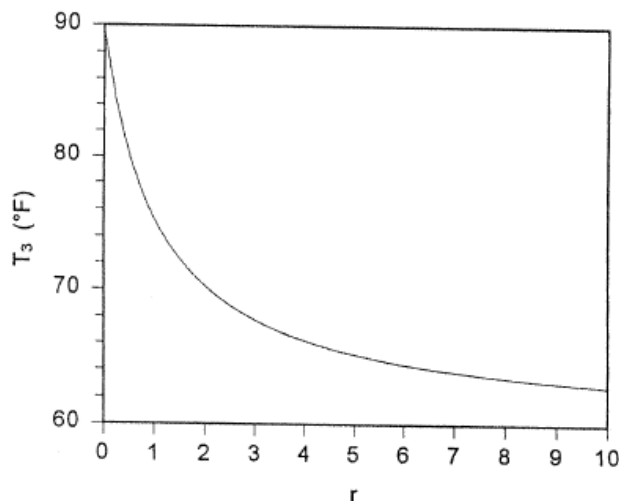
$\omega_2 = 0.02457 \text{ lb(v)/lb(a)}$

$\omega_3 = 0.01037 \text{ lb(v)/lb(a)}$

$\phi_3 = 0.6577 \text{ (65.77\%)}$

$T_3 = 70.26^\circ\text{F}$

PLOT:



As $r \rightarrow \infty$, $T_3 \rightarrow T_1 = 60^\circ\text{F}$, as expected.

PROBLEM 12.102

KNOWN: Operating data are provided for two moist air streams mixing adiabatically at steady state.

FIND: Determine the exergy destruction rate.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: (1) The control volume shown operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Each stream is modeled as an ideal gas mixture. (3) Let $T_0 = 55^\circ\text{F}$.

ANALYSIS: To determine the exergy destruction rate, we first fix state 3 using mass and energy balances. At steady state

$$\text{air: } \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\text{water: } \dot{m}_1 + \dot{m}_2 = \dot{m}_3 \Rightarrow \omega_1 \dot{m}_1 + \omega_2 \dot{m}_2 = \omega_3 \dot{m}_3$$

Thus

$$\omega_3 = \frac{\omega_1 \dot{m}_1 + \omega_2 \dot{m}_2}{\dot{m}_1 + \dot{m}_2} \quad (1)$$

At state 1, $P_{v1} = \phi_1 P_g(T_1) = 0.5 (0.5073) = 0.25365 \text{ lbf/in}^2$; $\omega_1 = .622 \left(\frac{P_{v1}}{P - P_{v1}} \right) = 0.01092 \frac{\text{lb(v)}}{\text{lb(a)}}$

To find ω_2 , use Eqs. 12.53 and 12.52, respectively, with $T_{wb2} = T_{a3}$.

$$\omega' = .622 \left[\frac{P_g(T_{wb2})}{P - P_g(T_{wb2})} \right] = .622 \left[\frac{0.5967}{14.696 - 0.5967} \right] = 0.02632$$

$$\omega_2 = \frac{c_p(T_{wb2} - T_2) + \omega' h_{fg@T_{wb2}}}{h_g(T_2) - h_f(T_{wb2})} = \frac{0.24(85 - 120) + (0.02632)(1043.45)}{1113.5 - 53.08}$$

$$= 0.01803 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Also

$$P_{v2} = \frac{\omega_2 P}{.622 + \omega_2} = \frac{(0.01803)(14.696)}{0.64} = 0.414 \text{ lbf/in}^2$$

The mass flow rates $\dot{m}_1$ and $\dot{m}_2$ are

$$\dot{m}_1 = \frac{(P - P_{v1})(AV)_1}{R_a T_1} = \frac{(14.696 - 0.25365) \text{ lbf/in}^2 (2770 \text{ ft}^3/\text{min}) \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right|}{\left(\frac{1545}{28.97} \right) \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot ^\circ\text{R}} (540^\circ\text{R})} = 200 \frac{\text{lb(a)}}{\text{min}}$$

$$\dot{m}_2 = \frac{(14.696 - 0.414)(3000)}{\left(\frac{1545}{28.97} \right) (580)} \left| \frac{144}{1} \right| = 199.5 \frac{\text{lb(a)}}{\text{min}}$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 399.5 \frac{\text{lb(a)}}{\text{min}}$$

At steady state, the energy rate balance reduces to

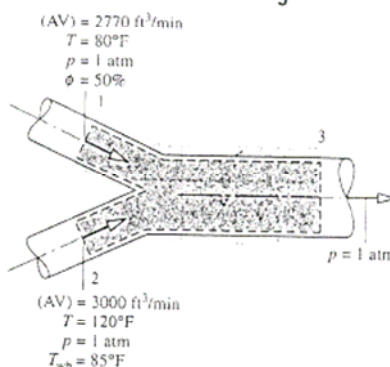
$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_1 h_{a1} + \dot{m}_1 h_{v1}] + [\dot{m}_2 h_{a2} + \dot{m}_2 h_{v2}] - [\dot{m}_3 h_{a3} + \dot{m}_3 h_{v3}]$$

Thus

$$0 = \dot{m}_1 [h_{a1} + \omega_1 h_{g1}] + \dot{m}_2 [h_{a2} + \omega_2 h_{g2}] - \dot{m}_3 [h_{a3} + \omega_3 h_{g3}]$$

or

$$h_{a3} + \omega_3 h_{g3} = \frac{\dot{m}_1}{\dot{m}_3} [h_{a1} + \omega_1 h_{g1}] + \frac{\dot{m}_2}{\dot{m}_3} [h_{a2} + \omega_2 h_{g2}] \quad (2)$$



Continued on next slide

Problem 12-102 continued

Using the result of Problem 12.74, $h_a + \omega h_g = 0.24 T + \omega (1061 + 0.444 T)$. Thus

$$h_{a1} + \omega_1 h_{g1} = 31.174 \text{ Btu/lb(a)}$$

$$h_{a2} + \omega_2 h_{g2} = 48.89 \text{ Btu/lb(a)}$$

From (2), and using Eq. (1) to get $\omega_3 = 0.01447 \text{ lb(v)/lb(a)}$

$$h_{a3} + \omega_3 h_{g3} = \left(\frac{200}{399.5}\right)(31.174) + \left(\frac{199.5}{399.5}\right)(48.89) = 40.021 \Rightarrow T_3 = 100.1^\circ \text{F} \leftarrow T_3$$

The exergy destruction rate is $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the entropy production rate from an entropy balance which at steady state takes the form

$$0 = \sum \dot{Q}_j / T_j + [\dot{m}_{a1} s_a(T_1, p_{a1}) + \dot{m}_{v1} s_v(T_1, p_{v1})] + [\dot{m}_{a2} s_a(T_2, p_{a2}) + \dot{m}_{v2} s_v(T_2, p_{v2})] - [\dot{m}_{a3} s_a(T_3, p_{a3}) + \dot{m}_{v3} s_v(T_3, p_{v3})] + \dot{\sigma}_{cv}$$

or

$$\begin{aligned} \dot{\sigma}_{cv} &= \dot{m}_{a3} [s_a(T_3, p_{a3}) + \omega_3 s_v(T_3, p_{v3})] - \dot{m}_{a1} [s_a(T_1, p_{a1}) + \omega_1 s_v(T_1, p_{v1})] - \dot{m}_{a2} [s_a(T_2, p_{a2}) + \omega_2 s_v(T_2, p_{v2})] \\ &= \dot{m}_{a1} [s_a(T_3, p_{a3}) - s_a(T_1, p_{a1})] + \dot{m}_{a2} [s_a(T_3, p_{a3}) - s_a(T_2, p_{a2})] + \dot{m}_{a3} \omega_3 s_v(T_3, p_{v3}) - \dot{m}_{a1} \omega_1 s_v(T_1, p_{v1}) - \dot{m}_{a2} \omega_2 s_v(T_2, p_{v2}) \end{aligned}$$

From Eq. 6.19, $s_v(T, p_v) = s_g(T) - \bar{R}/M_v \ln \phi$, and using Eq. 6.23 for the dry air

$$\begin{aligned} \dot{\sigma}_{cv} &= \dot{m}_{a1} \left[c_{pa} \ln \frac{T_3}{T_1} - \frac{\bar{R}}{M_a} \ln \frac{p_{a3}}{p_{a1}} \right] + \dot{m}_{a2} \left[c_{pa} \ln \frac{T_3}{T_2} - \frac{\bar{R}}{M_a} \ln \frac{p_{a3}}{p_{a2}} \right] + \dot{m}_{a3} \omega_3 \left[s_g(T_3) - \frac{\bar{R}}{M_v} \ln \phi_3 \right] - \\ &\quad \dot{m}_{a1} \omega_1 \left[s_g(T_1) - \frac{\bar{R}}{M_v} \ln \phi_1 \right] - \dot{m}_{a2} \omega_2 \left[s_g(T_2) - \frac{\bar{R}}{M_v} \ln \phi_2 \right] \quad (3) \end{aligned}$$

From above $p_{a1} = p - p_{v1} = 14.696 - 0.25365 = 14.442$ and $p_{a2} = p - p_{v2} = 14.282$. Now

$$p_{v3} = \frac{\omega_3 p}{0.622 + \omega_3} = 0.3341 \text{ lbf/in}^2 \Rightarrow p_{a3} = p - p_{v3} = 14.362 \text{ lbf/in}^2$$

Inserting values into Eq. (3)

$$\begin{aligned} \dot{\sigma}_{cv} &= 200 \left[0.24 \ln \frac{560.1}{540} - \frac{1.986}{28.97} \ln \frac{14.362}{14.442} \right] + 199.5 \left[0.24 \ln \frac{560.1}{580} - \frac{1.986}{28.97} \ln \frac{14.362}{14.282} \right] + \\ &\quad 399.5 (0.01447) \left[1.98195 - \frac{1.986}{18.02} \ln 0.3504 \right] - 200 (0.01092) \left[2.0356 - \frac{1.986}{18.02} \ln 0.5 \right] \\ &\quad - 199.5 (0.01803) \left[1.9336 - \frac{1.986}{18.02} \ln 0.244 \right] = 0.08079 \frac{\text{Btu/min}}{^\circ \text{R}} \end{aligned}$$

Finally

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = (555)(0.08079) = 44.84 \text{ Btu/min} \leftarrow \dot{E}_d$$

1. The following is a much simpler solution that uses the psychrometric functions in IT:

IT Code

```
T1 = 80 // °F
p = 1 // atm
phi1 = 0.5
AV1 = 2770 // ft³/min
va1 = va_Tw(T1, w1, p)
mdota1 = AV1 / va1
w1 = w_Tphi(T1, phi1, p)
```

```
T2 = 120 // °F
Twb2 = 85 // °F
phi12 = phi_Tw(T2, w2, p)
w2 = w_TTwb(T2, Twb2, p)
AV2 = 3000 // ft³/min
va2 = va_Tw(T2, w2, p)
```

```
mdota2 = AV2 / va2
mdota3 = mdota1 + mdota2
mdota3 * w3 = mdota1 * w1 + mdota2 * w2

mdota3 * h3 = mdota1 * h1 + mdota2 * h2
h1 = ha_Tw(T1, w1)
h2 = ha_Tw(T2, w2)
h3 = ha_Tw(T3, w3)
```

```
0 = mdota1 * s1 + mdota2 * s2 - mdota3 * s3 + sigmadot
s1 = sa_Tw(T1, w1, p)
s2 = sa_Tw(T2, w2, p)
s3 = sa_Tw(T3, w3, p)

Edotd = To * sigmadot
To = 80 + 460
```

IT Results

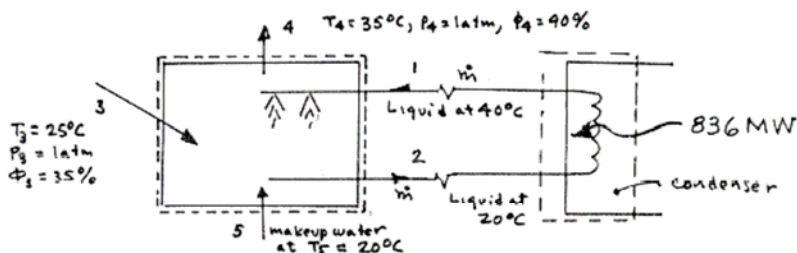
```
w1 = 0.01091 lb(v)/lb(a)
w2 = 0.01799 lb(v)/lb(a)
phi2 = 0.2441 (24.41%)
w3 = 0.01445 lb(v)/lb(a)
m_a1 = 200.1 lb(a)/min
m_a2 = 199.6 lb(a)/min
m_a3 = 399.7 lb(a)/min
h1 = 31.13 Btu/lb(a)
h2 = 48.8 Btu/lb(a)
h3 = 39.95 Btu/lb(a)
sigma_cv = 0.08154 Btu/min-°R
T3 = 100.1°F
E_d = 45.26 Btu/min
```

PROBLEM 12.103

KNOWN: Operating data are provided for a cooling tower that services a power plant.

FIND: Determine the mass flow rates of the entering atmospheric air and the make up water.

SCHEMATIC GIVEN DATA:



ENGINEERING MODEL: (1) The control volumes shown in the figure are at steady state with negligible $\dot{Q}_{cv}$, $\dot{W}_{cv}$, and kinetic/potential energy effects. (2) For the streams 1, 2, 5, $h \approx h_f(T)$.

ANALYSIS: The mass flow rates at 1, 2 are equal, $\dot{m}$. An energy balance on the cooling water side of the condenser is $\dot{Q}_{cv} = \dot{m}[h_1 - h_2]$, or

$$\dot{m} = \frac{\dot{Q}_{cv}}{h_1 - h_2} = \frac{836 \times 10^3 \text{ kJ/s}}{(167.6 - 84) \text{ kJ/kg}} = 10^4 \text{ kg/s}$$

Next, for a control volume enclosing the cooling tower, mass rate balances give $\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_a$ and

$$\dot{m}_1 + \dot{m}_3 + \dot{m}_5 = \dot{m}_2 + \dot{m}_4$$

Since $\dot{m}_1 = \dot{m}_2$

$$\dot{m}_3 = \dot{m}_4 - \dot{m}_5 = \dot{m}_a(\omega_4 - \omega_3) \quad (1)$$

The mass flow rate of the entering atmospheric air is

$$\dot{m}_3 = \dot{m}_a + \dot{m}_5 = \dot{m}_a(1 + \omega_3) \quad (2)$$

From Eqs (1) and (2) it is clear that $\dot{m}_a$, ω_3 , and ω_4 must be evaluated.

To find ω_3 and ω_4 , write $P_{v3} = \phi_3 P_g(T_3) = (0.35)(0.03169) = 0.01109 \text{ bars}$, $P_{v4} = \phi_4 P_g(T_4) = (0.9)(0.05628) = 0.05065 \text{ bar}$. Then

$$\omega_3 = 0.622 \left[\frac{0.01109}{1.01325 - 0.01109} \right] = 0.00688 \frac{\text{kg(v)}}{\text{kg(a)}}, \quad \omega_4 = 0.622 \left[\frac{0.05065}{1.01325 - 0.05065} \right] = 0.0327 \frac{\text{kg(v)}}{\text{kg(a)}}$$

The mass flow rate $\dot{m}_a$ can be found from an energy rate balance which at steady state reduces to

$$0 = \dot{Q}_{cv} + \dot{m}_3 h_3 + \dot{m}_5 h_5 + \dot{m}_1 h_1 + \dot{m}_4 h_4 - \dot{m}_2 h_2 - [\dot{m}_a h_a + \dot{m}_5 h_5]$$

Introducing Eq. (1)

$$0 = \dot{m}_a [h_3 + \omega_3 h_{g3}] + \dot{m}_1 [h_f(T_1) - h_f(T_2)] + \dot{m}_a (\omega_4 - \omega_3) h_f(T_5) - \dot{m}_a [h_a + \omega_4 h_{g4}]$$

Solving for $\dot{m}_a$

$$\dot{m}_a = \frac{\dot{m}_1 [h_f(T_1) - h_f(T_2)]}{(h_3 - h_a) + \omega_3 h_{g3} - \omega_4 h_{g4} + (\omega_4 - \omega_3) h_f(T_5)}$$

$$= \frac{10,000 [84 - 167.6]}{[298.18 - 308.23] + (0.00688)(2547.2) - (0.0327)(2565.3) + (0.0327 - 0.00688)(83.96)} = 11,261 \text{ kg(a)/s}$$

With ω_3 , ω_4 , and $\dot{m}_a$, Eq. (1) gives

$$\dot{m}_5 = \dot{m}_a (\omega_4 - \omega_3) = 11,261 (0.0327 - 0.00688) = 290.8 \text{ kg/s} \quad \leftarrow \text{Makeup}$$

And Eq. (2) gives

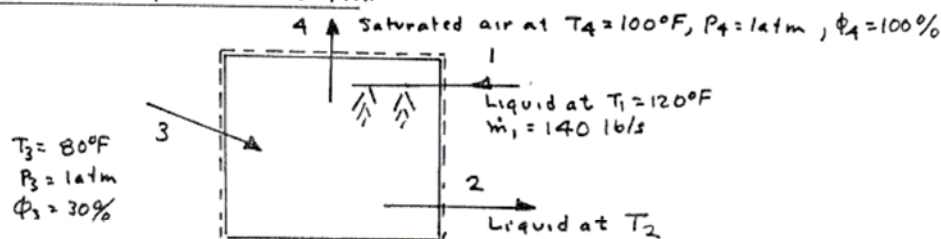
$$\dot{m}_3 = \dot{m}_a (1 + \omega_3) = 11,261 [1 + 0.00688] = 11,338.5 \text{ kg/s} \quad \leftarrow \text{Atm. air}$$

PROBLEM 12.104

KNOWN: Steady state operating data are provided for a cooling tower.

FIND: Plot the mass flow rate of dry air versus the temperature at which cooled water exits the tower.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) For streams 1 and 2 $h = h_f(T)$.

ANALYSIS: At steady state, mass rate balances give $\dot{m}_{a3} = \dot{m}_{a4} = \dot{m}_a$ and

$$\dot{m}_1 + \dot{m}_{v3} = \dot{m}_2 + \dot{m}_{v4} \Rightarrow \dot{m}_2 = \dot{m}_1 + \dot{m}_a(w_3 - w_4) \quad (1)$$

To find w_3 and w_4 , write $P_{v3} = \phi_3 P_g(T_3) = (0.30)(0.5073) = 0.15219 \text{ lbf/in}^2$, $P_{v4} = P_g(100^\circ\text{F}) = 0.9503 \text{ lbf/in}^2$. Then

$$w_3 = 0.622 \left[\frac{0.15219}{14.696 - 0.15219} \right] = 0.0065 \frac{\text{lb(v)}}{\text{lb(a)}}, \quad w_4 = 0.622 \left[\frac{0.9503}{14.696 - 0.9503} \right] = 0.0430 \frac{\text{lb(v)}}{\text{lb(a)}}$$

The mass flow rate $\dot{m}_a$ can be found from an energy rate balance which at steady state reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_3 h_{f3} + \dot{m}_{v3} h_{v3}] + \dot{m}_1 h_1 - \dot{m}_2 h_2 - [\dot{m}_4 h_{f4} + \dot{m}_{v4} h_{v4}]$$

Introducing Eq. (1)

$$0 = \dot{m}_a [h_{f3} + w_3 h_{g3}] + \dot{m}_1 h_f(T_1) - [\dot{m}_1 + \dot{m}_a(w_3 - w_4)] h_f(T_2) - \dot{m}_a (h_{f4} + w_4 h_{g4})$$

Solving for $\dot{m}_a$

$$\begin{aligned} \dot{m}_a &= \frac{\dot{m}_1 [h_f(T_1) - h_f(T_2)]}{(h_{f4} - h_{f3}) + w_4 h_{g4} - w_3 h_{g3} + (w_3 - w_4) h_f(T_2)} \\ &= \frac{140 [88 - h_f(T_2)]}{0.24(100 - 80) + (0.043)(105) - (0.0065)(1096.4) + (0.043 - 0.0065) h_f(T_2)} \\ &= \frac{140 [88 - h_f(T_2)]}{45.188 + (0.0365) h_f(T_2)} \left(\frac{\text{lb}}{\text{s}} \right) \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| \quad (1) \end{aligned}$$

Sample Calculation: When $T_2 = 80^\circ\text{F}$, $h_f(T_2) = 48.09 \text{ Btu/lb}$. Eq. (1) gives

$$\dot{m}_a = 4.28 \times 10^5 \text{ lb/h.}$$

The data for the required plot are obtained using IT, as follows:

Continued on next slide

Problem 12-104 continued

IT Code

```
T1 = 120 // °F
mdot1 = 140 // lb/s
T2 = 80 // °F
T3 = 80 // °F
p3 = 14.696 // lbf/in²
phi3 = 0.3
T4 = 100 // °F
p4 = p3
phi4 = 1.0
```

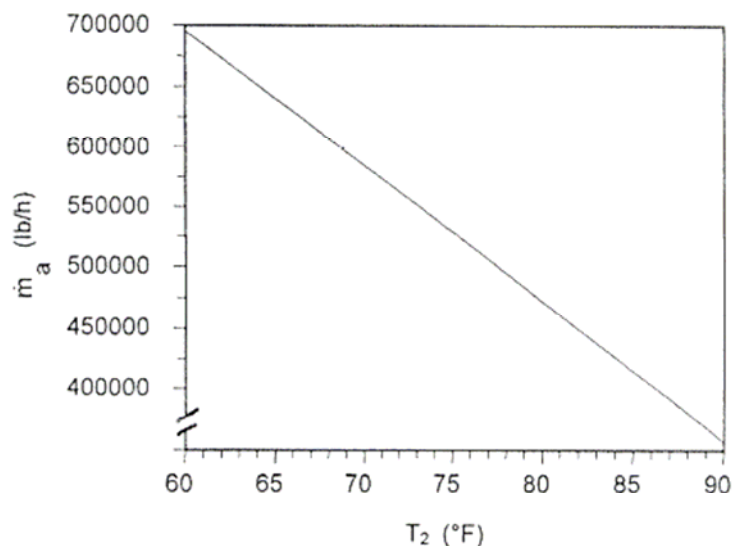
```
h1 = hsat_Px("Water/Steam", psat1, 0)
psat1 = Psat_T("Water/Steam", T1)
h2 = hsat_Px("Water/Steam", psat2, 0)
psat2 = Psat_T("Water/Steam", T2)
w3 = w_Tphi(T3, phi3, p3)
h3 = ha_Tw(T3, w3)
w4 = w_Tphi(T4, phi4, p4)
h4 = ha_Tw(T4, w4)
```

```
mdota3 = mdota4
mdot1 + w3 * mdota3 = mdot2 + w4 * mdota4
0 = mdota3 * h3 + mdot1 * h1 - mdota4 * h4 -
mdot2 * h2
mdota = mdota3 * 3600 // lb/h
```

IT Results for $T_2 = 80^\circ\text{F}$

```
 $\omega_3 = 0.006503 \text{ lb(v)/lb(a)}$ 
 $\omega_4 = 0.04265 \text{ lb(v)/lb(a)}$ 
 $\dot{m}_2 = 135.3 \text{ lb/s}$ 
 $\dot{m}_a = 4.72\text{E}5 \text{ lb/h}$ 
```

PLOT:



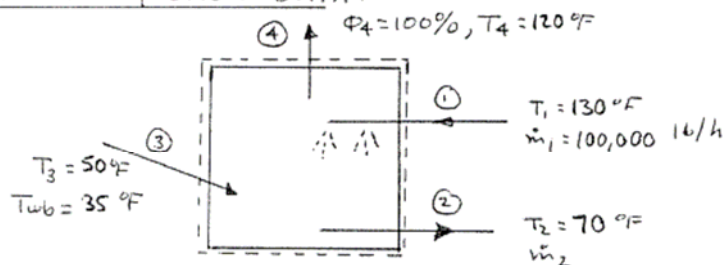
To achieve more cooling (lower liquid temperature exiting the tower), requires increased air flow, as expected.

PROBLEM 12.105

KNOWN: Operating data are provided for a cooling tower at steady state.

FIND: Determine the mass flow rate of the cooled water stream exiting.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) For streams 1 and 2, $h = h_f(T)$. (3) The pressure is constant at 1 atm.

ANALYSIS: Conservation of mass at steady state:

$$\text{(air)} \quad \dot{m}_3 = \dot{m}_4 \equiv \dot{m}_a$$

$$\text{(water)} \quad \dot{m}_1 + \dot{m}_3 = \dot{m}_2 + \dot{m}_4$$

Combining gives, $\dot{m}_2 = \dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4)$.

At steady state, an energy rate balance reduces to

$$0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 + \dot{m}_a [(h_a + \omega h_v)_3 - (h_a + \omega h_v)_4]$$

Collecting results, and using $h_1 = h_f(T_1)$, $h_2 = h_f(T_2)$,

$$0 = \dot{m}_1 [h_f(T_1) - h_f(T_2)] + \dot{m}_a [h_{a3} - h_{a4} + \omega_3 h_{g3} - \omega_4 h_{g4} - (\omega_3 - \omega_4) h_f(T_2)]$$

Solving

$$\dot{m}_a = \frac{\dot{m}_1 (h_f(T_2) - h_f(T_1))}{\underbrace{h_{a3} - h_{a4} + \omega_3 h_{g3} - \omega_4 h_{g4} - (\omega_3 - \omega_4) h_f(T_2)}_{c_p a (T_3 - T_4)}}$$

From Fig. A-9E, $\omega_3 = 0.0010 \text{ lb(v)/lb(a)}$. State 4 is off the chart, so with $P_g(T_4)$

$$\omega_4 = 0.622 \left[\frac{1.695}{14.7 - 1.695} \right] = 0.1303$$

Then, with steam table data

$$\begin{aligned} \dot{m}_a &= \frac{(100,000)(38.09 - 97.98)}{0.24[50 - 120] + 0.0010(1083.3) - 0.1303(1113.5) + 0.1293(38.09)} \\ &= 38,420 \text{ lb(a)/h} \end{aligned}$$

Accordingly

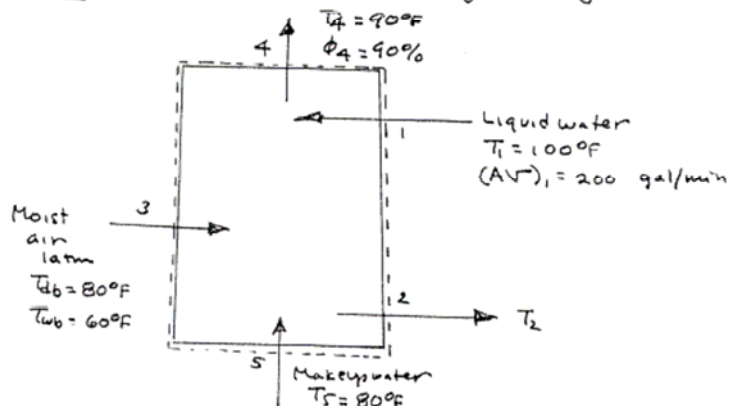
$$\begin{aligned} \dot{m}_2 &= \dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4) \\ &= 100,000 + 38,420(0.0010 - 0.1303) \\ &= 95,032 \frac{\text{lb}}{\text{h}} \end{aligned}$$

← $\dot{m}_2$

PROBLEM 12.106

KNOWN: Steady state operating data are provided for a cooling tower.

FIND: Plot the mass flow rates of the dry air and makeup water versus T_2 .



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv}$, $\dot{W}_{cv}$, and kinetic/potential energy effects are negligible. (3) Each liquid stream is regarded as a saturated liquid at the corresponding temperature. (4) The moist air streams are modeled as ideal gas mixtures at 1 atm. (5) The temperature T_2 is treated as a parameter, ranging $80^\circ\text{F} \leq T_2 \leq 100^\circ\text{F}$.

ANALYSIS: Following the analysis of Example 12.17, the basic relations are

$$\dot{m}_5 = \dot{m}_a (\omega_4 - \omega_3) \quad (1)$$

$$\dot{m}_a = \frac{\dot{m}_1 [h_{f1} - h_{f2}]}{(h_a + \omega h_g)_4 - (h_a + \omega h_g)_3 - (\omega_4 - \omega_3) h_{f5}} \quad (2)$$

Using T_{wb} , T_{db} at 3, the psychrometric chart gives $\omega_3 \approx 0.0066$, $(h_a + \omega h_g)_3 \approx 26.4 \text{ Btu/lb(a)}$. With T_4 , ϕ_4 the chart gives $\omega_4 \approx 0.028$. The value of $(h_a + \omega h_g)_4 = (0.24)(90) + (0.028)(1100.7) = 52.4 \text{ Btu/lb(a)}$.

With $(AV)_1$ and $v_f(100^\circ\text{F})$,

$$\dot{m}_1 = \frac{(200 \frac{\text{gal}}{\text{min}}) (0.13368 \frac{\text{ft}^3}{\text{gal}})}{0.01617 \frac{\text{ft}^3}{\text{lb}}} = 1657.5 \frac{\text{lb}}{\text{min}}$$

With these data, Eq. (1) becomes $\dot{m}_5 = \dot{m}_a (0.028 - 0.0066) = 0.0214 \dot{m}_a$ (3) and Eq. (2) becomes

$$\dot{m}_a = \frac{(1657.5 \frac{\text{lb}}{\text{min}}) [68.05 - h_{f2} \text{ Btu/lb}]}{(52.4 - 26.4) - (0.028 - 0.0066)(48.1)} = 66.38 [68.05 - h_{f2}] \quad (4)$$

Sample calculation: $T_2 = 80^\circ\text{F}$. Then, $h_{f2} = 48.09 \text{ Btu/lb}$. Eq. (4) gives $\dot{m}_a = 66.38 (68.05 - 48.09) = 1324.9 \text{ lb(a)/min}$. Eq. (3) gives $\dot{m}_5 = 28.35 \text{ lb/min}$.

Data for the required plots are obtained using IT, as follows:

Continued on next slide

Problem 12-106 continued

IT Code

```
T1 = 100 // °F
AV1 = 200 // gal/min
T2 = 80 // °F
T3 = 80 // °F
Twb3 = 60 // °F
T4 = 90 // °F
phi4 = 0.9
T5 = 80 // °F
p = 14.696 // lbf/in²
```

```
h1 = hsat_Px("Water/Steam", psat1, 0)
psat1 = Psat_T("Water/Steam", T1)
v1 = vsat_Px("Water/Steam", psat1, 0)
h2 = hsat_Px("Water/Steam", psat2, 0)
psat2 = Psat_T("Water/Steam", T2)
w3 = w_TTwb(T3, Twb3, p)
```

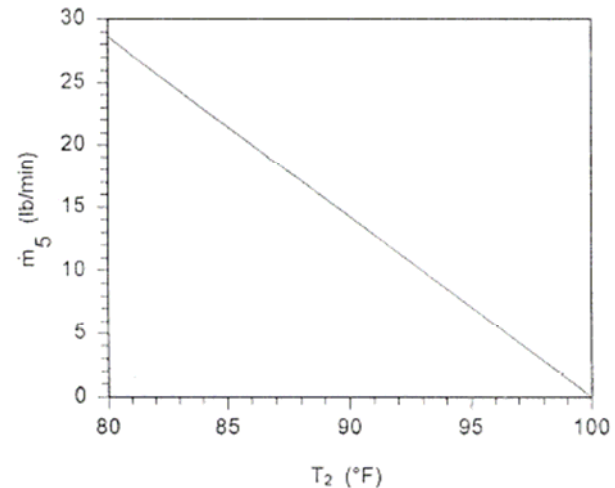
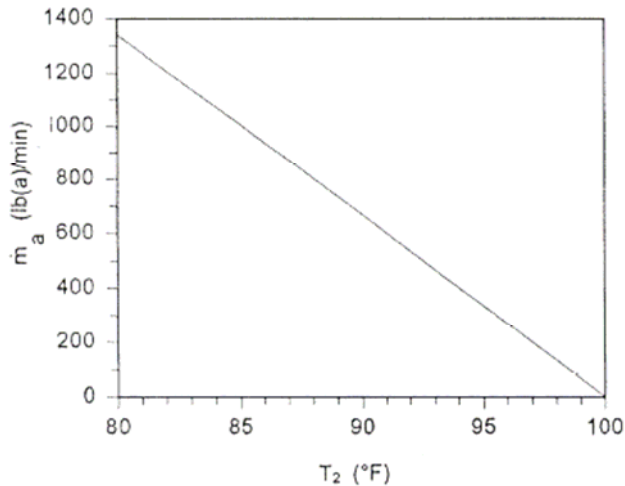
```
h3 = ha_Tw(T3, w3)
w4 = w_Tphi(T4, phi4, p)
h4 = ha_Tw(T4, w4)
h5 = hsat_Px("Water/Steam", psat5, 0)
psat5 = Psat_T("Water/Steam", T5)
```

```
mdot5 = mdota * (w4 - w3)
mdot1 = AV1 * 0.13368 / v1 // lb/min
0 = mdota * (h3 - h4) + mdot5 * h5 + mdot1 * (h1 - h2)
```

IT Results for $T_2 = 80^\circ\text{F}$

```
 $\omega_3 = 0.006453 \text{ lb(v)/lb(a)}$ 
 $\omega_4 = 0.02778 \text{ lb(v)/lb(a)}$ 
 $T_2 = 80^\circ\text{F}$ 
 $\dot{m}_a = 1343 \text{ lb(a)/min}$ 
 $\dot{m}_5 = 28.64 \text{ lb/min}$ 
```

PLOTS:



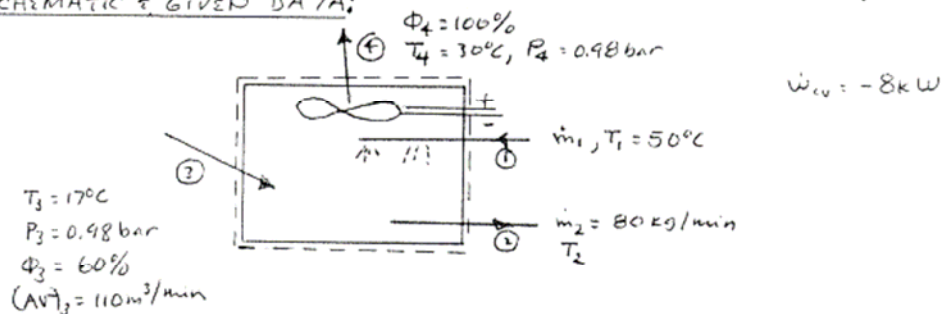
From the plots, we see that to achieve lower liquid temperature exiting the tower, we need increased air flow. Also, since there is more evaporation, greater make-up water flow is required. Both of these results are as expected.

PROBLEM 12.107

KNOWN: Steady state operating data are provided for a forced draft cooling tower.

FIND: Determine the mass flow rate of the entering liquid stream and the temperature of the cooled liquid stream exiting.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown is at steady state with $\dot{Q}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) For the streams at 1 and 2, $h = h_f(T)$.

ANALYSIS: Mass rate balances at steady state give, $\dot{m}_3 = \dot{m}_4 = \dot{m}_a$ and

$$\dot{m}_1 + \dot{m}_3 = \dot{m}_2 + \dot{m}_4 \Rightarrow \dot{m}_1 = \dot{m}_2 + \dot{m}_a(\omega_4 - \omega_3).$$

Calculating

$$\omega_3 = 0.622 \left[\frac{(0.6)(0.01938)}{0.98 - (0.6)(0.01938)} \right] = 0.0075 \frac{\text{kg(v)}}{\text{kg(a)}}, \quad \omega_4 = 0.622 \left[\frac{0.04246}{0.98 - 0.04246} \right] = 0.0282$$

$$\dot{m}_a = \frac{(\dot{A}V)_3}{v_{a3}} = \frac{(\dot{A}V)_3}{\left(\frac{R}{M_a}\right)\left(\frac{T_3}{P_{a3}}\right)} = \frac{(110 \text{ m}^3/\text{min})(0.965372 \times 10^5 \text{ N/m}^2)}{\left(\frac{8314 \text{ N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(290 \text{ K})} = 128.12 \frac{\text{kg(a)}}{\text{min}}$$

Thus

$$\begin{aligned} \dot{m}_1 &= \dot{m}_2 + \dot{m}_a(\omega_4 - \omega_3) = 80 \frac{\text{kg}}{\text{min}} + 128.12 \frac{\text{kg(a)}}{\text{min}} (0.0282 - 0.0075) \frac{\text{kg}}{\text{kg(a)}} \\ &= 82.65 \frac{\text{kg}}{\text{min}} \end{aligned}$$

An energy rate balance reduces at steady state to give

$$0 = -\dot{W}_{cv} + \dot{m}_a[(h_a + \omega h_v)_3 - (h_a + \omega h_v)_4] + \dot{m}_1 h_1 - \dot{m}_2 h_2$$

Or, with $h = h_f(T)$ at 1, 2

$$\begin{aligned} h_f(T_2) &= \frac{-\dot{W}_{cv} + \dot{m}_a[(h_a + \omega h_v)_3 - (h_a + \omega h_v)_4] + \dot{m}_1 h_f(T_1)}{\dot{m}_2} \\ &= \frac{-\dot{W}_{cv} + \dot{m}_a[c_p(T_3 - T_4) + \omega_3 h_{g3} - \omega_4 h_{g4}] + \dot{m}_1 h_f(T_1)}{\dot{m}_2} \\ &= \frac{-\left(\frac{-8 \text{ kJ}}{\text{s}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) + 128.12 [1.005(17 - 30) + 0.0075(2532.6) - 0.0282(2556.7)] + 82.65(209.7)}{80 \text{ kg/min}} \\ &= 116.3 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

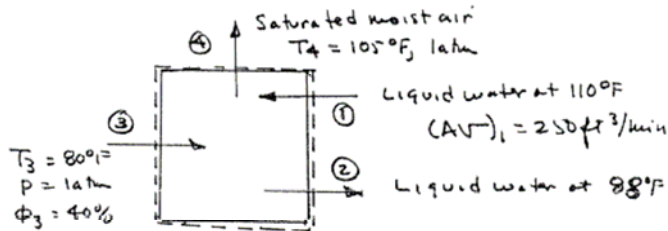
Interpolating in Table A-2 gives $T_2 \approx 28^\circ \text{C}$.

PROBLEM 12.108

KNOWN: Steady state operating data are provided for a cooling tower.

FIND: Determine (a) the mass flow rates of the dry air and cooled water, (b) the tower rate of exergy destruction.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv}$, $\dot{W}_{cv}$, and kinetic/potential energy effects are negligible. (3) Each liquid stream is regarded as a saturated liquid at the corresponding temperature. (4) The moist air streams are treated as ideal gas mixtures. (5) For the exergy reference environment, $T_0 = 53.7^\circ\text{R}$.

ANALYSIS: (a) Mass rate balances at steady state read $\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_a$

$$\dot{m}_2 = \dot{m}_1 + \dot{m}_3 - \dot{m}_4 = \dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4) \quad (1)$$

An energy rate balance reads

$$0 = \dot{m}_1 h_{f1} - \dot{m}_2 h_{f2} + \dot{m}_a [h_a + \omega h_g]_3 - \dot{m}_a [h_a + \omega h_g]_4$$

or, w. m Eq. (1)

$$0 = \dot{m}_1 h_{f1} - [\dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4)] h_{f2} + \dot{m}_a [h_a + \omega h_g]_3 - \dot{m}_a [h_a + \omega h_g]_4$$

or, solving for $\dot{m}_a$

$$\begin{aligned} \dot{m}_a &= \frac{\dot{m}_1 (h_{f1} - h_{f2})}{[h_a + \omega h_g]_4 - [h_a + \omega h_g]_3 + (\omega_3 - \omega_4) h_{f2}} \\ &= \frac{\dot{m}_1 (h_{f1} - h_{f2})}{c_p(T_4 - T_3) + \omega_4 h_{g4} - \omega_3 h_{g3} + (\omega_3 - \omega_4) h_{f2}} \quad (2) \end{aligned}$$

where

$$\omega_3 = \frac{0.622(\phi_3 P_{g3})}{P - \phi_3 P_{g3}} = \frac{0.622(0.4)(0.5073)}{14.7 - (0.4)(0.5073)} = 0.00871$$

$$\omega_4 = \frac{0.622(\phi_4 P_{g4})}{P - \phi_4 P_{g4}} = \frac{(0.622)(1)(1.103)}{14.7 - 1.103} = 0.05046$$

$$\dot{m}_1 = \frac{(AV)_1}{v_{f1}} = \frac{250 \text{ ft}^3/\text{min}}{0.01617 \text{ ft}^3/\text{lb}} = 15461 \frac{\text{lb}}{\text{min}}$$

Inserting values in Eq. (2)

$$\begin{aligned} \dot{m}_a &= \frac{15461 [78.02 - 56.07]}{0.24[105 - 80] + (0.05046)(1107.2) - 0.00871(1096.4) + (-0.04175)(56.07)} \\ &= 6790 \frac{\text{lb(a)}}{\text{min}} \quad \leftarrow \dot{m}_a \end{aligned}$$

Continued on next slide

Problem 12-108 continued

Then, with Eq. (1)

$$\begin{aligned}\dot{m}_2 &= 15461 + 6790(1.00871 - 0.05046) \\ &= 15,178 \frac{\text{lb}}{\text{min}} \quad \leftarrow \dot{m}_2\end{aligned}$$

(2) The rate of exergy destruction can be evaluated in terms of the entropy production rate: $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where from an entropy rate balance

$$\dot{\sigma}_{cv} = \dot{m}_2 s_f(T_2) - \dot{m}_1 s_f(T_1) + \dot{m}_a \left[s_a(T_4, P_{a4}) + \omega_4 s_g(T_4) \right] - \dot{m}_a \left[s_a(T_3, P_{a3}) + \omega_3 s_g(T_3, P_{13}) \right]$$

$\downarrow P_4 = P_{g4}$
 $= s_g(T_3) - R \ln \phi_3$
(Sec. 12.5.2)

Steam Table Data: (Btu/lb·°R)

$$\begin{aligned}s_f(T_1) &= 0.1473 & s_g(T_3) &= 2.0356 \\ s_f(T_2) &= 0.10801 & s_g(T_4) &= 1.9697\end{aligned}$$

$s_v(T_3, P_{13}) = s_g(T_3) - R \ln \phi_3$
 $= 2.0356 - \frac{1.986}{18} \ln 0.4$
 $= 2.1367$

Also

$$\begin{aligned}s_a(T_4, P_{a4}) - s_a(T_3, P_{a3}) &= c_p \ln \frac{T_4}{T_3} - \frac{\bar{R}}{M_a} \ln \frac{P_{a4}}{P_{a3}} = 0.24 \ln \left(\frac{565}{540} \right) - \frac{1.986}{28.97} \ln \left[\frac{14.7 - 1.103}{14.7 - (0.4)(1.073)} \right] \\ &= 0.01526 \text{ Btu/lb} \cdot \text{°R}\end{aligned}$$

Then

$$\begin{aligned}\textcircled{1} \quad \dot{\sigma}_{cv} &= 15,178(0.10801) - 15,461(0.1473) + 6790[0.01526 + 0.05046(1.9697) - 0.00871(2.1367)] \\ &= 14.01 \text{ Btu/min} \cdot \text{°R}\end{aligned}$$

Finally,

$$\begin{aligned}\dot{E}_d &= T_0 \dot{\sigma}_{cv} = (537^\circ\text{R}) \left(14.01 \frac{\text{Btu}}{\text{min} \cdot \text{°R}} \right) \\ &= 7523 \frac{\text{Btu}}{\text{min}}\end{aligned}$$

1. Entropy balance calculations tend to be sensitive to round off.

PROBLEM 13.1

KNOWN: A vessel contains a mixture of 60% O_2 and 40% CO on a mass basis.

FIND: Determine the percent excess or percent deficiency of O_2 , as appropriate.

ANALYSIS: For complete combustion of CO with the minimum amount of O_2 , $CO + \frac{1}{2} O_2 \rightarrow CO_2$. Consider a typical unit mass of mixture, then

	m_i	m_i	M_i	$n_i = m_i/M_i$	
O_2	0.6	0.6	32	0.01875	} : $\left(\frac{n_{O_2}}{n_{CO_2}}\right)_{act} = 1.31$
CO	0.4	0.4	28.01	0.01428	
				<u>0.03303</u>	

$\left(\frac{n_{O_2}}{n_{CO_2}}\right)_{theo} = 0.5$

$$\% \text{ excess } O_2 = \left(\frac{1.31 - 0.5}{0.5} \right) (100) = 162$$

PROBLEM 13.2

KNOWN: Ten grams of C_3H_8 burns with just enough O_2 for complete combustion.

FIND: Determine the mass of oxygen and the mass of products formed.

ANALYSIS: For complete combustion of C_3H_8 with the theoretical amount of O_2 : $C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$

To determine the mass of O_2 :

$$\left(\frac{5 \text{ mol } O_2}{1 \text{ mol } C_3H_8} \right) \left(\frac{32 \text{ g/mol}}{44.09 \text{ g/mol}} \right) = 3.629 \frac{\text{g}(O_2)}{\text{g}(C_3H_8)}$$

For $m_{C_3H_8} = 10 \text{ g}$

$$\left(3.629 \frac{\text{g}(O_2)}{\text{g}(C_3H_8)} \right) \times (10 \text{ g } C_3H_8) = 36.29 \text{ g}(O_2) \leftarrow m_{O_2}$$

For the combustion reaction

$$\textcircled{1} \quad m_{\text{prod}} = m_{C_3H_8} + m_{O_2} = 10 + 36.29 = 46.29 \text{ g}(\text{prod}) \leftarrow m_{\text{prod}}$$

1. Alternatively

$$\frac{3(44.01) + 8(18.02)}{1(44.09)} = 4.629 \frac{\text{g}(\text{prod})}{\text{g}(C_3H_8)}$$

thus

$$m_{\text{prod}} = \left(4.629 \frac{\text{g}(\text{prod})}{\text{g}(C_3H_8)} \right) \times (10 \text{ g } C_3H_8) = 46.29 \text{ g}(\text{prod})$$

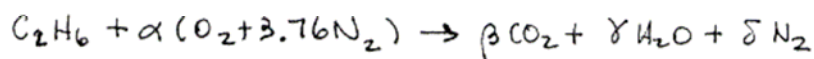
PROBLEM 13.3

KNOWN: C_2H_6 burns completely with theoretical air.

FIND: Determine the air-fuel ratio on a (a) molar basis, and (b) mass basis.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) N_2 is inert.

ANALYSIS: For complete combustion of ethane with theoretical air



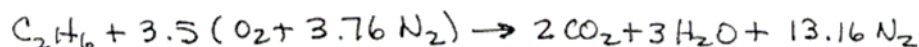
Carbon: $2 = \beta$

Hydrogen, H_2 : $3 = \gamma$

Oxygen, O_2 : $\alpha = \beta + \frac{\gamma}{2} = 3.5$

Nitrogen, N_2 : $\delta = \alpha(3.76) = 13.16$

Thus



$$(a) \quad \bar{AF} = \frac{(3.5)(4.76) \text{ kmol (air)}}{1 \text{ kmol (} C_2H_6 \text{)}} = 16.66 \quad \bar{AF}$$

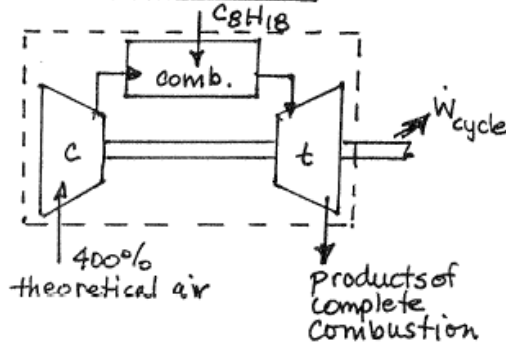
$$(b) \quad AF = \bar{AF} \left(\frac{M_{\text{air}}}{M_{\text{fuel}}} \right) = 16.66 \left(\frac{28.97}{30.07} \right) = 16.05 \quad AF$$

PROBLEM 13.4

KNOWN: A gas turbine burns C_8H_{18} completely with 400% of theoretical air.

FIND: Determine the amount of N_2 in the products, in kmol per kmol of fuel.

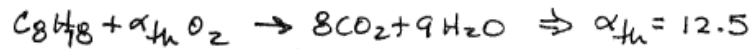
SCHEMATIC & GIVEN DATA:



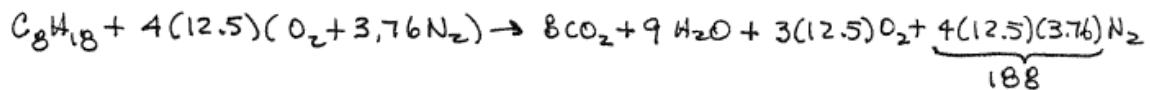
ENGINEERING MODEL:

- (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 is inert.

ANALYSIS: For theoretical combustion



For $\alpha_{act} = 4\alpha_{th}$ (400% of theoretical air)



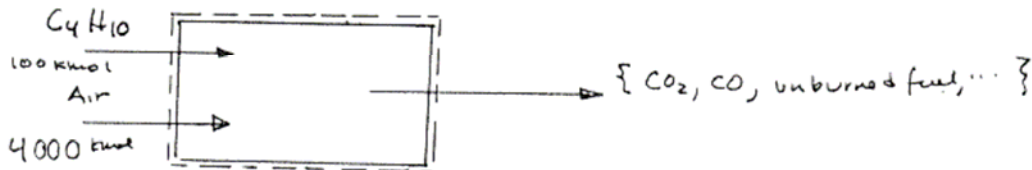
$$n_{N_2} = 188 \frac{\text{kmol}(N_2)}{\text{kmol}(\text{fuel})} \leftarrow n_{N_2}$$

PROBLEM 13.5

KNOWN: 100 kmol of C_4H_{10} and 4000 kmol of air enter a furnace. CO_2 , CO and unburned fuel are in the products of combustion.

FIND: Determine the percent excess or percent deficiency of air, as appropriate.

SCHEMATIC & GIVEN DATA:

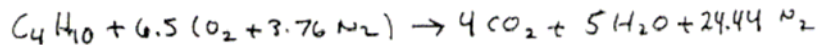


ENGINEERING MODEL: 3.76 kmol of N_2 accompany each kmol of O_2 in air. N_2 is inert.

ANALYSIS: The data provided are for actual operation. Thus

$$\overline{AF} = \frac{4000}{100} = 40.00 \frac{\text{kmol (a)}}{\text{kmol (fuel)}}$$

The balance equation for complete combustion with the theoretical amount of air is



The theoretical air/fuel ratio is

$$(\overline{AF})_{\text{theo}} = 6.5 \left(\frac{4.76}{1} \right) = 30.94 \frac{\text{kmol (a)}}{\text{kmol (fuel)}}$$

Accordingly

$$\% \text{ excess} = \left(\frac{40.00 - 30.94}{30.94} \right) (100) = 29.3\% \quad \leftarrow$$

PROBLEM 13.6

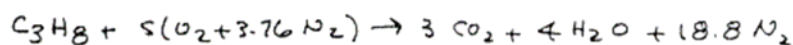
KNOWN: Three cases where C_3H_8 is burned with air.

FIND: Obtain the balanced chemical reaction for complete combustion.

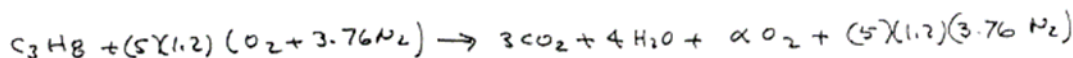
ENGINEERING: 3.76 moles of N_2 accompany each mole of O_2 in the air.

MODEL: N_2 is inert.

ANALYSIS: (a) Complete combustion with the theoretical amount of air.



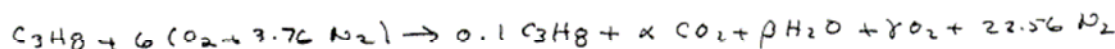
(b) Complete combustion with 20% excess air.



$$O: (6)(2) = 6 + 4 + 2\alpha \Rightarrow \alpha = 1$$

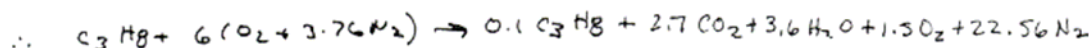


(c) Combustion with 20% excess air. 90% of the fuel is burned.



$$C: 3 = 0.3 + \alpha \Rightarrow \alpha = 2.7 \quad O: 12 = 2(2.7) + (3.6) + 2\gamma \Rightarrow \gamma = 1.5$$

$$H: 8 = 0.8 + 2\beta \Rightarrow \beta = 3.6$$



PROBLEM 13.7

KNOWN: C_4H_{10} burns completely with air. The equivalence ratio is known.

FIND: Determine (a) the balanced reaction equation, and (b) the percent excess air.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 is inert.

ANALYSIS:

(a) Theoretical combustion: $C_4H_{10} + \alpha_{th} O_2 \rightarrow 4CO_2 + 5H_2O \Rightarrow \alpha_{th} = 6.5$
The equivalence ratio is

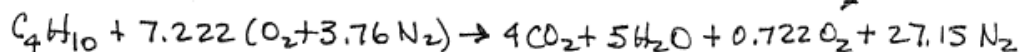
$$\frac{(\bar{F}A)_{actual}}{(\bar{F}A)_{theoretical}} = 0.9 \Rightarrow (\bar{F}A)_{actual} = 0.9(\bar{F}A)_{theoretical}$$

$$(\bar{F}A)_{theoretical} = \frac{1 \text{ kmol}(C_4H_{10})}{(6.5)(4.76) \text{ kmol}(air)} = 0.03232$$

$$(\bar{F}A)_{actual} = (0.9)(0.03232) = 0.02909$$

$$(\bar{A}F)_{actual} = \frac{1}{(\bar{F}A)_{actual}} = 34.376 \frac{\text{kmol}(air)}{\text{kmol}(C_4H_{10})}$$

$$\therefore \alpha_{act} = \frac{34.376}{4.76} = 7.222$$



balanced equation

$$(b) \quad \% \text{ excess air} = \left(\frac{\alpha_{act} - \alpha_{th}}{\alpha_{th}} \right) \times 100 = 11.1\% \quad \text{excess air}$$

PROBLEM 13.8

KNOWN: Methane burns completely with the stoichiometric amount of H_2O_2 .

FIND: Determine the balanced reaction equation.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air and N_2 is inert. (2) The amount of O_2 provided is the minimum amount for complete combustion.

ANALYSIS: On a per mole of fuel basis

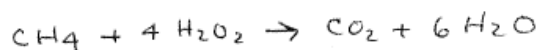


$$\text{C: } 1 = \beta$$

$$\text{H: } 4 + 2\alpha = 2\gamma$$

$$\text{O: } 2\alpha = 2\beta + \gamma \quad \left. \begin{array}{l} \text{C: } 1 = \beta \\ \text{H: } 4 + 2\alpha = 2\gamma \\ \text{O: } 2\alpha = 2\beta + \gamma \end{array} \right\} \text{ solving, } \begin{array}{l} \alpha = 4 \\ \gamma = 6 \end{array}$$

Accordingly, the balanced reaction equation is



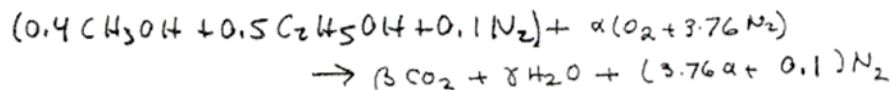
PROBLEM 13.9

KNOWN: A fuel mixture with a specified molar analysis burns completely with 33% excess air.

FIND: Determine (a) the balanced reaction equation, (b) the air-fuel ratio on a molar and a mass basis.

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

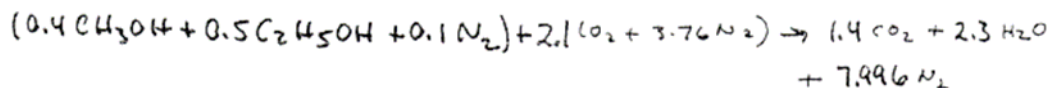
ANALYSIS: On the basis of 1 mole of fuel mixture, the reaction equation for complete combustion with the theoretical amount of air is



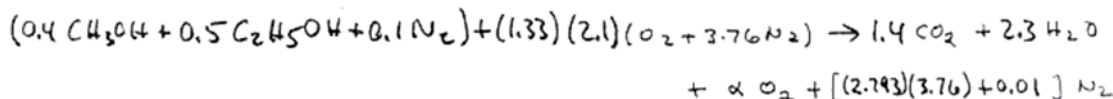
$$C: 0.4 + (2)(0.5) = \beta, \quad \beta = 1.4 \quad O: 0.4 + 0.5 + 2\alpha = 2(1.4) + 2.3, \quad \alpha = 2.1$$

$$H: (4)(0.4) + (6)(0.5) = 2\gamma, \quad \gamma = 2.3$$

Thus

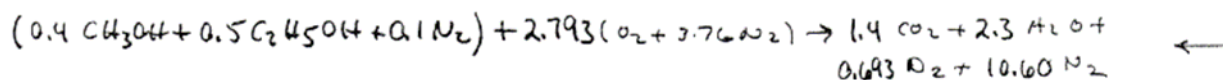


Thus, the reaction for complete combustion with 33% excess air is



$$O: 0.4 + 0.5 + 2(2.793) = 2.8 + 2.3 + 2\alpha, \quad \alpha = 0.693$$

Accordingly



(b) The air-fuel ratio

$$\overline{AF} = \frac{(2.793)(4.76)}{1} = 13.29 \frac{\text{kmol (air)}}{\text{kmol (fuel)}} \quad \leftarrow \overline{AF}$$

The fuel molecular weight is

$$M_f = (0.4)(32.04) + (0.5)(46.07) + (0.1)(28.01) = 38.652$$

So

$$AF = (\overline{AF}) \left(\frac{M_a}{M_f} \right) = (13.29) \left(\frac{28.97}{38.652} \right) = 9.961 \frac{\text{kg (air)}}{\text{kg (fuel)}} \quad \leftarrow AF$$

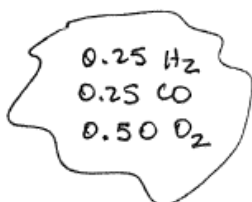
PROBLEM 13.10

KNOWN: A gas mixture with a known molar analysis reacts to form specified products.

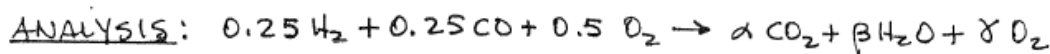
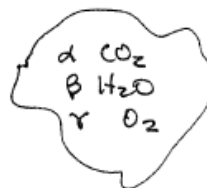
FIND: Determine the amount of each product in kg per kg of mixture.

SCHEMATIC & GIVEN DATA:

Initially:



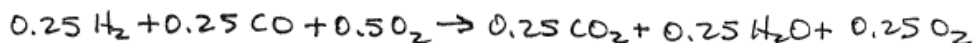
Finally:



$$\text{H}_2: 0.25 = \beta$$

$$\text{C}: 0.25 = \alpha$$

$$\text{O}_2: \frac{0.25}{2} + 0.5 = \alpha + \frac{\beta}{2} + \gamma \Rightarrow \gamma = 0.25$$



Converting the analysis of the initial mixture to a mass basis

	y_i	$\times$	M_i	$= m_i$	m_i
H ₂	0.25	$\times$	2.016	= 0.5040	0.0214
CO	0.25	$\times$	28.01	= 7.0025	0.2979
O ₂	0.5	$\times$	32	= 16.0000	0.6807
	1.0 kmol			23.5065 kg	1.0000

$$\frac{1}{23.5065} = 0.04254 \frac{\text{kmol(mix)}}{\text{kg(mix)}}$$

$$\text{CO}_2: \left(0.25 \frac{\text{kmol(CO}_2)}{\text{kmol(mix)}}\right) \left(0.04254 \frac{\text{kmol(mix)}}{\text{kg(mix)}}\right) \left(44.01 \frac{\text{kg(CO}_2)}{\text{kmol(CO}_2)}\right) = 0.468 \text{ kg} \leftarrow m_{\text{CO}_2}$$

$$\text{H}_2\text{O}: (0.25)(0.04254)(18.02) = 0.1916 \text{ kg} \leftarrow m_{\text{H}_2\text{O}}$$

$$\text{O}_2: (0.25)(0.04254)(32) = 0.3403 \text{ kg} \leftarrow m_{\text{O}_2}$$

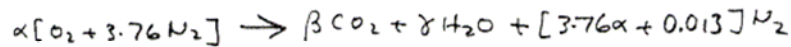
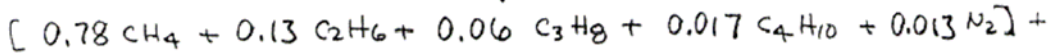
PROBLEM 13.11

KNOWN: A specified fuel mixture burns completely with 40% excess air.

FIND: Determine the molar flow rate of the air, if the molar flow rate of the fuel is 0.5 kmol/h.

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: On the basis of 1 mole of fuel mixture, complete combustion with the theoretical amount of air is



$$C: 0.78 + 2(0.13) + 3(0.06) + 4(0.017) = \beta, \quad \beta = 1.288$$

$$H: 4(0.78) + 6(0.13) + 8(0.06) + 10(0.017) = 2\gamma, \quad \gamma = 2.275$$

$$O: 2\alpha = 2(1.288) + 2.275, \quad \alpha = 4.851$$

Combustion with 40% excess air means that the air-fuel ratio is

$$\overline{AF} = \frac{1.4(4.851)(4.76)}{1} = 32.33 \frac{\text{kmol (air)}}{\text{kmol (fuel)}}$$

Then, with $\dot{n}_{\text{fuel}} = 0.5 \text{ kmol/h}$

$$\dot{n}_{\text{air}} = 16.17 \text{ kmol (air)/h}$$

← $\dot{n}_{\text{air}}$

PROBLEM 1312

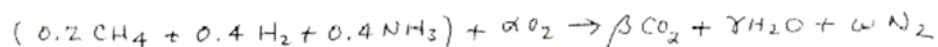
KNOWN: A fuel mixture having the molar analysis below burns completely with 150% of theoretical oxygen.

{ 20% CH₄, 40% H₂, 40% NH₃ }

FIND: Determine the balanced reaction equation.

ENGINEERING MODEL: Combustion is complete. N₂ is inert.

ANALYSIS: The first step is to determine the theoretical amount of O₂.
Based on one mole of fuel mixture

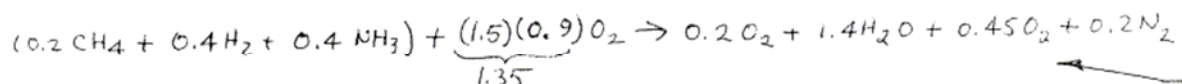


$$\text{C: } 0.2 = \beta$$

$$\text{H: } (0.2)(4) + (0.4)(2) + (0.4)(3) = 2\gamma, \gamma = 1.4$$

$$\text{O: } 2\alpha = 2\beta + \gamma = 2(0.2) + 1.4 = 1.8, \alpha = 0.9$$

Then, for 150% of theoretical O₂



PROBLEM 13.13

KNOWN: Coal with a specified mass analysis burns completely with 120% of theoretical air.

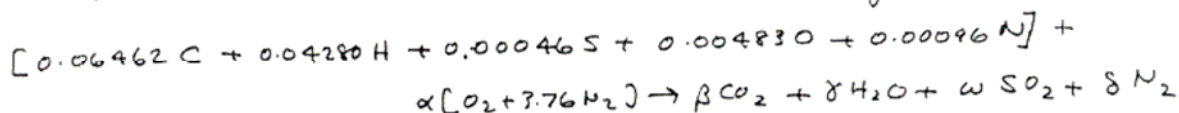
FIND: Determine (a) the balanced reaction equation, (b) the amount of SO_2 produced, in kg per kg of coal.

ENGINEERING MODEL: 3.76 kmol of N_2 accompany each kmol of O_2 in the combustion air. N_2 is inert.

ANALYSIS: (a) On the basis of one kg of coal mixture, the corresponding molar analysis is

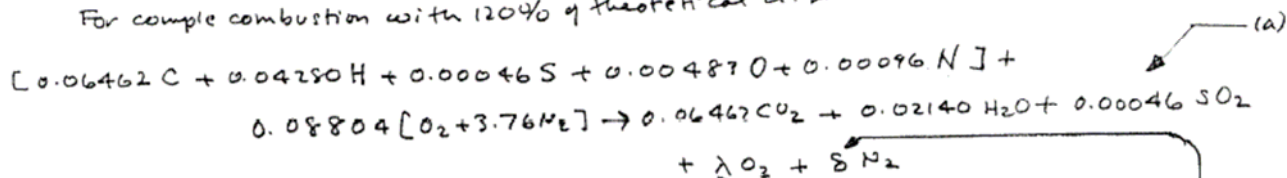
$$\begin{aligned} \text{C} &= \frac{0.7754}{12} = 0.06462 & \text{O} &= \frac{0.0772}{16} = 0.00483 \\ \text{H} &= \frac{0.0428}{1} = 0.04280 & \text{N} &= \frac{0.0134}{14} = 0.00096 \\ \text{S} &= \frac{0.0146}{32} = 0.00046 & \text{ASH} &= \text{balance} \end{aligned}$$

Complete combustion with the theoretical amount of air is



$$\begin{aligned} \text{C}: 0.06462 &= \beta \\ \text{H}: 0.04280 &= 2\gamma, \gamma = 0.02140 \\ \text{S}: 0.00046 &= \omega \\ \text{O}: 0.00483 + 2\alpha &= 2(0.06462) + 0.02140 + 2(0.00046), \alpha = 0.07337 \end{aligned}$$

For complete combustion with 120% of theoretical air



$$\text{O}: 0.00483 + 2(0.08804) = 2(0.06462) + 0.02140 + 2(0.00046) + 2\lambda \Rightarrow \lambda = 0.01468$$

$$\text{N}: 0.00096 + (0.08804)(2)(3.76) = 2\delta, \delta = 0.33151$$

(b) For each 1 kg of coal burned, 0.00046 kmol of SO_2 is formed.
Thus

$$\begin{aligned} \left[\begin{array}{c} \text{mass of } \text{SO}_2 \text{ formed} \\ \text{per kg of coal} \\ \text{burned} \end{array} \right] &= 0.00046 \text{ kmol} \left[64.06 \frac{\text{kg}(\text{SO}_2)}{\text{kmol}(\text{SO}_2)} \right] \\ &= 0.029 \frac{\text{kg}(\text{SO}_2)}{\text{kg}(\text{coal})} \quad (b) \end{aligned}$$

PROBLEM 13.14

KNOWN: Coal having the mass analysis given below burns completely with air.

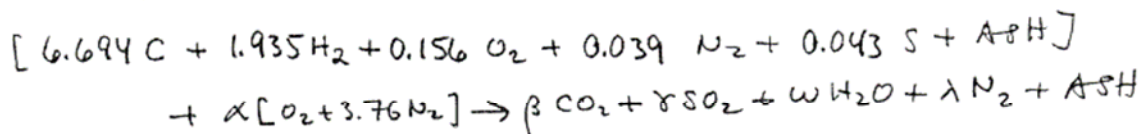
FIND: For complete combustion with 120% of the theoretical amount of air, determine the air-fuel ratio on a mass basis.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. 2. The ash is noncombustible.

ANALYSIS: On the basis of 100 kg of coal, the amounts of substances present, in kmol, are

	%	m_i	M_i	$n_i = m_i/M_i$
C	80.4	80.4	12.01	6.694
H_2	3.9	3.9	2.016	1.935
O_2	5.0	5.0	32.00	0.156
N_2	1.1	1.1	28.01	0.039
S	1.1	1.1	32.06	0.043
ASH	8.5	8.5	---	---

For complete combustion with the theoretical amount of air



Balancing:

$$C: 6.694 = \beta$$

$$H_2: 1.935 = w$$

$$S: 0.043 = \gamma$$

$$O: 2(0.156) + 2\alpha = 2\beta + 2\gamma + w$$

$$2\alpha = 2(6.694) + 2(0.043) + 1.935 - 2(0.156)$$

$$\alpha = 7.549$$

Accordingly, the reaction requires $(7.549)(4.76)$ kmol of air per 100 kg of coal

$$(AF)_{theo} = \frac{(7.549)(4.76)(28.97)}{100} = 10.410 \frac{kg(a)}{kg(coal)}$$

For 120% of the theoretical amount of air

$$(AF) = (1.2)(10.410) = 12.49 \frac{kg(a)}{kg(coal)} \quad \leftarrow AF$$

PROBLEM 13.15

KNOWN: Dried feedlot manure with the mass analysis below burns completely with 120% of theoretical air.

{ 42.7% C, 5.5% H₂, 31.3% O₂, 2.4% N₂, 0.3% S, 17.8% Ash }

FIND: Determine (a) the balanced reaction equation, and (b) the air-fuel ratio on a mass basis.

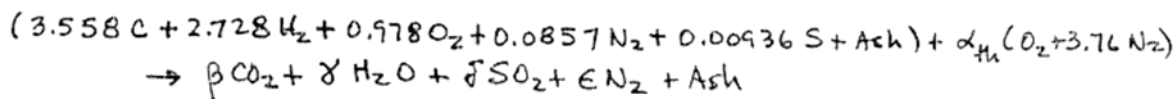
ENGINEERING

MODEL: (1) 3.76 kmol of N₂ accompany each kmol of O₂ in the air, and the N₂ is inert. (2) The ash is non-combustible.

ANALYSIS: (a) On the basis of 100 kg of manure, the amounts of the substances present, are respectively

	m_i (kg)	M_i	$n_i = m_i/M_i$ (kmol)
C:	42.7	12	3.558
H ₂ :	5.5	2.016	2.728
O ₂ :	31.3	32	0.978
N ₂ :	2.4	28.01	0.0857
S:	0.3	32.06	0.00936
Ash:	17.8	---	---
	100.0 kg		

for complete combustion with the theoretical amount of air



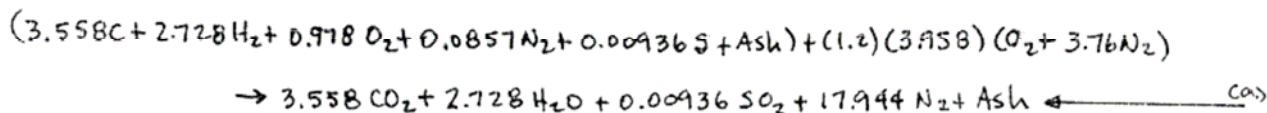
$$\text{C: } 3.558 = \beta$$

$$\text{H: } 2(2.728) = 2\gamma \Rightarrow \gamma = 2.728$$

$$\text{S: } 0.00936 = \delta$$

$$\text{O: } 2(0.978) + 2\alpha_{\text{th}} = 2(3.558) + 2.728 + 2(0.00936) \Rightarrow \alpha_{\text{th}} = 3.958$$

Then, for complete combustion with 120% of theoretical air



(b) The actual reaction requires (1.2)(3.958)(4.76) kmol of air per 100 kg of manure. Thus

$$\text{AF} = \frac{(1.2)(3.958)(4.76)(28.97)}{100} = 6.55 \frac{\text{kg (air)}}{\text{kg (manure)}} \quad \leftarrow \text{AF}$$

PROBLEM 13.16

KNOWN: Coal with the mass analysis below burns completely with the theoretical amount of air.

{ 71.1 % C, 5.1 % H₂, 9.0 % O₂, 1.4 % N₂, 5.8 % S, 7.6 % Ash }

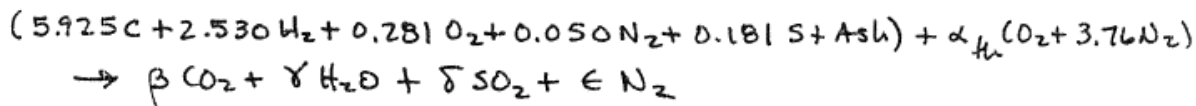
FIND: Determine (a) the amount of SO₂ produced, in kg per kg of coal, and (b) the air-fuel ratio on a mass basis.

ENGINEERING MODEL: (1) 3.76 kmol of N₂ accompany each kmol of O₂ in the air, and the N₂ is inert. (2) The ash is non-combustible.

ANALYSIS: On the basis of 100 kg of coal, the amounts of the substances present are, respectively

	m_i (kg)	M_i	$n_i = m_i/M_i$ (kmol)
C:	71.1	12	5.925
H ₂ :	5.1	2.016	2.530
O ₂ :	9.0	32	0.281
N ₂ :	1.4	28.01	0.050
S:	5.8	32.06	0.181
Ash:	7.6	----	----
	100 kg		

For complete combustion with the theoretical amount of air



$$\text{C: } 5.925 = \beta$$

$$\text{H}_2: \gamma = 2.530$$

$$\text{S: } 0.181 = \delta$$

$$\text{O: } (0.281)2 + \alpha_{\text{th}}(2) = (5.925)2 + 2.530 + (0.181)2 \Rightarrow \alpha_{\text{th}} = 7.09$$

$$\text{N: } (0.05)2 + (7.09)(3.76)(2) = \epsilon \cdot 2 \Rightarrow \epsilon = 26.71$$

(a) The amount of SO₂ produced is 0.181 kmol per 100 kg of coal. Thus

$$m_{\text{SO}_2} = \frac{(0.181)(64.06)}{100} = 0.116 \frac{\text{kg (SO}_2\text{)}}{\text{kg (coal)}} \leftarrow m_{\text{SO}_2}$$

(b) The amount of air required is (7.09)(4.76) kmol per 100 kg of coal. Thus

$$\text{AF} = \frac{(7.09)(4.76)(28.97)}{100} = 9.777 \frac{\text{kg (air)}}{\text{kg (coal)}} \leftarrow \text{AF}$$

PROBLEM 13.17

KNOWN: C_8H_{18} burns completely with 120% of theoretical air.

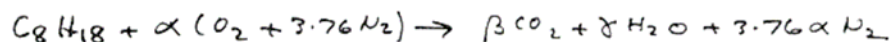
FIND: Determine (a) the air-fuel ratio on a molar and mass basis.

(b) the dew point temperature at 1 atm.

ENGINEERING

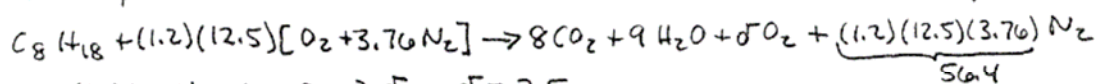
MODEL: 3.76 kmol of N_2 accompany each kmol of O_2 in the combustion air. N_2 is inert.

ANALYSIS: (a) For complete combustion with the theoretical amount of air



$$\begin{aligned} C: 8 &= \beta & O: 2\alpha &= 2(8) + \gamma, \alpha = 12.5 \\ H: 18 &= 2\gamma, \gamma &= 9 \end{aligned}$$

Then, complete combustion with 120% of the theoretical amount of air



$$O: 2(1.2)(12.5) = 16 + 9 + 2(5), 5 = 2.5$$

Accordingly,

$$\overline{AF} = \frac{(1.2)(12.5)(4.76)}{1} = 71.4 \frac{\text{kmol (air)}}{\text{kmol (fuel)}} \quad (a)$$

$$AF = \overline{AF} \left(\frac{M_{\text{air}}}{M_{\text{fuel}}} \right) = \frac{(71.4)(28.97)}{(8)(12.01) + 18(1.009)} = 18.12 \frac{\text{kg (air)}}{\text{kg (fuel)}}$$

114.152

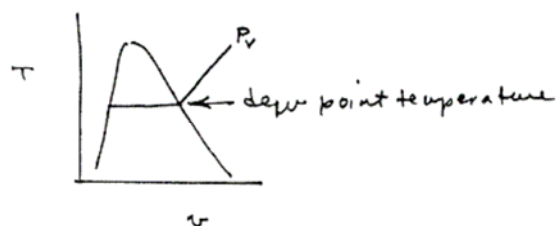
(b) The partial pressure of the water in the combustion products is

$P_v = Y_v P$, where

$$Y_v = \frac{9}{8 + 9 + 2.5 + 56.4} = \frac{9}{75.9} = 0.1186$$

So

$$P_v = (0.1186)(1.01325 \text{ bar}) = 0.12015 \text{ bar}$$



Then, from Table A-2

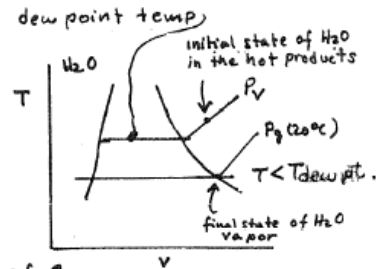
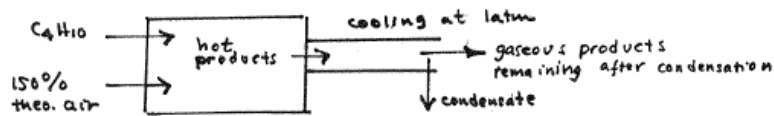
$$T_{\text{dew}} \approx 49.4^\circ\text{C} \quad (b)$$

Problem 13.18

KNOWN: C_4H_{10} burns completely with 150% of theoretical air. The products of combustion are cooled to temperature T at 1 atm, $20 \leq T \leq 60^\circ C$.

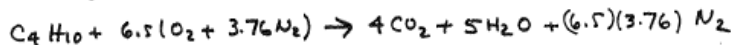
FIND: Plot the amount of water vapor condensed, in kmol per kmol of fuel.

SCHEMATIC & GIVEN DATA:

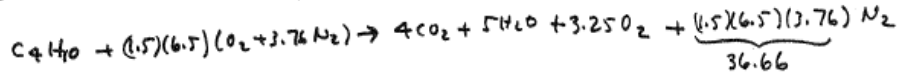


ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The gaseous products of combustion can be modeled as an ideal gas.

ANALYSIS: Complete combustion of C_4H_{10} with the theoretical amount of air is described by



Complete combustion with 150% of theoretical air is then



Condensation occurs when the combustion products are cooled below the dew point temperature. First, evaluate the partial pressure of water in the combustion products:

$$P_v = Y_v p = \left(\frac{5}{4 + 5 + 3.25 + 36.66} \right) (1.01325 \text{ bar}) = 0.10358 \text{ bar}$$

Interpolation in Table A-2 gives $T_{dew, pt} \approx 46.5^\circ C$. Accordingly, the amount of condensate is zero until the temperature T is less than $46.5^\circ C$.

According to the model introduced in Chap. 12, the gaseous products remaining after the products have been cooled to $T < T_{dew, pt}$ would have saturated water vapor at temperature T . That is, the partial pressure of water vapor in this mixture would equal $P_g(T)$. The partial pressure is given by

$$P_v = \left(\frac{n_v}{n_v + n_{dry}} \right) p \quad (1)$$

where n_{dry} accounts for the moles of "dry" products: CO_2 , O_2 , and N_2 . That is, $n_{dry} = 4 + 3.25 + 36.66 = 43.91$. Accordingly, with $p = 1 \text{ atm}$ and $P_v = P_g(T)$, Eq. (1) becomes

$$P_g(T) = \left[\frac{n_v}{n_v + 43.91} \right] (1.01325) \Rightarrow n_v = \frac{P_g(T)(43.91)}{1.01325 - P_g(T)} \quad \text{Amount condensed} = 5 - n_v \quad (2)$$

Sample Calculation: $T = 20^\circ C$, $P_g = 0.02339 \text{ bar}$, $n_v = 1.038$, Amt. Condensed = 3.962

The data for the required plots are obtained using IT, as follows:

Continued on next slide

Problem 13-18 continued

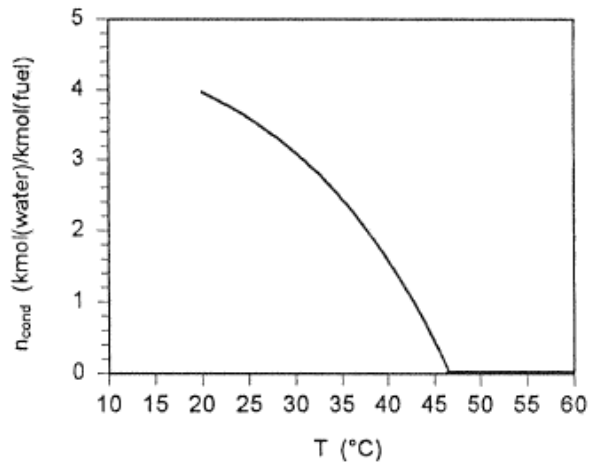
IT Code

```
T = 20 // °C
pv = 0.10358 // bar
Tdp = Tsat_P("Water/Steam", pv)
ncond = 5 - nv
nv = pg * 43.41 / (1.01325 - pg)
pg = Psat_T("Water/Steam", T)
```

IT Results for T = 20°C

```
Tdp = 46.5°C
nv = 1.026 bar
pg = 0.02339 bar
ncond = 3.974 kmol(water)/kmol(fuel)
```

Plot:



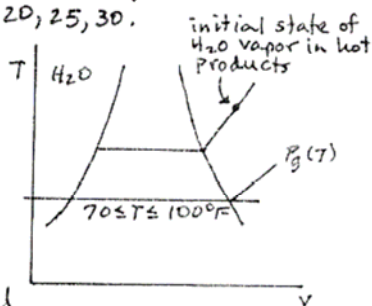
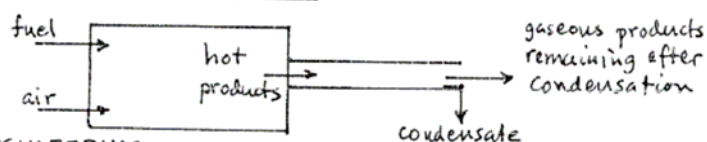
Note that condensation begins at the dew point temperature and increases as temperature is lowered further, as expected.

PROBLEM 13.19

KNOWN: C_2H_4 burns completely with air. The air-fuel ratio on a mass basis is AF , and the products of combustion are cooled to temperature T at 1 atm.

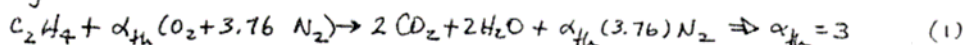
FIND: (a) Determine, for $AF=15$ and $T=70^\circ F$, the percent excess air and the amount of water vapor condensed. (b) Plot the amount of water vapor condensed versus T ranging from 70 to $100^\circ F$ for $AF = 15, 20, 25, 30$.

SCHEMATIC & GIVEN DATA:

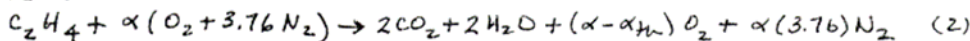


ENGINEERING MODEL: (1) 3.76 lbmol of N_2 accompany each lbmol of O_2 in the air, and the N_2 is inert. (2) The gaseous products of combustion can be modeled as an ideal gas mixture.

ANALYSIS: (a) Complete combustion with the theoretical amount of air is described by



For complete combustion with excess air



For $AF=15$

$$\bar{AF} = AF \left(\frac{M_{fuel}}{M_{air}} \right) = (15) \left(\frac{28.05}{28.97} \right) = 14.524 \frac{\text{lbmol (air)}}{\text{lbmol (fuel)}}$$

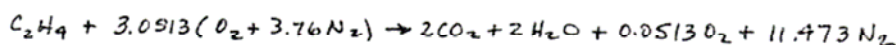
Thus

$$\alpha = \frac{\bar{AF}}{4.76} = \frac{14.524}{4.76} = 3.0513$$

Accordingly

$$\% \text{ excess} = \left(\frac{\alpha - \alpha_{th}}{\alpha_{th}} \right) 100 = \left(\frac{3.0513 - 3}{3} \right) 100 = 1.71\% \quad \leftarrow \% \text{ Excess air}$$

Now, the actual reaction equation is



According to the model introduced in Chap. 12, the gaseous products remaining after the products have been below their dew point would have saturated vapor at T present. In this case, the partial pressure of the water vapor remaining would be $p_g(T) = p_g(70^\circ F) = 0.3632 \text{ lbf/in}^2$. This pressure can be expressed as

$$p_v = \left(\frac{n_v}{n_v + n_{dry}} \right) p \quad (3)$$

where "dry" refers to the products CO_2 , O_2 , and N_2 . That is $n_{dry} = 2 + 0.0513 + 11.473 = 13.524$. With these results, Eq. (3) becomes

$$0.3632 = \left(\frac{n_v}{n_v + 13.524} \right) (14.696) \Rightarrow n_v = 0.3427 \text{ lbmol (vapor) / lbmol (fuel)}$$

Since 2 lbmol of H_2O are formed on combustion, the amount condensed is

$$m_{cond} = (2 - 0.3427) \text{ lbmol} \frac{18.02 \text{ lb}}{\text{lbmol}} = 29.86 \text{ lb (water) per lbmol of fuel} \quad \leftarrow m_{cond}$$

Continued on next slide

Problem 13-19 continued

(b) The data for the required plot are obtained using IT, as follows:

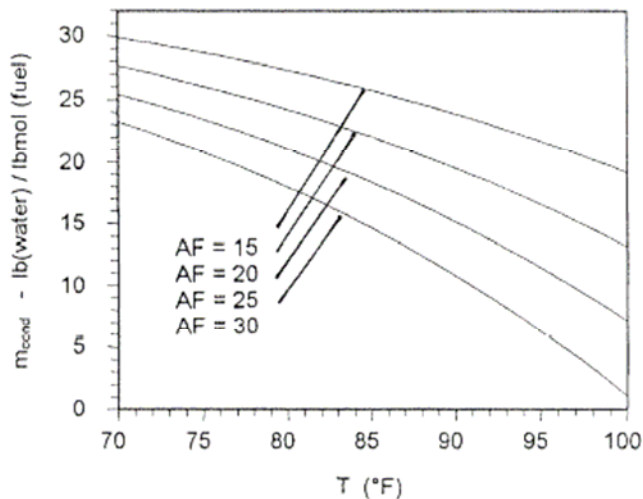
IT Code

```
T = 70 // °F
AF = 15 // kg(air) / kg(fuel)
alpha = AF * (28.05 / 28.97) / 4.76
pctXS = ((alpha - 3) / 3) * 100
ndry = 2 + (alpha - 3) + alpha * 3.76
pv = Psat_T("Water/Steam", T)
pv = (nv / (nv + ndry)) * 14.696
mcond = (2 - nv) * 18.02 // lb(water) / lbmol(fuel)
```

IT Results for AF = 15, T = 70°F

```
alpha = 3.051
% excess air = 1.706 %
ndry = 13.52 lbmol(dry prod.)/lbmol(fuel)
pv = 0.3632 lbf/in.2
nv = 0.3427 lbmol(water vap.)/lbmol(fuel)
mcond = 29.86 lb(water)/lbmol(fuel)
```

plot:



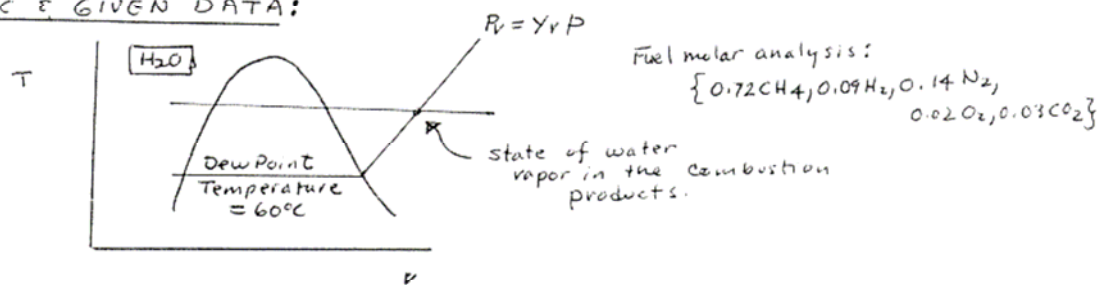
With higher AF (more air), there is more "dry" gas in the products. By Eq. (3) we see that n_v must increase as well. Thus, for fixed T there is less condensate formed as AF increases. For fixed AF, cooling to lower values of T causes more condensate to be formed, as expected.

PROBLEM 13.20

KNOWN: A fuel mixture with the molar analysis below burns completely with moist air to form gaseous products at 1 atm. The dew point temperature of the products is 60°C.

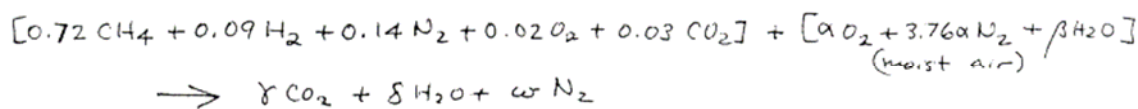
FIND: Determine the amount of water vapor present in the combustion air, in kmol/kmol(fuel).

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air and N_2 is inert. (2) The gaseous combustion products can be modeled as an ideal gas mixture.

ANALYSIS: First, the balanced reaction equation is obtained on the basis of one kmol of fuel mixture.



$$\text{C: } 0.72 + 0.03 = \gamma, \quad \gamma = 0.75$$

$$\text{H: } 4(0.72) + 2(0.09) + 2\beta = 2\delta \Rightarrow \delta = 1.53 + \beta$$

$$\text{O: } 2(0.02) + 2(0.03) + 2\alpha + \beta = 2\gamma + \delta \Rightarrow 2\alpha + \beta = 1.4 + \delta$$

$$\Rightarrow 2\alpha + \beta = 1.4 + (1.53 + \beta)$$

$$\alpha = 1.465$$

$$\text{N}_2: 0.14 + 3.76\alpha = \omega \Rightarrow \omega = 5.648$$

The combustion products form an ideal gas mixture in which the partial pressure P_v is $P_v = y_v P$, where

$$y_v = \frac{\delta}{\gamma + \delta + \omega} = \frac{\delta}{\delta + 0.75 + 5.648} = \frac{\delta}{\delta + 6.398}$$

Since the dewpoint temperature is known: $P_v = P_g(60^\circ\text{C}) = 0.1994 \text{ bar}$.
 Collecting results,

$$y_v = \frac{P_v}{P} = \frac{0.1994 \text{ bar}}{1.01325 \text{ bar}} = \frac{\delta}{\delta + 6.398} \Rightarrow \delta = 1.568$$

Returning to the H balance, $\delta = 1.53 + \beta$, or

$$\beta = 1.568 - 1.53 = 0.038 \frac{\text{kmol}(\text{H}_2\text{O})}{\text{kmol}(\text{fuel})}$$

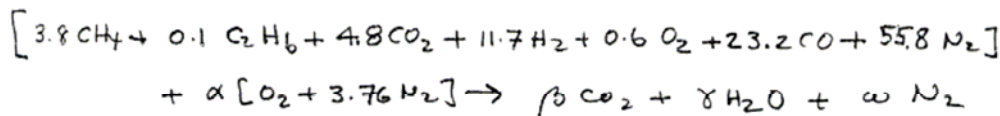
PROBLEM 13.21

KNOWN: The volumetric analysis of a producer gas is provided.

FIND: If the gas burns completely with theoretical amount of air determine (a) the molar analysis of the dry products, (b) the amount of water vapor condensed if the products are cooled to 70°F at 1 atm.

ENGINEERING MODEL: (1) 3.76 moles of N₂ accompany each mole of O₂ in the air, and N₂ is inert. (2) Ideal gas mixture principles apply to the products.

ANALYSIS: Basing the calculation on 100 moles of fuel mixture



$$\text{C: } 3.8 + 2(0.1) + 4.8 + 23.2 = \beta; \quad \beta = 32$$

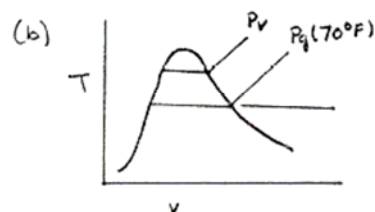
$$\text{H: } (3.8)(4) + 6(0.1) + 2(11.7) = 2\gamma; \quad \gamma = 19.6$$

$$\text{O: } (4.8)(2) + (0.6)(2) + 23.2 + 2\alpha = 2(32) + 19.6; \quad \alpha = 24.8$$

$$\text{N}_2: (55.8) + (24.8)(3.76) = \omega; \quad \omega = 149.05$$

(a) The total amount of dry products is $32 + 149.05 = 181.05$. Thus, the molar analysis of the dry products is

$$\% \text{ CO}_2 = \frac{32}{181.05} = 0.177 \text{ (17.7\%)}; \quad \% \text{ N}_2 = \frac{149.05}{181.05} = 0.823 \text{ (82.3\%)} \quad (a)$$



The partial pressure of the water vapor in the products is

$$P_v = \left(\frac{19.6}{32 + 149.05 + 19.6} \right) (14.7 \frac{\text{lb}}{\text{in}^2}) = 1.44 \frac{\text{lb}}{\text{in}^2}$$

Since $P_v > P_g(70^\circ\text{F}) = 0.3632 \frac{\text{lb}}{\text{in}^2}$, condensation would occur as the products are cooled to 70°F at $p = 1 \text{ atm}$.

The gas phase would consist of 181.05 moles of dry products plus n_v , which is the water vapor present. The partial pressure of the water vapor would be $P_g(70^\circ\text{F})$. Thus,

$$0.3632 \frac{\text{lb}}{\text{in}^2} = \left[\frac{n_v}{181.05 + n_v} \right] (14.7 \frac{\text{lb}}{\text{in}^2}) \Rightarrow n_v = 4.587$$

Accordingly, the amount of water that condenses per kmol of fuel is

$$\frac{(19.6 - 4.587) \text{ lbmol (H}_2\text{O)}}{100 \text{ lbmol (fuel)}} = 0.15 \frac{\text{lbmol (H}_2\text{O)}}{\text{lbmol (fuel)}} \quad (b)$$

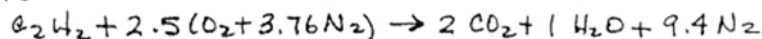
PROBLEM 13.22

KNOWN: C_2H_2 burns completely with 110% of theoretical air entering at $74^\circ F$, 1 atm, $\phi = 50\%$

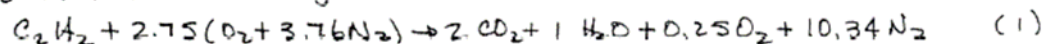
FIND: Obtain the balanced reaction equation and determine the dew point temperature of the products.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) N_2 and the H_2O in the air supply are inert. (3) Air and the combustion products, each at 1 atm, are modeled as ideal gas mixtures.

ANALYSIS: Complete combustion of C_2H_2 with the theoretical amount of dry air is



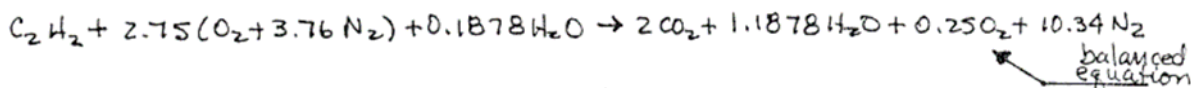
With 110% of theoretical dry air



For a moist air mixture at $74^\circ F$, 1 atm, $\phi = 50\%$ in which there is $(2.75)(4.76) = 13.09$ lbmol of dry air and n_v lbmol of water vapor, the partial pressure p_v is

$$p_v = \left(\frac{n_v}{n_a + n_v} \right) p = \left(\frac{n_v}{13.09 + n_v} \right) (14.696 \text{ lbf/in}^2) \quad (2)$$

Using $\phi = 50\%$ and data from Table A-2E: $p_v = \phi p_g = (0.5)(0.4158) = 0.2079 \frac{\text{lbf}}{\text{in}^2}$
Solving (2) for n_v we get $n_v = 0.1878$ lbmol. Thus



considering the combustion products, the mole fraction of water vapor in the products is

$$y_v = \frac{1.1878}{13.78} = 0.0862 \Rightarrow p_v = (0.0862)(14.696) = 1.267 \text{ lbf/in}^2$$

Interpolating in Table A-3E; $T_{\text{dew point}} \approx 110^\circ F \leftarrow T_{DP}$

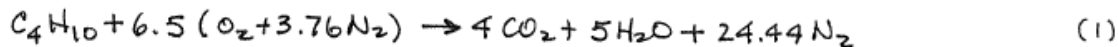
PROBLEM 13.23

KNOWN: C_4H_{10} burns completely with 160% of theoretical air at 20°C , 1 atm, and $\phi = 90\%$.

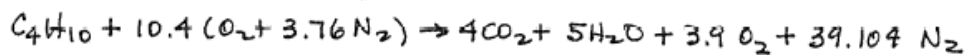
FIND: Determine (a) the balanced reaction equation, and (b) the dew point temperature of the products.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) N_2 and H_2O in the air supply are inert. (3) Air and the combustion products, each at 1 atm, are modeled as ideal gas mixtures.

ANALYSIS: Complete combustion of C_4H_{10} with the theoretical amount of dry air is



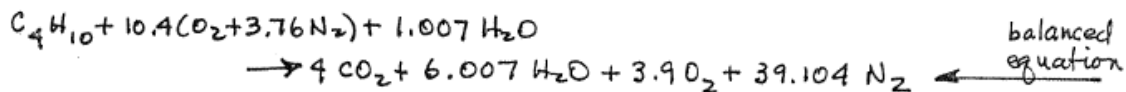
With 160% of theoretical dry air



For moist air at 20°C , 1 atm, $\phi = 90\%$ in which there is $(10.4)(3.76) = 49.504$ kmol of dry air and n_v kmol of water vapor, the partial pressure p_v is

$$p_v = \left(\frac{n_v}{49.504 + n_v} \right) P = \left(\frac{n_v}{49.504 + n_v} \right) (1.01325 \text{ bar}) \quad (2)$$

Using $\phi = 90\%$ and data from Table A-2: $p_v = \phi p_g = (0.9)(0.02339) = 0.02105 \text{ bar}$
Solving (2) for n_v we get $n_v = 1.007 \text{ kmol}$. Thus



Considering the combustion products, the mole fraction of water vapor is

$$y_v = \frac{6.007}{53.011} = 0.11332 \Rightarrow p_v = (0.11332)(1.01325) = 0.11482 \text{ bar}$$

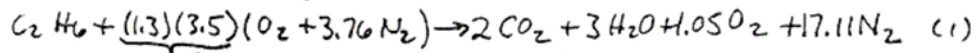
Interpolating in Table A-3; $T_{\text{dew point}} \approx 48^\circ\text{C} \leftarrow T_{DP}$

PROBLEM 13.24

KNOWN: C_2H_6 burns completely with 130% of theoretical air at $25^\circ C$, 0.85 bar, $\phi = 50\%$.
FIND: Obtain (a) the balanced reaction equation, (b) the dewpoint temperature.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 and H_2O in the air are inert. (3) The air is modeled as an ideal gas.

ANALYSIS: (a) The complete combustion of C_2H_6 with 130% of theoretical dry air is described by



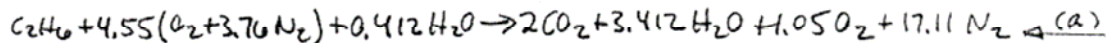
Consider next a moist air mixture at $25^\circ C$, 0.85 bar, $\phi = 50\%$ in which there is $4.55(4.76) = 21.66$ kmol of dry air and n_v kmol of water vapor. The partial pressure of the water vapor is

$$P_v = \left(\frac{n_v}{n_a + n_v} \right) p = \left(\frac{n_v}{21.66 + n_v} \right) (0.85)$$

Also, $P_v = \phi P_g(25^\circ C) = 0.5(0.03169 \text{ bar}) = 0.01585 \text{ bar}$. Collecting results

$$0.01585 = \left(\frac{n_v}{21.66 + n_v} \right) (0.85) \Rightarrow n_v = 0.412 \text{ kmol}$$

When water vapor is present in the combustion air, Eq. (1) reads

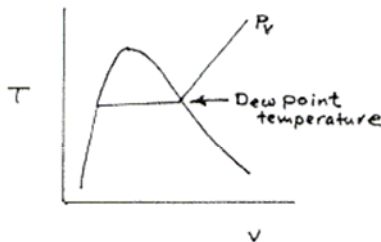


(b) To find the dewpoint temperature,

$$P_v = \left(\frac{n_v}{n_{\text{products}}} \right) p$$

$$= \left(\frac{3.412}{23.512} \right) (0.85 \text{ bar}) = 0.1230 \text{ bar}$$

Interpolation = Table A-2, $T_{\text{dew point}} \approx 50^\circ C \leftarrow (b)$

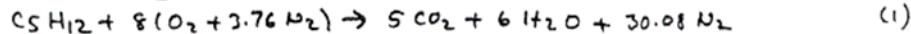


PROBLEM 13.25

KNOWN: C_5H_{12} burns completely with the theoretical amount of air at $75^\circ F$, 1 atm , $\phi = 75\%$
FIND: Determine (a) the balanced reaction equation, (b) the dew point temperature of the combustion products, (c) the amount of water condensed if the products are cooled to $75^\circ F$

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 and H_2O in the air are inert. (3) The air and products are modeled as ideal gases.

ANALYSIS: (a) Complete combustion of C_5H_{12} with the theoretical amount of dry air is described by



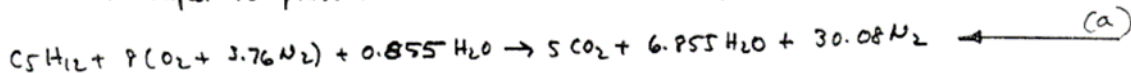
Consider next a moist air mixture at $75^\circ F$, 1 atm , $\phi = 75\%$ in which there is $8(4.76) = 38.08 \text{ lbmol}$ of dry air and $n_v \text{ lbmol}$ of water vapor. The partial pressure of the water vapor is

$$P_v = \left(\frac{n_v}{n_a + n_v} \right) P = \left(\frac{n_v}{38.08 + n_v} \right) (14.696 \text{ lbf/in}^2)$$

Also, $P_v = \phi P_g(75^\circ F) = 0.75(0.4302) = 0.3227 \text{ lbf/in}^2$. Collecting results

$$0.3227 = \left(\frac{n_v}{38.08 + n_v} \right) (14.696) \Rightarrow n_v = 0.855 \text{ lbmol}$$

When water vapor is present in the combustion air, Eq. (1) reads



(b) The partial pressure of the water vapor in the products is

$$P_v = \left(\frac{6.855}{5 + 6.855 + 30.08} \right) (14.696) = 2.402 \text{ lbf/in}^2$$

Interpolating in Table 3E with $P_v = P_g$, the dew point temperature is $132.2^\circ F$ (b)

(c) Since $75^\circ F$ is below the dew point temperature, condensation would occur for cooling to $75^\circ F$. At $75^\circ F$, the vapor phase would consist of n_v moles of water vapor and $n_{dry} = 5 + 30.08 = 35.08$ moles of dry products. The partial pressure of the vapor would equal $P_g(75^\circ F)$. Thus

$$P_g(75^\circ F) = \left(\frac{n_v}{n_v + n_{dry}} \right) P \quad ; \quad 0.4302 \frac{\text{lbf}}{\text{in}^2} = \left(\frac{n_v}{n_v + 35.08} \right) (14.696 \frac{\text{lbf}}{\text{in}^2})$$

Table 3E

Solving this equation, $n_v = 1.058 \text{ lbmol}$. The amount of water condensed per lbmol of fuel consumed is then

$$\left[\begin{array}{l} \text{amount of water} \\ \text{condensed per} \\ \text{lbmol of fuel} \\ \text{consumed} \end{array} \right] = 6.855 - 1.058 = 5.8 \frac{\text{kmol (water)}}{\text{kmol (fuel)}} \quad (c)$$

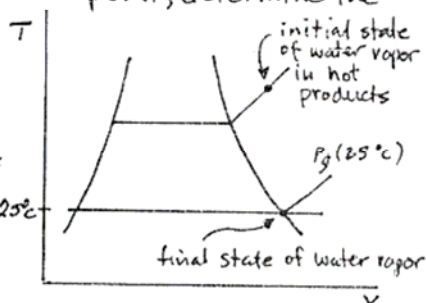
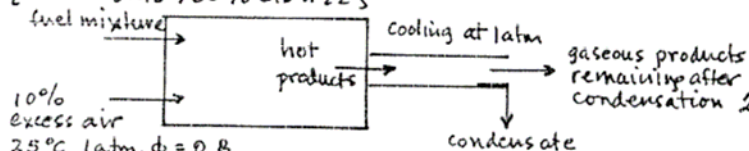
PROBLEM 13.26

KNOWN: A liquid fuel mixture with a known mass analysis is burned completely with excess air at a specified temperature and relative humidity.

FIND: (a) Determine the equivalent hydrocarbon composition, C_aH_b , with the same carbon-hydrogen ratio on a mass basis as the mixture.
(b) If the products are cooled below their dew point, determine the amount of water vapor condensed.

SCHEMATIC & GIVEN DATA:

{ 40% C_8H_{18} , 60% $C_{10}H_{22}$ }



ENGINEERING

MODEL: (1) 3.76 kmol of N_2 accompany each kmol of O_2 in the air. (2) The N_2 and H_2O in the supply air are inert. (3) The air and products are both modeled as ideal gas mixtures.

ANALYSIS: (a) Assuming 100 kg of liquid fuel

	m_i (kg)	M_i	$n_i = m_i/M_i$ (kmol)
C_8H_{18} :	40	114	0.3509
$C_{10}H_{22}$:	60	142	0.4225

thus, in 100 kg of fuel, there are the following amounts of C and H, respectively

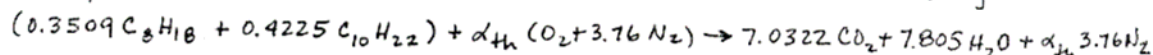
$$C: [0.3509 \text{ kmol } (C_8H_{18})] \left[8 \frac{\text{kmol } (C)}{\text{kmol } (C_8H_{18})} \right] + [0.4225] [10] = 7.0322 \text{ kmol } (C)$$

$$H: [0.3509] [18] + [0.4225] [22] = 15.61 \text{ kmol } (H)$$

The "equivalent" hydrocarbon composition is

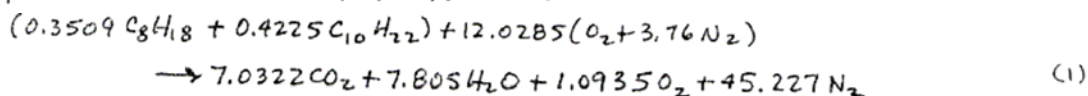
$$C_{7.0322} H_{15.61} \quad \leftarrow \text{(a)}$$

(b) Complete combustion with the theoretical amount of air is described by



$$d_{th} = 7.0322 + \frac{7.805}{2} = 10.935$$

Complete combustion with 10% excess is then



Consider next a moist air mixture at 25°C, 1 atm, $\phi = 80\%$ in which there is 12.0285 (4.76) = 57.256 kmol of dry air and n_v kmol of water vapor. The partial pressure of water vapor is

$$P_v = \left(\frac{n_v}{n_a + n_v} \right) P = \left(\frac{n_v}{57.256 + n_v} \right) (1.01325 \text{ bar})$$

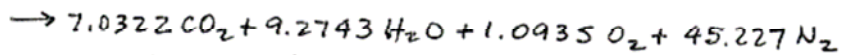
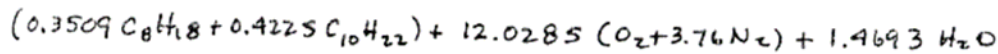
Also, $P_v = \phi P_g(25^\circ C) = 0.8 (0.03169) = 0.02535 \text{ bar}$. Collecting results

$$0.02535 = \left(\frac{n_v}{57.256 + n_v} \right) (1.01325) \Rightarrow n_v = 1.4693 \text{ kmol}$$

Continued on next slide

Problem 13-26 continued

When water vapor is present in the combustion air, Eq. (1) reads



According to the model introduced in chap. 12, the gaseous products remaining after the products have been cooled to 25°C would contain saturated water vapor at 25°C. That is, the partial pressure of the water vapor in this mixture would be $p_g(25^\circ\text{C}) = 0.03169 \text{ bar}$. The partial pressure is expressed as

$$p_v = \left(\frac{n'_v}{n_{\text{dry}} + n'_v} \right) p \quad (2)$$

where $n_{\text{dry}} = 7.0322 + 1.0935 + 45.227 = 53.353$. Accordingly, from Eq. (2)

$$0.03169 = \left(\frac{n'_v}{53.353 + n'_v} \right) (1.01325) \Rightarrow n'_v = 1.7225 \text{ kmol}$$

Since there is 9.2743 kmol of H_2O per 100 kg of fuel in the products, the amount of water that condenses is $9.2743 - 1.7225 = 7.5518 \text{ kmol}(\text{H}_2\text{O})$ per 100 kg of fuel. Thus

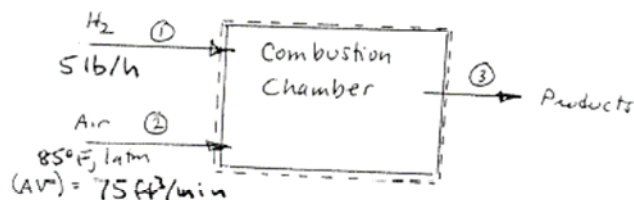
$$m_{\text{cond}} = \left[7.5518 \frac{\text{kmol}(\text{H}_2\text{O})}{100 \text{ kg}(\text{fuel})} \right] \left[\frac{18.02 \text{ kg}(\text{H}_2\text{O})}{1 \text{ kmol}(\text{H}_2\text{O})} \right] = 1.361 \frac{\text{kg}(\text{H}_2\text{O})}{\text{kg}(\text{fuel})} \leftarrow (b)$$

PROBLEM 13.27

KNOWN: H_2 enters a combustion chamber at a rate of 5 lb/h and burns with air entering at 85°F, 1 atm and a volumetric flow rate of 75 ft³/min.

FIND: Determine the percent of theoretical air used.

SCHEMATIC & GIVEN DATA

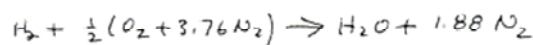


ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the entering air. N_2 is inert and the air is dry. (2) The air can be modeled as an ideal gas.

ANALYSIS: The percent theoretical air can be found from

$$\% \text{ theo air} = \frac{(\overline{AF})}{(\overline{AF})_{\text{theo}}} \times 100 \quad (1)$$

For the complete combustion of H_2 with the theoretical amount of air,



So, $(\overline{AF})_{\text{theo}} = 2.38$

For the actual reaction, the mass flow rate of the air is

$$\dot{m}_{\text{AIR}} = \frac{(\overline{AV})_2}{\nu_2} = \frac{(\overline{AV})_{2R}}{(\overline{R}/M)_2} = \frac{(75) \frac{\text{ft}^3}{\text{min}} (14.7) (144) \frac{\text{lb}_f}{\text{ft}^2}}{\left(\frac{1545}{28.97} \right) \frac{\text{ft} \cdot \text{lb}_f}{\text{lb} \cdot \text{R}} (545) \text{R}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 327.73 \frac{\text{lb (air)}}{\text{h}}$$

Thus

$$AF = \frac{327.73 \text{ lb (air)/h}}{5 \text{ lb (fuel)/h}} = 65.55 \frac{\text{lb (air)}}{\text{lb (fuel)}}$$

on a molar basis

$$\begin{aligned} \overline{AF} &= 65.55 \frac{\text{lb (air)}}{\text{lb (fuel)}} \left[\frac{2.018 \text{ lb (fuel) / lbmol (fuel)}}{28.97 \text{ lb (air) / lbmol (air)}} \right] \\ &= 4.57 \frac{\text{lbmol (air)}}{\text{lbmol (fuel)}} \end{aligned}$$

Finally, returning to Eq. (1)

$$\% \text{ theo air} = \left(\frac{4.57}{2.38} \right) (100) = 191.8 \quad \begin{array}{l} \text{\% theo. air} \\ \leftarrow \end{array}$$

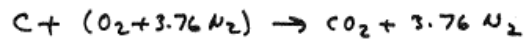
PROBLEM 13.28

KNOWN: Carbon burns with 80% theoretical air yielding CO_2 , CO , and N_2 only.

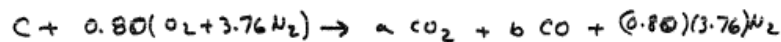
FIND: Determine (a) the balanced reaction equation, (b) $\overline{AF}$, (c) the analysis of the products on a molar basis.

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air, and N_2 is inert.

ANALYSIS: (a) Complete combustion of C with the theoretical amount of air is described by



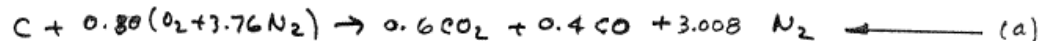
Thus, combustion with 80% theoretical air to produce CO_2 , CO , and N_2 is



$$\text{C: } 1 = a + b \Rightarrow b = 1 - a$$

$$\text{O: } (0.80)(2) = 2a + b \Rightarrow 1.6 = 2a + (1 - a) \Rightarrow a = 0.6, b = 0.4$$

Accordingly, the balanced reaction equation is



(b) The air fuel ratio is

$$\overline{AF} = 0.80(4.76)/1 = 3.808 \text{ kmol (air)/kmol (fuel)}$$

Then with Eq. 13-2

$$AF = \overline{AF} \left(\frac{M_{\text{air}}}{M_{\text{fuel}}} \right) = (3.808) \left(\frac{28.97}{12.01} \right) = 9.19 \frac{\text{kg (air)}}{\text{kg (fuel)}} \quad \leftarrow (b)$$

(c) The molar analysis of the products is

$$\% \text{CO}_2 = \left(\frac{0.6}{4.008} \right) (100) = 15\%, \quad \% \text{CO} = \frac{0.4}{4.008} = 10\%, \quad \% \text{N}_2 = 75\% \quad \leftarrow (c)$$

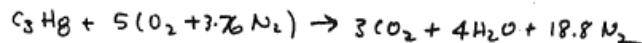
PROBLEM 13.29

KNOWN: C_3H_8 reacts with 80% of theoretical air yielding CO_2 , CO , H_2O and N_2 only.

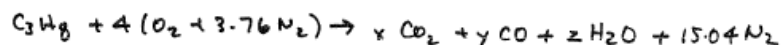
FIND: Determine (a) the balanced reaction equation, (b) AF, (c) the analysis of the products on a dry molar basis.

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air, and N_2 is inert.

ANALYSIS: (a) complete combustion of C_3H_8 with the theoretical amount of air is described by



Combustion with 80% of theoretical air is then

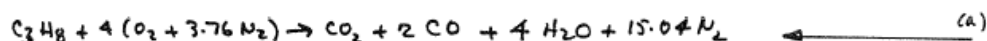


$$C: 3 = x + y \Rightarrow y = 3 - x$$

$$H: 8 = 2z \Rightarrow z = 4$$

$$O: 8 = 2x + y + 4 \Rightarrow 4 = 2x + (3 - x) \Rightarrow x = 1, y = 2$$

Accordingly



$$(b) \quad AF = \bar{AF} \left(\frac{M_{air}}{M_{fuel}} \right) = (4)(4.76) \left[\frac{28.97}{44.09} \right] = 12.51 \frac{kg(air)}{kg(fuel)} \quad \leftarrow (b)$$

(c)

The amount of dry products per mole of fuel is $n_{dry} = 1 + 2 + 15.04 = 18.04$. Thus, the analysis of the products on a dry basis is

$$y_{CO_2} = \frac{1}{18.04} = 0.0554 \text{ (5.54\%)}, y_{CO} = \frac{2}{18.04} = 0.1109 \text{ (11.09\%)}, y_{N_2} = \frac{15.04}{18.04} = 0.8337 \text{ (83.37\%)} \quad \leftarrow (c)$$

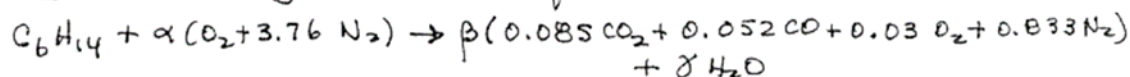
PROBLEM 13.30

KNOWN: C_6H_{14} burns with dry air. The dry molar analysis of the products is given.

FIND: Determine (a) the balanced reaction equation, (2) the percent theoretical air, (c) the dew point temperature of the products at 1 atm.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) the N_2 is inert. (3) The combustion products are modeled as an ideal gas mixture.

ANALYSIS: (a) Balancing the reaction equation for 1 kmol of hexane

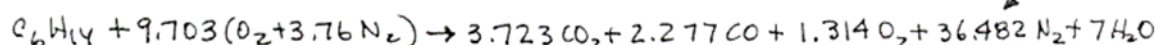


$$C: 6 = \beta(0.085 + 0.052) \Rightarrow \beta = 43.796$$

$$H_2: \frac{14}{2} = \gamma \Rightarrow \gamma = 7$$

$$\textcircled{1} \quad N_2: \alpha(3.76) = \beta(0.833) \Rightarrow \alpha = 9.703$$

Thus



(b) Theoretical combustion: $C_6H_{14} + \alpha_{th} O_2 \rightarrow 6 CO_2 + 7 H_2O \Rightarrow \alpha_{th} = 9.5$

$$\% \text{ theoretical air} = \left(\frac{\alpha}{\alpha_{th}} \right) \times 100 = (9.703/9.5) \times 100 = 102.1 \% \quad \leftarrow \begin{array}{l} \text{\% theo.} \\ \text{air} \end{array}$$

(c) Considering the combustion products, the total number of moles is 50.796 kmol. The fraction of water vapor is

$$y_v = \frac{7}{50.796} = 0.1378 \Rightarrow p_v = (0.1378)(1.01325 \text{ bar}) = 0.1396 \text{ bar}$$

From data in Table A-3; $T_{\text{dew point}} \approx 52.4^\circ C \quad \leftarrow T_{DP}$

$$1. \text{ Alternatively, for } O_2: \alpha = \beta(0.085 + 0.052/2 + 0.03) + \gamma/2$$

$$\Rightarrow \alpha = 9.675$$

The difference between this number and the value calculated above is experimental error and round-off.

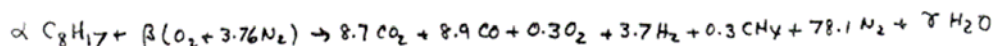
PROBLEM 13.31

KNOWN: The products of combustion of C_8H_{17} have the following dry molar analysis 8.7% CO_2 , 8.9% CO , 0.3% O_2 , 3.7% H_2 , 0.3% CH_4 , 78.1% N_2 .

FIND: Determine the equivalence ratio

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air, and N_2 is inert.

ANALYSIS: Basing the calculation on 100 moles of dry products



$$C: 8\alpha = 8.7 + 8.9 + 0.3 \Rightarrow \alpha = 2.2375$$

$$H: 17(2.2375) = 2(3.7) + 4(0.3) + 2\gamma \Rightarrow \gamma = 14.7188$$

$$N_2: 3.76\beta = 78.1 \Rightarrow \beta = 20.7713$$

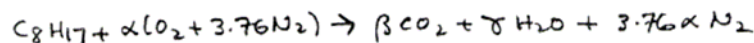
① check

$$O: \frac{(2)(20.7713)}{2} \stackrel{?}{=} (8.7)(2) + 8.9 + 2(0.3) + 14.7188$$

$$41.54 = 41.62$$

$$\text{The fuel air ratio is } FA = \frac{2.2375}{(20.7713)(4.76)} = 0.0226 \frac{\text{kmole(fuel)}}{\text{kmole(air)}}$$

The reaction equation for complete combustion with the theoretical amount of air is



$$C: 8 = \beta$$

$$H: 17 = 2\gamma, \gamma = 8.5$$

$$O: 2\alpha = 2\beta + \gamma = 16 + 8.5, \alpha = 12.25$$

$$\text{The fuel air ratio is } (FA)_{\text{theo}} = \frac{1}{(12.25)(4.76)} = 0.0171 \frac{\text{kmole(fuel)}}{\text{kmole(air)}}$$

The equivalence ratio:

$$\textcircled{2} \quad \left(\text{equivalence ratio} \right) = \frac{FA}{(FA)_{\text{theo}}} = \frac{0.0226}{0.0171} = 1.32$$

1. When using experimental data, an exact closure cannot be expected.
2. The reactants form a rich mixture.

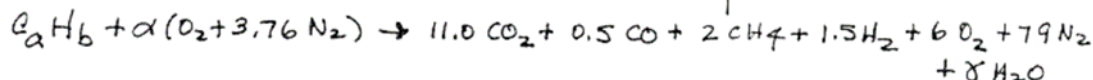
PROBLEM 13.32

KNOWN: A hydrocarbon represented as C_aH_b burns with dry air. The dry molar analysis of the products is given.

FIND: Determine the air-fuel ratio on a (a) molar basis, (b) mass basis.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 is inert, (3) Assume 100 kmol of dry product gas.

ANALYSIS: First, obtain the balanced reaction equation.



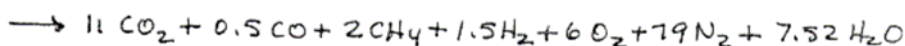
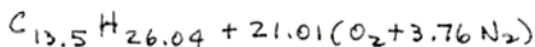
$$C: a = 11.0 + 0.5 + 2 = 13.5$$

$$H: b = (2)(4) + (1.5)(2) + (2)\gamma$$

$$N_2: \alpha(3.76) = 79 \Rightarrow \alpha = 21.01$$

$$O_2: 21.01 = 11.0 + \frac{0.5}{2} + 6 + \frac{\gamma}{2} \Rightarrow \gamma = 7.52$$

$$\text{With } \gamma = 7.52; b = 26.04$$



$$(a) \quad \overline{AF} = \frac{(21.01)(4.76)}{(1)} = 100 \quad \overline{AF}$$

$$(b) \quad AF = \left(100 \frac{\text{kmol air}}{\text{kmol fuel}}\right) \frac{(28.97)}{[(13.5)(12.01) + (26.04)(1.008)]} \frac{(\text{kg/kmol})_{\text{air}}}{(\text{kg/kmol})_{\text{fuel}}} = 15.38 \quad AF$$

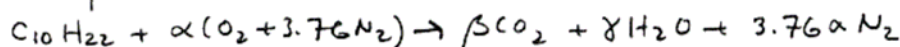
PROBLEM 13.33

KNOWN: $C_{10}H_{22}$ burns with 95% of theoretical air, producing CO_2 , CO , H_2O , N_2 .

FIND: Determine (a) $\overline{AF}$, (b) the analysis of the products on a dry basis.

ENGINEERING MODEL: 3.76 mole of N_2 accompany each mole of O_2 in the air, and N_2 is inert.

ANALYSIS: The complete combustion of $C_{10}H_{22}$ with the theoretical amount of air is

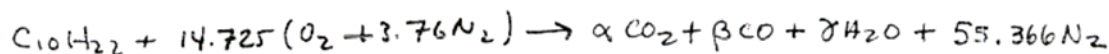


$$C: 10 = \beta$$

$$H: 22 = 2\gamma \Rightarrow \gamma = 11$$

$$O: 2\alpha = 2\beta + \gamma = 31, \alpha = 15.5$$

Combustion with 95% of theoretical air is then

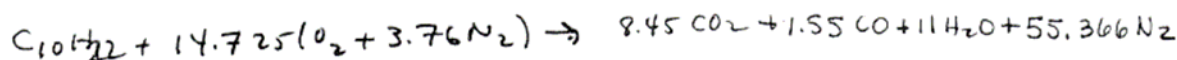


$$C: 10 = \alpha + \beta, \beta = 10 - \alpha$$

$$H: 22 = 2\gamma, \gamma = 11$$

$$O: (14.725)(2) = 2\alpha + (10 - \alpha) + 11 \Rightarrow \alpha = 8.45$$

The balanced reaction equation is then



$$(a) \quad \overline{AF} = \frac{14.725(4.76)}{1} = 70.09 \frac{\text{kmol(air)}}{\text{kmol(fuel)}}$$

(b) The amount of dry products is

$$n_{dry} = 8.45 + 1.55 + 55.366 = 65.366$$

Thus, the analysis on a dry basis is

$$y_{CO_2} = \frac{8.45}{65.366} = 0.129 \quad (12.9\%)$$

$$y_{CO} = \frac{1.55}{65.366} = 0.024 \quad (2.4\%)$$

$$y_{N_2} = \frac{55.366}{65.366} = 0.847 \quad (84.7\%)$$

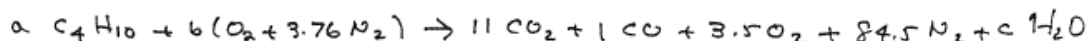
PROBLEM 13.34

KNOWN: C_4H_{10} burns with air to form products with the dry molar analysis: $\{11\% CO, 1\% CO, 3.5\% O_2, 84.5\% N_2\}$

FIND: Determine (a) the percent theoretical air, (b) the dewpoint temperature.

ENGINEERING MODEL: Each mole of O_2 in the combustion air is accompanied by 3.76 moles of N_2 . N_2 is inert.

ANALYSIS: (a) On the basis of 100 moles of dry products, the reaction is



$$C: 4a = 11 + 1, a = 3$$

$$H: (10)(3) = 2c, c = 15$$

$$O: 2b = 22 + 1 + 7.0 + 15, b = 22.5$$

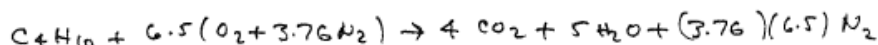
$$\text{Check } N: (3.76)(22.5) \stackrel{?}{=} 84.5$$

$$84.6 \neq 84.5 \text{ OK, close}$$

For this reaction,

$$\overline{AF} = \frac{22.5(4.76)}{3} = 35.7$$

For complete combustion with the theoretical amount of air



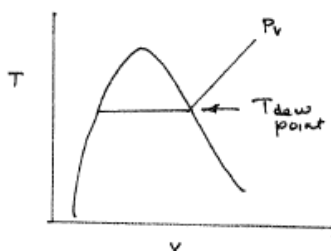
For this reaction

$$(\overline{AF})_{theo} = \frac{(6.5)(4.76)}{1} = 30.94$$

Then

$$\% \text{ theoretical air} = \left(\frac{35.7}{30.94} \right) (100) = 115\% \quad \leftarrow (a)$$

(b) To find the dew point temperature for $p = 1 \text{ bar}$



$$P_v = \left(\frac{n_v}{n_{prod}} \right) P$$

$$= \left(\frac{15}{115} \right) (1 \text{ bar})$$

$$= 0.1304 \text{ bar}$$

$$\text{Table 2E; } T_{dew, pt} \approx 51^\circ C \quad \leftarrow (b)$$

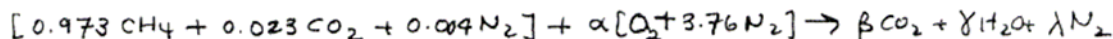
PROBLEM 13.35

KNOWN: A natural gas with a known volumetric analysis burns with air to give products for which a dry molar analysis is provided.

FIND: (a) Determine the percent of theoretical air used, (b) the dew point temperature.

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: Basing the calculation on one mole of fuel mixture, complete combustion with the theoretical amount of air is

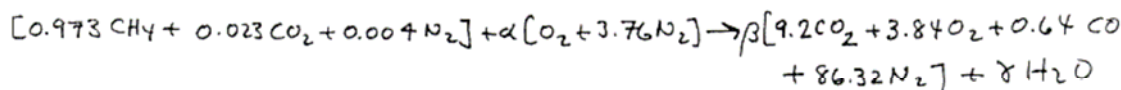


$$C: 0.973 + 0.023 = \beta \Rightarrow \beta = 0.996$$

$$H: 4(0.973) = 2\gamma, \quad \gamma = 1.946$$

$$O: 2(0.023) + 2\alpha = 2(0.996) + 1.946, \quad \alpha = 1.946$$

The actual reaction equation takes the form



$$C: (0.973 + 0.023) = \beta [9.2 + 0.64], \quad \beta = 0.1012$$

$$H: 4(0.973) = 2\gamma, \quad \gamma = 1.946$$

$$O: 2[0.023] + 2\alpha = \beta [(9.2)(2) + (3.84)(2) + 0.64] + 1.946, \quad \alpha = 2.302$$

CHECK:

$$N_2: (0.004) + (2.302)(3.76) = (0.1012)(86.32) \\ 8.66 \stackrel{?}{=} 8.74$$

①

(a) The actual $\overline{AF}$ value is obtained as

$$\overline{AF} = \frac{(2.302)(4.76)}{1}$$

whereas the theoretical value is

$$(\overline{AF})_{theo} = \frac{1.946(4.76)}{1}$$

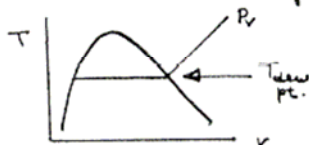
$$\left. \begin{array}{l} \overline{AF} = \frac{(2.302)(4.76)}{1} \\ (\overline{AF})_{theo} = \frac{1.946(4.76)}{1} \end{array} \right\} \begin{array}{l} \% \text{theo. air} = \left[\frac{\overline{AF}}{(\overline{AF})_{theo}} \right] 100 \\ = \left(\frac{2.302}{1.946} \right) (100) = 118 \end{array}$$

(b) The partial pressure of the water vapor is $P_v = y_v P$, where

$$y_v = \frac{1.946}{0.1012(100) + 1.946} = 0.1613$$

Then, with $p = 14.7 \text{ lbf/in}^2$, $P_v = 2.37 \text{ lbf/in}^2$. Interpolation in

Table A-3E with $P_g = 2.37 \text{ lbf/in}^2$ gives $T_{dew \text{ point}} = 132^\circ F \leftarrow T_{DP}$



PROBLEM 13.36

KNOWN: A fuel oil having the mass analysis { 85.7% C, 14.2% H, 0.1% inert } burns with air to give products with a dry molar analysis { 12.29% CO₂, 3.76% O₂, 83.95% N₂ }.

FIND: Determine the air-fuel ratio on a mass basis.

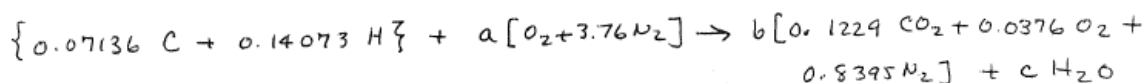
ENGINEERING MODEL: (1) 3.76 moles of N₂ accompany each mole of O₂ in the air, and the N₂ is inert. (2) The inert matter in the fuel is ignored in writing the reaction equation below.

ANALYSIS: Based on 1 kg of fuel oil, the mass analysis of the fuel allows the molar amounts of the combustibles to be evaluated:

$$C = \frac{0.857 \text{ kg(C) / kg(fuel)}}{12.01 \text{ kg(C) / kmol(C)}} = 0.07136 \frac{\text{kmol(C)}}{\text{kg(fuel)}}$$

$$H = \frac{0.142 \text{ kg(H) / kg(fuel)}}{1.009 \text{ kg(H) / kmol(H)}} = 0.14073 \frac{\text{kmol(H)}}{\text{kg(fuel)}}$$

On the basis of 1 kg of fuel oil, the reaction equation reads



$$\text{C: } 0.07136 = 0.1229 b \Rightarrow b = 0.5806$$

$$\text{H: } 0.14073 = 2c \Rightarrow c = 0.0704$$

$$\text{N}_2: 3.76a = 0.8395 b \Rightarrow a = 0.1296$$

Check for closure:

$$\begin{aligned} \text{O: } 2a &\stackrel{?}{=} b[2(0.1229) + 2(0.0376)] + 2c \\ 2(0.1296) &\stackrel{?}{=} 2(0.5806)[0.1229 + 0.0376] + (0.0704) \\ 0.2592 &\stackrel{?}{=} 0.2568 \quad (\text{good}) \end{aligned}$$

The air-fuel ratio is then

$$\text{AF} = \frac{(0.1296)(4.76)(28.97)}{1}$$

$$= 17.87$$

$$\frac{4}{\text{AF}}$$

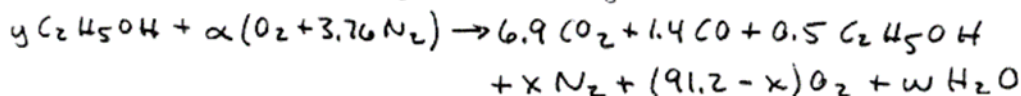
PROBLEM 13.37

Known: C_2H_5OH burns with air. A laboratory report gives only these percentages on a dry molar basis: 6.9% CO_2 , 1.4% CO , 0.5% C_2H_5OH .

FIND: Determine (a) the percentages of O_2 and N_2 in the dry molar analysis, (b) percent excess air.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (2) The dry products include only CO_2 , CO , C_2H_5OH , O_2 and N_2 .

ANALYSIS: a) On the basis of 100 moles of dry products



Note: For the dry products, $\underset{\text{N}_2}{6.9} + \underset{\text{O}_2}{1.4} + 0.5 + x + z = 100 \Rightarrow x + z = 91.2$ or $z = 91.2 - x$

$$C: 2y = 6.9 + 1.4 + 2(0.5), \quad y = 4.65$$

$$H: 6(4.65) = (0.5)(6) + 2\omega, \quad \omega = 12.45$$

$$0: (4.65) + 2x = (6.9)(2) + 1.4 + 0.5 + 2(91.2 - x) + 12.45$$

$$\Rightarrow \alpha = 102.95 - x$$

$$N: (2)(2)(3.76) = 2 \times$$

$$\Rightarrow 3.76 \alpha = x$$

$$\alpha = 102.95 - 3.76 \alpha$$

$$\alpha = 21.628$$

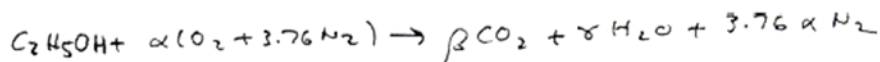
$$x = 81.322$$

and $91.2 - x = 9.88$

Accordingly,

$$\%N_2 = 81.32, \quad \%O_2 = 9.88 \quad \leftarrow$$

(b) For complete combustion with the theoretical amount of air



$$c: 2 = \beta$$

$$H: 6 = 2x, x = 3$$

$$0: 1 + 2a = 2(2) + 3$$

$\alpha = 3$

$$\left(\% \text{ Theoretical Air} \right) = \left(\frac{21.628 (4.76) / 4.65}{3 (4.76) / 1} \right) (100) = 155$$

% excess air = 55 $\longleftarrow$ (b)

PROBLEM 13.38

KNOWN: A fuel oil with the mass analysis { 87% C, 11% H, 1.4% S, 0.6% inert } burns with 120% of theoretical air. H and S are fully oxidized, but 95% of the Carbon is oxidized to CO_2 and the balance to CO.

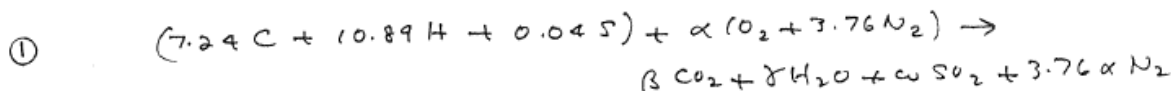
FIND: Determine (a) the balanced reaction equation, (b) for CO and SO_2 the amounts in kmol per 10⁶ kmol of combustion products.

ENGINEERING MODEL: There are 3.76 kmol of N_2 accompanying each kmol of O_2 in the combustion air. N_2 is inert.

ANALYSIS: (A) Based on 100 kg of fuel oil, the molar analysis is

$$\text{C} : \frac{87}{12.01} = 7.24, \text{H} : \frac{11}{1.01} = 10.89, \text{S} : \frac{1.4}{32.06} = 0.04 \text{ kmol}$$

To find the theoretical amount of air, the reaction equation for 100 kg of fuel oil is then



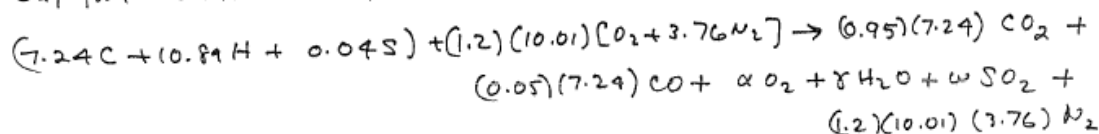
$$\text{C} : 7.24 = \beta$$

$$\text{H} : 10.89 = 2\gamma, \gamma = 5.45$$

$$\text{S} : 0.04 = \omega$$

$$\text{O} : 2\alpha = 2(7.24) + 5.45 + 2(0.04), \alpha = 10.01$$

Then, for the actual reaction

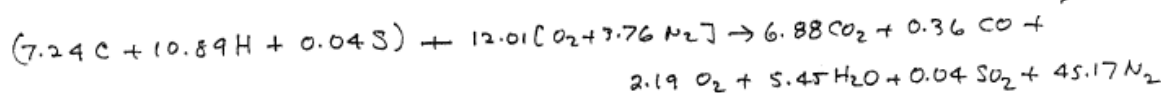


$$\text{H} : 10.89 = 2\gamma, \gamma = 5.45 \text{ (as above)}$$

$$\text{S} : 0.04 = \omega \text{ (as above)}$$

$$\text{O} : \underbrace{(1.2)(10.01)(2)}_{24.024} = 13.756 + 0.362 + 2\alpha + 5.45 + 2(0.04), \alpha = 2.19$$

The balanced reaction equation for 100 kg of fuel oil is



(b) Total moles of products = 60.09 kmol / 100 kg of fuel oil.

$$\text{ppm CO} = \left(\frac{0.36}{60.09} \right) (10^6) = 5991 \quad \leftarrow \text{(b)}$$

$$\text{ppm SO}_2 = \left(\frac{0.04}{60.09} \right) (10^6) = 666$$

1. The presence of the inert matter is not shown in the reaction equation of part (a). Also, in part (b) only the gaseous products are considered in the ppm calculations.

PROBLEM 13.39

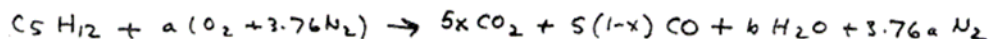
KNOWN: C_5H_{12} burns with air so that a fraction x of the carbon is converted to CO_2 and the rest appears as CO . No free O_2 appears in the products.

FIND: Plot $\bar{AF}$ and % theoretical air versus x .

ENGINEERING

MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

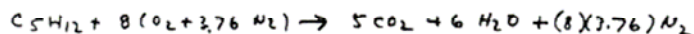
ANALYSIS: The reaction equation takes the form



$$H: 12 = 2b, \quad b = 6$$

$$O: 2a = 10x + 5(1-x) + 6 \Rightarrow 2a = 5x + 11 \Rightarrow a = (5x + 11)/2.$$

The complete combustion of C_5H_{12} with the theoretical amount of air is described by



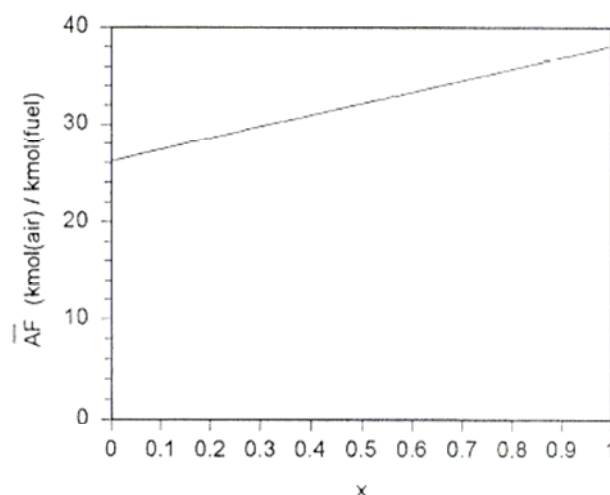
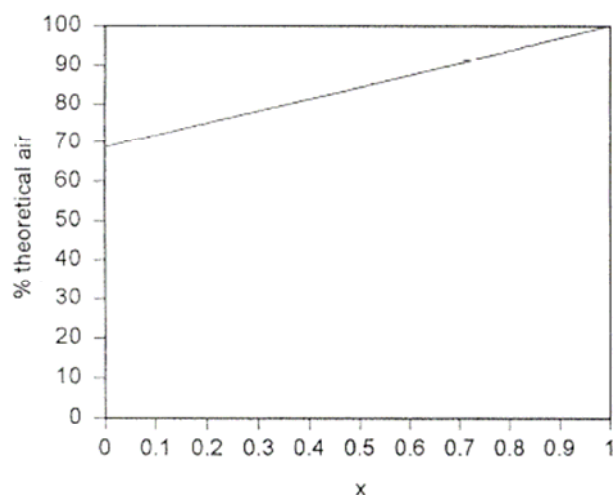
Accordingly, the percent of theoretical air required is

$$\% \text{ theo. air} = \left(\frac{\bar{AF}}{(\bar{AF})_{\text{theo}}} \right) (100) = \left(\frac{(5x + 11)/2}{8} \right) 100 = \left(\frac{5x + 11}{16} \right) (100) \\ = 31.25x + 68.75 \quad (1)$$

The air-fuel ratio is

$$\bar{AF} = \frac{(5x + 11)/2}{1} (4.76) = (2.5x + 5.5) 4.76 \quad \text{or} \quad \bar{AF} = 11.9x + 26.18 \quad (2)$$

Eqs. (1) and (2) are simple linear relations that can be plotted readily by hand or using computer software. The following plots are obtained using IT:



Note that with a deficiency of combustion air, the combustion is less complete, i.e. less CO_2 and more CO produced.

PROBLEM 13.40

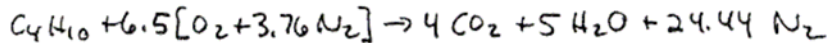
KNOWN: Fuel-air mixtures are specified.

FIND: Determine the equivalence ratio for each case and indicate if the mixture is lean or rich

ENGINEERING MODEL: Accompanying each mole of O_2 in the air is 3.76 moles of N_2 which is inert.

ANALYSIS: (a) $\{1 \text{ kmol } C_4H_{10}, 32 \text{ kmol air}\}$

The reaction equation for the theoretical amount of air reads



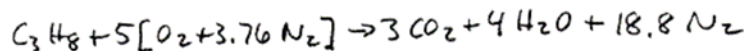
$$\Rightarrow (\overline{AF})_{THEO} = \frac{6.5(4.76)}{1} = 30.94$$

Then, the equivalence ratio is

$$\textcircled{1} \quad \text{Equivalence ratio} = \frac{(\overline{AF})_{THEO}}{(\overline{AF})_{ACT}} = \frac{30.94}{(32/1)} = 0.967 \quad (\text{Lean})$$

(b) $\{1 \text{ lb } C_3H_8, 14.5 \text{ lb air}\}$

The reaction equation for the theoretical amount of air reads



$$\Rightarrow (\overline{AF})_{THEO} = \frac{(5)(4.76)}{1} \left[\frac{28.97}{44.09} \right] = 15.64$$

Then, the equivalence ratio is

$$\text{Equivalence ratio} = \frac{(\overline{AF})_{THEO}}{(\overline{AF})_{ACT}} = \frac{15.64}{(14.5/1)} = 1.078 \quad (\text{Rich}) \quad \leftarrow$$

1. Alternatively,

$$(\text{equivalence ratio}) = \frac{(\overline{FA})_{act}}{(\overline{FA})_{theo}}$$

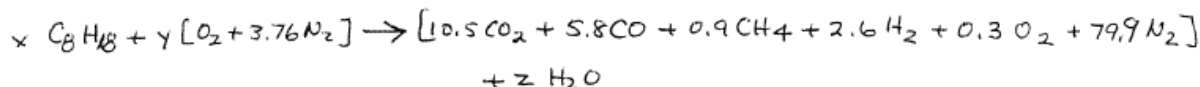
PROBLEM 13.41

KNOWN: C_8H_{18} burns with air to give products with the dry molar analysis: $\{10.5\% CO_2, 5.8\% CO, 0.9\% CH_4, 2.6\% H_2, 0.3\% O_2, 79.9\% N_2\}$

FIND: Determine the equivalence ratio.

ENGINEERING MODEL: Each mole of O_2 in the combustion air is accompanied by 3.76 moles of N_2 which is inert.

ANALYSIS: On the basis of 100 moles of dry products, the reaction equation is



$$C: 8x = 10.5 + 5.8 + 0.9, x = 2.15$$

$$H: (18)(2.15) = 4(0.9) + 2(2.6) + 2z, z = 14.95$$

$$N_2: 3.76y = 79.9, y = 21.25$$

Check for closure

$$O: 2y = 2(10.5) + 5.8 + 2(1.3) + 14.95, y = 21.18 \text{ (good)}$$

The air-fuel ratio is then

$$\overline{AF} = \frac{(21.18)(4.76)}{2.15} = 46.89$$

The balanced equation for the theoretical amount of air reads



Thus

$$(\overline{AF})_{THEO} = \frac{(12.5)(4.76)}{1} = 59.5$$

The equivalence ratio is

$$\textcircled{1} \quad \text{Equivalence ratio} = \frac{(\overline{AF})_{THEO}}{(\overline{AF})_{ACT}} = \frac{59.5}{46.89} = 1.269 \text{ (Rich)} \leftarrow$$

1. Alternatively

$$(\text{equivalence ratio}) = \frac{(\overline{FA})_{act}}{(\overline{FA})_{theo}}$$

PROBLEM 13.42

KNOWN: CH_4 burns with air to form products consisting of $\{\text{CO}_2, \text{CO}, \text{H}_2\text{O}, \text{N}_2\}$. The equivalence ratio is 1.25.

FIND: Determine the balanced reaction equation.

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the combustion air and N_2 is inert.

ANALYSIS: The balanced reaction equation for combustion with the theoretical amount of air gives $(\bar{A}/\bar{F})_{\text{THEO}} = 9.52$ (Eq. 13.4). For the actual reaction, the equivalence ratio is

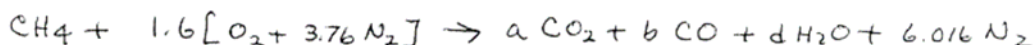
$$\textcircled{1} \quad \text{Equivalence ratio} = \frac{(\bar{A}/\bar{F})_{\text{THEO}}}{(\bar{A}/\bar{F})_{\text{ACT}}} = 1.25$$

Accordingly

$$(\bar{A}/\bar{F})_{\text{ACT}} = \frac{9.52}{1.25} = 7.616$$

The amount of O_2 in the reaction is then $7.616/4.76 = 1.6$.

Thus, the reaction equation has the form



$$\text{C: } 1 = a + b$$

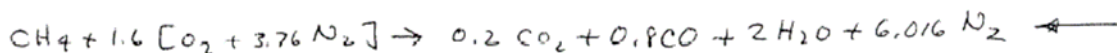
$$\text{H: } 4 = 2d, \quad d = 2$$

$$\text{O: } 2(1.6) = 2a + b + 2$$

$$1.2 = 2a + b$$

$$\begin{aligned} a &= 0.2 \\ b &= 0.8 \end{aligned}$$

Finally



1. Alternatively,

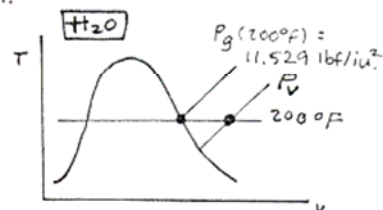
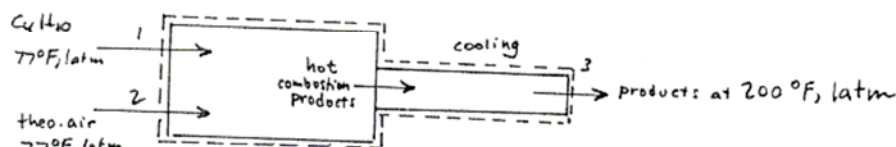
$$\left(\text{equivalence ratio} \right) = \frac{(\bar{F}/\bar{A})_{\text{act}}}{(\bar{F}/\bar{A})_{\text{theo}}}$$

PROBLEM 13.43

KNOWN: C_4H_{10} at $77^\circ F$, 1 atm enters a combustion chamber operating at steady state and burns completely with the theoretical amount of air entering at $77^\circ F$, 1 atm .

FIND: Determine the rate of heat transfer, in Btu per lbmol of fuel.

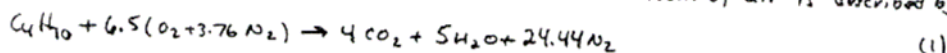
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the combustion air and the gaseous products of the combustion.

ANALYSIS: Complete combustion of C_4H_{10} with the theoretical amount of air is described by



Before applying the energy balance it is necessary to determine whether any of the water formed during combustion condenses as the products are cooled to $200^\circ F$. If the combustion products remain a gas throughout, the partial pressure of the water vapor is

$P_v = \left(\frac{5}{4 + 5 + 24.44} \right) \left(14.7 \frac{\text{lbf}}{\text{in}^2} \right) = 2.20 \frac{\text{lbf}}{\text{in}^2}$. Since $P_v < P_g(200^\circ F)$, the water formed remains a vapor as the products are cooled.

An energy rate balance reduces to give

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{C_4H_{10}})_1 + 6.5 [(\bar{h}_{O_2})_1 + 3.76 (\bar{h}_{N_2})_1] - [4 \bar{h}_{CO_2} + 5 \bar{h}_{H_2O} + 24.44 \bar{h}_{N_2}]_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = [4 (\bar{h}_f^\circ + \Delta \bar{h})_{CO_2} + 5 (\bar{h}_f^\circ + \Delta \bar{h})_{H_2O(g)} + 24.44 (\bar{h}_f^\circ + \Delta \bar{h})_{N_2}]_3 - (\bar{h}_f^\circ)_{C_4H_{10}}$$

With data from Tables A-23E and A-25E

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} &= \left\{ 4 [(-169,300) + (5165 - 4028)] + 5 [(-104,040) + (5250 - 4258)] + 24.44 [4586 - 3730] \right\} \\ &\quad - (-54,270) \\ &= -1,112,701 \text{ Btu/lbmol(fuel)} \end{aligned}$$

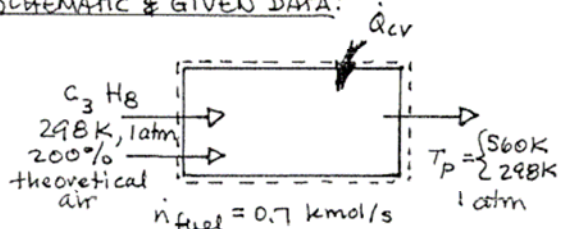
← $\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}}$

PROBLEM 13.44

KNOWN: Propane enters a combustion chamber with known temperature and pressure and a given molar flow rate, where it burns completely with 200% theoretical air. The temperature of the exiting combustion products is specified.

FIND: Determine the heat transfer rate for each of two exit temperatures.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

- (1) 3.76 moles of N_2 accompany each mole of O_2 in the air.
- (2) The N_2 is inert.
- (3) The products are modeled as an ideal gas mixture.
- (4) The control volume is at steady state with $w_{cv} = 0$.
- (5) Kinetic and potential energy effects are negligible.

ANALYSIS: Theoretical combustion: $C_3H_8 + 5(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 18.8N_2$

Actual Combustion: $C_3H_8 + 10(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 5O_2 + 37.6N_2$

$T_p = 560 K$ The energy rate balance can be written as

$$0 = \dot{Q}_{cv} + \dot{n}_{fuel} \left\{ (\bar{h}_f^\circ + \Delta \bar{h})_{fuel} + 10(\bar{h}_f^\circ + \Delta \bar{h})_{O_2, in} + 37.6(\bar{h}_f^\circ + \Delta \bar{h})_{N_2, in} - 3(\bar{h}_f^\circ + \Delta \bar{h})_{CO_2, out} - 4(\bar{h}_f^\circ + \Delta \bar{h})_{H_2O, out} - 5(\bar{h}_f^\circ + \Delta \bar{h})_{O_2, out} - 37.6(\bar{h}_f^\circ + \Delta \bar{h})_{N_2, out} \right\}$$

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 3 \left[\bar{h}_{CO_2}^\circ + \bar{h}_{CO_2}(560) - \bar{h}_{CO_2}(298) \right] + 4 \left[\bar{h}_{H_2O(g)}^\circ + \bar{h}_{H_2O(g)}(560) - \bar{h}_{H_2O}(298) \right] + 5 \left[\bar{h}_{O_2}^\circ + \bar{h}_{O_2}(560) - \bar{h}_{O_2}(298) \right] + 37.6 \left[\bar{h}_{N_2}^\circ + \bar{h}_{N_2}(560) - \bar{h}_{N_2}(298) \right] - (\bar{h}_f^\circ C_3H_8)$$

With data from Table A-23

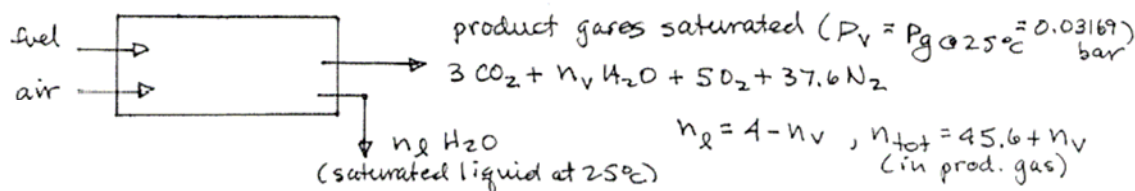
$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} &= 3 \left[-393,520 + (20,407 - 9,364) \right] + 4 \left[(-241,820 + (18,959 - 9,904)) \right] \\ &\quad + 5 \left[16,654 - 8,682 \right] + 37.6 \left[16,363 - 8,669 \right] - (-103,850) \\ &= 3 \left[-382,477 \right] + 4 \left[-232,765 \right] + 5 \left[7972 \right] + 37.6 \left[7694 \right] - (-103,850) \\ &= -1,645,299 \text{ kJ/kmol} \end{aligned}$$

$$\dot{Q}_{cv} = (0.7 \frac{\text{kmol}}{\text{s}}) (-1,645,299 \frac{\text{kJ}}{\text{kmol}}) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 1.152 \times 10^6 \text{ kW} \quad \left(T_p = 560 K \right)$$

$T_p = 298 K$ In this case, condensation might occur in the product gas mixture. To check this, determine the dew point temperature of the products, as follows: $n_v = 4$, $n_{tot} = 49.6 \Rightarrow y_v = 4/49.6 = 0.08065$

$P_v = (0.08065)(1.01325 \text{ bar}) = 0.08172 \text{ bar}$. From Table A-2 $T_{dew} \approx 42^\circ C$

Since $298 K = 25^\circ C$ is less than the dew point, there is condensation. The following model will be used:



Continued on next slide

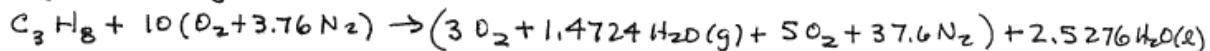
Problem 13-44 continued

To find n_v and n_g

$$y_v = \frac{P_v}{P} = \frac{0.03169}{1.01325} = 0.03128$$

and $y_v = \frac{n_v}{45.6 + n_v} \Rightarrow n_v = 1.4724$

Finally, collecting results



The energy balance becomes

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} &= 3 [\bar{h}_f^\circ CO_2] + 1.4724 [\bar{h}_f^\circ H_2O(g)] + 2.5276 [\bar{h}_f^\circ H_2O(l)] - [\bar{h}_f^\circ C_3H_8] \\ &= 3(-393,520) + 1.4724(-241,820) + 2.5276(-285,830) - (-103,850) \\ &= -2.155 \times 10^6 \text{ kJ/kmol fuel} \end{aligned}$$

$$\dot{Q}_{cv} = (0.7)(-2.155 \times 10^6) = -1.5085 \times 10^6 \text{ kW} \leftarrow \frac{\dot{Q}_{cv}}{(T_p = 298K)}$$

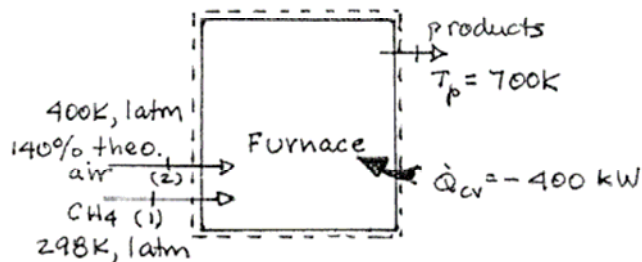
Note that cooling the gases to 298K and condensing some of the water vapor increases the heat transfer rate substantially.

PROBLEM 13.45

KNOWN: Methane enters a furnace with known conditions and burns completely with 140% of theoretical air entering at an elevated temperature. The temperature of the exiting products and the heat transfer rate to the surroundings are known.

FIND: Determine the mass flow rate of fuel.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

(1) 3.76 moles of N_2 accompany each mole of oxygen in the air. (2) The N_2 is inert. (3) The control volume is at steady state. (4) $W_{cv} = 0$ and kinetic and potential energy effects are negligible. (5) The ideal gas model is used for all streams.

ANALYSIS: Theoretical combustion: $CH_4 + 2(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 7.52N_2$

Actual combustion: $CH_4 + 2.8(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 0.8O_2 + 10.528N_2$

The energy rate balance reduces at steady state to

$$0 = \dot{Q}_{cv} + \dot{n}_{fuel} [(\bar{h}_{CH_4})_1 + 2.8(\bar{h}_{O_2})_2 + 10.528(\bar{h}_{N_2})_2 - (\bar{h}_{CO_2})_p - 2(\bar{h}_{H_2O})_p - 0.8(\bar{h}_{O_2})_p - 10.528(\bar{h}_{N_2})_p]$$

or

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} &= [\bar{h}_{f,CO_2}^o + \bar{h}_{CO_2}(700) - \bar{h}_{CO_2}(298)] + 2[\bar{h}_{f,H_2O(g)}^o + \bar{h}_{H_2O}(700) - \bar{h}_{H_2O}(298)] \\ &\quad + 0.8[\bar{h}_{f,O_2}^o + \bar{h}_{O_2}(700) - \bar{h}_{O_2}(298)] + 10.528[\bar{h}_{f,N_2}^o + \bar{h}_{N_2}(700) - \bar{h}_{N_2}(298)] \\ &\quad - [\bar{h}_{f,CH_4}^o] - 2.8[\bar{h}_{f,O_2}^o + \bar{h}_{O_2}(400) - \bar{h}_{O_2}(298)] \\ &\quad - 10.528[\bar{h}_{f,N_2}^o + \bar{h}_{N_2}(400) - \bar{h}_{N_2}(298)] \\ &= [-343,520 + (27,125 - 9,364)] + 2[-241,820 + (24,088 - 9,904)] \\ &\quad + 0.8[21,184 - 8,682] + 10.528[20,604 - 8,669] - [-74,850] \\ &\quad - 2.8[11,711 - 8,682] - 10.528[11,640 - 8,669] \\ &= -597,676 \text{ kJ/kmol fuel} \end{aligned}$$

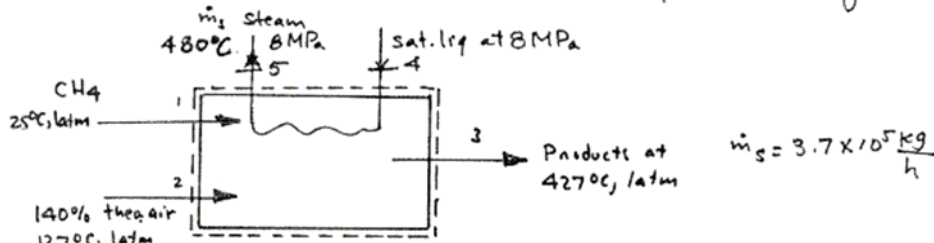
$$\dot{n}_{fuel} = \frac{-400 \text{ kW}}{-597,676 \text{ kJ/kmol fuel}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 6.693 \times 10^{-4} \frac{\text{kmol}}{\text{s}}$$

$$\dot{m}_{fuel} = \dot{n}_{fuel} M_{fuel} = (6.693 \times 10^{-4})(16.04) = 0.0107 \text{ kg/s} \leftarrow \dot{m}_{fuel}$$

PROBLEM 13.46

KNOWN: Steady state operating data are provided for a boiler fueled by methane.

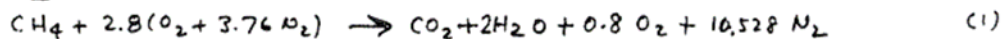
FIND: Determine the volumetric flow rate of the fuel.



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: Complete combustion of CH_4 with 140% of theoretical air is described by



An energy rate balance reduces to give

$$0 = \frac{\dot{m}_s}{\dot{n}_F} [h_4 - h_5] + (\bar{h}_{CH_4})_1 + [2.8 \bar{h}_{O_2} + 10.528 \bar{h}_{N_2}]_2 - [\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 0.8 \bar{h}_{O_2} + 10.528 \bar{h}_{N_2}]_3$$

$$\text{with } \bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$$

$$\frac{\dot{m}_s}{\dot{n}_F} [h_4 - h_5] = [\bar{h}_f^\circ + \bar{h}(700) - \bar{h}(298)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(700) - \bar{h}(298)]_{H_2O} + 0.8[\bar{h}(700) - \bar{h}(298)]_{O_2} + 10.528[\bar{h}(700) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{CH_4} - 2.8[\bar{h}(400) - \bar{h}(298)]_{O_2} - 10.528[\bar{h}(400) - \bar{h}(298)]_{N_2}$$

Inserting data from the steam tables and the ideal gas tables

$$\begin{aligned} \frac{\dot{m}_s}{\dot{n}_F} [316.6 - 3348.4] &= [-293,520 + 27,125 - 9,364] + 2[-241,920 + 24,088 - 9,904] + 0.8[21,184 - 8,682] + \\ &\quad 10.528[20,604 - 11,640] - (-74,850) - 2.8[11,711 - 8,682] \\ &= -660,287 \text{ kJ/kmol}(CH_4) \end{aligned}$$

Thus

$$\frac{\dot{m}_s}{\dot{n}_F} = \frac{660,287}{2031.8} = 324.98 \frac{\text{kg(steam)}}{\text{kmol(fuel)}}$$

or

$$\dot{n}_F = \frac{3.7 \times 10^5 \text{ kg/h}}{324.98 \text{ kg/kmol(fuel)}} = 1138.5 \frac{\text{kmol}}{\text{h}}$$

Since

$$\dot{n}_F = \frac{(AV)_F}{\bar{v}_F} = \frac{(AV)_F}{\bar{R}T_F/P_F} \Rightarrow (AV)_F = \dot{n}_F \left[\frac{\bar{R}T_F}{P_F} \right]$$

$$= (1138.5 \frac{\text{kmol}}{\text{h}}) \left(\frac{(8314 \frac{\text{N} \cdot \text{m}}{\text{kmol} \cdot \text{K}})(298\text{K})}{1.01325 \times 10^5 \text{ N/m}^2} \right) = 27,838 \frac{\text{m}^3}{\text{h}} \leftarrow \dot{V}_F$$

PROBLEM 13.47

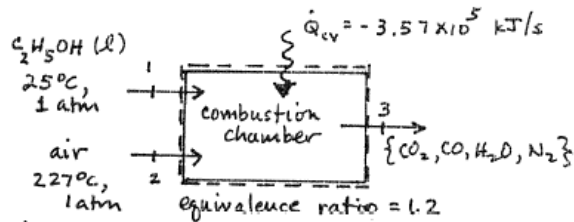
KNOWN: Liquid ethanol enters a combustion chamber and burns with entering separately with an equivalence ratio of 1.2. The mass flow rate of fuel and the heat transfer rate are specified as are the combustion products.

FIND: Determine (a) the exit temperature, and (b) the air-fuel ratio on a mass basis.

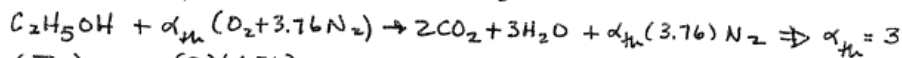
SCHEMATIC & GIVEN DATA:

ENGINEERING

MODEL: (1) The control volume shown is at steady state, and $\dot{W}_{cv} = 0$ with negligible kinetic and potential energy effects. (2) 3.76 kmol of N_2 accompany each kmol of O_2 in the air and the N_2 is inert. (3) The combustion air and product gases behave as ideal gas mixtures.



ANALYSIS: (a) For complete combustion of $C_2H_5OH(l)$ with theoretical air

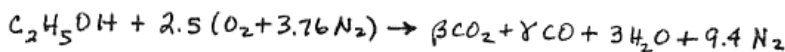


thus $(AF)_{theo} = \frac{(3)(4.76)}{1} = 14.28$

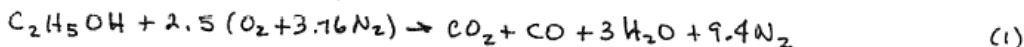
Now $equiv. ratio = \frac{(FA)_{act}}{(FA)_{theo}} = \frac{(AF)_{theo}}{(AF)_{act}} \Rightarrow (AF)_{act} = \frac{(AF)_{theo}}{equiv. ratio} = 11.9$

and $\alpha_{act} = \frac{(AF)_{act}}{(4.76)} = 2.5$

Thus, the actual combustion reaction is



$$\begin{aligned} C: 2 &= \beta + \gamma \\ O: 1 + 2.5(2) &= \beta(2) + \gamma + 3 \end{aligned} \quad \left. \begin{aligned} \beta &= 1 \\ \gamma &= 1 \end{aligned} \right\}$$



(a) An energy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_F} - \frac{\dot{W}_{cv}}{\dot{n}_F} + \bar{h}_{fuel} + [2.5 \bar{h}_{O_2} + 9.4 \bar{h}_{N_2}]_2 - [\bar{h}_{CO_2} + \bar{h}_{CO} + 3 \bar{h}_{H_2O} + 9.4 \bar{h}_{N_2}]_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2 , this becomes

$$\begin{aligned} 0 &= \frac{\dot{Q}_{cv}}{\dot{n}_F} + (\bar{h}_f^\circ)_{C_2H_5OH(l)} + 2.5 [\bar{h}(500) - \bar{h}(298)]_{O_2} + 9.4 [\bar{h}(500) - \bar{h}(298)]_{N_2} \\ &\quad - [\bar{h}_f^\circ + \bar{h}(T_3)]_{CO_2} - [\bar{h}_f^\circ + \bar{h}(T_3)]_{CO} - 3 [\bar{h}_f^\circ + \bar{h}(T_3)]_{H_2O(g)} - 9.4 [\bar{h}(T_3) - \bar{h}(298)]_{N_2} \end{aligned} \quad (2)$$

The molar flow rate of fuel is $\dot{n}_F = \dot{m}_F / M_{fuel} = \frac{25 \text{ kg/s}}{46.07 \text{ kg/kmol}} = 0.5427 \frac{\text{kmol}}{\text{s}}$

solving for $[\bar{h}(T_3)]_{CO_2} + [\bar{h}(T_3)]_{CO} + 3 [\bar{h}(T_3)]_{H_2O} + 9.4 [\bar{h}(T_3)]_{N_2}$ and inserting data from Tables A-23E and A-25E

$$\begin{aligned} &[\bar{h}(T_3)]_{CO_2} + [\bar{h}(T_3)]_{CO} + 3 [\bar{h}(T_3)]_{H_2O} + 9.4 [\bar{h}(T_3)]_{N_2} \\ &= \frac{\dot{Q}_{cv}}{\dot{n}_F} + (\bar{h}_f^\circ)_{C_2H_5OH(l)} + 2.5 [\bar{h}(500) - \bar{h}(298)]_{O_2} + 9.4 [\bar{h}(500) - \bar{h}(298)]_{N_2} \\ &\quad - [\bar{h}_f^\circ - \bar{h}(298)]_{CO_2} - [\bar{h}_f^\circ - \bar{h}(298)]_{CO} - 3 [\bar{h}_f^\circ - \bar{h}(298)]_{H_2O(g)} - 9.4 [-\bar{h}(298)]_{N_2} \end{aligned}$$

Continued on next slide

Problem 13-47 continued

$$= \frac{(-3.75 \times 10^5)}{(0.5427)} + 2.5 [14770 - 8669] + 9.4 [14581 - 8669] - [-393520 - 9364] - [-110530 - 8669] - 3 [-241820 - 9904] - 9.4 [-8669] = 460,857 \frac{\text{kJ}}{\text{kmol}(\text{fuel})}$$

This equation can be solved iteratively for T_3 using table data. The result is

① $T_3 \approx 994 \text{ K.}$ $\longleftarrow T_3$

(b) Using Eq. (13.2), the air-fuel ratio on a mass basis is

$$AF = \bar{A} \bar{F} \left(\frac{M_{\text{air}}}{M_{\text{fuel}}} \right) = 11.9 \left(\frac{28.97}{46.07} \right) = 7.483 \frac{\text{kg}(\text{air})}{\text{kg}(\text{fuel})} \quad \longleftarrow AF$$

1. To avoid iteration using table data, IT can be used. The IT solution follows.

IT Code

```
T1 = 25 + 273 // K
T2 = 227 + 273 // K
hfo_fuel = -277690 // from Table A-25
Qdot = -3.75E05
ndot_fuel = 25 / 46.07 // kmol/s

0 = Qdot / ndot_fuel + hfuel + 2.5 * hO2_2 +
    9.4 * hN2_2 - hCO2_3 - hCO_3 - 3 * hH2O_3 - 9.4 * hN2_3
hfuel = hfo_fuel
hO2_2 = h_T("O2", T2)
hN2_2 = h_T("N2", T2)
hCO2_3 = h_T("CO2", T3)
hCO_3 = h_T("CO", T3)
hH2O_3 = h_T("H2O", T3)
hN2_3 = h_T("N2", T3)
```

IT Result

$T_3 = 993.7 \text{ K}$

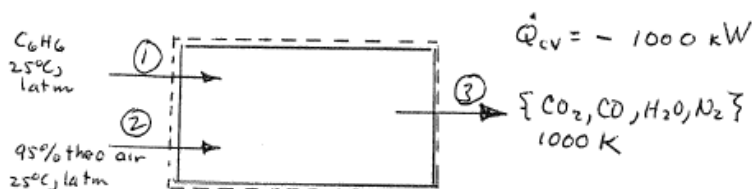
Note that the IT solution agrees favorably with the iterative solution. Also, note that IT evaluates the specific enthalpies in the bracketed terms in Eq. (13.2) directly.

PROBLEM 13.48

KNOWN: Steady state operating data are provided for a combustion chamber in which C_6H_6 burns with 95% theoretical air.

FIND: Determine the fuel mass flow rate for a heat transfer rate $\dot{Q}_{cv} = 1000 \text{ kW}$.

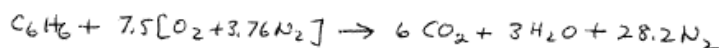
SCHEMATIC & GIVEN DATA:



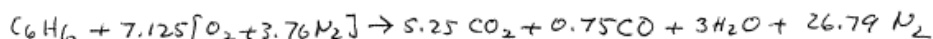
ENGINEERING

MODEL: (1) The control volume shown is at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic/potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the combustion air and N_2 is inert. (3) The ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: The balanced reaction equation for combustion with the theoretical amount of air is



The balanced reaction equation for combustion with 95% theoretical air is



An energy rate balance reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{fuel})_1 + [7.125\bar{h}_{O_2} + 26.79\bar{h}_{N_2}]_2 - [5.25\bar{h}_{CO_2} + 0.75\bar{h}_{CO} + 3\bar{h}_{H_2O} + 26.79\bar{h}_{N_2}]_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 5.25[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 0.75[\bar{h}_f^\circ + \Delta\bar{h}]_{CO} + 3[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 26.79[\bar{h}_f^\circ + \Delta\bar{h}]_{N_2} - (\bar{h}_f^\circ)_{fuel}$$

With data from the ideal gas tables

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} &= 5.25[-393520 + 42769 - 9364] + 0.75[-110530 + 30355 - 8669] + 3[-241820 + 35882 - 9904] \\ &\quad + 26.79[30129 - 8669] - (-82930) \\ &= -1,890,604 - 66,633 - 647,526 + 574,913 - 82,930 \\ &= -2,112,780 \text{ kJ/kmol(fuel)} \end{aligned}$$

Then, for $\dot{Q}_{cv} = -1000 \text{ kJ/s}$

$$\dot{n}_{fuel} = \frac{-10^3 \text{ kJ/s}}{-2,112,780 \text{ kJ/kmol(fuel)}} = 4.73 \times 10^{-4} \frac{\text{kmol(fuel)}}{\text{s}}$$

With $M = 78.11$ from Table A-1

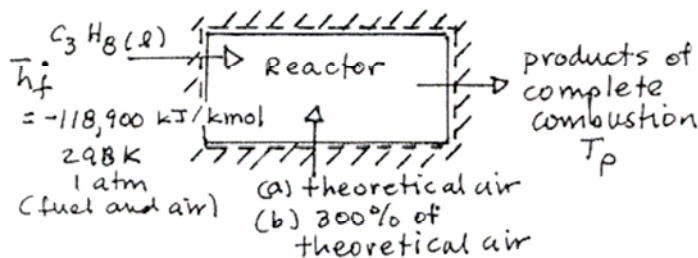
$$\dot{n}_{fuel} = \left(4.73 \times 10^{-4} \frac{\text{kmol(fuel)}}{\text{s}}\right) \left(78.11 \frac{\text{kg(fuel)}}{\text{kmol(fuel)}}\right) = 0.037 \frac{\text{kg(fuel)}}{\text{s}} \leftarrow \dot{n}_{fuel}$$

PROBLEM 13.49

KNOWN: Liquid C_3H_8 and air, each at known temperature and pressure, enter a well insulated reactor.

FIND: Determine the temperature of the products for complete combustion with (a) theoretical air, and (b) 300% of theoretical air.

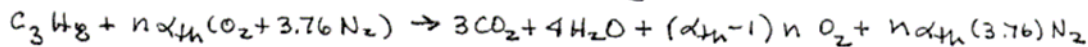
SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

- (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 is inert. (3) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (4) Kinetic and potential energy effects can be neglected. (5) All gaseous streams are modeled as ideal gas mixtures.

ANALYSIS: For $\alpha = n \alpha_{th}$ kmol of O_2 entering



For theoretical combustion: $C_3H_8 + 5(O_2 + 3.76 N_2) \rightarrow 3 CO_2 + 4 H_2O + 18.8 N_2$
 $\Rightarrow \alpha_{th} = 5$

Thus $C_3H_8 + 5n(O_2 + 3.76 N_2) \rightarrow 3n CO_2 + 4n H_2O + (5n - 1)n O_2 + 5n(3.76) N_2$ (1)

The energy rate balance reduces to

$$0 = [\dot{h}_f^{\circ} C_3H_8(l) + \Delta \dot{h}^{\circ}] + 5n [\dot{h}_f^{\circ} O_2 + \Delta \dot{h}^{\circ}]_{O_2, in} + 5n(3.76) [\dot{h}_f^{\circ} N_2 + \Delta \dot{h}^{\circ}]_{N_2, in} \\ - 3 [\dot{h}_f^{\circ} CO_2 + \dot{h}(T_p) - \dot{h}(298)] - 4 [\dot{h}_f^{\circ} H_2O(g) + \dot{h}(T_p) - \dot{h}(298)] \\ - (5n - 1)n [\dot{h}_f^{\circ} O_2 + \dot{h}(T_p) - \dot{h}(298)] - 5n(3.76) [\dot{h}_f^{\circ} N_2 + \dot{h}(T_p) - \dot{h}(298)]$$

Rearranging

$$3 \dot{h}_{CO_2}(T_p) + 4 \dot{h}_{H_2O}(T_p) + 5(n - 1) \dot{h}_{O_2}(T_p) + 18.8n \dot{h}_{N_2}(T_p) \\ = 3 [\dot{h}_f^{\circ} CO_2 - \dot{h}_{CO_2}(298)] + 4 [\dot{h}_f^{\circ} H_2O(g) - \dot{h}_{H_2O}(298)] + 5(n - 1) [-\dot{h}_{O_2}(298)] \\ + 18.8n \dot{h}_{N_2}(298) - \dot{h}_f^{\circ} C_3H_8(l) \\ = 3 [-393,520 - 9,364] + 4 [-241,820 - 9,904] + 5(n - 1) (-8,682) \\ + 18.8n (-8,669) - (-118,900) \\ = -206,387n - 2,053,238$$

① Using data from Table A-23 to iterate and find T_p , we get

(a) for $n = 1$; $T_p = 2380$ K (theoretical air) ← (a)

(b) for $n = 3$; $T_p = 1150$ K (300% of theoretical air) ← (b)

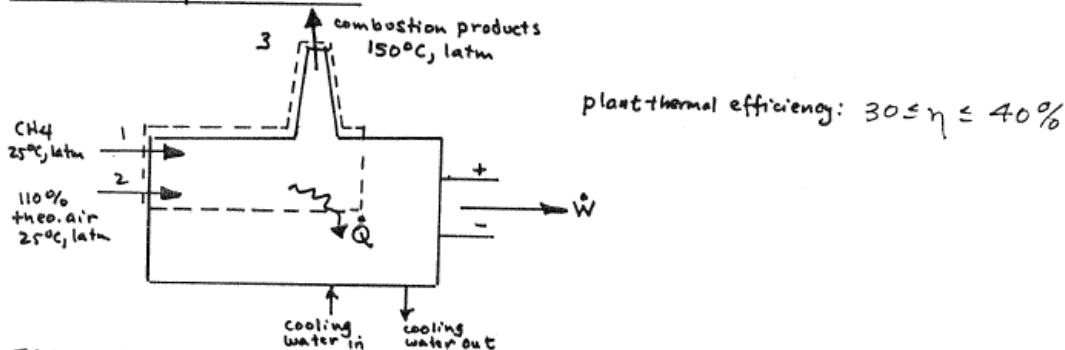
1. An iterative solution can be avoided by using Interactive Thermodynamics: IT to solve the problem.

PROBLEM 13.50

KNOWN: Steady state operating data are provided for a simple vapor plant.

FIND: Plot the mass flow rate of fuel required, in kg/h per MW of power developed, versus the plant thermal efficiency, η .

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure by the dashed line operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and products of combustion are modeled as ideal gases.

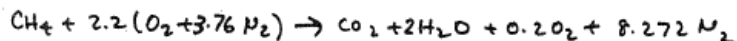
ANALYSIS: By definition, $\eta = \dot{W} / \dot{Q} \Rightarrow \dot{Q} = \dot{W} / \eta$. For N MW of power developed generated

$$\dot{Q} = \frac{N \times 10^3 \text{ kJ/s}}{\eta} \quad (1)$$

where η is expressed as a decimal.

To evaluate the mass flow rate of the fuel required, an energy rate balance, the chemical reaction equation, and this value for $\dot{Q}$.

Complete combustion of CH_4 with the theoretical amount of air is described by Eq. 13.4. Complete combustion with 110% of theoretical air is then



Noting that $\dot{Q}$ is positive in the direction of the arrow on the above figure, an energy rate balance at steady state reduces to give

$$\frac{\dot{Q}}{\dot{n}_{CH_4}} = (\bar{h}_{CH_4})_1 + (2.2 \bar{h}_{O_2} + 8.272 \bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2 \bar{h}_{H_2O} + 0.2 \bar{h}_{O_2} + 8.272 \bar{h}_{N_2})_3$$

$$\text{With } \bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$$

$$\frac{\dot{Q}}{\dot{n}_{CH_4}} = (\bar{h}_f^\circ)_{CH_4} + (2.2 \bar{h}_f^\circ)_{O_2} + 8.272 \bar{h}_f^\circ)_{N_2} - [(\bar{h}_f^\circ + \bar{h}(423) - \bar{h}(298))_{CO_2} + 2(\bar{h}_f^\circ + \bar{h}(423) - \bar{h}(298))_{H_2O(g)} + 0.2(\bar{h}_f^\circ + \bar{h}(423) - \bar{h}(298))_{O_2} + 8.272(\bar{h}_f^\circ + \bar{h}(423) - \bar{h}(298))_{N_2}]$$

Then, with data from the ideal gas tables

$$\begin{aligned} \frac{\dot{Q}}{\dot{n}_{CH_4}} &= (-74,850) - [-393,520 + 14,333 - 9364] - 2[-241,820 + 14,147 - 9,904] - 0.2[12,405 - 8,669] \\ &\quad - 8.272[12,313 - 8,669] \\ &= 757,967 \text{ kJ/kmol}(CH_4) \end{aligned}$$

Continued on next slide

Problem 13-50 continued

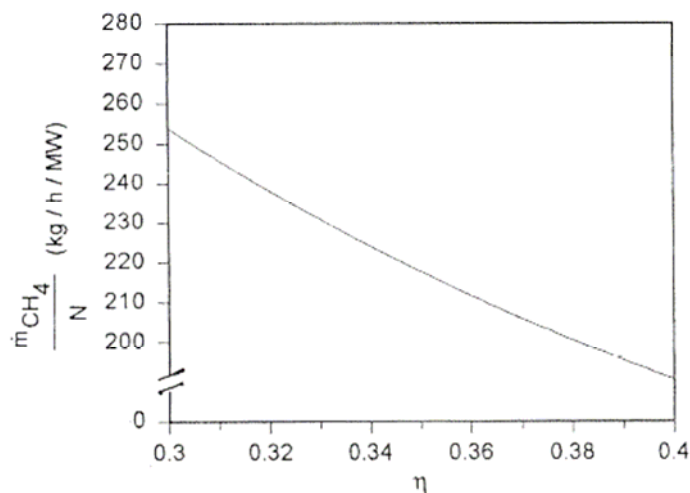
Combining this with (1)

$$\dot{m}_{CH_4} = \frac{N \times 10^3 \text{ kJ/s}}{\eta (757,967 \text{ kJ/kmol})}$$

$$\dot{m}_{CH_4} = \frac{N \times 10^3 \text{ kJ/s}}{\eta [757,967 \text{ kJ/kmol}] \left[\frac{16.04 \text{ kg}}{\text{kmol}} \right] \left| \frac{3600 \text{ s}}{\text{h}} \right|}$$

$$\frac{\dot{m}_{CH_4}}{N} = \frac{76.18}{\eta} \left(\frac{\text{kg/h}}{\text{MW}} \right) \quad (2)$$

Eq. (2) is a simple mathematical relation that can be plotted readily, by hand or using computer software. The following plot is obtained using IT:



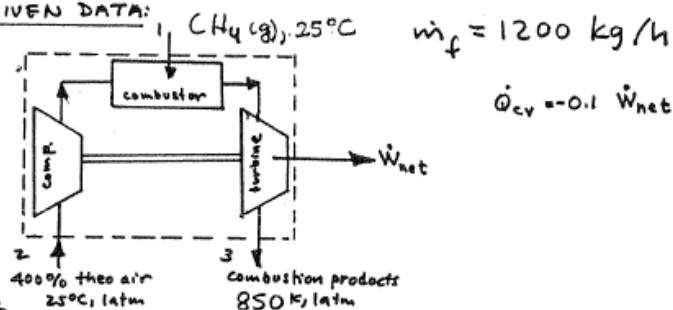
Note that as the thermal efficiency increases, less fuel is needed per MW of power developed, as expected.

PROBLEM 13.51

KNOWN: Steady state operating data are provided for a simple gas turbine power plant.

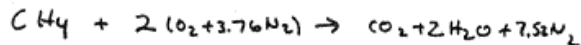
FIND: Determine the net power output for a given fuel mass flow rate.

SCHEMATIC & GIVEN DATA:

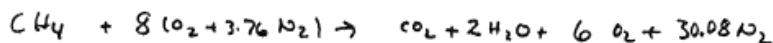


ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the fuel, combustion air, and the products of combustion. **ANALYSIS:** Complete combustion of CH_4 with the theoretical amount of air is described by



Complete combustion with 400% of theoretical air is then



An energy rate balance reduces at steady state to

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{CH_4})_1 + (2\bar{h}_{O_2} + 30.08\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 6\bar{h}_{O_2} + 30.08\bar{h}_{N_2})_3$$

Then, with $\dot{Q}_{cv} = -0.1 \dot{W}_{cv}$ and $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$\textcircled{1} \quad 1.1 \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} = (\bar{h}_f^\circ)_{CH_4} + (2\bar{h}_{f,O_2}^\circ + 30.08\bar{h}_{f,N_2}^\circ) - [\bar{h}_f^\circ + \bar{h}(850) - \bar{h}(298)]_{CO_2} - 2[\bar{h}_f^\circ + \bar{h}(850) - \bar{h}(298)]_{H_2O} - 6[\bar{h}(850) - \bar{h}(298)]_{O_2} - 30.08[\bar{h}(850) - \bar{h}(298)]_{N_2}$$

With data from the ideal gas tables and Table A-25

$$1.1 \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} = -74,850 - [-393,520 + 34,773 - 9,364] - 2[-241,820 + 29,846 - 9,904] - 6[26,218 - 8,682] - 30.08[25,292 - 8,669]$$

$$= 131,781 \text{ kJ/kmol fuel}$$

$$\dot{n}_{fuel} = \frac{(1200) \text{ kg/h}}{(16.04) \frac{\text{kg}}{\text{kmol}}} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 0.02078 \frac{\text{kmol fuel}}{\text{s}}$$

$$\dot{W}_{cv} = \frac{(0.02078) \text{ kmol/s}}{1.1} (131,781) \frac{\text{kJ}}{\text{kmol}} \left| \frac{1 \text{ MW}}{1000 \text{ kJ/s}} \right| = 2.49 \text{ MW}$$

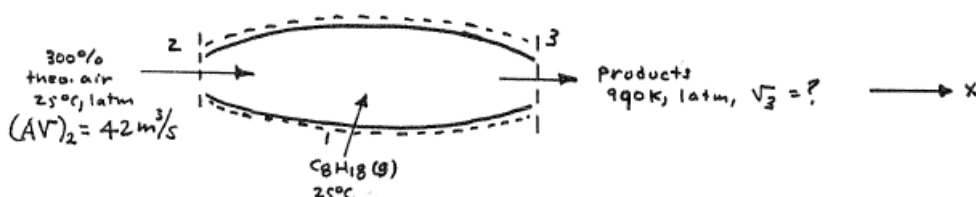
1. The fuel must enter the combustor at an elevated pressure because the combustion air pressure has been increased in passing through the compressor. However, with the ideal gas model the fuel enthalpy is determined by the temperature only.

PROBLEM 13.52

KNOWN: $C_8H_{18}(g)$ at $25^\circ C$, 1 atm enters a jet engine and burns completely with 300% of theoretical air entering at $25^\circ C$, 1 atm . Products exit at 990 K , 1 atm .

FIND: Determine the thrust produced by the engine.

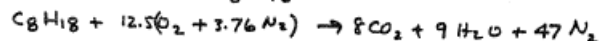
SCHEMATIC & GIVEN DATA:



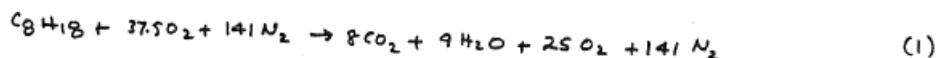
ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The kinetic energies of the incoming air and fuel can be neglected. Potential energy can be neglected throughout. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: Complete combustion of C_8H_{18} with the theoretical amount of air is described by



Complete combustion with 300% of theoretical air is then



An energy rate balance at steady state reduces to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{n}_{C_8H_{18}} \bar{h}_{C_8H_{18}}]_1 + [\dot{n}_{O_2} \bar{h}_{O_2} + \dot{n}_{N_2} \bar{h}_{N_2}]_2 - \left\{ [\dot{n}_{CO_2} \bar{h}_{CO_2} + \dot{n}_{H_2O} \bar{h}_{H_2O} + \dot{n}_{O_2} \bar{h}_{O_2} + \dot{n}_{N_2} \bar{h}_{N_2}] + \dot{m}_3 \frac{V_3^2}{2} \right\} \quad (2)$$

(fuel) (air) (products)

where $\dot{n}$ denotes a molar flow rate and $\dot{m}_3$ is the mass flow rate of the exiting products:

$$\dot{m}_3 = (\dot{n}_{CO_2} + \dot{n}_{H_2O} + \dot{n}_{O_2} + \dot{n}_{N_2})_3 = (M_{CO_2} \dot{n}_{CO_2} + M_{H_2O} \dot{n}_{H_2O} + M_{O_2} \dot{n}_{O_2} + M_{N_2} \dot{n}_{N_2})_3 \quad (3)$$

Dividing throughout by the molar flow rate of the fuel and solving for $V_3^2/2$, Eqs. (1)–(3) give

$$\frac{V_3^2}{2} = \frac{(\bar{h}_{C_8H_{18}})_1 + [37.5 \bar{h}_{O_2} + 141 \bar{h}_{N_2}]_2 - [8 \bar{h}_{CO_2} + 9 \bar{h}_{H_2O} + 25 \bar{h}_{O_2} + 141 \bar{h}_{N_2}]_3}{(8 M_{CO_2} + 9 M_{H_2O} + 25 M_{O_2} + 141 M_{N_2})}$$

With $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{V_3^2}{2} = \frac{(\bar{h}_f^\circ)_{C_8H_{18}} + 8[\bar{h}_f^\circ + \bar{h}(990) - \bar{h}(298)]_{CO_2} - 9[\bar{h}_f^\circ + \bar{h}(990) - \bar{h}(298)]_{H_2O} - 25[\bar{h}(990) - \bar{h}(298)]_{O_2} - 141[\bar{h}(990) - \bar{h}(298)]_{N_2}}{(8 M_{CO_2} + 9 M_{H_2O} + 25 M_{O_2} + 141 M_{N_2})}$$

With data from the ideal gas tables

$$\begin{aligned} \frac{V_3^2}{2} &= \frac{(-208,450) - 8[-393,520 + 43,226 - 9,364] - 9[-241,810 + 35,472 - 9,904] - 25[31,041 - 8,682] - 141[29,803 - 8,669]}{(8(44.01) + 9(18.02) + 25(32.00) + 141(28.01))} \\ &= \frac{-208,450 + 2,885,264 + 1,146,268 - 558,975 - 2,979,894}{5263.67} \\ &= 205.98 \frac{\text{kJ}}{\text{kg(products)}} \end{aligned}$$

Continued on next slide

Problem 13-52 continued

Solving for V_3 , and introducing appropriate conversion factors

$$V_3 = \sqrt{2 \left(205.98 \frac{\text{KJ}}{\text{Kg}} \right) \left| \frac{10^3 \text{ N}\cdot\text{m}}{\text{KJ}} \right| \left| \frac{1 \text{ Kg}\cdot\text{m/s}^2}{1 \text{ N}} \right|} = 641.84 \text{ m/s}$$

Ignoring the momentum transfer associated with the fuel and incoming combustion air, Eq. 9.31 gives the magnitude of the resultant force in the x direction acting on the control volume

$$F_x = \dot{m}_3 V_3 \quad (4)$$

where

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = \dot{m}_1 \left[1 + \left(\frac{\dot{m}_2}{\dot{m}_1} \right) \right] = \dot{m}_1 [1 + (FA)]$$

$$\dot{m}_1 = \frac{(AV)_1}{RT_1/P_1} = \frac{(47 \text{ m}^3/\text{s})(1.01325 \times 10^5 \text{ N/m}^2)}{\left(\frac{8314}{28.97} \frac{\text{N}\cdot\text{m}}{\text{Kg}\cdot\text{K}} \right)(298 \text{ K})} = 49.8 \frac{\text{Kg}}{\text{s}}$$

From the reaction equation the fuel-air ratio on a mass basis is

$$FA = \frac{[1 \text{ kmol (fuel)}][114.22 \text{ Kg/kmol}]}{[(37.5 + 141) \text{ kmol (air)}][28.97 \text{ Kg/kmol}]} = 0.0221 \frac{\text{Kg (fuel)}}{\text{Kg (air)}}$$

$$\Rightarrow \dot{m}_3 = (0.0221 \times 49.8 \frac{\text{Kg}}{\text{s}}) = 50.9 \frac{\text{Kg}}{\text{s}}$$

Accordingly, Eq. (4) gives

$$F_x = (50.9 \frac{\text{Kg}}{\text{s}}) \left(641.84 \frac{\text{m}}{\text{s}} \right) \left| \frac{1 \text{ N}}{1 \text{ Kg}\cdot\text{m/s}^2} \right|$$

$$= 32,670 \text{ N}$$

The thrust is the force developed by the control volume:

$$\text{Thrust} = -F_x = -32,670 \text{ N}$$

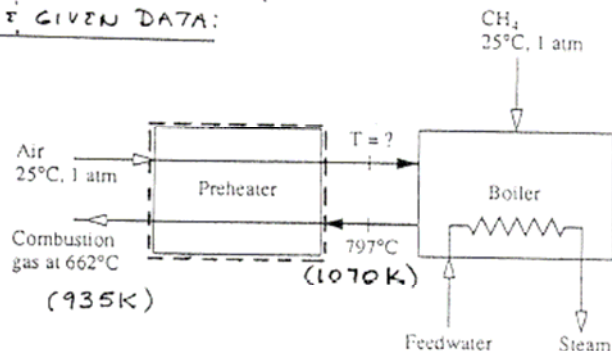
↑ acts to the left (see figure)

PROBLEM 13.53

KNOWN: Steady state operating data are provided for a boiler and preheater.

FIND: Determine the temperature of the combustion air entering the boiler from the preheater.

SCHEMATIC & GIVEN DATA:

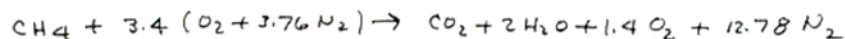


ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state.

(2) For the control volume, $\dot{Q}_{cv}$ and kinetic/potential energy effects are negligible.

(3) Combustion is complete with 170% of theoretical air. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The air and the combustion products can each be modeled as an ideal gas mixture.

ANALYSIS: Complete combustion with 170% of theoretical air is described by



An energy rate balance for the preheater reads

$$0 = [3.4 \bar{h}_{O_2} + 12.78 \bar{h}_{N_2}]_1 - [3.4 \bar{h}_{O_2} + 12.78 \bar{h}_{N_2}]_2 + [\bar{h}_{CO_2} + 2 \bar{h}_{H_2O} + 1.4 \bar{h}_{O_2} + 12.78 \bar{h}_{N_2}]_3 - [\bar{h}_{CO_2} + 2 \bar{h}_{H_2O} + 1.4 \bar{h}_{O_2} + 12.78 \bar{h}_{N_2}]_4$$

With data from the ideal gas tables

$$3.4 [\bar{h}(T_2) - 8682]_{O_2} + 12.78 [\bar{h}(T_2) - 8669]_{N_2} = 12.78 [32432 - 28016]_{CO_2} + [46602 - 39268]_{H_2O} + 2 [38802 - 33234]_{H_2O} + 1.4 [33842 - 29133]_{O_2}$$

$$\Rightarrow 3.4 \bar{h}_{O_2}(T_2) + 12.78 \bar{h}_{N_2}(T_2) = 221,808 \text{ kJ/kmol(fuel)}$$

① Solving iteratively using data from Table A-23, $T_2 = 197^\circ\text{C}$

1. Iteration using table data can be avoided by using IT, as follows:

IT Code

T1 = 25 // °C
T3 = 797 // °C
T4 = 662 // °C

hO2_1 = h_T("O2", T1)
hN2_1 = h_T("N2", T1)
hO2_2 = h_T("O2", T2)
hN2_2 = h_T("N2", T2)
hCO2_3 = h_T("CO2", T3)
hH2O_3 = h_T("H2O", T3)
hO2_3 = h_T("O2", T3)

hN2_3 = h_T("N2", T3)
hCO2_4 = h_T("CO2", T4)
hH2O_4 = h_T("H2O", T4)
hO2_4 = h_T("O2", T4)
hN2_4 = h_T("N2", T4)

$$0 = (3.4 * hO2_1 + 12.78 * hN2_1) - (3.4 * hO2_2 + 12.78 * hN2_2) + (hCO2_3 + 2 * hH2O_3 + 1.4 * hO2_3 + 12.78 * hN2_3) - (hCO2_4 + 2 * hH2O_4 + 1.4 * hO2_4 + 12.78 * hN2_4)$$

IT Result

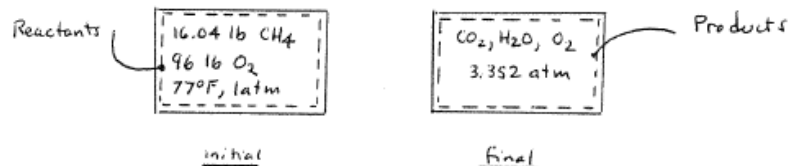
$T_2 = 196.4^\circ\text{C}$

PROBLEM 13.54

KNOWN: A rigid tank initially contains CH_4 and O_2 at a known condition. Complete combustion occurs and the final pressure is specified.

FIND: Determine the heat transfer in Btu.

SCHEMATIC & GIVEN DATA:

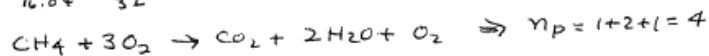


ENGINEERING MODEL: (1) The system is indicated in the above figure. (2) For the system, $W=0$ and kinetic/potential energy effects are absent. (3) The ideal gas model is applicable.

ANALYSIS: An energy balance reduces to give $Q = \Delta U$. To evaluate ΔU the final temperature is required. Using the ideal gas equation of state

$$P_R = \frac{n_R \bar{R} T_R}{V}, \quad P_P = \frac{n_P \bar{R} T_P}{V} \Rightarrow \frac{P_P}{P_R} = \frac{n_P}{n_R} \frac{T_P}{T_R} \Rightarrow \frac{P_P}{P_R} = \frac{T_P}{T_R}$$

where $n_R = \frac{16.04}{16.04} + \frac{96}{32} = 1 + 3 = 4$ and for complete combustion



Accordingly, $T_P = \left(\frac{3.352}{4}\right)(537^\circ\text{R}) = 1800^\circ\text{R}$.

Then, $Q = U_P - U_R$ or

$$Q = [\bar{u}_{\text{CO}_2} + 2\bar{u}_{\text{H}_2\text{O}} + \bar{u}_{\text{O}_2}]_P - [\bar{u}_{\text{CH}_4} + 3\bar{u}_{\text{O}_2}]_R$$

With $\bar{u} = \bar{h} - \bar{R}T$

$$Q = [(\bar{h}_{\text{CO}_2} - \bar{R}T_P) + 2(\bar{h}_{\text{H}_2\text{O}} - \bar{R}T_P) + (\bar{h}_{\text{O}_2} - \bar{R}T_P)]_P - [(\bar{h}_{\text{CH}_4} - \bar{R}T_R) + 3(\bar{h}_{\text{O}_2} - \bar{R}T_R)]_R$$

$$= [\bar{h}_{\text{CO}_2} + 2\bar{h}_{\text{H}_2\text{O}} + \bar{h}_{\text{O}_2}]_P - [\bar{h}_{\text{CH}_4} + 3\bar{h}_{\text{O}_2}]_R + 4\bar{R}[T_R - T_P]$$

Then, using $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$Q = \{[\bar{h}_f^\circ + \Delta\bar{h}]_{\text{CO}_2} + 2[\bar{h}_f^\circ + \Delta\bar{h}]_{\text{H}_2\text{O}} + [\Delta\bar{h}]_{\text{O}_2}\}_P - [(\bar{h}_f^\circ)_{\text{CH}_4}]_R + 4\bar{R}[T_R - T_P]$$

With data from the ideal gas tables

$$Q = [-169,300 + (18392 - 4028)] + 2[-104,040 + (15433 - 4258)] + [3486 - 3725] - [-32,410] + 4(1.986)[537 - 1800]$$

$$= -154,936 - 185,730 + 9761 + 32,210 - 10,033$$

$$= -308,728 \text{ Btu}$$

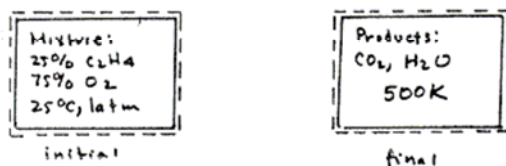
← Q

PROBLEM 13.55

KNOWN: A closed, rigid vessel contains a gaseous mixture at 25°C, 1 atm with the molar analysis { 25% C₂H₄, 75% O₂ }. The mixture burns completely and the products are cooled to 500 K.

FIND: Determine (a) the heat transfer from the vessel per kmol of C₂H₄, and (b) the final pressure, in atm.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system is shown in the accompanying figure. (2) For the system $W = 0$ and there are no effects of kinetic and potential energy. (3) The ideal gas model is applicable to the initial and final mixtures. (4) Combustion is complete.

ANALYSIS: Complete combustion of C₂H₄ with the theoretical amount of O₂ is described by



For these reactants on a molar basis: C₂H₄: 25%, O₂: 75%. Accordingly, it can be concluded that there is no excess O₂, and that the reaction undergone by the system is described by Eq. (1).

(a) An energy balance reduces to $\Delta U = Q - W$ or $Q = \Delta U = U_2 - U_1$. When expressed on a per mole of C₂H₄ basis, this is

$$\frac{Q}{n_{\text{C}_2\text{H}_4}} = \frac{U_2 - U_1}{n_{\text{C}_2\text{H}_4}} = [2\bar{u}_{\text{CO}_2} + 2\bar{u}_{\text{H}_2\text{O}}]_2 - [\bar{u}_{\text{C}_2\text{H}_4} + 3\bar{u}_{\text{O}_2}]_1 \quad (2)$$

For an ideal gas $u(T) = \bar{h}(T) - \bar{R}T$. Accordingly, Eq. (2) becomes

$$\begin{aligned} \frac{Q}{n_{\text{C}_2\text{H}_4}} &= [2(\bar{h}_{\text{CO}_2}(T_2) - \bar{R}T_2) + 2(\bar{h}_{\text{H}_2\text{O}}(T_2) - \bar{R}T_2)] - [\bar{h}_{\text{C}_2\text{H}_4}(T_1) - \bar{R}T_1 + 3(\bar{h}_{\text{O}_2}(T_1) - \bar{R}T_1)] \\ &= \{2\bar{h}_{\text{CO}_2}(T_2) + 2\bar{h}_{\text{H}_2\text{O}}(T_2) - \bar{h}_{\text{C}_2\text{H}_4}(T_1) - 3\bar{h}_{\text{O}_2}(T_1)\} - 4\bar{R}T_2 + 4\bar{R}T_1 \end{aligned}$$

or, with $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$\frac{Q}{n_{\text{C}_2\text{H}_4}} = 2[\bar{h}_f^\circ + \bar{h}(500) - \bar{h}(298)]_{\text{CO}_2} + 2[\bar{h}_f^\circ + \bar{h}(500) - \bar{h}(298)]_{\text{H}_2\text{O}} - (\bar{h}_f^\circ)_{\text{C}_2\text{H}_4} - 3(\bar{h}_f^\circ)_{\text{O}_2} - 4\bar{R}(T_2 - T_1)$$

With data from the ideal gas tables

$$\begin{aligned} \frac{Q}{n_{\text{C}_2\text{H}_4}} &= 2[-393,520 + 17,678 - 9,364] + 2[-241,820 + 16,828 - 9,909] - (52,280) - 4(8.314)[500 - 298] \\ &= -770,412 - 469,792 - 52,280 - 6718 \\ &= -1,299,202 \text{ kJ/kmol(C}_2\text{H}_4\text{)} \quad \leftarrow \quad (Q/n_{\text{C}_2\text{H}_4}) \end{aligned}$$

(b) Using the ideal gas equation of state

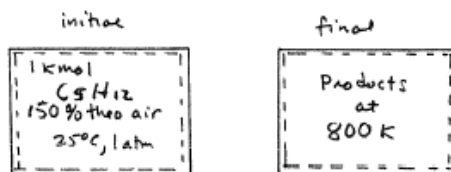
$$\begin{aligned} P_R &= \frac{nR\bar{R}T_R}{V} & \frac{P_P}{P_R} &= \frac{n_P T_P}{n_R T_R} = \left(\frac{4}{4}\right) \left(\frac{500\text{ K}}{298\text{ K}}\right) = 1.68 \\ P_P &= \frac{n_P \bar{R} T_P}{V} & \Rightarrow P_P &= 1.68 \text{ atm} \quad \leftarrow P_P \end{aligned}$$

PROBLEM 13.56

KNOWN: 1 kmol of C_5H_{12} and 150 % of theoretical air are contained in a closed, rigid vessel at $25^\circ C, 1 \text{ atm}$. The mixture burns completely forming products at 800 K .

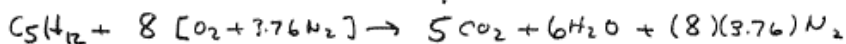
FIND: Determine (a) the heat transfer and (b) the final pressure.

SCHEMATIC & GIVEN DATA:

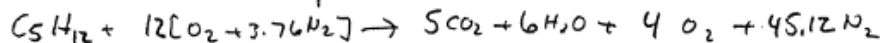


ENGINEERING MODEL: (1) The system is the contents of the vessel. (2) Combustion is complete. 3.67 kmol of N_2 accompany each kmol of O_2 in the combustion air. N_2 is inert. (3) The ideal gas model applies to both the initial and final gas mixtures.

ANALYSIS: The complete reaction of C_5H_{12} with the theoretical amount of air is



Complete reaction with 150 % of theoretical air is



Using the ideal gas model

$$P_R = \frac{n_R \bar{R} T_R}{V} \quad \left\{ \quad \frac{P_P}{P_R} = \left(\frac{n_P}{n_R} \right) \left(\frac{T_P}{T_R} \right) = \left(\frac{60.12}{58.12} \right) \left(\frac{800 \text{ K}}{298 \text{ K}} \right) = 2.777 \right.$$

$$P_P = \frac{n_P \bar{R} T_P}{V} \quad \Rightarrow \quad P_P = 2.777 \text{ atm} \quad \leftarrow (6)$$

Following the discussion of Sec. 13.2.2, an energy balance reduces to read

$$Q - W = \sum_P n (\bar{h}_f^\circ + \Delta \bar{h}) - \sum_R n (\bar{h}_f^\circ + \Delta \bar{h}) - \bar{R} T_P \sum_P n + \bar{R} T_R \sum_R n \quad (13.17b)$$

or

$$Q = 5 [\bar{h}_f^\circ + \Delta \bar{h}]_{CO_2} + 6 [\bar{h}_f^\circ + \Delta \bar{h}]_{H_2O} + 4 [\bar{h}_f^\circ + \Delta \bar{h}]_{O_2} + 45.12 [\bar{h}_f^\circ + \Delta \bar{h}]_{N_2} - (\bar{h}_f^\circ)_{C_5H_{12}} - \bar{R} T_P \sum_P n + \bar{R} T_R \sum_R n$$

$\underbrace{60.12}_{\text{CO}_2} \quad \underbrace{58.12}_{\text{N}_2}$

with ideal gas table data and $(\bar{h}_f^\circ)_{C_5H_{12}}$ from Table A-25

$$Q = 5 [-393,520 + 32,179 - 9364] + 6 [-241,820 + 27,896 - 9904] + 4 [24,323 - 8682] + 45.12 [23,714 - 8669] - (-146,440) - 8.314 [(800)(60.12) - (298)(58.12)]$$

$$= -2,563,732 \text{ kJ} \quad \leftarrow (a)$$

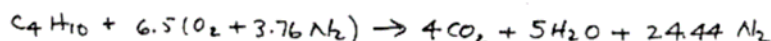
Problem 13.57

KNOWN: $C_4H_{10}(g)$ reacts completely with air. The reactants and products are at $25^\circ C, 1 \text{ atm}$.

FIND: Determine the enthalpy of combustion for (a) $H_2O(g)$ in the products, (b) $H_2O(l)$ in the products, express each in $\text{kJ per kmol of fuel}$ and kJ per kg of fuel .

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) Combustion is with the theoretical amount of air. (3) The combustion air and the combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of C_4H_{10} with the theoretical amount of air is described by



The enthalpy of combustion is

$$\bar{h}_{RP} = 4\bar{h}_{CO_2} + 5\bar{h}_{H_2O} - \bar{h}_{C_4H_{10}} - 6.5\bar{h}_{O_2}$$

Since the reactants and products are at $25^\circ C, 1 \text{ atm}$ this reduces simply to

$$\bar{h}_{RP}^\circ = 4(\bar{h}_f^\circ)_{CO_2} + 5(\bar{h}_f^\circ)_{H_2O} - (\bar{h}_f^\circ)_{C_4H_{10}}$$

(a) H_2O is a vapor. Using data from Table A-25

$$\bar{h}_{RP}^\circ = 4(-393,520) + 5(-241,820) - (-126,150) = -2.657 \times 10^6 \text{ kJ/kmol}(C_4H_{10})$$

For C_4H_{10} , $M = 58.12$, so

$$\bar{h}_{RP}^\circ = -\frac{2.657 \times 10^6}{58.12} = -45,716 \frac{\text{kJ}}{\text{kg}}$$

← (a)

(b) H_2O is a liquid. Using data from Table A-25

$$\bar{h}_{RP}^\circ = 4(-393,520) + 5(-285,830) - (-126,150) = -2.877 \times 10^6 \text{ kJ/kmol}(C_4H_{10})$$

and

$$\bar{h}_{RP}^\circ = -\frac{2.877 \times 10^6}{58.12} = -49,501 \frac{\text{kJ}}{\text{kg}}$$

← (b)

PROBLEM 13.58

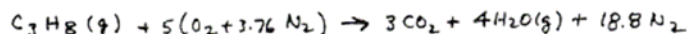
KNOWN: $C_3H_8(g)$ reacts completely with air. The reactants and products are each at 1 atm and temperature T . For $C_3H_8(g)$, $c_p = 0.41 \text{ Btu/lb} \cdot ^\circ\text{R}$.

FIND: Plot the enthalpy of combustion versus T for $77 \leq T \leq 500^\circ\text{F}$.

ENGINEERING

- MODEL:** (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) Combustion is with theoretical amount of air. (3) In the products, H_2O is a vapor. (4) The combustion air and combustion products can be modeled as ideal gases. (5) For $C_3H_8(g)$, c_p is constant.

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of air is described by



The enthalpy of combustion is

$$\begin{aligned} \bar{h}_{RP} &= 3\bar{h}_{CO_2} + 4\bar{h}_{H_2O(g)} - \bar{h}_{C_3H_8(g)} - 5\bar{h}_{O_2} \\ &= 3[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} + 4[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O(g)} \\ &\quad - [\bar{h}_f^\circ + \Delta\bar{h}]_{C_3H_8(g)} - 5[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{O_2} \\ &= \{3(\bar{h}_f^\circ)_{CO_2} + 4(\bar{h}_f^\circ)_{H_2O(g)} - (\bar{h}_f^\circ)_{C_3H_8(g)}\} + 3[\bar{h}(T) - \bar{h}(537)]_{CO_2} \\ &\quad + 4[\bar{h}(T) - \bar{h}(537)]_{H_2O(g)} - [\bar{c}_p(T - 537)]_{C_3H_8(g)} - 5[\bar{h}(T) - \bar{h}(537)]_{O_2} \end{aligned} \quad (1)$$

Sample Calculation: $T = 960^\circ\text{R}$ (500°F). With data from Table A-23E and A-25E

$$\begin{aligned} \bar{h}_{RP} &= \{3(-169,300) + 4(-104,040) - (-44680)\} + 3[\bar{h}(T) - 4028]_{CO_2} \\ &\quad + 4[\bar{h}(T) - 4258]_{H_2O(g)} - (0.41)(44.04)[T - 537] - 5[\bar{h}(T) - 3725]_{O_2} \\ &= -879,830 \frac{\text{kJ}}{\text{kmol(fuel)}} + 3[\bar{h}(T) - 4028]_{CO_2} + 4[\bar{h}(T) - 4258]_{H_2O(g)} \\ &\quad - \underbrace{(0.41)(44.04)[T - 537]}_{18.08} - 5[\bar{h}(T) - 3725]_{O_2} \\ &= -879,830 + 3[8244 - 4028] + 4[7738 - 4258] - 18.08[960 - 537] - 5[6786 - 3725] \\ &= -875,165 \text{ Btu/lbmol(fuel)} \end{aligned}$$

The data for the required plot are obtained using IT, as follows:

IT Code

$$T = 500 \text{ // } ^\circ\text{F}$$

$$h_{CO_2} = h_T("CO_2", T)$$

$$h_{H_2O} = h_T("H_2O", T)$$

$$h_{C_3H_8} = -44680 + (0.41) * (44.04) * (T - 77)$$

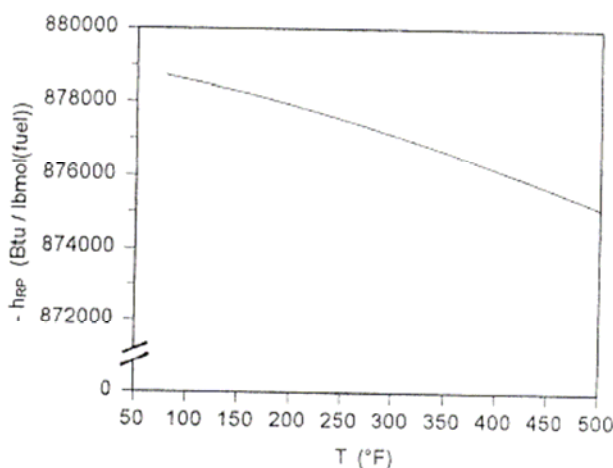
$$h_{O_2} = h_T("O_2", T)$$

$$-h_{RP} = 3 * h_{CO_2} + 4 * h_{H_2O} - h_{C_3H_8} - 5 * h_{O_2}$$

IT Results for $T = 500^\circ\text{F}$

$$-h_{RP} = 8.751 \times 10^5 \text{ Btu/lbmol(fuel)}$$

PLOT:



Note that h_{RP} varies only slightly with temperature in the range shown.

1. The results would differ only slightly if the variation of c_p with temperature were considered. IT includes a function for C_3H_8 that accounts for such variation.

PROBLEM 13.59

KNOWN: $\text{CH}_4(\text{g})$ reacts completely with air. The reactants and products are each at 1 atm and temperature T . For CH_4 ,

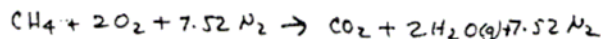
$$\bar{c}_p = 4.52 + 7.37(T/1000) \text{ Btu/lbmol} \cdot ^\circ\text{R}, \quad T \text{ in } ^\circ\text{R}.$$

FIND: Plot $\bar{h}_{\text{RP}}$ versus T for $537 \leq T \leq 1800^\circ\text{R}$.

ENGINEERING MODEL:

(1) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) Combustion is with the theoretical amount of air. N_2 is inert. (3) In the products, H_2O is a vapor. (4) The CH_4 , combustion air, and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of CH_4 with the theoretical amount of air is described by



The enthalpy of combustion is

$$\begin{aligned} \bar{h}_{\text{RP}} &= \bar{h}_{\text{CO}_2} + 2\bar{h}_{\text{H}_2\text{O}(\text{g})} - \bar{h}_{\text{CH}_4} - 2\bar{h}_{\text{O}_2} \\ &= [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{\text{CO}_2} + 2[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{\text{H}_2\text{O}(\text{g})} - [\bar{h}_f^\circ + \Delta\bar{h}]_{\text{CH}_4} - 2[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(298)]_{\text{O}_2} \\ &= \{\bar{h}_f^\circ\}_{\text{CO}_2} + 2\{\bar{h}_f^\circ\}_{\text{H}_2\text{O}(\text{g})} - \{\bar{h}_f^\circ\}_{\text{CH}_4} + [\bar{h}(T) - \bar{h}(537)]_{\text{CO}_2} + 2[\bar{h}(T) - \bar{h}(537)]_{\text{H}_2\text{O}(\text{g})} \\ &\quad - (\Delta\bar{h})_{\text{CH}_4} - 2[\bar{h}(T) - \bar{h}(298)]_{\text{O}_2} \quad (1) \end{aligned}$$

where

$$(\Delta\bar{h})_{\text{CH}_4} = \int_{537^\circ\text{R}}^T [4.52 + 7.37(\frac{T}{1000})] dT = 4.52[T - 537] + \frac{7.37}{2} \left[\frac{T^2 - (537)^2}{1000} \right] \quad (2)$$

Sample Calculation: $T = 900^\circ\text{R}$. With data for ideal gases from Table A-23E and $(\bar{h}_f^\circ)_{\text{CH}_4}$ from Table A-25E

$$\begin{aligned} \bar{h}_{\text{RP}} &= \{(-169,300) + 2(-104,040) - (-32,210)\} + [7598 - 4028] + 2[7231 - 4258] \\ &\quad - \{4.52[900 - 537] + \frac{7.37}{2} \left[\frac{900^2 - 537^2}{1000} \right]\} - 2[6339 - 3725] \\ &= -344,446 \text{ Btu/lbmol (fuel)} \end{aligned}$$

The data for the required plot are obtained using IT, as follows:

IT Code

$$T = 900 \text{ // } ^\circ\text{R}$$

$$h_{\text{CO}_2} = h_T(\text{"CO}_2", T)$$

$$h_{\text{H}_2\text{O}} = h_T(\text{"H}_2\text{O", T})$$

$$h_{\text{CH}_4} = -32210 + 4.52 * (T - 537) + (7.37 / 2000) * (T^2 - 537^2)$$

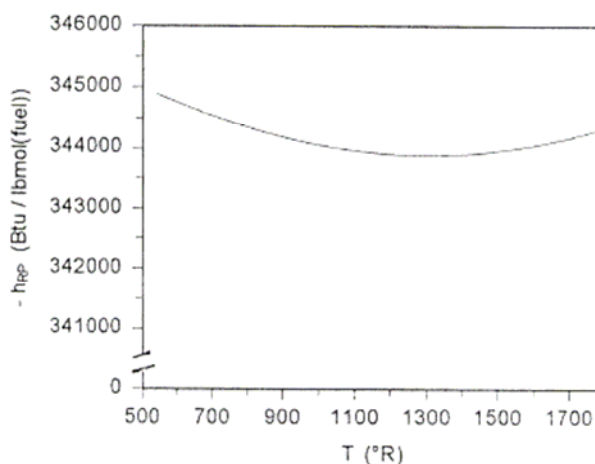
$$h_{\text{O}_2} = h_T(\text{"O}_2", T)$$

$$-h_{\text{RP}} = h_{\text{CO}_2} + 2 * h_{\text{H}_2\text{O}} - h_{\text{CH}_4} - 2 * h_{\text{O}_2}$$

IT result for $T = 900^\circ\text{R}$

$$-h_{\text{RP}} = 3.442 \times 10^5 \text{ Btu/lbmol(fuel)}$$

PLOT:



From the plot we see that h_{RP} varies only slightly with temperature over the range from 537 to 1800°R . Interestingly, the curve exhibits a minimum within this range.

PROBLEM 13.60

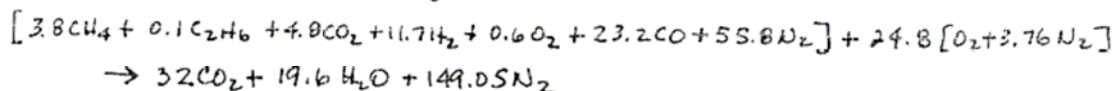
① KNOWN: The producer gas of Prob. 13.21 reacts completely with air. The reactants and products are at 77°F, 1 atm.

FIND: Determine the enthalpy of combustion for $\text{H}_2\text{O}(g)$ in the products.

ENGINEERING

MODEL: (1) 3.76 lbmol of N_2 accompany each lbmol of O_2 in the air, and the N_2 is inert. (2) Combustion is with the theoretical amount of air. (3) The combustion air and products can be modeled as ideal gases.

ANALYSIS: Complete combustion of the producer gas with the theoretical amount of air is described by (based on 100 lbmol of gas)



The enthalpy of combustion is

$$\bar{h}_{RP} = \sum_P n_e \bar{h}_e - \sum_R n_i \bar{h}_i$$

For combustion at $T = 77^\circ\text{F} = 537^\circ\text{R}$, $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}^\circ$. Also, for O_2 , H_2 , and N_2 , $\bar{h}_f^\circ = 0$. Thus

$$\bar{h}_{RP}^\circ = 32(\bar{h}_f^\circ)_{\text{CO}_2} + 19.6(\bar{h}_f^\circ)_{\text{H}_2\text{O}(g)} - 3.8(\bar{h}_f^\circ)_{\text{CH}_4} - 0.1(\bar{h}_f^\circ)_{\text{C}_2\text{H}_6} - 4.8(\bar{h}_f^\circ)_{\text{CO}_2} - 23.2(\bar{h}_f^\circ)_{\text{CO}}$$

With data from Table A-25E

$$\bar{h}_{RP}^\circ = 32(-169,300) + 19.6(-104,040) - 3.8(-32,210) - 0.1(-36,420) \\ - 4.8(-169,300) - 23.2(-47,540)$$

$$= -5415,176 \text{ Btu} / 100 \text{ lbmol fuel}$$

$$= -54,152 \text{ Btu} / \text{lbmol fuel} \quad \leftarrow \bar{h}_{RP}^\circ$$

PROBLEM 13.61

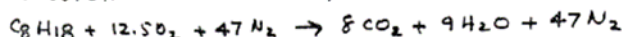
KNOWN: Specified fuels react completely with air. The reactants and products are each at 25°C, 1 atm.

FIND: Determine the higher heating value for (a) $C_8H_{18}(l)$, (b) $H_2(g)$, (c) $CH_3OH(l)$, (d) $C_4H_{10}(g)$, in kJ/kmol and kJ/kg. Compare with Table A-25 data.

ENGINEERING

MODEL: (1) Combustion is with the theoretical amount of air. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. (3) The combustion air and gaseous products of combustion can be modeled as ideal gases. The water formed on combustion is a liquid.

ANALYSIS: (a) $C_8H_{18}(l)$. The reaction equation for complete combustion with the theoretical amount of air is



The enthalpy of combustion at 25°C, 1 atm is

$$\begin{aligned} \bar{h}_{RP}^0 &= 8(\bar{h}_f^0)_{CO_2} + 9(\bar{h}_f^0)_{H_2O(l)} - (\bar{h}_f^0)_{C_8H_{18}} \\ &= 8(-393,520) + 9(-285,830) - (-249,910) = -5,470,720 \text{ kJ/kmol}(C_8H_{18}). \end{aligned}$$

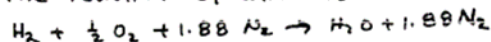
Accordingly

$$\overline{HHV} = 5,470,720 \text{ kJ/kmol}(C_8H_{18}) \quad \leftarrow (a)$$

$$HHV = \frac{5,470,720 \text{ kJ/kmol}(C_8H_{18})}{114.22 \text{ kg/kmol}(C_8H_{18})} = 47,896 \frac{\text{kJ}}{\text{kg}(C_8H_{18})}$$

This value agrees with the Table A-25 entry.

(b) $H_2(g)$. The reaction equation is



The enthalpy of combustion at 25°C, 1 atm is

$$\bar{h}_{RP}^0 = (\bar{h}_f^0)_{H_2O(l)} = -285,830 \text{ kJ/kmol}(H_2)$$

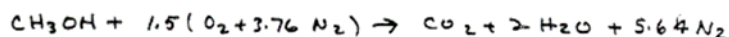
Accordingly

$$\overline{HHV} = 285,830 \text{ kJ/kmol}(H_2) \quad \leftarrow (b)$$

$$HHV = \frac{285,830}{2.016} = 141,781 \frac{\text{kJ}}{\text{kg}(H_2)}$$

This value agrees with the Table A-25 entry.

(c) $CH_3OH(l)$. The reaction equation is



The enthalpy of combustion at 25°C, 1 atm is

$$\begin{aligned} \bar{h}_{RP}^0 &= 2(\bar{h}_f^0)_{H_2O(l)} + (\bar{h}_f^0)_{CO_2} - (\bar{h}_f^0)_{CH_3OH(l)} \\ &= 2(-285,830) + (-393,520) - (-238,910) = -726,370 \text{ kJ/kmol}(CH_3OH) \end{aligned}$$

Accordingly

$$\overline{HHV} = 726,370 \text{ kJ/kmol}(CH_3OH) \quad \leftarrow (c)$$

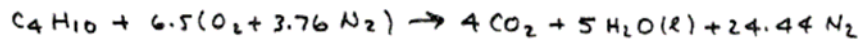
$$HHV = \frac{726,370}{32.05} = 22,664 \frac{\text{kJ}}{\text{kg}(CH_3OH)}$$

This value agrees with the Table A-25 entry.

Continued on next slide

Problem 13-61 continued

(d) C₄H₁₀(g): The reaction equation is



The enthalpy of combustion at 25°C, 1 atm is

$$\begin{aligned}\bar{h}_{\text{RP}}^0 &= 4(\bar{h}_f^0)_{\text{CO}_2} + 5(\bar{h}_f^0)_{\text{H}_2\text{O}(\ell)} - (\bar{h}_f^0)_{\text{C}_4\text{H}_{10}(\text{g})} \\ &= 4(-393,520) + 5(-285,830) - (-126,150) \\ &= -2,877,080 \text{ kJ/kmol (C}_4\text{H}_{10}\text{)}\end{aligned}$$

Accordingly

$$\overline{\text{HHV}} = 2,877,080 \text{ kJ/kmol (C}_4\text{H}_{10}\text{)} \quad \leftarrow \text{--- (d)}$$

$$\text{HHV} = \frac{2,877,080}{58.12} = 49,502 \frac{\text{kJ}}{\text{kg (C}_4\text{H}_{10}\text{)}} \quad \leftarrow$$

This value agrees with the Table A-25 entry.

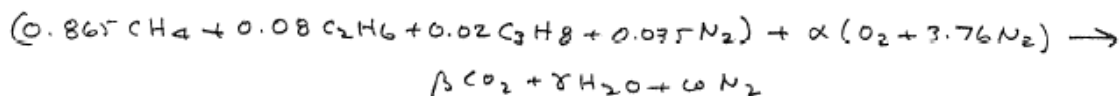
PROBLEM 13.62

KNOWN: A natural gas at 25°C, 1 atm has a molar analysis
 $\{86.5\% \text{CH}_4, 8\% \text{C}_2\text{H}_6, 2\% \text{C}_3\text{H}_8, 3.5\% \text{N}_2\}$

FIND: Determine $\overline{\text{LHV}}$ and LHV at 25°C, 1 atm.

ENGINEERING MODEL: (1) Combustion is with the theoretical amount of air. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) The fuel, combustion air, and combustion products can be modeled as ideal gas mixtures. (3) The water formed on combustion is a vapor.

ANALYSIS: On the basis of 1 kmol of natural gas



$$\text{C: } 0.865 + 2(0.08) + 3(0.02) = \beta, \quad \beta = 1.085.$$

$$\text{H: } 4(0.865) + 6(0.08) + 8(0.02) = 2\gamma, \quad \gamma = 2.05$$

$$\begin{aligned} \overline{\text{LHV}} &= 0.865(\bar{h}_f^\circ)_{\text{CH}_4} + 0.08(\bar{h}_f^\circ)_{\text{C}_2\text{H}_6} + 0.02(\bar{h}_f^\circ)_{\text{C}_3\text{H}_8} - 1.085(\bar{h}_f^\circ)_{\text{CO}_2} - 2.05(\bar{h}_f^\circ)_{\text{H}_2\text{O}(g)} \\ &= 0.865[-74,850] + 0.08[-84,680] + 0.02[-103,850] \\ &\quad - 1.085[-393,520] - 2.05[-241,820] \\ &= 849,104 \frac{\text{kJ}}{\text{kmol}} \end{aligned}$$

The molecular weight of the fuel mixture is

$$\begin{aligned} M &= 0.865(16.04) + 0.08(30.07) + 0.02(44.09) + 0.035(28.01) \\ &= 18.14 \end{aligned}$$

Then

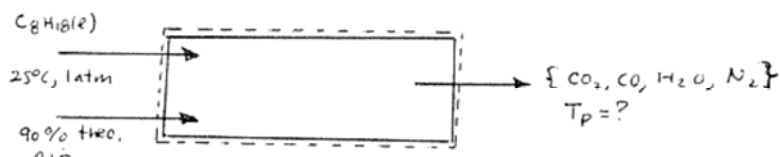
$$\text{LHV} = \frac{849,104}{18.14} = 46,808 \frac{\text{kJ}}{\text{kg}}$$

PROBLEM 13.63

KNOWN: Operating data are provided for an insulated reactor in which $C_8H_{18}(l)$ is burned with 90% of theoretical air.

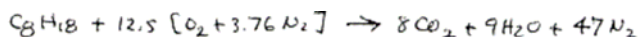
FIND: Determine the temperature of the exiting products.

SCHEMATIC & GIVEN DATA:

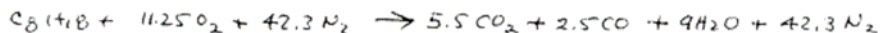


ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and kinetic/potential energy effects can be ignored. (3) The ideal gas model is applicable to the combustion air and the combustion products. (4) 3.76 moles of inert N_2 accompany each mole of O_2 in the combustion air. N_2 is inert.

ANALYSIS: The balanced reaction equation for complete combustion with the theoretical amount of air is



For combustion with 90% of theoretical air



An energy rate balance reduces to read $\bar{h}_p = \bar{h}_R$. With $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$, this takes the form

$$5.5 [\bar{h}_f^\circ + \Delta \bar{h}]_{CO_2} + 2.5 [\bar{h}_f^\circ + \Delta \bar{h}]_{CO} + 9 [\bar{h}_f^\circ + \Delta \bar{h}]_{H_2O} + 42.3 [\Delta \bar{h}]_{N_2} = [\bar{h}_f^\circ]_{C_8H_{18}(l)}$$

or

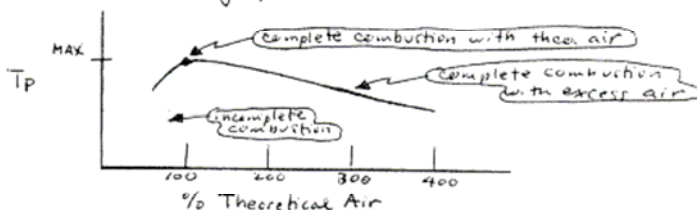
$$5.5 [\Delta \bar{h}]_{CO_2} + 2.5 [\Delta \bar{h}]_{CO} + 9 [\Delta \bar{h}]_{H_2O} + 42.3 [\Delta \bar{h}]_{N_2} = [\bar{h}_f^\circ]_{C_8H_{18}(l)} - 5.5 [\bar{h}_f^\circ]_{CO_2} - 2.5 [\bar{h}_f^\circ]_{CO} - 9 [\bar{h}_f^\circ]_{H_2O}$$

With data from the ideal gas tables

$$\begin{aligned} 5.5 [\bar{h}(T_p) - 9364]_{CO_2} + 2.5 [\bar{h}(T_p) - 8669]_{CO} + 9 [\bar{h}(T_p) - 9904]_{H_2O} + 42.3 [\bar{h}(T_p) - 8669]_{N_2} \\ = [-249,910] - 5.5 [-393,520] - 2.5 [-110,530] - 9 [-241,820] \\ = 4,367,155 \text{ kJ/kmol(fuel)} \end{aligned}$$

- ① The above expression is a single equation with just one unknown: T_p . Solving iteratively using table data, $T_p = 2286 \text{ K}$.

Discussion: In Example 13.8, for the case of complete combustion with the theoretical amount of air $T_p = 2395 \text{ K}$, and for complete combustion with 400% theoretical air $T_p = 962 \text{ K}$. Accordingly, in accordance with the discussion at the close of Sec. 13.3



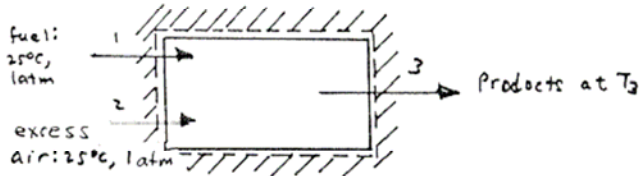
1. Iteration using table data can be avoided by using IT.

PROBLEM 13.64

KNOWN: Fuel at 25°C, 1 atm enters a reactor and burns completely with x percent excess air entering at 25°C, 1 atm.

FIND: Plot the adiabatic flame temperature versus x for (a) C, (b) H₂, and (c) C₈H₁₈(l).

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) For the control volume shown in the accompanying figure $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and the effects of kinetic and potential energy are negligible. (2) 3.76 moles of N₂ accompany each mole of O₂ in the air. N₂ is inert. (3) The ideal gas model is applicable to the combustion air and products.

ANALYSIS: To find the adiabatic flame temperature, use Eq. 13.21a

$$\sum_P n_e \bar{h}_e = \sum_R n_i \bar{h}_i \quad (1)$$

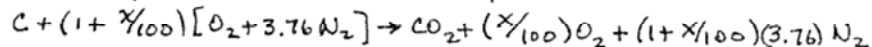
When using IT, the enthalpies are evaluated directly at the respective inlet and exit temperatures. When using table data, Eq. (1) takes the form

$$\sum_P n_e (\bar{h}_f^\circ + \Delta \bar{h})_e = \sum_R n_i (\bar{h}_f^\circ + \Delta \bar{h})_i$$

or

$$\sum_P n_e (\Delta \bar{h})_e = \sum_R n_i (\Delta \bar{h})_i + \sum_R n_i \bar{h}_{f,i}^\circ - \sum_P n_e \bar{h}_{f,e}^\circ \quad (2)$$

(a) **Fuel is C:** For complete combustion with x percent excess air



Applying Eq. (2) with $\Delta \bar{h}_i = 0$ and $\bar{h}_{f,i}^\circ = 0$ for O₂ and N₂ and C.

$$[\bar{h}_{CO_2}(T_3) - \bar{h}_{CO_2}(2981)] + (x/100)[\bar{h}_{O_2}(T_3) - \bar{h}_{O_2}(2981)] + (1 + x/100)(3.76)[\bar{h}_{N_2}(T_3) - \bar{h}_{N_2}(2981)] = 0$$

For x = 100%; we can solve iteratively using table data to get T₃ = 1506 K.

(b) **Fuel is H₂:** H₂ + 1/2(1 + x/100)[O₂ + 3.76 N₂] → H₂O + x/200 O₂ + 1/2(1 + x/100)(3.76) N₂

and

$$[\bar{h}(T_3) - \bar{h}(2981)]_{H_2O} + (x/200)[\bar{h}(T_3) - \bar{h}(2981)]_{O_2} + 1/2(1 + x/100)(3.76)[\bar{h}(T_3) - \bar{h}(2981)]_{N_2} = -(\bar{h}_f^\circ)_{H_2O(g)}$$

For x = 100%, we get T₃ = 1647 K

(c) **Fuel is C₈H₁₈(l):** C₈H₁₈ + 12.5(1 + x/100)[O₂ + 3.76 N₂] → 8CO₂ + 9H₂O + x/100 O₂ + 12.5(1 + x/100)(3.76) N₂

and

$$8[\bar{h}(T_3) - \bar{h}(2981)]_{CO_2} + 9[\bar{h}(T_3) - \bar{h}(2981)]_{H_2O} + x/100[\bar{h}(T_3) - \bar{h}(2981)]_{O_2} + 12.5(1 + x/100)(3.76)[\bar{h}(T_3) - \bar{h}(2981)]_{N_2} = (\bar{h}_f^\circ)_{C_8H_{18}(l)} - 8(\bar{h}_f^\circ)_{CO_2} - 9(\bar{h}_f^\circ)_{H_2O(g)}$$

Solving for x = 100%; T₃ = 1507 K.

Continued on next slide

Problem 13-64 continued

The data for the required plots are obtained using IT, as follows:

IT Code

$T = 25 + 273.15$ // K

$x = 1$

$PCT = x * 100$

Part (a)

$0 = -h_{CO2_3} - x * h_{O2_3} - (1+x) * 3.76 * h_{N2_3}$

$h_{CO2_3} = h_T("CO2", T_p)$

$h_{O2_3} = h_T("O2", T_p)$

$h_{N2_3} = h_T("N2", T_p)$

Result for $x = 100\%$: $T_p = 1507$ K

Part (b)

$0 = -h_{H2O_3} - (x/2) * h_{O2_3} - ((1+x)/2) * 3.76 * h_{N2_3}$

$h_{H2O_3} = h_T("H2O", T_p)$

$h_{O2_3} = h_T("O2", T_p)$

$h_{N2_3} = h_T("N2", T_p)$

Result for $x = 100\%$: $T_p = 1647$ K part (b)

Part (c)

$0 = h_{C8H18_1} - 8 * h_{CO2_3} - 9 * h_{H2O_3} - 12.5 * x * h_{O2_3} - 12.5 * (1+x) * 3.76 * h_{N2_3}$

$h_{C8H18_1} = -249910$ // Table A-25

$h_{CO2_3} = h_T("CO2", T_p)$

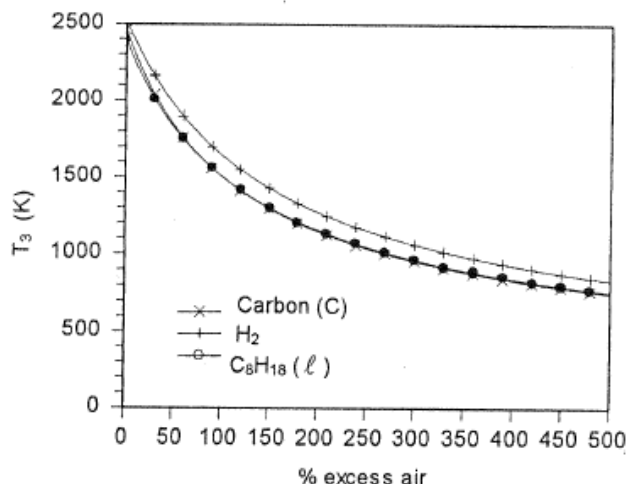
$h_{H2O_3} = h_T("H2O", T_p)$

$h_{O2_3} = h_T("O2", T_p)$

$h_{N2_3} = h_T("N2", T_p)$

Result for $x = 100\%$: $T_p = 1507$ K part(c)

Plots:



The excess air leads to more O_2 and N_2 in the products, thereby lowering the adiabatic flame temperature, as expected.

PROBLEM 13.65

KNOWN: $C_3H_8(g)$ at $25^\circ C, 1 \text{ atm}$ enters an insulated reactor and burns completely with air entering at $25^\circ C, 1 \text{ atm}$.

FIND: Plot the adiabatic flame temperature versus the percent of theoretical air varying from 100 to 400%.

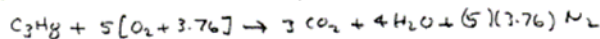
SCHEMATIC & GIVEN DATA:



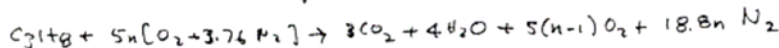
ENGINEERING

MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{W}_{cv} = 0$, $\dot{Q}_{cv}$ and kinetic/potential energy effects are negligible. (3) Combustion is complete, 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. The combustion air and combustion products can each be modeled as ideal gas mixtures.

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of air is described by



For n times the theoretical amount of air



An energy rate balance reduces with listed assumptions to read

$$0 = (\bar{h}_f^\circ)_{C_3H_8} - 3[\bar{h}_f^\circ + \Delta \bar{h}]_{CO_2} - 4[\bar{h}_f^\circ + \Delta \bar{h}]_{H_2O} - 5(n-1)[\Delta \bar{h}]_{O_2} - [\Delta \bar{h}]_{N_2} \quad (1)$$

or

$$0 = (\bar{h}_f^\circ)_{C_3H_8} - 3[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} - 4[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{H_2O} \\ - 5(n-1)[\bar{h}(T_3) - \bar{h}(298)]_{O_2} - 18.8n[\bar{h}(T_3) - \bar{h}(298)]_{N_2}$$

or

$$3\bar{h}_{CO_2}(T_3) + 4\bar{h}_{H_2O}(T_3) + 5(n-1)\bar{h}_{O_2}(T_3) + 18.8n\bar{h}_{N_2}(T_3) = (\bar{h}_f^\circ)_{C_3H_8} - 3[\bar{h}_f^\circ - \bar{h}(298)]_{CO_2} \\ - 4[\bar{h}_f^\circ - \bar{h}(298)]_{H_2O} + 5(n-1)\bar{h}(298)_{O_2} + 18.8n\bar{h}(298)_{N_2}$$

with ideal gas table data and $(\bar{h}_f^\circ)_{C_3H_8}$ from Table A-25

$$3\bar{h}_{CO_2}(T_3) + 4\bar{h}_{H_2O}(T_3) + 5(n-1)\bar{h}_{O_2}(T_3) + 18.8n\bar{h}_{N_2}(T_3) = -103,850 - 3[-393,520 - 9364] \\ - 4[-241,820 - 9904] - 5(n-1)(8682) - 18.8n(8669)$$

$$3\bar{h}_{CO_2}(T_3) + 4\bar{h}_{H_2O}(T_3) + 5(n-1)\bar{h}_{O_2}(T_3) + 18.8n\bar{h}_{N_2}(T_3) = 2,111,698 + 43,410(n-1) + 162,977n$$

Sample Calculation. Using Table A-23 data for the case $n=1.2$, we get $T_3 = 2125 \text{ K}$.

The data for the required plot are obtained using IT, as follows:

Continued on next slide

Problem 13-65 continued

IT Code

n = 120 // % theoretical air

$$0 = h_{fo_C3H8} - 3 * h_{CO2_3} - 4 * h_{H2O_3} - 5 * (n/100 - 1) * h_{O2_3} - 18.8 * n/100 * h_{N2_3}$$

hfo_C3H8 = -103850 // Table A-25

hCO2_3 = h_T("CO2", T3)

hH2O_3 = h_T("H2O", T3)

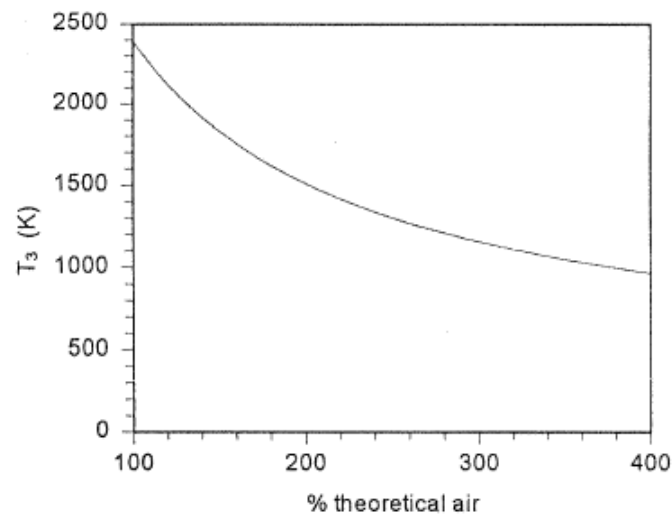
hO2_3 = h_T("O2", T3)

hN2_3 = h_T("N2", T3)

IT Result for n = 120%

T₃ = 2124 K

PLOT:



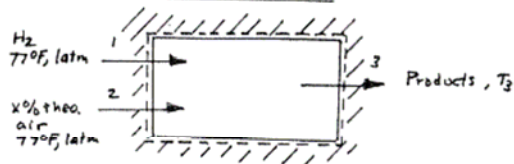
The excess air leads to more O₂ and N₂ in the products, thereby lowering the adiabatic flame temperature, as expected.

PROBLEM 13.66

KNOWN: H_2 at $77^\circ F$, 1 atm enters an insulated reactor and burns completely with $x\%$ of theoretical air entering at $77^\circ F$, 1 atm .

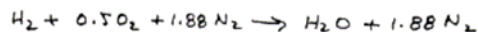
FIND: Plot the adiabatic flame temperature for x ranging from 100 to 400 %.

SCHEMATIC & GIVEN DATA:

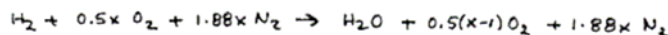


ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of H_2 with the theoretical amount of air is described by



Complete combustion with x percent of the theoretical amount of air is then



An energy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{M}_{H_2}} - \frac{\dot{W}_{cv}}{\dot{M}_{H_2}} + (\bar{h}_{H_2})_1 + (0.5x \bar{h}_{O_2} + 1.88x \bar{h}_{N_2})_2 - (\bar{h}_{H_2O} + 0.5(x-1) \bar{h}_{O_2} + 1.88x \bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for H_2 , O_2 , and N_2 , this becomes

$$0 = [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 0.5(x-1)[\bar{h}(T_3) - \bar{h}(537)]_{O_2} + 1.88x[\bar{h}(T_3) - \bar{h}(537)]_{N_2}$$

With data from the ideal gas tables

$$0 = [-104,040 + \bar{h}_{H_2O}(T_3) - 4258] + 0.5(x-1)[\bar{h}_{O_2}(T_3) - 3725.1] + 1.88x[\bar{h}_{N_2}(T_3) - 3729.5]$$

or

$$\bar{h}_{H_2O}(T_3) + 0.5(x-1)\bar{h}_{O_2}(T_3) + 1.88x\bar{h}_{N_2}(T_3) = 108298 + (x-1)(1862.55) + x(7011.46) \quad (1)$$

Sample Calculation: With data from Table A-23 for the case $x=2$, we get $T_3 \approx 2965^\circ R$

The data for the required plot are obtained using IT, as follows:

IT Code

$x = 200$ // % theoretical air

$$0 = h_{H_2O_3} + 0.5 * (x/100 - 1) * h_{O2_3} + 1.88 * x/100 * h_{N2_3}$$

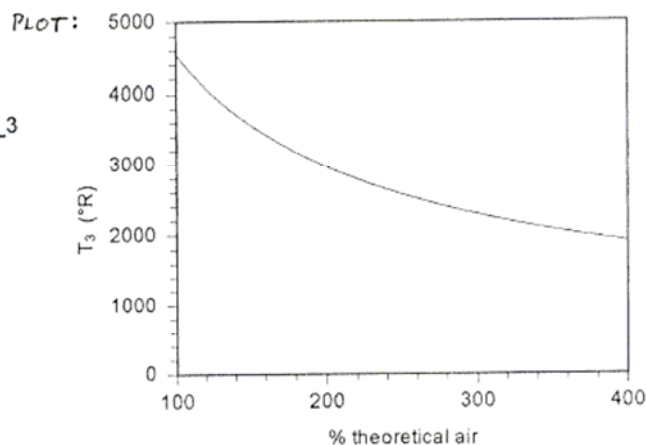
$$h_{H_2O_3} = h_T("H_2O", T_3)$$

$$h_{O2_3} = h_T("O_2", T_3)$$

$$h_{N2_3} = h_T("N_2", T_3)$$

IT Result for $x = 200\%$

$$T_3 = 2964^\circ R$$



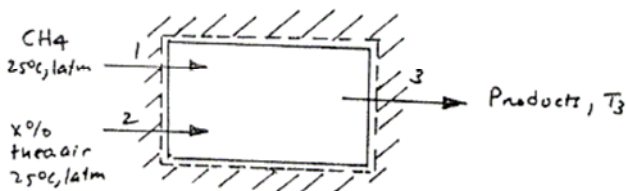
The excess air leads to more O_2 and N_2 in the products, thereby lowering the adiabatic flame temperature, as expected.

PROBLEM 13.67

KNOWN: CH₄ at 25°C, 1 atm enters an insulated reactor and burns completely with x% of theoretical air entering at 25°C, 1 atm.

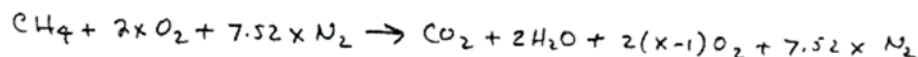
FIND: Plot the adiabatic flame temperature for x ranging from 100 to 400 %

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) 3.76 moles of N₂ accompany each mole of O₂ in the air. N₂ is inert. (3) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of CH₄ with the theoretical amount of air is given by Eq. 13.4. Complete combustion with x % of the theoretical amount of air is then



An energy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}^0}{\dot{n}_{\text{CH}_4}} - \frac{\dot{W}_{cv}^0}{\dot{n}_{\text{CH}_4}} + (\bar{h}_{\text{CH}_4})_1 + (2x\bar{h}_{\text{O}_2} + 7.52x\bar{h}_{\text{N}_2})_2 - (\bar{h}_{\text{CO}_2} + 2\bar{h}_{\text{H}_2\text{O}} + 2(x-1)\bar{h}_{\text{O}_2} + 7.52x\bar{h}_{\text{N}_2})_3$$

With $\bar{h} = \bar{h}_f^0 + \Delta\bar{h}$, and noting that $\bar{h}_f^0 = 0$ for O₂ and N₂, this becomes

$$0 = (\bar{h}_f^0)_{\text{CH}_4} + (0) - [(\bar{h}_f^0 + \bar{h}(T_3) - \bar{h}(298))\text{CO}_2 + 2(\bar{h}_f^0 + \bar{h}(T_3) - \bar{h}(298))\text{H}_2\text{O} + 2(x-1)(\bar{h}(T_3) - \bar{h}(298))\text{O}_2 + 7.52x(\bar{h}(T_3) - \bar{h}(298))\text{N}_2]$$

or

$$\bar{h}_{\text{CO}_2}(T_3) + 2\bar{h}_{\text{H}_2\text{O}}(T_3) + 2(x-1)\bar{h}_{\text{O}_2}(T_3) + 7.52x\bar{h}_{\text{N}_2}(T_3) = (\bar{h}_f^0)_{\text{CH}_4} - [\bar{h}_f^0 - \bar{h}(298)]\text{CO}_2 - 2[\bar{h}_f^0 - \bar{h}(298)]\text{H}_2\text{O} + 2(x-1)\bar{h}_{\text{O}_2}(298) + 7.52x\bar{h}_{\text{N}_2}(298)$$

With data from Tables A-23, A-25

$$\begin{aligned} \bar{h}_{\text{CO}_2}(T_3) + 2\bar{h}_{\text{H}_2\text{O}}(T_3) + 2(x-1)\bar{h}_{\text{O}_2}(T_3) + 7.52x\bar{h}_{\text{N}_2}(T_3) &= -74,850 - [-393,520 - 9,364] - \\ &\quad 2[-241,820 - 9,904] + \\ &\quad 2(x-1)[8682] + 7.52x[8669] \\ &= 831,482 + 17364(x-1) + 65191x \end{aligned}$$

Sample calculation: when $x = 2$, $T_3 \approx 1481 \text{ K}$

The data for the required plot are obtained using IT, as follows:

Continued on next slide

Problem 13-67 continued

IT Code

T1 = 25 + 273.15 // K

x = 200 // % theoretical air

0 = hCH4_1 - (hCO2_3 + 2 * hH2O_3 + 2 * (x/100 - 1) * hO2_3 + 7.52 * x/100 * hN2_3)

hCH4_1 = h_T("CH4", T1)

hCO2_3 = h_T("CO2", T3)

hH2O_3 = h_T("H2O", T3)

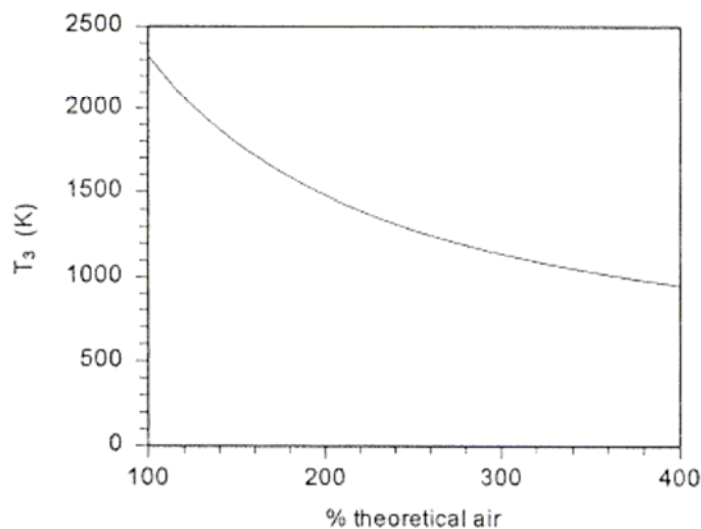
hO2_3 = h_T("O2", T3)

hN2_3 = h_T("N2", T3)

IT Results for x = 200%

T₃ = 1481 K

Plot:



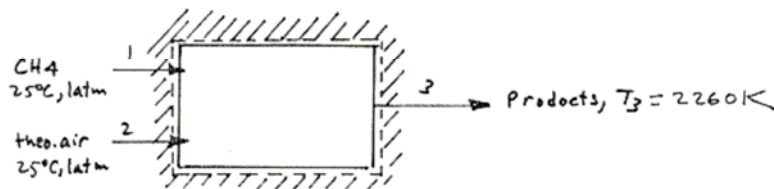
The excess air lead to more O₂ and N₂ in the products, thereby lowering the adiabatic flame temperature, as expected.

PROBLEM 13.68

KNOWN: CH_4 at 25°C , 1 atm enters an insulated reactor operating at steady state and burns with the theoretical amount of air entering at 25°C , 1 atm. The products, which contain CO_2 , CO , H_2O , O_2 , and N_2 , exit at 2260K .

FIND: Determine the fractions of the entering carbon that burn to CO_2 , CO .

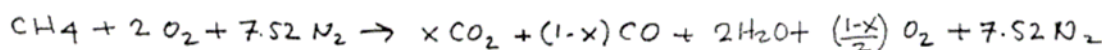
SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.

(2) Combustion is with the theoretical amount of air. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: The reaction equation takes the form



At steady state, an energy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}^0}{\dot{n}_{\text{CH}_4}} - \frac{\dot{W}_{cv}^0}{\dot{n}_{\text{CH}_4}} + (\bar{h}_{\text{CH}_4})_1 + [2\bar{h}_{\text{O}_2} + 7.52\bar{h}_{\text{N}_2}]_2 - [x\bar{h}_{\text{CO}_2} + (1-x)\bar{h}_{\text{CO}} + 2\bar{h}_{\text{H}_2\text{O}} + \left(\frac{1-x}{2}\right)\bar{h}_{\text{O}_2} + 7.52\bar{h}_{\text{N}_2}]_3$$

with $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{\text{CH}_4} + [0] - x[\bar{h}_f^\circ + \Delta\bar{h}]_{\text{CO}_2} - (1-x)[\bar{h}_f^\circ + \Delta\bar{h}]_{\text{CO}} - 2[\bar{h}_f^\circ + \Delta\bar{h}]_{\text{H}_2\text{O}} - \left(\frac{1-x}{2}\right)(\Delta\bar{h})_{\text{O}_2} - 7.52(\Delta\bar{h})_{\text{N}_2}$$

(with data from Tables A-23, 25)

$$0 = (-74,850) - x[-393,520 + (116,594 - 9364)] - (1-x)[-110,530 + (74,882 - 8669)] - 2[-241,820 + (96099 - 9904)] - \left(\frac{1-x}{2}\right)[77781 - 8682] - 7.52[74820 - 8669]$$

or

$$0 = (-74,850) - x[-286,290] - (1-x)[-44,317] - 2[-155,635] - \left(\frac{1-x}{2}\right)(69,099) - 492,944$$

$$0 = (-74,850) - (-44,317) - 2(155,635) - \frac{69,099}{2} - 492,944 - x(-286,290 + 44,317 - \frac{69,099}{2})$$

$$x = \frac{(-74,850) + 44,317 + 2(155,635) - 69,099/2 - 492,944}{(-286,290 + 44,317 - 69,099/2)}$$

$$= \frac{-246,757}{-276,523} = 0.892 \Rightarrow \begin{array}{l} 89.2\% \text{ of carbon to } \text{CO}_2 \\ 10.8\% \text{ of carbon to } \text{CO} \end{array}$$

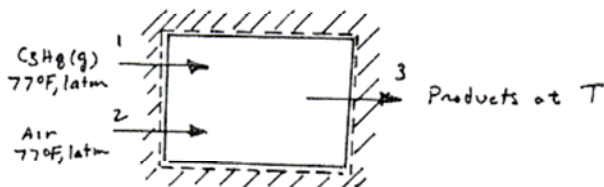
←

PROBLEM 13.69

KNOWN: $C_3H_8(g)$ at $77^\circ F$, 1 atm enters an insulated reactor and burns completely with air entering at $77^\circ F$, 1 atm .

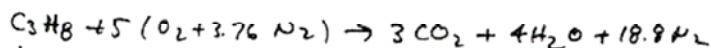
FIND: Determine the percent of theoretical air if products exit at (a) $1140^\circ F$, (b) $2240^\circ F$.

SCHEMATIC & GIVEN DATA:

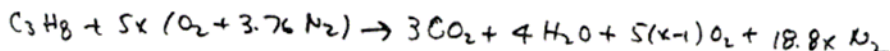


ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy, (2) Complete combustion occurs, (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert, (4) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of air is described by



Complete combustion with x times the theoretical amount of air is described by



An energy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{C_3H_8})_1 + (5x\bar{h}_{O_2} + 18.8x\bar{h}_{N_2})_2 - (3\bar{h}_{CO_2} + 4\bar{h}_{H_2O} + 5(x-1)\bar{h}_{O_2} + 18.8x\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{C_3H_8} + (0) - 3[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} - 4[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O} - 5(x-1)[\bar{h}(T) - \bar{h}(537)]_{O_2} - 18.8x[\bar{h}(T) - \bar{h}(537)]_{N_2}$$

Accordingly

$$x = \frac{(\bar{h}_f^\circ)_{C_3H_8} - 3[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} - 4[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O} + 5[\bar{h}(T) - \bar{h}(537)]_{O_2}}{5[\bar{h}(T) - \bar{h}(537)]_{O_2} + 18.8[\bar{h}(T) - \bar{h}(537)]_{N_2}}$$

(a) $T = 1600^\circ R (1140^\circ F)$. With data from the ideal gas tables and Table A-25

$$x = \frac{-44,680 - 3[-169,300 + 15829 - 4028] - 4[-104,040 + 13494 - 4258] + 5[11833 - 3725]}{5[11833 - 3725] + 18.8[11410 - 3730]} = 4.58 \quad (458\% \text{ of theoretical air})$$

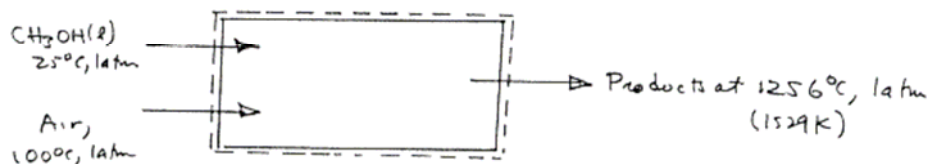
(b) $T = 2700^\circ R (2240^\circ F)$

$$x = \frac{-44,680 - 3[-169,300 + 30,581 - 4028] - 4[-104,040 + 24957 - 4258] + 5[21,183 - 3725]}{5[21,183 - 3725] + 18.8[20,246 - 3730]} = 2.02 \quad (202\% \text{ of theoretical air})$$

PROBLEM 13.70

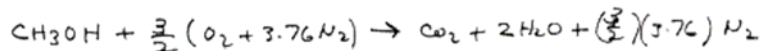
KNOWN: $\text{CH}_3\text{OH}(l)$ at 25°C , 1atm enters an insulated reactor operating at steady state and burns completely with air entering at 100°C , 1atm .

FIND: If combustion products exit at 1256°C , determine the percent excess air used.

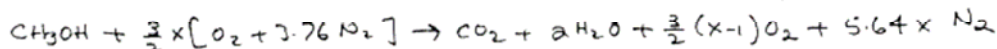


ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and kinetic/potential energy effects can be ignored. (3) Combustion is complete. 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (4) The combustion air and the combustion products can be modeled as ideal gas mixtures.

ANALYSIS: Complete combustion of CH_3OH with the theoretical amount of air is described by



Complete combustion with x times the theoretical amount of air is



With the given assumptions, an energy rate balance reduces to read

$$0 = (\bar{h}_f^\circ)_{\text{fuel}} + \frac{3}{2}x[\Delta\bar{h}]_{\text{O}_2} + 5.64x[\Delta\bar{h}]_{\text{N}_2} - [(\bar{h}_f^\circ + \Delta\bar{h})_{\text{CO}_2} + 2(\bar{h}_f^\circ + \Delta\bar{h})_{\text{H}_2\text{O}} + \frac{3}{2}(x-1)(\Delta\bar{h})_{\text{O}_2} + 5.64x(\Delta\bar{h})_{\text{N}_2}]$$

where $(\bar{h}_f^\circ)_{\text{O}_2} = (\bar{h}_f^\circ)_{\text{N}_2} = 0$ are not shown. That is

$$0 = (\bar{h}_f^\circ)_{\text{fuel}} + \frac{3}{2}x[\bar{h}(373) - \bar{h}(298)]_{\text{O}_2} + 5.64x[\bar{h}(373) - \bar{h}(298)]_{\text{N}_2} - [\bar{h}_f^\circ + \bar{h}(1529) - \bar{h}(298)]_{\text{CO}_2} - 2[\bar{h}_f^\circ + \bar{h}(1529) - \bar{h}(298)]_{\text{H}_2\text{O}} - \frac{3}{2}(x-1)[\bar{h}(1529) - \bar{h}(298)]_{\text{O}_2} - 5.64x[\bar{h}(1529) - \bar{h}(298)]_{\text{N}_2}$$

$$0 = (\bar{h}_f^\circ)_{\text{fuel}} - [\bar{h}_f^\circ + \bar{h}(1529) - \bar{h}(298)]_{\text{CO}_2} - 2[\bar{h}_f^\circ + \bar{h}(1529) - \bar{h}(298)]_{\text{H}_2\text{O}} - \frac{3}{2}x[\bar{h}(1529) - \bar{h}(298)]_{\text{O}_2} - 5.64x[\bar{h}(1529) - \bar{h}(373)]_{\text{N}_2} + \frac{3}{2}[\bar{h}(1529) - \bar{h}(298)]_{\text{O}_2}$$

Solving for x

$$x = \frac{(\bar{h}_f^\circ)_{\text{fuel}} - [\bar{h}_f^\circ + \bar{h}(1529) - \bar{h}(298)]_{\text{CO}_2} - 2[\bar{h}_f^\circ + \bar{h}(1529) - \bar{h}(298)]_{\text{H}_2\text{O}} + \frac{3}{2}[\bar{h}(1529) - \bar{h}(298)]_{\text{O}_2}}{5.64[\bar{h}(1529) - \bar{h}(373)]_{\text{N}_2} + \frac{3}{2}[\bar{h}(1529) - \bar{h}(373)]_{\text{O}_2}}$$

$$= \frac{-238,181 - [-393,520 + 72,773 - 93,647] - 2[-241,820 + 59,368 - 9,904] + \frac{3}{2}[50,353 - 8,682]}{5.64[48,106 - 10,851] + \frac{3}{2}[50,353 - 10,879]}$$

$$= 2.0 \quad (100\% \text{ excess air})$$

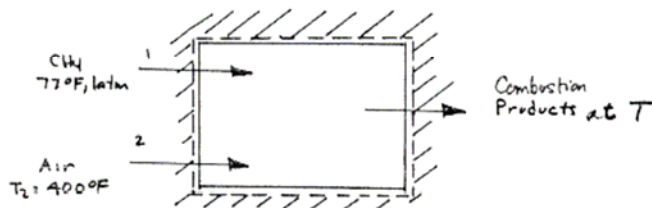
% excess
air

PROBLEM 13.71

KNOWN: Steady state operating data are provided for the combustor of a gas turbine power plant.

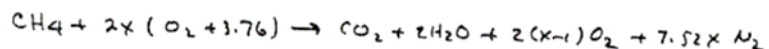
FIND: Plot the percent excess air versus the combustion product temperature for $1400 \leq T \leq 1800^\circ\text{F}$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Complete combustion with excess air occurs. (3) 7.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of CH_4 with excess air is described by



For combustion with the theoretical amount of air, $x=1$.

An energy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}_{CH_4}} - \frac{\dot{W}_{cv}}{\dot{m}_{CH_4}} + (\bar{h}_{CH_4})_1 + (2x\bar{h}_{O_2} + 7.52x\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2(x-1)\bar{h}_{O_2} + 7.52x\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2 , this becomes

$$\begin{aligned} 0 &= (\bar{h}_f^\circ)_{CH_4} + 2x[\bar{h}(860) - \bar{h}(537)]_{O_2} + 7.52x[\bar{h}(860) - \bar{h}(537)]_{N_2} - (\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537))_{CO_2} \\ &\quad - 2[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O} - 2(x-1)[\bar{h}(T) - \bar{h}(537)]_{O_2} - 7.52x[\bar{h}(T) - \bar{h}(537)]_{N_2} \\ &= (\bar{h}_f^\circ)_{CH_4} - (\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537))_{CO_2} - 2(\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537))_{H_2O} - 2x[\bar{h}(T) - \bar{h}(860)]_{O_2} \\ &\quad + 2[\bar{h}(T) - \bar{h}(537)]_{O_2} - 7.52x[\bar{h}(T) - \bar{h}(860)]_{N_2} \end{aligned}$$

Introducing data from the ideal gas tables and Table A-25, and solving for x

$$\begin{aligned} x &= \frac{(\bar{h}_f^\circ)_{CH_4} - [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O} + 2[\bar{h}(T) - \bar{h}(537)]_{O_2}}{2[\bar{h}(T) - \bar{h}(860)]_{O_2} + 7.52[\bar{h}(T) - \bar{h}(860)]_{N_2}} \\ &= \frac{(-32,210) - [-169,300 + \bar{h}(T) - 4028]_{CO_2} - 2[-104,040 + \bar{h}(T) - 4258] + 2[\bar{h}(T) - 3725]_{O_2}}{2[\bar{h}(T) - 6042]_{O_2} + 7.52[\bar{h}(T) - 5986]_{N_2}} \\ x &= \frac{-\bar{h}_{CO_2}(T) - 2\bar{h}_{H_2O}(T) + 2\bar{h}_{O_2}(T) + 350,264}{2\bar{h}_{O_2}(T) + 7.52\bar{h}_{N_2}(T) - 57099} \end{aligned}$$

Sample Calculation: $T = 1860^\circ\text{R}$ (1400°F)

$$x = \frac{-19173 - 2(16028) + 2(13987) + 350,264}{2(13987) + 7.52(13427) - 57099} = 4.55 \quad (355\% \text{ excess air})$$

Continued on next slide

Problem 13-71 continued

The data for the required plot are obtained using IT, as follows :

IT Code

T1 = 77 // °F

T2 = 400 // °F

T = 1400 // °F

0 = hCH4_1 + 2 * x * (hO2_2 + 3.76 * hN2_2) -
(hCO2_3 + 2 * hH2O_3 + 2 * (x - 1) * hO2_3 + 7.52 * x * hN2_3)

hCH4_1 = h_T("CH4", T1)

hO2_2 = h_T("O2", T2)

hN2_2 = h_T("N2", T2)

hCO2_3 = h_T("CO2", T)

hH2O_3 = h_T("H2O", T)

hO2_3 = h_T("O2", T)

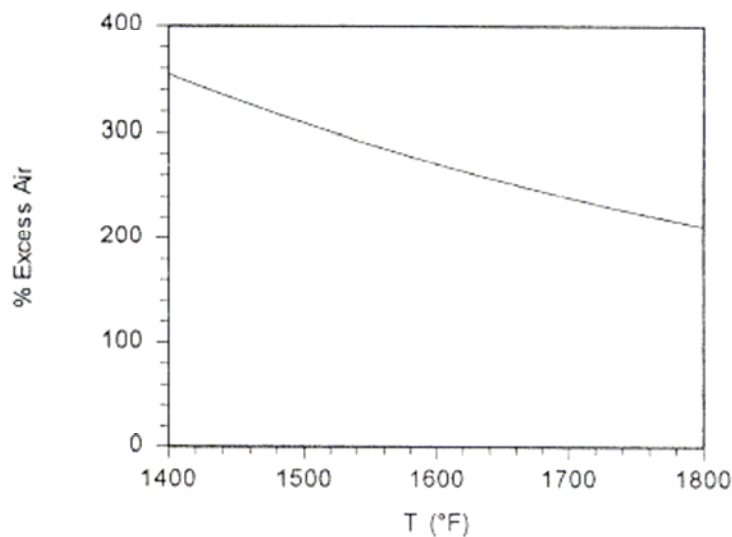
hN2_3 = h_T("N2", T)

PCTexcess = (x - 1) * 100

IT Results for T = 1400°F

% Excess = 354.7%

PLOT:



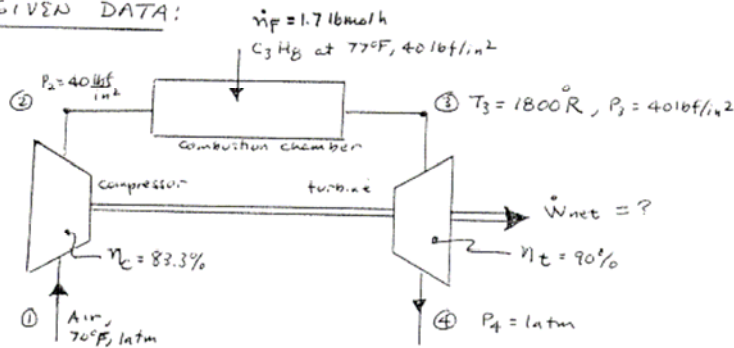
Notice that excess air leads to a decrease in the temperature of the Combustion products, as expected.

PROBLEM 13.72

KNOWN: Operating data are provided for a simple gas turbine operating at steady state. C_3H_8 burns completely in the combustion chamber.

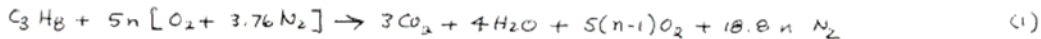
FIND: For a fuel consumption rate of 1.7 lbmol/h , determine (a) the percent of theoretical air, (b) the power developed, in hp.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Control volumes enclosing various components, as required, are at steady state. (2) The compressor, turbine, and combustion chamber are adiabatic. (3) The fuel, air, and combustion products are modeled as ideal gases. (4) Kinetic and potential energy effects are negligible. (5) 3.76 moles of inert N_2 accompany each mole of O_2 in the combustion air. (6) Combustion is complete.

ANALYSIS: (a) Letting n denote the multiple of the theoretical amount of air, the balanced reaction equation reads



An energy rate balance for the combustion chamber reads $\dot{h}_p = \dot{h}_R$. Solving this gives T_3 , but T_2 is required. To find T_2 , use the given compressor efficiency: $\eta_c = 0.833 = (h_{2s} - h_1) / (h_2 - h_1)$. To determine T_{2s} , consider an isentropic compression of the assumed air mixture from $T_1 = 530^\circ R$, $P_1 = 1 \text{ atm}$ to T_{2s} , 40 lbf/in^2 . Then,

$$0 = \bar{s}_2 - \bar{s}_1 = 1 \left[\bar{s}_{O_2}^\circ(T_{2s}) - \bar{s}_{O_2}^\circ(T_1) - \bar{R} \ln \frac{P_2}{P_1} \right] + 3.76 \left[\bar{s}_{N_2}^\circ(T_{2s}) - \bar{s}_{N_2}^\circ(T_1) - \bar{R} \ln \frac{P_2}{P_1} \right]$$

$$\Rightarrow \bar{s}_{O_2}^\circ(T_{2s}) + 3.76 \bar{s}_{N_2}^\circ(T_{2s}) = \bar{s}_{O_2}^\circ(T_1) + 3.76 \bar{s}_{N_2}^\circ(T_1) + 4.76 \bar{R} \ln \frac{P_2}{P_1}$$

$$= 48.889 + 3.76(45.61) + (4.76)(1.986) \ln \frac{40}{14.7}$$

$$= 230 \text{ Btu/lb} \cdot ^\circ R$$

① Solving iteratively, $T_{2s} = 705^\circ R$. Accordingly

$$\eta_c = 0.833 = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{1(\bar{h}_{O_2}(705) - \bar{h}_{O_2}(530)) + 3.76(\bar{h}_{N_2}(705) - \bar{h}_{N_2}(530))}{h_2 - h_1}$$

$$= \frac{1(4915 - 3676) + 3.76(4900 - 3681)}{h_2 - h_1} = 6990 \frac{\text{Btu}}{\text{lbmol}(O_2)} \quad (\text{note})$$

Then, with $h_2 - h_1 = 6990$, or

$$1[\bar{h}_{O_2}(T_2) - \bar{h}_{O_2}(T_1)] + 3.76[\bar{h}_{N_2}(T_2) - \bar{h}_{N_2}(T_1)] = 6990$$

$$\bar{h}_{O_2}(T_2) + 3.76 \bar{h}_{N_2}(T_2) = 6990 + 3676 + 3.76(7661) = 24507 \frac{\text{Btu}}{\text{lbmol}(O_2)}$$

② Solving for T_2 , $T_2 \approx 740^\circ R$.

Returning to the energy rate balance $\dot{h}_p = \dot{h}_R$, and using $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$

$$3[\bar{h}_f^\circ + \Delta \bar{h}]_{CO_2} + 4[\bar{h}_f^\circ + \Delta \bar{h}]_{H_2O} + 5(n-1)[\Delta \bar{h}]_{O_2} + 18.8n[\Delta \bar{h}]_{N_2} = [\bar{h}_f^\circ]_{C_3H_8} + 5n[\Delta \bar{h}]_{O_2} + 18.8n[\Delta \bar{h}]_{N_2}$$

Then with data from the ideal gas tables and Table A-2SE

Continued on next slide

Problem 13-72 continued

$$3[-169,300 + (18392 - 40203)] + 4[-104,040 + (15433 - 42583)] + 5(n-1)[13486 - 3721] + 18.8n[12956 - 3730] \\ = -44,680 + 5n[5167 - 3721] + 18.8n[5144 - 3730]$$

Solving, $n = 4.46 \Rightarrow 446\%$ of theoretical air.

(b) The net power developed is the difference between the turbine power output and the compressor power input. To find the turbine output

$$\eta_t = \frac{\dot{W}_t / \dot{n}_{\text{fuel}}}{(\dot{W}_t / \dot{n}_{\text{fuel}})_s} \Rightarrow \frac{\dot{W}_t}{\dot{n}_{\text{fuel}}} = \eta_t \left(\frac{\dot{W}_t}{\dot{n}_{\text{fuel}}} \right)_s \quad (2)$$

Accordingly, consider next an isentropic expansion of the combustion products from $(1800^\circ\text{R}, 40 \text{ lbf/in}^2)$ to $(T_{4s}, 1 \text{ atm})$: $\bar{s}_3 - \bar{s}_{4s} = 0$. That is

$$0 = 3 \left[\bar{s}^\circ(1800) - \bar{s}^\circ(T_{4s}) - \bar{R} \ln \frac{40}{14.7} \right]_{\text{CO}_2} + 4 \left[\bar{s}^\circ(1800) - \bar{s}^\circ(T_{4s}) - \bar{R} \ln \frac{40}{14.7} \right]_{\text{H}_2\text{O}} \\ + 17.3 \left[\bar{s}^\circ(1800) - \bar{s}^\circ(T_{4s}) - \bar{R} \ln \frac{40}{14.7} \right]_{\text{O}_2} + 83.848 \left[\bar{s}^\circ(1800) - \bar{s}^\circ(T_{4s}) - \bar{R} \ln \frac{40}{14.7} \right]_{\text{N}_2}$$

or, on rewriting this as

$$3 \bar{s}_{\text{CO}_2}^\circ(T_{4s}) + 4 \bar{s}_{\text{H}_2\text{O}}^\circ(T_{4s}) + 17.3 \bar{s}_{\text{O}_2}^\circ(T_{4s}) + 83.848 \bar{s}_{\text{N}_2}^\circ(T_{4s}) \\ = 3 \bar{s}_{\text{CO}_2}^\circ(1800) + 4 \bar{s}_{\text{H}_2\text{O}}^\circ(1800) + 17.3 \bar{s}_{\text{O}_2}^\circ(1800) + 83.848 \bar{s}_{\text{N}_2}^\circ(1800) - 108.148 \bar{R} \ln \frac{40}{14.7} \\ = 3(64,292) + 4(55,551) + 17.3(58,155) + 83.848(54,772) - 215 \\ = 5773.6 \frac{\text{Btu}}{(\text{lbmol})^\circ\text{R}}$$

② Solving $T_{4s} \approx 1400^\circ\text{R}$. The power that would be developed, per mole of fuel consumed, in an isentropic expansion is then

$$\left[\frac{\dot{W}_t}{\dot{n}_{\text{fuel}}} \right]_s = 3 [\bar{h}(1800) - \bar{h}(1400)]_{\text{CO}_2} + 4 [\bar{h}(1800) - \bar{h}(1400)]_{\text{H}_2\text{O}} + 17.3 [\bar{h}(1800) - \bar{h}(1400)]_{\text{O}_2} \\ + 83.848 [\bar{h}(1800) - \bar{h}(1400)]_{\text{N}_2} \\ = 3[18392 - 13345] + 4[15433 - 11625] + 17.3[13486 - 10210] + 83.848[12956 - 9897] \\ = 343,539 \frac{\text{Btu}}{(\text{lbmol})(\text{fuel})}$$

Eq. (2) gives

$$\left(\frac{\dot{W}_t}{\dot{n}_{\text{fuel}}} \right) = 0.9(343,539) = 309,185 \frac{\text{Btu}}{(\text{lbmol})(\text{fuel})}$$

For the compressor

$$\frac{\dot{W}_c}{\dot{n}_{\text{fuel}}} = (\bar{h}_2 - \bar{h}_1) \left(\frac{1 \text{ lbmol}(\text{O}_2)}{1 \text{ lbmol}(\text{fuel})} \right) = (6990) \left(\frac{5(4.46)}{1} \right) = 155,877 \frac{\text{Btu}}{(\text{lbmol})(\text{fuel})}$$

Finally, the net power output is

$$\dot{W}_{\text{net}} = \dot{n}_{\text{fuel}} \left[\frac{\dot{W}_t}{\dot{n}_{\text{fuel}}} - \frac{\dot{W}_c}{\dot{n}_{\text{fuel}}} \right] \\ = 1.7 \frac{\text{lbmol}(\text{fuel})}{\text{h}} [309,185 - 155,877] \left(\frac{\text{Btu}}{(\text{lbmol})(\text{fuel})} \right) \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 102.4 \text{ hp} \quad (b)$$

1. By ignoring the slight difference between the combustion air model and the air of Table A-22E, table data can be used to analyze the compressor. Thus, for example,

$$Pr_{2s} = (P_2/P_1) Pr_1 = (40/14.7)(1.3) = 3.537 \Rightarrow T_{2s} = 705^\circ\text{R}.$$

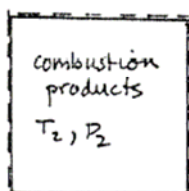
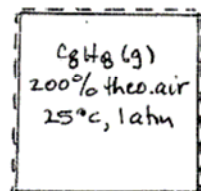
2. An iterative solution using table data can be avoided by using IT.

PROBLEM 13.73

KNOWN: A mixture of gaseous C_8H_{18} and 200% of theoretical air, initially at $25^\circ C$, 1 atm, reacts completely in a rigid vessel.

FIND: (a) If the vessel is well-insulated, determine the temperature and pressure of the combustion products. (b) If the combustion products are cooled to $25^\circ C$, determine the final pressure and the heat transfer per kmol of fuel.

SCHEMATIC & GIVEN DATA:

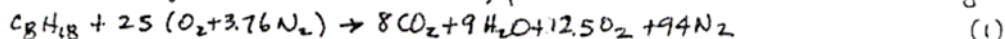


- (a) $Q=0$; $T_2=?$, $P_2=?$
 (b) $T_2=T_1$, $P_2=?$, $Q/n_F=?$

ENGINEERING

MODEL: (1) For the closed system shown, (a) $Q=0$, (b) $T_2=T_1$, and $W=0$. There are no effects of kinetic or potential energy. (2) Combustion is complete. (3) 3.76 kmol of N_2 accompany each kmol of O_2 , and the nitrogen is inert. (4) The initial fuel-air mixture and the gaseous products of combustion can be modeled as ideal gas mixtures.

ANALYSIS: (a) Assuming 1 kmol of fuel initially present, the reaction is described by



The energy balance reduces to $\Delta U + \Delta KE + \Delta PE = Q - W \Rightarrow \Delta U = 0$. Thus

$$0 = (8\bar{u}_{CO_2} + 9\bar{u}_{H_2O} + 12.5\bar{u}_{O_2} + 94\bar{u}_{N_2})_2 - (\bar{u}_{C_8H_{18}} + 25\bar{u}_{O_2} + 94\bar{u}_{N_2})_1$$

With $\bar{u} = \bar{h} - \bar{R}T$ this becomes

$$0 = (8\bar{h}_{CO_2} + 9\bar{h}_{H_2O} + 12.5\bar{h}_{O_2} + 94\bar{h}_{N_2})_2 - (\bar{h}_{C_8H_{18}} + 25\bar{h}_{O_2} + 94\bar{h}_{N_2})_1 - (8+9+12.5+94)\bar{R}T_2 + (1+25+94)\bar{R}T_1$$

Introducing $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = 8[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{CO_2} + 9[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{H_2O(g)} + 12.5[\bar{h}(T_2) - \bar{h}(298)]_{O_2} + 94[\bar{h}(T_2) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{C_8H_{18}(g)} - \bar{R}[123.5T_2 - 120T_1]$$

With data from Tables A-23 and A-25

$$8\bar{h}_{CO_2}(T_2) + 9\bar{h}_{H_2O}(T_2) + 12.5\bar{h}_{O_2}(T_2) + 94\bar{h}_{N_2}(T_2) - (123.5)(8.314)T_2 = (-8)(-393,520 - 9364) - 9(-241,820 - 9904) + 12.5(8682) + 94(8669) - 208,450 - (120)(8.314)(298) = 5,906,240$$

① Solving iteratively for T_2 , $T_2 \approx 1859$ K. (a) T_2

Using the ideal gas equation of state

$$\left. \begin{aligned} P_1 V &= n_1 \bar{R} T_1 \\ P_2 V &= n_2 \bar{R} T_2 \end{aligned} \right\} \frac{P_2}{P_1} = \frac{n_2}{n_1} \cdot \frac{T_2}{T_1} \Rightarrow P_2 = \frac{n_2}{n_1} \cdot \frac{T_2}{T_1} \cdot P_1 = \left(\frac{123.5}{120} \right) \left(\frac{1859}{298} \right) (1 \text{ atm}) = 6.42 \text{ atm} \quad \text{(a) } P_2$$

Continued on next slide

Problem 13-73 continued

(b) The reaction is described by Eq. (1). For 1 kmol of fuel, the products contain 9 kmol of H_2O and $n_{dry} = 114.5$ kmol. When the products cool to $25^\circ C$, condensation occurs. Thus, if n_v is the amount of water vapor and n_l the amount of saturated liquid water at the final state

$$n_v + n_l = 9 \text{ kmol}$$

Further, the partial pressure of the water vapor is

$$P_v = P_g(25^\circ C) = 0.03169 \text{ bar} = \left(\frac{n_v}{114.5 + n_v} \right) P_2 \quad (2)$$

Both P_2 and n_v are unknown. Another relation is obtained based on the assumption that the volume occupied by the liquid phase is negligible. Then

$$\frac{n_1 \bar{R} T}{P_1} = \frac{(n_{dry} + n_v) \bar{R} T}{P_2} \Rightarrow P_2 = \left(\frac{n_{dry} + n_v}{n_1} \right) P_1$$

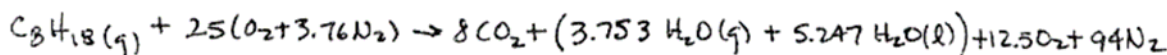
$$= \left(\frac{114.5 + n_v}{120} \right) (1.01325 \text{ bar}) \quad (3)$$

Solving (2) and (3) simultaneously

$$n_v = 3.753 \text{ kmol}, \quad P_2 = 0.9985 \text{ bar} \quad \leftarrow (b) P_2$$

$$n_l = 9 - n_v = 5.247 \text{ kmol}$$

Therefore



The energy balance reduces to $\Delta U = Q$. Thus

$$Q = (8 \bar{u}_{CO_2} + 3.753 \bar{u}_{H_2O(g)} + 5.247 \bar{u}_{H_2O(l)} + 12.5 \bar{u}_{O_2} + 94 \bar{u}_{N_2})_2 - (\bar{u}_{C_8H_{18}} + 25 \bar{u}_{O_2} + 94 \bar{u}_{N_2})_1$$

With $\bar{u} = \bar{h} - \bar{R}T$ for the gas components and $\bar{u} = \bar{h} - p\bar{v}^0$ for the liquid

$$Q = (8 \bar{h}_{CO_2} + 3.753 \bar{h}_{H_2O(g)} + 12.5 \bar{h}_{O_2} + 94 \bar{h}_{N_2})_2 - (\bar{h}_{C_8H_{18}} + 25 \bar{h}_{O_2} + 94 \bar{h}_{N_2})_1$$

$$+ 5.247 \bar{h}_{H_2O(l)} - (123.5 - 5.247 - 120)(8.314)(298)$$

Introducing $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$, noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2 , and setting the $\Delta \bar{h}$ terms to zero since $T_2 = T_1$,

$$Q = 8(\bar{h}_f^\circ)_{CO_2} + 3.753(\bar{h}_f^\circ)_{H_2O(g)} - (\bar{h}_f^\circ)_{C_8H_{18}(g)} + 5.247(\bar{h}_f^\circ)_{H_2O(l)} - (-1.747)(8.314)(298)$$

$$= 8(-393,520) + 3.753(-241,820) - (-208,450) + 5.247(-285,830) + 4328.3$$

$$= -5.343 \times 10^6 \text{ kJ (kmol fuel)} \quad \leftarrow (b) Q$$

1. Iteration using table data can be avoided by using (1).

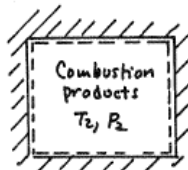
PROBLEM 13.74

KNOWN: A mixture of CH_4 and 200% of theoretical air, initially at 77°F , 1 atm, reacts completely in an insulated vessel. Two cases are considered: (a) reaction at constant volume, (b) reaction at constant pressure.

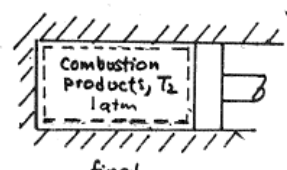
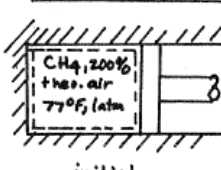
FIND: Determine the temperature of the combustion products.

SCHEMATIC & GIVEN DATA:

(a) Constant Volume



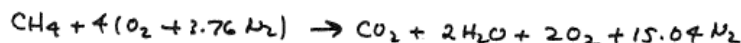
(b) Constant Pressure



ENGINEERING

MODEL: (1) For the systems shown in the accompanying figure $Q=0$ and there are no effects of kinetic or potential energy. (2) Pressure remains constant. (3) Combustion is complete. (4) 3.76 moles of N_2 accompany each mole of O_2 in the air. (5) The initial fuel-air mixture and the products of combustion can be modeled as ideal gases.

ANALYSIS: Basing the calculation on one mole of fuel initially present, the reaction is described by



(a) Volume is constant. Since volume is constant, $W=0$, and an energy balance reduces with assumption (1) to read $\Delta U=0$. Thus

$$0 = (\bar{u}_{\text{CO}_2} + 2\bar{u}_{\text{H}_2\text{O}} + 2\bar{u}_{\text{O}_2} + 15.04\bar{u}_{\text{N}_2})_2 - (\bar{u}_{\text{CH}_4} + 4\bar{u}_{\text{O}_2} + 15.04\bar{u}_{\text{N}_2})_1$$

With $\bar{u} = \bar{h} - RT$

$$0 = (\bar{h}_{\text{CO}_2} + 2\bar{h}_{\text{H}_2\text{O}} + 2\bar{h}_{\text{O}_2} + 15.04\bar{h}_{\text{N}_2})_2 - (\bar{h}_{\text{CH}_4} + 4\bar{h}_{\text{O}_2} + 15.04\bar{h}_{\text{N}_2})_1 - 20.04\bar{R}T_2 + 20.04\bar{R}T_1$$

Introducing $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{\text{CO}_2} + 2[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{\text{H}_2\text{O}} + 2[\bar{h}(T_2) - \bar{h}(537)]_{\text{O}_2} + 15.04[\bar{h}(T_2) - \bar{h}(537)]_{\text{N}_2} - (\bar{h}_f^\circ)_{\text{CH}_4} - 20.04\bar{R}T_1 + 20.04\bar{R}T_2$$

with data from the ideal gas tables and Table A-2SE

$$\begin{aligned} \bar{h}_{\text{CO}_2}(T_2) + 2\bar{h}_{\text{H}_2\text{O}}(T_2) + 2\bar{h}_{\text{O}_2}(T_2) + 15.04\bar{h}_{\text{N}_2}(T_2) - 20.04\bar{R}T_2 &= -[-169,300 - 4028] - \\ &= 2[-104,040 - 4258] + \\ &= 2(3725) + 15.04(3730) - 32,210 \\ &= (20.04)(1.986)(537) \\ &= 399,891 \end{aligned}$$

① Solving $T_2 \approx 3260^\circ\text{R}$.

(a)

Continued on next slide

Problem 13-74 continued

(b) Pressure is constant. Since pressure is constant, $W = \int p dV = p \Delta V$. So, with assumption (1) an energy balance reads $\Delta U = Q - P \Delta V$, or $\Delta H = 0$. That is

$$(\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2\bar{h}_{O_2} + 15.04\bar{h}_{N_2})_2 - (\bar{h}_{CH_4} + 4\bar{h}_{O_2} + 15.04\bar{h}_{N_2})_1 = 0$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{H_2O} + 2[\bar{h}(T_2) - \bar{h}(537)]_{O_2} + 15.04[\bar{h}(T_2) - \bar{h}(537)]_{N_2} - (\bar{h}_f^\circ)_{CH_4} = 0$$

With data from the ideal gas tables and Table A-25E

$$\begin{aligned} \bar{h}_{CO_2}(T_2) + 2\bar{h}_{H_2O}(T_2) + 2\bar{h}_{O_2}(T_2) + 15.04\bar{h}_{N_2}(T_2) &= -[\bar{h}_f^\circ - \bar{h}(537)]_{CO_2} - \\ &\quad 2[\bar{h}_f^\circ - \bar{h}(537)]_{H_2O} + \\ &\quad 2\bar{h}_{O_2}(537) + 15.04\bar{h}_{N_2}(537) + \\ &\quad (\bar{h}_f^\circ)_{CH_4} \\ &= -[-169,300 - 4028] - 2[-104,040 - 4258] \\ &\quad + 2(3725) + 15.04(3730) - 32,210 \\ &= 421,263 \end{aligned}$$

② ③ Solving, $T_2 \approx 2666^\circ R$ (b)

1. Note that with the ideal equation of state the final pressure can be obtained:

$$\left. \begin{aligned} P_1 V &= n_1 R T_1 \\ P_2 V &= n_2 R T_2 \end{aligned} \right\} \Rightarrow \frac{P_2}{P_1} = \frac{n_2}{n_1} \cdot \frac{T_2}{T_1} = \left(\frac{20.04}{20.04} \right) \left(\frac{2666}{537} \right) = 6.07$$

Thus, $P_2 = 6.07 \text{ atm}$.

2. Note that with the ideal gas equation of state, the volume ratio can be obtained:

$$\left. \begin{aligned} P V_1 &= n_1 R T_1 \\ P V_2 &= n_2 R T_2 \end{aligned} \right\} \Rightarrow \frac{V_2}{V_1} = \left(\frac{n_2}{n_1} \right) \left(\frac{T_2}{T_1} \right) = \left(\frac{20.04}{20.04} \right) \left(\frac{2666}{537} \right) = 4.96$$

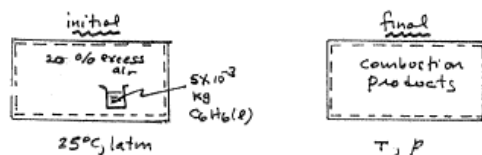
3. For both parts (a) and (b), iteration with table data can be avoided by using IT.

PROBLEM 13.75

KNOWN: 5×10^{-3} kg of liquid C_6H_6 and 20% excess air, initially at $25^\circ C$, later, reacts completely in a rigid insulated vessel.

FIND: Determine the final temperature and final pressure.

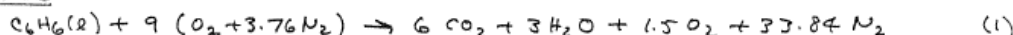
SCHEMATIC GIVEN DATA:



ENGINEERING

MODEL: (1) The system is shown in the figure above. (2) For the system, $Q=0$, $W=0$, and kinetic/potential energy effects are absent. (3) The reaction of the fuel and air is complete. 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (4) The volume occupied by the liquid fuel initially is much less than the volume of the gas phase. (5) The air and the combustion products can each be modeled as an ideal gas.

ANALYSIS: The reaction is described by



With assumption (1), the energy balance reduces to $\Delta U = 0$. Or

$$0 = (6\bar{u}_{CO_2} + 3\bar{u}_{H_2O} + 1.5\bar{u}_{O_2} + 33.84\bar{u}_{N_2}) - [\bar{u}_{C_6H_6} + 9\bar{u}_{O_2} + 33.84\bar{u}_{N_2}] \quad (2)$$

For the gases, the ideal gas model gives $\bar{u} = \bar{h} - \bar{R}T$. For the liquid fuel, $p \ll u$; so $\bar{h} \approx \bar{u}$. Thus Eq. (2) becomes

$$0 = 6[\bar{h} - \bar{R}T]_{CO_2} + 3[\bar{h} - \bar{R}T]_{H_2O} + 1.5[\bar{h} - \bar{R}T]_{O_2} + 33.84[\bar{h} - \bar{R}T]_{N_2} - \bar{h}_{C_6H_6} - 9[\bar{h} - \bar{R}T]_{O_2} - 33.84[\bar{h} - \bar{R}T]_{N_2}$$

or

$$0 = 6[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 3[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 1.5[\bar{h}_f^\circ + \Delta\bar{h}]_{O_2} + 33.84[\bar{h}_f^\circ + \Delta\bar{h}]_{N_2} - (\bar{h}_f^\circ)_{C_6H_6(l)} - 9[\bar{h}_f^\circ + \Delta\bar{h}]_{O_2} - 33.84[\bar{h}_f^\circ + \Delta\bar{h}]_{N_2} + 42.84\bar{R}T - 44.34\bar{R}T$$

With ideal gas table data and data from the literature for $(\bar{h}_f^\circ)_{C_6H_6(l)} = 49,100 \text{ kJ/kmol}$

$$0 = 6[-393,520 + \bar{h}_{CO_2}(T) - 9364] + 3[-241,820 + \bar{h}_{H_2O}(T) - 9904] + 1.5[\bar{h}_{O_2}(T) - 8682] + 33.84[\bar{h}_{N_2}(T) - 8669] - 49,100 + (42.84)(8.314)(298) - (44.34)(8.314)T$$

$$\Rightarrow 6\bar{h}_{CO_2}(T) + 3\bar{h}_{H_2O}(T) + 1.5\bar{h}_{O_2}(T) + 33.84\bar{h}_{N_2}(T) - 368.6T = 3,421,819 \text{ kJ/kmol (fuel)}$$

① Solving iteratively with table data $T \approx 2700 \text{ K}$ ($T = 2427^\circ C$)

Using the ideal gas equation of state for the gas phase only

$$pV = n_p RT$$

$$pV = n_R RT_i$$

↑ gas phase only
↑ ignore volume occupied by the liquid

$$\Rightarrow \frac{p}{R} = \left(\frac{n_p}{n_R}\right)\left(\frac{T}{T_i}\right) = \left(\frac{44.34}{42.84}\right)\left(\frac{2700}{298}\right) = 9.38$$

$$\Rightarrow p = 9.38 \text{ atm}$$

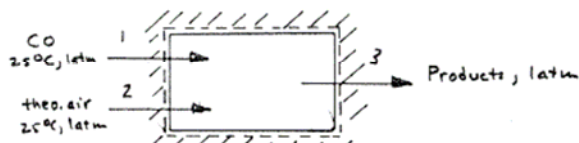
1. Iteration with table data can be avoided by using IT.

PROBLEM 13.76

KNOWN: CO at 25°C, 1 atm enters an insulated reactor and reacts completely with the theoretical amount of air entering in a separate stream at 25°C, 1 atm. The products exit at 1 atm.

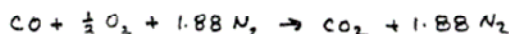
FIND: Determine the rate of entropy production per kmol of CO entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The reaction is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and products of combustion can be modeled as ideal gases.

ANALYSIS: The reaction is described by



It is necessary to determine the temperature of the combustion products. Thus, an energy rate balance at steady state reduces to give

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CO}} - \frac{\dot{W}_{cv}}{\dot{n}_{CO}} + (\bar{h}_{CO})_1 + \left(\frac{1}{2} \bar{h}_{O_2} + 1.88 \bar{h}_{N_2}\right)_2 \rightarrow (\bar{h}_{CO_2} + 1.88 \bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{CO} + (0) - (\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298))_{CO_2} - 1.88 [\bar{h}(T_3) - \bar{h}(298)]_{N_2}$$

or

$$\bar{h}_{CO_2}(T_3) + 1.88 \bar{h}_{N_2}(T_3) = (\bar{h}_f^\circ)_{CO} - (\bar{h}_f^\circ - \bar{h}(298))_{CO_2} + 1.88 \bar{h}_{N_2}(298)$$

With data from the ideal gas tables

$$\begin{aligned} \bar{h}_{CO_2}(T_3) + 1.88 \bar{h}_{N_2}(T_3) &= -110,530 - (-393,520 - 9364) + 1.88(8669) \\ &= 308,652 \end{aligned}$$

① Solving for T_3 , $T_3 \approx 2665$ K.

An entropy rate balance at steady state takes the form

$$0 = \sum \frac{\dot{Q}_j T_j}{\dot{n}_{CO}} + (\bar{s}_{CO})_1 + \left(\frac{1}{2} \bar{s}_{O_2} + 1.88 \bar{s}_{N_2}\right)_2 - (\bar{s}_{CO_2} + 1.88 \bar{s}_{N_2})_3 + \dot{S}_{cv}/\dot{n}_{CO}$$

or

$$\dot{S}_{cv}/\dot{n}_{CO} = (\bar{s}_{CO_2} + 1.88 \bar{s}_{N_2})_3 - (\bar{s}_{CO})_1 - \left(\frac{1}{2} \bar{s}_{O_2} + 1.88 \bar{s}_{N_2}\right)_2 \quad (1)$$

The CO enters separately at 298 K, 1 atm, so from Table A-24, $\bar{s}_{CO} = 197.54$ kJ/kmol·K. The O_2 and N_2 in the combustion air enters as a mixture at P_{ref} 1 atm. Thus, with Eq. 13.23 and absolute entropy data from Table A-25

$$\bar{s}_{O_2} = \bar{s}_{O_2}^\circ(298) - \bar{R} \ln \frac{y_{O_2} P_{ref}}{P_{ref}} = 205.03 - 8.314 \ln 0.21 = 218.01 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{N_2} = \bar{s}_{N_2}^\circ(298) - \bar{R} \ln \frac{y_{N_2} P_{ref}}{P_{ref}} = 191.5 - 8.314 \ln 0.79 = 193.46 \text{ kJ/kmol} \cdot \text{K}$$

The products exit as an ideal gas mixture at 1 atm, 2665 K with the following composition $y_{CO_2} = 1/2.88$, $y_{N_2} = 1.88/2.88$. Then, with absolute entropy data from Tables A-23 and 27

$$\bar{s}_{CO_2} = \bar{s}_{CO_2}^\circ(2665) - \bar{R} \ln \frac{y_{CO_2} P_{ref}}{P_{ref}} = 326.74 - 8.314 \ln \frac{1}{2.88} = 335.53 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{N_2} = \bar{s}_{N_2}^\circ(2665) - \bar{R} \ln \frac{y_{N_2} P_{ref}}{P_{ref}} = 262.42 - 8.314 \ln \frac{1.88}{2.88} = 265.97 \text{ kJ/kmol} \cdot \text{K}$$

Substituting values into Eq. (1)

$$\dot{S}_{cv}/\dot{n}_{CO} = 335.53 + 1.88(265.97) - 197.54 - \frac{1}{2}(218.01) - 1.88(193.46) = 165.3 \frac{\text{kJ}}{\text{kmol}(\text{CO}) \cdot \text{K}} \quad \leftarrow \frac{\dot{S}_{cv}}{\dot{n}_{CO}}$$

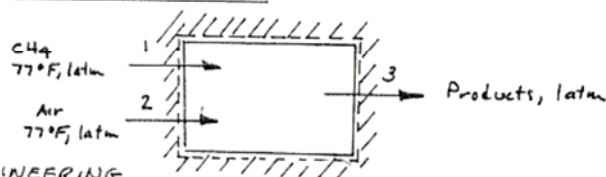
1. Iteration with table data can be avoided by using IT.

PROBLEM 13.77

KNOWN: CH_4 at 77°F , 1atm enters an insulated reactor and burns completely with air entering in a separate stream at 77°F , 1atm . The products exit at 1atm .

FIND: Determine the rate of entropy production, per 1bmol of CH_4 , for combustion with (a) the theoretical amount of air, (b) 200% of theoretical air.

SCHEMATIC & GIVEN DATA:

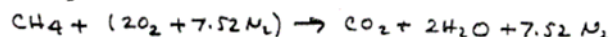


ENGINEERING

MODEL:

(1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The reaction is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and products of combustion can be modeled as ideal gases.

ANALYSIS: (a) Complete combustion of CH_4 with the theoretical amount of air is described by



To determine T_3 , write an energy rate balance at steady state:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{\text{CH}_4}} - \frac{\dot{W}_{cv}}{\dot{n}_{\text{CH}_4}} + (\bar{h}_{\text{CH}_4})_1 + (2\bar{h}_{\text{O}_2} + 7.52\bar{h}_{\text{N}_2})_2 - (\bar{h}_{\text{CO}_2} + 2\bar{h}_{\text{H}_2\text{O}} + 7.52\bar{h}_{\text{N}_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{\text{CH}_4} + (0) - [(\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537))_{\text{CO}_2} + 2(\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537))_{\text{H}_2\text{O}} + 7.52(\bar{h}(T_3) - \bar{h}(537))_{\text{N}_2}]$$

With data from the ideal gas tables and Table A-25E

$$\begin{aligned} \bar{h}_{\text{CO}_2}(T_3) + 2\bar{h}_{\text{H}_2\text{O}}(T_3) + 7.52\bar{h}_{\text{N}_2}(T_3) &= (\bar{h}_f^\circ)_{\text{CH}_4} - [\bar{h}_f^\circ - \bar{h}(537)]_{\text{CO}_2} - 2[\bar{h}_f^\circ - \bar{h}(537)]_{\text{H}_2\text{O}} + 7.52[\bar{h}(537)]_{\text{N}_2} \\ &= -33,210 - [-169,300 - 4028] - 2[-104,040 - 4255] + 7.52[3780] \\ &= 385,764 \end{aligned}$$

① Solving for T_3 , $T_3 \approx 4187^\circ\text{R}$.

An entropy rate balance at steady state takes the form

$$0 = \sum \frac{\dot{Q}_j T_j}{T_j} + (\dot{S}_{\text{CH}_4})_1 + (2\dot{S}_{\text{O}_2} + 7.52\dot{S}_{\text{N}_2})_2 - (\dot{S}_{\text{CO}_2} + 2\dot{S}_{\text{H}_2\text{O}} + 7.52\dot{S}_{\text{N}_2})_3 + \dot{Q}_{cv}/\dot{n}_{\text{CH}_4}$$

or

$$\dot{Q}_{cv}/\dot{n}_{\text{CH}_4} = (\dot{S}_{\text{CO}_2} + 2\dot{S}_{\text{H}_2\text{O}} + 7.52\dot{S}_{\text{N}_2})_3 - (\dot{S}_{\text{CH}_4})_1 - (2\dot{S}_{\text{O}_2} + 7.52\dot{S}_{\text{N}_2})_2 \quad (1)$$

The CH_4 enters separately at 537°R , 1atm , so from Table A-25E $\dot{S}_{\text{CH}_4} = 44.49 \text{ Btu/lbmol} \cdot ^\circ\text{R}$.

The O_2 and N_2 enters as a mixture at $P_{\text{ref}} = 1\text{atm}$. With Eq. 13.23 and $\dot{S}^\circ$ from Table A-23E

$$\dot{S}_{\text{O}_2} = \dot{S}_{\text{O}_2}^\circ(537) - \bar{R} \ln \frac{y_{\text{O}_2} P_{\text{ref}}}{P_{\text{ref}}} = 48.98 - 1.986 \ln 0.21 = 52.079 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

$$\dot{S}_{\text{N}_2} = \dot{S}_{\text{N}_2}^\circ(537) - \bar{R} \ln \frac{y_{\text{N}_2} P_{\text{ref}}}{P_{\text{ref}}} = 45.74 - 1.986 \ln 0.79 = 46.208 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

The products exit as an ideal gas mixture at 1atm , 4187°R with the following composition $y_{\text{CO}_2} = 1/10.52$, $y_{\text{H}_2\text{O}} = 2/10.52$, $y_{\text{N}_2} = 7.52/10.52$. Then, with $\dot{S}^\circ$ data from the ideal gas tables

$$\dot{S}_{\text{CO}_2} = \dot{S}_{\text{CO}_2}^\circ(4187) - \bar{R} \ln \frac{y_{\text{CO}_2} P_{\text{ref}}}{P_{\text{ref}}} = 76.073 - 1.986 \ln \frac{1}{10.52} = 80.747 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

$$\dot{S}_{\text{H}_2\text{O}} = \dot{S}_{\text{H}_2\text{O}}^\circ(4187) - \bar{R} \ln \frac{y_{\text{H}_2\text{O}} P_{\text{ref}}}{P_{\text{ref}}} = 65.101 - 1.986 \ln \frac{2}{10.52} = 68.402 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

$$\dot{S}_{\text{N}_2} = \dot{S}_{\text{N}_2}^\circ(4187) - \bar{R} \ln \frac{y_{\text{N}_2} P_{\text{ref}}}{P_{\text{ref}}} = 61.494 - 1.986 \ln \frac{7.52}{10.52} = 62.161 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

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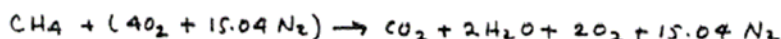
Problem 13-77 continued

Inserting values into Eq. (1)

$$\dot{Q}_{cv}/\dot{n}_{CH_4} = [80.747 + 2(68.402) + 7.52(62.461)] - 44.49 - [2(52.079) + 7.52(46.208)]$$

$$= 188.87 \text{ Btu/lbmol}(CH_4) \cdot ^\circ R \quad \leftarrow (a)$$

(b) Complete combustion of CH_4 with 200 percent of the theoretical amount of air is



To determine T_3 , write an energy rate balance at steady state:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} - \frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} + (\bar{h}_{CH_4})_1 + (4\bar{h}_{O_2} + 15.04\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2\bar{h}_{O_2} + 15.04\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$0 = (\bar{h}_f^\circ)_{CH_4} + (0) - [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 2[\bar{h}(T_3) - \bar{h}(537)]_{O_2} + 15.04[\bar{h}(T_3) - \bar{h}(537)]_{N_2}$$

With data from the ideal gas tables and Table A-25E

$$\bar{h}_{CO_2}(T_3) + 2\bar{h}_{H_2O}(T_3) + 2\bar{h}_{O_2}(T_3) + 15.04\bar{h}_{N_2}(T_3) = (\bar{h}_f^\circ)_{CH_4} - [\bar{h}_f^\circ - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^\circ - \bar{h}(537)]_{H_2O}$$

$$+ 2\bar{h}_{O_2}(537) + 15.04\bar{h}_{N_2}(537)$$

$$= -32,210 - [-169,300 - 4028] - 2[-104,040 - 4258]$$

$$+ 2(3725) + 15.04(3730)$$

$$= 421,264$$

① Solving for T_3 , $T_3 \approx 2666^\circ R$.

At steady state an entropy rate balance reduces to give

$$\dot{Q}_{cv}/\dot{n}_{CH_4} = (\bar{s}_{CO_2} + 2\bar{s}_{H_2O} + 2\bar{s}_{O_2} + 15.04\bar{s}_{N_2}) - \bar{s}_{CH_4} - (4\bar{s}_{O_2} + 15.04\bar{s}_{N_2}) \quad (2)$$

The values of $\bar{s}_{CH_4}$, $\bar{s}_{O_2}$ and $\bar{s}_{N_2}$ of the reactants are the same as in part (a). The products exit as an ideal gas mixture at 1 atm, 2666°R with the following composition

$y_{CO_2} = 1/20.04$, $y_{H_2O} = y_{O_2} = 2/20.04$, $y_{N_2} = 15.04/20.04$. Then with $\bar{s}^\circ$ from the ideal gas tables

$$\bar{s}_{CO_2} = \bar{s}_{CO_2}^\circ(2666) - \bar{R} \ln y_{CO_2} = 69.594 - 1.986 \ln \frac{1}{20.04} = 75.547 \text{ Btu/lbmol} \cdot ^\circ R$$

$$\bar{s}_{H_2O} = \bar{s}_{H_2O}^\circ(2666) - \bar{R} \ln y_{H_2O} = 59.694 - 1.986 \ln \frac{2}{20.04} = 64.271 \text{ Btu/lbmol} \cdot ^\circ R$$

$$\bar{s}_{O_2} = \bar{s}_{O_2}^\circ(2666) - \bar{R} \ln y_{O_2} = 61.506 - 1.986 \ln \frac{2}{20.04} = 66.083 \text{ Btu/lbmol} \cdot ^\circ R$$

$$\bar{s}_{N_2} = \bar{s}_{N_2}^\circ(2666) - \bar{R} \ln y_{N_2} = 57.644 - 1.986 \ln \frac{15.04}{20.04} = 58.214 \text{ Btu/lbmol} \cdot ^\circ R$$

Inserting values into Eq. (2)

$$\dot{Q}_{cv}/\dot{n}_{CH_4} = 75.547 + 2(64.271) + 2(66.083) + 15.04(58.214) - 44.49 - 4(52.079) - 15.04(46.208)$$

$$= 263.97 \text{ Btu/lbmol} \cdot ^\circ R \quad \leftarrow (b)$$

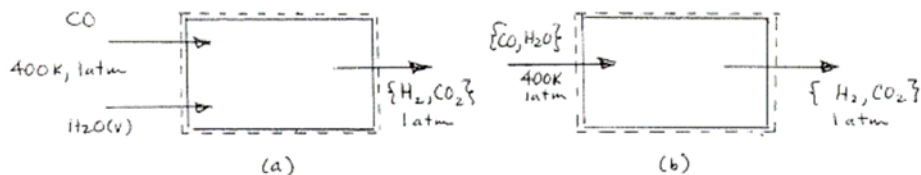
1. Iteration with table data can be avoided by using IT.

PROBLEM 13.78

KNOWN: Two cases are under consideration involving an insulated reactor at steady state: (a) carbon monoxide and water vapor enter in separate streams and form H_2 , CO_2 , (b) CO and $H_2O(v)$ enter as a mixture and form H_2 , CO_2 .

FIND: For each case determine the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volumes shown above are at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic/potential energy effects. (2) The ideal gas model is applicable to incoming and exiting streams.

ANALYSIS: To evaluate the entropy production rates, the temperature of the exiting mixture is required. To determine this, write an energy balance for the reaction



That is

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CO}} - \frac{\dot{W}_{cv}}{\dot{n}_{CO}} + [\bar{h}_{CO} + \bar{h}_{H_2O}]_R - [\bar{h}_{CO_2} + \bar{h}_{H_2}]_P \quad (1)$$

① Eq. 1 is solved using IT, as follows:

$$T_{in} = 400 \text{ // K}$$

$$h_{CO_in} = h_T("CO", T_{in})$$

$$h_{H_2O_in} = h_T("H_2O", T_{in})$$

$$h_{H_2_out} = h_T("H_2", T_p)$$

$$h_{CO_2_out} = h_T("CO_2", T_p)$$

$$IT \text{ result: } T_p = 922.5 \text{ K} \leftarrow T_p$$

$$0 = h_{CO_in} + h_{H_2O_in} - h_{H_2_out} - h_{CO_2_out}$$

Now, an entropy balance reduces to

$$0 = \frac{1}{\dot{n}_{CO}} \sum_j \dot{Q}_j \left(\frac{1}{T_j} \right) + \bar{s}_{CO} + \bar{s}_{H_2O(v)} - \bar{s}_{CO_2} - \bar{s}_{H_2} + \dot{\sigma}_{cv} / \dot{n}_{CO}$$

$$\text{or } \dot{\sigma}_{cv} / \dot{n}_{CO} = \bar{s}_{CO_2} + \bar{s}_{H_2} - \bar{s}_{CO} - \bar{s}_{H_2O(v)} \quad (2)$$

In terms of temperatures and pressures

$$\dot{\sigma}_{cv} / \dot{n}_{CO} = \bar{s}_{CO_2}(T_p, P_{CO_2-out}) + \bar{s}_{H_2}(T_p, P_{H_2-out}) - \bar{s}_{CO}(T_{in}, P_{CO-in}) - \bar{s}_{H_2O(v)}(T_{in}, P_{H_2O(v)-in})$$

With IT:

$$\begin{aligned} s_{CO_in} &= s_TP("CO", T_{in}, p_{CO_in}) \\ s_{H_2O_in} &= s_TP("H_2O", T_{in}, p_{H_2O_in}) \\ s_{CO_2_out} &= s_TP("CO_2", T_p, p_{CO_2_out}) \\ s_{H_2_out} &= s_TP("H_2", T_p, p_{H_2_out}) \end{aligned}$$

$$\text{sigmadot} = s_{CO_2_out} + s_{H_2_out} - s_{CO_in} - s_{H_2O_in}$$

Continued on next slide

Problem 13-78 continued

- (a) The CO and H₂O(v) enter separately, each at 400K, 1atm = 1.01325 bar. The products exit as an ideal gas mixture at 922.5K, 1.01325 bar, with the molar analysis $y_{H_2} = y_{CO_2} = 0.5$. Thus, with IT:

$$\begin{aligned}p_{CO_in} &= 1.01325 \\p_{H_2O_in} &= 1.01325 \\p_{CO_2_out} &= 0.5 * 1.01325 \\p_{H_2_out} &= 0.5 * 1.01325\end{aligned}$$

and the result is: $\dot{\sigma}_{cv} / \dot{n}_{cv} = 35.29 \text{ kJ/kmol(CO)·K}$ ← (a)

- (b) In this case, the CO and H₂O(v) enter as a mixture, with $y_{CO} = y_{H_2O} = 0.5$. Thus, with IT:

$$\begin{aligned}p_{CO_in} &= 0.5 * 1.01325 \\p_{H_2O_in} &= 0.5 * 1.01325\end{aligned}$$

and the result is: $\dot{\sigma}_{cv} / \dot{n}_{cv} = 23.77 \text{ kJ/kmol(CO)·K}$ ← (b)

Discussion: The entropy production rate in case (b) is less than for part (a) because the mixture has already been formed prior to entering the reactor. In case (a), the mixture forms within the reactor and so there is an accompanying entropy production with this type of irreversibility.

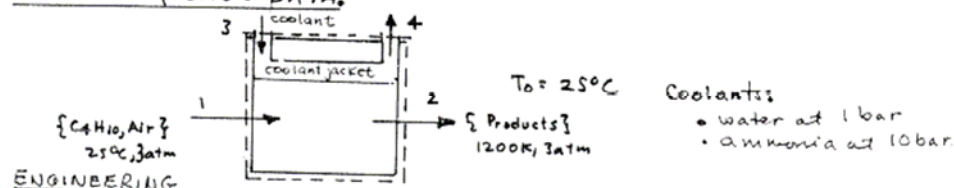
-
1. Only an IT solution is presented here, as Tables A-23 do not include data for H₂. Data for H₂ can be found in the same source as Table A-23.

PROBLEM 13.79

KNOWN: A gaseous mixture of C_4H_{10} and 80% excess air enters a coolant-jacketed reactor at $25^\circ C, 3 \text{ atm}$. Complete combustion occurs and products exit at $1200 \text{ K}, 3 \text{ atm}$. Saturated liquid coolant enters the jacket and saturated vapor exits.

FIND: Determine (a) the mass flow rate of the coolant, in kg per kmol of fuel, (b) the rate of entropy production, (c) and the rate of exergy destruction. Consider two coolants: water at 1 bar and ammonia at 10 bar.

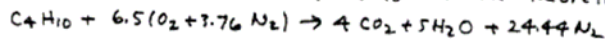
SCHEMATIC & GIVEN DATA:



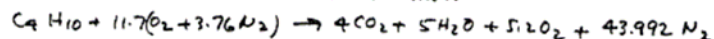
ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The incoming fuel-air mixture and exiting combustion products can be modeled as ideal gases.

ANALYSIS: The complete combustion of C_4H_{10} with the theoretical amount of air is



Complete combustion with 80% excess air is then



(a) An energy rate balance at steady reduces to the form

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_{cw} [h_3 - h_4] + \dot{n}_{C_4H_{10}} [(\bar{h}_{C_4H_{10}} + 11.7 \bar{h}_{O_2} + 43.992 \bar{h}_{N_2})_1 - (4 \bar{h}_{CO_2} + 5 \bar{h}_{H_2O} + 5.2 \bar{h}_{O_2} + 43.992 \bar{h}_{N_2})_2]$$

where $\dot{m}_{cw}$ is the mass flow rate of the coolant and $\dot{n}_{C_4H_{10}}$ is the molar flow rate of the C_4H_{10} . Accordingly

$$\frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = \frac{(\bar{h}_{C_4H_{10}} + 11.7 \bar{h}_{O_2} + 43.992 \bar{h}_{N_2})_1 - (4 \bar{h}_{CO_2} + 5 \bar{h}_{H_2O} + 5.2 \bar{h}_{O_2} + 43.992 \bar{h}_{N_2})_2}{h_4 - h_3}$$

With $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\begin{aligned} \frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} &= \frac{(\bar{h}_f^\circ)_{C_4H_{10}} \{4[\bar{h}_f^\circ + \bar{h}(1200) - \bar{h}(298)]_{CO_2} + 5[\bar{h}_f^\circ + \bar{h}(1200) - \bar{h}(298)]_{H_2O} + 5.2[\bar{h}(1200) - \bar{h}(298)]_{O_2} + 43.992[\bar{h}(1200) - \bar{h}(298)]_{N_2}\}}{h_g - h_f} \\ &= \frac{-126,150 - 4[-393,520 + 53,848 - 9364] - 5[-241,820 + 44,380 - 9904] - 5.2[38447 - 8682] - 43.992[76,777 - 8669]}{h_g - h_f} \\ &= \frac{915,409 \text{ kJ/kmol (fuel)}}{h_g - h_f} \end{aligned} \quad (1)$$

• For water at 1 bar, $h_{fg} = 2258 \frac{\text{kJ}}{\text{kg}} \Rightarrow \frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = \frac{915,409}{2258} = 405.4 \frac{\text{kg}}{\text{kmol (fuel)}}$

• For ammonia at 10 bar, $h_{fg} = 1165.4 \frac{\text{kJ}}{\text{kg}} \Rightarrow \frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = 785.5 \frac{\text{kg}}{\text{kmol (fuel)}}$

(a)

Continued on next slide

Problem 13-79 continued

(b) An entropy rate balance at steady state reduces to give

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_4H_{10}}} = (4 \bar{s}_{CO_2} + 5 \bar{s}_{H_2O} + 5.2 \bar{s}_{O_2} + 43.992 \bar{s}_{N_2}) - (\bar{s}_{C_4H_{10}} + 11.7 \bar{s}_{O_2} + 43.992 \bar{s}_{N_2}) + \frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} (s_4 - s_3) \quad (1)$$

The fuel and air enter as a mixture at 25°C, 3 atm with the composition, $y_{C_4H_{10}} = 1/56.692$, $y_{O_2} = 11.7/56.692$, $y_{N_2} = 43.992/56.692$. Accordingly

$$\bar{s}_{C_4H_{10}} = \bar{s}_{C_4H_{10}}^0(298) - \bar{R} \ln \frac{y_{C_4H_{10}} P}{P_{ref}} = 310.03 - 8.314 \ln \frac{3}{56.692} = 334.46 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{O_2} = \bar{s}_{O_2}^0(298) - \bar{R} \ln \frac{y_{O_2} P}{P_{ref}} = 205.03 - 8.314 \ln \frac{(11.7)(3)}{56.692} = 209.02 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{N_2} = \bar{s}_{N_2}^0(298) - \bar{R} \ln \frac{y_{N_2} P}{P_{ref}} = 191.5 - 8.314 \ln \frac{(43.992)(3)}{56.692} = 184.47 \text{ kJ/kmol} \cdot \text{K}$$

The products exit as a mixture at 1200 K, 3 atm with the composition, $y_{CO_2} = 4/58.192$, $y_{H_2O} = 5/58.192$, $y_{O_2} = 5.2/58.192$, $y_{N_2} = 43.992/58.192$. Accordingly

$$\bar{s}_{CO_2} = \bar{s}_{CO_2}^0(1200) - \bar{R} \ln \frac{y_{CO_2} P}{P_{ref}} = 279.207 - 8.314 \ln \frac{(4)(3)}{58.192} = 292.43 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{H_2O} = \bar{s}_{H_2O}^0(1200) - \bar{R} \ln \frac{y_{H_2O} P}{P_{ref}} = 240.333 - 8.314 \ln \frac{(5)(3)}{58.192} = 251.60 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{O_2} = \bar{s}_{O_2}^0(1200) - \bar{R} \ln \frac{y_{O_2} P}{P_{ref}} = 249.906 - 8.314 \ln \frac{(5.2)(3)}{58.192} = 260.85 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{N_2} = \bar{s}_{N_2}^0(1200) - \bar{R} \ln \frac{y_{N_2} P}{P_{ref}} = 234.115 - 8.314 \ln \frac{(43.992)(3)}{58.192} = 227.31 \text{ kJ/kmol} \cdot \text{K}$$

From Table A-3, $s_4 - s_3 = (7.3559 - 1.306) = 6.0499 \text{ kJ/kg} \cdot \text{K}$.

Substituting values into Eq. (1)

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_4H_{10}}} = 4(292.43) + 5(251.60) + 5.2(260.85) + 43.992(227.31) - 334.46 - 11.7(209.02) - 43.992(184.47) + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}} (s_4 - s_3)$$

$$= 2888.77 + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}} (s_4 - s_3)$$

$$= 2888.77 + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}} s_{fg}$$

The value for s_{fg} can be obtained directly from table data, but it is more instructive to rewrite the last expression using $s_{fg} = h_{fg}/T_{sat}$ from Sec. 6.3. Then

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{C_4H_{10}}} &= 2888.77 + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}} \cdot \frac{h_{fg}}{T_{sat}} = 2888.77 + \left[\frac{915,409}{h_{fg}} \right] \left[\frac{h_{fg}}{T_{sat}} \right] \\ &= 2888.77 + \frac{915,409}{T_{sat}} \end{aligned} \quad (2)$$

①

• Water at 1 bar, $T_{sat} = 373 \text{ K} \Rightarrow \frac{\dot{Q}_{cv}}{\dot{n}_{C_4H_{10}}} = 2888.77 + \frac{915,409}{373} = 5342.9 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$

• Ammonia at 10 bar, $T_{sat} = 298 \text{ K} \Rightarrow \frac{\dot{Q}_{cv}}{\dot{n}_{C_4H_{10}}} = 2888.77 + \frac{915,409}{298} = 5960.6 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$

(b)

Continued on next slide

Problem 13-79 continued

(c) The energy destruction is obtained using

$$\frac{\dot{E}_d}{\dot{n}_{\text{C}_2\text{H}_6}} = T_0 \frac{\dot{S}_{\text{cr}}}{\dot{n}_{\text{C}_2\text{H}_6}}$$

• water at 1 bar:

$$\frac{\dot{E}_d}{\dot{n}_{\text{C}_2\text{H}_6}} = (298\text{K})(5342.9) = 1.592 \times 10^6 \frac{\text{kJ}}{\text{kmol(fuel)}}$$

• ammonia at 10 bars:

$$\frac{\dot{E}_d}{\dot{n}_{\text{C}_2\text{H}_6}} = (298)(5960.6) = 1.776 \times 10^6 \frac{\text{kJ}}{\text{kmol(fuel)}}$$



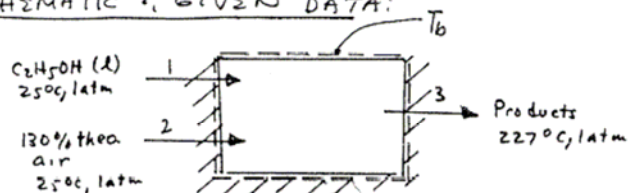
1. The difference between the entropy production rates is not due to the natures of the two coolants, but only to the temperature T_{sat} , as shown by Eq. (2). If each coolant were at the same temperature, the same entropy production rate would be determined for each. The choice of the coolant is determined by other considerations. For example, at a temperature of $T_{\text{sat}} = 50^\circ\text{C}$ the pressure of saturated water is just 0.1235 bar, whereas the pressure of saturated ammonia is 20.331 bar. Such extreme pressures are likely to be avoided in applications of the type under consideration, however. And as shown by part (a) the temperature T_{sat} also affects the mass flow rate.

PROBLEM 13.80

KNOWN: $C_2H_5OH(l)$ at $25^\circ C, 1 \text{ atm}$ enters a reactor and burns completely with 130% of theoretical air entering separately at $25^\circ C, 1 \text{ atm}$. Products exit at $227^\circ C, 1 \text{ atm}$. Heat transfer from the reactor occurs at temperature T_b .

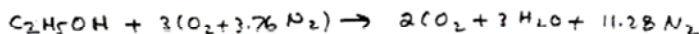
FIND: For T_b ranging from $25^\circ C$ to $2000^\circ C$, determine the rate of exergy destruction within the reactor.

SCHEMATIC & GIVEN DATA:

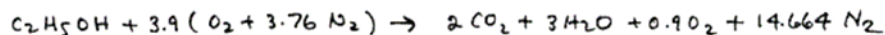


ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$, negligible kinetic and potential energy effects, and heat transfer occurring at temperature T_b . (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and products of combustion can be modeled as ideal gases. (5) For the exergy reference environment, $T_0 = 25^\circ C$ (298 K).

ANALYSIS: The rate of exergy destruction is given by $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance. The plan then is to evaluate $\dot{\sigma}_{cv}$. Complete combustion of C_2H_5OH with the theoretical amount of air is described by



Complete combustion with 130% of the theoretical amount of air is thus



With assumption 1, an entropy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (\bar{s}_{fuel})_1 + (3.9 \bar{s}_{O_2} + 14.664 \bar{s}_{N_2})_2 - (2 \bar{s}_{CO_2} + 3 \bar{s}_{H_2O} + 0.9 \bar{s}_{O_2} + 14.664 \bar{s}_{N_2})_3 + \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}}$$

Accordingly

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = -\frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_0} + (2 \bar{s}_{CO_2} + 3 \bar{s}_{H_2O} + 0.9 \bar{s}_{O_2} + 14.664 \bar{s}_{N_2})_3 - (\bar{s}_{fuel})_1 - (3.9 \bar{s}_{O_2} + 14.664 \bar{s}_{N_2})_2 \quad (1)$$

The term $\dot{Q}_{cv}/\dot{n}_{fuel}$ can be obtained from an energy rate balance which reduces with assumption 1 to give

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} &= (2 \bar{h}_{CO_2} + 3 \bar{h}_{H_2O} + 0.9 \bar{h}_{O_2} + 14.664 \bar{h}_{N_2})_3 - (\bar{h}_{fuel})_1 - (3.9 \bar{h}_{O_2} + 14.664 \bar{h}_{N_2})_2 \\ &= 2 [\bar{h}_f^\circ + \bar{h}(500) - \bar{h}(298)]_{CO_2} + 3 [\bar{h}_f^\circ + \bar{h}(500) - \bar{h}(298)]_{H_2O} + 0.9 [\bar{h}(500) - \bar{h}(298)]_{O_2} \\ &\quad + 14.664 [\bar{h}(500) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{fuel} - (0) \end{aligned}$$

With data from the ideal gas tables and Table A-25

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} &= 2 [-393,520 + 17,678 - 9364] + 3 [-241,820 + 16,828 - 9904] + 0.9 [14770 - 8682] \\ &\quad + 14.664 [14,581 - 8669] - (-277,690) \\ &= -1,05,237 \text{ kJ/kmol(fuel)} \end{aligned}$$

Continued on next slide

Problem 13-80 continued

Next, the specific entropies appearing in Eq. (i) are evaluated. Since the fuel enters at 25°C, 1 atm, $\bar{s}_{\text{fuel}}$ is obtained from Table A-25: $\bar{s}_{\text{fuel}} = 160.7 \text{ kJ/kmol} \cdot \text{K}$. The combustion air enters as a mixture with $y_{\text{O}_2} = 0.21$, $y_{\text{N}_2} = 0.79$. Thus, with $p = p_{\text{ref}}$ and $T = 298 \text{ K}$

$$\bar{s}_{\text{O}_2} = \bar{s}_{\text{O}_2}^{\circ}(298) - \bar{R} \ln \frac{y_{\text{O}_2} p_{\text{ref}}}{p_{\text{ref}}} = 205.033 - 8.314 \ln 0.21 = 218.01 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{\text{N}_2} = \bar{s}_{\text{N}_2}^{\circ}(298) - \bar{R} \ln \frac{y_{\text{N}_2} p_{\text{ref}}}{p_{\text{ref}}} = 191.502 - 8.314 \ln 0.79 = 193.462 \text{ kJ/kmol} \cdot \text{K}$$

The combustion products exit as a mixture at 500 K, 1 atm with the composition $y_{\text{CO}_2} = 2/20.564$, $y_{\text{H}_2\text{O}} = 3/20.564$, $y_{\text{O}_2} = 0.9/20.564$, $y_{\text{N}_2} = 14.664/20.564$. Thus, with $\bar{s}^{\circ}$ data from the ideal gas tables

$$\bar{s}_{\text{CO}_2} = \bar{s}_{\text{CO}_2}^{\circ}(500) - \bar{R} \ln \frac{y_{\text{CO}_2} p_{\text{ref}}}{p_{\text{ref}}} = 234.81 - 8.314 \ln \frac{2}{20.564} = 254.185 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{\text{H}_2\text{O}} = \bar{s}_{\text{H}_2\text{O}}^{\circ}(500) - \bar{R} \ln \frac{y_{\text{H}_2\text{O}} p_{\text{ref}}}{p_{\text{ref}}} = 206.413 - 8.314 \ln \frac{3}{20.564} = 222.417 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{\text{O}_2} = \bar{s}_{\text{O}_2}^{\circ}(500) - \bar{R} \ln \frac{y_{\text{O}_2} p_{\text{ref}}}{p_{\text{ref}}} = 220.529 - 8.314 \ln \frac{0.9}{20.564} = 246.603 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{\text{N}_2} = \bar{s}_{\text{N}_2}^{\circ}(500) - \bar{R} \ln \frac{y_{\text{N}_2} p_{\text{ref}}}{p_{\text{ref}}} = 206.670 - 8.314 \ln \frac{14.664}{20.564} = 209.441 \text{ kJ/kmol} \cdot \text{K}$$

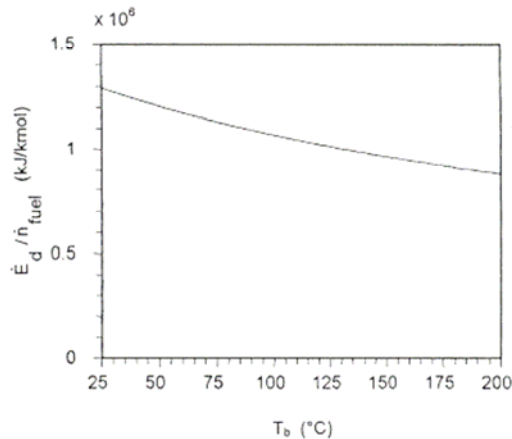
Collecting results

$$\frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{fuel}}} = - \left(\frac{1,105,237}{T_b} \right) + 2(254.185) + 3(222.417) + 0.9(246.603) + 14.664(209.441) - 160.7 - 3.9(218.01) - 14.664(193.462)$$

$$= \frac{1,105,237}{T_b} + 620.9 \frac{\text{kJ}}{\text{kmol}(\text{fuel}) \cdot \text{K}} \Rightarrow \frac{\dot{E}_d}{\dot{n}_{\text{fuel}}} = T_b \left[\frac{1,105,237}{T_b} + 620.9 \right] \frac{\text{kJ}}{\text{kmol}(\text{fuel})}$$

Sample Calculation: When $T_b = 298 \text{ K}$, $(\dot{E}_d/\dot{n}_{\text{fuel}}) = 1.29 \times 10^6 \text{ kJ/kmol}(\text{fuel})$

PLOT:



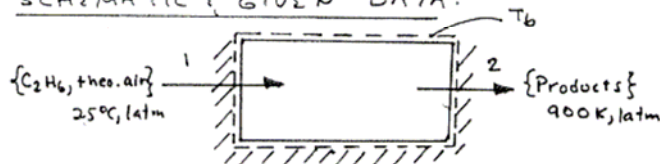
- ① At steady state, the net exergy carried into the reactor is a fixed value equal to the exergy destroyed within the reactor plus the exergy carried out accompanying heat transfer. Since the exergy carried out accompanying heat transfer increases as T_b increases: $[(1 - T_0/T_b) \dot{Q}_{\text{cv}}]$, the rate of exergy destruction within the reactor decreases as T_b increases, as shown in the figure.

PROBLEM 13.81

KNOWN: A gaseous mixture of C_2H_6 and the theoretical amount of air at $25^\circ C$, 1 atm enters a reactor and burns completely. Combustion products exit at $627^\circ C$, 1 atm. Heat transfer from the reactor takes place at T_b .

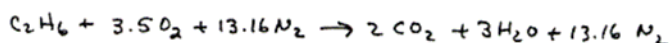
FIND: Determine the rate of exergy destruction, in kJ per kmol of C_2H_6 , for T_b ranging from 25 to $600^\circ C$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$, negligible effects of kinetic and potential energy, and heat transfer occurring at temperature T_b . (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The incoming and exiting mixtures can be modeled as ideal gases. (5) For the exergy reference environment, $T_0 = 298 K$.

ANALYSIS: The rate of exergy destruction is given by $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance. The plan then is to evaluate $\dot{\sigma}_{cv}$. Complete combustion of C_2H_6 with the theoretical amount of air is described by



With assumption 1, an entropy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (\bar{s}_{fuel} + 3.5 \bar{s}_{O_2} + 13.16 \bar{s}_{N_2})_1 - (2 \bar{s}_{CO_2} + 3 \bar{s}_{H_2O} + 13.16 \bar{s}_{N_2})_2 + \frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}}$$

Accordingly

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = - \frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (2 \bar{s}_{CO_2} + 3 \bar{s}_{H_2O} + 13.16 \bar{s}_{N_2})_2 - (\bar{s}_{fuel} + 3.5 \bar{s}_{O_2} + 13.16 \bar{s}_{N_2})_1 \quad (1)$$

The term $\dot{Q}_{cv}/\dot{n}_{fuel}$ can be obtained from an energy rate balance which reduces with assumption 1 to give

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} &= (2 \bar{h}_{CO_2} + 3 \bar{h}_{H_2O} + 13.16 \bar{h}_{N_2})_2 - (\bar{h}_{fuel} + 3.5 \bar{h}_{O_2} + 13.16 \bar{h}_{N_2})_1 \\ &= 2 [\bar{h}_f^\circ + \bar{h}(900) - \bar{h}(298)]_{CO_2} + 3 [\bar{h}_f^\circ + \bar{h}(900) - \bar{h}(298)]_{H_2O} + 13.16 [\bar{h}(900) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{fuel} \end{aligned}$$

With data from the ideal gas tables and Table A-25

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} &= 2 [-393,520 + 37,405 - 9364] + 3 [-241,820 + 31,828 - 9904] + 13.16 [26,890 - 8669] - (-84,680) \\ &= -1,066,178 \text{ kJ/kmol } (C_2H_6) \end{aligned}$$

Next, the specific entropies appearing in Eq. (1) are evaluated. The fuel and air enter as a mixture at $P = P_{ref}$ and $T = 298 K$ with the composition $y_{fuel} = 1/17.66$, $y_{O_2} = 3.5/17.66$, $y_{N_2} = 13.16/17.66$. Thus

$$\begin{aligned} \bar{s}_{fuel} &= \bar{s}_{fuel}^\circ(298) - \bar{R} \ln y_{fuel, \frac{P_{ref}}{P_{ref}}} = 229.49 - 8.314 \ln \frac{1}{17.66} = 253.36 \text{ kJ/kmol} \cdot K \\ \bar{s}_{O_2} &= \bar{s}_{O_2}^\circ(298) - \bar{R} \ln y_{O_2} = 205.03 - 8.314 \ln \frac{3.5}{17.66} = 218.49 \text{ kJ/kmol} \cdot K \\ \bar{s}_{N_2} &= \bar{s}_{N_2}^\circ(298) - \bar{R} \ln y_{N_2} = 191.5 - 8.314 \ln \frac{13.16}{17.66} = 193.95 \text{ kJ/kmol} \cdot K \end{aligned}$$

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Problem 13-81 continued

The combustion products exit as a mixture at 900 K, 1 atm with the composition $y_{\text{CO}_2} = 7/18.16$, $y_{\text{H}_2\text{O}} = 3/18.16$, $y_{\text{N}_2} = 13.16/18.16$. Thus

$$\bar{s}_{\text{CO}_2} = \bar{s}_{\text{CO}_2}^{\circ}(900) - \bar{R} \ln \frac{y_{\text{CO}_2}^{\text{Prod}}}{P_{\text{ref}}} = 267.559 - 8.314 \ln \frac{7}{18.16} = 281.90 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{\text{H}_2\text{O}} = \bar{s}_{\text{H}_2\text{O}}^{\circ}(900) - \bar{R} \ln \frac{y_{\text{H}_2\text{O}}}{P_{\text{ref}}} = 228.321 - 8.314 \ln \frac{3}{18.16} = 243.29 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{\text{N}_2} = \bar{s}_{\text{N}_2}^{\circ}(900) - \bar{R} \ln \frac{y_{\text{N}_2}}{P_{\text{ref}}} = 224.647 - 8.314 \ln \frac{13.16}{18.16} = 227.32 \text{ kJ/kmol} \cdot \text{K}$$

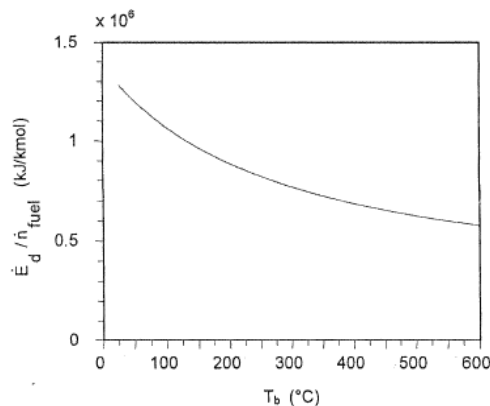
Collecting results

$$\frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{fuel}}} = - \frac{(-1,066,178)}{T_b} + 2(281.90) + 3(243.29) + 13.16(227.32) - 257.36 - 2.5(218.99) - 13.16(193.95) =$$

$$\Rightarrow (\dot{E}_d/\dot{n}_{\text{fuel}}) = T_0 \left[\frac{1,066,178}{T_b} + 714.7 \right] \frac{\text{kJ}}{\text{kmol}(\text{fuel})} \quad \frac{1,066,178}{T_b} + 714.7 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

Sample Calculation: when $T_b = 298 \text{ K}$, $(\dot{E}_d/\dot{n}_{\text{fuel}}) = 1.28 \times 10^6 \text{ kJ/kmol}(\text{fuel})$

PLOT:



- At steady state, the net exergy carried into the reactor is a fixed value equal to the exergy destroyed within the reactor plus the exergy carried out accompanying heat transfer. Since the exergy carried out accompanying heat transfer increases as T_b increases: $[1 - T_0/T_b] \dot{Q}_{\text{cv}}$, the rate of exergy destruction within the reactor decreases as T_b increases, as shown in the figure.

PROBLEM 13.82

KNOWN: The reaction is $\text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O}$. Reactants and products are at 25°C , 1atm .

FIND: Determine ΔG using (a) Gibbs function of formation data, (b) enthalpy of formation and absolute entropy data.

ANALYSIS: For $\text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O}$, ΔG is

$$\Delta G = \bar{g}_{\text{CO}_2} + 2\bar{g}_{\text{H}_2\text{O}} - \bar{g}_{\text{CH}_4} - 2\bar{g}_{\text{O}_2} \quad (1)$$

where each $\bar{g}$ is at 25°C , 1atm

(a) With $\bar{g}_f^\circ$ data from Table A-25, for H_2O as a gas, Eq. (1) gives

$$\Delta G = -394,380 + (2)(-228,590) - (-50,790) - (2)(0) = -800,170 \text{ kJ/kmol CH}_4 \quad (a)$$

(b) With $\bar{g} = \bar{h} - T\bar{s}$, Eq. (1) becomes

$$\begin{aligned} \Delta G &= (\bar{h} - T\bar{s})_{\text{CO}_2} + 2(\bar{h} - T\bar{s})_{\text{H}_2\text{O}} - (\bar{h} - T\bar{s})_{\text{CH}_4} - 2(\bar{h} - T\bar{s})_{\text{O}_2} \\ &= (\bar{h}_{\text{CO}_2} + 2\bar{h}_{\text{H}_2\text{O}} - \bar{h}_{\text{CH}_4} - 2\bar{h}_{\text{O}_2}) - T(\bar{s}_{\text{CO}_2} + 2\bar{s}_{\text{H}_2\text{O}} - \bar{s}_{\text{CH}_4} - 2\bar{s}_{\text{O}_2}) \end{aligned}$$

Inserting $\bar{h}_f^\circ$ and $\bar{s}^\circ$ data from Table A-25

$$\begin{aligned} \Delta G &= [-393,520 + (2)(-241,820) - (-74,850) - (2)(0)] \\ &\quad - 298.15 [213.69 + (2)(188.72) - 186.16 - (2)(205.03)] \\ &= -802,310 - 298.15(-5.09) \\ &= -800,792 \text{ kJ/kmol CH}_4 \quad (b) \end{aligned}$$

①

1. This is in agreement with the answer of part (a). The difference is due to round off.

PROBLEM 13.83

KNOWN: The reaction is $\text{H}_2(\text{g}) + \frac{1}{2}\text{O}_2(\text{g}) \rightarrow \text{H}_2\text{O}(\text{g})$. Reactants and products are at 77°F , 1 atm .

FIND: Determine ΔG using (a) Gibbs function of formation data, (b) enthalpy of formation and absolute entropy data.

ANALYSIS: For $\text{H}_2 + \frac{1}{2}\text{O}_2 \rightarrow \text{H}_2\text{O}$, ΔG is

$$\Delta G = \bar{g}_{\text{H}_2\text{O}(\text{g})} - \bar{g}_{\text{H}_2} - \frac{1}{2}\bar{g}_{\text{O}_2} \quad (1)$$

where each of the $\bar{g}$'s is evaluated at 77°F , 1 atm .

(a) With $\bar{g}_f^\circ$ data from Table A-25, Eq. (1) becomes

$$\begin{aligned} \Delta G &= (\bar{g}_f^\circ)_{\text{H}_2\text{O}(\text{g})} - (\bar{g}_f^\circ)_{\text{H}_2} - \frac{1}{2}(\bar{g}_f^\circ)_{\text{O}_2} \\ &= -98,350 - (0) - \frac{1}{2}(0) = -98,350 \text{ Btu/lbmol} \end{aligned} \quad (a)$$

(b) With $\bar{g} = \bar{h} - T\bar{s}$, Eq. (1) becomes

$$\begin{aligned} \Delta G &= (\bar{h} - T\bar{s})_{\text{H}_2\text{O}(\text{g})} - (\bar{h} - T\bar{s})_{\text{H}_2} - \frac{1}{2}(\bar{h} - T\bar{s})_{\text{O}_2} \\ &= (\bar{h}_{\text{H}_2\text{O}(\text{g})} - \bar{h}_{\text{H}_2} - \frac{1}{2}\bar{h}_{\text{O}_2}) - T(\bar{s}_{\text{H}_2\text{O}(\text{g})} - \bar{s}_{\text{H}_2} - \frac{1}{2}\bar{s}_{\text{O}_2}) \end{aligned}$$

Using $\bar{h}_f^\circ$ and $\bar{s}^\circ$ data from Table A-25

$$\Delta G = [-104,040 - 0 - \frac{1}{2}(0)] - 537[45.08 - 31.19 - \frac{1}{2}(49.981)]$$

$$\textcircled{1} \quad = -98,348 \text{ Btu/lbmol} \quad (b)$$

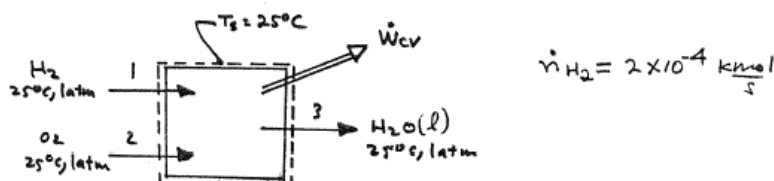
1. This agrees very closely with the result of part (a). The slight difference is due to round off.

PROBLEM 13.84

KNOWN: Separate streams of H_2 and O_2 at $25^\circ C$, 1 atm enter a fuel cell operating at steady state. A stream of liquid water exits at $25^\circ C$, 1 atm.

FIND: Determine the maximum theoretical power the cell can develop for a specified fuel flow rate and the accompanying rate of heat transfer.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) The cell temperature is $25^\circ C$.

ANALYSIS: The overall cell reaction is $H_2 + \frac{1}{2} O_2 \rightarrow H_2 O$. At steady state an energy rate balance reduces to the form

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2}} - \frac{\dot{W}_{cv}}{\dot{n}_{H_2}} + (\bar{h}_{H_2})_1 + \frac{1}{2} (\bar{h}_{O_2})_2 - (\bar{h}_{H_2O(l)})_3$$

or

$$\frac{\dot{W}_{cv}}{\dot{n}_{H_2}} = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2}} + (\bar{h}_{H_2})_1 + \frac{1}{2} (\bar{h}_{O_2})_2 - (\bar{h}_{H_2O(l)})_3 \quad (1)$$

At steady state an entropy rate balance takes the form

$$0 = \frac{(\dot{Q}_{cv}/\dot{n}_{H_2})}{T_s} + (\bar{s}_{H_2})_1 + \frac{1}{2} (\bar{s}_{O_2})_2 - (\bar{s}_{H_2O(l)})_3 + \frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}} \quad (2)$$

where $\dot{\sigma}_{cv}$ accounts for entropy production within the cell.

Combining Eqs. (1) and (2)

$$\frac{\dot{W}_{cv}}{\dot{n}_{H_2}} = \underbrace{[(\bar{h}_{H_2} + \frac{1}{2} \bar{h}_{O_2} - \bar{h}_{H_2O(l)})]_{(25^\circ C, 1 atm)} - T_s [(\bar{s}_{H_2} + \frac{1}{2} \bar{s}_{O_2} - \bar{s}_{H_2O(l)})]_{(25^\circ C, 1 atm)}}_{\text{fixed by } 25^\circ C, 1 atm} - T_s \frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}}$$

As the value of the underlined term is fixed by $25^\circ C$, 1 atm and $\dot{\sigma}_{cv} \geq 0$, it is evident that the maximum theoretical value is obtained when $\dot{\sigma}_{cv} = 0$:

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2}} \right)_{MAX} = [(\bar{h}_{H_2} + \frac{1}{2} \bar{h}_{O_2} - \bar{h}_{H_2O(l)})]_{(25^\circ C, 1 atm)} - T_s [(\bar{s}_{H_2} + \frac{1}{2} \bar{s}_{O_2} - \bar{s}_{H_2O(l)})] \quad (3)$$

This can be evaluated using enthalpy of formation data together with absolute entropy data. Alternatively, since $T_s = 25^\circ C$ and $\bar{g} = \bar{h} - T_s \bar{s}$, Eq. (3) can be expressed as

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2}} \right)_{MAX} = [\bar{g}_{H_2} + \frac{1}{2} \bar{g}_{O_2} - \bar{g}_{H_2O(l)}]_{(25^\circ C, 1 atm)}$$

Then, with $\bar{g}_f^\circ$ data from Table A-25

$$\begin{aligned} \left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2}} \right)_{MAX} &= 0 + \frac{1}{2} (0) - (-237,180) \\ &= 237,180 \text{ kJ/kmol}(H_2) \end{aligned}$$

Continued on next slide

Problem 13-84 continued

With the specified fuel flow rate

$$(\dot{W}_{cv})_{MAX} = \left[237,180 \frac{\text{kJ}}{\text{kmol}(\text{H}_2)} \right] \left[2 \times 10^{-4} \frac{\text{kmol}(\text{H}_2)}{\text{s}} \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 47.44 \text{ kW} \quad \leftarrow \dot{W}_{cv}$$

Returning to Eq. (1)

$$(\dot{Q}_{cv})_{MIN} = (\dot{W}_{cv})_{MAX} + \dot{n}_{\text{H}_2} \left[\left(\bar{h}_{\text{H}_2\text{O}(e)} \right)_3 - \left(\bar{h}_{\text{H}_2} \right)_1 - \frac{1}{2} \left(\bar{h}_{\text{O}_2} \right)_2 \right]$$

$$= 47.44 \text{ kW} + \left(2 \times 10^{-4} \frac{\text{kmol}(\text{H}_2)}{\text{s}} \right) \left[-285,830 \frac{\text{kJ}}{\text{kmol}} \right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

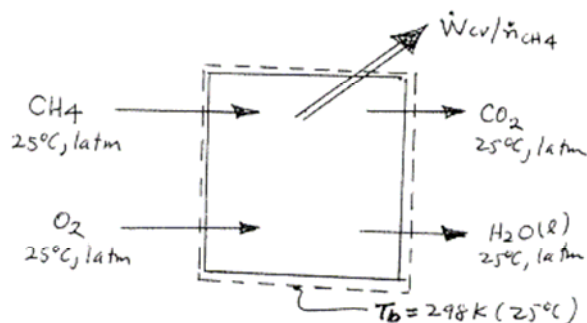
$$= -9.73 \text{ kW} \quad \leftarrow \dot{Q}_{cv}$$

PROBLEM 13.85

KNOWN: Operating data are provided for a CH_4/O_2 fuel cell at steady state.

FIND: Determine the maximum theoretical work that can be developed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown is at steady state. (2) Kinetic and potential energy effects are negligible. (3) Heat transfer takes place at $T_b = 298 \text{ K} (25^\circ\text{C})$. (4) At the condition specified, H_2O exits as a liquid.

ANALYSIS: The cell reaction is



Energy and entropy balances at steady state read, respectively.

$$\text{(energy)} \quad 0 = \frac{\dot{Q}_{cv}}{\dot{n}_{\text{CH}_4}} - \frac{\dot{W}_{cv}}{\dot{n}_{\text{CH}_4}} + (\bar{h}_{\text{CH}_4} + 2\bar{h}_{\text{O}_2} - \bar{h}_{\text{CO}_2} - 2\bar{h}_{\text{H}_2\text{O}})(298 \text{ K}, 1 \text{ atm})$$

$$\text{(entropy)} \quad 0 = \frac{\dot{Q}_{cv}/\dot{n}_{\text{CH}_4}}{T_b} + (\bar{s}_{\text{CH}_4} + 2\bar{s}_{\text{O}_2} - \bar{s}_{\text{CO}_2} - 2\bar{s}_{\text{H}_2\text{O}})(298 \text{ K}, 1 \text{ atm}) + \dot{Q}_{cv}/\dot{n}_{\text{CH}_4}$$

Eliminating the heat transfer term between these gives

$$\frac{\dot{W}_{cv}}{\dot{n}_{\text{CH}_4}} = (\bar{h}_{\text{CH}_4} + 2\bar{h}_{\text{O}_2} - \bar{h}_{\text{CO}_2} - 2\bar{h}_{\text{H}_2\text{O}})(298 \text{ K}, 1 \text{ atm}) - T_b [\bar{s}_{\text{CH}_4} + 2\bar{s}_{\text{O}_2} - \bar{s}_{\text{CO}_2} - 2\bar{s}_{\text{H}_2\text{O}}](298 \text{ K}, 1 \text{ atm}) - T_b \dot{Q}_{cv}/\dot{n}_{\text{CH}_4}$$

In terms of the Gibbs function

$$\frac{\dot{W}_{cv}}{\dot{n}_{\text{CH}_4}} = [\bar{g}_{\text{CH}_4} + 2\bar{g}_{\text{O}_2} - \bar{g}_{\text{CO}_2} - 2\bar{g}_{\text{H}_2\text{O}(l)}](298 \text{ K}, 1 \text{ atm}) - T_b \dot{Q}_{cv}/\dot{n}_{\text{CH}_4}$$

Accordingly, with data from Table A-25, and noting that the maximum value corresponds to the case of no entropy production,

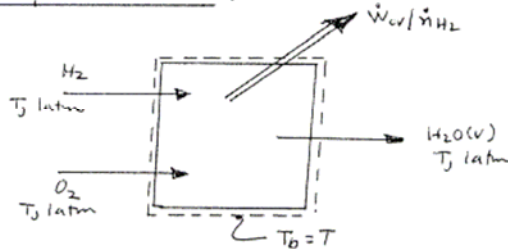
$$\begin{aligned} \left(\frac{\dot{W}_{cv}}{\dot{n}_{\text{CH}_4}} \right)_{\text{MAX}} &= -50,790 + 2(0) - (-394,380) - 2(-237,180) \\ &= 817,950 \frac{\text{kJ}}{\text{kmol}(\text{CH}_4)} \end{aligned}$$

PROBLEM 13.86

KNOWN: Operating data are provided for a H_2/O_2 fuel cell at steady state.

FIND: Determine the maximum theoretical work that can be developed if the cell operates isothermally at (a) 300 K, (b) 400 K, (c) 500 K.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown is at steady state. (2) Kinetic and potential energy effects are negligible. (3) Heat transfer occurs at the cell temperature.

ANALYSIS: The cell reaction is $H_2 + \frac{1}{2} O_2 \rightarrow H_2O$. Energy and entropy balances read respectively

$$\text{(energy)} \quad 0 = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2}} - \frac{\dot{W}_{cv}}{\dot{n}_{H_2}} + [\bar{h}_{H_2} + \frac{1}{2} \bar{h}_{O_2} - \bar{h}_{H_2O(v)}](T_j, \text{atm})$$

$$\text{(entropy)} \quad 0 = \frac{\dot{Q}_{cv}/\dot{n}_{H_2}}{T_b} + [\bar{s}_{H_2} + \frac{1}{2} \bar{s}_{O_2} - \bar{s}_{H_2O(v)}](T_j, \text{atm}) + \frac{\dot{Q}_{cv}}{\dot{n}_{H_2}}$$

Eliminating $\dot{Q}_{cv}$ from these expressions

$$\frac{\dot{W}_{cv}}{\dot{n}_{H_2}} = [\bar{h}_{H_2} + \frac{1}{2} \bar{h}_{O_2} - \bar{h}_{H_2O(v)}] - T[\bar{s}_{H_2} + \frac{1}{2} \bar{s}_{O_2} - \bar{s}_{H_2O(v)}] - T \frac{\dot{Q}_{cv}}{\dot{n}_{H_2}}$$

Accordingly, with $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$ and $\bar{s} = \bar{s}^\circ(T, \text{atm})$, and noting that the maximum value corresponds to the case of no entropy production

$$\begin{aligned} \left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2}} \right)_{\max} &= \left[(\bar{h}_f^\circ)_{H_2} + \frac{1}{2} (\bar{h}_f^\circ)_{O_2} - (\bar{h}_f^\circ)_{H_2O(v)} \right] + [\Delta \bar{h}]_{H_2} + \frac{1}{2} [\Delta \bar{h}]_{O_2} - [\Delta \bar{h}]_{H_2O} \\ &\quad - T \left[\bar{s}_{H_2}^\circ + \frac{1}{2} \bar{s}_{O_2}^\circ - \bar{s}_{H_2O(v)}^\circ \right](T) \\ &= -[\bar{h}_f^\circ]_{H_2O(v)} + [\Delta \bar{h}]_{H_2} + \frac{1}{2} [\Delta \bar{h}]_{O_2} - [\Delta \bar{h}]_{H_2O} - T \left[\bar{s}_{H_2}^\circ + \frac{1}{2} \bar{s}_{O_2}^\circ - \bar{s}_{H_2O(v)}^\circ \right](T) \quad (1) \end{aligned}$$

Eq. (1) is evaluated using the following IT code:

```
T = 300 // K
p = 1.01325 // bar
```

①

```
Wdot_max = - hfo_H2O + delh_H2 + .5 * delh_O2 - delh_H2O - T * (so_H2 + .5 * so_O2 - so_H2O)
hfo_H2O = h_T("H2O", 298.15)
delh_H2 = h_T("H2", T) - h_T("H2", 298.15)
delh_O2 = h_T("O2", T) - h_T("O2", 298.15)
delh_H2O = h_T("H2O", T) - h_T("H2O", 298.15)
so_H2 = s_TP("H2", T, p)
so_O2 = s_TP("O2", T, p)
so_H2O = s_TP("H2O", T, p)
```

The results are

Continued on next slide

Problem 13-86 continued

	<u>T = 300 K</u>	<u>T = 400 K</u>	<u>T = 500 K</u>
$[\Delta \bar{h}]_{\text{H}_2}$ (kJ/kmol)	53.39	2955	5877
$[\Delta \bar{h}]_{\text{H}_2\text{O}}$ (kJ/kmol)	62.04	3449	6918
$[\Delta \bar{h}]_{\text{O}_2}$ (kJ/kmol)	54.36	3030	6094
$\left[\bar{h}_f^\circ \right]_{\text{H}_2\text{O}}$ (kJ/kmol)	-2.418E5	-2.418E5	-2.418E5
$\bar{s}_{\text{H}_2}^\circ$ (kJ/kmol·K)	130.7	139.1	145.6
$\bar{s}_{\text{H}_2\text{O}}^\circ$ (kJ/kmol·K)	188.9	198.7	206.4
$\bar{s}_{\text{O}_2}^\circ$ (kJ/kmol·K)	205.2	213.8	220.6
$\left(\frac{\dot{W}_{\text{cv}}}{\dot{n}_{\text{H}_2}} \right)_{\text{max}}$ (kJ/kmol)	2.285E5	2.239E5	2.191E5

As T increases, $(\dot{W}_{\text{cv}}/\dot{n}_{\text{H}_2})_{\text{MAX}}$ decreases and the accompanying heat transfer increases. In principle, additional work could be developed from the heat transfer. The maximum obtainable corresponds to the case where the heat transfer is provided to a reversible power cycle operating between the cell at temperature T and the surroundings at T_0 ; the maximum would equal

$$\left[1 - \frac{T_0}{T} \right] (\dot{Q}_{\text{cv}}/\dot{n}_{\text{H}_2})$$

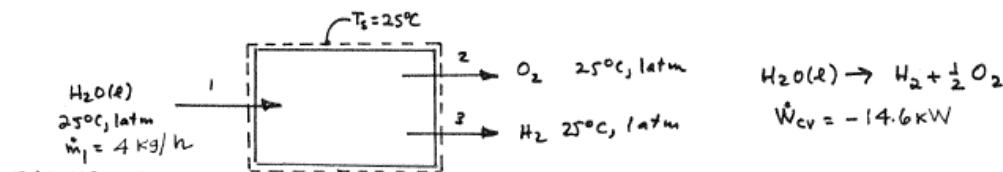
-
1. Only an IT solution is presented here, as Tables A-23 do not include data for H_2 . Data for H_2 can be found in the same source as Table A-23.

PROBLEM 13.87

KNOWN: At steady state a device takes in liquid water at 25°C , 1 atm with a mass flow rate of 5 kg/h and produces separate streams of H_2 and O_2 , each at 25°C , 1 atm. The device operates isothermally at 25°C . It is claimed that a power input of 16 kW is required.

FIND: Evaluate this claim

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) The device operates isothermally at 25°C .

ANALYSIS: The plan is to calculate the rate of entropy production on the basis that the given value for $\dot{W}_{cv}$ is correct. Thus, at steady state an entropy rate balance reduces to

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{\text{H}_2\text{O}}}{T_s} + [\bar{s}_{\text{H}_2\text{O}(l)}]_1 - [\bar{s}_{\text{H}_2}]_3 - \frac{1}{2} [\bar{s}_{\text{O}_2}]_2 + \dot{Q}_{cv}/\dot{n}_{\text{H}_2\text{O}} \quad (1)$$

The heat transfer rate can be evaluated from an energy rate balance at steady state:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} - \frac{\dot{W}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} + [\bar{h}_{\text{H}_2\text{O}(l)}]_1 - [\bar{h}_{\text{H}_2}]_3 - \frac{1}{2} [\bar{h}_{\text{O}_2}]_2 \quad (2)$$

Combining Eqs. (1) and (2)

$$T_s \left(\frac{\dot{Q}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} \right) = - \frac{\dot{W}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} - \left\{ [\bar{h}_{\text{H}_2}]_3 + \frac{1}{2} [\bar{h}_{\text{O}_2}]_2 - [\bar{h}_{\text{H}_2\text{O}(l)}]_1 \right\} + T_s \left\{ [\bar{s}_{\text{H}_2}]_3 + \frac{1}{2} [\bar{s}_{\text{O}_2}]_2 - [\bar{s}_{\text{H}_2\text{O}(l)}]_1 \right\}$$

Each of the $\bar{h}$ and $\bar{s}$ terms in this expression is evaluated at 25°C , 1 atm. Then, since $T_s = 25^\circ\text{C}$ and $\bar{q} = \bar{h} - T_s \bar{s}$, it can be rewritten as

$$T_s \left(\frac{\dot{Q}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} \right) = - \frac{\dot{W}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} - \left[(\bar{q}_{\text{H}_2})_3 + \frac{1}{2} (\bar{q}_{\text{O}_2})_2 - (\bar{q}_{\text{H}_2\text{O}(l)})_1 \right] (25^\circ\text{C}, 1 \text{ atm}) \quad (3)$$

Using $M = 18.02 \text{ kg/kmol}$ for water, the molar flow rate of H_2O is

$$\dot{n}_{\text{H}_2\text{O}} = \frac{\dot{m}_1}{M} = \frac{4 \text{ kg/h}}{18.02 \text{ kg/kmol}} = 0.222 \frac{\text{kmol}}{\text{h}}$$

Accordingly

$$\frac{\dot{W}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} = \frac{-(14.6 \text{ kJ/s}) | 3600 \text{ s/h} |}{(0.222 \text{ kmol/h})} = - 236,757 \frac{\text{kJ}}{\text{kmol}}$$

Then, with $\bar{q}_f^\circ$ data from Table A-25

$$\begin{aligned} T_s \left(\frac{\dot{Q}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} \right) &= 236,757 - \left[0 + \frac{1}{2} (0) - (-237,180) \right] \\ &= - 423 \frac{\text{kJ}}{\text{kmol}(\text{H}_2\text{O})} \end{aligned}$$

As $T_s > 0$, this implies $\dot{Q}_{cv}/\dot{n}_{\text{H}_2\text{O}} < 0$ which is impossible. The value claimed for $\dot{W}_{cv}$ cannot be correct.

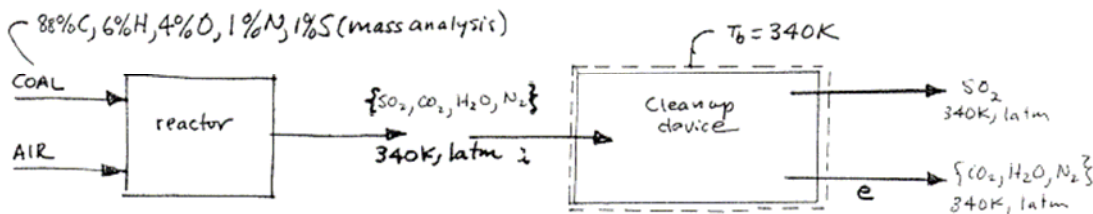
PROBLEM 13.88

KNOWN: Coal with a specified mass analysis burns completely with the theoretical amount of air.

FIND: Determine (a) the amount of SO_2 produced, (b) the air-fuel ratio on a mass basis. Also determine (c) the minimum theoretical power required to remove SO_2 from the combustion products via a specified device.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) The cleanup device operates at steady state at 340 K with negligible kinetic & potential energy effects. (3) Ideal gas principles apply.

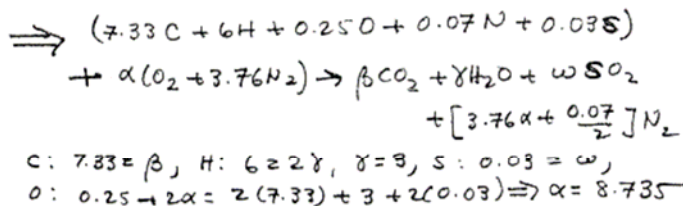
SCHEMATIC & GIVEN DATA:



ANALYSIS: On the basis of 100 kg of coal, the amounts of the substances present, in kmol, are

	m_i	M_i^*	$n_i = m_i / M_i^*$
C	88	12	7.33
H	6	1	6.00
O	4	16	0.25
N	1	14	0.07
S	1	32	0.03

* rounded to nearest integer



(a) The amount of SO_2 produced is 0.03 kmol per 100 kg of coal. Thus, with $M_{\text{SO}_2} = 64$

$$m_{\text{SO}_2} = \frac{(0.03)(64)}{100} = 0.0192 \frac{\text{kg}(\text{SO}_2)}{\text{kg}(\text{coal})}$$

(b) The amount of air required is $(8.735)(4.76)$ kmol per 100 kg of coal. Thus

$$\text{AF} = \frac{(8.735)(4.76)(28.97)}{100} = 12.05 \frac{\text{kg}(\text{air})}{\text{kg}(\text{coal})}$$

(c) Referring to the reaction equation, for a coal burn rate of 10 kg/s, the molar flow rates of the combustion products are

Based on a coal burn rate of 10 kg/s	
CO_2	0.733 kmol/s
H_2O	0.300 kmol/s
N_2	3.284 kmol/s
SO_2	0.003 kmol/s
$\dot{n}_i$	4.320 kmol/s

For the cleanup device, the molar analyses of the incoming and outgoing streams are

	i entering, $\dot{y}$	e exiting, $\dot{y}$
CO_2	0.1697	0.1698
H_2O	0.0694	0.0695
N_2	0.7602	0.7607
SO_2	0.0007	—

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Problem 13-88 continued

An energy rate balance reduces at steady state to $0 = \dot{Q}_{cv} - \dot{W}_c + \sum \dot{n}_i h_i - \sum \dot{n}_e h_e$. Then, since ideal gas principles apply, the enthalpy terms cancel because the entering and exiting streams are at the same temperature. Accordingly, $\dot{W}_{cv} = \dot{Q}_{cv}$.

An entropy rate balance reduces at steady state to

$$0 = \frac{\dot{Q}_{cv}}{T_b} + [\dot{n}_{CO_2} \bar{s}_{CO_2}(T, y_{CO_2} p) + \dot{n}_{H_2O} \bar{s}_{H_2O}(T, y_{H_2O} p) + \dot{n}_{N_2} \bar{s}_{N_2}(T, y_{N_2} p) + \dot{n}_{SO_2} \bar{s}_{SO_2}(T, y_{SO_2} p)] - [\dot{n}_{CO_2} \bar{s}_{CO_2}(T, y'_{CO_2} p) + \dot{n}_{H_2O} \bar{s}_{H_2O}(T, y'_{H_2O} p) + \dot{n}_{N_2} \bar{s}_{N_2}(T, y'_{N_2} p)] - \dot{n}_{SO_2} \bar{s}_{SO_2}(T, p) + \dot{\sigma}_{cv}$$

Collecting results and using Eq. 13.23 & $\bar{s}_i(T, y_i p) = \bar{s}_i^o(T) - \bar{R} \ln y_i p / p_{ref}$ and $p = p_{ref}$

$$\dot{W}_{cv} = T_b \left[\dot{n}_{CO_2} \left(-\bar{R} \ln \frac{y'_{CO_2}}{y_{CO_2}} \right) + \dot{n}_{H_2O} \left(-\bar{R} \ln \frac{y'_{H_2O}}{y_{H_2O}} \right) + \dot{n}_{N_2} \left(-\bar{R} \ln \frac{y'_{N_2}}{y_{N_2}} \right) + \dot{n}_{SO_2} \left(-\bar{R} \ln \frac{1}{y_{SO_2}} \right) - \dot{\sigma}_{cv} \right]$$

The minimum theoretical power input corresponds to $\dot{\sigma}_{cv} = 0$. Then, with y and y' from above

$$\begin{aligned} (\dot{W}_{cv})_{min} &= - \left(8.314 \frac{kJ}{kmol \cdot K} \right) (340 K) \left[0.733 \frac{kmol}{s} \ln \frac{0.1698}{0.1697} + 0.3 \ln \frac{0.0695}{0.0694} + 3.284 \ln \frac{0.7607}{0.7602} + 0.003 \ln \frac{1}{0.007} \right] \\ &= - 70.2 \frac{kJ}{s} = - 70.2 kW \end{aligned}$$



PROBLEM 13.89

KNOWN: An environment is specified in which the gas phase obeys the ideal gas model.

FIND: For each of several specified substances, determine the chemical exergy.

ENGINEERING MODEL: The reference environment consists of an ideal gas mixture having the analysis

Environment		
$T_0 = 298.15 \text{ K } (25^\circ\text{C}), p_0 = 1 \text{ atm}$		
Gas Phase:	Component	y^e (%)
	N_2	75.67
	O_2	20.35
	$\text{H}_2\text{O}(\text{g})$	3.12
	CO_2	0.03
	Other	0.83

ANALYSIS: (a) Carbon: $\text{C} + \text{O}_2 \rightarrow \text{CO}_2$. Eq. 13.36 takes the form

$$\bar{e}^{\text{CH}} = (\bar{g}_{\text{C}} + \bar{g}_{\text{O}_2} - \bar{g}_{\text{CO}_2})(T_0, p_0) + \bar{R} T_0 \ln \left[\frac{y_{\text{O}_2}^e}{y_{\text{CO}_2}^e} \right]$$

Then, with $\bar{g}_f^\circ$ data from Table A-25

$$\begin{aligned} \bar{e}^{\text{CH}} &= (0 + 0 - (-394,380)) + (8.314)(298.15) \ln \left[\frac{0.2035}{0.0003} \right] \\ &= 410,541 \text{ kJ/kmol} \end{aligned}$$

With $M = 12$ from Table A-1

$$e^{\text{CH}} = \frac{410,541}{12} = 34,212 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow (a)$$

(b) Hydrogen: $\text{H}_2 + \frac{1}{2} \text{O}_2 \rightarrow \text{H}_2\text{O}$. Eq. 13.36 takes the form

$$\bar{e}^{\text{CH}} = (\bar{g}_{\text{H}_2} + \frac{1}{2} \bar{g}_{\text{O}_2} - \bar{g}_{\text{H}_2\text{O}})(T_0, p_0) + \bar{R} T_0 \ln \left[\frac{(y_{\text{O}_2}^e)^{1/2}}{y_{\text{H}_2\text{O}}^e} \right]$$

Then, with $\bar{g}_f^\circ$ data from Table A-25

$$\begin{aligned} \bar{e}^{\text{CH}} &= (0 + \frac{1}{2}(0) - (-228,590)) + (8.314)(298.15) \ln \left[\frac{(0.2035)^{1/2}}{0.0312} \right] \\ &= 235,212 \text{ kJ/kmol} \end{aligned}$$

With $M = 2.018$

$$e^{\text{CH}} = \frac{235,212}{2.018} = 116,557 \text{ kJ/kg} \quad \leftarrow (b)$$

(c) Methane: $\text{CH}_4 + 2 \text{O}_2 \rightarrow \text{CO}_2 + 2 \text{H}_2\text{O}$. Eq. 13.36 takes the form

$$\bar{e}^{\text{CH}} = [\bar{g}_{\text{CH}_4} + 2\bar{g}_{\text{O}_2} - \bar{g}_{\text{CO}_2} - 2\bar{g}_{\text{H}_2\text{O}}] + \bar{R} T_0 \ln \left[\frac{(y_{\text{O}_2}^e)^2}{(y_{\text{CO}_2}^e)(y_{\text{H}_2\text{O}}^e)^2} \right]$$

Then, with $\bar{g}_f^\circ$ data from Table A-25

$$\begin{aligned} \bar{e}^{\text{CH}} &= [-50,790 + 2(0) - (-394,380) - 2(-228,590)] + (8.314)(298.15) \ln \left[\frac{(0.2035)^2}{(0.0003)(0.0312)^2} \right] \\ &= 830,174 \text{ kJ/kmol} \end{aligned}$$

With $M = 16.04$

$$e^{\text{CH}} = \frac{830,174}{16.04} = 51,756 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow (c)$$

Continued on next slide

Problem 13-89 continued

(d) Carbon Monoxide: $\text{CO} + \frac{1}{2} \text{O}_2 \rightarrow \text{CO}_2$. Eq. 13.38 takes the form

$$\bar{e}^{\text{CH}} = \bar{g}_{\text{CO}} + \frac{1}{2} \bar{g}_{\text{O}_2} - \bar{g}_{\text{CO}_2}(T_0, P_1) + \bar{R} T_0 \ln \left[\frac{(y_{\text{O}_2}^e)^{1/2}}{y_{\text{CO}_2}^e} \right]$$

With $\bar{g}_f^0$ data from Table A-25

$$\begin{aligned} \bar{e}^{\text{CH}} &= (-137,150) + \frac{1}{2} (0) - (-394,380) + (8.314)(298.15) \ln \left[\frac{(0.2035)^{1/2}}{0.0003} \right] \\ &= 275,364 \text{ kJ/kmol} \end{aligned}$$

With $M = 28.01$

$$e^{\text{CH}} = \frac{275,364}{28.01} = 9831 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow (d)$$

(e) Liquid Methanol: $\text{CH}_3\text{OH} + 1.5 \text{O}_2 \rightarrow \text{CO}_2 + 2 \text{H}_2\text{O}$

$$\begin{aligned} \bar{e}^{\text{CH}} &= (\bar{g}_{\text{CH}_3\text{OH}} + 1.5 \bar{g}_{\text{O}_2} - \bar{g}_{\text{CO}_2} - 2 \bar{g}_{\text{H}_2\text{O}}) + \bar{R} T_0 \ln \left[\frac{(y_{\text{O}_2}^e)^{1.5}}{y_{\text{CO}_2}^e (y_{\text{H}_2\text{O}}^e)^2} \right] \\ &= (-166,290) + 1.5(0) - (-394,380) - 2(-228,590) + (8.314)(298.15) \ln \left[\frac{(0.2035)^{1.5}}{0.0003(0.0312)^2} \right] \\ &= 716,647 \text{ kJ/kmol} \end{aligned}$$

With $M = 32.05$

$$e^{\text{CH}} = \frac{716,647}{32.05} = 22,360 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow (e)$$

(f) Nitrogen: With Eq. 13.40

$$\bar{e}^{\text{CH}} = \bar{R} T_0 \ln \left(\frac{1}{y_{\text{N}_2}^e} \right) = (8.314)(298.15) \ln \left(\frac{1}{0.7587} \right) = 691.1 \text{ kJ/kmol}$$

With $M = 28.01$

$$e^{\text{CH}} = \frac{691.1}{28.01} = 24.7 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow (f)$$

(g) Oxygen. With Eq. 13.40

$$\bar{e}^{\text{CH}} = \bar{R} T_0 \ln \left(\frac{1}{y_{\text{O}_2}^e} \right) = (8.314)(298.15) \ln \left(\frac{1}{0.2035} \right) = 3947 \frac{\text{kJ}}{\text{kmol}}$$

With $M = 32$

$$e^{\text{CH}} = \frac{3947}{32} = 123.3 \frac{\text{kJ}}{\text{kg}} \quad \leftarrow (g)$$

(h) Carbon dioxide. With Eq. 13.40

$$\bar{e}^{\text{CH}} = \bar{R} T_0 \ln \left(\frac{1}{y_{\text{CO}_2}^e} \right) = (8.314)(298.15) \ln \left(\frac{1}{0.0003} \right) = 20,108 \text{ kJ/kmol}$$

With $M = 44.01$

$$e^{\text{CH}} = \frac{20,108}{44.01} = 456.9 \text{ kJ/kg} \quad \leftarrow (h)$$

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Problem 13-89 continued

(i) Water. With Eq. 13.39

$$\bar{e}^{CH} = (\bar{g}_{H_2O(l)} - \bar{g}_{H_2O(g)})(T_0, P_0) + \bar{R} T_0 \ln\left(\frac{1}{y_{H_2O}^e}\right)$$

With $\bar{g}_f^\circ$ data from Table A-30

$$\textcircled{1} \quad \bar{e}^{CH} = (-237,180) - (-228,590) + (8.314)(298.15) \ln\left(\frac{1}{0.0312}\right) = 4.9 \text{ kJ/kmol}$$

1. This is a negligible amount that would be zero were the table values for $\bar{g}_f^\circ$ not rounded, suggesting that when $y_{H_2O}^e = 0.0312$ liquid water at T_0, P_0 could also be included within the environment with no inconsistency. This point is explored in Problem 13.90.

PROBLEM 13.90

KNOWN: An environment is specified in which there is a gas phase and a condensed water phase.

FIND: (a) Show that the chemical exergy of C_2H_6 can be determined as

$$\bar{e}^{CH} = \left[\bar{g}_F + \left(a + \frac{b}{4} \right) \bar{g}_{O_2} - a \bar{g}_{CO_2} - \frac{b}{2} \bar{g}_{H_2O(l)} \right] + \bar{R} T_0 \ln \left[\frac{(y_{O_2}^e)^{a+b/4}}{(y_{CO_2}^e)^a} \right]$$

(b) Using the result of part (a); repeat parts (a) - (c) of Problem 13.89.

ENGINEERING MODEL: The environment consists of a gas phase obeying the ideal gas model and a condensed water phase:

Environment		
$T_0 = 298.15 \text{ K (25°C)}, p_0 = 1 \text{ atm}$		
Condensed Phase: $H_2O(l)$ at T_0, p_0		
Gas Phase:	Component	y^e (%)
	N_2	75.67
	O_2	20.35
	$H_2O(g)$	3.12
	CO_2	0.03
	Other	0.83

ANALYSIS: The derivation proceeds as in Sec. 13.6 except we now think of the water formed in the cell reaction as being released to the environment as liquid water at T_0, p_0 . Accordingly, in Eqs. 13.30 through 13.33 $\bar{h}_{H_2O}$ and $\bar{s}_{H_2O}$ refer to liquid water at T_0, p_0 . Then, Eq. 13.34 applies only to the O_2 and CO_2 , giving in place of Eq. 13.35 the following expression

$$\begin{aligned} \bar{e}^{CH} = & \left[\bar{h}_F + \left(a + \frac{b}{4} \right) \bar{h}_{O_2} - a \bar{h}_{CO_2} - \frac{b}{2} \bar{h}_{H_2O} \right] (T_0, p_0) - \\ & T_0 \left[\bar{s}_F + \left(a + \frac{b}{4} \right) \bar{s}_{O_2} - a \bar{s}_{CO_2} - \frac{b}{2} \bar{s}_{H_2O} \right] (T_0, p_0) + \\ & \bar{R} T_0 \ln \left[\frac{(y_{O_2}^e)^{a+b/4}}{(y_{CO_2}^e)^a} \right] \end{aligned}$$

With $\bar{g} = \bar{h} - T\bar{s}$ this can be expressed in the form indicated above under **FIND**.

Thus, two expressions for $\bar{e}^{CH}$ are obtained. One is given by Eq. 13.36:

$$\bar{e}^{CH} = \left[\bar{g}_F + \left(a + \frac{b}{4} \right) \bar{g}_{O_2} - a \bar{g}_{CO_2} - \frac{b}{2} \bar{g}_{H_2O(g)} \right] (T_0, p_0) + \bar{R} T_0 \ln \left[\frac{(y_{O_2}^e)^{a+b/4}}{(y_{CO_2}^e)^a (y_{H_2O}^e)^{b/2}} \right]$$

The other is the result obtained here:

$$\bar{e}^{CH} = \left[\bar{g}_F + \left(a + \frac{b}{4} \right) \bar{g}_{O_2} - a \bar{g}_{CO_2} - \frac{b}{2} \bar{g}_{H_2O(l)} \right] (T_0, p_0) + \bar{R} T_0 \ln \left[\frac{(y_{O_2}^e)^{a+b/4}}{(y_{CO_2}^e)^a} \right]$$

Upon comparison, it is evident that the same numerical value for $\bar{e}^{CH}$ will be obtained only if $y_{H_2O}^e$ is specified so that

$$\bar{g}_{H_2O(g)} + \bar{R} T_0 \ln y_{H_2O}^e = \bar{g}_{H_2O(l)}$$

or

$$\ln y_{H_2O}^e = \frac{\bar{s}_{H_2O(l)} - \bar{s}_{H_2O(g)}}{\bar{R} T_0}$$

where each of the $\bar{g}$'s is evaluated at T_0, p_0 . Since $T_0 = 298.15 \text{ K}$ and $p_0 = 1 \text{ atm}$, Table A-25 gives

$$\ln y_{H_2O}^e = \frac{-237,180 - (-228,590)}{(8.314)(298.15)}$$

This gives, closely, $y_{H_2O}^e = 0.03126$ which departs from the table value only slightly owing to round off in the values used for $\bar{g}_F$.

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Problem 13-90 continued

(b) Using the result of part (a), repeat parts (a)-(c) of Problem 13.89.

(a) Carbon: $C + O_2 \rightarrow CO_2$. The result of part (a) takes the form

$$\bar{a}^{CH} = (\bar{g}_C + \bar{g}_{O_2} - \bar{g}_{CO_2}) + \bar{R}T_0 \ln \left[\frac{(y_{O_2}^e)}{y_{CO_2}^e} \right]$$

This clearly gives the same answer as in Problem 13.89(a): $\bar{a}^{CH} = 39,212 \text{ kJ/kg}$. (a)

(b) Hydrogen: $H_2 + \frac{1}{2} O_2 \rightarrow H_2O$. The result of part (a) takes the form

$$\bar{e}^{CH} = \left[\bar{g}_{H_2} + \frac{1}{2} \bar{g}_{O_2} - \bar{g}_{H_2O(l)} \right] + \bar{R}T_0 \ln \left((y_{O_2}^e)^{1/2} \right)$$

With $\bar{g}_f^0$ data from Table A-25

$$\begin{aligned} \bar{e}^{CH} &= [0 + \frac{1}{2}(0) - (-237,180)] + (8.314)(298.15) \ln((0.2035)^{1/2}) \\ &= 235,207 \text{ kJ/kmol} \end{aligned}$$

or with $M = 2.018$

$$\bar{e}^{CH} = 116,554 \text{ kJ/kg} \quad \text{--- (b)}$$

which is in excellent agreement with the result of Problem 13.89(b)

(c) Methane: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$. The result of part (a) takes the form

$$\bar{e}^{CH} = \left[\bar{g}_{CH_4} + 2\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O(l)} \right] + \bar{R}T_0 \ln \left(\frac{(y_{O_2}^e)^2}{(y_{CO_2}^e)} \right)$$

With $\bar{g}_f^0$ data from Table A-25

$$\begin{aligned} \bar{e}^{CH} &= [(-50,790) + 2(0) - (-394,380) - 2(-237,180)] + (8.314)(298.15) \ln \left[\frac{(0.2035)^2}{0.0003} \right] \\ &= 830,165 \text{ kJ/kmol} \end{aligned}$$

or with $M = 16.04$

$$\bar{e}^{CH} = 51,756 \frac{\text{kJ}}{\text{kg}} \quad \text{--- (c)}$$

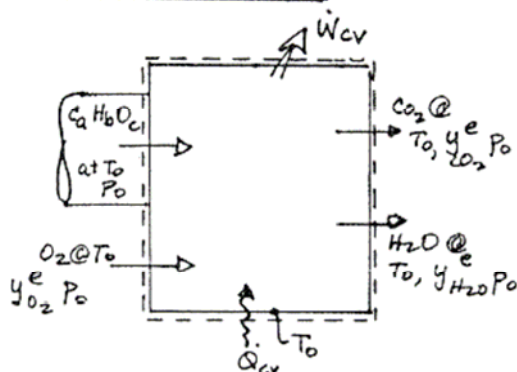
which is the result obtained in Problem 13.89(c)

PROBLEM 13.91

KNOWN: The table below lists compounds that can be represented as $C_a H_b O_c$.

FIND: Develop expressions for the chemical exergy of this group of compounds following the approach of Sec. 13.6.

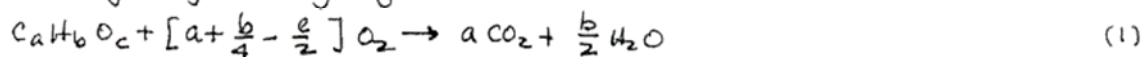
SCHEMATIC & GIVEN DATA:



	C	H ₂	C ₂ H ₆	CO	CO ₂	H ₂ O(liq.)
a	1	0	a	1	1	0
b	0	2	b	0	0	2
c	0	0	0	1	2	1

ENGINEERING MODEL: (1) The control volume is at steady state. (2) The O_2 enters at its condition in the reference environment. (3) The fuel and oxygen react completely to produce carbon dioxide and water vapor which exit in separate streams at their respective conditions within the environment.

ANALYSIS: Begin by writing Eq. 13.29 as follows



The energy rate balance reduces to give the parallel form to Eq. 13.30

$$\frac{\dot{W}_{cv}}{\dot{n}_F} = \frac{\dot{Q}_{cv}}{\dot{n}_F} + \bar{h}_F + \left(a + \frac{b}{4} - \frac{c}{2} \right) \bar{h}_{O_2} - a \bar{h}_{CO_2} - \frac{b}{2} \bar{h}_{H_2O} \quad (2)$$

An entropy balance reduces to give the parallel form to Eq. 13.31

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_F}{T_0} + \bar{s}_F + \left(a + \frac{b}{4} - \frac{c}{2} \right) \bar{s}_{O_2} - a \bar{s}_{CO_2} - \frac{b}{2} \bar{s}_{H_2O} + \frac{\dot{\sigma}_{cv}}{\dot{n}_F} \quad (3)$$

Combining (2) and (3), and setting the entropy production rate $\dot{\sigma}_{cv}$ to zero gives the expression for chemical exergy (parallel to Eq. 13.36)

$$\begin{aligned} \bar{e}^{ch} = & \left[\bar{g}_{C_a H_b O_c} + \left(a + \frac{b}{4} - \frac{c}{2} \right) \bar{g}_{O_2} - a \bar{g}_{CO_2} - \frac{b}{2} \bar{g}_{H_2O} \right] (T_0, P_0) \\ & + \bar{R} T_0 \ln \left[\frac{(y_{O_2}^e)^{a + b/4 - c/2}}{(y_{CO_2}^e)^a (y_{H_2O}^e)^{b/2}} \right] \end{aligned} \quad (4)$$

Note: ① When $c=0$, Eq. (4) reduces to Eq. 13.36 which applies for C_2H_2 and C_2H_6 .

② When $a=1, b=0, c=1$, we have CO, and Eq. (4) reduces to the corresponding result; Eq. 13.38.

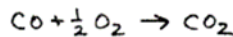
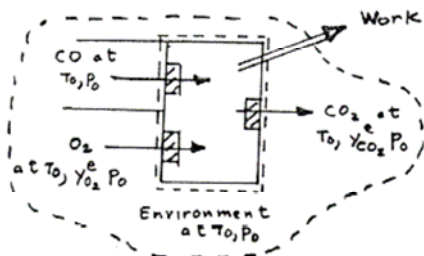
③ When $a=0, b=2, c=1$, we have $H_2O(l)$ and Eq. (4) reduces to the corresponding result; Eq. 13.39.

④ When $a=1, b=0, c=2$, we have CO_2 and Eq. (4) reduces to the corresponding result; Eq. 13.40.

PROBLEM 13.92

FIND: Showing all important steps, derive (a) Eq. 13.38, (b) Eq. 13.39, (c) Eq. 13.40, (d) Eq. 13.41a, b, (e) Eq. 13.44 a, b.

ANALYSIS: (a) Eq. 13.38. Paralleling the derivation of Sec. 13.6 consider this device operating at steady state.



energy rate balance:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{co}} - \frac{\dot{W}_{cv}}{\dot{n}_{co}} + \bar{h}_{co} + \frac{1}{2} \bar{h}_{O_2} - \bar{h}_{CO_2}$$

entropy rate balance:

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{co}}{T_0} + \bar{s}_{co}(T_0, P_0) + \frac{1}{2} \bar{s}_{O_2}(T_0, y_{O_2}^e P_0) - \bar{s}_{CO_2}(T_0, y_{CO_2}^e P_0) + \frac{\dot{Q}_{cv}}{\dot{n}_{co}}$$

Combining these two equations

$$\frac{\dot{W}_{cv}}{\dot{n}_{co}} = \left\{ \bar{h}_{co}(T_0) + \frac{1}{2} \bar{h}_{O_2}(T_0) - \bar{h}_{CO_2}(T_0) \right\} - T_0 \left\{ \bar{s}_{co}(T_0, P_0) + \frac{1}{2} \bar{s}_{O_2}(T_0, y_{O_2}^e P_0) - \bar{s}_{CO_2}(T_0, y_{CO_2}^e P_0) \right\} - T_0 \frac{\dot{Q}_{cv}}{\dot{n}_{co}}$$

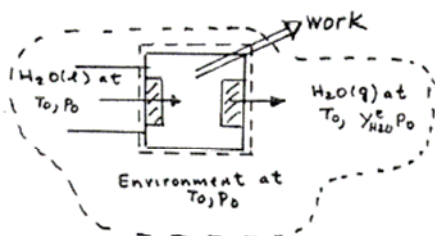
The maximum value for $\dot{W}_{cv}/\dot{n}_{co}$ corresponds to the case $\dot{Q}_{cv} = 0$. Then, using Eq. 13.34 for the $\bar{s}_{O_2}$ and $\bar{s}_{CO_2}$ terms

$$\bar{a}^{CH} = \left\{ \bar{h}_{co}(T_0) + \frac{1}{2} \bar{h}_{O_2}(T_0) - \bar{h}_{CO_2}(T_0) \right\} - T_0 \left\{ \bar{s}_{co}(T_0, P_0) + \frac{1}{2} \bar{s}_{O_2}(T_0, P_0) - \bar{s}_{CO_2}(T_0, P_0) \right\} + \bar{R} T_0 \ln \left[\frac{(y_{O_2}^e)^{1/2}}{y_{CO_2}^e} \right]$$

Finally, using $\bar{g} = \bar{h} - T\bar{s}$

$$\bar{e}^{CH} = (\bar{g}_{co} + \frac{1}{2} \bar{g}_{O_2} - \bar{g}_{CO_2})(T_0, P_0) + \bar{R} T_0 \ln \left[\frac{(x_{O_2}^e)^{1/2}}{x_{CO_2}^e} \right]$$

(b) Eq. 13.39. Consider this device operating at steady state



energy rate balance:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2O}} + \frac{\dot{W}_{cv}}{\dot{n}_{H_2O}} + \bar{h}_{H_2O(l)} - \bar{h}_{H_2O(g)}$$

entropy rate balance:

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{H_2O}}{T_0} + \bar{s}_{H_2O(l)}(T_0, P_0) - \bar{s}_{H_2O(g)}(T_0, y_{H_2O}^e P_0) + \frac{\dot{Q}_{cv}}{\dot{n}_{H_2O}}$$

Collecting these two equations

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2O}} \right)_{H_2O} = \left\{ \bar{h}_{H_2O(l)} - \bar{h}_{H_2O(g)} \right\} - T_0 \left\{ \bar{s}_{H_2O(l)}(T_0, P_0) - \bar{s}_{H_2O(g)}(T_0, y_{H_2O}^e P_0) \right\}$$

Using Eq. 13.34 for the water vapor

$$\bar{e}^{CH} = \left\{ \bar{h}_{H_2O(l)} - \bar{h}_{H_2O(g)} \right\} - T_0 \left\{ \bar{s}_{H_2O(l)}(T_0, P_0) - \bar{s}_{H_2O(g)}(T_0, P_0) \right\} - \bar{R} T_0 \ln y_{H_2O}^e$$

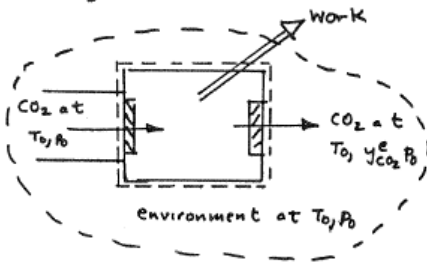
or with $\bar{g} = \bar{h} - T\bar{s}$

$$\bar{e}^{CH} = [\bar{g}_{H_2O(l)} - \bar{g}_{H_2O(g)}](T_0, P_0) + \bar{R} T_0 \ln y_{H_2O}^e$$

Continued on next slide

Problem 13-92 continued

(c) Eq. 13.40. Taking CO_2 as an example, consider this device operating at steady state



energy rate balance:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}_2}} - \frac{\dot{W}_{cv}}{\dot{n}_{\text{CO}_2}} + (\bar{h}_{\text{CO}_2})_{\text{in}} - (\bar{h}_{\text{CO}_2})_{\text{out}}$$

$= 0$ since $\bar{h}$ depends on T alone and the temperature is T_0 at inlet and exit

entropy rate balance:

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{\text{CO}_2}}{T_0} + \bar{s}_{\text{CO}_2}(T_0, P_0) - \bar{s}_{\text{CO}_2}(T_0, y_{\text{CO}_2}^e P_0) - \frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}_2}}$$

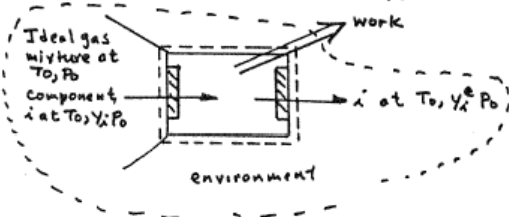
Combining these two equations

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_{\text{CO}_2}}\right)_{\text{MAX}} = -T_0 [\bar{s}_{\text{CO}_2}(T_0, P_0) - \bar{s}_{\text{CO}_2}(T_0, y_{\text{CO}_2}^e P_0)]$$

Using Eq. 13.34

$$\begin{aligned} \bar{e}^{\text{CH}} &= -T_0 [\bar{s}_{\text{CO}_2}(T_0, P_0) - (\bar{s}_{\text{CO}_2}(T_0, P_0) - \bar{R} \ln y_{\text{CO}_2}^e)] \\ &= \bar{R} T_0 \ln (1/y_{\text{CO}_2}^e) \end{aligned}$$

(d) Eq. 13.41a For each component i of an ideal gas mixture, consider a cell into which i passes at T_0 and the partial pressure $y_i P_0$ and from which i exits at T_0 and the partial pressure $y_i^e P_0$.



Energy rate balance:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_i} - \frac{\dot{W}_{cv}}{\dot{n}_i} + (\bar{h}_i)_{\text{in}} - (\bar{h}_i)_{\text{out}}$$

$= 0$ since $\bar{h}$ depends on T alone and the temperature is T_0 at the inlet and exit

Entropy rate balance:

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_i}{T_0} + \bar{s}_i(T_0, y_i P_0) - \bar{s}_i(T_0, y_i^e P_0) + \frac{\dot{Q}_{cv}}{\dot{n}_i}$$

Combining these equations

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_i}\right)_{\text{MAX}} = -T_0 (\bar{s}_i(T_0, y_i P_0) - \bar{s}_i(T_0, y_i^e P_0))$$

Using Eq. 13.34, the chemical exergy per mole of i is

$$\begin{aligned} \bar{e}_i^{\text{CH}} &= -T_0 [(\bar{s}_i(T_0, P_0) - \bar{R} \ln y_i) - (\bar{s}_i(T_0, P_0) - \bar{R} \ln y_i^e)] \\ &= \bar{R} T_0 \ln \frac{y_i}{y_i^e} \end{aligned}$$

Summing all such expressions for the mixture, the chemical exergy per mole of mixture is

$$\begin{aligned} \bar{e}^{\text{CH}} &= \sum_i y_i \bar{R} T_0 \ln \frac{y_i}{y_i^e} \\ &= \bar{R} T_0 \sum_i y_i \ln \frac{y_i}{y_i^e} \end{aligned}$$

Continued on next slide

Problem 13-92 continued

Eq. 13.41b. Expanding the last expression

$$\begin{aligned}\bar{e}^{CH} &= \bar{R} T_0 \sum_i (y_i \ln y_i - y_i \ln y_i^E) \\ &= \bar{R} T_0 \sum_i y_i \ln y_i + \sum_i y_i \underbrace{\left(\bar{R} T_0 \ln \left(\frac{1}{y_i^E} \right) \right)}_{\bar{e}_i^{CH} \text{ (Eq. 13.40)}} \\ &= \sum_i y_i \bar{e}_i^{CH} + \bar{R} T_0 \sum_i y_i \ln y_i \quad \leftarrow\end{aligned}$$

(e) Eq. 13.44a. Rewrite the underlined term of Eq. 13.43 as $\overline{HHV}$.

Introducing this into Eq. 13.43, gives Eq. 13.44a:

$$\begin{aligned}\left(\frac{\dot{W}_{cv}}{\dot{W}_F} \right)_{int, rev} &= \underbrace{\overline{HHV}(T_0, P_0) - T_0 \left[\bar{s}_F + \left(a + \frac{b}{4} \right) \bar{s}_{O_2} - a \bar{s}_{CO_2} - \frac{b}{2} \bar{s}_{H_2O(l)} \right]}_{\text{Then}}(T_0, P_0) \\ \bar{e}_F^{CH} &= \{ \dots \} + a \bar{a}_{CO_2}^{CH} + \frac{b}{2} \bar{a}_{H_2O(l)}^{CH} - \left(a + \frac{b}{2} \right) \bar{a}_{O_2}^{CH} \quad \leftarrow\end{aligned}$$

Eq. 13.44b. Using $\bar{g}(T_0, s_0) = \bar{h}(T_0, s_0) - T_0 \bar{s}(T_0, s_0)$, Eq. 13.43 becomes

$$\begin{aligned}\left(\frac{\dot{W}_{cv}}{\dot{W}_F} \right)_{int, rev} &= \underbrace{\left[\bar{g}_F + \left(a + \frac{b}{4} \right) \bar{g}_{O_2} - a \bar{g}_{CO_2} - \frac{b}{2} \bar{g}_{H_2O(l)} \right]}_{\text{Then, Eq. 13.42 becomes}}(T_0, P_0) \\ \bar{e}_F^{CH} &= \{ \dots \} + a \bar{a}_{CO_2}^{CH} + \frac{b}{2} \bar{a}_{H_2O(l)}^{CH} - \left(a + \frac{b}{2} \right) \bar{a}_{O_2}^{CH} \quad \leftarrow\end{aligned}$$

PROBLEM 13.93

KNOWN: Data from Tables A-25 and A-26

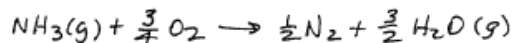
FIND: Using Eq. 13.45, determine the standard molar chemical energy $\bar{e}^{\text{CH}}$, in kJ/kmol, for (a) $\text{NH}_3(\text{g})$, (b) C_3H_8 .

ENGINEERING MODEL: Model I of Table A-26 applies.

ANALYSIS:

NH_3 .

Consider the reaction



For this reaction, Eq. 13.45 takes the form

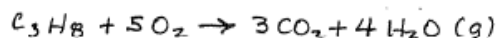
$$\begin{aligned}\bar{e}^{\text{CH}} &= [\bar{g}_{\text{NH}_3(\text{g})} + \frac{3}{4}\bar{g}_{\text{O}_2} - \frac{1}{2}\bar{g}_{\text{N}_2} - \frac{3}{2}\bar{g}_{\text{H}_2\text{O}(\text{g})}](T_0, P_0) \\ &\quad + \frac{1}{2}\bar{e}_{\text{N}_2}^{\text{CH}} + \frac{3}{2}\bar{e}_{\text{H}_2\text{O}(\text{g})}^{\text{CH}} - \frac{3}{4}\bar{e}_{\text{O}_2}^{\text{CH}}\end{aligned}$$

① With $\bar{g}$'s from Table A-25 and $\bar{e}^{\text{CH}}$'s from Table A-26 (Model I)

$$\begin{aligned}\bar{e}^{\text{CH}} &= [-16,590 + \frac{3}{4}(0) - \frac{1}{2}(0) - \frac{3}{2}(-228,590)] \\ &\quad + [(\frac{1}{2})(640) + (\frac{3}{2})(8635) - (\frac{3}{4})(3950)] \\ &= 326,295 + 10,310 = 336,605 \text{ kJ/kmol} \quad \leftarrow \text{(a)}\end{aligned}$$

This value agrees very closely with the Table A-26 entry.

C_3H_8 . Consider the reaction



For this reaction, Eq. 13.45 takes the form

$$\bar{e}^{\text{CH}} = [\bar{g}_{\text{C}_3\text{H}_8} + 5\bar{g}_{\text{O}_2} - 3\bar{g}_{\text{CO}_2} - 4\bar{g}_{\text{H}_2\text{O}}](T_0, P_0) + 3\bar{e}_{\text{CO}_2}^{\text{CH}} + 4\bar{e}_{\text{H}_2\text{O}(\text{g})}^{\text{CH}} - 5\bar{e}_{\text{O}_2}^{\text{CH}}$$

With $\bar{g}$'s from Table A-25 and $\bar{e}^{\text{CH}}$'s from Table A-26 (Model I)

$$\begin{aligned}\bar{e}^{\text{CH}} &= [-23,490 + (5)(0) - (3)(-394,380) - (4)(-228,590)] + [(3)(14,175) + (4)(8,635) - (5)(3950)] \\ &= 2,074,010 + 57,315 = 2,131,325 \text{ kJ/kmol} \quad \leftarrow \text{(b)}\end{aligned}$$

The value of $\bar{e}^{\text{CH}}$ for $\text{H}_2\text{O}_2(\text{g})$ is not provided in Table A-26. Nevertheless, such a calculated result would be a plausible value to use in the absence of other information.

1. Here we ignore the very slight difference in pressure of the two tables: Table A-25 gives data at 1 atm whereas Table A-26 (Model I) is based on 1.019 atm.

PROBLEM 13.94

FIND: Express the chemical exergies of several (a) gaseous hydrocarbons, C_aH_b , and (b) liquid hydrocarbons, C_aH_b , by an expression of the form

$$\textcircled{1} \quad \frac{\bar{e}^{\text{CH}}}{(\text{LHV})} = c_1 + c_2 \left[\frac{b}{a} \right] + \frac{c_3}{a} \quad (1)$$

where c_1, c_2 , and c_3 are constants, and (LHV) is the lower heating value.

ENGINEERING MODEL: The environment is described as in Problem 13.89.

ANALYSIS: The required values of $\bar{e}^{\text{CH}}$ can be evaluated from Eq. 13.36. Following the discussion of Sec. 13.2.3,

$$\text{LHV} = \bar{h}_{\text{C}_a\text{H}_b} + \left(a + \frac{b}{4}\right) \bar{h}_{\text{O}_2} - a \bar{h}_{\text{CO}_2} - \frac{b}{2} \bar{h}_{\text{H}_2\text{O}(g)}$$

where each $\bar{h}$ is evaluated at T_0, P_0 .

For each family of hydrocarbons, c_1, c_2 , and c_3 can be obtained from these data using a computer, together with suitable software. Expressions having the form of Eq. (1) for fairly large families of gaseous and liquid hydrocarbons obtained in this manner are presented on p. 174 (Problem 7-8) of the text: M.J. Moran, Availability Analysis: A Guide to Efficient Energy Use, ASME PRESS, New York, 1989. These expressions are determined, however, for a slightly different specification of the environment than described in Problem 13.89.

-
1. This expression provides a plausible means for estimating $\bar{e}^{\text{CH}}$ for hydrocarbon m mixtures represented as C_aH_b from measured values of the lower heating value.

PROBLEM 13.95

KNOWN: N_2 is at 200°F , 4 atm

- ① FIND: Determine the specific flow exergy using (a) the environment of Problem 13.89. (b) data from Table A-26.

ENGINEERING MODEL: N_2 is modeled as an ideal gas. (2) The effects of motion and gravity can be ignored.

ANALYSIS: In introducing ideal gas relations for h and s , Eq. 13.47 becomes

$$e_f = h - h_0 - T_0(s - s_0) + \frac{V^2}{2} + gz + e^{ch}$$

— ignore by assumption (2) —

$$= h(T) - h(T_0) - T_0(s^\circ(T) - s^\circ(T_0) - R \ln \frac{P}{P_0}) + e^{ch}$$

With data from Table A-23, $T = 660^\circ\text{R}$, $T_0 = 537^\circ\text{R}$, $P = 4\text{ atm}$ and $P_0 = 1\text{ atm}$

$$e_f = \frac{4585.8 - 3729.5 - 537[47.178 - 45.743 - 1.986 \ln 4]}{28.01} + e^{ch}$$

$$= 55.84 \frac{\text{Btu}}{\text{lb}} + e^{ch} \quad (1)$$

- (a) Evaluating e^{ch} relative to the environment of Problem 13.89, use Eq. 13.40

$$e^{ch} = R T_0 \ln \left(\frac{1}{y_e} \right) = \left(\frac{1.986}{28.01} \right) (537) \ln \left(\frac{1}{0.7567} \right)$$

$$= 10.61 \frac{\text{Btu}}{\text{lb}}$$

So, with Eq. (1)

$$e_f = 66.45 \frac{\text{Btu}}{\text{lb}} \quad \leftarrow (a)$$

- (b) Using data from Table A-26 for N_2 :

② Model I: $\bar{e}^{ch} = 640 \frac{\text{kJ}}{\text{kmol}} \Rightarrow e^{ch} = \left(640 \frac{\text{kJ}}{\text{kmol}} \right) \left(\frac{0.42992 \text{ Btu/lbmol}}{1 \text{ kJ/kmol}} \right) \left(\frac{1 \text{ lbmol}}{28.01 \text{ lb}} \right) = 9.82 \frac{\text{Btu}}{\text{lb}}$

Model II: $\bar{e}^{ch} = 720 \frac{\text{kJ}}{\text{kmol}} \Rightarrow e^{ch} = 720 \left(0.42992 \right) \left(\frac{1}{28.01} \right) = 11.05$

So with Eq. (1)

$$\text{Model I: } e_f = 65.66 \frac{\text{Btu}}{\text{lb}} \quad \leftarrow (b)$$

$$\text{Model II: } e_f = 66.89 \frac{\text{Btu}}{\text{lb}}$$

1. In any particular analysis, only one environmental model should be used.
2. We ignore the difference between $P_0 = 1\text{ atm}$ used in obtaining Eq. (1) and the Table A-26 value of P_0 for Model I.

PROBLEM 13.96

KNOWN: Water vapor is at 200°C, 1 bar

- ① FIND: Determine the specific flow EXERGY using (a) the environment of Problem 13.82, (b) data from Table A-26

ENGINEERING MODEL: (1) For liquid water, $h \sim h_f(T)$, $s \sim s_f(T)$. (2) the effects of motion and gravity can be ignored.

ANALYSIS: Equation 13.47 becomes with assumption (2)

$$e_f = h - h_o - T_o(s - s_o) + e^{ch}$$

Using Steam Table data with assumption (1)

$$\begin{aligned} e_f &= (2875.3 - 104.89) - 298.15(7.8343 - 0.3674) + e^{ch} \\ &= 544.2 + e^{ch} \quad \frac{\text{kJ}}{\text{kg}} \end{aligned} \quad (1)$$

- (a) Evaluating e^{ch} relative to the environment of Problem 13.82, use Eq. 13.39, together with $\bar{g}$'s from Table A-25

$$\begin{aligned} \bar{e}^{ch} &= [\bar{g}_{H_2O(l)} - \bar{g}_{H_2O(g)}](T_o, P_o) + \bar{R}T_o \ln \frac{1}{y_{H_2O}} \\ &= (-237,180) - (-228,590) + (8.314)(298.15) \ln \frac{1}{0.0312} \\ &= 4.9 \frac{\text{kJ}}{\text{kmol}} \end{aligned}$$

or

$$e^{ch} = \frac{4.9}{18.02} = 0.27 \frac{\text{kJ}}{\text{kg}}$$

This quantity is a negligible amount that could be ignored owing to round-off of table data. But, including it with Eq. (1) gives

$$e_f = 544.5 \frac{\text{kJ}}{\text{kg}} \quad (a)$$

- (b) Using data from Table A-26 for $H_2O(g)$

$$\text{② Model I: } \bar{e}^{ch} = 45 \frac{\text{kJ}}{\text{kmol}} \Rightarrow e^{ch} = \frac{45}{18.02} = 2.5 \frac{\text{kJ}}{\text{kg}}$$

$$\text{Model II: } \bar{e}^{ch} = 900 \frac{\text{kJ}}{\text{kmol}} \Rightarrow e^{ch} = \frac{900}{18.02} = 49.9 \frac{\text{kJ}}{\text{kg}}$$

So, with Eq. (1)

$$\text{Model I: } e_f = 546.7 \text{ kJ/kg} \quad (b)$$

$$\text{③ Model II: } e_f = 594.1 \text{ kJ/kg}$$

1. In any particular analysis, only one environmental model should be used.
2. We ignore the difference between $P_o = 1 \text{ atm}$ used in the other calculations and the Table A-26 value of P_o for Model I.
3. The Model I and part (a) values are in agreement. The Model II result is about 9% higher.

PROBLEM 13.97

KNOWN: An equimolar mixture of O_2 and N_2 is at $20^\circ C$, 1 atm.

① FIND: Determine the specific flow exergy using (a) the environment of Problem 13.89, (b) data from Table A-26.

ENGINEERING MODEL: (1) The ideal gas model is applicable. (2) Ignore the effects of motion and gravity.

ANALYSIS: With assumption (2), Equation 13.47 becomes

$$e_f = h - h_o - T_o (s - s_o) + e^{CH} \quad (1)$$

Using ideal gas mixture principles

$$\begin{aligned} \bar{h} - \bar{h}_o - T_o (\bar{s} - \bar{s}_o) &= [y_{O_2} (\bar{h}_{O_2}(T) - \bar{h}_{O_2}(T_o)) + y_{N_2} (\bar{h}_{N_2}(T) - \bar{h}_{N_2}(T_o))] \\ &\quad - T_o \left[y_{O_2} (\bar{s}_{O_2}^o(T) - \bar{s}_{O_2}^o(T_o) - \bar{R} \ln \frac{y_{O_2} P}{y_{O_2} P_o}) + y_{N_2} (\bar{s}_{N_2}^o(T) - \bar{s}_{N_2}^o(T_o) - \bar{R} \ln \frac{y_{N_2} P}{y_{N_2} P_o}) \right] \end{aligned}$$

Since $p = p_o$ the $\ln$ terms drop out, leaving

$$\begin{aligned} \bar{h} - \bar{h}_o - T_o (\bar{s} - \bar{s}_o) &= y_{O_2} (\bar{h}_{O_2}(T) - \bar{h}_{O_2}(T_o)) + y_{N_2} (\bar{h}_{N_2}(T) - \bar{h}_{N_2}(T_o)) \\ &\quad - T_o [y_{O_2} (\bar{s}_{O_2}^o(T) - \bar{s}_{O_2}^o(T_o)) + y_{N_2} (\bar{s}_{N_2}^o(T) - \bar{s}_{N_2}^o(T_o))] \end{aligned}$$

Then, with $y_{O_2} = y_{N_2} = 0.5$ and ideal gas table data

$$\begin{aligned} \bar{h} - \bar{h}_o - T_o (\bar{s} - \bar{s}_o) &= 0.5 \left\{ [(8521 - 8669) - 298(190.990 - 191.502)] + [(8533 - 8662) - 298(204.52 - 205.03)] \right\} \\ &= 2.6 \text{ kJ/kmol} \end{aligned} \quad (2)$$

The mixture molecular weight is

$$M = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = 0.5[32 + 28.01] = 30.01 \quad (3)$$

(a) Evaluating e^{CH} relative to the environment of Problem 13.89 using Eq. 13.41a

$$\bar{e}^{CH} = \bar{R} T_o \left[y_{O_2} \ln \frac{y_{O_2}}{y_{O_2}^e} + y_{N_2} \ln \frac{y_{N_2}}{y_{N_2}^e} \right]$$

With $y_{O_2} = y_{N_2} = 0.5$ and $y_{O_2}^e$ and $y_{N_2}^e$ from the table of Problem 13.89

$$\bar{e}^{CH} = (8.314)(298)(0.5) \left[\ln \frac{0.5}{0.2035} + \ln \frac{0.5}{0.7567} \right] = 600.3 \frac{\text{kJ}}{\text{kmol}} \quad (4)$$

Then, with Eqs 2-4, Eq. 1 gives

$$e_f = \frac{2.6 + 600.3}{30.01} = 20.1 \frac{\text{kJ}}{\text{kg}} \quad (a)$$

(b) Using data from Table A-26 and Eq. 13.41b

$$\begin{aligned} \bar{e}^{CH} &= y_{O_2} \bar{e}_{O_2}^{CH} + y_{N_2} \bar{e}_{N_2}^{CH} + \bar{R} T_o (y_{O_2} \ln y_{O_2} + y_{N_2} \ln y_{N_2}) \\ &= 0.5 [\bar{e}_{O_2}^{CH} + \bar{e}_{N_2}^{CH}] + \underbrace{(8.314)(298.15) \ln 0.5}_{-1718.2 \text{ kJ/kmol}} \end{aligned} \quad (5)$$

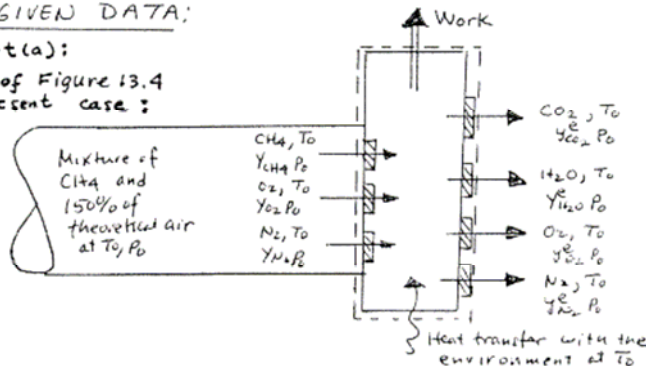
PROBLEM 13.98

KNOWN: A mixture of CH_4 and 150% of theoretical air is under consideration

FIND: Determine the specific flow exergy using (a) the environment of Problem 13.89, (b) data from Table A-26.

SCHEMATIC & GIVEN DATA:

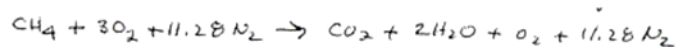
Figure for Part (a):
Counterpart of Figure 13.4
for the present case:



ENGINEERING MODEL: (1) In evaluating chemical exergy, the idealizations of Sec. 13.6 apply. (2) The effects of motion and gravity can be ignored. (3) Ideal gas principles apply.

ANALYSIS: (a) The plan is to parallel the development of Sec. 13.6:

The cell reaction is (Eq. 13.5)



The mole fractions of the mixture of CH_4 , O_2 , N_2 are

$$y_{CH_4} = \frac{1}{15.28} = 0.0654, \quad y_{O_2} = \frac{2}{15.28} = 0.1963, \quad y_{N_2} = \frac{11.28}{15.28} = 0.7382$$

An energy rate balance reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} - \frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} + \left[\bar{h}_{CH_4} + 3\bar{h}_{O_2} + 11.28\bar{h}_{N_2} \right]_{\text{(entering)}} - \left[\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + \bar{h}_{O_2} + 11.28\bar{h}_{N_2} \right]_{\text{(exiting)}}$$

or

$$\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} + (\bar{h}_{CH_4} + 3\bar{h}_{O_2} + 11.28\bar{h}_{N_2}) - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + \bar{h}_{O_2} + 11.28\bar{h}_{N_2}) \quad (1)$$

An entropy rate balance reads

$$0 = \frac{\dot{Q}_{cv} / \dot{n}_{CH_4}}{T_0} + \left[\bar{s}_{CH_4}(T_0, y_{CH_4} P_0) + 3 \bar{s}_{O_2}(T_0, y_{O_2} P_0) + 11.28 \bar{s}_{N_2}(T_0, y_{N_2} P_0) \right] - \left[\bar{s}_{CO_2}(T_0, y_{CO_2}^e P_0) + 2 \bar{s}_{H_2O}(T_0, y_{H_2O}^e P_0) + \bar{s}_{O_2}(T_0, y_{O_2}^e P_0) + 11.28 \bar{s}_{N_2}(T_0, y_{N_2}^e P_0) \right] + \dot{Q}_{cv} / \dot{n}_{CH_4}$$

Using Eq 13.34, as appropriate

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = -T_0 \left[(\bar{S}_{CH_4}(T_0, P_0) - \bar{R} \ln y_{CH_4}) + 3(\bar{S}_{O_2}(T_0, P_0) - \bar{R} \ln y_{O_2}) + 11.28(\bar{S}_{N_2}(T_0, P_0) - \bar{R} \ln y_{N_2}) \right. \\ \left. - [\bar{S}_{CO_2}(T_0, P_0) - \bar{R} \ln y_{CO_2}^e] - 2[\bar{S}_{H_2O}(T_0, P_0) - \bar{R} \ln y_{H_2O}^e] - [\bar{S}_{O_2}(T_0, P_0) - \bar{R} \ln y_{O_2}^e] \right. \\ \left. - 11.28[\bar{S}_{N_2}(T_0, P_0) - \bar{R} \ln y_{N_2}^e] \right] - T_0 \dot{S}_{cv} / \dot{n}_{CH_4} \quad (2)$$

Combining Eqs. (1), (2) gives the counterpart to Eq. 13.36.

Continued on next slide

Problem 13-98 continued

$$\bar{e}^{CH} = \left[\bar{g}_{CH_4} + 3\bar{g}_{O_2} + 11.28\bar{g}_{N_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O} - \bar{g}_{O_2} - 11.28\bar{g}_{N_2} \right] (T_0, P_0) \\ + R T_0 \ln \left\{ \frac{y_{CH_4} (y_{O_2})^3}{y_{CO_2}^2 (y_{H_2O})^2 y_{O_2}} \left[\frac{y_{N_2}}{y_{N_2}} \right]^{11.28} \right\}$$

Since $T_0 = 77^\circ F$, $P_0 = 1 \text{ atm}$, the $\bar{g}^0$'s can be read directly from Table A-30E. Accordingly

$$\bar{e}^{CH} = [-21,860 + 3(0) - (-169,680) - 2(-98,350) - (0)] + \\ (1.986 \times 537) \ln \left\{ \frac{(0.0654)(0.1963)^3}{(0.0003)(0.0312)^2 (0.2015)} \left[\frac{0.7382}{0.7567} \right]^{11.28} \right\} \\ \textcircled{1} \quad = 344,520 + 9329 = 353,849 \frac{\text{Btu}}{\text{lbmol}(\text{CH}_4)} \quad \leftarrow (a)$$

Since the mixture is at T_0, P_0 , and the effects of motion/gravity can be ignored, Equation 13.47 reduces to $\bar{e}_f = \bar{e}^{CH}$.

(b) For use with data from Table A-26, Eq. 13.41b may be invoked:

$$\bar{e}^{CH} = y_{CH_4} \bar{e}_{CH_4}^{CH} + y_{O_2} \bar{e}_{O_2}^{CH} + y_{N_2} \bar{e}_{N_2}^{CH} + R T_0 \left[y_{CH_4} \ln y_{CH_4} + y_{O_2} \ln y_{O_2} + y_{N_2} \ln y_{N_2} \right] \\ \text{The underlined term is} \\ = (8.314)(298.15) \left[0.0654 \ln(0.0654) + 0.1963 \ln(0.1963) + 0.7382 \ln(0.7382) \right] \\ = -1789.8 \frac{\text{kJ}}{\text{kmol}(\text{mix})}$$

$$\text{So} \\ \textcircled{2} \quad \bar{e}^{CH} = (0.0654) \bar{e}_{CH_4}^{CH} + (0.1963) \bar{e}_{O_2}^{CH} + (0.7382) \bar{e}_{N_2}^{CH} - 1789.8 \frac{\text{kJ}}{\text{kmol}(\text{mix})}$$

Model I:

$$\bar{e}^{CH} = (0.0654)(824,350) + (0.1963)(3950) + (0.7382)(640) - 1789.8 \\ = 53371 \text{ kJ/kmol}(\text{mix})$$

or, on a per mole of CH_4 basis,

$$\bar{e}^{CH} = \frac{53371 \text{ kJ/kmol}(\text{mix})}{0.0654 \text{ kmol}(\text{CH}_4)/\text{kmol}(\text{mix})} \left| \frac{0.42992 \text{ Btu/lbmol}}{1 \text{ kJ/kmol}} \right| = 350,845 \frac{\text{Btu}}{\text{lbmol}(\text{CH}_4)} \quad \leftarrow (b)$$

$$\text{Model II: } \bar{e}^{CH} = 354,395 \text{ Btu/lbmol}(\text{CH}_4)$$

1. The term involving Gibbs functions accounts for over 97% of the sum.
2. By comparing this calculation to the considerations of part (a), we see the relative simplicity of using standard chemical exergies.

Continued on next slide

Problem 13-98 continued

③ Model I:

$$\left. \begin{array}{l} \bar{e}_{O_2}^{CH} = 3950 \text{ kJ/kmol} \\ \bar{e}_{N_2}^{CH} = 640 \text{ kJ/kmol} \end{array} \right\} \begin{array}{l} 0.5(\bar{e}_{O_2}^{CH} + \bar{e}_{N_2}^{CH}) = \\ 0.5(3950 + 640) = 2295 \frac{\text{kJ}}{\text{kmol}} \end{array}$$

Model II

$$\left. \begin{array}{l} \bar{e}_{O_2}^{CH} = 3970 \text{ kJ/kmol} \\ \bar{e}_{N_2}^{CH} = 720 \text{ kJ/kmol} \end{array} \right\} \begin{array}{l} 0.5(\bar{e}_{O_2}^{CH} + \bar{e}_{N_2}^{CH}) = \\ 0.5(3970 + 720) = 2345 \frac{\text{kJ}}{\text{kmol}} \end{array}$$

Thus, combining these results with Eqs. (1), (2), (3), (5)

Model I:

$$e_f = \frac{2.6 + (2295 - 1718.2)}{30.01} = 19.3 \frac{\text{kJ}}{\text{kg}}$$

Model II:

$$e_f = \frac{2.6 + (2345 - 1718.2)}{30.01} = 21.0 \frac{\text{kJ}}{\text{kg}}$$

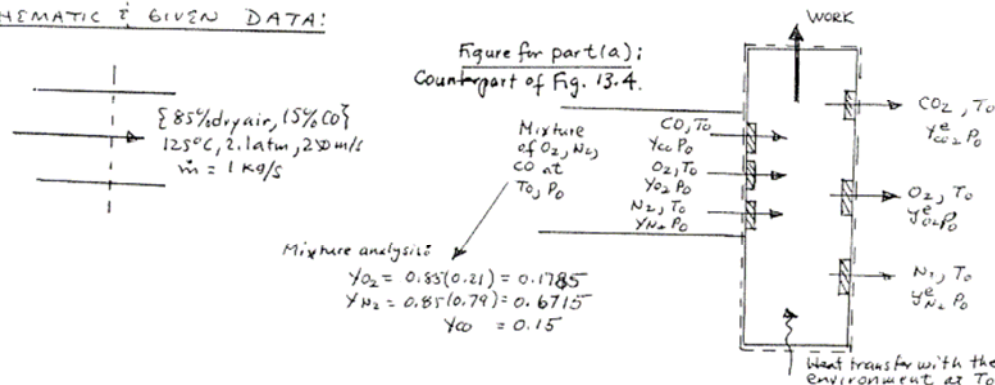
-
1. In any particular analysis, only one environmental model should be used.
 2. In this application, the chemical exergy is the largest contributor to e_f .
 3. We ignore the difference between $p_0 = 1 \text{ atm}$ used in the other calculations and the Table A-26 value of p_0 for Model I.

PROBLEM 13.99

KNOWN: A mixture having an analysis on a molar basis {85% dry air, 15% CO} enters a device at 125°C, 2.1 atm, 250 m/s with a mass flow rate of 1 kg/s.

FIND: Determine the rate that exergy enters using (a) the environment of Problem 13.89, (b) data from Table A-26.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) In evaluating the chemical exergy, the idealizations of Sec. 13.6 apply. (2) Dry air is modeled as 21% O_2 , 79% N_2 . (3) The effect of gravity can be ignored. (4) Ideal gas principles apply.

ANALYSIS: With assumption (3), Equation 13.47 reduces to

$$\underline{e_f} = \underline{h} - \underline{h}_0 - T_0(s - s_0) + \frac{V^2}{2} + a^{CH}$$

where the underlined term is the thermomechanical contribution, $\underline{e_f}^{TM}$, on a per kmol of mixture basis

$$\begin{aligned} \underline{e_f}^{TM} = & [0.15 (\bar{h}_{CO}(298K) - \bar{h}_{CO}(298K)) + 0.6715 (\bar{h}_{N_2}(298K) - \bar{h}_{N_2}(298K)) + 0.1785 (\bar{h}_{O_2}(298K) - \bar{h}_{O_2}(298K))] \\ & - 298 \left[0.15 (\bar{s}_{CO}^0(298K) - \bar{s}_{CO}^0(298K) - \bar{R} \ln \frac{P}{P_0}) + 0.6715 (\bar{s}_{N_2}^0(298K) - \bar{s}_{N_2}^0(298K) - \bar{R} \ln \frac{P}{P_0}) \right. \\ & \left. + 0.1785 (\bar{s}_{O_2}^0(298K) - \bar{s}_{O_2}^0(298K) - \bar{R} \ln \frac{P}{P_0}) \right] + M \frac{V^2}{2} \end{aligned}$$

where M denotes the mixture molecular weight:

$$M = (0.1785)(32) + (0.6715)(28.01) + (0.15)(28.01) = 28.72$$

Then, with data from the ideal gas tables

$$\begin{aligned} \underline{e_f}^{TM} = & [0.15 (11585 - 8669) + 0.6715 (11581 - 8669) + 0.1785 (11651 - 8682)] \\ & - 298 \left[0.15 (205.98 - 197.54 - 8.314 \ln \frac{2.1}{1}) + 0.6715 (199.92 - 191.50 - 8.314 \ln \frac{2.1}{1}) \right. \\ & \left. + 0.1785 (213.61 - 205.03 - 8.314 \ln \frac{2.1}{1}) \right] + \left(\frac{28.72 \text{ kg}}{2 \text{ kmol}} \right) \left(\frac{250 \text{ m}}{\text{s}} \right)^2 \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ = & [0.15 (2916) + 0.6715 (2912) + 0.1785 (2969)] - 298 [0.15 (2.2715) + 0.6715 (2.2515) \\ & + 0.1785 (2.4115)] + 897.5 \\ = & 3140 \frac{\text{kJ}}{\text{kmol (mix)}} \end{aligned}$$

or

$$\underline{e_f}^{TM} = \frac{3140}{28.72} = 109.3 \frac{\text{kJ}}{\text{kg (mix)}}$$

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Problem 13-99 continued

- (a) To find the chemical exergy relative to the environment of Problem 13.89, a procedure that parallels the one employed in Sec. 13.6 is used. The cell reaction is $\text{CO} + \frac{1}{2}\text{O}_2 \rightarrow \text{CO}_2$. Accordingly, for every mole of mixture, 0.15 moles of CO enter the cell and reacts with $0.15/2 = 0.075$ moles of O_2 , producing 0.15 moles of CO_2 . An energy balance on the cell reads

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{\text{mix}}} - \frac{\dot{W}_{cv}}{\dot{n}_{\text{mix}}} + [0.15 \bar{h}_{\text{CO}} + 0.1785 \bar{h}_{\text{O}_2} + 0.6715 \bar{h}_{\text{N}_2}] - [0.15 \bar{h}_{\text{CO}_2} + 0.1035 \bar{h}_{\text{O}_2} + 0.6715 \bar{h}_{\text{N}_2}]$$

An entropy balance reads

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{\text{mix}}}{T_0} + [0.15 \bar{s}_{\text{CO}}(T_0, y_{\text{CO}} P_0) + 0.1785 \bar{s}_{\text{O}_2}(T_0, y_{\text{O}_2} P_0) + 0.6715 \bar{s}_{\text{N}_2}(T_0, y_{\text{N}_2} P_0)] - [0.15 \bar{s}_{\text{CO}_2}(T_0, y_{\text{CO}_2}^e P_0) + 0.1035 \bar{s}_{\text{O}_2}(T_0, y_{\text{O}_2}^e P_0) + 0.6715 \bar{s}_{\text{N}_2}(T_0, y_{\text{N}_2}^e P_0)] + \frac{\dot{Q}_{cv}}{\dot{n}_{\text{mix}}}$$

Using Eq 13.34 as appropriate

$$-\frac{\dot{Q}_{cv}/\dot{n}_{\text{mix}}}{T_0} = 0.15 [\bar{s}_{\text{CO}}(T_0, P_0) - \bar{R} \ln y_{\text{CO}}] + 0.1785 [\bar{s}_{\text{O}_2}(T_0, P_0) - \bar{R} \ln y_{\text{O}_2}] + 0.6715 [\bar{s}_{\text{N}_2}(T_0, P_0) - \bar{R} \ln y_{\text{N}_2}] - [0.15 [\bar{s}_{\text{CO}_2}(T_0, P_0) - \bar{R} \ln y_{\text{CO}_2}^e] + 0.1035 [\bar{s}_{\text{O}_2}(T_0, P_0) - \bar{R} \ln y_{\text{O}_2}^e] + 0.6715 [\bar{s}_{\text{N}_2}(T_0, P_0) - \bar{R} \ln y_{\text{N}_2}^e]] + \frac{\dot{Q}_{cv}}{\dot{n}_{\text{mix}}}$$

Combining these results gives the counterpart to Eq 13.36

$$\begin{aligned} \bar{e}^{\text{CH}} &= 0.15 [\bar{g}_{\text{CO}} + \frac{1}{2} \bar{g}_{\text{O}_2} - \bar{g}_{\text{CO}_2}](T_0, P_0) + \bar{R} T_0 \left[0.15 \ln y_{\text{CO}} + 0.1785 \ln y_{\text{O}_2} - 0.15 \ln y_{\text{CO}_2}^e - 0.1035 \ln y_{\text{O}_2}^e + 0.6715 \ln \frac{y_{\text{N}_2}}{y_{\text{N}_2}^e} \right] \\ &= 0.15 [\bar{g}_{\text{CO}} + \frac{1}{2} \bar{g}_{\text{O}_2} - \bar{g}_{\text{CO}_2}](T_0, P_0) + \bar{R} T_0 \left[0.15 \ln \left(\frac{y_{\text{CO}}}{y_{\text{CO}_2}^e} \right) + 0.6715 \ln \frac{y_{\text{N}_2}}{y_{\text{N}_2}^e} + 0.1785 \ln y_{\text{O}_2} - 0.1035 \ln y_{\text{O}_2}^e \right] \end{aligned}$$

Since $T_0 = 25^\circ\text{C}$, $P_0 = 1 \text{ atm}$, the $\bar{g}$'s can be read directly from Table A-25.

Accordingly, the chemical exergy per mole of mixture is

$$\begin{aligned} \textcircled{1} \quad \bar{e}^{\text{CH}} &= 0.15 [(-137,150) + \frac{1}{2}(0) - (-394,380)] \\ &\quad + (8.314)(298) \left[0.15 \ln \left(\frac{0.15}{0.0003} \right) + 0.6715 \ln \left(\frac{0.6715}{0.7567} \right) + 0.1785 \ln(0.1785) - 0.1035 \ln(0.2035) \right] \\ &= 40,342 \text{ kJ/kmol(mix)} \end{aligned}$$

or on a unit mass of mixture basis

$$\bar{e}^{\text{CH}} = \frac{40,342}{28.72} = 1404.7 \frac{\text{kJ}}{\text{kg(mix)}}$$

Adding the $\bar{e}_f^{\text{TM}}$ and $\bar{e}^{\text{CH}}$ contributions

$$\textcircled{2} \quad \bar{e}_f = 109.3 + 1404.7 = 1514 \text{ kJ/kg}$$

For a mass flow rate of 1 kg/s

$$\dot{E}_f = \left(1 \frac{\text{kg}}{\text{s}} \right) \left(1514 \frac{\text{kJ}}{\text{kg}} \right) \left(\frac{1 \text{ kW}}{1 \text{ kJ/s}} \right) = 1514 \text{ kW} \quad \leftarrow (a)$$

Continued on next slide

Problem 13-99 continued

(b) For use with data from Table A-26, Eq. 13.41b may be invoked

$$\bar{e}^{CH} = y_{CO} \bar{e}_{CO}^{CH} + y_{O_2} \bar{e}_{O_2}^{CH} + y_{N_2} \bar{e}_{N_2}^{CH} + \bar{R}T_0 [y_{CO} \ln y_{CO} + y_{O_2} \ln y_{O_2} + y_{N_2} \ln y_{N_2}]$$

The underlined term is

$$= (8.314)(298.15) [0.15 \ln 0.15 + 0.1785 \ln 0.1785 + 0.6715 \ln 0.6715]$$

$$= -2130.8 \frac{\text{kJ}}{\text{kmol(mix)}}$$

So

$$\textcircled{3} \quad \bar{e}^{CH} = 0.15 \bar{e}_{CO}^{CH} + 0.1785 \bar{e}_{O_2}^{CH} + 0.6715 \bar{e}_{N_2}^{CH} - 2130.8$$

Model I

$$\bar{e}^{CH} = 0.15(269,410) + 0.1785(3950) + 0.6715(640) - 2130.8$$

$$= 39416 \text{ kJ/kmol(mix)}$$

or

$$e^{CH} = \frac{39416}{28.72} = 1372.4 \frac{\text{kJ}}{\text{kg(mix)}}$$

The sum is

$$e_f = 109.3 + 1372.4 = 1481.7 \frac{\text{kJ}}{\text{kg(mix)}}$$

For a mass flowrate of 1 kg/s

$$E_f = 1481.7 \text{ kW}$$

(6)

Model II

$$\bar{e}^{CH} = 40326 \text{ kJ/kmol}$$

$$e^{CH} = 1404.1 \text{ kJ/kg}$$

$$\dot{E}_f = 1513.4 \text{ kW}$$

(6)

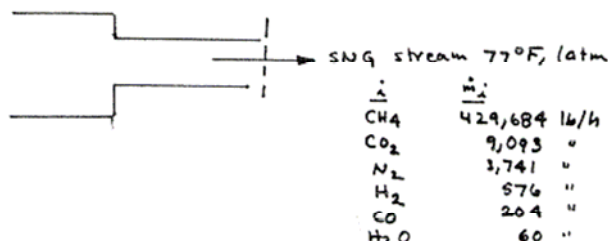
1. The term involving Gibbs functions accounts for over 95% of the sum.
2. In the present application, the chemical contribution dominates.
3. By comparing this calculation to the considerations of part (a), we see the relative simplicity of using standard chemical exergies.

PROBLEM 13.100

KNOWN: An SNG stream is flowing at 77°F, 1 atm. The mass flow rates of the components are specified.

FIND: Determine the flow exergy rate.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The SNG stream can be modeled as an ideal gas. (2) The effects of motion and gravity can be neglected. (3) The environment is described in Problem 13.89.

ANALYSIS: Since the stream is at 77°F, 1 atm, the thermomechanical contribution to the flow exergy vanishes, leaving only the chemical contribution. Thus

$$\dot{E}_f = \sum_i \dot{n}_i \bar{e}_i^{CH}$$

where $\dot{n}_i$ is the molar flow rate of component i and $\bar{e}_i^{CH}$ is its chemical exergy. For CO₂, N₂, H₂O, which appear in the environment,

$$\bar{e}_i^{CH} = \bar{R} T_0 \ln \frac{y_i}{y_i^e} \quad (1)$$

(See Problem 13.84d for elaboration.) As carbon monoxide does not appear in the environment, its contribution is

$$\begin{aligned} \bar{e}_{CO}^{CH} &= (\bar{g}_{CO} + \frac{1}{2} \bar{g}_{O_2} - \bar{g}_{CO_2})(T_0, P_0) + \bar{R} T_0 \ln \left[\frac{y_{CO} (y_{O_2}^e)^{1/2}}{y_{CO_2}^e} \right] \\ &= (\bar{g}_{CO} + \frac{1}{2} \bar{g}_{O_2} - \bar{g}_{CO_2})(T_0, P_0) + \bar{R} T_0 \ln \left[\frac{(y_{O_2}^e)^{1/2}}{y_{CO_2}^e} \right] + \bar{R} T_0 \ln y_{CO} \end{aligned} \quad (2)$$

(See Sec. 13.6 for discussion.) with data from Table A-25E

$$\bar{e}_{CO}^{CH} = 118,384 \frac{\text{Btu}}{\text{lbmol(CO)}} + \bar{R} T_0 \ln y_{CO} \quad (3)$$

where y_{CO} is the mole fraction of CO in the SNG stream. With similar reasoning

$$\bar{e}_{H_2}^{CH} = 101,122 \frac{\text{Btu}}{\text{lbmol(H}_2\text{)}} + \bar{R} T_0 \ln y_{H_2} \quad (4)$$

$$\bar{e}_{CH_4}^{CH} = 356,908 \frac{\text{Btu}}{\text{lbmol(CH}_4\text{)}} + \bar{R} T_0 \ln y_{CH_4} \quad (5)$$

The required molar flow rates and mole fractions are obtained using the mass flow rate data provided:

i	$\dot{M}_i$	$\dot{n}_i$ (lbmol/h)	$y_i = \dot{n}_i / \dot{n}$
CH ₄	16.04	26788.28	0.97680
CO ₂	44.01	206.61	0.00753
N ₂	28.01	133.56	0.00487
H ₂	2.018	285.43	0.01041
CO	28.01	7.28	0.00027
H ₂ O	18.02	3.33	0.00012

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Problem 13-100 continued

Using known data, Eq. (1) gives the following for CO_2 , N_2 , H_2O

$$\bar{e}_{\text{CO}_2}^{\text{CH}} = (1.986)(537) \ln \frac{0.00753}{0.0003} = 3437 \text{ Btu/lbmol}(\text{CO}_2)$$

$$\bar{e}_{\text{N}_2}^{\text{CH}} = (1.986)(537) \ln \frac{0.00487}{0.7567} = -5381 \text{ Btu/lbmol}(\text{N}_2)$$

$$\bar{e}_{\text{H}_2\text{O}}^{\text{CH}} = (1.986)(537) \ln \frac{0.00012}{0.00312} = -3475 \text{ Btu/lbmol}(\text{H}_2\text{O})$$

Also, Eqs. (3), (4), and (5) give the following for CO , H_2 , CH_4

$$\bar{e}_{\text{CO}}^{\text{CH}} = 118,384 + (1.986)(537) \ln(0.00027) = 109,621 \text{ Btu/lbmol}(\text{CO})$$

$$\bar{e}_{\text{H}_2}^{\text{CH}} = 101,122 + (1.986)(537) \ln(0.01041) = 96,254 \text{ Btu/lbmol}(\text{H}_2)$$

$$\bar{e}_{\text{CH}_4}^{\text{CH}} = 356,908 + (1.986)(537) \ln(0.97680) = 356,883 \text{ Btu/lbmol}(\text{CH}_4)$$

In summary

①	i	$(\dot{n}_i \bar{e}_i^{\text{CH}}) \times 10^{-5} \text{ (Btu/h)}$
	CH ₄	95602
	CO ₂	7
	N ₂	-7
	H ₂	275
	CO	8
	H ₂ O	---
		95885

Accordingly, $\dot{E}_f = 95885 \times 10^5 \text{ Btu/h}$ ←

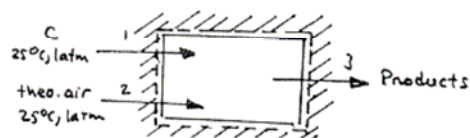
1. The CH₄ contribution clearly dominates.

PROBLEM 13.101

KNOWN: C at 25°C, 1 atm enters an insulated reactor and burns completely with the theoretical amount of air entering at 25°C, 1 atm.

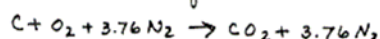
FIND: Determine (a) the rate of exergy destruction, and (b) evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases. (4) The environment is described in Problem 13.89. (5) The combustion air enters at the condition of the environment. (6) The products exit at 1 atm.

ANALYSIS: As a first step, an energy rate balance is applied to determine the temperature of the products of the reaction



That is

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_C} - \frac{\dot{W}_{cv}}{\dot{n}_C} + (\bar{h}_C)_1 + [\bar{h}_{O_2} + 3.76 \bar{h}_{N_2}]_2 - [\bar{h}_{CO_2} + 3.76 \bar{h}_{N_2}]_3$$

Then, with $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for C, N_2 , and O_2

$$0 = [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 3.76 [\bar{h}(T_3) - \bar{h}(298)]_{N_2}$$

or

$$\bar{h}_{CO_2}(T_3) + 3.76 \bar{h}_{N_2}(T_3) = -[\bar{h}_f^\circ - \bar{h}(298)]_{CO_2} + 3.76 \bar{h}_{N_2}(298)$$

With data from Table A-23

$$\bar{h}_{CO_2}(T_3) + 3.76 \bar{h}_{N_2}(T_3) = -[-393,520 - 9364] + 3.76 [8669] = 435,479 \Rightarrow T_3 \approx 2460 \text{ K}$$

(a) In this case where there is no heat transfer or work, the rate of exergy destruction is the difference between the exergy carried in with the fuel and the exergy carried out with the combustion products:

$$\dot{E}_D = \dot{E}_F - \dot{E}_{\text{products}}$$

① Also note that the combustion air enters at the condition of the environment, and thus with no exergy.

As the fuel enters at 25°C, 1 atm, there is no thermomechanical contribution, so $(\dot{E}_F/\dot{n}_C) = \bar{e}^{CH} = 410,541 \text{ kJ/kmol(C)}$, which is calculated in Problem 13.89(a)

The exergy of the products can be evaluated using Eq. 13.47. Since CO_2 and N_2 are present with the environment, Eq. 13.41a can be used to obtain $\bar{e}^{CH}$. Thus, with the dead gas principles

$$\begin{aligned} \frac{\dot{E}_{\text{products}}}{\dot{n}_C} &= 1 [\bar{h}(2460) - \bar{h}(298)]_{CO_2} + 3.76 [\bar{h}(2460) - \bar{h}(298)]_{N_2} \\ &- T_0 \left[1 \left[\bar{s}_{CO_2}^\circ(2460) - \bar{s}_{CO_2}^\circ(298) - R \ln \frac{Y_{CO_2} P}{Y_{CO_2} P_0} \right] + 3.76 \left[\bar{s}_{N_2}^\circ(2460) - \bar{s}_{N_2}^\circ(298) - R \ln \frac{Y_{N_2} P}{Y_{N_2} P_0} \right] \right] \\ &+ \left\{ R T_0 \left[1 \ln \frac{Y_{CO_2}}{Y_{CO_2}^\circ} + 3.76 \ln \frac{Y_{N_2}}{Y_{N_2}^\circ} \right] \right\} \end{aligned}$$

Continued on next slide

Problem 13-101 continued

Since $p = p_0$, the log terms drop out. Then, with data from Table A-23

$$\begin{aligned}\frac{\dot{E}_{\text{products}}}{\dot{n}_C} &= 1[128,833 - 43,664] + 3.76[81,515 - 86,697] - 298 \left\{ 1[321.814 - 213.685] + 3.76[259.480 - 191.502] \right\} \\ &\quad + 8.314(298) \left[1 \ln \frac{(1/4.76)}{0.003} + 3.76 \ln \frac{(3.76/4.76)}{0.7567} \right] \\ &= 301,612 \text{ kJ/kmol(C)}\end{aligned}$$

The rate of exergy destruction per unit mole of Carbon is

$$\frac{\dot{E}_d}{\dot{n}_C} = 410,541 - 301,612 = 108,929 \frac{\text{kJ}}{\text{kmol(C)}}$$

(b) An exergetic efficiency can be written as

$$\textcircled{2} \quad \epsilon = \frac{\dot{E}_{\text{products}}}{\dot{E}_F} = \frac{301,612}{410,541} = 0.735 \text{ (73.5\%)}$$

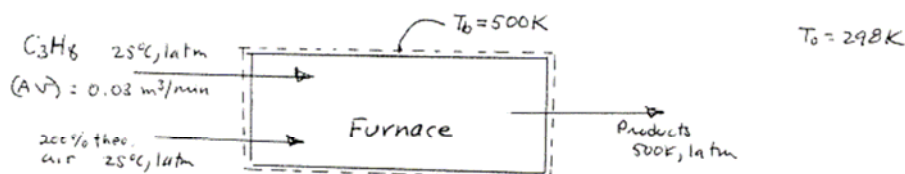
1. Throughout Chap. 13 combustion air is modeled as a binary mixture: 79% N_2 , 21% O_2 . As these mole fractions differ from those listed in Problem 13.89 for N_2 and O_2 , a small nonzero value for the exergy of the combustion air can be calculated using Eq. 13.41a; this is $\approx 479 \text{ kJ/kmol(C)}$.
2. Combustion is a significant source of irreversibility.

PROBLEM 13.102

KNOWN: Steady state operating data are provided for a furnace fueled by propane gas.

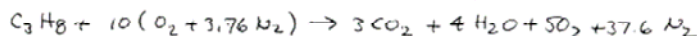
FIND: Compare the rate of exergy transfer accompanying heat transfer from the furnace with the rate of exergy destruction within the furnace.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic/potential energy effects. (2) 3.76 moles of inert N_2 accompany each mole of O_2 in the combustion air. (3) The fuel, air, and combustion products can be modeled as ideal gases. (4) For the environment, $T_0 = 298 \text{ K}$.

ANALYSIS: The balanced reaction equation for complete combustion of C_3H_8 with 200% of the theoretical amount of air is



An energy rate balance for the furnace reads

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{fuel} + 10\bar{h}_{O_2} + 37.6\bar{h}_{N_2}) - (3\bar{h}_{CO_2} + 4\bar{h}_{H_2O} + 5\bar{h}_{O_2} + 37.6\bar{h}_{N_2})$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and data from Table A-23

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} &= 3[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 4[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 5[\bar{h}_f^\circ + \Delta\bar{h}]_{O_2} + 37.6[\bar{h}_f^\circ + \Delta\bar{h}]_{N_2} - (\bar{h}_f^\circ)_{fuel} \\ &= 3[-393,520 + (17678 - 4364)] + 4[-241,820 + (16828 - 9904)] \\ &\quad + 5[14770 - 8682] + 37.6[14,581 - 8669] - (-103,830) \\ &= -1,738,621 \text{ kJ/kmol(fuel)} \end{aligned}$$

Using the ideal gas equation of state, the fuel molar flow rate is

$$\dot{n}_{fuel} = \frac{(\dot{A}V)}{v} = \frac{(0.03 \text{ m}^3/\text{min})(1.01325 \times 10^5 \text{ Pa}/2)}{(8314 \frac{\text{J}}{\text{kmol} \cdot \text{K}})(298 \text{ K})} = 0.00123 \frac{\text{kmol(fuel)}}{\text{min}}$$

So,

$$\dot{Q}_{cv} = -(1,738,621)(0.00123) = -2138.5 \frac{\text{kJ}}{\text{min}}$$

The accompanying exergy transfer is then

$$\begin{aligned} \left[\text{Rate of exergy transfer} \right]_{\text{accompanying heat transfer}} &= \left[1 - \frac{T_0}{T_b} \right] \dot{Q}_{cv} \\ &= \left[1 - \frac{298}{500} \right] (-2138.5 \frac{\text{kJ}}{\text{min}}) \\ &= -864 \frac{\text{kJ}}{\text{min}} \end{aligned}$$

Continued on next slide

Problem 13-102 continued

The rate of exergy destruction can be found in terms of the entropy production rate as $\dot{E}_d = T_0 \dot{\sigma}_{cv}$. The entropy production rate is obtained from an entropy balance:

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + \bar{s}_{fuel}(298K, p) + 10 \bar{s}_{O_2}(298, 0.21p) + 37.6(298, 0.79p) - [3 \bar{s}_{CO_2}(500K, y_{CO_2}p) + 4 \bar{s}_{H_2O}(500K, y_{H_2O}p) + 5 \bar{s}_{O_2}(500K, y_{O_2}p) + 37.6 \bar{s}_{N_2}(500K, y_{N_2}p)] + \dot{Q}_{cv}/\dot{n}_{fuel}$$

Reactants:

$$\bar{s}_{fuel}(298K, latm) = 269.91 \text{ kJ/kmol} \cdot K$$

$$\bar{s}_{O_2}(298, 0.21p) = \bar{s}^0(298K) - R \ln 0.21 = 205.07 - 8.314 \ln(0.21) = 218.01$$

$$\bar{s}_{N_2}(298, 0.79p) = 191.5 - 8.314 \ln(0.79) = 193.46$$

Products: $y_{CO_2} = 3/49.6$, $y_{O_2} = 5/49.6$, $y_{H_2O} = 4/49.6$, $y_{N_2} = 37.6/49.6$

$$\bar{s}_{CO_2}(500K, y_{CO_2}p) = \bar{s}_{CO_2}^0(500K) - R \ln y_{CO_2} = 234.819 - 8.314 \ln(3/49.6) = 258.14 \text{ kJ/kmol} \cdot K$$

$$\bar{s}_{H_2O}(500, y_{H_2O}p) = 206.413 - 8.314 \ln(4/49.6) = 227.35$$

$$\bar{s}_{O_2}(500, y_{O_2}p) = 220.589 - 8.314 \ln(5/49.6) = 239.67$$

$$\bar{s}_{N_2}(500, y_{N_2}p) = 206.63 - 8.314 \ln(37.6/49.6) = 209.93$$

Inserting values into the entropy rate balance

$$0 = \frac{-1,738,621 \text{ kJ/kmol(fuel)}}{500K} + 269.91 + 10(218.01) + 37.6(193.46) - 3(258.14) - 4(227.35) - 5(239.67) - 37.6(209.93) + \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}}$$

$$\Rightarrow \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 4491. \frac{\text{kJ/kmol(fuel)}}{K}$$

$$\Rightarrow \dot{E}_d = (298K) \left(1.23 \times 10^{-3} \frac{\text{kmol(fuel)}}{\text{min}} \right) \left(4491 \frac{\text{kJ}}{\text{kmol(fuel)}} \right) = 1646 \frac{\text{kJ}}{\text{min}}$$

Comparing the rates of exergy transfer and destruction, we get

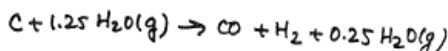
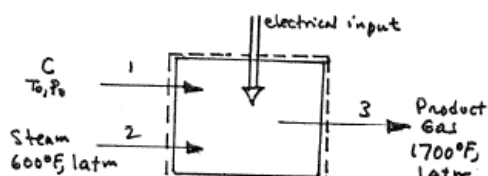
$$\frac{\dot{E}_d}{|\dot{E}_g|} = \frac{1646}{864} = 1.9 \text{ (approximately twice as big)}$$

PROBLEM 13.103

KNOWN: Steady-state operating data are provided for a coal gasification reactor making use of the carbon-steam process.

FIND: Determine per lbmol of Carbon entering (a) the required electrical input, (b) the exergy entering with the carbon, (c) the exergy entering with the steam, (d) the exergy exiting with the product gas, and (e) the exergy destruction.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{\Phi}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The combustion products can be modeled as an ideal gas mixture. (3) The environment is described in Problem 13.89.

ANALYSIS: (a) An energy rate balance reduces at steady state to

$$0 = \frac{\dot{\Phi}_{cv}}{\dot{n}_C} - \frac{\dot{W}_{cv}}{\dot{n}_C} + (\bar{h}_C)_1 + 1.25(\bar{h}_{H_2O})_1 - (\bar{h}_{CO} + \bar{h}_{H_2} + 0.25\bar{h}_{H_2O})_2$$

Noting that $(\bar{h}_C)_1 = (\bar{h}_f^\circ)_C(77^\circ F, 1 \text{ atm}) = 0$, and using IT to evaluate the other enthalpy values, $\dot{W}_{cv}/\dot{n}_C$ is found as follows:

T0 = 77 // °F
p0 = 1 // atm
T2 = 600 // °F
p2 = 1 // atm
T3 = 1700 // °F
p3 = 1 // atm

hC_in = 0
hH2O_in = h_T("H2O", T2)
hCO_out = h_T("CO", T3)
hH2_out = h_T("H2", T3)
hH2O_out = h_T("H2O", T3)

IT result:

$$\dot{W}_{cv}/\dot{n}_C = -7.848 \times 10^4 \text{ Btu/lbmol (C)} \leftarrow (a) \dot{W}_{cv}/\dot{n}_C$$

ndotC = 1

$$\dot{W}_{dotcv} = \text{ndotC} * (hC_in + 1.25 * hH2O_in - (hCO_out + hH2_out + 0.25 * hH2O_out))$$

(b) Since the carbon enters a T_0, p_0 , there is no thermomechanical contribution. The chemical exergy has been evaluated in Prob. 13.89 (a): $e^{CH} = 176,500$ Btu/lbmol (c), where the units have been converted from SI to English units.

(c) The exergy of the entering steam is given by Eq. 13.47. However, since water is a liquid at T_0, p_0 the chemical exergy of water relative to the environment of Problem 13.89 is zero (for discussion, see Problem 13.89 (i)). Thus, with steam table data, using $h_0 \approx h_f(T_0), s_0 \approx s_f(T_0)$, we get

$$(e_f)_2 = (h_2 - h_0) - T_0(s - s_0) = (1335.2 - 48.1) - 537(1.9787 - 0.08775) = 277.34 \text{ Btu/lb(steam)}$$

Continued on next slide

Problem 13-103 continued

Since 1.25 lbmol(steam) enters per lbmol(C)

$$\frac{\dot{E}_{\text{steam}}}{\dot{n}_c} = \left(277.34 \frac{\text{Btu}}{\text{lb(steam)}} \right) \left(\frac{18.02 \text{ lb}}{1 \text{ lbmol}} \right) \left(1.25 \frac{\text{lbmol(steam)}}{\text{lbmol(C)}} \right) = 6247 \frac{\text{Btu}}{\text{lbmol(C)}} \leftarrow (c)$$

(d) The combustion products exit as an ideal gas mixture at 2160°R, 1 atm. The thermomechanical contribution, per lbmol of C, is

$$\begin{aligned} \bar{h} - \bar{h}_o - T_o(\bar{s} - \bar{s}_o) = & 1 \left(\bar{h}(2160) - \bar{h}(537) - 537 \left(\bar{s}^o(2160) - \bar{s}^o(537) - 537 \ln \frac{y_{\text{CO}} P}{y_{\text{CO}} P_o} \right) \right)_{\text{CO}} + \\ & 1 \left(\bar{h}(2160) - \bar{h}(537) - 537 \left(\bar{s}^o(2160) - \bar{s}^o(537) - 537 \ln \frac{y_{\text{H}_2} P}{y_{\text{H}_2} P_o} \right) \right)_{\text{H}_2} + \\ & 0.25 \left(\bar{h}(2160) - \bar{h}(537) - 537 \left(\bar{s}^o(2160) - \bar{s}^o(537) - 537 \ln \frac{y_{\text{H}_2\text{O}} P}{y_{\text{H}_2\text{O}} P_o} \right) \right)_{\text{H}_2\text{O}} \end{aligned}$$

Using IT to evaluate the properties:

$$\begin{aligned} \text{efCO}_{\text{out}} &= h_T(\text{"CO"}, T3) - h_T(\text{"CO"}, 77) - 537 * (s_{\text{TP}}(\text{"CO"}, T3, 1) - s_{\text{TP}}(\text{"CO"}, 77, 1)) \\ \text{efH}_2_{\text{out}} &= h_T(\text{"H}_2", T3) - h_T(\text{"H}_2", 77) - 537 * (s_{\text{TP}}(\text{"H}_2", T3, 1) - s_{\text{TP}}(\text{"H}_2", 77, 1)) \\ \text{efH}_2\text{O}_{\text{out}} &= h_T(\text{"H}_2\text{O"}, T3) - h_T(\text{"H}_2\text{O"}, 77) - 537 * (s_{\text{TP}}(\text{"H}_2\text{O"}, T3, 1) - s_{\text{TP}}(\text{"H}_2\text{O"}, 77, 1)) \\ \text{ef3} &= \text{efCO}_{\text{out}} + \text{efH}_2_{\text{out}} + 0.25 * \text{efH}_2\text{O}_{\text{out}} \end{aligned}$$

The result is: $e_3 = 15,000 \text{ Btu/lbmol(C)}$

The chemical contribution to the exergy of the products is obtained by summing the contribution of each of the three components. First, for the water vapor

$$\bar{e}_{\text{H}_2\text{O}}^{\text{ch}} = \bar{R} T_o \ln \frac{y_{\text{H}_2\text{O}}}{y_{\text{H}_2\text{O}}^e} = (1.986)(537) \ln \left(\frac{0.25/2.25}{0.0312} \right) = 1354.6 \text{ Btu/lbmol(H}_2\text{O)}$$

For CO as a member of an ideal gas mixture

$$\bar{e}_{\text{CO}}^{\text{ch}} = \left[\bar{g}_{\text{CO}} + \frac{1}{2} \bar{g}_{\text{O}_2} - \bar{g}_{\text{CO}_2} \right] (T_o, P_o) + \bar{R} T_o \ln \left[\frac{y_{\text{CO}} (y_{\text{O}_2}^e)^{1/2}}{y_{\text{CO}_2}^e} \right]$$

With data from Table A-25

$$\begin{aligned} \bar{e}_{\text{CO}}^{\text{ch}} &= [-59,010 + \frac{1}{2}(0) - (-169,680)] + (1.986)(537) \ln \left[\frac{(1/2.25)(.2035)^{1/2}}{0.0003} \right] \\ &= 117,607.2 \text{ Btu/lbmol(CO)} \end{aligned}$$

Similarly, for H₂ as a member of an ideal gas mixture

$$\begin{aligned} \bar{e}_{\text{H}_2}^{\text{ch}} &= \left(\bar{g}_{\text{H}_2} + \frac{1}{2} \bar{g}_{\text{O}_2} - \bar{g}_{\text{H}_2\text{O}(g)} \right) (T_o, P_o) + \bar{R} T_o \ln \left[\frac{y_{\text{H}_2} (y_{\text{O}_2}^e)^{1/2}}{y_{\text{H}_2\text{O}}^e} \right] \\ &= (0 + \frac{1}{2}(0) - (-98,338)) + (1.986)(537) \ln \left(\frac{(1/2.25)(.2035)^{1/2}}{0.0312} \right) \\ &= 100,334.0 \text{ Btu/lbmol(H}_2\text{)} \end{aligned}$$

Accordingly, the chemical exergy of the mixture is

$$\bar{e}^{\text{ch}} = (1)(117,607.2) + (1)(100,334.0) + 0.25(1354.6) = 218,280 \text{ Btu/lbmol(C)}$$

Collecting results the exergy of the exiting products is

$$(\dot{E}_{\text{products}}/\dot{n}_c) = 15,000 + 218,280 = 233,280 \text{ Btu/lbmol(C)}$$

Continued on next slide

Problem 13-103 continued

(e) The rate of exergy destruction is the difference between the exergy entering and exiting. That is

$$\textcircled{1} \quad (\dot{E}_d/n_c) = 78,480 + 176,500 + 6247 - 233,280 = 27,947 \frac{\text{Btu}}{\text{lbmol(C)}} \leftarrow (e)$$

(electrical work) (carbon) (steam) (products)

1. An exergetic efficiency can be written as

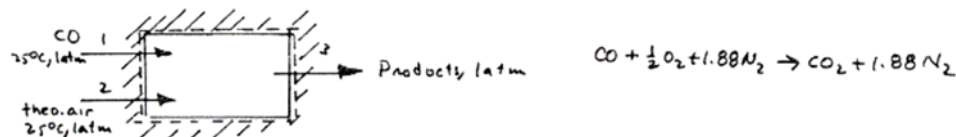
$$\epsilon = \frac{\dot{E}_{\text{products}}}{\dot{E}_{\text{in}}} = \frac{233,280}{(78,480 + 176,500 + 6247)} = 0.893 \quad (89.3\%)$$

PROBLEM 13.104

KNOWN: CO at 25°C, 1 atm enters an insulated reactor and burns with the theoretical amount of air entering separately at 25°C, 1 atm. Products exit at 1 atm.

FIND: Determine per mole of CO entering (a) the exergy entering with CO, (b) the exergy exiting with the products, (c) the rate of exergy destruction, (d) Evaluate an exergetic efficiency for the reactor.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The reaction is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and products of combustion can be modeled as ideal gases. (5) The environment is as described in Problem 13.89. (6) The combustion air enters at the condition of the environment.

ANALYSIS: The solution to Problem 13.76 gives $T_p = 2665\text{ K}$ and $\dot{Q}_{cv}/\dot{n}_{CO} = 165.3\text{ kJ/kmol(CO)} \cdot \text{K}$. The rate of exergy destruction is then

$$\frac{\dot{E}_d}{\dot{n}_{CO}} = T_0 \frac{\dot{Q}_{cv}}{\dot{n}_{CO}} = (298)(165.3) = 49,259 \frac{\text{kJ}}{\text{kmol(CO)}} \quad \leftarrow (c)$$

Since CO enters at T_0, P_0 , there is no thermomechanical contribution to its exergy. The chemical contribution, evaluated in Problem 13.89(d), is

$$\frac{\dot{E}_{CO}}{\dot{n}_{CO}} = 275,364 \frac{\text{kJ}}{\text{kmol(CO)}} \quad \leftarrow (a)$$

- ① Regarding the combustion air as entering with no exergy, the exergy of the products is the difference between the exergy entering with CO and the exergy destroyed:

$$\frac{\dot{E}_{\text{products}}}{\dot{n}_{CO}} = \frac{\dot{E}_{CO}}{\dot{n}_{CO}} - \frac{\dot{E}_d}{\dot{n}_{CO}} = (275,364 - 49,259) = 226,105 \frac{\text{kJ}}{\text{kmol(CO)}} \quad \leftarrow (b)$$

An exergetic efficiency can be written as

$$\epsilon = \frac{\dot{E}_{\text{products}}}{\dot{E}_{CO}} = \frac{226,105}{275,364} = 0.821 \text{ (82.1\%)} \quad \leftarrow (d)$$

- Throughout Chap. 13, combustion air is modeled as a binary mixture of N_2 and O_2 : 79% N_2 , 21% O_2 . As these mole fractions differ from those for N_2, O_2 in Problem 13.89, a small nonzero value for the exergy of combustion air can be calculated using Eq. 13.41a: $\approx 240\text{ kJ/kmol(CO)}$. This has been ignored in the analysis given.
- Alternatively, the exergy of the products can be found using Eq. 13.47, as follows: The thermomechanical contribution is

$$\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = 1 \left[\bar{h}(T_3) - \bar{h}(298) - T_0(\bar{s}(T_3) - \bar{s}(298)) - \bar{R} \ln \frac{y_{CO_2} P}{y_{CO} P_0} \right]_{CO} + 1.88 \left[\bar{h}(T_3) - \bar{h}(298) - T_0(\bar{s}(T_3) - \bar{s}(298)) - \bar{R} \ln \frac{y_{N_2} P}{y_{N_2} P_0} \right]_{N_2}$$

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Problem 13-104 continued

With data from Table A-23

$$\begin{aligned}\bar{h} - \bar{h}_o - T_o(\bar{s} - \bar{s}_o) &= 1 [141,459 - 9364 - 298(326.742 - 213.685)] \\ &\quad + 1.88 [89040 - 8669 - 298(262.420 - 191.502)] \\ &= 209,770 \text{ kJ/kmol(CO)}\end{aligned}$$

The chemical contribution, found with Eq. 13.41, is

$$\begin{aligned}\bar{e}^{ch} &= \bar{R} T_o \left[1 \ln \frac{y_{CO_2}}{y_{CO_2}^e} + 1.88 \ln \frac{y_{H_2O}}{y_{H_2O}^e} \right] \\ &= (8.314)(298) \left[1 \ln \left(\frac{1/2.88}{0.0003} \right) + 1.88 \ln \left(\frac{1.88/2.88}{0.7567} \right) \right] \\ &= 16,789 \text{ kJ/kmol(CO)}\end{aligned}$$

Adding these contributions

$$\frac{\dot{E}_{\text{products}}}{\dot{n}_{\text{CO}}} = 209,770 + 16,789 = 226,559 \text{ kJ/kmol(CO)}$$

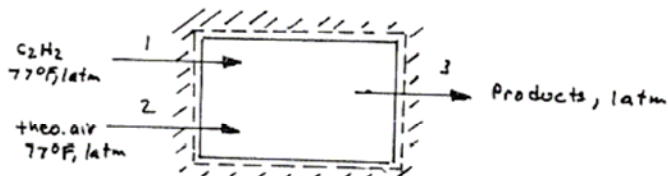
This calculation shows that, for the present application, the thermomechanical exergy is the dominant contribution to the exergy of the combustion products.

PROBLEM 13.105

KNOWN: C_2H_2 at $77^\circ F$, 1 atm enters an insulated reactor and burns completely with 180% of theoretical air at $77^\circ F$, 1 atm . Products at 1 atm exit.

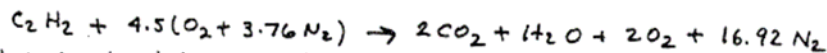
FIND: Determine, per mole of fuel, (a) the exergy of the fuel entering the reactor, (b) the exergy exiting with the products, (c) the rate of exergy destruction. Also, evaluate an exergetic efficiency for the reactor.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases. (4) The environment is as described in Problem 13.89. (5) The combustion air enters at the condition of the environment.

ANALYSIS: Complete combustion of C_2H_2 with 180% of the theoretical amount of air is described by



The first step is to determine the temperature of the exiting products using an energy rate balance at steady state:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{fuel})_1 + [4.5\bar{h}_{O_2} + 16.92\bar{h}_{N_2}]_2 - [2\bar{h}_{CO_2} + \bar{h}_{H_2O} + 2\bar{h}_{O_2} + 16.92\bar{h}_{N_2}]_3$$

Then, using $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{C_2H_2} + [0] - 2[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} - [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} - 2[\bar{h}(T_3) - \bar{h}(537)]_{O_2}$$

or with data from the ideal gas tables and Table A-25E

$$\begin{aligned} 2\bar{h}_{CO_2}(T_3) + \bar{h}_{H_2O}(T_3) + 2\bar{h}_{O_2}(T_3) + 16.92\bar{h}_{N_2}(T_3) &= (\bar{h}_f^\circ)_{C_2H_2} - 2[\bar{h}_f^\circ - \bar{h}(537)]_{CO_2} \\ &\quad - [\bar{h}_f^\circ - \bar{h}(537)]_{H_2O} + 2\bar{h}_{O_2}(537) + 16.92\bar{h}_{N_2}(537) \\ &= 97,540 - 2[-169,300 - 4028] - \\ &\quad [-104,040 - 4258] + 2[3725] + \\ &\quad 16.92[3730] = 623056 \Rightarrow T_3 = 3456^\circ R \end{aligned}$$

(a) Since C_2H_2 enters at T_0, P_0 , the exergy entering with the fuel is chemical exergy, obtained using Eq. 13.36 as follows:

$$\begin{aligned} \bar{e}_{CH}^{CH} &= [\bar{g}_{C_2H_2} + 2.5\bar{g}_{O_2} - 2\bar{g}_{CO_2} - \bar{g}_{H_2O(g)}](T_0, P_0) + \bar{R}T_0 \ln \left[\frac{(y_{O_2}^e)^{2.5}}{(y_{CO_2}^e)^2 (y_{H_2O}^e)} \right] \\ &= [87,990 + 2.5(0) - 2(-169,680) - (-98,350)] + (1.986)(537) \ln \left[\frac{(0.2035)^{2.5}}{(0.0003)^2 (0.0312)} \right] \\ &= 542,455 \text{ Btu/lbmol}(C_2H_2) \quad \text{--- (a)} \end{aligned}$$

Continued on next slide

Problem 13-105 continued

(b) The exergy exiting with the products can be evaluated using Eq. 13.47 with $\bar{e}^{ch}$ obtained from Eq. 13.41. Accordingly, when expressed on a per lbmol of fuel basis, we get

$$\begin{aligned}\bar{e}^{ch} &= \bar{R}T_0 \left[2 \ln \frac{y_{CO_2}}{y_{CO_2}^0} + \ln \frac{y_{H_2O}}{y_{H_2O}^0} + 2 \ln \frac{y_{O_2}}{y_{O_2}^0} + 16.92 \ln \frac{y_{N_2}}{y_{N_2}^0} \right] \\ &= (1.986)(537) \left[2 \ln \left(\frac{2/21.92}{0.0003} \right) + \ln \left(\frac{1/21.92}{0.0312} \right) + 2 \ln \left(\frac{2/21.92}{0.2035} \right) + 16.92 \ln \left(\frac{16.92/21.92}{0.7567} \right) \right] \\ &= 11,248 \text{ Btu/lbmol}(C_2H_2)\end{aligned}$$

And

$$\begin{aligned}\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) &= 2 \left[\bar{h}(3456) - \bar{h}(537) - 537(\bar{s}^0(3456) - \bar{s}^0(537) - \bar{R} \ln \frac{y_{CO_2}^0 P}{y_{CO_2} P}) \right]_{CO_2} + \\ &\quad \left[\bar{h}(3456) - \bar{h}(537) - 537(\bar{s}^0(3456) - \bar{s}^0(537) - \bar{R} \ln \frac{y_{H_2O}^0 P}{y_{H_2O} P}) \right]_{H_2O} + \\ &\quad 2 \left[\bar{h}(3456) - \bar{h}(537) - 537(\bar{s}^0(3456) - \bar{s}^0(537) - \bar{R} \ln \frac{y_{O_2}^0 P}{y_{O_2} P}) \right]_{O_2} + \\ &\quad 16.92 \left[\bar{h}(3456) - \bar{h}(537) - 537(\bar{s}^0(3456) - \bar{s}^0(537) - \bar{R} \ln \frac{y_{N_2}^0 P}{y_{N_2} P}) \right]_{N_2} \\ &= 2 \left[41330 - 4028 - 537(73.280 - 51.032) \right] + \left[33791 - 4258 - 537(62.724 - 45.079) \right] + \\ &\quad 2 \left[27879 - 3725 - 537(63.801 - 48.982) \right] + 16.92 \left[26639 - 3730 - 537(59.836 - 45.749) \right] \\ &= 362,730 \text{ Btu/lbmol}(C_2H_2)\end{aligned}$$

Adding these contributions

$$\dot{E}_{\text{products}}/\dot{n}_{C_2H_2} = 362,730 + 11,248 = 373,978 \text{ Btu/lbmol}(C_2H_2) \quad (6)$$

- (c) The rate of exergy destruction equals the difference between the exergy entering and the exergy exiting. As the combustion air enters at the condition of the environment, and thus with zero exergy, the exergy entering is just the result of part (a). The exergy exiting is the result of part (b). Accordingly

$$\frac{\dot{E}_d}{\dot{n}_{C_2H_2}} = 542,455 - 373,978 = 168,477 \text{ Btu/lbmol}(C_2H_2) \quad (c)$$

An exergetic efficiency can be expressed as

$$\epsilon = \frac{\dot{E}_{\text{products}}}{\dot{E}_{\text{fuel}}} = \frac{373,978}{542,455} = 0.689 \text{ (68.9)\%}$$

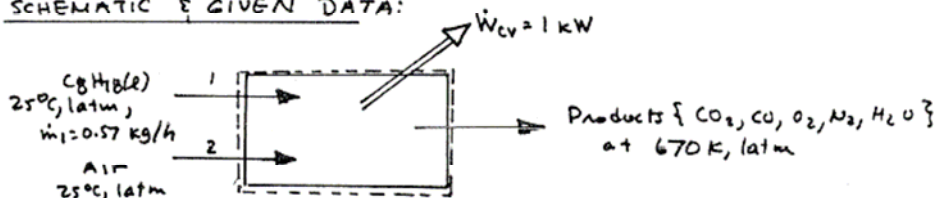
1. Throughout Chap. 13 combustion air is modeled as 79% N_2 , 21% O_2 . As these values differ from the percentages listed for N_2 , O_2 in Problem 13.89, a small nonzero value for the chemical exergy of the combustion air can be calculated via Eq. 13.41a. This is ignored in the present analysis.

PROBLEM 13.106

KNOWN: $C_8H_{18}(l)$ at $25^\circ C$, 1 atm and a mass flow rate of 0.57 kg/h enters an engine and burns with air entering at $25^\circ C$, 1 atm . Products exit at 670 K , 1 atm with a dry molar analysis: $11.4\% \text{ CO}_2$, $2.9\% \text{ CO}$, $1.6\% \text{ O}_2$, $84.1\% \text{ N}_2$. The engine develops power at a rate of 1 kW .

FIND: Determine (a) the rate of heat transfer, (b) Evaluate an exergetic efficiency for the engine.

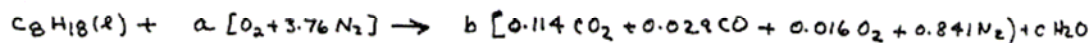
SCHEMATIC & GIVEN DATA:



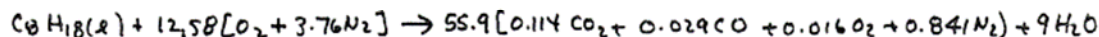
ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases. (4) The environment is described as in Problem 13.89. (5) The combustion air enters at the condition of the environment.

ANALYSIS: The reaction equation takes the form



From a carbon balance $b = 55.9$. From a hydrogen balance $c = 9$. From an oxygen balance $a = 12.58$. (Checking the nitrogen balance for closure: $3.76a = 0.841 b$ or $47.3 \approx 47.01$.) Accordingly, the balance reaction equation is



(a) From an energy rate balance at steady state

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{fuel})_1 + [12.58 \bar{h}_{O_2} + (12.58 \times 3.76) \bar{h}_{N_2}]_1 - \{ 55.9 (0.114 \bar{h}_{CO_2} + 0.029 \bar{h}_{CO} + 0.016 \bar{h}_{O_2} + 0.841 \bar{h}_{N_2}) \}_3 + 9 (\bar{h}_{H_2O})_3 \}$$

Introducing $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} - (\bar{h}_f^\circ)_{C_8H_{18}(l)} + 55.9 [0.114 [\bar{h}_f^\circ + \bar{h}(670) - \bar{h}(298)]_{CO_2} + 0.029 [\bar{h}_f^\circ + \bar{h}(670) - \bar{h}(298)]_{CO} + 0.016 [\bar{h}(670) - \bar{h}(298)]_{O_2} + 0.841 [\bar{h}(670) - \bar{h}(298)]_{N_2}] + 9 [\bar{h}_f^\circ + \bar{h}(670) - \bar{h}(298)]_{H_2O}$$

$$= \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} - (-249,910) + 55.9 [0.114 [-393,520 + 25648 - 9364] + 0.029 [-110,530 + 19,758 - 8669] + 0.016 [20,197 - 8682] + 0.841 [17,685 - 8669]] + 9 [-241,820 + 22,970 - 9904]$$

$$= \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} - 3,845,872 \frac{\text{kJ}}{\text{kmol } (C_8H_{18})}$$

Continued on next slide

Problem 13-106 continued

The molar flow rate of the fuel is $\dot{n}_{\text{fuel}} = \dot{m}_{\text{fuel}} / M_{\text{fuel}}$. Thus

$$\dot{Q}_{\text{cv}} = 1 \text{ kW} - \left(\frac{0.57 \text{ kg/h}}{114.22 \text{ kg/kmol}} \right) \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left(3,845,872 \frac{\text{kJ}}{\text{kmol}} \right) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= -4.33 \text{ kW} \quad \leftarrow$$

(b) Paralleling the analysis of Example 13.15, an exergetic efficiency takes the form

$$\epsilon = \frac{\dot{W}_{\text{cv}}}{\dot{E}_F}$$

where $\dot{E}_F$ is the rate exergy enters the engine with the fuel.

Since the fuel enters at T_0, p_0 , there is no thermomechanical contribution to its exergy. The chemical exergy is evaluated in Example 13.12(a) as $\bar{e}^{\text{CH}} = 5,407,843 \text{ kJ/kmol} (\text{C}_8\text{H}_{18})$. Then

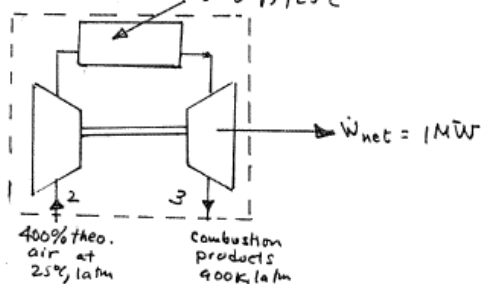
$$\epsilon = \frac{1 \text{ kW}}{\left(5,407,843 \frac{\text{kJ}}{\text{kmol}} \right) \left(\frac{0.57 \text{ kmol}}{114.22 \text{ h}} \right) \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|} = 0.133 \text{ (13.3\%)} \quad \leftarrow$$

PROBLEM 13.107

KNOWN: Steady state operating data are provided for a gas turbine power plant.

FIND: Evaluate an exergetic efficiency for the power plant.

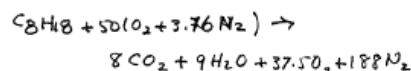
SCHEMATIC & GIVEN DATA: $C_8H_{18}(g), 25^\circ C$



$$\dot{Q}_{cv} = -0.15 \text{ MW}$$

From solution to Problem 13.44,

$$\dot{n}_{fuel} = 7.57 \frac{\text{kmol}}{\text{h}}$$



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (3) The ideal gas model is applicable to the fuel, combustion air, and the products of combustion. (4) The exergy of the air entering the control volume at 2 can be ignored. (5) Exergy values are evaluated relative to the environment of Problem 13.89.

ANALYSIS: An exergy rate balance for the control volume reads

$$0 = \dot{E}_{fuel} + \dot{E}_{air,2} - \dot{E}_{prod,3} - \dot{W}_{net} - \sum [1 - T_0/T_j] |\dot{Q}_j| - \dot{E}_d$$

Rearranging

$$\dot{E}_{fuel} + \dot{E}_{air,2} = \dot{W}_{net} + \dot{E}_{prod,3} + \sum [1 - T_0/T_j] |\dot{Q}_j| + \dot{E}_d$$

(i) (ii) (iii)

That is, the exergy entering the control volume with the fuel and combustion air exits as (i) net power out, (ii) with the combustion products, (iii) with heat transfer or is destroyed by irreversibilities within the control volume. An exergetic efficiency can be written as

$$\epsilon = \frac{\dot{W}_{net} + \dot{E}_{prod,3}}{\dot{E}_{fuel}} \quad (1)$$

- ① In writing this, $\dot{E}_{air,2}$ has been ignored (assumption 4) and the exergy exiting with heat transfer has been taken as a loss. The exergy of the products $\dot{E}_{prod,3}$ has been included in the numerator, together with $\dot{W}_{net}$, in recognition of its considerable utility (see Sects. 9.7, 9.10 for discussion).

The value of $\bar{e}_{fuel}^{CH}$ is found as in Example 13.12(a) except here the octane is considered to be a gas. The result is

$$\bar{e}_{fuel}^{CH} = 5,418,553 \text{ kJ/kmol(fuel)}$$

Since the fuel enters at $25^\circ C$ and ideal gas principles are assumed to apply, Equation 13.47 reads

Continued on next slide

Problem 13-107 continued

$$\bar{e}_{f, \text{fuel}} = \underbrace{h(T_0) - h(T_0)}_{=0} - T_0 \left[\underbrace{\bar{s}^0(T_0) - \bar{s}^0(T_0)}_{=0} - \bar{R} \ln \frac{P}{P_0} \right] + \bar{e}^{\text{ch}}$$

That is, only the term $\bar{R} T_0 \ln P/P_0$ remains of the thermomechanical contribution. This term is ignored relative to $\bar{e}^{\text{ch}}$ (sample calculations indicate that the term would be on the order of 0.1 to 0.2% of $\bar{e}^{\text{ch}}$).

For the combustion gases exiting at 3, $\bar{e}_{f,3}$ can be evaluated as in Example 13.14. First, the molar analysis of the products is found:

$$\text{CO}_2 : \frac{8}{242.5} = 0.033$$

$$\text{O}_2 : \frac{37.5}{242.5} = 0.155$$

$$\text{H}_2\text{O} : \frac{9}{242.5} = 0.037$$

$$\text{N}_2 : \frac{188}{242.5} = 0.775$$

Then

$$\begin{aligned} \bar{e}_3^{\text{ch}} &= (8.314)(298) \left[8 \ln \left(\frac{3.3}{0.03} \right) + 9 \ln \left(\frac{3.7}{3.12} \right) + 37.5 \ln \left(\frac{15.5}{20.95} \right) + 188 \ln \left(\frac{77.5}{75.67} \right) \right] \\ &= 82,800 \text{ kJ/kmol (fuel)} \end{aligned}$$

(per mole of fuel)

And

$$\begin{aligned} \bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) &= \left[8 [37,405 - 9364 - 298(263.559 - 213.685)] + \right. \\ &\quad \left. 9 [31,828 - 9904 - 298(228.321 - 188.720)] + \right. \\ &\quad \left. 37.5 [27,928 - 8682 - 298(239.822 - 205.033)] + \right. \\ &\quad \left. 188 [26,890 - 8669 - 298(224.647 - 191.502)] \right] \\ &= 2,098,114 \text{ kJ/kmol (fuel)} \end{aligned}$$

Summing

$$\bar{e}_{f,3} = 2,098,114 + 82,800 = 2,180,914 \text{ kJ/kmol (fuel)}$$

Returning to Eq. (1)

$$\begin{aligned} \epsilon &= \frac{10^3 \text{ kJ/s} + \left(\frac{7.57}{3600} \right) (2,180,914)}{\left(\frac{7.57}{3600} \right) (5,418,553)} = \frac{5586}{11390} \\ &= 0.49 \quad (49\%) \end{aligned}$$

1. If all heat transfer from the turbine takes place at temperature T_0 , the net exergy exit with heat transfer is evaluated as

$$\left[1 - \frac{T_0}{T_b} \right] \dot{Q}_{\text{ex}} = \left[1 - \frac{T_0}{T_b} \right] (150 \text{ kJ/s})$$

As a sample calculation, consider $T_b = 500\text{K}$ then

$$\left[1 - \frac{298}{500} \right] (150 \text{ kJ/s}) = 61 \text{ kJ/s}$$

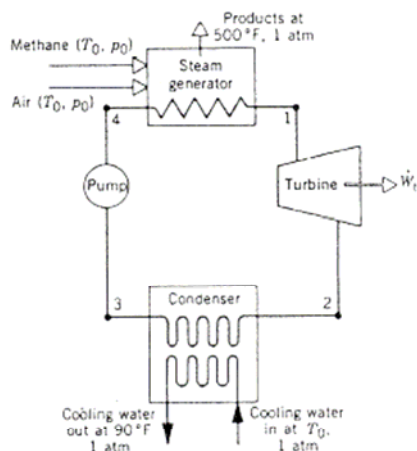
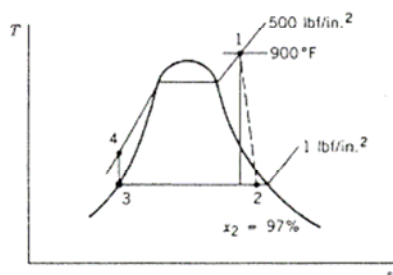
If this amount would be included with the numerator of Eq. (1), the value of ϵ would be $\approx 49.6\%$.

PROBLEM 13.108

KNOWN: CH_4 enters the steam generator of a vapor power plant and burns completely with 200% of theoretical air at 77°F , 1 atm. Additional steady-state operating data are provided.

FIND: Determine (a) the balanced reaction equation, (b) the vapor mass flow rate and (c) the cooling water mass flowrate, each per mole of fuel. (d) Also, find as a percentage of the exergy entering with the fuel each of the following: exergy exiting with the stack gases, exergy destroyed in the steam generator, power developed by the turbine, exergy destroyed in the turbine, exergy exiting with the cooling water, exergy destroyed in the condenser.

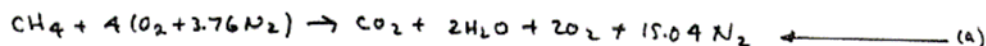
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) Each component can be modeled as a control volume at steady state with negligible energy loss by heat transfer. Kinetic and potential energy effects can be ignored. (2) For the vapor power plant, pump work can be neglected and saturated liquid exits the condenser. (3) Combustion is complete. (4) 3.76 moles of N_2 accompany each mole of air. N_2 is inert. (5) The environment is described as in Problem 13.89. (6) The combustion air and combustion products can be modeled as ideal gases. (7) The air enters at the condition of the environment.

ANALYSIS: (a) The balanced reaction equation for complete combustion with 200% of the theoretical amount of air is



(b) An energy rate balance at steady state for a control volume enclosing the steam generator reads

$$0 = \frac{\dot{Q}_{\text{SG}}}{\dot{n}_{\text{CH}_4}} - \frac{\dot{W}_{\text{SG}}}{\dot{n}_{\text{CH}_4}} + \bar{h}_{\text{CH}_4} + (4\bar{h}_{\text{O}_2} + 15.04\bar{h}_{\text{N}_2}) - [\bar{h}_{\text{CO}_2} + 2\bar{h}_{\text{H}_2\text{O}} + 2\bar{h}_{\text{O}_2} + 15.04\bar{h}_{\text{N}_2}] + \frac{\dot{m}}{\dot{n}_{\text{CH}_4}}(h_4 - h_1)$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{\text{CH}_4} + (0) - [\bar{h}_f^\circ + \bar{h}(960) - \bar{h}(537)]_{\text{CO}_2} - 2[\bar{h}_f^\circ + \bar{h}(960) - \bar{h}(537)]_{\text{H}_2\text{O}} - 2[\bar{h}(960) - \bar{h}(537)]_{\text{O}_2} - 15.04[\bar{h}(960) - \bar{h}(537)]_{\text{N}_2} + \frac{\dot{m}}{\dot{n}_{\text{CH}_4}}(h_4 - h_1)$$

$$\text{or} \quad \frac{\dot{m}}{\dot{n}_{\text{CH}_4}} = \frac{(\bar{h}_f^\circ)_{\text{CH}_4} - [\bar{h}_f^\circ + \bar{h}(960) - \bar{h}(537)]_{\text{CO}_2} - 2[\bar{h}_f^\circ + \bar{h}(960) - \bar{h}(537)]_{\text{H}_2\text{O}} - 2[\bar{h}(960) - \bar{h}(537)]_{\text{O}_2} - 15.04[\bar{h}(960) - \bar{h}(537)]_{\text{N}_2}}{h_1 - h_4}$$

Continued on next slide

Problem 13-108 continued

From Table A-4E, $h_1 = 1466.5 \text{ Btu/lb}$, $s_1 = 1.6987 \text{ Btu/lb} \cdot \text{OR}$. From Table A-3E, $h_3 = 69.74 \text{ Btu/lb}$, $s_3 = 0.1327 \text{ Btu/lb} \cdot \text{OR}$. Then, with $s_4 = s_3$ and $P_4 = 500 \text{ lb/in}^2$, Table A-5E gives $h_4 = 71.33 \text{ Btu/lb}$. Then, with data from the ideal gas tables and Table A-25E

$$\frac{\dot{m}_v}{\dot{m}_{\text{CH}_4}} = \frac{-32,210 - [-169,300 + 8244 - 4028] - 2[-104,040 + 7738 - 4258] - 2[6786 - 3725] - 15.04[6693 - 3730]}{(1466.5 - 71.33)}$$

$$= 203.1 \text{ lb(vapor)/lbmol(CH}_4\text{)} \quad \leftarrow (b)$$

(c) An energy rate balance at steady state for a control volume enclosing the condenser reads

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}_v [h_2 - h_3] + \dot{m}_{\text{cw}} [h_{\text{cw},\text{in}} - h_{\text{cw},\text{out}}]$$

To find h_2 write

$$h_2 = h_f + x_2(h_g - h_f) = 69.74 + (0.97)(1036) = 1074.66 \text{ Btu/lb}$$

Then, evaluating $h_{\text{cw}} \approx h_f(T)$

$$\frac{\dot{m}_{\text{cw}}}{\dot{m}_{\text{CH}_4}} = \left(\frac{\dot{m}_v}{\dot{m}_{\text{CH}_4}} \right) \left[\frac{h_2 - h_3}{h_{\text{cw},\text{out}} - h_{\text{cw},\text{in}}} \right] = (203.1) \left[\frac{1074.66 - 69.74}{58.07 - 45.09} \right] = 15,724 \frac{\text{lb(cw)}}{\text{lbmol(CH}_4\text{)}} \quad (c)$$

(d) As the fuel enters at T_0, P_0 , the exergy entering with the fuel is its chemical exergy, found from Eq. 13.36:

$$\begin{aligned} \bar{e}_{\text{CH}_4}^{\text{ch}} &= (\bar{g}_{\text{CH}_4} + 2\bar{g}_{\text{O}_2} - \bar{g}_{\text{CO}_2} - 2\bar{g}_{\text{H}_2\text{O}})(T_0, P_0) + R T_0 \ln \left[\frac{(y_{\text{O}_2}^e)^2}{(y_{\text{CO}_2}^e)(y_{\text{H}_2\text{O}}^e)^2} \right] \\ &= (-21,860 + 2(0) - (-169,680) - 2(-98,320)) + (1.986)(537) \ln \left[\frac{(0.2035)^2}{(0.0003)(0.0712)^2} \right] \\ &= 357,171 \text{ Btu/lbmol} \end{aligned}$$

Stack Gases: The exergy exiting with the stack gases can be found using Eq. 13.47 where $\bar{e}^{\text{ch}}$ is obtained with Eq. 13.41a. Thus, on a per lbmol of fuel basis

$$\begin{aligned} \bar{e}^{\text{ch}} &= (1.986)(537) \left[\ln \left(\frac{1/20.04}{0.0003} \right) + 2 \ln \left(\frac{2/20.04}{0.0712} \right) + 2 \ln \left(\frac{2/20.04}{0.2035} \right) + 15.04 \ln \left(\frac{15.04/20.04}{0.7567} \right) \right] \\ &= 62,924 \text{ Btu/lbmol(CH}_4\text{)} \end{aligned}$$

And

$$\begin{aligned} \bar{h} - \bar{h} - T_0(\bar{s} - \bar{s}_0) &= [9243.8 - 4027.5 - 537(56.765 - 51.032)]_{\text{CO}_2} + 2[7738 - 4258 - 537(49.843 - 45.079)]_{\text{H}_2\text{O}} \\ &\quad + 2[6786 - 3725.1 - 537(53.170 - 48.982)]_{\text{O}_2} + 15.04[6693.1 - 3729.5 - 537(49.808 - 45.743)]_{\text{N}_2} \\ &= 16,046.8 \text{ Btu/lbmol(CH}_4\text{)} \end{aligned}$$

Adding these contributions, $\dot{E}_{\text{stack}}/\dot{m}_{\text{CH}_4} = 22,329 \text{ Btu/lbmol(CH}_4\text{)}$. Thus

$$\left(\frac{\% \text{ of fuel exergy exiting with stack gases}}{\% \text{ of fuel exergy exiting with stack gases}} \right) = \left(\frac{22,329}{357,171} \right) (100) = 6.25 \%$$

Steam Generator: The exergy destroyed in the steam generator equals the difference between the total exergy entering and the total exergy exiting. Regarding the exergy of the entering combustion air as negligible

$$\frac{\dot{E}_d}{\dot{m}_{\text{CH}_4}} = \bar{e}_{\text{CH}_4}^{\text{ch}} + \frac{\dot{m}_v}{\dot{m}_{\text{CH}_4}} [(h_4 - h_1) - T_0(s_4 - s_1)] - \frac{\dot{E}_{\text{stack gas}}}{\dot{m}_{\text{CH}_4}}$$

Continued on next slide

Problem 13-108 continued

Thus

$$\frac{\dot{E}_d}{\dot{n}_{\text{CH}_4}} = 357,171 - 203.1 [1466.5 - 71.33 - 537(1.6987 - 0.1327)] - 22,329$$

$$= 222,278 \text{ Btu/lbmol (CH}_4\text{)}$$

And

$$\textcircled{1} \left[\begin{array}{l} \% \text{ of fuel exergy} \\ \text{destroyed in the} \\ \text{steam generator} \end{array} \right] = \left(\frac{222,278}{357,171} \right) (100) = 62.23\%$$

Turbine. The power developed by the turbine is obtained from an energy rate balance at steady state for a control volume enclosing the turbine:

$$\frac{\dot{W}_{\text{cv}}}{\dot{n}_{\text{CH}_4}} = \left(\frac{\dot{m}}{\dot{n}_{\text{CH}_4}} \right) (h_1 - h_2) = (203.1)(1466.5 - 1074.66) = 79,583 \text{ Btu/lbmol (CH}_4\text{)}$$

The rate of exergy destruction is

$$\frac{\dot{E}_d}{\dot{n}_{\text{CH}_4}} = T_0 \left(\frac{\dot{m}}{\dot{n}_{\text{CH}_4}} \right) (s_2 - s_1) = (537)(203.1) [0.1327 + 0.97(1.8453)] - 1.6987]$$

$$= 24,424 \text{ Btu/lbmol (CH}_4\text{)}$$

Accordingly

$$\left[\begin{array}{l} \% \text{ of fuel exergy} \\ \text{exiting as developed power} \end{array} \right] = \left(\frac{79,583}{357,171} \right) (100) = 22.28$$

$$\left[\begin{array}{l} \% \text{ of fuel exergy} \\ \text{destroyed in the turbine} \end{array} \right] = \left(\frac{24,424}{357,171} \right) (100) = 6.84$$

Condenser. The net exergy exiting with the cooling water is

$$\frac{\dot{E}_{\text{cw}}}{\dot{n}_{\text{CH}_4}} = \frac{\dot{m}_{\text{cw}}}{\dot{n}_{\text{CH}_4}} [(h_{\text{cw,out}} - h_{\text{cw,in}}) - T_0 (s_{\text{cw,out}} - s_{\text{cw,in}})]$$

$$= 15,724 [(58.07 - 45.09) - 537 (0.1117 - 0.08775)]$$

$$= 1869 \text{ Btu/lbmol (CH}_4\text{)}$$

The exergy destroyed in the condenser is

$$\frac{\dot{E}_d}{\dot{n}_{\text{CH}_4}} = T_0 \left[\frac{\dot{m}_{\text{cw}}}{\dot{n}_{\text{CH}_4}} (s_{\text{cw,out}} - s_{\text{cw,in}}) + \frac{\dot{m}}{\dot{n}_{\text{CH}_4}} (s_3 - s_2) \right]$$

$$= 537 [15,724 (0.1117 - 0.08775) + 203.1 (0.1327 - 1.9226)]$$

$$= 7013 \text{ Btu/lbmol (CH}_4\text{)}$$

Accordingly

$$\left[\begin{array}{l} \% \text{ of fuel exergy} \\ \text{carried out with cooling water} \end{array} \right] = \left(\frac{1869}{357,171} \right) (100) = 0.52$$

$$\left[\begin{array}{l} \% \text{ of fuel exergy} \\ \text{destroyed in the condenser} \end{array} \right] = \left(\frac{7013}{357,171} \right) (100) = 1.96$$

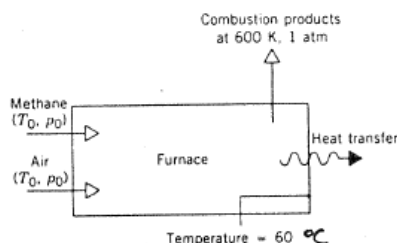
1. The steam generator is, by far, the most significant site of irreversibility.

PROBLEM 13.109

KNOWN: CH_4 at 25°C , 1 atm enters a furnace and burns completely with 200% theoretical air entering at 25°C , 1 atm. Combustion products exit at 600 K , 1 atm and the furnace delivers energy by heat transfer at $T_s = 333\text{ K}$ (60°C).

FIND: Determine in kJ per kmol of fuel (a) the exergy entering with the fuel, (b) the exergy exiting with the products, (c) the rate of exergy destruction. Also, evaluate an exergetic efficiency for the furnace.

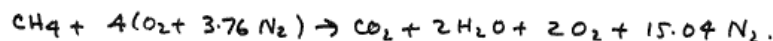
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and combustion products can be modeled as ideal gases. (5) The environment is as described in Problem 13.89. (6) The combustion air enters at the condition of the environment.

ANALYSIS: The balanced reaction equation for complete combustion with 200% theoretical air is



(a) The fuel enters at T_0, p_0 . Accordingly, there is no thermomechanical contribution, and the exergy entering with the fuel is $\bar{e}^{\text{ch}}$, which is determined in Problem 13.89(c) as $\bar{e}_{\text{CH}_4}^{\text{ch}} = 830,174 \text{ kJ/kmol}$. (a)

(b) The exergy exiting with the combustion products can be evaluated using Eq. 13.47 where $\bar{e}^{\text{ch}}$ is obtained with Eq. 13.41a. Thus, on a per kmol of CH_4 basis

$$\begin{aligned} \bar{e}^{\text{ch}} &= \bar{R}T_0 \left[\ln \frac{y_{\text{CO}_2}}{y_{\text{CO}_2}^e} + 2 \ln \frac{y_{\text{H}_2\text{O}}}{y_{\text{H}_2\text{O}}^e} + 2 \ln \frac{y_{\text{O}_2}}{y_{\text{O}_2}^e} + 15.04 \ln \frac{y_{\text{N}_2}}{y_{\text{N}_2}^e} \right] \\ &= (8.314)(298) \left[\ln \left(\frac{1/20.04}{0.0003} \right) + 2 \ln \left(\frac{2/20.04}{0.0812} \right) + 2 \ln \left(\frac{2/20.04}{0.2035} \right) + 15.04 \ln \left(\frac{15.04/20.04}{0.7567} \right) \right] \\ &= 14595 \text{ kJ/kmol}(\text{CH}_4) \end{aligned}$$

And

$$\begin{aligned} \bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) &= [22,280 - 9364 - 298(243.199 - 213.685)] \text{CO}_2 + \\ &\quad 2[20,402 - 9904 - 298(212.420 - 188.720)] \text{H}_2\text{O} + \\ &\quad 2[17,929 - 8682 - 298(226.346 - 205.033)] \text{O}_2 + \\ &\quad 15.04[17,563 - 8669 - 298(212.066 - 191.572)] \text{N}_2 \\ &= 58085 \text{ kJ/kmol}(\text{CH}_4) \end{aligned}$$

Continued on next slide

Problem 13-109 continued

Adding these contributions

$$\frac{\dot{E}_{\text{products}}}{\dot{n}_{\text{CH}_4}} = 58085 + 14595 = 72,680 \text{ kJ/kmol}(\text{CH}_4) \quad \leftarrow (b)$$

(c) At steady state, the rate exergy is destroyed equals the difference between the rate exergy enters and the rate exergy exits. As the air enters at the condition of the environment, and thus with zero exergy, exergy enters only with the fuel. Exergy exits with the combustion products and accompanying the heat transfer. The heat transfer can be evaluated using an energy rate balance:

$$0 = \frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{CH}_4}} - \frac{\dot{W}_{\text{cv}}}{\dot{n}_{\text{CH}_4}} + \bar{h}_{\text{CH}_4} + (4\bar{h}_{\text{O}_2} + 15.04\bar{h}_{\text{N}_2}) - (\bar{h}_{\text{CO}_2} + 2\bar{h}_{\text{H}_2\text{O}} + 2\bar{h}_{\text{O}_2} + 15.04\bar{h}_{\text{N}_2})$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{CH}_4}} = [\bar{h}_f^\circ + \bar{h}(600) - \bar{h}(298)]_{\text{CO}_2} + 2[\bar{h}_f^\circ + \bar{h}(600) - \bar{h}(298)]_{\text{H}_2\text{O}} + 2[\bar{h}(600) - \bar{h}(298)]_{\text{O}_2} + 15.04[\bar{h}(600) - \bar{h}(298)]_{\text{N}_2} - (\bar{h}_f^\circ)_{\text{CH}_4}$$

$$= [-393,520 + 23,280 - 9364] + 2[-241,820 + 20,402 - 9904] + 2[17929 - 8682] + 15.04[17,563 - 8669] - (-74,850)$$

$$= -616,138 \text{ kJ/kmol}(\text{CH}_4)$$

The accompanying exergy transfer is

$$\frac{\dot{E}_q}{\dot{n}_{\text{CH}_4}} = \left[1 - \frac{T_0}{T_s}\right] \frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{CH}_4}} = \left[1 - \frac{298}{333}\right] (-616,138) = -64,759 \text{ kJ/kmol}(\text{CH}_4)$$

Where the minus sign denotes a transfer from the furnace.

In summary

$$\dot{E}_d / \dot{n}_{\text{CH}_4} = \underset{\text{(fuel)}}{830,174} - \underset{\text{(heat transfer)}}{64,759} - \underset{\text{(products)}}{72,680} = 692,735 \text{ kJ/kmol}(\text{CH}_4) \quad \leftarrow (c)$$

As the object of the furnace is to provide an energy transfer by heat, an exergetic efficiency that gauges the extent to which the exergy entering the furnace with the fuel is obtained as exergy accompanying heat transfer is

$$\epsilon = \frac{|\dot{E}_q / \dot{n}_{\text{CH}_4}|}{\bar{e}_{\text{CH}_4}} = \frac{64,759}{830,174} = 0.078 \text{ (7.8\%)} \quad \leftarrow \%$$

①

- Owing to the significant destruction of exergy during combustion, such a furnace does not make an effective use of the exergy supplied with the fuel.

PROBLEM 13.111

KNOWN: A moist air stream is at temperature T , pressure P , with dry air and water vapor mole fractions y_a and y_v , respectively.

FIND: Show that the flow exergy can be expressed as

$$\begin{aligned}\bar{e}_f = T_0 & \left\{ (y_a \bar{c}_{pa} + y_v \bar{c}_{pv}) \left[\left(\frac{T}{T_0} \right) \right. \right. \\ & \left. \left. - 1 - \ln \left(\frac{T}{T_0} \right) \right] + \bar{R} \ln \left(\frac{P}{P_0} \right) \right\} \\ & + \bar{R} T_0 \left[y_a \ln \left(\frac{y_a}{y_a^e} \right) + y_v \ln \left(\frac{y_v}{y_v^e} \right) \right]\end{aligned}$$

relative to an environment modeled as an ideal gas mixture of water vapor and dry air at T_0 , P_0 , and mole fractions of dry air and water vapor, respectively, of y_a^e , y_v^e .

ANALYSIS: The flow exergy can be determined using Eq. 13.47, where $\bar{e}^{ch}$ is obtained using Eq. 13.41a.

When expressed on a per mole of mixture basis

$$\bar{e}^{ch} = \bar{R} T_0 \left[y_a \ln \frac{y_a}{y_a^e} + y_v \ln \frac{y_v}{y_v^e} \right]$$

And

$$\begin{aligned}\bar{h} - \bar{h}_0 - T_0 (\bar{s} - \bar{s}_0) = & y_a \left[\bar{h}_a(T) - \bar{h}_a(T_0) - T_0 (\bar{s}_a^0(T) - \bar{s}_a^0(T_0) - \bar{R} \ln \frac{y_a P}{y_a^e P_0}) \right] + \\ & y_v \left[\bar{h}_v(T) - \bar{h}_v(T_0) - T_0 (\bar{s}_v^0(T) - \bar{s}_v^0(T_0) - \bar{R} \ln \frac{y_v P}{y_v^e P_0}) \right]\end{aligned}$$

Evaluating the enthalpy and entropy terms on the basis of mean values for the specific heats $\bar{c}_{pa}$ and $\bar{c}_{pv}$, this expression becomes

$$\begin{aligned}\bar{h} - \bar{h}_0 - T_0 (\bar{s} - \bar{s}_0) = & y_a \left[\bar{c}_{pa} [T - T_0] - T_0 \left(\bar{c}_{pa} \ln T/T_0 - \bar{R} \ln P/P_0 \right) \right] + \\ & y_v \left[\bar{c}_{pv} [T - T_0] - T_0 \left(\bar{c}_{pv} \ln T/T_0 - \bar{R} \ln P/P_0 \right) \right] \\ = & T_0 \left\{ (y_a \bar{c}_{pa} + y_v \bar{c}_{pv}) \left[\frac{T}{T_0} - 1 - \ln \frac{T}{T_0} \right] + \bar{R} \underbrace{(y_a + y_v)}_{=1} \ln \frac{P}{P_0} \right\}\end{aligned}$$

Adding these contributions

$$\bar{e}_f = T_0 \left\{ (y_a \bar{c}_{pa} + y_v \bar{c}_{pv}) \left[\frac{T}{T_0} - 1 - \ln \frac{T}{T_0} \right] + \bar{R} \ln \frac{P}{P_0} \right\} + \bar{R} T_0 \left[y_a \ln \frac{y_a}{y_a^e} + y_v \ln \frac{y_v}{y_v^e} \right]$$

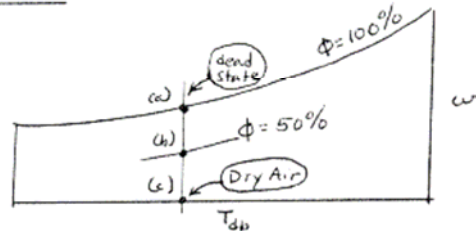
which is the result to be obtained.

PROBLEM 13.112

KNOWN: Three cases involving air at 20°C , 1atm are specified:
(a) saturated air, (b) $\phi = 50\%$, (c) dry air.

FIND: Using the result of Problem 13.103, evaluate e_f relative to an environment consisting of moist air at 20°C , 1atm , $\phi = 100\%$.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The idealizations of Problem 13.111 apply. (2) The effects of motion and gravity can be ignored.

ANALYSIS: Since $T = T_0$ and $P = P_0$, the result of Problem 13.111 becomes, simply

$$\bar{e}_f = \bar{R} T_0 \left[y_a \ln \left(\frac{y_a}{y_a^e} \right) + y_v \ln \left(\frac{y_v}{y_v^e} \right) \right]$$

Also, for moist air at T, p, ϕ

$$\begin{cases} y_a = \frac{P_a}{P} = \frac{P - P_v}{P} = 1 - \frac{P_v}{P} = 1 - \phi \left[\frac{P_g}{P} \right] \\ y_v = 1 - y_a = \phi \left[\frac{P_g}{P} \right] \end{cases}$$

$$\phi = 1 \Rightarrow \begin{cases} y_a^e = 1 - \frac{P_g(T_0)}{P_0} \\ y_v^e = \frac{P_g(T_0)}{P_0} \end{cases}$$

$$(a) \quad \phi = 1 \Rightarrow y_a = y_a^e, y_v = y_v^e \Rightarrow \bar{e}_f = 0 \quad \leftarrow (a)$$

$$(b) \quad \phi = 0.50, P_g = 0.02339 \text{ bar at } T_0 = 20^\circ\text{C}.$$

$$y_v^e = \frac{0.02339}{1.01325} = 0.0231, y_a^e = 0.9769$$

$$y_v = 0.5 [0.0231] = 0.01155, y_a = 0.98845$$

$$\begin{aligned} \bar{e}_f &= \left(8.314 \frac{\text{kJ}}{\text{kmol}(\text{mol}) \text{K}} \right) (293 \text{ K}) \left[0.98845 \ln \left(\frac{0.98845}{0.9769} \right) + 0.01155 \ln \left(\frac{0.01155}{0.0231} \right) \right] \\ &= 8.8 \frac{\text{kJ}}{\text{kmol}(\text{mol})} \end{aligned}$$

$$\text{For the mixture, } M = y_a M_a + y_v M_v = (0.98845)(28.97) + (0.01155)(18.02) = 28.84$$

$$\Rightarrow e_f = (8.8/28.84) = 0.305 \text{ kJ/kg(mix)} \quad \leftarrow (b)$$

(c) Dry air, $y_v = 0, y_a = 1.0$, and a special case of the form Eq (13.40) applies:

$$e_f = R T_0 \ln \left[\frac{1}{y_a^e} \right] = \frac{(8.314)(293)}{28.97} \ln \left[\frac{1}{0.9769} \right]$$

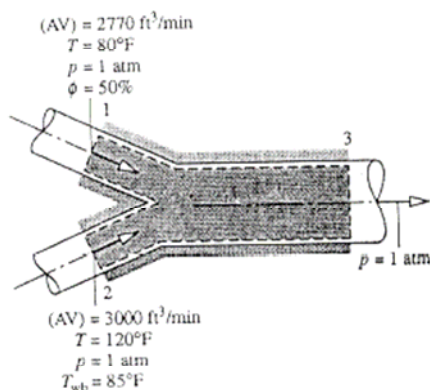
$$= 1.965 \text{ kJ/kg(a)} \quad \leftarrow (c)$$

PROBLEM 13.113

KNOWN: Steady-state operating data are provided for a device in which two moist air streams are mixed to form a single exiting moist air stream.

FIND: Using the result of Problem 13.111, determine the flow exergy at 1, 2, and 3 (see figure below), and the rate of exergy destruction.

SCHEMATIC & GIVEN DATA:



From the solution to Problem 12.102

$$T_3 = 100.1^\circ\text{C}$$

$$\dot{m}_{a1} = 200 \text{ lb/min}$$

$$\dot{m}_{a2} = 199.5 \text{ lb/min}$$

$$\dot{m}_{a3} = 399.5 \text{ lb/min}$$

$$\omega_1 = 0.01092$$

$$\omega_2 = 0.01803 \quad \phi_2 = 24.4\%$$

$$\omega_3 = 0.01447$$

ENGINEERING

MODEL: (1) See solution to Problem 12.102. (2) The environment is a mixture of dry air and water vapor at 95°F, 1 atm with $y_v^e = 0.022$, $y_a^e = 0.978$, $c_{pa} = 0.24 \text{ Btu/lb} \cdot ^\circ\text{R}$, $c_{pv} = 0.44 \text{ Btu/lb} \cdot ^\circ\text{R}$.

ANALYSIS: Since $p = p_0$ for each stream, $\ln(p/p_0)$ drops out of the expression of Problem 13.111. Moreover, the expression can be written on a unit mass of dry air basis as

$$\textcircled{1} \quad e_f = T_0 \left\{ [c_{pa} + \omega c_{pv}] \left[\frac{T}{T_0} - 1 - \ln \frac{T}{T_0} \right] + \frac{\bar{R}}{M_a} \left[(1 + \tilde{\omega}) \ln \left(\frac{1 + \tilde{\omega}^e}{1 + \tilde{\omega}} \right) + \tilde{\omega} \ln \left(\frac{\tilde{\omega}}{\tilde{\omega}^e} \right) \right] \right\} \quad (1)$$

where $\tilde{\omega} = (M_a/M_v) \omega = 1.6078 \omega$. Also

$$\omega = \frac{m_v}{m_a} = \frac{M_v y_v}{M_a y_a} = 0.622 \frac{y_v}{y_a}$$

Thus, $\tilde{\omega}^e = y_v^e/y_a^e = 0.02249$.

Using Eq. (1)

$$e_{f1} = 555 \left\{ [0.24 + 0.01092(0.44)] \left[\frac{540}{555} - 1 - \ln \frac{540}{555} \right] + \left(\frac{1.986}{28.97} \right) \left[1.01756 \ln \left(\frac{1.02249}{1.01756} \right) + 0.01756 \ln \left(\frac{0.01756}{0.02249} \right) \right] \right\}$$

$$= 0.0723 \text{ Btu/lb(a)}$$

$$e_{f2} = 555 \left\{ [0.24 + 0.01803(0.44)] \left[\frac{580}{555} - 1 - \ln \frac{580}{555} \right] + \left(\frac{1.986}{28.97} \right) \left[1.02899 \ln \left(\frac{1.02249}{1.02899} \right) + 0.02899 \ln \left(\frac{0.02899}{0.02249} \right) \right] \right\}$$

$$= 0.1675 \text{ Btu/lb(a)}$$

$$e_{f3} = 555 \left\{ [0.24 + 0.01447(0.44)] \left[\frac{560}{555} - 1 - \ln \frac{560}{555} \right] + \left(\frac{1.986}{28.97} \right) \left[1.02326 \ln \left(\frac{1.02249}{1.02326} \right) + 0.02326 \ln \left(\frac{0.02326}{0.02249} \right) \right] \right\}$$

$$= 0.00552 \text{ Btu/lb(a)}$$

An exergy rate balances reduces to give, $\dot{E}_d = (200)(0.0723) + (199.5)(0.1675) - (399.5)(0.00552) = 45.67 \text{ Btu/min}$, which agrees with the result of Problem 12.96.

1. See the development of Eq. (6-5b), p. 140 of M.J. Moran, *Availability Analysis*, ASME Press, New York, 1989.

PROBLEM 13.114

A full discussion of this class of problems including a solution to Problem 13.114 is provided in

M.J.Moran, Availability Analysis, ASME Press, New York, 1989

See pp. 139-143.

$$\dot{E}_d = 232.6 \text{ Btu/min} \leftarrow$$

Ideal Gas Properties of Hydrogen, H₂ : Special Note

In several problems of this chapter, it is necessary to evaluate the specific enthalpy of hydrogen, H₂, modeled as an ideal gas. This can be done using data from the two following tables, obtained from the same source as Tables A-23, or by means of Interactive Thermodynamics : IT.

SI . Ideal Gas Properties of Hydrogen, H₂

T (K), $\bar{h}$ and $\bar{u}$ (kJ/kmol), $\bar{s}^\circ$ (kJ/kmol·K)

[$\bar{h}_f^\circ = 0$ kJ/kmol]

T	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	T	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$
0	0	0	0	1440	42,808	30,835	177.410
260	7,370	5,209	126.636	1480	44,091	31,786	178.291
270	7,657	5,412	127.719	1520	45,384	32,746	179.153
280	7,945	5,617	128.765	1560	46,683	33,713	179.995
290	8,233	5,822	129.775	1600	47,990	34,687	180.820
298	8,468	5,989	130.574	1640	49,303	35,668	181.632
300	8,522	6,027	130.754	1680	50,662	36,654	182.428
320	9,100	6,440	132.621	1720	51,947	37,646	183.208
340	9,680	6,853	134.378	1760	53,279	38,645	183.973
360	10,262	7,268	136.039	1800	54,618	39,652	184.724
380	10,843	7,684	137.612	1840	55,962	40,663	185.463
400	11,426	8,100	139.106	1880	57,311	41,680	186.190
420	12,010	8,518	140.529	1920	58,668	42,705	186.904
440	12,594	8,936	141.888	1960	60,031	43,735	187.607
460	13,179	9,355	143.187	2000	61,400	44,771	188.297
480	13,764	9,773	144.432	2050	63,119	46,074	189.148
500	14,350	10,193	145.628	2100	64,847	47,386	189.979
520	14,935	10,611	146.775	2150	66,584	48,708	190.796
560	16,107	11,451	148.945	2200	68,328	50,037	191.598
600	17,280	12,291	150.968	2250	70,080	51,373	192.385
640	18,453	13,133	152.863	2300	71,839	52,716	193.159
680	19,630	13,976	154.645	2350	73,608	54,069	193.921
720	20,807	14,821	156.328	2400	75,383	55,429	194.669
760	21,988	15,669	157.923	2450	77,168	56,798	195.403
800	23,171	16,520	159.440	2500	78,960	58,175	196.125
840	24,359	17,375	160.891	2550	80,755	59,554	196.837
880	25,551	18,235	162.277	2600	82,558	60,941	197.539
920	26,747	19,098	163.607	2650	84,386	62,335	198.229
960	27,948	19,966	164.884	2700	86,186	63,737	198.907
1000	29,154	20,839	166.114	2750	88,008	65,144	199.575
1040	30,364	21,717	167.300	2800	89,838	66,558	200.234
1080	31,580	22,601	168.449	2850	91,671	67,976	200.885
1120	32,802	23,490	169.560	2900	93,512	69,401	201.527
1160	34,028	24,384	170.636	2950	95,358	70,831	202.157
1200	35,262	25,284	171.682	3000	97,211	72,268	202.778
1240	36,502	26,192	172.698	3050	99,065	73,707	203.391
1280	37,749	27,106	173.687	3100	100,926	75,152	203.995
1320	39,002	28,027	174.652	3150	102,793	76,604	204.592
1360	40,263	28,955	175.593	3200	104,667	78,061	205.181
1400	41,530	29,889	176.510	3250	106,545	79,523	205.765

English. Ideal Gas Properties of Hydrogen, H₂

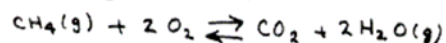
$T(^{\circ}\text{R})$, $\bar{h}$ and $\bar{u}$ (Btu/lbmol), $\bar{s}^{\circ}$ (Btu/lbmol $\cdot^{\circ}\text{R}$)

$[\bar{h}_f^{\circ} = 0 \text{ Btu/lbmol}]$

T	$\bar{h}$	$\bar{u}$	$\bar{s}^{\circ}$	T	$\bar{h}$	$\bar{u}$	$\bar{s}^{\circ}$
300	2063.5	1467.7	27.337	1400	9673.8	6893.6	37.883
320	2189.4	1553.9	27.742	1500	10381.5	7402.7	38.372
340	2317.2	1642.0	28.130	1600	11092.5	7915.1	38.830
360	2446.8	1731.9	28.501	1700	11807.4	8431.4	39.264
380	2577.8	1823.2	28.856	1800	12526.8	8952.2	39.675
400	2710.2	1915.8	29.195	1900	13250.9	9477.8	40.067
420	2843.7	2009.6	29.520	2000	13980.1	10008.4	40.441
440	2978.1	2104.3	29.833	2100	14714.5	10544.2	40.799
460	3113.5	2200.0	30.133	2200	15454.4	11085.5	41.143
480	3249.4	2296.2	30.424	2300	16199.8	11632.3	41.475
500	3386.1	2393.2	30.703	2400	16950.6	12184.5	41.794
520	3523.2	2490.6	30.972	2500	17707.3	12742.6	42.104
537	3640.3	2573.9	31.194	2600	18469.7	13306.4	42.403
540	3660.9	2588.5	31.232	2700	19237.8	13876.0	42.692
560	3798.8	2686.7	31.482	2800	20011.8	14451.4	42.973
580	3937.1	2785.3	31.724	2900	20791.5	15032.5	43.247
600	4075.6	2884.1	31.959	3000	21576.9	15619.3	43.514
620	4214.3	2983.1	32.187	3100	22367.7	16211.5	43.773
640	4353.1	3082.1	32.407	3200	23164.1	16809.3	44.026
660	4492.1	3181.4	32.621	3300	23965.5	17412.1	44.273
680	4631.1	3280.7	32.829	3400	24771.9	18019.9	44.513
700	4770.2	3380.1	33.031	3500	25582.9	18632.4	44.748
720	4909.5	3479.6	33.226	3600	26398.5	19249.4	44.978
740	5048.8	3579.2	33.417	3700	27218.5	19870.8	45.203
760	5188.1	3678.8	33.603	3800	28042.8	20496.5	45.423
780	5327.6	3778.6	33.784	3900	28871.1	21126.2	45.638
800	5467.1	3878.4	33.961	4000	29703.5	21760.0	45.849
820	5606.7	3978.3	34.134	4100	30539.8	22397.7	46.056
840	5746.3	4078.2	34.302	4200	31379.8	23039.2	46.257
860	5885.9	4178.0	34.466	4300	32223.5	23684.3	46.456
880	6025.6	4278.0	34.627	4400	33070.9	24333.1	46.651
900	6165.3	4378.0	34.784	4500	33921.6	24985.2	46.842
920	6305.1	4478.1	34.938	4600	34775.7	25640.7	47.030
940	6444.9	4578.1	35.087	4700	35633.0	26299.4	47.215
960	6584.7	4678.3	35.235	4800	36493.4	26961.2	47.396
980	6724.6	4778.4	35.379	4900	35356.9	27626.1	47.574
1000	6864.5	4878.6	35.520	5000	38223.3	28294.0	47.749
1100	7564.6	5380.1	36.188	5100	39092.8	28964.9	47.921
1200	8265.8	5882.8	36.798	5200	39965.1	29638.6	48.090
1300	8968.7	6387.1	37.360	5300	40840.2	30315.1	48.257

PROBLEM 14.1

KNOWN: The reaction is



FIND: Determine the change in Gibbs function ΔG° at 25°C using (a) Gibbs function of formation data, (b) enthalpy of formation and absolute entropy data.

ANALYSIS: ΔG° is given by

$$\Delta G^\circ = \bar{g}_{\text{CO}_2}^\circ + 2 \bar{g}_{\text{H}_2\text{O}(\text{g})}^\circ - \bar{g}_{\text{CH}_4(\text{g})}^\circ - 2 \bar{g}_{\text{O}_2}^\circ \quad (1)$$

(a) With data from Table A-25

$$\begin{aligned} \Delta G^\circ &= [(-394,380) + 2(-228,590) - (-50,790) - 2(0)] \text{ kJ/kmol} \\ &= -800,770 \text{ kJ/kmol} \end{aligned} \quad \leftarrow (a)$$

(b) Using $\bar{g} = \bar{h} - T\bar{s}$ Eq. (1) becomes

$$\begin{aligned} \Delta G^\circ &= [\bar{h} - T\bar{s}]_{\text{CO}_2}^\circ + 2[\bar{h} - T\bar{s}]_{\text{H}_2\text{O}(\text{g})}^\circ - [\bar{h} - T\bar{s}]_{\text{CH}_4(\text{g})}^\circ - 2[\bar{h} - T\bar{s}]_{\text{O}_2}^\circ \\ &= [\bar{h}_{\text{CO}_2}^\circ + 2\bar{h}_{\text{H}_2\text{O}(\text{g})}^\circ - \bar{h}_{\text{CH}_4(\text{g})}^\circ - 2\bar{h}_{\text{O}_2}^\circ] - (298.15 \text{ K}) [\bar{s}_{\text{CO}_2}^\circ + 2\bar{s}_{\text{H}_2\text{O}(\text{g})}^\circ - \bar{s}_{\text{CH}_4(\text{g})}^\circ - 2\bar{s}_{\text{O}_2}^\circ] \end{aligned}$$

With data from Table A-25

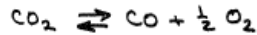
$$\begin{aligned} \Delta G^\circ &= [(-393,520) + 2(-241,820) - (-74,850) - 2(0)] \\ &\quad - (298.15)(213.69 + 2(188.72) - 186.16 - 2(205.03)) \end{aligned}$$

$$\textcircled{1} \quad = -800,792 \text{ kJ/kmol} \quad \leftarrow (b)$$

1. The slight difference in values calculated results from round-off in data values listed in Table A-25.

PROBLEM 14.2

KNOWN: The reaction is



FIND: Determine $\log_{10} K$ at (a) 500 K (b) 1800°R

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: From Eq. 14.31 $\ln K = -\Delta G^\circ / RT$ where

$$\Delta G^\circ = \bar{g}_{\text{CO}}^\circ + \frac{1}{2} \bar{g}_{\text{O}_2}^\circ - \bar{g}_{\text{CO}_2}^\circ$$

With $\bar{g} = \bar{h} - T\bar{s}$ where $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$

$$\Delta G^\circ = \left[[\bar{h}_f^\circ + \Delta \bar{h}]_{\text{CO}} + \frac{1}{2} [\bar{h}_f^\circ + \Delta \bar{h}]_{\text{O}_2} - [\bar{h}_f^\circ + \Delta \bar{h}]_{\text{CO}_2} \right] - T \left[\bar{s}_{\text{CO}}^\circ + \frac{1}{2} \bar{s}_{\text{O}_2}^\circ - \bar{s}_{\text{CO}_2}^\circ \right] \quad (1)$$

(a) T = 500 K with data from ideal gas tables

$$\begin{aligned} \Delta G^\circ &= \left[(-110,530) + 14,600 - 8669 \right] + \frac{1}{2} [0 + 14,770 - 8682] - \left[(-393,520) + 17,678 - 9364 \right] \\ &\quad - 500 \left[212.719 + \frac{1}{2} (220.589) - 254.814 \right] = 239,551 \text{ kJ/kmol} \end{aligned}$$

Then

$$\ln K = -239551 / (8.314 \times 500) \Rightarrow \log_{10} K = -25.027$$

Table Value:

$$\log_{10} K = -25.025$$

(b) T = 1800°R = 1000 K with data from ideal gas tables

$$\begin{aligned} \Delta G^\circ &= \left[(-110,530) + 30,355 - 8669 \right] + \frac{1}{2} [0 + 31,389 - 8682] - \left[(-393,520) + 42,769 - 9364 \right] \\ &\quad - 1000 \left[234.421 + \frac{1}{2} (243.471) - 269.215 \right] = 195,683 \text{ kJ/kmol} \end{aligned}$$

Then

$$\ln K = \frac{-195,683}{(8.314)(1000)} \Rightarrow \log_{10} K = -10.222$$

Table Value:

$$\log_{10} K = -10.221$$

PROBLEM 14.3

KNOWN: The reaction is



FIND: Determine $\log_{10} K$ at (a) 298 K, (b) 1000 K. Compare with Table A-27 data.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: From Eq. 14.31 $\ln K = -\Delta G^\circ / \bar{R}T$ where

$$\Delta G^\circ = \bar{g}_{\text{CO}_2}^\circ + \bar{g}_{\text{H}_2}^\circ - \bar{g}_{\text{CO}}^\circ - \bar{g}_{\text{H}_2\text{O}(\text{g})}^\circ$$

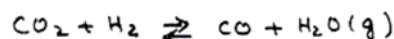
(a) 298 K With data from Table A-25

$$\begin{aligned} \Delta G^\circ &= (-394,380) + (0) - (-137,150) - (-228,590) \\ &= -28,640 \text{ kJ/kmol} \end{aligned}$$

Then

$$\ln K = \frac{28,640}{(8.314)(298.15)} \Rightarrow \log_{10} K = 5.018$$

Table A-27 gives $\log_{10} K$ for the reaction



as $\log_{10} K = -5.018$, which in light of Eq. 14.34 and the accompanying discussion is in agreement with the calculated value.

(b) 1000 K With data from the ideal gas tables

$$\begin{aligned} \textcircled{1} \quad \Delta G^\circ &= [\bar{h}_f^\circ + \Delta \bar{h}]_{\text{CO}_2} + [\bar{h}_f^\circ + \Delta \bar{h}]_{\text{H}_2} - [\bar{h}_f^\circ + \Delta \bar{h}]_{\text{CO}} - [\bar{h}_f^\circ + \Delta \bar{h}]_{\text{H}_2\text{O}(\text{g})} \\ &\quad - T [\bar{s}_{\text{CO}_2}^\circ + \bar{s}_{\text{H}_2}^\circ - \bar{s}_{\text{CO}}^\circ - \bar{s}_{\text{H}_2\text{O}(\text{g})}^\circ] \\ &= [(-393,520 + 42,769 - 9364) + [0 + 29,154 - 8468] - [-110,530 + 30,355 - 8669] \\ &\quad - [-241,820 + 35882 - 9904]] - 1000 [269.215 + 166.114 - 234.421 - 232.597] \\ &= -3054 \text{ kJ/kmol} \end{aligned}$$

Then

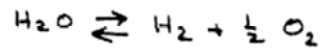
$$\ln K = \frac{3054}{(8.314)(1000)} \Rightarrow \log_{10} K = 0.1595$$

Table A-27 at 1000 K for $\text{CO}_2 + \text{H}_2 \rightleftharpoons \text{CO} + \text{H}_2\text{O}$, $\log_{10} K = -0.159$.
Thus, for $\text{CO} + \text{H}_2\text{O} \rightleftharpoons \text{CO}_2 + \text{H}_2$, $\log_{10} K = +0.159$.

1. IT can be used alternatively.

PROBLEM 14.4

KNOWN: The reaction is



FIND: Calculate $\log_{10} K$ at (a) 298 K, (b) 3600°R and compare with values from Table A-27.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: From Eq. 14.31 $\ln K = -\Delta G^\circ / RT$ where

$$\Delta G^\circ = \bar{g}^\circ_{\text{H}_2} + \frac{1}{2} \bar{g}^\circ_{\text{O}_2} - \bar{g}^\circ_{\text{H}_2\text{O}(g)} \quad (1)$$

(a) At 298.15 K, Table A-25 data give

$$\Delta G^\circ = 0 + \frac{1}{2}(0) - (-228,590) = 228,590 \text{ kJ/kmol}$$

Then

$$\ln K = \frac{-228,590}{(8.314)(298.15)} \Rightarrow \log_{10} K = -40.05 \quad \leftarrow (a)$$

Table A-27 gives $\log_{10} K = -40.048$.

(b) At 3600°R = 2000 K

$$\Delta G^\circ = \left[(\bar{h}_f^\circ + \Delta \bar{h})_{\text{H}_2} + \frac{1}{2} (\bar{h}_f^\circ + \Delta \bar{h})_{\text{O}_2} - (\bar{h}_f^\circ + \Delta \bar{h})_{\text{H}_2\text{O}(g)} \right] - T \left[\bar{s}^\circ_{\text{H}_2} + \frac{1}{2} \bar{s}^\circ_{\text{O}_2} - \bar{s}^\circ_{\text{H}_2\text{O}(g)} \right]$$

Then, with ideal gas table data (or using IT)

$$\begin{aligned} \Delta G^\circ &= \left\{ [0 + (61,400 - 8468)] + \frac{1}{2} [0 + (67,881 - 8682)] - [-241,820 + (82,593 - 9909)] \right\} \\ &\quad - 2000 \left[188.297 + \frac{1}{2} (268.655) - 264.571 \right] \\ &= 135,555.5 \text{ kJ/kmol} \end{aligned}$$

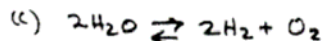
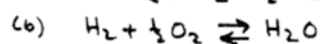
Then

$$\ln K = \frac{-135,555.5}{(8.314)(2000)} \Rightarrow \log_{10} K = -3.5405 \quad \leftarrow (b)$$

which checks the value from Table A-27.

PROBLEM 14.5

KNOWN: Reactions are



FIND: Using Table A-27, determine $\log_{10} K$ at 2500K

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: For reaction (a) Table A-27 gives $\log_{10} K = -2.224$. ← (a)

Using Eq. 14.34, the accompanying discussion indicates that for reaction (b)

$$\log_{10} K(b) = -\log_{10} K(a) = +2.224. \quad \leftarrow (b)$$

For reaction (c), $(\Delta G^\circ)_{(c)} = 2(\Delta G^\circ)_{(a)}$. Then, with Eq. 14.31

$$\ln K_{(c)} = \frac{-\Delta G^\circ_{(c)}}{RT} = -\frac{2(\Delta G^\circ)_{(a)}}{RT}$$

or

$$\ln K_{(c)} = 2 \ln K_{(a)} \Rightarrow \log_{10} K_{(c)} = 2 \log_{10} K_{(a)} = -4.448 \quad \leftarrow (c)$$

PROBLEM 14.6

KNOWN: For interpolation in Table A-27, $\log_{10} K$ is nearly linear in $1/T$:

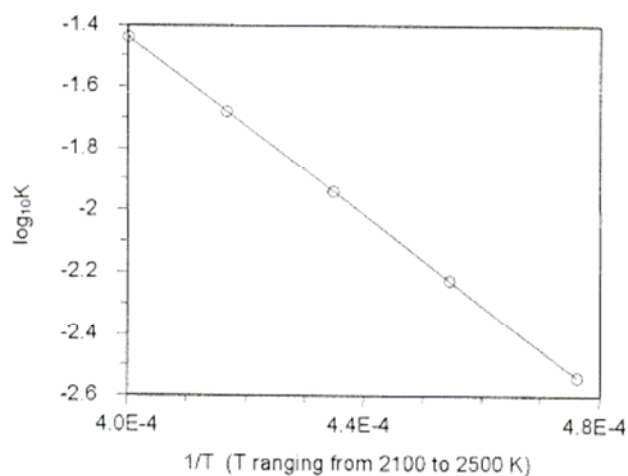
$$\log_{10} K = c_1 + c_2/T$$

where c_1 and c_2 are constants.

FIND: (a) Verify the interpolation rule by plotting $\log_{10} K$ vs $1/T$ for 2000 K $\leq$ 2500 K.

(b) Evaluate c_1, c_2 for any pair of adjacent table entries in the temperature interval of (a)

ANALYSIS: (a) For $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$ as an example



(b) The linearity of $\log_{10} K$ vs. $1/T$ suggests a more accurate scheme for interpolating in Table A-27 than simple linear interpolation. First, let us determine c_1 and c_2 based on $T=2400$ and $T=2500$ K and the corresponding $\log_{10} K$ data for $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$ from Table A-27:

$$\left. \begin{array}{l} T=2400\text{K}: -1.679 = c_1 + c_2/2400 \\ T=2500\text{K}: -1.440 = c_1 + c_2/2500 \end{array} \right\} \Rightarrow \begin{array}{l} c_1 = 4.296 \\ c_2 = -14340 \end{array}$$

Thus, $\log_{10} K = 4.296 - 14340/T$ ($2400 \leq T \leq 2500$ K) (b)

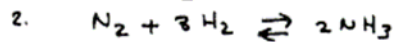
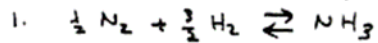
Let us use this result to estimate $\log_{10} K$ at $T=2450$ K. That is

$$\log_{10} K = 4.296 - 14340/2450 = -1.5571$$

For comparison, the value obtain by direct linear interpolation is -1.5595, which differs from the curve fit value by about 0.15%.

PROBLEM 14.7

KNOWN: Two alternative ways of expressing the ammonia synthesis reaction:



FIND: Determine the relationship between the ideal gas equilibrium constants K_1 and K_2 .

ENGINEERING MODEL: (1) Ideal gas principles apply. (2) The temperature is T_0 .

ANALYSIS: For the two reactions the corresponding ΔG° expressions are

$$(\Delta G^\circ)_{(1)} = \bar{g}^\circ_{\text{NH}_3} - \frac{1}{2} \bar{g}^\circ_{\text{N}_2} - \frac{3}{2} \bar{g}^\circ_{\text{H}_2}$$

$$(\Delta G^\circ)_{(2)} = 2 \bar{g}^\circ_{\text{NH}_3} - \bar{g}^\circ_{\text{N}_2} - 3 \bar{g}^\circ_{\text{H}_2}$$

That is,

$$(\Delta G^\circ)_{(2)} = 2(\Delta G^\circ)_{(1)} \quad (1)$$

Using Eq. 14.31

$$\ln K_{(1)} = - \frac{(\Delta G^\circ)_{(1)}}{RT} \quad (2)$$

$$\ln K_{(2)} = - \frac{(\Delta G^\circ)_{(2)}}{RT} \quad (3)$$

Inserting Eq. (1) into Eq. (3) and using Eq. (2)

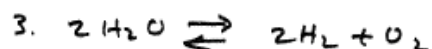
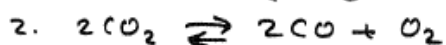
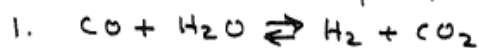
$$\ln K_{(2)} = - 2 \frac{(\Delta G^\circ)_{(1)}}{RT}$$

$$= 2 \ln K_{(1)}$$

$$\Rightarrow K_{(2)} = K_{(1)}^2$$

PROBLEM 14.8

KNOWN: Reactions are specified as



FIND: Show that the equilibrium constants are related by the expression $K_1 = (K_3/K_2)^{1/2}$.

ENGINEERING MODEL: (1) Ideal gas principles apply. (2) The temperature is T .

ANALYSIS: Using Eq. 14.31 the equilibrium constants corresponding to the three reactions have the forms

$$\ln K_1 = -\frac{(\Delta G^\circ)_{(1)}}{RT}, \quad \ln K_2 = -\frac{(\Delta G^\circ)_{(2)}}{RT}, \quad \ln K_3 = -\frac{(\Delta G^\circ)_{(3)}}{RT}$$

where

$$\begin{cases} (\Delta G^\circ)_{(1)} = \bar{g}_{\text{H}_2}^\circ + \bar{g}_{\text{CO}_2}^\circ - \bar{g}_{\text{CO}}^\circ - \bar{g}_{\text{H}_2\text{O}}^\circ \\ (\Delta G^\circ)_{(2)} = 2\bar{g}_{\text{CO}}^\circ + \bar{g}_{\text{O}_2}^\circ - 2\bar{g}_{\text{CO}_2}^\circ \\ (\Delta G^\circ)_{(3)} = 2\bar{g}_{\text{H}_2}^\circ + \bar{g}_{\text{O}_2}^\circ - 2\bar{g}_{\text{H}_2\text{O}}^\circ \end{cases} \quad (1)$$

From Eqs. (1) it follows that

$$(\Delta G^\circ)_{(1)} = \frac{1}{2} [(\Delta G^\circ)_{(3)} - (\Delta G^\circ)_{(2)}]$$

Accordingly

$$\frac{(\Delta G^\circ)_{(1)}}{RT} = \frac{1}{2} \left[\frac{(\Delta G^\circ)_{(3)}}{RT} - \frac{(\Delta G^\circ)_{(2)}}{RT} \right]$$

or

$$\ln K_1 = \frac{1}{2} [\ln K_3 - \ln K_2]$$

Finally

$$K_1 = \left(\frac{K_3}{K_2} \right)^{1/2}$$



PROBLEM 14.9

KNOWN: Reactions are specified as

1. $\text{CO}_2 + \text{H}_2 \rightleftharpoons \text{CO} + \text{H}_2\text{O}$
2. $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$
3. $\text{H}_2\text{O} \rightleftharpoons \text{H}_2 + \frac{1}{2} \text{O}_2$

FIND: (a) Show that $K_1 = K_2 / K_3$. (b) Use the result from part (a) and data from Table A-27 to evaluate $\log_{10} K_1$ at 298 K. (c) Check the result of part (b) using Eq. 14.31 together with $\bar{g}_f^\circ$ data from Table A-25.

ENGINEERING MODEL: (1) Ideal gas principles apply. (2) The temperature is T.

ANALYSIS: (a) For these reactions the respective ΔG° expressions are

$$(\Delta G^\circ)_{(1)} = \bar{g}^\circ_{\text{CO}} + \bar{g}^\circ_{\text{H}_2\text{O}} - \bar{g}^\circ_{\text{CO}_2} - \bar{g}^\circ_{\text{H}_2}$$

$$(\Delta G^\circ)_{(2)} = \bar{g}^\circ_{\text{CO}} + \frac{1}{2} \bar{g}^\circ_{\text{O}_2} - \bar{g}^\circ_{\text{CO}_2}$$

$$(\Delta G^\circ)_{(3)} = \bar{g}^\circ_{\text{H}_2} + \frac{1}{2} \bar{g}^\circ_{\text{O}_2} - \bar{g}^\circ_{\text{H}_2\text{O}}$$

Thus

$$(\Delta G^\circ)_{(1)} = (\Delta G^\circ)_{(2)} - (\Delta G^\circ)_{(3)}$$

$$\frac{(\Delta G^\circ)_{(1)}}{RT} = \frac{(\Delta G^\circ)_{(2)}}{RT} - \frac{(\Delta G^\circ)_{(3)}}{RT} \Rightarrow -\frac{(\Delta G^\circ)_{(1)}}{RT} = -\left[\frac{(\Delta G^\circ)_{(2)}}{RT} - \frac{(\Delta G^\circ)_{(3)}}{RT} \right]$$

With Eq. 14.31

$$\ln K_1 = \ln K_2 - \ln K_3 \Rightarrow K_1 = \frac{K_2}{K_3} \quad \leftarrow (a)$$

(b) From part (a)

$$\log_{10} K_1 = \log_{10} K_2 - \log_{10} K_3$$

With values from Table A-27 at 298 K for reactions 2 and 3

$$\log_{10} K_1 = (-45.066) - (-40.048) = -5.018 \quad \leftarrow (b)$$

(c) With $\bar{g}_f^\circ$ data from Table A-25.

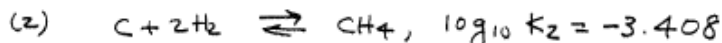
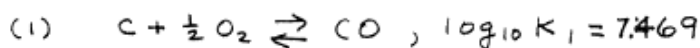
$$\begin{aligned} (\Delta G^\circ)_{(1)} &= (-137,150) + (-228,590) - (-394,380) - (0) \\ &= -28,640 \text{ kJ/kmol} \end{aligned}$$

Then

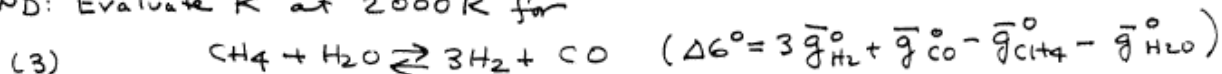
$$\ln K_1 = -\frac{(\Delta G^\circ)_{(1)}}{RT} = \frac{-28,640}{(8.314)(298.15)} \Rightarrow \log_{10} K_1 = -5.018 \quad \leftarrow (c)$$

PROBLEM 14.10

KNOWN: At 2000 K



FIND: Evaluate K at 2000 K for



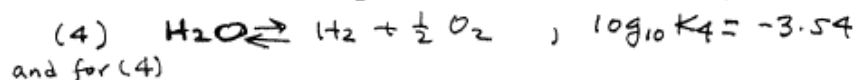
ENGINEERING MODEL: The principles of Sec. 14.3 apply.

ANALYSIS: For (1), (2)

$$(1)' \quad (\Delta G^\circ)_{(1)} = \bar{g}_{CO}^\circ - \bar{g}_C^\circ - \frac{1}{2} \bar{g}_{O_2}^\circ$$

$$(2)' \quad (\Delta G^\circ)_{(2)} = \bar{g}_{CH_4}^\circ - \bar{g}_C^\circ - 2\bar{g}_{H_2}^\circ$$

Also, for the following reaction Table A.27 gives



and for (4)

$$(4)' \quad (\Delta G^\circ)_{(4)} = \bar{g}_{H_2}^\circ + \frac{1}{2} \bar{g}_{O_2}^\circ - \bar{g}_{H_2O}^\circ$$

Subtracting (2)' from (1)'

$$(\Delta G^\circ)_{(1)} - (\Delta G^\circ)_{(2)} = \bar{g}_{CO}^\circ - \frac{1}{2} \bar{g}_{O_2}^\circ - \bar{g}_{CH_4}^\circ + 2\bar{g}_{H_2}^\circ$$

adding (4)'

$$(\Delta G^\circ)_{(1)} - (\Delta G^\circ)_{(2)} + (\Delta G^\circ)_{(4)} = \underbrace{\bar{g}_{CO}^\circ + 3\bar{g}_{H_2}^\circ - \bar{g}_{CH_4}^\circ - \bar{g}_{H_2O}^\circ}_{= \Delta G^\circ_{(3)}}$$

Accordingly,

$$\frac{\Delta G^\circ_{(3)}}{RT} = \frac{\Delta G^\circ_{(1)}}{RT} - \frac{\Delta G^\circ_{(2)}}{RT} + \frac{\Delta G^\circ_{(4)}}{RT}$$

$$\Rightarrow \ln K_3 = \ln K_1 - \ln K_2 + \ln K_4$$

or

$$\log_{10} K_3 = \log_{10} K_1 - \log_{10} K_2 + \log_{10} K_4$$

$$= 7.469 - (-3.408) + (-3.54) = 7.337$$

$$\Rightarrow K_3 = 2.173 \times 10^7$$

PROBLEM 14.11

KNOWN (a) One kmol of N_2O_4 dissociates to form an equilibrium mixture of N_2O_4 and NO_2 at $25^\circ C$, 2 atm. For $N_2O_4 \rightleftharpoons 2NO_2$, $\Delta G^\circ = 5400 \text{ kJ/kmol}$ at $25^\circ C$.

(b) One kmol of CH_4 dissociates to form an equilibrium mixture at 1000 K , 5 atm. For $C + 2H_2 \rightleftharpoons CH_4$, $\log_{10} K = 1.011$ at 1000 K .

FIND: In each case determine the equilibrium composition.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: (a) Applying conservation of mass

$1 N_2O_4 \rightarrow (1-x) N_2O_4 + y NO_2 \Rightarrow N: 2(1-x) + y = 2$, or $y = 2x$ where $(1-x)$ is the amount of N_2O_4 present in the mixture. The total number of moles of mixture is $n = (1-x) + 2x = 1+x$. At equilibrium $N_2O_4 \rightleftharpoons 2NO_2$, so Eq. 14.35 takes the form

$$K = \left[\frac{2x}{1-x} \right]^2 \left[\frac{P/P_{ref}}{1+x} \right]^{2-1}$$

$$= \frac{2[2x]^2}{(1-x)(1+x)} = \frac{8x^2}{1-x^2} \quad (1)$$

Using Eq. 14.31 and the known ΔG° value

$$\ln K = \frac{-5400}{(8.314)(298.15)} \Rightarrow K = 0.11322 \quad (2)$$

Combining Eqs. (1) and (2)

$$\frac{8x^2}{1-x^2} = 0.11322$$

$$\Rightarrow 8.11322x^2 = 0.11322 \Rightarrow x = 0.118$$

Finally, the equilibrium mixture is $\{0.882 N_2O_4, 0.236 NO_2\}$ (a)

(b) Mass balance: $1 CH_4 \rightarrow (1-x) CH_4 + 2x H_2 + x C$, where $n = 1+2x$. Then Eq. 14.35 reads for $C + 2H_2 \rightleftharpoons CH_4$

$$K = \frac{(1-x)}{(x)(2x)^2} \left(\frac{P/P_{ref}}{n} \right)^{1-1-2} = \frac{(1-x)}{(x)(2x)^2} \left(\frac{5}{1+2x} \right)^{-2}$$

or

$$K = \frac{(1-x)}{x} \left(\frac{1+2x}{2x} \right)^2 \frac{1}{25} \quad \text{when } \log_{10} K = 1.011$$

$$\Rightarrow 256.413 = \left(\frac{1-x}{x} \right) \left(\frac{1+2x}{2x} \right)^2$$

Using an equation solver or iteration with a hand calculator, $x = 0.1088$, and the equilibrium mixture is (b)

$\{0.8912 CH_4, 0.2176 H_2, 0.1088 C\}$

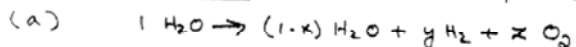
PROBLEM 14.12

KNOWN (a) One lbmol of H_2O dissociates to form an equilibrium mixture of H_2O , H_2 , O_2 at 5200°R , 1.25 atm .

(b) One lbmol of CO_2 dissociates to form an equilibrium mixture of CO_2 , CO , O_2 at 5200°R , 1.25 atm .

FIND: For each case determine the extent to which dissociation occurs.

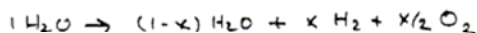
ENGINEERING MODEL: Ideal gas mixture principles apply.



H: $2 = 2(1-x) + 2y \Rightarrow y = x$

O: $1 = (1-x) + 2z \Rightarrow z = x/2$

Thus



where $1-x$ is the amount of H_2O , in kmol, present in the mixture. The amount of mixture is $n = (1-x) + x + x/2 = (2+x)/2$.

At equilibrium $\text{H}_2\text{O} \rightleftharpoons \text{H}_2 + 1/2 \text{O}_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x][x/2]^{1/2}}{[1-x]} \left[\frac{P/P_{\text{ref}}}{(2+x)/2} \right]^{1+1/2-1}$$

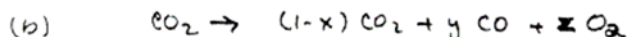
$$= \left(\frac{x}{1-x} \right) \left(\frac{x}{2+x} \right)^{1/2} (P/P_{\text{ref}})^{1/2}$$

or

$$K^2 = \left(\frac{x}{1-x} \right)^2 \left(\frac{x}{2+x} \right) (P/P_{\text{ref}}) \quad (1)$$

① At 5200°R , Table A-27 gives for $\text{H}_2\text{O} \rightleftharpoons \text{H}_2 + 1/2 \text{O}_2$, $\log_{10} K = -1.51311$ or $K = 0.030682$. Thus, Eq. (1) becomes with $P/P_{\text{ref}} = 1.25$

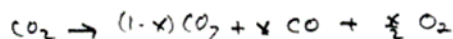
$\Rightarrow 7.5313 \times 10^{-4} = \left(\frac{x}{1-x} \right)^2 \left(\frac{x}{2+x} \right)$. Using an equation solver or iteration with a hand calculator, $x = 0.108$. Thus, the extent of dissociation is 0.892. (a)



C: $1 = (1-x) + y \Rightarrow y = x$

O: $2 = 2(1-x) + y + 2z \Rightarrow z = x/2$

Thus



where $1-x$ is the amount of CO_2 , in kmol, present in the mixture. The number of moles of mixture is $n = (1-x) + x + x/2 = (2+x)/2$

At equilibrium, $\text{CO}_2 \rightleftharpoons \text{CO} + 1/2 \text{O}_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x][x/2]^{1/2}}{[1-x]} \left[\frac{P/P_{\text{ref}}}{(2+x)/2} \right]^{1+1/2-1}$$

$$= \left(\frac{x}{1-x} \right) \left(\frac{x}{2+x} \right)^{1/2} (P/P_{\text{ref}})^{1/2} \Rightarrow K^2 = \left(\frac{x}{1-x} \right)^2 \left(\frac{x}{2+x} \right) (P/P_{\text{ref}}) \quad (1)$$

At 5200°R , Table A-27 gives for $\text{CO}_2 \rightleftharpoons \text{CO} + 1/2 \text{O}_2$, $\log_{10} K = -0.668556$. Thus Eq. (1) gives

① $3.6811 \times 10^{-2} = \left(\frac{x}{1-x} \right)^2 \left(\frac{x}{2+x} \right)$. Using an equation solver or iteration with a hand calculator, $x = 0.336$. The extent of dissociation is 0.664. (b)

1. $\log_{10} K$ is obtained from Table A-27 using linear interpolation. Alternatively, the interpolation procedure of Problem 14.6 can be used.

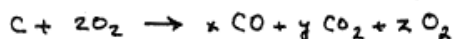
PROBLEM 14.13

KNOWN: One lbmol of C reacts with 2 lbmol of O_2 to form an equilibrium mixture of CO_2 , CO , and O_2 at $5400^\circ R$, 1 atm.

FIND: Determine the equilibrium composition.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: The reaction takes the form

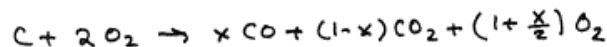


Balancing

$$C: 1 = x + y \Rightarrow y = 1 - x$$

$$O: 4 = x + 2y + 2z \Rightarrow 4 = x + 2(1 - x) + 2z \Rightarrow z = 1 + \frac{x}{2} = \frac{2+x}{2}$$

Thus



The amount of mixture, in lbmol, is $n = x + (1-x) + \left(1 + \frac{x}{2}\right) = \frac{(4+x)}{2}$.

At equilibrium $CO_2 \rightleftharpoons CO + \frac{1}{2} O_2$. Accordingly, Eq. 14.85 takes the form

$$K = \frac{[x] \left[\frac{2+x}{2}\right]^{1/2}}{[1-x]} \left[\frac{P/P_{ref}}{(4+x)/2}\right]^{(1+1/2-1)}$$

$$= \frac{x}{1-x} \left[\frac{2+x}{4+x}\right]^{1/2}$$

At $5400^\circ R$, Table A-27 gives $\log_{10} K = -0.485 \Rightarrow K = 0.32734$. Thus

$$0.32734 = \frac{x}{1-x} \left[\frac{2+x}{4+x}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $x = 0.309$.

The equilibrium mixture is

$$\{0.309 CO, 0.691 CO_2, 1.1545 O_2\}$$

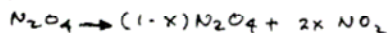
PROBLEM 14.14

KNOWN: Three cases are provided involving the formation of oxides of nitrogen.

FIND: Provide information about the equilibrium mixture in each case.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: (a) One kmol of N_2O_4 dissociates at $25^\circ C$, 1 atm to form an equilibrium mixture of $\{N_2O_4, NO_2\}$ in which the amount of N_2O_4 present is 0.8154. Determine the amount of N_2O_4 that would be present in an equilibrium mixture at $25^\circ C$, 0.5 atm.



The amount of mixture is $n = (1-x) + 2x = 1+x$.

At equilibrium $N_2O_4 \rightleftharpoons 2NO_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[2x]^2}{[1-x]} \left(\frac{P/P_{ref}}{1+x} \right)^{2-1}$$

$$K = \frac{4x^2}{1-x^2} \left(\frac{P/P_{ref}}{1+x} \right)$$

As K depends on T which is $25^\circ C$ in both cases, $K_1 = K_2$ or

$$\frac{4x_1^2}{1-x_1^2} \left[\frac{P_1}{P_{ref}} \right] = \frac{4x_2^2}{1-x_2^2} \left[\frac{P_2}{P_{ref}} \right]$$

where 1 denotes the case $P_1 = 1 \text{ atm}$ and 2 denotes the case $P_2 = 0.5$. Thus

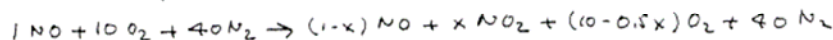
$$\frac{x_1^2}{1-x_1^2} = \frac{0.5x_2^2}{1-x_2^2}$$

From the given information, $1-x_1 = 0.8154 \Rightarrow x_1 = 0.1846$. Accordingly

$$\frac{x_2^2}{1-x_2^2} = 0.070559. \text{ Solving this quadratic equation, } x_2 = 0.2567.$$

The amount of N_2O_4 present is then 0.7433 kmol. (a)

(b) 1 kmol NO , 10 kmol O_2 , 40 kmol N_2 react to form an equilibrium mixture of $\{NO_2, NO, O_2\}$ at 500 K, 0.1 atm. If for $NO + \frac{1}{2}O_2 \rightleftharpoons NO_2$ $K = 120$ at 500 K, determine the equilibrium mixture composition. The reaction is described by



where $n = 51 - 0.5x$. Equation 14.35 takes the form

$$K = \frac{x}{(1-x)[10-0.5x]^{1/2}} \left[\frac{P/P_{ref}}{51-0.5x} \right]^{1-1-1/2} = \left(\frac{x}{1-x} \right) \left[\frac{51-0.5x}{10-0.5x} \right]^{1/2} \left(\frac{1}{0.1} \right)^{1/2}$$

The value of K is given as 120. So

$$37.9473 = \left(\frac{x}{1-x} \right) \left(\frac{51-0.5x}{10-0.5x} \right)^{1/2}. \text{ Using an equation solver or iteration}$$

with a hand calculator, $x = 0.943$. The equilibrium mixture is $\{0.057 NO, 0.943 NO_2, 9.5285 O_2, 40 N_2\}$

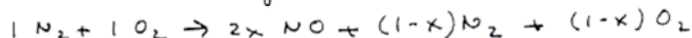
In this analysis, N_2 has been regarded as inert. If N_2 also dissociated, the procedures of Section 14.4.4 would be applied.

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Problem 14.14 continued

- (c) An equimolar mixture of O_2 and N_2 reacts to form an equilibrium mixture of O_2 , N_2 , NO . Plot the mole fraction of NO versus T ranging from 1200 to 2000 K.

The reaction is described by



The amount of mixture is $n = 2x + (1-x) + (1-x) = 2$.

Thus, the mole fraction of NO in the equilibrium mixture is

$$Y_{NO} = \frac{2x}{2} = x$$

For $\frac{1}{2} N_2 + \frac{1}{2} O_2 \rightleftharpoons NO$, Equation 14.31 takes the form

$$K = \frac{2x}{[(1-x)(1-x)]^{1/2} \left(\frac{P/P_{ref}}{n} \right)^{1-\frac{1}{2}-\frac{1}{2}}} = \frac{2x}{(1-x)}$$

Here the term $\left(\frac{P/P_{ref}}{n} \right)$ drops out of the evaluation since $1-\frac{1}{2}-\frac{1}{2} = 0$.

Solving,

$$x = \frac{K(T)}{2 + K(T)}$$

Sample calculation: $T = 1200$ K. From Table A-27, $\log_{10} K = -3.275 \Rightarrow K = 0.0005309$ and $x = 2.654 \times 10^{-4}$

To obtain data for the required plot, use IT, as follows:

IT Code

T = 1200 // K

p = 1 // atm

// 1 N2 + 1 O2 ==> 2x NO + (1-x) N2 + (1-x) O2

nNO = 2 * x

nN2 = 1 - x

nO2 = 1 - x

ntot = 2 * x + 2 * (1 - x)

yNO = nNO / ntot

yN2 = nN2 / ntot

yO2 = nO2 / ntot

pref = 1 // atm

// For the reaction $\frac{1}{2} N_2 + \frac{1}{2} O_2 \rightleftharpoons NO$

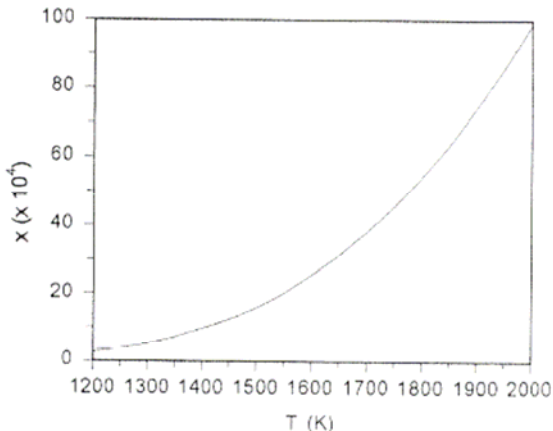
$K = (y_{NO} / (y_{N2} * y_{O2})^{0.5}) * (p / pref)^0$

// Data from Table A-27 are stored in EQNO.LUT.

$\log(K) = \text{LOOKUPVAL}(\text{EQNO}, 1, T, 3)$

IT Results

T (K)	K	x
1200	.0005309	0.0002654
1300	.001015	0.0005073
1400	.001941	0.0009695
1500	.003155	0.001575
1600	.005129	0.002558
1700	.007656	0.003813
1800	.01091	0.005428
1900	.01503	0.007460
2000	.02000	0.009900



note that at $T = 1200$ K, very little NO is formed, whereas at $T = 2000$ K, nearly all of the N_2 has combined with oxygen to form NO .

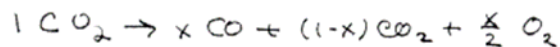
PROBLEM 14.15

KNOWN: One kmol of CO dissociates to form an equilibrium mixture of $\{\text{CO}_2, \text{CO}, \text{O}_2\}$ at T, p

FIND: (a) For $T=3000\text{K}$, plot the amount of CO present, in kmol, versus pressure for $1 \leq p \leq 10\text{atm}$.
(b) For $p=1\text{atm}$, plot the amount of CO present, in kmol, versus temperature for $2000 \leq T \leq 3500\text{K}$

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: The reaction takes the form



The amount of mixture is $n = x + (1-x) + \frac{x}{2} = 1 + \frac{x}{2}$

At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{ O}_2$, so

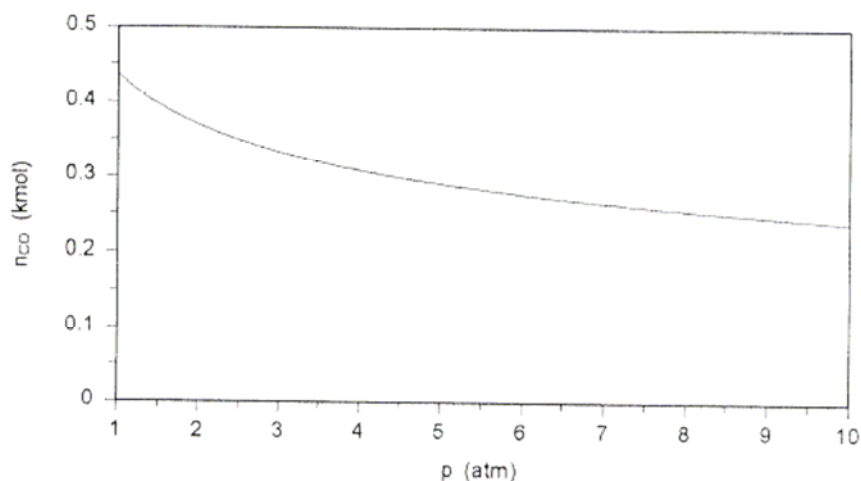
$$\textcircled{1} \quad K = \frac{x(x/2)^{1/2}}{1-x} \left[\frac{P/P_{\text{ref}}}{1+x/2} \right]^{1+\frac{1}{2}-1} = \frac{x}{1-x} \left[\frac{x}{2+x} \right]^{1/2} (P/P_{\text{ref}})^{1/2} \quad (1)$$

(a) $T=3000\text{K}$. Table A-27 gives $\log_{10} K = -0.485$ or $K=0.32734$. Then Eq.(1) becomes

$$\frac{x}{1-x} \left[\frac{x}{2+x} \right]^{1/2} = \frac{0.32734}{(P/P_{\text{ref}})^{1/2}}$$

Solving and plotting, we get

$p(\text{atm})$	n_{CO}
1.000	0.436
2.000	0.370
3.000	0.333
4.000	0.309
5.000	0.291
6.000	0.277
7.000	0.265
8.000	0.256
9.000	0.247
10.000	0.240



Thus, at fixed temperature the amount of CO in the equilibrium mixture decreases as pressure increases.

Continued on next slide

Problem 14-15 continued

(b) $p = 1 \text{ atm}$. Eq. (1) becomes $\frac{x}{1-x} \left[\frac{x}{2+x} \right]^{1/2} = K(T)$.

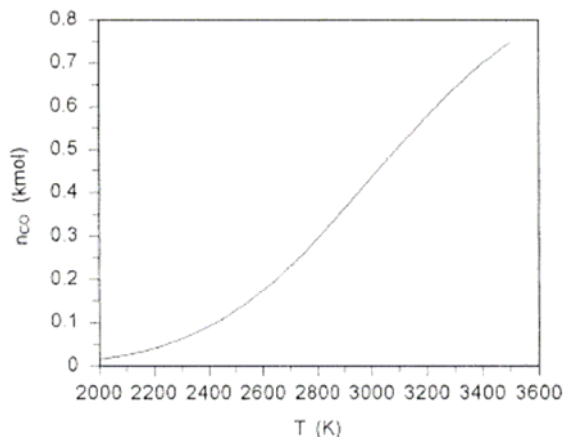
Sample calculation. From Table A-27 at $T = 3000 \text{ K}$, $\log_{10} K = -0.485$. Thus $K = 0.32734$. Solving, $x = 0.4362$.

The data for the required plot are obtained using IT, as follows:

IT Code	IT Results
T = 3000 // K	T(K)
p = 1 // atm	n _{co} (kmol)
// CO2 ==> x CO + (1-x) CO2 + x/2 O2	2000 0.01494
nCO = x	2100 0.02524
nCO2 = 1 - x	2200 0.04048
nO2 = x / 2	2300 0.06207
ntot = x + (1-x) + x / 2	2400 0.09116
yCO = nCO / ntot	2500 0.1287
yCO2 = nCO2 / ntot	2600 0.1754
yO2 = nO2 / ntot	2700 0.2309
pref = 1 // atm	2800 0.2946
	2900 0.3638
	3000 0.4362
	3100 0.5084
	3200 0.5775
	3300 0.6418
	3400 0.6983
	3500 0.7480

K = ((yCO * yO2^0.5) / yCO2) * (p / pref)^0.5
 // Data from Table A-27 are stored in
 EQCO2A.LUT.
 log(K) = LOOKUPVAL(EQCO2A,1,T,3)

PLOT:



When pressure is held constant, the amount of CO present at equilibrium increases greatly with T over the range of values in this problem.

1. Eq. (1) can be written as

$$\frac{K^2}{p/p_{ref}} = \left(\frac{x}{1-x} \right)^2 \left(\frac{x}{2+x} \right) \Rightarrow x^3 + 3 \left[\frac{\alpha}{1-\alpha} \right] x - 2 \left[\frac{\alpha}{1-\alpha} \right] = 0$$

$\equiv r$

The real root of this cubic equation is

$$x = \left[r + r[1+r]^{1/2} \right]^{1/3} + \left[r - r[1+r]^{1/2} \right]^{1/3} \quad \text{where } r \equiv \frac{\alpha}{1-\alpha}$$

PROBLEM 14.16

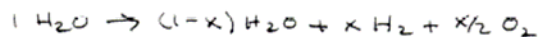
KNOWN: 1 lbmol of H_2O dissociates to form an equilibrium mixture of H_2O , H_2 , O_2 at T, p .

FIND: (a) For $T = 5400^\circ R$, plot the amount of H_2 present in the mixture versus pressure for $1 \leq p \leq 10 \text{ atm}$.
(b) For $p = 1 \text{ atm}$, plot the amount of H_2 present versus Temperature, $3600^\circ R \leq T \leq 6300^\circ R$.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS:

The reaction takes the form



The amount of mixture is $n = (1-x) + x + x/2 = (2+x)/2$.

At equilibrium $H_2O \rightleftharpoons H_2 + \frac{1}{2} O_2$. Then Eq. 14.35 takes the form

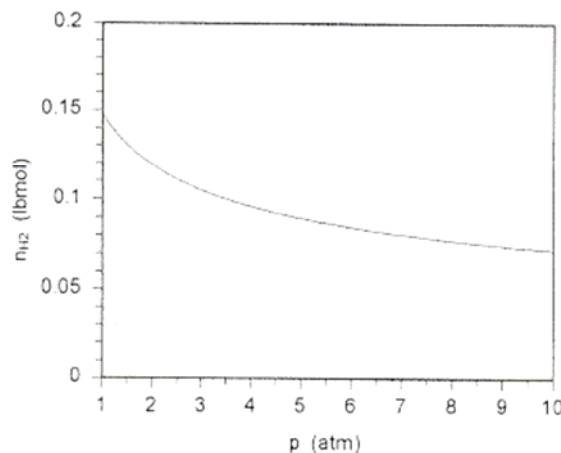
$$\textcircled{1} \quad K = \frac{x(x/2)^{1/2}}{(1-x)} \left[\frac{p/p_{ref}}{(2+x)/2} \right]^{1+1/2-1} = \left(\frac{x}{1-x} \right) \left[\frac{x}{2+x} \right]^{1/2} [p/p_{ref}]^{1/2} \quad (1)$$

(a) $T = 5400^\circ R$. Table A-27 gives $\log_{10} K = -1.343$, $K = 0.04539$. So, Eq. (1) becomes

$$\left(\frac{x}{1-x} \right) \left(\frac{x}{2+x} \right)^{1/2} = \frac{0.04539}{[p/p_{ref}]^{1/2}}$$

Solving and plotting

p (atm)	n_{H_2} (lbmol)
1	0.1476
2	0.1192
3	0.1050
4	0.09592
5	0.08938
6	0.08435
7	0.08031
8	0.07696
9	0.07411
10	0.07165



Thus, at fixed temperature the amount of H_2 in the equilibrium mixture decreases as pressure increases.

1. See note 1 of the solution to problem 14.15

Continued on next slide

Problem 14-16 continued

(b) $p = 1 \text{ atm}$

Equation (1) takes the form

$$K(T) = \frac{x}{1-x} \left[\frac{x}{2+x} \right]^{1/2}$$

Sample calculation. From Table A-27 at $T = 5400^\circ\text{R}$, $\log_{10} K = -1.343 \Rightarrow K = 0.04539$.
Solving, $x = 0.1476$.

The data for the required plot are obtained using IT, as follows:

IT Code

$T = 5400 \text{ // } ^\circ\text{R}$

$p = 1 \text{ // atm}$

// $\text{H}_2\text{O} \leftrightarrow (1-x)\text{H}_2\text{O} + x\text{H}_2 + x/2\text{O}_2$

$n_{\text{H}_2\text{O}} = 1 - x$

$n_{\text{H}_2} = x$

$n_{\text{O}_2} = x/2$

$n_{\text{tot}} = (1-x) + x + x/2$

$y_{\text{H}_2\text{O}} = n_{\text{H}_2\text{O}} / n_{\text{tot}}$

$y_{\text{H}_2} = n_{\text{H}_2} / n_{\text{tot}}$

$y_{\text{O}_2} = n_{\text{O}_2} / n_{\text{tot}}$

$p_{\text{ref}} = 1 \text{ // atm}$

// For the reaction $\text{H}_2\text{O} \leftrightarrow \text{H}_2 + 1/2\text{O}_2$

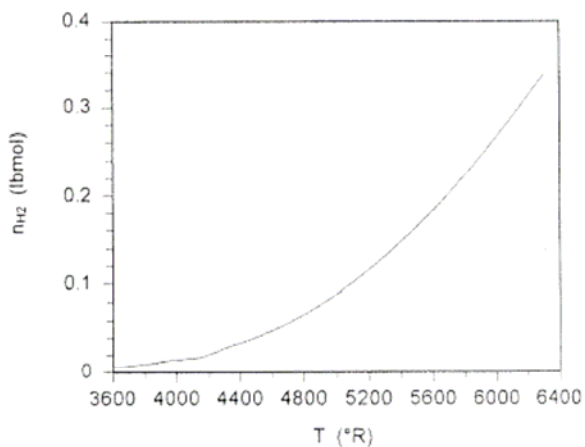
$K = ((y_{\text{H}_2} \cdot y_{\text{O}_2}^{0.5}) / y_{\text{H}_2\text{O}}) \cdot (p/p_{\text{ref}})^{0.5}$

//Data from Table A-27 are stored in EQH2O.LUT.

$\log(K) = \text{LOOKUPVAL}(\text{EQH2O}, 2, T, 3)$

T (°R)	K	n_{H_2} (lbmol)
3600	0.0002884	0.005485
3750	0.0005258	0.008174
3900	0.0009183	0.01183
4050	0.001332	0.01514
4200	0.002056	0.02016
4350	0.003922	0.03084
4500	0.005970	0.04061
4650	0.008814	0.05232
4800	0.01272	0.06629
4950	0.01797	0.08271
5100	0.02491	0.1017
5250	0.03391	0.1233
5400	0.04539	0.1476
5550	0.05961	0.1740
5700	0.07733	0.2030
5850	0.09897	0.2341
6000	0.1251	0.2671
6150	0.1566	0.3018
6300	0.1941	0.3380

PLOT:



Thus, at fixed pressure the amount of H_2 present in the equilibrium mixture increases as temperature increases.

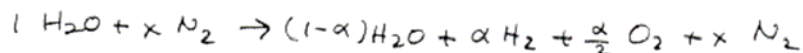
PROBLEM 14.17

KNOWN: 1 lbmol of H_2O plus x lbmol of N_2 forms an equilibrium mixture at $5400^\circ R$ and 1 atm consisting of H_2O , H_2 , O_2 and N_2

FIND: Plot the amount of H_2 present in the mixture versus x , $0 \leq x \leq 2$.

ENGINEERING MODEL: (1) Ideal gas principles apply. (2) N_2 is inert.

ANALYSIS: The reaction is



The amount of mixture is $n = (1-\alpha) + \alpha + \frac{\alpha}{2} + x = 1 + \frac{\alpha}{2} + x$

For $H_2O \rightleftharpoons H_2 + \frac{1}{2} O_2$, Eq. 14.35 takes the form

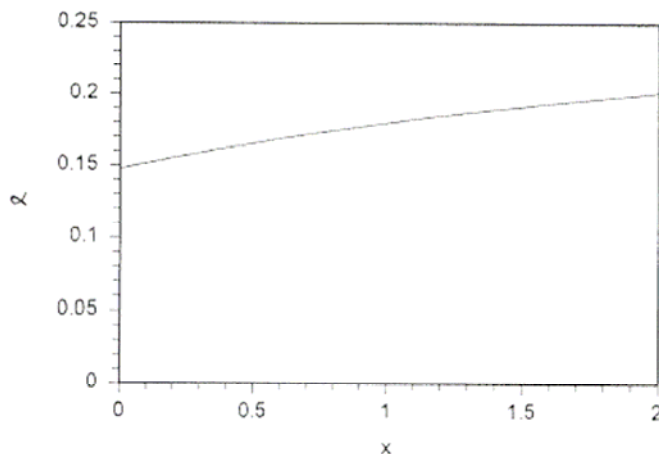
$$K = \frac{\alpha [\alpha/2]^{1/2}}{1-\alpha} \left[\frac{P/P_{ref}}{1 + \frac{\alpha}{2} + x} \right]^{1+1/2-1} = \frac{\alpha}{1-\alpha} \left[\frac{\alpha/2}{1 + \frac{\alpha}{2} + x} \right]^{1/2} \underbrace{(P/P_{ref})^{1/2}}_{=1}$$

From Table A-27 at $5400^\circ R$, $\log_{10} K = -1.343$, $K = 0.04539$. So

$$\frac{\alpha}{1-\alpha} \left[\frac{\alpha/2}{1 + \frac{\alpha}{2} + x} \right]^{1/2} = 0.04539$$

Solving and plotting

x	α
0.000	0.148
0.100	0.152
0.200	0.155
0.300	0.159
0.400	0.162
0.500	0.166
0.600	0.169
0.700	0.172
0.800	0.174
0.900	0.177
1.000	0.180
1.100	0.182
1.200	0.185
1.300	0.187
1.400	0.189
1.500	0.191
1.600	0.193
1.700	0.195
1.800	0.197
1.900	0.199
2.000	0.201



Thus, at $5400^\circ R$, 1 atm the amount of H_2 in the equilibrium mixture increases as the amount of the inert N_2 increases.

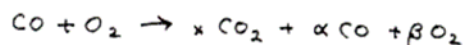
PROBLEM 14.18

KNOWN: An equimolar mixture of CO and O₂ reacts to form an equilibrium mixture of CO₂, CO, and O₂ at 3000 K and pressure p.

FIND: Determine the effect of pressure on the composition of the mixture.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: The reaction takes the form

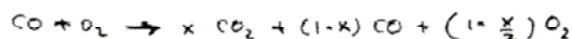


Balancing

$$\text{C: } 1 = x + \alpha, \quad \alpha = 1 - x$$

$$\text{O: } 3 = 2x + (1-x) + 2\beta, \quad \beta = 1 - \frac{x}{2} = \frac{2-x}{2}$$

Thus



The amount of mixture is $n = x + (1-x) + \left(1 - \frac{x}{2}\right) = 2 - \frac{x}{2} = \frac{4-x}{2}$

At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$. Accordingly, Eq. 14.35 takes the form

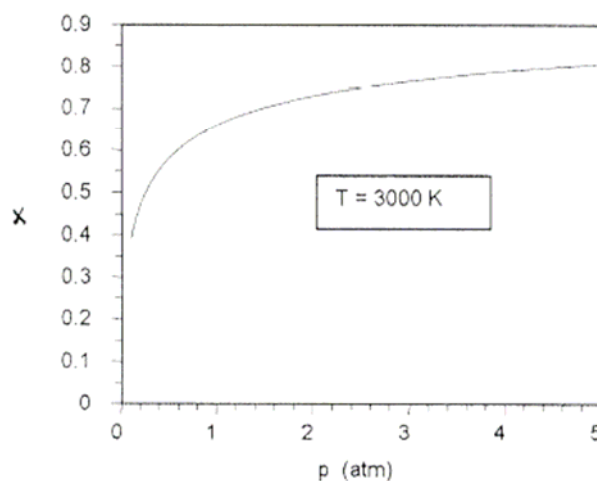
$$K = \frac{\left[\frac{1-x}{2}\right] \left[\frac{2-x}{2}\right]^{1/2}}{x} \left[\frac{p/p_{\text{ref}}}{4-x}\right]^{1+1/2-1}$$

$$K = \left[\frac{1-x}{x}\right] \left[\frac{2-x}{4-x}\right]^{1/2} \left[\frac{p}{p_{\text{ref}}}\right]^{1/2}$$

At 3000 K, Table A-27 gives $\log_{10} K = -0.485 \Rightarrow K = 0.32724$. Thus

$$0.32724 = \left[\frac{1-x}{x}\right] \left[\frac{2-x}{4-x}\right]^{1/2} \left[\frac{p}{p_{\text{ref}}}\right]^{1/2} \quad (1)$$

The following plot is obtained using IT, with $p_{\text{ref}} = 1 \text{ atm}$:



From the plot we conclude that the amount of CO₂ present at equilibrium decrease as pressure decreases at T = 3000 K.

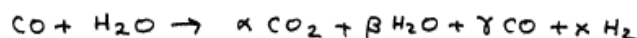
PROBLEM 14.19

KNOWN: An equimolar mixture of CO and H₂O(g) reacts to form an equilibrium mixture of CO₂, CO, H₂O, and H₂ at 2000 K, 1 atm.

FIND: Determine whether the amount of H₂ formed increases or decreases as temperature decreases. Also determine the effect of pressure on the amount of H₂ formed.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: The reaction takes the form

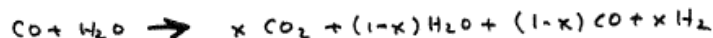


$$\text{C: } 1 = \alpha + \gamma \Rightarrow \gamma = 1 - \alpha$$

$$\text{O: } 2 = 2\alpha + \beta + \gamma$$

$$\text{H: } 2 = 2\beta + 2x \Rightarrow \beta = 1 - x$$

Finally, $\alpha = x$, $\gamma = 1 - x$, $\beta = 1 - x$. Thus



The amount of mixture is $n = x + (1-x) + (1-x) + x = 2$

At equilibrium $\text{CO}_2 + \text{H}_2 \rightleftharpoons \text{CO} + \text{H}_2\text{O}$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[1-x][1-x]}{[x][x]} \left[\frac{P/P_{\text{ref}}}{n} \right]^{1+1-1-1}$$

or

$$K = \left(\frac{1-x}{x} \right)^2$$

Thus

$$\sqrt{K} = \frac{1-x}{x} \quad (1)$$

Solving Eq. (1)

$$x = \frac{1}{1 + \sqrt{K}} \quad (2)$$

Observe that pressure drops out of the final expression. Accordingly, pressure has no effect on the amount of H₂ formed.

Referring to Table A-27 at 2000 K, $\log_{10} K = 0.656 \Rightarrow K = 4.52898$, whereas at 1900 K $\log_{10} K = 0.619 \Rightarrow 4.15911$. It follows from Eq. (2), therefore, that x increases as T decreases.

← (b)

← (a)

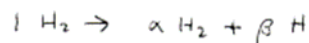
PROBLEM 14.20

KNOWN: Nine percent of H_2 dissociates into H at $p=10 \text{ atm}$, T .

FIND: Determine T .

ENGINEERING MODEL: Ideal gas mixture principles apply.

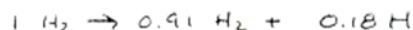
ANALYSIS: The reaction is



where $\alpha = 0.91$. Then

$$H: 2 = 2(0.91) + \beta \Rightarrow \beta = 0.18$$

So



The amount of mixture is $n = 0.91 + 0.18 = 1.09$

At equilibrium $H_2 \rightleftharpoons 2H$, so Eq. 14.35 takes the form

$$K = \frac{(0.18)^2}{(0.91)} \left[\frac{10}{1.09} \right]^{2-1} = 0.3266$$

① Then $\log_{10} K = -0.486$. Interpolating in Table A.27, $T = 3488 \text{ K}$ ←

Consider the case of 10% dissociation: $1 H_2 \rightarrow 0.9 H_2 + 0.2 H$. Then

$$K = \frac{(0.2)^2}{(0.9)} \left[\frac{10}{1.10} \right]^{2-1} = 0.404 \Rightarrow \log_{10} K = -0.394 \Rightarrow T > 3488 \text{ K}$$

By inspection of Table A.27, T increases as the percentage of H_2 dissociating increases (i.e., as the amount of H_2 in the equilibrium mixture decreases).

-
1. Linear interpolation is used here. Alternatively, the approach of Problem 14.6 can be invoked.

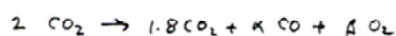
PROBLEM 14.21

KNOWN: 2 kmol of CO_2 dissociate to form an equilibrium mixture of CO_2 , CO , and O_2 in which 1.8 kmol is present at temperature T and pressure p .

FIND: plot T versus p for $0.5 \leq p \leq 10$ atm.

ENGINEERING MODEL: Ideal gas mixture principles apply.

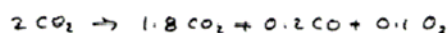
ANALYSIS: The reaction takes the form



$$\text{C: } 2 = 1.8 + \alpha \Rightarrow \alpha = 0.2$$

$$\text{O: } 4 = (1.8)(2) + 0.2 + 2\beta \Rightarrow \beta = 0.1$$

Thus



The amount of mixture is $n = 2.1$.

At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[0.2][0.1]^{1/2}}{1.8} \left[\frac{p/p_{\text{ref}}}{2.1} \right]^{1+1/2-1} \quad (1)$$

Sample calculation. $T = 2400$ K. From Table A-27, $\log_{10} K = -1.679 \Rightarrow K = 0.02094$.

Solving for p , with $p_{\text{ref}} = 1$

$$p = \left(\frac{2.1}{0.1} \right) \left[\frac{(0.02094)(1.8)}{0.2} \right]^2 = 0.7459 \text{ atm}$$

The data for the required plot are obtained using IT, as follows:

IT Code

$p = 0.5$ // atm

// $2 \text{CO}_2 \rightarrow 1.8 \text{CO}_2 + 0.2 \text{CO} + 0.1 \text{O}_2$

$n_{\text{CO}_2} = 1.8$

$n_{\text{CO}} = 2 - n_{\text{CO}_2}$

$n_{\text{O}_2} = 2 - n_{\text{CO}_2} - n_{\text{CO}} / 2$

$n_{\text{tot}} = n_{\text{CO}_2} + 2 + 0.1$

$y_{\text{CO}_2} = n_{\text{CO}_2} / n_{\text{tot}}$

$y_{\text{CO}} = n_{\text{CO}} / n_{\text{tot}}$

$y_{\text{O}_2} = n_{\text{O}_2} / n_{\text{tot}}$

$p_{\text{ref}} = 1$ // atm

// For the reaction $\text{CO}_2 \rightleftharpoons \text{CO} + 1/2 \text{O}_2$

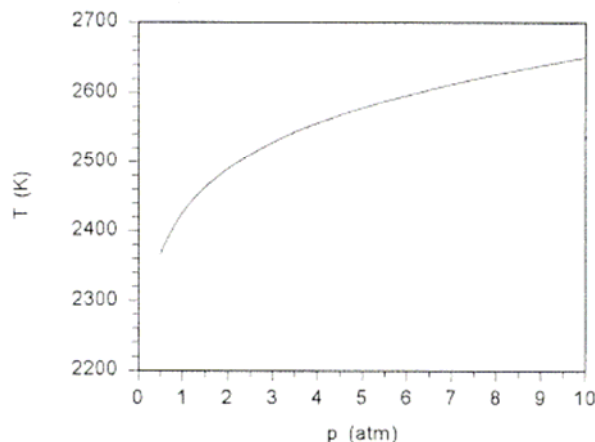
$K = ((y_{\text{CO}} * y_{\text{O}_2}^{0.5}) / y_{\text{CO}_2}) * (p / p_{\text{ref}})^{0.5}$

// Data from Table A-27 are stored in EQCO2A.LUT.

$\log(K) = \text{LOOKUPVAL}(\text{EQCO2A}, 1, T, 3)$

IT Results

K	T (K)	p (atm)
0.01714	2367	0.5
0.02425	2427	1.0
0.02970	2463	1.5
0.03429	2490	2.0
0.03834	2511	2.5
0.04200	2529	3.0
0.04536	2544	3.5
0.04849	2557	4.0
0.05143	2568	4.5
0.05422	2579	5.0
0.05686	2588	5.5
0.05939	2597	6.0
0.06182	2605	6.5
0.06415	2613	7.0
0.06640	2620	7.5
0.06858	2627	8.0
0.07069	2634	8.5
0.07274	2640	9.0
0.07473	2645	9.5
0.07667	2651	10.0



For fixed composition, temperature increases as pressure increases.

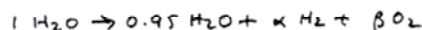
PROBLEM 14.22

KNOWN: One kmol of $\text{H}_2\text{O(g)}$ dissociates to form an equilibrium mixture of $\text{H}_2\text{O(g)}$, H_2 , and O_2 in which the amount of water vapor present is 0.95 kmol at temperature T and pressure p .

FIND: Plot T versus p for $1 \leq p \leq 10$ atm.

ENGINEERING MODEL: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The reaction takes the form



$$\text{H: } 2 = 1.9 + 2\alpha, \quad \alpha = 0.05$$

$$\text{O: } 1 = 0.95 + 2\beta, \quad \beta = 0.025$$

Thus



The amount of mixture is 1.025 kmol.

At equilibrium $\text{H}_2\text{O} \rightleftharpoons \text{H}_2 + \frac{1}{2} \text{O}_2$. Accordingly, Eq. (14.35) takes the form

$$K = \frac{[0.05][0.025]^{1/2}}{0.95} \left[\frac{p/p_{\text{ref}}}{1.025} \right]^{1+1/2-1}$$

Sample calculation. $T = 2600 \text{ K}$. From Table A-27, $\log_{10} K = -2.038 \Rightarrow K = 0.0091622$.

Solving for p , with $p_{\text{ref}} = 1 \text{ atm}$

$$p = \left(\frac{1.025}{0.025} \right) \left[\frac{(0.0091622)(0.95)}{0.05} \right]^2 = 1.242 \text{ atm}$$

The data for the required plot are obtained using IT, as follows:

IT Code

$p = 1 \text{ // atm}$

$\text{// H}_2\text{O} \rightarrow 0.95 \text{ H}_2\text{O} + (1 - 0.95) \text{ H}_2 + ((1 - 0.95) / 2) \text{ O}_2$

$n\text{H}_2\text{O} = 0.95$

$n\text{H}_2 = 1 - n\text{H}_2\text{O}$

$n\text{O}_2 = (1 - n\text{H}_2\text{O}) / 2$

$n_{\text{tot}} = 0.95 + (1 - 0.95) + (1 - 0.95) / 2$

$y\text{H}_2\text{O} = n\text{H}_2\text{O} / n_{\text{tot}}$

$y\text{H}_2 = n\text{H}_2 / n_{\text{tot}}$

$y\text{O}_2 = n\text{O}_2 / n_{\text{tot}}$

$p_{\text{ref}} = 1 \text{ // atm}$

$\text{// For the reaction H}_2\text{O} \leftrightarrow \text{H}_2 + 1/2 \text{ O}_2$

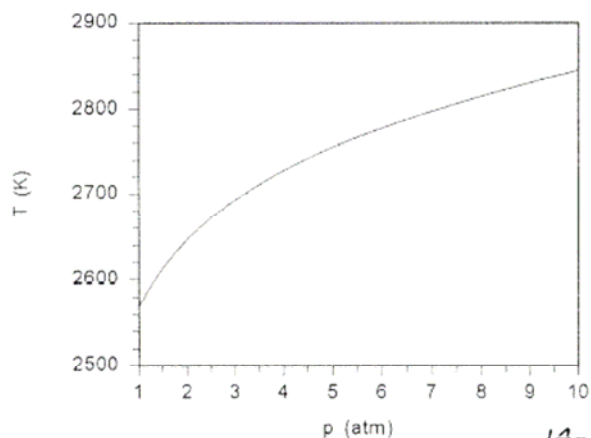
$K = (y\text{H}_2 * y\text{O}_2^{0.5}) / y\text{H}_2\text{O} * (p / p_{\text{ref}})^{0.5}$

$\text{// Data from Table A-27 are stored in EQH}_2\text{O.LUT.}$

$\log(K) = \text{LOOKUPVAL}(\text{EQH}_2\text{O}, 1, T, 3)$

IT Results

T (K)	K	p (atm)
2568	0.00822	1.0
2613	0.01007	1.5
2646	0.01162	2.0
2672	0.01300	2.5
2693	0.01424	3.0
2711	0.01538	3.5
2728	0.01644	4.0
2743	0.01744	4.5
2756	0.01838	5.0
2767	0.01928	5.5
2778	0.02013	6.0
2788	0.02096	6.5
2797	0.02175	7.0
2806	0.02251	7.5
2815	0.02325	8.0
2823	0.02396	8.5
2831	0.02466	9.0
2838	0.02533	9.5
2845	0.02599	10.0



For fixed composition, temperature increases as pressure increases.

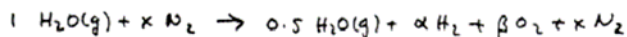
PROBLEM 14.23

KNOWN: A vessel initially containing 1 kmol of $\text{H}_2\text{O}(g)$ and x kmol of N_2 forms an equilibrium mixture at 1 atm and temperature T consisting of $\text{H}_2\text{O}(g)$, H_2 , O_2 , N_2 in which 0.5 kmol of $\text{H}_2\text{O}(g)$ is present.

FIND: Plot x versus T for $3000 \leq T \leq 3500$ K.

ENGINEERING MODEL: (1) The equilibrium mixture is modeled as an ideal gas mixture. (2) N_2 is inert.

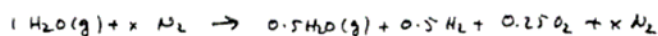
ANALYSIS: The reaction takes the form



$$\text{H: } 2 = (0.5)(2) + 2\alpha \Rightarrow \alpha = 0.5$$

$$\text{O: } 1 = 0.5 + 2\beta \Rightarrow \beta = 0.25$$

Thus



The amount of mixture is $n = 0.5 + 0.5 + 0.25 + x = 1.25 + x$.

At equilibrium $\text{H}_2\text{O} \rightleftharpoons \text{H}_2 + \frac{1}{2} \text{O}_2$. Accordingly Eq 14.35 takes the form

$$K = \frac{[0.5][0.25]^{1/2}}{[0.5]} \left[\frac{p/p_{\text{ref}}}{1.25+x} \right]^{1/2}$$

Since $p/p_{\text{ref}} = 1$, this reduces to

$$K = \frac{0.25}{(1.25+x)^{1/2}} \Rightarrow x = \left[\frac{0.25}{\{K(T)\}^2} - 1.25 \right]$$

Sample calculations. From Table A-27 at $T = 3000$ K, $\log_{10} K = -1.343 \Rightarrow K = 0.045294$.

Thus $x = 120.07$.

The data for the required plot are obtained using IT, as follows:

IT Code

$p = 1$ // atm
 $T = 3000$ // K

// $\text{H}_2\text{O} + x \text{ N}_2 \rightarrow 0.5 \text{ H}_2\text{O} + \alpha \text{ H}_2 + \beta \text{ O}_2 + x \text{ N}_2$

$n_{\text{H}_2\text{O}} = 0.5$

$\alpha = 1 - 0.5$

$n_{\text{H}_2} = \alpha$

$\beta = (1 - 0.5) / 2$

$n_{\text{O}_2} = \beta$

$n_{\text{tot}} = 0.5 + \alpha + \beta + x$

$y_{\text{H}_2\text{O}} = n_{\text{H}_2\text{O}} / n_{\text{tot}}$

$y_{\text{H}_2} = n_{\text{H}_2} / n_{\text{tot}}$

$y_{\text{O}_2} = n_{\text{O}_2} / n_{\text{tot}}$

$p_{\text{ref}} = 1$ // atm

// For the reaction $\text{H}_2\text{O} \rightleftharpoons \text{H}_2 + \frac{1}{2} \text{O}_2$

$K = (y_{\text{H}_2} \cdot y_{\text{O}_2}^{0.5} / y_{\text{H}_2\text{O}}) \cdot (p / p_{\text{ref}})^{0.5}$

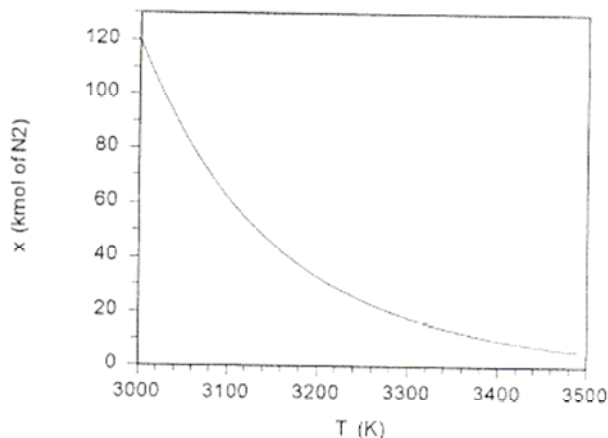
// Data from Table A-27 are stored in EQH2O.LUT.

$\log(K) = \text{LOOKUPVAL}(\text{EQH2O}, 1, T, 3)$

For fixed composition in terms of H_2O , H_2 , and O_2 , the amount of inert N_2 required decreases as the temperature increases at fixed pressure.

IT Results

T (K)	K	x (kmol of N_2)
3000	0.04539	120.1
3050	0.05346	86.24
3100	0.06295	61.84
3150	0.07345	45.09
3200	0.08570	32.79
3250	0.09897	24.27
3300	0.1143	17.89
3350	0.1309	13.34
3400	0.1500	9.866
3450	0.1706	7.339



PROBLEM 14.24

KNOWN: A vessel initially contains 2 lbmol of N_2 and 1 lbmol of O_2 . An equilibrium mixture of N_2 , O_2 and NO forms at 1 atm, T .

FIND: Plot the amount of NO formed versus T .

ENGINEERING MODEL: The ideal gas model applies.

ANALYSIS: The reaction takes the form



$$N: 4 = 2x + z \Rightarrow z = 4 - 2x$$

$$O: 2 = 2y + z \Rightarrow z = 2y + (4 - 2x) \Rightarrow y = x - 1$$

At equilibrium $\frac{1}{2}O_2 + \frac{1}{2}N_2 \rightleftharpoons NO$. Accordingly

$$K = \frac{[4-2x]}{[x-1]^{1/2}[x]^{1/2}} \left[\frac{p/p_{ref}}{n} \right]^{-1/2-1/2} \Rightarrow K(T) = \frac{[4-2x]}{[x(x-1)]^{1/2}} \quad (1)$$

This expression is a quadratic equation that can be solved analytically for x as a function of $K(T)$. Then, x can be calculated for given values of T using data for $K(T)$ from Table A-27. Alternatively, data for the required plot are obtained using IT, as follows:

// Code

p = 1 // atm

T = 5400 // °R

// 2 N2 + 1 O2 ==> x N2 + (x - 1) O2 + 2 * (2 - x) NO

nN2 = x

nO2 = x - 1

nNO = 2 * (2 - x)

ntot = x + (x - 1) + 2 * (2 - x)

yN2 = nN2 / ntot

yO2 = nO2 / ntot

yNO = nNO / ntot

pref = 1 // atm

// For the reaction $\frac{1}{2}N_2 + \frac{1}{2}O_2 \rightleftharpoons NO$

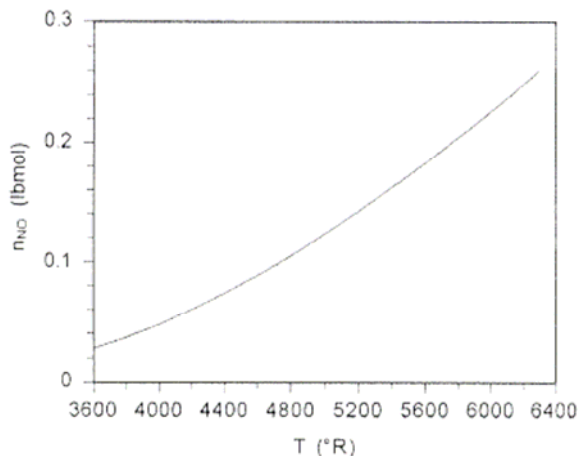
$K = (yNO / (yN2^{0.5} * yO2^{0.5})) * (p / pref)^0$

// Data from Table A-27 are stored in EQNO.LUT.

log(K) = LOOKUPVAL(EQNO, 2, T, 3)

IT results

T (°R)	K	n _{NO} (lbmol)
3600	0.02000	0.02799
3800	0.02663	0.03713
4000	0.03441	0.04779
4200	0.04342	0.06002
4400	0.05366	0.07379
4600	0.06510	0.08898
4800	0.07774	0.1056
5000	0.09146	0.1233
5200	0.1063	0.1423
5400	0.1222	0.1622
5600	0.1388	0.1828
5800	0.1566	0.2044
6000	0.1750	0.2264
6200	0.1942	0.2489



As temperature increases at fixed pressure, the amount of NO in the equilibrium mixture increases.

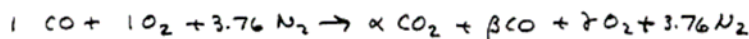
PROBLEM 14.25

KNOWN: A vessel initially containing 1 kmol of CO and 4.76 kmols of dry air forms an equilibrium mixture of CO_2 , CO, O_2 , and N_2 at 3000K, 1 atm.

FIND: Determine the equilibrium composition.

ENGINEERING MODEL: (1) The equilibrium mixture is modeled as an ideal gas mixture. (2) The N_2 is inert.

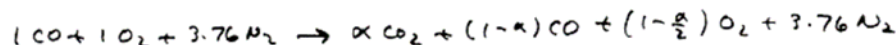
ANALYSIS: The reaction takes the form



$$\text{C: } 1 = \alpha + \beta \Rightarrow \beta = 1 - \alpha$$

$$\text{O: } 3 = 2\alpha + 2\gamma \Rightarrow 3 = 2\alpha + (1 - \alpha) + 2\gamma \Rightarrow \gamma = 1 - \frac{\alpha}{2}$$

Thus



The amount of mixture is $n = \alpha + (1 - \alpha) + \left(1 - \frac{\alpha}{2}\right) + 3.76 = 5.76 - \frac{\alpha}{2}$.

At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$. Accordingly Eq 14.35 takes the form

$$K = \frac{[\alpha] \left[\frac{2-\alpha}{2}\right]^{1/2}}{\alpha} \left[\frac{P/P_{\text{ref}}}{(5.76-\alpha)/2}\right]^{1/2} \Rightarrow K = \frac{1-\alpha}{\alpha} \left[\frac{2-\alpha}{11.52-\alpha}\right]^{1/2}$$

From Table A-27 at 3000K, $\log_{10} K = -0.485 \Rightarrow K = 0.32734$. Thus

$$0.32734 = \frac{1-\alpha}{\alpha} \left[\frac{2-\alpha}{11.52-\alpha}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $\alpha = 0.528$.

The equilibrium composition is

$$\{0.528 \text{ CO}_2, 0.472 \text{ CO}, 0.736 \text{ O}_2, 3.76 \text{ N}_2\}$$

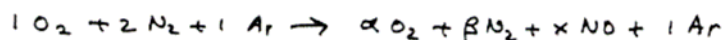
PROBLEM 14.26

KNOWN: A vessel initially containing 1 kmol O_2 , 2 kmol N_2 , 1 kmol Ar forms an equilibrium mixture of O_2 , N_2 , NO, and Ar at 3000 K, 1 atm.

FIND: Determine the equilibrium composition.

ENGINEERING MODEL: (1) The equilibrium mixture is modeled as an ideal gas.
(2) Argon is inert.

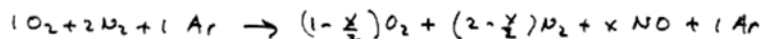
ANALYSIS: The reaction takes the form



$$O: 2 = 2\alpha + x, \quad x = 2 - 2\alpha$$

$$N: 4 = 2\beta + x, \quad \beta = 2 - x/2$$

Thus



The amount of mixture is $n = (1 - \frac{x}{2}) + (2 - \frac{x}{2}) + x + 1 = 4$.

At equilibrium $\frac{1}{2} O_2 + \frac{1}{2} N_2 \rightleftharpoons NO$. Accordingly, Eq. 14.35 takes the form

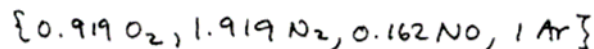
$$\textcircled{1} \quad K = \frac{x}{[1 - \frac{x}{2}]^{1/2} [2 - \frac{x}{2}]^{1/2}} \left[\frac{P/P_{ref}}{4} \right]^{1 - 1/2 - 1/2} \Rightarrow K = \frac{x}{[1 - \frac{x}{2}]^{1/2} [2 - \frac{x}{2}]^{1/2}}$$

At 3000 K, Table A-27 gives $\log_{10} K = -0.913 \Rightarrow K = 0.12218$. Thus

$$0.12218 = \frac{x}{[(1 - \frac{x}{2})(2 - \frac{x}{2})]^{1/2}}$$

Using an equation solver or iteration with a hand calculator, $x = 0.162$.

The equilibrium composition is



1. Note that (P/P_{ref}) drops out of this expression.

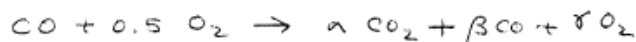
PROBLEM 14.27

KNOWN! One kmol CO and 0.5 kmol O₂ react to form {CO₂, O₂, CO} at T, p. When p = 1 atm, 0.35 kmol of CO is present in the equilibrium mixture.

FIND: Determine the amount of CO present in an equilibrium mixture at T, p = 10 atm.

ENGINEERING MODEL: The equilibrium mixture is modeled as an ideal gas.

ANALYSIS: The reaction takes the form



$$\text{C: } 1 = \alpha + \beta \Rightarrow \alpha = 1 - \beta$$

$$\text{O: } 2 = 2(1 - \beta) + \beta + 2\gamma$$

$$\Rightarrow \gamma = \beta/2$$

For this case, $n = (1 - \beta) + \beta + \beta/2 = 1 + (\beta/2)$. For CO₂ $\rightleftharpoons$ CO + $\frac{1}{2}$ O₂
Eq. 14.35 reads

$$K = \frac{\beta (\beta/2)^{1/2}}{(1 - \beta)} \left[\frac{p/P_{\text{ref}}}{1 + (\beta/2)} \right]^{1/2} \quad (1)$$

Thus, when $\beta = 0.35$, and $p = P_{\text{ref}}$, we get

$$K = \frac{(0.35)(0.175)^{1/2}}{(0.65)} \left[\frac{1}{1.175} \right]^{1/2} = 0.2078$$

Since T remains unchanged, K remains unchanged. Then, with $p = 10 \text{ atm}$, Eq. (1) reads

$$\begin{aligned} 0.2078 &= \frac{\beta (\beta/2)^{1/2}}{1 - \beta} \left[\frac{10}{1 + (\beta/2)} \right]^{1/2} \\ &= \frac{\beta}{1 - \beta} \left[\frac{10\beta}{2 + \beta} \right]^{1/2} \end{aligned}$$

① Using an equation solver or iteration with a hand calculator,
 $\beta = 0.1844$ ←

-
1. Keeping temperature constant and increasing pressure results in a greater amount of CO₂ formed.

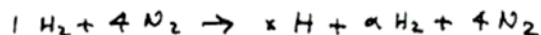
PROBLEM 14.28

KNOWN: A vessel initially contains 1 kmol H_2 and 4 kmol N_2 . An equilibrium mixture of H_2 , H , and N_2 forms at 3000 K, 1 atm.

FIND: Determine the equilibrium composition. If pressure is increased while keeping temperature fixed, determine if the amount of H would increase or decrease.

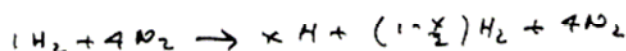
ENGINEERING MODEL: (1) The equilibrium mixture is modeled as an ideal gas. (2) N_2 is inert.

ANALYSIS: The reaction takes the form



$$H: 2 = x + 2\alpha \Rightarrow \alpha = 1 - \frac{x}{2}$$

Thus



The amount of mixture is $n = x + (1 - \frac{x}{2}) + 4 = 5 + \frac{x}{2}$.

At equilibrium $H_2 \rightleftharpoons 2H$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x]^2}{[1 - \frac{x}{2}]} \left[\frac{P/P_{ref}}{10 + \frac{x}{2}} \right]^{2-1} \Rightarrow K = \frac{4x^2 (P/P_{ref})}{(2-x)(10+x)} \quad (1)$$

Eq (1) gives upon rearrangement

$$(\gamma + 1)x^2 + 8x - 20 = 0$$

where $\gamma = [4(P/P_{ref})/K]$. Solving this quadratic equation

$$x = \frac{-8 + \sqrt{64 + 80(\gamma + 1)}}{2(\gamma + 1)} \quad (2)$$

From Table A.27 at 3000 K $\log_{10} K = -1.606 \Rightarrow K = 0.024774$. With $P/P_{ref} = 1$, $\gamma = 161.46$. Thus, Eq. (2) gives $x = 0.3271$. The equilibrium mixture is then

$$\{ 0.3271 H, 0.9365 H_2, 4 N_2 \}$$

To determine the effect on x owing to an increase in pressure at fixed temperature, consider $P/P_{ref} = 1.1$. Then $\gamma = 177.61$ and Eq. (2) gives $x = 0.313$. Accordingly x tends to decrease as p increases.

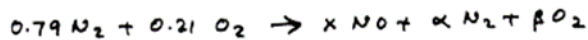
PROBLEM 14.29

KNOWN: An equilibrium mixture of N_2 , O_2 , and NO at $4342^\circ R$, 1 atm forms from dry air.

FIND: Determine the mole fraction of NO in the equilibrium mixture. Determine if the amount of NO increases or decreases as T decreases at fixed P .

ENGINEERING MODEL: (1) The equilibrium mixture is modeled as an ideal gas mixture.
(2) The analysis is based on 1 lbmol of air initially.

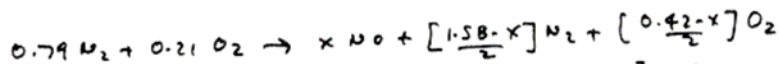
ANALYSIS: The reaction takes the form



$$N: (0.79)(2) = x + 2\alpha \Rightarrow \alpha = (1.58 - x)/2$$

$$O: (0.21)(2) = x + 2\beta \Rightarrow \beta = (0.42 - x)/2$$

Thus



The amount of mixture is $n = x + \left[\frac{1.58 - x}{2} \right] + \left[\frac{0.42 - x}{2} \right] = 1$

At equilibrium $\frac{1}{2} O_2 + \frac{1}{2} N_2 \rightleftharpoons NO$. According to Eq. 14.35 takes the form

$$K = \frac{x}{\left[\frac{1.58 - x}{2} \right]^{1/2} \left[\frac{0.42 - x}{2} \right]^{1/2}} \left[\frac{P/P_{ref}}{1} \right]^{1 - 1/2 - 1/2} = \frac{2x}{[1.58 - x]^{1/2} [0.42 - x]^{1/2}} \quad (1)$$

or

$$K^2 = \frac{4x^2}{(1.58 - x)(0.42 - x)} \Rightarrow \gamma x^2 + 2x - 0.6676 = 0 \quad (1)$$

where $\gamma = \frac{4}{K^2} - 1$. Solving this quadratic equation

$$x = \frac{-2 + \sqrt{4 + 2.6544\gamma}}{2\gamma} \quad (2)$$

At $4342^\circ R$, Table A-27 gives $\log_{10} K = -1.2956 \Rightarrow K = 0.05062$. Thus $\gamma = 1560$ and Eq. (2) gives $x = 0.02$. The mole fraction of NO in the equilibrium mixture is then

$$y_{NO} = \frac{0.02}{1} = 0.02$$

To determine the effect of a higher temperature on x , consider $T = 4500^\circ R$. Then Table A-27 gives $\log_{10} K = -1.227 \Rightarrow K = 0.05929$. Thus $\gamma = 1136.9$ and Eq. (2) gives $x = 0.0233$. Thus, as temperature decreases the amount of NO also decreases.

- Note that (P/P_{ref}) drops out of this expression, and thus of the calculation of the equilibrium composition.

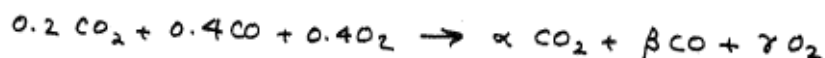
PROBLEM 14.30

KNOWN: A gaseous mixture with a specified molar analysis is heated at a pressure of 1.5 bar and forms an equilibrium mixture of CO_2 , CO , and O_2 at 2000 K.

FIND: Determine the molar analysis of the equilibrium mixture.

ENGINEERING MODEL: (1) The analysis is based on 1 kmol of initial mixture. (2) The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: On the basis of 1 kmol of initial mixture, the reaction is

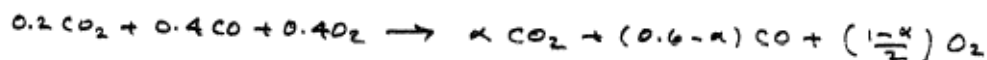


$$\text{C: } 0.2 + 0.4 = \alpha + \beta \Rightarrow \beta = 0.6 - \alpha$$

$$\text{O: } 2(0.2) + 0.4 + 2(0.4) = 2\alpha + \beta + 2\gamma$$

$$1.6 = 2\alpha + (0.6 - \alpha) + 2\gamma \Rightarrow \gamma = \frac{1 - \alpha}{2}$$

Then



The amount of mixture is $n = \alpha + (0.6 - \alpha) + \left(\frac{1 - \alpha}{2}\right) = (2.2 - \alpha)/2$.

At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$. Accordingly Eq 14.35 takes the form

$$K = \frac{\left[\frac{0.6 - \alpha}{\alpha}\right] \left[\frac{1 - \alpha}{2.2 - \alpha}\right]^{1/2}}{\left[\frac{1 - \alpha}{2.2 - \alpha}\right]^{1/2}} \left[\frac{P/P_{\text{ref}}}{(2.2 - \alpha)/2}\right]^{1 + 1/2 - 1}$$

$$P/P_{\text{ref}} = 1.5/1.01325 \Rightarrow$$

$$K = \left[\frac{0.6 - \alpha}{\alpha}\right] \left[\frac{1 - \alpha}{2.2 - \alpha}\right]^{1/2} [1.21671]$$

From Table A-27 at 2000 K, $\log_{10} K = -0.485 \Rightarrow K = 0.32734$. Thus

$$0.27127 = \left[\frac{0.6 - \alpha}{\alpha}\right] \left[\frac{1 - \alpha}{2.2 - \alpha}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $\alpha = 0.408$.

The analysis of the equilibrium mixture in terms of mole fractions is

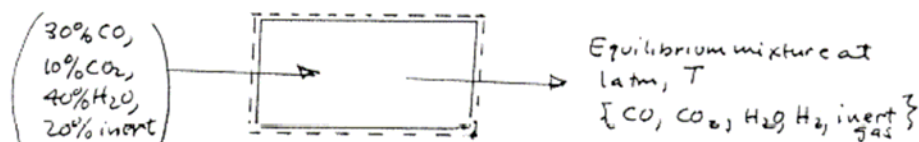
$$\begin{cases} y_{\text{CO}_2} = \frac{0.408}{0.896} = 0.4554 & (45.54\%) \\ y_{\text{CO}} = \frac{0.192}{0.896} = 0.2143 & (21.43\%) \\ y_{\text{O}_2} = \frac{0.296}{0.896} = 0.3304 & (33.04\%) \end{cases}$$

PROBLEM 14.31

KNOWN: An ideal gas mixture with a molar analysis {30% CO, 10% CO₂, 40% H₂O, 20% inert} enters a reactor operating at steady state. An equilibrium mixture of CO, CO₂, H₂O, H₂, and inert gas exits the reactor at 1 atm.

FIND: (a) If the exiting mixture is at 1200 K, determine the ratio of moles of H₂ exiting to moles of H₂O entering.
(b) If y_{CO} = 0.075 in the exiting mixture, determine the mixture temperature.

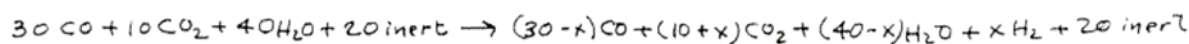
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown is at steady state. (2) The exiting mixture is modeled as an ideal gas mixture.

ANALYSIS: Based on 100 kmol of entering gas, at steady state



For the exiting mixture, $n = (30-x) + (10+x) + (40-x) + x + 20 = 100 \text{ kmol}$

At equilibrium for $\text{CO} + \text{H}_2\text{O} \rightleftharpoons \text{CO}_2 + \text{H}_2$, and so

$$\textcircled{1} \quad K = \frac{(10+x)(x)}{(30-x)(40-x)} \left[\frac{P/P_{\text{ref}}}{n} \right]^{1+1-1-1} = \frac{(10+x)(x)}{(30-x)(40-x)} \quad (1)$$

(a) When $T = 1200 \text{ K}$, Table A-27 gives $\log_{10} K = -0.135 \Rightarrow K = 0.7328$.

Rearranging Eq. (1), it can be expressed as a quadratic equation:

$$x^2 + 229.401x - 3291.02 = 0. \text{ Solving with the quadratic formula, } x = 13.547$$

$$\text{Thus, } \frac{13.547 \text{ kmol H}_2 \text{ (produced)}}{40 \text{ kmol H}_2\text{O (feed)}} = 0.339 \quad (a)$$

(b) When $y_{\text{CO}} = 0.075$, then

$$y_{\text{CO}} = \frac{(30-x)}{100} = 0.075 \Rightarrow x = 22.5 \quad \text{Eq. (1)} \quad K = \frac{(32.5)(22.5)}{(7.5)(17.5)} \Rightarrow \log_{10} K = 0.74597$$

For use with Table A-27, $\log_{10} K = -0.74597$. Inspection shows that $500 < T < 1000 \text{ K}$. Using the interpolation rule of Problem 14.6:

$$\log_{10} K = C_1 + \frac{C_2}{T}$$

$$\begin{array}{lcl} 500 \text{ K:} & -2.139 = C_1 + \frac{C_2}{500} \\ 1000 \text{ K:} & -0.159 = C_1 + \frac{C_2}{1000} \end{array} \quad \left. \vphantom{\begin{array}{l} 500 \text{ K:} \\ 1000 \text{ K:} \end{array}} \right\} \begin{array}{l} C_1 = 1.821 \\ C_2 = -1980 \text{ K} \end{array}$$

$$\text{Thus, } -0.74597 = 1.821 - \frac{1980}{T} \Rightarrow \frac{1980}{T} = 2.56697 \Rightarrow T = 771 \text{ K} \quad (b)$$

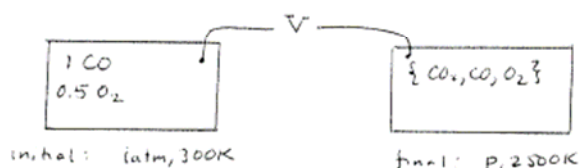
1. Note that (P/P_{ref}) drops out of this expression, and thus plays no role in the subsequent evaluations.

PROBLEM 14.32

KNOWN: One kmol CO and 0.5 kmol O_2 , initially at 1 atm and 300 K, reacts to form an equilibrium mixture of $\{CO_2, CO, O_2\}$ at 2500 K and the same total volume.

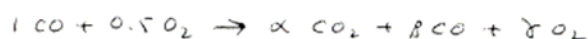
FIND: Determine the final pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The ideal gas model applies.

ANALYSIS: The reaction has the form



$$C: 1 = \alpha + \beta \Rightarrow \alpha = 1 - \beta, \quad O: 2 = 2(1 - \beta) + \beta + 2\gamma, \quad \gamma = \beta/2$$

The amount of mixture is $n = (1 - \beta) + \beta + \beta/2 = 1 + \beta/2$.

The ideal gas equation of state for the initial mixture gives $p'V = n'RT'$, where the prime indicates the initial values. For the final mixture $pV = nRT$. Accordingly

$$\frac{p}{p'} = \frac{nT}{n'T'} \Rightarrow \frac{p}{n} = \frac{p'}{n'} \frac{T}{T'} = \left(\frac{1 \text{ atm}}{1.5 \text{ kmol}} \right) \left(\frac{2500 \text{ K}}{300 \text{ K}} \right) = 5.556 \frac{\text{atm}}{\text{kmol}} \quad (1)$$

For $CO_2 \rightleftharpoons CO + \frac{1}{2} O_2$, Eq. 14.35 takes the form

$$K = \frac{\beta [9/2]^{1/2}}{1 - \beta} \left[\frac{p}{n} \right]^{1/2}$$

$\uparrow$
 $5.556 \frac{\text{atm}}{\text{kmol}} \quad (\text{Eq. (1)})$

With $K = 0.0363$ from Table A-27 at 2500 K

$$0.0363 = \frac{\beta}{1 - \beta} \left[\frac{5.556}{2} \beta \right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $\beta = 0.074$. Thus

$$n = 1 + \frac{0.074}{2} = 1.037 \text{ kmol}$$

Returning to Eq. (1)

$$p = (5.556)(1.037) = 5.761 \text{ atm}$$

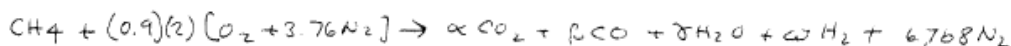
PROBLEM 14.33

KNOWN: CH_4 burns with 90% of theoretical air to form an equilibrium mixture of CO_2 , CO , $\text{H}_2\text{O}(g)$, H_2 , N_2 at 1000K , 1atm .

FIND: Determine the composition of the equilibrium mixture, per kmol of mixture.

ENGINEERING MODEL: (1) The ideal gas model applies. (2) N_2 is inert.

ANALYSIS: The reaction takes the form



$$\text{C: } 1 = \alpha + \beta \Rightarrow \alpha = 1 - \beta$$

$$\text{H: } 4 = 2\gamma + 2\omega, \quad 2 = \gamma + \omega$$

$$\text{O: } 3.6 = 2(1 - \beta) + \beta + \gamma \Rightarrow \gamma = 1.6 + \beta \Rightarrow \omega = 0.4 - \beta$$

At equilibrium, $\text{CO}_2 + \text{H}_2 \rightleftharpoons \text{CO} + \text{H}_2\text{O}$. Table A-27 gives for $T = 1000\text{K}$, $K = 0.693$.
And therefore

$$\textcircled{1} \quad 0.693 = \frac{[\beta][1.6 + \beta]}{[1 - \beta][0.4 - \beta]} \left[\frac{P/P_{\text{ref}}}{n} \right]^{\overbrace{1+1-1-1}^0}$$

Using an equation solver or iteration with a hand calculator, $\beta = 0.1065$. So, the composition of the equilibrium mixture per kmol of CH_4 is

$$\{0.8935 \text{CO}_2, 0.1065 \text{CO}, 1.7065 \text{H}_2\text{O}, 0.2935 \text{H}_2, 6.768 \text{N}_2\}$$

The amount of mixture per kmol of CH_4 is 9.768 kmol . So, the molar analysis of the equilibrium mixture is

$$9.15\% \text{CO}_2, 1.09\% \text{CO}, 17.47\% \text{H}_2\text{O}(g), 3\% \text{H}_2, 69.29\% \text{N}_2$$

-
1. Note that (P/P_{ref}) drops out of this expression and thus out of the subsequent evaluations.

PROBLEM 14.34

KNOWN: C_8H_{18} burns with air to form an equilibrium mixture of $\{CO_2, H_2O, CO, H_2, N_2\}$ at 1700 K , 1 atm .

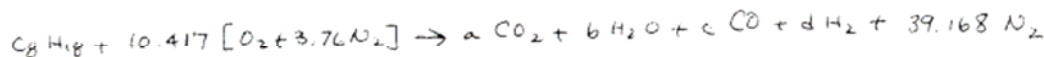
FIND: If the equivalence ratio is 1.2, determine the composition of the equilibrium mixture in kmol per kmol of fuel.

ENGINEERING MODEL: (1) The ideal gas model applies. (2) N_2 is inert.

ANALYSIS: With $(AF)_{theo} = 12.5(4.76)$ from Example 13.4

$$\text{Equivalence ratio} = \frac{(AF)_{theo}}{AF} \Rightarrow AF = \frac{12.5(4.76)}{1.2} = (10.417)(4.76)$$

Thus



$$C: 8 = a + c \Rightarrow a = 8 - c$$

$$H: 18 = 2b + 2d \Rightarrow d = 9 - b$$

$$O: 20.834 = 2a + b + c \\ = 2(8 - c) + b + c \Rightarrow b = 4.834 + c \quad \text{and} \quad d = 9 - [4.834 + c] \\ = 4.166 - c$$

At equilibrium, $CO_2 + H_2 \rightleftharpoons CO + H_2O$. Table A-27 gives for $T = 1700\text{ K}$, $K = 3.388$. And therefore

$$\textcircled{1} \quad 3.388 = \frac{[c][4.834 + c]}{[8 - c][4.166 - c]} \left[\frac{P/P_{ref}}{n} \right]^{\frac{0}{(1+1)-(1-1)}}$$

This can be solved analytically by expressing it as a quadratic equation and using the quadratic formula. Alternatively, an equation solver can be used, or iteration with a hand calculator can be employed. We get $c = 2.883$.

The composition of the equilibrium mixture, per kmol of C_8H_{18} , is then

$$\{5.117 CO_2, 7.717 H_2O(g), 2.883 CO, 1.283 H_2, 39.168 N_2\} \leftarrow$$

-
1. Note that (P/P_{ref}) drops out of this expression and thus from the following evaluations.

PROBLEM 14.35

KNOWN: C_2H_2 at $25^\circ C$, 1 atm, burns with 40% excess air at $25^\circ C$, 1 atm, $\phi = 80\%$.
An equilibrium mixture of $\{CO_2, H_2O, O_2, NO, N_2\}$ is formed at $2200 K$, 0.9 atm.

FIND: Determine the composition of the equilibrium mixture, per kmol of C_2H_2 .

ENGINEERING MODEL: The ideal gas model applies.

ANALYSIS: The balanced reaction equation for complete combustion of C_2H_2 with the theoretical amount of dry air is $C_2H_2 + \frac{5}{2}(O_2 + 3.76 N_2) \rightarrow 2CO_2 + H_2O + (\frac{5}{2})3.76 N_2$.

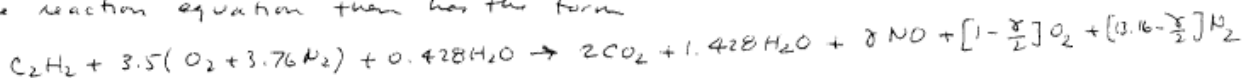
The amount of dry air for combustion with 40% excess air is, then $(1.4)(2.5)(4.76)$ per kmol of C_2H_2 . That is, $(3.5)(4.76) = 16.66 \text{ kmol (air)/kmol (fuel)}$.

The amount of water vapor accompanying the dry air is found using Chap. 12 concepts: $P_v = \phi P_g(25^\circ C) = (0.8)(0.03169 \text{ bar}) = 0.025352 \text{ bar}$.

$P_a = P - P_v = 1.01325 - 0.02535 = 0.987898 \text{ bar}$. Then, referring to the development of Eqs. 12.41, we get

$$\frac{n_v}{n_a} = \frac{P_v}{P_a} \Rightarrow n_v = \left(\frac{0.025352}{0.987898} \right) \left(16.66 \frac{\text{kmol (dry air)}}{\text{kmol (fuel)}} \right) = 0.428 \frac{\text{kmol (H}_2\text{O)}}{\text{kmol (fuel)}}$$

The reaction equation then has the form



At equilibrium, $\frac{1}{2} O_2 + \frac{1}{2} N_2 \rightleftharpoons NO$. Table A-27 gives for $2200 K$, $K = 0.0328$.

And therefore

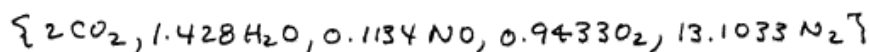
$$\textcircled{1} \quad 0.0328 = \frac{\delta}{\left[1 - \frac{\delta}{2}\right]^{1/2} \left[13.16 - \frac{\delta}{2}\right]^{1/2}} \left[\frac{P/P_{atm}}{n} \right]^{1 - \frac{1}{2} - \frac{1}{2}}$$

$$0.0328 = \frac{\delta}{\left[\left(1 - \frac{\delta}{2}\right)\left(13.16 - \frac{\delta}{2}\right)\right]^{1/2}}$$

Squaring both sides this can be expressed as a quadratic equation and solved using the quadratic formula. Alternatively, an equation solver can be used, or iteration with a hand calculator employed.

We get $\delta = 0.1134$.

The composition of the equilibrium mixture, per kmol of C_2H_2 , is



1. Note that (P/P_{atm}) drops out of this expression.

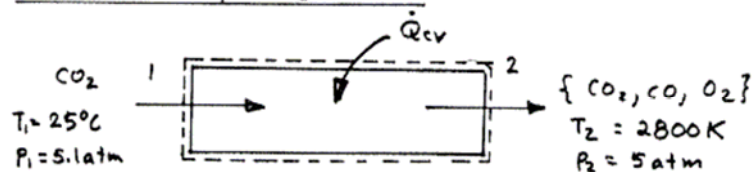
PROBLEM 14.36

KNOWN: CO_2 gas at 25°C , 5 atm enters a heat exchanger operating at steady state.

An equilibrium mixture of CO_2 , CO , and O_2 exits at 2800 K , 5 atm .

FIND: Determine the composition of the exiting mixture and the heat transfer, each per kmol of CO_2 entering.

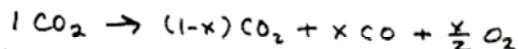
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ideal gas model applies to the incoming CO_2 and the exiting equilibrium mixture.

ANALYSIS: The reaction has the form



The amount of mixture is $n = (1-x) + x + \frac{x}{2} = (2+x)/2$. At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{ O}_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x][x/2]^{1/2}}{[1-x]} \left[\frac{P/P_{\text{ref}}}{(x+2)/2} \right]^{1/2} = \left[\frac{x}{1-x} \right] \left[\frac{5x}{x+2} \right]^{1/2}$$

From Table A-27 at 2800 K , $\log_{10} K = -0.825 \Rightarrow K = 0.14962$. Solving, $x = 0.1867$.

The equilibrium mixture has the following composition in kmol per kmol of CO_2 entering

$$\{ 0.8133 \text{ CO}_2, 0.1867 \text{ CO}, 0.09335 \text{ O}_2 \}$$

An energy rate balance reduces at steady state to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}_2}} - \frac{\dot{W}_{cv}}{\dot{n}_{\text{CO}_2}} + 1 \cdot \bar{h}_{\text{CO}_2}(T_1) - [0.8133 \bar{h}_{\text{CO}_2}(T_2) + 0.1867 \bar{h}_{\text{CO}}(T_2) + 0.09335 \bar{h}_{\text{O}_2}(T_2)]$$

or

$$\frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}_2}} = 0.8133 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{CO}_2} + 0.1867 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{CO}} + 0.09335 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{O}_2} - [\bar{h}_f^\circ]_{\text{CO}_2}$$

With data from the ideal gas tables

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}_2}} &= 0.8133 [-393,520 + 149,808 - 93,64] + 0.1867 [-110,530 + 94,784 - 8,669] + \\ &\quad 0.09335 [98,826 - 8,682] - [-393,520] \\ &= 191,550 \text{ kJ/kmol(CO}_2\text{)} \end{aligned}$$

Continued on next slide

Problem 14-36 continued

IT can be used as an alternative to iteration using a hand calculator and to using table data in the energy balance. The IT program follows.

IT Code

```
T1 = 25 + 273.15 // K
p1 = 5.1 // atm
T2 = 2800 // K
p2 = 5 // atm

// CO2 ==> (1 - x) CO2 + x CO + x/2 O2
ndot1 = 1 // per kmol entering
ndotCO2_2 = 1 - x
ndotCO_2 = x
ndotO2_2 = x/2
ndot2 = ndotCO2_2 + ndotCO_2 + ndotO2_2
yCO2 = ndotCO2_2 / ndot2
yCO = ndotCO_2 / ndot2
yO2 = ndotO2_2 / ndot2
pref = 1 // atm

// For the reaction CO2 <==> CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p2 / pref)^0.5
//Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T2,3)

0 = Qdot + ndot1*hCO2_1 - ndotCO2_2*hCO2_2 - ndotCO_2*hCO_2 -
ndotO2_2*hO2_2
hCO2_1 = h_T("CO2",T1)
hCO2_2 = h_T("CO2",T2)
hCO_2 = h_T("CO",T2)
hO2_2 = h_T("O2",T2)
```

IT Results

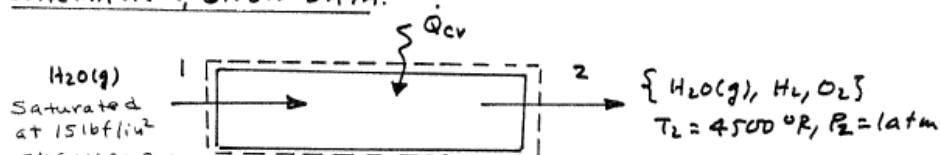
```
K = 0.1496
 $\dot{n}_{\text{CO}_2,2} / \dot{n}_{\text{CO}_2,1} = 0.8136 \text{ kmol / kmol (CO}_2 \text{ entering)}$ 
 $\dot{n}_{\text{CO},2} / \dot{n}_{\text{CO}_2,1} = 0.1864 \text{ kmol / kmol (CO}_2 \text{ entering)}$ 
 $\dot{n}_{\text{O}_2,2} / \dot{n}_{\text{CO}_2,1} = 0.09322 \text{ kmol / kmol (CO}_2 \text{ entering)}$ 
 $\dot{Q}_{\text{cv}} / \dot{n}_{\text{CO}_2,1} = 1.916\text{E5 kJ/kmol (CO}_2 \text{ entering)}$ 
```


PROBLEM 14.37

KNOWN: Saturated water vapor at 15 lbf/in^2 enters a heat exchanger operating at steady state. An equilibrium mixture of $\text{H}_2\text{O(g)}$, H_2 , and O_2 exits at 4500°R , 1 atm .

FIND: Determine the composition of the exiting mixture and the heat transfer, each per 1 lbmol of steam entering.

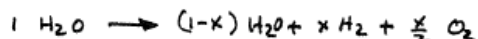
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The incoming steam and exiting equilibrium mixture can be modeled as ideal gases.

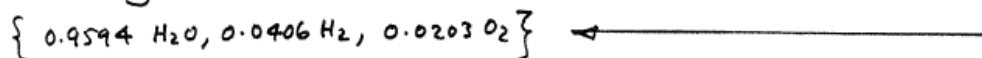
ANALYSIS: The reaction has the form



The amount of mixture is $n = (1-x) + x + \frac{x}{2} = \frac{(x+2)}{2}$. At equilibrium $\text{H}_2\text{O} \rightleftharpoons \text{H}_2 + \frac{1}{2} \text{O}_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x] [\frac{x}{2}]^{1/2}}{[1-x]} \left[\frac{P/P_{\text{ref}}}{(x+2)/2} \right]^{1/2} = \left[\frac{x}{1-x} \right] \left[\frac{x}{x+2} \right]^{1/2}$$

From Table A-27 at 4500°R $\log_{10} K = -2.224 \Rightarrow K = 0.00597$. Solving $x = 0.0406$. The equilibrium mixture has the following composition in 1 lbmol per 1 lbmol of steam entering



An energy rate balance reduces at steady state to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} - \frac{\dot{W}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} + \bar{h}_{\text{H}_2\text{O}}(T_1) - [0.9594 \bar{h}_{\text{H}_2\text{O}}(T_2) + 0.0406 \bar{h}_{\text{H}_2}(T_2) + 0.0203 \bar{h}_{\text{O}_2}(T_2)]$$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} = 0.9594 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{\text{H}_2\text{O}} + 0.0406 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{\text{H}_2} + 0.0203 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{\text{O}_2} - [\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(537)]_{\text{H}_2\text{O}}$$

With ideal gas table data

$$T_1 = T_{\text{sat at } 15 \text{ lbf/in}^2} = 213^\circ\text{F} = 673^\circ\text{R}$$

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{\text{H}_2\text{O}}} &= 0.9594 [-104,040 + 46,836 - 4258] + 0.0406 [33921.6 - 3640.3] + \\ &\quad 0.0203 [37,412 - 3725.1] - [-104,040 + 5356 - 4258] \\ &= 45,888 \text{ Btu/lbmol (H}_2\text{O)} \end{aligned}$$

Continued on next slide

Problem 14-37 continued

PROBLEM 14.37 (Cont'd.)

Alternative IT solution

IT Code

```
p1 = 15 // lbf/in2, saturated vapor
T1 = Tsat_P("Steam",p1)
T2 = 4500 // °R
p2 = 1 // atm

// H2O → (1 - x) H2O + x H2 + x/2 O2
ndot1 = 1 // Do calculations on the basis of 1 lbmol of H2O entering.
ndotH2O_2 = 1 - x
ndotH2_2 = x
ndotO2_2 = x/2
ndot2 = ndotH2O_2 + ndotH2_2 + ndotO2_2
yH2O = ndotH2O_2 / ndot2
yH2 = ndotH2_2 / ndot2
yO2 = ndotO2_2 / ndot2
pref = 1 // atm

// For the reaction H2O ↔ H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p2 / pref)^0.5
//Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,2,T2,3)

0 = Qdot + ndot1 * hH2O_1 - (ndotH2O_2 * hH2O_2 + ndotH2_2 * hH2_2 + ndotO2_2 * hO2_2)
// Note: Use the ideal gas function for H2O in order to get the correct
// reference value for H2O in the calculations. This results in only a slight error.
hH2O_1 = h_T("H2O", T1)
hH2O_2 = h_T("H2O", T2)
hH2_2 = h_T("H2", T2)
hO2_2 = h_T("O2", T2)
```

IT Results

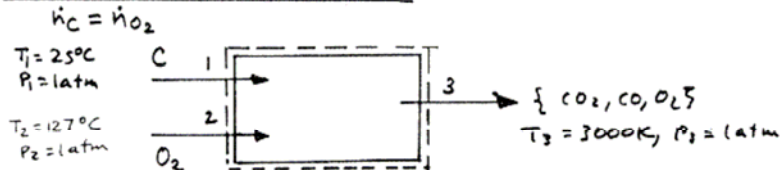
```
K = 0.00597
 $\dot{n}_{\text{H}_2\text{O},2} / \dot{n}_{\text{H}_2\text{O},1} = 0.9594 \text{ lbmol/lbmol}(\text{H}_2\text{O entering})$ 
 $\dot{n}_{\text{H}_2,2} / \dot{n}_{\text{H}_2\text{O},1} = 0.04061 \text{ lbmol/lbmol}(\text{H}_2\text{O entering})$ 
 $\dot{n}_{\text{O}_2,2} / \dot{n}_{\text{H}_2\text{O},1} = 0.0203 \text{ lbmol/lbmol}(\text{H}_2\text{O entering})$ 
 $\dot{Q}_{\text{cv}} / \dot{n}_{\text{H}_2\text{O},1} = 4.582 \times 10^4 \text{ Btu/lbmol}(\text{H}_2\text{O entering})$ 
```

PROBLEM 14.38

KNOWN: Carbon at 25°C, 1 atm and O₂ at 127°C, 1 atm enter a reactor operating at steady state with equal molar flow rates. An equilibrium mixture of CO₂, CO, and O₂ exits at 3000 K, 1 atm.

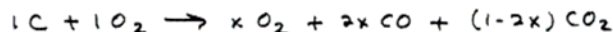
FIND: Determine the composition of the exiting mixture and the heat transfer, each per kmol of Carbon entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ideal gas model applies to the equilibrium mixture.

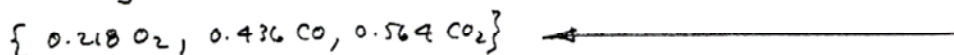
ANALYSIS: The reaction takes the form



The amount of mixture is $n = x + 2x + (1-2x) = 1+x$. At equilibrium $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[2x][x]^{1/2}}{[1-2x]} \left[\frac{P/P_{ref}}{1+x} \right]^{-1/2} = \frac{2x}{1-2x} \left[\frac{x}{1+x} \right]^{1/2}$$

At 3000 K, Table A.27 gives $\log_{10} K = -0.485 \Rightarrow K = 0.3273$. Solving $x = 0.218$. The equilibrium mixture has the following composition in kmol per kmol of Carbon entering



An energy rate balance reduces at steady state to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_C} - \frac{\dot{W}_{cv}}{\dot{n}_C} + \bar{h}_C(T_1, P_1) + \bar{h}_{O_2}(T_2, P_2) - [0.218 \bar{h}_{O_2}(T_3) + 0.436 \bar{h}_{CO}(T_3) + 0.564 \bar{h}_{CO_2}(T_3)]$$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{n}_C} = 0.218 [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{O_2} + 0.436 [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO} + 0.564 [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} - [\bar{h}(T_2) - \bar{h}(298)]_{O_2}$$

With data from the ideal gas tables

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_C} &= 0.218 [106,780 - 8682] + 0.436 [-110,530 + 102,210 - 8669] + \\ &\quad 0.564 [-393,520 + 164,226 - 9364] - [11,711 - 8682] \\ &= -124,782 \text{ kJ/kmol(C)} \end{aligned}$$

IT can be used as an alternative to iteration using a hand calculator and to using table data in the energy balance. The IT program follows.

Continued on next slide

Problem 14-38 continued

4 14.38 (cont'd.) Alternative IT solution

IT Code

```
p1 = 1 // atm
T1 = 25 + 273.15 // K
p2 = 1 // atm
T2 = 127 + 273.15 // K
ndot1 = 1 // Do all calculations on the basis of 1 kmol of carbon entering.
ndot2 = ndot1 // Entering streams have equal molar flow rates.
T3 = 2727 + 273.15 // K
p3 = 1 // atm
```

```
// C + O2 → x O2 + 2x CO + (1 - 2x) CO2
ndotO2_3 = x
ndotCO_3 = 2*x
ndotCO2_3 = 1 - 2 * x
ndot3 = ndotO2_3 + ndotCO_3 + ndotCO2_3
yO2 = ndotO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yCO2 = ndotCO2_3 / ndot3
pref = 1 // atm
```

```
// For the reaction CO2 ↔ CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQCO2a.LUT.
log(K) = LOOKUPVAL(EQCO2a,1,T3,3)
```

```
0 = Qdot + ndot1*hC + ndot2*hO2_2 - (ndotCO2_3*hCO2_3 +
ndotCO_3*hCO_3 + ndotO2_3*hO2_3)
hC = 0 // See Table A-25.
hO2_2 = h_T("O2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
```

IT Results

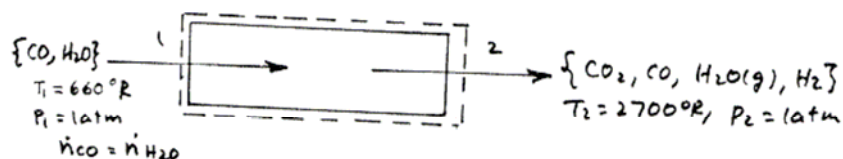
```
K = 0.3275
 $\dot{n}_{\text{CO}_2,3} / \dot{n}_1 = 0.5637 \text{ kmol/kmol(C)}$ 
 $\dot{n}_{\text{CO},3} / \dot{n}_1 = 0.4363 \text{ kmol/kmol(C)}$ 
 $\dot{n}_{\text{O}_2,3} / \dot{n}_1 = 0.2181 \text{ kmol/kmol(C)}$ 
 $\dot{Q}_{\text{cv}} / \dot{n}_1 = -1.246 \times 10^5 \text{ kJ/kmol(C)}$ 
```

PROBLEM 14.39

KNOWN: An equimolar mixture of CO and H₂O(g) at 200°F, 1 atm enters a reactor operating at steady state. An equilibrium mixture of CO₂, CO, H₂O(g), and H₂ exits at 2700°R, 1 atm.

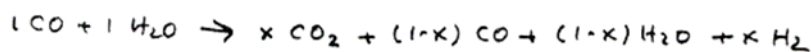
FIND: Determine the heat transfer per lbmol of CO entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ideal gas model applies to the incoming and outgoing mixtures.

ANALYSIS: The reaction takes the form



The amount of mixture is $n = x + (1-x) + (1-x) + x = 2$. At equilibrium $\text{CO}_2 + \text{H}_2 \rightleftharpoons \text{CO} + \text{H}_2\text{O}$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[1-x][1-x]}{[x][x]} \left[\frac{P/P_{ref}}{2} \right]^{1+1-1-1} = \left(\frac{1-x}{x} \right)^2 \Rightarrow x = \frac{1}{1+\sqrt{K}}$$

At 2700°R, Table A-27 gives $\log_{10} K = 0.4035 \Rightarrow K = 2.5322$. Solving, $x = 0.386$.

At steady state an energy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CO}} - \frac{\dot{W}_{cv}}{\dot{n}_{CO}} + \bar{h}_{CO}(T_1) + \bar{h}_{H_2O}(T_1) - [0.386 \bar{h}_{CO_2}(T_2) + 0.614 \bar{h}_{CO}(T_2) + 0.614 \bar{h}_{H_2O}(T_2) + 0.386 \bar{h}_{H_2}(T_2)]$$

Thus

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{CO}} = & 0.386 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{CO_2} + 0.614 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{CO} + \\ & 0.614 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{H_2O} + 0.386 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{H_2} - \\ & \{ 1 [\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(537)]_{CO} + 1 [\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(537)]_{H_2O} \} \end{aligned}$$

With data from the ideal gas tables

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{CO}} = & 0.386 [-169,300 + 30,581 - 4027.5] + 0.614 [-47,540 + 20,434 - 3725.1] + \\ & 0.614 [-104,040 + 24,957 - 4258] + 0.386 [19237.8 - 3640.3] - \\ & \{ 1 [-47,540 + 4586.6 - 3725.1] + 1 [-104,040 + 5250 - 4258] \} \\ = & 30,547 \text{ Btu/lbmol(CO)} \end{aligned}$$

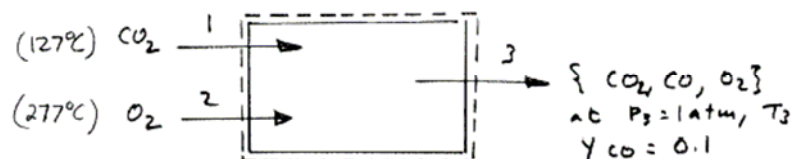
Alternatively, a solution using IT can be developed.

PROBLEM 14.40

KNOWN: CO_2 and O_2 in a 1:2 molar ratio enter a reactor operating at steady state in separate streams at 1 atm and 127°C , 277°C respectively. An equilibrium mixture of CO_2 , CO , and O_2 exits at 1 atm. The mole fraction of CO in the exiting mixture is 0.1

FIND: Determine the heat transfer per kmol of CO_2 entering.

SCHEMATIC & GIVEN DATA:

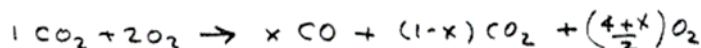


$$P_1 = P_2 = 1 \text{ atm}$$

$$\dot{n}_{\text{O}_2} / \dot{n}_{\text{CO}_2} = 2$$

ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{\text{cv}} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas.

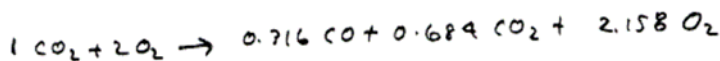
ANALYSIS: The first step is to determine T_3 using equilibrium principles. The reaction takes the form



The amount of mixture is $n = x + (1-x) + \left(\frac{4+x}{2}\right) = \frac{6+x}{2}$. From the given data $y_{\text{CO}} = 0.1$. Thus

$$0.1 = \frac{x}{(6+x)/2} \Rightarrow x = 0.316$$

Thus



At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{ O}_2$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[0.316][2.158]^{1/2}}{0.684} \left[\frac{P/P_{\text{ref}}}{3.158} \right]^{1/2} = \left(\frac{0.316}{0.684} \right) \left(\frac{2.158}{3.158} \right)^{1/2} = 0.3819$$

Thus $\log_{10} K = -0.418$. Interpolation in Table A-27 gives $T_3 = 3044 \text{ K}$.

At steady state an energy rate balance reduces to give

$$0 = \frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{CO}_2}} - \frac{\dot{W}_{\text{cv}}}{\dot{n}_{\text{CO}_2}} + \bar{h}_{\text{CO}_2}(T_1) + 2\bar{h}_{\text{O}_2}(T_1) - [0.316 \bar{h}_{\text{CO}}(T_3) + 0.684 \bar{h}_{\text{CO}_2}(T_3) + 2.158 \bar{h}_{\text{O}_2}(T_3)]$$

$$\text{or } \frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{CO}_2}} = 0.316 [\bar{h}_f^0 + \bar{h}(T_3) - \bar{h}(298)]_{\text{CO}} + 0.684 [\bar{h}_f^0 + \bar{h}(T_3) - \bar{h}(298)]_{\text{CO}_2} + 2.158 [\bar{h}_f^0 + \bar{h}(T_3) - \bar{h}(298)]_{\text{O}_2} - [\bar{h}_f^0 + \bar{h}(T_1) - \bar{h}(298)]_{\text{CO}_2} - 2[\bar{h}(T_1) - \bar{h}(298)]_{\text{O}_2}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{CO}_2}} = 0.316 [-110,530 + 103,849 - 8669] + 0.684 [-393,520 + 164,967 - 9364] + 2.158 [108,538 - 8682] - [(-393,520) + 13,372 - 9364] - 2[14,770 - 8682]$$

$$= 425,239 \text{ kJ/kmol(CO}_2\text{)}$$

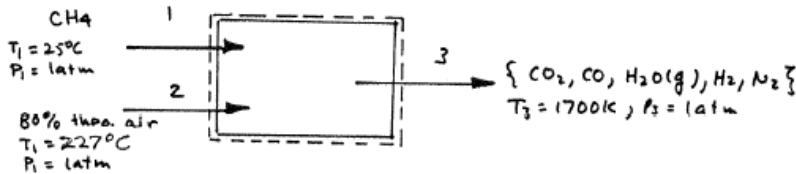
$\dot{Q}_{\text{cv}} / \dot{n}_{\text{CO}_2}$

PROBLEM 14.41

KNOWN: CH_4 at 25°C , 1 atm enters a reactor operating at steady state and burns with 80% of theoretical air entering at 227°C , 1 atm. An equilibrium mixture of CO_2 , CO , $\text{H}_2\text{O}(\text{g})$, H_2 and N_2 exits at 1700K , 1 atm.

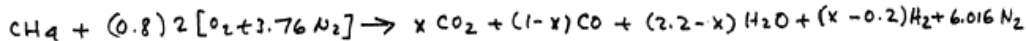
FIND: Determine the composition of the exiting mixture and the heat transfer, each per kmol of CH_4 entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume, shown in the accompanying figure operates at steady state with $\dot{W}=0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas. (3) N_2 is inert.

ANALYSIS: The balanced reaction equation for complete combustion of CH_4 with the theoretical amount of air is given by Eq. 13.4. Accordingly, combustion with 80% of theoretical air to form CO_2 , CO , $\text{H}_2\text{O}(\text{g})$, H_2 , and N_2 takes the form

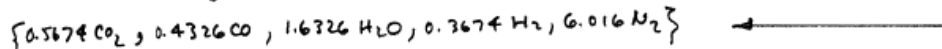


The amount of mixture is $n = x + (1-x) + (2.2-x) + (x-0.2) + 6.016 = 9.016$.

At equilibrium $\text{CO}_2 + \text{H}_2 \rightleftharpoons \text{CO} + \text{H}_2\text{O}$. Accordingly, Eq. 14.35 takes the form

$$\textcircled{1} \quad K = \frac{[1-x][2.2-x]}{[x][x-0.2]} \left[\frac{P/P_{\text{ref}}}{9.016} \right]^{1+1-1-1} = \left(\frac{1-x}{x} \right) \left(\frac{2.2-x}{x-0.2} \right)$$

At 1700K , Table A.27 gives $\log_{10} K = 0.530 \Rightarrow K = 3.3884$. Solving, $x = 0.5674$. Thus, the composition of the exiting equilibrium mixture in kmol per kmol of CH_4 is



At steady state an energy rate balance reduces to read

$$0 = \frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{CH}_4}} - \frac{\dot{W}_{\text{cv}}}{\dot{n}_{\text{CH}_4}} + \bar{h}_{\text{CH}_4}(T_1) + [1.6 \bar{h}_{\text{O}_2}(T_1) + 6.016 \bar{h}_{\text{N}_2}(T_1)] - [0.5674 \bar{h}_{\text{CO}_2} + 0.4326 \bar{h}_{\text{CO}} + 1.6326 \bar{h}_{\text{H}_2\text{O}} + 0.3674 \bar{h}_{\text{H}_2} + 6.016 \bar{h}_{\text{N}_2}]$$

Thus

$$\frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{CH}_4}} = 0.5674 [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{\text{CO}_2} + 0.4326 [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{\text{CO}} + 1.6326 [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{\text{H}_2\text{O}} + 0.3674 [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{\text{H}_2} + 6.016 [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{\text{N}_2} - [\bar{h}_f^\circ]_{\text{CH}_4} - 1.6 [\bar{h}(T_1) - \bar{h}(298)]_{\text{O}_2} - 6.016 [\bar{h}(T_1) - \bar{h}(298)]_{\text{N}_2}$$

With data from the ideal gas tables

$$\begin{aligned} \frac{\dot{Q}_{\text{cv}}}{\dot{n}_{\text{CH}_4}} &= 0.5674 [-393,520 + 82,856 - 9364] + 0.4326 [-110,530 + 54,609 - 8669] + \\ &\quad 1.6326 [-241,820 + 67,589 - 9904] + 0.3674 [51805 - 8460] + \\ &\quad 6.016 [54,1099 - 8669] - [-74,850] - 1.6 [14,770 - 8682] - 6.016 [14,581 - 8669] \\ &= -191,558 \text{ kJ/kmol CCH}_4 \end{aligned}$$

1. (P/P_{ref}) drops out of this expression.

Continued on next slide

Problem 14-41 continued

PROBLEM 14.41 (Cont'd.) Alternative IT solution

IT Code

```

T1 = 25 + 273.15 // K
p1 = 1 // atm
T2 = 227 + 273.15 // K
p2 = 1 // atm
T3 = 1427 + 273.15 // K
p3 = 1 // atm
ndotCH4 = 1 // Do calculations on the basis of 1 kmol of CH4 entering.

// CH4 + 1.6 (O2 + 3.76 N2) → x CO2 + (1 - x) CO + (2.2 - x) H2O + (x - 0.2) H2 + 6.016 N2
ndotCO2_3 = x
ndotCO_3 = 1 - x
ndotH2O_3 = 2.2 - x
ndotH2_3 = x - 0.2
ndotN2_3 = 6.016
ndot3 = ndotCO2_3 + ndotCO_3 + ndotH2O_3 + ndotH2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
pref = 1 // atm

// For the reaction CO2 + H2 ↔ CO + H2O
K = (yCO * yH2O) / (yCO2 * yH2) * (p3 / pref)^0
//Data from Table A-27 are stored in EQWATGAS.LUT.
log(K) = LOOKUPVAL(EQWATGAS,1,T3,3)

0 = Qdot + ndotCH4*hCH4_1 + 1.6*hO2_2 + 6.016*hN2_2 - (ndotCO2_3*hCO2_3 +
    ndotCO_3*hCO_3 + ndotH2O_3*hH2O_3 + ndotH2_3*hH2_3 + 6.016*hN2_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hN2_3 = h_T("N2",T3)

```

IT Results

```

K = 3.389
 $\dot{n}_{\text{CO}_2,3} / \dot{n}_{\text{CH}_4} = 0.4326 \text{ kmol/kmol}(\text{CH}_4)$ 
 $\dot{n}_{\text{CO}_2,3} / \dot{n}_{\text{CH}_4} = 0.5674 \text{ kmol/kmol}(\text{CH}_4)$ 
 $\dot{n}_{\text{H}_2,3} / \dot{n}_{\text{CH}_4} = 0.3674 \text{ kmol/kmol}(\text{CH}_4)$ 
 $\dot{n}_{\text{H}_2\text{O},3} / \dot{n}_{\text{CH}_4} = 1.633 \text{ kmol/kmol}(\text{CH}_4)$ 
 $\dot{n}_{\text{N}_2,3} / \dot{n}_{\text{CH}_4} = 6.016 \text{ kmol/kmol}(\text{CH}_4)$ 
Qdot = -1.917 x 105 kJ/kmol(CH4)

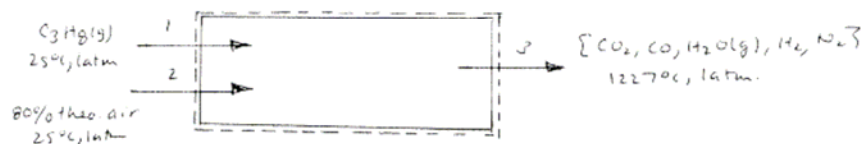
```

PROBLEM 14.42

KNOWN: C_3H_8 at $25^\circ C$, 1 atm enters a reactor at steady state and burns with 80% of theoretical air entering separately at $25^\circ C$, 1 atm. An equilibrium mixture of $\{CO_2, CO, H_2O(g), H_2, N_2\}$ exits at $1227^\circ C$, 1 atm.

FIND: Determine the heat transfer, in kJ per kmol of C_3H_8 .

SCHEMATIC & GIVEN DATA:

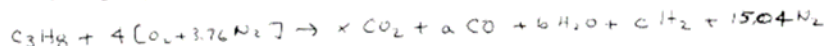


ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The exiting equilibrium mixture is modeled as an ideal gas mixture. (3) N_2 is inert.

ANALYSIS: The complete combustion of C_3H_8 with the theoretical amount of air is described by



Accordingly, the reaction of C_3H_8 with 80% of theoretical air to form the specified mixture is



$$C: 3 = x + a \Rightarrow a = 3 - x$$

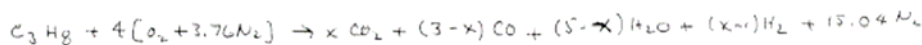
$$H: 8 = 2b + 2c$$

$$O: 8 = 2x + a + b$$

$$= 2x + (3 - x) + b \Rightarrow b = 5 - x$$

$$\Rightarrow \begin{aligned} c &= 4 - b \\ &= 4 - (5 - x) \\ &= x - 1 \end{aligned}$$

That is



At equilibrium $CO_2 + H_2 \rightleftharpoons CO + H_2O$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[3 - x][5 - x]}{[x][x - 1]} \left[\frac{P/P_{atm}}{n} \right]^0$$

At 1500 K, Table A-27 gives $\log_{10} K = 0.4031 \Rightarrow K = 2.5322$. Solving $x = 1.818$. Accordingly



An energy rate balance at steady state reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} - \frac{\dot{W}_{cv}}{\dot{n}_{C_3H_8}} + \bar{h}_{C_3H_8}(T_1) + [4\bar{h}_{O_2}(T_1) + 15.04\bar{h}_{N_2}(T_1)] - [1.818\bar{h}_{CO_2}(T_2) + 1.182\bar{h}_{CO}(T_2) + 3.182\bar{h}_{H_2O}(T_2) + 0.818\bar{h}_{H_2}(T_2) + 15.04\bar{h}_{N_2}(T_2)]$$

Rearranging and noting that $\bar{h}_f^0 = 0$ for H_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 1.818 [\bar{h}_f^0 + \bar{h}(T_2) - \bar{h}(298)]_{CO_2} + 1.182 [\bar{h}_f^0 + \bar{h}(T_2) - \bar{h}(298)]_{CO} + 3.182 [\bar{h}_f^0 + \bar{h}(T_2) - \bar{h}(298)]_{H_2O} + 0.818 [\bar{h}(T_2) - \bar{h}(298)]_{H_2} + 15.04 [\bar{h}(T_2) - \bar{h}(298)]_{N_2} - [\bar{h}_f^0]_{C_3H_8}$$

With data from the ideal gas tables

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} &= 1.818 [-393,520 + 71,078 - 9364] + 1.182 [-110,530 + 47,517 - 8669] + 3.182 [-241,820 + 57,999 \\ &\quad - 9909] + 0.818 [44,738 - 8468] + 15.04 [47,073 - 8669] - (-103,853) \\ &= -543,269 \frac{\text{kJ}}{\text{kmol}(C_3H_8)} \end{aligned}$$

Continued on next slide

Problem 14-42 continued

PROBLEM 14.42 (Cont'd.) Alternative IT solution

IT can be used as an alternative to iteration with a hand calculator and to using table data in the energy balance. The IT program follows.

IT Code

```
T1 = 25 + 273.15 // K
T2 = 25 + 273.15 // K
T3 = 1227 + 273.15 // K
p3 = 1 // atm
ndotfuel = 1 // Do calculations on the basis of 1 kmol of C3H8 entering.

// C3H8 + 4 (O2 + 3.76 N2) -> x CO2 + (3 - x) CO + (5 - x) H2O + (x - 1) H2 + 15.04 N2
ndotCO2_3 = x
ndotCO_3 = 3 - x
ndotH2O_3 = 5 - x
ndotH2_3 = x - 1
ndotN2_3 = 15.04
ndot3 = ndotCO2_3 + ndotCO_3 + ndotH2O_3 + ndotH2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
pref = 1 // atm

// For the reaction CO2 + H2 <=> CO + H2O
K = ((yCO * yH2O) / (yCO2 * yH2)) * (p3 / pref)^0
// Data from Table A-27 are stored in EQWATGAS.LUT.
log(K) = LOOKUPVAL(EQWATGAS,1,T3,3)

0 = Qdot + ndotfuel*hC3H8 + 4*hO2_2 + 15.04*hN2_2 - (ndotCO2_3*hCO2_3 +
ndotCO_3*hCO_3 + ndotH2O_3*hH2O_3 + ndotH2_3*hH2_3 + ndotN2_3*hN2_3)
hC3H8 = h_T("C3H8",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

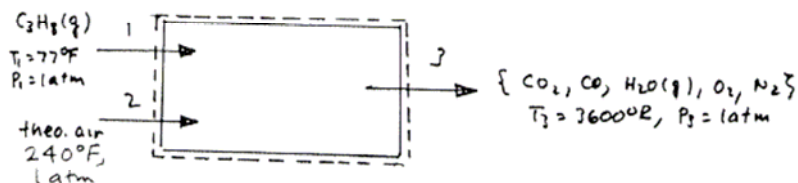
```
K = 2.533
n_CO,3 / n_fuel = 1.182 kmol/kmol(C3H8)
n_CO2,3 / n_fuel = 1.818 kmol/kmol(C3H8)
n_H2,3 / n_fuel = 0.8175 kmol/kmol(C3H8)
n_H2O,3 / n_fuel = 3.182 kmol/kmol(C3H8)
n_N2,3 / n_fuel = 15.04 kmol/kmol(C3H8)
Q_cv / n_fuel = -5.933 x 10^5 kJ/kmol(C3H8)
```

PROBLEM 14.43

KNOWN: $C_3H_8(g)$ at $77^\circ F$, 1 atm enters a reactor operating at steady state and burns with the theoretical amount of air entering at $240^\circ F$, 1 atm . An equilibrium mixture of CO_2 , CO , $H_2O(g)$, O_2 , and N_2 exits at $3600^\circ R$, 1 atm .

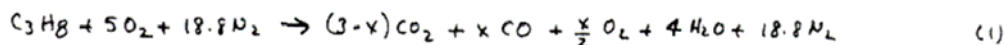
FIND: Determine the heat transfer per lbmol of fuel entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The exiting equilibrium mixture is modeled as an ideal gas mixture. (3) N_2 is inert.

ANALYSIS: The reaction of $C_3H_8(g)$ with the theoretical amount of air to form CO_2 , CO , $H_2O(g)$, O_2 , and N_2 is given by

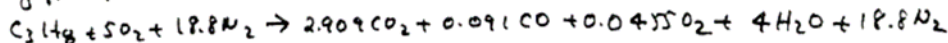


The amount of mixture is $n = (3-x) + x + \frac{x}{2} + 4 + 18.8 = 25.8 + \frac{x}{2}$

At equilibrium $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$. Accordingly, Eq. (1) takes the form

$$K = \frac{[x][\frac{x}{2}]^{1/2}}{[3-x]} \left[\frac{P/P_{ref}}{51.6+x} \right]^{1/2} = \frac{x}{3-x} \left[\frac{x}{51.6+x} \right]^{1/2}$$

At $3600^\circ R$, Table A-27 gives $\log_{10} K = -2.884 \Rightarrow K = 0.001306$. Solving, $x = 0.091$. Accordingly, Eq. (1) becomes



An energy rate balance at steady state reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} - \frac{\dot{W}_{cv}}{\dot{n}_{C_3H_8}} + \bar{h}_{C_3H_8}(T_1) + [5\bar{h}_{O_2}(T_1) + 18.8\bar{h}_{N_2}(T_1)] - [2.909\bar{h}_{CO_2}(T_2) + 0.091\bar{h}_{CO}(T_2) + 0.0455\bar{h}_{O_2}(T_2) + 4\bar{h}_{H_2O}(T_2) + 18.8\bar{h}_{N_2}(T_2)]$$

Rearranging and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 2.909[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{CO_2} + 0.091[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{CO} + 0.0455[\bar{h}(T_2) - \bar{h}(537)]_{O_2} + 4[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{H_2O} + 18.8[\bar{h}(T_2) - \bar{h}(537)]_{N_2} - [\bar{h}_f^\circ]_{C_3H_8} - 5[\bar{h}(T_1) - \bar{h}(537)]_{O_2} - 18.8[\bar{h}(T_1) - \bar{h}(537)]_{N_2}$$

With data from the ideal gas tables

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} &= 2.909[-169,800 + 43,411 - 4027.5] + 0.091[-47,540 + 28127 - 3725.1] + \\ &+ 0.0455[29,174 - 3725.1] + 4[-109,040 + 35541 - 4258] + \\ &+ 18.8[27875 - 3729.5] - [-44,680] - 5[4879 - 3725] - 18.8[4865 - 3730] \\ &= -198,396 \text{ Btu/lbmol}(C_3H_8) \end{aligned}$$

Continued on next slide

Problem 14-43 continued

PROBLEM 14.43 (Cont'd.)

Alternative IT solution

IT can be used as an alternative to iteration with a hand calculator and to using table data in the energy balance. The IT program follows.

IT Code

```
T1 = 77 + 459.67 // °R
T2 = 240 + 459.67 // °R
T3 = 3140 + 460 // °R
p3 = 1 // atm
ndotfuel = 1 // Do calculations on the basis of 1 lbmol of C3H8 entering.

// C3H8 + 5 (O2 + 3.76 N2) → (3 - x) CO2 + x CO + x/2 O2 + 4 H2O + 18.8 N2
ndot_CO2 = 3 - x
ndot_CO = x
ndot_O2 = x / 2
ndot_H2O = 4
ndot_N2 = 18.8
ndot3 = ndot_CO2 + ndot_CO + ndot_O2 + ndot_H2O + ndot_N2
yCO2 = ndot_CO2 / ndot3
yCO = ndot_CO / ndot3
yO2 = ndot_O2 / ndot3
pref = 1 // atm

// For the reaction CO2 ↔ CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T3,3)

0 = Qdot + ndotfuel*hC3H8 + 5*hO2_2 + 18.8*hN2_2 - (ndot_CO2*hCO2_3 +
ndot_CO*hCO_3 + ndot_O2*hO2_3 + ndot_H2O*hH2O_3 + 18.8*hN2_3)
hC3H8 = h_T("C3H8",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

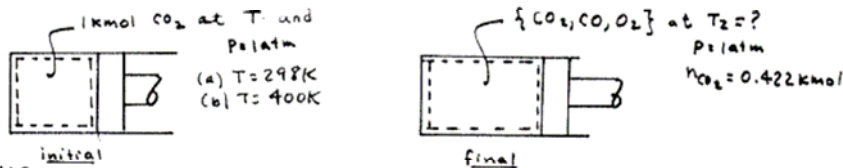
```
K = 0.001306
 $\dot{n}_{\text{CO}_2,3} / \dot{n}_{\text{fuel}} = 0.09071 \text{ lbmol/lbmol}(\text{C}_3\text{H}_8)$ 
 $\dot{n}_{\text{CO}_2,3} / \dot{n}_{\text{fuel}} = 2.909 \text{ lbmol/lbmol}(\text{C}_3\text{H}_8)$ 
 $\dot{n}_{\text{O}_2,3} / \dot{n}_{\text{fuel}} = 0.04536 \text{ lbmol/lbmol}(\text{C}_3\text{H}_8)$ 
 $\dot{Q}_{\text{cv}} / \dot{n}_{\text{fuel}} = -1.985\text{E}5 \text{ Btu/lbmol}(\text{C}_3\text{H}_8)$ 
```


PROBLEM 14.44

KNOWN: One kmol of CO_2 initially at T , 1 atm is heated at constant pressure until a final state is attained consisting of an equilibrium mixture of CO_2 , CO , and O_2 in which 0.422 kmol of CO_2 is present.

FIND: Determine the heat transfer and the work for (a) $T = 298\text{K}$, (b) $T = 400\text{K}$.

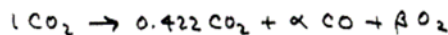
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system is shown in the accompanying figure by the dashed line. (2) Changes in kinetic and potential energy are negligible. (3) The final mixture is an equilibrium mixture. (4) Pressure remains constant during the process. (4) Ideal gas model applies to the CO_2 initially present and to the final equilibrium mixture.

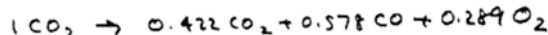
ANALYSIS: The first step is to determine T_2 : The reaction takes the form



$$\text{C: } 1 = 0.422 + \alpha \Rightarrow \alpha = 0.578$$

$$\text{O: } 2 = 2(0.422) + 0.578(2) + 2\beta \Rightarrow \beta = 0.289$$

Thus



At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + 1/2 \text{O}_2$. Accordingly Eq. 14.35 takes the form

$$K = \frac{(0.578)(0.289)^{1/2}}{0.422} \left[\frac{P/P_0}{1.299} \right]^{1/2} \Rightarrow K = \frac{(0.578)}{(0.422)} \left[\frac{0.289}{1.299} \right]^{1/2} \Rightarrow \log_{10} K = -0.1881$$

Interpolating in Table A-27, $T_2 \approx 3200\text{K}$.

(a) $T = 298\text{K}$

The work is given by $W = \int_1^2 p dV = p(V_2 - V_1)$. Using the ideal gas equation of state, $PV_1 = n_1 \bar{R}T_1$, $PV_2 = n_2 \bar{R}T_2$. Thus

$$W = \bar{R} [n_2 T_2 - n_1 T_1] = (8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}) [(0.289)(3200) - (1)(298)] \text{ kmol} \cdot \text{K} \\ = 31816 \text{ kJ} \leftarrow W$$

An energy balance gives $\Delta U = Q - W$. With $W = p(V_2 - V_1)$ this yields

$$Q = (U_2 - U_1) + p(V_2 - V_1) = (U_2 + pV_2) - (U_1 + pV_1) = H_2 - H_1$$

Accordingly

$$Q = [0.422 \bar{h}_{\text{CO}_2}(T_2) + 0.578 \bar{h}_{\text{CO}}(T_2) + 0.289 \bar{h}_{\text{O}_2}(T_2)] - 1 \bar{h}_{\text{CO}_2}(T_1) \\ = 4.22 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{CO}_2} + 0.578 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{CO}} + \\ 0.289 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{O}_2} - (\bar{h}_f^\circ)_{\text{CO}_2}$$

With data from the ideal gas tables

$$Q = 4.22 [-393,520 + 174,695 - 9364] + 0.578 [-110,530 + 109,667 - 8669] + \\ 0.289 [114,809 - 8682] - [-393,520] = 322,386 \text{ kJ} \leftarrow Q$$

Continued on next slide

Problem 14-44 continued

(b) $T = 400 \text{ K}$

As in part (a)

$$\begin{aligned} W &= \bar{R} (n_2 T_2 - n_1 T_1) \\ &= (8314) ((1.289)(3200) - (1)(400)) \\ &= 30,968 \text{ KJ} \end{aligned} \quad \leftarrow W$$

And as in part (a), $Q = H_2 - H_1$. The only difference is the evaluation of the initial CO_2 enthalpy:

$$\begin{aligned} [\bar{h}_f^\circ + \bar{h}(400) - \bar{h}(298)]_{\text{CO}_2} &= [-393,520 + 13,372 - 9364] \\ &= -389,512 \text{ KJ / kmol}(\text{CO}_2) \end{aligned}$$

Then

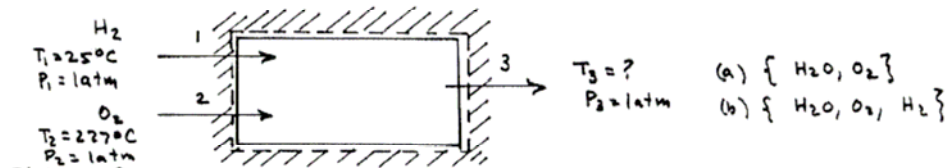
$$Q = 318,378 \text{ KJ} \quad \leftarrow Q$$

PROBLEM 14.45

KNOWN: H_2 at 25°C , 1 atm enters an insulated reactor operating at steady state and reacts with 250% excess oxygen entering at 227°C , 1 atm. Products of combustion exit at 1 atm.

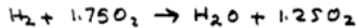
FIND: Determine the temperature if (a) combustion is complete, (b) an equilibrium mixture of H_2O , H_2 , and O_2 exits.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) In part (a), combustion is complete. In part (b), an equilibrium mixture exits. (3) The ideal gas model is applicable.

ANALYSIS: (a) For complete combustion with 250% excess O_2 , the reaction equation is



An energy rate balance at steady state reduces to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{n}_{H_2} \bar{h}_{H_2}]_1 + [1.75 \dot{n}_{O_2} \bar{h}_{O_2}]_2 - [\dot{n}_{H_2O} \bar{h}_{H_2O} + 1.25 \dot{n}_{O_2} \bar{h}_{O_2}]_3$$

$$\text{Thus } 0 = [\bar{h}_f^\circ]_{H_2} + 1.75 [\bar{h}_f^\circ]_{O_2} + \dot{n} [\bar{h}(T_2) - \bar{h}(298)]_{O_2} - [\dot{n} \bar{h}_f^\circ + \dot{n} (\bar{h}(T_3) - \bar{h}(298))]_{H_2O} - 1.25 [\dot{n} \bar{h}_f^\circ + \dot{n} (\bar{h}(T_3) - \bar{h}(298))]_{O_2}$$

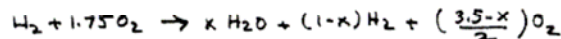
or

$$\begin{aligned} \bar{h}_{H_2O}(T_3) + 1.25 \bar{h}_{O_2}(T_3) &= -[\bar{h}_f^\circ]_{H_2O} - \bar{h}(298)_{H_2O} + 1.75 [\bar{h}(T_2) - \bar{h}(298)]_{O_2} + 1.25 \bar{h}_{O_2}(298) \\ &= -[-241,820 - 9904] + 1.75 [14,770 - 8682] + 1.25 (8682) \\ &= 273,231 \text{ kJ/kmol} \end{aligned}$$

Solving iteratively with data from Table A-23, $T_3 = 3033 \text{ K}$.

(a) T_3

(b) For this case the reaction takes the form



The amount of product mixture is $n = x + (1-x) + \left(\frac{3.5-x}{2}\right) = \left(\frac{5.5-x}{2}\right)$. At equilibrium

$H_2O \rightleftharpoons H_2 + \frac{1}{2} O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[1-x] \left[\frac{3.5-x}{2}\right]^{1/2}}{x \left[\frac{5.5-x}{2}\right]^{3/2}} \left[\frac{P/P_{ref}}{(5.5-x)/2}\right]^{1/2} = \frac{1-x}{x} \left[\frac{3.5-x}{5.5-x}\right]^{1/2} \quad (1)$$

Another equation relating x and T_3 is obtained from an energy rate equation at steady state which reduces to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{n}_{H_2} \bar{h}_{H_2}]_1 + 1.75 [\dot{n}_{O_2} \bar{h}_{O_2}]_2 - [x \dot{n}_{H_2O} \bar{h}_{H_2O} + (1-x) \dot{n}_{H_2} \bar{h}_{H_2} + \left(\frac{3.5-x}{2}\right) \dot{n}_{O_2} \bar{h}_{O_2}]_3$$

Noting that $\bar{h}_f^\circ = 0$ for H_2 and O_2 this becomes

$$0 = 1.75 [\dot{n}_{O_2} (500) - \dot{n}_{O_2} (298)] - \left\{ x [\dot{n}_{H_2O} \bar{h}_{H_2O}(T_3) - \dot{n}_{H_2O} (298)] + (1-x) [\dot{n}_{H_2} \bar{h}_{H_2}(T_3) - \dot{n}_{H_2} (298)] + \left(\frac{3.5-x}{2}\right) [\dot{n}_{O_2} \bar{h}_{O_2}(T_3) - \dot{n}_{O_2} (298)] \right\}$$

or

$$\begin{aligned} x [-241,820 + \bar{h}_{H_2O}(T_3) - 9904] + (1-x) [\bar{h}_{H_2}(T_3) - 8468] + \left(\frac{3.5-x}{2}\right) [\bar{h}_{O_2}(T_3) - 8682] \\ = 1.75 [14,770 - 8682] \end{aligned} \quad (2)$$

10,654

Continued on next slide

Problem 14-45 continued

Eqs. (1), (2) are simultaneous equations in x and T_3 . Solving iteratively with table data we get

$$x = 0.9933, T_3 = 2926 \text{ K} \quad \text{(b) } x, T_3$$

ALTERNATIVE IT SOLUTION

The following IT solution provides an alternative to iteration using table data by hand, as shown previously:

IT Code

```
T1 = 25 + 273.15 // K
T2 = 227 + 273.15 // K
p3 = 1 // atm
ndot1 = 1 // Do calculations on the basis of 1 kmol of H2 entering.

/* Use this section for part (a), comment out part (b).
// For part (a): H2 + 1.75 O2 → H2O + 1.25 O2
hr = h_T("H2",T1) + 1.75 * h_T("O2",T2)
hp = h_T("H2O",T3) + 1.25 * h_T("O2",T3)
hp = hr
*/

// For part (b) H2 + 1.75 O2 → x H2O + (1 - x) H2 + ((3.5 - x)/2) O2
ndotH2O_3 = x
ndotH2_3 = 1 - x
ndotO2_3 = (3.5 - x) / 2
ndot3 = (5.5 - x) / 2
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1 // atm

// For the reaction H2O ↔ H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,1,T3,3)

hr = h_T("H2",T1) + 1.75 * h_T("O2",T2)
hp = ndotH2O_3 * hH2O_3 + ndotH2_3 * hH2_3 + ndotO2_3 * hO2_3
hp = hr
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hO2_3 = h_T("O2",T3)
```

IT Results

Part (a): $T_3 = 3033 \text{ K}$

Part (b): $x = 0.9533, T_3 = 2926 \text{ K}$

PROBLEM 14.46

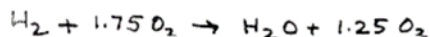
KNOWN: H_2 at 25°C , 1 atm enters an insulated reactor operating at steady state and reacts with 250% excess oxygen entering at 227°C , 1 atm. Products of combustion exit at 1 atm.

FIND: Determine the rate of entropy production per kmol of H_2 entering if (a) combustion is complete, (b) an equilibrium mixture of H_2O , H_2 , and O_2 exits.

SCHEMATIC & GIVEN DATA: See Problem 14.45 solution.

ENGINEERING MODEL: See Problem 14.45 solution.

ANALYSIS: (a) Complete combustion of H_2 with 250% excess O_2 is described by



The solution to Problem 14.45(a) gives 3033 K as the temperature of the products.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{Q}_{cv}}{\dot{n}_{H_2}} = (\bar{s}_{H_2O} + 1.25 \bar{s}_{O_2})_2 - (\bar{s}_{H_2})_1 - 1.75 (\bar{s}_{O_2})_2 \quad (1)$$

As H_2 enters at 25°C , 1 atm, Table A-23 gives $\bar{s}_{H_2} = 130.57 \text{ kJ/kmol}\cdot\text{K}$. O_2 enters at 500 K, 1 atm. Thus, with Eq. 13.22 and data from Table A-23 $\bar{s}_{O_2} = \bar{s}_{O_2}^\circ(500) = 220.589 \text{ kJ/kmol}\cdot\text{K}$. The products exit as a mixture at 3033 K, 1 atm with composition $y_{O_2} = 1.25/2.25$, $y_{H_2O} = 1/2.25$. Thus, with Eq. 13.23 and data from Table A-23

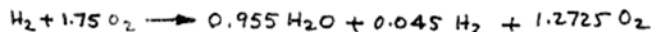
$$\bar{s}_{H_2O} = \bar{s}_{H_2O}^\circ(3033) - \bar{R} \ln \frac{y_{H_2O} P_{ref}}{P_{ref}} = 286.88 - 8.314 \ln \frac{1}{2.25} = 293.62 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{O_2} = \bar{s}_{O_2}^\circ(3033) - \bar{R} \ln y_{O_2} = 284.84 - 8.314 \ln \frac{1.25}{2.25} = 289.73 \text{ kJ/kmol}\cdot\text{K}$$

Inserting values into Eq. (1)

$$\frac{\dot{Q}_{cv}}{\dot{n}_{H_2}} = (293.62) + (1.25)(289.73) - 130.57 - 1.75(220.589) = 139.18 \frac{\text{kJ}}{\text{kmol}(H_2)\cdot\text{K}} \quad (a)$$

(b) The reaction is described by



The temperature of the products is obtained in the solution to Problem 14.45(b) as 2926 K.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{Q}_{cv}}{\dot{n}_{H_2}} = (0.955 \bar{s}_{H_2O} + 0.045 \bar{s}_{H_2} + 1.2725 \bar{s}_{O_2})_3 - (\bar{s}_{H_2})_1 - (\bar{s}_{O_2})_2 \quad (2)$$

The values of $(\bar{s}_{H_2})_1$ and $(\bar{s}_{O_2})_2$ are the same as in part (a) above. The products exit as a mixture at 2926 K, 1 atm with the composition $y_{H_2O} = 0.955/2.2725$, $y_{H_2} = 0.045/2.2725$, $y_{O_2} = 1.2725/2.2725$. Thus, with Eq. 13.23 and data from the ideal gas tables

$$\bar{s}_{H_2O} = \bar{s}_{H_2O}^\circ(2926) - \bar{R} \ln y_{H_2O} = 284.88 - 8.314 \ln \frac{0.955}{2.2725} = 292.09 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{H_2} = \bar{s}_{H_2}^\circ(2926) - \bar{R} \ln y_{H_2} = 201.85 - 8.314 \ln \frac{0.045}{2.2725} = 234.46 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{O_2} = \bar{s}_{O_2}^\circ(2926) - \bar{R} \ln y_{O_2} = 283.40 - 8.314 \ln \frac{1.2725}{2.2725} = 288.22 \text{ kJ/kmol}\cdot\text{K}$$

Inserting values into Eq. (2)

$$\textcircled{1} \frac{\dot{Q}_{cv}}{\dot{n}_{H_2}} = (0.955)(292.09) + (0.045)(234.46) + (1.2725)(288.22) - 130.57 - 1.75(220.589) = 139.66 \frac{\text{kJ}}{\text{kmol}(H_2)\cdot\text{K}} \quad (b)$$

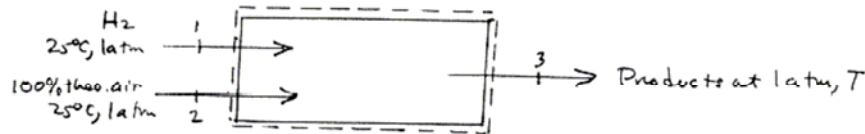
1. The greater entropy production of part (b) suggests that complete combustion would not occur under the specified conditions.

PROBLEM 14.47

KNOWN: H_2 at 25°C , 1 atm enters an insulated reactor operating at steady state and reacts with 100% of theoretical air entering at 25°C , 1 atm. The products of combustion exit at 1 atm and temperature T .

FIND: Determine T if (a) combustion is complete, (b) an equilibrium mixture of H_2O , H_2 , O_2 , and N_2 exits.

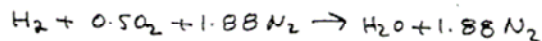
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown above is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The ideal gas model is applicable. (4) In part (a) combustion is complete. In part (b) an equilibrium mixture is formed.

ANALYSIS: (a) The complete reaction with 100% theoretical air is



An energy rate balance reduces with given assumptions to read $H_R = H_P$, or

$$0 = [\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 1.88[\bar{h}_f^\circ + \Delta\bar{h}]_{N_2} - [\bar{h}_f^\circ]_{H_2} - 1.88[\bar{h}_f^\circ]$$

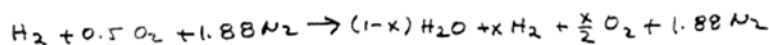
Inserting ideal gas table data

$$0 = [-241,820 + \bar{h}_{H_2O}(T) - 9904] + 1.88[\bar{h}_{N_2}(T) - 8669] = 0$$

or

$$\bar{h}_{H_2O}(T) + 1.88\bar{h}_{N_2}(T) = 268,022 \text{ kJ/kmol. Solving iteratively with table data, } T = 2525 \text{ K.}$$

(b) If an equilibrium mixture is formed, the reaction equation reads



where $n = (1-x) + x + x/2 + 1.88 = 2.88 + x/2$. For $H_2O \rightleftharpoons \frac{1}{2}O_2 + H_2$ and $P/P_{ref} = 1$

$$K = \frac{(x/2)^{1/2}(x)}{(1-x)} \left[\frac{P/P_{ref}}{n} \right]^{\frac{1}{2} - 1} = \left(\frac{x}{1-x} \right) \left(\frac{x/2}{2.88 + x/2} \right)^{1/2} \quad (1)$$

An energy rate balance reduces to $H_R = H_P$, or

$$0 = (1-x)[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(298)]_{H_2O} + x[\bar{h}(T) - \bar{h}(298)]_{H_2} + \frac{x}{2}[\bar{h}(T) - \bar{h}(298)]_{O_2} + 1.88[\bar{h}(T) - \bar{h}(298)]_{N_2}$$

with ideal gas table data

$$0 = (1-x)[-241,820 + \bar{h}_{H_2O}(T) - 9904] + x[\bar{h}_{H_2}(T) - 8468] + \frac{x}{2}[\bar{h}_{O_2}(T) - 8682] + 1.88[\bar{h}_{N_2}(T) - 8669] \quad (2)$$

Equations (1), (2) are simultaneous equations for x , T . Solving iteratively

① with table data, $x = 0.045$, $T = 2433 \text{ K}$.

1. Because of dissociation — case (b) — the temperature of the products is less than in the complete combustion case (a).

Continued on next slide

Problem 14-47 continued

Alternative Solution Using IT

IT Code

```
T1 = 25 + 273.15 // K
T2 = 25 + 273.15 // K
p3 = 1 // atm
ndot1 = 1 // Do all calculations on the basis of 1 kmol of H2 entering.

// For part (a): H2 + 0.5 O2 + 1.88 N2 → H2O + 1.88 N2
hr = h_T("H2",T1) + 0.5 * h_T("O2",T2) + 1.88 * h_T("N2",T2)
hp = h_T("H2O",Tp) + 1.88 * h_T("N2",Tp)
hp = hr

/* Use this section for part (b) and comment out part (a).
// For part (b): H2 + 0.5 O2 + 1.88 N2 → (1 - x) H2O + x H2 + x/2 O2 + 1.88 N2
ndotH2O_3 = 1 - x
ndotH2_3 = x
ndotO2_3 = x/2
ndotN2_3 = 1.88
ndot3 = ndotH2O_3 + ndotH2_3 + ndotO2_3 + ndotN2_3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1 // atm

// For the reaction H2O ↔ H2 + 1/2 O2

K = ((yH2 * yO2^0.5) / yH2O) * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,1,T3,3)

hr = h_T("H2",T1) + 0.5 * h_T("O2",T2) + 1.88 * h_T("N2",T2)
hp = (1-x)*hH2O_3 + x*hH2_3 + (x/2)*hO2_3 + 1.88*hN2_3
hp = hr
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
*/
```

IT Results

Part (a): $T_3 = 2526 \text{ K}$

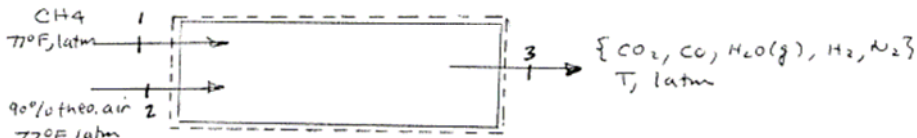
Part(b): $x = 0.04563$, $K = 0.004239$, $T_3 = 2432 \text{ K}$

PROBLEM 14.48

KNOWN: CH₄ at 77°F, 1 atm enters an insulated reactor and burns with 90% of theoretical air entering separately at 77°F, 1 atm. An equilibrium mixture {CO₂, CO, H₂O(g), H₂, N₂} exits at 1 atm.

FIND: Determine the temperature of the exiting mixture.

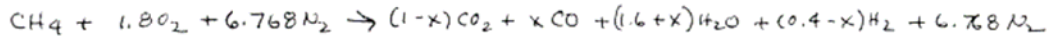
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = \dot{Q}_{cv} = 0$ and negligible kinetic/potential energy effects. (2) The exiting equilibrium mixture is modeled as an ideal gas mixture. (3) N₂ is inert.

ANALYSIS: The complete combustion of CH₄ with the theoretical amount of air is given by Eq. 13.4. Accordingly, combustion with 90% of theoretical air to form the specified mixture is described by



At equilibrium $\text{CO}_2 + \text{H}_2 \rightleftharpoons \text{CO} + \text{H}_2\text{O}$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x][1.6+x]}{[1-x][0.4-x]} \left[\frac{P/P_{atm}}{n} \right]^0 = \frac{x(1.6+x)}{(1-x)(0.4-x)} \quad (1)$$

An energy rate balance at steady state reduces to

$$[1-x][\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]\text{CO}_2 + x[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]\text{CO} + (1.6+x)[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]\text{H}_2\text{O} \\ + [0.4-x][\bar{h}(T) - \bar{h}(537)]_{\text{H}_2} + 6.768[\bar{h}(T) - \bar{h}(537)]\text{N}_2 - (\bar{h}_f^\circ)_{\text{CH}_4}$$

Inserting table data

$$0 = [1-x][-169,300 + \bar{h}_{\text{CO}_2} - 4028] + x[-47,540 + \bar{h}_{\text{CO}} - 3725] + (1.6+x)[-104,040 + \bar{h}_{\text{H}_2\text{O}}(T) - 4756] \\ + [0.4-x][\bar{h}_{\text{H}_2} - 3640] + 6.768[\bar{h}_{\text{N}_2} - 3730] - (-32,210)$$

or

$$0 = [1-x][-173,328 + \bar{h}_{\text{CO}_2}] + x[-51,265 + \bar{h}_{\text{CO}}] + [1.6+x][-108,298 + \bar{h}_{\text{H}_2\text{O}}] + \\ [0.4-x][\bar{h}_{\text{H}_2} - 3640] + 6.768[\bar{h}_{\text{N}_2} - 3730] + 32,210 \quad (2)$$

Equations (1), (2) are simultaneous in x, T . When solved iteratively

- ① using Table data, $T = 4000^\circ\text{R}$, $x = 0.2693$.

1. For the products to include both CO and H₂, inspecting the reaction equation shows that x must be in the interval $0 < x < 0.4$.

Continued on next slide

Problem 14-48 continued

Alternative Solution Using IT

IT Code

```
T1 = 77 + 459.67 // °R
T2 = 77 + 459.67 // °R
p3 = 1 // atm
ndot_CH4 = 1 // Do calculations on the basis of 1 kmol of CH4 entering.

// CH4 + 1.8 O2 + 6.768 N2 → (1 - x) CO2 + x CO + (1.6 + x) H2O + (0.4 - x) H2 + 6.768 N2
ndotCO2_3 = 1 - x
ndotCO_3 = x
ndotH2O_3 = 1.6 + x
ndotH2_3 = 0.4 - x
ndotN2_3 = 6.768
ndotO2_3 = 1.8
ndot3 = ndotCO2_3 + ndotCO_3 + ndotH2O_3 + ndotH2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
pref = 1 // atm

// For the reaction: CO2 + H2 ↔ CO + H2O
K = (yCO * yH2O) / (yCO2 * yH2)
// Data from Table A-27 are stored in EQWATGAS.LUT.
log(K) = LOOKUPVAL(EQWATGAS,2,T3,3)

0 = hCH4_1 + 1.8*hO2_2 + 6.768*hN2_2 - (1-x)*hCO2_3 - x*hCO_3 - (1.6+x)*hH2O_3 -
    (0.4-x)*hH2_3 - 6.768*hN2_3
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

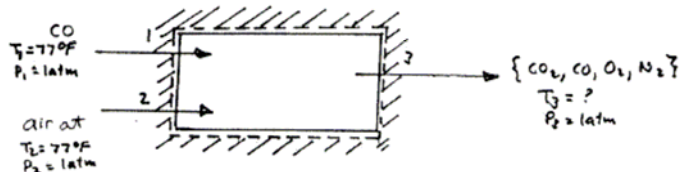
```
K = 5.276
x = 0.2694
T3 = 4003°R
```


PROBLEM 14.49

KNOWN: CO at 77°F, 1 atm enters an insulated reactor operating at steady state and burns with air entering at 77°F, 1 atm. An equilibrium mixture of CO₂, CO, O₂, N₂ exits at 1 atm, T.

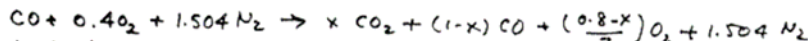
FIND: Determine T if the combustion occurs with (a) 80% theoretical air, (b) 100% theoretical air.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas as is the entering combustion air. (3) N₂ is inert.

ANALYSIS: (a) The reaction of CO with 80% of theoretical air to form {CO₂, CO, O₂, N₂} is described by



The amount of mixture is $n = x + (1-x) + (0.8-x) + 1.504 = (5.808-x)/2$. At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2}\text{O}_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[1-x][\frac{0.8-x}{2}]^{1/2}}{x} \left[\frac{P/P_{ref}}{(5.808-x)/2} \right]^{1/2} = \frac{1-x}{x} \left[\frac{0.8-x}{5.808-x} \right]^{1/2} \quad (1)$$

Another equation relating x and T_3 is obtained from an energy rate equation at steady state which reduces to give

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CO}} - \frac{\dot{W}_{cv}}{\dot{n}_{CO}} + [h_{CO}]_1 + [0.4\bar{h}_{O_2} + 1.504\bar{h}_{N_2}]_2 - [x\bar{h}_{CO_2} + (1-x)\bar{h}_{CO} + (0.8-x)\bar{h}_{O_2} + 1.504\bar{h}_{N_2}]_3$$

Then, with $\bar{h}_f^0 = 0$ for O₂ and N₂ this becomes

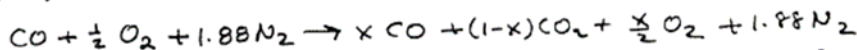
$$0 = [\bar{h}_f^0]_{CO} - \{x[\bar{h}_f^0 + \bar{h}(T_3) - \bar{h}(537)]\}_{CO_2} + (1-x)[\bar{h}_f^0 + \bar{h}(T_3) - \bar{h}(537)]_{CO} + (0.8-x)[\bar{h}(T_3) - \bar{h}(537)]_{O_2} + 1.504[\bar{h}(T_3) - \bar{h}(537)]_{N_2}$$

or with data from Table A-23E

$$x[-169,300 + \bar{h}_{CO_2}(T_3) - 4027.5] + (1-x)[-47,540 + \bar{h}_{CO}(T_3) - 3725.1] + (0.8-x)[\bar{h}_{O_2}(T_3) - 3725.1] + 1.504[\bar{h}_{N_2}(T_3) - 3729.3] = -47,540 \quad (2)$$

For the products to include both CO₂ and O₂, inspection of the reaction equation shows that x must be in the interval $0 < x < 0.8$. Eqs. (1), (2) are simultaneous in x and T_3 . Solving iteratively with table data, $T_3 = 4380^\circ\text{R}$, $x = 0.766$.

(b) The reaction of CO with the theoretical amount of air to produce CO₂, CO, O₂, and N₂ is



For the products to include both CO and CO₂, it is necessary for x to be in the interval $0 < x < 1$.

The amount of mixture is $n = x + (1-x) + \frac{x}{2} + 1.88 = (5.76+x)/2$. At equilibrium, $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2}\text{O}_2$. Thus, Eq. 14.35 takes the form

$$K(T_3) = \frac{[x][x/2]^{1/2}}{[1-x]} \left[\frac{P_3/P_{ref}}{(5.76+x)/2} \right]^{1/2} = \left[\frac{x}{1-x} \right] \left[\frac{x}{5.76+x} \right]^{1/2} \quad (1)$$

Continued on next slide

Problem 14-49 continued

Another equation involving T_3 and x can be obtained from an energy rate balance which reduces at steady state to

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}_{co}} - \frac{\dot{W}_{cv}}{\dot{m}_{co}} + [0.5\bar{h}_{O_2} + 1.88\bar{h}_{N_2}]_2 + [\bar{h}_{co}]_1 - [x\bar{h}_{co} + (1-x)\bar{h}_{co_2} + \frac{x}{2}\bar{h}_{O_2} + 1.88\bar{h}_{N_2}]_3$$

Then, with $\bar{h}_f^0 = 0$ for O_2 and N_2 this becomes

$$x[\bar{h}_f^0 + \bar{h}(T_3) - \bar{h}(298)]_{co} + (1-x)[\bar{h}_f^0 + \bar{h}(T_3) - \bar{h}(298)]_{co_2} + \frac{x}{2}[\bar{h}(T_3) - \bar{h}(298)]_{O_2} + 1.88[\bar{h}(T_3) - \bar{h}(298)]_{N_2} = (\bar{h}_f^0)_{co}$$

Noting that $77^\circ F$ corresponds to $25^\circ C$, we use data from the SI table Table A-23 to obtain

$$x[-110,530 + \bar{h}(T_3) - 8669]_{co} + (1-x)[-393,520 + \bar{h}(T_3) - 9364]_{co_2} + \frac{x}{2}[\bar{h}(T_3) - 8682]_{O_2} + 1.88[\bar{h}(T_3) - 8669]_{N_2} = -110,530$$

or

$$x[\bar{h}(T_3) - 119,149] + (1-x)[\bar{h}(T_3) - 402,884] + \frac{x}{2}[\bar{h}(T_3) - 8682] + 1.88[\bar{h}(T_3) - 8669] = -110,530 \quad (2)$$

Equations (1), (2) are simultaneous equations for x , T_3 . Solving

① iteratively with table data, $x = 0.125$, $T_3 = 2399 K (4318^\circ R)$

Continued on next slide

Problem 14-49 continued

IT can be used as an alternative to an iterative solution by hand using table data. The program follows.

IT Code

```
T1 = 77 + 459.67 // °R
T2 = 77 + 459.67 // °R
p3 = 1 // atm
ndotCO_1 = 1 // Do calculations on the basis of 1 lbmol of CO entering.

// CO + a (0.5) (O2 + 3.76 N2) → x CO2 + (1 - x) CO + ((a - x) / 2) O2 + a 1.88 N2

ndotCO2_3 = x
ndotCO_3 = 1 - x
ndotO2_3 = (a - x) / 2
ndotN2_3 = a * 1.88
ndot_3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot_3
yO2 = ndotO2_3 / ndot_3
yCO = ndotCO_3 / ndot_3
pref = 1 // atm
// Part (a):
a = 0.8
// For Part(b): Set a = 1.

// For the reaction CO2 <==> CO + 1/2 O2
K = (((yCO) * (yO2)^0.5) / (yCO2)) * (p3 / pref)^.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T3,3)

0 = hCO_1 + a*0.5*hO2_2 + a*1.88*hN2_2 - x*hCO2_3 - (1-x)*hCO_3 -
    ((a-x)/2)*hO2_3 - a*1.88*hN2_3
hCO_1 = h_T("CO",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

Part (a): 80% theoretical air: $x = 0.7658$, $T_3 = 4380^\circ\text{R}$

Part (b): Theoretical air: $x = 0.8751$ (lbmol of CO out per lbmol of CO in)
 $T_3 = 4318^\circ\text{R}$

PROBLEM 14.50

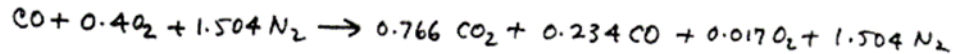
KNOWN: See solution to Problem 14.49.

FIND: For each case, determine the exergy destruction per kmol of CO entering.

SCHEMATIC & GIVEN DATA: See solution to Problem 14.49.

ENGINEERING MODEL: (1) See solution to Problem 14.49. (2) $T_0 = 537^\circ\text{R}$.

ANALYSIS: (a) From the solution to Problem 14.49, the reaction is



where the products are at 4380°R .

At steady state an entropy rate balance reduces to give

$$\frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}}} = (0.766 \bar{s}_{\text{CO}_2} + 0.234 \bar{s}_{\text{CO}} + 0.017 \bar{s}_{\text{O}_2} + 1.504 \bar{s}_{\text{N}_2})_3 - (\bar{s}_{\text{CO}})_1 - (0.4 \bar{s}_{\text{O}_2} + 1.504 \bar{s}_{\text{N}_2})_2 \quad (1)$$

The carbon monoxide enters at 77°F , 1 atm. Thus from Table A-23E $(\bar{s}_{\text{CO}})_1 = \bar{s}_{\text{CO}}^\circ(537^\circ\text{R}) = 47.27 \text{ Btu/lbmol}\cdot^\circ\text{R}$. The combustion air enters at 77°F , 1 atm, so with Eq. 13.23

$$\bar{s}_{\text{O}_2} = \bar{s}_{\text{O}_2}^\circ(537) - \bar{R} \ln y_{\text{O}_2} = 48.48 - 1.986 \ln 0.21 = 52.08 \text{ Btu/lbmol}\cdot^\circ\text{R}$$

$$\bar{s}_{\text{N}_2} = \bar{s}_{\text{N}_2}^\circ(537) - \bar{R} \ln y_{\text{N}_2} = 45.74 - 1.986 \ln 0.79 = 46.21 \text{ Btu/lbmol}\cdot^\circ\text{R}$$

The products exit at 4380°R , 1 atm with the composition, $y_{\text{CO}_2} = \frac{0.766}{2.521}$, $y_{\text{CO}} = \frac{0.234}{2.521}$, $y_{\text{O}_2} = \frac{0.017}{2.521}$, $y_{\text{N}_2} = \frac{1.504}{2.521}$. Thus, with Eq. 13.23 and data from Table A-23E

$$\bar{s}_{\text{CO}_2} = \bar{s}_{\text{CO}_2}^\circ(4380) - \bar{R} \ln y_{\text{CO}_2} = 76.736 - 1.986 \ln \frac{0.766}{2.521} = 79.10 \text{ Btu/lbmol}\cdot^\circ\text{R}$$

$$\bar{s}_{\text{CO}} = \bar{s}_{\text{CO}}^\circ(4380) - \bar{R} \ln y_{\text{CO}} = 63.567 - 1.986 \ln \frac{0.234}{2.521} = 68.29 \text{ Btu/lbmol}\cdot^\circ\text{R}$$

$$\bar{s}_{\text{O}_2} = \bar{s}_{\text{O}_2}^\circ(4380) - \bar{R} \ln y_{\text{O}_2} = 65.958 - 1.986 \ln \frac{0.017}{2.521} = 75.89 \text{ Btu/lbmol}\cdot^\circ\text{R}$$

$$\bar{s}_{\text{N}_2} = \bar{s}_{\text{N}_2}^\circ(4380) - \bar{R} \ln y_{\text{N}_2} = 61.887 - 1.986 \ln \frac{1.504}{2.521} = 62.91 \text{ Btu/lbmol}\cdot^\circ\text{R}$$

Inserting values into Eq. (1)

$$\begin{aligned} \frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}}} &= (0.766)(79.10) + (0.234)(68.29) + (0.017)(75.89) + (1.504)(62.91) - (47.27) - (0.4)(52.08) - (1.504)(46.21) \\ &= 34.88 \text{ Btu/lbmol}\cdot^\circ\text{R} \end{aligned}$$

Calculating the exergy destruction in SI units, as required

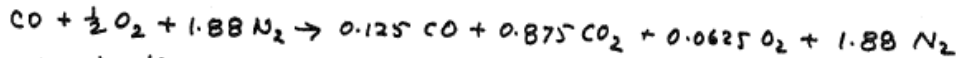
$$\begin{aligned} \frac{\dot{E}_d}{\dot{n}_{\text{CO}}} &= T_0 \frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}}} = (537^\circ\text{R}) \left(34.88 \frac{\text{Btu}}{\text{lbmol}\cdot^\circ\text{R}} \right) \left| \frac{1 \text{ kJ}}{0.9478 \text{ Btu}} \right| \left| \frac{2.2046 \text{ lb}}{1 \text{ kg}} \right| \\ &= 43,568 \frac{\text{kJ}}{\text{kmol}(\text{CO})} \end{aligned}$$

← (a)

Continued on next slide

Problem 14-50 continued

(b) From the solution to Problem 14.49(b)



where the combustion products are at 2399 K.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}}} = (0.125 \bar{s}_{\text{CO}} + 0.875 \bar{s}_{\text{CO}_2} + 0.0625 \bar{s}_{\text{O}_2} + 1.88 \bar{s}_{\text{N}_2})_2 - (\bar{s}_{\text{CO}})_1 - (\frac{1}{2} \bar{s}_{\text{O}_2} + 1.88 \bar{s}_{\text{N}_2})_1 \quad (1)$$

- ① The CO enters at 25°C, 1 atm. Thus $(\bar{s}_{\text{CO}})_1 = \bar{s}_{\text{CO}}^\circ(298) = 197.54 \text{ kJ/kmol} \cdot \text{K}$ from Table A-23. The air enters at 25°C, 1 atm, so with Eq. 13.23 and data from Table A-23

$$\bar{s}_{\text{O}_2} = \bar{s}_{\text{O}_2}^\circ(298) - \bar{R} \ln y_{\text{O}_2} = 205.08 - 8.314 \ln 0.21 = 218.01 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{\text{N}_2} = \bar{s}_{\text{N}_2}^\circ(298) - \bar{R} \ln y_{\text{N}_2} = 191.5 - 8.314 \ln 0.79 = 193.46 \text{ kJ/kmol} \cdot \text{K}$$

The combustion products exit at 2399 K, 1 atm with composition $y_{\text{CO}} = \frac{0.125}{2.9425}$, $y_{\text{O}_2} = \frac{0.0625}{2.9425}$, $y_{\text{N}_2} = \frac{1.88}{2.9425}$. Thus with Eq. 13.23 and data from Table A-23

$$\bar{s}_{\text{CO}} = \bar{s}_{\text{CO}}^\circ(2399) - \bar{R} \ln y_{\text{CO}} = 265.238 - \bar{R} \ln \frac{0.125}{2.9425} = 291.499 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{\text{CO}_2} = \bar{s}_{\text{CO}_2}^\circ(2399) - \bar{R} \ln y_{\text{CO}_2} = 320.276 - \bar{R} \ln \frac{0.875}{2.9425} = 330.359 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{\text{O}_2} = \bar{s}_{\text{O}_2}^\circ(2399) - \bar{R} \ln y_{\text{O}_2} = 275.609 - \bar{R} \ln \frac{0.0625}{2.9425} = 307.633 \text{ kJ/kmol} \cdot \text{K}$$

$$\bar{s}_{\text{N}_2} = \bar{s}_{\text{N}_2}^\circ(2399) - \bar{R} \ln y_{\text{N}_2} = 258.565 - \bar{R} \ln \frac{1.88}{2.9425} = 262.290 \text{ kJ/kmol} \cdot \text{K}$$

Inserting values in Eq. (1)

$$\frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}}} = (0.125)(291.499) + (0.875)(330.359) + (0.0625)(307.633) + (1.88)(262.290) - (197.54) - (0.5)(218.01) - 1.88(193.46)$$

$$= 167.583 \text{ kJ/kmol}(\text{CO}) \cdot \text{K}$$

Then

$$\textcircled{2} \quad \frac{\dot{E}_d}{\dot{n}_{\text{CO}}} = T_0 \frac{\dot{Q}_{cv}}{\dot{n}_{\text{CO}}} = (298 \text{ K}) \left(\frac{167.583 \text{ kJ}}{\text{kmol}(\text{CO}) \cdot \text{K}} \right) = 49,940 \frac{\text{kJ}}{\text{kmol}(\text{CO})} \leftarrow (b)$$

1. Working in SI units as in Problem 14.49(b)

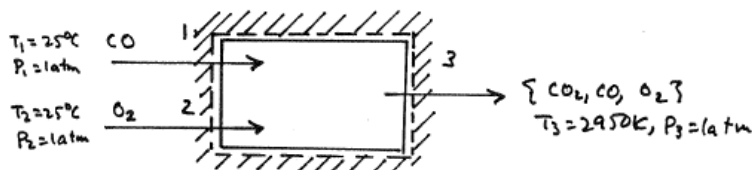
2. The exergy destruction of part (b) is greater than in part (a) because more air is used, and this entails more mixing within the control volume.

PROBLEM 14.51

KNOWN: CO at 25°C, 1 atm enters an insulated reactor and burns with excess O₂ entering at 25°C, 1 atm. An equilibrium mixture of CO₂, CO, and O₂ exits at 1 atm, 2950 K.

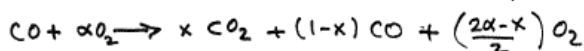
FIND: Determine the percent excess O₂.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas.

ANALYSIS: The reaction of CO with O₂ to produce CO₂, CO, and O₂ is



The amount of mixture is $n = x + (1-x) + \left(\frac{2\alpha-x}{2}\right) = \frac{2(1+\alpha)-x}{2}$. At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$. Accordingly Eq. 14.35 takes the form

$$K = \frac{[1-x] \left[\frac{2\alpha-x}{2}\right]^{1/2}}{x} \left[\frac{P_3/P_{atm}}{(2(1+\alpha)-x)/2}\right]^{1/2} = \left(\frac{1-x}{x}\right) \left(\frac{2\alpha-x}{2(1+\alpha)-x}\right)^{1/2}$$

At 2950 K Table A-27 gives $\log_{10} K = -0.567 \Rightarrow K = 0.271019$. Thus

$$0.07345 = \left[\frac{1-x}{x}\right]^2 \left(\frac{2\alpha-x}{2(1+\alpha)-x}\right) \quad (1)$$

Another equation relating α and x can be obtained from an energy rate equation which reduces at steady state to

$$0 = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{\dot{n}_{\text{CO}}} + [\bar{h}_{\text{CO}}]_1 + \alpha [\bar{h}_{\text{O}_2}]_2 - [x \bar{h}_{\text{CO}_2} + (1-x) \bar{h}_{\text{CO}} + \left(\frac{2\alpha-x}{2}\right) \bar{h}_{\text{O}_2}]_3$$

Then, with $\bar{h}_f^\circ = 0$ for O₂ this gives

$$x [\bar{h}_f^\circ + \bar{h}_{\text{CO}_2}(2950) - \bar{h}_{\text{CO}_2}(298)] + (1-x) [\bar{h}_f^\circ + \bar{h}_{\text{CO}}(2950) - \bar{h}_{\text{CO}}(298)] + \left(\frac{2\alpha-x}{2}\right) [\bar{h}_{\text{O}_2}(2950) - \bar{h}_{\text{O}_2}(298)] = (\bar{h}_f^\circ)_{\text{CO}}$$

or with table data

$$x [-393,520 + 159,117 - 9364] + (1-x) [-110,530 + 100,852 - 8669] + \left(\frac{2\alpha-x}{2}\right) [104,785 - 8682] = -110,530$$

$$x [-243,767] + (1-x) [-18,847] + \left(\frac{2\alpha-x}{2}\right) (96,103) = -110,530$$

On further reduction

$$\alpha = 2.8404x - 0.954 \quad (2)$$

Combining Eqs. (1) and (2)

$$0.07345 = \left[\frac{1-x}{x}\right]^2 \left[\frac{4.6808x - 1.908}{4.6808x + 0.092}\right] \Rightarrow x = 0.702$$

Using this value for x , Eq. (2) gives $\alpha = 1.04$. The theoretical amount of O₂ required is 0.5 kmol/kmol(CO). Accordingly, the percent excess O₂ is 108%.

Continued on next slide

Problem 14-51 continued

Alternative Solution Using IT

IT Code

```
T1 = 25 + 273.15 // K
T2 = 25 + 273.15 // K
T3 = 2950 // K
p3 = 1 // atm
ndotCO_1 = 1 // Do calculations on the basis of 1kmol of CO entering.
```

```
// CO + (1+XS) (0.5) O2 → x CO2 + (1 - x) CO + ((1 + XS - x) / 2) O2
ndotCO2_3 = x
ndotCO_3 = 1 - x
ndotO2_3 = (1 + XS - x) / 2
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1 // atm
```

```
// For the reaction CO2 ↔ CO + 1/2 O2
K = (((yCO) * (yO2)^0.5) / (yCO2)) * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)
```

```
0 = hCO_1 + (1+XS)*(0.5)*hO2_2 - x*hCO2_3 - (1-x)*hCO_3 - ((1+XS-x)/2)*hO2_3
hCO_1 = h_T("CO",T1)
hO2_2 = h_T("O2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
PCTXS = XS * 100
```

IT Results

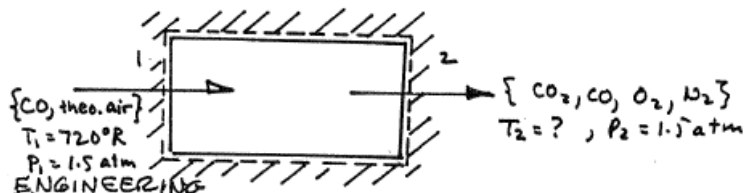
```
x = 0.7019
% excess oxygen = 107.6%
```


PROBLEM 14.52

KNOWN: A mixture of CO and the theoretical amount of air at 720°R , 1.5 atm enters an insulated reactor operating at steady state. An equilibrium mixture of CO_2 , CO, O_2 , and N_2 exits at 1.5 atm.

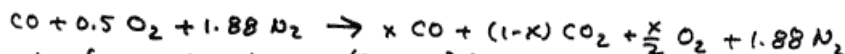
FIND: Determine the temperature of the exiting mixture.

SCHEMATIC & GIVEN DATA:



MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ingoing and outgoing mixtures are modeled as ideal gas mixtures. (3) N_2 is inert.

ANALYSIS: The reaction is described by



The amount of mixture is $n = (5.76 + x)/2$. At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + 1/2 \text{O}_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_2) = \frac{[x][x/2]}{[1-x]} \left[\frac{P_2/P_{\text{ref}}}{(5.76+x)/2} \right]^{1/2} = \left[\frac{x}{1-x} \right] \left[\frac{1.5x}{5.76+x} \right]^{1/2} \quad (1)$$

Another equation involving T_2 and x can be obtained from an energy rate balance which reduces at steady state to

$$0 = \frac{\dot{Q}_{cv}^0}{\dot{n}_{\text{CO}}} - \frac{\dot{W}_{cv}^0}{\dot{n}_{\text{CO}}} + [0.5 \bar{h}_{\text{O}_2} + 1.88 \bar{h}_{\text{N}_2} + \bar{h}_{\text{CO}}]_1 - [x \bar{h}_{\text{CO}} + (1-x) \bar{h}_{\text{CO}_2} + \frac{x}{2} \bar{h}_{\text{O}_2} + 1.88 \bar{h}_{\text{N}_2}]_2$$

or, with $\bar{h}_f^0 = 0$ for O_2 and N_2 , and data from the ideal gas tables

$$x[-47,540 + \bar{h}_{\text{CO}}(T_2) - 3725.1] + (1-x)[-169,300 + \bar{h}_{\text{CO}_2}(T_2) - 4027.5] + \frac{x}{2}[\bar{h}_{\text{O}_2}(T_2) - 3725.1] + 1.88[\bar{h}_{\text{N}_2}(T_2) - 3729.5] = 0.5[5022.9 - 3725.1] + 1.88[5004.5 - 3729.5] + [-47,540 + 5006.1 - 3725.1]$$

or

$$x[\bar{h}_{\text{CO}}(T_2) - 51,265] + (1-x)[\bar{h}_{\text{CO}_2}(T_2) - 173,328] + \frac{x}{2}[\bar{h}_{\text{O}_2}(T_2) - 3725.1] + 1.88[\bar{h}_{\text{N}_2}(T_2) - 3729.5] = -43,213 \quad (2)$$

Equations (1), (2) are simultaneous in x and T_2 . Inspection of the reaction equation shows that for CO and CO_2 to be in the products x must be in the interval $0 < x < 1$. Solving (1), (2) iteratively using table data, $x = 0.1335$, $T_2 = 4420^\circ\text{R}$.

The solution presented on the next page uses IT to avoid the iterative solution of simultaneous equations involving table data.

Continued on next slide

Problem 14-52 continued

Alternative Solution Using IT

IT Code

```
T1 = 260 + 459.67 // °R
p1 = 1.5 // atm
p2 = 1.5 // atm
ndotCO_1 = 1 // Do calculations on the basis of 1 lbmol of CO entering.

// CO + 0.5 O2 + 1.88 N2 ==> x CO2 + (1 - x) CO + (1 - x)/2 O2 + 1.88 N2
ndotCO2_2 = x
ndotCO_2 = 1 - x
ndotO2_2 = (1 - x) / 2
ndotN2_2 = 1.88
ndot2 = ndotCO2_2 + ndotCO_2 + ndotO2_2 + ndotN2_2
yCO2 = ndotCO2_2 / ndot2
yCO = ndotCO_2 / ndot2
yO2 = ndotO2_2 / ndot2
pref = 1 // atm

// For the reaction CO2 <==> CO + 1/2 O2
K = (((yCO)*(yO2^0.5)) / (yCO2)) * (p2 / pref)^0.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T2,3)

0 = hCO_1 + 0.5*hO2_1 + 1.88*hN2_1 - x*hCO2_2 - (1-x)*hCO_2 - ((1-x)/2)*hO2_2 - 1.88*hN2_2
hCO_1 = h_T("CO",T1)
hO2_1 = h_T("O2",T1)
hN2_1 = h_T("N2",T1)
hCO2_2 = h_T("CO2",T2)
hCO_2 = h_T("CO",T2)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
```

IT Results

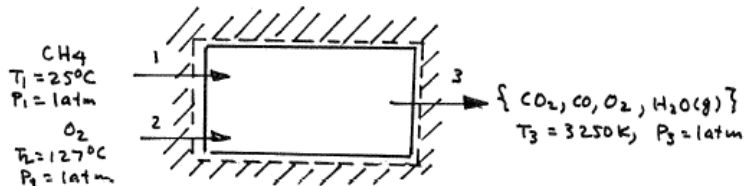
```
K = 0.02862
x = 0.8659
T2 = 4422°R
```

PROBLEM 14.53

KNOWN: CH_4 at 25°C , 1atm enters an insulated reactor operating at steady state and burns with O_2 entering at 127°C , 1atm . An equilibrium mixture of CO_2 , CO , O_2 , and $\text{H}_2\text{O}(\text{g})$ exits at 3250K , 1atm .

FIND: Determine the rate O_2 enters, in $\text{kmol per kmol of CH}_4$.

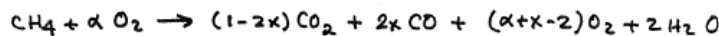
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown on the accompanying figure is at steady state with $\dot{Q}_{\text{cv}} = \dot{W}_{\text{cv}} = 0$ and negligible effects of kinetic and potential energy. (2) The mixture at 3 and incoming O_2 can be modeled as ideal gases.

ANALYSIS: The equation describing the reaction of CH_4 with O_2 to form CO_2 , CO , O_2 , H_2O is



The amount of mixture is $n = (1-2x) + 2x + (\alpha+x-2) + 2 = 1 + \alpha + x$.

At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2}\text{O}_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[2x][\alpha+x-2]^{1/2}}{[1-2x]} \left[\frac{P/\text{atm}}{1+\alpha+x} \right]^{1/2} = \left[\frac{2x}{1-2x} \right] \left[\frac{\alpha+x-2}{\alpha+x+1} \right]^{1/2} \quad (1)$$

At 3250K , Table A-27 gives $\log_{10} K = -0.1215 \Rightarrow K = 0.75596$. Squaring both sides of Eq. (1) and solving for α

$$\alpha = \frac{[1 - f(x)]x - (f(x) + 2)}{f(x) - 1} \quad \text{where } f(x) \equiv \left[\frac{(1-2x)K}{2x} \right]^2 \quad (2)$$

Another equation involving α and x can be obtained from an energy rate balance which reduces at steady state to give

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + [\dot{n}_{\text{CH}_4} \bar{h}_{\text{CH}_4}]_1 + \alpha [\dot{n}_{\text{O}_2} \bar{h}_{\text{O}_2}]_2 - [(1-2x)\dot{n}_{\text{CO}_2} \bar{h}_{\text{CO}_2} + 2x\dot{n}_{\text{CO}} \bar{h}_{\text{CO}} + (\alpha+x-2)\dot{n}_{\text{O}_2} \bar{h}_{\text{O}_2} + 2\dot{n}_{\text{H}_2\text{O}} \bar{h}_{\text{H}_2\text{O}}]_3$$

Then with $\bar{h}_f^\circ = 0$ for O_2 and data from the ideal gas tables

$$0 = (\bar{h}_f^\circ)_{\text{CH}_4} + \alpha (\bar{h}(T_2) - \bar{h}(298))_{\text{O}_2} - [(1-2x) [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{\text{CO}_2} + 2x [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{\text{CO}} + (\alpha+x-2) [\bar{h}(T_3) - \bar{h}(298)]_{\text{O}_2} + 2 [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{\text{H}_2\text{O}}]$$

$$0 = [-74,850] + \alpha [14,711 - 8,682] - (1-2x) [-393,520 + 173,822 - 9364] - 2x [-110,530 + 111,534 - 9669] - (\alpha+x-2) [116,827 - 8682] - 2 [-241,820 + 150,272 - 9904]$$

Upon reduction

$$105,116\alpha + 542,939x = 569,406 \quad (3)$$

Equations (2), (3) are simultaneous solutions in x and α . By inspecting the reaction equation, for the products to contain CO and CO_2 x must be in the interval $0 < x < 0.5$. For O_2 to be in the products, $\alpha > 2$. Solving iteratively using table data

$$x = 0.2671, \alpha = 4.0373 \text{ kmol}(\text{O}_2) / \text{kmol}(\text{CH}_4) \leftarrow$$

Continued on next slide

Problem 14-53 continued

Iterative solution of simultaneous equations involving table is avoided by using IT, as follows:

IT Code

```
T1 = 25 + 273.15 // K
T2 = 127 + 273.15 // K
T3 = 3250 // K
p3 = 1 // atm
ndotCH4_1 = 1 // Do calculations on the basis of 1 kmol of CH4 entering.

// CH4 + A O2 ==> x CO2 + (1 - x) CO + (A - (3 + x) / 2) O2 + 2 H2O
ndotCO2_3 = x
ndotCO_3 = 1 - x
ndotO2_3 = A - (3 + x)/2
ndotH2O_3 = 2
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotH2O_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1 // atm

// For the reaction CO2 <==> CO + 1/2 O2
K = (((yCO) * (yO2^0.5)) / (yCO2)) * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)

0 = hCH4_1 + A*hO2_2 - (x*hCO2_3 + (1-x)*hCO_3 + (A-(3+x)/2)*hO2_3 + 2*hH2O_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
```

IT Result

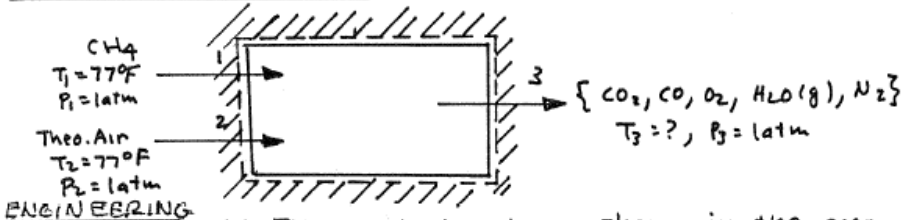
$$\dot{n}_{\text{O}_2} / \dot{n}_{\text{CH}_4} = 4.037 \text{ kmol/kmol}$$

PROBLEM 14.54

KNOWN: CH₄ at 77°F, 1 atm enters an insulated reactor operating at steady state and burns with the theoretical amount of air entering at 77°F, 1 atm. An equilibrium mixture of CO₂, CO, O₂, H₂O(g), and N₂ exits at 1 atm.

FIND: Determine (a) the temperature of the exiting mixture, (b) the exergy destruction in Btu per lbmol of CH₄ entering.

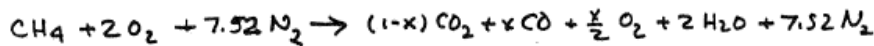
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The incoming air and exiting mixture can be modeled as ideal gases. (3) N₂ is inert. (4) T₀ = 537°R.

ANALYSIS: (a) Combustion of methane with the theoretical amount of air to produce CO₂, CO, O₂, H₂O(g), and N₂ is described by



The amount of mixture is $n = (1-x) + x + \frac{x}{2} + 2 + 7.52 = (21.04 + x)/2$. At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + 1/2\text{O}_2$. According to Eq. 14.35 takes the form

$$K(T_3) = \frac{[x][x/2]^{1/2}}{[1-x]} \left[\frac{P/P_{ref}}{(21.04+x)/2} \right]^{1/2} = \frac{x}{1-x} \left[\frac{x}{21.04+x} \right]^{1/2} \quad (1)$$

Another equation in x and T_3 can be obtained from an energy rate equation which reduces at steady state to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + (\dot{n}_{\text{CH}_4})_1 \bar{h}_{\text{CH}_4} + (2\dot{n}_{\text{O}_2} + 7.52\dot{n}_{\text{N}_2})_2 - [(1-x)\dot{n}_{\text{CO}_2} + x\dot{n}_{\text{CO}} + \frac{x}{2}\dot{n}_{\text{O}_2} + 2\dot{n}_{\text{H}_2\text{O}} + 7.52\dot{n}_{\text{N}_2}]_3$$

Then, with $\bar{h}_f^\circ = 0$ for O₂ and N₂

$$(1-x) [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{\text{CO}_2} + x [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{\text{CO}} + \frac{x}{2} [\bar{h}(T_3) - \bar{h}(537)]_{\text{O}_2} + 2 [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{\text{H}_2\text{O}} + 7.52 [\bar{h}(T_3) - \bar{h}(537)]_{\text{N}_2} = (\bar{h}_f^\circ)_{\text{CH}_4}$$

With data from the ideal gas tables

$$(1-x) [-169,300 + \bar{h}_{\text{CO}_2}(T_3) - 4027.5] + x [-47,540 + \bar{h}_{\text{CO}}(T_3) - 3725.1] + \frac{x}{2} [\bar{h}_{\text{O}_2}(T_3) - 3725.1] + 2 [-104,040 + \bar{h}_{\text{H}_2\text{O}}(T_3) - 4258] + 7.52 [\bar{h}_{\text{N}_2}(T_3) - 3729.5] = -32,210$$

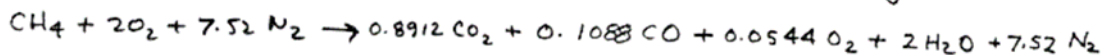
$$(1-x) [\bar{h}_{\text{CO}_2}(T_3) - 173,328] + x [\bar{h}_{\text{CO}}(T_3) - 51,265] + \frac{x}{2} [\bar{h}_{\text{O}_2}(T_3) - 3725] + 2 [\bar{h}_{\text{H}_2\text{O}}(T_3) - 108,298] + 7.52 [\bar{h}_{\text{N}_2}(T_3) - 3730] = -32,210 \quad (2)$$

Eqs. (1), (2) are simultaneous in x and T_3 . By inspection of the reaction equation, for CO and CO₂ to be in the products x must be in the interval $0 < x < 1$. Solving iteratively using table data $x = 0.1088$, $T_3 = 4066^\circ\text{R}$.

Continued on next slide

Problem 14-54 continued

(b) From the solution to part (a), the reaction is described by



where the combustion products are at 4066 °R.

The rate of exergy destruction is $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy rate balance:

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{\text{CH}_4}} = [0.8912 \bar{s}_{\text{CO}_2} + 0.1088 \bar{s}_{\text{CO}} + 0.0544 \bar{s}_{\text{O}_2} + 2 \bar{s}_{\text{H}_2\text{O}} + 7.52 \bar{s}_{\text{N}_2}]_3 - [\bar{s}_{\text{CH}_4}]_1 - [2 \bar{s}_{\text{O}_2} + 7.52 \bar{s}_{\text{N}_2}]_2$$

The methane enters at 77 °F, 1 atm. Thus, from Table A-25, $\bar{s}_{\text{CH}_4} = 44.49$ Btu/lbmol·°R. The air enters at 2500, 1 atm, so with Eq. 13.23 and data from Table A-25

$$\bar{s}_{\text{O}_2} = \bar{s}_{\text{O}_2}^\circ(537) - \bar{R} \ln y_{\text{O}_2} = 48.98 - 1.986 \ln 0.21 = 52.08 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

$$\bar{s}_{\text{N}_2} = \bar{s}_{\text{N}_2}^\circ(537) - \bar{R} \ln y_{\text{N}_2} = 45.74 - 1.986 \ln 0.79 = 46.21 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

The combustion products exit at 4066 °R, 1 atm with composition $y_{\text{CO}_2} = (0.8912/10.5744)$, $y_{\text{CO}} = (0.1088/10.5744)$, $y_{\text{O}_2} = (0.0544/10.5744)$, $y_{\text{H}_2\text{O}} = (2/10.5744)$, $y_{\text{N}_2} = (7.52/10.5744)$. Thus, with data from the ideal gas tables

$$\bar{s}_{\text{CO}_2} = \bar{s}_{\text{CO}_2}^\circ(4066) - \bar{R} \ln y_{\text{CO}_2} = 75.643 - 1.986 \ln (0.8912/10.5744) = 80.556 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

$$\bar{s}_{\text{CO}} = \bar{s}_{\text{CO}}^\circ(4066) - \bar{R} \ln y_{\text{CO}} = 62.915 - 1.986 \ln (0.1088/10.5744) = 72.004 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

$$\bar{s}_{\text{O}_2} = \bar{s}_{\text{O}_2}^\circ(4066) - \bar{R} \ln y_{\text{O}_2} = 65.274 - 1.986 \ln (0.0544/10.5744) = 75.740 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

$$\bar{s}_{\text{H}_2\text{O}} = \bar{s}_{\text{H}_2\text{O}}^\circ(4066) - \bar{R} \ln y_{\text{H}_2\text{O}} = 64.784 - 1.986 \ln (2/10.5744) = 68.041 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

$$\bar{s}_{\text{N}_2} = \bar{s}_{\text{N}_2}^\circ(4066) - \bar{R} \ln y_{\text{N}_2} = 61.238 - 1.986 \ln (7.52/10.5744) = 61.915 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

Using calculated data

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{\text{CH}_4}} = \{ (0.8912)(80.556) + (0.1088)(72.004) + (0.0544)(75.740) + 2(68.041) + 7.52(61.915) \} - 44.49 - (2)(52.08) + 7.52(46.21) = 189.28 \text{ Btu/lbmol} \cdot ^\circ\text{R}$$

Finally, the exergy destruction is

$$\frac{\dot{E}_d}{\dot{n}_{\text{CH}_4}} = T_0 \left(\frac{\dot{\sigma}_{cv}}{\dot{n}_{\text{CH}_4}} \right) = (537^\circ\text{R}) (189.28 \frac{\text{Btu}}{\text{lbmol} \cdot ^\circ\text{R}}) = 101,643 \frac{\text{Btu}}{\text{lbmol}(\text{CH}_4)} \leftarrow$$

An iterative solution of simultaneous equations with table data can be avoided using IT, as shown on the next page.

Continued on next slide

Problem 14-54 continued

Alternative Solution Using IT.

IT Code

```

T1 = 77 + 459.67 // °R
p1 = 1 // atm
T2 = 77 + 459.67 // °R
p2 = 1 // atm
p3 = 1 // atm
To = 537 // °R
ndotCH4_1 = 1 // Do calculations on the basis of 1 lbmol of CH4 entering.

// CH4 + 2 * (O2 + 3.76 N2) -> (1 - x) CO2 + x CO + x/2 O2 + 2 H2O + 7.52 N2
ndotCO2_3 = 1 - x
ndotCO_3 = x
ndotO2_3 = x/2
ndotH2O_3 = 2
ndotN2_3 = 7.52
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotH2O_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yN2 = ndotN2_3 / ndot3
pref = 1 // atm

// For the reaction CO2 <=> CO + 1/2 O2
K = (yCO * yO2^0.5) / yCO2 * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T3,3)

0 = hCH4_1 + 2*hO2_2 + 7.52*hN2_2 - ((1-x)*hCO2_3 + x*hCO_3 + (x/2)*hO2_3 + 2*hH2O_3
+ 7.52*hN2_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
hN2_3 = h_T("N2",T3)

Edotd = To * (sigmadot / ndotCH4_1)
sigmadot / ndotCH4_1 = (1-x)*sCO2_3 + x*sCO_3 + (x/2)*sO2_3 + 2*sH2O_3 + 7.52*sN2_3 -
sCH4_1 - 2*sO2_2 - 7.52*sN2_2
sCH4_1 = s_Tp("CH4",T1,p1)
pO2_2 = .21 * p2
sO2_2 = s_Tp("O2",T2,pO2_2)
pN2_2 = .79 * p2
sN2_2 = s_Tp("N2",T2,pN2_2)
pCO2_3 = yCO2 * p3
sCO2_3 = s_Tp("CO2",T3,pCO2_3)
pCO_3 = yCO * p3
sCO_3 = s_Tp("CO",T3,pCO_3)
pO2_3 = yO2 * p3
sO2_3 = s_Tp("O2",T3,pO2_3)
pH2O_3 = yH2O * p3
sH2O_3 = s_Tp("H2O",T3,pH2O_3)
pN2_3 = yN2 * p3
sN2_3 = s_Tp("N2",T3,pN2_3)

```

IT Results

```

x = 0.109
T3 = 4067°R
σcv / ṁCH4 = 189.3 Btu/lbmol(CH4)·°R
Ėd / ṁCH4 = 1.016 x 105 Btu/lbmol(CH4)

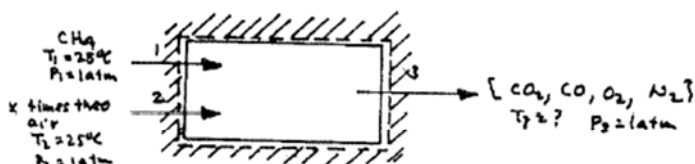
```


PROBLEM 14.55

KNOWN: $\text{CH}_4(\text{g})$ at 25°C , 1 atm enters an insulated reactor operating at steady state and burns with x times the theoretical amount of air entering at 25°C , 1 atm. An equilibrium mixture of CO_2 , CO , O_2 and N_2 exits at 1 atm.

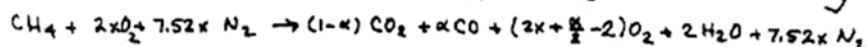
FINDS: For x ranging from 1 to 4, determine the temperature of the exiting equilibrium mixture.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture can be modeled as an ideal gas. (3) N_2 is inert. (4) $x \geq 1$.

ANALYSIS: The reaction of CH_4 with x times the theoretical amount of air to produce CO_2 , CO , O_2 , and N_2 is described by



For CO and CO_2 to appear in the products, it is necessary for α to be in the interval $0 < \alpha < 1$. The amount of mixture, n , is

$$n = (1-\alpha) + \alpha + (2x + \frac{\alpha}{2} - 2) + 2 + 7.52x = (2 + 19.04x + \alpha)/2$$

At equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2}\text{O}_2$. Accordingly, Eq. 14.35 becomes

$$K(T) = \frac{(\alpha) \left[\frac{4x + \alpha - 4}{2} \right]^{1/2}}{(1-\alpha)} \left[\frac{P/P_{\text{ref}}}{(2 + 19.04x + \alpha)/2} \right]^{1/2} = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{4x + \alpha - 4}{2 + 19.04x + \alpha} \right)^{1/2} \quad (1)$$

Another equation relating T_2 and α for specified x is obtained using the energy rate balance which at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}_{\text{CH}_4}} - \frac{\dot{W}_{cv}}{\dot{m}_{\text{CH}_4}} + (\bar{h}_{\text{CH}_4})_1 + [2x\bar{h}_{\text{O}_2} + 7.52x\bar{h}_{\text{N}_2}]_2 - [(1-\alpha)\bar{h}_{\text{CO}_2} + \alpha\bar{h}_{\text{CO}} + (2x + \frac{\alpha}{2} - 2)\bar{h}_{\text{O}_2} + 2\bar{h}_{\text{H}_2\text{O}} + 7.52x\bar{h}_{\text{N}_2}]_3$$

Then, with $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{\text{CH}_4} - (1-\alpha) [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{CO}_2} - \alpha [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{CO}} - (2x + \frac{\alpha}{2} - 2) [\bar{h}(T_2) - \bar{h}(298)]_{\text{O}_2} - 2 [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{H}_2\text{O}} - 7.52x [\bar{h}(T_2) - \bar{h}(298)]_{\text{N}_2}$$

With data from the ideal gas tables

$$0 = (-74850) - (1-\alpha) [-393,520 + \bar{h}_{\text{CO}_2}(T_2) - 9364] - \alpha [-110,530 + \bar{h}_{\text{CO}}(T_2) - 8669] - (2x + \frac{\alpha}{2} - 2) [\bar{h}_{\text{O}_2}(T_2) - 8682] - 2 [-241,820 + \bar{h}_{\text{H}_2\text{O}}(T_2) - 9904] + 7.52x [\bar{h}_{\text{N}_2}(T_2) - 8669]$$

or

$$0 = (-74850) - (1-\alpha) [\bar{h}_{\text{CO}_2}(T_2) - 402,884] - \alpha [\bar{h}_{\text{CO}}(T_2) - 119,199] - (2x + \frac{\alpha}{2} - 2) [\bar{h}_{\text{O}_2}(T_2) - 8682] - [\bar{h}_{\text{H}_2\text{O}}(T_2) - 251,724] - 7.52x [\bar{h}_{\text{N}_2}(T_2) - 8669] \quad (2)$$

Eqs. (1), (2) are simultaneous in T_2 , α , and x . For each specified value for x : $1 \leq x \leq 4$, the equations can be solved for T_2 and α , as shown in the table following the IT solution.

Continued on next slide

Problem 14-55 continued

```
T1 = 25 + 273.15 // K
T2 = 25 + 273.15 // K
p3 = 1 // atm
x = 1
ndotCH4_1 = 1 // Do calculations on the basis of 1 kmol of CH4 entering.
```

```
// CH4 + (2 * x) O2 + (7.52 * x) N2 ==>
// (1 - a) CO2 + a CO + (2 * x + a / 2 - 2) O2 + 2 H2O + (7.52 * x) N2
ndotCO2_3 = 1 - a
ndotCO_3 = a
ndotO2_3 = 2 * x + a / 2 - 2
ndotH2O_3 = 2
ndotN2_3 = 7.52 * x
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotH2O_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1 // atm
```

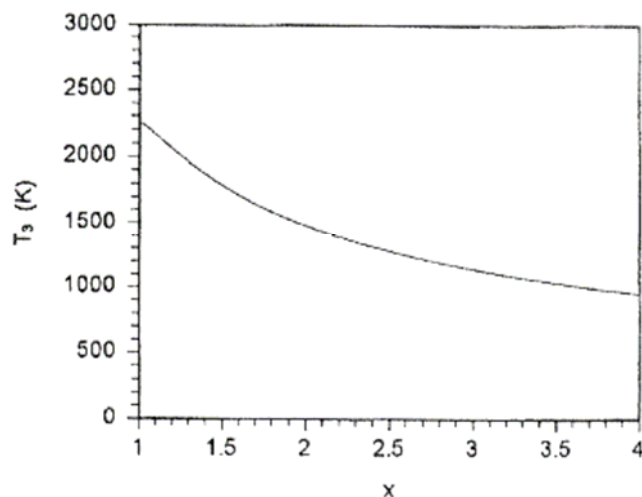
```
// For the reaction: CO2 <==> CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)
```

```
0 = hCH4_1 + 2*x*hO2_2 + 7.52*x*hN2_2 - ((1-a)*hCO2_3 + a*hCO_3 + (2*x+a/2-2)*hO2_3 +
2*hH2O_3 + 7.52*x*hN2_3)
```

```
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

x	a	T ₃ (K)
1.0	0.109	2259
1.5	7.025×10^{-4}	1789
2.0	1.036×10^{-5}	1481
2.5	2.471×10^{-7}	1280
3.0	8.101×10^{-9}	1138
3.5	3.934×10^{-10}	1032
4.0	5.312×10^{-12}	950.7



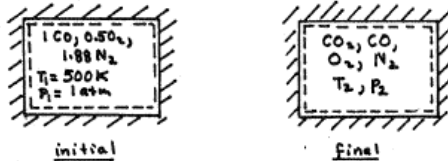
As the amount of air supplied increases, the temperature of the combustion products and the amount of CO decrease.

PROBLEM 14.56

KNOWN: A mixture of 1 kmol CO, 0.5 kmol O₂, 1.88 kmol N₂ is initially at 227°C, 1 atm in a closed, rigid, insulated vessel. An equilibrium mixture of CO₂, CO, O₂, and N₂ eventually forms.

FIND: Determine the final pressure.

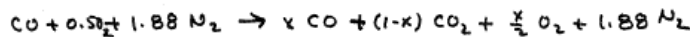
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system is shown in the accompanying figure. (2) The ideal gas model applies at the initial and final states. (3) For the system $Q = W = 0$ and kinetic and potential energy effects are negligible. (4) N₂ is inert.

ANALYSIS: The reaction takes the form



The initial amount of mixture is $n_1 = 1 + 0.5 + 1.88 = 3.38$ kmol. The final amount of mixture is $n_2 = x + (1-x) + \frac{x}{2} + 1.88 = (5.76 + x)/2$.

The ideal gas equation of state gives

$$\left. \begin{aligned} P_1 V &= n_1 \bar{R} T_1 \\ P_2 V &= n_2 \bar{R} T_2 \end{aligned} \right\} : \frac{P_2}{P_1} = \frac{n_2 T_2}{n_1 T_1} \quad (1)$$

As P_1, T_1 , and n_1 are known, P_2 can be determined once n_2 and T_2 are known.

At equilibrium, $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2}\text{O}_2$. Accordingly Eq. 14.35 takes the form

$$K(T_2) = \frac{[x][\frac{x}{2}]}{[1-x]} \left[\frac{P_2/P_{\text{ref}}}{n_2} \right]^{1/2} \quad (2)$$

Since $P_1 = P_{\text{ref}}$, use of Eq. (1) in Eq. (2) gives

$$K(T_2) = \left[\frac{x}{1-x} \right] \left[\frac{x T_2}{2 n_1 T_1} \right]^{1/2} \quad (3)$$

Another equation involving x and T_2 can be obtained from an energy balance which reduces to $\Delta U = 0$. That is,

$$[x \bar{u}_{\text{CO}} + (1-x) \bar{u}_{\text{CO}_2} + \frac{x}{2} \bar{u}_{\text{O}_2} + 1.88 \bar{u}_{\text{N}_2}]_2 - [\bar{u}_{\text{CO}} + 0.5 \bar{u}_{\text{O}_2} + 1.88 \bar{u}_{\text{N}_2}]_1 = 0$$

With $\bar{h} = \bar{u} + \bar{R}T$, $\bar{u} = \bar{h} - \bar{R}T$

$$[x [\bar{h}_{\text{CO}} - \bar{R}T] + (1-x) [\bar{h}_{\text{CO}_2} - \bar{R}T] + \frac{x}{2} [\bar{h}_{\text{O}_2} - \bar{R}T]]_2 - [\bar{h}_{\text{CO}} - \bar{R}T]_1 + 0.5 [\bar{h}_{\text{O}_2} - \bar{R}T]_1 + 1.88 [\bar{u}_{\text{N}_2}(T_2) - \bar{u}_{\text{N}_2}(T_1)] = 0$$

or

$$[x \bar{h}_{\text{CO}} + (1-x) \bar{h}_{\text{CO}_2} + \frac{x}{2} \bar{h}_{\text{O}_2}]_2 - [\bar{h}_{\text{CO}} + 0.5 \bar{h}_{\text{O}_2}]_1 + 1.88 [\bar{u}_{\text{N}_2}(T_2) - \bar{u}_{\text{N}_2}(T_1)] + \frac{3}{2} \bar{R} T_1 - (1 + \frac{x}{2}) \bar{R} T_2 = 0$$

For CO, CO₂, and O₂, $\bar{h} = \bar{h}_f^\circ + \Delta \bar{h}$. Thus

$$x [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{CO}} + (1-x) [\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{\text{CO}_2} + \frac{x}{2} [\bar{h}(T_2) - \bar{h}(298)]_{\text{O}_2} - [\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(298)]_{\text{CO}} - 0.5 [\bar{h}(T_1) - \bar{h}(298)]_{\text{O}_2} + 1.88 [\bar{u}_{\text{N}_2}(T_2) - \bar{u}_{\text{N}_2}(T_1)] + \frac{3}{2} \bar{R} T_1 - (1 + \frac{x}{2}) \bar{R} T_2 = 0$$

With data from Table A-23

$$\begin{aligned} & x [-110,530 + \bar{h}_{\text{CO}}(T_2) - 8669] + (1-x) [-393,520 + \bar{h}_{\text{CO}_2}(T_2) - 9364] + \\ & \frac{x}{2} [\bar{h}_{\text{O}_2}(T_2) - 8682] - [-110,530 + 14,600 - 8669] - 0.5 [14,770 - 8682] + \\ & 1.88 [\bar{u}_{\text{N}_2}(T_2) - 10,413] + \frac{3}{2} (8.314)(500) - (1 + \frac{x}{2}) \bar{R} T_2 = 0 \end{aligned}$$

Continued on next slide

Problem 14-56 continued

Thus

$$x [\bar{h}_{\text{CO}}(T_2) - 119,199] + (1-x) [\bar{h}_{\text{CO}_2}(T_2) - 402,814] + \frac{x}{2} [\bar{h}_{\text{O}_2}(T_2) - 8682] + 1.88 \bar{u}_{\text{N}_2}(T_2) - (1+x) \bar{R} T_2 + 88195.3 = 0 \quad (4)$$

Eqs. (3), (4) are simultaneous in x and T_2 . Inspection of the reaction equation shows that for CO and CO₂ to be in the products x must be in the interval $0 < x < 1$. Solving iteratively using table data, $x = 0.2262$, $T_2 = 2761 \text{ K}$. Thus

$$n_2 = \frac{5.76 + 0.2262}{2} = 2.993$$

Eq. (1) then gives

$$P_2 = \left(\frac{2.993}{3.38} \right) \left(\frac{2761}{500} \right) (1 \text{ atm}) = 4.89 \text{ atm} \quad \leftarrow$$

The following IT solution provides an alternative to the iterative approach involving the use of table data:

IT Code

```
nCO_1 = 1 // kmol
nO2_1 = 0.5 // kmol
T1 = 227 + 273.15 // K
p1 = 1 // atm
n1 = nCO_1 + nO2_1 + nN2

// CO + 0.5 O2 + 1.88 N2 -> x CO + (1-x) CO2 + x/2 O2 + 1.88 N2
nCO = x
nCO2 = 1 - x
nO2 = x/2
nN2 = 1.88
n2 = nCO + nCO2 + nO2 + nN2
yCO = nCO / n2
yO2 = nO2 / n2
yCO2 = nCO2 / n2
pref = 1 // atm

p2 / p1 = (n2 / n1) * (T2 / T1)

// For the reaction CO2 <=> CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p2 / pref)^0.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = Lookupval(EQCO2A,1,T2,3)

U2 / nCO_1 = (x * uCO_2 + (1-x) * uCO2_2 + (x/2) * uO2_2 + 1.88 * uN2_2)
U1 / nCO_1 = (uCO_1 + 0.5 * uO2_1 + 1.88 * uN2_1)
U2 = U1
uCO_1 = u_T("CO",T1)
uO2_1 = u_T("O2",T1)
uN2_1 = u_T("N2",T1)
uCO_2 = u_T("CO",T2)
uCO2_2 = u_T("CO2",T2)
uO2_2 = u_T("O2",T2)
uN2_2 = u_T("N2",T2)
```

IT Results

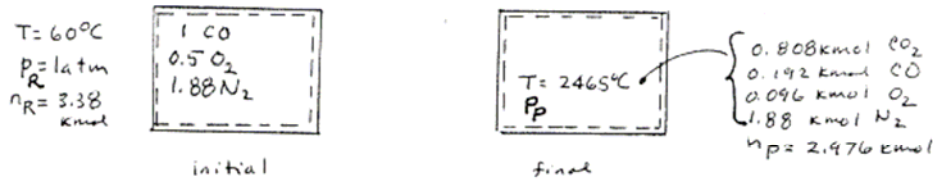
```
n2 = 2.993 kmol
x = 0.2262
T2 = 2760 K
p2 = 4.887 atm
```

PROBLEM 14.57

KNOWN: A mixture of CO and the theoretical amount of air reacts in an insulated vessel to form an equilibrium mixture of CO_2 , CO, O_2 for which a molar analysis is provided. A measured value for the final temperature is indicated as well.

FIND: Check the consistency of the given data.

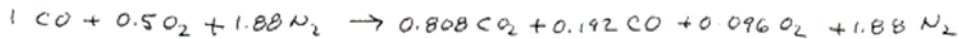
SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system is shown in the figure above. (2) For the system $Q = W = 0$ and kinetic/potential energy effects are negligible. (3) Ideal gas principles apply. (4) N_2 is inert.

ANALYSIS: Using the data above the balanced reaction equation is



Carbon and oxygen balances verify this expression is correct.

Next, an energy balance reads $\Delta U = \cancel{Q} - \cancel{W}$. Paralleling the development of Eq. 13.17b, this can be rewritten as

$$0 = \sum_P n(\bar{h}_f^\circ + \Delta \bar{h}) - \sum_R n(\bar{h}_f^\circ + \Delta \bar{h}) - \bar{R} T_P \sum_P n + \bar{R} T_R \sum_R n. \text{ With table data this becomes}$$

$$0 \stackrel{?}{=} 0.808[-393,520 + (145,971 - 9364)] + 0.192[-110,530 + (92485 - 8669)] + 0.096[96,379 - 8682] + 1.88[91,729 - 9684] - 1[-110,530 + (9685 - 8669)] - 0.5[9709 - 8682] - 8.314[(2738)(2.976) - (333)(3.38)] = 561 \text{ kJ}$$

Accordingly, the energy balance requirement is closely satisfied.

Finally, the equilibrium constraint has to be considered. This requires the final mixture pressure, obtained using the ideal gas equation of state at fixed volume: $PV = nRT$. Thus

$$\frac{P_P}{P_R} = \frac{n_P T_P}{n_R T_R} \Rightarrow \frac{P_P}{n_P} = \frac{P_R T_P}{n_R T_R} = \frac{(1 \text{ atm})}{(3.38 \text{ kmol})} \left[\frac{2738 \text{ K}}{333 \text{ K}} \right] = 2.433 \Rightarrow P_P = 2.433 \text{ atm.}$$

Then, at equilibrium $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$, so

$$K(2738 \text{ K}) = \frac{[0.192][0.096]^{1/2}}{[0.808]} \left[2.433 \right]^{1 - 1/2 - 1} = \left[\frac{0.192}{0.808} \right] [(0.096)(2.433)]^{1/2} = 0.1148$$

Then, from Table A-27 at 2738 K, $\log_{10} K = -0.9428$, $K = 0.1141$. Thus, the equilibrium condition is also closely satisfied.

Accordingly, the given data are consistent.

PROBLEM 14.58

KNOWN: At 2000 K, $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$

FIND: Estimate the enthalpy of reaction using the van't Hoff equation and equilibrium constant data. Compare with the value calculated using enthalpy data.

ENGINEERING MODEL: The ideal gas model is applicable.

ANALYSIS: Rearranging Eq. 14.48b and using $\ln K = 2.303 \log_{10} K$

$$\Delta H = \bar{R} T^2 \frac{d \ln K}{dT} = 2.303 \bar{R} T^2 \frac{d \log_{10} K}{dT}$$

With the interpolation rule of Problem 14.6

$$\log_{10} K = C_1 + \frac{C_2}{T} \Rightarrow \frac{d \log_{10} K}{dT} = -\frac{C_2}{T^2}$$

Collecting results

$$\Delta H = -2.303 \bar{R} C_2 \quad (1)$$

Then, in the interval $1900 \leq T \leq 2000 \text{ K}$, using data from Table A-27

$$\log_{10} K = C_1 + \frac{C_2}{T}$$

$$\begin{aligned} @ 1900 \text{ K}: & -3.267 = C_1 + C_2/1900 \\ @ 2100 \text{ K}: & -2.539 = C_1 + C_2/2100 \end{aligned} \quad \left. \vphantom{\begin{aligned} @ 1900 \text{ K}: \\ @ 2100 \text{ K}: \end{aligned}} \right\} C_2 = -14523 \text{ K}$$

Using this value for C_2 , Eq. (1) gives

$$\begin{aligned} \Delta H &= -2.303 \left(8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \right) (-14523 \text{ K}) \\ &= 278,074 \text{ kJ/kmol} \end{aligned} \quad (2)$$

With data from the ideal gas tables

$$\begin{aligned} \Delta H &= \bar{h}_{\text{CO}} + \frac{1}{2} \bar{h}_{\text{O}_2} - \bar{h}_{\text{CO}_2} \\ &= [\bar{h}_f^\circ + \bar{h}(2000) - \bar{h}(298)]_{\text{CO}} + \frac{1}{2} [\bar{h}_f^\circ + \bar{h}(2000) - \bar{h}(298)]_{\text{O}_2} - [\bar{h}_f^\circ + \bar{h}(2000) - \bar{h}(298)]_{\text{CO}_2} \\ &= [-110,530 + 65,408 - 8669] + \frac{1}{2} [67,881 - 8682] - [-393,520 + 100,804 - 9364] \\ &= 277,889 \text{ kJ/kmol} \end{aligned} \quad (3)$$

The results given by Eqs. (2), (3) differ by less than 0.1%

PROBLEM 14.59

KNOWN: At 2000 K, $\text{H}_2\text{O} \rightleftharpoons \text{H}_2 + \frac{1}{2}\text{O}_2$

FIND: Estimate the enthalpy of reaction using the van't Hoff and equilibrium constant data; compare with the value calculated using enthalpy data.

ENGINEERING MODEL: The ideal gas model is applicable.

ANALYSIS: Rearranging Eq. 14.43b and using $\ln K = 2.303 \log_{10} K$

$$\Delta T = RT^2 \frac{d \ln K}{dT} = 2.303 \bar{R} T^2 \frac{d \log_{10} K}{dT}$$

With the interpolation rule of Problem 14.6

$$\log_{10} = C_1 + \frac{C_2}{T}, \quad \frac{d \log_{10}}{dT} = -\frac{C_2}{T^2}$$

Collecting results

$$\Delta H = -2.303 \bar{R} C_2 \quad (1)$$

Then, in the interval $1900 \leq T \leq 2100 \text{ K}$, using data from Table A-27

$$@ 1900 \text{ K}: -3.886 = C_1 + C_2/1900 \quad \left\{ \begin{array}{l} C_2 = -13,147 \text{ K} \end{array} \right.$$

$$@ 2100 \text{ K}: -3.227 = C_1 + C_2/2100$$

Using this value for C_2 , Eq. (1) gives

$$\Delta H = -2.303 (8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}) (-13,147 \text{ K}) = 251,727 \frac{\text{kJ}}{\text{kmol}} \quad (2)$$

With data from the ideal gas tables

$$\Delta H = \bar{h}_{\text{H}_2} + \frac{1}{2} \bar{h}_{\text{O}_2} - \bar{h}_{\text{H}_2\text{O}}$$

$$= [\cancel{\bar{h}_f^0} + \bar{h}(2000) - \bar{h}(298)]_{\text{H}_2} + \frac{1}{2} [\cancel{\bar{h}_f^0} + \bar{h}(2000) - \bar{h}(298)]_{\text{O}_2} - [\bar{h}_f^0 + \bar{h}(2000) - \bar{h}(298)]_{\text{H}_2\text{O}}$$

$$= [61,400 - 8468] + \frac{1}{2} [67,881 - 8682] - [-241,820 + 82,593 - 9904]$$

$$= 251,663 \frac{\text{kJ}}{\text{kmol}} \quad (3)$$

The results given by Eqs. (2), (3) differ by less than 0.03%.

PROBLEM 14.60

KNOWN: At 2800 K, $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$

FIND: Estimate the equilibrium constant at 2800 K using K at 2000 K from Table A-27, together with the Van't Hoff equation and enthalpy data. Compare with the value obtained from Table A-27 at 2800 K.

ENGINEERING MODEL: The ideal gas model is applicable.

ANALYSIS: The van't Hoff equation, Eq. 14.47b, gives upon rearrangement

$$\frac{d \ln K}{dT} = \frac{\Delta H}{RT^2} \quad (1)$$

Evaluating ΔH as the average of the values at 2000 K and 3000 K, Eq. (1) gives upon integration

$$\ln \frac{K_2}{K_1} = -\frac{(\Delta H)_{\text{ave}}}{R} \left[\frac{1}{T_2} - \frac{1}{T_1} \right] \quad (2)$$

For $\text{CO}_2 \rightarrow \text{CO} + \frac{1}{2} \text{O}_2$, data from the ideal gas tables gives

$$\begin{aligned} \Delta H &= \bar{h}_{\text{CO}} + \frac{1}{2} \bar{h}_{\text{O}_2} - \bar{h}_{\text{CO}_2} \\ &= [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(298)]_{\text{CO}} + \frac{1}{2} [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(298)]_{\text{O}_2} - [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(298)]_{\text{CO}_2} \\ &= [\bar{h}_{\text{CO}}(T) + \frac{1}{2} \bar{h}_{\text{O}_2}(T) - \bar{h}_{\text{CO}_2}(T)] + [\bar{h}_f^\circ - \bar{h}(298)]_{\text{CO}} - \frac{[\bar{h}(298)]_{\text{O}_2}}{2} - [\bar{h}_f^\circ - \bar{h}(298)]_{\text{CO}_2} \\ &= [\bar{h}_{\text{CO}}(T) + \frac{1}{2} \bar{h}_{\text{O}_2}(T) - \bar{h}_{\text{CO}_2}(T)] + [-110,530 - 8669] - 4341 - [-393,520 - 4364] \\ &= [\bar{h}_{\text{CO}}(T) + \frac{1}{2} \bar{h}_{\text{O}_2}(T) - \bar{h}_{\text{CO}_2}(T)] + 279,344 \end{aligned}$$

At $T = 2000 \text{ K}$:

$$\Delta H = [65,409 + \frac{67881}{2} - 100,804] + 279,344 = 277,889 \text{ kJ/kmol}$$

At $T = 2800 \text{ K}$:

$$\Delta H = [94,784 + \frac{98826}{2} - 149808] + 279,344 = 273,737 \text{ kJ/kmol}$$

Then $(\Delta H)_{\text{ave}} = 275,811 \text{ kJ/kmol}$, and Eq. (2) gives

$$\ln \frac{K_2}{K_1} = \frac{-275,811 \text{ kJ/kmol}}{8.314 \text{ kJ/kmol} \cdot \text{K}} \left[\frac{1}{2800} - \frac{1}{2000} \right] \text{ K}^{-1}$$

$$\Rightarrow \frac{K_2}{K_1} = 114.34$$

From Table A-27, at 2000 K, $\log_{10} K_1 = -2.884$, $K_1 = 1.3061 \times 10^{-3}$. Thus

$$K_2 = 114.34 (1.3061 \times 10^{-3}) = 0.1493$$

For comparison, Table A-27, at 2800 K, $\log_{10} K_2 = -0.825$, so $K_2 = 0.1496$. The estimated value is within 0.2 % of the table value.

PROBLEM 14.61

KNOWN: At 2800 K, $\text{H}_2\text{O} \rightleftharpoons \text{H}_2 + \frac{1}{2}\text{O}_2$

FIND: Estimate the equilibrium constant at 2800 K using K at 2500 K from Table A-27, together with the van't Hoff equation and enthalpy data. Compare with the value obtained from Table A-27 at 2800 K.

ENGINEERING MODEL: The ideal gas model is applicable.

ANALYSIS: The van't Hoff equation, Eq. 14.436, gives upon rearrangement

$$\frac{d \ln K}{dT} = \frac{\Delta H}{RT^2} \quad (1)$$

Evaluating ΔH as the average of the values at 2500 K and 2800 K, Eq. (1) gives upon integration

$$\ln \frac{K_2}{K_1} = - \frac{(\Delta H)_{\text{ave}}}{R} \left[\frac{1}{T_2} - \frac{1}{T_1} \right] \quad (2)$$

For $\text{H}_2\text{O} \rightarrow \text{H}_2 + \frac{1}{2}\text{O}_2$, data from the ideal gas tables

$$\begin{aligned} \Delta H &= \bar{h}_{\text{H}_2} + \frac{1}{2} \bar{h}_{\text{O}_2} - \bar{h}_{\text{H}_2\text{O}} \\ &= [\bar{h}_f^\circ + \bar{h}(T) - h(298)]_{\text{H}_2} + \frac{1}{2} [\bar{h}_f^\circ + \bar{h}(T) - h(298)]_{\text{O}_2} - [\bar{h}_f^\circ + \bar{h}(T) - h(298)]_{\text{H}_2\text{O}} \\ &= \bar{h}_{\text{H}_2}(T) + \frac{1}{2} \bar{h}_{\text{O}_2}(T) - \bar{h}_{\text{H}_2\text{O}}(T) - \bar{h}_{\text{H}_2}(298) - \frac{1}{2} \bar{h}_{\text{O}_2}(298) - (\bar{h}_f^\circ - \bar{h}(298))_{\text{H}_2\text{O}} \\ &= \bar{h}_{\text{H}_2}(T) + \frac{1}{2} \bar{h}_{\text{O}_2}(T) - \bar{h}_{\text{H}_2\text{O}}(T) - 8468 - 4341 - (-241,820 - 9904) \\ &= \bar{h}_{\text{H}_2}(T) + \frac{1}{2} \bar{h}_{\text{O}_2}(T) - \bar{h}_{\text{H}_2\text{O}}(T) + 238,915 \end{aligned}$$

At 2500 K:

$$\Delta H = 78,960 + \frac{1}{2}(87,057) - 108,864 + 238,915 = 252,536 \text{ kJ/kmol}$$

At 2800 K:

$$\Delta H = 89,838 + \frac{1}{2}(98,826) - 125,198 + 238,915 = 252,968 \text{ kJ/kmol}$$

Thus, $(\Delta H)_{\text{ave}} = 252,752 \text{ kJ/kmol}$, and Eq. (2) gives

$$\ln \frac{K_2}{K_1} = - \frac{252,752}{8.314} \left[\frac{1}{2800} - \frac{1}{2500} \right] \Rightarrow \frac{K_2}{K_1} = 3.6799$$

From Table A-27, at 2500 K, $\log_{10} K_1 = -2.2224$, $K_1 = 5.992 \times 10^{-4}$. Thus

$$K_2 = (5.992 \times 10^{-4})(3.6799) = 0.02205$$

For comparison, Table A-27, at 2800 K, $\log_{10} K_2 = -1.658$, so $K_2 = 0.02198$. The estimated value is within about 0.3% of the table value.

PROBLEM 14.62

KNOWN: For $C + 2H_2 \rightleftharpoons CH_4$ at $25^\circ C$ $\log_{10} K = 8.9$.

FIND: Estimate $\log_{10} K$ at $500^\circ C$.

ENGINEERING MODEL: (1) The Van't Hoff equation is applicable. (2) ΔH is constant.

ANALYSIS: The van't Hoff equation is given by

$$\frac{d \ln K}{dT} = \frac{\Delta H}{RT^2}$$

If ΔH is constant

$$\ln \frac{K_2}{K_1} = -\frac{\Delta H}{R} \left[\frac{1}{T_2} - \frac{1}{T_1} \right] \quad (1)$$

For $C + 2H_2 \rightleftharpoons CH_4$, ΔH is given by

$$\Delta H = \bar{h}_{CH_4} - \bar{h}_C - 2\bar{h}_{H_2}$$

At $298 K$, Table A-25 gives

$$\Delta H = (-74,850) - (0) - 2(0) = -74,850 \text{ kJ/kmol}$$

Substituting values into Eq. (1),

$$\ln \frac{K_2}{K_1} = \frac{74,850}{8.314} \left[\frac{1}{773} - \frac{1}{298} \right]$$

$\Rightarrow$

$$\log_{10} \frac{K_2}{K_1} = -8.06$$

or

$$\log_{10} K_2 = 8.9 - 8.06 = 0.84$$

①

1. For a discussion of the equilibrium constant and the heterogeneous system in which the pure solid or liquid phase of one (or more) of the substances is present in the equilibrium mixture, see Concepts of Thermodynamics by E.F. Obert, McGraw-Hill, 1960, Sec. 14-13.

PROBLEM 14.63

KNOWN: For $\text{Cs} \rightleftharpoons \text{Cs}^+ + \text{e}^-$, $K = 0.78$ and 15.63 at 1600 K , 2000 K , respectively.
FIND: Estimate ΔH at 1800 K .

ENGINEERING MODEL: (1) The Van't Hoff equation is applicable. (2) ΔH varies only gradually with temperature from 1600 to 2000 K .

ANALYSIS: The van't Hoff equation is

$$\frac{d \ln K}{dT} = \frac{\Delta H}{RT^2}$$

Then

$$\ln \frac{K_2}{K_1} = - \frac{(\Delta H)_{\text{ave}}}{R} \left[\frac{1}{T_2} - \frac{1}{T_1} \right]$$

where $(\Delta H)_{\text{ave}}$ is the average value of the enthalpy of ionization over the temperature range. Thus

$$\begin{aligned} (\Delta H)_{\text{ave}} &= \frac{\bar{R} \ln \frac{K_2}{K_1}}{\left(\frac{1}{T_1} - \frac{1}{T_2} \right)} \\ &= \frac{(8.314) \left(\ln \frac{15.63}{0.78} \right)}{\left(\frac{1}{1600} - \frac{1}{2000} \right)} = 199,380 \frac{\text{kJ}}{\text{kmol}} \end{aligned}$$

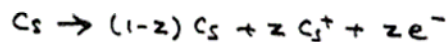
PROBLEM 14.64

KNOWN: An equilibrium mixture at 2000K, 1atm consists of Cs , Cs^+ , e^- . $K = 15.63$

FIND: If 1 kmol of Cs is present initially, determine the percent ionization of Cs .

ENGINEERING MODEL: Equilibrium can be analyzed using ideal gas equilibrium concepts.

ANALYSIS: The ionization of Cs to form a mixture of Cs , Cs^+ , and e^- is described by



The amount of mixture is $n = (1-z) + z + z = 1+z$

At equilibrium $\text{Cs} \rightleftharpoons \text{Cs}^+ + e^-$, so Eq. 14.35 takes the form

$$K = \frac{[z][z]}{[1-z]} \left[\frac{P/P_{\text{ref}}}{1+z} \right]^{1+1-1} = \frac{z^2}{1-z^2} \Rightarrow z = \left[\frac{K}{1+K} \right]^{1/2}$$

With $K = 15.6$, this becomes

$$z = \left[\frac{15.6}{16.6} \right]^{1/2} = 0.969$$

Thus, the amount of Cs in the mixture is 0.031 kmol. Cs is 96.9% dissociated. $\leftarrow$

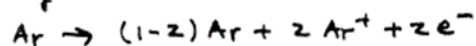
PROBLEM 14.65

KNOWN: At 18,000 °R and pressure P , an equilibrium mixture consists of Ar, Ar^+ , e^- . For $\text{Ar} \rightleftharpoons \text{Ar}^+ + e^-$, $K = 4.2 \times 10^{-4}$ at 18,000 °R.

FIND: Plot the percent ionization of Ar for $0.01 \leq p \leq 0.05$ atm.

ENGINEERING MODEL: Equilibrium can be modeled using ideal gas mixture concepts.

ANALYSIS: The ionization of Ar to form a mixture of Ar, Ar^+ , and e^- is described by



The amount of mixture is $n = (1-z) + z + z = 1 + z$.

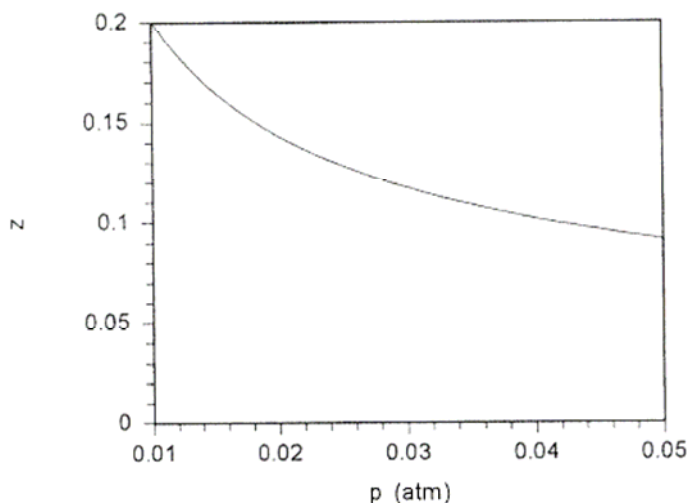
At equilibrium $\text{Ar} \rightleftharpoons \text{Ar}^+ + e^-$, so Eq. 14.35 takes the form

$$K = \frac{[z][z]}{[1-z]} \left[\frac{P/P_{\text{ref}}}{1+z} \right]^{1+1-1} = \left(\frac{z^2}{1-z} \right) (P/P_{\text{ref}})$$

Thus

$$z = \left[\frac{K/(P/P_{\text{ref}})}{1 + K/(P/P_{\text{ref}})} \right]^{1/2} = \left[\frac{1}{\frac{(P/P_{\text{ref}})}{K} + 1} \right]^{1/2}$$

($= 4.2 \times 10^{-4}$)



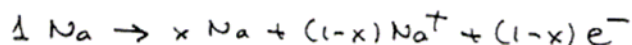
PROBLEM 14.66

KNOWN: 1 kmol of Na ionizes to form an equilibrium mixture of Na, Na⁺, e⁻ at 2000K and pressure p in which the amount of Na present is x kmol. For $\text{Na} \rightleftharpoons \text{Na}^+ + \text{e}^-$ at 2000K, $K = 0.668$.

FIND: Plot p versus x for $0.25 \leq x \leq 0.3$ kmol.

ENGINEERING MODEL: Equilibrium can be modeled using ideal gas mixture concepts.

ANALYSIS: The ionization of Na to form a mixture of Na, Na⁺, e⁻ is described by

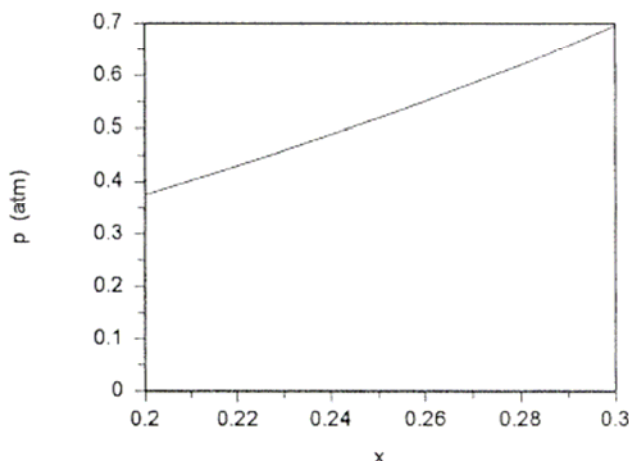


The amount of mixture is $n = x + (1-x) + (1-x) = 2-x$.

At equilibrium $\text{Na} \rightleftharpoons \text{Na}^+ + \text{e}^-$, so Eq. 14.35 takes the form

$$K = \frac{(1-x)(1-x)}{x} \left[\frac{p/p_{\text{ref}}}{2-x} \right]^{1+1-1} = \frac{(1-x)^2}{x(2-x)} (p/p_{\text{ref}})$$

Since $K = 0.668$, $\frac{p}{p_{\text{ref}}} = 0.668 \frac{x(2-x)}{(1-x)^2}$. Thus for $p_{\text{ref}} = 1 \text{ atm}$



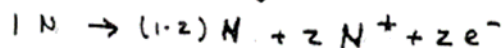
PROBLEM 14.67

KNOWN: 1 kmol of N ionizes to form an equilibrium mixture of N, N^+ , and e^- at 12,000 K, 6 atm in which the amount of N is 0.95 kmol.

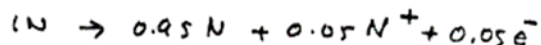
FIND: For $N \rightleftharpoons N^+ + e^-$, determine K .

ENGINEERING MODEL: Equilibrium can be modeled as an ideal gas mixture concepts.

ANALYSIS: The ionization of N to form an equilibrium mixture of N, N^+ , and e^- is described by



Since $1-z = 0.95$ kmol, $z = 0.05$ kmol. Accordingly



The amount of mixture is $n = 1.05$ kmol.

At equilibrium $N \rightleftharpoons N^+ + e^-$, so Eq. 14.35 takes the form

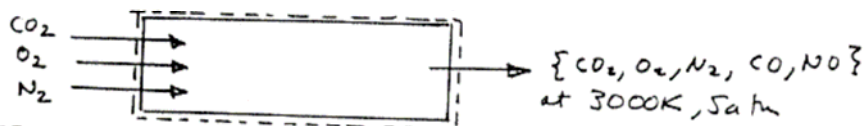
$$K = \frac{[0.05][0.05]}{[0.95]} \left[\frac{6}{1.05} \right]^{1+1-1} = 0.01504$$

PROBLEM 14.68

KNOWN: CO_2 , O_2 , N_2 enter a reactor operating at steady state with equal molar flow rates. An equilibrium mixture of $\{\text{CO}_2, \text{O}_2, \text{N}_2, \text{CO}, \text{NO}\}$ exits at 3000K , 5atm .

FIND: Determine the molar analysis of the equilibrium mixture.

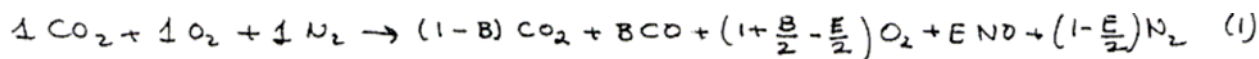
SCHEMATIC & GIVEN DATA



ENGINEERING

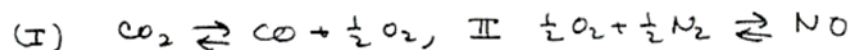
MODEL: (1) The control volume shown above is at steady state.
(2) Ideal gas principles apply.

ANALYSIS: On the basis of 1 mole each of CO , O_2 , N_2 entering, at steady state the balanced reaction equation has the form



where B, N are unknowns. For the products, $n = 3 + B/2$.

At equilibrium, these independent reactions must be satisfied



For reaction (I) with $K_1 = 0.3273$ from Table A-27 at 3000K ,

$$0.3273 = \frac{B \left(1 + \frac{B}{2} - \frac{E}{2}\right)^{1/2}}{(1-B)} \left[\frac{P/P_{\text{ref}}}{n} \right]^{1/2} = \left(\frac{B}{1-B} \right) \left[\frac{1 + \frac{B}{2} - \frac{E}{2}}{3 + \frac{B}{2}} \right]^{1/2} (5)^{1/2} \quad (2)$$

For reaction (II) with $K_2 = 0.1222$ from Table A-27 at 3000K

$$0.1222 = \frac{E}{\left[1 + \frac{B}{2} - \frac{E}{2}\right]^{1/2} \left[1 - \frac{E}{2}\right]^{1/2}} \left[\frac{P/P_{\text{ref}}}{n} \right]^0 = \frac{2E}{[2+B-E][2-E]}^{1/2} \quad (3)$$

Equations (2), (3) are two simultaneous equations for B and E . Solving,

$B = 0.2017$, $E = 0.1208$. Then, referring to Eq. (1)

$$\left. \begin{array}{l} (1-B) \text{ CO}_2 : 0.7983 \\ B \text{ CO} : 0.2017 \\ \left(1 + \frac{B}{2} - \frac{E}{2}\right) \text{ O}_2 : 1.0405 \\ E \text{ NO} : 0.1208 \\ \left(1 - \frac{E}{2}\right) \text{ N}_2 : 0.9396 \\ \hline 3.1009 \end{array} \right\} \quad \begin{array}{l} y_{\text{CO}_2} = 0.257 \\ y_{\text{CO}} = 0.065 \\ y_{\text{O}_2} = 0.336 \\ y_{\text{NO}} = 0.039 \\ y_{\text{N}_2} = 0.303 \end{array} \quad \leftarrow$$

Continued on next slide

Problem 14-68 continued

Alternative Solution Using IT

IT Code

T = 3000 // K

p = 5 // atm

// $\text{CO}_2 + \text{O}_2 + \text{N}_2 \rightarrow (1 - b) \text{CO}_2 + b \text{CO} + (1 + b/2 - e/2) \text{O}_2 + e \text{NO} + (1 - e/2) \text{N}_2$

nCO2 = 1 - b

nCO = b

nO2 = 1 + b/2 - e/2

nNO = e

nN2 = 1 - e/2

ntot = nCO2 + nCO + nO2 + nNO + nN2

yCO = nCO / ntot

yO2 = nO2 / ntot

yCO2 = nCO2 / ntot

yNO = nNO / ntot

yN2 = nN2 / ntot

pref = 1 // atm

// For $\text{CO}_2 \leftrightarrow \text{CO} + 0.5 \text{O}_2$

K1 = (yCO * yO2^0.5 / yCO2) * (p / pref)^0.5

// For $1/2 \text{O}_2 + 1/2 \text{N}_2 \leftrightarrow \text{NO}$

K2 = (yNO / (yO2^0.5 * yN2^0.5))

// Look up log10(K1) in EQCO2A.LUT.

log(K1) = LOOKUPVAL(EQCO2A,1,T,3)

// Look up log10(K2) in EQNO.LUT.

log(K2) = LOOKUPVAL(EQNO,1,T,3)

IT Results

K₁ = 0.3273

K₂ = 0.1222

y_{CO} = 0.06506

y_{CO2} = 0.2574

y_{N2} = 0.303

y_{NO} = 0.03896

y_{O2} = 0.3355

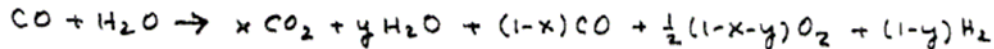
PROBLEM 14.69

KNOWN: An equimolar mixture of CO and H₂O(g) enters a heat exchanger. An equilibrium mixture of CO, CO₂, O₂, H₂O, and H₂ exits at 2500 K, 1 atm.

FIND: Determine the molar analysis of the equilibrium mixture.

ENGINEERING MODEL: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The overall reaction has the form



The total number of moles n in the product mixture is

$$n = x + y + (1-x) + \frac{1}{2}(1-x-y) + (1-y) = (5-x-y)/2$$

At equilibrium, two independent reactions relate the components of the product mixture:

1. $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$
2. $\text{CO}_2 + \text{H}_2 \rightleftharpoons \text{CO} + \text{H}_2\text{O}$

From Table A-27 at 2500 K the respective values of $\log_{10} K$ are

$$\log_{10} K_1 = -1.44 \Rightarrow K_1 = 0.03631$$

$$\log_{10} K_2 = +0.784 \Rightarrow K_2 = 6.08135$$

The forms taken by Eq 14.35 when $P/P_{\text{ref}} = 1$ are, respectively

$$K_1 = \frac{[(1-x)] \left[\frac{1-x-y}{2} \right]^{1/2}}{x} \left[\frac{P/P_{\text{ref}}}{(5-x-y)/2} \right]^{1/2} = \frac{(1-x) \left[\frac{1-x-y}{2} \right]^{1/2}}{x \left[\frac{5-x-y}{2} \right]^{1/2}}$$

$$K_2 = \frac{[(1-x)] [y]}{[x] [1-y]} \left[\frac{P/P_{\text{ref}}}{n} \right]^0 = \left[\frac{1-x}{x} \right] \left[\frac{y}{1-y} \right]$$

Solving this pair of simultaneous equations for x and y

$$\begin{aligned} x &= 0.28808 \\ y &= 0.71105 \end{aligned}$$

The equilibrium mixture on a molar basis is then

$$\{ 0.28808 \text{ CO}_2, 0.71105 \text{ H}_2\text{O}, 0.71192 \text{ CO}, 0.000435 \text{ O}_2, 0.28895 \text{ H}_2 \}$$

Continued on next slide

Problem 14-69 continued

Alternative Solution Using IT

IT Code

T = 2227 + 273.15 // K

p = 1 // atm

// $\text{CO}_2 + \text{H}_2\text{O} \rightarrow x \text{CO}_2 + y \text{H}_2\text{O} + (1-x) \text{CO} + 1/2 (1-x-y) \text{O}_2 + (1-y) \text{H}_2$

nCO2 = x

nH2O = y

nCO = 1 - x

nO2 = (1/2) * (1 - x - y)

nH2 = 1 - y

ntot = nCO2 + nH2O + nCO + nO2 + nH2

yCO2 = nCO2 / ntot

yH2O = nH2O / ntot

yCO = nCO / ntot

yO2 = nO2 / ntot

yH2 = nH2 / ntot

pref = 1 // atm

// For the reaction: $\text{CO}_2 \leftrightarrow \text{CO} + 1/2 \text{O}_2$

$K_1 = ((y_{\text{CO}} * y_{\text{O}_2}^{0.5}) / y_{\text{CO}_2}) * (p / \text{pref})^{0.5}$

// Look up log10(K2) in EQCO2A.LUT.

log(K1) = LOOKUPVAL(EQCO2A,1,T,3)

// For the reaction: $\text{CO}_2 + \text{H}_2 \leftrightarrow \text{CO} + \text{H}_2\text{O}$

$K_2 = (y_{\text{CO}} * y_{\text{H}_2\text{O}}) / (y_{\text{CO}_2} * y_{\text{H}_2})$

// Look up log10(K1) in EQWATGAS.LUT.

log(K2) = LOOKUPVAL(EQWATGAS,1,T,3)

IT Results

$K_1 = 0.03634$

$K_2 = 6.082$

$n_{\text{CO}} = 0.7119 \text{ kmol}$

$n_{\text{CO}_2} = 0.2881 \text{ kmol}$

$n_{\text{H}_2} = 0.2889 \text{ kmol}$

$n_{\text{H}_2\text{O}} = 0.7111 \text{ kmol}$

$n_{\text{O}_2} = 0.0004324 \text{ kmol}$

$x = 0.2881$

$y = 0.7111$

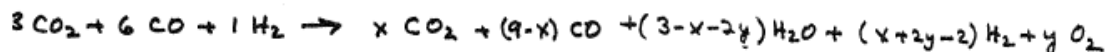
PROBLEM 14.70

KNOWN: A closed vessel initially contains 3 lbmol CO_2 , 6 lbmol CO , and 1 lbmol H_2 . An equilibrium mixture is formed at 4680°R , 1 atm consisting of CO_2 , CO , H_2O , H_2 , O_2 .

FIND: Determine the composition of the equilibrium mixture.

ENGINEERING MODEL: The equilibrium mixture is modeled as an ideal gas mixture.

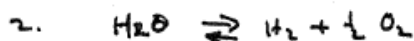
ANALYSIS: The overall reaction has the form



The total number of moles n in the product mixture is

$$n = x + (9-x) + (3-x-2y) + (x+2y-2) + y = 10+y$$

At equilibrium, two independent reactions relate the components of the product mixture:



From Table A-27 at 4680°R the respective values of $\log_{10} K$ are

$$\log_{10} K_1 = -1.219 \Rightarrow K_1 = 0.06039$$

$$\log_{10} K_2 = -2.021 \Rightarrow K_2 = 0.00953$$

The forms taken by Eq. 14.35 when $P/P_{\text{ref}} = 1$ are, respectively

$$K_1 = \frac{(9-x)[y]^{1/2}}{[x]} \left[\frac{P/P_{\text{ref}}}{10+y} \right]^{1/2} = \left(\frac{9-x}{x} \right) \left(\frac{y}{10+y} \right)^{1/2}$$

$$K_2 = \frac{(x+2y-2)[y]^{1/2}}{[3-x-2y]} \left[\frac{P/P_{\text{ref}}}{10+y} \right]^{1/2} = \left[\frac{x+2y-2}{3-x-2y} \right] \left[\frac{y}{10+y} \right]^{1/2}$$

Solving this pair of simultaneous equations for x and y

$$x = 2.30559$$

$$y = 0.00433$$

The equilibrium mixture on a molar basis is then

$$\left\{ 2.30559 \text{ CO}_2, 6.69441 \text{ CO}, 0.68575 \text{ H}_2\text{O}, 0.31425 \text{ H}_2, 0.00433 \text{ H}_2 \right\}$$

Continued on next slide

Problem 14-70 continued

Alternative Solution Using IT

IT Code

T = 4220 + 459.67 // °R

p = 1 // atm

// $3 \text{ CO}_2 + 6 \text{ CO} + 1 \text{ H}_2 \rightarrow x \text{ CO}_2 + (9 - x) \text{ CO} + (3 - x - 2y) \text{ H}_2\text{O} + (x + 2y - 2) \text{ H}_2 + y \text{ O}_2$

nCO2 = x

nCO = 9 - x

nH2O = 3 - x - 2*y

nH2 = x + 2*y - 2

nO2 = y

ntot = nCO2 + nCO + nH2O + nH2 + nO2

yCO2 = nCO2 / ntot

yCO = nCO / ntot

yH2O = nH2O / ntot

yH2 = nH2 / ntot

yO2 = nO2 / ntot

pref = 1 // atm

// For the reaction: $\text{CO}_2 \leftrightarrow \text{CO} + 1/2 \text{ O}_2$

K1 = ((yCO * yO2^0.5) / yCO2) * (p / pref)^0.5

// Look up log10(K1) in EQCO2A.LUT.

log(K1) = LOOKUPVAL(EQCO2A,2,T,3)

// For the reaction: $\text{H}_2\text{O} \leftrightarrow \text{H}_2 + 1/2 \text{ O}_2$

K2 = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5

// Look up log10(K2) in EQH2O.LUT.

log(K2) = LOOKUPVAL(EQH2O,2,T,3)

IT Results

K₁ = 0.06034

K₂ = 0.00952

n_{CO} = 6.694 lbmol

n_{CO2} = 2.306 lbmol

n_{H2} = 0.3142 lbmol

n_{H2O} = 0.6858

n_{O2} = 0.00432

x = 2.306

y = 0.432

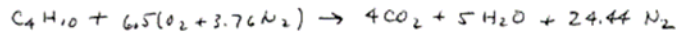
PROBLEM 14.71

KNOWN: C_4H_{10} burns with 100% excess air to form an equilibrium mixture at 1400K, 20 atm consisting of CO_2 , O_2 , $H_2O(g)$, N_2 , NO , NO_2 . For the reaction $N_2 + 2O_2 \rightleftharpoons 2NO_2$, at 1400K, $K = 8.4 \times 10^{-10}$.

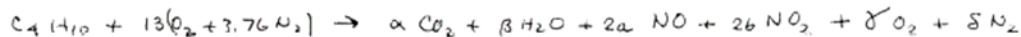
FIND: Determine the balanced reaction equation.

① **ENGINEERING MODEL:** The ideal gas model applies to the equilibrium mixture.

ANALYSIS: The balanced reaction equation for complete combustion with the theoretical amount of air is



For combustion with 100% excess air



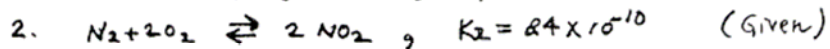
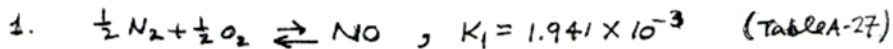
$$C: 4 = \alpha \quad H: 10 = 2\beta, \beta = 5$$

$$O: 26 = 8 + 5 + 2a + 4b + 2\gamma \Rightarrow \gamma = 6.5 - a - 2b$$

$$N: 2(48.88) = 2a + 2b + 2\delta \Rightarrow \delta = 48.88 - a - b$$

$$\text{The total amount of mixture is } n = 64.38 - b$$

At equilibrium there are two reactions applicable:



Accordingly, there are two constraints that determine a and b :

$$1.941 \times 10^{-3} = \frac{2a}{[48.88 - a - b][6.5 - a - 2b]}^{1/2} \left[\frac{P/P_{ref}}{n} \right]^{1 - \frac{1}{2} - \frac{1}{2}}$$

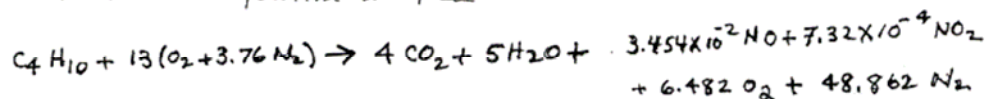
$$= \frac{2a}{[48.88 - a - b][6.5 - a - 2b]}^{1/2} \quad (1)$$

$$8.4 \times 10^{-10} = \frac{[2b]^2}{[48.88 - a - b][6.5 - a - 2b]^2} \left[\frac{20/1}{64.38 - b} \right]^{2 - 1 - 2}$$

$$= \left[\frac{2b}{6.5 - a - 2b} \right]^2 \left[\frac{64.38 - b}{48.88 - a - b} \right] \left[\frac{1}{20} \right] \quad (2)$$

Solving Eqs. (1), (2) simultaneously results in $a = 1.727 \times 10^{-2}$, $b = 3.660 \times 10^{-4}$

The reaction equation is then



1. Using Kay's rule (Sec 11.8) to evaluate T_c for the mixture, it can be confirmed that, because of the relatively high mixture temperature: 1400K, T_R for the mixture is large enough for the ideal gas model to be applicable.

Continued on next slide

Problem 14-71 continued

Alternative Solution Using IT

IT Code

T = 1400 // K
p = 20 // atm
K2 = 8.4E-10

// C₄H₁₀ + 13 (O₂ + 3.76 N₂) →
4 CO₂ + 5 H₂O + x NO + y NO₂ + (6.5 - x/2 - y) O₂ + (48.88 - x/2 - y/2) N₂
nCO₂ = 4
nH₂O = 5
nNO = x
nNO₂ = y
nO₂ = 6.5 - x/2 - y
nN₂ = 48.88 - x/2 - y/2
ntot = nCO₂ + nH₂O + nNO + nNO₂ + nO₂ + nN₂
yNO = nNO / ntot
yO₂ = nO₂ / ntot
yNO₂ = nNO₂ / ntot
yN₂ = nN₂ / ntot
pref = 1 // atm

// For the reaction: 1/2 N₂ + 1/2 O₂ ↔ NO
K1 = (yNO / (yN₂^{0.5} * yO₂^{0.5}))
// Data from Table A-27 are stored in EQNO.LUT.
log(K1) = LOOKUPVAL(EQNO,1,T,3)

// For the reaction: N₂ + 2 O₂ ↔ 2 NO₂
K2 = (yNO₂² / (yN₂ * yO₂²)) * (p / pref)⁻¹

IT Results

K₁ = 0.001941
nCO₂ = 4 kmol/kmol(C₄H₁₀)
nH₂O = 5 kmol/kmol(C₄H₁₀)
nNO = 0.03454 kmol/kmol(C₄H₁₀)
nNO₂ = 0.0007319 kmol/kmol(C₄H₁₀)
nO₂ = 6.482 kmol/kmol(C₄H₁₀)
nN₂ = 48.86 kmol/kmol(C₄H₁₀)

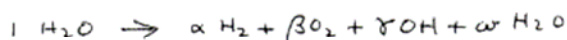
PROBLEM 14.72

KNOWN: One lbmol of $\text{H}_2\text{O}(g)$ dissociates to form an equilibrium mixture at 5000°R , 1 atm consisting of $\text{H}_2\text{O}(g)$, H_2 , O_2 , OH

FIND: Determine the equilibrium composition.

ENGINEERING MODEL: The ideal gas model applies to the equilibrium mixture.

ANALYSIS: The dissociation reaction has the form

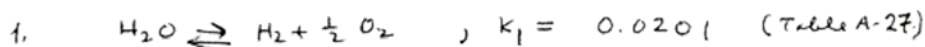


$$\text{H: } 2 = 2\alpha + \gamma + 2\omega \Rightarrow \alpha = 1 - \omega - \gamma/2 \quad \alpha = 1 - (1 - 2\beta - \gamma) - \gamma/2$$

$$\text{O: } 1 = 2\beta + \gamma + \omega \Rightarrow \omega = 1 - 2\beta - \gamma \quad = 2\beta + \gamma/2$$

The total amount of mixture is $n = (2\beta + \gamma/2) + \beta + \gamma + (1 - 2\beta - \gamma) = 1 + \beta + \frac{\gamma}{2}$.

At equilibrium there are two applicable reactions



Accordingly, there are two constraints that determine β and γ :

$$\begin{aligned} 0.0201 &= \frac{[2\beta + \gamma/2][\beta]}{[1 - 2\beta - \gamma]}^{1/2} \left[\frac{P/P_{\text{ref}}}{1 + \beta + \gamma/2} \right]^{1 + \frac{1}{2} - 1} \\ &= \left[\frac{2\beta + \gamma/2}{1 - 2\beta - \gamma} \right] \left[\frac{\beta}{1 + \beta + \gamma/2} \right]^{1/2} \end{aligned} \quad (1)$$

$$\begin{aligned} 0.0215 &= \frac{[\gamma][2\beta + \gamma/2]}{[1 - 2\beta - \gamma]}^{1/2} \left[\frac{P/P_{\text{ref}}}{1 + \beta + \gamma/2} \right]^{1 + \frac{1}{2} - 1} \\ &= \left[\frac{\gamma}{1 - 2\beta - \gamma} \right] \left[\frac{2\beta + \gamma/2}{1 + \beta + \gamma/2} \right]^{1/2} \end{aligned} \quad (2)$$

Solving Eqs (1), (2) simultaneously results in $\beta = 0.0341$, $\gamma = 0.06$. The equilibrium composition is then, per lbmol of H_2O initially present

$$\{ 0.0982 \text{ H}_2, 0.0341 \text{ O}_2, 0.06 \text{ OH}, 0.8718 \text{ H}_2\text{O}(g) \}$$

Continued on next slide

Problem 14-72 continued

Alternative Solution Using IT

IT Code

```
T = 5000 // °R
p = 1 // atm

// H2O → a H2 + b O2 + c OH + d H2O
1 = a + c/2 + d // H2 balance
1/2 = b + c/2 + d/2 // O2 balance
ntot = a + b + c + d
yH2 = a / ntot
yO2 = b / ntot
yOH = c / ntot
yH2O = d / ntot
pref = 1 // atm

// For the reaction: H2O ↔ H2 + 1/2 O2
K1 = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5
// Data from Table A-27 are stored in EQH2O.LUT
log(K1) = LOOKUPVAL(EQH2O,2,T,3)

// For the reaction: H2O ↔ OH + 1/2 H2
K2 = ((yOH * yH2^0.5) / yH2O) * (p / pref)^0.5
// Data from Table A-27 are stored in EQH2O_OH.LUT
log(K2) = LOOKUPVAL(EQH2O_OH,2,T,3)
```

IT Results

```
K1 = 0.0201
K2 = 0.02147
a = 0.09828 lbmol
b = 0.03376 lbmol
c = 0.06153 lbmol
d = 0.8709 lbmol
```

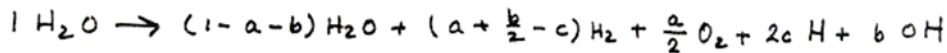
PROBLEM 14.73

KNOWN: Steam enters a heat exchanger. An equilibrium mixture of H_2O , H_2 , O_2 , H , and OH exits at 1 atm and (a) 2800 K, (b) 3000 K.

FIND: Determine the molar analysis of the equilibrium mixture.

ENGINEERING MODEL: The equilibrium mixture is modeled as an ideal gas mixture.

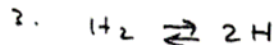
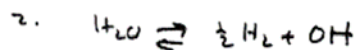
ANALYSIS: The overall reaction has the form



The total number of moles n in the product mixture is

$$n = (1-a-b) + (a + \frac{b}{2} - c) + \frac{a}{2} + 2c + b = 1 + \frac{a}{2} + \frac{b}{2} + c$$

At equilibrium, three independent reactions relate the components of the product mixture:



(a) From Table A-27 at 2800 K, the respective values of $\log_{10} K$ are

$$\log_{10} K_1 = -1.658 \Rightarrow K_1 = 0.02198$$

$$\log_{10} K_2 = -1.624 \Rightarrow K_2 = 0.02377$$

$$\log_{10} K_3 = -2.178 \Rightarrow K_3 = 0.00664$$

The forms taken by Eq. 14.35 when $P/P_{\text{ref}} = 1$ are, respectively

$$K_1 = \frac{(a + \frac{b}{2} - c)(\frac{a}{2})^{1/2}}{(1-a-b)} \left(\frac{P/P_{\text{ref}}}{1 + \frac{a}{2} + \frac{b}{2} + c} \right)^{1/2} = \left(\frac{a + \frac{b}{2} - c}{1-a-b} \right) \left(\frac{a/2}{1 + \frac{a}{2} + \frac{b}{2} + c} \right)^{1/2}$$

$$K_2 = \frac{(a + \frac{b}{2} - c)^{1/2} (b)}{1-a-b} \left(\frac{P/P_{\text{ref}}}{1 + \frac{a}{2} + \frac{b}{2} + c} \right)^{1/2} = \left(\frac{b}{1-a-b} \right) \left(\frac{a + \frac{b}{2} - c}{1 + \frac{a}{2} + \frac{b}{2} + c} \right)^{1/2}$$

$$K_3 = \frac{[2c]^2}{(a - \frac{b}{2} - 2) \left(\frac{P/P_{\text{ref}}}{1 + \frac{a}{2} + \frac{b}{2} + c} \right)^{1/2}} = \frac{4c^2}{(a + \frac{b}{2} - c)(1 + \frac{a}{2} + \frac{b}{2} + c)}$$

Solving this set of simultaneous equations for a , b , and c :

$$a = 0.076754, b = 0.0679, c = 0.013334$$

The equilibrium mixture on a molar basis is then

$$\{0.855346 \text{ H}_2\text{O}, 0.09737 \text{ H}_2, 0.038377 \text{ O}_2, 0.026668 \text{ H}, 0.0679 \text{ OH}\}$$

Continued on next slide

Problem 14-73 continued

(b) From Table A-27 at 3000 K the respective values of $\log_{10} K$ are

$$\log_{10} K_1 = -1.343 \Rightarrow K_1 = 0.04539$$

$$\log_{10} K_2 = -1.265 \Rightarrow K_2 = 0.05433$$

$$\log_{10} K_3 = -1.606 \Rightarrow K_3 = 0.02477$$

Solving the set of equations considered in part (a) gives

$$a = 0.12568, b = 0.11232, c = 0.03282$$

The equilibrium mixture on a molar basis is then

$$\{ 0.762 \text{ H}_2\text{O}, 0.14902 \text{ H}_2, 0.06284 \text{ O}_2, 0.06564 \text{ H}, 0.11232 \text{ OH} \}$$



Alternative Solution Using IT

IT Code

```
T = 3000 // K
```

```
p = 1 // atm
```

```
// H2O -> a H2O + b H2 + c O2 + d H + e OH
```

```
1 = a + b + d/2 // H2 balance
```

```
1/2 = a/2 + c + e/2 // O2 balance
```

```
ntot = a + b + c + d + e
```

```
yH2O = a / ntot
```

```
yH2 = b / ntot
```

```
yO2 = c / ntot
```

```
yH = d / ntot
```

```
yOH = e / ntot
```

```
pref = 1 // atm
```

```
// H2O <=> H2 + 1/2 O2 // Reaction 1
```

```
// H2O <=> 1/2 H2 + OH // Reaction 2
```

```
// H2 <=> 2 H // Reaction 3
```

```
K1 = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5
```

```
K2 = ((yH2^0.5 * yOH) / yH2O) * (p / pref)^0.5
```

```
K3 = (yH^2 / yH2) * (p / pref)^1
```

```
// Data from Table A-27 are stored in EQH2O.LUT, EQH2O_OH.LUT, and EQH2.LUT.
```

```
// Look up log10(K) in the appropriate tables.
```

```
log(K1) = LOOKUPVAL(EQH2O,1,T,3)
```

```
log(K2) = LOOKUPVAL(EQH2O_OH,1,T,3)
```

```
log(K3) = LOOKUPVAL(EQH2,1,T,3)
```

IT Results

Part (a): T = 2800 K

$n_{\text{H}_2\text{O}} = 0.8719 \text{ kmol}$

$n_{\text{H}_2} = 0.1136 \text{ kmol}$

$n_{\text{O}_2} = 0.03162 \text{ kmol}$

$n_{\text{H}} = 0.02894 \text{ kmol}$

$n_{\text{OH}} = 0.06481 \text{ kmol}$

Part (b): T = 3000 K

$n_{\text{H}_2\text{O}} = 0.7884 \text{ kmol}$

$n_{\text{H}_2} = 0.1755 \text{ kmol}$

$n_{\text{O}_2} = 0.04983 \text{ kmol}$

$n_{\text{H}} = 0.07216 \text{ kmol}$

$n_{\text{OH}} = 0.1119 \text{ kmol}$

PROBLEM 14.74

KNOWN: The system is a two-phase liquid-vapor mixture of (a) water at 100°C , (b) R134a at 20°C .

FIND: Using tabulated property data show that $g_f = g_g$.

ANALYSIS: (a) At 100°C , Table A-2 gives

$$h_f = 419.04 \text{ kJ/kg}$$

$$h_g = 2676.1 \text{ kJ/kg}$$

$$s_f = 1.3069 \text{ kJ/kg}\cdot\text{K}$$

$$s_g = 7.3549 \text{ kJ/kg}\cdot\text{K}$$

Then, with $g = h - Ts$

$$g_f = h_f - Ts_f = 419.04 - (373.15)(1.3069) = -68.63 \text{ kJ/kg}$$

$$\textcircled{1} \quad g_g = h_g - Ts_g = 2676.1 - (373.15)(7.3549) = -68.38 \text{ kJ/kg}$$

(b) At 20°C , Table A-10 gives

$$h_f = 77.26 \text{ kJ/kg}$$

$$h_g = 258.36 \text{ kJ/kg}$$

$$s_f = 0.2924 \text{ kJ/kg}\cdot\text{K}$$

$$s_g = 0.9102 \text{ kJ/kg}\cdot\text{K}$$

Then, with $g = h - Ts$

$$g_f = 77.26 - (293.15)(0.2924) = -8.457 \text{ kJ/kg}$$

$$g_g = 258.36 - (293.15)(0.9102) = -8.465 \text{ kJ/kg}$$

1. As developed in Sec. 6.3.1, $s_g - s_f = (h_g - h_f)/T$ for a change in phase from saturated liquid to saturated vapor at T . Thus $h_g - Ts_g = h_f - Ts_f$, or $g_g = g_f$. The slight difference in the calculated values of g_g and g_f owes to table roundoff and signifies a slight inconsistency in table values.

PROBLEM 14.75

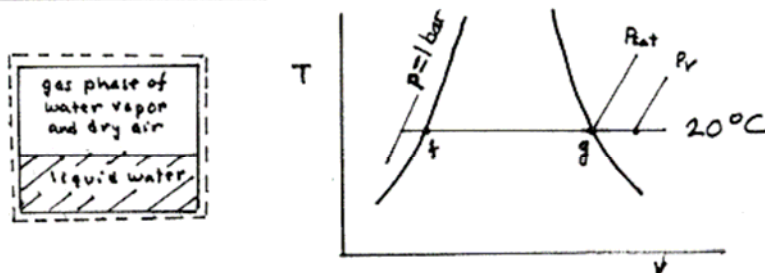
See the respective problems in Chapter 11 for solutions.

PROBLEM 14.76

KNOWN: A phase of liquid water only is in equilibrium with moist air at 20°C, 1 bar.

FIND: Determine the departure, in percent, of the partial pressure of the water vapor from the saturation pressure of pure water at 20°C.

SCHEMATIC & GIVEN DATA



ENGINEERING MODEL: (1) The gas phase can be modeled as an ideal gas. (2) The liquid phase involves water only.

ANALYSIS: Proceeding as in Example 14.10, the following result is obtained

$$\frac{P_v}{P_{sat}} = \exp \frac{\bar{v}_f (P - P_{sat})}{RT} \quad (1)$$

With data from Table A-2 at 20°C, $\bar{v}_f = 1.0018 \times 10^{-3} \text{ m}^3/\text{kg}$, $P_{sat} = 0.02339 \text{ bar}$, so

$$\begin{aligned} \frac{\bar{v}_f (P - P_{sat})}{RT} &= \frac{[1.0018 \times 10^{-3} \text{ m}^3/\text{kg}] (18.02 \text{ kg/kmol}) (1 - 0.02339) \times 10^5 \text{ N/m}^2}{[8314 \text{ N} \cdot \text{m}/(\text{kmol} \cdot \text{K})] [293.15 \text{ K}]} \\ &= 7.23 \times 10^{-4} \end{aligned}$$

Equation (1) then gives

$$\begin{aligned} \frac{P_v}{P_{sat}} &= \exp (7.23 \times 10^{-4}) \\ &= 1.00072 \end{aligned}$$

When expressed as a percentage the departure of P_v from P_{sat} is

$$\left(\frac{P_v - P_{sat}}{P_{sat}} \right) (100) = (1.00072 - 1) (100) = 0.072\%$$

1. Review note 1 of Example 14.10
2. Review note 2 of Example 14.10

PROBLEM 14.77

Known: Graphite and diamond exist in equilibrium at 25°C , p .

FIND: Derive an expression for estimating the pressure p in terms of $\bar{V}$, $\bar{g}$ and the isothermal compressibility K of each phase at 25°C , 1atm .

ENGINEERING MODEL: The isothermal compressibility K is constant with pressure.

ANALYSIS: By Eq 14.59, at equilibrium $\bar{g}(25^\circ\text{C}, p) = \bar{g}'(25^\circ\text{C}, p)$ where $\bar{g}$ and $\bar{g}'$ denote the molar Gibbs functions of graphite and diamond, respectively.

To evaluate the molar Gibbs functions relative to $T_{ref} = 25^\circ\text{C}$, $P_{ref} = 1 \text{ atm}$, begin with Eq. 11.23. Since T is fixed

$$d\bar{g} = \bar{v} dp \quad (4)$$

Also, at fixed T the differential of $\bar{v}(T, p)$ reduces to

$$d\bar{v} = \underbrace{\left(\frac{\partial \bar{v}}{\partial P}\right)_T}_{L = -K\bar{v} \text{ (Eq. 11.63)}} dP + \cancel{\left(\frac{\partial \bar{v}}{\partial T}\right)_P} dT = -K\bar{v} dP$$

50

$$\frac{d \ln G_1}{G_1} = -K dp \Rightarrow \text{Integration at fixed } T \text{ and constant } K, \ln \left[\frac{\bar{v}(T_{ref}, p)}{\bar{v}(T_{ref}, p_{ref})} \right] = -K(p - p_{ref})$$

or

$$\bar{v}(T_{ref}, p) = \bar{v}(T_{ref}, p_{ref}) \exp(-K(p - p_{ref})) \quad (2)$$

Combining Eqs (1), (2), and integrating

$$\begin{aligned} \bar{g}(T_{ref}, p) - \bar{g}(T_{ref}, p_{ref}) &= \bar{v}(T_{ref}, p_{ref}) \frac{\exp(\kappa p_{ref})}{-\kappa} \left[\exp(-\kappa p) \right]_{p_{ref}}^p \\ &= -\bar{v}(T_{ref}, p_{ref}) \frac{1}{\kappa} \left[\exp(-\kappa(p - p_{ref})) - 1 \right] \end{aligned}$$

Then, with $T_{ref} = 25^\circ\text{C}$, $p_{ref} = 1\text{ atm}$ and the condition $\bar{g} = \bar{g}'$

$$\frac{\bar{g}(z_{50^\circ}, lat_m) - \bar{u}(z_{50^\circ}, lat_m)}{K} \left[\exp(-K(p - lat_m)) - 1 \right] = \frac{\bar{g}'(z_{50^\circ}, lat_m) - \bar{u}'(z_{50^\circ}, lat_m)}{K'} \left[\exp(-K'(p - lat_m)) - 1 \right] \quad (3)$$

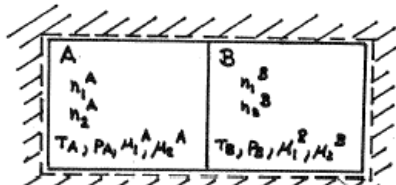
Equation (3) has a single unknown: the pressure p . Thus Eq (3) gives p implicitly in terms of $\bar{g}$, $\bar{v}$ and K for each phase at 25°C, 1 atm.

PROBLEM 14.78

KNOWN: An isolated system has two phases, A and B. Each phase consists of the same two substances, 1 and 2.

FIND: Show that conditions for equilibrium are 1: $T_A = T_B$,
2: $P_A = P_B$, 3: $\mu_1^A = \mu_1^B$, $\mu_2^A = \mu_2^B$.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system shown in the accompanying figure is isolated.

(2) Eq. 11.114(a) is applicable to each phase. (3) A and B do not react with one another.

ANALYSIS: According to Eq. 14.3 any process of an isolated system must satisfy $dS]_{U,V} \geq 0$. This indicates that the entropy of an isolated increases during an irreversible process. Each step of the process results in an increase in the entropy of the system and brings the system closer to equilibrium. The equilibrium state is the one having the maximum value of S . Therefore, when

$$dS]_{U,V} = 0 \quad (1)$$

there is equilibrium.

For the isolated system under present consideration, $dS = dS_A + dS_B$.

Using Eq. 11.114a to evaluate dS_A and dS_B , respectively, we get

$$dS = \left[\frac{dU_A}{T_A} + \frac{P_A}{T_A} dV_A - \frac{\mu_1^A}{T_A} dn_1^A - \frac{\mu_2^A}{T_A} dn_2^A \right] + \left[\frac{dU_B}{T_B} + \frac{P_B}{T_B} dV_B - \frac{\mu_1^B}{T_B} dn_1^B - \frac{\mu_2^B}{T_B} dn_2^B \right] \quad (2)$$

For the isolated system, the total energy, total volume, and total mass for each substance are constant:

$$dU_A + dU_B = 0, \quad dV_A + dV_B = 0, \quad dn_1^A + dn_1^B = 0, \quad dn_2^A + dn_2^B = 0 \quad (3)$$

Introducing Eqs. (3) into Eq. (2)

$$dS = \left[\frac{1}{T_A} - \frac{1}{T_B} \right] dU_A + \left[\frac{P_A}{T_A} - \frac{P_B}{T_B} \right] dV_A - \left[\frac{\mu_1^A}{T_A} - \frac{\mu_1^B}{T_B} \right] dn_1^A - \left[\frac{\mu_2^A}{T_A} - \frac{\mu_2^B}{T_B} \right] dn_2^A \quad (4)$$

Eq. (4) shows that in seeking the maximum in S the independent variables are U_A , V_A , n_1^A , and n_2^A . Necessary conditions for a maximum are

$$\frac{\partial S}{\partial U_A} = \frac{\partial S}{\partial V_A} = \frac{\partial S}{\partial n_1^A} = \frac{\partial S}{\partial n_2^A} = 0$$

The first of these conditions requires

$$\frac{1}{T_A} - \frac{1}{T_B} = 0 \Rightarrow T_A = T_B \quad \leftarrow$$

From the second

$$\frac{P_A}{T_A} - \frac{P_B}{T_B} = 0 \Rightarrow P_A = P_B \quad \leftarrow$$

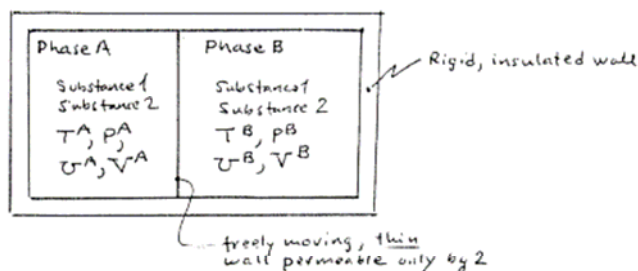
With the remaining two conditions

$$\frac{\mu_1^A}{T_A} - \frac{\mu_1^B}{T_B} = 0 \Rightarrow \mu_1^A = \mu_1^B, \quad \frac{\mu_2^A}{T_A} - \frac{\mu_2^B}{T_B} = 0 \Rightarrow \mu_2^A = \mu_2^B \quad \leftarrow$$

PROBLEM 14.79

KNOWN: An isolated system has two phases, A and B, each of which consists of the same two substances, 1 and 2. The phases are separated by a freely moving, thin wall permeable only by substance 2.

FIND: Determine the necessary conditions for equilibrium.



ENGINEERING MODEL: The system consists of phase A plus phase B. (2) For the system, $\dot{Q} = \dot{W} = 0$ and there are no kinetic/potential energy effects. (3) The phases are separated by a freely moving, thin wall permeable only by 2.

ANALYSIS: The appropriate equilibrium criteria for this case is obtained via Equation 14.3 as $dS]_{U,V} = 0$, where $dS = dS^A + dS^B$. Invoking Eq 11.114a

$$dU^A = T^A dS^A - p^A dV^A + \cancel{\mu_1^A d\eta_1^A} + \mu_2^A d\eta_2^A$$

$$dU^B = T^B dS^B - p^B dV^B + \cancel{\mu_1^B d\eta_1^B} + \mu_2^B d\eta_2^B$$

Since the wall is permeable only by substance 2, the amount of substance 1 present in each of the phases cannot change, and so these terms have been dropped in the expression above.

Since the container is rigid and insulated

$$\text{total volume is constant: } dV^B = -dV^A$$

$$\text{total energy is constant: } dU^B = -dU^A$$

Collecting results:

$$dS]_{U,V} = \left[\frac{dU^A}{T^A} + \frac{p^A}{T^A} dV^A - \frac{\mu_2^A}{T^A} d\eta_2^A \right] + \left[\frac{dU^B}{T^B} + \frac{p^B}{T^B} dV^B - \frac{\mu_2^B}{T^B} d\eta_2^B \right]$$

$$= dU^A \left[\frac{1}{T^A} - \frac{1}{T^B} \right] + dV^A \left[\frac{p^A}{T^A} - \frac{p^B}{T^B} \right] - d\eta_2^A \left[\frac{\mu_2^A}{T^A} - \frac{\mu_2^B}{T^B} \right]$$

Since U^A, V^A, η_2^A can be varied independently, it follows that when $dS]_{U,V} = 0$ the terms in the parentheses must vanish. Therefore, the criteria of equilibrium are

$$\textcircled{1} \quad \begin{aligned} T^A &= T^B & (\text{temperature is the same in each phase}) \\ p^A &= p^B & (\text{pressure is the same in each phase}) \\ \mu_2^A &= \mu_2^B & (\text{chemical potential of substance 2 is the same in each phase}) \end{aligned}$$

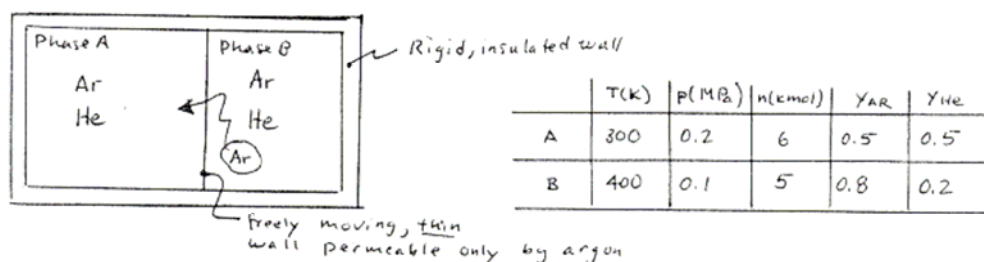
1. There is no restriction on μ_1^A or μ_1^B .

PROBLEM 14.80

KNOWN: An isolated system has two gas phases, A and B, each of which consists of argon and helium. The phases are separated by a freely moving, thin wall permeable only to argon. Initial data is provided.

FIND: Using the result of Problem 14.79, determine the final equilibrium temperature, pressure, and composition in the two phases.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The idealizations of Problem 14.79 apply. (2) Ideal gas principles apply to the gas mixtures.

ANALYSIS: An energy balance requires $\Delta U = \Delta U^A + \Delta U^B = 0$, where

$$\Delta U^A = [n_{a,f}^A \bar{u}_a(T_f) + n_{h,f}^A \bar{u}_h(T_f)] - [n_{a,i}^A \bar{u}_a(T^A) + n_{h,i}^A \bar{u}_h(T^A)]$$

$$\Delta U^B = [n_{a,f}^B \bar{u}_a(T_f) + n_{h,f}^B \bar{u}_h(T_f)] - [n_{a,i}^B \bar{u}_a(T^B) + n_{h,i}^B \bar{u}_h(T^B)]$$

So,

$$0 = [n_{a,f}^A + n_{a,f}^B] \bar{u}_a(T_f) - n_{a,i}^A \bar{u}_a(T^A) - n_{a,i}^B \bar{u}_a(T^B) + n_{h,i}^A [\bar{u}_h(T_f) - \bar{u}_h(T^A)] + n_{h,i}^B [\bar{u}_h(T_f) - \bar{u}_h(T^B)]$$

With $n_{a,f}^A + n_{a,f}^B = n_{a,i}^A + n_{a,i}^B$, where $n_{a,i}^A = 3$, $n_{a,i}^B = 4$, $n_{a,i}^{\text{TOT}} = 7$.

$$0 = n_{a,i}^A [\bar{u}_a(T_f) - \bar{u}_a(T^A)] + n_{a,i}^B [\bar{u}_a(T_f) - \bar{u}_a(T^B)] + n_{h,i}^A [\bar{u}_h(T_f) - \bar{u}_h(T^A)] + n_{h,i}^B [\bar{u}_h(T_f) - \bar{u}_h(T^B)]$$

For monatomic gases $\Delta \bar{u} = c_v \Delta T$, where c_v is a constant. So, this becomes after arrangement

$$T_f = \frac{(n_{a,i}^A + n_{a,i}^B) T^A + (n_{a,i}^B + n_{h,i}^B) T^B}{(n_{a,i}^A + n_{h,i}^A) + (n_{a,i}^B + n_{h,i}^B)} = \frac{(6)(300\text{K}) + 5(400\text{K})}{11} = 345.5\text{K} \leftarrow T_f$$

The total volume is unchanged. Thus $(V^A + V^B)_f = (V^A + V^B)_i$:

$$n_{\text{TOT}} \frac{\bar{R} T_f}{P_f} = \frac{n^A \bar{R} T^A}{P^A} + \frac{n^B \bar{R} T^B}{P^B} \Rightarrow P_f = \frac{n_{\text{TOT}} T_f}{\frac{n^A T^A}{P^A} + \frac{n^B T^B}{P^B}} = \frac{(11)(345.5)}{\frac{(6)(300)}{0.2} + \frac{(5)(400)}{0.1}} = 0.131\text{MPa} \leftarrow P_f$$

The final constraint is $\mu_a^A = \mu_a^B$ at the final state. With Eq. 14.17, this becomes $y_{a,f}^A = y_{a,f}^B$. That is

$$\frac{n_{a,f}^A}{n_{a,f}^A + n_{h,f}^A} = \frac{n_{a,f}^B}{n_{a,f}^B + n_{h,f}^B} \Rightarrow \frac{n_{a,f}^A}{n_{a,f}^A + 3} = \frac{(7 - n_{a,f}^A)}{(7 - n_{a,f}^A) + 1} \Rightarrow 4 n_{a,f}^A = 21 \Rightarrow n_{a,f}^A = 5.25$$

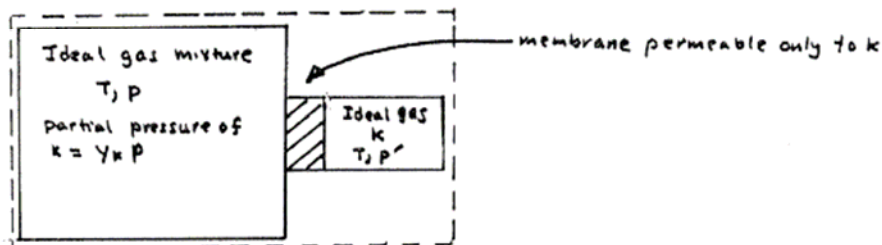
Accordingly at the final equilibrium state Phase A contains 5.25 kmol of Argon and 3.0 kmol of Helium. Phase B contains 1.75 kmol of Argon and 1.0 kmol of Helium.

PROBLEM 14-81

KNOWN: An ideal gas mixture at T and p containing substance k is separated by a semipermeable membrane from a gas phase of pure k at T and p' .

FIND: Determine the relationship between p and p' for there to be no net transfer through the membrane.

SCHEMATIC & GIVEN DATA:



ENGINEERING

MODEL: (1) The system is shown in the accompanying figure. (2) The gas mixture and the pure phase can be modeled as ideal gases. (3) The membrane allows only k to pass.

ANALYSIS: Following the discussion of Sec. 14.6, there would be no net transfer of k when the chemical potential of k in the mixture equals the chemical potential of pure k :

$$\mu_{k, \text{mixture}} = \mu_{k, \text{pure}} \quad (1)$$

With Eq. 14.12, $\mu_{k, \text{pure}} = \bar{g}_k(T, p')$, and with Eq. 14.16, $\mu_{k, \text{mixture}} = \bar{g}_k(T, y_k p)$. Thus Eq. (1) becomes

$$\bar{g}_k(T, y_k p) = \bar{g}_k(T, p') \Rightarrow p' = y_k p \quad \leftarrow$$

That is, the pressure p' of the pure phase must equal the partial pressure of k in the mixture.

PROBLEM 14.82

KNOWN: A system involves (a) one component, (b) two components, (c) three components.

FIND: Determine the maximum number of homogeneous phases that can exist at equilibrium.

ENGINEERING MODEL: No reaction takes place.

ANALYSIS: Equation 14.68 summarizes Gibbs' phase rule, on rearrangement

$$P = 2 + N - F$$

where N is the number of components and P is the number of phases. The value of P is greatest when F , the degrees of freedom is least: zero.

Thus, when $F = 0$

$$P = 2 + N$$

(1)

(a) $N = 1 \Rightarrow P = 3$. (b) $N = 2 \Rightarrow P = 4$. (c) $N = 3, P = 5$.

PROBLEM 12.83

KNOWN: Various systems are specified.

FIND: Determine the number of degrees of freedom.

ENGINEERING MODEL: No reaction occurs.

ANALYSIS: Equation 14.68 Summarizes Gibbs' phase rule:

$$F = 2 + N - P \quad (1)$$

where F is the number of degrees of freedom, N is the number of components, and P is the number of phases.

(a) ice and liquid water : $N=1, P=2$

$$F = 2 + 1 - 2 = 1$$

(b) ice, liquid water, water vapor : $N=1, P=3$

$$F = 2 + 1 - 3 = 0$$

(c) liquid water, water vapor : $N=1, P=2$

$$F = 2 + 1 - 2 = 1$$

(d) water vapor only : $N=1, P=1$

$$F = 2 + 1 - 1 = 2$$

(e) water vapor, dry air : $N=2, P=1$ (dry air regarded as a single component)

$$F = 2 + 2 - 1 = 3$$

(f) liquid water, water vapor, dry air : $N=2, P=2$ (dry air regarded as a single component)

$$F = 2 + 2 - 2 = 2$$

(g) ice, water vapor, dry air : $N=2, P=2$ (dry air regarded as a single component)

$$F = 2 + 2 - 2 = 2$$

(h) N_2 and O_2 at $20^\circ C$, 1 atm : $N=2, P=1$.

$$F = 2 + 2 - 1 = 3$$

(i) liquid and vapor phases, each containing ammonia and water : $N=2, P=2$.

$$F = 2 + 2 - 2 = 2$$

(j) liquid mercury, liquid water, vapor phase of mercury and water : $N=2, P=3$

$$F = 2 + 2 - 3 = 1$$

Note: Water and mercury are immiscible, and thus form two liquid phases.

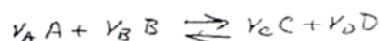
(k) liquid acetone and a vapor phase of acetone and N_2 : $N=2, P=2$

$$F = 2 + 2 - 2 = 2$$

PROBLEM 14.84

FIND: Develop the phase rule for chemically reacting systems.

ANALYSIS: If there is a chemical reaction within the system, the constituents are not completely independent. For example, if the constituents A, B, C, D undergo the reaction



there is an additional independent equation. Accordingly, the number of independent equations is $\{N(P-1)+1\}$. The number of variables is $P[N-1]+2$, as on p. 685. Accordingly, the number of degrees of freedom is

$$\begin{aligned} F &= [P(N-1)+2] - [N(P-1)+1] \\ &= (N-1) - P + 2 \end{aligned}$$

If a number of independent reactions occur, say r reactions, the phase rule takes the form

$$F = (N-r) - P + 2$$

PROBLEM 14.85

KNOWN: The gas phase reaction: $\text{CH}_4 + \text{H}_2\text{O} \rightleftharpoons \text{CO} + 3\text{H}_2$

FIND: Determine the number of degrees of freedom.

ENGINEERING MODEL: There is a single independent reaction.

ANALYSIS: Applying the result of Problem 14.84, the number of degrees of freedom is

$$F = (N - r) - P + 2$$

No. of components = 4

No. of independent reactions = 1

No. of phases = 1

Thus

① $F = (4 - 1) - 1 + 2 = 4$ $\longleftarrow$

1. To specify this system, one might give values for T , p and the mole fractions of any two of the four components.

PROBLEM 14.86

KNOWN: Two cases are identified involving a two-phase liquid-vapor ammonia-water system at temperature T and pressure p :
 (a) $T = 20^\circ\text{C}$, mole fraction of NH_3 in the liquid phase is 80%
 (b) $T = 40^\circ\text{C}$, $p = 12 \text{ bar}$.

FIND: (a) Determine p , in bar, and the mole fraction of NH_3 in the vapor phase. (b) Determine the mole fraction of NH_3 in each of the phases.

ENGINEERING MODEL: Raoult's law applies.

ANALYSIS: (a) For the liquid phase $y_{\text{NH}_3} = 0.80$. Thus, it follows that $y_{\text{H}_2\text{O}} = 0.20$. By Raoult's law, the partial pressures in the vapor phase are

$$P_{\text{NH}_3} = 0.8 P_{\text{sat}, \text{NH}_3}(20^\circ\text{C}) = 0.8 (8.5762 \text{ bars}) = 6.861 \text{ bar} \quad \leftarrow \text{Table A-13}$$

$$P_{\text{H}_2\text{O}} = 0.2 P_{\text{sat}, \text{H}_2\text{O}}(20^\circ\text{C}) = 0.2 (0.02339 \text{ bars}) = 0.005 \text{ bar} \quad \leftarrow \text{Table A-2}$$

Then for the vapor phase

$$p = P_{\text{NH}_3} + P_{\text{H}_2\text{O}} = 6.861 + 0.005 = 6.866 \text{ bar} \quad \leftarrow p$$

Also, for the vapor phase, $P_{\text{NH}_3} = x_{\text{NH}_3} p$. So

$$\textcircled{1} \quad x_{\text{NH}_3} = \frac{P_{\text{NH}_3}}{p} = \frac{6.861}{6.866} = 0.999 \quad \leftarrow x_{\text{NH}_3}$$

(b) The partial pressures of the vapor phase sum to give the system pressure:

$$12 \text{ bars} = P_{\text{NH}_3} + P_{\text{H}_2\text{O}} \quad (1)$$

According to Raoult's law

$$P_{\text{NH}_3} = y_{\text{NH}_3} P_{\text{sat}, \text{NH}_3}(40^\circ\text{C}) = y_{\text{NH}_3} (15.549 \text{ bar}) \quad (2) \quad \leftarrow \text{Table A-13}$$

$$P_{\text{H}_2\text{O}} = y_{\text{H}_2\text{O}} P_{\text{sat}, \text{H}_2\text{O}}(40^\circ\text{C}) = y_{\text{H}_2\text{O}} (0.07384 \text{ bar}) \quad (3) \quad \leftarrow \text{Table A-2}$$

Also

$$1 = y_{\text{NH}_3} + y_{\text{H}_2\text{O}} \quad (4)$$

Eqs. (1)-(4) give

$$12 \text{ bar} = y_{\text{NH}_3} (15.549 \text{ bars}) + (1 - y_{\text{NH}_3}) (0.07384 \text{ bar})$$

$$\text{Solving, } y_{\text{NH}_3} = 0.771. \quad \leftarrow y_{\text{NH}_3}$$

For the gas phase, $P_{\text{NH}_3} = x_{\text{NH}_3} p$. Thus, Eq. (2) with $y_{\text{NH}_3} = 0.771$ gives

$$(0.771)(15.549 \text{ bar}) = x_{\text{NH}_3} (12 \text{ bar})$$

$$\textcircled{1} \quad \Rightarrow x_{\text{NH}_3} = \frac{(0.771)(15.549)}{12} = 0.999 \quad \leftarrow x_{\text{NH}_3}$$

1. In each case the vapor phase is nearly all ammonia. The small amount of water vapor present in the gas phase is important owing to the possibility of freezing at some points of the cycle.